

# Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.2-Quartic/1.2.2.3-d+e-  
 $x^2-m-a+b-x^2+c-x^4-p$

Nasser M. Abbasi

September 20, 2021

Compiled on September 20, 2021 at 2:17pm

## Contents

<b>1</b>	<b>Introduction</b>	<b>3</b>
<b>2</b>	<b>detailed summary tables of results</b>	<b>17</b>
<b>3</b>	<b>Listing of integrals</b>	<b>63</b>
<b>4</b>	<b>Appendix</b>	<b>1101</b>



# Chapter 1

## Introduction

### Local contents

1.1	Listing of CAS systems tested . . . . .	4
1.2	Results . . . . .	5
1.3	Performance . . . . .	8
1.4	list of integrals that has no closed form antiderivative . . . . .	10
1.5	list of integrals solved by CAS but has no known antiderivative . . . . .	11
1.6	list of integrals solved by CAS but failed verification . . . . .	12
1.7	Timing . . . . .	12
1.8	Verification . . . . .	13
1.9	Important notes about some of the results . . . . .	13
1.10	Design of the test system . . . . .	15

This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [ 212 ]. This is test number [ 27 ].

## 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. IntegrateAlgebraic under Mathematica 12.3.1 on windows 10. [https://github.com/stblake/algebraic\\_integration](https://github.com/stblake/algebraic_integration). September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 ( 212 )	0.00 ( 0 )
Mathematica	100.00 ( 212 )	0.00 ( 0 )
Maple	100.00 ( 212 )	0.00 ( 0 )
Fricas	97.17 ( 206 )	2.83 ( 6 )
Mupad	83.96 ( 178 )	16.04 ( 34 )
Giac	81.60 ( 173 )	18.40 ( 39 )
Sympy	75.00 ( 159 )	% 25.00 ( 53 )
Maxima	51.42 ( 109 )	48.58 ( 103 )
IntegrateAlgebraic	10.85 ( 23 )	89.15 ( 189 )

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

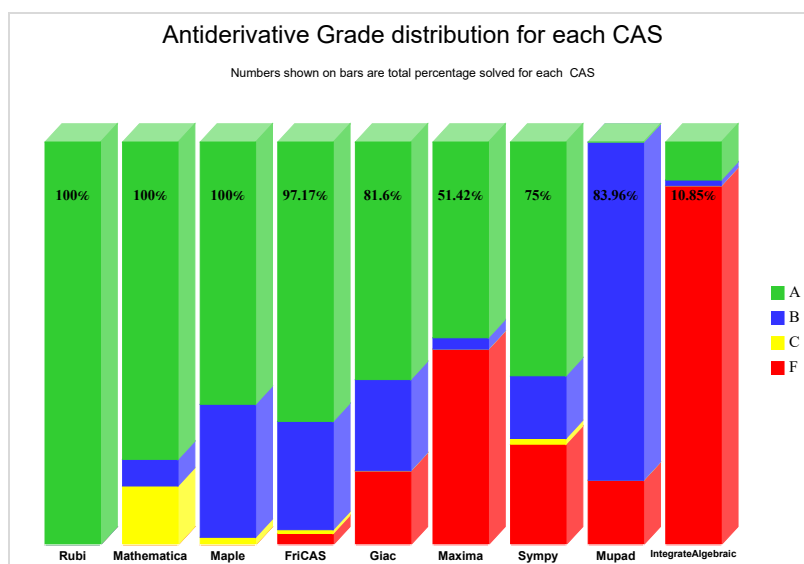
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

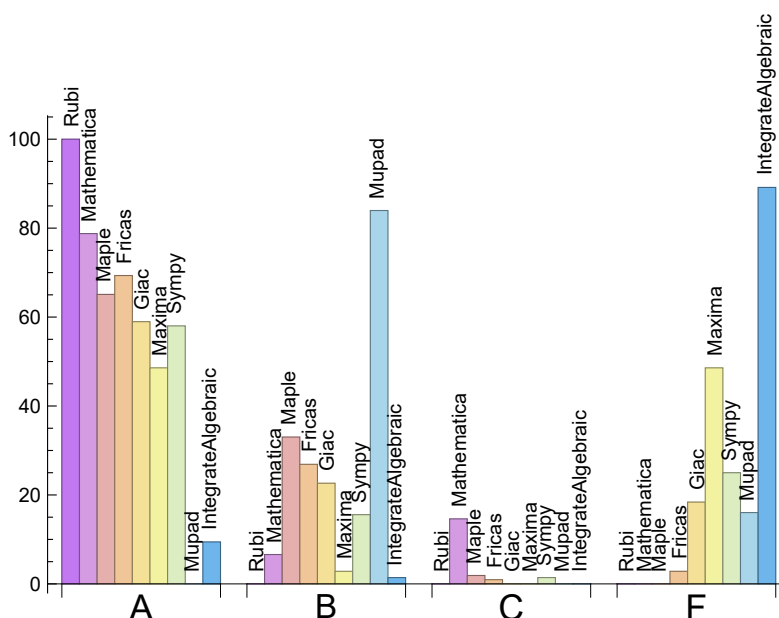
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	78.77	6.60	14.62	0.00
Fricas	69.34	26.89	0.94	2.83
Maple	65.09	33.02	1.89	0.00
Giac	58.96	22.64	0.00	18.40
Sympy	58.02	15.57	1.42	25.00
Maxima	48.58	2.83	0.00	48.58
IntegrateAlgebraic	9.43	1.42	0.00	89.15
Mupad	N/A	83.96	0.00	16.04

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	0	0.00 %	0.00 %	0.00 %
Fricas	6	0.00 %	100.00 %	0.00 %
IntegrateAlgebraic	189	100.00 %	0.00 %	0.00 %
Giac	39	48.72 %	10.26 %	41.03 %
Maxima	103	93.20 %	0.00 %	6.80 %
Sympy	53	52.83 %	35.85 %	11.32 %
Mupad	34	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

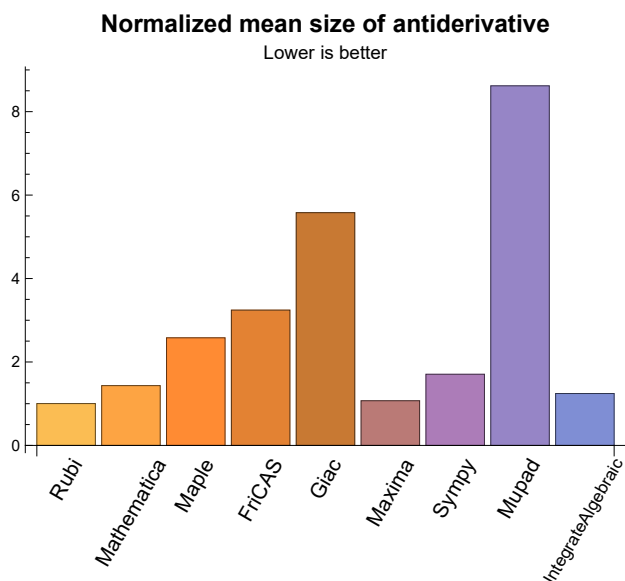
### 1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

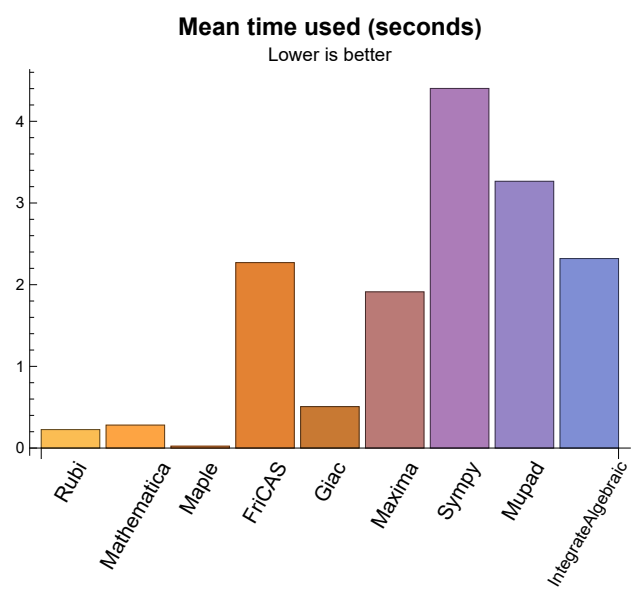
System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.23	126.33	1.00	79.50	1.00
Mathematica	0.28	146.85	1.43	99.00	1.00
Maple	0.02	345.21	2.58	118.00	1.26
Maxima	1.91	121.50	1.07	84.00	0.99
Fricas	2.27	637.53	3.24	151.50	1.87
Sympy	4.40	162.87	1.71	88.00	1.14
Giac	0.51	780.17	5.58	80.00	0.99
Mupad	3.27	3907.56	8.62	72.00	0.95
IntegrateAlgebraic	2.32	126.96	1.24	95.00	1.16

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.







## 1.4 list of integrals that has no closed form antiderivative

{

## 1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {86, 87, 140, 165, 166, 209, 210, 211, 212}

**IntegrateAlgebraic** {}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

### 1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

### 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user slelievre at [https://ask.sagemath.org/question/57123/could-we-have-a-leaf\\_count-function-in-base-sagemath/](https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/)

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

### 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

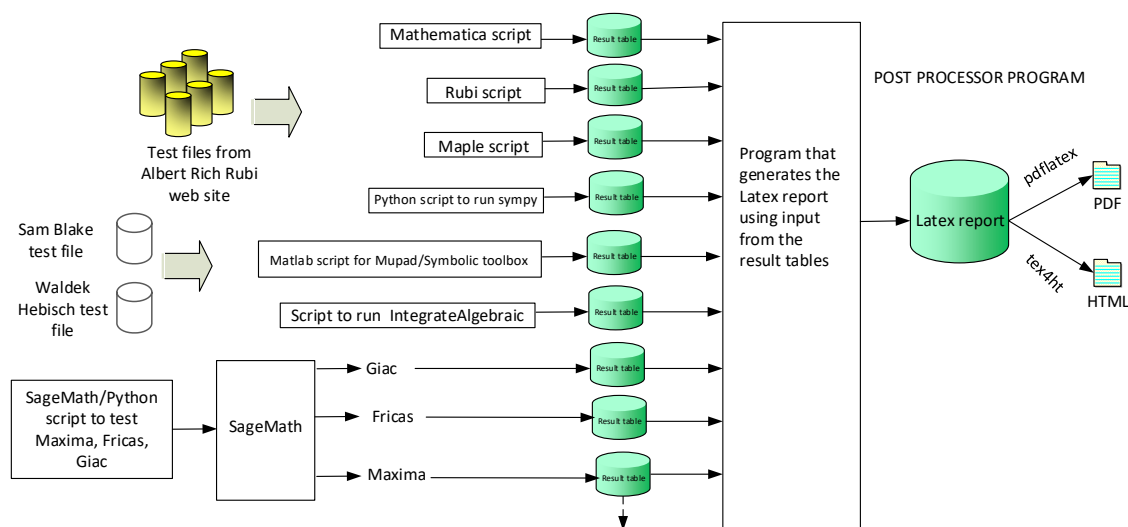
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives  $\sin(x) \sim 2/2$

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



**One record (line) per one integral result. The line is CSV comma separated. This is description of each record**

1. integer. the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"  
*The following field present only in Rubi and Mathematica Tables*
13. integer. 1 if result was verified or 0 if not verified.  
*The following fields present only in Rubi Tables*
14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,...}" which is list of the rules used by Rubi

**High level overview of the CAS independent integration test build system**

Nasser M. Abbasi  
May 11, 2021





# Chapter 2

## detailed summary tables of results

### Local contents

2.1	List of integrals sorted by grade for each CAS . . . . .	18
2.2	Detailed conclusion table per each integral for all CAS systems . . . . .	22
2.3	Detailed conclusion table specific for Rubi results . . . . .	58

## 2.1 List of integrals sorted by grade for each CAS

### Local contents

2.1.1	Rubi	19
2.1.2	Mathematica	19
2.1.3	Maple	19
2.1.4	Maxima	20
2.1.5	FriCAS	20
2.1.6	Sympy	20
2.1.7	Giac	21
2.1.8	Mupad	21
2.1.9	IntegrateAlgebraic	21

### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212 }

B grade: { }

C grade: { }

F grade: { }

### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 22, 23, 24, 25, 29, 30, 31, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 82, 83, 84, 85, 92, 93, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208 }

B grade: { 14, 15, 16, 17, 18, 19, 20, 21, 26, 27, 28, 53, 68, 76 }

C grade: { 32, 33, 34, 36, 37, 61, 80, 81, 86, 87, 88, 89, 90, 91, 94, 95, 96, 97, 98, 99, 100, 140, 141, 142, 143, 165, 166, 209, 210, 211, 212 }

F grade: { }

### 2.1.3 Maple

A grade: { 1, 2, 7, 8, 13, 14, 15, 16, 17, 18, 19, 20, 21, 25, 26, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 60, 61, 63, 64, 65, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 86, 87, 90, 91, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 141, 142, 143, 147, 148, 149, 153, 154, 155, 157, 158, 159, 160, 161, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 183, 184, 185, 190, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208 }

B grade: { 3, 4, 5, 6, 9, 10, 11, 12, 22, 23, 24, 27, 28, 29, 41, 42, 43, 55, 56, 57, 58, 59, 62, 66, 67, 68, 69, 70, 71, 79, 84, 85, 88, 89, 92, 93, 118, 125, 137, 138, 139, 140, 144, 145, 146, 150, 151, 152, 156, 162, 163, 164, 165, 166, 179, 180, 181, 182, 186, 187, 188, 189, 191, 192, 193, 194, 195, 196, 197, 198 }

C grade: { 209, 210, 211, 212 }

F grade: { }

### 2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 25, 30, 31, 32, 35, 39, 40, 44, 45, 46, 49, 54, 60, 61, 62, 64, 65, 72, 73, 74, 77, 80, 81, 82, 84, 86, 87, 96, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 199, 200, 201, 202, 203, 204, 207, 208 }

B grade: { 7, 53, 76, 83, 205, 206 }

C grade: { }

F grade: { 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 33, 34, 36, 37, 38, 41, 42, 43, 47, 48, 50, 51, 52, 55, 56, 57, 58, 59, 63, 66, 67, 68, 69, 70, 71, 75, 78, 79, 85, 88, 89, 90, 91, 92, 93, 94, 95, 97, 98, 99, 100, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 209, 210, 211, 212 }

### 2.1.5 FriCAS

A grade: { 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 86, 87, 96, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 122, 128, 131, 132, 133, 134, 135, 137, 138, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 157, 158, 159, 160, 162, 163, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 184, 185, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208 }

B grade: { 1, 2, 3, 4, 7, 53, 76, 82, 83, 84, 85, 88, 89, 92, 93, 94, 95, 97, 98, 99, 100, 118, 119, 120, 121, 123, 124, 125, 126, 127, 129, 136, 139, 140, 153, 154, 155, 156, 161, 164, 165, 166, 181, 182, 183, 187, 188, 189, 190, 193, 194, 195, 196, 209, 210, 211, 212 }

C grade: { 90, 91 }

F grade: { 130, 186, 191, 192, 197, 198 }

### 2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 85, 88, 89, 94, 97, 101, 102, 103, 104, 108, 109, 110, 111, 112, 115, 116, 117, 118, 119, 120, 121, 122, 125, 126, 127, 128, 131, 132, 133, 167, 168, 169, 170, 173, 174, 175, 176, 177, 178, 181, 182, 189, 190, 196, 202, 203 }

B grade: { 7, 26, 30, 53, 76, 83, 84, 92, 93, 105, 106, 107, 113, 114, 134, 135, 136, 156, 157, 158, 159, 171, 172, 179, 180, 184, 185, 199, 200, 201, 204, 205, 206 }

C grade: { 86, 87, 96 }

F grade: { 90, 91, 95, 98, 99, 100, 123, 124, 129, 130, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 160, 161, 162, 163, 164, 165, 166, 183, 186, 187, 188, 191, 192, 193, 194, 195, 197, 198, 207, 208, 209, 210, 211, 212 }

### 2.1.7 Giac

A grade: { 1, 2, 5, 6, 8, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 85, 86, 87, 90, 91, 96, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 137, 139, 162, 163, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208 }

B grade: { 3, 4, 7, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 35, 38, 41, 53, 76, 83, 84, 88, 89, 92, 93, 131, 132, 133, 134, 138, 156, 157, 158, 159, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196 }

C grade: { }

F grade: { 9, 10, 56, 68, 94, 95, 97, 98, 99, 100, 135, 136, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 160, 161, 164, 165, 166, 197, 198, 209, 210, 211, 212 }

### 2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 156, 157, 158, 159, 160, 161, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 205, 206, 207, 208 }

C grade: { }

F grade: { 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 162, 163, 164, 165, 166, 199, 200, 201, 202, 203, 204, 209, 210, 211, 212 }

### 2.1.9 IntegrateAlgebraic

A grade: { 137, 138, 139, 140, 162, 166, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212 }

B grade: { 163, 164, 165 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column N.S. in the table below, which stands for **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$ . To help make the table fit, Mathematica was abbreviated to MMA and IntegrateAlgebraic to I.A.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	247	247	183	260	221	767	109	241	599	0
N.S.	1	1.00	0.74	1.05	0.89	3.11	0.44	0.98	2.43	0.00
time (sec)	N/A	0.151	0.078	0.009	2.395	0.869	0.691	0.188	0.377	0.001
Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	247	247	184	260	221	767	110	241	603	0
N.S.	1	1.00	0.74	1.05	0.89	3.11	0.45	0.98	2.44	0.00
time (sec)	N/A	0.138	0.045	0.003	2.342	0.661	0.680	0.172	0.257	0.001
Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	95	122	109	755	110	230	579	0
N.S.	1	1.00	1.10	1.42	1.27	8.78	1.28	2.67	6.73	0.00
time (sec)	N/A	0.045	0.030	0.005	2.288	0.902	0.726	0.182	4.643	0.001
Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	95	122	109	755	110	228	579	0
N.S.	1	1.00	1.10	1.42	1.27	8.78	1.28	2.65	6.73	0.00
time (sec)	N/A	0.040	0.023	0.003	2.339	0.566	0.943	0.325	4.578	0.001
Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	33	122	39	33	41	52	29	0
N.S.	1	1.00	0.82	3.05	0.98	0.82	1.02	1.30	0.72	0.00
time (sec)	N/A	0.020	0.014	0.006	2.388	0.690	0.124	0.201	0.090	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	44	82	39	42	49	40	21	0
N.S.	1	1.00	0.86	1.61	0.76	0.82	0.96	0.78	0.41	0.00
time (sec)	N/A	0.021	0.014	0.003	2.417	0.552	0.125	0.170	4.433	0.000
Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	32	13	25	29	32	29	12	0
N.S.	1	1.00	2.00	0.81	1.56	1.81	2.00	1.81	0.75	0.00
time (sec)	N/A	0.003	0.015	0.002	2.393	0.772	0.115	0.159	0.092	0.000
Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	12	12	15	12	12	0
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.94	0.75	0.75	0.00
time (sec)	N/A	0.003	0.005	0.003	2.310	0.862	0.113	0.159	0.027	0.000
Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	60	254	100	148	138	0	57	0
N.S.	1	1.00	0.80	3.39	1.33	1.97	1.84	0.00	0.76	0.00
time (sec)	N/A	0.037	0.021	0.005	2.305	0.944	0.394	0.000	4.793	0.001
Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	91	254	70	151	131	0	43	0
N.S.	1	1.00	0.86	2.40	0.66	1.42	1.24	0.00	0.41	0.00
time (sec)	N/A	0.047	0.022	0.004	2.369	0.944	0.457	0.000	4.757	0.001
Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	60	290	74	137	87	222	57	0
N.S.	1	1.00	0.80	3.87	0.99	1.83	1.16	2.96	0.76	0.00
time (sec)	N/A	0.050	0.035	0.010	2.476	1.498	0.223	0.171	4.406	0.001

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	75	290	62	140	80	222	41	0
N.S.	1	1.00	0.83	3.22	0.69	1.56	0.89	2.47	0.46	0.00
time (sec)	N/A	0.047	0.023	0.004	2.412	0.530	0.234	0.224	0.086	0.001
Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	25	18	17	17	22	19	9	0
N.S.	1	1.00	1.92	1.38	1.31	1.31	1.69	1.46	0.69	0.00
time (sec)	N/A	0.006	0.006	0.006	2.350	0.785	0.197	0.159	0.040	0.000
Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	F	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	181	71	0	162	122	1642	94	0
N.S.	1	1.00	2.21	0.87	0.00	1.98	1.49	20.02	1.15	0.00
time (sec)	N/A	0.100	0.110	0.039	0.000	0.669	0.538	1.119	4.433	0.001
Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	F	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	181	71	0	162	122	1642	98	0
N.S.	1	1.00	2.21	0.87	0.00	1.98	1.49	20.02	1.20	0.00
time (sec)	N/A	0.110	0.111	0.037	0.000	0.743	0.561	1.087	4.515	0.001
Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	F	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	189	75	0	176	110	1676	30	0
N.S.	1	1.00	2.42	0.96	0.00	2.26	1.41	21.49	0.38	0.00
time (sec)	N/A	0.098	0.105	0.034	0.000	0.840	0.572	1.122	0.128	0.001
Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	F	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	189	75	0	179	121	1676	88	0
N.S.	1	1.00	2.20	0.87	0.00	2.08	1.41	19.49	1.02	0.00
time (sec)	N/A	0.104	0.107	0.032	0.000	0.757	0.550	1.139	4.394	0.001



Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	F	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	182	88	0	172	121	1642	99	0
N.S.	1	1.00	2.33	1.13	0.00	2.21	1.55	21.05	1.27	0.00
time (sec)	N/A	0.052	0.121	0.023	0.000	0.695	0.583	1.163	0.087	0.001
Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	F	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	182	69	0	173	110	1642	57	0
N.S.	1	1.00	2.33	0.88	0.00	2.22	1.41	21.05	0.73	0.00
time (sec)	N/A	0.048	0.125	0.025	0.000	0.590	0.568	1.254	4.436	0.001
Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	F	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	190	61	0	168	112	1676	29	0
N.S.	1	1.00	2.71	0.87	0.00	2.40	1.60	23.94	0.41	0.00
time (sec)	N/A	0.044	0.130	0.023	0.000	0.671	0.598	1.127	4.442	0.001
Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	F	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	190	61	0	168	112	1676	29	0
N.S.	1	1.00	2.71	0.87	0.00	2.40	1.60	23.94	0.41	0.00
time (sec)	N/A	0.047	0.128	0.024	0.000	0.823	0.609	1.081	0.110	0.001
Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	134	250	582	0	244	158	2202	129	0
N.S.	1	1.00	1.87	4.34	0.00	1.82	1.18	16.43	0.96	0.00
time (sec)	N/A	0.101	0.161	0.081	0.000	0.824	0.862	1.372	0.181	0.001
Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	248	582	0	232	160	2202	232	0
N.S.	1	1.00	1.91	4.48	0.00	1.78	1.23	16.94	1.78	0.00
time (sec)	N/A	0.166	0.120	0.027	0.000	0.645	0.772	1.402	4.522	0.001

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	248	582	0	232	160	2202	232	0
N.S.	1	1.00	1.91	4.48	0.00	1.78	1.23	16.94	1.78	0.00
time (sec)	N/A	0.131	0.044	0.013	0.000	0.754	0.794	1.350	0.129	0.001
Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	26	25	25	26	25	12	0
N.S.	1	1.00	1.00	0.90	0.86	0.86	0.90	0.86	0.41	0.00
time (sec)	N/A	0.026	0.019	0.012	1.043	0.678	0.470	0.245	4.410	0.001
Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	138	52	0	164	117	51	55	0
N.S.	1	1.00	2.30	0.87	0.00	2.73	1.95	0.85	0.92	0.00
time (sec)	N/A	0.069	0.202	0.007	0.000	0.849	0.464	0.176	0.075	0.001
Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	126	277	0	110	95	77	66	0
N.S.	1	1.00	2.03	4.47	0.00	1.77	1.53	1.24	1.06	0.00
time (sec)	N/A	0.058	0.060	0.045	0.000	0.829	0.378	0.308	4.385	0.000
Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	134	277	0	120	83	80	24	0
N.S.	1	1.00	2.03	4.20	0.00	1.82	1.26	1.21	0.36	0.00
time (sec)	N/A	0.058	0.059	0.032	0.000	0.805	0.385	0.312	4.407	0.001
Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	83	136	0	31	42	39	29	0
N.S.	1	1.00	1.84	3.02	0.00	0.69	0.93	0.87	0.64	0.00
time (sec)	N/A	0.059	0.077	0.052	0.000	0.800	0.147	0.170	0.087	0.001

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	17	12	11	19	22	11	19	0
N.S.	1	1.00	1.13	0.80	0.73	1.27	1.47	0.73	1.27	0.00
time (sec)	N/A	0.009	0.007	0.009	2.495	0.708	0.122	0.151	0.066	0.000
Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	12	11	11	14	11	11	0
N.S.	1	1.00	1.00	0.86	0.79	0.79	1.00	0.79	0.79	0.00
time (sec)	N/A	0.007	0.005	0.002	2.299	0.691	0.118	0.162	0.026	0.000
Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	97	34	33	33	44	33	29	0
N.S.	1	1.00	2.55	0.89	0.87	0.87	1.16	0.87	0.76	0.00
time (sec)	N/A	0.035	0.184	0.008	2.392	0.919	0.135	0.174	0.086	0.000
Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	99	40	0	29	42	45	29	0
N.S.	1	1.00	2.06	0.83	0.00	0.60	0.88	0.94	0.60	0.00
time (sec)	N/A	0.040	0.104	0.033	0.000	0.914	0.129	0.190	4.391	0.001
Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	97	40	0	33	44	52	29	0
N.S.	1	1.00	2.11	0.87	0.00	0.72	0.96	1.13	0.63	0.00
time (sec)	N/A	0.043	0.224	0.032	0.000	1.771	0.131	0.257	4.355	0.000
Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	17	18	17	15	14	46	15	0
N.S.	1	1.00	0.81	0.86	0.81	0.71	0.67	2.19	0.71	0.00
time (sec)	N/A	0.013	0.006	0.006	2.240	0.714	0.113	0.158	4.288	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	101	40	0	31	42	52	29	0
N.S.	1	1.00	2.20	0.87	0.00	0.67	0.91	1.13	0.63	0.00
time (sec)	N/A	0.041	0.275	0.033	0.000	0.634	0.144	0.242	4.372	0.000
Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	99	40	0	26	29	46	21	0
N.S.	1	1.00	2.25	0.91	0.00	0.59	0.66	1.05	0.48	0.00
time (sec)	N/A	0.034	0.101	0.036	0.000	0.732	0.135	0.175	0.057	0.001
Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	14	20	0	15	12	42	15	0
N.S.	1	1.00	0.61	0.87	0.00	0.65	0.52	1.83	0.65	0.00
time (sec)	N/A	0.026	0.007	0.035	0.000	0.710	0.115	0.193	4.347	0.001
Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	12	11	12	12	8	12	12	0
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.73	1.09	1.09	0.00
time (sec)	N/A	0.005	0.005	0.006	0.929	0.580	0.091	0.158	4.298	0.000
Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	29	30	29	25	26	33	14	0
N.S.	1	1.00	0.74	0.77	0.74	0.64	0.67	0.85	0.36	0.00
time (sec)	N/A	0.018	0.006	0.009	1.045	0.789	0.126	0.171	0.297	0.000
Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	42	82	0	47	46	77	20	0
N.S.	1	1.00	0.95	1.86	0.00	1.07	1.05	1.75	0.45	0.00
time (sec)	N/A	0.035	0.015	0.043	0.000	0.784	0.123	0.340	0.224	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	127	279	0	109	94	73	63	0
N.S.	1	1.00	1.92	4.23	0.00	1.65	1.42	1.11	0.95	0.00
time (sec)	N/A	0.029	0.073	0.019	0.000	0.811	0.378	0.314	0.068	0.002
Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	84	136	0	28	39	39	30	0
N.S.	1	1.00	1.83	2.96	0.00	0.61	0.85	0.85	0.65	0.00
time (sec)	N/A	0.031	0.073	0.017	0.000	0.628	0.132	0.176	4.380	0.001
Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	12	10	9	17	14	9	17	0
N.S.	1	1.00	1.33	1.11	1.00	1.89	1.56	1.00	1.89	0.00
time (sec)	N/A	0.009	0.007	0.007	2.356	0.913	0.118	0.174	4.363	0.001
Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	11	11	11	7	11	11	0
N.S.	1	1.00	1.00	1.00	1.00	1.00	0.64	1.00	1.00	0.00
time (sec)	N/A	0.005	0.004	0.008	1.079	0.713	0.090	0.157	4.299	0.001
Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	26	25	25	26	25	12	0
N.S.	1	1.00	1.00	0.90	0.86	0.86	0.90	0.86	0.41	0.00
time (sec)	N/A	0.016	0.006	0.004	1.002	0.868	0.114	0.154	0.063	0.000
Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	42	39	0	45	46	34	20	0
N.S.	1	1.00	0.84	0.78	0.00	0.90	0.92	0.68	0.40	0.00
time (sec)	N/A	0.023	0.014	0.010	0.000	0.728	0.114	0.172	4.369	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	42	39	0	43	46	41	20	0
N.S.	1	1.00	0.84	0.78	0.00	0.86	0.92	0.82	0.40	0.00
time (sec)	N/A	0.023	0.015	0.011	0.000	0.809	0.120	0.262	0.074	0.000
Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	28	27	27	22	34	15	0
N.S.	1	1.00	1.00	0.90	0.87	0.87	0.71	1.10	0.48	0.00
time (sec)	N/A	0.015	0.005	0.004	1.061	0.664	0.112	0.157	0.068	0.001
Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	42	39	0	45	46	41	20	0
N.S.	1	1.00	0.84	0.78	0.00	0.90	0.92	0.82	0.40	0.00
time (sec)	N/A	0.023	0.014	0.011	0.000	0.990	0.116	0.242	4.350	0.000
Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	42	39	0	45	46	40	20	0
N.S.	1	1.00	0.84	0.78	0.00	0.90	0.92	0.80	0.40	0.00
time (sec)	N/A	0.024	0.020	0.010	0.000	0.656	0.119	0.178	0.068	0.000
Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	42	39	0	45	46	41	20	0
N.S.	1	1.00	0.84	0.78	0.00	0.90	0.92	0.82	0.40	0.00
time (sec)	N/A	0.022	0.016	0.010	0.000	0.662	0.130	0.206	4.390	0.000
Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	32	12	25	29	32	29	11	0
N.S.	1	1.00	2.29	0.86	1.79	2.07	2.29	2.07	0.79	0.00
time (sec)	N/A	0.006	0.009	0.002	2.355	0.607	0.107	0.158	4.328	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	31	30	29	27	29	33	15	0
N.S.	1	1.00	0.79	0.77	0.74	0.69	0.74	0.85	0.38	0.00
time (sec)	N/A	0.017	0.006	0.009	0.957	0.472	0.122	0.153	0.101	0.000
Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	42	82	0	45	46	77	20	0
N.S.	1	1.00	0.88	1.71	0.00	0.94	0.96	1.60	0.42	0.00
time (sec)	N/A	0.039	0.019	0.017	0.000	0.590	0.118	0.322	0.127	0.000
Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	124	277	0	101	88	0	73	0
N.S.	1	1.00	2.00	4.47	0.00	1.63	1.42	0.00	1.18	0.00
time (sec)	N/A	0.056	0.058	0.043	0.000	1.663	0.376	0.000	0.065	0.000
Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	83	136	0	31	41	26	29	0
N.S.	1	1.00	1.69	2.78	0.00	0.63	0.84	0.53	0.59	0.00
time (sec)	N/A	0.088	0.139	0.050	0.000	0.812	0.124	0.177	0.083	0.000
Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	81	110	0	31	41	26	29	0
N.S.	1	1.00	1.88	2.56	0.00	0.72	0.95	0.60	0.67	0.00
time (sec)	N/A	0.050	0.070	0.048	0.000	0.670	0.141	0.186	0.085	0.001
Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	83	104	0	31	41	26	29	0
N.S.	1	1.00	1.69	2.12	0.00	0.63	0.84	0.53	0.59	0.00
time (sec)	N/A	0.062	0.104	0.043	0.000	0.943	0.127	0.163	4.391	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	2	2	2	3	2	2	2	2	2	0
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	1.00	1.00	0.00
time (sec)	N/A	0.002	0.003	0.002	2.419	1.101	0.100	0.155	4.332	0.000
Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	99	34	33	31	41	26	29	0
N.S.	1	1.00	2.61	0.89	0.87	0.82	1.08	0.68	0.76	0.00
time (sec)	N/A	0.027	0.195	0.006	2.403	0.918	0.123	0.162	0.077	0.000
Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	30	88	39	29	39	39	29	0
N.S.	1	1.00	0.86	2.51	1.11	0.83	1.11	1.11	0.83	0.00
time (sec)	N/A	0.018	0.015	0.004	2.418	1.221	0.123	0.192	4.368	0.000
Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	12	20	0	7	7	30	7	0
N.S.	1	1.00	0.52	0.87	0.00	0.30	0.30	1.30	0.30	0.00
time (sec)	N/A	0.020	0.007	0.019	0.000	1.154	0.110	0.168	4.315	0.000
Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	10	16	10	10	7	11	10	0
N.S.	1	1.00	0.91	1.45	0.91	0.91	0.64	1.00	0.91	0.00
time (sec)	N/A	0.003	0.004	0.005	1.063	1.212	0.087	0.152	4.341	0.000
Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	29	22	21	21	19	43	12	0
N.S.	1	1.00	0.45	0.34	0.32	0.32	0.29	0.66	0.18	0.00
time (sec)	N/A	0.032	0.006	0.006	0.992	1.204	0.113	0.165	0.256	0.000



Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	40	70	0	36	39	39	18	0
N.S.	1	1.00	0.93	1.63	0.00	0.84	0.91	0.91	0.42	0.00
time (sec)	N/A	0.033	0.014	0.040	0.000	1.149	0.114	0.215	4.395	0.000
Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	40	82	0	39	39	39	18	0
N.S.	1	1.00	0.87	1.78	0.00	0.85	0.85	0.85	0.39	0.00
time (sec)	N/A	0.036	0.014	0.036	0.000	1.007	0.119	0.244	4.474	0.000
Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F	A	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	125	279	0	100	87	0	76	0
N.S.	1	1.00	2.02	4.50	0.00	1.61	1.40	0.00	1.23	0.00
time (sec)	N/A	0.029	0.073	0.018	0.000	1.208	0.348	0.000	4.338	0.000
Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	87	136	0	31	42	26	31	0
N.S.	1	1.00	1.74	2.72	0.00	0.62	0.84	0.52	0.62	0.00
time (sec)	N/A	0.040	0.135	0.017	0.000	0.796	0.130	0.167	0.079	0.001
Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	82	111	0	31	42	26	31	0
N.S.	1	1.00	1.86	2.52	0.00	0.70	0.95	0.59	0.70	0.00
time (sec)	N/A	0.029	0.071	0.017	0.000	0.933	0.131	0.162	0.079	0.001
Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	10	104	0	13	10	26	13	0
N.S.	1	1.00	0.26	2.67	0.00	0.33	0.26	0.67	0.33	0.00
time (sec)	N/A	0.033	0.007	0.018	0.000	0.894	0.119	0.183	4.308	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	9	10	9	9	5	7	9	0
N.S.	1	1.00	1.00	1.11	1.00	1.00	0.56	0.78	1.00	0.00
time (sec)	N/A	0.004	0.004	0.007	0.998	0.850	0.095	0.181	0.030	0.000
Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	22	21	21	19	35	10	0
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.76	1.40	0.40	0.00
time (sec)	N/A	0.013	0.006	0.004	1.038	1.558	0.116	0.149	0.060	0.000
Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	40	62	34	34	39	34	18	0
N.S.	1	1.00	0.87	1.35	0.74	0.74	0.85	0.74	0.39	0.00
time (sec)	N/A	0.019	0.013	0.003	2.263	0.999	0.113	0.150	0.060	0.000
Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	40	35	0	39	39	39	18	0
N.S.	1	1.00	0.87	0.76	0.00	0.85	0.85	0.85	0.39	0.00
time (sec)	N/A	0.021	0.013	0.012	0.000	0.778	0.121	0.179	4.305	0.000
Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	2	2	19	3	13	13	12	15	2	0
N.S.	1	1.00	9.50	1.50	6.50	6.50	6.00	7.50	1.00	0.00
time (sec)	N/A	0.002	0.002	0.001	1.072	1.073	0.109	0.148	4.305	0.000
Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	40	34	55	39	39	39	18	0
N.S.	1	1.00	1.05	0.89	1.45	1.03	1.03	1.03	0.47	0.00
time (sec)	N/A	0.029	0.014	0.004	2.458	1.113	0.116	0.182	0.112	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	40	70	0	39	39	39	18	0
N.S.	1	1.00	0.85	1.49	0.00	0.83	0.83	0.83	0.38	0.00
time (sec)	N/A	0.036	0.018	0.018	0.000	1.263	0.118	0.322	4.323	0.000
Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	40	82	0	39	39	39	18	0
N.S.	1	1.00	0.87	1.78	0.00	0.85	0.85	0.85	0.39	0.00
time (sec)	N/A	0.035	0.016	0.018	0.000	0.780	0.142	0.225	4.388	0.000
Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	99	34	33	33	46	33	29	0
N.S.	1	1.00	2.30	0.79	0.77	0.77	1.07	0.77	0.67	0.00
time (sec)	N/A	0.035	0.102	0.008	2.354	1.212	0.142	0.160	4.378	0.001
Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	99	34	33	33	46	33	29	0
N.S.	1	1.00	2.30	0.79	0.77	0.77	1.07	0.77	0.67	0.00
time (sec)	N/A	0.032	0.033	0.004	2.487	0.924	0.150	0.177	0.002	0.001
Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	27	28	23	34	22	25	17	0
N.S.	1	1.00	1.29	1.33	1.10	1.62	1.05	1.19	0.81	0.00
time (sec)	N/A	0.005	0.010	0.009	1.104	1.031	0.129	0.173	0.033	0.000
Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	53	26	38	49	53	44	17	0
N.S.	1	1.00	1.89	0.93	1.36	1.75	1.89	1.57	0.61	0.00
time (sec)	N/A	0.013	0.019	0.006	2.356	1.005	0.606	0.174	4.385	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	72	56	51	55	474	60	290	0
N.S.	1	1.00	2.00	1.56	1.42	1.53	13.17	1.67	8.06	0.00
time (sec)	N/A	0.040	0.041	0.008	2.407	0.997	1.502	0.155	4.389	0.000
Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	73	104	0	137	46	41	117	0
N.S.	1	1.00	0.99	1.41	0.00	1.85	0.62	0.55	1.58	0.00
time (sec)	N/A	0.045	0.098	0.025	0.000	1.049	0.209	0.158	0.108	0.000
Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	83	83	97	114	69	69	740	69	827	0
N.S.	1	1.00	1.17	1.37	0.83	0.83	8.92	0.83	9.96	0.00
time (sec)	N/A	0.055	0.127	0.005	2.427	1.039	1.264	0.154	4.496	0.001
Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	119	119	147	168	105	185	874	109	897	0
N.S.	1	1.00	1.24	1.41	0.88	1.55	7.34	0.92	7.54	0.00
time (sec)	N/A	0.091	0.249	0.014	2.349	0.803	1.895	0.157	4.495	0.000
Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	234	234	111	710	0	3406	122	544	771	0
N.S.	1	1.00	0.47	3.03	0.00	14.56	0.52	2.32	3.29	0.00
time (sec)	N/A	0.230	0.116	0.099	0.000	1.285	1.322	0.878	4.490	0.000
Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	316	316	165	756	0	4346	165	988	1491	0
N.S.	1	1.00	0.52	2.39	0.00	13.75	0.52	3.13	4.72	0.00
time (sec)	N/A	0.289	0.216	0.313	0.000	1.149	1.801	0.946	4.499	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	C	F(-2)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	53	199	0	97	0	122	121	0
N.S.	1	1.00	0.33	1.24	0.00	0.61	0.00	0.76	0.76	0.00
time (sec)	N/A	0.146	0.045	0.089	0.000	1.008	0.000	0.378	4.956	0.001
Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	C	F(-2)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	172	172	53	199	0	97	0	126	121	0
N.S.	1	1.00	0.31	1.16	0.00	0.56	0.00	0.73	0.70	0.00
time (sec)	N/A	0.137	0.035	0.088	0.000	1.129	0.000	0.328	4.953	0.001
Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	137	285	0	451	1469	1501	1227	0
N.S.	1	1.00	0.86	1.78	0.00	2.82	9.18	9.38	7.67	0.00
time (sec)	N/A	0.119	0.090	0.023	0.000	1.216	2.862	0.322	1.067	0.001
Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	136	283	0	455	1467	1501	1227	0
N.S.	1	1.00	0.85	1.77	0.00	2.84	9.17	9.38	7.67	0.00
time (sec)	N/A	0.103	0.057	0.019	0.000	1.102	2.729	0.353	5.246	0.001
Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	115	92	0	517	27	0	133	0
N.S.	1	1.00	1.01	0.81	0.00	4.54	0.24	0.00	1.17	0.00
time (sec)	N/A	0.076	0.172	0.037	0.000	0.846	0.253	0.000	4.484	0.001
Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	F(-2)	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	115	96	0	251	0	0	159	0
N.S.	1	1.00	0.94	0.79	0.00	2.06	0.00	0.00	1.30	0.00
time (sec)	N/A	0.080	0.153	0.048	0.000	1.066	0.000	0.000	5.060	0.001

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	115	89	88	264	143	92	133	0
N.S.	1	1.00	0.93	0.72	0.71	2.13	1.15	0.74	1.07	0.00
time (sec)	N/A	0.077	0.131	0.026	2.287	1.288	0.313	0.177	0.237	0.001
Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	130	190	0	4551	172	0	1007	0
N.S.	1	1.00	0.96	1.40	0.00	33.46	1.26	0.00	7.40	0.00
time (sec)	N/A	0.104	0.150	0.026	0.000	2.412	1.907	0.000	4.587	0.001
Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	F(-2)	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	138	198	0	1141	0	0	1155	0
N.S.	1	1.00	0.86	1.24	0.00	7.13	0.00	0.00	7.22	0.00
time (sec)	N/A	0.116	0.135	0.041	0.000	1.678	0.000	0.000	4.986	0.001
Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	F(-2)	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	414	414	247	404	0	1457	0	0	3285	0
N.S.	1	1.00	0.60	0.98	0.00	3.52	0.00	0.00	7.93	0.00
time (sec)	N/A	0.453	0.202	0.064	0.000	1.514	0.000	0.000	5.219	0.001
Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	F(-2)	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	234	234	163	320	0	1469	0	0	1575	0
N.S.	1	1.00	0.70	1.37	0.00	6.28	0.00	0.00	6.73	0.00
time (sec)	N/A	0.172	0.186	0.071	0.000	2.806	0.000	0.000	5.290	0.001
Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	106	97	94	98	110	94	95	0
N.S.	1	1.00	1.00	0.92	0.89	0.92	1.04	0.89	0.90	0.00
time (sec)	N/A	0.083	0.020	0.002	1.051	0.868	0.093	0.153	4.350	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	79	72	71	73	78	71	71	0
N.S.	1	1.00	1.00	0.91	0.90	0.92	0.99	0.90	0.90	0.00
time (sec)	N/A	0.056	0.016	0.002	1.037	0.873	0.088	0.149	0.030	0.000
Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	56	49	48	50	56	50	49	0
N.S.	1	1.00	1.00	0.88	0.86	0.89	1.00	0.89	0.88	0.00
time (sec)	N/A	0.032	0.011	0.000	0.967	0.570	0.077	0.199	0.024	0.000
Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	27	26	26	29	28	26	0
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.88	0.81	0.00
time (sec)	N/A	0.014	0.002	0.001	1.061	0.807	0.076	0.178	0.042	0.000
Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	55	57	47	131	104	44	45	0
N.S.	1	1.00	1.00	1.04	0.85	2.38	1.89	0.80	0.82	0.00
time (sec)	N/A	0.035	0.036	0.008	2.546	1.106	0.324	0.169	0.069	0.001
Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	78	82	74	222	138	62	68	0
N.S.	1	1.00	1.05	1.11	1.00	3.00	1.86	0.84	0.92	0.00
time (sec)	N/A	0.052	0.051	0.011	2.238	1.109	0.510	0.157	4.442	0.000
Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	92	99	102	306	219	77	97	0
N.S.	1	1.00	0.99	1.06	1.10	3.29	2.35	0.83	1.04	0.00
time (sec)	N/A	0.067	0.064	0.009	2.559	0.894	0.755	0.160	4.481	0.001

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	113	122	137	424	204	100	129	0
N.S.	1	1.00	0.92	0.99	1.11	3.45	1.66	0.81	1.05	0.00
time (sec)	N/A	0.114	0.081	0.009	2.355	0.999	0.952	0.154	4.483	0.001
Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	133	130	129	131	144	128	127	0
N.S.	1	1.00	1.00	0.98	0.97	0.98	1.08	0.96	0.95	0.00
time (sec)	N/A	0.107	0.022	0.002	1.066	0.786	0.095	0.159	0.058	0.000
Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	97	90	89	91	104	91	89	0
N.S.	1	1.00	1.00	0.93	0.92	0.94	1.07	0.94	0.92	0.00
time (sec)	N/A	0.068	0.018	0.002	1.027	0.751	0.088	0.149	0.048	0.000
Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	60	51	50	50	60	53	50	0
N.S.	1	1.00	1.00	0.85	0.83	0.83	1.00	0.88	0.83	0.00
time (sec)	N/A	0.030	0.003	0.002	1.037	1.274	0.076	0.152	0.026	0.000
Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	22	21	21	22	21	21	0
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84	0.00
time (sec)	N/A	0.008	0.001	0.000	1.004	0.369	0.065	0.166	0.028	0.000
Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	97	136	113	268	236	105	141	0
N.S.	1	1.00	0.90	1.26	1.05	2.48	2.19	0.97	1.31	0.00
time (sec)	N/A	0.077	0.079	0.005	2.453	1.268	0.498	0.156	4.394	0.000



Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	134	170	142	394	314	128	183	0
N.S.	1	1.00	1.02	1.30	1.08	3.01	2.40	0.98	1.40	0.00
time (sec)	N/A	0.188	0.108	0.011	2.283	1.602	0.932	0.171	4.399	0.001
Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	155	155	154	211	167	516	257	145	164	0
N.S.	1	1.00	0.99	1.36	1.08	3.33	1.66	0.94	1.06	0.00
time (sec)	N/A	0.252	0.110	0.011	2.313	0.885	1.713	0.172	4.410	0.001
Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	184	184	174	262	205	662	292	167	199	0
N.S.	1	1.00	0.95	1.42	1.11	3.60	1.59	0.91	1.08	0.00
time (sec)	N/A	0.297	0.138	0.014	2.392	0.868	2.612	0.164	4.486	0.001
Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	200	231	244	806	335	198	240	0
N.S.	1	1.00	0.90	1.04	1.09	3.61	1.50	0.89	1.08	0.00
time (sec)	N/A	0.339	0.189	0.012	2.413	0.958	4.114	0.252	4.492	0.001
Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	437	437	444	741	432	2878	500	498	4022	0
N.S.	1	1.00	1.02	1.70	0.99	6.59	1.14	1.14	9.20	0.00
time (sec)	N/A	0.453	0.339	0.011	2.453	11.045	3.752	0.185	5.081	0.001
Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	370	370	360	572	342	2133	350	405	2712	0
N.S.	1	1.00	0.97	1.55	0.92	5.76	0.95	1.09	7.33	0.00
time (sec)	N/A	0.501	0.277	0.004	2.484	2.980	2.273	0.205	4.877	0.001

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	297	297	269	412	288	1480	238	318	1479	0
N.S.	1	1.00	0.91	1.39	0.97	4.98	0.80	1.07	4.98	0.00
time (sec)	N/A	0.293	0.257	0.004	2.361	1.362	1.478	0.179	4.794	0.001
Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	247	247	183	260	221	767	109	245	599	0
N.S.	1	1.00	0.74	1.05	0.89	3.11	0.44	0.99	2.43	0.00
time (sec)	N/A	0.152	0.054	0.003	2.533	0.982	0.680	0.176	4.682	0.001
Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	185	185	134	128	169	121	20	179	33	0
N.S.	1	1.00	0.72	0.69	0.91	0.65	0.11	0.97	0.18	0.00
time (sec)	N/A	0.111	0.018	0.003	2.439	1.759	0.174	0.181	4.409	0.000
Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	336	336	234	363	268	4084	0	339	4802	0
N.S.	1	1.00	0.70	1.08	0.80	12.15	0.00	1.01	14.29	0.00
time (sec)	N/A	0.270	0.154	0.007	2.385	2.564	0.000	0.207	5.706	0.001
Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	453	453	362	650	403	8409	0	517	16369	0
N.S.	1	1.00	0.80	1.43	0.89	18.56	0.00	1.14	36.13	0.00
time (sec)	N/A	0.384	0.468	0.012	2.445	42.205	0.000	0.253	6.548	0.001
Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	363	363	371	624	292	2116	352	425	2560	0
N.S.	1	1.00	1.02	1.72	0.80	5.83	0.97	1.17	7.05	0.00
time (sec)	N/A	0.410	0.260	0.011	2.364	1.132	3.371	0.188	4.940	0.001

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	349	349	295	464	324	1596	275	350	1565	0
N.S.	1	1.00	0.85	1.33	0.93	4.57	0.79	1.00	4.48	0.00
time (sec)	N/A	0.313	0.167	0.009	2.589	1.773	2.069	0.190	4.786	0.001
Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	275	275	267	303	253	873	136	273	637	0
N.S.	1	1.00	0.97	1.10	0.92	3.17	0.49	0.99	2.32	0.00
time (sec)	N/A	0.203	0.273	0.006	2.267	1.103	1.032	0.440	0.396	0.001
Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	202	183	143	189	173	39	194	58	0
N.S.	1	1.00	0.91	0.71	0.94	0.86	0.19	0.96	0.29	0.00
time (sec)	N/A	0.133	0.115	0.005	2.430	1.091	0.348	0.182	0.084	0.000
Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	689	689	429	873	506	9892	0	603	17945	0
N.S.	1	1.00	0.62	1.27	0.73	14.36	0.00	0.88	26.04	0.00
time (sec)	N/A	0.623	0.295	0.017	2.449	45.347	0.000	0.209	6.781	0.001
Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	864	864	540	1169	732	0	0	855	28923	0
N.S.	1	1.00	0.62	1.35	0.85	0.00	0.00	0.99	33.48	0.00
time (sec)	N/A	0.906	0.584	0.020	2.607	0.000	0.000	0.247	8.330	0.001
Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	51	42	56	116	75	144	42	0
N.S.	1	1.00	1.00	0.82	1.10	2.27	1.47	2.82	0.82	0.00
time (sec)	N/A	0.041	0.023	0.003	2.248	0.832	0.241	0.213	0.091	0.001

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	38	31	45	90	58	123	28	0
N.S.	1	1.00	1.00	0.82	1.18	2.37	1.53	3.24	0.74	0.00
time (sec)	N/A	0.034	0.018	0.003	2.453	0.868	0.201	0.232	0.055	0.001
Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	22	36	73	34	118	21	0
N.S.	1	1.00	1.00	0.76	1.24	2.52	1.17	4.07	0.72	0.00
time (sec)	N/A	0.023	0.009	0.002	2.451	1.676	0.183	0.206	4.432	0.001
Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	16	31	68	46	116	16	0
N.S.	1	1.00	1.00	0.67	1.29	2.83	1.92	4.83	0.67	0.00
time (sec)	N/A	0.012	0.005	0.002	2.351	1.094	0.155	0.290	0.058	0.001
Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	65	55	71	189	226	0	74	0
N.S.	1	1.00	0.90	0.76	0.99	2.62	3.14	0.00	1.03	0.00
time (sec)	N/A	0.057	0.037	0.011	2.445	0.883	0.452	0.000	0.159	0.001
Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	76	73	92	278	257	0	96	0
N.S.	1	1.00	0.85	0.82	1.03	3.12	2.89	0.00	1.08	0.00
time (sec)	N/A	0.083	0.059	0.011	2.493	1.235	0.715	0.000	0.163	0.001
Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	61	1442	0	199	0	24	-1	86
N.S.	1	1.00	0.98	23.26	0.00	3.21	0.00	0.39	-0.02	1.39
time (sec)	N/A	0.044	0.029	0.059	0.000	0.854	0.000	0.246	0.000	0.116

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	38	986	0	138	0	131	-1	61
N.S.	1	1.00	1.00	25.95	0.00	3.63	0.00	3.45	-0.03	1.61
time (sec)	N/A	0.026	0.155	0.024	0.000	0.557	0.000	0.528	0.000	0.072
Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	108	441	0	209	0	1	-1	84
N.S.	1	1.00	1.77	7.23	0.00	3.43	0.00	0.02	-0.02	1.38
time (sec)	N/A	0.039	0.126	0.022	0.000	1.082	0.000	0.328	0.000	0.148
Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	80	80	345	911	0	279	0	0	-1	94
N.S.	1	1.00	4.31	11.39	0.00	3.49	0.00	0.00	-0.01	1.18
time (sec)	N/A	0.069	3.343	0.028	0.000	1.741	0.000	0.000	0.000	0.200
Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	153	98	132	0	251	0	0	-1	0
N.S.	1	1.00	0.64	0.86	0.00	1.64	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.055	0.165	0.073	0.000	1.134	0.000	0.000	0.000	3.027
Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	86	107	0	223	0	0	-1	0
N.S.	1	1.00	0.78	0.97	0.00	2.03	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.035	0.072	0.022	0.000	1.085	0.000	0.000	0.000	2.548
Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	50	69	0	121	0	0	-1	0
N.S.	1	1.00	0.77	1.06	0.00	1.86	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.022	0.043	0.024	0.000	1.023	0.000	0.000	0.000	1.335

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	78	249	0	152	0	0	-1	0
N.S.	1	1.00	1.00	3.19	0.00	1.95	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.036	0.052	0.055	0.000	0.848	0.000	0.000	0.000	1.616
Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	125	111	488	0	297	0	0	-1	0
N.S.	1	1.00	0.89	3.90	0.00	2.38	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.051	0.085	0.056	0.000	0.889	0.000	0.000	0.000	2.620
Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	168	168	123	711	0	365	0	0	-1	0
N.S.	1	1.00	0.73	4.23	0.00	2.17	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.086	0.107	0.059	0.000	0.737	0.000	0.000	0.000	2.963
Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	152	123	105	0	265	0	0	-1	0
N.S.	1	1.00	0.81	0.69	0.00	1.74	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.055	0.200	0.020	0.000	1.318	0.000	0.000	0.000	3.072
Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	110	85	0	236	0	0	-1	0
N.S.	1	1.00	1.01	0.78	0.00	2.17	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.035	0.110	0.014	0.000	1.038	0.000	0.000	0.000	2.580
Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	67	54	0	125	0	0	-1	0
N.S.	1	1.00	1.05	0.84	0.00	1.95	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.025	0.040	0.014	0.000	1.067	0.000	0.000	0.000	1.359

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	77	267	0	155	0	0	-1	0
N.S.	1	1.00	1.00	3.47	0.00	2.01	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.036	0.050	0.063	0.000	1.081	0.000	0.000	0.000	1.595
Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	110	510	0	302	0	0	-1	0
N.S.	1	1.00	0.89	4.11	0.00	2.44	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.054	0.082	0.042	0.000	1.139	0.000	0.000	0.000	2.681
Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	122	739	0	376	0	0	-1	0
N.S.	1	1.00	0.73	4.43	0.00	2.25	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.087	0.106	0.053	0.000	1.157	0.000	0.000	0.000	2.940
Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	38	25	0	73	0	0	-1	0
N.S.	1	1.00	1.27	0.83	0.00	2.43	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.010	0.023	0.013	0.000	0.690	0.000	0.000	0.000	0.710
Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	40	34	33	0	65	0	0	-1	0
N.S.	1	1.67	1.42	1.38	0.00	2.71	0.00	0.00	-0.04	0.00
time (sec)	N/A	0.011	0.020	0.011	0.000	1.028	0.000	0.000	0.000	0.686
Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	72	71	59	0	137	0	0	-1	0
N.S.	1	0.99	0.97	0.81	0.00	1.88	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.118	0.056	0.002	0.000	1.419	0.000	0.000	0.000	4.666

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	121	226	0	446	345	10312	182	0
N.S.	1	1.00	1.00	1.87	0.00	3.69	2.85	85.22	1.50	0.00
time (sec)	N/A	0.160	0.075	0.010	0.000	0.482	0.996	5.854	4.532	0.001
Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	84	142	0	311	275	8680	113	0
N.S.	1	1.00	0.98	1.65	0.00	3.62	3.20	100.93	1.31	0.00
time (sec)	N/A	0.107	0.046	0.005	0.000	1.697	0.720	5.304	4.522	0.001
Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	63	79	0	210	212	7051	52	0
N.S.	1	1.00	0.98	1.23	0.00	3.28	3.31	110.17	0.81	0.00
time (sec)	N/A	0.078	0.055	0.004	0.000	1.112	0.485	4.818	0.069	0.001
Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	48	33	0	134	124	3276	38	0
N.S.	1	1.00	0.98	0.67	0.00	2.73	2.53	66.86	0.78	0.00
time (sec)	N/A	0.029	0.012	0.002	0.000	0.626	0.325	6.095	4.491	0.001
Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F(-1)	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	133	155	0	895	0	0	3901	0
N.S.	1	1.00	0.98	1.14	0.00	6.58	0.00	0.00	28.68	0.00
time (sec)	N/A	0.178	0.201	0.013	0.000	1.287	0.000	0.000	5.403	0.001
Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F(-1)	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	187	187	177	319	0	1765	0	0	6267	0
N.S.	1	1.00	0.95	1.71	0.00	9.44	0.00	0.00	33.51	0.00
time (sec)	N/A	0.278	0.411	0.013	0.000	3.708	0.000	0.000	6.453	0.001



Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	139	134	7043	0	1079	0	54	-1	179
N.S.	1	1.00	0.96	50.67	0.00	7.76	0.00	0.39	-0.01	1.29
time (sec)	N/A	0.276	0.257	0.063	0.000	2.745	0.000	2.394	0.000	0.377
Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	103	4308	0	940	0	27	-1	217
N.S.	1	1.00	0.95	39.89	0.00	8.70	0.00	0.25	-0.01	2.01
time (sec)	N/A	0.126	0.086	0.023	0.000	1.041	0.000	2.385	0.000	0.264
Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	76	2252	0	432	0	0	-1	192
N.S.	1	1.00	1.00	29.63	0.00	5.68	0.00	0.00	-0.01	2.53
time (sec)	N/A	0.070	0.065	0.023	0.000	1.094	0.000	0.000	0.000	0.185
Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	106	106	418	771	0	701	0	0	-1	221
N.S.	1	1.00	3.94	7.27	0.00	6.61	0.00	0.00	-0.01	2.08
time (sec)	N/A	0.117	1.049	0.021	0.000	1.270	0.000	0.000	0.000	0.367
Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	149	149	1058	1637	0	1063	0	0	-1	256
N.S.	1	1.00	7.10	10.99	0.00	7.13	0.00	0.00	-0.01	1.72
time (sec)	N/A	0.269	4.136	0.021	0.000	2.645	0.000	0.000	0.000	0.567
Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	135	135	136	135	148	156	142	131	0
N.S.	1	1.00	1.00	1.01	1.00	1.10	1.16	1.05	0.97	0.00
time (sec)	N/A	0.126	0.037	0.001	0.962	0.869	0.113	0.154	0.063	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	104	103	102	111	112	108	101	0
N.S.	1	1.00	1.01	1.00	0.99	1.08	1.09	1.05	0.98	0.00
time (sec)	N/A	0.095	0.028	0.001	1.034	0.600	0.291	0.155	4.628	0.000
Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	73	70	69	76	78	76	70	0
N.S.	1	1.00	1.00	0.96	0.95	1.04	1.07	1.04	0.96	0.00
time (sec)	N/A	0.060	0.020	0.001	1.074	0.792	0.108	0.149	4.587	0.000
Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	37	36	40	39	43	38	0
N.S.	1	1.00	1.00	0.88	0.86	0.95	0.93	1.02	0.90	0.00
time (sec)	N/A	0.027	0.009	0.001	0.904	0.928	0.101	0.148	0.044	0.000
Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	65	84	58	159	117	56	57	0
N.S.	1	1.00	0.98	1.27	0.88	2.41	1.77	0.85	0.86	0.00
time (sec)	N/A	0.045	0.052	0.004	2.409	1.005	0.729	0.151	0.085	0.001
Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	88	118	84	268	153	75	77	0
N.S.	1	1.00	1.06	1.42	1.01	3.23	1.84	0.90	0.93	0.00
time (sec)	N/A	0.093	0.056	0.009	2.245	1.025	1.233	0.170	4.670	0.001
Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	110	131	121	391	196	101	112	0
N.S.	1	1.00	0.96	1.14	1.05	3.40	1.70	0.88	0.97	0.00
time (sec)	N/A	0.107	0.097	0.008	2.253	1.369	2.266	0.232	4.847	0.001

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	142	158	162	530	241	134	144	0
N.S.	1	1.00	0.95	1.05	1.08	3.53	1.61	0.89	0.96	0.00
time (sec)	N/A	0.205	0.133	0.009	2.507	0.596	4.409	0.159	4.509	0.001
Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	223	219	218	261	272	255	220	0
N.S.	1	1.00	1.00	0.98	0.98	1.17	1.22	1.14	0.99	0.00
time (sec)	N/A	0.199	0.087	0.001	1.044	0.668	0.220	0.157	4.484	0.000
Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	155	155	156	155	147	181	192	181	148	0
N.S.	1	1.00	1.01	1.00	0.95	1.17	1.24	1.17	0.95	0.00
time (sec)	N/A	0.141	0.054	0.000	1.136	0.557	0.155	0.169	4.516	0.000
Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	96	91	90	100	107	106	90	0
N.S.	1	1.00	1.00	0.95	0.94	1.04	1.11	1.10	0.94	0.00
time (sec)	N/A	0.068	0.024	0.000	1.009	0.767	0.247	0.170	0.038	0.000
Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	49	42	45	43	48	43	42	0
N.S.	1	1.00	1.00	0.86	0.92	0.88	0.98	0.88	0.86	0.00
time (sec)	N/A	0.025	0.006	0.001	1.103	0.750	0.154	0.144	0.022	0.000
Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	144	267	176	406	371	185	229	0
N.S.	1	1.00	1.01	1.87	1.23	2.84	2.59	1.29	1.60	0.00
time (sec)	N/A	0.140	0.066	0.005	2.377	0.680	1.530	0.162	4.469	0.001

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	183	320	205	600	484	207	293	0
N.S.	1	1.00	1.10	1.93	1.23	3.61	2.92	1.25	1.77	0.00
time (sec)	N/A	0.298	0.101	0.011	2.416	0.915	3.786	0.180	4.563	0.001
Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	201	201	217	402	245	794	398	244	257	0
N.S.	1	1.00	1.08	2.00	1.22	3.95	1.98	1.21	1.28	0.00
time (sec)	N/A	0.419	0.113	0.013	2.363	0.712	17.717	0.182	0.118	0.001
Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	250	250	267	506	300	1016	457	296	308	0
N.S.	1	1.00	1.07	2.02	1.20	4.06	1.83	1.18	1.23	0.00
time (sec)	N/A	0.543	0.148	0.014	2.390	0.805	94.000	0.182	4.599	0.001
Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	317	317	345	412	366	1266	0	364	375	0
N.S.	1	1.00	1.09	1.30	1.15	3.99	0.00	1.15	1.18	0.00
time (sec)	N/A	0.650	0.225	0.013	2.519	0.641	0.000	0.195	4.574	0.001
Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	88	118	84	268	153	75	77	0
N.S.	1	1.00	1.06	1.42	1.01	3.23	1.84	0.90	0.93	0.00
time (sec)	N/A	0.093	0.017	0.000	2.342	0.687	0.820	0.162	0.002	0.001
Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	88	118	84	268	153	75	77	0
N.S.	1	1.00	1.06	1.42	1.01	3.23	1.84	0.90	0.93	0.00
time (sec)	N/A	0.084	0.017	0.009	2.369	0.619	0.862	0.154	0.115	0.001

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	459	459	570	1888	0	0	0	9285	29551	0
N.S.	1	1.00	1.24	4.11	0.00	0.00	0.00	20.23	64.38	0.00
time (sec)	N/A	1.537	0.687	0.049	0.000	0.000	0.000	1.628	9.313	0.001
Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	316	316	402	1211	0	9584	0	6407	17954	0
N.S.	1	1.00	1.27	3.83	0.00	30.33	0.00	20.28	56.82	0.00
time (sec)	N/A	0.786	0.550	0.037	0.000	34.095	0.000	1.355	7.290	0.001
Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	238	238	269	695	0	4690	0	4107	9600	0
N.S.	1	1.00	1.13	2.92	0.00	19.71	0.00	17.26	40.34	0.00
time (sec)	N/A	0.635	0.322	0.028	0.000	3.221	0.000	1.136	6.484	0.001
Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	174	174	172	328	0	1525	314	1402	4109	0
N.S.	1	1.00	0.99	1.89	0.00	8.76	1.80	8.06	23.61	0.00
time (sec)	N/A	0.202	0.139	0.020	0.000	0.881	20.947	0.872	5.382	0.001
Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	129	116	0	613	87	1024	763	0
N.S.	1	1.00	0.86	0.77	0.00	4.09	0.58	6.83	5.09	0.00
time (sec)	N/A	0.098	0.085	0.014	0.000	0.407	1.272	0.599	0.514	0.000
Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	254	254	274	480	0	0	0	7650	23640	0
N.S.	1	1.00	1.08	1.89	0.00	0.00	0.00	30.12	93.07	0.00
time (sec)	N/A	0.586	0.272	0.022	0.000	0.000	0.000	2.535	9.446	0.001

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	429	429	354	1141	0	0	0	13225	91169	0
N.S.	1	1.00	0.83	2.66	0.00	0.00	0.00	30.83	212.52	0.00
time (sec)	N/A	1.415	0.753	0.029	0.000	0.000	0.000	2.510	10.280	0.001
Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	563	563	540	1846	0	12117	0	8983	29030	0
N.S.	1	1.00	0.96	3.28	0.00	21.52	0.00	15.96	51.56	0.00
time (sec)	N/A	3.519	1.630	0.050	0.000	111.890	0.000	2.459	8.793	0.001
Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	386	386	415	1223	0	7338	0	6390	18785	0
N.S.	1	1.00	1.08	3.17	0.00	19.01	0.00	16.55	48.67	0.00
time (sec)	N/A	2.079	1.111	0.042	0.000	13.152	0.000	1.846	9.845	0.001
Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	293	293	310	1761	0	4573	0	4433	12350	0
N.S.	1	1.00	1.06	6.01	0.00	15.61	0.00	15.13	42.15	0.00
time (sec)	N/A	0.789	0.749	0.085	0.000	3.549	0.000	1.764	9.387	0.001
Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	243	733	0	2309	394	2682	6404	0
N.S.	1	1.00	0.96	2.91	0.00	9.16	1.56	10.64	25.41	0.00
time (sec)	N/A	0.517	0.425	0.060	0.000	0.845	170.284	0.602	6.257	0.000
Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	660	660	708	3841	0	0	0	0	237586	0
N.S.	1	1.00	1.07	5.82	0.00	0.00	0.00	0.00	359.98	0.00
time (sec)	N/A	2.873	2.789	0.064	0.000	0.000	0.000	0.000	16.455	0.001

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1077	1077	1020	5709	0	0	0	0	97073	0
N.S.	1	1.00	0.95	5.30	0.00	0.00	0.00	0.00	90.13	0.00
time (sec)	N/A	12.639	5.843	0.084	0.000	0.000	0.000	0.000	17.810	0.001
Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	215	215	190	283	261	370	505	180	-1	189
N.S.	1	1.00	0.88	1.32	1.21	1.72	2.35	0.84	-0.00	0.88
time (sec)	N/A	0.161	0.388	0.010	1.123	1.645	63.830	0.226	0.000	0.409
Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	175	175	157	229	207	304	413	145	-1	153
N.S.	1	1.00	0.90	1.31	1.18	1.74	2.36	0.83	-0.01	0.87
time (sec)	N/A	0.122	0.320	0.010	1.018	0.981	31.095	0.222	0.000	0.290
Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	121	175	153	232	272	106	-1	117
N.S.	1	1.00	0.92	1.33	1.16	1.76	2.06	0.80	-0.01	0.89
time (sec)	N/A	0.109	0.234	0.010	0.984	0.997	12.265	0.219	0.000	0.191
Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	82	122	100	174	230	79	-1	85
N.S.	1	1.00	0.85	1.26	1.03	1.79	2.37	0.81	-0.01	0.88
time (sec)	N/A	0.061	0.064	0.009	1.066	1.297	7.045	0.193	0.000	0.117
Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	98	112	97	249	134	80	-1	89
N.S.	1	1.00	1.10	1.26	1.09	2.80	1.51	0.90	-0.01	1.00
time (sec)	N/A	0.073	0.105	0.009	1.130	0.847	9.984	0.205	0.000	0.164

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	112	124	135	289	450	88	-1	95
N.S.	1	1.00	1.11	1.23	1.34	2.86	4.46	0.87	-0.01	0.94
time (sec)	N/A	0.071	0.190	0.008	1.007	0.851	18.951	0.225	0.000	0.199
Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	67	66	173	93	639	75	133	69
N.S.	1	1.00	0.78	0.77	2.01	1.08	7.43	0.87	1.55	0.80
time (sec)	N/A	0.107	0.052	0.005	1.156	1.066	45.985	0.211	4.704	0.208
Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	101	100	227	136	1989	113	154	103
N.S.	1	1.00	0.80	0.79	1.80	1.08	15.79	0.90	1.22	0.82
time (sec)	N/A	0.146	0.093	0.004	1.199	0.952	119.187	0.270	4.667	0.285
Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	165	164	132	136	281	177	0	148	189	139
N.S.	1	0.99	0.80	0.82	1.70	1.07	0.00	0.90	1.15	0.84
time (sec)	N/A	0.210	0.117	0.006	1.202	1.148	0.000	0.228	4.752	0.404
Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	210	210	167	172	335	224	0	189	226	175
N.S.	1	1.00	0.80	0.82	1.60	1.07	0.00	0.90	1.08	0.83
time (sec)	N/A	0.222	0.142	0.008	1.111	1.310	0.000	0.235	4.760	0.500
Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	65	65	685	327	0	323	0	0	-1	77
N.S.	1	1.00	10.54	5.03	0.00	4.97	0.00	0.00	-0.02	1.18
time (sec)	N/A	0.145	3.068	0.166	0.000	1.641	0.000	0.000	0.000	12.085



Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	63	63	876	311	0	112	0	0	-1	77
N.S.	1	1.00	13.90	4.94	0.00	1.78	0.00	0.00	-0.02	1.22
time (sec)	N/A	0.141	7.827	0.161	0.000	1.361	0.000	0.000	0.000	11.973
Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	72	72	623	336	0	328	0	0	-1	81
N.S.	1	1.00	8.65	4.67	0.00	4.56	0.00	0.00	-0.01	1.12
time (sec)	N/A	0.133	1.732	0.161	0.000	1.289	0.000	0.000	0.000	12.222
Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	70	70	881	337	0	114	0	0	-1	81
N.S.	1	1.00	12.59	4.81	0.00	1.63	0.00	0.00	-0.01	1.16
time (sec)	N/A	0.129	6.386	0.154	0.000	1.273	0.000	0.000	0.000	12.017

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [128] had the largest ratio of [.7778]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	9	6	1.00	17	0.353
2	A	9	6	1.00	18	0.333
3	A	3	3	1.00	18	0.167
4	A	3	3	1.00	19	0.158
5	A	5	3	1.00	17	0.176
6	A	3	2	1.00	17	0.118
7	A	2	2	1.00	17	0.118
8	A	2	2	1.00	17	0.118
9	A	5	3	1.00	27	0.111
10	A	3	2	1.00	28	0.071
11	A	5	3	1.00	21	0.143
12	A	3	2	1.00	22	0.091
13	A	3	3	1.00	15	0.200
14	A	5	3	1.00	26	0.115
15	A	5	3	1.00	26	0.115
16	A	5	3	1.00	27	0.111
17	A	5	3	1.00	27	0.111
18	A	3	2	1.00	27	0.074
19	A	3	2	1.00	27	0.074
20	A	3	2	1.00	28	0.071
21	A	3	2	1.00	28	0.071
22	A	3	2	1.00	30	0.067
23	A	5	3	1.00	29	0.103
24	A	6	4	1.00	29	0.138
25	A	3	2	1.00	32	0.062
26	A	5	3	1.00	31	0.097
27	A	5	3	1.00	22	0.136
28	A	5	3	1.00	23	0.130
29	A	3	2	1.00	22	0.091
30	A	3	2	1.00	22	0.091
31	A	3	3	1.00	22	0.136
32	A	5	3	1.00	22	0.136
33	A	5	3	1.00	22	0.136
34	A	5	3	1.00	20	0.150
35	A	5	3	1.00	17	0.176
36	A	5	3	1.00	22	0.136
37	A	5	3	1.00	22	0.136
38	A	5	3	1.00	22	0.136
39	A	2	2	1.00	22	0.091

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
40	A	7	3	1.00	22	0.136
41	A	5	3	1.00	22	0.136
42	A	3	2	1.00	22	0.091
43	A	3	2	1.00	22	0.091
44	A	3	2	1.00	22	0.091
45	A	2	2	1.00	22	0.091
46	A	3	2	1.00	22	0.091
47	A	3	2	1.00	22	0.091
48	A	3	2	1.00	20	0.100
49	A	3	2	1.00	17	0.118
50	A	3	2	1.00	22	0.091
51	A	3	2	1.00	22	0.091
52	A	3	2	1.00	22	0.091
53	A	3	3	1.00	22	0.136
54	A	7	3	1.00	22	0.136
55	A	5	3	1.00	22	0.136
56	A	5	3	1.00	18	0.167
57	A	3	2	1.00	18	0.111
58	A	3	2	1.00	18	0.111
59	A	3	2	1.00	18	0.111
60	A	2	2	1.00	18	0.111
61	A	5	3	1.00	16	0.188
62	A	5	3	1.00	13	0.231
63	A	5	3	1.00	18	0.167
64	A	2	2	1.00	18	0.111
65	A	7	3	1.00	18	0.167
66	A	5	3	1.00	18	0.167
67	A	5	3	1.00	18	0.167
68	A	3	2	1.00	20	0.100
69	A	3	2	1.00	20	0.100
70	A	3	2	1.00	20	0.100
71	A	3	2	1.00	20	0.100
72	A	2	2	1.00	20	0.100
73	A	3	2	1.00	18	0.111
74	A	3	2	1.00	15	0.133
75	A	3	2	1.00	20	0.100
76	A	3	3	1.00	20	0.150
77	A	5	3	1.00	20	0.150
78	A	5	3	1.00	20	0.150
79	A	5	3	1.00	20	0.150
80	A	5	3	1.00	23	0.130
81	A	5	3	1.00	22	0.136
82	A	3	3	1.00	20	0.150
83	A	3	2	1.00	22	0.091
84	A	3	2	1.00	22	0.091
85	A	3	2	1.00	18	0.111
86	A	9	5	1.00	18	0.278
87	A	10	6	1.00	18	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	9	5	1.00	18	0.278
89	A	10	6	1.00	18	0.333
90	A	9	5	1.00	29	0.172
91	A	9	5	1.00	26	0.192
92	A	9	5	1.00	24	0.208
93	A	9	5	1.00	22	0.227
94	A	9	5	1.00	25	0.200
95	A	9	5	1.00	31	0.161
96	A	9	5	1.00	32	0.156
97	A	9	5	1.00	23	0.217
98	A	9	5	1.00	25	0.200
99	A	9	5	1.00	29	0.172
100	A	9	5	1.00	32	0.156
101	A	2	1	1.00	17	0.059
102	A	2	1	1.00	17	0.059
103	A	2	1	1.00	17	0.059
104	A	2	1	1.00	15	0.067
105	A	3	2	1.00	17	0.118
106	A	3	3	1.00	17	0.176
107	A	3	3	1.00	17	0.176
108	A	4	4	1.00	17	0.235
109	A	2	1	1.00	19	0.053
110	A	2	1	1.00	19	0.053
111	A	2	1	1.00	17	0.059
112	A	2	1	1.00	9	0.111
113	A	3	2	1.00	19	0.105
114	A	4	3	1.00	19	0.158
115	A	5	4	1.00	19	0.210
116	A	5	5	1.00	19	0.263
117	A	5	5	1.00	19	0.263
118	A	11	7	1.00	19	0.368
119	A	11	7	1.00	19	0.368
120	A	11	7	1.00	19	0.368
121	A	9	6	1.00	17	0.353
122	A	9	6	1.00	9	0.667
123	A	12	8	1.00	19	0.421
124	A	14	9	1.00	19	0.474
125	A	11	8	1.00	19	0.421
126	A	11	8	1.00	19	0.421
127	A	10	7	1.00	17	0.412
128	A	10	7	1.00	9	0.778
129	A	22	9	1.00	19	0.474
130	A	24	10	1.00	19	0.526
131	A	4	3	1.00	24	0.125
132	A	4	3	1.00	24	0.125
133	A	3	3	1.00	24	0.125
134	A	2	2	1.00	22	0.091
135	A	5	5	1.00	24	0.208

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
136	A	6	6	1.00	24	0.250
137	A	6	6	1.00	26	0.231
138	A	3	3	1.00	26	0.115
139	A	4	4	1.00	26	0.154
140	A	6	6	1.00	26	0.231
141	A	5	5	1.00	28	0.179
142	A	4	4	1.00	28	0.143
143	A	3	3	1.00	28	0.107
144	A	3	3	1.00	28	0.107
145	A	4	4	1.00	28	0.143
146	A	6	6	1.00	28	0.214
147	A	5	5	1.00	29	0.172
148	A	4	4	1.00	29	0.138
149	A	3	3	1.00	29	0.103
150	A	3	3	1.00	29	0.103
151	A	4	4	1.00	29	0.138
152	A	6	6	1.00	29	0.207
153	A	2	2	1.00	19	0.105
154	A	3	3	1.67	19	0.158
155	A	7	5	0.99	31	0.161
156	A	4	3	1.00	39	0.077
157	A	4	3	1.00	39	0.077
158	A	3	3	1.00	39	0.077
159	A	2	2	1.00	37	0.054
160	A	5	5	1.00	39	0.128
161	A	6	6	1.00	39	0.154
162	A	7	7	1.00	41	0.171
163	A	6	6	1.00	41	0.146
164	A	3	3	1.00	41	0.073
165	A	4	4	1.00	41	0.098
166	A	6	6	1.00	41	0.146
167	A	2	1	1.00	22	0.045
168	A	2	1	1.00	22	0.045
169	A	2	1	1.00	22	0.045
170	A	2	1	1.00	20	0.050
171	A	3	2	1.00	22	0.091
172	A	3	3	1.00	22	0.136
173	A	3	3	1.00	22	0.136
174	A	4	4	1.00	22	0.182
175	A	2	1	1.00	24	0.042
176	A	2	1	1.00	24	0.042
177	A	2	1	1.00	22	0.045
178	A	2	1	1.00	14	0.071
179	A	3	2	1.00	24	0.083
180	A	4	3	1.00	24	0.125
181	A	5	4	1.00	24	0.167
182	A	5	4	1.00	24	0.167
183	A	5	4	1.00	24	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
184	A	3	3	1.00	22	0.136
185	A	3	3	1.00	23	0.130
186	A	5	3	1.00	24	0.125
187	A	5	3	1.00	24	0.125
188	A	5	3	1.00	24	0.125
189	A	3	2	1.00	22	0.091
190	A	3	2	1.00	14	0.143
191	A	6	3	1.00	24	0.125
192	A	8	4	1.00	24	0.167
193	A	4	3	1.00	24	0.125
194	A	4	3	1.00	24	0.125
195	A	4	3	1.00	22	0.136
196	A	4	3	1.00	14	0.214
197	A	10	4	1.00	24	0.167
198	A	12	5	1.00	24	0.208
199	A	7	5	1.00	24	0.208
200	A	6	5	1.00	24	0.208
201	A	5	5	1.00	24	0.208
202	A	4	4	1.00	24	0.167
203	A	4	4	1.00	24	0.167
204	A	4	4	1.00	24	0.167
205	A	4	4	1.00	24	0.167
206	A	5	5	1.00	24	0.208
207	A	6	5	0.99	24	0.208
208	A	7	5	1.00	24	0.208
209	A	2	2	1.00	40	0.050
210	A	2	2	1.00	40	0.050
211	A	2	2	1.00	46	0.043
212	A	2	2	1.00	46	0.043

# Chapter 3

## Listing of integrals

### Local contents

3.1	$\int \frac{c+dx^2}{a+bx^4} dx$	64
3.2	$\int \frac{c-dx^2}{a+bx^4} dx$	68
3.3	$\int \frac{c+dx^2}{a-bx^4} dx$	72
3.4	$\int \frac{c-dx^2}{a-bx^4} dx$	75
3.5	$\int \frac{2+3x^2}{4+9x^4} dx$	78
3.6	$\int \frac{2-3x^2}{4+9x^4} dx$	81
3.7	$\int \frac{2+3x^2}{4-9x^4} dx$	84
3.8	$\int \frac{2-3x^2}{4-9x^4} dx$	87
3.9	$\int \frac{\sqrt{a}\sqrt{b+bx^2}}{a+bx^4} dx$	90
3.10	$\int \frac{\sqrt{a}\sqrt{b-bx^2}}{a+bx^4} dx$	93
3.11	$\int \frac{d+ex^2}{d^2+e^2x^4} dx$	96
3.12	$\int \frac{d-ex^2}{d^2+e^2x^4} dx$	99
3.13	$\int \frac{5+2x^2}{-1+x^4} dx$	102
3.14	$\int \frac{d+ex^2}{d^2+bx^2+e^2x^4} dx$	104
3.15	$\int \frac{d+ex^2}{d^2+fx^2+e^2x^4} dx$	108
3.16	$\int \frac{d+ex^2}{d^2-bx^2+e^2x^4} dx$	112
3.17	$\int \frac{d+ex^2}{d^2-fx^2+e^2x^4} dx$	116
3.18	$\int \frac{d-ex^2}{d^2+bx^2+e^2x^4} dx$	120
3.19	$\int \frac{d-ex^2}{d^2+fx^2+e^2x^4} dx$	124
3.20	$\int \frac{d-ex^2}{d^2-bx^2+e^2x^4} dx$	128
3.21	$\int \frac{d-ex^2}{d^2-fx^2+e^2x^4} dx$	132
3.22	$\int \frac{d-ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4} dx$	136
3.23	$\int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4} dx$	140
3.24	$\int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)} dx$	144
3.25	$\int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4} dx$	149

3.26	$\int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4} dx$	151
3.27	$\int \frac{1+2x^2}{1+bx^2+4x^4} dx$	154
3.28	$\int \frac{1+2x^2}{1-bx^2+4x^4} dx$	157
3.29	$\int \frac{1+2x^2}{1+6x^2+4x^4} dx$	160
3.30	$\int \frac{1+2x^2}{1+5x^2+4x^4} dx$	163
3.31	$\int \frac{1+2x^2}{1+4x^2+4x^4} dx$	165
3.32	$\int \frac{1+2x^2}{1+3x^2+4x^4} dx$	168
3.33	$\int \frac{1+2x^2}{1+2x^2+4x^4} dx$	171
3.34	$\int \frac{1+2x^2}{1+x^2+4x^4} dx$	174
3.35	$\int \frac{1+2x^2}{1+4x^4} dx$	177
3.36	$\int \frac{1+2x^2}{1-x^2+4x^4} dx$	180
3.37	$\int \frac{1+2x^2}{1-2x^2+4x^4} dx$	183
3.38	$\int \frac{1+2x^2}{1-3x^2+4x^4} dx$	186
3.39	$\int \frac{1+2x^2}{1-4x^2+4x^4} dx$	189
3.40	$\int \frac{1+2x^2}{1-5x^2+4x^4} dx$	191
3.41	$\int \frac{1+2x^2}{1-6x^2+4x^4} dx$	194
3.42	$\int \frac{1-2x^2}{1+bx^2+4x^4} dx$	197
3.43	$\int \frac{1-2x^2}{1+6x^2+4x^4} dx$	200
3.44	$\int \frac{1-2x^2}{1+5x^2+4x^4} dx$	203
3.45	$\int \frac{1-2x^2}{1+4x^2+4x^4} dx$	205
3.46	$\int \frac{1-2x^2}{1+3x^2+4x^4} dx$	207
3.47	$\int \frac{1-2x^2}{1+2x^2+4x^4} dx$	209
3.48	$\int \frac{1-2x^2}{1+x^2+4x^4} dx$	212
3.49	$\int \frac{1-2x^2}{1+4x^4} dx$	215
3.50	$\int \frac{1-2x^2}{1-x^2+4x^4} dx$	218
3.51	$\int \frac{1-2x^2}{1-2x^2+4x^4} dx$	221
3.52	$\int \frac{1-2x^2}{1-3x^2+4x^4} dx$	224
3.53	$\int \frac{1-2x^2}{1-4x^2+4x^4} dx$	227
3.54	$\int \frac{1-2x^2}{1-5x^2+4x^4} dx$	230
3.55	$\int \frac{1-2x^2}{1-6x^2+4x^4} dx$	233
3.56	$\int \frac{1+x^2}{1+bx^2+x^4} dx$	236
3.57	$\int \frac{1+x^2}{1+5x^2+x^4} dx$	239
3.58	$\int \frac{1+x^2}{1+4x^2+x^4} dx$	242
3.59	$\int \frac{1+x^2}{1+3x^2+x^4} dx$	245
3.60	$\int \frac{1+x^2}{1+2x^2+x^4} dx$	248
3.61	$\int \frac{1+x^2}{1+x^2+x^4} dx$	250
3.62	$\int \frac{1+x^2}{1+x^4} dx$	253
3.63	$\int \frac{1+x^2}{1-x^2+x^4} dx$	256



3.64	$\int \frac{1+x^2}{1-2x^2+x^4} dx$	259
3.65	$\int \frac{1+x^2}{1-3x^2+x^4} dx$	261
3.66	$\int \frac{1+x^2}{1-4x^2+x^4} dx$	264
3.67	$\int \frac{1+x^2}{1-5x^2+x^4} dx$	267
3.68	$\int \frac{1-x^2}{1+bx^2+x^4} dx$	270
3.69	$\int \frac{1-x^2}{1+5x^2+x^4} dx$	273
3.70	$\int \frac{1-x^2}{1+4x^2+x^4} dx$	276
3.71	$\int \frac{1-x^2}{1+3x^2+x^4} dx$	279
3.72	$\int \frac{1-x^2}{1+2x^2+x^4} dx$	282
3.73	$\int \frac{1-x^2}{1+x^2+x^4} dx$	284
3.74	$\int \frac{1-x^2}{1+x^4} dx$	287
3.75	$\int \frac{1-x^2}{1-x^2+x^4} dx$	290
3.76	$\int \frac{1-x^2}{1-2x^2+x^4} dx$	293
3.77	$\int \frac{1-x^2}{1-3x^2+x^4} dx$	296
3.78	$\int \frac{1-x^2}{1-4x^2+x^4} dx$	299
3.79	$\int \frac{1-x^2}{1-5x^2+x^4} dx$	302
3.80	$\int \frac{-1-3x^2}{1+2x^2+9x^4} dx$	305
3.81	$\int \frac{1+3x^2}{-1-2x^2-9x^4} dx$	308
3.82	$\int \frac{3+2x^2}{1-2x^2+x^4} dx$	311
3.83	$\int \frac{2+3x^2}{5-8x^2+3x^4} dx$	314
3.84	$\int \frac{d+ex^2}{5-8x^2+3x^4} dx$	317
3.85	$\int \frac{3+x^2}{1+3x^2+x^4} dx$	320
3.86	$\int \frac{a+bx^2}{1+x^2+x^4} dx$	323
3.87	$\int \frac{a+bx^2}{(1+x^2+x^4)^2} dx$	327
3.88	$\int \frac{a+bx^2}{2+x^2+x^4} dx$	331
3.89	$\int \frac{a+bx^2}{(2+x^2+x^4)^2} dx$	336
3.90	$\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4} dx$	343
3.91	$\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4} dx$	346
3.92	$\int \frac{\sqrt{2}-x^2}{1+bx^2+x^4} dx$	349
3.93	$\int \frac{\sqrt{2}+x^2}{1+bx^2+x^4} dx$	354
3.94	$\int \frac{2a-x^2}{a^2-ax^2+x^4} dx$	359
3.95	$\int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}x^2+x^4} dx$	365
3.96	$\int \frac{2b^{2/3}+x^2}{b^{4/3}+b^{2/3}x^2+x^4} dx$	368
3.97	$\int \frac{A+Bx^2}{a^2-ax^2+x^4} dx$	372
3.98	$\int \frac{A+Bx^2}{a-\sqrt{a}x^2+x^4} dx$	380
3.99	$\int \frac{A+Bx^2}{a-\sqrt{ac}x^2+cx^4} dx$	384
3.100	$\int \frac{A+Bx^2}{a-\sqrt{a}\sqrt{c}x^2+cx^4} dx$	389

3.101	$\int (d + ex^2)^4 (a + cx^4) dx$	394
3.102	$\int (d + ex^2)^3 (a + cx^4) dx$	397
3.103	$\int (d + ex^2)^2 (a + cx^4) dx$	399
3.104	$\int (d + ex^2) (a + cx^4) dx$	401
3.105	$\int \frac{a+cx^4}{d+ex^2} dx$	403
3.106	$\int \frac{a+cx^4}{(d+ex^2)^2} dx$	406
3.107	$\int \frac{a+cx^4}{(d+ex^2)^3} dx$	409
3.108	$\int \frac{a+cx^4}{(d+ex^2)^4} dx$	412
3.109	$\int (d + ex^2)^3 (a + cx^4)^2 dx$	416
3.110	$\int (d + ex^2)^2 (a + cx^4)^2 dx$	419
3.111	$\int (d + ex^2) (a + cx^4)^2 dx$	422
3.112	$\int (a + cx^4)^2 dx$	424
3.113	$\int \frac{(a+cx^4)^2}{d+ex^2} dx$	426
3.114	$\int \frac{(a+cx^4)^2}{(d+ex^2)^2} dx$	429
3.115	$\int \frac{(a+cx^4)^2}{(d+ex^2)^3} dx$	433
3.116	$\int \frac{(a+cx^4)^2}{(d+ex^2)^4} dx$	437
3.117	$\int \frac{(a+cx^4)^2}{(d+ex^2)^5} dx$	441
3.118	$\int \frac{(d+ex^2)^4}{a+cx^4} dx$	445
3.119	$\int \frac{(d+ex^2)^3}{a+cx^4} dx$	452
3.120	$\int \frac{(d+ex^2)^2}{a+cx^4} dx$	458
3.121	$\int \frac{d+ex^2}{a+cx^4} dx$	463
3.122	$\int \frac{1}{a+cx^4} dx$	467
3.123	$\int \frac{1}{(d+ex^2)(a+cx^4)} dx$	471
3.124	$\int \frac{1}{(d+ex^2)^2(a+cx^4)} dx$	478
3.125	$\int \frac{(d+ex^2)^3}{(a+cx^4)^2} dx$	491
3.126	$\int \frac{(d+ex^2)^2}{(a+cx^4)^2} dx$	497
3.127	$\int \frac{d+ex^2}{(a+cx^4)^2} dx$	502
3.128	$\int \frac{1}{(a+cx^4)^2} dx$	506
3.129	$\int \frac{1}{(d+ex^2)(a+cx^4)^2} dx$	510
3.130	$\int \frac{1}{(d+ex^2)^2(a+cx^4)^2} dx$	525
3.131	$\int \frac{(d+ex^2)^4}{d^2-e^2x^4} dx$	541

3.132	$\int \frac{(d+ex^2)^3}{d^2-e^2x^4} dx$	544
3.133	$\int \frac{(d+ex^2)^2}{d^2-e^2x^4} dx$	547
3.134	$\int \frac{d+ex^2}{d^2-e^2x^4} dx$	550
3.135	$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)} dx$	553
3.136	$\int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)} dx$	556
3.137	$\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4} dx$	560
3.138	$\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4} dx$	564
3.139	$\int \frac{1}{\sqrt{d+ex^2}(d^2-e^2x^4)} dx$	567
3.140	$\int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)} dx$	570
3.141	$\int \frac{(a+bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}} dx$	574
3.142	$\int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}} dx$	577
3.143	$\int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}} dx$	580
3.144	$\int \frac{1}{\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} dx$	583
3.145	$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}} dx$	586
3.146	$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{a^2-b^2x^4}} dx$	590
3.147	$\int \frac{(a-bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}} dx$	594
3.148	$\int \frac{(a-bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}} dx$	597
3.149	$\int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}} dx$	600
3.150	$\int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}} dx$	603
3.151	$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{a^2-b^2x^4}} dx$	606
3.152	$\int \frac{1}{(a-bx^2)^{5/2}\sqrt{a^2-b^2x^4}} dx$	610
3.153	$\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}} dx$	614
3.154	$\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}} dx$	617
3.155	$\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}} dx$	620
3.156	$\int \frac{(d+ex^2)^4}{-cd^2+bde+be^2x^2+ce^2x^4} dx$	623
3.157	$\int \frac{(d+ex^2)^3}{-cd^2+bde+be^2x^2+ce^2x^4} dx$	630
3.158	$\int \frac{(d+ex^2)^2}{-cd^2+bde+be^2x^2+ce^2x^4} dx$	637
3.159	$\int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4} dx$	643
3.160	$\int \frac{1}{(d+ex^2)(-cd^2+bde+be^2x^2+ce^2x^4)} dx$	647
3.161	$\int \frac{1}{(d+ex^2)^2(-cd^2+bde+be^2x^2+ce^2x^4)} dx$	652
3.162	$\int \frac{(d+ex^2)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4} dx$	663

3.163	$\int \frac{(d+ex^2)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4} dx$	667
3.164	$\int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4} dx$	672
3.165	$\int \frac{1}{\sqrt{d+ex^2}(-cd^2+bde+be^2x^2+ce^2x^4)} dx$	676
3.166	$\int \frac{1}{(d+ex^2)^{3/2}(-cd^2+bde+be^2x^2+ce^2x^4)} dx$	680
3.167	$\int (d+ex^2)^4 (a+bx^2+cx^4) dx$	685
3.168	$\int (d+ex^2)^3 (a+bx^2+cx^4) dx$	688
3.169	$\int (d+ex^2)^2 (a+bx^2+cx^4) dx$	691
3.170	$\int (d+ex^2) (a+bx^2+cx^4) dx$	693
3.171	$\int \frac{a+bx^2+cx^4}{d+ex^2} dx$	695
3.172	$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2} dx$	698
3.173	$\int \frac{a+bx^2+cx^4}{(d+ex^2)^3} dx$	701
3.174	$\int \frac{a+bx^2+cx^4}{(d+ex^2)^4} dx$	704
3.175	$\int (d+ex^2)^3 (a+bx^2+cx^4)^2 dx$	708
3.176	$\int (d+ex^2)^2 (a+bx^2+cx^4)^2 dx$	711
3.177	$\int (d+ex^2) (a+bx^2+cx^4)^2 dx$	714
3.178	$\int (a+bx^2+cx^4)^2 dx$	717
3.179	$\int \frac{(a+bx^2+cx^4)^2}{d+ex^2} dx$	719
3.180	$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^2} dx$	722
3.181	$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^3} dx$	726
3.182	$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^4} dx$	730
3.183	$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^5} dx$	734
3.184	$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2} dx$	738
3.185	$\int \frac{a+x^2(b+cx^2)}{(d+ex^2)^2} dx$	741
3.186	$\int \frac{(d+ex^2)^4}{a+bx^2+cx^4} dx$	744
3.187	$\int \frac{(d+ex^2)^3}{a+bx^2+cx^4} dx$	762
3.188	$\int \frac{(d+ex^2)^2}{a+bx^2+cx^4} dx$	778
3.189	$\int \frac{d+ex^2}{a+bx^2+cx^4} dx$	788
3.190	$\int \frac{1}{a+bx^2+cx^4} dx$	793
3.191	$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)} dx$	797
3.192	$\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)} dx$	812
3.193	$\int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2} dx$	853

3.194	$\int \frac{(d+ex^2)^2}{(a+bx^2+cx^4)^2} dx$	875
3.195	$\int \frac{d+ex^2}{(a+bx^2+cx^4)^2} dx$	890
3.196	$\int \frac{1}{(a+bx^2+cx^4)^2} dx$	902
3.197	$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)^2} dx$	910
3.198	$\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)^2} dx$	1005
3.199	$\int (d+ex^2)^{5/2} (a+bx^2+cx^4) dx$	1051
3.200	$\int (d+ex^2)^{3/2} (a+bx^2+cx^4) dx$	1055
3.201	$\int \sqrt{d+ex^2} (a+bx^2+cx^4) dx$	1059
3.202	$\int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}} dx$	1063
3.203	$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{3/2}} dx$	1066
3.204	$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{5/2}} dx$	1069
3.205	$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{7/2}} dx$	1073
3.206	$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{9/2}} dx$	1077
3.207	$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{11/2}} dx$	1081
3.208	$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{13/2}} dx$	1085
3.209	$\int \frac{1-\sqrt{3}+x}{(1+\sqrt{3}+x)\sqrt{-4+4\sqrt{3}x^2+x^4}} dx$	1089
3.210	$\int \frac{1+\sqrt{3}+x}{(1-\sqrt{3}+x)\sqrt{-4-4\sqrt{3}x^2+x^4}} dx$	1092
3.211	$\int \frac{1-\sqrt{3}+2x}{(1+\sqrt{3}+2x)\sqrt{-1+4\sqrt{3}x^2+4x^4}} dx$	1095
3.212	$\int \frac{1+\sqrt{3}+2x}{(1-\sqrt{3}+2x)\sqrt{-1-4\sqrt{3}x^2+4x^4}} dx$	1098

$$3.1 \quad \int \frac{c+dx^2}{a+bx^4} dx$$

**Optimal.** Leaf size=247

$$\frac{(\sqrt{bc} - \sqrt{ad}) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2)}{4\sqrt{2} a^{3/4} b^{3/4}} + \frac{(\sqrt{bc} - \sqrt{ad}) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2)}{4\sqrt{2} a^{3/4} b^{3/4}} - \frac{(\sqrt{ad} + \sqrt{bc}) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2} a^{3/4} b^{3/4}}$$

**Rubi [A]** time = 0.15, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$ , Rules used = {1168, 1162, 617, 204, 1165, 628}

$$\frac{(\sqrt{bc} - \sqrt{ad}) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2)}{4\sqrt{2} a^{3/4} b^{3/4}} + \frac{(\sqrt{bc} - \sqrt{ad}) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2)}{4\sqrt{2} a^{3/4} b^{3/4}} - \frac{(\sqrt{ad} + \sqrt{bc}) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4} b^{3/4}} + \frac{(\sqrt{ad} + \sqrt{bc}) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2} a^{3/4} b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a + b*x^4), x]
```

```
[Out] -((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c - Sqrt[a]*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sqrt[a]*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4))
```

#### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

#### Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

#### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

#### Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

### Rubi steps

$$\begin{aligned} \int \frac{c + dx^2}{a + bx^4} dx &= \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}} - d\right) \int \frac{\sqrt{a}\sqrt{b-bx^2}}{a+bx^4} dx}{2b} + \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}} + d\right) \int \frac{\sqrt{a}\sqrt{b+bx^2}}{a+bx^4} dx}{2b} \\ &= \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}} + d\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{4b} + \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}} - d\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{4b} - \frac{(\sqrt{bc} - \sqrt{a}d) \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}} + 2x}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}} dx}{4\sqrt{2}a^{3/4}b^{3/4}} \\ &= -\frac{(\sqrt{bc} - \sqrt{a}d) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2)}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} - \sqrt{a}d) \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2)}{4\sqrt{2}a^{3/4}b^{3/4}} \\ &= -\frac{(\sqrt{bc} + \sqrt{a}d) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} + \sqrt{a}d) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} - \frac{(\sqrt{bc} - \sqrt{a}d) \log}{4} \end{aligned}$$

**Mathematica [A]** time = 0.08, size = 183, normalized size = 0.74

$$\frac{-(\sqrt{bc} - \sqrt{a}d) \left( \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2) - \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2) \right) - 2(\sqrt{a}d + \sqrt{bc}) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt{a}}\right) + 2(\sqrt{a}d + \sqrt{bc}) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt{a}} + 1\right)}{4\sqrt{2}a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(a + b*x^4), x]
```

```
[Out] (-2*(Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c - Sqrt[a]*d)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx^2}{a + bx^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)/(a + b*x^4), x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)/(a + b*x^4), x]
```

**fricas [B]** time = 0.87, size = 767, normalized size = 3.11

```
----
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4))/(a^3*b^3)) + 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4))/(a^3*b^3))
```

$$2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*\sqrt{-(a*b*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))} + 2*c*d)/(a*b))} + 1/4*\sqrt{-(a*b*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))} + 2*c*d)/(a*b)}*\log(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))} + a*b^2*c^3 - a^2*b*c*d^2)*\sqrt{-(a*b*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))} + 2*c*d)/(a*b))} + 1/4*\sqrt{(a*b*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))} - 2*c*d)/(a*b)}*\log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))} - a*b^2*c^3 + a^2*b*c*d^2)*\sqrt{(a*b*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))} - 2*c*d)/(a*b)} - 1/4*\sqrt{(a*b*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))} - 2*c*d)/(a*b)}*\log(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))} - a*b^2*c^3 + a^2*b*c*d^2)*\sqrt{(a*b*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))} - 2*c*d)/(a*b))}$$

**giac** [A] time = 0.19, size = 241, normalized size = 0.98

$$\frac{\sqrt{2} \left( (ab^3)^{\frac{1}{4}} b^2 c + (ab^3)^{\frac{3}{4}} d \right) \arctan \left( \frac{\sqrt{2} \left( 2x + \sqrt{2} \left( \frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{a}{b} \right)^{\frac{1}{4}}} \right)}{4 ab^3} + \frac{\sqrt{2} \left( (ab^3)^{\frac{1}{4}} b^2 c + (ab^3)^{\frac{3}{4}} d \right) \arctan \left( \frac{\sqrt{2} \left( 2x - \sqrt{2} \left( \frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{a}{b} \right)^{\frac{1}{4}}} \right)}{4 ab^3} + \frac{\sqrt{2} \left( (ab^3)^{\frac{1}{4}} b^2 c - (ab^3)^{\frac{3}{4}} d \right) \log \left( x^2 + \sqrt{2} x \left( \frac{a}{b} \right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}} \right)}{8 ab^3} - \frac{\sqrt{2} \left( (ab^3)^{\frac{1}{4}} b^2 c - (ab^3)^{\frac{3}{4}} d \right) \log \left( x^2 - \sqrt{2} x \left( \frac{a}{b} \right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}} \right)}{8 ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2+c)/(b\*x^4+a),x, algorithm="giac")

[Out]  $1/4*\sqrt{2}*((a*b^3)^{(1/4)}*b^2*c + (a*b^3)^{(3/4)}*d)*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(a*b^3) + 1/4*\sqrt{2}*((a*b^3)^{(1/4)}*b^2*c + (a*b^3)^{(3/4)}*d)*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(a*b^3) + 1/8*\sqrt{2}*((a*b^3)^{(1/4)}*b^2*c - (a*b^3)^{(3/4)}*d)*\log(x^2 + \sqrt{2}*(a/b)^{(1/4)} + \sqrt{a/b})/(a*b^3) - 1/8*\sqrt{2}*((a*b^3)^{(1/4)}*b^2*c - (a*b^3)^{(3/4)}*d)*\log(x^2 - \sqrt{2}*(a/b)^{(1/4)} + \sqrt{a/b})/(a*b^3)$

**maple** [A] time = 0.01, size = 260, normalized size = 1.05

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} c \arctan\left(\frac{\sqrt{2}x-1}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4a} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} c \arctan\left(\frac{\sqrt{2}x+1}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4a} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} c \ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}\right)}{8a} + \frac{\sqrt{2} d \arctan\left(\frac{\sqrt{2}x-1}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b} + \frac{\sqrt{2} d \arctan\left(\frac{\sqrt{2}x+1}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b} + \frac{\sqrt{2} d \ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^2+c)/(b\*x^4+a),x)

[Out]  $1/8*c*(a/b)^{(1/4)}/a^2^{(1/2)}*\ln((x^2+(a/b)^{(1/4)}*x^2^{(1/2)}+(a/b)^{(1/2)})/(x^2-(a/b)^{(1/4)}*x^2^{(1/2)}+(a/b)^{(1/2)}))+1/4*c*(a/b)^{(1/4)}/a^2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x+1)+1/4*c*(a/b)^{(1/4)}/a^2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x-1)+1/8*d/b/(a/b)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/b)^{(1/4)}*x^2^{(1/2)}+(a/b)^{(1/2)})/(x^2+(a/b)^{(1/4)}*x^2^{(1/2)}+(a/b)^{(1/2)}))+1/4*d/b/(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x+1)+1/4*d/b/(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x-1)$

**maxima** [A] time = 2.40, size = 221, normalized size = 0.89

$$\frac{\sqrt{2}(\sqrt{b}c + \sqrt{a}d) \arctan\left(\frac{\sqrt{2}(2\sqrt{b}x + \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}})}{2\sqrt{a}\sqrt{b}}\right)}{4\sqrt{a}\sqrt{a}\sqrt{b}\sqrt{b}} + \frac{\sqrt{2}(\sqrt{b}c + \sqrt{a}d) \arctan\left(\frac{\sqrt{2}(2\sqrt{b}x - \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}})}{2\sqrt{a}\sqrt{b}}\right)}{4\sqrt{a}\sqrt{a}\sqrt{b}\sqrt{b}} + \frac{\sqrt{2}(\sqrt{b}c - \sqrt{a}d) \log(\sqrt{b}x^2 + \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a})}{8a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}(\sqrt{b}c - \sqrt{a}d) \log(\sqrt{b}x^2 - \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a})}{8a^{\frac{3}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2+c)/(b\*x^4+a),x, algorithm="maxima")

[Out]  $1/4*\sqrt{2}*(\sqrt{b}*c + \sqrt{a}*d)*\arctan(1/2*\sqrt{2}*(2*\sqrt{b}*x + \sqrt{2}*(a/b)^{(1/4)}*b^{(1/4)})/\sqrt{(\sqrt{a}*\sqrt{b})})/(\sqrt{a}*\sqrt{(\sqrt{a}*\sqrt{b})})*\sqrt{b} + 1/4*\sqrt{2}*(\sqrt{b}*c + \sqrt{a}*d)*\arctan(1/2*\sqrt{2}*(2*\sqrt{b}*(x - \sqrt{2}*(a/b)^{(1/4)}*b^{(1/4)})/\sqrt{(\sqrt{a}*\sqrt{b})})/(\sqrt{a}*\sqrt{(\sqrt{a}*\sqrt{b})})*\sqrt{b}$



\*x - sqrt(2)\*a^(1/4)\*b^(1/4))/sqrt(sqrt(a)\*sqrt(b)))/(sqrt(a)\*sqrt(sqrt(a)\*sqrt(b))\*sqrt(b)) + 1/8\*sqrt(2)\*(sqrt(b)\*c - sqrt(a)\*d)\*log(sqrt(b)\*x^2 + sqrt(2)\*a^(1/4)\*b^(1/4)\*x + sqrt(a))/(a^(3/4)\*b^(3/4)) - 1/8\*sqrt(2)\*(sqrt(b)\*c - sqrt(a)\*d)\*log(sqrt(b)\*x^2 - sqrt(2)\*a^(1/4)\*b^(1/4)\*x + sqrt(a))/(a^(3/4)\*b^(3/4))

**mupad [B]** time = 0.38, size = 599, normalized size = 2.43

$$-2 \operatorname{atanh} \left( \frac{8b^3c^2x\sqrt{\frac{b^2\sqrt{-3}b^3 - c^2\sqrt{-3}b^3 - cd}{16a^3b^3}} - \frac{cd}{8a^3}}{2b^2c^2d - 2abb^2 + \frac{2b^2\sqrt{-3}b^3 - 2cd\sqrt{-3}b^3}{2}} \right) \sqrt{\frac{b^2\sqrt{-3}b^3 - a^2\sqrt{-3}b^3 + 2a^2b^2cd}{16a^3b^3}} - 2 \operatorname{atanh} \left( \frac{8b^3c^2x\sqrt{\frac{b^2\sqrt{-3}b^3 - c^2\sqrt{-3}b^3 - cd}{16a^3b^3}} - \frac{cd}{8a^3}}{2b^2c^2d - 2abb^2 - \frac{2b^2\sqrt{-3}b^3 + 2cd\sqrt{-3}b^3}{2}} \right) \sqrt{\frac{a^2\sqrt{-3}b^3 - b^2\sqrt{-3}b^3 + 2a^2b^2cd}{16a^3b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^2)/(a + b\*x^4), x)

[Out] - 2\*atanh((8\*b^3\*c^2\*x\*((d^2\*(-a^3\*b^3)^(1/2))/(16\*a^2\*b^3) - (c^2\*(-a^3\*b^3)^(1/2))/(16\*a^3\*b^2) - (c\*d)/(8\*a\*b))^(1/2))/(2\*b^2\*c^2\*d - 2\*a\*b\*d^3 + (2\*b\*c^3\*(-a^3\*b^3)^(1/2))/a^2 - (2\*c\*d^2\*(-a^3\*b^3)^(1/2))/a) - (8\*a\*b^2\*d^2\*x\*((d^2\*(-a^3\*b^3)^(1/2))/(16\*a^2\*b^3) - (c^2\*(-a^3\*b^3)^(1/2))/(16\*a^3\*b^2) - (c\*d)/(8\*a\*b))^(1/2))/(2\*b^2\*c^2\*d - 2\*a\*b\*d^3 + (2\*b\*c^3\*(-a^3\*b^3)^(1/2))/a^2 - (2\*c\*d^2\*(-a^3\*b^3)^(1/2))/a))\*(-(b\*c^2\*(-a^3\*b^3)^(1/2) - a\*d^2\*(-a^3\*b^3)^(1/2) + 2\*a^2\*b^2\*c\*d)/(16\*a^3\*b^3))^(1/2) - 2\*atanh((8\*b^3\*c^2\*x\*((c^2\*(-a^3\*b^3)^(1/2))/(16\*a^3\*b^2) - (c\*d)/(8\*a\*b) - (d^2\*(-a^3\*b^3)^(1/2))/(16\*a^2\*b^3))^(1/2))/(2\*b^2\*c^2\*d - 2\*a\*b\*d^3 - (2\*b\*c^3\*(-a^3\*b^3)^(1/2))/a^2 + (2\*c\*d^2\*(-a^3\*b^3)^(1/2))/a) - (8\*a\*b^2\*d^2\*x\*((c^2\*(-a^3\*b^3)^(1/2))/(16\*a^3\*b^2) - (c\*d)/(8\*a\*b) - (d^2\*(-a^3\*b^3)^(1/2))/(16\*a^2\*b^3))^(1/2))/(2\*b^2\*c^2\*d - 2\*a\*b\*d^3 - (2\*b\*c^3\*(-a^3\*b^3)^(1/2))/a^2 + (2\*c\*d^2\*(-a^3\*b^3)^(1/2))/a))\*(-(a\*d^2\*(-a^3\*b^3)^(1/2) - b\*c^2\*(-a^3\*b^3)^(1/2) + 2\*a^2\*b^2\*c\*d)/(16\*a^3\*b^3))^(1/2)

**sympy [A]** time = 0.69, size = 109, normalized size = 0.44

$$\operatorname{RootSum} \left( 256t^4a^3b^3 + 64t^2a^2b^2cd + a^2d^4 + 2abc^2d^2 + b^2c^4, \left( t \mapsto t \log \left( x + \frac{64t^3a^3b^2d + 12ta^2bcd^2 - 4tab^2c^3}{a^2d^4 - b^2c^4} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*2+c)/(b\*x\*\*4+a), x)

[Out] RootSum(256\*\_t\*\*4\*a\*\*3\*b\*\*3 + 64\*\_t\*\*2\*a\*\*2\*b\*\*2\*c\*d + a\*\*2\*d\*\*4 + 2\*a\*b\*c\*\*2\*d\*\*2 + b\*\*2\*c\*\*4, Lambda(\_t, \_t\*log(x + (64\*\_t\*\*3\*a\*\*3\*b\*\*2\*d + 12\*\_t\*a\*\*2\*b\*c\*d\*\*2 - 4\*\_t\*a\*b\*\*2\*c\*\*3)/(a\*\*2\*d\*\*4 - b\*\*2\*c\*\*4))))

$$3.2 \quad \int \frac{c-dx^2}{a+bx^4} dx$$

**Optimal.** Leaf size=247

$$\frac{(\sqrt{a}d + \sqrt{b}c) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2)}{4\sqrt{2} a^{3/4} b^{3/4}} + \frac{(\sqrt{a}d + \sqrt{b}c) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2)}{4\sqrt{2} a^{3/4} b^{3/4}} - \frac{(\sqrt{b}c - \sqrt{a}d) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4} b^{3/4}} + \frac{(\sqrt{b}c - \sqrt{a}d) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2} a^{3/4} b^{3/4}}$$

**Rubi [A]** time = 0.14, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {1168, 1162, 617, 204, 1165, 628}

$$\frac{(\sqrt{a}d + \sqrt{b}c) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2)}{4\sqrt{2} a^{3/4} b^{3/4}} + \frac{(\sqrt{a}d + \sqrt{b}c) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2)}{4\sqrt{2} a^{3/4} b^{3/4}} - \frac{(\sqrt{b}c - \sqrt{a}d) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4} b^{3/4}} + \frac{(\sqrt{b}c - \sqrt{a}d) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2} a^{3/4} b^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(c - d\*x^2)/(a + b\*x^4), x]

[Out] -((Sqrt[b]\*c - Sqrt[a]\*d)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/(2\*Sqrt[2]\*a^(3/4)\*b^(3/4)) + ((Sqrt[b]\*c - Sqrt[a]\*d)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/(2\*Sqrt[2]\*a^(3/4)\*b^(3/4)) - ((Sqrt[b]\*c + Sqrt[a]\*d)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/(4\*Sqrt[2]\*a^(3/4)\*b^(3/4)) + ((Sqrt[b]\*c + Sqrt[a]\*d)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/(4\*Sqrt[2]\*a^(3/4)\*b^(3/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 1162

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

#### Rule 1165

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

#### Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

### Rubi steps

$$\begin{aligned} \int \frac{c - dx^2}{a + bx^4} dx &= \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}} - d\right) \int \frac{\sqrt{a}\sqrt{b+bx^2}}{a+bx^4} dx}{2b} + \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}} + d\right) \int \frac{\sqrt{a}\sqrt{b-bx^2}}{a+bx^4} dx}{2b} \\ &= \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}} - d\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{4b} + \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}} + d\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{4b} - \frac{(\sqrt{bc} + \sqrt{a}d) \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}} + 2x}{-\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}} dx}{4\sqrt{2}a^{3/4}b^{3/4}} \\ &= -\frac{(\sqrt{bc} + \sqrt{a}d) \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} + \sqrt{a}d) \log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} \\ &= -\frac{(\sqrt{bc} - \sqrt{a}d) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} - \sqrt{a}d) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} - \frac{(\sqrt{bc} + \sqrt{a}d) \log\left(\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}} + 2x\right)}{4\sqrt{2}a^{3/4}b^{3/4}} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 184, normalized size = 0.74

$$\frac{-(\sqrt{ad} + \sqrt{bc}) \left( \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2\right) - \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2\right) \right) + (2\sqrt{ad} - 2\sqrt{bc}) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) + 2(\sqrt{bc} - \sqrt{ad}) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}} + 1\right)}{4\sqrt{2}a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - d*x^2)/(a + b*x^4), x]
```

```
[Out] ((-2*Sqrt[b]*c + 2*Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(
Sqrt[b]*c - Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c
+ Sqrt[a]*d)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(4*Sqrt[2]*a^(3/4)*b^(
3/4))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c - dx^2}{a + bx^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c - d*x^2)/(a + b*x^4), x]
```

```
[Out] IntegrateAlgebraic[(c - d*x^2)/(a + b*x^4), x]
```

**fricas [B]** time = 0.66, size = 767, normalized size = 3.11

```
-----
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)/(b*x^4+a), x, algorithm="fricas")
```

```
[Out] -1/4*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d
)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2
d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))
```

$$\begin{aligned} & *d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*\sqrt{((a*b*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)} / (a^3*b^3)) + 2*c*d)/(a*b))} + 1/4*\sqrt{((a*b*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)} / (a^3*b^3)) + 2*c*d)/(a*b))} * \log(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)} / (a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*\sqrt{((a*b*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)} / (a^3*b^3)) + 2*c*d)/(a*b))} + 1/4*\sqrt{-(a*b*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)} / (a^3*b^3)) - 2*c*d)/(a*b)} * \log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)} / (a^3*b^3)) - a*b^2*c^3 + a^2*b*c*d^2)*\sqrt{-(a*b*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)} / (a^3*b^3)) - 2*c*d)/(a*b)} - 1/4*\sqrt{-(a*b*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)} / (a^3*b^3)) - 2*c*d)/(a*b)} * \log(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)} / (a^3*b^3)) - a*b^2*c^3 + a^2*b*c*d^2)*\sqrt{-(a*b*\sqrt{-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)} / (a^3*b^3)) - 2*c*d)/(a*b)} \end{aligned}$$

**giac** [A] time = 0.17, size = 241, normalized size = 0.98

$$\frac{\sqrt{2} \left( (ab^3)^{\frac{1}{4}} b^2 c - (ab^3)^{\frac{3}{4}} d \right) \arctan \left( \frac{\sqrt{2} \left( 2x + \sqrt{2} \left( \frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{a}{b} \right)^{\frac{1}{4}}} \right)}{4 ab^3} + \frac{\sqrt{2} \left( (ab^3)^{\frac{1}{4}} b^2 c - (ab^3)^{\frac{3}{4}} d \right) \arctan \left( \frac{\sqrt{2} \left( 2x - \sqrt{2} \left( \frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{a}{b} \right)^{\frac{1}{4}}} \right)}{4 ab^3} + \frac{\sqrt{2} \left( (ab^3)^{\frac{1}{4}} b^2 c + (ab^3)^{\frac{3}{4}} d \right) \log \left( x^2 + \sqrt{2} x \left( \frac{a}{b} \right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}} \right)}{8 ab^3} - \frac{\sqrt{2} \left( (ab^3)^{\frac{1}{4}} b^2 c + (ab^3)^{\frac{3}{4}} d \right) \log \left( x^2 - \sqrt{2} x \left( \frac{a}{b} \right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}} \right)}{8 ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-d\*x^2+c)/(b\*x^4+a),x, algorithm="giac")

[Out]  $1/4*\sqrt{2}*((a*b^3)^{(1/4)}*b^2*c - (a*b^3)^{(3/4)}*d)*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(a*b^3) + 1/4*\sqrt{2}*((a*b^3)^{(1/4)}*b^2*c - (a*b^3)^{(3/4)}*d)*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(a*b^3) + 1/8*\sqrt{2}*((a*b^3)^{(1/4)}*b^2*c + (a*b^3)^{(3/4)}*d)*\log(x^2 + \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(a*b^3) - 1/8*\sqrt{2}*((a*b^3)^{(1/4)}*b^2*c + (a*b^3)^{(3/4)}*d)*\log(x^2 - \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(a*b^3)$

**maple** [A] time = 0.00, size = 260, normalized size = 1.05

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} c \arctan\left(\frac{\sqrt{2}x-1}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4a} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} c \arctan\left(\frac{\sqrt{2}x+1}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4a} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} c \ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}\right)}{8a} - \frac{\sqrt{2} d \arctan\left(\frac{\sqrt{2}x-1}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}} b} - \frac{\sqrt{2} d \arctan\left(\frac{\sqrt{2}x+1}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}} b} - \frac{\sqrt{2} d \ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-d\*x^2+c)/(b\*x^4+a),x)

[Out]  $1/8*(a/b)^{(1/4)}*2^{(1/2)}/a*c*\ln((x^2+(a/b)^{(1/4)}*2^{(1/2)}*x+(a/b)^{(1/2)})/(x^2-(a/b)^{(1/4)}*2^{(1/2)}*x+(a/b)^{(1/2)}))+1/4*(a/b)^{(1/4)}*2^{(1/2)}/a*c*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x+1)+1/4*(a/b)^{(1/4)}*2^{(1/2)}/a*c*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x-1)-1/8/(a/b)^{(1/4)}*2^{(1/2)}/b*d*\ln((x^2-(a/b)^{(1/4)}*2^{(1/2)}*x+(a/b)^{(1/2)})/(x^2+(a/b)^{(1/4)}*2^{(1/2)}*x+(a/b)^{(1/2)}))-1/4/(a/b)^{(1/4)}*2^{(1/2)}/b*d*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x+1)-1/4/(a/b)^{(1/4)}*2^{(1/2)}/b*d*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x-1)$

**maxima** [A] time = 2.34, size = 221, normalized size = 0.89

$$\frac{\sqrt{2}(\sqrt{b}c - \sqrt{a}d) \arctan\left(\frac{\sqrt{2}(2\sqrt{b}x + \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}})}{2\sqrt{a}\sqrt{b}}\right)}{4\sqrt{a}\sqrt{a}\sqrt{b}\sqrt{b}} + \frac{\sqrt{2}(\sqrt{b}c - \sqrt{a}d) \arctan\left(\frac{\sqrt{2}(2\sqrt{b}x - \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}})}{2\sqrt{a}\sqrt{b}}\right)}{4\sqrt{a}\sqrt{a}\sqrt{b}\sqrt{b}} + \frac{\sqrt{2}(\sqrt{b}c + \sqrt{a}d) \log(\sqrt{b}x^2 + \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a})}{8a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}(\sqrt{b}c + \sqrt{a}d) \log(\sqrt{b}x^2 - \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a})}{8a^{\frac{3}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-d\*x^2+c)/(b\*x^4+a),x, algorithm="maxima")

[Out]  $1/4*\sqrt{2}*(\sqrt{b}*c - \sqrt{a}*d)*\arctan(1/2*\sqrt{2}*(2*\sqrt{b}*x + \sqrt{2}*(a/b)^{(1/4)}*b^{(1/4)})/\sqrt{a}*\sqrt{b})/(\sqrt{a}*\sqrt{a}*\sqrt{b}*\sqrt{b}) + 1/4*\sqrt{2}*(\sqrt{b}*c - \sqrt{a}*d)*\arctan(1/2*\sqrt{2}*(2*\sqrt{b}*x - \sqrt{2}*(a/b)^{(1/4)}*b^{(1/4)})/\sqrt{a}*\sqrt{b})/(\sqrt{a}*\sqrt{a}*\sqrt{b}*\sqrt{b})$

$*x - \sqrt{2} * a^{(1/4)} * b^{(1/4)} / \sqrt{\sqrt{a} * \sqrt{b}} / (\sqrt{a} * \sqrt{\sqrt{a} * \sqrt{b}} + 1/8 * \sqrt{2} * (\sqrt{b} * c + \sqrt{a} * d) * \log(\sqrt{b} * x^2 + \sqrt{2} * a^{(1/4)} * b^{(1/4)} * x + \sqrt{a})) / (a^{(3/4)} * b^{(3/4)}) - 1/8 * \sqrt{2} * (\sqrt{b} * c + \sqrt{a} * d) * \log(\sqrt{b} * x^2 - \sqrt{2} * a^{(1/4)} * b^{(1/4)} * x + \sqrt{a})) / (a^{(3/4)} * b^{(3/4)})$

**mupad [B]** time = 0.26, size = 603, normalized size = 2.44

$$2 \operatorname{atanh} \left( \frac{8 b^3 c^2 x \sqrt{\frac{d}{2a}} - \frac{c^2 \sqrt{a^3 b^3}}{16 a^2 b^3} + \frac{d \sqrt{a^3 b^3}}{16 a^2 b^3}}{2 b^2 c^2 d - 2 a b d^2 - \frac{2 b^3 \sqrt{a^3 b^3}}{a^2} + \frac{2 c d \sqrt{a^3 b^3}}{a}} - \frac{8 a b^2 d^2 x \sqrt{\frac{d}{2a}} - \frac{c^2 \sqrt{a^3 b^3}}{16 a^2 b^3} + \frac{d \sqrt{a^3 b^3}}{16 a^2 b^3}}{2 b^2 c^2 d - 2 a b d^2 - \frac{2 b^3 \sqrt{a^3 b^3}}{a^2} + \frac{2 c d \sqrt{a^3 b^3}}{a}} \right) \sqrt{\frac{d^2 \sqrt{-a^3 b^3} - b c^2 \sqrt{-a^3 b^3} + 2 a^2 b^2 c d}{16 a^2 b^3}} + 2 \operatorname{atanh} \left( \frac{8 b^3 c^2 x \sqrt{\frac{d}{2a}} + \frac{c^2 \sqrt{a^3 b^3}}{16 a^2 b^3} - \frac{d \sqrt{a^3 b^3}}{16 a^2 b^3}}{2 b^2 c^2 d - 2 a b d^2 + \frac{2 b^3 \sqrt{a^3 b^3}}{a^2} - \frac{2 c d \sqrt{a^3 b^3}}{a}} - \frac{8 a b^2 d^2 x \sqrt{\frac{d}{2a}} + \frac{c^2 \sqrt{a^3 b^3}}{16 a^2 b^3} - \frac{d \sqrt{a^3 b^3}}{16 a^2 b^3}}{2 b^2 c^2 d - 2 a b d^2 + \frac{2 b^3 \sqrt{a^3 b^3}}{a^2} - \frac{2 c d \sqrt{a^3 b^3}}{a}} \right) \sqrt{\frac{b c^2 \sqrt{-a^3 b^3} - a d^2 \sqrt{-a^3 b^3} + 2 a^2 b^2 c d}{16 a^2 b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c - d*x^2)/(a + b*x^4), x)`

[Out]  $2 * \operatorname{atanh}((8 * b^3 * c^2 * x * ((c * d) / (8 * a * b) - (c^2 * (-a^3 * b^3)^{(1/2)}) / (16 * a^3 * b^2) + (d^2 * (-a^3 * b^3)^{(1/2)}) / (16 * a^2 * b^3))^{(1/2)}) / (2 * b^2 * c^2 * d - 2 * a * b * d^3 - (2 * b * c^3 * (-a^3 * b^3)^{(1/2)}) / a^2 + (2 * c * d^2 * (-a^3 * b^3)^{(1/2)}) / a) - (8 * a * b^2 * d^2 * x * ((c * d) / (8 * a * b) - (c^2 * (-a^3 * b^3)^{(1/2)}) / (16 * a^3 * b^2) + (d^2 * (-a^3 * b^3)^{(1/2)}) / (16 * a^2 * b^3))^{(1/2)}) / (2 * b^2 * c^2 * d - 2 * a * b * d^3 - (2 * b * c^3 * (-a^3 * b^3)^{(1/2)}) / a^2 + (2 * c * d^2 * (-a^3 * b^3)^{(1/2)}) / a)) * ((a * d^2 * (-a^3 * b^3)^{(1/2)} - b * c^2 * (-a^3 * b^3)^{(1/2)} + 2 * a^2 * b^2 * c * d) / (16 * a^3 * b^3))^{(1/2)} + 2 * \operatorname{atanh}((8 * b^3 * c^2 * x * ((c * d) / (8 * a * b) + (c^2 * (-a^3 * b^3)^{(1/2)}) / (16 * a^3 * b^2) - (d^2 * (-a^3 * b^3)^{(1/2)}) / (16 * a^2 * b^3))^{(1/2)}) / (2 * b^2 * c^2 * d - 2 * a * b * d^3 + (2 * b * c^3 * (-a^3 * b^3)^{(1/2)}) / a^2 - (2 * c * d^2 * (-a^3 * b^3)^{(1/2)}) / a) - (8 * a * b^2 * d^2 * x * ((c * d) / (8 * a * b) + (c^2 * (-a^3 * b^3)^{(1/2)}) / (16 * a^3 * b^2) - (d^2 * (-a^3 * b^3)^{(1/2)}) / (16 * a^2 * b^3))^{(1/2)}) / (2 * b^2 * c^2 * d - 2 * a * b * d^3 + (2 * b * c^3 * (-a^3 * b^3)^{(1/2)}) / a^2 - (2 * c * d^2 * (-a^3 * b^3)^{(1/2)}) / a)) * ((b * c^2 * (-a^3 * b^3)^{(1/2)} - a * d^2 * (-a^3 * b^3)^{(1/2)} + 2 * a^2 * b^2 * c * d) / (16 * a^3 * b^3))^{(1/2)}$

**sympy [A]** time = 0.68, size = 110, normalized size = 0.45

$$- \operatorname{RootSum} \left( 256 t^4 a^3 b^3 - 64 t^2 a^2 b^2 c d + a^2 d^4 + 2 a b c^2 d^2 + b^2 c^4, \left( t \mapsto t \log \left( x + \frac{64 t^3 a^3 b^2 d - 12 t a^2 b c d^2 + 4 t a b^2 c^3}{a^2 d^4 - b^2 c^4} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-d*x**2+c)/(b*x**4+a), x)`

[Out]  $- \operatorname{RootSum}(256 * \_t^{**4} * a^{**3} * b^{**3} - 64 * \_t^{**2} * a^{**2} * b^{**2} * c * d + a^{**2} * d^{**4} + 2 * a * b * c^{**2} * d^{**2} + b^{**2} * c^{**4}, \operatorname{Lambda}(\_t, \_t * \log(x + (64 * \_t^{**3} * a^{**3} * b^{**2} * d - 12 * \_t * a^{**2} * b * c * d^{**2} + 4 * \_t * a * b^{**2} * c^{**3}) / (a^{**2} * d^{**4} - b^{**2} * c^{**4}))))$

$$3.3 \quad \int \frac{c+dx^2}{a-bx^4} dx$$

**Optimal.** Leaf size=86

$$\frac{(\sqrt{b}c - \sqrt{a}d) \tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{(\sqrt{a}d + \sqrt{b}c) \tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

**Rubi [A]** time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1167, 205, 208}

$$\frac{(\sqrt{b}c - \sqrt{a}d) \tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{(\sqrt{a}d + \sqrt{b}c) \tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^2)/(a - b\*x^4), x]

[Out] ((Sqrt[b]\*c - Sqrt[a]\*d)\*ArcTan[(b^(1/4)\*x)/a^(1/4)]/(2\*a^(3/4)\*b^(3/4)) + ((Sqrt[b]\*c + Sqrt[a]\*d)\*ArcTanh[(b^(1/4)\*x)/a^(1/4)]/(2\*a^(3/4)\*b^(3/4)))

**Rule 205**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 208**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

**Rule 1167**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-(a\*c), 2]}, Dist[e/2 + (c\*d)/(2\*q), Int[1/(-q + c\*x^2), x], x] + Dist[e/2 - (c\*d)/(2\*q), Int[1/(q + c\*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 - a\*e^2, 0] && PosQ[-(a\*c)]

**Rubi steps**

$$\begin{aligned} \int \frac{c+dx^2}{a-bx^4} dx &= \frac{1}{2} \left( -\frac{\sqrt{b}c}{\sqrt{a}} + d \right) \int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2} dx + \frac{1}{2} \left( \frac{\sqrt{b}c}{\sqrt{a}} + d \right) \int \frac{1}{\sqrt{a}\sqrt{b}-bx^2} dx \\ &= \frac{(\sqrt{b}c - \sqrt{a}d) \tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{(\sqrt{b}c + \sqrt{a}d) \tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 95, normalized size = 1.10

$$\frac{2(\sqrt{b}c - \sqrt{a}d) \tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) - (\sqrt{a}d + \sqrt{b}c) (\log(\sqrt[4]{a} - \sqrt[4]{b}x) - \log(\sqrt[4]{a} + \sqrt[4]{b}x))}{4a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^2)/(a - b\*x^4), x]

[Out] (2\*(Sqrt[b]\*c - Sqrt[a]\*d)\*ArcTan[(b^(1/4)\*x)/a^(1/4)] - (Sqrt[b]\*c + Sqrt[a]\*d)\*(Log[a^(1/4) - b^(1/4)\*x] - Log[a^(1/4) + b^(1/4)\*x]))/(4\*a^(3/4)\*b^(3/4))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx^2}{a - bx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(c + d\*x^2)/(a - b\*x^4), x]

[Out] IntegrateAlgebraic[(c + d\*x^2)/(a - b\*x^4), x]

**fricas [B]** time = 0.90, size = 755, normalized size = 8.78



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2+c)/(-b\*x^4+a), x, algorithm="fricas")

[Out] 1/4\*sqrt((a\*b\*sqrt((b^2\*c^4 + 2\*a\*b\*c^2\*d^2 + a^2\*d^4)/(a^3\*b^3)) + 2\*c\*d)/(a\*b))\*log(-(b^2\*c^4 - a^2\*d^4)\*x + (a^3\*b^2\*d\*sqrt((b^2\*c^4 + 2\*a\*b\*c^2\*d^2 + a^2\*d^4)/(a^3\*b^3)) - a\*b^2\*c^3 - a^2\*b\*c\*d^2)\*sqrt((a\*b\*sqrt((b^2\*c^4 + 2\*a\*b\*c^2\*d^2 + a^2\*d^4)/(a^3\*b^3)) + 2\*c\*d)/(a\*b))) - 1/4\*sqrt((a\*b\*sqrt((b^2\*c^4 + 2\*a\*b\*c^2\*d^2 + a^2\*d^4)/(a^3\*b^3)) + 2\*c\*d)/(a\*b))\*log(-(b^2\*c^4 - a^2\*d^4)\*x - (a^3\*b^2\*d\*sqrt((b^2\*c^4 + 2\*a\*b\*c^2\*d^2 + a^2\*d^4)/(a^3\*b^3)) - a\*b^2\*c^3 - a^2\*b\*c\*d^2)\*sqrt((a\*b\*sqrt((b^2\*c^4 + 2\*a\*b\*c^2\*d^2 + a^2\*d^4)/(a^3\*b^3)) + 2\*c\*d)/(a\*b))) - 1/4\*sqrt(-(a\*b\*sqrt((b^2\*c^4 + 2\*a\*b\*c^2\*d^2 + a^2\*d^4)/(a^3\*b^3)) - 2\*c\*d)/(a\*b))\*log(-(b^2\*c^4 - a^2\*d^4)\*x + (a^3\*b^2\*d\*sqrt((b^2\*c^4 + 2\*a\*b\*c^2\*d^2 + a^2\*d^4)/(a^3\*b^3)) + a\*b^2\*c^3 + a^2\*b\*c\*d^2)\*sqrt(-(a\*b\*sqrt((b^2\*c^4 + 2\*a\*b\*c^2\*d^2 + a^2\*d^4)/(a^3\*b^3)) - 2\*c\*d)/(a\*b))) + 1/4\*sqrt(-(a\*b\*sqrt((b^2\*c^4 + 2\*a\*b\*c^2\*d^2 + a^2\*d^4)/(a^3\*b^3)) - 2\*c\*d)/(a\*b))\*log(-(b^2\*c^4 - a^2\*d^4)\*x - (a^3\*b^2\*d\*sqrt((b^2\*c^4 + 2\*a\*b\*c^2\*d^2 + a^2\*d^4)/(a^3\*b^3)) + a\*b^2\*c^3 + a^2\*b\*c\*d^2)\*sqrt(-(a\*b\*sqrt((b^2\*c^4 + 2\*a\*b\*c^2\*d^2 + a^2\*d^4)/(a^3\*b^3)) - 2\*c\*d)/(a\*b)))

**giac [B]** time = 0.18, size = 230, normalized size = 2.67

$$\frac{\sqrt{2}(b^2c + \sqrt{-abd}) \arctan\left(\frac{\sqrt{2}(2x + \sqrt{2}(-\frac{a}{b})^{\frac{1}{4}})}{2(-\frac{a}{b})^{\frac{1}{4}}}\right)}{4(-ab^3)^{\frac{3}{4}}} - \frac{\sqrt{2}(b^2c - \sqrt{-abd}) \arctan\left(\frac{\sqrt{2}(2x - \sqrt{2}(-\frac{a}{b})^{\frac{1}{4}})}{2(-\frac{a}{b})^{\frac{1}{4}}}\right)}{4(-ab^3)^{\frac{3}{4}}} - \frac{\sqrt{2}(b^2c - \sqrt{-abd}) \log\left(x^2 + \sqrt{2}x(-\frac{a}{b})^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{8(-ab^3)^{\frac{3}{4}}} + \frac{\sqrt{2}(b^2c - \sqrt{-abd}) \log\left(x^2 - \sqrt{2}x(-\frac{a}{b})^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{8(-ab^3)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2+c)/(-b\*x^4+a), x, algorithm="giac")

[Out] -1/4\*sqrt(2)\*(b^2\*c + sqrt(-a\*b)\*b\*d)\*arctan(1/2\*sqrt(2)\*(2\*x + sqrt(2))\*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a\*b^3)^(3/4) - 1/4\*sqrt(2)\*(b^2\*c - sqrt(-a\*b)\*b\*d)\*arctan(1/2\*sqrt(2)\*(2\*x - sqrt(2))\*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a\*b^3)^(3/4) - 1/8\*sqrt(2)\*(b^2\*c - sqrt(-a\*b)\*b\*d)\*log(x^2 + sqrt(2)\*x\*(-a/b)^(1/4) + sqrt(-a/b))/(-a\*b^3)^(3/4) + 1/8\*sqrt(2)\*(b^2\*c - sqrt(-a\*b)\*b\*d)\*log(x^2 - sqrt(2)\*x\*(-a/b)^(1/4) + sqrt(-a/b))/(-a\*b^3)^(3/4)

**maple [B]** time = 0.00, size = 122, normalized size = 1.42

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} c \arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} c \ln\left(\frac{x + \left(\frac{a}{b}\right)^{\frac{1}{4}}}{x - \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4a} - \frac{d \arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}} b} + \frac{d \ln\left(\frac{x + \left(\frac{a}{b}\right)^{\frac{1}{4}}}{x - \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^2+c)/(-b*x^4+a),x)`

[Out]  $\frac{1}{4}c*(a/b)^{(1/4)}/a*\ln((x+(a/b)^{(1/4)})/(x-(a/b)^{(1/4)}))+1/2*c*(a/b)^{(1/4)}/a*\arctan(x/(a/b)^{(1/4)})-1/2*d/b/(a/b)^{(1/4)}*\arctan(x/(a/b)^{(1/4)})+1/4*d/b/(a/b)^{(1/4)}*\ln((x+(a/b)^{(1/4)})/(x-(a/b)^{(1/4)}))$

**maxima** [A] time = 2.29, size = 109, normalized size = 1.27

$$\frac{(\sqrt{b}c - \sqrt{a}d) \arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{(\sqrt{b}c + \sqrt{a}d) \log\left(\frac{\sqrt{b}x - \sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x + \sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^2+c)/(-b*x^4+a),x, algorithm="maxima")`

[Out]  $\frac{1}{2}*(\sqrt{b}*c - \sqrt{a}*d)*\arctan(\sqrt{b}*x/\sqrt{\sqrt{a}*c*\sqrt{b}})/(\sqrt{a}*c*\sqrt{\sqrt{a}*c*\sqrt{b}}*\sqrt{b}) - \frac{1}{4}*(\sqrt{b}*c + \sqrt{a}*d)*\log((\sqrt{b}*x - \sqrt{\sqrt{a}*c*\sqrt{b}})/(\sqrt{b}*x + \sqrt{\sqrt{a}*c*\sqrt{b}}))/(\sqrt{a}*c*\sqrt{\sqrt{a}*c*\sqrt{b}}*\sqrt{b})$

**mupad** [B] time = 4.64, size = 579, normalized size = 6.73

$$2\operatorname{atanh}\left(\frac{8b^3c^2x\sqrt{\frac{cd}{8ab} - \frac{c^2\sqrt{cd}}{16a^2b^2}} - \frac{d^2\sqrt{cd}}{16a^2b^2}}{2b^2c^2d + 2abd^3 - \frac{2cd^2\sqrt{cd}}{a}} + \frac{8a^2d^2x\sqrt{\frac{cd}{8ab} - \frac{c^2\sqrt{cd}}{16a^2b^2}} - \frac{d^2\sqrt{cd}}{16a^2b^2}}{2b^2c^2d + 2abd^3 - \frac{2cd^2\sqrt{cd}}{a}}\right)\sqrt{\frac{ad^2\sqrt{a^3b^3} + b^2\sqrt{a^3b^3} - 2a^2b^2cd}{16a^2b^3}} + 2\operatorname{atanh}\left(\frac{8b^3c^2x\sqrt{\frac{cd}{8ab} + \frac{c^2\sqrt{cd}}{16a^2b^2}} + \frac{d^2\sqrt{cd}}{16a^2b^2}}{2b^2c^2d + 2abd^3 + \frac{2cd^2\sqrt{cd}}{a}} + \frac{8a^2d^2x\sqrt{\frac{cd}{8ab} + \frac{c^2\sqrt{cd}}{16a^2b^2}} - \frac{d^2\sqrt{cd}}{16a^2b^2}}{2b^2c^2d + 2abd^3 + \frac{2cd^2\sqrt{cd}}{a}}\right)\sqrt{\frac{ad^2\sqrt{a^3b^3} + b^2\sqrt{a^3b^3} + 2a^2b^2cd}{16a^2b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d*x^2)/(a - b*x^4),x)`

[Out]  $2*\operatorname{atanh}((8*b^3*c^2*x*((c*d)/(8*a*b) - (c^2*(a^3*b^3)^{(1/2)})/(16*a^3*b^2) - (d^2*(a^3*b^3)^{(1/2)})/(16*a^2*b^3))^{(1/2)})/(2*b^2*c^2*d + 2*a*b*d^3 - (2*b*c^3*(a^3*b^3)^{(1/2)})/a^2 - (2*c*d^2*(a^3*b^3)^{(1/2)})/a) + (8*a*b^2*d^2*x*((c*d)/(8*a*b) - (c^2*(a^3*b^3)^{(1/2)})/(16*a^3*b^2) - (d^2*(a^3*b^3)^{(1/2)})/(16*a^2*b^3))^{(1/2)})/(2*b^2*c^2*d + 2*a*b*d^3 - (2*b*c^3*(a^3*b^3)^{(1/2)})/a^2 - (2*c*d^2*(a^3*b^3)^{(1/2)})/a))*(-(a*d^2*(a^3*b^3)^{(1/2)} + b*c^2*(a^3*b^3)^{(1/2)} - 2*a^2*b^2*c*d)/(16*a^3*b^3))^{(1/2)} + 2*\operatorname{atanh}((8*b^3*c^2*x*((c*d)/(8*a*b) + (c^2*(a^3*b^3)^{(1/2)})/(16*a^3*b^2) + (d^2*(a^3*b^3)^{(1/2)})/(16*a^2*b^3))^{(1/2)})/(2*b^2*c^2*d + 2*a*b*d^3 + (2*b*c^3*(a^3*b^3)^{(1/2)})/a^2 + (2*c*d^2*(a^3*b^3)^{(1/2)})/a) + (8*a*b^2*d^2*x*((c*d)/(8*a*b) + (c^2*(a^3*b^3)^{(1/2)})/(16*a^3*b^2) + (d^2*(a^3*b^3)^{(1/2)})/(16*a^2*b^3))^{(1/2)})/(16*a^3*b^2) + (d^2*(a^3*b^3)^{(1/2)})/(16*a^2*b^3))^{(1/2)})/(2*b^2*c^2*d + 2*a*b*d^3 + (2*b*c^3*(a^3*b^3)^{(1/2)})/a^2 + (2*c*d^2*(a^3*b^3)^{(1/2)})/a))*((a*d^2*(a^3*b^3)^{(1/2)} + b*c^2*(a^3*b^3)^{(1/2)} + 2*a^2*b^2*c*d)/(16*a^3*b^3))^{(1/2)}$

**sympy** [A] time = 0.73, size = 110, normalized size = 1.28

$$-\operatorname{RootSum}\left(256t^4a^3b^3 - 64t^2a^2b^2cd - a^2d^4 + 2abc^2d^2 - b^2c^4, \left(t \mapsto t \log\left(x + \frac{-64t^3a^3b^2d + 12ta^2bcd^2 + 4tab^2c^3}{a^2d^4 - b^2c^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**2+c)/(-b*x**4+a),x)`

[Out]  $-\operatorname{RootSum}(256*_t**4*a**3*b**3 - 64*_t**2*a**2*b**2*c*d - a**2*d**4 + 2*a*b*c**2*d**2 - b**2*c**4, \operatorname{Lambda}(_t, _t*\log(x + (-64*_t**3*a**3*b**2*d + 12*_t*a**2*b*c*d**2 + 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))$



$$3.4 \quad \int \frac{c-dx^2}{a-bx^4} dx$$

**Optimal.** Leaf size=86

$$\frac{(\sqrt{a}d + \sqrt{b}c) \tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{(\sqrt{b}c - \sqrt{a}d) \tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

**Rubi [A]** time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {1167, 205, 208}

$$\frac{(\sqrt{a}d + \sqrt{b}c) \tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{(\sqrt{b}c - \sqrt{a}d) \tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(c - d\*x^2)/(a - b\*x^4), x]

[Out] ((Sqrt[b]\*c + Sqrt[a]\*d)\*ArcTan[(b^(1/4)\*x)/a^(1/4)]/(2\*a^(3/4)\*b^(3/4)) + ((Sqrt[b]\*c - Sqrt[a]\*d)\*ArcTanh[(b^(1/4)\*x)/a^(1/4)]/(2\*a^(3/4)\*b^(3/4)))

**Rule 205**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 208**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

**Rule 1167**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[-(a\*c), 2]}, Dist[e/2 + (c\*d)/(2\*q), Int[1/(-q + c\*x^2), x], x] + Dist[e/2 - (c\*d)/(2\*q), Int[1/(q + c\*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 - a\*e^2, 0] && PosQ[-(a\*c)]

**Rubi steps**

$$\begin{aligned} \int \frac{c-dx^2}{a-bx^4} dx &= \frac{1}{2} \left( -\frac{\sqrt{b}c}{\sqrt{a}} - d \right) \int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2} dx + \frac{1}{2} \left( \frac{\sqrt{b}c}{\sqrt{a}} - d \right) \int \frac{1}{\sqrt{a}\sqrt{b}-bx^2} dx \\ &= \frac{(\sqrt{b}c + \sqrt{a}d) \tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{(\sqrt{b}c - \sqrt{a}d) \tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 95, normalized size = 1.10

$$\frac{2(\sqrt{a}d + \sqrt{b}c) \tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) - (\sqrt{b}c - \sqrt{a}d) (\log(\sqrt[4]{a} - \sqrt[4]{b}x) - \log(\sqrt[4]{a} + \sqrt[4]{b}x))}{4a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c - d\*x^2)/(a - b\*x^4), x]

[Out] (2\*(Sqrt[b]\*c + Sqrt[a]\*d)\*ArcTan[(b^(1/4)\*x)/a^(1/4)] - (Sqrt[b]\*c - Sqrt[a]\*d)\*(Log[a^(1/4) - b^(1/4)\*x] - Log[a^(1/4) + b^(1/4)\*x]))/(4\*a^(3/4)\*b^(3/4))

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

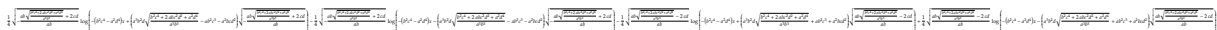
$$\int \frac{c - dx^2}{a - bx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(c - d\*x^2)/(a - b\*x^4), x]

[Out] IntegrateAlgebraic[(c - d\*x^2)/(a - b\*x^4), x]

**fricas** [B] time = 0.57, size = 755, normalized size = 8.78



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-d\*x^2+c)/(-b\*x^4+a), x, algorithm="fricas")

[Out] 1/4\*sqrt(-(a\*b\*sqrt((b^2\*c^4 + 2\*a\*b\*c^2\*d^2 + a^2\*d^4)/(a^3\*b^3)) + 2\*c\*d)/(a\*b))\*log(-(b^2\*c^4 - a^2\*d^4)\*x + (a^3\*b^2\*d\*sqrt((b^2\*c^4 + 2\*a\*b\*c^2\*d^2 + a^2\*d^4)/(a^3\*b^3)) - a\*b^2\*c^3 - a^2\*b\*c\*d^2)\*sqrt(-(a\*b\*sqrt((b^2\*c^4 + 2\*a\*b\*c^2\*d^2 + a^2\*d^4)/(a^3\*b^3)) + 2\*c\*d)/(a\*b))) - 1/4\*sqrt(-(a\*b\*sqrt((b^2\*c^4 + 2\*a\*b\*c^2\*d^2 + a^2\*d^4)/(a^3\*b^3)) + 2\*c\*d)/(a\*b))\*log(-(b^2\*c^4 - a^2\*d^4)\*x - (a^3\*b^2\*d\*sqrt((b^2\*c^4 + 2\*a\*b\*c^2\*d^2 + a^2\*d^4)/(a^3\*b^3)) - a\*b^2\*c^3 - a^2\*b\*c\*d^2)\*sqrt(-(a\*b\*sqrt((b^2\*c^4 + 2\*a\*b\*c^2\*d^2 + a^2\*d^4)/(a^3\*b^3)) + 2\*c\*d)/(a\*b))) - 1/4\*sqrt((a\*b\*sqrt((b^2\*c^4 + 2\*a\*b\*c^2\*d^2 + a^2\*d^4)/(a^3\*b^3)) - 2\*c\*d)/(a\*b))\*log(-(b^2\*c^4 - a^2\*d^4)\*x + (a^3\*b^2\*d\*sqrt((b^2\*c^4 + 2\*a\*b\*c^2\*d^2 + a^2\*d^4)/(a^3\*b^3)) + a\*b^2\*c^3 + a^2\*b\*c\*d^2)\*sqrt((a\*b\*sqrt((b^2\*c^4 + 2\*a\*b\*c^2\*d^2 + a^2\*d^4)/(a^3\*b^3)) - 2\*c\*d)/(a\*b))) + 1/4\*sqrt((a\*b\*sqrt((b^2\*c^4 + 2\*a\*b\*c^2\*d^2 + a^2\*d^4)/(a^3\*b^3)) - 2\*c\*d)/(a\*b))\*log(-(b^2\*c^4 - a^2\*d^4)\*x - (a^3\*b^2\*d\*sqrt((b^2\*c^4 + 2\*a\*b\*c^2\*d^2 + a^2\*d^4)/(a^3\*b^3)) + a\*b^2\*c^3 + a^2\*b\*c\*d^2)\*sqrt((a\*b\*sqrt((b^2\*c^4 + 2\*a\*b\*c^2\*d^2 + a^2\*d^4)/(a^3\*b^3)) - 2\*c\*d)/(a\*b)))

**giac** [B] time = 0.32, size = 228, normalized size = 2.65

$$\frac{\sqrt{2}(b^2c - \sqrt{-ab}bd) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4(-ab^3)^{\frac{3}{4}}} - \frac{\sqrt{2}(b^2c + \sqrt{-ab}bd) \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4(-ab^3)^{\frac{3}{4}}} - \frac{\sqrt{2}(b^2c + \sqrt{-ab}bd) \log\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8(-ab^3)^{\frac{3}{4}}} + \frac{\sqrt{2}(b^2c + \sqrt{-ab}bd) \log\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8(-ab^3)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-d\*x^2+c)/(-b\*x^4+a), x, algorithm="giac")

[Out] -1/4\*sqrt(2)\*(b^2\*c - sqrt(-a\*b)\*b\*d)\*arctan(1/2\*sqrt(2)\*(2\*x + sqrt(2))\*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a\*b^3)^(3/4) - 1/4\*sqrt(2)\*(b^2\*c + sqrt(-a\*b)\*b\*d)\*arctan(1/2\*sqrt(2)\*(2\*x - sqrt(2))\*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a\*b^3)^(3/4) - 1/8\*sqrt(2)\*(b^2\*c + sqrt(-a\*b)\*b\*d)\*log(x^2 + sqrt(2)\*x\*(-a/b)^(1/4) + sqrt(-a/b))/(-a\*b^3)^(3/4) + 1/8\*sqrt(2)\*(b^2\*c + sqrt(-a\*b)\*b\*d)\*log(x^2 - sqrt(2)\*x\*(-a/b)^(1/4) + sqrt(-a/b))/(-a\*b^3)^(3/4)

**maple** [B] time = 0.00, size = 122, normalized size = 1.42

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} c \arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} c \ln\left(\frac{x + \left(\frac{a}{b}\right)^{\frac{1}{4}}}{x - \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4a} + \frac{d \arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}} b} - \frac{d \ln\left(\frac{x + \left(\frac{a}{b}\right)^{\frac{1}{4}}}{x - \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-d*x^2+c)/(-b*x^4+a),x)`

[Out]  $\frac{1}{4}*(a/b)^{(1/4)}/a*c*\ln((x+(a/b)^{(1/4)})/(x-(a/b)^{(1/4)}))+1/2*(a/b)^{(1/4)}/a*c*\arctan(1/(a/b)^{(1/4)}*x)+1/2/(a/b)^{(1/4)}/b*d*\arctan(1/(a/b)^{(1/4)}*x)-1/4/(a/b)^{(1/4)}/b*d*\ln((x+(a/b)^{(1/4)})/(x-(a/b)^{(1/4)}))$

**maxima** [A] time = 2.34, size = 109, normalized size = 1.27

$$\frac{(\sqrt{b}c + \sqrt{a}d) \arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{(\sqrt{b}c - \sqrt{a}d) \log\left(\frac{\sqrt{b}x - \sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x + \sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-d*x^2+c)/(-b*x^4+a),x, algorithm="maxima")`

[Out]  $\frac{1}{2}*(\sqrt{b}*c + \sqrt{a}*d)*\arctan(\sqrt{b}*x/\sqrt{\sqrt{a}\sqrt{b}})/(\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}) - \frac{1}{4}*(\sqrt{b}*c - \sqrt{a}*d)*\log((\sqrt{b}*x - \sqrt{\sqrt{a}\sqrt{b}})/(\sqrt{b}*x + \sqrt{\sqrt{a}\sqrt{b}}))/(\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b})$

**mupad** [B] time = 4.58, size = 579, normalized size = 6.73

$$-2\operatorname{atanh}\left(\frac{8b^3c^2x\sqrt{\frac{cd}{8ab} - \frac{c^2\sqrt{a^3b^3}}{16a^2b^3}} + \frac{8a^2d^2x\sqrt{\frac{cd}{8ab} - \frac{c^2\sqrt{a^3b^3}}{16a^2b^3}}}{2b^2c^2d + 2abd^2 + \frac{21c^2\sqrt{a^3b^3}}{2} + \frac{21cd\sqrt{a^3b^3}}{2}}\right) \sqrt{\frac{a^2\sqrt{a^3b^3} + b^2\sqrt{a^3b^3} + 2a^2b^2cd}{16a^2b^3}} - 2\operatorname{atanh}\left(\frac{8b^3c^2x\sqrt{\frac{cd}{8ab} + \frac{c^2\sqrt{a^3b^3}}{16a^2b^3}} + \frac{8a^2d^2x\sqrt{\frac{cd}{8ab} + \frac{c^2\sqrt{a^3b^3}}{16a^2b^3}}}{2b^2c^2d + 2abd^2 - \frac{21c^2\sqrt{a^3b^3}}{2} - \frac{21cd\sqrt{a^3b^3}}{2}}\right) \sqrt{\frac{a^2\sqrt{a^3b^3} + b^2\sqrt{a^3b^3} - 2a^2b^2cd}{16a^2b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c - d*x^2)/(a - b*x^4),x)`

[Out]  $-2*\operatorname{atanh}((8*b^3*c^2*x*(-(c*d)/(8*a*b) - (c^2*(a^3*b^3)^{(1/2)})/(16*a^3*b^2)) - (d^2*(a^3*b^3)^{(1/2)})/(16*a^2*b^3))^{(1/2)})/(2*b^2*c^2*d + 2*a*b*d^3 + (2*b*c^3*(a^3*b^3)^{(1/2)})/a^2 + (2*c*d^2*(a^3*b^3)^{(1/2)})/a) + (8*a*b^2*d^2*x*(-(c*d)/(8*a*b) - (c^2*(a^3*b^3)^{(1/2)})/(16*a^3*b^2) - (d^2*(a^3*b^3)^{(1/2)})/(16*a^2*b^3))^{(1/2)})/(2*b^2*c^2*d + 2*a*b*d^3 + (2*b*c^3*(a^3*b^3)^{(1/2)})/a^2 + (2*c*d^2*(a^3*b^3)^{(1/2)})/a))*(-(a*d^2*(a^3*b^3)^{(1/2)} + b*c^2*(a^3*b^3)^{(1/2)} + 2*a^2*b^2*c*d)/(16*a^3*b^3))^{(1/2)} - 2*\operatorname{atanh}((8*b^3*c^2*x*((c^2*(a^3*b^3)^{(1/2)})/(16*a^3*b^2) - (c*d)/(8*a*b) + (d^2*(a^3*b^3)^{(1/2)})/(16*a^2*b^3))^{(1/2)})/(2*b^2*c^2*d + 2*a*b*d^3 - (2*b*c^3*(a^3*b^3)^{(1/2)})/a^2 - (2*c*d^2*(a^3*b^3)^{(1/2)})/a) + (8*a*b^2*d^2*x*((c^2*(a^3*b^3)^{(1/2)})/(16*a^3*b^2) - (c*d)/(8*a*b) + (d^2*(a^3*b^3)^{(1/2)})/(16*a^2*b^3))^{(1/2)})/(2*b^2*c^2*d + 2*a*b*d^3 - (2*b*c^3*(a^3*b^3)^{(1/2)})/a^2 - (2*c*d^2*(a^3*b^3)^{(1/2)})/a))*((a*d^2*(a^3*b^3)^{(1/2)} + b*c^2*(a^3*b^3)^{(1/2)} - 2*a^2*b^2*c*d)/(16*a^3*b^3))^{(1/2)}$

**sympy** [A] time = 0.94, size = 110, normalized size = 1.28

$$\operatorname{RootSum}\left(256t^4a^3b^3 + 64t^2a^2b^2cd - a^2d^4 + 2abc^2d^2 - b^2c^4, \left(t \mapsto t \log\left(x + \frac{-64t^3a^3b^2d - 12ta^2bcd^2 - 4tab^2c^3}{a^2d^4 - b^2c^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-d*x**2+c)/(-b*x**4+a),x)`

[Out]  $\operatorname{RootSum}(256*_t**4*a**3*b**3 + 64*_t**2*a**2*b**2*c*d - a**2*d**4 + 2*a*b*c**2*d**2 - b**2*c**4, \operatorname{Lambda}(_t, _t*\log(x + (-64*_t**3*a**3*b**2*d - 12*_t*a**2*b*c*d**2 - 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))$

$$3.5 \quad \int \frac{2+3x^2}{4+9x^4} dx$$

**Optimal.** Leaf size=40

$$\frac{\tan^{-1}(\sqrt{3}x+1)}{2\sqrt{3}} - \frac{\tan^{-1}(1-\sqrt{3}x)}{2\sqrt{3}}$$

**Rubi [A]** time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {1162, 617, 204}

$$\frac{\tan^{-1}(\sqrt{3}x+1)}{2\sqrt{3}} - \frac{\tan^{-1}(1-\sqrt{3}x)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x^2)/(4 + 9\*x^4), x]

[Out] -ArcTan[1 - Sqrt[3]\*x]/(2\*Sqrt[3]) + ArcTan[1 + Sqrt[3]\*x]/(2\*Sqrt[3])

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 617**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

**Rule 1162**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

**Rubi steps**

$$\begin{aligned} \int \frac{2+3x^2}{4+9x^4} dx &= \frac{1}{6} \int \frac{1}{\frac{2}{3} - \frac{2x}{\sqrt{3}} + x^2} dx + \frac{1}{6} \int \frac{1}{\frac{2}{3} + \frac{2x}{\sqrt{3}} + x^2} dx \\ &= \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1 - \sqrt{3}x\right)}{2\sqrt{3}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1 + \sqrt{3}x\right)}{2\sqrt{3}} \\ &= -\frac{\tan^{-1}(1 - \sqrt{3}x)}{2\sqrt{3}} + \frac{\tan^{-1}(1 + \sqrt{3}x)}{2\sqrt{3}} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 33, normalized size = 0.82

$$\frac{\tan^{-1}(\sqrt{3}x+1) - \tan^{-1}(1-\sqrt{3}x)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x^2)/(4 + 9\*x^4), x]

[Out] (-ArcTan[1 - Sqrt[3]\*x] + ArcTan[1 + Sqrt[3]\*x])/(2\*Sqrt[3])

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 + 3x^2}{4 + 9x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + 3\*x^2)/(4 + 9\*x^4), x]

[Out] IntegrateAlgebraic[(2 + 3\*x^2)/(4 + 9\*x^4), x]

**fricas** [A] time = 0.69, size = 33, normalized size = 0.82

$$\frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{4} \sqrt{3} (3x^3 + 2x)\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{2} \sqrt{3} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2+2)/(9\*x^4+4), x, algorithm="fricas")

[Out] 1/6\*sqrt(3)\*arctan(1/4\*sqrt(3)\*(3\*x^3 + 2\*x)) + 1/6\*sqrt(3)\*arctan(1/2\*sqrt(3)\*x)

**giac** [A] time = 0.20, size = 52, normalized size = 1.30

$$\frac{1}{6} \sqrt{3} \arctan\left(\frac{9}{8} \sqrt{2} \left(\frac{4}{9}\right)^{\frac{3}{4}} \left(2x + \sqrt{2} \left(\frac{4}{9}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{9}{8} \sqrt{2} \left(\frac{4}{9}\right)^{\frac{3}{4}} \left(2x - \sqrt{2} \left(\frac{4}{9}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2+2)/(9\*x^4+4), x, algorithm="giac")

[Out] 1/6\*sqrt(3)\*arctan(9/8\*sqrt(2)\*(4/9)^(3/4)\*(2\*x + sqrt(2)\*(4/9)^(1/4))) + 1/6\*sqrt(3)\*arctan(9/8\*sqrt(2)\*(4/9)^(3/4)\*(2\*x - sqrt(2)\*(4/9)^(1/4)))

**maple** [B] time = 0.01, size = 122, normalized size = 3.05

$$\frac{\sqrt{6} \sqrt{2} \arctan\left(\frac{\sqrt{6} \sqrt{2} x}{2} - 1\right)}{12} + \frac{\sqrt{6} \sqrt{2} \arctan\left(\frac{\sqrt{6} \sqrt{2} x}{2} + 1\right)}{12} + \frac{\sqrt{6} \sqrt{2} \ln\left(\frac{x^2 - \frac{\sqrt{6} \sqrt{2} x}{3} + \frac{2}{3}}{x^2 + \frac{\sqrt{6} \sqrt{2} x}{3} + \frac{2}{3}}\right)}{48} + \frac{\sqrt{6} \sqrt{2} \ln\left(\frac{x^2 + \frac{\sqrt{6} \sqrt{2} x}{3} + \frac{2}{3}}{x^2 - \frac{\sqrt{6} \sqrt{2} x}{3} + \frac{2}{3}}\right)}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x^2+2)/(9\*x^4+4), x)

[Out] 1/12\*6^(1/2)\*2^(1/2)\*arctan(1/2\*6^(1/2)\*x\*2^(1/2)-1)+1/48\*6^(1/2)\*2^(1/2)\*ln((x^2+1/3\*6^(1/2)\*x\*2^(1/2)+2/3)/(x^2-1/3\*6^(1/2)\*x\*2^(1/2)+2/3))+1/12\*6^(1/2)\*2^(1/2)\*arctan(1/2\*6^(1/2)\*x\*2^(1/2)+1)+1/48\*6^(1/2)\*2^(1/2)\*ln((x^2-1/3\*6^(1/2)\*x\*2^(1/2)+2/3)/(x^2+1/3\*6^(1/2)\*x\*2^(1/2)+2/3))

**maxima** [A] time = 2.39, size = 39, normalized size = 0.98

$$\frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (3x + \sqrt{3})\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (3x - \sqrt{3})\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2+2)/(9\*x^4+4),x, algorithm="maxima")

[Out] 1/6\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(3\*x + sqrt(3))) + 1/6\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(3\*x - sqrt(3)))

**mupad [B]** time = 0.09, size = 29, normalized size = 0.72

$$\frac{\sqrt{3} \left( \operatorname{atan} \left( \frac{3\sqrt{3}x^3}{4} + \frac{\sqrt{3}x}{2} \right) + \operatorname{atan} \left( \frac{\sqrt{3}x}{2} \right) \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x^2 + 2)/(9\*x^4 + 4),x)

[Out] (3^(1/2)\*(atan((3^(1/2)\*x)/2 + (3\*3^(1/2)\*x^3)/4) + atan((3^(1/2)\*x)/2))/6

**sympy [A]** time = 0.12, size = 41, normalized size = 1.02

$$\frac{\sqrt{3} \left( 2 \operatorname{atan} \left( \frac{\sqrt{3}x}{2} \right) + 2 \operatorname{atan} \left( \frac{3\sqrt{3}x^3}{4} + \frac{\sqrt{3}x}{2} \right) \right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x\*\*2+2)/(9\*x\*\*4+4),x)

[Out] sqrt(3)\*(2\*atan(sqrt(3)\*x/2) + 2\*atan(3\*sqrt(3)\*x\*\*3/4 + sqrt(3)\*x/2))/12

$$3.6 \quad \int \frac{2-3x^2}{4+9x^4} dx$$

**Optimal.** Leaf size=51

$$\frac{\log(3x^2 + 2\sqrt{3}x + 2)}{4\sqrt{3}} - \frac{\log(3x^2 - 2\sqrt{3}x + 2)}{4\sqrt{3}}$$

**Rubi [A]** time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {1165, 628}

$$\frac{\log(3x^2 + 2\sqrt{3}x + 2)}{4\sqrt{3}} - \frac{\log(3x^2 - 2\sqrt{3}x + 2)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(2 - 3\*x^2)/(4 + 9\*x^4), x]

[Out] -Log[2 - 2\*Sqrt[3]\*x + 3\*x^2]/(4\*Sqrt[3]) + Log[2 + 2\*Sqrt[3]\*x + 3\*x^2]/(4\*Sqrt[3])

Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[d\*Log[RemoveContent[a + b\*x + c\*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1165

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

Rubi steps

$$\begin{aligned} \int \frac{2-3x^2}{4+9x^4} dx &= -\frac{\int \frac{\frac{2}{\sqrt{3}}+2x}{-\frac{2}{3}-\frac{2x}{\sqrt{3}}-x^2} dx}{4\sqrt{3}} - \frac{\int \frac{\frac{2}{\sqrt{3}}-2x}{-\frac{2}{3}+\frac{2x}{\sqrt{3}}-x^2} dx}{4\sqrt{3}} \\ &= -\frac{\log(2 - 2\sqrt{3}x + 3x^2)}{4\sqrt{3}} + \frac{\log(2 + 2\sqrt{3}x + 3x^2)}{4\sqrt{3}} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 44, normalized size = 0.86

$$\frac{\log(3x^2 + 2\sqrt{3}x + 2) - \log(-3x^2 + 2\sqrt{3}x - 2)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 - 3\*x^2)/(4 + 9\*x^4), x]

[Out] (-Log[-2 + 2\*Sqrt[3]\*x - 3\*x^2] + Log[2 + 2\*Sqrt[3]\*x + 3\*x^2])/(4\*Sqrt[3])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 - 3x^2}{4 + 9x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 - 3\*x^2)/(4 + 9\*x^4), x]

[Out] IntegrateAlgebraic[(2 - 3\*x^2)/(4 + 9\*x^4), x]

fricas [A] time = 0.55, size = 42, normalized size = 0.82

$$\frac{1}{12} \sqrt{3} \log \left( \frac{9x^4 + 24x^2 + 4\sqrt{3}(3x^3 + 2x) + 4}{9x^4 + 4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3\*x^2+2)/(9\*x^4+4), x, algorithm="fricas")

[Out] 1/12\*sqrt(3)\*log((9\*x^4 + 24\*x^2 + 4\*sqrt(3)\*(3\*x^3 + 2\*x) + 4)/(9\*x^4 + 4))

giac [A] time = 0.17, size = 40, normalized size = 0.78

$$\frac{1}{12} \sqrt{3} \log \left( x^2 + \sqrt{2} \left( \frac{4}{9} \right)^{\frac{1}{4}} x + \frac{2}{3} \right) - \frac{1}{12} \sqrt{3} \log \left( x^2 - \sqrt{2} \left( \frac{4}{9} \right)^{\frac{1}{4}} x + \frac{2}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3\*x^2+2)/(9\*x^4+4), x, algorithm="giac")

[Out] 1/12\*sqrt(3)\*log(x^2 + sqrt(2)\*(4/9)^(1/4)\*x + 2/3) - 1/12\*sqrt(3)\*log(x^2 - sqrt(2)\*(4/9)^(1/4)\*x + 2/3)

maple [B] time = 0.00, size = 82, normalized size = 1.61

$$-\frac{\sqrt{6} \sqrt{2} \ln \left( \frac{x^2 - \frac{\sqrt{6} \sqrt{2} x + \frac{2}{3}}{3}}{x^2 + \frac{\sqrt{6} \sqrt{2} x + \frac{2}{3}}{3}} \right)}{48} + \frac{\sqrt{6} \sqrt{2} \ln \left( \frac{x^2 + \frac{\sqrt{6} \sqrt{2} x + \frac{2}{3}}{3}}{x^2 - \frac{\sqrt{6} \sqrt{2} x + \frac{2}{3}}{3}} \right)}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-3\*x^2+2)/(9\*x^4+4), x)

[Out] 1/48\*6^(1/2)\*2^(1/2)\*ln((x^2+1/3\*6^(1/2)\*2^(1/2)\*x+2/3)/(x^2-1/3\*6^(1/2)\*2^(1/2)\*x+2/3))-1/48\*6^(1/2)\*2^(1/2)\*ln((x^2-1/3\*6^(1/2)\*2^(1/2)\*x+2/3)/(x^2+1/3\*6^(1/2)\*2^(1/2)\*x+2/3))

maxima [A] time = 2.42, size = 39, normalized size = 0.76

$$\frac{1}{12} \sqrt{3} \log(3x^2 + 2\sqrt{3}x + 2) - \frac{1}{12} \sqrt{3} \log(3x^2 - 2\sqrt{3}x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3\*x^2+2)/(9\*x^4+4), x, algorithm="maxima")

[Out] 1/12\*sqrt(3)\*log(3\*x^2 + 2\*sqrt(3)\*x + 2) - 1/12\*sqrt(3)\*log(3\*x^2 - 2\*sqrt(3)\*x + 2)



**mupad [B]** time = 4.43, size = 21, normalized size = 0.41

$$\frac{\sqrt{3} \operatorname{atanh}\left(\frac{2\sqrt{3}x}{3x^2+2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(3*x^2 - 2)/(9*x^4 + 4), x)`

[Out] `(3^(1/2)*atanh((2*3^(1/2)*x)/(3*x^2 + 2)))/6`

**sympy [A]** time = 0.12, size = 49, normalized size = 0.96

$$-\frac{\sqrt{3} \log\left(x^2 - \frac{2\sqrt{3}x}{3} + \frac{2}{3}\right)}{12} + \frac{\sqrt{3} \log\left(x^2 + \frac{2\sqrt{3}x}{3} + \frac{2}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-3*x**2+2)/(9*x**4+4), x)`

[Out] `-sqrt(3)*log(x**2 - 2*sqrt(3)*x/3 + 2/3)/12 + sqrt(3)*log(x**2 + 2*sqrt(3)*x/3 + 2/3)/12`

$$3.7 \quad \int \frac{2+3x^2}{4-9x^4} dx$$

Optimal. Leaf size=16

$$\frac{\tanh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {26, 206}

$$\frac{\tanh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x^2)/(4 - 9\*x^4), x]

[Out] ArcTanh[Sqrt[3/2]\*x]/Sqrt[6]

Rule 26

Int[(u\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_)^(j\_.))^(p\_.), x\_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b\*x^n)^m, x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[j, 2\*n] && EqQ[p, -m] && EqQ[b^2\*c + a^2\*d, 0] && GtQ[a, 0] && LtQ[d, 0]

Rule 206

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])]/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{2+3x^2}{4-9x^4} dx &= \int \frac{1}{2-3x^2} dx \\ &= \frac{\tanh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 2.00

$$\frac{\log(3x + \sqrt{6}) - \log(\sqrt{6} - 3x)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x^2)/(4 - 9\*x^4), x]

[Out] (-Log[Sqrt[6] - 3\*x] + Log[Sqrt[6] + 3\*x])/(2\*Sqrt[6])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2+3x^2}{4-9x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + 3\*x^2)/(4 - 9\*x^4), x]

[Out] IntegrateAlgebraic[(2 + 3\*x^2)/(4 - 9\*x^4), x]

**fricas** [B] time = 0.77, size = 29, normalized size = 1.81

$$\frac{1}{12} \sqrt{6} \log\left(\frac{3x^2 + 2\sqrt{6}x + 2}{3x^2 - 2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2+2)/(-9\*x^4+4), x, algorithm="fricas")

[Out] 1/12\*sqrt(6)\*log((3\*x^2 + 2\*sqrt(6)\*x + 2)/(3\*x^2 - 2))

**giac** [B] time = 0.16, size = 29, normalized size = 1.81

$$\frac{1}{12} \sqrt{6} \log\left(\left|x + \frac{1}{3} \sqrt{6}\right|\right) - \frac{1}{12} \sqrt{6} \log\left(\left|x - \frac{1}{3} \sqrt{6}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2+2)/(-9\*x^4+4), x, algorithm="giac")

[Out] 1/12\*sqrt(6)\*log(abs(x + 1/3\*sqrt(6))) - 1/12\*sqrt(6)\*log(abs(x - 1/3\*sqrt(6)))

**maple** [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{\sqrt{6} \operatorname{arctanh}\left(\frac{\sqrt{6}x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x^2+2)/(-9\*x^4+4), x)

[Out] 1/6\*arctanh(1/2\*6^(1/2)\*x)\*6^(1/2)

**maxima** [B] time = 2.39, size = 25, normalized size = 1.56

$$-\frac{1}{12} \sqrt{6} \log\left(\frac{3x - \sqrt{6}}{3x + \sqrt{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2+2)/(-9\*x^4+4), x, algorithm="maxima")

[Out] -1/12\*sqrt(6)\*log((3\*x - sqrt(6))/(3\*x + sqrt(6)))

**mupad** [B] time = 0.09, size = 12, normalized size = 0.75

$$\frac{\sqrt{6} \operatorname{atanh}\left(\frac{\sqrt{6}x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(3\*x^2 + 2)/(9\*x^4 - 4), x)

[Out] (6^(1/2)\*atanh((6^(1/2)\*x)/2))/6

sympy [B] time = 0.11, size = 32, normalized size = 2.00

$$-\frac{\sqrt{6} \log\left(x - \frac{\sqrt{6}}{3}\right)}{12} + \frac{\sqrt{6} \log\left(x + \frac{\sqrt{6}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x\*\*2+2)/(-9\*x\*\*4+4),x)

[Out] -sqrt(6)\*log(x - sqrt(6)/3)/12 + sqrt(6)\*log(x + sqrt(6)/3)/12

$$3.8 \quad \int \frac{2-3x^2}{4-9x^4} dx$$

Optimal. Leaf size=16

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

**Rubi [A]** time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {26, 203}

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[(2 - 3\*x^2)/(4 - 9\*x^4), x]

[Out] ArcTan[Sqrt[3/2]\*x]/Sqrt[6]

Rule 26

Int[(u\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_)^(j\_.))^(p\_.), x\_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b\*x^n)^m, x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[j, 2\*n] && EqQ[p, -m] && EqQ[b^2\*c + a^2\*d, 0] && GtQ[a, 0] && LtQ[d, 0]

Rule 203

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{2-3x^2}{4-9x^4} dx &= \int \frac{1}{2+3x^2} dx \\ &= \frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 16, normalized size = 1.00

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 - 3\*x^2)/(4 - 9\*x^4), x]

[Out] ArcTan[Sqrt[3/2]\*x]/Sqrt[6]

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2-3x^2}{4-9x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 - 3\*x^2)/(4 - 9\*x^4), x]

[Out] IntegrateAlgebraic[(2 - 3\*x^2)/(4 - 9\*x^4), x]

**fricas** [A] time = 0.86, size = 12, normalized size = 0.75

$$\frac{1}{6} \sqrt{6} \arctan\left(\frac{1}{2} \sqrt{6} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3\*x^2+2)/(-9\*x^4+4), x, algorithm="fricas")

[Out] 1/6\*sqrt(6)\*arctan(1/2\*sqrt(6)\*x)

**giac** [A] time = 0.16, size = 12, normalized size = 0.75

$$\frac{1}{6} \sqrt{6} \arctan\left(\frac{1}{2} \sqrt{6} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3\*x^2+2)/(-9\*x^4+4), x, algorithm="giac")

[Out] 1/6\*sqrt(6)\*arctan(1/2\*sqrt(6)\*x)

**maple** [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{\sqrt{6} \arctan\left(\frac{\sqrt{6} x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-3\*x^2+2)/(-9\*x^4+4), x)

[Out] 1/6\*arctan(1/2\*6^(1/2)\*x)\*6^(1/2)

**maxima** [A] time = 2.31, size = 12, normalized size = 0.75

$$\frac{1}{6} \sqrt{6} \arctan\left(\frac{1}{2} \sqrt{6} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3\*x^2+2)/(-9\*x^4+4), x, algorithm="maxima")

[Out] 1/6\*sqrt(6)\*arctan(1/2\*sqrt(6)\*x)

**mupad** [B] time = 0.03, size = 12, normalized size = 0.75

$$\frac{\sqrt{6} \operatorname{atan}\left(\frac{\sqrt{6} x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x^2 - 2)/(9\*x^4 - 4), x)

[Out] (6^(1/2)\*atan((6^(1/2)\*x)/2))/6

sympy [A] time = 0.11, size = 15, normalized size = 0.94

$$\frac{\sqrt{6} \operatorname{atan}\left(\frac{\sqrt{6}x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3\*x\*\*2+2)/(-9\*x\*\*4+4), x)

[Out] sqrt(6)\*atan(sqrt(6)\*x/2)/6

$$3.9 \quad \int \frac{\sqrt{a} \sqrt{b+bx^2}}{a+bx^4} dx$$

**Optimal.** Leaf size=75

$$\frac{\sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}} + 1\right)}{\sqrt{2} \sqrt[4]{a}} - \frac{\sqrt[4]{b} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{\sqrt{2} \sqrt[4]{a}}$$

**Rubi [A]** time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {1162, 617, 204}

$$\frac{\sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}} + 1\right)}{\sqrt{2} \sqrt[4]{a}} - \frac{\sqrt[4]{b} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{\sqrt{2} \sqrt[4]{a}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a]\*Sqrt[b] + b\*x^2)/(a + b\*x^4),x]

[Out] -((b^(1/4)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)]/(Sqrt[2]\*a^(1/4))) + (b^(1/4)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)]/(Sqrt[2]\*a^(1/4)))

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 617**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

**Rule 1162**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

**Rubi steps**

$$\begin{aligned} \int \frac{\sqrt{a} \sqrt{b+bx^2}}{a+bx^4} dx &= \frac{1}{2} \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2} \sqrt[4]{a} x}{\sqrt[4]{b}} + x^2} dx + \frac{1}{2} \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2} \sqrt[4]{a} x}{\sqrt[4]{b}} + x^2} dx \\ &= \frac{\sqrt[4]{b} \operatorname{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{\sqrt{2} \sqrt[4]{a}} - \frac{\sqrt[4]{b} \operatorname{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1 + \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{\sqrt{2} \sqrt[4]{a}} \\ &= -\frac{\sqrt[4]{b} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{\sqrt{2} \sqrt[4]{a}} + \frac{\sqrt[4]{b} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{\sqrt{2} \sqrt[4]{a}} \end{aligned}$$



**Mathematica [A]** time = 0.02, size = 60, normalized size = 0.80

$$\frac{\sqrt[4]{b} \left( \tan^{-1} \left( \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}} + 1 \right) - \tan^{-1} \left( 1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}} \right) \right)}{\sqrt{2} \sqrt[4]{a}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a]\*Sqrt[b] + b\*x^2)/(a + b\*x^4), x]

[Out] (b^(1/4)\*(-ArcTan[1 - (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)] + ArcTan[1 + (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)]))/(Sqrt[2]\*a^(1/4))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a} \sqrt{b} + bx^2}{a + bx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(Sqrt[a]\*Sqrt[b] + b\*x^2)/(a + b\*x^4), x]

[Out] IntegrateAlgebraic[(Sqrt[a]\*Sqrt[b] + b\*x^2)/(a + b\*x^4), x]

**fricas [A]** time = 0.94, size = 148, normalized size = 1.97

$$\left[ \frac{1}{2} \sqrt{\frac{1}{2}} \sqrt{\frac{\sqrt{b}}{\sqrt{a}}} \log \left( \frac{bx^4 - 4\sqrt{a}\sqrt{b}x^2 + 4\sqrt{\frac{1}{2}}(\sqrt{a}\sqrt{b}x^3 - ax)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}} + a}{bx^4 + a} \right), \sqrt{\frac{1}{2}} \sqrt{\frac{\sqrt{b}}{\sqrt{a}}} \arctan \left( \sqrt{\frac{1}{2}} x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}} \right) + \sqrt{\frac{1}{2}} \sqrt{\frac{\sqrt{b}}{\sqrt{a}}} \arctan \left( \frac{\sqrt{\frac{1}{2}}(\sqrt{a}\sqrt{b}x^3 + ax)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}{a} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a^(1/2)\*b^(1/2))/(b\*x^4+a), x, algorithm="fricas")

[Out] [1/2\*sqrt(1/2)\*sqrt(-sqrt(b)/sqrt(a))\*log((b\*x^4 - 4\*sqrt(a)\*sqrt(b)\*x^2 + 4\*sqrt(1/2)\*(sqrt(a)\*sqrt(b)\*x^3 - a\*x)\*sqrt(-sqrt(b)/sqrt(a)) + a)/(b\*x^4 + a), sqrt(1/2)\*sqrt(sqrt(b)/sqrt(a))\*arctan(sqrt(1/2)\*x\*sqrt(sqrt(b)/sqrt(a))) + sqrt(1/2)\*sqrt(sqrt(b)/sqrt(a))\*arctan(sqrt(1/2)\*(sqrt(a)\*sqrt(b)\*x^3 + a\*x)\*sqrt(sqrt(b)/sqrt(a))/a)]

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a^(1/2)\*b^(1/2))/(b\*x^4+a), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Value

**maple [B]** time = 0.00, size = 254, normalized size = 3.39

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{b} \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\sqrt{a}} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{b} \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\sqrt{a}} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{b} \ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}\right)}{8\sqrt{a}} + \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}} + \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}} + \frac{\sqrt{2} \ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a^(1/2)\*b^(1/2))/(b\*x^4+a), x)

[Out] 1/8/a^(1/2)\*b^(1/2)\*(a/b)^(1/4)\*2^(1/2)\*ln((x^2+(a/b)^(1/4)\*2^(1/2)\*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)\*2^(1/2)\*x+(a/b)^(1/2)))+1/4/a^(1/2)\*b^(1/2)\*(a/b)^(1/4)

$$\begin{aligned} & (1/4)*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x+1)+1/4/a^{(1/2)}*b^{(1/2)}*(a/b)^{(1/4)} \\ & *2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x-1)+1/8/(a/b)^{(1/4)}*2^{(1/2)}*\ln((x^2- \\ & (a/b)^{(1/4)}*2^{(1/2)}*x+(a/b)^{(1/2)})/(x^2+(a/b)^{(1/4)}*2^{(1/2)}*x+(a/b)^{(1/2)})) \\ & +1/4/(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x+1)+1/4/(a/b)^{(1/4)}*2^{(1/2)} \\ & *\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x-1) \end{aligned}$$

**maxima** [A] time = 2.31, size = 100, normalized size = 1.33

$$\frac{\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a^(1/2)\*b^(1/2))/(b\*x^4+a),x, algorithm="maxima")

[Out] 1/2\*sqrt(2)\*sqrt(b)\*arctan(1/2\*sqrt(2)\*(2\*sqrt(b)\*x + sqrt(2)\*a^(1/4)\*b^(1/4))/sqrt(sqrt(a)\*sqrt(b)))/sqrt(sqrt(a)\*sqrt(b)) + 1/2\*sqrt(2)\*sqrt(b)\*arctan(1/2\*sqrt(2)\*(2\*sqrt(b)\*x - sqrt(2)\*a^(1/4)\*b^(1/4))/sqrt(sqrt(a)\*sqrt(b)))/sqrt(sqrt(a)\*sqrt(b))

**mupad** [B] time = 4.79, size = 57, normalized size = 0.76

$$\frac{\sqrt{2}b^{1/4}\left(2\operatorname{atan}\left(\frac{\sqrt{2}b^{1/4}x}{2a^{1/4}}\right)+2\operatorname{atan}\left(\frac{\sqrt{2}b^{3/4}x^3}{2a^{3/4}}+\frac{\sqrt{2}b^{1/4}x}{2a^{1/4}}\right)\right)}{4a^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2 + a^(1/2)\*b^(1/2))/(a + b\*x^4),x)

[Out] (2^(1/2)\*b^(1/4)\*(2\*atan((2^(1/2)\*b^(1/4)\*x)/(2\*a^(1/4))) + 2\*atan((2^(1/2)\*b^(3/4)\*x^3)/(2\*a^(3/4)) + (2^(1/2)\*b^(1/4)\*x)/(2\*a^(1/4)))))/(4\*a^(1/4))

**sympy** [A] time = 0.39, size = 138, normalized size = 1.84

$$\frac{\sqrt{2}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\log\left(-\frac{\sqrt{2}\sqrt{a}x\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}-\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4} + \frac{\sqrt{2}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{\sqrt{2}\sqrt{a}x\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}-\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a\*\*(1/2)\*b\*\*(1/2))/(b\*x\*\*4+a),x)

[Out] -sqrt(2)\*sqrt(-sqrt(b)/sqrt(a))\*log(-sqrt(2)\*sqrt(a)\*x\*sqrt(-sqrt(b)/sqrt(a)))/sqrt(b) - sqrt(a)/sqrt(b) + x\*\*2)/4 + sqrt(2)\*sqrt(-sqrt(b)/sqrt(a))\*log(sqrt(2)\*sqrt(a)\*x\*sqrt(-sqrt(b)/sqrt(a)))/sqrt(b) - sqrt(a)/sqrt(b) + x\*\*2)/4

$$3.10 \quad \int \frac{\sqrt{a} \sqrt{b-bx^2}}{a+bx^4} dx$$

**Optimal.** Leaf size=106

$$\frac{\sqrt[4]{b} \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2)}{2\sqrt{2} \sqrt[4]{a}} - \frac{\sqrt[4]{b} \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2)}{2\sqrt{2} \sqrt[4]{a}}$$

**Rubi [A]** time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {1165, 628}

$$\frac{\sqrt[4]{b} \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2)}{2\sqrt{2} \sqrt[4]{a}} - \frac{\sqrt[4]{b} \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2)}{2\sqrt{2} \sqrt[4]{a}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a]\*Sqrt[b] - b\*x^2)/(a + b\*x^4), x]

[Out] -(b^(1/4)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/(2\*Sqrt[2]\*a^(1/4)) + (b^(1/4)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/(2\*Sqrt[2]\*a^(1/4))

**Rule 628**

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[d\*Log[RemoveContent[a + b\*x + c\*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

**Rule 1165**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

**Rubi steps**

$$\begin{aligned} \int \frac{\sqrt{a} \sqrt{b-bx^2}}{a+bx^4} dx &= -\frac{\sqrt[4]{b} \int \frac{\frac{\sqrt{2} \sqrt[4]{a}}{\sqrt[4]{b}} + 2x}{-\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2} \sqrt[4]{a} x}{\sqrt[4]{b}} - x^2} dx}{2\sqrt{2} \sqrt[4]{a}} - \frac{\sqrt[4]{b} \int \frac{\frac{\sqrt{2} \sqrt[4]{a}}{\sqrt[4]{b}} - 2x}{-\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2} \sqrt[4]{a} x}{\sqrt[4]{b}} - x^2} dx}{2\sqrt{2} \sqrt[4]{a}} \\ &= -\frac{\sqrt[4]{b} \log(\sqrt{a} - \sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{b} x^2)}{2\sqrt{2} \sqrt[4]{a}} + \frac{\sqrt[4]{b} \log(\sqrt{a} + \sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{b} x^2)}{2\sqrt{2} \sqrt[4]{a}} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 91, normalized size = 0.86

$$\frac{\sqrt[4]{b} (\log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2) - \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x - \sqrt{a} - \sqrt{b} x^2))}{2\sqrt{2} \sqrt[4]{a}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a]\*Sqrt[b] - b\*x^2)/(a + b\*x^4), x]

[Out] (b^(1/4)\*(-Log[-Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*x - Sqrt[b]\*x^2] + Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2]))/(2\*Sqrt[2]\*a^(1/4))

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a} \sqrt{b} - bx^2}{a + bx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(Sqrt[a]\*Sqrt[b] - b\*x^2)/(a + b\*x^4), x]

[Out] IntegrateAlgebraic[(Sqrt[a]\*Sqrt[b] - b\*x^2)/(a + b\*x^4), x]

**fricas** [A] time = 0.94, size = 151, normalized size = 1.42

$$\left[ \frac{1}{2} \sqrt{\frac{1}{2}} \sqrt{\frac{\sqrt{b}}{\sqrt{a}}} \log \left( \frac{bx^4 + 4\sqrt{a}\sqrt{b}x^2 + 4\sqrt{\frac{1}{2}}(\sqrt{a}\sqrt{b}x^3 + ax)\sqrt{\frac{\sqrt{b}}{\sqrt{a}} + a}}{bx^4 + a} \right), -\sqrt{\frac{1}{2}} \sqrt{\frac{\sqrt{b}}{\sqrt{a}}} \arctan \left( \sqrt{\frac{1}{2}} x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}} \right) + \sqrt{\frac{1}{2}} \sqrt{\frac{\sqrt{b}}{\sqrt{a}}} \arctan \left( \frac{\sqrt{\frac{1}{2}}(\sqrt{a}\sqrt{b}x^3 - ax)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}{a} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2+a^(1/2)\*b^(1/2))/(b\*x^4+a), x, algorithm="fricas")

[Out] [1/2\*sqrt(1/2)\*sqrt(sqrt(b)/sqrt(a))\*log((b\*x^4 + 4\*sqrt(a)\*sqrt(b)\*x^2 + 4\*sqrt(1/2)\*(sqrt(a)\*sqrt(b)\*x^3 + a\*x)\*sqrt(sqrt(b)/sqrt(a)) + a)/(b\*x^4 + a)), -sqrt(1/2)\*sqrt(-sqrt(b)/sqrt(a))\*arctan(sqrt(1/2)\*x\*sqrt(-sqrt(b)/sqrt(a))) + sqrt(1/2)\*sqrt(-sqrt(b)/sqrt(a))\*arctan(sqrt(1/2)\*(sqrt(a)\*sqrt(b)\*x^3 - a\*x)\*sqrt(-sqrt(b)/sqrt(a))/a)]

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2+a^(1/2)\*b^(1/2))/(b\*x^4+a), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Value

**maple** [B] time = 0.00, size = 254, normalized size = 2.40

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{b} \arctan\left(\frac{\sqrt{2}x-1}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\sqrt{a}} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{b} \arctan\left(\frac{\sqrt{2}x+1}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\sqrt{a}} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} \sqrt{b} \ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}\right)}{8\sqrt{a}} - \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}x-1}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}} - \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}x+1}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}} - \frac{\sqrt{2} \ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^2+a^(1/2)\*b^(1/2))/(b\*x^4+a), x)

[Out] 1/8\*(a/b)^(1/4)\*2^(1/2)/a^(1/2)\*b^(1/2)\*ln((x^2+(a/b)^(1/4)\*2^(1/2)\*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)\*2^(1/2)\*x+(a/b)^(1/2)))+1/4\*(a/b)^(1/4)\*2^(1/2)/a^(1/2)\*b^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x+1)+1/4\*(a/b)^(1/4)\*2^(1/2)/a^(1/2)\*b^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x-1)-1/8/(a/b)^(1/4)\*2^(1/2)\*ln((x^2-(a/b)^(1/4)\*2^(1/2)\*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)\*2^(1/2)\*x+(a/b)^(1/2)))-1/4/(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x+1)-1/4/(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x-1)

**maxima** [A] time = 2.37, size = 70, normalized size = 0.66

$$\frac{\sqrt{2} b^{\frac{1}{4}} \log\left(\sqrt{b} x^2 + \sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} x + \sqrt{a}\right)}{4 a^{\frac{1}{4}}} - \frac{\sqrt{2} b^{\frac{1}{4}} \log\left(\sqrt{b} x^2 - \sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} x + \sqrt{a}\right)}{4 a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2+a^(1/2)\*b^(1/2))/(b\*x^4+a),x, algorithm="maxima")

[Out]  $\frac{1}{4}\sqrt{2}b^{1/4}\log(\sqrt{b}x^2 + \sqrt{2}a^{1/4}b^{1/4}x + \sqrt{a})/a^{1/4} - \frac{1}{4}\sqrt{2}b^{1/4}\log(\sqrt{b}x^2 - \sqrt{2}a^{1/4}b^{1/4}x + \sqrt{a})/a^{1/4}$

**mupad [B]** time = 4.76, size = 43, normalized size = 0.41

$$\frac{\sqrt{2} b^{1/4} \operatorname{atanh}\left(\frac{2 \sqrt{2} a^{1/4} b^{11/4} x}{2 \sqrt{a} b^{5/2} + 2 b^3 x^2}\right)}{2 a^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(b\*x^2 - a^(1/2)\*b^(1/2))/(a + b\*x^4),x)

[Out]  $(2^{1/2}b^{1/4}\operatorname{atanh}((2*2^{1/2}a^{1/4}b^{11/4}x)/(2*a^{1/2}b^{5/2} + 2*b^3*x^2)))/(2*a^{1/4})$

**sympy [A]** time = 0.46, size = 131, normalized size = 1.24

$$-\frac{\sqrt{2} \sqrt{\frac{\sqrt{b}}{\sqrt{a}}} \log\left(-\frac{\sqrt{2} \sqrt{a} x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}} + \frac{\sqrt{a}}{\sqrt{b}} + x^2\right)}{4} + \frac{\sqrt{2} \sqrt{\frac{\sqrt{b}}{\sqrt{a}}} \log\left(\frac{\sqrt{2} \sqrt{a} x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}} + \frac{\sqrt{a}}{\sqrt{b}} + x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*2+a\*\*(1/2)\*b\*\*(1/2))/(b\*x\*\*4+a),x)

[Out]  $-\sqrt{2}\sqrt{\sqrt{b}/\sqrt{a}}\log(-\sqrt{2}\sqrt{a}x\sqrt{\sqrt{b}/\sqrt{a}}/\sqrt{b} + \sqrt{a}/\sqrt{b} + x^2)/4 + \sqrt{2}\sqrt{\sqrt{b}/\sqrt{a}}\log(\sqrt{2}\sqrt{a}x\sqrt{\sqrt{b}/\sqrt{a}}/\sqrt{b} + \sqrt{a}/\sqrt{b} + x^2)/4$

$$3.11 \quad \int \frac{d+ex^2}{d^2+e^2x^4} dx$$

**Optimal.** Leaf size=75

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}\sqrt{e}} - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

**Rubi [A]** time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {1162, 617, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}\sqrt{e}} - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)/(d^2 + e^2\*x^4),x]

[Out] -(ArcTan[1 - (Sqrt[2]\*Sqrt[e]\*x)/Sqrt[d]]/(Sqrt[2]\*Sqrt[d]\*Sqrt[e])) + ArcTan[1 + (Sqrt[2]\*Sqrt[e]\*x)/Sqrt[d]]/(Sqrt[2]\*Sqrt[d]\*Sqrt[e])

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 1162

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

Rubi steps

$$\begin{aligned} \int \frac{d+ex^2}{d^2+e^2x^4} dx &= \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}+x^2} dx}{2e} + \frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}+x^2} dx}{2e} \\ &= \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1+\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}} \\ &= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}} \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 60, normalized size = 0.80

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}+1\right)-\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)/(d^2 + e^2\*x^4), x]

[Out] (-ArcTan[1 - (Sqrt[2]\*Sqrt[e]\*x)/Sqrt[d]] + ArcTan[1 + (Sqrt[2]\*Sqrt[e]\*x)/Sqrt[d]])/(Sqrt[2]\*Sqrt[d]\*Sqrt[e])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{d^2 + e^2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)/(d^2 + e^2\*x^4), x]

[Out] IntegrateAlgebraic[(d + e\*x^2)/(d^2 + e^2\*x^4), x]

**fricas [A]** time = 1.50, size = 137, normalized size = 1.83

$$\left[ \frac{\sqrt{2}\sqrt{-de}\log\left(\frac{e^2x^4-4dex^2-2\sqrt{2}(ex^3-dx)\sqrt{-de}+d^2}{e^2x^4+d^2}\right)}{4de}, \frac{\sqrt{2}\sqrt{de}\arctan\left(\frac{\sqrt{2}\sqrt{de}x}{2d}\right)+\sqrt{2}\sqrt{de}\arctan\left(\frac{\sqrt{2}(ex^3+dx)\sqrt{de}}{2d^2}\right)}{2de} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(e^2\*x^4+d^2), x, algorithm="fricas")

[Out] [-1/4\*sqrt(2)\*sqrt(-d\*e)\*log((e^2\*x^4 - 4\*d\*e\*x^2 - 2\*sqrt(2)\*(e\*x^3 - d\*x)\*sqrt(-d\*e) + d^2)/(e^2\*x^4 + d^2))/(d\*e), 1/2\*(sqrt(2)\*sqrt(d\*e)\*arctan(1/2\*sqrt(2)\*sqrt(d\*e)\*x/d) + sqrt(2)\*sqrt(d\*e)\*arctan(1/2\*sqrt(2)\*(e\*x^3 + d\*x)\*sqrt(d\*e)/d^2))/(d\*e)]

**giac [B]** time = 0.17, size = 222, normalized size = 2.96

$$\frac{\sqrt{2}\left(\frac{d^2}{e}\right)^{\frac{1}{4}}de^{\frac{1}{2}}+\left(\frac{d^2}{e}\right)^{\frac{1}{4}}e^{\frac{1}{2}}\arctan\left(\frac{\sqrt{2}\left(\frac{d^2}{e}\right)^{\frac{1}{4}}\sqrt{2x+\sqrt{\frac{d^2}{e}}}}{2\left(\frac{d^2}{e}\right)^{\frac{1}{4}}}\right)e^{d-6}}{4d^2}+\frac{\sqrt{2}\left(\frac{d^2}{e}\right)^{\frac{1}{4}}de^{\frac{1}{2}}+\left(\frac{d^2}{e}\right)^{\frac{1}{4}}e^{\frac{1}{2}}\arctan\left(\frac{\sqrt{2}\left(\frac{d^2}{e}\right)^{\frac{1}{4}}\sqrt{2x+\sqrt{\frac{d^2}{e}}}}{2\left(\frac{d^2}{e}\right)^{\frac{1}{4}}}\right)e^{d-6}}{4d^2}+\frac{\sqrt{2}\left(\frac{d^2}{e}\right)^{\frac{1}{4}}de^{\frac{1}{2}}-\left(\frac{d^2}{e}\right)^{\frac{1}{4}}e^{\frac{1}{2}}\log\left(\sqrt{2}\left(\frac{d^2}{e}\right)^{\frac{1}{4}}\sqrt{2x+\sqrt{\frac{d^2}{e}}}\right)}{8d^2}+\frac{\sqrt{2}\left(\frac{d^2}{e}\right)^{\frac{1}{4}}de^{\frac{1}{2}}-\left(\frac{d^2}{e}\right)^{\frac{1}{4}}e^{\frac{1}{2}}\log\left(-\sqrt{2}\left(\frac{d^2}{e}\right)^{\frac{1}{4}}\sqrt{2x+\sqrt{\frac{d^2}{e}}}\right)}{8d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(e^2\*x^4+d^2), x, algorithm="giac")

[Out] 1/4\*sqrt(2)\*((d^2)^(1/4)\*d\*e^(11/2) + (d^2)^(3/4)\*e^(11/2))\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*(d^2)^(1/4)\*e^(-1/2) + 2\*x)\*e^(1/2)/(d^2)^(1/4))\*e^(-6)/d^2 + 1/4\*sqrt(2)\*((d^2)^(1/4)\*d\*e^(11/2) + (d^2)^(3/4)\*e^(11/2))\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*(d^2)^(1/4)\*e^(-1/2) - 2\*x)\*e^(1/2)/(d^2)^(1/4))\*e^(-6)/d^2 + 1/8\*sqrt(2)\*((d^2)^(1/4)\*d\*e^(11/2) - (d^2)^(3/4)\*e^(11/2))\*e^(-6)\*log(sqrt(2)\*(d^2)^(1/4)\*x\*e^(-1/2) + x^2 + sqrt(d^2)\*e^(-1))/d^2 - 1/8\*sqrt(2)\*((d^2)^(1/4)\*d\*e^(11/2) - (d^2)^(3/4)\*e^(11/2))\*e^(-6)\*log(-sqrt(2)\*(d^2)^(1/4)\*x\*e^(-1/2) + x^2 + sqrt(d^2)\*e^(-1))/d^2

**maple [B]** time = 0.01, size = 290, normalized size = 3.87

$$\frac{\left(\frac{d^2}{e}\right)^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\left(\frac{d^2}{e}\right)^{\frac{1}{4}}}-1\right)}{4d}+\frac{\left(\frac{d^2}{e}\right)^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\left(\frac{d^2}{e}\right)^{\frac{1}{4}}}+1\right)}{4d}+\frac{\left(\frac{d^2}{e}\right)^{\frac{1}{4}}\sqrt{2}\ln\left(\frac{x^2+\left(\frac{d^2}{e}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{d^2}{e}}}{x^2-\left(\frac{d^2}{e}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{d^2}{e}}}\right)}{8d}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\left(\frac{d^2}{e}\right)^{\frac{1}{4}}}-1\right)}{4\left(\frac{d^2}{e}\right)^{\frac{1}{4}}e}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\left(\frac{d^2}{e}\right)^{\frac{1}{4}}}+1\right)}{4\left(\frac{d^2}{e}\right)^{\frac{1}{4}}e}+\frac{\sqrt{2}\ln\left(\frac{x^2+\left(\frac{d^2}{e}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{d^2}{e}}}{x^2-\left(\frac{d^2}{e}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{d^2}{e}}}\right)}{8\left(\frac{d^2}{e}\right)^{\frac{1}{4}}e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)/(e^2*x^4+d^2),x)`

[Out]  $\frac{1}{8} \frac{1}{d} \left( \frac{d^2}{e^2} \right)^{1/4} 2^{1/2} \ln \left( \frac{x^2 + \left( \frac{d^2}{e^2} \right)^{1/4} x 2^{1/2} + \left( \frac{d^2}{e^2} \right)^{1/2}}{x^2 - \left( \frac{d^2}{e^2} \right)^{1/4} x 2^{1/2} + \left( \frac{d^2}{e^2} \right)^{1/2}} \right) + \frac{1}{4} \frac{1}{d} \left( \frac{d^2}{e^2} \right)^{1/4} 2^{1/2} \arctan \left( \frac{2^{1/2}}{\left( \frac{d^2}{e^2} \right)^{1/4} x + 1} \right) + \frac{1}{4} \frac{1}{d} \left( \frac{d^2}{e^2} \right)^{1/4} 2^{1/2} \arctan \left( \frac{2^{1/2}}{\left( \frac{d^2}{e^2} \right)^{1/4} x - 1} \right) + \frac{1}{8} \frac{1}{e} \left( \frac{d^2}{e^2} \right)^{1/4} 2^{1/2} \ln \left( \frac{x^2 - \left( \frac{d^2}{e^2} \right)^{1/4} x 2^{1/2} + \left( \frac{d^2}{e^2} \right)^{1/2}}{x^2 + \left( \frac{d^2}{e^2} \right)^{1/4} x 2^{1/2} + \left( \frac{d^2}{e^2} \right)^{1/2}} \right) + \frac{1}{4} \frac{1}{e} \left( \frac{d^2}{e^2} \right)^{1/4} 2^{1/2} \arctan \left( \frac{2^{1/2}}{\left( \frac{d^2}{e^2} \right)^{1/4} x + 1} \right) + \frac{1}{4} \frac{1}{e} \left( \frac{d^2}{e^2} \right)^{1/4} 2^{1/2} \arctan \left( \frac{2^{1/2}}{\left( \frac{d^2}{e^2} \right)^{1/4} x - 1} \right)$

**maxima** [A] time = 2.48, size = 74, normalized size = 0.99

$$\frac{\sqrt{2} \arctan \left( \frac{\sqrt{2}(2ex + \sqrt{2}\sqrt{d}\sqrt{e})}{2\sqrt{de}} \right)}{2\sqrt{de}} + \frac{\sqrt{2} \arctan \left( \frac{\sqrt{2}(2ex - \sqrt{2}\sqrt{d}\sqrt{e})}{2\sqrt{de}} \right)}{2\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)/(e^2*x^4+d^2),x, algorithm="maxima")`

[Out]  $\frac{1}{2} \sqrt{2} \arctan \left( \frac{1}{2} \sqrt{2} (2ex + \sqrt{2}\sqrt{d}\sqrt{e}) / \sqrt{de} \right) / \sqrt{de} + \frac{1}{2} \sqrt{2} \arctan \left( \frac{1}{2} \sqrt{2} (2ex - \sqrt{2}\sqrt{d}\sqrt{e}) / \sqrt{de} \right) / \sqrt{de}$

**mupad** [B] time = 4.41, size = 57, normalized size = 0.76

$$\frac{\sqrt{2} \left( 2 \operatorname{atan} \left( \frac{\sqrt{2}\sqrt{e}x}{2\sqrt{d}} \right) + 2 \operatorname{atan} \left( \frac{\sqrt{2}e^{3/2}x^3}{2d^{3/2}} + \frac{\sqrt{2}\sqrt{e}x}{2\sqrt{d}} \right) \right)}{4\sqrt{d}\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x^2)/(d^2 + e^2*x^4),x)`

[Out]  $\frac{2^{1/2} \left( 2 \operatorname{atan} \left( \frac{2^{1/2} e^{1/2} x}{2d^{1/2}} \right) + 2 \operatorname{atan} \left( \frac{2^{1/2} e^{3/2} x^3}{2d^{3/2}} + \frac{2^{1/2} e^{1/2} x}{2d^{1/2}} \right) \right)}{4d^{1/2} e^{1/2}}$

**sympy** [A] time = 0.22, size = 87, normalized size = 1.16

$$-\frac{\sqrt{2} \sqrt{-\frac{1}{de}} \log \left( -\sqrt{2} dx \sqrt{-\frac{1}{de}} - \frac{d}{e} + x^2 \right)}{4} + \frac{\sqrt{2} \sqrt{-\frac{1}{de}} \log \left( \sqrt{2} dx \sqrt{-\frac{1}{de}} - \frac{d}{e} + x^2 \right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)/(e**2*x**4+d**2),x)`

[Out]  $-\sqrt{2} \sqrt{-1/(d*e)} \log(-\sqrt{2} d*x*\sqrt{-1/(d*e)} - d/e + x**2)/4 + \sqrt{2} \sqrt{-1/(d*e)} \log(\sqrt{2} d*x*\sqrt{-1/(d*e)} - d/e + x**2)/4$



$$3.12 \quad \int \frac{d-ex^2}{d^2+e^2x^4} dx$$

**Optimal.** Leaf size=90

$$\frac{\log(\sqrt{2}\sqrt{d}\sqrt{e}x+d+ex^2)}{2\sqrt{2}\sqrt{d}\sqrt{e}} - \frac{\log(-\sqrt{2}\sqrt{d}\sqrt{e}x+d+ex^2)}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

**Rubi [A]** time = 0.05, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {1165, 628}

$$\frac{\log(\sqrt{2}\sqrt{d}\sqrt{e}x+d+ex^2)}{2\sqrt{2}\sqrt{d}\sqrt{e}} - \frac{\log(-\sqrt{2}\sqrt{d}\sqrt{e}x+d+ex^2)}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[(d - e\*x^2)/(d^2 + e^2\*x^4), x]

[Out] -Log[d - Sqrt[2]\*Sqrt[d]\*Sqrt[e]\*x + e\*x^2]/(2\*Sqrt[2]\*Sqrt[d]\*Sqrt[e]) + Log[d + Sqrt[2]\*Sqrt[d]\*Sqrt[e]\*x + e\*x^2]/(2\*Sqrt[2]\*Sqrt[d]\*Sqrt[e])

**Rule 628**

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

**Rule 1165**

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

**Rubi steps**

$$\begin{aligned} \int \frac{d-ex^2}{d^2+e^2x^4} dx &= \int \frac{\frac{\sqrt{2}\sqrt{d}+2x}{\sqrt{e}}}{-\frac{d}{e}-\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}-x^2} dx - \int \frac{\frac{\sqrt{2}\sqrt{d}-2x}{\sqrt{e}}}{-\frac{d}{e}+\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}-x^2} dx \\ &= -\frac{\log(d-\sqrt{2}\sqrt{d}\sqrt{e}x+ex^2)}{2\sqrt{2}\sqrt{d}\sqrt{e}} + \frac{\log(d+\sqrt{2}\sqrt{d}\sqrt{e}x+ex^2)}{2\sqrt{2}\sqrt{d}\sqrt{e}} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 75, normalized size = 0.83

$$\frac{\log(\sqrt{2}\sqrt{d}\sqrt{e}x+d+ex^2) - \log(\sqrt{2}\sqrt{d}\sqrt{e}x-d-ex^2)}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Integrate[(d - e\*x^2)/(d^2 + e^2\*x^4), x]

[Out] (-Log[-d + Sqrt[2]\*Sqrt[d]\*Sqrt[e]\*x - e\*x^2] + Log[d + Sqrt[2]\*Sqrt[d]\*Sqrt[e]\*x + e\*x^2])/ (2\*Sqrt[2]\*Sqrt[d]\*Sqrt[e])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d - ex^2}{d^2 + e^2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d - e\*x^2)/(d^2 + e^2\*x^4), x]

[Out] IntegrateAlgebraic[(d - e\*x^2)/(d^2 + e^2\*x^4), x]

**fricas [A]** time = 0.53, size = 140, normalized size = 1.56

$$\left[ \frac{\sqrt{2} \sqrt{de} \log\left(\frac{e^2x^4 + 4dex^2 + 2\sqrt{2}(ex^3 + dx)\sqrt{de} + d^2}{e^2x^4 + d^2}\right)}{4de}, \frac{\sqrt{2} \sqrt{-de} \arctan\left(\frac{\sqrt{2} \sqrt{-de} x}{2d}\right) - \sqrt{2} \sqrt{-de} \arctan\left(\frac{\sqrt{2}(ex^3 - dx)\sqrt{-de}}{2d^2}\right)}{2de} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*x^2+d)/(e^2\*x^4+d^2), x, algorithm="fricas")

[Out] [1/4\*sqrt(2)\*sqrt(d\*e)\*log((e^2\*x^4 + 4\*d\*e\*x^2 + 2\*sqrt(2)\*(e\*x^3 + d\*x))\*sqrt(d\*e) + d^2)/(e^2\*x^4 + d^2))/(d\*e), -1/2\*(sqrt(2)\*sqrt(-d\*e)\*arctan(1/2\*sqrt(2)\*sqrt(-d\*e)\*x/d) - sqrt(2)\*sqrt(-d\*e)\*arctan(1/2\*sqrt(2)\*(e\*x^3 - d\*x)\*sqrt(-d\*e)/d^2))/(d\*e)]

**giac [B]** time = 0.22, size = 222, normalized size = 2.47

$$\frac{\sqrt{2} \left( \left( \frac{d^2}{e^2} \right)^{\frac{1}{4}} de^{\frac{11}{2}} - \left( \frac{d^2}{e^2} \right)^{\frac{1}{4}} e^{\frac{11}{2}} \right) \arctan\left(\frac{\sqrt{2} \left( \left( \frac{d^2}{e^2} \right)^{\frac{1}{4}} x^{\frac{1}{2}} - \left( \frac{d^2}{e^2} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{d^2}{e^2} \right)^{\frac{1}{4}}}\right) d^{-6} + \sqrt{2} \left( \left( \frac{d^2}{e^2} \right)^{\frac{1}{4}} de^{\frac{11}{2}} - \left( \frac{d^2}{e^2} \right)^{\frac{1}{4}} e^{\frac{11}{2}} \right) \arctan\left(-\frac{\sqrt{2} \left( \left( \frac{d^2}{e^2} \right)^{\frac{1}{4}} x^{\frac{1}{2}} - \left( \frac{d^2}{e^2} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{d^2}{e^2} \right)^{\frac{1}{4}}}\right) d^{-6}}{4d^2} + \frac{\sqrt{2} \left( \left( \frac{d^2}{e^2} \right)^{\frac{1}{4}} de^{\frac{11}{2}} + \left( \frac{d^2}{e^2} \right)^{\frac{1}{4}} e^{\frac{11}{2}} \right) d^{-6} \log\left(\sqrt{2} \left( \frac{d^2}{e^2} \right)^{\frac{1}{4}} x e^{-\frac{1}{2}} + x^2 + \sqrt{d^2 e^{-1}}\right) - \sqrt{2} \left( \left( \frac{d^2}{e^2} \right)^{\frac{1}{4}} de^{\frac{11}{2}} + \left( \frac{d^2}{e^2} \right)^{\frac{1}{4}} e^{\frac{11}{2}} \right) d^{-6} \log\left(-\sqrt{2} \left( \frac{d^2}{e^2} \right)^{\frac{1}{4}} x e^{-\frac{1}{2}} + x^2 + \sqrt{d^2 e^{-1}}\right)}{8d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*x^2+d)/(e^2\*x^4+d^2), x, algorithm="giac")

[Out] 1/4\*sqrt(2)\*((d^2)^(1/4)\*d\*e^(11/2) - (d^2)^(3/4)\*e^(11/2))\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*(d^2)^(1/4)\*e^(-1/2) + 2\*x)\*e^(1/2)/(d^2)^(1/4))\*e^(-6)/d^2 + 1/4\*sqrt(2)\*((d^2)^(1/4)\*d\*e^(11/2) - (d^2)^(3/4)\*e^(11/2))\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*(d^2)^(1/4)\*e^(-1/2) - 2\*x)\*e^(1/2)/(d^2)^(1/4))\*e^(-6)/d^2 + 1/8\*sqrt(2)\*((d^2)^(1/4)\*d\*e^(11/2) + (d^2)^(3/4)\*e^(11/2))\*e^(-6)\*log(sqrt(2)\*(d^2)^(1/4)\*x\*e^(-1/2) + x^2 + sqrt(d^2)\*e^(-1))/d^2 - 1/8\*sqrt(2)\*((d^2)^(1/4)\*d\*e^(11/2) + (d^2)^(3/4)\*e^(11/2))\*e^(-6)\*log(-sqrt(2)\*(d^2)^(1/4)\*x\*e^(-1/2) + x^2 + sqrt(d^2)\*e^(-1))/d^2

**maple [B]** time = 0.00, size = 290, normalized size = 3.22

$$\frac{\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2} x}{\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}}-1\right)}{4d} + \frac{\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2} x}{\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}}+1\right)}{4d} + \frac{\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{x^2+\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}} \sqrt{2} x+\sqrt{\frac{d^2}{e^2}}}{x^2-\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}} \sqrt{2} x+\sqrt{\frac{d^2}{e^2}}}\right)}{8d} - \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2} x}{\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}}-1\right)}{4\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}} e} - \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2} x}{\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}}+1\right)}{4\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}} e} - \frac{\sqrt{2} \ln\left(\frac{x^2-\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}} \sqrt{2} x+\sqrt{\frac{d^2}{e^2}}}{x^2+\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}} \sqrt{2} x+\sqrt{\frac{d^2}{e^2}}}\right)}{8\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}} e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e\*x^2+d)/(e^2\*x^4+d^2), x)

[Out] 1/8\*(d^2/e^2)^(1/4)\*2^(1/2)/d\*ln((x^2+(d^2/e^2)^(1/4)\*2^(1/2)\*x+(d^2/e^2)^(1/2))/(x^2-(d^2/e^2)^(1/4)\*2^(1/2)\*x+(d^2/e^2)^(1/2)))+1/4\*(d^2/e^2)^(1/4)\*2^(1/2)/d\*arctan(2^(1/2)/(d^2/e^2)^(1/4)\*x+1)+1/4\*(d^2/e^2)^(1/4)\*2^(1/2)/d\*arctan(2^(1/2)/(d^2/e^2)^(1/4)\*x-1)-1/8/(d^2/e^2)^(1/4)\*2^(1/2)/e\*ln((x^2-(d^2/e^2)^(1/4)\*2^(1/2)\*x+(d^2/e^2)^(1/2))/(x^2+(d^2/e^2)^(1/4)\*2^(1/2)\*x+(d^2/e^2)^(1/2)))-1/4/(d^2/e^2)^(1/4)\*2^(1/2)/e\*arctan(2^(1/2)/(d^2/e^2)^(1/4)\*x+1)-1/4/(d^2/e^2)^(1/4)\*2^(1/2)/e\*arctan(2^(1/2)/(d^2/e^2)^(1/4)\*x-1)

**maxima** [A] time = 2.41, size = 62, normalized size = 0.69

$$\frac{\sqrt{2} \log\left(ex^2 + \sqrt{2} \sqrt{d} \sqrt{e} x + d\right)}{4 \sqrt{d} \sqrt{e}} - \frac{\sqrt{2} \log\left(ex^2 - \sqrt{2} \sqrt{d} \sqrt{e} x + d\right)}{4 \sqrt{d} \sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*x^2+d)/(e^2\*x^4+d^2),x, algorithm="maxima")

[Out] 1/4\*sqrt(2)\*log(e\*x^2 + sqrt(2)\*sqrt(d)\*sqrt(e)\*x + d)/(sqrt(d)\*sqrt(e)) - 1/4\*sqrt(2)\*log(e\*x^2 - sqrt(2)\*sqrt(d)\*sqrt(e)\*x + d)/(sqrt(d)\*sqrt(e))

**mupad** [B] time = 0.09, size = 41, normalized size = 0.46

$$\frac{\sqrt{2} \operatorname{atanh}\left(\frac{2 \sqrt{2} \sqrt{d} e^{7/2} x}{2 e^4 x^2 + 2 d e^3}\right)}{2 \sqrt{d} \sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d - e\*x^2)/(d^2 + e^2\*x^4),x)

[Out] (2^(1/2)\*atanh((2\*2^(1/2)\*d^(1/2)\*e^(7/2)\*x)/(2\*d\*e^3 + 2\*e^4\*x^2)))/(2\*d^(1/2)\*e^(1/2))

**sympy** [A] time = 0.23, size = 80, normalized size = 0.89

$$-\frac{\sqrt{2} \sqrt{\frac{1}{de}} \log\left(-\sqrt{2} dx \sqrt{\frac{1}{de}} + \frac{d}{e} + x^2\right)}{4} + \frac{\sqrt{2} \sqrt{\frac{1}{de}} \log\left(\sqrt{2} dx \sqrt{\frac{1}{de}} + \frac{d}{e} + x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*x\*\*2+d)/(e\*\*2\*x\*\*4+d\*\*2),x)

[Out] -sqrt(2)\*sqrt(1/(d\*e))\*log(-sqrt(2)\*d\*x\*sqrt(1/(d\*e)) + d/e + x\*\*2)/4 + sqrt(2)\*sqrt(1/(d\*e))\*log(sqrt(2)\*d\*x\*sqrt(1/(d\*e)) + d/e + x\*\*2)/4

$$3.13 \quad \int \frac{5+2x^2}{-1+x^4} dx$$

Optimal. Leaf size=13

$$-\frac{3}{2} \tan^{-1}(x) - \frac{7}{2} \tanh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1167, 207, 203}

$$-\frac{3}{2} \tan^{-1}(x) - \frac{7}{2} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(5 + 2\*x^2)/(-1 + x^4), x]

[Out] (-3\*ArcTan[x])/2 - (7\*ArcTanh[x])/2

Rule 203

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 207

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1167

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-(a\*c), 2]}, Dist[e/2 + (c\*d)/(2\*q), Int[1/(-q + c\*x^2), x], x] + Dist[e/2 - (c\*d)/(2\*q), Int[1/(q + c\*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 - a\*e^2, 0] && PosQ[-(a\*c)]

Rubi steps

$$\begin{aligned} \int \frac{5+2x^2}{-1+x^4} dx &= -\left(\frac{3}{2} \int \frac{1}{1+x^2} dx\right) + \frac{7}{2} \int \frac{1}{-1+x^2} dx \\ &= -\frac{3}{2} \tan^{-1}(x) - \frac{7}{2} \tanh^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.92

$$\frac{7}{4} \log(1-x) - \frac{7}{4} \log(x+1) - \frac{3}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(5 + 2\*x^2)/(-1 + x^4), x]

[Out] (-3\*ArcTan[x])/2 + (7\*Log[1 - x])/4 - (7\*Log[1 + x])/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5+2x^2}{-1+x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(5 + 2\*x^2)/(-1 + x^4), x]

[Out] IntegrateAlgebraic[(5 + 2\*x^2)/(-1 + x^4), x]

**fricas** [A] time = 0.79, size = 17, normalized size = 1.31

$$-\frac{3}{2} \arctan(x) - \frac{7}{4} \log(x + 1) + \frac{7}{4} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+5)/(x^4-1), x, algorithm="fricas")

[Out] -3/2\*arctan(x) - 7/4\*log(x + 1) + 7/4\*log(x - 1)

**giac** [B] time = 0.16, size = 19, normalized size = 1.46

$$-\frac{3}{2} \arctan(x) - \frac{7}{4} \log(|x + 1|) + \frac{7}{4} \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+5)/(x^4-1), x, algorithm="giac")

[Out] -3/2\*arctan(x) - 7/4\*log(abs(x + 1)) + 7/4\*log(abs(x - 1))

**maple** [A] time = 0.01, size = 18, normalized size = 1.38

$$-\frac{3 \arctan(x)}{2} - \frac{7 \ln(x + 1)}{4} + \frac{7 \ln(x - 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2+5)/(x^4-1), x)

[Out] 7/4\*ln(x-1)-7/4\*ln(x+1)-3/2\*arctan(x)

**maxima** [A] time = 2.35, size = 17, normalized size = 1.31

$$-\frac{3}{2} \arctan(x) - \frac{7}{4} \log(x + 1) + \frac{7}{4} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+5)/(x^4-1), x, algorithm="maxima")

[Out] -3/2\*arctan(x) - 7/4\*log(x + 1) + 7/4\*log(x - 1)

**mupad** [B] time = 0.04, size = 9, normalized size = 0.69

$$-\frac{3 \operatorname{atan}(x)}{2} - \frac{7 \operatorname{atanh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 + 5)/(x^4 - 1), x)

[Out] - (3\*atan(x))/2 - (7\*atanh(x))/2

**sympy** [A] time = 0.20, size = 22, normalized size = 1.69

$$\frac{7 \log(x - 1)}{4} - \frac{7 \log(x + 1)}{4} - \frac{3 \operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2+5)/(x\*\*4-1), x)

[Out] 7\*log(x - 1)/4 - 7\*log(x + 1)/4 - 3\*atan(x)/2

$$3.14 \quad \int \frac{d+ex^2}{d^2+bx^2+e^2x^4} dx$$

**Optimal.** Leaf size=82

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de-b}+2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}} - \frac{\tan^{-1}\left(\frac{\sqrt{2de-b}-2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}$$

**Rubi [A]** time = 0.10, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de-b}+2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}} - \frac{\tan^{-1}\left(\frac{\sqrt{2de-b}-2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)/(d^2 + b\*x^2 + e^2\*x^4), x]

[Out] -(ArcTan[(Sqrt[-b + 2\*d\*e] - 2\*e\*x)/Sqrt[b + 2\*d\*e]]/Sqrt[b + 2\*d\*e]) + ArcTan[(Sqrt[-b + 2\*d\*e] + 2\*e\*x)/Sqrt[b + 2\*d\*e]]/Sqrt[b + 2\*d\*e]

Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 1161

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

Rubi steps

$$\begin{aligned} \int \frac{d+ex^2}{d^2+bx^2+e^2x^4} dx &= \frac{\int \frac{1}{\frac{d}{e} - \frac{\sqrt{-b+2de}x}{e} + x^2} dx}{2e} + \frac{\int \frac{1}{\frac{d}{e} + \frac{\sqrt{-b+2de}x}{e} + x^2} dx}{2e} \\ &= \frac{\text{Subst}\left(\int \frac{1}{\frac{-b+2de}{e^2} - x^2} dx, x, -\frac{\sqrt{-b+2de}}{e} + 2x\right)}{e} - \frac{\text{Subst}\left(\int \frac{1}{\frac{-b+2de}{e^2} - x^2} dx, x, \frac{\sqrt{-b+2de}}{e} + 2x\right)}{e} \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{-b+2de}-2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}} + \frac{\tan^{-1}\left(\frac{\sqrt{-b+2de}+2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}} \end{aligned}$$

**Mathematica [B]** time = 0.11, size = 181, normalized size = 2.21

$$\frac{\left(\sqrt{b^2-4d^2e^2}-b+2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{b-\sqrt{b^2-4d^2e^2}}}\right)+\left(\sqrt{b^2-4d^2e^2}+b-2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{b^2-4d^2e^2}+b}}\right)}{\sqrt{b-\sqrt{b^2-4d^2e^2}}+\sqrt{\sqrt{b^2-4d^2e^2}+b}}\sqrt{2}\sqrt{b^2-4d^2e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)/(d^2 + b\*x^2 + e^2\*x^4), x]

[Out] (((-b + 2\*d\*e + Sqrt[b^2 - 4\*d^2\*e^2])\*ArcTan[(Sqrt[2]\*e\*x)/Sqrt[b - Sqrt[b^2 - 4\*d^2\*e^2]]])/Sqrt[b - Sqrt[b^2 - 4\*d^2\*e^2]] + ((b - 2\*d\*e + Sqrt[b^2 - 4\*d^2\*e^2])\*ArcTan[(Sqrt[2]\*e\*x)/Sqrt[b + Sqrt[b^2 - 4\*d^2\*e^2]]])/Sqrt[b + Sqrt[b^2 - 4\*d^2\*e^2]])/(Sqrt[2]\*Sqrt[b^2 - 4\*d^2\*e^2])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{d^2 + bx^2 + e^2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)/(d^2 + b\*x^2 + e^2\*x^4), x]

[Out] IntegrateAlgebraic[(d + e\*x^2)/(d^2 + b\*x^2 + e^2\*x^4), x]

**fricas [A]** time = 0.67, size = 162, normalized size = 1.98

$$\left[ \frac{\sqrt{-2de-b} \log\left(\frac{e^2x^4 - (4de+b)x^2 + d^2 - 2(ex^3 - dx)\sqrt{-2de-b}}{e^2x^4 + bx^2 + d^2}\right)}{2(2de+b)}, \frac{\sqrt{2de+b} \arctan\left(\frac{ex}{\sqrt{2de+b}}\right) + \sqrt{2de+b} \arctan\left(\frac{(e^2x^3 + (de+b)x)\sqrt{2de+b}}{2d^2e + bd}\right)}{2de+b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(e^2\*x^4+b\*x^2+d^2), x, algorithm="fricas")

[Out] [-1/2\*sqrt(-2\*d\*e - b)\*log((e^2\*x^4 - (4\*d\*e + b)\*x^2 + d^2 - 2\*(e\*x^3 - d\*x)\*sqrt(-2\*d\*e - b))/(e^2\*x^4 + b\*x^2 + d^2))/(2\*d\*e + b), (sqrt(2\*d\*e + b)\*arctan(e\*x/sqrt(2\*d\*e + b)) + sqrt(2\*d\*e + b)\*arctan((e^2\*x^3 + (d\*e + b)\*x)\*sqrt(2\*d\*e + b)/(2\*d^2\*e + b\*d)))/(2\*d\*e + b)]

**giac [B]** time = 1.12, size = 1642, normalized size = 20.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(e^2\*x^4+b\*x^2+d^2), x, algorithm="giac")

[Out] 1/4\*(16\*sqrt(2)\*sqrt(b\*e^2 + sqrt(-4\*d^2\*e^2 + b^2)\*e^2)\*d^4\*e^4 - 8\*sqrt(2)\*sqrt(b\*e^2 + sqrt(-4\*d^2\*e^2 + b^2)\*e^2)\*b^2\*d^2\*e^2 + 4\*sqrt(2)\*sqrt(-4\*d^2\*e^2 + b^2)\*sqrt(b\*e^2 + sqrt(-4\*d^2\*e^2 + b^2)\*e^2)\*b\*d^2\*e^2 + sqrt(2)\*sqrt(b\*e^2 + sqrt(-4\*d^2\*e^2 + b^2)\*e^2)\*b^4 - 32\*d^4\*e^6 + 8\*sqrt(2)\*sqrt(b\*e^2 + sqrt(-4\*d^2\*e^2 + b^2)\*e^2)\*b\*d^2\*e^4 + 16\*b^2\*d^2\*e^4 - 2\*sqrt(2)\*sqrt(b\*e^2 + sqrt(-4\*d^2\*e^2 + b^2)\*e^2)\*b^3\*e^2 - 2\*b^4\*e^2 - sqrt(2)\*sqrt(-4\*d^2\*e^2 + b^2)\*sqrt(b\*e^2 + sqrt(-4\*d^2\*e^2 + b^2)\*e^2)\*b^3 + 2\*sqrt(2)\*sqrt(-4\*d^2\*e^2 + b^2)\*sqrt(b\*e^2 + sqrt(-4\*d^2\*e^2 + b^2)\*e^2)\*b^2\*e^2 - 4\*sqrt(2)\*sqrt(b\*e^2 + sqrt(-4\*d^2\*e^2 + b^2)\*e^2)\*d^2\*e^6 - 8\*b\*d^2\*e^6 + sqrt(2)\*sqrt(b\*e^2 + sqrt(-4\*d^2\*e^2 + b^2)\*e^2)\*b^2\*e^4 + 2\*b^3\*e^4 + 8\*(4\*d^2\*e^2 - b^2)\*d^2\*e^4 - 2\*(4\*d^2\*e^2 - b^2)\*b^2\*e^2 - sqrt(2)\*sqrt(-4\*d^2\*e^2 + b^2)\*sqrt(b\*e^2 + sqrt(-4\*d^2\*e^2 + b^2)\*e^2)\*b\*e^4 + 2\*(4\*d^2\*e^2 - b^2)\*b\*e^4 - 2\*(4\*sqrt(2)\*sqrt(-4\*d^2\*e^2 + b^2)\*sqrt(b\*e^2 + sqrt(-4\*d^2

$$\begin{aligned} & *e^2 + b^2)*e^2)*d^3*e^2 - \sqrt{2}*\sqrt{-4*d^2*e^2 + b^2}*\sqrt{b*e^2 + \sqrt{-4*d^2*e^2 + b^2}}*e^2)*b^2*d + 2*\sqrt{2}*\sqrt{-4*d^2*e^2 + b^2}*\sqrt{b*e^2 + \sqrt{-4*d^2*e^2 + b^2}}*e^2)*b*d*e^2 - 8*d^3*e^6 + 2*b^2*d*e^4 - \sqrt{2}*\sqrt{-4*d^2*e^2 + b^2}*\sqrt{b*e^2 + \sqrt{-4*d^2*e^2 + b^2}}*e^2)*d*e^4 + 2*(4*d^2*e^2 - b^2)*d*e^4)*e)*\arctan(2*\sqrt{1/2}*x*e/\sqrt{b + \sqrt{-4*d^2*e^2 + b^2}})))/(16*d^5*e^6 - 8*b^2*d^3*e^4 + b^4*d*e^2 + 8*b*d^3*e^6 - 2*b^3*d*e^4 - 4*d^3*e^8 + b^2*d*e^6) + 1/4*(16*\sqrt{2}*\sqrt{b*e^2 - \sqrt{-4*d^2*e^2 + b^2}}*e^2)*d^4*e^4 - 8*\sqrt{2}*\sqrt{b*e^2 - \sqrt{-4*d^2*e^2 + b^2}}*e^2)*b^2*d^2*e^2 - 4*\sqrt{2}*\sqrt{-4*d^2*e^2 + b^2}*\sqrt{b*e^2 - \sqrt{-4*d^2*e^2 + b^2}}*e^2)*b*d^2*e^2 + \sqrt{2}*\sqrt{b*e^2 - \sqrt{-4*d^2*e^2 + b^2}}*e^2)*b^4 + 32*d^4*e^6 + 8*\sqrt{2}*\sqrt{b*e^2 - \sqrt{-4*d^2*e^2 + b^2}}*e^2)*b*d^2*e^4 - 16*b^2*d^2*e^4 - 2*\sqrt{2}*\sqrt{b*e^2 - \sqrt{-4*d^2*e^2 + b^2}}*e^2)*b^3*e^2 + 2*b^4*e^2 + \sqrt{2}*\sqrt{-4*d^2*e^2 + b^2}*\sqrt{b*e^2 - \sqrt{-4*d^2*e^2 + b^2}}*e^2)*b^3 - 2*\sqrt{2}*\sqrt{-4*d^2*e^2 + b^2}*\sqrt{b*e^2 - \sqrt{-4*d^2*e^2 + b^2}}*e^2)*b^2*e^2 - 4*\sqrt{2}*\sqrt{b*e^2 - \sqrt{-4*d^2*e^2 + b^2}}*e^2)*d^2*e^6 + 8*b*d^2*e^6 + \sqrt{2}*\sqrt{b*e^2 - \sqrt{-4*d^2*e^2 + b^2}}*e^2)*b^2*e^4 - 2*b^3*e^4 - 8*(4*d^2*e^2 - b^2)*d^2*e^4 + 2*(4*d^2*e^2 - b^2)*b^2*e^2 + \sqrt{2}*\sqrt{-4*d^2*e^2 + b^2}*\sqrt{b*e^2 - \sqrt{-4*d^2*e^2 + b^2}}*e^2)*b*e^4 - 2*(4*d^2*e^2 - b^2)*b*e^4 + 2*(4*\sqrt{2}*\sqrt{-4*d^2*e^2 + b^2}*\sqrt{b*e^2 - \sqrt{-4*d^2*e^2 + b^2}}*e^2)*d^3*e^2 - \sqrt{2}*\sqrt{-4*d^2*e^2 + b^2}*\sqrt{b*e^2 - \sqrt{-4*d^2*e^2 + b^2}}*e^2)*b^2*d + 2*\sqrt{2}*\sqrt{-4*d^2*e^2 + b^2}*\sqrt{b*e^2 - \sqrt{-4*d^2*e^2 + b^2}}*e^2)*b*d*e^2 - 8*d^3*e^6 + 2*b^2*d*e^4 - \sqrt{2}*\sqrt{-4*d^2*e^2 + b^2}*\sqrt{b*e^2 - \sqrt{-4*d^2*e^2 + b^2}}*e^2)*d*e^4 + 2*(4*d^2*e^2 - b^2)*d*e^4)*e)*\arctan(2*\sqrt{1/2}*x*e/\sqrt{b - \sqrt{-4*d^2*e^2 + b^2}})))/(16*d^5*e^6 - 8*b^2*d^3*e^4 + b^4*d*e^2 + 8*b*d^3*e^6 - 2*b^3*d*e^4 - 4*d^3*e^8 + b^2*d*e^6) \end{aligned}$$

**maple [A]** time = 0.04, size = 71, normalized size = 0.87

$$-\frac{\arctan\left(\frac{-2ex+\sqrt{2de-b}}{\sqrt{2de+b}}\right)}{\sqrt{2de+b}} + \frac{\arctan\left(\frac{2ex+\sqrt{2de-b}}{\sqrt{2de+b}}\right)}{\sqrt{2de+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)/(e^2\*x^4+b\*x^2+d^2),x)

[Out] -arctan((-2\*e\*x+(2\*d\*e-b)^(1/2))/(2\*d\*e+b)^(1/2))/(2\*d\*e+b)^(1/2)+arctan((2\*e\*x+(2\*d\*e-b)^(1/2))/(2\*d\*e+b)^(1/2))/(2\*d\*e+b)^(1/2)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2 + d}{e^2x^4 + bx^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(e^2\*x^4+b\*x^2+d^2),x, algorithm="maxima")

[Out] integrate((e\*x^2 + d)/(e^2\*x^4 + b\*x^2 + d^2), x)

**mupad [B]** time = 4.43, size = 94, normalized size = 1.15

$$\frac{\operatorname{atan}\left(\frac{ex}{\sqrt{b+2de}}\right) + \operatorname{atan}\left(\frac{b^2x - \frac{x(b+2de)^2}{2} + \frac{bx(b+2de)}{2} + 2be^2x^3 - e^2x^3(b+2de)}{(bd-2d^2e)\sqrt{b+2de}}\right)}{\sqrt{b+2de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)/(b\*x^2 + d^2 + e^2\*x^4),x)



```
[Out] (atan((e*x)/(b + 2*d*e)^(1/2)) + atan((b^2*x - (x*(b + 2*d*e)^2)/2 + (b*x*(b + 2*d*e))/2 + 2*b*e^2*x^3 - e^2*x^3*(b + 2*d*e))/((b*d - 2*d^2*e)*(b + 2*d*e)^(1/2))))/(b + 2*d*e)^(1/2)
```

**sympy [A]** time = 0.54, size = 122, normalized size = 1.49

$$\frac{\sqrt{-\frac{1}{b+2de}} \log\left(-\frac{d}{e} + x^2 + \frac{x\left(-b\sqrt{-\frac{1}{b+2de}} - 2de\sqrt{-\frac{1}{b+2de}}\right)}{e}\right)}{2} + \frac{\sqrt{-\frac{1}{b+2de}} \log\left(-\frac{d}{e} + x^2 + \frac{x\left(b\sqrt{-\frac{1}{b+2de}} + 2de\sqrt{-\frac{1}{b+2de}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(e**2*x**4+b*x**2+d**2),x)
```

```
[Out] -sqrt(-1/(b + 2*d*e))*log(-d/e + x**2 + x*(-b*sqrt(-1/(b + 2*d*e)) - 2*d*e*sqrt(-1/(b + 2*d*e)))/e)/2 + sqrt(-1/(b + 2*d*e))*log(-d/e + x**2 + x*(b*sqrt(-1/(b + 2*d*e)) + 2*d*e*sqrt(-1/(b + 2*d*e)))/e)/2
```

$$3.15 \quad \int \frac{d+ex^2}{d^2+fx^2+e^2x^4} dx$$

Optimal. Leaf size=82

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de-f}+2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}} - \frac{\tan^{-1}\left(\frac{\sqrt{2de-f}-2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}$$

**Rubi [A]** time = 0.11, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de-f}+2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}} - \frac{\tan^{-1}\left(\frac{\sqrt{2de-f}-2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)/(d^2 + f\*x^2 + e^2\*x^4), x]

[Out] -(ArcTan[(Sqrt[2\*d\*e - f] - 2\*e\*x)/Sqrt[2\*d\*e + f]]/Sqrt[2\*d\*e + f]) + ArcTan[(Sqrt[2\*d\*e - f] + 2\*e\*x)/Sqrt[2\*d\*e + f]]/Sqrt[2\*d\*e + f]

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 1161

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

Rubi steps

$$\begin{aligned} \int \frac{d+ex^2}{d^2+fx^2+e^2x^4} dx &= \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2de-f}x}{e}+x^2} dx}{2e} + \frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2de-f}x}{e}+x^2} dx}{2e} \\ &= \frac{\text{Subst}\left(\int \frac{1}{-\frac{2de+f}{e^2}-x^2} dx, x, -\frac{\sqrt{2de-f}}{e}+2x\right)}{e} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{2de+f}{e^2}-x^2} dx, x, \frac{\sqrt{2de-f}}{e}+2x\right)}{e} \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{2de-f}-2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}} + \frac{\tan^{-1}\left(\frac{\sqrt{2de-f}+2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}} \end{aligned}$$

**Mathematica [B]** time = 0.11, size = 181, normalized size = 2.21

$$\frac{(\sqrt{f^2-4d^2e^2}+2de-f)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{f-\sqrt{f^2-4d^2e^2}}}\right)}{\sqrt{f-\sqrt{f^2-4d^2e^2}}} + \frac{(\sqrt{f^2-4d^2e^2}-2de+f)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{f^2-4d^2e^2}+f}}\right)}{\sqrt{\sqrt{f^2-4d^2e^2}+f}}$$

$$\sqrt{2}\sqrt{f^2-4d^2e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)/(d^2 + f\*x^2 + e^2\*x^4), x]

[Out] (((2\*d\*e - f + Sqrt[-4\*d^2\*e^2 + f^2])\*ArcTan[(Sqrt[2]\*e\*x)/Sqrt[f - Sqrt[-4\*d^2\*e^2 + f^2]]])/Sqrt[f - Sqrt[-4\*d^2\*e^2 + f^2]] + ((-2\*d\*e + f + Sqrt[-4\*d^2\*e^2 + f^2])\*ArcTan[(Sqrt[2]\*e\*x)/Sqrt[f + Sqrt[-4\*d^2\*e^2 + f^2]]])/Sqrt[f + Sqrt[-4\*d^2\*e^2 + f^2]])/(Sqrt[2]\*Sqrt[-4\*d^2\*e^2 + f^2])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{d^2 + fx^2 + e^2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)/(d^2 + f\*x^2 + e^2\*x^4), x]

[Out] IntegrateAlgebraic[(d + e\*x^2)/(d^2 + f\*x^2 + e^2\*x^4), x]

**fricas [A]** time = 0.74, size = 162, normalized size = 1.98

$$\left[ \frac{\sqrt{-2de-f} \log\left(\frac{e^2x^4 - (4de+f)x^2 + d^2 - 2(ex^3 - dx)\sqrt{-2de-f}}{e^2x^4 + fx^2 + d^2}\right)}{2(2de+f)}, \frac{\sqrt{2de+f} \arctan\left(\frac{ex}{\sqrt{2de+f}}\right) + \sqrt{2de+f} \arctan\left(\frac{(e^2x^3 + (de+f)x)\sqrt{2de+f}}{2d^2e + df}\right)}{2de+f} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(e^2\*x^4+f\*x^2+d^2), x, algorithm="fricas")

[Out] [-1/2\*sqrt(-2\*d\*e - f)\*log((e^2\*x^4 - (4\*d\*e + f)\*x^2 + d^2 - 2\*(e\*x^3 - d\*x)\*sqrt(-2\*d\*e - f))/(e^2\*x^4 + f\*x^2 + d^2))/(2\*d\*e + f), (sqrt(2\*d\*e + f)\*arctan(e\*x/sqrt(2\*d\*e + f)) + sqrt(2\*d\*e + f)\*arctan((e^2\*x^3 + (d\*e + f)\*x)\*sqrt(2\*d\*e + f)/(2\*d^2\*e + d\*f)))/(2\*d\*e + f)]

**giac [B]** time = 1.09, size = 1642, normalized size = 20.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(e^2\*x^4+f\*x^2+d^2), x, algorithm="giac")

[Out] 1/4\*(16\*sqrt(2)\*sqrt(f\*e^2 + sqrt(-4\*d^2\*e^2 + f^2)\*e^2)\*d^4\*e^4 - 8\*sqrt(2)\*sqrt(f\*e^2 + sqrt(-4\*d^2\*e^2 + f^2)\*e^2)\*d^2\*f^2\*e^2 + 4\*sqrt(2)\*sqrt(-4\*d^2\*e^2 + f^2)\*sqrt(f\*e^2 + sqrt(-4\*d^2\*e^2 + f^2)\*e^2)\*d^2\*f\*e^2 + sqrt(2)\*sqrt(f\*e^2 + sqrt(-4\*d^2\*e^2 + f^2)\*e^2)\*f^4 - 32\*d^4\*e^6 + 8\*sqrt(2)\*sqrt(f\*e^2 + sqrt(-4\*d^2\*e^2 + f^2)\*e^2)\*d^2\*f\*e^4 + 16\*d^2\*f^2\*e^4 - 2\*sqrt(2)\*sqrt(f\*e^2 + sqrt(-4\*d^2\*e^2 + f^2)\*e^2)\*f^3\*e^2 - 2\*f^4\*e^2 - sqrt(2)\*sqrt(-4\*d^2\*e^2 + f^2)\*sqrt(f\*e^2 + sqrt(-4\*d^2\*e^2 + f^2)\*e^2)\*f^3 + 2\*sqrt(2)\*sqrt(-4\*d^2\*e^2 + f^2)\*sqrt(f\*e^2 + sqrt(-4\*d^2\*e^2 + f^2)\*e^2)\*f^2\*e^2 - 4\*sqrt(2)\*sqrt(f\*e^2 + sqrt(-4\*d^2\*e^2 + f^2)\*e^2)\*d^2\*e^6 - 8\*d^2\*f\*e^6 + 8\*(4\*d^2\*e^2 - f^2)\*d^2\*e^4 + sqrt(2)\*sqrt(f\*e^2 + sqrt(-4\*d^2\*e^2 + f^2)\*e^2)\*f^2\*e^4 + 2\*f^3\*e^4 - 2\*(4\*d^2\*e^2 - f^2)\*f^2\*e^2 - sqrt(2)\*sqrt(-4\*d^2\*e^2 + f^2)\*sqrt(f\*e^2 + sqrt(-4\*d^2\*e^2 + f^2)\*e^2)\*f\*e^4 + 2\*(4\*d^2\*e^2

$$\begin{aligned}
& -f^2) * f * e^4 - 2 * (4 * \sqrt{2}) * \sqrt{-4 * d^2 * e^2 + f^2} * \sqrt{f * e^2 + \sqrt{-4 * d^2 * e^2 + f^2}} * e^2 \\
& - \sqrt{2} * \sqrt{-4 * d^2 * e^2 + f^2} * \sqrt{f * e^2 + \sqrt{-4 * d^2 * e^2 + f^2}} * e^2) * d^3 * e^2 - \sqrt{2} * \sqrt{-4 * d^2 * e^2 + f^2} * \sqrt{f * e^2 + \sqrt{-4 * d^2 * e^2 + f^2}} * e^2) * d * f^2 \\
& + 2 * \sqrt{2} * \sqrt{-4 * d^2 * e^2 + f^2} * \sqrt{f * e^2 + \sqrt{-4 * d^2 * e^2 + f^2}} * e^2) * d * f * e^2 - 8 * d^3 * e^6 + 2 * d * f^2 * e^4 - \sqrt{2} * \sqrt{-4 * d^2 * e^2 + f^2} * \sqrt{f * e^2 + \sqrt{-4 * d^2 * e^2 + f^2}} * e^2) * d * e^4 \\
& + 2 * (4 * d^2 * e^2 - f^2) * d * e^4) * e) * \arctan(2 * \sqrt{1/2}) * x * e / \sqrt{f + \sqrt{-4 * d^2 * e^2 + f^2}}) / (16 * d^5 * e^6 - 8 * d^3 * f^2 * e^4 + d * f^4 * e^2 + 8 * d^3 * f * e^6 - 2 * d * f^3 * e^4 - 4 * d^3 * e^8 + d * f^2 * e^6) + 1/4 * (16 * \sqrt{2}) * \sqrt{f * e^2 - \sqrt{-4 * d^2 * e^2 + f^2}} * e^2) * d^4 * e^4 - 8 * \sqrt{2} * \sqrt{f * e^2 - \sqrt{-4 * d^2 * e^2 + f^2}} * e^2) * d^2 * f^2 * e^2 - 4 * \sqrt{2} * \sqrt{-4 * d^2 * e^2 + f^2} * \sqrt{f * e^2 - \sqrt{-4 * d^2 * e^2 + f^2}} * e^2) * d^2 * f * e^2 + \sqrt{2} * \sqrt{f * e^2 - \sqrt{-4 * d^2 * e^2 + f^2}} * e^2) * f^4 + 32 * d^4 * e^6 + 8 * \sqrt{2} * \sqrt{f * e^2 - \sqrt{-4 * d^2 * e^2 + f^2}} * e^2) * d^2 * f * e^4 - 16 * d^2 * f^2 * e^4 - 2 * \sqrt{2} * \sqrt{f * e^2 - \sqrt{-4 * d^2 * e^2 + f^2}} * e^2) * f^3 * e^2 + 2 * f^4 * e^2 + \sqrt{2} * \sqrt{-4 * d^2 * e^2 + f^2} * \sqrt{f * e^2 - \sqrt{-4 * d^2 * e^2 + f^2}} * e^2) * f^3 - 2 * \sqrt{2} * \sqrt{-4 * d^2 * e^2 + f^2} * \sqrt{f * e^2 - \sqrt{-4 * d^2 * e^2 + f^2}} * e^2) * f^2 * e^2 - 4 * \sqrt{2} * \sqrt{f * e^2 - \sqrt{-4 * d^2 * e^2 + f^2}} * e^2) * d^2 * e^6 + 8 * d^2 * f * e^6 - 8 * (4 * d^2 * e^2 - f^2) * d^2 * e^4 + \sqrt{2} * \sqrt{f * e^2 - \sqrt{-4 * d^2 * e^2 + f^2}} * e^2) * f^2 * e^4 - 2 * f^3 * e^4 + 2 * (4 * d^2 * e^2 - f^2) * f^2 * e^2 + \sqrt{2} * \sqrt{-4 * d^2 * e^2 + f^2} * \sqrt{f * e^2 - \sqrt{-4 * d^2 * e^2 + f^2}} * e^2) * f^2 * e^4 - 2 * (4 * d^2 * e^2 - f^2) * f * e^4 + 2 * (4 * \sqrt{2}) * \sqrt{-4 * d^2 * e^2 + f^2} * \sqrt{f * e^2 - \sqrt{-4 * d^2 * e^2 + f^2}} * e^2) * d^3 * e^2 - \sqrt{2} * \sqrt{-4 * d^2 * e^2 + f^2} * \sqrt{f * e^2 - \sqrt{-4 * d^2 * e^2 + f^2}} * e^2) * d * f^2 + 2 * \sqrt{2} * \sqrt{-4 * d^2 * e^2 + f^2} * \sqrt{f * e^2 - \sqrt{-4 * d^2 * e^2 + f^2}} * e^2) * d * f * e^2 - 8 * d^3 * e^6 + 2 * d * f^2 * e^4 - \sqrt{2} * \sqrt{-4 * d^2 * e^2 + f^2} * \sqrt{f * e^2 - \sqrt{-4 * d^2 * e^2 + f^2}} * e^2) * d * e^4 + 2 * (4 * d^2 * e^2 - f^2) * d * e^4) * e) * \arctan(2 * \sqrt{1/2}) * x * e / \sqrt{f - \sqrt{-4 * d^2 * e^2 + f^2}}) / (16 * d^5 * e^6 - 8 * d^3 * f^2 * e^4 + d * f^4 * e^2 + 8 * d^3 * f * e^6 - 2 * d * f^3 * e^4 - 4 * d^3 * e^8 + d * f^2 * e^6)
\end{aligned}$$

**maple [A]** time = 0.04, size = 71, normalized size = 0.87

$$-\frac{\arctan\left(\frac{-2ex + \sqrt{2de-f}}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}} + \frac{\arctan\left(\frac{2ex + \sqrt{2de-f}}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)/(e^2\*x^4+f\*x^2+d^2),x)

[Out] -arctan((-2\*e\*x+(2\*d\*e-f)^(1/2))/(2\*d\*e+f)^(1/2))/(2\*d\*e+f)^(1/2)+arctan((2\*e\*x+(2\*d\*e-f)^(1/2))/(2\*d\*e+f)^(1/2))/(2\*d\*e+f)^(1/2)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2 + d}{e^2x^4 + fx^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(e^2\*x^4+f\*x^2+d^2),x, algorithm="maxima")

[Out] integrate((e\*x^2 + d)/(e^2\*x^4 + f\*x^2 + d^2), x)

**mupad [B]** time = 4.52, size = 98, normalized size = 1.20

$$\frac{\operatorname{atan}\left(\frac{f^2x - \frac{x(f+2de)^2}{2} + \frac{fx(f+2de)}{2} + 2e^2fx^3 - e^2x^3(f+2de)}{(2df - d(f+2de))\sqrt{f+2de}}\right) + \operatorname{atan}\left(\frac{ex}{\sqrt{f+2de}}\right)}{\sqrt{f+2de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x^2)/(f*x^2 + d^2 + e^2*x^4),x)`

[Out] `(atan((f^2*x - (x*(f + 2*d*e)^2)/2 + (f*x*(f + 2*d*e))/2 + 2*e^2*f*x^3 - e^2*x^3*(f + 2*d*e))/((2*d*f - d*(f + 2*d*e))*(f + 2*d*e)^(1/2))) + atan((e*x)/(f + 2*d*e)^(1/2)))/(f + 2*d*e)^(1/2)`

**sympy [A]** time = 0.56, size = 122, normalized size = 1.49

$$\frac{\sqrt{-\frac{1}{2de+f}} \log\left(-\frac{d}{e} + x^2 + \frac{x\left(-2de\sqrt{-\frac{1}{2de+f}} - f\sqrt{-\frac{1}{2de+f}}\right)}{e}\right)}{2} + \frac{\sqrt{-\frac{1}{2de+f}} \log\left(-\frac{d}{e} + x^2 + \frac{x\left(2de\sqrt{-\frac{1}{2de+f}} + f\sqrt{-\frac{1}{2de+f}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)/(e**2*x**4+f*x**2+d**2),x)`

[Out] `-sqrt(-1/(2*d*e + f))*log(-d/e + x**2 + x*(-2*d*e*sqrt(-1/(2*d*e + f)) - f*sqrt(-1/(2*d*e + f)))/e)/2 + sqrt(-1/(2*d*e + f))*log(-d/e + x**2 + x*(2*d*e*sqrt(-1/(2*d*e + f)) + f*sqrt(-1/(2*d*e + f)))/e)/2`

$$3.16 \quad \int \frac{d+ex^2}{d^2-bx^2+e^2x^4} dx$$

Optimal. Leaf size=78

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}-2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}+2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}$$

**Rubi [A]** time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}-2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}+2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)/(d^2 - b\*x^2 + e^2\*x^4), x]

[Out] ArcTanh[(Sqrt[b + 2\*d\*e] - 2\*e\*x)/Sqrt[b - 2\*d\*e]]/Sqrt[b - 2\*d\*e] - ArcTanh[(Sqrt[b + 2\*d\*e] + 2\*e\*x)/Sqrt[b - 2\*d\*e]]/Sqrt[b - 2\*d\*e]

Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 1161

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

Rubi steps

$$\begin{aligned} \int \frac{d+ex^2}{d^2-bx^2+e^2x^4} dx &= \frac{\int \frac{1}{\frac{d}{e} - \frac{\sqrt{b+2de}x}{e} + x^2} dx}{2e} + \frac{\int \frac{1}{\frac{d}{e} + \frac{\sqrt{b+2de}x}{e} + x^2} dx}{2e} \\ &= \frac{\text{Subst}\left(\int \frac{1}{\frac{b-2de}{e^2} - x^2} dx, x, -\frac{\sqrt{b+2de}}{e} + 2x\right)}{e} - \frac{\text{Subst}\left(\int \frac{1}{\frac{b-2de}{e^2} - x^2} dx, x, \frac{\sqrt{b+2de}}{e} + 2x\right)}{e} \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}-2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}+2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}} \end{aligned}$$

**Mathematica [B]** time = 0.11, size = 189, normalized size = 2.42

$$\frac{\left(\sqrt{b^2-4d^2e^2}+b+2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{-\sqrt{b^2-4d^2e^2}-b}}\right)+\left(\sqrt{b^2-4d^2e^2}-b-2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{b^2-4d^2e^2}-b}}\right)}{\sqrt{2}\sqrt{b^2-4d^2e^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)/(d^2 - b\*x^2 + e^2\*x^4), x]

[Out] (((b + 2\*d\*e + Sqrt[b^2 - 4\*d^2\*e^2])\*ArcTan[(Sqrt[2]\*e\*x)/Sqrt[-b - Sqrt[b^2 - 4\*d^2\*e^2]]])/Sqrt[-b - Sqrt[b^2 - 4\*d^2\*e^2]] + ((-b - 2\*d\*e + Sqrt[b^2 - 4\*d^2\*e^2])\*ArcTan[(Sqrt[2]\*e\*x)/Sqrt[-b + Sqrt[b^2 - 4\*d^2\*e^2]]])/Sqrt[-b + Sqrt[b^2 - 4\*d^2\*e^2]])/(Sqrt[2]\*Sqrt[b^2 - 4\*d^2\*e^2])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{d^2 - bx^2 + e^2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)/(d^2 - b\*x^2 + e^2\*x^4), x]

[Out] IntegrateAlgebraic[(d + e\*x^2)/(d^2 - b\*x^2 + e^2\*x^4), x]

**fricas [A]** time = 0.84, size = 176, normalized size = 2.26

$$\left[ \frac{\sqrt{-2de+b} \log\left(\frac{e^2x^4-(4de-b)x^2+d^2-2(ex^3-dx)\sqrt{-2de+b}}{e^2x^4-bx^2+d^2}\right)}{2(2de-b)}, \frac{\sqrt{2de-b} \arctan\left(\frac{ex}{\sqrt{2de-b}}\right) + \sqrt{2de-b} \arctan\left(\frac{(e^2x^3+(de-b)x)\sqrt{2de-b}}{2d^2e-bd}\right)}{2de-b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(e^2\*x^4-b\*x^2+d^2), x, algorithm="fricas")

[Out] [-1/2\*sqrt(-2\*d\*e + b)\*log((e^2\*x^4 - (4\*d\*e - b)\*x^2 + d^2 - 2\*(e\*x^3 - d\*x)\*sqrt(-2\*d\*e + b))/(e^2\*x^4 - b\*x^2 + d^2))/(2\*d\*e - b), (sqrt(2\*d\*e - b)\*arctan(e\*x/sqrt(2\*d\*e - b)) + sqrt(2\*d\*e - b)\*arctan((e^2\*x^3 + (d\*e - b)\*x)\*sqrt(2\*d\*e - b)/(2\*d^2\*e - b\*d)))/(2\*d\*e - b)]

**giac [B]** time = 1.12, size = 1676, normalized size = 21.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(e^2\*x^4-b\*x^2+d^2), x, algorithm="giac")

[Out] 1/4\*(16\*sqrt(2)\*sqrt(-b\*e^2 - sqrt(-4\*d^2\*e^2 + b^2)\*e^2)\*d^4\*e^4 - 8\*sqrt(2)\*sqrt(-b\*e^2 - sqrt(-4\*d^2\*e^2 + b^2)\*e^2)\*b^2\*d^2\*e^2 + 4\*sqrt(2)\*sqrt(-4\*d^2\*e^2 + b^2)\*sqrt(-b\*e^2 - sqrt(-4\*d^2\*e^2 + b^2)\*e^2)\*b\*d^2\*e^2 + sqrt(2)\*sqrt(-b\*e^2 - sqrt(-4\*d^2\*e^2 + b^2)\*e^2)\*b^4 + 32\*d^4\*e^6 - 8\*sqrt(2)\*sqrt(-b\*e^2 - sqrt(-4\*d^2\*e^2 + b^2)\*e^2)\*b\*d^2\*e^4 - 16\*b^2\*d^2\*e^4 + 2\*sqrt(2)\*sqrt(-b\*e^2 - sqrt(-4\*d^2\*e^2 + b^2)\*e^2)\*b^3\*e^2 + 2\*b^4\*e^2 - sqrt(2)\*sqrt(-4\*d^2\*e^2 + b^2)\*sqrt(-b\*e^2 - sqrt(-4\*d^2\*e^2 + b^2)\*e^2)\*b^3 - 2\*sqrt(2)\*sqrt(-4\*d^2\*e^2 + b^2)\*sqrt(-b\*e^2 - sqrt(-4\*d^2\*e^2 + b^2)\*e^2)\*b^2\*e^2 - 4\*sqrt(2)\*sqrt(-b\*e^2 - sqrt(-4\*d^2\*e^2 + b^2)\*e^2)\*d^2\*e^6 - 8\*b\*d^2\*e^6 + sqrt(2)\*sqrt(-b\*e^2 - sqrt(-4\*d^2\*e^2 + b^2)\*e^2)\*b^2\*e^4 + 2\*b^3\*e^4 - 8\*(4\*d^2\*e^2 - b^2)\*d^2\*e^4 + 2\*(4\*d^2\*e^2 - b^2)\*b^2\*e^2 - sqrt(2)\*sqrt(-4\*d^2\*e^2 + b^2)\*sqrt(-b\*e^2 - sqrt(-4\*d^2\*e^2 + b^2)\*e^2)\*b\*e^4 + 2\*(4\*d^2\*e^2 - b^2)\*b\*e^4 + 2\*(4\*sqrt(2)\*sqrt(-4\*d^2\*e^2 + b^2)\*sqrt(-b\*e^2 -

$$\begin{aligned} & \sqrt{-4d^2e^2 + b^2}e^2 * d^3e^2 - \sqrt{2} * \sqrt{-4d^2e^2 + b^2} * \sqrt{-be^2 - \sqrt{-4d^2e^2 + b^2}e^2} * b^2d - 2 * \sqrt{2} * \sqrt{-4d^2e^2 + b^2} * \sqrt{2} * \sqrt{-be^2 - \sqrt{-4d^2e^2 + b^2}e^2} * b^2d - 2 * \sqrt{2} * \sqrt{-4d^2e^2 + b^2} * \sqrt{2} * \sqrt{-be^2 - \sqrt{-4d^2e^2 + b^2}e^2} * b^2d \\ & - 8d^3e^6 + 2b^2d * e^4 - \sqrt{2} * \sqrt{-4d^2e^2 + b^2} * \sqrt{-be^2 - \sqrt{-4d^2e^2 + b^2}e^2} * d * e^4 + 2 * (4d^2e^2 - b^2) * d * e^4 * e * \arctan(2 * \sqrt{1/2} * x / \sqrt{-(b + \sqrt{-4d^2e^2 + b^2})} * e^{-2})) / (16d^5e^6 - 8b^2d^3e^4 + b^4d * e^2 - 8 * b * d^3e^6 + 2b^3d * e^4 - 4d^3e^8 + b^2d * e^6) + 1/4 * (16 * \sqrt{2} * \sqrt{-be^2 + \sqrt{-4d^2e^2 + b^2}e^2} * d^4e^4 - 8 * \sqrt{2} * \sqrt{-be^2 + \sqrt{-4d^2e^2 + b^2}e^2} * b^2d^2e^2 - 4 * \sqrt{2} * \sqrt{-4d^2e^2 + b^2} * \sqrt{-be^2 + \sqrt{-4d^2e^2 + b^2}e^2} * b * d^2e^2 + \sqrt{2} * \sqrt{-be^2 + \sqrt{-4d^2e^2 + b^2}e^2} * b^4 - 32d^4e^6 - 8 * \sqrt{2} * \sqrt{-be^2 + \sqrt{-4d^2e^2 + b^2}e^2} * b * d^2e^4 + 16b^2d^2e^4 + 2 * \sqrt{2} * \sqrt{-be^2 + \sqrt{-4d^2e^2 + b^2}e^2} * b^3e^2 - 2b^4e^2 + \sqrt{2} * \sqrt{-4d^2e^2 + b^2} * \sqrt{-be^2 + \sqrt{-4d^2e^2 + b^2}e^2} * b^3 + 2 * \sqrt{2} * \sqrt{-4d^2e^2 + b^2} * \sqrt{-be^2 + \sqrt{-4d^2e^2 + b^2}e^2} * b^2e^2 - 4 * \sqrt{2} * \sqrt{-be^2 + \sqrt{-4d^2e^2 + b^2}e^2} * d^2e^6 + 8 * b * d^2e^6 + \sqrt{2} * \sqrt{-be^2 + \sqrt{-4d^2e^2 + b^2}e^2} * b^2e^4 - 2b^3e^4 + 8 * (4d^2e^2 - b^2) * d^2e^4 - 2 * (4d^2e^2 - b^2) * b^2e^2 + \sqrt{2} * \sqrt{-4d^2e^2 + b^2} * \sqrt{-be^2 + \sqrt{-4d^2e^2 + b^2}e^2} * b * e^4 - 2 * (4d^2e^2 - b^2) * b * e^4 - 2 * (4 * \sqrt{2} * \sqrt{-4d^2e^2 + b^2} * \sqrt{-be^2 + \sqrt{-4d^2e^2 + b^2}e^2} * d^3e^2 - \sqrt{2} * \sqrt{-4d^2e^2 + b^2} * \sqrt{-be^2 + \sqrt{-4d^2e^2 + b^2}e^2} * b^2d - 2 * \sqrt{2} * \sqrt{-4d^2e^2 + b^2} * \sqrt{-be^2 + \sqrt{-4d^2e^2 + b^2}e^2} * b * d * e^2 - 8d^3e^6 + 2b^2d * e^4 - \sqrt{2} * \sqrt{-4d^2e^2 + b^2} * \sqrt{-be^2 + \sqrt{-4d^2e^2 + b^2}e^2} * d * e^4 + 2 * (4d^2e^2 - b^2) * d * e^4) * e * \arctan(2 * \sqrt{1/2} * x / \sqrt{-(b - \sqrt{-4d^2e^2 + b^2})} * e^{-2})) / (16d^5e^6 - 8b^2d^3e^4 + b^4d * e^2 - 8 * b * d^3e^6 + 2b^3d * e^4 - 4d^3e^8 + b^2d * e^6) \end{aligned}$$

**maple [A]** time = 0.03, size = 75, normalized size = 0.96

$$-\frac{\arctan\left(\frac{-2ex + \sqrt{2de+b}}{\sqrt{2de-b}}\right)}{\sqrt{2de-b}} + \frac{\arctan\left(\frac{2ex + \sqrt{2de+b}}{\sqrt{2de-b}}\right)}{\sqrt{2de-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)/(e^2\*x^4-b\*x^2+d^2),x)

[Out]  $-1/(2de-b)^{1/2} * \arctan((-2ex + (2de+b)^{1/2}) / (2de-b)^{1/2}) + 1/(2de-b)^{1/2} * \arctan((2ex + (2de+b)^{1/2}) / (2de-b)^{1/2})$

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2 + d}{e^2x^4 - bx^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(e^2\*x^4-b\*x^2+d^2),x, algorithm="maxima")

[Out] integrate((e\*x^2 + d)/(e^2\*x^4 - b\*x^2 + d^2), x)

**mupad [B]** time = 0.13, size = 30, normalized size = 0.38

$$\frac{\operatorname{atanh}\left(\frac{x\sqrt{b-2de}}{d-ex^2}\right)}{\sqrt{b-2de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)/(d^2 - b\*x^2 + e^2\*x^4),x)



[Out]  $\operatorname{atanh}\left(\frac{x\sqrt{b-2de}}{d-ex^2}\right)\sqrt{b-2de}$

**sympy [A]** time = 0.57, size = 110, normalized size = 1.41

$$\frac{\sqrt{\frac{1}{b-2de}} \log\left(-\frac{d}{e} + x^2 + \frac{x\left(-b\sqrt{\frac{1}{b-2de}} + 2de\sqrt{\frac{1}{b-2de}}\right)}{e}\right)}{2} - \frac{\sqrt{\frac{1}{b-2de}} \log\left(-\frac{d}{e} + x^2 + \frac{x\left(b\sqrt{\frac{1}{b-2de}} - 2de\sqrt{\frac{1}{b-2de}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)/(e**2*x**4-b*x**2+d**2), x)`

[Out]  $\sqrt{\frac{1}{b-2de}} \log\left(-\frac{d}{e} + x^2 + \frac{x\left(-b\sqrt{\frac{1}{b-2de}} + 2de\sqrt{\frac{1}{b-2de}}\right)}{e}\right) + 2de\sqrt{\frac{1}{b-2de}} \log\left(-\frac{d}{e} + x^2 + \frac{x\left(b\sqrt{\frac{1}{b-2de}} - 2de\sqrt{\frac{1}{b-2de}}\right)}{e}\right) - 2de\sqrt{\frac{1}{b-2de}} \log\left(-\frac{d}{e} + x^2 + \frac{x\left(-b\sqrt{\frac{1}{b-2de}} + 2de\sqrt{\frac{1}{b-2de}}\right)}{e}\right) - 2de\sqrt{\frac{1}{b-2de}} \log\left(-\frac{d}{e} + x^2 + \frac{x\left(b\sqrt{\frac{1}{b-2de}} - 2de\sqrt{\frac{1}{b-2de}}\right)}{e}\right)$

$$3.17 \quad \int \frac{d+ex^2}{d^2-fx^2+e^2x^4} dx$$

Optimal. Leaf size=86

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de+f}+2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}} - \frac{\tan^{-1}\left(\frac{\sqrt{2de+f}-2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}$$

**Rubi [A]** time = 0.10, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de+f}+2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}} - \frac{\tan^{-1}\left(\frac{\sqrt{2de+f}-2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)/(d^2 - f\*x^2 + e^2\*x^4), x]

[Out] -(ArcTan[(Sqrt[2\*d\*e + f] - 2\*e\*x)/Sqrt[2\*d\*e - f]]/Sqrt[2\*d\*e - f]) + ArcTan[(Sqrt[2\*d\*e + f] + 2\*e\*x)/Sqrt[2\*d\*e - f]]/Sqrt[2\*d\*e - f]

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 1161

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

Rubi steps

$$\begin{aligned} \int \frac{d+ex^2}{d^2-fx^2+e^2x^4} dx &= \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2de+f}x}{e}+x^2} dx}{2e} + \frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2de+f}x}{e}+x^2} dx}{2e} \\ &= \frac{\text{Subst}\left(\int \frac{1}{-\frac{2de-f}{e^2}-x^2} dx, x, -\frac{\sqrt{2de+f}}{e} + 2x\right)}{e} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{2de-f}{e^2}-x^2} dx, x, \frac{\sqrt{2de+f}}{e} + 2x\right)}{e} \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{2de+f}-2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}} + \frac{\tan^{-1}\left(\frac{\sqrt{2de+f}+2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}} \end{aligned}$$

**Mathematica [B]** time = 0.11, size = 189, normalized size = 2.20

$$\frac{(\sqrt{f^2-4d^2e^2}+2de+f) \tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{-\sqrt{f^2-4d^2e^2}-f}}\right) + (\sqrt{f^2-4d^2e^2}-2de-f) \tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{f^2-4d^2e^2}-f}}\right)}{\sqrt{2}\sqrt{f^2-4d^2e^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)/(d^2 - f\*x^2 + e^2\*x^4), x]

[Out] (((2\*d\*e + f + Sqrt[-4\*d^2\*e^2 + f^2])\*ArcTan[(Sqrt[2]\*e\*x)/Sqrt[-f - Sqrt[-4\*d^2\*e^2 + f^2]]])/Sqrt[-f - Sqrt[-4\*d^2\*e^2 + f^2]] + ((-2\*d\*e - f + Sqrt[-4\*d^2\*e^2 + f^2])\*ArcTan[(Sqrt[2]\*e\*x)/Sqrt[-f + Sqrt[-4\*d^2\*e^2 + f^2]]])/Sqrt[-f + Sqrt[-4\*d^2\*e^2 + f^2]])/(Sqrt[2]\*Sqrt[-4\*d^2\*e^2 + f^2])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{d^2 - fx^2 + e^2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)/(d^2 - f\*x^2 + e^2\*x^4), x]

[Out] IntegrateAlgebraic[(d + e\*x^2)/(d^2 - f\*x^2 + e^2\*x^4), x]

**fricas [A]** time = 0.76, size = 179, normalized size = 2.08

$$\left[ \frac{\sqrt{-2de+f} \log\left(\frac{e^2x^4-(4de-f)x^2+d^2-2(ex^3-dx)\sqrt{-2de+f}}{e^2x^4-fx^2+d^2}\right)}{2(2de-f)}, -\frac{\sqrt{2de-f} \arctan\left(-\frac{ex}{\sqrt{2de-f}}\right) + \sqrt{2de-f} \arctan\left(-\frac{(e^2x^3+(de-f)x)\sqrt{2de-f}}{2d^2e-df}\right)}{2de-f} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(e^2\*x^4-f\*x^2+d^2), x, algorithm="fricas")

[Out] [-1/2\*sqrt(-2\*d\*e + f)\*log((e^2\*x^4 - (4\*d\*e - f)\*x^2 + d^2 - 2\*(e\*x^3 - d\*x)\*sqrt(-2\*d\*e + f))/(e^2\*x^4 - f\*x^2 + d^2))/(2\*d\*e - f), -(sqrt(2\*d\*e - f)\*arctan(-e\*x/sqrt(2\*d\*e - f)) + sqrt(2\*d\*e - f)\*arctan(-(e^2\*x^3 + (d\*e - f)\*x)\*sqrt(2\*d\*e - f)/(2\*d^2\*e - d\*f)))/(2\*d\*e - f)]

**giac [B]** time = 1.14, size = 1676, normalized size = 19.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(e^2\*x^4-f\*x^2+d^2), x, algorithm="giac")

[Out] 1/4\*(16\*sqrt(2)\*sqrt(-f\*e^2 - sqrt(-4\*d^2\*e^2 + f^2)\*e^2)\*d^4\*e^4 - 8\*sqrt(2)\*sqrt(-f\*e^2 - sqrt(-4\*d^2\*e^2 + f^2)\*e^2)\*d^2\*f^2\*e^2 + 4\*sqrt(2)\*sqrt(-4\*d^2\*e^2 + f^2)\*sqrt(-f\*e^2 - sqrt(-4\*d^2\*e^2 + f^2)\*e^2)\*d^2\*f\*e^2 + sqrt(2)\*sqrt(-f\*e^2 - sqrt(-4\*d^2\*e^2 + f^2)\*e^2)\*f^4 + 32\*d^4\*e^6 - 8\*sqrt(2)\*sqrt(-f\*e^2 - sqrt(-4\*d^2\*e^2 + f^2)\*e^2)\*d^2\*f\*e^4 - 16\*d^2\*f^2\*e^4 + 2\*sqrt(2)\*sqrt(-f\*e^2 - sqrt(-4\*d^2\*e^2 + f^2)\*e^2)\*f^3\*e^2 + 2\*f^4\*e^2 - sqrt(2)\*sqrt(-4\*d^2\*e^2 + f^2)\*sqrt(-f\*e^2 - sqrt(-4\*d^2\*e^2 + f^2)\*e^2)\*f^3 - 2\*sqrt(2)\*sqrt(-4\*d^2\*e^2 + f^2)\*sqrt(-f\*e^2 - sqrt(-4\*d^2\*e^2 + f^2)\*e^2)\*f^2\*e^2 - 4\*sqrt(2)\*sqrt(-f\*e^2 - sqrt(-4\*d^2\*e^2 + f^2)\*e^2)\*d^2\*e^6 - 8\*d^2\*f\*e^6 - 8\*(4\*d^2\*e^2 - f^2)\*d^2\*e^4 + sqrt(2)\*sqrt(-f\*e^2 - sqrt(-4\*d^2\*e^2 + f^2)\*e^2)\*f^2\*e^4 + 2\*f^3\*e^4 + 2\*(4\*d^2\*e^2 - f^2)\*f^2\*e^2 - sqrt(2)\*sqrt(-4\*d^2\*e^2 + f^2)\*sqrt(-f\*e^2 - sqrt(-4\*d^2\*e^2 + f^2)\*e^2)\*f\*e^4 + 2\*

$(4*d^2*e^2 - f^2)*f*e^4 + 2*(4*\sqrt{2}*\sqrt{-4*d^2*e^2 + f^2}*\sqrt{-f*e^2 - \sqrt{-4*d^2*e^2 + f^2}*e^2}*d^3*e^2 - \sqrt{2}*\sqrt{-4*d^2*e^2 + f^2}*\sqrt{-f*e^2 - \sqrt{-4*d^2*e^2 + f^2}*e^2}*d*f^2 - 2*\sqrt{2}*\sqrt{-4*d^2*e^2 + f^2}*\sqrt{-f*e^2 - \sqrt{-4*d^2*e^2 + f^2}*e^2}*d*f*e^2 - 8*d^3*e^6 + 2*d*f^2*e^4 - \sqrt{2}*\sqrt{-4*d^2*e^2 + f^2}*\sqrt{-f*e^2 - \sqrt{-4*d^2*e^2 + f^2}*e^2})*d*e^4 + 2*(4*d^2*e^2 - f^2)*d*e^4)*e)*\arctan(2*\sqrt{1/2}*x/\sqrt{-(f + \sqrt{-4*d^2*e^2 + f^2})*e^{-2}})/(16*d^5*e^6 - 8*d^3*f^2*e^4 + d*f^4*e^2 - 8*d^3*f*e^6 + 2*d*f^3*e^4 - 4*d^3*e^8 + d*f^2*e^6) + 1/4*(16*\sqrt{2}*\sqrt{-f*e^2 + \sqrt{-4*d^2*e^2 + f^2}*e^2}*d^4*e^4 - 8*\sqrt{2}*\sqrt{-f*e^2 + \sqrt{-4*d^2*e^2 + f^2}*e^2}*d^2*f^2*e^2 - 4*\sqrt{2}*\sqrt{-4*d^2*e^2 + f^2}*\sqrt{-f*e^2 + \sqrt{-4*d^2*e^2 + f^2}*e^2}*d^2*f*e^2 + \sqrt{2}*\sqrt{-f*e^2 + \sqrt{-4*d^2*e^2 + f^2}*e^2}*f^4 - 32*d^4*e^6 - 8*\sqrt{2}*\sqrt{-f*e^2 + \sqrt{-4*d^2*e^2 + f^2}*e^2}*d^2*f*e^4 + 16*d^2*f^2*e^4 + 2*\sqrt{2}*\sqrt{-f*e^2 + \sqrt{-4*d^2*e^2 + f^2}*e^2}*f^3*e^2 - 2*f^4*e^2 + \sqrt{2}*\sqrt{-4*d^2*e^2 + f^2}*\sqrt{-f*e^2 + \sqrt{-4*d^2*e^2 + f^2}*e^2}*f^3 + 2*\sqrt{2}*\sqrt{-4*d^2*e^2 + f^2}*\sqrt{-f*e^2 + \sqrt{-4*d^2*e^2 + f^2}*e^2}*f^2*e^2 - 4*\sqrt{2}*\sqrt{-f*e^2 + \sqrt{-4*d^2*e^2 + f^2}*e^2}*d^2*e^6 + 8*d^2*f*e^6 + 8*(4*d^2*e^2 - f^2)*d^2*e^4 + \sqrt{2}*\sqrt{-f*e^2 + \sqrt{-4*d^2*e^2 + f^2}*e^2}*f^2*e^4 - 2*f^3*e^4 - 2*(4*d^2*e^2 - f^2)*f^2*e^2 + \sqrt{2}*\sqrt{-4*d^2*e^2 + f^2}*\sqrt{-f*e^2 + \sqrt{-4*d^2*e^2 + f^2}*e^2}*f*e^4 - 2*(4*d^2*e^2 - f^2)*f*e^4 - 2*(4*\sqrt{2}*\sqrt{-4*d^2*e^2 + f^2}*\sqrt{-f*e^2 + \sqrt{-4*d^2*e^2 + f^2}*e^2})*d^3*e^2 - \sqrt{2}*\sqrt{-4*d^2*e^2 + f^2}*\sqrt{-f*e^2 + \sqrt{-4*d^2*e^2 + f^2}*e^2}*d*f^2 - 2*\sqrt{2}*\sqrt{-4*d^2*e^2 + f^2}*\sqrt{-f*e^2 + \sqrt{-4*d^2*e^2 + f^2}*e^2}*d*f*e^2 - 8*d^3*e^6 + 2*d*f^2*e^4 - \sqrt{2}*\sqrt{-4*d^2*e^2 + f^2}*\sqrt{-f*e^2 + \sqrt{-4*d^2*e^2 + f^2}*e^2}*d*e^4 + 2*(4*d^2*e^2 - f^2)*d*e^4)*e)*\arctan(2*\sqrt{1/2}*x/\sqrt{-(f - \sqrt{-4*d^2*e^2 + f^2})*e^{-2}})/(16*d^5*e^6 - 8*d^3*f^2*e^4 + d*f^4*e^2 - 8*d^3*f*e^6 + 2*d*f^3*e^4 - 4*d^3*e^8 + d*f^2*e^6)$

**maple** [A] time = 0.03, size = 75, normalized size = 0.87

$$-\frac{\arctan\left(\frac{-2ex+\sqrt{2de+f}}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}} + \frac{\arctan\left(\frac{2ex+\sqrt{2de+f}}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)/(e^2\*x^4-f\*x^2+d^2),x)

[Out] -arctan((-2\*e\*x+(2\*d\*e+f)^(1/2))/(2\*d\*e-f)^(1/2))/(2\*d\*e-f)^(1/2)+arctan((2\*e\*x+(2\*d\*e+f)^(1/2))/(2\*d\*e-f)^(1/2))/(2\*d\*e-f)^(1/2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2 + d}{e^2x^4 - fx^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(e^2\*x^4-f\*x^2+d^2),x, algorithm="maxima")

[Out] integrate((e\*x^2 + d)/(e^2\*x^4 - f\*x^2 + d^2), x)

**mupad** [B] time = 4.39, size = 88, normalized size = 1.02

$$\frac{\operatorname{atan}\left(\frac{e^2x^3\sqrt{2de-f}-fx\sqrt{2de-f}+dex\sqrt{2de-f}}{d(f-2de)}\right) - \operatorname{atan}\left(\frac{ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x^2)/(d^2 - f*x^2 + e^2*x^4),x)`

[Out]  $-(\operatorname{atan}((e^2x^3(2de - f)^{1/2} - fxx(2de - f)^{1/2} + de*x(2de - f)^{1/2}))/d*(f - 2de)) - \operatorname{atan}((ex)/(2de - f)^{1/2}))/2$

**sympy [A]** time = 0.55, size = 121, normalized size = 1.41

$$\frac{\sqrt{-\frac{1}{2de-f}} \log\left(-\frac{d}{e} + x^2 + \frac{x\left(-2de\sqrt{-\frac{1}{2de-f}} + f\sqrt{-\frac{1}{2de-f}}\right)}{e}\right)}{2} + \frac{\sqrt{-\frac{1}{2de-f}} \log\left(-\frac{d}{e} + x^2 + \frac{x\left(2de\sqrt{-\frac{1}{2de-f}} - f\sqrt{-\frac{1}{2de-f}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)/(e**2*x**4-f*x**2+d**2),x)`

[Out]  $-\sqrt{-1/(2de - f)}*\log(-d/e + x**2 + x*(-2*d*e*\sqrt{-1/(2*d*e - f)} + f*\sqrt{-1/(2*d*e - f)}))/e)/2 + \sqrt{-1/(2*d*e - f)}*\log(-d/e + x**2 + x*(2*d*e*\sqrt{-1/(2*d*e - f)} - f*\sqrt{-1/(2*d*e - f)}))/e)/2$

$$3.18 \quad \int \frac{d-ex^2}{d^2+bx^2+e^2x^4} dx$$

Optimal. Leaf size=78

$$\frac{\log\left(x\sqrt{2de-b}+d+ex^2\right)}{2\sqrt{2de-b}} - \frac{\log\left(-x\sqrt{2de-b}+d+ex^2\right)}{2\sqrt{2de-b}}$$

**Rubi [A]** time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$ , Rules used = {1164, 628}

$$\frac{\log\left(x\sqrt{2de-b}+d+ex^2\right)}{2\sqrt{2de-b}} - \frac{\log\left(-x\sqrt{2de-b}+d+ex^2\right)}{2\sqrt{2de-b}}$$

Antiderivative was successfully verified.

[In] Int[(d - e\*x^2)/(d^2 + b\*x^2 + e^2\*x^4), x]

[Out] -Log[d - Sqrt[-b + 2\*d\*e]\*x + e\*x^2]/(2\*Sqrt[-b + 2\*d\*e]) + Log[d + Sqrt[-b + 2\*d\*e]\*x + e\*x^2]/(2\*Sqrt[-b + 2\*d\*e])

Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1164

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(-2\*d)/e - b/c, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && !GtQ[b^2 - 4\*a\*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{d-ex^2}{d^2+bx^2+e^2x^4} dx &= -\frac{\int \frac{\frac{\sqrt{-b+2de}+2x}{e}}{\frac{-d-\sqrt{-b+2de}x}{e}-x^2} dx}{2\sqrt{-b+2de}} - \frac{\int \frac{\frac{\sqrt{-b+2de}-2x}{e}}{\frac{-d+\sqrt{-b+2de}x}{e}-x^2} dx}{2\sqrt{-b+2de}} \\ &= -\frac{\log\left(d-\sqrt{-b+2de}x+ex^2\right)}{2\sqrt{-b+2de}} + \frac{\log\left(d+\sqrt{-b+2de}x+ex^2\right)}{2\sqrt{-b+2de}} \end{aligned}$$

**Mathematica [B]** time = 0.12, size = 182, normalized size = 2.33

$$\frac{\left(-\sqrt{b^2-4d^2e^2}+b+2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{b-\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{b-\sqrt{b^2-4d^2e^2}}} - \frac{\left(\sqrt{b^2-4d^2e^2}+b+2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{b^2-4d^2e^2}+b}}\right)}{\sqrt{\sqrt{b^2-4d^2e^2}+b}}$$

Antiderivative was successfully verified.

[In] Integrate[(d - e\*x^2)/(d^2 + b\*x^2 + e^2\*x^4), x]

```
[Out] (((b + 2*d*e - Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b - Sqrt[b^2 - 4*d^2*e^2]]])/Sqrt[b - Sqrt[b^2 - 4*d^2*e^2]] - ((b + 2*d*e + Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b + Sqrt[b^2 - 4*d^2*e^2]]])/Sqrt[b + Sqrt[b^2 - 4*d^2*e^2]])/(Sqrt[2]*Sqrt[b^2 - 4*d^2*e^2])
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d - ex^2}{d^2 + bx^2 + e^2x^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d - e*x^2)/(d^2 + b*x^2 + e^2*x^4), x]
```

```
[Out] IntegrateAlgebraic[(d - e*x^2)/(d^2 + b*x^2 + e^2*x^4), x]
```

**fricas** [A] time = 0.70, size = 172, normalized size = 2.21

$$\left[ \frac{\log\left(\frac{e^2x^4 + (4de-b)x^2 + d^2 + 2(ex^3 + dx)\sqrt{2de-b}}{e^2x^4 + bx^2 + d^2}\right)}{2\sqrt{2de-b}}, -\frac{\sqrt{-2de+b} \arctan\left(\frac{\sqrt{-2de+b}ex}{2de-b}\right) - \sqrt{-2de+b} \arctan\left(\frac{(e^2x^3 - (de-b)x)\sqrt{-2de+b}}{2d^2e-bd}\right)}{2de-b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+b*x^2+d^2), x, algorithm="fricas")
```

```
[Out] [1/2*log((e^2*x^4 + (4*d*e - b)*x^2 + d^2 + 2*(e*x^3 + d*x)*sqrt(2*d*e - b))/(e^2*x^4 + b*x^2 + d^2))/sqrt(2*d*e - b), -(sqrt(-2*d*e + b)*arctan(sqrt(-2*d*e + b)*e*x/(2*d*e - b)) - sqrt(-2*d*e + b)*arctan((e^2*x^3 - (d*e - b)*x)*sqrt(-2*d*e + b)/(2*d^2*e - b*d)))/(2*d*e - b)]
```

**giac** [B] time = 1.16, size = 1642, normalized size = 21.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+b*x^2+d^2), x, algorithm="giac")
```

```
[Out] 1/4*(16*sqrt(2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*d^4*e^4 - 8*sqrt(2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b^2*d^2*e^2 + 4*sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b*d^2*e^2 + sqrt(2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b^4 - 32*d^4*e^6 + 8*sqrt(2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b*d^2*e^4 + 16*b^2*d^2*e^4 - 2*sqrt(2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b^3*e^2 - 2*b^4*e^2 - sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b^3 + 2*sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b^2*e^2 - 4*sqrt(2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*d^2*e^6 - 8*b*d^2*e^6 + sqrt(2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b^2*e^4 + 2*b^3*e^4 + 8*(4*d^2*e^2 - b^2)*d^2*e^4 - 2*(4*d^2*e^2 - b^2)*b^2*e^2 - sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b*e^4 + 2*(4*d^2*e^2 - b^2)*b*e^4 + 2*(4*sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*d^3*e^2 - sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b^2*d + 2*sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b*d*e^2 - 8*d^3*e^6 + 2*b^2*d*e^4 - sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*d*e^4 + 2*(4*d^2*e^2 - b^2)*d*e^4)*e)*arctan(2*sqrt(1/2)*x*e/sqrt(b + sqrt(-4*d^2*e^2 + b^2)))/(16*d^5*e^6 - 8*b^2*d^3*e^4 + b^4*d*e^2 + 8*b*d^3*e^6 - 2*b^3*d*e^4 - 4*d^3*e^8 + b^2*d*e^6) + 1/4*(16*sqrt(2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*d^4*e^4 - 8*sqrt(2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b^2*d^2*e^2 - 4*sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b*d^2*e^2 + sqrt(2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b^4 + 32*d^4*e^6 + 8*sqrt(2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b*d^2*e^4
```

$$\begin{aligned}
& - 16b^2d^2e^4 - 2\sqrt{2}\sqrt{2}\sqrt{b^2e^2 - \sqrt{-4d^2e^2 + b^2}}e^2 * b^3 * \\
& e^2 + 2b^4e^2 + \sqrt{2}\sqrt{2}\sqrt{-4d^2e^2 + b^2}\sqrt{b^2e^2 - \sqrt{-4d^2e^2 + b^2}}e^2 * b^3 - 2\sqrt{2}\sqrt{2}\sqrt{-4d^2e^2 + b^2}\sqrt{b^2e^2 - \sqrt{-4d^2e^2 + b^2}}e^2 * b^2 * e^2 - 4\sqrt{2}\sqrt{2}\sqrt{b^2e^2 - \sqrt{-4d^2e^2 + b^2}} \\
& * e^2 * d^2 * e^6 + 8b * d^2 * e^6 + \sqrt{2}\sqrt{2}\sqrt{b^2e^2 - \sqrt{-4d^2e^2 + b^2}}e^2 * b^2 * e^4 - 2b^3 * e^4 - 8(4d^2e^2 - b^2) * d^2 * e^4 + 2(4d^2e^2 - b^2) \\
& * b^2 * e^2 + \sqrt{2}\sqrt{2}\sqrt{-4d^2e^2 + b^2}\sqrt{b^2e^2 - \sqrt{-4d^2e^2 + b^2}}e^2 * b * e^4 - 2(4d^2e^2 - b^2) * b * e^4 - 2(4\sqrt{2}\sqrt{2}\sqrt{-4d^2e^2 + b^2}\sqrt{b^2e^2 - \sqrt{-4d^2e^2 + b^2}}e^2 * d^3 * e^2 - \sqrt{2}\sqrt{2}\sqrt{-4d^2e^2 + b^2} \\
& * \sqrt{b^2e^2 - \sqrt{-4d^2e^2 + b^2}}e^2 * b^2 * d + 2\sqrt{2}\sqrt{2}\sqrt{-4d^2e^2 + b^2}\sqrt{b^2e^2 - \sqrt{-4d^2e^2 + b^2}}e^2 * b * d * e^2 - 8d^3 * e^6 + 2b^2 * d * e^4 - \sqrt{2}\sqrt{2}\sqrt{-4d^2e^2 + b^2}\sqrt{b^2e^2 - \sqrt{-4d^2e^2 + b^2}}e^2 * d * e^4 + 2(4d^2e^2 - b^2) * d * e^4 * e * \arctan(2\sqrt{1/2} * \\
& x * e / \sqrt{b - \sqrt{-4d^2e^2 + b^2}}) / (16d^5e^6 - 8b^2d^3e^4 + b^4d * e^2 + 8b * d^3 * e^6 - 2b^3 * d * e^4 - 4d^3 * e^8 + b^2 * d * e^6)
\end{aligned}$$

**maple [A]** time = 0.02, size = 88, normalized size = 1.13

$$-\frac{\sqrt{2de-b} \ln\left(e x^2 + d + \sqrt{2de-b} x\right)}{-4de+2b} + \frac{\sqrt{2de-b} \ln\left(-e x^2 - d + \sqrt{2de-b} x\right)}{-4de+2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e\*x^2+d)/(e^2\*x^4+b\*x^2+d^2), x)

[Out] 1/(-4\*d\*e+2\*b)\*(2\*d\*e-b)^(1/2)\*ln(-e\*x^2+x\*(2\*d\*e-b)^(1/2)-d)-1/(-4\*d\*e+2\*b)\*(2\*d\*e-b)^(1/2)\*ln(d+e\*x^2+x\*(2\*d\*e-b)^(1/2))

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ex^2 - d}{e^2x^4 + bx^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*x^2+d)/(e^2\*x^4+b\*x^2+d^2), x, algorithm="maxima")

[Out] -integrate((e\*x^2 - d)/(e^2\*x^4 + b\*x^2 + d^2), x)

**mupad [B]** time = 0.09, size = 99, normalized size = 1.27

$$\frac{\operatorname{atan}\left(\frac{bx(b-2de)+2be^2x^3+4d^2e^2x-e^2x^3(b-2de)+3dex(b-2de)}{(2ed^2+bd)\sqrt{b-2de}}\right) - \operatorname{atan}\left(\frac{ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d - e\*x^2)/(b\*x^2 + d^2 + e^2\*x^4), x)

[Out] (atan((b\*x\*(b - 2\*d\*e) + 2\*b\*e^2\*x^3 + 4\*d^2\*e^2\*x - e^2\*x^3\*(b - 2\*d\*e) + 3\*d\*e\*x\*(b - 2\*d\*e))/((b\*d + 2\*d^2\*e)\*(b - 2\*d\*e)^(1/2)))) - atan((e\*x)/(b - 2\*d\*e)^(1/2)))/(b - 2\*d\*e)^(1/2)

**sympy [A]** time = 0.58, size = 121, normalized size = 1.55

$$\frac{\sqrt{-\frac{1}{b-2de}} \log\left(\frac{d}{e} + x^2 + \frac{x\left(-b\sqrt{-\frac{1}{b-2de}} + 2de\sqrt{-\frac{1}{b-2de}}\right)}{e}\right)}{2} - \frac{\sqrt{-\frac{1}{b-2de}} \log\left(\frac{d}{e} + x^2 + \frac{x\left(b\sqrt{-\frac{1}{b-2de}} - 2de\sqrt{-\frac{1}{b-2de}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((-e*x**2+d)/(e**2*x**4+b*x**2+d**2),x)
```

```
[Out] sqrt(-1/(b - 2*d*e))*log(d/e + x**2 + x*(-b*sqrt(-1/(b - 2*d*e)) + 2*d*e*sqrt(-1/(b - 2*d*e))))/e)/2 - sqrt(-1/(b - 2*d*e))*log(d/e + x**2 + x*(b*sqrt(-1/(b - 2*d*e)) - 2*d*e*sqrt(-1/(b - 2*d*e))))/e)/2
```

$$3.19 \quad \int \frac{d-ex^2}{d^2+fx^2+e^2x^4} dx$$

**Optimal.** Leaf size=78

$$\frac{\log\left(x\sqrt{2de-f} + d + ex^2\right)}{2\sqrt{2de-f}} - \frac{\log\left(-x\sqrt{2de-f} + d + ex^2\right)}{2\sqrt{2de-f}}$$

**Rubi [A]** time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$ , Rules used = {1164, 628}

$$\frac{\log\left(x\sqrt{2de-f} + d + ex^2\right)}{2\sqrt{2de-f}} - \frac{\log\left(-x\sqrt{2de-f} + d + ex^2\right)}{2\sqrt{2de-f}}$$

Antiderivative was successfully verified.

[In] Int[(d - e\*x^2)/(d^2 + f\*x^2 + e^2\*x^4), x]

[Out] -Log[d - Sqrt[2\*d\*e - f]\*x + e\*x^2]/(2\*Sqrt[2\*d\*e - f]) + Log[d + Sqrt[2\*d\*e - f]\*x + e\*x^2]/(2\*Sqrt[2\*d\*e - f])

**Rule 628**

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

**Rule 1164**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(-2\*d)/e - b/c, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && !GtQ[b^2 - 4\*a\*c, 0]

**Rubi steps**

$$\begin{aligned} \int \frac{d-ex^2}{d^2+fx^2+e^2x^4} dx &= -\frac{\int \frac{\frac{\sqrt{2de-f}}{e}+2x}{-\frac{d}{e}-\frac{\sqrt{2de-f}x}{e}-x^2} dx}{2\sqrt{2de-f}} - \frac{\int \frac{\frac{\sqrt{2de-f}}{e}-2x}{-\frac{d}{e}+\frac{\sqrt{2de-f}x}{e}-x^2} dx}{2\sqrt{2de-f}} \\ &= -\frac{\log\left(d - \sqrt{2de-f}x + ex^2\right)}{2\sqrt{2de-f}} + \frac{\log\left(d + \sqrt{2de-f}x + ex^2\right)}{2\sqrt{2de-f}} \end{aligned}$$

**Mathematica [B]** time = 0.12, size = 182, normalized size = 2.33

$$\frac{(-\sqrt{f^2-4d^2e^2}+2de+f) \tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{f-\sqrt{f^2-4d^2e^2}}}\right)}{\sqrt{f-\sqrt{f^2-4d^2e^2}}} - \frac{(\sqrt{f^2-4d^2e^2}+2de+f) \tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{f^2-4d^2e^2}+f}}\right)}{\sqrt{\sqrt{f^2-4d^2e^2}+f}}$$

$$\sqrt{2}\sqrt{f^2-4d^2e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d - e\*x^2)/(d^2 + f\*x^2 + e^2\*x^4), x]

```
[Out] (((2*d*e + f - Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[f - Sqrt[-4*d^2*e^2 + f^2]]])/Sqrt[f - Sqrt[-4*d^2*e^2 + f^2]] - ((2*d*e + f + Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[f + Sqrt[-4*d^2*e^2 + f^2]]])/Sqrt[f + Sqrt[-4*d^2*e^2 + f^2]])/(Sqrt[2]*Sqrt[-4*d^2*e^2 + f^2])
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d - ex^2}{d^2 + fx^2 + e^2x^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d - e*x^2)/(d^2 + f*x^2 + e^2*x^4),x]
```

```
[Out] IntegrateAlgebraic[(d - e*x^2)/(d^2 + f*x^2 + e^2*x^4), x]
```

**fricas [A]** time = 0.59, size = 173, normalized size = 2.22

$$\left[ \frac{\log\left(\frac{e^2x^4 + (4de-f)x^2 + d^2 + 2(ex^3 + dx)\sqrt{2de-f}}{e^2x^4 + fx^2 + d^2}\right)}{2\sqrt{2de-f}}, \frac{\sqrt{-2de+f} \arctan\left(-\frac{\sqrt{-2de+f}ex}{2de-f}\right) - \sqrt{-2de+f} \arctan\left(-\frac{(e^2x^3 - (de-f)x)\sqrt{-2de+f}}{2d^2e - df}\right)}{2de-f} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((e^2*x^4 + (4*d*e - f)*x^2 + d^2 + 2*(e*x^3 + d*x)*sqrt(2*d*e - f))/(e^2*x^4 + f*x^2 + d^2))/sqrt(2*d*e - f), (sqrt(-2*d*e + f)*arctan(-sqrt(-2*d*e + f)*e*x/(2*d*e - f)) - sqrt(-2*d*e + f)*arctan(-(e^2*x^3 - (d*e - f)*x)*sqrt(-2*d*e + f)/(2*d^2*e - d*f)))/(2*d*e - f)]
```

**giac [B]** time = 1.25, size = 1642, normalized size = 21.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="giac")
```

```
[Out] 1/4*(16*sqrt(2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d^4*e^4 - 8*sqrt(2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*f^2*e^2 + 4*sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*f*e^2 + sqrt(2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*f^4 - 32*d^4*e^6 + 8*sqrt(2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*f*e^4 + 16*d^2*f^2*e^4 - 2*sqrt(2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*f^3*e^2 - 2*f^4*e^2 - sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*f^3 + 2*sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*f^2*e^2 - 4*sqrt(2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*e^6 - 8*d^2*f*e^6 + 8*(4*d^2*e^2 - f^2)*d^2*e^4 + sqrt(2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*f^2*e^4 + 2*f^3*e^4 - 2*(4*d^2*e^2 - f^2)*f^2*e^2 - sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*f*e^4 + 2*(4*d^2*e^2 - f^2)*f*e^4 + 2*(4*sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d^3*e^2 - sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d*f^2 + 2*sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d*f*e^2 - 8*d^3*e^6 + 2*d*f^2*e^4 - sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d*e^4 + 2*(4*d^2*e^2 - f^2)*d*e^4)*e)*arctan(2*sqrt(1/2)*x*e/sqrt(f + sqrt(-4*d^2*e^2 + f^2)))/(16*d^5*e^6 - 8*d^3*f^2*e^4 + d*f^4*e^2 + 8*d^3*f*e^6 - 2*d*f^3*e^4 - 4*d^3*e^8 + d*f^2*e^6) + 1/4*(16*sqrt(2)*sqrt(f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*d^4*e^4 - 8*sqrt(2)*sqrt(f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*f^2*e^2 - 4*sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*f*e^2 + sqrt(2)*sqrt(f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*f^4 + 32*d^4*e^6 + 8*sqrt(2)*sqrt(f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*f*e^4
```

$$\begin{aligned}
& -16d^2f^2e^4 - 2\sqrt{2}\sqrt{f^2e^2 - \sqrt{-4d^2e^2 + f^2}}e^2 * f^3e^2 + 2f^4e^2 + \sqrt{2}\sqrt{-4d^2e^2 + f^2}\sqrt{f^2e^2 - \sqrt{-4d^2e^2 + f^2}}e^2 * f^3 - 2\sqrt{2}\sqrt{-4d^2e^2 + f^2}\sqrt{f^2e^2 - \sqrt{-4d^2e^2 + f^2}}e^2 * f^2e^2 - 4\sqrt{2}\sqrt{f^2e^2 - \sqrt{-4d^2e^2 + f^2}}e^2 * d^2e^6 + 8d^2f^2e^6 - 8(4d^2e^2 - f^2)d^2e^4 + \sqrt{2}\sqrt{f^2e^2 - \sqrt{-4d^2e^2 + f^2}}e^2 * f^2e^4 - 2f^3e^4 + 2(4d^2e^2 - f^2) * f^2e^2 + \sqrt{2}\sqrt{-4d^2e^2 + f^2}\sqrt{f^2e^2 - \sqrt{-4d^2e^2 + f^2}}e^2 * f^4 - 2(4d^2e^2 - f^2)f^4 - 2(4\sqrt{2}\sqrt{-4d^2e^2 + f^2}\sqrt{f^2e^2 - \sqrt{-4d^2e^2 + f^2}}e^2 * d^3e^2 - \sqrt{2}\sqrt{-4d^2e^2 + f^2}\sqrt{f^2e^2 - \sqrt{-4d^2e^2 + f^2}}e^2 * d * f^2 + 2\sqrt{2}\sqrt{-4d^2e^2 + f^2}\sqrt{f^2e^2 - \sqrt{-4d^2e^2 + f^2}}e^2 * d * f^2e^2 - 8d^3e^6 + 2d * f^2e^4 - \sqrt{2}\sqrt{-4d^2e^2 + f^2}\sqrt{f^2e^2 - \sqrt{-4d^2e^2 + f^2}}e^2 * d * e^4 + 2(4d^2e^2 - f^2)d * e^4) * e * \arctan(2\sqrt{1/2} * x * e / \sqrt{f - \sqrt{-4d^2e^2 + f^2}}) / (16d^5e^6 - 8d^3f^2e^4 + d * f^4e^2 + 8d^3f^2e^6 - 2d * f^3e^4 - 4d^3e^8 + d * f^2e^6)
\end{aligned}$$

**maple [A]** time = 0.02, size = 69, normalized size = 0.88

$$\frac{\ln\left(e x^2 + d + \sqrt{2de - f} x\right)}{2\sqrt{2de - f}} - \frac{\ln\left(-e x^2 - d + \sqrt{2de - f} x\right)}{2\sqrt{2de - f}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e\*x^2+d)/(e^2\*x^4+f\*x^2+d^2),x)

[Out] 1/2\*ln(d+e\*x^2+x\*(2\*d\*e-f)^(1/2))/(2\*d\*e-f)^(1/2)-1/2/(2\*d\*e-f)^(1/2)\*ln(-e\*x^2+x\*(2\*d\*e-f)^(1/2)-d)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ex^2 - d}{e^2x^4 + fx^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*x^2+d)/(e^2\*x^4+f\*x^2+d^2),x, algorithm="maxima")

[Out] -integrate((e\*x^2 - d)/(e^2\*x^4 + f\*x^2 + d^2), x)

**mupad [B]** time = 4.44, size = 57, normalized size = 0.73

$$\frac{\operatorname{atan}\left(\frac{fx^{1i}-dex^{2i}}{d\sqrt{2de-f}+ex^2\sqrt{2de-f}}\right)1i}{\sqrt{2de-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d - e\*x^2)/(f\*x^2 + d^2 + e^2\*x^4),x)

[Out] (atan((f\*x\*1i - d\*e\*x\*2i)/(d\*(2\*d\*e - f)^(1/2) + e\*x^2\*(2\*d\*e - f)^(1/2)))\*1i)/(2\*d\*e - f)^(1/2)

**sympy [A]** time = 0.57, size = 110, normalized size = 1.41

$$-\frac{\sqrt{\frac{1}{2de-f}} \log\left(\frac{d}{e} + x^2 + \frac{x\left(-2de\sqrt{\frac{1}{2de-f}} + f\sqrt{\frac{1}{2de-f}}\right)}{e}\right)}{2} + \frac{\sqrt{\frac{1}{2de-f}} \log\left(\frac{d}{e} + x^2 + \frac{x\left(2de\sqrt{\frac{1}{2de-f}} - f\sqrt{\frac{1}{2de-f}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*x\*\*2+d)/(e\*\*2\*x\*\*4+f\*x\*\*2+d\*\*2),x)

```
[Out] -sqrt(1/(2*d*e - f))*log(d/e + x**2 + x*(-2*d*e*sqrt(1/(2*d*e - f)) + f*sqrt(1/(2*d*e - f)))/e)/2 + sqrt(1/(2*d*e - f))*log(d/e + x**2 + x*(2*d*e*sqrt(1/(2*d*e - f)) - f*sqrt(1/(2*d*e - f)))/e)/2
```

$$3.20 \quad \int \frac{d-ex^2}{d^2-bx^2+e^2x^4} dx$$

**Optimal.** Leaf size=70

$$\frac{\log\left(x\sqrt{b+2de}+d+ex^2\right)}{2\sqrt{b+2de}} - \frac{\log\left(-x\sqrt{b+2de}+d+ex^2\right)}{2\sqrt{b+2de}}$$

**Rubi [A]** time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {1164, 628}

$$\frac{\log\left(x\sqrt{b+2de}+d+ex^2\right)}{2\sqrt{b+2de}} - \frac{\log\left(-x\sqrt{b+2de}+d+ex^2\right)}{2\sqrt{b+2de}}$$

Antiderivative was successfully verified.

[In] Int[(d - e\*x^2)/(d^2 - b\*x^2 + e^2\*x^4), x]

[Out] -Log[d - Sqrt[b + 2\*d\*e]\*x + e\*x^2]/(2\*Sqrt[b + 2\*d\*e]) + Log[d + Sqrt[b + 2\*d\*e]\*x + e\*x^2]/(2\*Sqrt[b + 2\*d\*e])

Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1164

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(-2\*d)/e - b/c, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && !GtQ[b^2 - 4\*a\*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{d-ex^2}{d^2-bx^2+e^2x^4} dx &= -\frac{\int \frac{\frac{\sqrt{b+2de}}{e}+2x}{-\frac{d}{e}-\frac{\sqrt{b+2de}x}{e}-x^2} dx}{2\sqrt{b+2de}} - \frac{\int \frac{\frac{\sqrt{b+2de}}{e}-2x}{-\frac{d}{e}+\frac{\sqrt{b+2de}x}{e}-x^2} dx}{2\sqrt{b+2de}} \\ &= -\frac{\log\left(d-\sqrt{b+2de}x+ex^2\right)}{2\sqrt{b+2de}} + \frac{\log\left(d+\sqrt{b+2de}x+ex^2\right)}{2\sqrt{b+2de}} \end{aligned}$$

**Mathematica [B]** time = 0.13, size = 190, normalized size = 2.71

$$\frac{\left(-\sqrt{b^2-4d^2e^2}+b-2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{b^2-4d^2e^2}-b}}\right)}{\sqrt{\sqrt{b^2-4d^2e^2}-b}} - \frac{\left(\sqrt{b^2-4d^2e^2}+b-2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{-\sqrt{b^2-4d^2e^2}-b}}\right)}{\sqrt{-\sqrt{b^2-4d^2e^2}-b}}}{\sqrt{2}\sqrt{b^2-4d^2e^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d - e\*x^2)/(d^2 - b\*x^2 + e^2\*x^4), x]



$$\begin{aligned} &^2e^2 + b^2)e^2)*b*d^2e^4 + 16*b^2*d^2e^4 + 2*\sqrt{2}*\sqrt{-b*e^2 + \sqrt{-4*d^2e^2 + b^2}*e^2}*b^3e^2 - 2*b^4e^2 + \sqrt{2}*\sqrt{-4*d^2e^2 + b^2}*\sqrt{-b*e^2 + \sqrt{-4*d^2e^2 + b^2}*e^2}*b^3 + 2*\sqrt{2}*\sqrt{-4*d^2e^2 + b^2}*\sqrt{-b*e^2 + \sqrt{-4*d^2e^2 + b^2}*e^2}*b^2e^2 - 4*\sqrt{2}*\sqrt{-b*e^2 + \sqrt{-4*d^2e^2 + b^2}*e^2}*d^2e^6 + 8*b*d^2e^6 + \sqrt{2}*\sqrt{-b*e^2 + \sqrt{-4*d^2e^2 + b^2}*e^2}*b^2e^4 - 2*b^3e^4 + 8*(4*d^2e^2 - b^2)*d^2e^4 - 2*(4*d^2e^2 - b^2)*b^2e^2 + \sqrt{2}*\sqrt{-4*d^2e^2 + b^2}*\sqrt{-b*e^2 + \sqrt{-4*d^2e^2 + b^2}*e^2}*b*e^4 - 2*(4*d^2e^2 - b^2)*b*e^4 + 2*(4*\sqrt{2}*\sqrt{-4*d^2e^2 + b^2}*\sqrt{-b*e^2 + \sqrt{-4*d^2e^2 + b^2}*e^2})*d^3e^2 - \sqrt{2}*\sqrt{-4*d^2e^2 + b^2}*\sqrt{-b*e^2 + \sqrt{-4*d^2e^2 + b^2}*e^2}*b^2*d - 2*\sqrt{2}*\sqrt{-4*d^2e^2 + b^2}*\sqrt{-b*e^2 + \sqrt{-4*d^2e^2 + b^2}*e^2}*b*d*e^2 - 8*d^3e^6 + 2*b^2*d*e^4 - \sqrt{2}*\sqrt{-4*d^2e^2 + b^2}*\sqrt{-b*e^2 + \sqrt{-4*d^2e^2 + b^2}*e^2}*d*e^4 + 2*(4*d^2e^2 - b^2)*d*e^4)*e)*\arctan(2*\sqrt{1/2}*x/\sqrt{-(b - \sqrt{-4*d^2e^2 + b^2})*e^(-2)})/(16*d^5e^6 - 8*b^2*d^3e^4 + b^4*d*e^2 - 8*b*d^3e^6 + 2*b^3*d*e^4 - 4*d^3e^8 + b^2*d*e^6) \end{aligned}$$

**maple [A]** time = 0.02, size = 61, normalized size = 0.87

$$\frac{\ln\left(e x^2 + d + \sqrt{2de + b} x\right)}{2\sqrt{2de + b}} - \frac{\ln\left(-e x^2 - d + \sqrt{2de + b} x\right)}{2\sqrt{2de + b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e\*x^2+d)/(e^2\*x^4-b\*x^2+d^2),x)

[Out] -1/2/(2\*d\*e+b)^(1/2)\*ln(-e\*x^2+x\*(2\*d\*e+b)^(1/2)-d)+1/2\*ln(d+e\*x^2+x\*(2\*d\*e+b)^(1/2))/(2\*d\*e+b)^(1/2)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ex^2 - d}{e^2x^4 - bx^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*x^2+d)/(e^2\*x^4-b\*x^2+d^2),x, algorithm="maxima")

[Out] -integrate((e\*x^2 - d)/(e^2\*x^4 - b\*x^2 + d^2), x)

**mupad [B]** time = 4.44, size = 29, normalized size = 0.41

$$\frac{\operatorname{atanh}\left(\frac{x\sqrt{b+2de}}{ex^2+d}\right)}{\sqrt{b+2de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d - e\*x^2)/(d^2 - b\*x^2 + e^2\*x^4),x)

[Out] atanh((x\*(b + 2\*d\*e)^(1/2))/(d + e\*x^2))/(b + 2\*d\*e)^(1/2)

**sympy [A]** time = 0.60, size = 112, normalized size = 1.60

$$-\frac{\sqrt{\frac{1}{b+2de}} \log\left(\frac{d}{e} + x^2 + \frac{x\left(-b\sqrt{\frac{1}{b+2de}} - 2de\sqrt{\frac{1}{b+2de}}\right)}{e}\right)}{2} + \frac{\sqrt{\frac{1}{b+2de}} \log\left(\frac{d}{e} + x^2 + \frac{x\left(b\sqrt{\frac{1}{b+2de}} + 2de\sqrt{\frac{1}{b+2de}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*x\*\*2+d)/(e\*\*2\*x\*\*4-b\*x\*\*2+d\*\*2),x)



```
[Out] -sqrt(1/(b + 2*d*e))*log(d/e + x**2 + x*(-b*sqrt(1/(b + 2*d*e)) - 2*d*e*sqrt(1/(b + 2*d*e)))/e)/2 + sqrt(1/(b + 2*d*e))*log(d/e + x**2 + x*(b*sqrt(1/(b + 2*d*e)) + 2*d*e*sqrt(1/(b + 2*d*e)))/e)/2
```

$$3.21 \quad \int \frac{d-ex^2}{d^2-fx^2+e^2x^4} dx$$

**Optimal.** Leaf size=70

$$\frac{\log\left(x\sqrt{2de+f}+d+ex^2\right)}{2\sqrt{2de+f}} - \frac{\log\left(-x\sqrt{2de+f}+d+ex^2\right)}{2\sqrt{2de+f}}$$

**Rubi [A]** time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {1164, 628}

$$\frac{\log\left(x\sqrt{2de+f}+d+ex^2\right)}{2\sqrt{2de+f}} - \frac{\log\left(-x\sqrt{2de+f}+d+ex^2\right)}{2\sqrt{2de+f}}$$

Antiderivative was successfully verified.

[In] Int[(d - e\*x^2)/(d^2 - f\*x^2 + e^2\*x^4), x]

[Out] -Log[d - Sqrt[2\*d\*e + f]\*x + e\*x^2]/(2\*Sqrt[2\*d\*e + f]) + Log[d + Sqrt[2\*d\*e + f]\*x + e\*x^2]/(2\*Sqrt[2\*d\*e + f])

**Rule 628**

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

**Rule 1164**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(-2\*d)/e - b/c, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && !GtQ[b^2 - 4\*a\*c, 0]

**Rubi steps**

$$\begin{aligned} \int \frac{d-ex^2}{d^2-fx^2+e^2x^4} dx &= -\frac{\int \frac{\frac{\sqrt{2de+f}}{e}+2x}{-\frac{d}{e}-\frac{\sqrt{2de+f}x}{e}-x^2} dx}{2\sqrt{2de+f}} - \frac{\int \frac{\frac{\sqrt{2de+f}}{e}-2x}{-\frac{d}{e}+\frac{\sqrt{2de+f}x}{e}-x^2} dx}{2\sqrt{2de+f}} \\ &= -\frac{\log\left(d-\sqrt{2de+f}x+ex^2\right)}{2\sqrt{2de+f}} + \frac{\log\left(d+\sqrt{2de+f}x+ex^2\right)}{2\sqrt{2de+f}} \end{aligned}$$

**Mathematica [B]** time = 0.13, size = 190, normalized size = 2.71

$$\frac{(-\sqrt{f^2-4d^2e^2}-2de+f) \tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{f^2-4d^2e^2}-f}}\right)}{\sqrt{\sqrt{f^2-4d^2e^2}-f}} - \frac{(\sqrt{f^2-4d^2e^2}-2de+f) \tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{-\sqrt{f^2-4d^2e^2}-f}}\right)}{\sqrt{-\sqrt{f^2-4d^2e^2}-f}}}{\sqrt{2}\sqrt{f^2-4d^2e^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d - e\*x^2)/(d^2 - f\*x^2 + e^2\*x^4), x]



$$\begin{aligned} & \left( 2e^2 + f^2 \right) e^2 \cdot d^2 f e^4 + 16d^2 f^2 e^4 + 2\sqrt{2} \sqrt{-f e^2 + \sqrt{-4d^2 e^2 + f^2}} e^2 \cdot f^3 e^2 - 2f^4 e^2 + \sqrt{2} \sqrt{-4d^2 e^2 + f^2} \sqrt{-f e^2 + \sqrt{-4d^2 e^2 + f^2}} e^2 \cdot f^3 + 2\sqrt{2} \sqrt{-4d^2 e^2 + f^2} \sqrt{-f e^2 + \sqrt{-4d^2 e^2 + f^2}} e^2 \cdot f^2 e^2 - 4\sqrt{2} \sqrt{-f e^2 + \sqrt{-4d^2 e^2 + f^2}} e^2 \cdot d^2 e^6 + 8d^2 f e^6 + 8(4d^2 e^2 - f^2) d^2 e^4 + \sqrt{2} \sqrt{-f e^2 + \sqrt{-4d^2 e^2 + f^2}} e^2 \cdot f^2 e^4 - 2f^3 e^4 - 2(4d^2 e^2 - f^2) f^2 e^2 + \sqrt{2} \sqrt{-4d^2 e^2 + f^2} \sqrt{-f e^2 + \sqrt{-4d^2 e^2 + f^2}} e^2 \cdot f e^4 - 2(4d^2 e^2 - f^2) f e^4 + 2(4\sqrt{2} \sqrt{-4d^2 e^2 + f^2} \sqrt{-f e^2 + \sqrt{-4d^2 e^2 + f^2}} e^2) \cdot d^3 e^2 - \sqrt{2} \sqrt{-4d^2 e^2 + f^2} \sqrt{-f e^2 + \sqrt{-4d^2 e^2 + f^2}} e^2 \cdot d f^2 - 2\sqrt{2} \sqrt{-4d^2 e^2 + f^2} \sqrt{-f e^2 + \sqrt{-4d^2 e^2 + f^2}} e^2 \cdot d f e^2 - 8d^3 e^6 + 2d f^2 e^4 - \sqrt{2} \sqrt{-4d^2 e^2 + f^2} \sqrt{-f e^2 + \sqrt{-4d^2 e^2 + f^2}} e^2 \cdot d e^4 + 2(4d^2 e^2 - f^2) d e^4 \cdot e \cdot \arctan\left( 2\sqrt{1/2} \cdot x / \sqrt{-(f - \sqrt{-4d^2 e^2 + f^2})} \cdot e^{-2} \right) / (16d^5 e^6 - 8d^3 f^2 e^4 + d f^4 e^2 - 8d^3 f e^6 + 2d f^3 e^4 - 4d^3 e^8 + d f^2 e^6) \end{aligned}$$

**maple [A]** time = 0.02, size = 61, normalized size = 0.87

$$\frac{\ln\left(e x^2 + d + \sqrt{2de + f} x\right)}{2\sqrt{2de + f}} - \frac{\ln\left(-e x^2 - d + \sqrt{2de + f} x\right)}{2\sqrt{2de + f}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e\*x^2+d)/(e^2\*x^4-f\*x^2+d^2),x)

[Out] 1/2\*ln(d+e\*x^2+x\*(2\*d\*e+f)^(1/2))/(2\*d\*e+f)^(1/2)-1/2/(2\*d\*e+f)^(1/2)\*ln(-e\*x^2+x\*(2\*d\*e+f)^(1/2)-d)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ex^2 - d}{e^2x^4 - fx^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*x^2+d)/(e^2\*x^4-f\*x^2+d^2),x, algorithm="maxima")

[Out] -integrate((e\*x^2 - d)/(e^2\*x^4 - f\*x^2 + d^2), x)

**mupad [B]** time = 0.11, size = 29, normalized size = 0.41

$$\frac{\operatorname{atanh}\left(\frac{x\sqrt{f+2de}}{ex^2+d}\right)}{\sqrt{f+2de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d - e\*x^2)/(d^2 - f\*x^2 + e^2\*x^4),x)

[Out] atanh((x\*(f + 2\*d\*e)^(1/2))/(d + e\*x^2))/(f + 2\*d\*e)^(1/2)

**sympy [A]** time = 0.61, size = 112, normalized size = 1.60

$$\frac{\sqrt{\frac{1}{2de+f}} \log\left(\frac{d}{e} + x^2 + \frac{x\left(-2de\sqrt{\frac{1}{2de+f}} - f\sqrt{\frac{1}{2de+f}}\right)}{e}\right)}{2} + \frac{\sqrt{\frac{1}{2de+f}} \log\left(\frac{d}{e} + x^2 + \frac{x\left(2de\sqrt{\frac{1}{2de+f}} + f\sqrt{\frac{1}{2de+f}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*x\*\*2+d)/(e\*\*2\*x\*\*4-f\*x\*\*2+d\*\*2),x)

```
[Out] -sqrt(1/(2*d*e + f))*log(d/e + x**2 + x*(-2*d*e*sqrt(1/(2*d*e + f)) - f*sqrt(1/(2*d*e + f)))/e)/2 + sqrt(1/(2*d*e + f))*log(d/e + x**2 + x*(2*d*e*sqrt(1/(2*d*e + f)) + f*sqrt(1/(2*d*e + f)))/e)/2
```

$$3.22 \quad \int \frac{d-ex^2}{\frac{cd^2}{e^2} + bx^2 + cx^4} dx$$

**Optimal.** Leaf size=134

$$\frac{e^{3/2} \log(\sqrt{e} x \sqrt{2cd - be} + \sqrt{c} d + \sqrt{c} ex^2)}{2\sqrt{c} \sqrt{2cd - be}} - \frac{e^{3/2} \log(-\sqrt{e} x \sqrt{2cd - be} + \sqrt{c} d + \sqrt{c} ex^2)}{2\sqrt{c} \sqrt{2cd - be}}$$

**Rubi [A]** time = 0.10, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {1164, 628}

$$\frac{e^{3/2} \log(\sqrt{e} x \sqrt{2cd - be} + \sqrt{c} d + \sqrt{c} ex^2)}{2\sqrt{c} \sqrt{2cd - be}} - \frac{e^{3/2} \log(-\sqrt{e} x \sqrt{2cd - be} + \sqrt{c} d + \sqrt{c} ex^2)}{2\sqrt{c} \sqrt{2cd - be}}$$

Antiderivative was successfully verified.

[In] Int[(d - e\*x^2)/((c\*d^2)/e^2 + b\*x^2 + c\*x^4), x]

[Out] -(e^(3/2)\*Log[Sqrt[c]\*d - Sqrt[e]\*Sqrt[2\*c\*d - b\*e]\*x + Sqrt[c]\*e\*x^2])/(2\*Sqrt[c]\*Sqrt[2\*c\*d - b\*e]) + (e^(3/2)\*Log[Sqrt[c]\*d + Sqrt[e]\*Sqrt[2\*c\*d - b\*e]\*x + Sqrt[c]\*e\*x^2])/(2\*Sqrt[c]\*Sqrt[2\*c\*d - b\*e])

**Rule 628**

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

**Rule 1164**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(-2\*d)/e - b/c, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && !GtQ[b^2 - 4\*a\*c, 0]

**Rubi steps**

$$\int \frac{d - ex^2}{\frac{cd^2}{e^2} + bx^2 + cx^4} dx = \frac{e^{3/2} \int \frac{\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}} + 2x}{-\frac{d}{e} - \frac{\sqrt{2cd-be}x}{\sqrt{c}\sqrt{e}} - x^2} dx}{2\sqrt{c} \sqrt{2cd - be}} - \frac{e^{3/2} \int \frac{\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}} - 2x}{-\frac{d}{e} + \frac{\sqrt{2cd-be}x}{\sqrt{c}\sqrt{e}} - x^2} dx}{2\sqrt{c} \sqrt{2cd - be}}$$

$$= \frac{e^{3/2} \log(\sqrt{c} d - \sqrt{e} \sqrt{2cd - be} x + \sqrt{c} ex^2)}{2\sqrt{c} \sqrt{2cd - be}} + \frac{e^{3/2} \log(\sqrt{c} d + \sqrt{e} \sqrt{2cd - be} x + \sqrt{c} ex^2)}{2\sqrt{c} \sqrt{2cd - be}}$$

**Mathematica [A]** time = 0.16, size = 250, normalized size = 1.87

$$\frac{e^{3/2} \left( \frac{(\sqrt{b^2 e^2 - 4c^2 d^2} - be - 2cd) \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} \sqrt{e} x}{\sqrt{be - \sqrt{b^2 e^2 - 4c^2 d^2}}}\right)}{\sqrt{be - \sqrt{b^2 e^2 - 4c^2 d^2}}} - \frac{(\sqrt{b^2 e^2 - 4c^2 d^2} + be + 2cd) \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} \sqrt{e} x}{\sqrt{\sqrt{b^2 e^2 - 4c^2 d^2} + be}}\right)}{\sqrt{\sqrt{b^2 e^2 - 4c^2 d^2} + be}} \right)}{\sqrt{2} \sqrt{c} \sqrt{b^2 e^2 - 4c^2 d^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d - e\*x^2)/((c\*d^2)/e^2 + b\*x^2 + c\*x^4), x]

[Out]  $(e^{(3/2)} * (-(((-2*c*d - b*e + \sqrt{-4*c^2*d^2 + b^2*e^2}) * \text{ArcTan}[(\sqrt{2} * \text{Sqrt}[c] * \text{Sqrt}[e] * x) / \text{Sqrt}[b*e - \text{Sqrt}[-4*c^2*d^2 + b^2*e^2]])]) / \text{Sqrt}[b*e - \text{Sqrt}[-4*c^2*d^2 + b^2*e^2]]) - ((2*c*d + b*e + \sqrt{-4*c^2*d^2 + b^2*e^2}) * \text{ArcTan}[(\sqrt{2} * \text{Sqrt}[c] * \text{Sqrt}[e] * x) / \text{Sqrt}[b*e + \text{Sqrt}[-4*c^2*d^2 + b^2*e^2]])]) / \text{Sqrt}[b*e + \text{Sqrt}[-4*c^2*d^2 + b^2*e^2]])) / (\text{Sqrt}[2] * \text{Sqrt}[c] * \text{Sqrt}[-4*c^2*d^2 + b^2*e^2])$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d - ex^2}{\frac{cd^2}{e^2} + bx^2 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d - e\*x^2)/((c\*d^2)/e^2 + b\*x^2 + c\*x^4), x]

[Out] IntegrateAlgebraic[(d - e\*x^2)/((c\*d^2)/e^2 + b\*x^2 + c\*x^4), x]

fricas [A] time = 0.82, size = 244, normalized size = 1.82

$$\left[ \frac{1}{2} e^{\sqrt{\frac{e}{2c^2d - bce}}} \log \left( \frac{ce^2x^4 + cd^2 + (4cde - be^2)x^2 + 2((2c^2de - bce^2)x^3 + (2c^2d^2 - bcde)x) \sqrt{\frac{e}{2c^2d - bce}}}{ce^2x^4 + be^2x^2 + cd^2} \right), -e^{\sqrt{\frac{e}{2c^2d - bce}}} \arctan \left( cx \sqrt{\frac{e}{2c^2d - bce}} \right) + e^{\sqrt{\frac{e}{2c^2d - bce}}} \arctan \left( \frac{(cex^3 - (cd - be)x) \sqrt{\frac{e}{2c^2d - bce}}}{d} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*x^2+d)/(c\*d^2/e^2+b\*x^2+c\*x^4), x, algorithm="fricas")

[Out]  $[1/2 * e * \text{sqrt}(e / (2 * c^2 * d - b * c * e)) * \log((c * e^2 * x^4 + c * d^2 + (4 * c * d * e - b * e^2) * x^2 + 2 * ((2 * c^2 * d * e - b * c * e^2) * x^3 + (2 * c^2 * d^2 - b * c * d * e) * x) * \text{sqrt}(e / (2 * c^2 * d - b * c * e)))) / (c * e^2 * x^4 + b * e^2 * x^2 + c * d^2), -e * \text{sqrt}(-e / (2 * c^2 * d - b * c * e)) * \arctan(c * x * \text{sqrt}(-e / (2 * c^2 * d - b * c * e))) + e * \text{sqrt}(-e / (2 * c^2 * d - b * c * e)) * \text{arctan}((c * e * x^3 - (c * d - b * e) * x) * \text{sqrt}(-e / (2 * c^2 * d - b * c * e))) / d]$

giac [B] time = 1.37, size = 2202, normalized size = 16.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*x^2+d)/(c\*d^2/e^2+b\*x^2+c\*x^4), x, algorithm="giac")

[Out]  $-1/4 * (32 * c^5 * d^4 * e^4 - 16 * \text{sqrt}(2) * \text{sqrt}(b * c * e^4 + \text{sqrt}(-4 * c^2 * d^2 * e^2 + b^2 * e^4) * c * e^2) * c^4 * d^4 * e^2 - 16 * b^2 * c^3 * d^2 * e^6 + 8 * b * c^4 * d^2 * e^6 + 8 * \text{sqrt}(2) * \text{sqrt}(b * c * e^4 + \text{sqrt}(-4 * c^2 * d^2 * e^2 + b^2 * e^4) * c * e^2) * b^2 * c^2 * d^2 * e^4 - 8 * \text{sqrt}(2) * \text{sqrt}(b * c * e^4 + \text{sqrt}(-4 * c^2 * d^2 * e^2 + b^2 * e^4) * c * e^2) * b * c^3 * d^2 * e^4 + 4 * \text{sqrt}(2) * \text{sqrt}(b * c * e^4 + \text{sqrt}(-4 * c^2 * d^2 * e^2 + b^2 * e^4) * c * e^2) * c^4 * d^2 * e^4 - 4 * \text{sqrt}(2) * \text{sqrt}(-4 * c^2 * d^2 * e^2 + b^2 * e^4) * \text{sqrt}(b * c * e^4 + \text{sqrt}(-4 * c^2 * d^2 * e^2 + b^2 * e^4) * c * e^2) * b * c^2 * d^2 * e^2 - 8 * (4 * c^2 * d^2 * e^2 - b^2 * e^4) * c^3 * d^2 * e^2 + 2 * b^4 * c * e^8 - 2 * b^3 * c^2 * e^8 - \text{sqrt}(2) * \text{sqrt}(b * c * e^4 + \text{sqrt}(-4 * c^2 * d^2 * e^2 + b^2 * e^4) * c * e^2) * b^4 * e^6 + 2 * \text{sqrt}(2) * \text{sqrt}(b * c * e^4 + \text{sqrt}(-4 * c^2 * d^2 * e^2 + b^2 * e^4) * c * e^2) * b^3 * c * e^6 - \text{sqrt}(2) * \text{sqrt}(b * c * e^4 + \text{sqrt}(-4 * c^2 * d^2 * e^2 + b^2 * e^4) * c * e^2) * b^2 * c^2 * e^6 + \text{sqrt}(2) * \text{sqrt}(-4 * c^2 * d^2 * e^2 + b^2 * e^4) * \text{sqrt}(b * c * e^4 + \text{sqrt}(-4 * c^2 * d^2 * e^2 + b^2 * e^4) * c * e^2) * b^3 * e^4 - 2 * \text{sqrt}(2) * \text{sqrt}(-4 * c^2 * d^2 * e^2 + b^2 * e^4) * \text{sqrt}(b * c * e^4 + \text{sqrt}(-4 * c^2 * d^2 * e^2 + b^2 * e^4) * c * e^2) * b^2 * c * e^4 + 2 * (4 * c^2 * d^2 * e^2 - b^2 * e^4) * b^2 * c * e^4 - 2 * (4 * c^2 * d^2 * e^2 - b^2 * e^4) * b * c^2 * e^4 + 2 * (8 * c^5 * d^3 * e^4 - 4 * \text{sqrt}(2) * \text{sqrt}(-4 * c^2 * d^2 * e^2 + b^2 * e^4) * \text{sqrt}(b * c * e^4 + \text{sqrt}(-4 * c^2 * d^2 * e^2 + b^2 * e^4) * c * e^2)) * c^3 * d^3 - 2 * b^2 * c^3 * d * e^6 + \text{sqrt}(2) * \text{sqrt}(-4 * c^2 * d^2 * e^2 + b^2 * e^4) * \text{sqrt}(b * c * e^4 + \text{sqrt}(-4 * c^2 * d^2 * e^2 + b^2 * e^4) * c * e^2) * b^2 * c * d * e^2 - 2 * \text{sqrt}$

```
(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2 + b^2
*e^4)*c*e^2)*b*c^2*d*e^2 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c
e^4 + sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*c^3*d*e^2 - 2*(4*c^2*d^2*e^2 -
b^2*e^4)*c^3*d*e^2)*e)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(-4*c^2*d^2*e^(-2)
) + b^2))/c))/((16*c^5*d^5*e^2 - 8*b^2*c^3*d^3*e^4 + 8*b*c^4*d^3*e^4 - 4*c^
5*d^3*e^4 + b^4*c*d*e^6 - 2*b^3*c^2*d*e^6 + b^2*c^3*d*e^6)*abs(c)) + 1/4*(3
2*c^5*d^4*e^4 + 16*sqrt(2)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*
e^2)*c^4*d^4*e^2 - 16*b^2*c^3*d^2*e^6 + 8*b*c^4*d^2*e^6 - 8*sqrt(2)*sqrt(b*
c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b^2*c^2*d^2*e^4 + 8*sqrt(2)*s
qrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b*c^3*d^2*e^4 - 4*sqrt(
2)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*c^4*d^2*e^4 - 4*sqr
t(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^
2*e^4)*c*e^2)*b*c^2*d^2*e^2 - 8*(4*c^2*d^2*e^2 - b^2*e^4)*c^3*d^2*e^2 + 2*b
^4*c*e^8 - 2*b^3*c^2*e^8 + sqrt(2)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2
*e^4)*c*e^2)*b^4*e^6 - 2*sqrt(2)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e
^4)*c*e^2)*b^3*c*e^6 + sqrt(2)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4
)*c*e^2)*b^2*c^2*e^6 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4
- sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b^3*e^4 - 2*sqrt(2)*sqrt(-4*c^2*d^
2*e^2 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b^2*c*
e^4 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^
2*e^2 + b^2*e^4)*c*e^2)*b*c^2*e^4 + 2*(4*c^2*d^2*e^2 - b^2*e^4)*b^2*c*e^4 -
2*(4*c^2*d^2*e^2 - b^2*e^4)*b*c^2*e^4 + 2*(8*c^5*d^3*e^4 - 4*sqrt(2)*sqrt(-
4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^
2)*c^3*d^3 - 2*b^2*c^3*d*e^6 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(
b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b^2*c*d*e^2 - 2*sqrt(2)*sqr
t(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c
*e^2)*b*c^2*d*e^2 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 - s
qrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*c^3*d*e^2 - 2*(4*c^2*d^2*e^2 - b^2*e^4
)*c^3*d*e^2)*e)*arctan(2*sqrt(1/2)*x/sqrt((b - sqrt(-4*c^2*d^2*e^(-2) + b^2
))/c))/((16*c^5*d^5*e^2 - 8*b^2*c^3*d^3*e^4 + 8*b*c^4*d^3*e^4 - 4*c^5*d^3*e
^4 + b^4*c*d*e^6 - 2*b^3*c^2*d*e^6 + b^2*c^3*d*e^6)*abs(c))
```

maple [B] time = 0.08, size = 582, normalized size = 4.34

$$\frac{\sqrt{2} b^4 \operatorname{arctanh}\left(\frac{\sqrt{2} c d}{\sqrt{(b^2 + \sqrt{(b^2 - 2 a d)(b^2 + 2 a d)}}}\right)}{2 \sqrt{(b^2 - 2 a d)(b^2 + 2 a d)^2} \sqrt{(b^2 + \sqrt{(b^2 - 2 a d)(b^2 + 2 a d)}})} c - \frac{\sqrt{2} b^4 \operatorname{arctan}\left(\frac{\sqrt{2} c d}{\sqrt{(b^2 + \sqrt{(b^2 - 2 a d)(b^2 + 2 a d)}}}\right)}{2 \sqrt{(b^2 - 2 a d)(b^2 + 2 a d)^2} \sqrt{(b^2 + \sqrt{(b^2 - 2 a d)(b^2 + 2 a d)}})} c - \frac{\sqrt{2} c d^2 \operatorname{arctanh}\left(\frac{\sqrt{2} c d}{\sqrt{(b^2 + \sqrt{(b^2 - 2 a d)(b^2 + 2 a d)}}}\right)}{\sqrt{(b^2 - 2 a d)(b^2 + 2 a d)^2} \sqrt{(b^2 + \sqrt{(b^2 - 2 a d)(b^2 + 2 a d)}})} c - \frac{\sqrt{2} c d^2 \operatorname{arctan}\left(\frac{\sqrt{2} c d}{\sqrt{(b^2 + \sqrt{(b^2 - 2 a d)(b^2 + 2 a d)}}}\right)}{\sqrt{(b^2 - 2 a d)(b^2 + 2 a d)^2} \sqrt{(b^2 + \sqrt{(b^2 - 2 a d)(b^2 + 2 a d)}})} c + \frac{\sqrt{2} c^2 \operatorname{arctanh}\left(\frac{\sqrt{2} c d}{\sqrt{(b^2 + \sqrt{(b^2 - 2 a d)(b^2 + 2 a d)}}}\right)}{2 \sqrt{(b^2 - 2 a d)(b^2 + 2 a d)^2} \sqrt{(b^2 + \sqrt{(b^2 - 2 a d)(b^2 + 2 a d)}})} c - \frac{\sqrt{2} c^2 \operatorname{arctan}\left(\frac{\sqrt{2} c d}{\sqrt{(b^2 + \sqrt{(b^2 - 2 a d)(b^2 + 2 a d)}}}\right)}{2 \sqrt{(b^2 - 2 a d)(b^2 + 2 a d)^2} \sqrt{(b^2 + \sqrt{(b^2 - 2 a d)(b^2 + 2 a d)}})} c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e\*x^2+d)/(c\*d^2/e^2+b\*x^2+c\*x^4), x)

```
[Out] -1/2*e^4/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((-e^2*b+(e^2*(b*e-2*c
*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctanh(c*e*x^2^(1/2)/((-e^2*b+(e^2*(b*e-2
*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*b-e^3*c/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(
1/2)*2^(1/2)/((-e^2*b+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctanh
(c*e*x^2^(1/2)/((-e^2*b+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*d+1/
2*e^2*2^(1/2)/((-e^2*b+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan
h(c*e*x^2^(1/2)/((-e^2*b+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))-1/2
*e^4/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((e^2*b+(e^2*(b*e-2*c*d)*
(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(c*e*x^2^(1/2)/((e^2*b+(e^2*(b*e-2*c*d)*
(b*e+2*c*d))^(1/2))*c)^(1/2))*b-e^3*c/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^
(1/2)/((e^2*b+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(c*e*x^2^
(1/2)/((e^2*b+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*d-1/2*e^2*2^(1
/2)/((e^2*b+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(c*e*x^2^(1
/2)/((e^2*b+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ex^2 - d}{cx^4 + bx^2 + \frac{cd^2}{e^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((-e\*x^2+d)/(c\*d^2/e^2+b\*x^2+c\*x^4),x, algorithm="maxima")

[Out] -integrate((e\*x^2 - d)/(c\*x^4 + b\*x^2 + c\*d^2/e^2), x)

**mupad [B]** time = 0.18, size = 129, normalized size = 0.96

$$\frac{e^{3/2} \left( \operatorname{atan} \left( \frac{\sqrt{e} x \sqrt{bce-2c^2d}}{be-2cd} \right) + \operatorname{atan} \left( \frac{ce^{3/2} x^3 \sqrt{bce-2c^2d} + be^{3/2} x \sqrt{bce-2c^2d} - cd \sqrt{e} x \sqrt{bce-2c^2d}}{d(2c^2d-bce)} \right) \right)}{\sqrt{bce-2c^2d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d - e\*x^2)/(b\*x^2 + c\*x^4 + (c\*d^2)/e^2),x)

[Out]  $-(e^{3/2}) * (\operatorname{atan}((e^{1/2}) * x * (b * c * e - 2 * c^2 * d)^{1/2}) / (b * e - 2 * c * d)) + \operatorname{atan}((c * e^{3/2}) * x^3 * (b * c * e - 2 * c^2 * d)^{1/2} + b * e^{3/2} * x * (b * c * e - 2 * c^2 * d)^{1/2} - c * d * e^{1/2} * x * (b * c * e - 2 * c^2 * d)^{1/2}) / (d * (2 * c^2 * d - b * c * e))) / (b * c * e - 2 * c^2 * d)^{1/2}$

**sympy [A]** time = 0.86, size = 158, normalized size = 1.18

$$\frac{\sqrt{-\frac{e^3}{c(be-2cd)}} \log\left(\frac{d}{e} + x^2 + \frac{x\left(-be\sqrt{-\frac{e^3}{c(be-2cd)}} + 2cd\sqrt{-\frac{e^3}{c(be-2cd)}}\right)}{e^2}\right)}{2} - \frac{\sqrt{-\frac{e^3}{c(be-2cd)}} \log\left(\frac{d}{e} + x^2 + \frac{x\left(be\sqrt{-\frac{e^3}{c(be-2cd)}} - 2cd\sqrt{-\frac{e^3}{c(be-2cd)}}\right)}{e^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*x\*\*2+d)/(c\*d\*\*2/e\*\*2+b\*x\*\*2+c\*x\*\*4),x)

[Out]  $\sqrt{-e^{3/2}/(c*(b*e - 2*c*d))} * \log(d/e + x^2 + x*(-b*e*\sqrt{-e^{3/2}/(c*(b*e - 2*c*d))} + 2*c*d*\sqrt{-e^{3/2}/(c*(b*e - 2*c*d))})/e^{3/2})/2 - \sqrt{-e^{3/2}/(c*(b*e - 2*c*d))} * \log(d/e + x^2 + x*(b*e*\sqrt{-e^{3/2}/(c*(b*e - 2*c*d))} - 2*c*d*\sqrt{-e^{3/2}/(c*(b*e - 2*c*d))})/e^{3/2})/2$

$$3.23 \quad \int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4} dx$$

**Optimal.** Leaf size=130

$$\frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{ex}}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}} - \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{ex}}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}$$

**Rubi [A]** time = 0.17, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {1161, 618, 204}

$$\frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{ex}}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}} - \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{ex}}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)/((c\*d^2)/e^2 + b\*x^2 + c\*x^4), x]

[Out] -((e^(3/2)\*ArcTan[(Sqrt[2\*c\*d - b\*e] - 2\*Sqrt[c]\*Sqrt[e]\*x)/Sqrt[2\*c\*d + b\*e]]/(Sqrt[c]\*Sqrt[2\*c\*d + b\*e])) + (e^(3/2)\*ArcTan[(Sqrt[2\*c\*d - b\*e] + 2\*Sqrt[c]\*Sqrt[e]\*x)/Sqrt[2\*c\*d + b\*e]]/(Sqrt[c]\*Sqrt[2\*c\*d + b\*e]))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 1161

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

#### Rubi steps

$$\begin{aligned} \int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4} dx &= \frac{e \int \frac{1}{\frac{d}{e} - \frac{\sqrt{2cd-be}x}{\sqrt{c}\sqrt{e}} + x^2} dx}{2c} + \frac{e \int \frac{1}{\frac{d}{e} + \frac{\sqrt{2cd-be}x}{\sqrt{c}\sqrt{e}} + x^2} dx}{2c} \\ &= \frac{e \text{Subst}\left(\int \frac{1}{-\frac{b}{c} - \frac{2d}{e} - x^2} dx, x, -\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}} + 2x\right)}{c} - \frac{e \text{Subst}\left(\int \frac{1}{-\frac{b}{c} - \frac{2d}{e} - x^2} dx, x, \frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}} + 2x\right)}{c} \\ &= -\frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{ex}}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}} + \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{ex}}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}} \end{aligned}$$





$$\frac{2}{((b^2e^2 + ((b^2e - 2cd)(b^2e + 2cd))e^2)^{1/2})c^{1/2}c^2e^x + 1/2e^{1/2}} \\ / ((b^2e^2 + ((b^2e - 2cd)(b^2e + 2cd))e^2)^{1/2})c^{1/2}e^2 \arctan(2^{1/2} / ((b^2e^2 + ((b^2e - 2cd)(b^2e + 2cd))e^2)^{1/2})c^{1/2}c^2e^x)$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2 + d}{cx^4 + bx^2 + \frac{cd^2}{e^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(c\*d^2/e^2+b\*x^2+c\*x^4),x, algorithm="maxima")

[Out] integrate((e\*x^2 + d)/(c\*x^4 + b\*x^2 + c\*d^2/e^2), x)

**mupad** [B] time = 4.52, size = 232, normalized size = 1.78

$$\frac{e^{3/2} \left( \operatorname{atan} \left( \frac{c \sqrt{e} x}{\sqrt{c(b e + 2 c d)}} \right) - \operatorname{atan} \left( \frac{(2 d c^2 + b e c) \left( x \left( \frac{\sqrt{e} (c d e^7 - 4 c^3 d^2 e^7)}{d \sqrt{c(b e + 2 c d)} (b e - 2 c d)} + \frac{e^{3/2} (2 c^2 d e^6 - b c e^7)}{c d \sqrt{2 d c^2 + b e c} (b e - 2 c d)} \right) + \frac{\sqrt{e} x^3 \left( c e^8 - \frac{2 b c^2 e^9}{2 d c^2 + b e c} \right)}{d \sqrt{c(b e + 2 c d)} (b e - 2 c d)} \right)}{c e^7} \right)}{\sqrt{2 d c^2 + b e c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)/(b\*x^2 + c\*x^4 + (c\*d^2)/e^2),x)

[Out] (e^(3/2))\*atan((c\*e^(1/2)\*x)/(c\*(b\*e + 2\*c\*d))^(1/2)) - atan(((2\*c^2\*d + b\*c\*e)\*(x\*((e^(1/2)\*(c\*d\*e^7 - (4\*c^3\*d^2\*e^7)/(2\*c^2\*d + b\*c\*e)))/(d\*(c\*(b\*e + 2\*c\*d))^(1/2)\*(b\*e - 2\*c\*d)) + (e^(3/2)\*(2\*c^2\*d\*e^6 - b\*c\*e^7))/(c\*d\*(2\*c^2\*d + b\*c\*e)^(1/2)\*(b\*e - 2\*c\*d))) + (e^(1/2)\*x^3\*(c\*e^8 - (2\*b\*c^2\*e^9)/(2\*c^2\*d + b\*c\*e)))/(d\*(c\*(b\*e + 2\*c\*d))^(1/2)\*(b\*e - 2\*c\*d))))/(c\*e^7)))/(2\*c^2\*d + b\*c\*e)^(1/2)

**sympy** [A] time = 0.77, size = 160, normalized size = 1.23

$$\frac{\sqrt{-\frac{e^3}{c(b e + 2 c d)}} \log \left( -\frac{d}{e} + x^2 + \frac{x \left( -b e \sqrt{-\frac{e^3}{c(b e + 2 c d)}} - 2 c d \sqrt{-\frac{e^3}{c(b e + 2 c d)}} \right)}{e^2} \right)}{2} + \frac{\sqrt{-\frac{e^3}{c(b e + 2 c d)}} \log \left( -\frac{d}{e} + x^2 + \frac{x \left( b e \sqrt{-\frac{e^3}{c(b e + 2 c d)}} + 2 c d \sqrt{-\frac{e^3}{c(b e + 2 c d)}} \right)}{e^2} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)/(c\*d\*\*2/e\*\*2+b\*x\*\*2+c\*x\*\*4),x)

[Out] -sqrt(-e\*\*3/(c\*(b\*e + 2\*c\*d)))\*log(-d/e + x\*\*2 + x\*(-b\*e\*sqrt(-e\*\*3/(c\*(b\*e + 2\*c\*d)))) - 2\*c\*d\*sqrt(-e\*\*3/(c\*(b\*e + 2\*c\*d))))/e\*\*2)/2 + sqrt(-e\*\*3/(c\*(b\*e + 2\*c\*d)))\*log(-d/e + x\*\*2 + x\*(b\*e\*sqrt(-e\*\*3/(c\*(b\*e + 2\*c\*d)))) + 2\*c\*d\*sqrt(-e\*\*3/(c\*(b\*e + 2\*c\*d))))/e\*\*2)/2

$$3.24 \quad \int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)} dx$$

**Optimal.** Leaf size=130

$$\frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{ex}}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}} - \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{ex}}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}$$

**Rubi [A]** time = 0.13, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {1990, 1161, 618, 204}

$$\frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{ex}}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}} - \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{ex}}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)/(b\*x^2 + c\*(d^2/e^2 + x^4)), x]

[Out] -((e^(3/2)\*ArcTan[(Sqrt[2\*c\*d - b\*e] - 2\*Sqrt[c]\*Sqrt[e]\*x)/Sqrt[2\*c\*d + b\*e]])/(Sqrt[c]\*Sqrt[2\*c\*d + b\*e])) + (e^(3/2)\*ArcTan[(Sqrt[2\*c\*d - b\*e] + 2\*Sqrt[c]\*Sqrt[e]\*x)/Sqrt[2\*c\*d + b\*e]])/(Sqrt[c]\*Sqrt[2\*c\*d + b\*e]))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 1161

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

#### Rule 1990

Int[(u\_)^(q\_.)\*(v\_)^(p\_.), x\_Symbol] := Int[ExpandToSum[u, x]^q\*ExpandToSum[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] && !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])

#### Rubi steps

$$\begin{aligned}
\int \frac{d + ex^2}{bx^2 + c\left(\frac{d^2}{e^2} + x^4\right)} dx &= \int \frac{d + ex^2}{\frac{cd^2}{e^2} + bx^2 + cx^4} dx \\
&= \frac{e \int \frac{1}{\frac{d}{e} - \frac{\sqrt{2cd-be}x}{\sqrt{c}\sqrt{e}} + x^2} dx}{2c} + \frac{e \int \frac{1}{\frac{d}{e} + \frac{\sqrt{2cd-be}x}{\sqrt{c}\sqrt{e}} + x^2} dx}{2c} \\
&= \frac{e \operatorname{Subst}\left(\int \frac{1}{-\frac{b}{c} - \frac{2d}{e} - x^2} dx, x, -\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}} + 2x\right)}{c} - \frac{e \operatorname{Subst}\left(\int \frac{1}{-\frac{b}{c} - \frac{2d}{e} - x^2} dx, x, \frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}} + 2x\right)}{c} \\
&= -\frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{2cd-be} - 2\sqrt{c}\sqrt{e}x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}} + \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{2cd-be} + 2\sqrt{c}\sqrt{e}x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 248, normalized size = 1.91

$$\frac{e^{3/2} \left( \frac{\left(\sqrt{b^2e^2 - 4c^2d^2} - be + 2cd\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{be - \sqrt{b^2e^2 - 4c^2d^2}}}\right)}{\sqrt{be - \sqrt{b^2e^2 - 4c^2d^2}}} + \frac{\left(\sqrt{b^2e^2 - 4c^2d^2} + be - 2cd\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{\sqrt{b^2e^2 - 4c^2d^2} + be}}\right)}{\sqrt{\sqrt{b^2e^2 - 4c^2d^2} + be}} \right)}{\sqrt{2}\sqrt{c}\sqrt{b^2e^2 - 4c^2d^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)/(b\*x^2 + c\*(d^2/e^2 + x^4)), x]

[Out] (e^(3/2)\*(((2\*c\*d - b\*e + Sqrt[-4\*c^2\*d^2 + b^2\*e^2])\*ArcTan[(Sqrt[2]\*Sqrt[c]\*Sqrt[e]\*x)/Sqrt[b\*e - Sqrt[-4\*c^2\*d^2 + b^2\*e^2]]])/Sqrt[b\*e - Sqrt[-4\*c^2\*d^2 + b^2\*e^2]] + ((-2\*c\*d + b\*e + Sqrt[-4\*c^2\*d^2 + b^2\*e^2])\*ArcTan[(Sqrt[2]\*Sqrt[c]\*Sqrt[e]\*x)/Sqrt[b\*e + Sqrt[-4\*c^2\*d^2 + b^2\*e^2]]])/Sqrt[b\*e + Sqrt[-4\*c^2\*d^2 + b^2\*e^2]])/(Sqrt[2]\*Sqrt[c]\*Sqrt[-4\*c^2\*d^2 + b^2\*e^2])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{bx^2 + c\left(\frac{d^2}{e^2} + x^4\right)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)/(b\*x^2 + c\*(d^2/e^2 + x^4)), x]

[Out] IntegrateAlgebraic[(d + e\*x^2)/(b\*x^2 + c\*(d^2/e^2 + x^4)), x]

**fricas [A]** time = 0.75, size = 232, normalized size = 1.78

$$\left[ \frac{1}{2} e \sqrt{\frac{e}{2c^2d + bce}} \log\left(\frac{ce^2x^4 + cd^2 - (4cde + be^2)x^2 + 2((2c^2de + bce^2)x^3 - (2c^2d^2 + bcde)x)\sqrt{\frac{e}{2c^2d + bce}}}{c^2e^2x^4 + be^2x^2 + cd^2}\right), e \sqrt{\frac{e}{2c^2d + bce}} \arctan\left(cx\sqrt{\frac{e}{2c^2d + bce}}\right) + e \sqrt{\frac{e}{2c^2d + bce}} \arctan\left(\frac{(cex^3 + (cd + be)x)\sqrt{\frac{e}{2c^2d + bce}}}{d}\right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(b\*x^2+c\*(d^2/e^2+x^4)), x, algorithm="fricas")

[Out] [1/2\*e\*sqrt(-e/(2\*c^2\*d + b\*c\*e))\*log((c\*e^2\*x^4 + c\*d^2 - (4\*c\*d\*e + b\*e^2)\*x^2 + 2\*((2\*c^2\*d\*e + b\*c\*e^2)\*x^3 - (2\*c^2\*d^2 + b\*c\*d\*e)\*x)\*sqrt(-e/(2\*c^2\*d + b\*c\*e)))/(c\*e^2\*x^4 + b\*e^2\*x^2 + c\*d^2), e\*sqrt(e/(2\*c^2\*d + b\*c\*e))\*arctan(c\*x\*sqrt(e/(2\*c^2\*d + b\*c\*e))) + e\*sqrt(e/(2\*c^2\*d + b\*c\*e))\*arctan((c\*e\*x^3 + (c\*d + b\*e)\*x)\*sqrt(e/(2\*c^2\*d + b\*c\*e))/d)]





**maple [B]** time = 0.01, size = 582, normalized size = 4.48

$$\frac{\sqrt{2} b^d \operatorname{arctanh}\left(\frac{\sqrt{2} a x}{\sqrt{(b^2 + \sqrt{(b-2ad)(b+2ad)})^2}}\right)}{2\sqrt{(b-2ad)(b+2ad)}\sqrt{(b^2 + \sqrt{(b-2ad)(b+2ad)})^2}} + \frac{\sqrt{2} b^d \operatorname{arctan}\left(\frac{\sqrt{2} a x}{\sqrt{(b^2 + \sqrt{(b-2ad)(b+2ad)})^2}}\right)}{2\sqrt{(b-2ad)(b+2ad)}\sqrt{(b^2 + \sqrt{(b-2ad)(b+2ad)})^2}} - \frac{\sqrt{2} c d^2 \operatorname{arctanh}\left(\frac{\sqrt{2} a x}{\sqrt{(b^2 + \sqrt{(b-2ad)(b+2ad)})^2}}\right)}{\sqrt{(b-2ad)(b+2ad)}\sqrt{(b^2 + \sqrt{(b-2ad)(b+2ad)})^2}} - \frac{\sqrt{2} c d^2 \operatorname{arctan}\left(\frac{\sqrt{2} a x}{\sqrt{(b^2 + \sqrt{(b-2ad)(b+2ad)})^2}}\right)}{\sqrt{(b-2ad)(b+2ad)}\sqrt{(b^2 + \sqrt{(b-2ad)(b+2ad)})^2}} + \frac{\sqrt{2} e^d \operatorname{arctanh}\left(\frac{\sqrt{2} a x}{\sqrt{(b^2 + \sqrt{(b-2ad)(b+2ad)})^2}}\right)}{2\sqrt{(b^2 + \sqrt{(b-2ad)(b+2ad)})^2}} + \frac{\sqrt{2} e^d \operatorname{arctan}\left(\frac{\sqrt{2} a x}{\sqrt{(b^2 + \sqrt{(b-2ad)(b+2ad)})^2}}\right)}{2\sqrt{(b^2 + \sqrt{(b-2ad)(b+2ad)})^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)/(b\*x^2+c\*(d^2/e^2+x^4)),x)

[Out]  $\frac{1}{2} e^4 / ((b e - 2 c d) (b e + 2 c d) e^2)^{(1/2)} * 2^{(1/2)} / ((-b e^2 + ((b e - 2 c d) (b e + 2 c d) e^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b e^2 + ((b e - 2 c d) (b e + 2 c d) e^2)^{(1/2)}) * c)^{(1/2)} * c * e * x) * b - e^3 c / ((b e - 2 c d) (b e + 2 c d) e^2)^{(1/2)} * 2^{(1/2)} / ((-b e^2 + ((b e - 2 c d) (b e + 2 c d) e^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b e^2 + ((b e - 2 c d) (b e + 2 c d) e^2)^{(1/2)}) * c)^{(1/2)} * c * e * x) * d - 1/2 * e^2 * 2^{(1/2)} / ((-b e^2 + ((b e - 2 c d) (b e + 2 c d) e^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b e^2 + ((b e - 2 c d) (b e + 2 c d) e^2)^{(1/2)}) * c)^{(1/2)} * c * e * x) + 1/2 / ((b e - 2 c d) (b e + 2 c d) e^2)^{(1/2)} * 2^{(1/2)} / ((b e^2 + ((b e - 2 c d) (b e + 2 c d) e^2)^{(1/2)}) * c)^{(1/2)} * b e^4 * \operatorname{arctan}(2^{(1/2)} / ((b e^2 + ((b e - 2 c d) (b e + 2 c d) e^2)^{(1/2)}) * c)^{(1/2)} * c * e * x) - 1 / ((b e - 2 c d) (b e + 2 c d) e^2)^{(1/2)} * 2^{(1/2)} / ((b e^2 + ((b e - 2 c d) (b e + 2 c d) e^2)^{(1/2)}) * c)^{(1/2)} * c * d * e^3 * \operatorname{arctan}(2^{(1/2)} / ((b e^2 + ((b e - 2 c d) (b e + 2 c d) e^2)^{(1/2)}) * c)^{(1/2)} * c * e * x) + 1/2 * 2^{(1/2)} / ((b e^2 + ((b e - 2 c d) (b e + 2 c d) e^2)^{(1/2)}) * c)^{(1/2)} * e^2 * \operatorname{arctan}(2^{(1/2)} / ((b e^2 + ((b e - 2 c d) (b e + 2 c d) e^2)^{(1/2)}) * c)^{(1/2)} * c * e * x)$

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e x^2 + d}{b x^2 + \left(x^4 + \frac{d^2}{e^2}\right) c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(b\*x^2+c\*(d^2/e^2+x^4)),x, algorithm="maxima")

[Out] integrate((e\*x^2 + d)/(b\*x^2 + (x^4 + d^2/e^2)\*c), x)

**mupad [B]** time = 0.13, size = 232, normalized size = 1.78

$$\frac{e^{3/2} \left( \operatorname{atan}\left(\frac{c \sqrt{e} x}{\sqrt{c(b e + 2 c d)}}\right) - \operatorname{atan}\left(\frac{(2 d c^2 + b e c) \left( x \left( \frac{\sqrt{e} \left( c d e^7 - \frac{4 c^3 d^2 e^7}{2 d c^2 + b e c} \right) + \frac{e^{3/2} (2 c^2 d e^6 - b c e^7)}{c d \sqrt{2 d c^2 + b e c} (b e - 2 c d)} \right) + \frac{\sqrt{e} x^3 \left( c e^8 - \frac{2 b c^2 e^9}{2 d c^2 + b e c} \right)}{d \sqrt{c(b e + 2 c d) (b e - 2 c d)}} \right)}{c e^7} \right)}{\sqrt{2 d c^2 + b e c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)/(b\*x^2 + c\*(x^4 + d^2/e^2)),x)

[Out]  $(e^{(3/2)} * (\operatorname{atan}((c * e^{(1/2)} * x) / (c * (b * e + 2 * c * d)))^{(1/2)}) - \operatorname{atan}(((2 * c^2 * d + b * c * e) * (x * ((e^{(1/2)} * (c * d * e^7 - (4 * c^3 * d^2 * e^7) / (2 * c^2 * d + b * c * e))) / (d * (c * (b * e + 2 * c * d))^{(1/2)} * (b * e - 2 * c * d)) + (e^{(3/2)} * (2 * c^2 * d * e^6 - b * c * e^7)) / (c * d * (2 * c^2 * d + b * c * e)^{(1/2)} * (b * e - 2 * c * d))) + (e^{(1/2)} * x^3 * (c * e^8 - (2 * b * c^2 * e^9) / (2 * c^2 * d + b * c * e))) / (d * (c * (b * e + 2 * c * d))^{(1/2)} * (b * e - 2 * c * d)))) / (c * e^7))) / (2 * c^2 * d + b * c * e)^{(1/2)}$

**sympy [A]** time = 0.79, size = 160, normalized size = 1.23

$$\frac{\sqrt{-\frac{e^3}{c(b e + 2 c d)}} \log\left(-\frac{d}{e} + x^2 + \frac{x \left( -b e \sqrt{-\frac{e^3}{c(b e + 2 c d)}} - 2 c d \sqrt{-\frac{e^3}{c(b e + 2 c d)}} \right)}{e^2}\right)}{2} + \frac{\sqrt{\frac{e^3}{c(b e + 2 c d)}} \log\left(-\frac{d}{e} + x^2 + \frac{x \left( b e \sqrt{-\frac{e^3}{c(b e + 2 c d)}} + 2 c d \sqrt{-\frac{e^3}{c(b e + 2 c d)}} \right)}{e^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(b*x**2+c*(d**2/e**2+x**4)),x)
```

```
[Out] -sqrt(-e**3/(c*(b*e + 2*c*d)))*log(-d/e + x**2 + x*(-b*e*sqrt(-e**3/(c*(b*e + 2*c*d)))) - 2*c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/e**2)/2 + sqrt(-e**3/(c*(b*e + 2*c*d)))*log(-d/e + x**2 + x*(b*e*sqrt(-e**3/(c*(b*e + 2*c*d)))) + 2*c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/e**2)/2
```

$$3.25 \quad \int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4} dx$$

**Optimal.** Leaf size=29

$$\frac{1}{2} \log(a + bx^2 + x) - \frac{1}{2} \log(a + bx^2 - x)$$

**Rubi [A]** time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$ , Rules used = {1164, 628}

$$\frac{1}{2} \log(a + bx^2 + x) - \frac{1}{2} \log(a + bx^2 - x)$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)/(a^2 + (-1 + 2\*a\*b)\*x^2 + b^2\*x^4), x]

[Out] -Log[a - x + b\*x^2]/2 + Log[a + x + b\*x^2]/2

**Rule 628**

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

**Rule 1164**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(-2\*d)/e - b/c, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && !GtQ[b^2 - 4\*a\*c, 0]

**Rubi steps**

$$\begin{aligned} \int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4} dx &= -\left(\frac{1}{2} \int \frac{\frac{1}{b} + 2x}{-\frac{a}{b} - \frac{x}{b} - x^2} dx\right) - \frac{1}{2} \int \frac{\frac{1}{b} - 2x}{-\frac{a}{b} + \frac{x}{b} - x^2} dx \\ &= -\frac{1}{2} \log(a - x + bx^2) + \frac{1}{2} \log(a + x + bx^2) \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 29, normalized size = 1.00

$$\frac{1}{2} \log(a + bx^2 + x) - \frac{1}{2} \log(a + bx^2 - x)$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)/(a^2 + (-1 + 2\*a\*b)\*x^2 + b^2\*x^4), x]

[Out] -1/2\*Log[a - x + b\*x^2] + Log[a + x + b\*x^2]/2

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a - b\*x^2)/(a^2 + (-1 + 2\*a\*b)\*x^2 + b^2\*x^4), x]  
 [Out] IntegrateAlgebraic[(a - b\*x^2)/(a^2 + (-1 + 2\*a\*b)\*x^2 + b^2\*x^4), x]  
**fricas** [A] time = 0.68, size = 25, normalized size = 0.86

$$\frac{1}{2} \log(bx^2 + a + x) - \frac{1}{2} \log(bx^2 + a - x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2+a)/(a^2+(2\*a\*b-1)\*x^2+b^2\*x^4), x, algorithm="fricas")  
 [Out] 1/2\*log(b\*x^2 + a + x) - 1/2\*log(b\*x^2 + a - x)  
**giac** [A] time = 0.24, size = 25, normalized size = 0.86

$$\frac{1}{2} \log(bx^2 + a + x) - \frac{1}{2} \log(bx^2 + a - x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2+a)/(a^2+(2\*a\*b-1)\*x^2+b^2\*x^4), x, algorithm="giac")  
 [Out] 1/2\*log(b\*x^2 + a + x) - 1/2\*log(b\*x^2 + a - x)  
**maple** [A] time = 0.01, size = 26, normalized size = 0.90

$$-\frac{\ln(bx^2 + a - x)}{2} + \frac{\ln(bx^2 + a + x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^2+a)/(a^2+(2\*a\*b-1)\*x^2+b^2\*x^4), x)  
 [Out] -1/2\*ln(b\*x^2+a-x)+1/2\*ln(b\*x^2+a+x)  
**maxima** [A] time = 1.04, size = 25, normalized size = 0.86

$$\frac{1}{2} \log(bx^2 + a + x) - \frac{1}{2} \log(bx^2 + a - x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2+a)/(a^2+(2\*a\*b-1)\*x^2+b^2\*x^4), x, algorithm="maxima")  
 [Out] 1/2\*log(b\*x^2 + a + x) - 1/2\*log(b\*x^2 + a - x)  
**mupad** [B] time = 4.41, size = 12, normalized size = 0.41

$$\operatorname{atanh}\left(\frac{x}{bx^2 + a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - b\*x^2)/(x^2\*(2\*a\*b - 1) + a^2 + b^2\*x^4), x)  
 [Out] atanh(x/(a + b\*x^2))  
**sympy** [A] time = 0.47, size = 26, normalized size = 0.90

$$-\frac{\log\left(\frac{a}{b} + x^2 - \frac{x}{b}\right)}{2} + \frac{\log\left(\frac{a}{b} + x^2 + \frac{x}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*2+a)/(a\*\*2+(2\*a\*b-1)\*x\*\*2+b\*\*2\*x\*\*4), x)  
 [Out] -log(a/b + x\*\*2 - x/b)/2 + log(a/b + x\*\*2 + x/b)/2

$$3.26 \quad \int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4} dx$$

Optimal. Leaf size=60

$$\frac{\tanh^{-1}\left(\frac{1-2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}} - \frac{\tanh^{-1}\left(\frac{2bx+1}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}$$

Rubi [A] time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{1-2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}} - \frac{\tanh^{-1}\left(\frac{2bx+1}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(a^2 + (-1 + 2\*a\*b)\*x^2 + b^2\*x^4), x]

[Out] ArcTanh[(1 - 2\*b\*x)/Sqrt[1 - 4\*a\*b]]/Sqrt[1 - 4\*a\*b] - ArcTanh[(1 + 2\*b\*x)/Sqrt[1 - 4\*a\*b]]/Sqrt[1 - 4\*a\*b]

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 1161

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

Rubi steps

$$\begin{aligned} \int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4} dx &= \frac{\int \frac{1}{\frac{a}{b}-\frac{x}{b}+x^2} dx}{2b} + \frac{\int \frac{1}{\frac{a}{b}+\frac{x}{b}+x^2} dx}{2b} \\ &= -\frac{\text{Subst}\left(\int \frac{1}{\frac{1-4ab}{b^2}-x^2} dx, x, -\frac{1}{b} + 2x\right)}{b} - \frac{\text{Subst}\left(\int \frac{1}{\frac{1-4ab}{b^2}-x^2} dx, x, \frac{1}{b} + 2x\right)}{b} \\ &= \frac{\tanh^{-1}\left(\frac{1-2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}} - \frac{\tanh^{-1}\left(\frac{1+2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}} \end{aligned}$$

**Mathematica [B]** time = 0.20, size = 138, normalized size = 2.30

$$\frac{(\sqrt{1-4ab}+1) \tan^{-1}\left(\frac{bx}{\sqrt{ab-\frac{1}{2}}\sqrt{1-4ab-\frac{1}{2}}}\right) + (\sqrt{1-4ab}-1) \tan^{-1}\left(\frac{\sqrt{2}bx}{\sqrt{2ab+\sqrt{1-4ab}-1}}\right)}{\frac{\sqrt{2ab-\sqrt{1-4ab}-1}}{\sqrt{2-8ab}} + \frac{\sqrt{2ab+\sqrt{1-4ab}-1}}{\sqrt{2-8ab}}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(a^2 + (-1 + 2\*a\*b)\*x^2 + b^2\*x^4), x]

[Out] (((1 + Sqrt[1 - 4\*a\*b])\*ArcTan[(b\*x)/Sqrt[-1/2 + a\*b - Sqrt[1 - 4\*a\*b]/2]])/Sqrt[-1 + 2\*a\*b - Sqrt[1 - 4\*a\*b]] + ((-1 + Sqrt[1 - 4\*a\*b])\*ArcTan[(Sqrt[2]\*b\*x)/Sqrt[-1 + 2\*a\*b + Sqrt[1 - 4\*a\*b]])]/Sqrt[-1 + 2\*a\*b + Sqrt[1 - 4\*a\*b]])/Sqrt[2 - 8\*a\*b]

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2}{a^2 + (-1 + 2ab)x^2 + b^2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b\*x^2)/(a^2 + (-1 + 2\*a\*b)\*x^2 + b^2\*x^4), x]

[Out] IntegrateAlgebraic[(a + b\*x^2)/(a^2 + (-1 + 2\*a\*b)\*x^2 + b^2\*x^4), x]

**fricas [A]** time = 0.85, size = 164, normalized size = 2.73

$$\left[ \frac{\sqrt{-4ab+1} \log\left(\frac{b^2x^4 - (6ab-1)x^2 + a^2 - 2(bx^3 - ax)\sqrt{-4ab+1}}{b^2x^4 + (2ab-1)x^2 + a^2}\right)}{2(4ab-1)}, \frac{\sqrt{4ab-1} \arctan\left(\frac{bx}{\sqrt{4ab-1}}\right) + \sqrt{4ab-1} \arctan\left(\frac{(b^2x^3 + (3ab-1)x)\sqrt{4ab-1}}{4a^2b-a}\right)}{4ab-1} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/(a^2+(2\*a\*b-1)\*x^2+b^2\*x^4), x, algorithm="fricas")

[Out] [-1/2\*sqrt(-4\*a\*b + 1)\*log((b^2\*x^4 - (6\*a\*b - 1)\*x^2 + a^2 - 2\*(b\*x^3 - a\*x)\*sqrt(-4\*a\*b + 1))/(b^2\*x^4 + (2\*a\*b - 1)\*x^2 + a^2))/(4\*a\*b - 1), (sqrt(4\*a\*b - 1)\*arctan(b\*x/sqrt(4\*a\*b - 1)) + sqrt(4\*a\*b - 1)\*arctan((b^2\*x^3 + (3\*a\*b - 1)\*x)\*sqrt(4\*a\*b - 1)/(4\*a^2\*b - a)))/(4\*a\*b - 1)]

**giac [A]** time = 0.18, size = 51, normalized size = 0.85

$$\frac{\arctan\left(\frac{2bx+1}{\sqrt{4ab-1}}\right)}{\sqrt{4ab-1}} + \frac{\arctan\left(\frac{2bx-1}{\sqrt{4ab-1}}\right)}{\sqrt{4ab-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/(a^2+(2\*a\*b-1)\*x^2+b^2\*x^4), x, algorithm="giac")

[Out] arctan((2\*b\*x + 1)/sqrt(4\*a\*b - 1))/sqrt(4\*a\*b - 1) + arctan((2\*b\*x - 1)/sqrt(4\*a\*b - 1))/sqrt(4\*a\*b - 1)

**maple [A]** time = 0.01, size = 52, normalized size = 0.87

$$\frac{\arctan\left(\frac{2bx-1}{\sqrt{4ab-1}}\right)}{\sqrt{4ab-1}} + \frac{\arctan\left(\frac{2bx+1}{\sqrt{4ab-1}}\right)}{\sqrt{4ab-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x)`

[Out]  $1/(4*a*b-1)^{(1/2)}*\arctan((2*b*x-1)/(4*a*b-1)^{(1/2)})+1/(4*a*b-1)^{(1/2)}*\arctan((2*b*x+1)/(4*a*b-1)^{(1/2)})$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(a\*b-0.25>0)', see `assume?` for more details)Is a\*b-0.25 positive or negative?

**mupad** [B] time = 0.07, size = 55, normalized size = 0.92

$$\frac{\operatorname{atan}\left(\frac{bx}{\sqrt{4ab-1}}\right) + \operatorname{atan}\left(\frac{\frac{3x(4ab-1)}{4} - \frac{x}{4} + b^2x^3}{a\sqrt{4ab-1}}\right)}{\sqrt{4ab-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^2)/(x^2*(2*a*b - 1) + a^2 + b^2*x^4),x)`

[Out]  $(\operatorname{atan}((b*x)/(4*a*b - 1)^{(1/2)}) + \operatorname{atan}(((3*x*(4*a*b - 1))/4 - x/4 + b^2*x^3)/(a*(4*a*b - 1)^{(1/2)})))/(4*a*b - 1)^{(1/2)}$

**sympy** [B] time = 0.46, size = 117, normalized size = 1.95

$$\frac{\sqrt{-\frac{1}{4ab-1}} \log\left(-\frac{a}{b} + x^2 + \frac{x\left(-4ab\sqrt{-\frac{1}{4ab-1}} + \sqrt{-\frac{1}{4ab-1}}\right)}{b}\right)}{2} + \frac{\sqrt{-\frac{1}{4ab-1}} \log\left(-\frac{a}{b} + x^2 + \frac{x\left(4ab\sqrt{-\frac{1}{4ab-1}} - \sqrt{-\frac{1}{4ab-1}}\right)}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)/(a**2+(2*a*b-1)*x**2+b**2*x**4),x)`

[Out]  $-\operatorname{sqrt}(-1/(4*a*b - 1))*\log(-a/b + x**2 + x*(-4*a*b*\operatorname{sqrt}(-1/(4*a*b - 1)) + \operatorname{sqrt}(-1/(4*a*b - 1)))/b)/2 + \operatorname{sqrt}(-1/(4*a*b - 1))*\log(-a/b + x**2 + x*(4*a*b*\operatorname{sqrt}(-1/(4*a*b - 1)) - \operatorname{sqrt}(-1/(4*a*b - 1)))/b)/2$

$$3.27 \quad \int \frac{1+2x^2}{1+bx^2+4x^4} dx$$

Optimal. Leaf size=62

$$\frac{\tan^{-1}\left(\frac{\sqrt{4-b}+4x}{\sqrt{b+4}}\right)}{\sqrt{b+4}} - \frac{\tan^{-1}\left(\frac{\sqrt{4-b}-4x}{\sqrt{b+4}}\right)}{\sqrt{b+4}}$$

**Rubi [A]** time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{4-b}+4x}{\sqrt{b+4}}\right)}{\sqrt{b+4}} - \frac{\tan^{-1}\left(\frac{\sqrt{4-b}-4x}{\sqrt{b+4}}\right)}{\sqrt{b+4}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2\*x^2)/(1 + b\*x^2 + 4\*x^4), x]

[Out] -(ArcTan[(Sqrt[4 - b] - 4\*x)/Sqrt[4 + b]]/Sqrt[4 + b]) + ArcTan[(Sqrt[4 - b] + 4\*x)/Sqrt[4 + b]]/Sqrt[4 + b]

#### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 1161

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

#### Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1+bx^2+4x^4} dx &= \frac{1}{4} \int \frac{1}{\frac{1}{2} - \frac{1}{2}\sqrt{4-b}x + x^2} dx + \frac{1}{4} \int \frac{1}{\frac{1}{2} + \frac{1}{2}\sqrt{4-b}x + x^2} dx \\ &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{1}{\frac{1}{4}(-4-b) - x^2} dx, x, -\frac{\sqrt{4-b}}{2} + 2x\right)\right) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{\frac{1}{4}(-4-b) - x^2} dx, x, \right. \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{4-b}-4x}{\sqrt{4+b}}\right)}{\sqrt{4+b}} + \frac{\tan^{-1}\left(\frac{\sqrt{4-b}+4x}{\sqrt{4+b}}\right)}{\sqrt{4+b}} \end{aligned}$$



**Mathematica [B]** time = 0.06, size = 126, normalized size = 2.03

$$\frac{\frac{(\sqrt{b^2-16}-b+4)\tan^{-1}\left(\frac{2\sqrt{2}x}{\sqrt{b-\sqrt{b^2-16}}}\right)}{\sqrt{b-\sqrt{b^2-16}}} + \frac{(\sqrt{b^2-16}+b-4)\tan^{-1}\left(\frac{2\sqrt{2}x}{\sqrt{\sqrt{b^2-16}+b}}\right)}{\sqrt{\sqrt{b^2-16}+b}}}{\sqrt{2}\sqrt{b^2-16}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2\*x^2)/(1 + b\*x^2 + 4\*x^4), x]

[Out] (((4 - b + Sqrt[-16 + b^2])\*ArcTan[(2\*Sqrt[2]\*x)/Sqrt[b - Sqrt[-16 + b^2]]])/Sqrt[b - Sqrt[-16 + b^2]] + ((-4 + b + Sqrt[-16 + b^2])\*ArcTan[(2\*Sqrt[2]\*x)/Sqrt[b + Sqrt[-16 + b^2]]])/Sqrt[b + Sqrt[-16 + b^2]])/(Sqrt[2]\*Sqrt[-16 + b^2])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + 2x^2}{1 + bx^2 + 4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 2\*x^2)/(1 + b\*x^2 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(1 + 2\*x^2)/(1 + b\*x^2 + 4\*x^4), x]

**fricas [A]** time = 0.83, size = 110, normalized size = 1.77

$$\left[ \frac{\sqrt{-b-4} \log\left(\frac{4x^4-(b+8)x^2-2(2x^3-x)\sqrt{-b-4}+1}{4x^4+bx^2+1}\right)}{2(b+4)}, \frac{\sqrt{b+4} \arctan\left(\frac{4x^3+(b+2)x}{\sqrt{b+4}}\right) + \sqrt{b+4} \arctan\left(\frac{2x}{\sqrt{b+4}}\right)}{b+4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4+b\*x^2+1), x, algorithm="fricas")

[Out] [-1/2\*sqrt(-b - 4)\*log((4\*x^4 - (b + 8)\*x^2 - 2\*(2\*x^3 - x)\*sqrt(-b - 4) + 1)/(4\*x^4 + b\*x^2 + 1))/(b + 4), (sqrt(b + 4)\*arctan((4\*x^3 + (b + 2)\*x)/sqrt(b + 4)) + sqrt(b + 4)\*arctan(2\*x/sqrt(b + 4)))/(b + 4)]

**giac [A]** time = 0.31, size = 77, normalized size = 1.24

$$\frac{\sqrt{b+4}(b-8)\arctan\left(\frac{4\sqrt{\frac{1}{2}}x}{\sqrt{b+\sqrt{b^2-16}}}\right)}{b^2-4b-32} + \frac{\sqrt{b+4}(b-8)\arctan\left(\frac{4\sqrt{\frac{1}{2}}x}{\sqrt{b-\sqrt{b^2-16}}}\right)}{b^2-4b-32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4+b\*x^2+1), x, algorithm="giac")

[Out] sqrt(b + 4)\*(b - 8)\*arctan(4\*sqrt(1/2)\*x/sqrt(b + sqrt(b^2 - 16)))/(b^2 - 4\*b - 32) + sqrt(b + 4)\*(b - 8)\*arctan(4\*sqrt(1/2)\*x/sqrt(b - sqrt(b^2 - 16)))/(b^2 - 4\*b - 32)

**maple [B]** time = 0.04, size = 277, normalized size = 4.47

$$\frac{b \arctan\left(\frac{4x}{\sqrt{2b-2\sqrt{(b-4)(b+4)}}}\right)}{\sqrt{(b-4)(b+4)}\sqrt{2b-2\sqrt{(b-4)(b+4)}}} + \frac{b \arctan\left(\frac{4x}{\sqrt{2b+2\sqrt{(b-4)(b+4)}}}\right)}{\sqrt{(b-4)(b+4)}\sqrt{2b+2\sqrt{(b-4)(b+4)}}} + \frac{4 \arctan\left(\frac{4x}{\sqrt{2b-2\sqrt{(b-4)(b+4)}}}\right)}{\sqrt{(b-4)(b+4)}\sqrt{2b-2\sqrt{(b-4)(b+4)}}} + \frac{\arctan\left(\frac{4x}{\sqrt{2b-2\sqrt{(b-4)(b+4)}}}\right)}{\sqrt{(b-4)(b+4)}\sqrt{2b-2\sqrt{(b-4)(b+4)}}} - \frac{4 \arctan\left(\frac{4x}{\sqrt{2b+2\sqrt{(b-4)(b+4)}}}\right)}{\sqrt{(b-4)(b+4)}\sqrt{2b+2\sqrt{(b-4)(b+4)}}} + \frac{\arctan\left(\frac{4x}{\sqrt{2b+2\sqrt{(b-4)(b+4)}}}\right)}{\sqrt{(b-4)(b+4)}\sqrt{2b+2\sqrt{(b-4)(b+4)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2+1)/(4*x^4+b*x^2+1),x)`

[Out] 
$$\frac{4/((b-4)*(4+b))^{1/2}/(-2*((b-4)*(4+b))^{1/2}+2*b)^{1/2}*\arctan(4*x/(-2*((b-4)*(4+b))^{1/2}+2*b)^{1/2})+1/(-2*((b-4)*(4+b))^{1/2}+2*b)^{1/2}*\arctan(4*x/(-2*((b-4)*(4+b))^{1/2}+2*b)^{1/2})-1/((b-4)*(4+b))^{1/2}/(-2*((b-4)*(4+b))^{1/2}+2*b)^{1/2}*\arctan(4*x/(-2*((b-4)*(4+b))^{1/2}+2*b)^{1/2})*b-4/((b-4)*(4+b))^{1/2}/(2*((b-4)*(4+b))^{1/2}+2*b)^{1/2}*\arctan(4*x/(2*((b-4)*(4+b))^{1/2}+2*b)^{1/2})+1/(2*((b-4)*(4+b))^{1/2}+2*b)^{1/2}*\arctan(4*x/(2*((b-4)*(4+b))^{1/2}+2*b)^{1/2})+1/((b-4)*(4+b))^{1/2}/(2*((b-4)*(4+b))^{1/2}+2*b)^{1/2}*\arctan(4*x/(2*((b-4)*(4+b))^{1/2}+2*b)^{1/2})*b}{1}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2 + 1}{4x^4 + bx^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="maxima")`

[Out] `integrate((2*x^2 + 1)/(4*x^4 + b*x^2 + 1), x)`

**mupad** [B] time = 4.39, size = 66, normalized size = 1.06

$$\frac{\operatorname{atan}\left(\frac{-b^3x-4b^2x^3-2b^2x+16bx+64x^3+32x}{(b^2-16)\sqrt{b+4}}\right) - \operatorname{atan}\left(\frac{2x}{\sqrt{b+4}}\right)}{\sqrt{b+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 + 1)/(b*x^2 + 4*x^4 + 1),x)`

[Out] 
$$-\left(\operatorname{atan}\left(\frac{32*x + 16*b*x - 2*b^2*x - b^3*x + 64*x^3 - 4*b^2*x^3}{(b^2 - 16)*(b + 4)^{1/2}}\right)\right) - \operatorname{atan}\left(\frac{2*x}{(b + 4)^{1/2}}\right)/(b + 4)^{1/2}$$

**sympy** [A] time = 0.38, size = 95, normalized size = 1.53

$$\frac{\sqrt{-\frac{1}{b+4}} \log\left(x^2 + x\left(-\frac{b\sqrt{-\frac{1}{b+4}}}{2} - 2\sqrt{-\frac{1}{b+4}}\right) - \frac{1}{2}\right)}{2} + \frac{\sqrt{-\frac{1}{b+4}} \log\left(x^2 + x\left(\frac{b\sqrt{-\frac{1}{b+4}}}{2} + 2\sqrt{-\frac{1}{b+4}}\right) - \frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2+1)/(4*x**4+b*x**2+1),x)`

[Out] 
$$-\sqrt{-1/(b+4)}*\log(x**2 + x*(-b*\sqrt{-1/(b+4)}/2 - 2*\sqrt{-1/(b+4)}) - 1/2)/2 + \sqrt{-1/(b+4)}*\log(x**2 + x*(b*\sqrt{-1/(b+4)}/2 + 2*\sqrt{-1/(b+4)}) - 1/2)/2$$

$$3.28 \quad \int \frac{1+2x^2}{1-bx^2+4x^4} dx$$

Optimal. Leaf size=66

$$\frac{\tan^{-1}\left(\frac{\sqrt{b+4}+4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}} - \frac{\tan^{-1}\left(\frac{\sqrt{b+4}-4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}$$

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$ , Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b+4}+4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}} - \frac{\tan^{-1}\left(\frac{\sqrt{b+4}-4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2\*x^2)/(1 - b\*x^2 + 4\*x^4), x]

[Out] -(ArcTan[(Sqrt[4 + b] - 4\*x)/Sqrt[4 - b]]/Sqrt[4 - b]) + ArcTan[(Sqrt[4 + b] + 4\*x)/Sqrt[4 - b]]/Sqrt[4 - b]

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 1161

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1-bx^2+4x^4} dx &= \frac{1}{4} \int \frac{1}{\frac{1}{2} - \frac{1}{2}\sqrt{4+b}x + x^2} dx + \frac{1}{4} \int \frac{1}{\frac{1}{2} + \frac{1}{2}\sqrt{4+b}x + x^2} dx \\ &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{1}{\frac{1}{4}(-4+b) - x^2} dx, x, -\frac{\sqrt{4+b}}{2} + 2x\right)\right) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{\frac{1}{4}(-4+b) - x^2} dx, x, \frac{\sqrt{4+b}}{2} + 2x\right) \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{4+b}-4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}} + \frac{\tan^{-1}\left(\frac{\sqrt{4+b}+4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}} \end{aligned}$$

**Mathematica [B]** time = 0.06, size = 134, normalized size = 2.03

$$\frac{\left(\sqrt{b^2-16}+b+4\right)\tan^{-1}\left(\frac{2\sqrt{2}x}{\sqrt{-\sqrt{b^2-16}-b}}\right)}{\sqrt{-\sqrt{b^2-16}-b}} + \frac{\left(\sqrt{b^2-16}-b-4\right)\tan^{-1}\left(\frac{2\sqrt{2}x}{\sqrt{\sqrt{b^2-16}-b}}\right)}{\sqrt{\sqrt{b^2-16}-b}}$$

$$\frac{\hspace{10em}}{\sqrt{2}\sqrt{b^2-16}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2\*x^2)/(1 - b\*x^2 + 4\*x^4), x]

[Out] (((4 + b + Sqrt[-16 + b^2])\*ArcTan[(2\*Sqrt[2]\*x)/Sqrt[-b - Sqrt[-16 + b^2]]])/Sqrt[-b - Sqrt[-16 + b^2]] + ((-4 - b + Sqrt[-16 + b^2])\*ArcTan[(2\*Sqrt[2]\*x)/Sqrt[-b + Sqrt[-16 + b^2]]])/Sqrt[-b + Sqrt[-16 + b^2]])/(Sqrt[2]\*Sqrt[-16 + b^2])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + 2x^2}{1 - bx^2 + 4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 2\*x^2)/(1 - b\*x^2 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(1 + 2\*x^2)/(1 - b\*x^2 + 4\*x^4), x]

**fricas [A]** time = 0.80, size = 120, normalized size = 1.82

$$\left[ \frac{\log\left(\frac{4x^4+(b-8)x^2-2(2x^3-x)\sqrt{b-4}+1}{4x^4-bx^2+1}\right)}{2\sqrt{b-4}}, \frac{\sqrt{-b+4}\arctan\left(\frac{(4x^3-(b-2)x)\sqrt{-b+4}}{b-4}\right) + \sqrt{-b+4}\arctan\left(\frac{2\sqrt{-b+4}x}{b-4}\right)}{b-4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4-b\*x^2+1), x, algorithm="fricas")

[Out] [1/2\*log((4\*x^4 + (b - 8)\*x^2 - 2\*(2\*x^3 - x)\*sqrt(b - 4) + 1)/(4\*x^4 - b\*x^2 + 1))/sqrt(b - 4), (sqrt(-b + 4)\*arctan((4\*x^3 - (b - 2)\*x)\*sqrt(-b + 4)/(b - 4)) + sqrt(-b + 4)\*arctan(2\*sqrt(-b + 4)\*x/(b - 4)))/(b - 4)]

**giac [A]** time = 0.31, size = 80, normalized size = 1.21

$$\frac{(b+8)\sqrt{-b+4}\arctan\left(\frac{x}{\sqrt{-\frac{1}{8}b+\frac{1}{8}\sqrt{b^2-16}}}\right)}{b^2+4b-32} - \frac{(b+8)\sqrt{-b+4}\arctan\left(\frac{x}{\sqrt{-\frac{1}{8}b-\frac{1}{8}\sqrt{b^2-16}}}\right)}{b^2+4b-32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4-b\*x^2+1), x, algorithm="giac")

[Out] (b + 8)\*sqrt(-b + 4)\*arctan(x/sqrt(-1/8\*b + 1/8\*sqrt(b^2 - 16)))/(b^2 + 4\*b - 32) - (b + 8)\*sqrt(-b + 4)\*arctan(x/sqrt(-1/8\*b - 1/8\*sqrt(b^2 - 16)))/(b^2 + 4\*b - 32)

**maple [B]** time = 0.03, size = 277, normalized size = 4.20

$$\frac{b\arctan\left(\frac{4x}{\sqrt{-2b-2\sqrt{(b-4)(b+4)}}}\right)}{\sqrt{(b-4)(b+4)}\sqrt{-2b-2\sqrt{(b-4)(b+4)}}} - \frac{b\arctan\left(\frac{4x}{\sqrt{-2b+2\sqrt{(b-4)(b+4)}}}\right)}{\sqrt{(b-4)(b+4)}\sqrt{-2b+2\sqrt{(b-4)(b+4)}}} + \frac{4\arctan\left(\frac{4x}{\sqrt{-2b-2\sqrt{(b-4)(b+4)}}}\right)}{\sqrt{(b-4)(b+4)}\sqrt{-2b-2\sqrt{(b-4)(b+4)}}} + \frac{\arctan\left(\frac{4x}{\sqrt{-2b-2\sqrt{(b-4)(b+4)}}}\right)}{\sqrt{-2b-2\sqrt{(b-4)(b+4)}}} - \frac{4\arctan\left(\frac{4x}{\sqrt{-2b+2\sqrt{(b-4)(b+4)}}}\right)}{\sqrt{(b-4)(b+4)}\sqrt{-2b+2\sqrt{(b-4)(b+4)}}} + \frac{\arctan\left(\frac{4x}{\sqrt{-2b+2\sqrt{(b-4)(b+4)}}}\right)}{\sqrt{-2b+2\sqrt{(b-4)(b+4)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2+1)/(4\*x^4-b\*x^2+1),x)

[Out] 
$$-4/((b-4)*(b+4))^{1/2}/(2*((b-4)*(b+4))^{1/2}-2*b)^{1/2}*\arctan(4*x/(2*((b-4)*(b+4))^{1/2}-2*b)^{1/2})+1/(2*((b-4)*(b+4))^{1/2}-2*b)^{1/2}*\arctan(4*x/(2*((b-4)*(b+4))^{1/2}-2*b)^{1/2})-1/((b-4)*(b+4))^{1/2}/(2*((b-4)*(b+4))^{1/2}-2*b)^{1/2}*\arctan(4*x/(2*((b-4)*(b+4))^{1/2}-2*b)^{1/2})*b+4/((b-4)*(b+4))^{1/2}/(-2*((b-4)*(b+4))^{1/2}-2*b)^{1/2}*\arctan(4*x/(-2*((b-4)*(b+4))^{1/2}-2*b)^{1/2})+1/(-2*((b-4)*(b+4))^{1/2}-2*b)^{1/2}*\arctan(4*x/(-2*((b-4)*(b+4))^{1/2}-2*b)^{1/2})+1/((b-4)*(b+4))^{1/2}/(-2*((b-4)*(b+4))^{1/2}-2*b)^{1/2}*\arctan(4*x/(-2*((b-4)*(b+4))^{1/2}-2*b)^{1/2})*b$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2 + 1}{4x^4 - bx^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4-b\*x^2+1),x, algorithm="maxima")

[Out] integrate((2\*x^2 + 1)/(4\*x^4 - b\*x^2 + 1), x)

**mupad** [B] time = 4.41, size = 24, normalized size = 0.36

$$\frac{\operatorname{atanh}\left(\frac{x\sqrt{b-4}}{2x^2-1}\right)}{\sqrt{b-4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 + 1)/(4\*x^4 - b\*x^2 + 1),x)

[Out] 
$$-\operatorname{atanh}\left(\frac{x*(b-4)^{1/2}}{(2*x^2-1)}\right)/(b-4)^{1/2}$$

**sympy** [A] time = 0.39, size = 83, normalized size = 1.26

$$\frac{\sqrt{\frac{1}{b-4}} \log\left(x^2 + x\left(-\frac{b\sqrt{\frac{1}{b-4}}}{2} + 2\sqrt{\frac{1}{b-4}}\right) - \frac{1}{2}\right)}{2} - \frac{\sqrt{\frac{1}{b-4}} \log\left(x^2 + x\left(\frac{b\sqrt{\frac{1}{b-4}}}{2} - 2\sqrt{\frac{1}{b-4}}\right) - \frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2+1)/(4\*x\*\*4-b\*x\*\*2+1),x)

[Out] 
$$\sqrt{1/(b-4)}*\log(x**2 + x*(-b*\sqrt{1/(b-4)}/2 + 2*\sqrt{1/(b-4)})) - 1/2)/2 - \sqrt{1/(b-4)}*\log(x**2 + x*(b*\sqrt{1/(b-4)}/2 - 2*\sqrt{1/(b-4)})) - 1/2)/2$$

$$3.29 \quad \int \frac{1+2x^2}{1+6x^2+4x^4} dx$$

**Optimal.** Leaf size=45

$$\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{10}} + \frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{10}}$$

**Rubi [A]** time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {1163, 203}

$$\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{10}} + \frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{10}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2\*x^2)/(1 + 6\*x^2 + 4\*x^4),x]

[Out] ArcTan[(2\*x)/Sqrt[3 - Sqrt[5]]]/Sqrt[10] + ArcTan[(2\*x)/Sqrt[3 + Sqrt[5]]]/Sqrt[10]

Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[Rt[b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1163

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[e/2 + (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 - q/2 + c\*x^2), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && GtQ[b^2 - 4\*a\*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1+6x^2+4x^4} dx &= \frac{1}{5}(5-\sqrt{5}) \int \frac{1}{3-\sqrt{5}+4x^2} dx + \frac{1}{5}(5+\sqrt{5}) \int \frac{1}{3+\sqrt{5}+4x^2} dx \\ &= \frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{10}} + \frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{10}} \end{aligned}$$

**Mathematica [A]** time = 0.08, size = 83, normalized size = 1.84

$$\frac{(\sqrt{5}-1) \tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{2\sqrt{5}(3-\sqrt{5})} + \frac{(1+\sqrt{5}) \tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{2\sqrt{5}(3+\sqrt{5})}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2\*x^2)/(1 + 6\*x^2 + 4\*x^4),x]

[Out]  $((-1 + \sqrt{5}) \operatorname{ArcTan}[(2x)/\sqrt{3 - \sqrt{5}}]) / (2\sqrt{5(3 - \sqrt{5})}) + ((1 + \sqrt{5}) \operatorname{ArcTan}[(2x)/\sqrt{3 + \sqrt{5}}]) / (2\sqrt{5(3 + \sqrt{5})})$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + 2x^2}{1 + 6x^2 + 4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 2\*x^2)/(1 + 6\*x^2 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(1 + 2\*x^2)/(1 + 6\*x^2 + 4\*x^4), x]

**fricas** [A] time = 0.80, size = 31, normalized size = 0.69

$$\frac{1}{10} \sqrt{10} \arctan\left(\frac{2}{5} \sqrt{10} (x^3 + 2x)\right) + \frac{1}{10} \sqrt{10} \arctan\left(\frac{1}{5} \sqrt{10} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4+6\*x^2+1), x, algorithm="fricas")

[Out]  $1/10 \sqrt{10} \arctan(2/5 \sqrt{10} (x^3 + 2x)) + 1/10 \sqrt{10} \arctan(1/5 \sqrt{10} x)$

**giac** [A] time = 0.17, size = 39, normalized size = 0.87

$$\frac{1}{10} \sqrt{10} \arctan\left(\frac{4x}{\sqrt{10} + \sqrt{2}}\right) + \frac{1}{10} \sqrt{10} \arctan\left(\frac{4x}{\sqrt{10} - \sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4+6\*x^2+1), x, algorithm="giac")

[Out]  $1/10 \sqrt{10} \arctan(4x/(\sqrt{10} + \sqrt{2})) + 1/10 \sqrt{10} \arctan(4x/(\sqrt{10} - \sqrt{2}))$

**maple** [B] time = 0.05, size = 136, normalized size = 3.02

$$\frac{2\sqrt{5} \arctan\left(\frac{8x}{2\sqrt{10}-2\sqrt{2}}\right)}{5(2\sqrt{10}-2\sqrt{2})} + \frac{2 \arctan\left(\frac{8x}{2\sqrt{10}-2\sqrt{2}}\right)}{2\sqrt{10}-2\sqrt{2}} + \frac{2\sqrt{5} \arctan\left(\frac{8x}{2\sqrt{10}+2\sqrt{2}}\right)}{5(2\sqrt{10}+2\sqrt{2})} + \frac{2 \arctan\left(\frac{8x}{2\sqrt{10}+2\sqrt{2}}\right)}{2\sqrt{10}+2\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2+1)/(4\*x^4+6\*x^2+1), x)

[Out]  $2/5 \cdot 5^{(1/2)} / (2 \cdot 10^{(1/2)} + 2 \cdot 2^{(1/2)}) \cdot \arctan(8x / (2 \cdot 10^{(1/2)} + 2 \cdot 2^{(1/2)})) + 2 / (2 \cdot 10^{(1/2)} + 2 \cdot 2^{(1/2)}) \cdot \arctan(8x / (2 \cdot 10^{(1/2)} + 2 \cdot 2^{(1/2)})) - 2/5 \cdot 5^{(1/2)} / (2 \cdot 10^{(1/2)} - 2 \cdot 2^{(1/2)}) \cdot \arctan(8x / (2 \cdot 10^{(1/2)} - 2 \cdot 2^{(1/2)})) + 2 / (2 \cdot 10^{(1/2)} - 2 \cdot 2^{(1/2)}) \cdot \arctan(8x / (2 \cdot 10^{(1/2)} - 2 \cdot 2^{(1/2)}))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2 + 1}{4x^4 + 6x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4+6\*x^2+1), x, algorithm="maxima")

[Out] integrate((2\*x^2 + 1)/(4\*x^4 + 6\*x^2 + 1), x)

**mupad [B]** time = 0.09, size = 29, normalized size = 0.64

$$\frac{\sqrt{10} \left( \operatorname{atan}\left(\frac{2\sqrt{10}x^3}{5} + \frac{4\sqrt{10}x}{5}\right) + \operatorname{atan}\left(\frac{\sqrt{10}x}{5}\right) \right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 + 1)/(6\*x^2 + 4\*x^4 + 1),x)

[Out] (10^(1/2)\*(atan((4\*10^(1/2)\*x)/5 + (2\*10^(1/2)\*x^3)/5) + atan((10^(1/2)\*x)/5))/10

**sympy [A]** time = 0.15, size = 42, normalized size = 0.93

$$\frac{\sqrt{10} \left( 2 \operatorname{atan}\left(\frac{\sqrt{10}x}{5}\right) + 2 \operatorname{atan}\left(\frac{2\sqrt{10}x^3}{5} + \frac{4\sqrt{10}x}{5}\right) \right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2+1)/(4\*x\*\*4+6\*x\*\*2+1),x)

[Out] sqrt(10)\*(2\*atan(sqrt(10)\*x/5) + 2\*atan(2\*sqrt(10)\*x\*\*3/5 + 4\*sqrt(10)\*x/5))/20



$$3.30 \quad \int \frac{1+2x^2}{1+5x^2+4x^4} dx$$

Optimal. Leaf size=15

$$\frac{1}{3} \tan^{-1}(x) + \frac{1}{3} \tan^{-1}(2x)$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {1163, 203}

$$\frac{1}{3} \tan^{-1}(x) + \frac{1}{3} \tan^{-1}(2x)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2\*x^2)/(1 + 5\*x^2 + 4\*x^4), x]

[Out] ArcTan[x]/3 + ArcTan[2\*x]/3

Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]]/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1163

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[e/2 + (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 - q/2 + c\*x^2), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && GtQ[b^2 - 4\*a\*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1+5x^2+4x^4} dx &= \frac{2}{3} \int \frac{1}{1+4x^2} dx + \frac{4}{3} \int \frac{1}{4+4x^2} dx \\ &= \frac{1}{3} \tan^{-1}(x) + \frac{1}{3} \tan^{-1}(2x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.13

$$-\frac{1}{3} \tan^{-1}\left(\frac{3x}{2x^2-1}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2\*x^2)/(1 + 5\*x^2 + 4\*x^4), x]

[Out] -1/3\*ArcTan[(3\*x)/(-1 + 2\*x^2)]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+2x^2}{1+5x^2+4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 2\*x^2)/(1 + 5\*x^2 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(1 + 2\*x^2)/(1 + 5\*x^2 + 4\*x^4), x]

**fricas** [A] time = 0.71, size = 19, normalized size = 1.27

$$\frac{1}{3} \arctan\left(\frac{4}{3}x^3 + \frac{7}{3}x\right) + \frac{1}{3} \arctan\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4+5\*x^2+1),x, algorithm="fricas")

[Out] 1/3\*arctan(4/3\*x^3 + 7/3\*x) + 1/3\*arctan(2/3\*x)

**giac** [A] time = 0.15, size = 11, normalized size = 0.73

$$\frac{1}{3} \arctan(2x) + \frac{1}{3} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4+5\*x^2+1),x, algorithm="giac")

[Out] 1/3\*arctan(2\*x) + 1/3\*arctan(x)

**maple** [A] time = 0.01, size = 12, normalized size = 0.80

$$\frac{\arctan(x)}{3} + \frac{\arctan(2x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2+1)/(4\*x^4+5\*x^2+1),x)

[Out] 1/3\*arctan(x)+1/3\*arctan(2\*x)

**maxima** [A] time = 2.49, size = 11, normalized size = 0.73

$$\frac{1}{3} \arctan(2x) + \frac{1}{3} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4+5\*x^2+1),x, algorithm="maxima")

[Out] 1/3\*arctan(2\*x) + 1/3\*arctan(x)

**mupad** [B] time = 0.07, size = 19, normalized size = 1.27

$$\frac{\operatorname{atan}\left(\frac{2x}{3}\right)}{3} + \frac{\operatorname{atan}\left(\frac{4x^3}{3} + \frac{7x}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 + 1)/(5\*x^2 + 4\*x^4 + 1),x)

[Out] atan((2\*x)/3)/3 + atan((7\*x)/3 + (4\*x^3)/3)/3

**sympy** [B] time = 0.12, size = 22, normalized size = 1.47

$$\frac{\operatorname{atan}\left(\frac{2x}{3}\right)}{3} + \frac{\operatorname{atan}\left(\frac{4x^3}{3} + \frac{7x}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2+1)/(4\*x\*\*4+5\*x\*\*2+1),x)

[Out] atan(2\*x/3)/3 + atan(4\*x\*\*3/3 + 7\*x/3)/3

$$3.31 \quad \int \frac{1+2x^2}{1+4x^2+4x^4} dx$$

Optimal. Leaf size=14

$$\frac{\tan^{-1}(\sqrt{2}x)}{\sqrt{2}}$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {28, 21, 203}

$$\frac{\tan^{-1}(\sqrt{2}x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2\*x^2)/(1 + 4\*x^2 + 4\*x^4), x]

[Out] ArcTan[Sqrt[2]\*x]/Sqrt[2]

Rule 21

Int[(u\_.)\*((a\_) + (b\_.)\*(v\_))^(m\_.)\*((c\_) + (d\_.)\*(v\_))^(n\_.), x\_Symbol] :> Dist[(b/d)^m, Int[u\*(c + d\*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b\*c - a\*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d\*x, a + b\*x])

Rule 28

Int[(u\_.)\*((a\_) + (c\_.)\*(x\_)^(n2\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] :> Dist[1/c^p, Int[u\*(b/2 + c\*x^n)^(2\*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2\*n] && EqQ[b^2 - 4\*a\*c, 0] && IntegerQ[p]

Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1+4x^2+4x^4} dx &= 4 \int \frac{1+2x^2}{(2+4x^2)^2} dx \\ &= \int \frac{1}{1+2x^2} dx \\ &= \frac{\tan^{-1}(\sqrt{2}x)}{\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{\tan^{-1}(\sqrt{2}x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2\*x^2)/(1 + 4\*x^2 + 4\*x^4), x]

[Out] ArcTan[Sqrt[2]\*x]/Sqrt[2]

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + 2x^2}{1 + 4x^2 + 4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 2\*x^2)/(1 + 4\*x^2 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(1 + 2\*x^2)/(1 + 4\*x^2 + 4\*x^4), x]

**fricas** [A] time = 0.69, size = 11, normalized size = 0.79

$$\frac{1}{2} \sqrt{2} \arctan(\sqrt{2} x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4+4\*x^2+1), x, algorithm="fricas")

[Out] 1/2\*sqrt(2)\*arctan(sqrt(2)\*x)

**giac** [A] time = 0.16, size = 11, normalized size = 0.79

$$\frac{1}{2} \sqrt{2} \arctan(\sqrt{2} x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4+4\*x^2+1), x, algorithm="giac")

[Out] 1/2\*sqrt(2)\*arctan(sqrt(2)\*x)

**maple** [A] time = 0.00, size = 12, normalized size = 0.86

$$\frac{\sqrt{2} \arctan(\sqrt{2} x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2+1)/(4\*x^4+4\*x^2+1), x)

[Out] 1/2\*arctan(2^(1/2)\*x)\*2^(1/2)

**maxima** [A] time = 2.30, size = 11, normalized size = 0.79

$$\frac{1}{2} \sqrt{2} \arctan(\sqrt{2} x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4+4\*x^2+1), x, algorithm="maxima")

[Out] 1/2\*sqrt(2)\*arctan(sqrt(2)\*x)

**mupad** [B] time = 0.03, size = 11, normalized size = 0.79

$$\frac{\sqrt{2} \operatorname{atan}(\sqrt{2} x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(4*x^2 + 4*x^4 + 1),x)
```

```
[Out] (2^(1/2)*atan(2^(1/2)*x))/2
```

**sympy** [A] time = 0.12, size = 14, normalized size = 1.00

$$\frac{\sqrt{2} \operatorname{atan}(\sqrt{2} x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4+4*x**2+1),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*x)/2
```

$$3.32 \quad \int \frac{1+2x^2}{1+3x^2+4x^4} dx$$

Optimal. Leaf size=38

$$\frac{\tan^{-1}\left(\frac{4x+1}{\sqrt{7}}\right)}{\sqrt{7}} - \frac{\tan^{-1}\left(\frac{1-4x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{4x+1}{\sqrt{7}}\right)}{\sqrt{7}} - \frac{\tan^{-1}\left(\frac{1-4x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2\*x^2)/(1 + 3\*x^2 + 4\*x^4),x]

[Out] -(ArcTan[(1 - 4\*x)/Sqrt[7]]/Sqrt[7]) + ArcTan[(1 + 4\*x)/Sqrt[7]]/Sqrt[7]

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 1161

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1+3x^2+4x^4} dx &= \frac{1}{4} \int \frac{1}{\frac{1}{2} - \frac{x}{2} + x^2} dx + \frac{1}{4} \int \frac{1}{\frac{1}{2} + \frac{x}{2} + x^2} dx \\ &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{1}{-\frac{7}{4} - x^2} dx, x, -\frac{1}{2} + 2x\right)\right) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{-\frac{7}{4} - x^2} dx, x, \frac{1}{2} + 2x\right) \\ &= -\frac{\tan^{-1}\left(\frac{1-4x}{\sqrt{7}}\right)}{\sqrt{7}} + \frac{\tan^{-1}\left(\frac{1+4x}{\sqrt{7}}\right)}{\sqrt{7}} \end{aligned}$$

**Mathematica [C]** time = 0.18, size = 97, normalized size = 2.55

$$\frac{(\sqrt{7} - i) \tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}(3-i\sqrt{7})}}\right)}{\sqrt{42 - 14i\sqrt{7}}} + \frac{(\sqrt{7} + i) \tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}(3+i\sqrt{7})}}\right)}{\sqrt{42 + 14i\sqrt{7}}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2\*x^2)/(1 + 3\*x^2 + 4\*x^4), x]

[Out] ((-I + Sqrt[7])\*ArcTan[(2\*x)/Sqrt[(3 - I\*Sqrt[7])/2]])/Sqrt[42 - (14\*I)\*Sqrt[7]] + ((I + Sqrt[7])\*ArcTan[(2\*x)/Sqrt[(3 + I\*Sqrt[7])/2]])/Sqrt[42 + (14\*I)\*Sqrt[7]]

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + 2x^2}{1 + 3x^2 + 4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 2\*x^2)/(1 + 3\*x^2 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(1 + 2\*x^2)/(1 + 3\*x^2 + 4\*x^4), x]

**fricas [A]** time = 0.92, size = 33, normalized size = 0.87

$$\frac{1}{7} \sqrt{7} \arctan\left(\frac{1}{7} \sqrt{7} (4x^3 + 5x)\right) + \frac{1}{7} \sqrt{7} \arctan\left(\frac{2}{7} \sqrt{7} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4+3\*x^2+1), x, algorithm="fricas")

[Out] 1/7\*sqrt(7)\*arctan(1/7\*sqrt(7)\*(4\*x^3 + 5\*x)) + 1/7\*sqrt(7)\*arctan(2/7\*sqrt(7)\*x)

**giac [A]** time = 0.17, size = 33, normalized size = 0.87

$$\frac{1}{7} \sqrt{7} \arctan\left(\frac{1}{7} \sqrt{7} (4x + 1)\right) + \frac{1}{7} \sqrt{7} \arctan\left(\frac{1}{7} \sqrt{7} (4x - 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4+3\*x^2+1), x, algorithm="giac")

[Out] 1/7\*sqrt(7)\*arctan(1/7\*sqrt(7)\*(4\*x + 1)) + 1/7\*sqrt(7)\*arctan(1/7\*sqrt(7)\*(4\*x - 1))

**maple [A]** time = 0.01, size = 34, normalized size = 0.89

$$\frac{\sqrt{7} \arctan\left(\frac{(4x+1)\sqrt{7}}{7}\right)}{7} + \frac{\sqrt{7} \arctan\left(\frac{(4x-1)\sqrt{7}}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2+1)/(4\*x^4+3\*x^2+1), x)

[Out] 1/7\*7^(1/2)\*arctan(1/7\*(4\*x-1)\*7^(1/2))+1/7\*arctan(1/7\*(1+4\*x)\*7^(1/2))\*7^(1/2)

**maxima [A]** time = 2.39, size = 33, normalized size = 0.87

$$\frac{1}{7} \sqrt{7} \arctan\left(\frac{1}{7} \sqrt{7}(4x+1)\right) + \frac{1}{7} \sqrt{7} \arctan\left(\frac{1}{7} \sqrt{7}(4x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4+3\*x^2+1),x, algorithm="maxima")

[Out] 1/7\*sqrt(7)\*arctan(1/7\*sqrt(7)\*(4\*x + 1)) + 1/7\*sqrt(7)\*arctan(1/7\*sqrt(7)\*(4\*x - 1))

**mupad [B]** time = 0.09, size = 29, normalized size = 0.76

$$\frac{\sqrt{7} \left( \operatorname{atan}\left(\frac{4\sqrt{7}x^3}{7} + \frac{5\sqrt{7}x}{7}\right) + \operatorname{atan}\left(\frac{2\sqrt{7}x}{7}\right) \right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 + 1)/(3\*x^2 + 4\*x^4 + 1),x)

[Out] (7^(1/2)\*(atan((5\*7^(1/2)\*x)/7 + (4\*7^(1/2)\*x^3)/7) + atan((2\*7^(1/2)\*x)/7))/7

**sympy [A]** time = 0.14, size = 44, normalized size = 1.16

$$\frac{\sqrt{7} \left( 2 \operatorname{atan}\left(\frac{2\sqrt{7}x}{7}\right) + 2 \operatorname{atan}\left(\frac{4\sqrt{7}x^3}{7} + \frac{5\sqrt{7}x}{7}\right) \right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2+1)/(4\*x\*\*4+3\*x\*\*2+1),x)

[Out] sqrt(7)\*(2\*atan(2\*sqrt(7)\*x/7) + 2\*atan(4\*sqrt(7)\*x\*\*3/7 + 5\*sqrt(7)\*x/7))/14



$$3.33 \quad \int \frac{1+2x^2}{1+2x^2+4x^4} dx$$

**Optimal.** Leaf size=48

$$\frac{\tan^{-1}\left(\frac{2\sqrt{2}x+1}{\sqrt{3}}\right)}{\sqrt{6}} - \frac{\tan^{-1}\left(\frac{1-2\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}$$

**Rubi [A]** time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{2}x+1}{\sqrt{3}}\right)}{\sqrt{6}} - \frac{\tan^{-1}\left(\frac{1-2\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2\*x^2)/(1 + 2\*x^2 + 4\*x^4), x]

[Out] -(ArcTan[(1 - 2\*Sqrt[2]\*x)/Sqrt[3]]/Sqrt[6]) + ArcTan[(1 + 2\*Sqrt[2]\*x)/Sqrt[3]]/Sqrt[6]

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 1161

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1+2x^2+4x^4} dx &= \frac{1}{4} \int \frac{1}{\frac{1}{2} - \frac{x}{\sqrt{2}} + x^2} dx + \frac{1}{4} \int \frac{1}{\frac{1}{2} + \frac{x}{\sqrt{2}} + x^2} dx \\ &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{1}{-\frac{3}{2} - x^2} dx, x, -\frac{1}{\sqrt{2}} + 2x\right)\right) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{-\frac{3}{2} - x^2} dx, x, \frac{1}{\sqrt{2}} + 2x\right) \\ &= -\frac{\tan^{-1}\left(\frac{1-2\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}} + \frac{\tan^{-1}\left(\frac{1+2\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}} \end{aligned}$$

**Mathematica** [C] time = 0.10, size = 99, normalized size = 2.06

$$\frac{(\sqrt{3} - i) \tan^{-1}\left(\frac{2x}{\sqrt{1-i\sqrt{3}}}\right)}{2\sqrt{3}(1-i\sqrt{3})} + \frac{(\sqrt{3} + i) \tan^{-1}\left(\frac{2x}{\sqrt{1+i\sqrt{3}}}\right)}{2\sqrt{3}(1+i\sqrt{3})}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2\*x^2)/(1 + 2\*x^2 + 4\*x^4), x]

[Out] ((-I + Sqrt[3])\*ArcTan[(2\*x)/Sqrt[1 - I\*Sqrt[3]]])/(2\*Sqrt[3\*(1 - I\*Sqrt[3])]) + ((I + Sqrt[3])\*ArcTan[(2\*x)/Sqrt[1 + I\*Sqrt[3]]])/(2\*Sqrt[3\*(1 + I\*Sqrt[3])])

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + 2x^2}{1 + 2x^2 + 4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 2\*x^2)/(1 + 2\*x^2 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(1 + 2\*x^2)/(1 + 2\*x^2 + 4\*x^4), x]

**fricas** [A] time = 0.91, size = 29, normalized size = 0.60

$$\frac{1}{6} \sqrt{6} \arctan\left(\frac{2}{3} \sqrt{6}(x^3 + x)\right) + \frac{1}{6} \sqrt{6} \arctan\left(\frac{1}{3} \sqrt{6}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4+2\*x^2+1), x, algorithm="fricas")

[Out] 1/6\*sqrt(6)\*arctan(2/3\*sqrt(6)\*(x^3 + x)) + 1/6\*sqrt(6)\*arctan(1/3\*sqrt(6)\*x)

**giac** [A] time = 0.19, size = 45, normalized size = 0.94

$$\frac{1}{6} \sqrt{6} \arctan\left(\frac{4}{3} \sqrt{3} \left(\frac{1}{4}\right)^{\frac{3}{4}} \left(2x + \left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{6} \sqrt{6} \arctan\left(\frac{4}{3} \sqrt{3} \left(\frac{1}{4}\right)^{\frac{3}{4}} \left(2x - \left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4+2\*x^2+1), x, algorithm="giac")

[Out] 1/6\*sqrt(6)\*arctan(4/3\*sqrt(3)\*(1/4)^(3/4)\*(2\*x + (1/4)^(1/4))) + 1/6\*sqrt(6)\*arctan(4/3\*sqrt(3)\*(1/4)^(3/4)\*(2\*x - (1/4)^(1/4)))

**maple** [A] time = 0.03, size = 40, normalized size = 0.83

$$\frac{\sqrt{6} \arctan\left(\frac{(4x-\sqrt{2})\sqrt{6}}{6}\right)}{6} + \frac{\sqrt{6} \arctan\left(\frac{(4x+\sqrt{2})\sqrt{6}}{6}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2+1)/(4\*x^4+2\*x^2+1), x)

[Out] 1/6\*6^(1/2)\*arctan(1/6\*(4\*x+2^(1/2))\*6^(1/2))+1/6\*6^(1/2)\*arctan(1/6\*(4\*x-2^(1/2))\*6^(1/2))

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2 + 1}{4x^4 + 2x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4+2\*x^2+1),x, algorithm="maxima")

[Out] integrate((2\*x^2 + 1)/(4\*x^4 + 2\*x^2 + 1), x)

**mupad [B]** time = 4.39, size = 29, normalized size = 0.60

$$\frac{\sqrt{6} \left( \operatorname{atan} \left( \frac{2\sqrt{6}x^3}{3} + \frac{2\sqrt{6}x}{3} \right) + \operatorname{atan} \left( \frac{\sqrt{6}x}{3} \right) \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 + 1)/(2\*x^2 + 4\*x^4 + 1),x)

[Out] (6^(1/2)\*(atan((2\*6^(1/2)\*x)/3 + (2\*6^(1/2)\*x^3)/3) + atan((6^(1/2)\*x)/3)) /6

**sympy [A]** time = 0.13, size = 42, normalized size = 0.88

$$\frac{\sqrt{6} \left( 2 \operatorname{atan} \left( \frac{\sqrt{6}x}{3} \right) + 2 \operatorname{atan} \left( \frac{2\sqrt{6}x^3}{3} + \frac{2\sqrt{6}x}{3} \right) \right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2+1)/(4\*x\*\*4+2\*x\*\*2+1),x)

[Out] sqrt(6)\*(2\*atan(sqrt(6)\*x/3) + 2\*atan(2\*sqrt(6)\*x\*\*3/3 + 2\*sqrt(6)\*x/3))/12

$$3.34 \quad \int \frac{1+2x^2}{1+x^2+4x^4} dx$$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{4x+\sqrt{3}}{\sqrt{5}}\right)}{\sqrt{5}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}-4x}{\sqrt{5}}\right)}{\sqrt{5}}$$

**Rubi [A]** time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{4x+\sqrt{3}}{\sqrt{5}}\right)}{\sqrt{5}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}-4x}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2\*x^2)/(1 + x^2 + 4\*x^4), x]

[Out] -(ArcTan[(Sqrt[3] - 4\*x)/Sqrt[5]]/Sqrt[5]) + ArcTan[(Sqrt[3] + 4\*x)/Sqrt[5]]/Sqrt[5]

#### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 1161

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

#### Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1+x^2+4x^4} dx &= \frac{1}{4} \int \frac{1}{\frac{1}{2} - \frac{\sqrt{3}x}{2} + x^2} dx + \frac{1}{4} \int \frac{1}{\frac{1}{2} + \frac{\sqrt{3}x}{2} + x^2} dx \\ &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{1}{-\frac{5}{4} - x^2} dx, x, -\frac{\sqrt{3}}{2} + 2x\right)\right) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{-\frac{5}{4} - x^2} dx, x, \frac{\sqrt{3}}{2} + 2x\right) \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{3}-4x}{\sqrt{5}}\right)}{\sqrt{5}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}+4x}{\sqrt{5}}\right)}{\sqrt{5}} \end{aligned}$$

**Mathematica [C]** time = 0.22, size = 97, normalized size = 2.11

$$\frac{(\sqrt{15} - 3i) \tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}(1-i\sqrt{15})}}\right)}{\sqrt{30 - 30i\sqrt{15}}} + \frac{(\sqrt{15} + 3i) \tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}(1+i\sqrt{15})}}\right)}{\sqrt{30 + 30i\sqrt{15}}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2\*x^2)/(1 + x^2 + 4\*x^4), x]

[Out] ((-3\*I + Sqrt[15])\*ArcTan[(2\*x)/Sqrt[(1 - I\*Sqrt[15])/2]])/Sqrt[30 - (30\*I)\*Sqrt[15]] + ((3\*I + Sqrt[15])\*ArcTan[(2\*x)/Sqrt[(1 + I\*Sqrt[15])/2]])/Sqrt[30 + (30\*I)\*Sqrt[15]]

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + 2x^2}{1 + x^2 + 4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 2\*x^2)/(1 + x^2 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(1 + 2\*x^2)/(1 + x^2 + 4\*x^4), x]

**fricas [A]** time = 1.77, size = 33, normalized size = 0.72

$$\frac{1}{5} \sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5} (4x^3 + 3x)\right) + \frac{1}{5} \sqrt{5} \arctan\left(\frac{2}{5} \sqrt{5} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4+x^2+1), x, algorithm="fricas")

[Out] 1/5\*sqrt(5)\*arctan(1/5\*sqrt(5)\*(4\*x^3 + 3\*x)) + 1/5\*sqrt(5)\*arctan(2/5\*sqrt(5)\*x)

**giac [A]** time = 0.26, size = 52, normalized size = 1.13

$$\frac{1}{5} \sqrt{5} \arctan\left(\frac{2}{5} \sqrt{10} \left(\frac{1}{4}\right)^{\frac{3}{4}} \left(4x + \sqrt{6} \left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{5} \sqrt{5} \arctan\left(\frac{2}{5} \sqrt{10} \left(\frac{1}{4}\right)^{\frac{3}{4}} \left(4x - \sqrt{6} \left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4+x^2+1), x, algorithm="giac")

[Out] 1/5\*sqrt(5)\*arctan(2/5\*sqrt(10)\*(1/4)^(3/4)\*(4\*x + sqrt(6)\*(1/4)^(1/4))) + 1/5\*sqrt(5)\*arctan(2/5\*sqrt(10)\*(1/4)^(3/4)\*(4\*x - sqrt(6)\*(1/4)^(1/4)))

**maple [A]** time = 0.03, size = 40, normalized size = 0.87

$$\frac{\sqrt{5} \arctan\left(\frac{(4x-\sqrt{3})\sqrt{5}}{5}\right)}{5} + \frac{\sqrt{5} \arctan\left(\frac{(4x+\sqrt{3})\sqrt{5}}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2+1)/(4\*x^4+x^2+1), x)

[Out] 1/5\*arctan(1/5\*(4\*x+3^(1/2))\*5^(1/2))\*5^(1/2)+1/5\*5^(1/2)\*arctan(1/5\*(4\*x-3^(1/2))\*5^(1/2))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2 + 1}{4x^4 + x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4+x^2+1),x, algorithm="maxima")

[Out] integrate((2\*x^2 + 1)/(4\*x^4 + x^2 + 1), x)

**mupad** [B] time = 4.36, size = 29, normalized size = 0.63

$$\frac{\sqrt{5} \left( \operatorname{atan} \left( \frac{4\sqrt{5}x^3}{5} + \frac{3\sqrt{5}x}{5} \right) + \operatorname{atan} \left( \frac{2\sqrt{5}x}{5} \right) \right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 + 1)/(x^2 + 4\*x^4 + 1),x)

[Out] (5^(1/2)\*(atan((3\*5^(1/2)\*x)/5 + (4\*5^(1/2)\*x^3)/5) + atan((2\*5^(1/2)\*x)/5))/5

**sympy** [A] time = 0.13, size = 44, normalized size = 0.96

$$\frac{\sqrt{5} \left( 2 \operatorname{atan} \left( \frac{2\sqrt{5}x}{5} \right) + 2 \operatorname{atan} \left( \frac{4\sqrt{5}x^3}{5} + \frac{3\sqrt{5}x}{5} \right) \right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2+1)/(4\*x\*\*4+x\*\*2+1),x)

[Out] sqrt(5)\*(2\*atan(2\*sqrt(5)\*x/5) + 2\*atan(4\*sqrt(5)\*x\*\*3/5 + 3\*sqrt(5)\*x/5))/10

$$3.35 \quad \int \frac{1+2x^2}{1+4x^4} dx$$

**Optimal.** Leaf size=21

$$\frac{1}{2} \tan^{-1}(2x+1) - \frac{1}{2} \tan^{-1}(1-2x)$$

**Rubi [A]** time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {1162, 617, 204}

$$\frac{1}{2} \tan^{-1}(2x+1) - \frac{1}{2} \tan^{-1}(1-2x)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2\*x^2)/(1 + 4\*x^4), x]

[Out] -ArcTan[1 - 2\*x]/2 + ArcTan[1 + 2\*x]/2

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 1162

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

#### Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1+4x^4} dx &= \frac{1}{4} \int \frac{1}{\frac{1}{2}-x+x^2} dx + \frac{1}{4} \int \frac{1}{\frac{1}{2}+x+x^2} dx \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{1}{-1-x^2} dx, x, 1-2x \right) - \frac{1}{2} \text{Subst} \left( \int \frac{1}{-1-x^2} dx, x, 1+2x \right) \\ &= -\frac{1}{2} \tan^{-1}(1-2x) + \frac{1}{2} \tan^{-1}(1+2x) \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 17, normalized size = 0.81

$$-\frac{1}{2} \tan^{-1} \left( \frac{2x}{2x^2-1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2\*x^2)/(1 + 4\*x^4), x]

[Out]  $-1/2 \cdot \text{ArcTan}[(2x)/(-1 + 2x^2)]$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + 2x^2}{1 + 4x^4} dx$$

Verification is not applicable to the result.

[In] `IntegrateAlgebraic[(1 + 2*x^2)/(1 + 4*x^4), x]`

[Out] `IntegrateAlgebraic[(1 + 2*x^2)/(1 + 4*x^4), x]`

**fricas** [A] time = 0.71, size = 15, normalized size = 0.71

$$\frac{1}{2} \arctan(2x^3 + x) + \frac{1}{2} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2+1)/(4*x^4+1), x, algorithm="fricas")`

[Out] `1/2*arctan(2*x^3 + x) + 1/2*arctan(x)`

**giac** [B] time = 0.16, size = 46, normalized size = 2.19

$$\frac{1}{2} \arctan\left(2\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2x + \sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{2} \arctan\left(2\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2x - \sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2+1)/(4*x^4+1), x, algorithm="giac")`

[Out] `1/2*arctan(2*sqrt(2)*(1/4)^(3/4)*(2*x + sqrt(2)*(1/4)^(1/4))) + 1/2*arctan(2*sqrt(2)*(1/4)^(3/4)*(2*x - sqrt(2)*(1/4)^(1/4)))`

**maple** [A] time = 0.01, size = 18, normalized size = 0.86

$$\frac{\arctan(2x + 1)}{2} + \frac{\arctan(2x - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2+1)/(4*x^4+1), x)`

[Out] `1/2*arctan(2*x-1)+1/2*arctan(2*x+1)`

**maxima** [A] time = 2.24, size = 17, normalized size = 0.81

$$\frac{1}{2} \arctan(2x + 1) + \frac{1}{2} \arctan(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2+1)/(4*x^4+1), x, algorithm="maxima")`

[Out] `1/2*arctan(2*x + 1) + 1/2*arctan(2*x - 1)`

**mupad** [B] time = 4.29, size = 15, normalized size = 0.71

$$\frac{\text{atan}(2x^3 + x)}{2} + \frac{\text{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] int((2*x^2 + 1)/(4*x^4 + 1),x)
```

```
[Out] atan(x + 2*x^3)/2 + atan(x)/2
```

**sympy** [A] time = 0.11, size = 14, normalized size = 0.67

$$\frac{\operatorname{atan}(x)}{2} + \frac{\operatorname{atan}(2x^3 + x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4+1),x)
```

```
[Out] atan(x)/2 + atan(2*x**3 + x)/2
```

$$3.36 \quad \int \frac{1+2x^2}{1-x^2+4x^4} dx$$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{4x+\sqrt{5}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt{5}-4x}{\sqrt{3}}\right)}{\sqrt{3}}$$

**Rubi [A]** time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{4x+\sqrt{5}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt{5}-4x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2\*x^2)/(1 - x^2 + 4\*x^4), x]

[Out] -(ArcTan[(Sqrt[5] - 4\*x)/Sqrt[3]]/Sqrt[3]) + ArcTan[(Sqrt[5] + 4\*x)/Sqrt[3]]/Sqrt[3]

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 1161

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

#### Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1-x^2+4x^4} dx &= \frac{1}{4} \int \frac{1}{\frac{1}{2} - \frac{\sqrt{5}x}{2} + x^2} dx + \frac{1}{4} \int \frac{1}{\frac{1}{2} + \frac{\sqrt{5}x}{2} + x^2} dx \\ &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{1}{-\frac{3}{4} - x^2} dx, x, -\frac{\sqrt{5}}{2} + 2x\right)\right) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{-\frac{3}{4} - x^2} dx, x, \frac{\sqrt{5}}{2} + 2x\right) \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{5}-4x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt{5}+4x}{\sqrt{3}}\right)}{\sqrt{3}} \end{aligned}$$

**Mathematica [C]** time = 0.27, size = 101, normalized size = 2.20

$$\frac{(\sqrt{15} - 5i) \tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}(-1-i\sqrt{15})}}\right)}{\sqrt{30}(-1-i\sqrt{15})} + \frac{(\sqrt{15} + 5i) \tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}(-1+i\sqrt{15})}}\right)}{\sqrt{30}(-1+i\sqrt{15})}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2\*x^2)/(1 - x^2 + 4\*x^4), x]

[Out] ((-5\*I + Sqrt[15])\*ArcTan[(2\*x)/Sqrt[(-1 - I\*Sqrt[15])/2]])/Sqrt[30\*(-1 - I\*Sqrt[15])] + ((5\*I + Sqrt[15])\*ArcTan[(2\*x)/Sqrt[(-1 + I\*Sqrt[15])/2]])/Sqrt[30\*(-1 + I\*Sqrt[15])]

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + 2x^2}{1 - x^2 + 4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 2\*x^2)/(1 - x^2 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(1 + 2\*x^2)/(1 - x^2 + 4\*x^4), x]

**fricas [A]** time = 0.63, size = 31, normalized size = 0.67

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (4x^3 + x)\right) + \frac{1}{3} \sqrt{3} \arctan\left(\frac{2}{3} \sqrt{3} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4-x^2+1), x, algorithm="fricas")

[Out] 1/3\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(4\*x^3 + x)) + 1/3\*sqrt(3)\*arctan(2/3\*sqrt(3)\*x)

**giac [A]** time = 0.24, size = 52, normalized size = 1.13

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{2}{3} \sqrt{6} \left(\frac{1}{4}\right)^{\frac{3}{4}} \left(4x + \sqrt{10} \left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{3} \sqrt{3} \arctan\left(\frac{2}{3} \sqrt{6} \left(\frac{1}{4}\right)^{\frac{3}{4}} \left(4x - \sqrt{10} \left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4-x^2+1), x, algorithm="giac")

[Out] 1/3\*sqrt(3)\*arctan(2/3\*sqrt(6)\*(1/4)^(3/4)\*(4\*x + sqrt(10)\*(1/4)^(1/4))) + 1/3\*sqrt(3)\*arctan(2/3\*sqrt(6)\*(1/4)^(3/4)\*(4\*x - sqrt(10)\*(1/4)^(1/4)))

**maple [A]** time = 0.03, size = 40, normalized size = 0.87

$$\frac{\sqrt{3} \arctan\left(\frac{(4x-\sqrt{5})\sqrt{3}}{3}\right)}{3} + \frac{\sqrt{3} \arctan\left(\frac{(4x+\sqrt{5})\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2+1)/(4\*x^4-x^2+1), x)

[Out] 1/3\*arctan(1/3\*(4\*x+5^(1/2))\*3^(1/2))\*3^(1/2)+1/3\*3^(1/2)\*arctan(1/3\*(4\*x-5^(1/2))\*3^(1/2))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2 + 1}{4x^4 - x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4-x^2+1),x, algorithm="maxima")

[Out] integrate((2\*x^2 + 1)/(4\*x^4 - x^2 + 1), x)

**mupad** [B] time = 4.37, size = 29, normalized size = 0.63

$$\frac{\sqrt{3} \left( \operatorname{atan}\left(\frac{4\sqrt{3}x^3}{3} + \frac{\sqrt{3}x}{3}\right) + \operatorname{atan}\left(\frac{2\sqrt{3}x}{3}\right) \right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 + 1)/(4\*x^4 - x^2 + 1),x)

[Out] (3^(1/2)\*(atan((3^(1/2)\*x)/3 + (4\*3^(1/2)\*x^3)/3) + atan((2\*3^(1/2)\*x)/3)) /3

**sympy** [A] time = 0.14, size = 42, normalized size = 0.91

$$\frac{\sqrt{3} \left( 2 \operatorname{atan}\left(\frac{2\sqrt{3}x}{3}\right) + 2 \operatorname{atan}\left(\frac{4\sqrt{3}x^3}{3} + \frac{\sqrt{3}x}{3}\right) \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2+1)/(4\*x\*\*4-x\*\*2+1),x)

[Out] sqrt(3)\*(2\*atan(2\*sqrt(3)\*x/3) + 2\*atan(4\*sqrt(3)\*x\*\*3/3 + sqrt(3)\*x/3))/6

$$3.37 \quad \int \frac{1+2x^2}{1-2x^2+4x^4} dx$$

**Optimal.** Leaf size=44

$$\frac{\tan^{-1}(2\sqrt{2}x + \sqrt{3})}{\sqrt{2}} - \frac{\tan^{-1}(\sqrt{3} - 2\sqrt{2}x)}{\sqrt{2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}(2\sqrt{2}x + \sqrt{3})}{\sqrt{2}} - \frac{\tan^{-1}(\sqrt{3} - 2\sqrt{2}x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2\*x^2)/(1 - 2\*x^2 + 4\*x^4), x]

[Out] -(ArcTan[Sqrt[3] - 2\*Sqrt[2]\*x]/Sqrt[2]) + ArcTan[Sqrt[3] + 2\*Sqrt[2]\*x]/Sqrt[2]

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 1161

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1-2x^2+4x^4} dx &= \frac{1}{4} \int \frac{1}{\frac{1}{2} - \sqrt{\frac{3}{2}}x + x^2} dx + \frac{1}{4} \int \frac{1}{\frac{1}{2} + \sqrt{\frac{3}{2}}x + x^2} dx \\ &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{1}{-\frac{1}{2} - x^2} dx, x, -\sqrt{\frac{3}{2}} + 2x\right)\right) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{-\frac{1}{2} - x^2} dx, x, \sqrt{\frac{3}{2}} + 2x\right) \\ &= -\frac{\tan^{-1}(\sqrt{3} - 2\sqrt{2}x)}{\sqrt{2}} + \frac{\tan^{-1}(\sqrt{3} + 2\sqrt{2}x)}{\sqrt{2}} \end{aligned}$$

**Mathematica** [C] time = 0.10, size = 99, normalized size = 2.25

$$\frac{(\sqrt{3} - 3i) \tan^{-1}\left(\frac{2x}{\sqrt{-1-i\sqrt{3}}}\right)}{2\sqrt{3}(-1-i\sqrt{3})} + \frac{(\sqrt{3} + 3i) \tan^{-1}\left(\frac{2x}{\sqrt{-1+i\sqrt{3}}}\right)}{2\sqrt{3}(-1+i\sqrt{3})}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2\*x^2)/(1 - 2\*x^2 + 4\*x^4), x]

[Out] ((-3\*I + Sqrt[3])\*ArcTan[(2\*x)/Sqrt[-1 - I\*Sqrt[3]]])/(2\*Sqrt[3\*(-1 - I\*Sqrt[3])]) + ((3\*I + Sqrt[3])\*ArcTan[(2\*x)/Sqrt[-1 + I\*Sqrt[3]]])/(2\*Sqrt[3\*(-1 + I\*Sqrt[3])])

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + 2x^2}{1 - 2x^2 + 4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 2\*x^2)/(1 - 2\*x^2 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(1 + 2\*x^2)/(1 - 2\*x^2 + 4\*x^4), x]

**fricas** [A] time = 0.73, size = 26, normalized size = 0.59

$$\frac{1}{2} \sqrt{2} \arctan\left(2 \sqrt{2} x^3\right) + \frac{1}{2} \sqrt{2} \arctan\left(\sqrt{2} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4-2\*x^2+1), x, algorithm="fricas")

[Out] 1/2\*sqrt(2)\*arctan(2\*sqrt(2)\*x^3) + 1/2\*sqrt(2)\*arctan(sqrt(2)\*x)

**giac** [A] time = 0.17, size = 46, normalized size = 1.05

$$\frac{1}{2} \sqrt{2} \arctan\left(4 \left(\frac{1}{4}\right)^{\frac{3}{4}} \left(2x + \sqrt{3} \left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{2} \sqrt{2} \arctan\left(4 \left(\frac{1}{4}\right)^{\frac{3}{4}} \left(2x - \sqrt{3} \left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4-2\*x^2+1), x, algorithm="giac")

[Out] 1/2\*sqrt(2)\*arctan(4\*(1/4)^(3/4)\*(2\*x + sqrt(3)\*(1/4)^(1/4))) + 1/2\*sqrt(2)\*arctan(4\*(1/4)^(3/4)\*(2\*x - sqrt(3)\*(1/4)^(1/4)))

**maple** [A] time = 0.04, size = 40, normalized size = 0.91

$$\frac{\sqrt{2} \arctan\left(\frac{(4x-\sqrt{6})\sqrt{2}}{2}\right)}{2} + \frac{\sqrt{2} \arctan\left(\frac{(4x+\sqrt{6})\sqrt{2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2+1)/(4\*x^4-2\*x^2+1), x)

[Out] 1/2\*2^(1/2)\*arctan(1/2\*(4\*x+6^(1/2))\*2^(1/2))+1/2\*2^(1/2)\*arctan(1/2\*(4\*x-6^(1/2))\*2^(1/2))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2 + 1}{4x^4 - 2x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4-2\*x^2+1),x, algorithm="maxima")

[Out] integrate((2\*x^2 + 1)/(4\*x^4 - 2\*x^2 + 1), x)

**mupad** [B] time = 0.06, size = 21, normalized size = 0.48

$$\frac{\sqrt{2} \left( \operatorname{atan}(\sqrt{2} x) + \operatorname{atan}(2\sqrt{2} x^3) \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 + 1)/(4\*x^4 - 2\*x^2 + 1),x)

[Out] (2^(1/2)\*(atan(2^(1/2)\*x) + atan(2\*2^(1/2)\*x^3)))/2

**sympy** [A] time = 0.13, size = 29, normalized size = 0.66

$$\frac{\sqrt{2} \left( 2 \operatorname{atan}(\sqrt{2} x) + 2 \operatorname{atan}(2\sqrt{2} x^3) \right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2+1)/(4\*x\*\*4-2\*x\*\*2+1),x)

[Out] sqrt(2)\*(2\*atan(sqrt(2)\*x) + 2\*atan(2\*sqrt(2)\*x\*\*3))/4

$$3.38 \quad \int \frac{1+2x^2}{1-3x^2+4x^4} dx$$

Optimal. Leaf size=23

$$\tan^{-1}(4x + \sqrt{7}) - \tan^{-1}(\sqrt{7} - 4x)$$

**Rubi [A]** time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {1161, 618, 204}

$$\tan^{-1}(4x + \sqrt{7}) - \tan^{-1}(\sqrt{7} - 4x)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2\*x^2)/(1 - 3\*x^2 + 4\*x^4), x]

[Out] -ArcTan[Sqrt[7] - 4\*x] + ArcTan[Sqrt[7] + 4\*x]

#### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 1161

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

#### Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1-3x^2+4x^4} dx &= \frac{1}{4} \int \frac{1}{\frac{1}{2} - \frac{\sqrt{7}x}{2} + x^2} dx + \frac{1}{4} \int \frac{1}{\frac{1}{2} + \frac{\sqrt{7}x}{2} + x^2} dx \\ &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{1}{-\frac{1}{4} - x^2} dx, x, -\frac{\sqrt{7}}{2} + 2x\right)\right) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{-\frac{1}{4} - x^2} dx, x, \frac{\sqrt{7}}{2} + 2x\right) \\ &= -\tan^{-1}(\sqrt{7} - 4x) + \tan^{-1}(\sqrt{7} + 4x) \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 14, normalized size = 0.61

$$-\tan^{-1}\left(\frac{x}{2x^2-1}\right)$$

Antiderivative was successfully verified.



[In] Integrate[(1 + 2\*x^2)/(1 - 3\*x^2 + 4\*x^4), x]

[Out] -ArcTan[x/(-1 + 2\*x^2)]

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + 2x^2}{1 - 3x^2 + 4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 2\*x^2)/(1 - 3\*x^2 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(1 + 2\*x^2)/(1 - 3\*x^2 + 4\*x^4), x]

**fricas** [A] time = 0.71, size = 15, normalized size = 0.65

$$\arctan(4x^3 - x) + \arctan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4-3\*x^2+1), x, algorithm="fricas")

[Out] arctan(4\*x^3 - x) + arctan(2\*x)

**giac** [B] time = 0.19, size = 42, normalized size = 1.83

$$\arctan\left(2\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(4x + \sqrt{14}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right) + \arctan\left(2\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(4x - \sqrt{14}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4-3\*x^2+1), x, algorithm="giac")

[Out] arctan(2\*sqrt(2)\*(1/4)^(3/4)\*(4\*x + sqrt(14)\*(1/4)^(1/4))) + arctan(2\*sqrt(2)\*(1/4)^(3/4)\*(4\*x - sqrt(14)\*(1/4)^(1/4)))

**maple** [A] time = 0.04, size = 20, normalized size = 0.87

$$\arctan(4x - \sqrt{7}) + \arctan(4x + \sqrt{7})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2+1)/(4\*x^4-3\*x^2+1), x)

[Out] arctan(4\*x-7^(1/2))+arctan(4\*x+7^(1/2))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2 + 1}{4x^4 - 3x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4-3\*x^2+1), x, algorithm="maxima")

[Out] integrate((2\*x^2 + 1)/(4\*x^4 - 3\*x^2 + 1), x)

**mupad** [B] time = 4.35, size = 15, normalized size = 0.65

$$\operatorname{atan}(2x) - \operatorname{atan}(x - 4x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(4*x^4 - 3*x^2 + 1),x)
```

```
[Out] atan(2*x) - atan(x - 4*x^3)
```

**sympy** [A] time = 0.11, size = 12, normalized size = 0.52

$$\operatorname{atan}(2x) + \operatorname{atan}(4x^3 - x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4-3*x**2+1),x)
```

```
[Out] atan(2*x) + atan(4*x**3 - x)
```

$$3.39 \quad \int \frac{1+2x^2}{1-4x^2+4x^4} dx$$

Optimal. Leaf size=11

$$\frac{x}{1-2x^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {28, 383}

$$\frac{x}{1-2x^2}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2\*x^2)/(1 - 4\*x^2 + 4\*x^4), x]

[Out] x/(1 - 2\*x^2)

Rule 28

Int[(u\_.)\*((a\_.) + (c\_.)\*(x\_)^(n2\_.)) + (b\_.)\*(x\_)^(n\_.)]^(p\_.), x\_Symbol] :> Dist[1/c^p, Int[u\*(b/2 + c\*x^n)^(2\*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2\*n] && EqQ[b^2 - 4\*a\*c, 0] && IntegerQ[p]

Rule 383

Int[((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Simp[c\*x\*(a + b\*x^n)^(p + 1)/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a\*d - b\*c\*(n\*(p + 1) + 1), 0]

Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1-4x^2+4x^4} dx &= 4 \int \frac{1+2x^2}{(-2+4x^2)^2} dx \\ &= \frac{x}{1-2x^2} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 12, normalized size = 1.09

$$-\frac{x}{2x^2-1}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2\*x^2)/(1 - 4\*x^2 + 4\*x^4), x]

[Out] -(x/(-1 + 2\*x^2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+2x^2}{1-4x^2+4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 2\*x^2)/(1 - 4\*x^2 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(1 + 2\*x^2)/(1 - 4\*x^2 + 4\*x^4), x]

**fricas** [A] time = 0.58, size = 12, normalized size = 1.09

$$-\frac{x}{2x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4-4\*x^2+1),x, algorithm="fricas")

[Out] -x/(2\*x^2 - 1)

**giac** [A] time = 0.16, size = 12, normalized size = 1.09

$$-\frac{x}{2x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4-4\*x^2+1),x, algorithm="giac")

[Out] -x/(2\*x^2 - 1)

**maple** [A] time = 0.01, size = 11, normalized size = 1.00

$$-\frac{x}{2\left(x^2 - \frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2+1)/(4\*x^4-4\*x^2+1),x)

[Out] -1/2\*x/(x^2-1/2)

**maxima** [A] time = 0.93, size = 12, normalized size = 1.09

$$-\frac{x}{2x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4-4\*x^2+1),x, algorithm="maxima")

[Out] -x/(2\*x^2 - 1)

**mupad** [B] time = 4.30, size = 12, normalized size = 1.09

$$-\frac{x}{2\left(x^2 - \frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 + 1)/(4\*x^4 - 4\*x^2 + 1),x)

[Out] -x/(2\*(x^2 - 1/2))

**sympy** [A] time = 0.09, size = 8, normalized size = 0.73

$$-\frac{x}{2x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2+1)/(4\*x\*\*4-4\*x\*\*2+1),x)

[Out] -x/(2\*x\*\*2 - 1)

$$3.40 \quad \int \frac{1+2x^2}{1-5x^2+4x^4} dx$$

Optimal. Leaf size=39

$$-\frac{1}{2} \log(1-2x) + \frac{1}{2} \log(1-x) - \frac{1}{2} \log(x+1) + \frac{1}{2} \log(2x+1)$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {1161, 616, 31}

$$-\frac{1}{2} \log(1-2x) + \frac{1}{2} \log(1-x) - \frac{1}{2} \log(x+1) + \frac{1}{2} \log(2x+1)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2\*x^2)/(1 - 5\*x^2 + 4\*x^4), x]

[Out] -Log[1 - 2\*x]/2 + Log[1 - x]/2 - Log[1 + x]/2 + Log[1 + 2\*x]/2

Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 616

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(n\_), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c\*x, x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c\*x, x], x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && PosQ[b^2 - 4\*a\*c] && PerfectSquareQ[b^2 - 4\*a\*c]

Rule 1161

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1-5x^2+4x^4} dx &= \frac{1}{4} \int \frac{1}{\frac{1}{2} - \frac{3x}{2} + x^2} dx + \frac{1}{4} \int \frac{1}{\frac{1}{2} + \frac{3x}{2} + x^2} dx \\ &= \frac{1}{2} \int \frac{1}{-1+x} dx - \frac{1}{2} \int \frac{1}{-\frac{1}{2}+x} dx + \frac{1}{2} \int \frac{1}{\frac{1}{2}+x} dx - \frac{1}{2} \int \frac{1}{1+x} dx \\ &= -\frac{1}{2} \log(1-2x) + \frac{1}{2} \log(1-x) - \frac{1}{2} \log(1+x) + \frac{1}{2} \log(1+2x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.74

$$\frac{1}{2} \log(-2x^2 + x + 1) - \frac{1}{2} \log(-2x^2 - x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2\*x^2)/(1 - 5\*x^2 + 4\*x^4), x]

[Out] -1/2\*Log[1 - x - 2\*x^2] + Log[1 + x - 2\*x^2]/2

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + 2x^2}{1 - 5x^2 + 4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 2\*x^2)/(1 - 5\*x^2 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(1 + 2\*x^2)/(1 - 5\*x^2 + 4\*x^4), x]

**fricas [A]** time = 0.79, size = 25, normalized size = 0.64

$$-\frac{1}{2} \log(2x^2 + x - 1) + \frac{1}{2} \log(2x^2 - x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4-5\*x^2+1), x, algorithm="fricas")

[Out] -1/2\*log(2\*x^2 + x - 1) + 1/2\*log(2\*x^2 - x - 1)

**giac [A]** time = 0.17, size = 33, normalized size = 0.85

$$\frac{1}{2} \log(|2x + 1|) - \frac{1}{2} \log(|2x - 1|) - \frac{1}{2} \log(|x + 1|) + \frac{1}{2} \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4-5\*x^2+1), x, algorithm="giac")

[Out] 1/2\*log(abs(2\*x + 1)) - 1/2\*log(abs(2\*x - 1)) - 1/2\*log(abs(x + 1)) + 1/2\*log(abs(x - 1))

**maple [A]** time = 0.01, size = 30, normalized size = 0.77

$$-\frac{\ln(x + 1)}{2} + \frac{\ln(2x + 1)}{2} + \frac{\ln(x - 1)}{2} - \frac{\ln(2x - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2+1)/(4\*x^4-5\*x^2+1), x)

[Out] -1/2\*ln(2\*x-1)+1/2\*ln(2\*x+1)-1/2\*ln(x+1)+1/2\*ln(x-1)

**maxima [A]** time = 1.04, size = 29, normalized size = 0.74

$$\frac{1}{2} \log(2x + 1) - \frac{1}{2} \log(2x - 1) - \frac{1}{2} \log(x + 1) + \frac{1}{2} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4-5\*x^2+1), x, algorithm="maxima")

[Out] 1/2\*log(2\*x + 1) - 1/2\*log(2\*x - 1) - 1/2\*log(x + 1) + 1/2\*log(x - 1)

**mupad [B]** time = 0.30, size = 14, normalized size = 0.36

$$-\operatorname{atanh}\left(\frac{x}{2x^2 - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 + 1)/(4*x^4 - 5*x^2 + 1),x)`

[Out] `-atanh(x/(2*x^2 - 1))`

**sympy [A]** time = 0.13, size = 26, normalized size = 0.67

$$\frac{\log\left(x^2 - \frac{x}{2} - \frac{1}{2}\right)}{2} - \frac{\log\left(x^2 + \frac{x}{2} - \frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2+1)/(4*x**4-5*x**2+1),x)`

[Out] `log(x**2 - x/2 - 1/2)/2 - log(x**2 + x/2 - 1/2)/2`

$$3.41 \quad \int \frac{1+2x^2}{1-6x^2+4x^4} dx$$

**Optimal.** Leaf size=44

$$\frac{\tanh^{-1}(\sqrt{5} - 2\sqrt{2}x)}{\sqrt{2}} - \frac{\tanh^{-1}(2\sqrt{2}x + \sqrt{5})}{\sqrt{2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}(\sqrt{5} - 2\sqrt{2}x)}{\sqrt{2}} - \frac{\tanh^{-1}(2\sqrt{2}x + \sqrt{5})}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2\*x^2)/(1 - 6\*x^2 + 4\*x^4), x]

[Out] ArcTanh[Sqrt[5] - 2\*Sqrt[2]\*x]/Sqrt[2] - ArcTanh[Sqrt[5] + 2\*Sqrt[2]\*x]/Sqrt[2]

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 618**

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

**Rule 1161**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

**Rubi steps**

$$\begin{aligned} \int \frac{1+2x^2}{1-6x^2+4x^4} dx &= \frac{1}{4} \int \frac{1}{\frac{1}{2} - \sqrt{\frac{5}{2}}x + x^2} dx + \frac{1}{4} \int \frac{1}{\frac{1}{2} + \sqrt{\frac{5}{2}}x + x^2} dx \\ &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{1}{\frac{1}{2} - x^2} dx, x, -\sqrt{\frac{5}{2}} + 2x\right)\right) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{\frac{1}{2} - x^2} dx, x, \sqrt{\frac{5}{2}} + 2x\right) \\ &= \frac{\tanh^{-1}(\sqrt{5} - 2\sqrt{2}x)}{\sqrt{2}} - \frac{\tanh^{-1}(\sqrt{5} + 2\sqrt{2}x)}{\sqrt{2}} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 42, normalized size = 0.95

$$\frac{\log(-2x^2 + \sqrt{2}x + 1) - \log(2x^2 + \sqrt{2}x - 1)}{2\sqrt{2}}$$



Antiderivative was successfully verified.

[In] Integrate[(1 + 2\*x^2)/(1 - 6\*x^2 + 4\*x^4), x]

[Out] (Log[1 + Sqrt[2]\*x - 2\*x^2] - Log[-1 + Sqrt[2]\*x + 2\*x^2])/(2\*Sqrt[2])

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + 2x^2}{1 - 6x^2 + 4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 2\*x^2)/(1 - 6\*x^2 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(1 + 2\*x^2)/(1 - 6\*x^2 + 4\*x^4), x]

**fricas** [A] time = 0.78, size = 47, normalized size = 1.07

$$\frac{1}{4} \sqrt{2} \log\left(\frac{4x^4 - 2x^2 - 2\sqrt{2}(2x^3 - x) + 1}{4x^4 - 6x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4-6\*x^2+1), x, algorithm="fricas")

[Out] 1/4\*sqrt(2)\*log((4\*x^4 - 2\*x^2 - 2\*sqrt(2)\*(2\*x^3 - x) + 1)/(4\*x^4 - 6\*x^2 + 1))

**giac** [B] time = 0.34, size = 77, normalized size = 1.75

$$-\frac{1}{4}\sqrt{2}\log\left(x + \frac{1}{4}\sqrt{10} + \frac{1}{4}\sqrt{2}\right) + \frac{1}{4}\sqrt{2}\log\left(x + \frac{1}{4}\sqrt{10} - \frac{1}{4}\sqrt{2}\right) - \frac{1}{4}\sqrt{2}\log\left(x - \frac{1}{4}\sqrt{10} + \frac{1}{4}\sqrt{2}\right) + \frac{1}{4}\sqrt{2}\log\left(x - \frac{1}{4}\sqrt{10} - \frac{1}{4}\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4-6\*x^2+1), x, algorithm="giac")

[Out] -1/4\*sqrt(2)\*log(abs(x + 1/4\*sqrt(10) + 1/4\*sqrt(2))) + 1/4\*sqrt(2)\*log(abs(x + 1/4\*sqrt(10) - 1/4\*sqrt(2))) - 1/4\*sqrt(2)\*log(abs(x - 1/4\*sqrt(10) + 1/4\*sqrt(2))) + 1/4\*sqrt(2)\*log(abs(x - 1/4\*sqrt(10) - 1/4\*sqrt(2)))

**maple** [B] time = 0.04, size = 82, normalized size = 1.86

$$\frac{2(-5 + \sqrt{5})\sqrt{5} \operatorname{arctanh}\left(\frac{8x}{2\sqrt{10}-2\sqrt{2}}\right)}{5(2\sqrt{10}-2\sqrt{2})} - \frac{2(5 + \sqrt{5})\sqrt{5} \operatorname{arctanh}\left(\frac{8x}{2\sqrt{10}+2\sqrt{2}}\right)}{5(2\sqrt{10}+2\sqrt{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2+1)/(4\*x^4-6\*x^2+1), x)

[Out] -2/5\*(-5+5^(1/2))\*5^(1/2)/(2\*10^(1/2)-2\*2^(1/2))\*arctanh(8/(2\*10^(1/2)-2\*2^(1/2))\*x)-2/5\*(5+5^(1/2))\*5^(1/2)/(2\*10^(1/2)+2\*2^(1/2))\*arctanh(8/(2\*10^(1/2)+2\*2^(1/2))\*x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2 + 1}{4x^4 - 6x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+1)/(4\*x^4-6\*x^2+1), x, algorithm="maxima")

[Out] integrate((2\*x^2 + 1)/(4\*x^4 - 6\*x^2 + 1), x)

**mupad [B]** time = 0.22, size = 20, normalized size = 0.45

$$-\frac{\sqrt{2} \operatorname{atanh}\left(\frac{\sqrt{2}x}{2x^2-1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 + 1)/(4\*x^4 - 6\*x^2 + 1),x)

[Out] -(2^(1/2)\*atanh((2^(1/2)\*x)/(2\*x^2 - 1)))/2

**sympy [A]** time = 0.12, size = 46, normalized size = 1.05

$$\frac{\sqrt{2} \log\left(x^2 - \frac{\sqrt{2}x}{2} - \frac{1}{2}\right)}{4} - \frac{\sqrt{2} \log\left(x^2 + \frac{\sqrt{2}x}{2} - \frac{1}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2+1)/(4\*x\*\*4-6\*x\*\*2+1),x)

[Out] sqrt(2)\*log(x\*\*2 - sqrt(2)\*x/2 - 1/2)/4 - sqrt(2)\*log(x\*\*2 + sqrt(2)\*x/2 - 1/2)/4

$$3.42 \quad \int \frac{1-2x^2}{1+bx^2+4x^4} dx$$

Optimal. Leaf size=66

$$\frac{\log(\sqrt{4-b}x + 2x^2 + 1)}{2\sqrt{4-b}} - \frac{\log(-\sqrt{4-b}x + 2x^2 + 1)}{2\sqrt{4-b}}$$

**Rubi [A]** time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {1164, 628}

$$\frac{\log(\sqrt{4-b}x + 2x^2 + 1)}{2\sqrt{4-b}} - \frac{\log(-\sqrt{4-b}x + 2x^2 + 1)}{2\sqrt{4-b}}$$

Antiderivative was successfully verified.

[In] Int[(1 - 2\*x^2)/(1 + b\*x^2 + 4\*x^4), x]

[Out] -Log[1 - Sqrt[4 - b]\*x + 2\*x^2]/(2\*Sqrt[4 - b]) + Log[1 + Sqrt[4 - b]\*x + 2\*x^2]/(2\*Sqrt[4 - b])

Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[d\*Log[RemoveContent[a + b\*x + c\*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1164

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_.) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(-2\*d)/e - b/c, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && !GtQ[b^2 - 4\*a\*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-2x^2}{1+bx^2+4x^4} dx &= -\frac{\int \frac{\frac{\sqrt{4-b}}{2}+2x}{-\frac{1}{2}-\frac{1}{2}\sqrt{4-b}x-x^2} dx}{2\sqrt{4-b}} - \frac{\int \frac{\frac{\sqrt{4-b}}{2}-2x}{-\frac{1}{2}+\frac{1}{2}\sqrt{4-b}x-x^2} dx}{2\sqrt{4-b}} \\ &= -\frac{\log(1-\sqrt{4-b}x+2x^2)}{2\sqrt{4-b}} + \frac{\log(1+\sqrt{4-b}x+2x^2)}{2\sqrt{4-b}} \end{aligned}$$

**Mathematica [A]** time = 0.07, size = 127, normalized size = 1.92

$$\frac{(-\sqrt{b^2-16}+b+4)\tan^{-1}\left(\frac{2\sqrt{2}x}{\sqrt{b-\sqrt{b^2-16}}}\right)}{\sqrt{b-\sqrt{b^2-16}}} - \frac{(\sqrt{b^2-16}+b+4)\tan^{-1}\left(\frac{2\sqrt{2}x}{\sqrt{\sqrt{b^2-16}+b}}\right)}{\sqrt{\sqrt{b^2-16}+b}}$$

$$\frac{\quad}{\sqrt{2}\sqrt{b^2-16}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2\*x^2)/(1 + b\*x^2 + 4\*x^4), x]

[Out]  $\frac{((4 + b - \sqrt{-16 + b^2}) \operatorname{ArcTan}[(2\sqrt{2}x)/\sqrt{b - \sqrt{-16 + b^2}}])/\sqrt{b - \sqrt{-16 + b^2}} - ((4 + b + \sqrt{-16 + b^2}) \operatorname{ArcTan}[(2\sqrt{2}x)/\sqrt{b + \sqrt{-16 + b^2}}])/\sqrt{b + \sqrt{-16 + b^2}}}{(\sqrt{2} \sqrt{-16 + b^2})}$

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 - 2x^2}{1 + bx^2 + 4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - 2\*x^2)/(1 + b\*x^2 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(1 - 2\*x^2)/(1 + b\*x^2 + 4\*x^4), x]

**fricas [A]** time = 0.81, size = 109, normalized size = 1.65

$$\left[ -\frac{\sqrt{-b+4} \log\left(\frac{4x^4 - (b-8)x^2 + 2(2x^3+x)\sqrt{-b+4} + 1}{4x^4 + bx^2 + 1}\right)}{2(b-4)}, \frac{\sqrt{b-4} \arctan\left(\frac{4x^3 + (b-2)x}{\sqrt{b-4}}\right) - \sqrt{b-4} \arctan\left(\frac{2x}{\sqrt{b-4}}\right)}{b-4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4+b\*x^2+1), x, algorithm="fricas")

[Out]  $[-1/2 \sqrt{-b+4} \log((4x^4 - (b-8)x^2 + 2(2x^3+x)\sqrt{-b+4} + 1)/(4x^4 + bx^2 + 1))/(b-4), (\sqrt{b-4} \arctan((4x^3 + (b-2)x)/\sqrt{b-4}) - \sqrt{b-4} \arctan(2x/\sqrt{b-4}))/ (b-4)]$

**giac [A]** time = 0.31, size = 73, normalized size = 1.11

$$-\frac{\sqrt{b-4} b \arctan\left(\frac{4\sqrt{\frac{1}{2}}x}{\sqrt{b+\sqrt{b^2-16}}}\right)}{b^2-4b} - \frac{\sqrt{b-4} b \arctan\left(\frac{4\sqrt{\frac{1}{2}}x}{\sqrt{b-\sqrt{b^2-16}}}\right)}{b^2-4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4+b\*x^2+1), x, algorithm="giac")

[Out]  $-\sqrt{b-4} b \arctan(4\sqrt{1/2}x/\sqrt{b+\sqrt{b^2-16}})/(b^2-4b) - \sqrt{b-4} b \arctan(4\sqrt{1/2}x/\sqrt{b-\sqrt{b^2-16}})/(b^2-4b)$

**maple [B]** time = 0.02, size = 279, normalized size = 4.23

$$\frac{b \arctan\left(\frac{4x}{\sqrt{2b-2\sqrt{(b-4)(b+4)}}}\right)}{\sqrt{(b-4)(b+4)}\sqrt{2b-2\sqrt{(b-4)(b+4)}}} - \frac{b \arctan\left(\frac{4x}{\sqrt{2b+2\sqrt{(b-4)(b+4)}}}\right)}{\sqrt{(b-4)(b+4)}\sqrt{2b+2\sqrt{(b-4)(b+4)}}} + \frac{4 \arctan\left(\frac{4x}{\sqrt{2b-2\sqrt{(b-4)(b+4)}}}\right)}{\sqrt{(b-4)(b+4)}\sqrt{2b-2\sqrt{(b-4)(b+4)}}} - \frac{\arctan\left(\frac{4x}{\sqrt{2b-2\sqrt{(b-4)(b+4)}}}\right)}{\sqrt{2b-2\sqrt{(b-4)(b+4)}}} - \frac{4 \arctan\left(\frac{4x}{\sqrt{2b+2\sqrt{(b-4)(b+4)}}}\right)}{\sqrt{(b-4)(b+4)}\sqrt{2b+2\sqrt{(b-4)(b+4)}}} - \frac{\arctan\left(\frac{4x}{\sqrt{2b+2\sqrt{(b-4)(b+4)}}}\right)}{\sqrt{2b+2\sqrt{(b-4)(b+4)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2\*x^2+1)/(4\*x^4+b\*x^2+1), x)

[Out]  $\frac{4}{((b-4)(b+4))^{1/2}} \frac{1}{(2b-2((b-4)(b+4))^{1/2})^{1/2}} \arctan\left(\frac{4}{(2b-2((b-4)(b+4))^{1/2})^{1/2}} x\right) - \frac{1}{(2b-2((b-4)(b+4))^{1/2})^{1/2}} \arctan\left(\frac{4}{(2b-2((b-4)(b+4))^{1/2})^{1/2}} x\right) + \frac{1}{((b-4)(b+4))^{1/2}} \frac{1}{(2b-2((b-4)(b+4))^{1/2})^{1/2}} \arctan\left(\frac{4}{(2b-2((b-4)(b+4))^{1/2})^{1/2}} x\right) - \frac{4}{((b-4)(b+4))^{1/2}} \frac{1}{(2b+2((b-4)(b+4))^{1/2})^{1/2}} \arctan\left(\frac{4}{(2b+2((b-4)(b+4))^{1/2})^{1/2}} x\right) - \frac{1}{(2b+2((b-4)(b+4))^{1/2})^{1/2}} \arctan\left(\frac{4}{(2b+2((b-4)(b+4))^{1/2})^{1/2}} x\right) - \frac{1}{((b-4)(b+4))^{1/2}} \frac{1}{(2b+2((b-4)(b+4))^{1/2})^{1/2}} \arctan\left(\frac{4}{(2b+2((b-4)(b+4))^{1/2})^{1/2}} x\right) - \frac{1}{(2b+2((b-4)(b+4))^{1/2})^{1/2}} \arctan\left(\frac{4}{(2b+2((b-4)(b+4))^{1/2})^{1/2}} x\right)$

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x^2 - 1}{4x^4 + bx^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4+b\*x^2+1),x, algorithm="maxima")

[Out] -integrate((2\*x^2 - 1)/(4\*x^4 + b\*x^2 + 1), x)

**mupad [B]** time = 0.07, size = 63, normalized size = 0.95

$$\frac{\operatorname{atan}\left(\frac{2x}{\sqrt{b-4}}\right) - \operatorname{atan}\left(\frac{b^3x+4b^2x^3-2b^2x-16bx-64x^3+32x}{(b-4)^{3/2}(b+4)}\right)}{\sqrt{b-4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(2\*x^2 - 1)/(b\*x^2 + 4\*x^4 + 1),x)

[Out] -(atan((2\*x)/(b - 4)^(1/2)) - atan((32\*x - 16\*b\*x - 2\*b^2\*x + b^3\*x - 64\*x^3 + 4\*b^2\*x^3)/((b - 4)^(3/2)\*(b + 4))))/(b - 4)^(1/2)

**sympy [A]** time = 0.38, size = 94, normalized size = 1.42

$$\frac{\sqrt{-\frac{1}{b-4}} \log\left(x^2 + x\left(-\frac{b\sqrt{-\frac{1}{b-4}}}{2} + 2\sqrt{-\frac{1}{b-4}}\right) + \frac{1}{2}\right)}{2} - \frac{\sqrt{-\frac{1}{b-4}} \log\left(x^2 + x\left(\frac{b\sqrt{-\frac{1}{b-4}}}{2} - 2\sqrt{-\frac{1}{b-4}}\right) + \frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x\*\*2+1)/(4\*x\*\*4+b\*x\*\*2+1),x)

[Out] sqrt(-1/(b - 4))\*log(x\*\*2 + x\*(-b\*sqrt(-1/(b - 4)))/2 + 2\*sqrt(-1/(b - 4))) + 1/2)/2 - sqrt(-1/(b - 4))\*log(x\*\*2 + x\*(b\*sqrt(-1/(b - 4)))/2 - 2\*sqrt(-1/(b - 4))) + 1/2)/2

$$3.43 \quad \int \frac{1-2x^2}{1+6x^2+4x^4} dx$$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {1163, 203}

$$\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 - 2\*x^2)/(1 + 6\*x^2 + 4\*x^4), x]

[Out] ArcTan[(2\*x)/Sqrt[3 - Sqrt[5]]]/Sqrt[2] - ArcTan[(2\*x)/Sqrt[3 + Sqrt[5]]]/Sqrt[2]

Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1163

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[e/2 + (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 - q/2 + c\*x^2), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && GtQ[b^2 - 4\*a\*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-2x^2}{1+6x^2+4x^4} dx &= (-1-\sqrt{5}) \int \frac{1}{3+\sqrt{5}+4x^2} dx + (-1+\sqrt{5}) \int \frac{1}{3-\sqrt{5}+4x^2} dx \\ &= \frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{2}} \end{aligned}$$

**Mathematica [A]** time = 0.07, size = 84, normalized size = 1.83

$$\frac{-\left((\sqrt{5}-5)\sqrt{3+\sqrt{5}}\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)\right)-\sqrt{3-\sqrt{5}}(5+\sqrt{5})\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2\*x^2)/(1 + 6\*x^2 + 4\*x^4), x]

[Out]  $(-((-5 + \sqrt{5})\sqrt{3 + \sqrt{5}}\operatorname{ArcTan}[(2x)/\sqrt{3 - \sqrt{5}}]) - \sqrt{3 - \sqrt{5}}(5 + \sqrt{5})\operatorname{ArcTan}[(2x)/\sqrt{3 + \sqrt{5}}]))/(4\sqrt{5})$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 - 2x^2}{1 + 6x^2 + 4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - 2\*x^2)/(1 + 6\*x^2 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(1 - 2\*x^2)/(1 + 6\*x^2 + 4\*x^4), x]

**fricas** [A] time = 0.63, size = 28, normalized size = 0.61

$$\frac{1}{2}\sqrt{2}\arctan\left(2\sqrt{2}(x^3 + x)\right) - \frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4+6\*x^2+1), x, algorithm="fricas")

[Out]  $1/2\sqrt{2}\arctan(2\sqrt{2}(x^3 + x)) - 1/2\sqrt{2}\arctan(\sqrt{2}x)$

**giac** [A] time = 0.18, size = 39, normalized size = 0.85

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{4x}{\sqrt{10} + \sqrt{2}}\right) + \frac{1}{2}\sqrt{2}\arctan\left(\frac{4x}{\sqrt{10} - \sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4+6\*x^2+1), x, algorithm="giac")

[Out]  $-1/2\sqrt{2}\arctan(4x/(\sqrt{10} + \sqrt{2})) + 1/2\sqrt{2}\arctan(4x/(\sqrt{10} - \sqrt{2}))$

**maple** [B] time = 0.02, size = 136, normalized size = 2.96

$$\frac{2\sqrt{5}\arctan\left(\frac{8x}{2\sqrt{10}-2\sqrt{2}}\right)}{2\sqrt{10}-2\sqrt{2}} - \frac{2\arctan\left(\frac{8x}{2\sqrt{10}-2\sqrt{2}}\right)}{2\sqrt{10}-2\sqrt{2}} - \frac{2\sqrt{5}\arctan\left(\frac{8x}{2\sqrt{10}+2\sqrt{2}}\right)}{2\sqrt{10}+2\sqrt{2}} - \frac{2\arctan\left(\frac{8x}{2\sqrt{10}+2\sqrt{2}}\right)}{2\sqrt{10}+2\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2\*x^2+1)/(4\*x^4+6\*x^2+1), x)

[Out]  $-2*5^{(1/2)}/(2*10^{(1/2)}+2*2^{(1/2)})\arctan(8/(2*10^{(1/2)}+2*2^{(1/2)})x)-2/(2*10^{(1/2)}+2*2^{(1/2)})\arctan(8/(2*10^{(1/2)}+2*2^{(1/2)})x)+2*5^{(1/2)}/(2*10^{(1/2)}-2*2^{(1/2)})\arctan(8/(2*10^{(1/2)}-2*2^{(1/2)})x)-2/(2*10^{(1/2)}-2*2^{(1/2)})\arctan(8/(2*10^{(1/2)}-2*2^{(1/2)})x)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x^2 - 1}{4x^4 + 6x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4+6\*x^2+1), x, algorithm="maxima")

[Out] -integrate((2\*x^2 - 1)/(4\*x^4 + 6\*x^2 + 1), x)

**mupad [B]** time = 4.38, size = 30, normalized size = 0.65

$$\frac{\sqrt{2} \left( \operatorname{atan}\left(2\sqrt{2}x^3 + 2\sqrt{2}x\right) - \operatorname{atan}\left(\sqrt{2}x\right) \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(2*x^2 - 1)/(6*x^2 + 4*x^4 + 1),x)`

[Out] `(2^(1/2)*(atan(2*2^(1/2)*x + 2*2^(1/2)*x^3) - atan(2^(1/2)*x)))/2`

**sympy [A]** time = 0.13, size = 39, normalized size = 0.85

$$\frac{\sqrt{2} \left( 2 \operatorname{atan}\left(\sqrt{2}x\right) - 2 \operatorname{atan}\left(2\sqrt{2}x^3 + 2\sqrt{2}x\right) \right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2*x**2+1)/(4*x**4+6*x**2+1),x)`

[Out] `-sqrt(2)*(2*atan(sqrt(2)*x) - 2*atan(2*sqrt(2)*x**3 + 2*sqrt(2)*x))/4`



$$3.44 \quad \int \frac{1-2x^2}{1+5x^2+4x^4} dx$$

**Optimal.** Leaf size=9

$$\tan^{-1}(2x) - \tan^{-1}(x)$$

**Rubi [A]** time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {1163, 203}

$$\tan^{-1}(2x) - \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 - 2\*x^2)/(1 + 5\*x^2 + 4\*x^4), x]

[Out] -ArcTan[x] + ArcTan[2\*x]

**Rule 203**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

**Rule 1163**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[e/2 + (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 - q/2 + c\*x^2), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && GtQ[b^2 - 4\*a\*c, 0]

**Rubi steps**

$$\begin{aligned} \int \frac{1-2x^2}{1+5x^2+4x^4} dx &= 2 \int \frac{1}{1+4x^2} dx - 4 \int \frac{1}{4+4x^2} dx \\ &= -\tan^{-1}(x) + \tan^{-1}(2x) \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 12, normalized size = 1.33

$$\tan^{-1}\left(\frac{x}{2x^2+1}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2\*x^2)/(1 + 5\*x^2 + 4\*x^4), x]

[Out] ArcTan[x/(1 + 2\*x^2)]

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-2x^2}{1+5x^2+4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - 2\*x^2)/(1 + 5\*x^2 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(1 - 2\*x^2)/(1 + 5\*x^2 + 4\*x^4), x]

**fricas** [A] time = 0.91, size = 17, normalized size = 1.89

$$\arctan(4x^3 + 3x) - \arctan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4+5\*x^2+1),x, algorithm="fricas")

[Out] arctan(4\*x^3 + 3\*x) - arctan(2\*x)

**giac** [A] time = 0.17, size = 9, normalized size = 1.00

$$\arctan(2x) - \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4+5\*x^2+1),x, algorithm="giac")

[Out] arctan(2\*x) - arctan(x)

**maple** [A] time = 0.01, size = 10, normalized size = 1.11

$$-\arctan(x) + \arctan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2\*x^2+1)/(4\*x^4+5\*x^2+1),x)

[Out] -arctan(x)+arctan(2\*x)

**maxima** [A] time = 2.36, size = 9, normalized size = 1.00

$$\arctan(2x) - \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4+5\*x^2+1),x, algorithm="maxima")

[Out] arctan(2\*x) - arctan(x)

**mupad** [B] time = 4.36, size = 17, normalized size = 1.89

$$\operatorname{atan}(4x^3 + 3x) - \operatorname{atan}(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(2\*x^2 - 1)/(5\*x^2 + 4\*x^4 + 1),x)

[Out] atan(3\*x + 4\*x^3) - atan(2\*x)

**sympy** [A] time = 0.12, size = 14, normalized size = 1.56

$$-\operatorname{atan}(2x) + \operatorname{atan}(4x^3 + 3x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x\*\*2+1)/(4\*x\*\*4+5\*x\*\*2+1),x)

[Out] -atan(2\*x) + atan(4\*x\*\*3 + 3\*x)

$$3.45 \quad \int \frac{1-2x^2}{1+4x^2+4x^4} dx$$

**Optimal.** Leaf size=11

$$\frac{x}{2x^2 + 1}$$

**Rubi [A]** time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {28, 383}

$$\frac{x}{2x^2 + 1}$$

Antiderivative was successfully verified.

[In] Int[(1 - 2\*x^2)/(1 + 4\*x^2 + 4\*x^4), x]

[Out] x/(1 + 2\*x^2)

**Rule 28**

Int[(u\_)\*((a\_) + (c\_)\*(x\_)^(n2\_)) + (b\_)\*(x\_)^(n\_)]^(p\_), x\_Symbol] := Dist[1/c^p, Int[u\*(b/2 + c\*x^n)^(2\*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2\*n] && EqQ[b^2 - 4\*a\*c, 0] && IntegerQ[p]

**Rule 383**

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[(c\*x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a\*d - b\*c\*(n\*(p + 1) + 1), 0]

**Rubi steps**

$$\begin{aligned} \int \frac{1-2x^2}{1+4x^2+4x^4} dx &= 4 \int \frac{1-2x^2}{(2+4x^2)^2} dx \\ &= \frac{x}{1+2x^2} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 11, normalized size = 1.00

$$\frac{x}{2x^2 + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2\*x^2)/(1 + 4\*x^2 + 4\*x^4), x]

[Out] x/(1 + 2\*x^2)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-2x^2}{1+4x^2+4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - 2\*x^2)/(1 + 4\*x^2 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(1 - 2\*x^2)/(1 + 4\*x^2 + 4\*x^4), x]

**fricas** [A] time = 0.71, size = 11, normalized size = 1.00

$$\frac{x}{2x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4+4\*x^2+1),x, algorithm="fricas")

[Out] x/(2\*x^2 + 1)

**giac** [A] time = 0.16, size = 11, normalized size = 1.00

$$\frac{x}{2x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4+4\*x^2+1),x, algorithm="giac")

[Out] x/(2\*x^2 + 1)

**maple** [A] time = 0.01, size = 11, normalized size = 1.00

$$\frac{x}{2x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2\*x^2+1)/(4\*x^4+4\*x^2+1),x)

[Out] 1/2\*x/(x^2+1/2)

**maxima** [A] time = 1.08, size = 11, normalized size = 1.00

$$\frac{x}{2x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4+4\*x^2+1),x, algorithm="maxima")

[Out] x/(2\*x^2 + 1)

**mupad** [B] time = 4.30, size = 11, normalized size = 1.00

$$\frac{x}{2\left(x^2 + \frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(2\*x^2 - 1)/(4\*x^2 + 4\*x^4 + 1),x)

[Out] x/(2\*(x^2 + 1/2))

**sympy** [A] time = 0.09, size = 7, normalized size = 0.64

$$\frac{x}{2x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x\*\*2+1)/(4\*x\*\*4+4\*x\*\*2+1),x)

[Out] x/(2\*x\*\*2 + 1)

$$3.46 \quad \int \frac{1-2x^2}{1+3x^2+4x^4} dx$$

Optimal. Leaf size=29

$$\frac{1}{2} \log(2x^2 + x + 1) - \frac{1}{2} \log(2x^2 - x + 1)$$

**Rubi [A]** time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {1164, 628}

$$\frac{1}{2} \log(2x^2 + x + 1) - \frac{1}{2} \log(2x^2 - x + 1)$$

Antiderivative was successfully verified.

[In] Int[(1 - 2\*x^2)/(1 + 3\*x^2 + 4\*x^4), x]

[Out] -Log[1 - x + 2\*x^2]/2 + Log[1 + x + 2\*x^2]/2

Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1164

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(-2\*d)/e - b/c, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && !GtQ[b^2 - 4\*a\*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-2x^2}{1+3x^2+4x^4} dx &= -\left(\frac{1}{2} \int \frac{\frac{1}{2}+2x}{-\frac{1}{2}-\frac{x}{2}-x^2} dx\right) - \frac{1}{2} \int \frac{\frac{1}{2}-2x}{-\frac{1}{2}+\frac{x}{2}-x^2} dx \\ &= -\frac{1}{2} \log(1-x+2x^2) + \frac{1}{2} \log(1+x+2x^2) \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 29, normalized size = 1.00

$$\frac{1}{2} \log(2x^2 + x + 1) - \frac{1}{2} \log(2x^2 - x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2\*x^2)/(1 + 3\*x^2 + 4\*x^4), x]

[Out] -1/2\*Log[1 - x + 2\*x^2] + Log[1 + x + 2\*x^2]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-2x^2}{1+3x^2+4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - 2\*x^2)/(1 + 3\*x^2 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(1 - 2\*x^2)/(1 + 3\*x^2 + 4\*x^4), x]

**fricas** [A] time = 0.87, size = 25, normalized size = 0.86

$$\frac{1}{2} \log(2x^2 + x + 1) - \frac{1}{2} \log(2x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4+3\*x^2+1), x, algorithm="fricas")

[Out] 1/2\*log(2\*x^2 + x + 1) - 1/2\*log(2\*x^2 - x + 1)

**giac** [A] time = 0.15, size = 25, normalized size = 0.86

$$\frac{1}{2} \log(2x^2 + x + 1) - \frac{1}{2} \log(2x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4+3\*x^2+1), x, algorithm="giac")

[Out] 1/2\*log(2\*x^2 + x + 1) - 1/2\*log(2\*x^2 - x + 1)

**maple** [A] time = 0.00, size = 26, normalized size = 0.90

$$-\frac{\ln(2x^2 - x + 1)}{2} + \frac{\ln(2x^2 + x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2\*x^2+1)/(4\*x^4+3\*x^2+1), x)

[Out] -1/2\*ln(2\*x^2-x+1)+1/2\*ln(2\*x^2+x+1)

**maxima** [A] time = 1.00, size = 25, normalized size = 0.86

$$\frac{1}{2} \log(2x^2 + x + 1) - \frac{1}{2} \log(2x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4+3\*x^2+1), x, algorithm="maxima")

[Out] 1/2\*log(2\*x^2 + x + 1) - 1/2\*log(2\*x^2 - x + 1)

**mupad** [B] time = 0.06, size = 12, normalized size = 0.41

$$\operatorname{atanh}\left(\frac{x}{2x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(2\*x^2 - 1)/(3\*x^2 + 4\*x^4 + 1), x)

[Out] atanh(x/(2\*x^2 + 1))

**sympy** [A] time = 0.11, size = 26, normalized size = 0.90

$$-\frac{\log\left(x^2 - \frac{x}{2} + \frac{1}{2}\right)}{2} + \frac{\log\left(x^2 + \frac{x}{2} + \frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x\*\*2+1)/(4\*x\*\*4+3\*x\*\*2+1), x)

[Out] -log(x\*\*2 - x/2 + 1/2)/2 + log(x\*\*2 + x/2 + 1/2)/2

$$3.47 \quad \int \frac{1-2x^2}{1+2x^2+4x^4} dx$$

**Optimal.** Leaf size=50

$$\frac{\log(2x^2 + \sqrt{2}x + 1)}{2\sqrt{2}} - \frac{\log(2x^2 - \sqrt{2}x + 1)}{2\sqrt{2}}$$

**Rubi [A]** time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {1164, 628}

$$\frac{\log(2x^2 + \sqrt{2}x + 1)}{2\sqrt{2}} - \frac{\log(2x^2 - \sqrt{2}x + 1)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 - 2\*x^2)/(1 + 2\*x^2 + 4\*x^4), x]

[Out] -Log[1 - Sqrt[2]\*x + 2\*x^2]/(2\*Sqrt[2]) + Log[1 + Sqrt[2]\*x + 2\*x^2]/(2\*Sqrt[2])

Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[d\*Log[RemoveContent[a + b\*x + c\*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1164

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_.) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(-2\*d)/e - b/c, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && !GtQ[b^2 - 4\*a\*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-2x^2}{1+2x^2+4x^4} dx &= -\frac{\int \frac{\frac{1}{\sqrt{2}}+2x}{-\frac{1}{2}-\frac{x}{\sqrt{2}}-x^2} dx}{2\sqrt{2}} - \frac{\int \frac{\frac{1}{\sqrt{2}}-2x}{-\frac{1}{2}+\frac{x}{\sqrt{2}}-x^2} dx}{2\sqrt{2}} \\ &= -\frac{\log(1 - \sqrt{2}x + 2x^2)}{2\sqrt{2}} + \frac{\log(1 + \sqrt{2}x + 2x^2)}{2\sqrt{2}} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 42, normalized size = 0.84

$$\frac{\log(2x^2 + \sqrt{2}x + 1) - \log(-2x^2 + \sqrt{2}x - 1)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2\*x^2)/(1 + 2\*x^2 + 4\*x^4), x]

[Out] (-Log[-1 + Sqrt[2]\*x - 2\*x^2] + Log[1 + Sqrt[2]\*x + 2\*x^2])/(2\*Sqrt[2])

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 - 2x^2}{1 + 2x^2 + 4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - 2\*x^2)/(1 + 2\*x^2 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(1 - 2\*x^2)/(1 + 2\*x^2 + 4\*x^4), x]

**fricas** [A] time = 0.73, size = 45, normalized size = 0.90

$$\frac{1}{4} \sqrt{2} \log \left( \frac{4x^4 + 6x^2 + 2\sqrt{2}(2x^3 + x) + 1}{4x^4 + 2x^2 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4+2\*x^2+1), x, algorithm="fricas")

[Out] 1/4\*sqrt(2)\*log((4\*x^4 + 6\*x^2 + 2\*sqrt(2)\*(2\*x^3 + x) + 1)/(4\*x^4 + 2\*x^2 + 1))

**giac** [A] time = 0.17, size = 34, normalized size = 0.68

$$\frac{1}{4} \sqrt{2} \log \left( x^2 + \left( \frac{1}{4} \right)^{\frac{1}{4}} x + \frac{1}{2} \right) - \frac{1}{4} \sqrt{2} \log \left( x^2 - \left( \frac{1}{4} \right)^{\frac{1}{4}} x + \frac{1}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4+2\*x^2+1), x, algorithm="giac")

[Out] 1/4\*sqrt(2)\*log(x^2 + (1/4)^(1/4)\*x + 1/2) - 1/4\*sqrt(2)\*log(x^2 - (1/4)^(1/4)\*x + 1/2)

**maple** [A] time = 0.01, size = 39, normalized size = 0.78

$$-\frac{\sqrt{2} \ln(2x^2 - \sqrt{2}x + 1)}{4} + \frac{\sqrt{2} \ln(2x^2 + \sqrt{2}x + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2\*x^2+1)/(4\*x^4+2\*x^2+1), x)

[Out] -1/4\*ln(1+2\*x^2-2^(1/2)\*x)\*2^(1/2)+1/4\*ln(1+2\*x^2+2^(1/2)\*x)\*2^(1/2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x^2 - 1}{4x^4 + 2x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4+2\*x^2+1), x, algorithm="maxima")

[Out] -integrate((2\*x^2 - 1)/(4\*x^4 + 2\*x^2 + 1), x)

**mupad** [B] time = 4.37, size = 20, normalized size = 0.40

$$\frac{\sqrt{2} \operatorname{atanh} \left( \frac{\sqrt{2}x}{2x^2+1} \right)}{2}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(2*x^2 - 1)/(2*x^2 + 4*x^4 + 1), x)`

[Out]  $(2^{(1/2)}*\operatorname{atanh}(2^{(1/2)}*x)/(2*x^2 + 1))/2$

**sympy** [A] time = 0.11, size = 46, normalized size = 0.92

$$-\frac{\sqrt{2} \log\left(x^2 - \frac{\sqrt{2}x}{2} + \frac{1}{2}\right)}{4} + \frac{\sqrt{2} \log\left(x^2 + \frac{\sqrt{2}x}{2} + \frac{1}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2*x**2+1)/(4*x**4+2*x**2+1), x)`

[Out]  $-\operatorname{sqrt}(2)*\log(x**2 - \operatorname{sqrt}(2)*x/2 + 1/2)/4 + \operatorname{sqrt}(2)*\log(x**2 + \operatorname{sqrt}(2)*x/2 + 1/2)/4$

$$3.48 \quad \int \frac{1-2x^2}{1+x^2+4x^4} dx$$

**Optimal.** Leaf size=50

$$\frac{\log(2x^2 + \sqrt{3}x + 1)}{2\sqrt{3}} - \frac{\log(2x^2 - \sqrt{3}x + 1)}{2\sqrt{3}}$$

**Rubi [A]** time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {1164, 628}

$$\frac{\log(2x^2 + \sqrt{3}x + 1)}{2\sqrt{3}} - \frac{\log(2x^2 - \sqrt{3}x + 1)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 - 2\*x^2)/(1 + x^2 + 4\*x^4), x]

[Out] -Log[1 - Sqrt[3]\*x + 2\*x^2]/(2\*Sqrt[3]) + Log[1 + Sqrt[3]\*x + 2\*x^2]/(2\*Sqrt[3])

Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1164

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(-2\*d)/e - b/c, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && !GtQ[b^2 - 4\*a\*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-2x^2}{1+x^2+4x^4} dx &= \int \frac{\frac{\sqrt{3}}{2}+2x}{-\frac{1}{2}-\frac{\sqrt{3}x}{2}-x^2} dx - \int \frac{\frac{\sqrt{3}}{2}-2x}{-\frac{1}{2}+\frac{\sqrt{3}x}{2}-x^2} dx \\ &= -\frac{\log(1 - \sqrt{3}x + 2x^2)}{2\sqrt{3}} + \frac{\log(1 + \sqrt{3}x + 2x^2)}{2\sqrt{3}} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 42, normalized size = 0.84

$$\frac{\log(2x^2 + \sqrt{3}x + 1) - \log(-2x^2 + \sqrt{3}x - 1)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2\*x^2)/(1 + x^2 + 4\*x^4), x]

[Out] (-Log[-1 + Sqrt[3]\*x - 2\*x^2] + Log[1 + Sqrt[3]\*x + 2\*x^2])/(2\*Sqrt[3])

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 - 2x^2}{1 + x^2 + 4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - 2\*x^2)/(1 + x^2 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(1 - 2\*x^2)/(1 + x^2 + 4\*x^4), x]

**fricas** [A] time = 0.81, size = 43, normalized size = 0.86

$$\frac{1}{6} \sqrt{3} \log\left(\frac{4x^4 + 7x^2 + 2\sqrt{3}(2x^3 + x) + 1}{4x^4 + x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4+x^2+1), x, algorithm="fricas")

[Out] 1/6\*sqrt(3)\*log((4\*x^4 + 7\*x^2 + 2\*sqrt(3)\*(2\*x^3 + x) + 1)/(4\*x^4 + x^2 + 1))

**giac** [A] time = 0.26, size = 41, normalized size = 0.82

$$\frac{1}{6} \sqrt{3} \log\left(x^2 + \frac{1}{2} \sqrt{6} \left(\frac{1}{4}\right)^{\frac{1}{4}} x + \frac{1}{2}\right) - \frac{1}{6} \sqrt{3} \log\left(x^2 - \frac{1}{2} \sqrt{6} \left(\frac{1}{4}\right)^{\frac{1}{4}} x + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4+x^2+1), x, algorithm="giac")

[Out] 1/6\*sqrt(3)\*log(x^2 + 1/2\*sqrt(6)\*(1/4)^(1/4)\*x + 1/2) - 1/6\*sqrt(3)\*log(x^2 - 1/2\*sqrt(6)\*(1/4)^(1/4)\*x + 1/2)

**maple** [A] time = 0.01, size = 39, normalized size = 0.78

$$-\frac{\sqrt{3} \ln(2x^2 - \sqrt{3}x + 1)}{6} + \frac{\sqrt{3} \ln(2x^2 + \sqrt{3}x + 1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2\*x^2+1)/(4\*x^4+x^2+1), x)

[Out] -1/6\*ln(1+2\*x^2-3^(1/2)\*x)\*3^(1/2)+1/6\*ln(1+2\*x^2+3^(1/2)\*x)\*3^(1/2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x^2 - 1}{4x^4 + x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4+x^2+1), x, algorithm="maxima")

[Out] -integrate((2\*x^2 - 1)/(4\*x^4 + x^2 + 1), x)

**mupad** [B] time = 0.07, size = 20, normalized size = 0.40

$$\frac{\sqrt{3} \operatorname{atanh}\left(\frac{\sqrt{3}x}{2x^2+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(2*x^2 - 1)/(x^2 + 4*x^4 + 1), x)`

[Out]  $(3^{1/2} * \operatorname{atanh}((3^{1/2} * x) / (2 * x^2 + 1))) / 3$

sympy [A] time = 0.12, size = 46, normalized size = 0.92

$$-\frac{\sqrt{3} \log\left(x^2 - \frac{\sqrt{3}x}{2} + \frac{1}{2}\right)}{6} + \frac{\sqrt{3} \log\left(x^2 + \frac{\sqrt{3}x}{2} + \frac{1}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2*x**2+1)/(4*x**4+x**2+1), x)`

[Out]  $-\operatorname{sqrt}(3) * \log(x^2 - \operatorname{sqrt}(3) * x / 2 + 1/2) / 6 + \operatorname{sqrt}(3) * \log(x^2 + \operatorname{sqrt}(3) * x / 2 + 1/2) / 6$

$$3.49 \quad \int \frac{1-2x^2}{1+4x^4} dx$$

Optimal. Leaf size=31

$$\frac{1}{4} \log(2x^2 + 2x + 1) - \frac{1}{4} \log(2x^2 - 2x + 1)$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {1165, 628}

$$\frac{1}{4} \log(2x^2 + 2x + 1) - \frac{1}{4} \log(2x^2 - 2x + 1)$$

Antiderivative was successfully verified.

[In] Int[(1 - 2\*x^2)/(1 + 4\*x^4), x]

[Out] -Log[1 - 2\*x + 2\*x^2]/4 + Log[1 + 2\*x + 2\*x^2]/4

Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[d\*Log[RemoveContent[a + b\*x + c\*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1165

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

Rubi steps

$$\begin{aligned} \int \frac{1-2x^2}{1+4x^4} dx &= -\left(\frac{1}{4} \int \frac{1+2x}{-\frac{1}{2}-x-x^2} dx\right) - \frac{1}{4} \int \frac{1-2x}{-\frac{1}{2}+x-x^2} dx \\ &= -\frac{1}{4} \log(1-2x+2x^2) + \frac{1}{4} \log(1+2x+2x^2) \end{aligned}$$

Mathematica [A] time = 0.00, size = 31, normalized size = 1.00

$$\frac{1}{4} \log(2x^2 + 2x + 1) - \frac{1}{4} \log(2x^2 - 2x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2\*x^2)/(1 + 4\*x^4), x]

[Out] -1/4\*Log[1 - 2\*x + 2\*x^2] + Log[1 + 2\*x + 2\*x^2]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-2x^2}{1+4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - 2\*x^2)/(1 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(1 - 2\*x^2)/(1 + 4\*x^4), x]

**fricas** [A] time = 0.66, size = 27, normalized size = 0.87

$$\frac{1}{4} \log(2x^2 + 2x + 1) - \frac{1}{4} \log(2x^2 - 2x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4+1), x, algorithm="fricas")

[Out] 1/4\*log(2\*x^2 + 2\*x + 1) - 1/4\*log(2\*x^2 - 2\*x + 1)

**giac** [A] time = 0.16, size = 34, normalized size = 1.10

$$\frac{1}{4} \log\left(x^2 + \sqrt{2} \left(\frac{1}{4}\right)^{\frac{1}{4}} x + \frac{1}{2}\right) - \frac{1}{4} \log\left(x^2 - \sqrt{2} \left(\frac{1}{4}\right)^{\frac{1}{4}} x + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4+1), x, algorithm="giac")

[Out] 1/4\*log(x^2 + sqrt(2)\*(1/4)^(1/4)\*x + 1/2) - 1/4\*log(x^2 - sqrt(2)\*(1/4)^(1/4)\*x + 1/2)

**maple** [A] time = 0.00, size = 28, normalized size = 0.90

$$-\frac{\ln(2x^2 - 2x + 1)}{4} + \frac{\ln(2x^2 + 2x + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2\*x^2+1)/(4\*x^4+1), x)

[Out] -1/4\*ln(2\*x^2-2\*x+1)+1/4\*ln(2\*x^2+2\*x+1)

**maxima** [A] time = 1.06, size = 27, normalized size = 0.87

$$\frac{1}{4} \log(2x^2 + 2x + 1) - \frac{1}{4} \log(2x^2 - 2x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4+1), x, algorithm="maxima")

[Out] 1/4\*log(2\*x^2 + 2\*x + 1) - 1/4\*log(2\*x^2 - 2\*x + 1)

**mupad** [B] time = 0.07, size = 15, normalized size = 0.48

$$\frac{\operatorname{atanh}\left(\frac{2x}{2x^2+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(2\*x^2 - 1)/(4\*x^4 + 1), x)

[Out] atanh((2\*x)/(2\*x^2 + 1))/2

**sympy** [A] time = 0.11, size = 22, normalized size = 0.71

$$-\frac{\log\left(x^2 - x + \frac{1}{2}\right)}{4} + \frac{\log\left(x^2 + x + \frac{1}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4+1),x)
```

```
[Out] -log(x**2 - x + 1/2)/4 + log(x**2 + x + 1/2)/4
```

$$3.50 \quad \int \frac{1-2x^2}{1-x^2+4x^4} dx$$

**Optimal.** Leaf size=50

$$\frac{\log(2x^2 + \sqrt{5}x + 1)}{2\sqrt{5}} - \frac{\log(2x^2 - \sqrt{5}x + 1)}{2\sqrt{5}}$$

**Rubi [A]** time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {1164, 628}

$$\frac{\log(2x^2 + \sqrt{5}x + 1)}{2\sqrt{5}} - \frac{\log(2x^2 - \sqrt{5}x + 1)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[(1 - 2\*x^2)/(1 - x^2 + 4\*x^4), x]

[Out] -Log[1 - Sqrt[5]\*x + 2\*x^2]/(2\*Sqrt[5]) + Log[1 + Sqrt[5]\*x + 2\*x^2]/(2\*Sqrt[5])

Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1164

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(-2\*d)/e - b/c, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && !GtQ[b^2 - 4\*a\*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-2x^2}{1-x^2+4x^4} dx &= \int \frac{\frac{\sqrt{5}}{2}+2x}{-\frac{1}{2}-\frac{\sqrt{5}x}{2}-x^2} dx - \int \frac{\frac{\sqrt{5}}{2}-2x}{-\frac{1}{2}+\frac{\sqrt{5}x}{2}-x^2} dx \\ &= -\frac{\log(1 - \sqrt{5}x + 2x^2)}{2\sqrt{5}} + \frac{\log(1 + \sqrt{5}x + 2x^2)}{2\sqrt{5}} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 42, normalized size = 0.84

$$\frac{\log(2x^2 + \sqrt{5}x + 1) - \log(-2x^2 + \sqrt{5}x - 1)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2\*x^2)/(1 - x^2 + 4\*x^4), x]

[Out] (-Log[-1 + Sqrt[5]\*x - 2\*x^2] + Log[1 + Sqrt[5]\*x + 2\*x^2])/(2\*Sqrt[5])



**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 - 2x^2}{1 - x^2 + 4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - 2\*x^2)/(1 - x^2 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(1 - 2\*x^2)/(1 - x^2 + 4\*x^4), x]

**fricas** [A] time = 0.99, size = 45, normalized size = 0.90

$$\frac{1}{10} \sqrt{5} \log\left(\frac{4x^4 + 9x^2 + 2\sqrt{5}(2x^3 + x) + 1}{4x^4 - x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4-x^2+1), x, algorithm="fricas")

[Out] 1/10\*sqrt(5)\*log((4\*x^4 + 9\*x^2 + 2\*sqrt(5)\*(2\*x^3 + x) + 1)/(4\*x^4 - x^2 + 1))

**giac** [A] time = 0.24, size = 41, normalized size = 0.82

$$\frac{1}{10} \sqrt{5} \log\left(x^2 + \frac{1}{2} \sqrt{10} \left(\frac{1}{4}\right)^{\frac{1}{4}} x + \frac{1}{2}\right) - \frac{1}{10} \sqrt{5} \log\left(x^2 - \frac{1}{2} \sqrt{10} \left(\frac{1}{4}\right)^{\frac{1}{4}} x + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4-x^2+1), x, algorithm="giac")

[Out] 1/10\*sqrt(5)\*log(x^2 + 1/2\*sqrt(10)\*(1/4)^(1/4)\*x + 1/2) - 1/10\*sqrt(5)\*log(x^2 - 1/2\*sqrt(10)\*(1/4)^(1/4)\*x + 1/2)

**maple** [A] time = 0.01, size = 39, normalized size = 0.78

$$-\frac{\sqrt{5} \ln(2x^2 - \sqrt{5}x + 1)}{10} + \frac{\sqrt{5} \ln(2x^2 + \sqrt{5}x + 1)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2\*x^2+1)/(4\*x^4-x^2+1), x)

[Out] -1/10\*ln(1+2\*x^2-5^(1/2)\*x)\*5^(1/2)+1/10\*ln(1+2\*x^2+5^(1/2)\*x)\*5^(1/2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x^2 - 1}{4x^4 - x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4-x^2+1), x, algorithm="maxima")

[Out] -integrate((2\*x^2 - 1)/(4\*x^4 - x^2 + 1), x)

**mupad** [B] time = 4.35, size = 20, normalized size = 0.40

$$\frac{\sqrt{5} \operatorname{atanh}\left(\frac{\sqrt{5}x}{2x^2+1}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(2*x^2 - 1)/(4*x^4 - x^2 + 1),x)`

[Out]  $(5^{(1/2)}*\operatorname{atanh}((5^{(1/2)}*x)/(2*x^2 + 1)))/5$

sympy [A] time = 0.12, size = 46, normalized size = 0.92

$$-\frac{\sqrt{5} \log\left(x^2 - \frac{\sqrt{5}x}{2} + \frac{1}{2}\right)}{10} + \frac{\sqrt{5} \log\left(x^2 + \frac{\sqrt{5}x}{2} + \frac{1}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2*x**2+1)/(4*x**4-x**2+1),x)`

[Out]  $-\operatorname{sqrt}(5)*\log(x**2 - \operatorname{sqrt}(5)*x/2 + 1/2)/10 + \operatorname{sqrt}(5)*\log(x**2 + \operatorname{sqrt}(5)*x/2 + 1/2)/10$

$$3.51 \quad \int \frac{1-2x^2}{1-2x^2+4x^4} dx$$

**Optimal.** Leaf size=50

$$\frac{\log(2x^2 + \sqrt{6}x + 1)}{2\sqrt{6}} - \frac{\log(2x^2 - \sqrt{6}x + 1)}{2\sqrt{6}}$$

**Rubi [A]** time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {1164, 628}

$$\frac{\log(2x^2 + \sqrt{6}x + 1)}{2\sqrt{6}} - \frac{\log(2x^2 - \sqrt{6}x + 1)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[(1 - 2\*x^2)/(1 - 2\*x^2 + 4\*x^4), x]

[Out] -Log[1 - Sqrt[6]\*x + 2\*x^2]/(2\*Sqrt[6]) + Log[1 + Sqrt[6]\*x + 2\*x^2]/(2\*Sqrt[6])

Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1164

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(-2\*d)/e - b/c, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && !GtQ[b^2 - 4\*a\*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-2x^2}{1-2x^2+4x^4} dx &= -\frac{\int \frac{\sqrt{\frac{3}{2}+2x}}{-\frac{1}{2}-\sqrt{\frac{3}{2}}x-x^2} dx}{2\sqrt{6}} - \frac{\int \frac{\sqrt{\frac{3}{2}-2x}}{-\frac{1}{2}+\sqrt{\frac{3}{2}}x-x^2} dx}{2\sqrt{6}} \\ &= -\frac{\log(1 - \sqrt{6}x + 2x^2)}{2\sqrt{6}} + \frac{\log(1 + \sqrt{6}x + 2x^2)}{2\sqrt{6}} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 42, normalized size = 0.84

$$\frac{\log(2x^2 + \sqrt{6}x + 1) - \log(-2x^2 + \sqrt{6}x - 1)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2\*x^2)/(1 - 2\*x^2 + 4\*x^4), x]

[Out] (-Log[-1 + Sqrt[6]\*x - 2\*x^2] + Log[1 + Sqrt[6]\*x + 2\*x^2])/(2\*Sqrt[6])

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 - 2x^2}{1 - 2x^2 + 4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - 2\*x^2)/(1 - 2\*x^2 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(1 - 2\*x^2)/(1 - 2\*x^2 + 4\*x^4), x]

**fricas** [A] time = 0.66, size = 45, normalized size = 0.90

$$\frac{1}{12} \sqrt{6} \log \left( \frac{4x^4 + 10x^2 + 2\sqrt{6}(2x^3 + x) + 1}{4x^4 - 2x^2 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4-2\*x^2+1), x, algorithm="fricas")

[Out] 1/12\*sqrt(6)\*log((4\*x^4 + 10\*x^2 + 2\*sqrt(6)\*(2\*x^3 + x) + 1)/(4\*x^4 - 2\*x^2 + 1))

**giac** [A] time = 0.18, size = 40, normalized size = 0.80

$$\frac{1}{12} \sqrt{6} \log \left( x^2 + \sqrt{3} \left( \frac{1}{4} \right)^{\frac{1}{4}} x + \frac{1}{2} \right) - \frac{1}{12} \sqrt{6} \log \left( x^2 - \sqrt{3} \left( \frac{1}{4} \right)^{\frac{1}{4}} x + \frac{1}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4-2\*x^2+1), x, algorithm="giac")

[Out] 1/12\*sqrt(6)\*log(x^2 + sqrt(3)\*(1/4)^(1/4)\*x + 1/2) - 1/12\*sqrt(6)\*log(x^2 - sqrt(3)\*(1/4)^(1/4)\*x + 1/2)

**maple** [A] time = 0.01, size = 39, normalized size = 0.78

$$-\frac{\sqrt{6} \ln(2x^2 - \sqrt{6}x + 1)}{12} + \frac{\sqrt{6} \ln(2x^2 + \sqrt{6}x + 1)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2\*x^2+1)/(4\*x^4-2\*x^2+1), x)

[Out] -1/12\*ln(1+2\*x^2-6^(1/2)\*x)\*6^(1/2)+1/12\*ln(1+2\*x^2+6^(1/2)\*x)\*6^(1/2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x^2 - 1}{4x^4 - 2x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4-2\*x^2+1), x, algorithm="maxima")

[Out] -integrate((2\*x^2 - 1)/(4\*x^4 - 2\*x^2 + 1), x)

**mupad** [B] time = 0.07, size = 20, normalized size = 0.40

$$\frac{\sqrt{6} \operatorname{atanh} \left( \frac{\sqrt{6}x}{2x^2+1} \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(2*x^2 - 1)/(4*x^4 - 2*x^2 + 1), x)`

[Out]  $(6^{(1/2)} * \operatorname{atanh}((6^{(1/2)} * x) / (2 * x^2 + 1))) / 6$

sympy [A] time = 0.12, size = 46, normalized size = 0.92

$$-\frac{\sqrt{6} \log\left(x^2 - \frac{\sqrt{6}x}{2} + \frac{1}{2}\right)}{12} + \frac{\sqrt{6} \log\left(x^2 + \frac{\sqrt{6}x}{2} + \frac{1}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2*x**2+1)/(4*x**4-2*x**2+1), x)`

[Out]  $-\operatorname{sqrt}(6) * \log(x**2 - \operatorname{sqrt}(6) * x / 2 + 1 / 2) / 12 + \operatorname{sqrt}(6) * \log(x**2 + \operatorname{sqrt}(6) * x / 2 + 1 / 2) / 12$

$$3.52 \quad \int \frac{1-2x^2}{1-3x^2+4x^4} dx$$

Optimal. Leaf size=50

$$\frac{\log(2x^2 + \sqrt{7}x + 1)}{2\sqrt{7}} - \frac{\log(2x^2 - \sqrt{7}x + 1)}{2\sqrt{7}}$$

**Rubi [A]** time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {1164, 628}

$$\frac{\log(2x^2 + \sqrt{7}x + 1)}{2\sqrt{7}} - \frac{\log(2x^2 - \sqrt{7}x + 1)}{2\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Int[(1 - 2\*x^2)/(1 - 3\*x^2 + 4\*x^4), x]

[Out] -Log[1 - Sqrt[7]\*x + 2\*x^2]/(2\*Sqrt[7]) + Log[1 + Sqrt[7]\*x + 2\*x^2]/(2\*Sqrt[7])

Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1164

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(-2\*d)/e - b/c, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && !GtQ[b^2 - 4\*a\*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-2x^2}{1-3x^2+4x^4} dx &= \int \frac{\frac{\sqrt{7}}{2}+2x}{-\frac{1}{2}-\frac{\sqrt{7}x}{2}-x^2} dx - \int \frac{\frac{\sqrt{7}}{2}-2x}{-\frac{1}{2}+\frac{\sqrt{7}x}{2}-x^2} dx \\ &= -\frac{\log(1 - \sqrt{7}x + 2x^2)}{2\sqrt{7}} + \frac{\log(1 + \sqrt{7}x + 2x^2)}{2\sqrt{7}} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 42, normalized size = 0.84

$$\frac{\log(2x^2 + \sqrt{7}x + 1) - \log(-2x^2 + \sqrt{7}x - 1)}{2\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2\*x^2)/(1 - 3\*x^2 + 4\*x^4), x]

[Out] (-Log[-1 + Sqrt[7]\*x - 2\*x^2] + Log[1 + Sqrt[7]\*x + 2\*x^2])/(2\*Sqrt[7])

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 - 2x^2}{1 - 3x^2 + 4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - 2\*x^2)/(1 - 3\*x^2 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(1 - 2\*x^2)/(1 - 3\*x^2 + 4\*x^4), x]

**fricas** [A] time = 0.66, size = 45, normalized size = 0.90

$$\frac{1}{14} \sqrt{7} \log\left(\frac{4x^4 + 11x^2 + 2\sqrt{7}(2x^3 + x) + 1}{4x^4 - 3x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4-3\*x^2+1), x, algorithm="fricas")

[Out] 1/14\*sqrt(7)\*log((4\*x^4 + 11\*x^2 + 2\*sqrt(7)\*(2\*x^3 + x) + 1)/(4\*x^4 - 3\*x^2 + 1))

**giac** [A] time = 0.21, size = 41, normalized size = 0.82

$$\frac{1}{14} \sqrt{7} \log\left(x^2 + \frac{1}{2} \sqrt{14} \left(\frac{1}{4}\right)^{\frac{1}{4}} x + \frac{1}{2}\right) - \frac{1}{14} \sqrt{7} \log\left(x^2 - \frac{1}{2} \sqrt{14} \left(\frac{1}{4}\right)^{\frac{1}{4}} x + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4-3\*x^2+1), x, algorithm="giac")

[Out] 1/14\*sqrt(7)\*log(x^2 + 1/2\*sqrt(14)\*(1/4)^(1/4)\*x + 1/2) - 1/14\*sqrt(7)\*log(x^2 - 1/2\*sqrt(14)\*(1/4)^(1/4)\*x + 1/2)

**maple** [A] time = 0.01, size = 39, normalized size = 0.78

$$-\frac{\sqrt{7} \ln(2x^2 - \sqrt{7} x + 1)}{14} + \frac{\sqrt{7} \ln(2x^2 + \sqrt{7} x + 1)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2\*x^2+1)/(4\*x^4-3\*x^2+1), x)

[Out] -1/14\*ln(1+2\*x^2-x\*7^(1/2))\*7^(1/2)+1/14\*ln(1+2\*x^2+x\*7^(1/2))\*7^(1/2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x^2 - 1}{4x^4 - 3x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4-3\*x^2+1), x, algorithm="maxima")

[Out] -integrate((2\*x^2 - 1)/(4\*x^4 - 3\*x^2 + 1), x)

**mupad** [B] time = 4.39, size = 20, normalized size = 0.40

$$\frac{\sqrt{7} \operatorname{atanh}\left(\frac{\sqrt{7} x}{2x^2+1}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(2*x^2 - 1)/(4*x^4 - 3*x^2 + 1),x)`

[Out]  $(7^{(1/2)}*\operatorname{atanh}((7^{(1/2)}*x)/(2*x^2 + 1)))/7$

sympy [A] time = 0.13, size = 46, normalized size = 0.92

$$-\frac{\sqrt{7} \log\left(x^2 - \frac{\sqrt{7}x}{2} + \frac{1}{2}\right)}{14} + \frac{\sqrt{7} \log\left(x^2 + \frac{\sqrt{7}x}{2} + \frac{1}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2*x**2+1)/(4*x**4-3*x**2+1),x)`

[Out]  $-\operatorname{sqrt}(7)*\log(x**2 - \operatorname{sqrt}(7)*x/2 + 1/2)/14 + \operatorname{sqrt}(7)*\log(x**2 + \operatorname{sqrt}(7)*x/2 + 1/2)/14$



$$3.53 \quad \int \frac{1-2x^2}{1-4x^2+4x^4} dx$$

Optimal. Leaf size=14

$$\frac{\tanh^{-1}(\sqrt{2}x)}{\sqrt{2}}$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {28, 21, 206}

$$\frac{\tanh^{-1}(\sqrt{2}x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 - 2\*x^2)/(1 - 4\*x^2 + 4\*x^4), x]

[Out] ArcTanh[Sqrt[2]\*x]/Sqrt[2]

Rule 21

Int[(u\_.)\*((a\_) + (b\_.)\*(v\_))^(m\_.)\*((c\_) + (d\_.)\*(v\_))^(n\_.), x\_Symbol] :> Dist[(b/d)^m, Int[u\*(c + d\*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b\*c - a\*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d\*x, a + b\*x])

Rule 28

Int[(u\_.)\*((a\_) + (c\_.)\*(x\_)^(n2\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] :> Dist[1/c^p, Int[u\*(b/2 + c\*x^n)^(2\*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2\*n] && EqQ[b^2 - 4\*a\*c, 0] && IntegerQ[p]

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1-2x^2}{1-4x^2+4x^4} dx &= 4 \int \frac{1-2x^2}{(-2+4x^2)^2} dx \\ &= \int \frac{1}{1-2x^2} dx \\ &= \frac{\tanh^{-1}(\sqrt{2}x)}{\sqrt{2}} \end{aligned}$$

Mathematica [B] time = 0.01, size = 32, normalized size = 2.29

$$\frac{\log(2x + \sqrt{2}) - \log(\sqrt{2} - 2x)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2\*x^2)/(1 - 4\*x^2 + 4\*x^4), x]

[Out] (-Log[Sqrt[2] - 2\*x] + Log[Sqrt[2] + 2\*x])/(2\*Sqrt[2])

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 - 2x^2}{1 - 4x^2 + 4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - 2\*x^2)/(1 - 4\*x^2 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(1 - 2\*x^2)/(1 - 4\*x^2 + 4\*x^4), x]

**fricas** [B] time = 0.61, size = 29, normalized size = 2.07

$$\frac{1}{4} \sqrt{2} \log\left(\frac{2x^2 + 2\sqrt{2}x + 1}{2x^2 - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4-4\*x^2+1), x, algorithm="fricas")

[Out] 1/4\*sqrt(2)\*log((2\*x^2 + 2\*sqrt(2)\*x + 1)/(2\*x^2 - 1))

**giac** [B] time = 0.16, size = 29, normalized size = 2.07

$$\frac{1}{4} \sqrt{2} \log\left(\left|x + \frac{1}{2} \sqrt{2}\right|\right) - \frac{1}{4} \sqrt{2} \log\left(\left|x - \frac{1}{2} \sqrt{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4-4\*x^2+1), x, algorithm="giac")

[Out] 1/4\*sqrt(2)\*log(abs(x + 1/2\*sqrt(2))) - 1/4\*sqrt(2)\*log(abs(x - 1/2\*sqrt(2)))

**maple** [A] time = 0.00, size = 12, normalized size = 0.86

$$\frac{\sqrt{2} \operatorname{arctanh}(\sqrt{2} x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2\*x^2+1)/(4\*x^4-4\*x^2+1), x)

[Out] 1/2\*arctanh(2^(1/2)\*x)\*2^(1/2)

**maxima** [B] time = 2.35, size = 25, normalized size = 1.79

$$-\frac{1}{4} \sqrt{2} \log\left(\frac{2x - \sqrt{2}}{2x + \sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4-4\*x^2+1), x, algorithm="maxima")

[Out] -1/4\*sqrt(2)\*log((2\*x - sqrt(2))/(2\*x + sqrt(2)))

**mupad** [B] time = 4.33, size = 11, normalized size = 0.79

$$\frac{\sqrt{2} \operatorname{atanh}(\sqrt{2} x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(2*x^2 - 1)/(4*x^4 - 4*x^2 + 1), x)`

[Out]  $(2^{(1/2)} * \operatorname{atanh}(2^{(1/2)} * x)) / 2$

**sympy [B]** time = 0.11, size = 32, normalized size = 2.29

$$-\frac{\sqrt{2} \log\left(x - \frac{\sqrt{2}}{2}\right)}{4} + \frac{\sqrt{2} \log\left(x + \frac{\sqrt{2}}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2*x**2+1)/(4*x**4-4*x**2+1), x)`

[Out]  $-\operatorname{sqrt}(2) * \log(x - \operatorname{sqrt}(2)/2) / 4 + \operatorname{sqrt}(2) * \log(x + \operatorname{sqrt}(2)/2) / 4$

$$3.54 \quad \int \frac{1-2x^2}{1-5x^2+4x^4} dx$$

Optimal. Leaf size=39

$$-\frac{1}{6} \log(1-2x) - \frac{1}{6} \log(1-x) + \frac{1}{6} \log(x+1) + \frac{1}{6} \log(2x+1)$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {1161, 616, 31}

$$-\frac{1}{6} \log(1-2x) - \frac{1}{6} \log(1-x) + \frac{1}{6} \log(x+1) + \frac{1}{6} \log(2x+1)$$

Antiderivative was successfully verified.

[In] Int[(1 - 2\*x^2)/(1 - 5\*x^2 + 4\*x^4), x]

[Out] -Log[1 - 2\*x]/6 - Log[1 - x]/6 + Log[1 + x]/6 + Log[1 + 2\*x]/6

Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 616

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(n\_), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c\*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c\*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && PosQ[b^2 - 4\*a\*c] && PerfectSquareQ[b^2 - 4\*a\*c]

Rule 1161

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

Rubi steps

$$\begin{aligned} \int \frac{1-2x^2}{1-5x^2+4x^4} dx &= -\left(\frac{1}{4} \int \frac{1}{-\frac{1}{2}-\frac{x}{2}+x^2} dx\right) - \frac{1}{4} \int \frac{1}{-\frac{1}{2}+\frac{x}{2}+x^2} dx \\ &= -\left(\frac{1}{6} \int \frac{1}{-1+x} dx\right) - \frac{1}{6} \int \frac{1}{-\frac{1}{2}+x} dx + \frac{1}{6} \int \frac{1}{\frac{1}{2}+x} dx + \frac{1}{6} \int \frac{1}{1+x} dx \\ &= -\frac{1}{6} \log(1-2x) - \frac{1}{6} \log(1-x) + \frac{1}{6} \log(1+x) + \frac{1}{6} \log(1+2x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.79

$$\frac{1}{6} \log(2x^2 + 3x + 1) - \frac{1}{6} \log(2x^2 - 3x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2\*x^2)/(1 - 5\*x^2 + 4\*x^4), x]

[Out] -1/6\*Log[1 - 3\*x + 2\*x^2] + Log[1 + 3\*x + 2\*x^2]/6

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 - 2x^2}{1 - 5x^2 + 4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - 2\*x^2)/(1 - 5\*x^2 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(1 - 2\*x^2)/(1 - 5\*x^2 + 4\*x^4), x]

**fricas** [A] time = 0.47, size = 27, normalized size = 0.69

$$\frac{1}{6} \log(2x^2 + 3x + 1) - \frac{1}{6} \log(2x^2 - 3x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4-5\*x^2+1), x, algorithm="fricas")

[Out] 1/6\*log(2\*x^2 + 3\*x + 1) - 1/6\*log(2\*x^2 - 3\*x + 1)

**giac** [A] time = 0.15, size = 33, normalized size = 0.85

$$\frac{1}{6} \log(|2x + 1|) - \frac{1}{6} \log(|2x - 1|) + \frac{1}{6} \log(|x + 1|) - \frac{1}{6} \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4-5\*x^2+1), x, algorithm="giac")

[Out] 1/6\*log(abs(2\*x + 1)) - 1/6\*log(abs(2\*x - 1)) + 1/6\*log(abs(x + 1)) - 1/6\*log(abs(x - 1))

**maple** [A] time = 0.01, size = 30, normalized size = 0.77

$$\frac{\ln(x + 1)}{6} + \frac{\ln(2x + 1)}{6} - \frac{\ln(x - 1)}{6} - \frac{\ln(2x - 1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2\*x^2+1)/(4\*x^4-5\*x^2+1), x)

[Out] -1/6\*ln(2\*x-1)+1/6\*ln(2\*x+1)+1/6\*ln(x+1)-1/6\*ln(x-1)

**maxima** [A] time = 0.96, size = 29, normalized size = 0.74

$$\frac{1}{6} \log(2x + 1) - \frac{1}{6} \log(2x - 1) + \frac{1}{6} \log(x + 1) - \frac{1}{6} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4-5\*x^2+1), x, algorithm="maxima")

[Out] 1/6\*log(2\*x + 1) - 1/6\*log(2\*x - 1) + 1/6\*log(x + 1) - 1/6\*log(x - 1)

**mupad** [B] time = 0.10, size = 15, normalized size = 0.38

$$\frac{\operatorname{atanh}\left(\frac{3x}{2x^2+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(2*x^2 - 1)/(4*x^4 - 5*x^2 + 1),x)`

[Out] `atanh((3*x)/(2*x^2 + 1))/3`

sympy [A] time = 0.12, size = 29, normalized size = 0.74

$$-\frac{\log\left(x^2 - \frac{3x}{2} + \frac{1}{2}\right)}{6} + \frac{\log\left(x^2 + \frac{3x}{2} + \frac{1}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2*x**2+1)/(4*x**4-5*x**2+1),x)`

[Out] `-log(x**2 - 3*x/2 + 1/2)/6 + log(x**2 + 3*x/2 + 1/2)/6`

$$3.55 \quad \int \frac{1-2x^2}{1-6x^2+4x^4} dx$$

Optimal. Leaf size=48

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{2}x+1}{\sqrt{5}}\right)}{\sqrt{10}} - \frac{\tanh^{-1}\left(\frac{1-2\sqrt{2}x}{\sqrt{5}}\right)}{\sqrt{10}}$$

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{2}x+1}{\sqrt{5}}\right)}{\sqrt{10}} - \frac{\tanh^{-1}\left(\frac{1-2\sqrt{2}x}{\sqrt{5}}\right)}{\sqrt{10}}$$

Antiderivative was successfully verified.

[In] Int[(1 - 2\*x^2)/(1 - 6\*x^2 + 4\*x^4), x]

[Out] -(ArcTanh[(1 - 2\*Sqrt[2]\*x)/Sqrt[5]]/Sqrt[10]) + ArcTanh[(1 + 2\*Sqrt[2]\*x)/Sqrt[5]]/Sqrt[10]

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 1161

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

Rubi steps

$$\begin{aligned} \int \frac{1-2x^2}{1-6x^2+4x^4} dx &= -\left(\frac{1}{4} \int \frac{1}{-\frac{1}{2} - \frac{x}{\sqrt{2}} + x^2} dx\right) - \frac{1}{4} \int \frac{1}{-\frac{1}{2} + \frac{x}{\sqrt{2}} + x^2} dx \\ &= \frac{1}{2} \text{Subst}\left(\int \frac{1}{\frac{5}{2} - x^2} dx, x, -\frac{1}{\sqrt{2}} + 2x\right) + \frac{1}{2} \text{Subst}\left(\int \frac{1}{\frac{5}{2} - x^2} dx, x, \frac{1}{\sqrt{2}} + 2x\right) \\ &= -\frac{\tanh^{-1}\left(\frac{1-2\sqrt{2}x}{\sqrt{5}}\right)}{\sqrt{10}} + \frac{\tanh^{-1}\left(\frac{1+2\sqrt{2}x}{\sqrt{5}}\right)}{\sqrt{10}} \end{aligned}$$

**Mathematica** [A] time = 0.02, size = 42, normalized size = 0.88

$$\frac{\log(2x^2 + \sqrt{10}x + 1) - \log(-2x^2 + \sqrt{10}x - 1)}{2\sqrt{10}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2\*x^2)/(1 - 6\*x^2 + 4\*x^4), x]

[Out] (-Log[-1 + Sqrt[10]\*x - 2\*x^2] + Log[1 + Sqrt[10]\*x + 2\*x^2])/(2\*Sqrt[10])

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 - 2x^2}{1 - 6x^2 + 4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - 2\*x^2)/(1 - 6\*x^2 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(1 - 2\*x^2)/(1 - 6\*x^2 + 4\*x^4), x]

**fricas** [A] time = 0.59, size = 45, normalized size = 0.94

$$\frac{1}{20} \sqrt{10} \log\left(\frac{4x^4 + 14x^2 + 2\sqrt{10}(2x^3 + x) + 1}{4x^4 - 6x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4-6\*x^2+1), x, algorithm="fricas")

[Out] 1/20\*sqrt(10)\*log((4\*x^4 + 14\*x^2 + 2\*sqrt(10)\*(2\*x^3 + x) + 1)/(4\*x^4 - 6\*x^2 + 1))

**giac** [A] time = 0.32, size = 77, normalized size = 1.60

$$\frac{1}{20} \sqrt{10} \log\left(\left|x + \frac{1}{4} \sqrt{10} + \frac{1}{4} \sqrt{2}\right|\right) + \frac{1}{20} \sqrt{10} \log\left(\left|x + \frac{1}{4} \sqrt{10} - \frac{1}{4} \sqrt{2}\right|\right) - \frac{1}{20} \sqrt{10} \log\left(\left|x - \frac{1}{4} \sqrt{10} + \frac{1}{4} \sqrt{2}\right|\right) - \frac{1}{20} \sqrt{10} \log\left(\left|x - \frac{1}{4} \sqrt{10} - \frac{1}{4} \sqrt{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4-6\*x^2+1), x, algorithm="giac")

[Out] 1/20\*sqrt(10)\*log(abs(x + 1/4\*sqrt(10) + 1/4\*sqrt(2))) + 1/20\*sqrt(10)\*log(abs(x + 1/4\*sqrt(10) - 1/4\*sqrt(2))) - 1/20\*sqrt(10)\*log(abs(x - 1/4\*sqrt(10) + 1/4\*sqrt(2))) - 1/20\*sqrt(10)\*log(abs(x - 1/4\*sqrt(10) - 1/4\*sqrt(2)))

**maple** [B] time = 0.02, size = 82, normalized size = 1.71

$$\frac{2(\sqrt{5} - 1)\sqrt{5} \operatorname{arctanh}\left(\frac{8x}{2\sqrt{10} - 2\sqrt{2}}\right)}{5(2\sqrt{10} - 2\sqrt{2})} + \frac{2(\sqrt{5} + 1)\sqrt{5} \operatorname{arctanh}\left(\frac{8x}{2\sqrt{10} + 2\sqrt{2}}\right)}{5(2\sqrt{10} + 2\sqrt{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2\*x^2+1)/(4\*x^4-6\*x^2+1), x)

[Out] 2/5\*(5^(1/2)-1)\*5^(1/2)/(2\*10^(1/2)-2\*2^(1/2))\*arctanh(8/(2\*10^(1/2)-2\*2^(1/2))\*x)+2/5\*(5^(1/2)+1)\*5^(1/2)/(2\*10^(1/2)+2\*2^(1/2))\*arctanh(8/(2\*10^(1/2)+2\*2^(1/2))\*x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x^2 - 1}{4x^4 - 6x^2 + 1} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x^2+1)/(4\*x^4-6\*x^2+1),x, algorithm="maxima")

[Out] -integrate((2\*x^2 - 1)/(4\*x^4 - 6\*x^2 + 1), x)

**mupad [B]** time = 0.13, size = 20, normalized size = 0.42

$$\frac{\sqrt{10} \operatorname{atanh}\left(\frac{\sqrt{10} x}{2x^2+1}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(2\*x^2 - 1)/(4\*x^4 - 6\*x^2 + 1),x)

[Out] (10^(1/2)\*atanh((10^(1/2)\*x)/(2\*x^2 + 1)))/10

**sympy [A]** time = 0.12, size = 46, normalized size = 0.96

$$-\frac{\sqrt{10} \log\left(x^2 - \frac{\sqrt{10}x}{2} + \frac{1}{2}\right)}{20} + \frac{\sqrt{10} \log\left(x^2 + \frac{\sqrt{10}x}{2} + \frac{1}{2}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*x\*\*2+1)/(4\*x\*\*4-6\*x\*\*2+1),x)

[Out] -sqrt(10)\*log(x\*\*2 - sqrt(10)\*x/2 + 1/2)/20 + sqrt(10)\*log(x\*\*2 + sqrt(10)\*x/2 + 1/2)/20

$$3.56 \quad \int \frac{1+x^2}{1+bx^2+x^4} dx$$

Optimal. Leaf size=62

$$\frac{\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{b+2}}\right)}{\sqrt{b+2}} - \frac{\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{\sqrt{b+2}}$$

**Rubi [A]** time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{b+2}}\right)}{\sqrt{b+2}} - \frac{\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{\sqrt{b+2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 + b\*x^2 + x^4), x]

[Out] -(ArcTan[(Sqrt[2 - b] - 2\*x)/Sqrt[2 + b]]/Sqrt[2 + b]) + ArcTan[(Sqrt[2 - b] + 2\*x)/Sqrt[2 + b]]/Sqrt[2 + b]

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 1161

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

#### Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{1+bx^2+x^4} dx &= \frac{1}{2} \int \frac{1}{1-\sqrt{2-b}x+x^2} dx + \frac{1}{2} \int \frac{1}{1+\sqrt{2-b}x+x^2} dx \\ &= -\text{Subst}\left(\int \frac{1}{-2-b-x^2} dx, x, -\sqrt{2-b}+2x\right) - \text{Subst}\left(\int \frac{1}{-2-b-x^2} dx, x, \sqrt{2-b}+2x\right) \\ &= \frac{\tan^{-1}\left(\frac{-\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{\sqrt{2+b}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{\sqrt{2+b}} \end{aligned}$$

**Mathematica [A]** time = 0.06, size = 124, normalized size = 2.00

$$\frac{\left(\sqrt{b^2-4}-b+2\right)\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b-\sqrt{b^2-4}}}\right)+\left(\sqrt{b^2-4}+b-2\right)\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{b^2-4}+b}}\right)}{\sqrt{b-\sqrt{b^2-4}}+\sqrt{\sqrt{b^2-4}+b}}$$

$$\frac{\sqrt{2}\sqrt{b^2-4}}{\sqrt{b-\sqrt{b^2-4}}+\sqrt{\sqrt{b^2-4}+b}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 + b\*x^2 + x^4), x]

[Out] (((2 - b + Sqrt[-4 + b^2])\*ArcTan[(Sqrt[2]\*x)/Sqrt[b - Sqrt[-4 + b^2]]])/Sqrt[b - Sqrt[-4 + b^2]] + ((-2 + b + Sqrt[-4 + b^2])\*ArcTan[(Sqrt[2]\*x)/Sqrt[b + Sqrt[-4 + b^2]]])/Sqrt[b + Sqrt[-4 + b^2]])/(Sqrt[2]\*Sqrt[-4 + b^2])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^2}{1+bx^2+x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^2)/(1 + b\*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(1 + x^2)/(1 + b\*x^2 + x^4), x]

**fricas [A]** time = 1.66, size = 101, normalized size = 1.63

$$\left[ \frac{\sqrt{-b-2} \log\left(\frac{x^4-(b+4)x^2-2(x^3-x)\sqrt{-b-2}+1}{x^4+bx^2+1}\right)}{2(b+2)}, \frac{\sqrt{b+2} \arctan\left(\frac{x^3+(b+1)x}{\sqrt{b+2}}\right) + \sqrt{b+2} \arctan\left(\frac{x}{\sqrt{b+2}}\right)}{b+2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+b\*x^2+1), x, algorithm="fricas")

[Out] [-1/2\*sqrt(-b - 2)\*log((x^4 - (b + 4)\*x^2 - 2\*(x^3 - x)\*sqrt(-b - 2) + 1)/(x^4 + b\*x^2 + 1))/(b + 2), (sqrt(b + 2)\*arctan((x^3 + (b + 1)\*x)/sqrt(b + 2)) + sqrt(b + 2)\*arctan(x/sqrt(b + 2)))/(b + 2)]

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+b\*x^2+1), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [b]=[0]Precision problem choosing root in common\_EXT, current precision 14Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [b]=[0]Precision problem choosing root in common\_EXT, current precision 14Undef/Unsigned Inf encountered in limitLimit: Max order reached or unable to make series expansion Error: Bad Argument Value

**maple [B]** time = 0.04, size = 277, normalized size = 4.47

$$\frac{b \arctan\left(\frac{2x}{\sqrt{2b-2\sqrt{(b-2)(b+2)}}}\right)}{\sqrt{(b-2)(b+2)}\sqrt{2b-2\sqrt{(b-2)(b+2)}}} + \frac{b \arctan\left(\frac{2x}{\sqrt{2b+2\sqrt{(b-2)(b+2)}}}\right)}{\sqrt{(b-2)(b+2)}\sqrt{2b+2\sqrt{(b-2)(b+2)}}} + \frac{2 \arctan\left(\frac{2x}{\sqrt{2b-2\sqrt{(b-2)(b+2)}}}\right)}{\sqrt{(b-2)(b+2)}\sqrt{2b-2\sqrt{(b-2)(b+2)}}} + \frac{\arctan\left(\frac{2x}{\sqrt{2b-2\sqrt{(b-2)(b+2)}}}\right)}{\sqrt{2b-2\sqrt{(b-2)(b+2)}}} - \frac{2 \arctan\left(\frac{2x}{\sqrt{2b+2\sqrt{(b-2)(b+2)}}}\right)}{\sqrt{(b-2)(b+2)}\sqrt{2b+2\sqrt{(b-2)(b+2)}}} + \frac{\arctan\left(\frac{2x}{\sqrt{2b+2\sqrt{(b-2)(b+2)}}}\right)}{\sqrt{2b+2\sqrt{(b-2)(b+2)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+1)/(x^4+b*x^2+1),x)`

[Out] 
$$\begin{aligned} & -2/((b-2)*(2+b))^{(1/2)}/(2*((b-2)*(2+b))^{(1/2)}+2*b)^{(1/2)}*\arctan(2*x/(2*((b-2)*(2+b))^{(1/2)}+2*b)^{(1/2)}) \\ & +1/(2*((b-2)*(2+b))^{(1/2)}+2*b)^{(1/2)}*\arctan(2*x/(2*((b-2)*(2+b))^{(1/2)}+2*b)^{(1/2)}) \\ & +1/((b-2)*(2+b))^{(1/2)}/(2*((b-2)*(2+b))^{(1/2)}+2*b)^{(1/2)}*\arctan(2*x/(2*((b-2)*(2+b))^{(1/2)}+2*b)^{(1/2)}) \\ & *b+2/((b-2)*(2+b))^{(1/2)}/(-2*((b-2)*(2+b))^{(1/2)}+2*b)^{(1/2)}*\arctan(2*x/(-2*((b-2)*(2+b))^{(1/2)}+2*b)^{(1/2)}) \\ & +1/(-2*((b-2)*(2+b))^{(1/2)}+2*b)^{(1/2)}*\arctan(2*x/(-2*((b-2)*(2+b))^{(1/2)}+2*b)^{(1/2)}) \\ & -1/((b-2)*(2+b))^{(1/2)}/(-2*((b-2)*(2+b))^{(1/2)}+2*b)^{(1/2)}*\arctan(2*x/(-2*((b-2)*(2+b))^{(1/2)}+2*b)^{(1/2)}) *b \end{aligned}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + 1}{x^4 + bx^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)/(x^4+b*x^2+1),x, algorithm="maxima")`

[Out] `integrate((x^2 + 1)/(x^4 + b*x^2 + 1), x)`

**mupad** [B] time = 0.06, size = 73, normalized size = 1.18

$$\frac{\operatorname{atan}\left(\frac{x}{\sqrt{b+2}}\right) + \operatorname{atan}\left((b+2)\left(x\left(\frac{1}{\sqrt{b+2}} + \frac{\frac{4}{b+2}-1}{(b-2)\sqrt{b+2}}\right) + \frac{x^3\left(\frac{2b}{b+2}-1\right)}{(b-2)\sqrt{b+2}}\right)\right)}{\sqrt{b+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + 1)/(b*x^2 + x^4 + 1),x)`

[Out] 
$$\begin{aligned} & (\operatorname{atan}(x/(b+2)^{(1/2)}) + \operatorname{atan}((b+2)*(x*(1/(b+2)^{(1/2)} + (4/(b+2) - 1) \\ & /((b-2)*(b+2)^{(1/2)}))) + (x^3*((2*b)/(b+2) - 1))/((b-2)*(b+2)^{(1/2)} \\ & )))))/(b+2)^{(1/2)} \end{aligned}$$

**sympy** [A] time = 0.38, size = 88, normalized size = 1.42

$$-\frac{\sqrt{-\frac{1}{b+2}} \log\left(x^2 + x\left(-b\sqrt{-\frac{1}{b+2}} - 2\sqrt{-\frac{1}{b+2}}\right) - 1\right)}{2} + \frac{\sqrt{-\frac{1}{b+2}} \log\left(x^2 + x\left(b\sqrt{-\frac{1}{b+2}} + 2\sqrt{-\frac{1}{b+2}}\right) - 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+1)/(x**4+b*x**2+1),x)`

[Out] 
$$\begin{aligned} & -\sqrt{-1/(b+2)}*\log(x**2 + x*(-b*\sqrt{-1/(b+2)} - 2*\sqrt{-1/(b+2)})) - \\ & 1)/2 + \sqrt{-1/(b+2)}*\log(x**2 + x*(b*\sqrt{-1/(b+2)} + 2*\sqrt{-1/(b+2)})) - \\ & 1)/2 \end{aligned}$$

$$3.57 \quad \int \frac{1+x^2}{1+5x^2+x^4} dx$$

**Optimal.** Leaf size=49

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{7}} + \frac{\tan^{-1}\left(\sqrt{\frac{1}{2}(5+\sqrt{21})}x\right)}{\sqrt{7}}$$

**Rubi [A]** time = 0.09, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {1163, 203}

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{7}} + \frac{\tan^{-1}\left(\sqrt{\frac{1}{2}(5+\sqrt{21})}x\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 + 5\*x^2 + x^4), x]

[Out] ArcTan[Sqrt[2/(5 + Sqrt[21])] \* x] / Sqrt[7] + ArcTan[Sqrt[(5 + Sqrt[21])/2] \* x] / Sqrt[7]

Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1163

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[e/2 + (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 - q/2 + c\*x^2), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && GtQ[b^2 - 4\*a\*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{1+5x^2+x^4} dx &= \frac{1}{14} (7 - \sqrt{21}) \int \frac{1}{\frac{5}{2} - \frac{\sqrt{21}}{2} + x^2} dx + \frac{1}{14} (7 + \sqrt{21}) \int \frac{1}{\frac{5}{2} + \frac{\sqrt{21}}{2} + x^2} dx \\ &= \frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{7}} + \frac{\tan^{-1}\left(\sqrt{\frac{1}{2}(5+\sqrt{21})}x\right)}{\sqrt{7}} \end{aligned}$$

**Mathematica [A]** time = 0.14, size = 83, normalized size = 1.69

$$\frac{(\sqrt{21} - 3) \tan^{-1}\left(\sqrt{\frac{2}{5-\sqrt{21}}}x\right)}{\sqrt{42(5-\sqrt{21})}} + \frac{(3 + \sqrt{21}) \tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{42(5+\sqrt{21})}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 + 5\*x^2 + x^4), x]

[Out]  $((-3 + \text{Sqrt}[21]) \cdot \text{ArcTan}[\text{Sqrt}[2/(5 - \text{Sqrt}[21])] \cdot x]) / \text{Sqrt}[42 \cdot (5 - \text{Sqrt}[21])] + ((3 + \text{Sqrt}[21]) \cdot \text{ArcTan}[\text{Sqrt}[2/(5 + \text{Sqrt}[21])] \cdot x]) / \text{Sqrt}[42 \cdot (5 + \text{Sqrt}[21])]$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^2}{1+5x^2+x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^2)/(1 + 5\*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(1 + x^2)/(1 + 5\*x^2 + x^4), x]

**fricas** [A] time = 0.81, size = 31, normalized size = 0.63

$$\frac{1}{7} \sqrt{7} \arctan\left(\frac{1}{7} \sqrt{7} (x^3 + 6x)\right) + \frac{1}{7} \sqrt{7} \arctan\left(\frac{1}{7} \sqrt{7} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+5\*x^2+1), x, algorithm="fricas")

[Out]  $1/7 \cdot \text{sqrt}(7) \cdot \arctan(1/7 \cdot \text{sqrt}(7) \cdot (x^3 + 6x)) + 1/7 \cdot \text{sqrt}(7) \cdot \arctan(1/7 \cdot \text{sqrt}(7) \cdot x)$

**giac** [A] time = 0.18, size = 26, normalized size = 0.53

$$\frac{1}{14} \sqrt{7} \left( \pi \text{sgn}(x) + 2 \arctan\left(\frac{\sqrt{7}(x^2 - 1)}{7x}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+5\*x^2+1), x, algorithm="giac")

[Out]  $1/14 \cdot \text{sqrt}(7) \cdot (\pi \cdot \text{sgn}(x) + 2 \cdot \arctan(1/7 \cdot \text{sqrt}(7) \cdot (x^2 - 1)/x))$

**maple** [B] time = 0.05, size = 136, normalized size = 2.78

$$-\frac{2\sqrt{21} \arctan\left(\frac{4x}{2\sqrt{7}-2\sqrt{3}}\right)}{7(2\sqrt{7}-2\sqrt{3})} + \frac{2 \arctan\left(\frac{4x}{2\sqrt{7}-2\sqrt{3}}\right)}{2\sqrt{7}-2\sqrt{3}} + \frac{2\sqrt{21} \arctan\left(\frac{4x}{2\sqrt{7}+2\sqrt{3}}\right)}{7(2\sqrt{7}+2\sqrt{3})} + \frac{2 \arctan\left(\frac{4x}{2\sqrt{7}+2\sqrt{3}}\right)}{2\sqrt{7}+2\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^4+5\*x^2+1), x)

[Out]  $-2/7 \cdot 21^{(1/2)} / (2 \cdot 7^{(1/2)} - 2 \cdot 3^{(1/2)}) \cdot \arctan(4x / (2 \cdot 7^{(1/2)} - 2 \cdot 3^{(1/2)})) + 2 / (2 \cdot 7^{(1/2)} - 2 \cdot 3^{(1/2)}) \cdot \arctan(4x / (2 \cdot 7^{(1/2)} - 2 \cdot 3^{(1/2)})) + 2/7 \cdot 21^{(1/2)} / (2 \cdot 7^{(1/2)} + 2 \cdot 3^{(1/2)}) \cdot \arctan(4x / (2 \cdot 7^{(1/2)} + 2 \cdot 3^{(1/2)})) + 2 / (2 \cdot 7^{(1/2)} + 2 \cdot 3^{(1/2)}) \cdot \arctan(4x / (2 \cdot 7^{(1/2)} + 2 \cdot 3^{(1/2)}))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + 1}{x^4 + 5x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+5\*x^2+1), x, algorithm="maxima")

[Out] integrate((x^2 + 1)/(x^4 + 5\*x^2 + 1), x)

**mupad [B]** time = 0.08, size = 29, normalized size = 0.59

$$\frac{\sqrt{7} \left( \operatorname{atan} \left( \frac{\sqrt{7} x^3}{7} + \frac{6\sqrt{7} x}{7} \right) + \operatorname{atan} \left( \frac{\sqrt{7} x}{7} \right) \right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 1)/(5\*x^2 + x^4 + 1), x)

[Out] (7^(1/2)\*(atan((6\*7^(1/2)\*x)/7 + (7^(1/2)\*x^3)/7) + atan((7^(1/2)\*x)/7))/7

**sympy [A]** time = 0.12, size = 41, normalized size = 0.84

$$\frac{\sqrt{7} \left( 2 \operatorname{atan} \left( \frac{\sqrt{7} x}{7} \right) + 2 \operatorname{atan} \left( \frac{\sqrt{7} x^3}{7} + \frac{6\sqrt{7} x}{7} \right) \right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*2+1)/(x\*\*4+5\*x\*\*2+1), x)

[Out] sqrt(7)\*(2\*atan(sqrt(7)\*x/7) + 2\*atan(sqrt(7)\*x\*\*3/7 + 6\*sqrt(7)\*x/7))/14

$$3.58 \quad \int \frac{1+x^2}{1+4x^2+x^4} dx$$

**Optimal.** Leaf size=43

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{6}} + \frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{6}}$$

**Rubi [A]** time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {1163, 203}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{6}} + \frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 + 4\*x^2 + x^4), x]

[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/Sqrt[6] + ArcTan[x/Sqrt[2 + Sqrt[3]]]/Sqrt[6]

Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1163

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[e/2 + (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 - q/2 + c\*x^2), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && GtQ[b^2 - 4\*a\*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{1+4x^2+x^4} dx &= \frac{1}{6}(3-\sqrt{3}) \int \frac{1}{2-\sqrt{3}+x^2} dx + \frac{1}{6}(3+\sqrt{3}) \int \frac{1}{2+\sqrt{3}+x^2} dx \\ &= \frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{6}} + \frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{6}} \end{aligned}$$

**Mathematica [A]** time = 0.07, size = 81, normalized size = 1.88

$$\frac{(\sqrt{3}-1) \tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}(2-\sqrt{3})} + \frac{(1+\sqrt{3}) \tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}(2+\sqrt{3})}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 + 4\*x^2 + x^4), x]



[Out]  $((-1 + \sqrt{3}) \operatorname{ArcTan}[x/\sqrt{2 - \sqrt{3}}]) / (2\sqrt{3}(2 - \sqrt{3})) + ((1 + \sqrt{3}) \operatorname{ArcTan}[x/\sqrt{2 + \sqrt{3}}]) / (2\sqrt{3}(2 + \sqrt{3}))$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + x^2}{1 + 4x^2 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^2)/(1 + 4\*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(1 + x^2)/(1 + 4\*x^2 + x^4), x]

**fricas** [A] time = 0.67, size = 31, normalized size = 0.72

$$\frac{1}{6} \sqrt{6} \arctan\left(\frac{1}{6} \sqrt{6} (x^3 + 5x)\right) + \frac{1}{6} \sqrt{6} \arctan\left(\frac{1}{6} \sqrt{6} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+4\*x^2+1), x, algorithm="fricas")

[Out]  $1/6 \sqrt{6} \arctan(1/6 \sqrt{6} (x^3 + 5x)) + 1/6 \sqrt{6} \arctan(1/6 \sqrt{6} x)$

**giac** [A] time = 0.19, size = 26, normalized size = 0.60

$$\frac{1}{12} \sqrt{6} \left( \pi \operatorname{sgn}(x) + 2 \arctan\left(\frac{\sqrt{6}(x^2 - 1)}{6x}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+4\*x^2+1), x, algorithm="giac")

[Out]  $1/12 \sqrt{6} (\pi \operatorname{sgn}(x) + 2 \arctan(1/6 \sqrt{6} (x^2 - 1)/x))$

**maple** [B] time = 0.05, size = 110, normalized size = 2.56

$$-\frac{\sqrt{3} \arctan\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{3(\sqrt{6}-\sqrt{2})} + \frac{\arctan\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{\sqrt{6}-\sqrt{2}} + \frac{\sqrt{3} \arctan\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{3\sqrt{6}+3\sqrt{2}} + \frac{\arctan\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{\sqrt{6}+\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^4+4\*x^2+1), x)

[Out]  $1/3 \cdot 3^{1/2} / (6^{1/2} + 2^{1/2}) \arctan(2x / (6^{1/2} + 2^{1/2})) + 1 / (6^{1/2} + 2^{1/2}) \arctan(2x / (6^{1/2} + 2^{1/2})) - 1/3 \cdot 3^{1/2} / (6^{1/2} - 2^{1/2}) \arctan(2x / (6^{1/2} - 2^{1/2})) + 1 / (6^{1/2} - 2^{1/2}) \arctan(2x / (6^{1/2} - 2^{1/2}))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + 1}{x^4 + 4x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+4\*x^2+1), x, algorithm="maxima")

[Out] integrate((x^2 + 1)/(x^4 + 4\*x^2 + 1), x)

**mupad [B]** time = 0.08, size = 29, normalized size = 0.67

$$\frac{\sqrt{6} \left( \operatorname{atan} \left( \frac{\sqrt{6} x^3}{6} + \frac{5\sqrt{6} x}{6} \right) + \operatorname{atan} \left( \frac{\sqrt{6} x}{6} \right) \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + 1)/(4*x^2 + x^4 + 1), x)`

[Out] `(6^(1/2)*(atan((5*6^(1/2)*x)/6 + (6^(1/2)*x^3)/6) + atan((6^(1/2)*x)/6)))/6`

**sympy [A]** time = 0.14, size = 41, normalized size = 0.95

$$\frac{\sqrt{6} \left( 2 \operatorname{atan} \left( \frac{\sqrt{6} x}{6} \right) + 2 \operatorname{atan} \left( \frac{\sqrt{6} x^3}{6} + \frac{5\sqrt{6} x}{6} \right) \right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+1)/(x**4+4*x**2+1), x)`

[Out] `sqrt(6)*(2*atan(sqrt(6)*x/6) + 2*atan(sqrt(6)*x**3/6 + 5*sqrt(6)*x/6))/12`

$$3.59 \quad \int \frac{1+x^2}{1+3x^2+x^4} dx$$

Optimal. Leaf size=49

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt{5}} + \frac{\tan^{-1}\left(\sqrt{\frac{1}{2}(3+\sqrt{5})}x\right)}{\sqrt{5}}$$

**Rubi [A]** time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {1163, 203}

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt{5}} + \frac{\tan^{-1}\left(\sqrt{\frac{1}{2}(3+\sqrt{5})}x\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 + 3\*x^2 + x^4), x]

[Out] ArcTan[Sqrt[2/(3 + Sqrt[5])]\*x]/Sqrt[5] + ArcTan[Sqrt[(3 + Sqrt[5])/2]\*x]/Sqrt[5]

Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1163

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[e/2 + (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 - q/2 + c\*x^2), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && GtQ[b^2 - 4\*a\*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{1+3x^2+x^4} dx &= \frac{1}{10} (5-\sqrt{5}) \int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2} dx + \frac{1}{10} (5+\sqrt{5}) \int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2} dx \\ &= \frac{\tan^{-1}\left(\sqrt{\frac{2}{3-\sqrt{5}}}x\right)}{\sqrt{5}} + \frac{\tan^{-1}\left(\sqrt{\frac{1}{2}(3+\sqrt{5})}x\right)}{\sqrt{5}} \end{aligned}$$

**Mathematica [A]** time = 0.10, size = 83, normalized size = 1.69

$$\frac{(\sqrt{5}-1)\tan^{-1}\left(\sqrt{\frac{2}{3-\sqrt{5}}}x\right)}{\sqrt{10(3-\sqrt{5})}} + \frac{(1+\sqrt{5})\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt{10(3+\sqrt{5})}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 + 3\*x^2 + x^4), x]

[Out]  $((-1 + \sqrt{5}) \cdot \text{ArcTan}[\sqrt{2/(3 - \sqrt{5})}] \cdot x) / \sqrt{10(3 - \sqrt{5})}) + ((1 + \sqrt{5}) \cdot \text{ArcTan}[\sqrt{2/(3 + \sqrt{5})}] \cdot x) / \sqrt{10(3 + \sqrt{5})})$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^2}{1+3x^2+x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^2)/(1 + 3\*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(1 + x^2)/(1 + 3\*x^2 + x^4), x]

**fricas** [A] time = 0.94, size = 31, normalized size = 0.63

$$\frac{1}{5} \sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5} (x^3 + 4x)\right) + \frac{1}{5} \sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+3\*x^2+1), x, algorithm="fricas")

[Out]  $1/5 \cdot \sqrt{5} \cdot \arctan(1/5 \cdot \sqrt{5} \cdot (x^3 + 4x)) + 1/5 \cdot \sqrt{5} \cdot \arctan(1/5 \cdot \sqrt{5} \cdot x)$

**giac** [A] time = 0.16, size = 26, normalized size = 0.53

$$\frac{1}{10} \sqrt{5} \left( \pi \operatorname{sgn}(x) + 2 \arctan\left(\frac{\sqrt{5}(x^2 - 1)}{5x}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+3\*x^2+1), x, algorithm="giac")

[Out]  $1/10 \cdot \sqrt{5} \cdot (\pi \cdot \operatorname{sgn}(x) + 2 \cdot \arctan(1/5 \cdot \sqrt{5} \cdot (x^2 - 1)/x))$

**maple** [B] time = 0.04, size = 104, normalized size = 2.12

$$-\frac{2\sqrt{5} \arctan\left(\frac{4x}{2\sqrt{5}-2}\right)}{5(2\sqrt{5}-2)} + \frac{2 \arctan\left(\frac{4x}{2\sqrt{5}-2}\right)}{2\sqrt{5}-2} + \frac{2\sqrt{5} \arctan\left(\frac{4x}{2\sqrt{5}+2}\right)}{5(2\sqrt{5}+2)} + \frac{2 \arctan\left(\frac{4x}{2\sqrt{5}+2}\right)}{2\sqrt{5}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^4+3\*x^2+1), x)

[Out]  $2/5 \cdot 5^{(1/2)} / (2 \cdot 5^{(1/2)} + 2) \cdot \arctan(4x / (2 \cdot 5^{(1/2)} + 2)) + 2 / (2 \cdot 5^{(1/2)} + 2) \cdot \arctan(4x / (2 \cdot 5^{(1/2)} + 2)) - 2/5 \cdot 5^{(1/2)} / (2 \cdot 5^{(1/2)} - 2) \cdot \arctan(4x / (2 \cdot 5^{(1/2)} - 2)) + 2 / (2 \cdot 5^{(1/2)} - 2) \cdot \arctan(4x / (2 \cdot 5^{(1/2)} - 2))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{x^4+3x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+3\*x^2+1), x, algorithm="maxima")

[Out] integrate((x^2 + 1)/(x^4 + 3\*x^2 + 1), x)

**mupad [B]** time = 4.39, size = 29, normalized size = 0.59

$$\frac{\sqrt{5} \left( \operatorname{atan} \left( \frac{\sqrt{5} x^3}{5} + \frac{4\sqrt{5} x}{5} \right) + \operatorname{atan} \left( \frac{\sqrt{5} x}{5} \right) \right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 1)/(3\*x^2 + x^4 + 1), x)

[Out] (5^(1/2)\*(atan((4\*5^(1/2)\*x)/5 + (5^(1/2)\*x^3)/5) + atan((5^(1/2)\*x)/5))/5

**sympy [A]** time = 0.13, size = 41, normalized size = 0.84

$$\frac{\sqrt{5} \left( 2 \operatorname{atan} \left( \frac{\sqrt{5} x}{5} \right) + 2 \operatorname{atan} \left( \frac{\sqrt{5} x^3}{5} + \frac{4\sqrt{5} x}{5} \right) \right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*2+1)/(x\*\*4+3\*x\*\*2+1), x)

[Out] sqrt(5)\*(2\*atan(sqrt(5)\*x/5) + 2\*atan(sqrt(5)\*x\*\*3/5 + 4\*sqrt(5)\*x/5))/10

$$3.60 \quad \int \frac{1+x^2}{1+2x^2+x^4} dx$$

Optimal. Leaf size=2

$$\tan^{-1}(x)$$

**Rubi [A]** time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {28, 203}

$$\tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 + 2\*x^2 + x^4), x]

[Out] ArcTan[x]

Rule 28

Int[(u\_.)\*((a\_.) + (c\_.)\*(x\_)^(n2\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] :> Dist[1/c^p, Int[u\*(b/2 + c\*x^n)^(2\*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2\*n] && EqQ[b^2 - 4\*a\*c, 0] && IntegerQ[p]

Rule 203

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\int \frac{1+x^2}{1+2x^2+x^4} dx = \int \frac{1}{1+x^2} dx = \tan^{-1}(x)$$

**Mathematica [A]** time = 0.00, size = 2, normalized size = 1.00

$$\tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 + 2\*x^2 + x^4), x]

[Out] ArcTan[x]

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^2}{1+2x^2+x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^2)/(1 + 2\*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(1 + x^2)/(1 + 2\*x^2 + x^4), x]

**fricas [A]** time = 1.10, size = 2, normalized size = 1.00

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+2\*x^2+1),x, algorithm="fricas")

[Out] arctan(x)

**giac** [A] time = 0.16, size = 2, normalized size = 1.00

arctan(x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+2\*x^2+1),x, algorithm="giac")

[Out] arctan(x)

**maple** [A] time = 0.00, size = 3, normalized size = 1.50

arctan(x)

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^4+2\*x^2+1),x)

[Out] arctan(x)

**maxima** [A] time = 2.42, size = 2, normalized size = 1.00

arctan(x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+2\*x^2+1),x, algorithm="maxima")

[Out] arctan(x)

**mupad** [B] time = 4.33, size = 2, normalized size = 1.00

atan(x)

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 1)/(2\*x^2 + x^4 + 1),x)

[Out] atan(x)

**sympy** [A] time = 0.10, size = 2, normalized size = 1.00

atan(x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*2+1)/(x\*\*4+2\*x\*\*2+1),x)

[Out] atan(x)

$$3.61 \quad \int \frac{1+x^2}{1+x^2+x^4} dx$$

Optimal. Leaf size=38

$$\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

**Rubi [A]** time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 + x^2 + x^4), x]

[Out] -(ArcTan[(1 - 2\*x)/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 + 2\*x)/Sqrt[3]]/Sqrt[3]

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 1161

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{1+x^2+x^4} dx &= \frac{1}{2} \int \frac{1}{1-x+x^2} dx + \frac{1}{2} \int \frac{1}{1+x+x^2} dx \\ &= -\text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, -1+2x\right) - \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1+2x\right) \\ &= \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} \end{aligned}$$

**Mathematica [C]** time = 0.19, size = 99, normalized size = 2.61

$$\frac{(\sqrt{3}-i) \tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}(1-i\sqrt{3})}}\right)}{\sqrt{6(1-i\sqrt{3})}} + \frac{(\sqrt{3}+i) \tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}(1+i\sqrt{3})}}\right)}{\sqrt{6(1+i\sqrt{3})}}$$



Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 + x^2 + x^4), x]

[Out]  $\frac{((-I + \sqrt{3}) \operatorname{ArcTan}[x/\sqrt{(1 - I\sqrt{3})/2}])/\sqrt{6(1 - I\sqrt{3})}}{1} + \frac{((I + \sqrt{3}) \operatorname{ArcTan}[x/\sqrt{(1 + I\sqrt{3})/2}])/\sqrt{6(1 + I\sqrt{3})}}{1}$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + x^2}{1 + x^2 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^2)/(1 + x^2 + x^4), x]

[Out] IntegrateAlgebraic[(1 + x^2)/(1 + x^2 + x^4), x]

**fricas** [A] time = 0.92, size = 31, normalized size = 0.82

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (x^3 + 2x)\right) + \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+x^2+1), x, algorithm="fricas")

[Out]  $\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (x^3 + 2x)\right) + \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} x\right)$

**giac** [A] time = 0.16, size = 26, normalized size = 0.68

$$\frac{1}{6} \sqrt{3} \left( \pi \operatorname{sgn}(x) + 2 \arctan\left(\frac{\sqrt{3}(x^2 - 1)}{3x}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+x^2+1), x, algorithm="giac")

[Out]  $\frac{1}{6} \sqrt{3} (\pi \operatorname{sgn}(x) + 2 \arctan(\frac{1}{3} \sqrt{3} (x^2 - 1)/x))$

**maple** [A] time = 0.01, size = 34, normalized size = 0.89

$$\frac{\sqrt{3} \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3} + \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^4+x^2+1), x)

[Out]  $\frac{1}{3} \arctan\left(\frac{1}{3} (2x+1) \sqrt{3}\right) \sqrt{3} + \frac{1}{3} \arctan\left(\frac{1}{3} (2x-1) \sqrt{3}\right) \sqrt{3}$

**maxima** [A] time = 2.40, size = 33, normalized size = 0.87

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x + 1)\right) + \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+x^2+1), x, algorithm="maxima")

[Out]  $\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)$

**mupad [B]** time = 0.08, size = 29, normalized size = 0.76

$$\frac{\sqrt{3} \left( \operatorname{atan}\left(\frac{\sqrt{3}x^3}{3} + \frac{2\sqrt{3}x}{3}\right) + \operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right) \right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + 1)/(x^2 + x^4 + 1), x)`

[Out]  $(3^{1/2}) \cdot (\operatorname{atan}((2 \cdot 3^{1/2})x)/3 + (3^{1/2})x^3/3) + \operatorname{atan}((3^{1/2})x/3))/3$

**sympy [A]** time = 0.12, size = 41, normalized size = 1.08

$$\frac{\sqrt{3} \left( 2 \operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right) + 2 \operatorname{atan}\left(\frac{\sqrt{3}x^3}{3} + \frac{2\sqrt{3}x}{3}\right) \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+1)/(x**4+x**2+1), x)`

[Out]  $\sqrt{3} \cdot (2 \cdot \operatorname{atan}(\sqrt{3}x/3) + 2 \cdot \operatorname{atan}(\sqrt{3}x^3/3 + 2\sqrt{3}x/3))/6$

$$3.62 \quad \int \frac{1+x^2}{1+x^4} dx$$

**Optimal.** Leaf size=35

$$\frac{\tan^{-1}(\sqrt{2}x+1)}{\sqrt{2}} - \frac{\tan^{-1}(1-\sqrt{2}x)}{\sqrt{2}}$$

**Rubi [A]** time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {1162, 617, 204}

$$\frac{\tan^{-1}(\sqrt{2}x+1)}{\sqrt{2}} - \frac{\tan^{-1}(1-\sqrt{2}x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 + x^4), x]

[Out] -(ArcTan[1 - Sqrt[2]\*x]/Sqrt[2]) + ArcTan[1 + Sqrt[2]\*x]/Sqrt[2]

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 617**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

**Rule 1162**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

**Rubi steps**

$$\begin{aligned} \int \frac{1+x^2}{1+x^4} dx &= \frac{1}{2} \int \frac{1}{1-\sqrt{2}x+x^2} dx + \frac{1}{2} \int \frac{1}{1+\sqrt{2}x+x^2} dx \\ &= \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1-\sqrt{2}x\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1+\sqrt{2}x\right)}{\sqrt{2}} \\ &= -\frac{\tan^{-1}(1-\sqrt{2}x)}{\sqrt{2}} + \frac{\tan^{-1}(1+\sqrt{2}x)}{\sqrt{2}} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 30, normalized size = 0.86

$$\frac{\tan^{-1}(\sqrt{2}x+1) - \tan^{-1}(1-\sqrt{2}x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 + x^4), x]

[Out] (-ArcTan[1 - Sqrt[2]\*x] + ArcTan[1 + Sqrt[2]\*x])/Sqrt[2]

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^2}{1+x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^2)/(1 + x^4), x]

[Out] IntegrateAlgebraic[(1 + x^2)/(1 + x^4), x]

**fricas** [A] time = 1.22, size = 29, normalized size = 0.83

$$\frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(x^3 + x)\right) + \frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+1), x, algorithm="fricas")

[Out] 1/2\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(x^3 + x)) + 1/2\*sqrt(2)\*arctan(1/2\*sqrt(2)\*x)

**giac** [A] time = 0.19, size = 39, normalized size = 1.11

$$\frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(2x + \sqrt{2})\right) + \frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(2x - \sqrt{2})\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+1), x, algorithm="giac")

[Out] 1/2\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(2\*x + sqrt(2))) + 1/2\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(2\*x - sqrt(2)))

**maple** [B] time = 0.00, size = 88, normalized size = 2.51

$$\frac{\sqrt{2} \arctan(\sqrt{2}x - 1)}{2} + \frac{\sqrt{2} \arctan(\sqrt{2}x + 1)}{2} + \frac{\sqrt{2} \ln\left(\frac{x^2 - \sqrt{2}x + 1}{x^2 + \sqrt{2}x + 1}\right)}{8} + \frac{\sqrt{2} \ln\left(\frac{x^2 + \sqrt{2}x + 1}{x^2 - \sqrt{2}x + 1}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^4+1), x)

[Out] 1/2\*arctan(-1+2^(1/2)\*x)\*2^(1/2)+1/8\*2^(1/2)\*ln((1+x^2+2^(1/2)\*x)/(1+x^2-2^(1/2)\*x))+1/2\*arctan(1+2^(1/2)\*x)\*2^(1/2)+1/8\*2^(1/2)\*ln((1+x^2-2^(1/2)\*x)/(1+x^2+2^(1/2)\*x))

**maxima** [A] time = 2.42, size = 39, normalized size = 1.11

$$\frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(2x + \sqrt{2})\right) + \frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(2x - \sqrt{2})\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+1), x, algorithm="maxima")

[Out]  $\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x + \sqrt{2})\right) + \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x - \sqrt{2})\right)$

**mupad [B]** time = 4.37, size = 29, normalized size = 0.83

$$\frac{\sqrt{2} \left( \operatorname{atan}\left(\frac{\sqrt{2}x^3}{2} + \frac{\sqrt{2}x}{2}\right) + \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right) \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + 1)/(x^4 + 1), x)`

[Out]  $(2^{1/2} * (\operatorname{atan}((2^{1/2}) * x) / 2 + (2^{1/2}) * x^3 / 2) + \operatorname{atan}((2^{1/2}) * x) / 2)) / 2$

**sympy [A]** time = 0.12, size = 39, normalized size = 1.11

$$\frac{\sqrt{2} \left( 2 \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right) + 2 \operatorname{atan}\left(\frac{\sqrt{2}x^3}{2} + \frac{\sqrt{2}x}{2}\right) \right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+1)/(x**4+1), x)`

[Out]  $\sqrt{2} * (2 * \operatorname{atan}(\sqrt{2} * x / 2) + 2 * \operatorname{atan}(\sqrt{2} * x^3 / 2 + \sqrt{2} * x / 2)) / 4$

$$3.63 \quad \int \frac{1+x^2}{1-x^2+x^4} dx$$

Optimal. Leaf size=23

$$\tan^{-1}(2x + \sqrt{3}) - \tan^{-1}(\sqrt{3} - 2x)$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1161, 618, 204}

$$\tan^{-1}(2x + \sqrt{3}) - \tan^{-1}(\sqrt{3} - 2x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 - x^2 + x^4), x]

[Out] -ArcTan[Sqrt[3] - 2\*x] + ArcTan[Sqrt[3] + 2\*x]

#### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 1161

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

#### Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{1-x^2+x^4} dx &= \frac{1}{2} \int \frac{1}{1-\sqrt{3}x+x^2} dx + \frac{1}{2} \int \frac{1}{1+\sqrt{3}x+x^2} dx \\ &= -\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, -\sqrt{3}+2x\right) - \text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, \sqrt{3}+2x\right) \\ &= -\tan^{-1}(\sqrt{3}-2x) + \tan^{-1}(\sqrt{3}+2x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 0.52

$$-\tan^{-1}\left(\frac{x}{x^2-1}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 - x^2 + x^4), x]

[Out] -ArcTan[x/(-1 + x^2)]

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^2}{1-x^2+x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^2)/(1 - x^2 + x^4), x]

[Out] IntegrateAlgebraic[(1 + x^2)/(1 - x^2 + x^4), x]

**fricas** [A] time = 1.15, size = 7, normalized size = 0.30

$$\arctan(x^3) + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4-x^2+1), x, algorithm="fricas")

[Out] arctan(x^3) + arctan(x)

**giac** [A] time = 0.17, size = 30, normalized size = 1.30

$$\frac{1}{4} \pi \operatorname{sgn}(x) + \frac{1}{2} \arctan\left(\frac{x^4 - 3x^2 + 1}{2(x^3 - x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4-x^2+1), x, algorithm="giac")

[Out] 1/4\*pi\*sgn(x) + 1/2\*arctan(1/2\*(x^4 - 3\*x^2 + 1)/(x^3 - x))

**maple** [A] time = 0.02, size = 20, normalized size = 0.87

$$\arctan(2x - \sqrt{3}) + \arctan(2x + \sqrt{3})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^4-x^2+1), x)

[Out] arctan(2\*x-3^(1/2))+arctan(2\*x+3^(1/2))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{x^4-x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4-x^2+1), x, algorithm="maxima")

[Out] integrate((x^2 + 1)/(x^4 - x^2 + 1), x)

**mupad** [B] time = 4.31, size = 7, normalized size = 0.30

$$\operatorname{atan}(x^3) + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 1)/(x^4 - x^2 + 1), x)

[Out] atan(x^3) + atan(x)

sympy [A] time = 0.11, size = 7, normalized size = 0.30

$$\operatorname{atan}(x) + \operatorname{atan}(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4-x**2+1),x)
```

```
[Out] atan(x) + atan(x**3)
```



$$3.64 \quad \int \frac{1+x^2}{1-2x^2+x^4} dx$$

Optimal. Leaf size=11

$$\frac{x}{1-x^2}$$

**Rubi [A]** time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {28, 383}

$$\frac{x}{1-x^2}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 - 2\*x^2 + x^4), x]

[Out] x/(1 - x^2)

Rule 28

Int[(u\_)\*((a\_) + (c\_)\*(x\_)^(n2\_)) + (b\_)\*(x\_)^(n\_)]^(p\_), x\_Symbol] :> Dist[1/c^p, Int[u\*(b/2 + c\*x^n)^(2\*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2\*n] && EqQ[b^2 - 4\*a\*c, 0] && IntegerQ[p]

Rule 383

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] :> Simp[c\*x\*(a + b\*x^n)^(p + 1)/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a\*d - b\*c\*(n\*(p + 1) + 1), 0]

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{1-2x^2+x^4} dx &= \int \frac{1+x^2}{(-1+x^2)^2} dx \\ &= \frac{x}{1-x^2} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 10, normalized size = 0.91

$$-\frac{x}{x^2-1}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 - 2\*x^2 + x^4), x]

[Out] -(x/(-1 + x^2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^2}{1-2x^2+x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^2)/(1 - 2\*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(1 + x^2)/(1 - 2\*x^2 + x^4), x]

**fricas** [A] time = 1.21, size = 10, normalized size = 0.91

$$-\frac{x}{x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4-2\*x^2+1),x, algorithm="fricas")

[Out] -x/(x^2 - 1)

**giac** [A] time = 0.15, size = 11, normalized size = 1.00

$$-\frac{1}{x - \frac{1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4-2\*x^2+1),x, algorithm="giac")

[Out] -1/(x - 1/x)

**maple** [A] time = 0.00, size = 16, normalized size = 1.45

$$-\frac{1}{2(x+1)} - \frac{1}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^4-2\*x^2+1),x)

[Out] -1/2/(x+1)-1/2/(x-1)

**maxima** [A] time = 1.06, size = 10, normalized size = 0.91

$$-\frac{x}{x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4-2\*x^2+1),x, algorithm="maxima")

[Out] -x/(x^2 - 1)

**mupad** [B] time = 4.34, size = 10, normalized size = 0.91

$$-\frac{x}{x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 1)/(x^4 - 2\*x^2 + 1),x)

[Out] -x/(x^2 - 1)

**sympy** [A] time = 0.09, size = 7, normalized size = 0.64

$$-\frac{x}{x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*2+1)/(x\*\*4-2\*x\*\*2+1),x)

[Out] -x/(x\*\*2 - 1)

$$3.65 \quad \int \frac{1+x^2}{1-3x^2+x^4} dx$$

**Optimal.** Leaf size=65

$$\frac{1}{2} \log(-2x - \sqrt{5} + 1) + \frac{1}{2} \log(-2x + \sqrt{5} + 1) - \frac{1}{2} \log(2x - \sqrt{5} + 1) - \frac{1}{2} \log(2x + \sqrt{5} + 1)$$

**Rubi [A]** time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1161, 616, 31}

$$\frac{1}{2} \log(-2x - \sqrt{5} + 1) + \frac{1}{2} \log(-2x + \sqrt{5} + 1) - \frac{1}{2} \log(2x - \sqrt{5} + 1) - \frac{1}{2} \log(2x + \sqrt{5} + 1)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 - 3\*x^2 + x^4), x]

[Out] Log[1 - Sqrt[5] - 2\*x]/2 + Log[1 + Sqrt[5] - 2\*x]/2 - Log[1 - Sqrt[5] + 2\*x]/2 - Log[1 + Sqrt[5] + 2\*x]/2

**Rule 31**

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

**Rule 616**

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(n\_), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c\*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c\*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && PosQ[b^2 - 4\*a\*c] && PerfectSquareQ[b^2 - 4\*a\*c]

**Rule 1161**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

**Rubi steps**

$$\begin{aligned} \int \frac{1+x^2}{1-3x^2+x^4} dx &= \frac{1}{2} \int \frac{1}{1-\sqrt{5}x+x^2} dx + \frac{1}{2} \int \frac{1}{1+\sqrt{5}x+x^2} dx \\ &= \frac{1}{2} \int \frac{1}{\frac{1}{2}(-1-\sqrt{5})+x} dx - \frac{1}{2} \int \frac{1}{\frac{1}{2}(1-\sqrt{5})+x} dx + \frac{1}{2} \int \frac{1}{\frac{1}{2}(-1+\sqrt{5})+x} dx - \frac{1}{2} \int \frac{1}{\frac{1}{2}(1+\sqrt{5})+x} dx \\ &= \frac{1}{2} \log(1-\sqrt{5}-2x) + \frac{1}{2} \log(1+\sqrt{5}-2x) - \frac{1}{2} \log(1-\sqrt{5}+2x) - \frac{1}{2} \log(1+\sqrt{5}+2x) \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 29, normalized size = 0.45

$$\frac{1}{2} \log(-x^2 + x + 1) - \frac{1}{2} \log(-x^2 - x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 - 3\*x^2 + x^4), x]

[Out] -1/2\*Log[1 - x - x^2] + Log[1 + x - x^2]/2

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^2}{1-3x^2+x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^2)/(1 - 3\*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(1 + x^2)/(1 - 3\*x^2 + x^4), x]

**fricas** [A] time = 1.20, size = 21, normalized size = 0.32

$$-\frac{1}{2} \log(x^2 + x - 1) + \frac{1}{2} \log(x^2 - x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4-3\*x^2+1), x, algorithm="fricas")

[Out] -1/2\*log(x^2 + x - 1) + 1/2\*log(x^2 - x - 1)

**giac** [A] time = 0.17, size = 43, normalized size = 0.66

$$-\frac{1}{4} \log\left(x + \frac{1}{x - \frac{1}{x}} - \frac{1}{x} + 2\right) + \frac{1}{4} \log\left(x + \frac{1}{x - \frac{1}{x}} - \frac{1}{x} - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4-3\*x^2+1), x, algorithm="giac")

[Out] -1/4\*log(abs(x + 1/(x - 1/x) - 1/x + 2)) + 1/4\*log(abs(x + 1/(x - 1/x) - 1/x - 2))

**maple** [A] time = 0.01, size = 22, normalized size = 0.34

$$\frac{\ln(x^2 - x - 1)}{2} - \frac{\ln(x^2 + x - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^4-3\*x^2+1), x)

[Out] -1/2\*ln(x^2+x-1)+1/2\*ln(x^2-x-1)

**maxima** [A] time = 0.99, size = 21, normalized size = 0.32

$$-\frac{1}{2} \log(x^2 + x - 1) + \frac{1}{2} \log(x^2 - x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4-3\*x^2+1), x, algorithm="maxima")

[Out] -1/2\*log(x^2 + x - 1) + 1/2\*log(x^2 - x - 1)

**mupad** [B] time = 0.26, size = 12, normalized size = 0.18

$$-\operatorname{atanh}\left(\frac{x}{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + 1)/(x^4 - 3*x^2 + 1), x)`

[Out] `-atanh(x/(x^2 - 1))`

**sympy** [A] time = 0.11, size = 19, normalized size = 0.29

$$\frac{\log(x^2 - x - 1)}{2} - \frac{\log(x^2 + x - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+1)/(x**4-3*x**2+1), x)`

[Out] `log(x**2 - x - 1)/2 - log(x**2 + x - 1)/2`

$$3.66 \quad \int \frac{1+x^2}{1-4x^2+x^4} dx$$

Optimal. Leaf size=43

$$\frac{\tanh^{-1}(\sqrt{3} - \sqrt{2}x)}{\sqrt{2}} - \frac{\tanh^{-1}(\sqrt{2}x + \sqrt{3})}{\sqrt{2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}(\sqrt{3} - \sqrt{2}x)}{\sqrt{2}} - \frac{\tanh^{-1}(\sqrt{2}x + \sqrt{3})}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 - 4\*x^2 + x^4), x]

[Out] ArcTanh[Sqrt[3] - Sqrt[2]\*x]/Sqrt[2] - ArcTanh[Sqrt[3] + Sqrt[2]\*x]/Sqrt[2]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 1161

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

#### Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{1-4x^2+x^4} dx &= \frac{1}{2} \int \frac{1}{1-\sqrt{6}x+x^2} dx + \frac{1}{2} \int \frac{1}{1+\sqrt{6}x+x^2} dx \\ &= -\text{Subst}\left(\int \frac{1}{2-x^2} dx, x, -\sqrt{6}+2x\right) - \text{Subst}\left(\int \frac{1}{2-x^2} dx, x, \sqrt{6}+2x\right) \\ &= \frac{\tanh^{-1}(\sqrt{3} - \sqrt{2}x)}{\sqrt{2}} - \frac{\tanh^{-1}(\sqrt{3} + \sqrt{2}x)}{\sqrt{2}} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 40, normalized size = 0.93

$$\frac{\log(-x^2 + \sqrt{2}x + 1) - \log(x^2 + \sqrt{2}x - 1)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 - 4\*x^2 + x^4), x]

[Out] (Log[1 + Sqrt[2]\*x - x^2] - Log[-1 + Sqrt[2]\*x + x^2])/(2\*Sqrt[2])

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + x^2}{1 - 4x^2 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^2)/(1 - 4\*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(1 + x^2)/(1 - 4\*x^2 + x^4), x]

**fricas** [A] time = 1.15, size = 36, normalized size = 0.84

$$\frac{1}{4} \sqrt{2} \log\left(\frac{x^4 - 2\sqrt{2}(x^3 - x) + 1}{x^4 - 4x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4-4\*x^2+1), x, algorithm="fricas")

[Out] 1/4\*sqrt(2)\*log((x^4 - 2\*sqrt(2)\*(x^3 - x) + 1)/(x^4 - 4\*x^2 + 1))

**giac** [A] time = 0.21, size = 39, normalized size = 0.91

$$\frac{1}{4} \sqrt{2} \log\left(\frac{\left|2x - 2\sqrt{2} - \frac{2}{x}\right|}{\left|2x + 2\sqrt{2} - \frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4-4\*x^2+1), x, algorithm="giac")

[Out] 1/4\*sqrt(2)\*log(abs(2\*x - 2\*sqrt(2) - 2/x)/abs(2\*x + 2\*sqrt(2) - 2/x))

**maple** [B] time = 0.04, size = 70, normalized size = 1.63

$$-\frac{(-3 + \sqrt{3}) \sqrt{3} \operatorname{arctanh}\left(\frac{2x}{\sqrt{6} - \sqrt{2}}\right)}{3(\sqrt{6} - \sqrt{2})} - \frac{(\sqrt{3} + 3) \sqrt{3} \operatorname{arctanh}\left(\frac{2x}{\sqrt{6} + \sqrt{2}}\right)}{3(\sqrt{6} + \sqrt{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^4-4\*x^2+1), x)

[Out] -1/3\*(-3+3^(1/2))\*3^(1/2)/(6^(1/2)-2^(1/2))\*arctanh(2/(6^(1/2)-2^(1/2))\*x) - 1/3\*(3^(1/2)+3)\*3^(1/2)/(6^(1/2)+2^(1/2))\*arctanh(2/(6^(1/2)+2^(1/2))\*x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + 1}{x^4 - 4x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4-4\*x^2+1), x, algorithm="maxima")

[Out] integrate((x^2 + 1)/(x^4 - 4\*x^2 + 1), x)

**mupad [B]** time = 4.40, size = 18, normalized size = 0.42

$$-\frac{\sqrt{2} \operatorname{atanh}\left(\frac{\sqrt{2}x}{x^2-1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + 1)/(x^4 - 4*x^2 + 1), x)`

[Out] `-(2^(1/2)*atanh((2^(1/2)*x)/(x^2 - 1)))/2`

**sympy [A]** time = 0.11, size = 39, normalized size = 0.91

$$\frac{\sqrt{2} \log(x^2 - \sqrt{2}x - 1)}{4} - \frac{\sqrt{2} \log(x^2 + \sqrt{2}x - 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+1)/(x**4-4*x**2+1), x)`

[Out] `sqrt(2)*log(x**2 - sqrt(2)*x - 1)/4 - sqrt(2)*log(x**2 + sqrt(2)*x - 1)/4`



$$3.67 \quad \int \frac{1+x^2}{1-5x^2+x^4} dx$$

**Optimal.** Leaf size=46

$$\frac{\tanh^{-1}\left(\frac{\sqrt{7}-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{2x+\sqrt{7}}{\sqrt{3}}\right)}{\sqrt{3}}$$

**Rubi [A]** time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{7}-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{2x+\sqrt{7}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 - 5\*x^2 + x^4), x]

[Out] ArcTanh[(Sqrt[7] - 2\*x)/Sqrt[3]]/Sqrt[3] - ArcTanh[(Sqrt[7] + 2\*x)/Sqrt[3]]/Sqrt[3]

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 618**

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

**Rule 1161**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

**Rubi steps**

$$\begin{aligned} \int \frac{1+x^2}{1-5x^2+x^4} dx &= \frac{1}{2} \int \frac{1}{1-\sqrt{7}x+x^2} dx + \frac{1}{2} \int \frac{1}{1+\sqrt{7}x+x^2} dx \\ &= -\text{Subst}\left(\int \frac{1}{3-x^2} dx, x, -\sqrt{7}+2x\right) - \text{Subst}\left(\int \frac{1}{3-x^2} dx, x, \sqrt{7}+2x\right) \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{7}-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{7}+2x}{\sqrt{3}}\right)}{\sqrt{3}} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 40, normalized size = 0.87

$$\frac{\log(-x^2 + \sqrt{3}x + 1) - \log(x^2 + \sqrt{3}x - 1)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 - 5\*x^2 + x^4),x]

[Out] (Log[1 + Sqrt[3]\*x - x^2] - Log[-1 + Sqrt[3]\*x + x^2])/(2\*Sqrt[3])

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^2}{1-5x^2+x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^2)/(1 - 5\*x^2 + x^4),x]

[Out] IntegrateAlgebraic[(1 + x^2)/(1 - 5\*x^2 + x^4), x]

**fricas** [A] time = 1.01, size = 39, normalized size = 0.85

$$\frac{1}{6} \sqrt{3} \log \left( \frac{x^4 + x^2 - 2\sqrt{3}(x^3 - x) + 1}{x^4 - 5x^2 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4-5\*x^2+1),x, algorithm="fricas")

[Out] 1/6\*sqrt(3)\*log((x^4 + x^2 - 2\*sqrt(3)\*(x^3 - x) + 1)/(x^4 - 5\*x^2 + 1))

**giac** [A] time = 0.24, size = 39, normalized size = 0.85

$$\frac{1}{6} \sqrt{3} \log \left( \frac{\left| 2x - 2\sqrt{3} - \frac{2}{x} \right|}{\left| 2x + 2\sqrt{3} - \frac{2}{x} \right|} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4-5\*x^2+1),x, algorithm="giac")

[Out] 1/6\*sqrt(3)\*log(abs(2\*x - 2\*sqrt(3) - 2/x)/abs(2\*x + 2\*sqrt(3) - 2/x))

**maple** [B] time = 0.04, size = 82, normalized size = 1.78

$$\frac{2\sqrt{21}(-7 + \sqrt{21}) \operatorname{arctanh}\left(\frac{4x}{2\sqrt{7}-2\sqrt{3}}\right)}{21(2\sqrt{7}-2\sqrt{3})} - \frac{2(7 + \sqrt{21})\sqrt{21} \operatorname{arctanh}\left(\frac{4x}{2\sqrt{7}+2\sqrt{3}}\right)}{21(2\sqrt{7}+2\sqrt{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^4-5\*x^2+1),x)

[Out] -2/21\*(7+21^(1/2))\*21^(1/2)/(2\*7^(1/2)+2\*3^(1/2))\*arctanh(4/(2\*7^(1/2)+2\*3^(1/2))\*x)-2/21\*21^(1/2)\*(-7+21^(1/2))/(2\*7^(1/2)-2\*3^(1/2))\*arctanh(4/(2\*7^(1/2)-2\*3^(1/2))\*x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{x^4-5x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4-5\*x^2+1),x, algorithm="maxima")

[Out] integrate((x^2 + 1)/(x^4 - 5\*x^2 + 1), x)

**mupad [B]** time = 4.47, size = 18, normalized size = 0.39

$$-\frac{\sqrt{3} \operatorname{atanh}\left(\frac{\sqrt{3}x}{x^2-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 1)/(x^4 - 5\*x^2 + 1), x)

[Out] -(3^(1/2)\*atanh((3^(1/2)\*x)/(x^2 - 1)))/3

**sympy [A]** time = 0.12, size = 39, normalized size = 0.85

$$\frac{\sqrt{3} \log(x^2 - \sqrt{3}x - 1)}{6} - \frac{\sqrt{3} \log(x^2 + \sqrt{3}x - 1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*2+1)/(x\*\*4-5\*x\*\*2+1), x)

[Out] sqrt(3)\*log(x\*\*2 - sqrt(3)\*x - 1)/6 - sqrt(3)\*log(x\*\*2 + sqrt(3)\*x - 1)/6

$$3.68 \quad \int \frac{1-x^2}{1+bx^2+x^4} dx$$

**Optimal.** Leaf size=62

$$\frac{\log(\sqrt{2-b}x+x^2+1)}{2\sqrt{2-b}} - \frac{\log(-\sqrt{2-b}x+x^2+1)}{2\sqrt{2-b}}$$

**Rubi [A]** time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {1164, 628}

$$\frac{\log(\sqrt{2-b}x+x^2+1)}{2\sqrt{2-b}} - \frac{\log(-\sqrt{2-b}x+x^2+1)}{2\sqrt{2-b}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 + b\*x^2 + x^4), x]

[Out] -Log[1 - Sqrt[2 - b]\*x + x^2]/(2\*Sqrt[2 - b]) + Log[1 + Sqrt[2 - b]\*x + x^2]/(2\*Sqrt[2 - b])

**Rule 628**

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

**Rule 1164**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(-2\*d)/e - b/c, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && !GtQ[b^2 - 4\*a\*c, 0]

**Rubi steps**

$$\begin{aligned} \int \frac{1-x^2}{1+bx^2+x^4} dx &= -\frac{\int \frac{\sqrt{2-b}+2x}{-1-\sqrt{2-b}x-x^2} dx}{2\sqrt{2-b}} - \frac{\int \frac{\sqrt{2-b}-2x}{-1+\sqrt{2-b}x-x^2} dx}{2\sqrt{2-b}} \\ &= -\frac{\log(1-\sqrt{2-b}x+x^2)}{2\sqrt{2-b}} + \frac{\log(1+\sqrt{2-b}x+x^2)}{2\sqrt{2-b}} \end{aligned}$$

**Mathematica [B]** time = 0.07, size = 125, normalized size = 2.02

$$\frac{(-\sqrt{b^2-4}+b+2) \tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b-\sqrt{b^2-4}}}\right) - (\sqrt{b^2-4}+b+2) \tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{b^2-4}+b}}\right)}{\sqrt{b-\sqrt{b^2-4}} \sqrt{\sqrt{b^2-4}+b}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 + b\*x^2 + x^4), x]



$(b+2)^{(1/2)} \cdot x + 1 / ((b-2) \cdot (b+2)^{(1/2)} / (2 \cdot b - 2 \cdot ((b-2) \cdot (b+2)^{(1/2)}))^{(1/2)}) \cdot b \cdot \arctan(2 / (2 \cdot b - 2 \cdot ((b-2) \cdot (b+2)^{(1/2)}))^{(1/2)} \cdot x)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 - 1}{x^4 + bx^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+b\*x^2+1),x, algorithm="maxima")

[Out] -integrate((x^2 - 1)/(x^4 + b\*x^2 + 1), x)

**mupad** [B] time = 4.34, size = 76, normalized size = 1.23

$$\frac{\operatorname{atan}\left(\frac{x}{\sqrt{b-2}}\right) - \operatorname{atan}\left((b-2) \left(x \left(\frac{1}{\sqrt{b-2}} + \frac{\frac{4}{b-2} + 1}{\sqrt{b-2}(b+2)}\right) + \frac{x^3 \left(\frac{2b-1}{b-2}\right)}{\sqrt{b-2}(b+2)}\right)\right)}{\sqrt{b-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2 - 1)/(b\*x^2 + x^4 + 1),x)

[Out] -(atan(x/(b - 2)^(1/2)) - atan((b - 2)\*(x\*(1/(b - 2)^(1/2) + (4/(b - 2) + 1)/((b - 2)^(1/2)\*(b + 2)))) + (x^3\*((2\*b)/(b - 2) - 1))/((b - 2)^(1/2)\*(b + 2))))/(b - 2)^(1/2)

**sympy** [A] time = 0.35, size = 87, normalized size = 1.40

$$\frac{\sqrt{-\frac{1}{b-2}} \log\left(x^2 + x\left(-b\sqrt{-\frac{1}{b-2}} + 2\sqrt{-\frac{1}{b-2}}\right) + 1\right)}{2} - \frac{\sqrt{-\frac{1}{b-2}} \log\left(x^2 + x\left(b\sqrt{-\frac{1}{b-2}} - 2\sqrt{-\frac{1}{b-2}}\right) + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*2+1)/(x\*\*4+b\*x\*\*2+1),x)

[Out] sqrt(-1/(b - 2))\*log(x\*\*2 + x\*(-b\*sqrt(-1/(b - 2)) + 2\*sqrt(-1/(b - 2)))) + 1)/2 - sqrt(-1/(b - 2))\*log(x\*\*2 + x\*(b\*sqrt(-1/(b - 2)) - 2\*sqrt(-1/(b - 2)))) + 1)/2

$$3.69 \quad \int \frac{1-x^2}{1+5x^2+x^4} dx$$

Optimal. Leaf size=50

$$\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}(5+\sqrt{21})}x\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{3}}$$

**Rubi [A]** time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {1163, 203}

$$\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}(5+\sqrt{21})}x\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 + 5\*x^2 + x^4), x]

[Out] -(ArcTan[Sqrt[2/(5 + Sqrt[21])]x]/Sqrt[3]) + ArcTan[Sqrt[(5 + Sqrt[21])/2]x]/Sqrt[3]

Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1163

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[e/2 + (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 - q/2 + c\*x^2), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && GtQ[b^2 - 4\*a\*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-x^2}{1+5x^2+x^4} dx &= \frac{1}{6}(-3+\sqrt{21}) \int \frac{1}{\frac{5}{2}-\frac{\sqrt{21}}{2}+x^2} dx - \frac{1}{6}(3+\sqrt{21}) \int \frac{1}{\frac{5}{2}+\frac{\sqrt{21}}{2}+x^2} dx \\ &= -\frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\sqrt{\frac{1}{2}(5+\sqrt{21})}x\right)}{\sqrt{3}} \end{aligned}$$

**Mathematica [A]** time = 0.14, size = 87, normalized size = 1.74

$$\frac{(7-\sqrt{21})\tan^{-1}\left(\sqrt{\frac{2}{5-\sqrt{21}}}x\right)}{\sqrt{42}(5-\sqrt{21})} + \frac{(-7-\sqrt{21})\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{42}(5+\sqrt{21})}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 + 5\*x^2 + x^4), x]

[Out]  $((7 - \sqrt{21}) \cdot \text{ArcTan}[\sqrt{2/(5 - \sqrt{21})}] \cdot x) / \sqrt{42 \cdot (5 - \sqrt{21})} + ((-7 - \sqrt{21}) \cdot \text{ArcTan}[\sqrt{2/(5 + \sqrt{21})}] \cdot x) / \sqrt{42 \cdot (5 + \sqrt{21})}$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 - x^2}{1 + 5x^2 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - x^2)/(1 + 5\*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(1 - x^2)/(1 + 5\*x^2 + x^4), x]

**fricas** [A] time = 0.80, size = 31, normalized size = 0.62

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (x^3 + 4x)\right) - \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+5\*x^2+1), x, algorithm="fricas")

[Out]  $1/3 \cdot \sqrt{3} \cdot \arctan(1/3 \cdot \sqrt{3} \cdot (x^3 + 4x)) - 1/3 \cdot \sqrt{3} \cdot \arctan(1/3 \cdot \sqrt{3} \cdot x)$

**giac** [A] time = 0.17, size = 26, normalized size = 0.52

$$\frac{1}{6} \sqrt{3} \left( \pi \operatorname{sgn}(x) - 2 \arctan\left(\frac{\sqrt{3}(x^2 + 1)}{3x}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+5\*x^2+1), x, algorithm="giac")

[Out]  $1/6 \cdot \sqrt{3} \cdot (\pi \cdot \operatorname{sgn}(x) - 2 \cdot \arctan(1/3 \cdot \sqrt{3} \cdot (x^2 + 1)/x))$

**maple** [B] time = 0.02, size = 136, normalized size = 2.72

$$\frac{2\sqrt{21} \arctan\left(\frac{4x}{2\sqrt{7}-2\sqrt{3}}\right)}{3(2\sqrt{7}-2\sqrt{3})} - \frac{2 \arctan\left(\frac{4x}{2\sqrt{7}-2\sqrt{3}}\right)}{2\sqrt{7}-2\sqrt{3}} - \frac{2\sqrt{21} \arctan\left(\frac{4x}{2\sqrt{7}+2\sqrt{3}}\right)}{3(2\sqrt{7}+2\sqrt{3})} - \frac{2 \arctan\left(\frac{4x}{2\sqrt{7}+2\sqrt{3}}\right)}{2\sqrt{7}+2\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(x^4+5\*x^2+1), x)

[Out]  $2/3 \cdot 21^{(1/2)} / (2 \cdot 7^{(1/2)} - 2 \cdot 3^{(1/2)}) \cdot \arctan(4 / (2 \cdot 7^{(1/2)} - 2 \cdot 3^{(1/2)}) \cdot x) - 2 / (2 \cdot 7^{(1/2)} - 2 \cdot 3^{(1/2)}) \cdot \arctan(4 / (2 \cdot 7^{(1/2)} - 2 \cdot 3^{(1/2)}) \cdot x) - 2/3 \cdot 21^{(1/2)} / (2 \cdot 7^{(1/2)} + 2 \cdot 3^{(1/2)}) \cdot \arctan(4 / (2 \cdot 7^{(1/2)} + 2 \cdot 3^{(1/2)}) \cdot x) - 2 / (2 \cdot 7^{(1/2)} + 2 \cdot 3^{(1/2)}) \cdot \arctan(4 / (2 \cdot 7^{(1/2)} + 2 \cdot 3^{(1/2)}) \cdot x)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{x^2 - 1}{x^4 + 5x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+5\*x^2+1), x, algorithm="maxima")

[Out] -integrate((x^2 - 1)/(x^4 + 5\*x^2 + 1), x)



**mupad [B]** time = 0.08, size = 31, normalized size = 0.62

$$\frac{\sqrt{3} \left( \operatorname{atan} \left( \frac{\sqrt{3} x^3}{3} + \frac{4\sqrt{3} x}{3} \right) - \operatorname{atan} \left( \frac{\sqrt{3} x}{3} \right) \right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2 - 1)/(5\*x^2 + x^4 + 1), x)

[Out] (3^(1/2)\*(atan((4\*3^(1/2)\*x)/3 + (3^(1/2)\*x^3)/3) - atan((3^(1/2)\*x)/3))/3

**sympy [A]** time = 0.13, size = 42, normalized size = 0.84

$$\frac{\sqrt{3} \left( 2 \operatorname{atan} \left( \frac{\sqrt{3} x}{3} \right) - 2 \operatorname{atan} \left( \frac{\sqrt{3} x^3}{3} + \frac{4\sqrt{3} x}{3} \right) \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*2+1)/(x\*\*4+5\*x\*\*2+1), x)

[Out] -sqrt(3)\*(2\*atan(sqrt(3)\*x/3) - 2\*atan(sqrt(3)\*x\*\*3/3 + 4\*sqrt(3)\*x/3))/6

$$3.70 \quad \int \frac{1-x^2}{1+4x^2+x^4} dx$$

**Optimal.** Leaf size=44

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {1163, 203}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 + 4\*x^2 + x^4),x]

[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/Sqrt[2] - ArcTan[x/Sqrt[2 + Sqrt[3]]]/Sqrt[2]

Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[Rt[b, 2]\*x]/Rt[a, 2])]/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1163

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[e/2 + (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 - q/2 + c\*x^2), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && GtQ[b^2 - 4\*a\*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-x^2}{1+4x^2+x^4} dx &= \frac{1}{2}(-1-\sqrt{3}) \int \frac{1}{2+\sqrt{3}+x^2} dx + \frac{1}{2}(-1+\sqrt{3}) \int \frac{1}{2-\sqrt{3}+x^2} dx \\ &= \frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2}} \end{aligned}$$

**Mathematica [A]** time = 0.07, size = 82, normalized size = 1.86

$$\frac{-\left((\sqrt{3}-3)\sqrt{2+\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)\right)-\sqrt{2-\sqrt{3}}(3+\sqrt{3})\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 + 4\*x^2 + x^4),x]

[Out]  $(-((-3 + \sqrt{3})\sqrt{2 + \sqrt{3}}\operatorname{ArcTan}[x/\sqrt{2 - \sqrt{3}}]) - \sqrt{2 - \sqrt{3}}(3 + \sqrt{3})\operatorname{ArcTan}[x/\sqrt{2 + \sqrt{3}}]))/(2\sqrt{3})$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 - x^2}{1 + 4x^2 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - x^2)/(1 + 4\*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(1 - x^2)/(1 + 4\*x^2 + x^4), x]

**fricas** [A] time = 0.93, size = 31, normalized size = 0.70

$$\frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} (x^3 + 3x)\right) - \frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+4\*x^2+1), x, algorithm="fricas")

[Out]  $1/2*\sqrt{2}*\arctan(1/2*\sqrt{2}*(x^3 + 3*x)) - 1/2*\sqrt{2}*\arctan(1/2*\sqrt{2}*(x))$

**giac** [A] time = 0.16, size = 26, normalized size = 0.59

$$\frac{1}{4} \sqrt{2} \left( \pi \operatorname{sgn}(x) - 2 \arctan\left(\frac{\sqrt{2}(x^2 + 1)}{2x}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+4\*x^2+1), x, algorithm="giac")

[Out]  $1/4*\sqrt{2}*(\pi*\operatorname{sgn}(x) - 2*\arctan(1/2*\sqrt{2}*(x^2 + 1)/x))$

**maple** [B] time = 0.02, size = 111, normalized size = 2.52

$$\frac{\sqrt{3} \arctan\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{\sqrt{6}-\sqrt{2}} - \frac{\arctan\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{\sqrt{6}-\sqrt{2}} - \frac{\sqrt{3} \arctan\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{\sqrt{6}+\sqrt{2}} - \frac{\arctan\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{\sqrt{6}+\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(x^4+4\*x^2+1), x)

[Out]  $-3^{(1/2)}/(6^{(1/2)}+2^{(1/2)})*\arctan(2/(6^{(1/2)}+2^{(1/2)})*x)-1/(6^{(1/2)}+2^{(1/2)})*\arctan(2/(6^{(1/2)}+2^{(1/2)})*x)+3^{(1/2)}/(6^{(1/2)}-2^{(1/2)})*\arctan(2/(6^{(1/2)}-2^{(1/2)})*x)-1/(6^{(1/2)}-2^{(1/2)})*\arctan(2/(6^{(1/2)}-2^{(1/2)})*x)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 - 1}{x^4 + 4x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+4\*x^2+1), x, algorithm="maxima")

[Out]  $-\operatorname{integrate}((x^2 - 1)/(x^4 + 4*x^2 + 1), x)$

**mupad [B]** time = 0.08, size = 31, normalized size = 0.70

$$\frac{\sqrt{2} \left( \operatorname{atan} \left( \frac{\sqrt{2} x^3}{2} + \frac{3\sqrt{2} x}{2} \right) - \operatorname{atan} \left( \frac{\sqrt{2} x}{2} \right) \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x^2 - 1)/(4*x^2 + x^4 + 1), x)`

[Out]  $(2^{(1/2)} * (\operatorname{atan}((3 * 2^{(1/2)} * x) / 2 + (2^{(1/2)} * x^3) / 2) - \operatorname{atan}((2^{(1/2)} * x) / 2))) / 2$

**sympy [A]** time = 0.13, size = 42, normalized size = 0.95

$$\frac{\sqrt{2} \left( 2 \operatorname{atan} \left( \frac{\sqrt{2} x}{2} \right) - 2 \operatorname{atan} \left( \frac{\sqrt{2} x^3}{2} + \frac{3\sqrt{2} x}{2} \right) \right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**2+1)/(x**4+4*x**2+1), x)`

[Out]  $-\operatorname{sqrt}(2) * (2 * \operatorname{atan}(\operatorname{sqrt}(2) * x / 2) - 2 * \operatorname{atan}(\operatorname{sqrt}(2) * x ** 3 / 2 + 3 * \operatorname{sqrt}(2) * x / 2)) / 4$

$$3.71 \quad \int \frac{1-x^2}{1+3x^2+x^4} dx$$

**Optimal.** Leaf size=39

$$\tan^{-1}\left(\sqrt{\frac{1}{2}}(3+\sqrt{5})x\right) - \tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)$$

**Rubi [A]** time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {1163, 203}

$$\tan^{-1}\left(\sqrt{\frac{1}{2}}(3+\sqrt{5})x\right) - \tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 + 3\*x^2 + x^4), x]

[Out] -ArcTan[Sqrt[2/(3 + Sqrt[5])]\*x] + ArcTan[Sqrt[(3 + Sqrt[5])/2]\*x]

**Rule 203**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

**Rule 1163**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[e/2 + (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 - q/2 + c\*x^2), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && GtQ[b^2 - 4\*a\*c, 0]

**Rubi steps**

$$\begin{aligned} \int \frac{1-x^2}{1+3x^2+x^4} dx &= \frac{1}{2}(-1-\sqrt{5}) \int \frac{1}{\frac{3}{2} + \frac{\sqrt{5}}{2} + x^2} dx + \frac{1}{2}(-1+\sqrt{5}) \int \frac{1}{\frac{3}{2} - \frac{\sqrt{5}}{2} + x^2} dx \\ &= -\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right) + \tan^{-1}\left(\sqrt{\frac{1}{2}}(3+\sqrt{5})x\right) \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 10, normalized size = 0.26

$$\tan^{-1}\left(\frac{x}{x^2+1}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 + 3\*x^2 + x^4), x]

[Out] ArcTan[x/(1 + x^2)]

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^2}{1+3x^2+x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - x^2)/(1 + 3\*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(1 - x^2)/(1 + 3\*x^2 + x^4), x]

**fricas** [A] time = 0.89, size = 13, normalized size = 0.33

$$\arctan(x^3 + 2x) - \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+3\*x^2+1),x, algorithm="fricas")

[Out] arctan(x^3 + 2\*x) - arctan(x)

**giac** [A] time = 0.18, size = 26, normalized size = 0.67

$$\frac{1}{4} \pi \operatorname{sgn}(x) - \frac{1}{2} \arctan\left(\frac{x^4 + x^2 + 1}{2(x^3 + x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+3\*x^2+1),x, algorithm="giac")

[Out] 1/4\*pi\*sgn(x) - 1/2\*arctan(1/2\*(x^4 + x^2 + 1)/(x^3 + x))

**maple** [B] time = 0.02, size = 104, normalized size = 2.67

$$\frac{2\sqrt{5} \arctan\left(\frac{4x}{2\sqrt{5}-2}\right)}{2\sqrt{5}-2} - \frac{2 \arctan\left(\frac{4x}{2\sqrt{5}-2}\right)}{2\sqrt{5}-2} - \frac{2\sqrt{5} \arctan\left(\frac{4x}{2\sqrt{5}+2}\right)}{2\sqrt{5}+2} - \frac{2 \arctan\left(\frac{4x}{2\sqrt{5}+2}\right)}{2\sqrt{5}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(x^4+3\*x^2+1), x)

[Out] -2\*5^(1/2)/(2\*5^(1/2)+2)\*arctan(4/(2\*5^(1/2)+2)\*x)-2/(2\*5^(1/2)+2)\*arctan(4/(2\*5^(1/2)+2)\*x)+2\*5^(1/2)/(2\*5^(1/2)-2)\*arctan(4/(2\*5^(1/2)-2)\*x)-2/(2\*5^(1/2)-2)\*arctan(4/(2\*5^(1/2)-2)\*x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{x^2 - 1}{x^4 + 3x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+3\*x^2+1),x, algorithm="maxima")

[Out] -integrate((x^2 - 1)/(x^4 + 3\*x^2 + 1), x)

**mupad** [B] time = 4.31, size = 13, normalized size = 0.33

$$\operatorname{atan}(x^3 + 2x) - \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2 - 1)/(3\*x^2 + x^4 + 1), x)

[Out] atan(2\*x + x^3) - atan(x)

**sympy** [A] time = 0.12, size = 10, normalized size = 0.26

$$- \operatorname{atan}(x) + \operatorname{atan}(x^3 + 2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4+3*x**2+1),x)
```

```
[Out] -atan(x) + atan(x**3 + 2*x)
```

$$3.72 \quad \int \frac{1-x^2}{1+2x^2+x^4} dx$$

Optimal. Leaf size=9

$$\frac{x}{x^2+1}$$

**Rubi [A]** time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {28, 383}

$$\frac{x}{x^2+1}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 + 2\*x^2 + x^4), x]

[Out] x/(1 + x^2)

Rule 28

Int[(u\_.)\*((a\_.) + (c\_.)\*(x\_)^(n2\_.)) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] :> Dist[1/c^p, Int[u\*(b/2 + c\*x^n)^(2\*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2\*n] && EqQ[b^2 - 4\*a\*c, 0] && IntegerQ[p]

Rule 383

Int[((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Simp[(c\*x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a\*d - b\*c\*(n\*(p + 1) + 1), 0]

Rubi steps

$$\begin{aligned} \int \frac{1-x^2}{1+2x^2+x^4} dx &= \int \frac{1-x^2}{(1+x^2)^2} dx \\ &= \frac{x}{1+x^2} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 9, normalized size = 1.00

$$\frac{x}{x^2+1}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 + 2\*x^2 + x^4), x]

[Out] x/(1 + x^2)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^2}{1+2x^2+x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - x^2)/(1 + 2\*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(1 - x^2)/(1 + 2\*x^2 + x^4), x]



**fricas** [A] time = 0.85, size = 9, normalized size = 1.00

$$\frac{x}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+2\*x^2+1),x, algorithm="fricas")

[Out] x/(x^2 + 1)

**giac** [A] time = 0.18, size = 7, normalized size = 0.78

$$\frac{1}{x + \frac{1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+2\*x^2+1),x, algorithm="giac")

[Out] 1/(x + 1/x)

**maple** [A] time = 0.01, size = 10, normalized size = 1.11

$$\frac{x}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(x^4+2\*x^2+1),x)

[Out] 1/(x^2+1)\*x

**maxima** [A] time = 1.00, size = 9, normalized size = 1.00

$$\frac{x}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+2\*x^2+1),x, algorithm="maxima")

[Out] x/(x^2 + 1)

**mupad** [B] time = 0.03, size = 9, normalized size = 1.00

$$\frac{x}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2 - 1)/(2\*x^2 + x^4 + 1),x)

[Out] x/(x^2 + 1)

**sympy** [A] time = 0.09, size = 5, normalized size = 0.56

$$\frac{x}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*2+1)/(x\*\*4+2\*x\*\*2+1),x)

[Out] x/(x\*\*2 + 1)

$$3.73 \quad \int \frac{1-x^2}{1+x^2+x^4} dx$$

Optimal. Leaf size=25

$$\frac{1}{2} \log(x^2 + x + 1) - \frac{1}{2} \log(x^2 - x + 1)$$

**Rubi [A]** time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {1164, 628}

$$\frac{1}{2} \log(x^2 + x + 1) - \frac{1}{2} \log(x^2 - x + 1)$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 + x^2 + x^4), x]

[Out] -Log[1 - x + x^2]/2 + Log[1 + x + x^2]/2

Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1164

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(-2\*d)/e - b/c, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && !GtQ[b^2 - 4\*a\*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-x^2}{1+x^2+x^4} dx &= -\left(\frac{1}{2} \int \frac{1+2x}{-1-x-x^2} dx\right) - \frac{1}{2} \int \frac{1-2x}{-1+x-x^2} dx \\ &= -\frac{1}{2} \log(1-x+x^2) + \frac{1}{2} \log(1+x+x^2) \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{2} \log(x^2 + x + 1) - \frac{1}{2} \log(x^2 - x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 + x^2 + x^4), x]

[Out] -1/2\*Log[1 - x + x^2] + Log[1 + x + x^2]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^2}{1+x^2+x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - x^2)/(1 + x^2 + x^4), x]

[Out] IntegrateAlgebraic[(1 - x^2)/(1 + x^2 + x^4), x]

**fricas** [A] time = 1.56, size = 21, normalized size = 0.84

$$\frac{1}{2} \log(x^2 + x + 1) - \frac{1}{2} \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+x^2+1), x, algorithm="fricas")

[Out] 1/2\*log(x^2 + x + 1) - 1/2\*log(x^2 - x + 1)

**giac** [A] time = 0.15, size = 35, normalized size = 1.40

$$\frac{1}{4} \log\left(x + \frac{1}{x + \frac{1}{x}} + \frac{1}{x} + 2\right) - \frac{1}{4} \log\left(x + \frac{1}{x + \frac{1}{x}} + \frac{1}{x} - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+x^2+1), x, algorithm="giac")

[Out] 1/4\*log(abs(x + 1/(x + 1/x) + 1/x + 2)) - 1/4\*log(abs(x + 1/(x + 1/x) + 1/x - 2))

**maple** [A] time = 0.00, size = 22, normalized size = 0.88

$$-\frac{\ln(x^2 - x + 1)}{2} + \frac{\ln(x^2 + x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(x^4+x^2+1), x)

[Out] -1/2\*ln(x^2-x+1)+1/2\*ln(x^2+x+1)

**maxima** [A] time = 1.04, size = 21, normalized size = 0.84

$$\frac{1}{2} \log(x^2 + x + 1) - \frac{1}{2} \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+x^2+1), x, algorithm="maxima")

[Out] 1/2\*log(x^2 + x + 1) - 1/2\*log(x^2 - x + 1)

**mupad** [B] time = 0.06, size = 10, normalized size = 0.40

$$\operatorname{atanh}\left(\frac{x}{x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2 - 1)/(x^2 + x^4 + 1), x)

[Out] atanh(x/(x^2 + 1))

**sympy** [A] time = 0.12, size = 19, normalized size = 0.76

$$-\frac{\log(x^2 - x + 1)}{2} + \frac{\log(x^2 + x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4+x**2+1),x)
```

```
[Out] -log(x**2 - x + 1)/2 + log(x**2 + x + 1)/2
```

$$3.74 \quad \int \frac{1-x^2}{1+x^4} dx$$

**Optimal.** Leaf size=46

$$\frac{\log(x^2 + \sqrt{2}x + 1)}{2\sqrt{2}} - \frac{\log(x^2 - \sqrt{2}x + 1)}{2\sqrt{2}}$$

**Rubi [A]** time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {1165, 628}

$$\frac{\log(x^2 + \sqrt{2}x + 1)}{2\sqrt{2}} - \frac{\log(x^2 - \sqrt{2}x + 1)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 + x^4), x]

[Out] -Log[1 - Sqrt[2]\*x + x^2]/(2\*Sqrt[2]) + Log[1 + Sqrt[2]\*x + x^2]/(2\*Sqrt[2])

Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] :> Simp[d\*Log[RemoveContent[a + b\*x + c\*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1165

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

Rubi steps

$$\begin{aligned} \int \frac{1-x^2}{1+x^4} dx &= -\frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2} dx}{2\sqrt{2}} - \frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2} dx}{2\sqrt{2}} \\ &= -\frac{\log(1 - \sqrt{2}x + x^2)}{2\sqrt{2}} + \frac{\log(1 + \sqrt{2}x + x^2)}{2\sqrt{2}} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 40, normalized size = 0.87

$$\frac{\log(x^2 + \sqrt{2}x + 1) - \log(-x^2 + \sqrt{2}x - 1)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 + x^4), x]

[Out] (-Log[-1 + Sqrt[2]\*x - x^2] + Log[1 + Sqrt[2]\*x + x^2])/(2\*Sqrt[2])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^2}{1+x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - x^2)/(1 + x^4), x]

[Out] IntegrateAlgebraic[(1 - x^2)/(1 + x^4), x]

**fricas** [A] time = 1.00, size = 34, normalized size = 0.74

$$\frac{1}{4} \sqrt{2} \log\left(\frac{x^4 + 4x^2 + 2\sqrt{2}(x^3 + x) + 1}{x^4 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+1), x, algorithm="fricas")

[Out] 1/4\*sqrt(2)\*log((x^4 + 4\*x^2 + 2\*sqrt(2)\*(x^3 + x) + 1)/(x^4 + 1))

**giac** [A] time = 0.15, size = 34, normalized size = 0.74

$$\frac{1}{4} \sqrt{2} \log(x^2 + \sqrt{2}x + 1) - \frac{1}{4} \sqrt{2} \log(x^2 - \sqrt{2}x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+1), x, algorithm="giac")

[Out] 1/4\*sqrt(2)\*log(x^2 + sqrt(2)\*x + 1) - 1/4\*sqrt(2)\*log(x^2 - sqrt(2)\*x + 1)

**maple** [A] time = 0.00, size = 62, normalized size = 1.35

$$-\frac{\sqrt{2} \ln\left(\frac{x^2 - \sqrt{2}x + 1}{x^2 + \sqrt{2}x + 1}\right)}{8} + \frac{\sqrt{2} \ln\left(\frac{x^2 + \sqrt{2}x + 1}{x^2 - \sqrt{2}x + 1}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(x^4+1), x)

[Out] 1/8\*2^(1/2)\*ln((x^2+2^(1/2)\*x+1)/(x^2-2^(1/2)\*x+1))-1/8\*2^(1/2)\*ln((x^2-2^(1/2)\*x+1)/(x^2+2^(1/2)\*x+1))

**maxima** [A] time = 2.26, size = 34, normalized size = 0.74

$$\frac{1}{4} \sqrt{2} \log(x^2 + \sqrt{2}x + 1) - \frac{1}{4} \sqrt{2} \log(x^2 - \sqrt{2}x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4+1), x, algorithm="maxima")

[Out] 1/4\*sqrt(2)\*log(x^2 + sqrt(2)\*x + 1) - 1/4\*sqrt(2)\*log(x^2 - sqrt(2)\*x + 1)

**mupad** [B] time = 0.06, size = 18, normalized size = 0.39

$$\frac{\sqrt{2} \operatorname{atanh}\left(\frac{\sqrt{2}x}{x^2+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2 - 1)/(x^4 + 1), x)

[Out] (2^(1/2)\*atanh((2^(1/2)\*x)/(x^2 + 1)))/2

sympy [A] time = 0.11, size = 39, normalized size = 0.85

$$-\frac{\sqrt{2} \log(x^2 - \sqrt{2}x + 1)}{4} + \frac{\sqrt{2} \log(x^2 + \sqrt{2}x + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*2+1)/(x\*\*4+1),x)

[Out] -sqrt(2)\*log(x\*\*2 - sqrt(2)\*x + 1)/4 + sqrt(2)\*log(x\*\*2 + sqrt(2)\*x + 1)/4

$$3.75 \quad \int \frac{1-x^2}{1-x^2+x^4} dx$$

**Optimal.** Leaf size=46

$$\frac{\log(x^2 + \sqrt{3}x + 1)}{2\sqrt{3}} - \frac{\log(x^2 - \sqrt{3}x + 1)}{2\sqrt{3}}$$

**Rubi [A]** time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {1164, 628}

$$\frac{\log(x^2 + \sqrt{3}x + 1)}{2\sqrt{3}} - \frac{\log(x^2 - \sqrt{3}x + 1)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 - x^2 + x^4), x]

[Out] -Log[1 - Sqrt[3]\*x + x^2]/(2\*Sqrt[3]) + Log[1 + Sqrt[3]\*x + x^2]/(2\*Sqrt[3])

Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1164

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(-2\*d)/e - b/c, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && !GtQ[b^2 - 4\*a\*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-x^2}{1-x^2+x^4} dx &= -\frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2} dx}{2\sqrt{3}} - \frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2} dx}{2\sqrt{3}} \\ &= -\frac{\log(1 - \sqrt{3}x + x^2)}{2\sqrt{3}} + \frac{\log(1 + \sqrt{3}x + x^2)}{2\sqrt{3}} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 40, normalized size = 0.87

$$\frac{\log(x^2 + \sqrt{3}x + 1) - \log(-x^2 + \sqrt{3}x - 1)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 - x^2 + x^4), x]

[Out] (-Log[-1 + Sqrt[3]\*x - x^2] + Log[1 + Sqrt[3]\*x + x^2])/(2\*Sqrt[3])



**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^2}{1-x^2+x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - x^2)/(1 - x^2 + x^4), x]

[Out] IntegrateAlgebraic[(1 - x^2)/(1 - x^2 + x^4), x]

**fricas** [A] time = 0.78, size = 39, normalized size = 0.85

$$\frac{1}{6} \sqrt{3} \log \left( \frac{x^4 + 5x^2 + 2\sqrt{3}(x^3 + x) + 1}{x^4 - x^2 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-x^2+1), x, algorithm="fricas")

[Out] 1/6\*sqrt(3)\*log((x^4 + 5\*x^2 + 2\*sqrt(3)\*(x^3 + x) + 1)/(x^4 - x^2 + 1))

**giac** [A] time = 0.18, size = 39, normalized size = 0.85

$$-\frac{1}{6} \sqrt{3} \log \left( \frac{\left| 2x - 2\sqrt{3} + \frac{2}{x} \right|}{\left| 2x + 2\sqrt{3} + \frac{2}{x} \right|} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-x^2+1), x, algorithm="giac")

[Out] -1/6\*sqrt(3)\*log(abs(2\*x - 2\*sqrt(3) + 2/x)/abs(2\*x + 2\*sqrt(3) + 2/x))

**maple** [A] time = 0.01, size = 35, normalized size = 0.76

$$-\frac{\sqrt{3} \ln(x^2 - \sqrt{3}x + 1)}{6} + \frac{\sqrt{3} \ln(x^2 + \sqrt{3}x + 1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(x^4-x^2+1), x)

[Out] -1/6\*3^(1/2)\*ln(x^2-3^(1/2)\*x+1)+1/6\*3^(1/2)\*ln(x^2+3^(1/2)\*x+1)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 - 1}{x^4 - x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-x^2+1), x, algorithm="maxima")

[Out] -integrate((x^2 - 1)/(x^4 - x^2 + 1), x)

**mupad** [B] time = 4.31, size = 18, normalized size = 0.39

$$\frac{\sqrt{3} \operatorname{atanh} \left( \frac{\sqrt{3}x}{x^2+1} \right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x^2 - 1)/(x^4 - x^2 + 1),x)`

[Out]  $(3^{1/2} \operatorname{atanh}((3^{1/2}x)/(x^2 + 1)))/3$

**sympy [A]** time = 0.12, size = 39, normalized size = 0.85

$$-\frac{\sqrt{3} \log(x^2 - \sqrt{3}x + 1)}{6} + \frac{\sqrt{3} \log(x^2 + \sqrt{3}x + 1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**2+1)/(x**4-x**2+1),x)`

[Out]  $-\sqrt{3} \log(x^2 - \sqrt{3}x + 1)/6 + \sqrt{3} \log(x^2 + \sqrt{3}x + 1)/6$

$$3.76 \quad \int \frac{1-x^2}{1-2x^2+x^4} dx$$

**Optimal.** Leaf size=2

$$\tanh^{-1}(x)$$

**Rubi [A]** time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {28, 21, 207}

$$\tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 - 2\*x^2 + x^4), x]

[Out] ArcTanh[x]

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :=
  Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
  && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
  a + b*x])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :=
  Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
  EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1-x^2}{1-2x^2+x^4} dx &= \int \frac{1-x^2}{(-1+x^2)^2} dx \\ &= -\int \frac{1}{-1+x^2} dx \\ &= \tanh^{-1}(x) \end{aligned}$$

**Mathematica [B]** time = 0.00, size = 19, normalized size = 9.50

$$\frac{1}{2} \log(x+1) - \frac{1}{2} \log(1-x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 - 2\*x^2 + x^4), x]

[Out] -1/2\*Log[1 - x] + Log[1 + x]/2

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^2}{1-2x^2+x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - x^2)/(1 - 2\*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(1 - x^2)/(1 - 2\*x^2 + x^4), x]

**fricas** [B] time = 1.07, size = 13, normalized size = 6.50

$$\frac{1}{2} \log(x+1) - \frac{1}{2} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-2\*x^2+1), x, algorithm="fricas")

[Out] 1/2\*log(x + 1) - 1/2\*log(x - 1)

**giac** [B] time = 0.15, size = 15, normalized size = 7.50

$$\frac{1}{2} \log(|x+1|) - \frac{1}{2} \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-2\*x^2+1), x, algorithm="giac")

[Out] 1/2\*log(abs(x + 1)) - 1/2\*log(abs(x - 1))

**maple** [A] time = 0.00, size = 3, normalized size = 1.50

$$\operatorname{arctanh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(x^4-2\*x^2+1), x)

[Out] arctanh(x)

**maxima** [B] time = 1.07, size = 13, normalized size = 6.50

$$\frac{1}{2} \log(x+1) - \frac{1}{2} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-2\*x^2+1), x, algorithm="maxima")

[Out] 1/2\*log(x + 1) - 1/2\*log(x - 1)

**mupad** [B] time = 4.30, size = 2, normalized size = 1.00

$$\operatorname{atanh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2 - 1)/(x^4 - 2\*x^2 + 1), x)

[Out] atanh(x)

sympy [B] time = 0.11, size = 12, normalized size = 6.00

$$-\frac{\log(x-1)}{2} + \frac{\log(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4-2*x**2+1), x)
```

```
[Out] -log(x - 1)/2 + log(x + 1)/2
```

$$3.77 \quad \int \frac{1-x^2}{1-3x^2+x^4} dx$$

**Optimal.** Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{2x+1}{\sqrt{5}}\right)}{\sqrt{5}} - \frac{\tanh^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)}{\sqrt{5}}$$

**Rubi [A]** time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{2x+1}{\sqrt{5}}\right)}{\sqrt{5}} - \frac{\tanh^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 - 3\*x^2 + x^4), x]

[Out] -(ArcTanh[(1 - 2\*x)/Sqrt[5]]/Sqrt[5]) + ArcTanh[(1 + 2\*x)/Sqrt[5]]/Sqrt[5]

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 618**

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

**Rule 1161**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

**Rubi steps**

$$\begin{aligned} \int \frac{1-x^2}{1-3x^2+x^4} dx &= -\left(\frac{1}{2} \int \frac{1}{-1-x+x^2} dx\right) - \frac{1}{2} \int \frac{1}{-1+x+x^2} dx \\ &= \text{Subst}\left(\int \frac{1}{5-x^2} dx, x, -1+2x\right) + \text{Subst}\left(\int \frac{1}{5-x^2} dx, x, 1+2x\right) \\ &= \frac{\tanh^{-1}\left(\frac{-1+2x}{\sqrt{5}}\right)}{\sqrt{5}} + \frac{\tanh^{-1}\left(\frac{1+2x}{\sqrt{5}}\right)}{\sqrt{5}} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 40, normalized size = 1.05

$$\frac{\log(x^2 + \sqrt{5}x + 1) - \log(-x^2 + \sqrt{5}x - 1)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 - 3\*x^2 + x^4), x]

[Out] (-Log[-1 + Sqrt[5]\*x - x^2] + Log[1 + Sqrt[5]\*x + x^2])/(2\*Sqrt[5])

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^2}{1-3x^2+x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - x^2)/(1 - 3\*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(1 - x^2)/(1 - 3\*x^2 + x^4), x]

**fricas** [A] time = 1.11, size = 39, normalized size = 1.03

$$\frac{1}{10} \sqrt{5} \log\left(\frac{x^4 + 7x^2 + 2\sqrt{5}(x^3 + x) + 1}{x^4 - 3x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-3\*x^2+1), x, algorithm="fricas")

[Out] 1/10\*sqrt(5)\*log((x^4 + 7\*x^2 + 2\*sqrt(5)\*(x^3 + x) + 1)/(x^4 - 3\*x^2 + 1))

**giac** [A] time = 0.18, size = 39, normalized size = 1.03

$$-\frac{1}{10} \sqrt{5} \log\left(\frac{\left|2x - 2\sqrt{5} + \frac{2}{x}\right|}{\left|2x + 2\sqrt{5} + \frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-3\*x^2+1), x, algorithm="giac")

[Out] -1/10\*sqrt(5)\*log(abs(2\*x - 2\*sqrt(5) + 2/x)/abs(2\*x + 2\*sqrt(5) + 2/x))

**maple** [A] time = 0.00, size = 34, normalized size = 0.89

$$\frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(2x+1)\sqrt{5}}{5}\right)}{5} + \frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(2x-1)\sqrt{5}}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(x^4-3\*x^2+1), x)

[Out] 1/5\*arctanh(1/5\*(2\*x+1)\*5^(1/2))\*5^(1/2)+1/5\*5^(1/2)\*arctanh(1/5\*(2\*x-1)\*5^(1/2))

**maxima** [A] time = 2.46, size = 55, normalized size = 1.45

$$-\frac{1}{10} \sqrt{5} \log\left(\frac{2x - \sqrt{5} + 1}{2x + \sqrt{5} + 1}\right) - \frac{1}{10} \sqrt{5} \log\left(\frac{2x - \sqrt{5} - 1}{2x + \sqrt{5} - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-3\*x^2+1), x, algorithm="maxima")

[Out] -1/10\*sqrt(5)\*log((2\*x - sqrt(5) + 1)/(2\*x + sqrt(5) + 1)) - 1/10\*sqrt(5)\*log((2\*x - sqrt(5) - 1)/(2\*x + sqrt(5) - 1))

**mupad [B]** time = 0.11, size = 18, normalized size = 0.47

$$\frac{\sqrt{5} \operatorname{atanh}\left(\frac{\sqrt{5}x}{x^2+1}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x^2 - 1)/(x^4 - 3*x^2 + 1),x)`

[Out] `(5^(1/2)*atanh((5^(1/2)*x)/(x^2 + 1)))/5`

**sympy [A]** time = 0.12, size = 39, normalized size = 1.03

$$-\frac{\sqrt{5} \log(x^2 - \sqrt{5}x + 1)}{10} + \frac{\sqrt{5} \log(x^2 + \sqrt{5}x + 1)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**2+1)/(x**4-3*x**2+1),x)`

[Out] `-sqrt(5)*log(x**2 - sqrt(5)*x + 1)/10 + sqrt(5)*log(x**2 + sqrt(5)*x + 1)/10`



$$3.78 \quad \int \frac{1-x^2}{1-4x^2+x^4} dx$$

**Optimal.** Leaf size=47

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x+1}{\sqrt{3}}\right)}{\sqrt{6}} - \frac{\tanh^{-1}\left(\frac{1-\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}$$

**Rubi [A]** time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x+1}{\sqrt{3}}\right)}{\sqrt{6}} - \frac{\tanh^{-1}\left(\frac{1-\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 - 4\*x^2 + x^4), x]

[Out] -(ArcTanh[(1 - Sqrt[2]\*x)/Sqrt[3]]/Sqrt[6]) + ArcTanh[(1 + Sqrt[2]\*x)/Sqrt[3]]/Sqrt[6]

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 618**

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

**Rule 1161**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

**Rubi steps**

$$\begin{aligned} \int \frac{1-x^2}{1-4x^2+x^4} dx &= -\left(\frac{1}{2} \int \frac{1}{-1-\sqrt{2}x+x^2} dx\right) - \frac{1}{2} \int \frac{1}{-1+\sqrt{2}x+x^2} dx \\ &= \text{Subst}\left(\int \frac{1}{6-x^2} dx, x, -\sqrt{2}+2x\right) + \text{Subst}\left(\int \frac{1}{6-x^2} dx, x, \sqrt{2}+2x\right) \\ &= \frac{\tanh^{-1}\left(\frac{-1+\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}} + \frac{\tanh^{-1}\left(\frac{1+\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 40, normalized size = 0.85

$$\frac{\log(x^2 + \sqrt{6}x + 1) - \log(-x^2 + \sqrt{6}x - 1)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 - 4\*x^2 + x^4), x]

[Out] (-Log[-1 + Sqrt[6]\*x - x^2] + Log[1 + Sqrt[6]\*x + x^2])/(2\*Sqrt[6])

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^2}{1-4x^2+x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - x^2)/(1 - 4\*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(1 - x^2)/(1 - 4\*x^2 + x^4), x]

**fricas** [A] time = 1.26, size = 39, normalized size = 0.83

$$\frac{1}{12} \sqrt{6} \log \left( \frac{x^4 + 8x^2 + 2\sqrt{6}(x^3 + x) + 1}{x^4 - 4x^2 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-4\*x^2+1), x, algorithm="fricas")

[Out] 1/12\*sqrt(6)\*log((x^4 + 8\*x^2 + 2\*sqrt(6)\*(x^3 + x) + 1)/(x^4 - 4\*x^2 + 1))

**giac** [A] time = 0.32, size = 39, normalized size = 0.83

$$-\frac{1}{12} \sqrt{6} \log \left( \frac{\left| 2x - 2\sqrt{6} + \frac{2}{x} \right|}{\left| 2x + 2\sqrt{6} + \frac{2}{x} \right|} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-4\*x^2+1), x, algorithm="giac")

[Out] -1/12\*sqrt(6)\*log(abs(2\*x - 2\*sqrt(6) + 2/x)/abs(2\*x + 2\*sqrt(6) + 2/x))

**maple** [A] time = 0.02, size = 70, normalized size = 1.49

$$\frac{(\sqrt{3}-1)\sqrt{3} \operatorname{arctanh}\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{3\sqrt{6}-3\sqrt{2}} + \frac{(1+\sqrt{3})\sqrt{3} \operatorname{arctanh}\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{3\sqrt{6}+3\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(x^4-4\*x^2+1), x)

[Out] 1/3\*(3^(1/2)-1)\*3^(1/2)/(6^(1/2)-2^(1/2))\*arctanh(2/(6^(1/2)-2^(1/2))\*x)+1/3\*(1+3^(1/2))\*3^(1/2)/(6^(1/2)+2^(1/2))\*arctanh(2/(6^(1/2)+2^(1/2))\*x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2-1}{x^4-4x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-4\*x^2+1), x, algorithm="maxima")

[Out] -integrate((x^2 - 1)/(x^4 - 4\*x^2 + 1), x)

**mupad** [B] time = 4.32, size = 18, normalized size = 0.38

$$\frac{\sqrt{6} \operatorname{atanh}\left(\frac{\sqrt{6}x}{x^2+1}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x^2 - 1)/(x^4 - 4*x^2 + 1), x)`

[Out] `(6^(1/2)*atanh((6^(1/2)*x)/(x^2 + 1)))/6`

**sympy** [A] time = 0.12, size = 39, normalized size = 0.83

$$-\frac{\sqrt{6} \log(x^2 - \sqrt{6}x + 1)}{12} + \frac{\sqrt{6} \log(x^2 + \sqrt{6}x + 1)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**2+1)/(x**4-4*x**2+1), x)`

[Out] `-sqrt(6)*log(x**2 - sqrt(6)*x + 1)/12 + sqrt(6)*log(x**2 + sqrt(6)*x + 1)/12`

$$3.79 \quad \int \frac{1-x^2}{1-5x^2+x^4} dx$$

**Optimal.** Leaf size=46

$$\frac{\tanh^{-1}\left(\frac{2x+\sqrt{3}}{\sqrt{7}}\right)}{\sqrt{7}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

**Rubi [A]** time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{2x+\sqrt{3}}{\sqrt{7}}\right)}{\sqrt{7}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 - 5\*x^2 + x^4),x]

[Out] -(ArcTanh[(Sqrt[3] - 2\*x)/Sqrt[7]]/Sqrt[7]) + ArcTanh[(Sqrt[3] + 2\*x)/Sqrt[7]]/Sqrt[7]

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 618**

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

**Rule 1161**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

**Rubi steps**

$$\begin{aligned} \int \frac{1-x^2}{1-5x^2+x^4} dx &= -\left(\frac{1}{2} \int \frac{1}{-1-\sqrt{3}x+x^2} dx\right) - \frac{1}{2} \int \frac{1}{-1+\sqrt{3}x+x^2} dx \\ &= \text{Subst}\left(\int \frac{1}{7-x^2} dx, x, -\sqrt{3}+2x\right) + \text{Subst}\left(\int \frac{1}{7-x^2} dx, x, \sqrt{3}+2x\right) \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{3}-2x}{\sqrt{7}}\right)}{\sqrt{7}} + \frac{\tanh^{-1}\left(\frac{\sqrt{3}+2x}{\sqrt{7}}\right)}{\sqrt{7}} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 40, normalized size = 0.87

$$\frac{\log(x^2 + \sqrt{7}x + 1) - \log(-x^2 + \sqrt{7}x - 1)}{2\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 - 5\*x^2 + x^4), x]

[Out] (-Log[-1 + Sqrt[7]\*x - x^2] + Log[1 + Sqrt[7]\*x + x^2])/(2\*Sqrt[7])

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^2}{1-5x^2+x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - x^2)/(1 - 5\*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(1 - x^2)/(1 - 5\*x^2 + x^4), x]

**fricas** [A] time = 0.78, size = 39, normalized size = 0.85

$$\frac{1}{14} \sqrt{7} \log\left(\frac{x^4 + 9x^2 + 2\sqrt{7}(x^3 + x) + 1}{x^4 - 5x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-5\*x^2+1), x, algorithm="fricas")

[Out] 1/14\*sqrt(7)\*log((x^4 + 9\*x^2 + 2\*sqrt(7)\*(x^3 + x) + 1)/(x^4 - 5\*x^2 + 1))

**giac** [A] time = 0.22, size = 39, normalized size = 0.85

$$-\frac{1}{14} \sqrt{7} \log\left(\frac{\left|2x - 2\sqrt{7} + \frac{2}{x}\right|}{\left|2x + 2\sqrt{7} + \frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-5\*x^2+1), x, algorithm="giac")

[Out] -1/14\*sqrt(7)\*log(abs(2\*x - 2\*sqrt(7) + 2/x)/abs(2\*x + 2\*sqrt(7) + 2/x))

**maple** [B] time = 0.02, size = 82, normalized size = 1.78

$$\frac{2(-3 + \sqrt{21})\sqrt{21} \operatorname{arctanh}\left(\frac{4x}{2\sqrt{7}-2\sqrt{3}}\right)}{21(2\sqrt{7}-2\sqrt{3})} + \frac{2(3 + \sqrt{21})\sqrt{21} \operatorname{arctanh}\left(\frac{4x}{2\sqrt{7}+2\sqrt{3}}\right)}{21(2\sqrt{7}+2\sqrt{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(x^4-5\*x^2+1), x)

[Out] 2/21\*(3+21^(1/2))\*21^(1/2)/(2\*7^(1/2)+2\*3^(1/2))\*arctanh(4/(2\*7^(1/2)+2\*3^(1/2))\*x)+2/21\*(-3+21^(1/2))\*21^(1/2)/(2\*7^(1/2)-2\*3^(1/2))\*arctanh(4/(2\*7^(1/2)-2\*3^(1/2))\*x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2-1}{x^4-5x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^4-5\*x^2+1), x, algorithm="maxima")

[Out] -integrate((x^2 - 1)/(x^4 - 5\*x^2 + 1), x)

**mupad [B]** time = 4.39, size = 18, normalized size = 0.39

$$\frac{\sqrt{7} \operatorname{atanh}\left(\frac{\sqrt{7}x}{x^2+1}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x^2 - 1)/(x^4 - 5*x^2 + 1),x)`

[Out] `(7^(1/2)*atanh((7^(1/2)*x)/(x^2 + 1)))/7`

**sympy [A]** time = 0.14, size = 39, normalized size = 0.85

$$-\frac{\sqrt{7} \log(x^2 - \sqrt{7}x + 1)}{14} + \frac{\sqrt{7} \log(x^2 + \sqrt{7}x + 1)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**2+1)/(x**4-5*x**2+1),x)`

[Out] `-sqrt(7)*log(x**2 - sqrt(7)*x + 1)/14 + sqrt(7)*log(x**2 + sqrt(7)*x + 1)/14`

$$3.80 \quad \int \frac{-1-3x^2}{1+2x^2+9x^4} dx$$

Optimal. Leaf size=43

$$\frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)}{2\sqrt{2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$ , Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[-((1 + 3\*x^2)/(1 + 2\*x^2 + 9\*x^4)),x]

[Out] ArcTan[(1 - 3\*x)/Sqrt[2]]/(2\*Sqrt[2]) - ArcTan[(1 + 3\*x)/Sqrt[2]]/(2\*Sqrt[2])

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 1161

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

#### Rubi steps

$$\begin{aligned} \int \frac{-1-3x^2}{1+2x^2+9x^4} dx &= -\left(\frac{1}{6} \int \frac{1}{\frac{1}{3} - \frac{2x}{3} + x^2} dx\right) - \frac{1}{6} \int \frac{1}{\frac{1}{3} + \frac{2x}{3} + x^2} dx \\ &= \frac{1}{3} \text{Subst}\left(\int \frac{1}{-\frac{8}{9} - x^2} dx, x, -\frac{2}{3} + 2x\right) + \frac{1}{3} \text{Subst}\left(\int \frac{1}{-\frac{8}{9} - x^2} dx, x, \frac{2}{3} + 2x\right) \\ &= \frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(\frac{1+3x}{\sqrt{2}}\right)}{2\sqrt{2}} \end{aligned}$$

**Mathematica** [C] time = 0.10, size = 99, normalized size = 2.30

$$\frac{(\sqrt{2} - i) \tan^{-1}\left(\frac{3x}{\sqrt{1-2i\sqrt{2}}}\right)}{2\sqrt{2}(1-2i\sqrt{2})} - \frac{(\sqrt{2} + i) \tan^{-1}\left(\frac{3x}{\sqrt{1+2i\sqrt{2}}}\right)}{2\sqrt{2}(1+2i\sqrt{2})}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 - 3\*x^2)/(1 + 2\*x^2 + 9\*x^4), x]

[Out] -1/2\*((-I + Sqrt[2])\*ArcTan[(3\*x)/Sqrt[1 - (2\*I)\*Sqrt[2]]])/Sqrt[2\*(1 - (2\*I)\*Sqrt[2])] - ((I + Sqrt[2])\*ArcTan[(3\*x)/Sqrt[1 + (2\*I)\*Sqrt[2]]])/(2\*Sqrt[2\*(1 + (2\*I)\*Sqrt[2])])

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1 - 3x^2}{1 + 2x^2 + 9x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-1 - 3\*x^2)/(1 + 2\*x^2 + 9\*x^4), x]

[Out] IntegrateAlgebraic[(-1 - 3\*x^2)/(1 + 2\*x^2 + 9\*x^4), x]

**fricas** [A] time = 1.21, size = 33, normalized size = 0.77

$$-\frac{1}{4}\sqrt{2} \arctan\left(\frac{1}{4}\sqrt{2}(9x^3 + 5x)\right) - \frac{1}{4}\sqrt{2} \arctan\left(\frac{3}{4}\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3\*x^2-1)/(9\*x^4+2\*x^2+1), x, algorithm="fricas")

[Out] -1/4\*sqrt(2)\*arctan(1/4\*sqrt(2)\*(9\*x^3 + 5\*x)) - 1/4\*sqrt(2)\*arctan(3/4\*sqrt(2)\*x)

**giac** [A] time = 0.16, size = 33, normalized size = 0.77

$$-\frac{1}{4}\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}(3x + 1)\right) - \frac{1}{4}\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}(3x - 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3\*x^2-1)/(9\*x^4+2\*x^2+1), x, algorithm="giac")

[Out] -1/4\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(3\*x + 1)) - 1/4\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(3\*x - 1))

**maple** [A] time = 0.01, size = 34, normalized size = 0.79

$$\frac{\sqrt{2} \arctan\left(\frac{(6x-2)\sqrt{2}}{4}\right)}{4} - \frac{\sqrt{2} \arctan\left(\frac{(6x+2)\sqrt{2}}{4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-3\*x^2-1)/(9\*x^4+2\*x^2+1), x)

[Out] -1/4\*2^(1/2)\*arctan(1/4\*(6\*x+2)\*2^(1/2))-1/4\*2^(1/2)\*arctan(1/4\*(6\*x-2)\*2^(1/2))



**maxima** [A] time = 2.35, size = 33, normalized size = 0.77

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+1)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3\*x^2-1)/(9\*x^4+2\*x^2+1),x, algorithm="maxima")

[Out] -1/4\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(3\*x + 1)) - 1/4\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(3\*x - 1))

**mupad** [B] time = 4.38, size = 29, normalized size = 0.67

$$\frac{\sqrt{2}\left(\operatorname{atan}\left(\frac{9\sqrt{2}x^3}{4}+\frac{5\sqrt{2}x}{4}\right)+\operatorname{atan}\left(\frac{3\sqrt{2}x}{4}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(3\*x^2 + 1)/(2\*x^2 + 9\*x^4 + 1),x)

[Out] -(2^(1/2)\*(atan((5\*2^(1/2)\*x)/4 + (9\*2^(1/2)\*x^3)/4) + atan((3\*2^(1/2)\*x)/4)))/4

**sympy** [A] time = 0.14, size = 46, normalized size = 1.07

$$\frac{\sqrt{2}\left(2\operatorname{atan}\left(\frac{3\sqrt{2}x}{4}\right)+2\operatorname{atan}\left(\frac{9\sqrt{2}x^3}{4}+\frac{5\sqrt{2}x}{4}\right)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3\*x\*\*2-1)/(9\*x\*\*4+2\*x\*\*2+1),x)

[Out] -sqrt(2)\*(2\*atan(3\*sqrt(2)\*x/4) + 2\*atan(9\*sqrt(2)\*x\*\*3/4 + 5\*sqrt(2)\*x/4))/8

$$3.81 \quad \int \frac{1+3x^2}{-1-2x^2-9x^4} dx$$

Optimal. Leaf size=43

$$\frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 3\*x^2)/(-1 - 2\*x^2 - 9\*x^4),x]

[Out] ArcTan[(1 - 3\*x)/Sqrt[2]]/(2\*Sqrt[2]) - ArcTan[(1 + 3\*x)/Sqrt[2]]/(2\*Sqrt[2])

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[Rt[-b, 2]\*x]/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 1161

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

#### Rubi steps

$$\begin{aligned} \int \frac{1+3x^2}{-1-2x^2-9x^4} dx &= -\left(\frac{1}{6} \int \frac{1}{\frac{1}{3} - \frac{2x}{3} + x^2} dx\right) - \frac{1}{6} \int \frac{1}{\frac{1}{3} + \frac{2x}{3} + x^2} dx \\ &= \frac{1}{3} \text{Subst}\left(\int \frac{1}{-\frac{8}{9} - x^2} dx, x, -\frac{2}{3} + 2x\right) + \frac{1}{3} \text{Subst}\left(\int \frac{1}{-\frac{8}{9} - x^2} dx, x, \frac{2}{3} + 2x\right) \\ &= \frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(\frac{1+3x}{\sqrt{2}}\right)}{2\sqrt{2}} \end{aligned}$$

**Mathematica [C]** time = 0.03, size = 99, normalized size = 2.30

$$\frac{(\sqrt{2} - i) \tan^{-1}\left(\frac{3x}{\sqrt{1-2i\sqrt{2}}}\right)}{2\sqrt{2}(1-2i\sqrt{2})} - \frac{(\sqrt{2} + i) \tan^{-1}\left(\frac{3x}{\sqrt{1+2i\sqrt{2}}}\right)}{2\sqrt{2}(1+2i\sqrt{2})}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 3\*x^2)/(-1 - 2\*x^2 - 9\*x^4), x]

[Out] -1/2\*((-I + Sqrt[2])\*ArcTan[(3\*x)/Sqrt[1 - (2\*I)\*Sqrt[2]]])/Sqrt[2\*(1 - (2\*I)\*Sqrt[2])] - ((I + Sqrt[2])\*ArcTan[(3\*x)/Sqrt[1 + (2\*I)\*Sqrt[2]]])/(2\*Sqrt[2\*(1 + (2\*I)\*Sqrt[2])])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + 3x^2}{-1 - 2x^2 - 9x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 3\*x^2)/(-1 - 2\*x^2 - 9\*x^4), x]

[Out] IntegrateAlgebraic[(1 + 3\*x^2)/(-1 - 2\*x^2 - 9\*x^4), x]

**fricas [A]** time = 0.92, size = 33, normalized size = 0.77

$$-\frac{1}{4}\sqrt{2} \arctan\left(\frac{1}{4}\sqrt{2}(9x^3 + 5x)\right) - \frac{1}{4}\sqrt{2} \arctan\left(\frac{3}{4}\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2+1)/(-9\*x^4-2\*x^2-1), x, algorithm="fricas")

[Out] -1/4\*sqrt(2)\*arctan(1/4\*sqrt(2)\*(9\*x^3 + 5\*x)) - 1/4\*sqrt(2)\*arctan(3/4\*sqrt(2)\*x)

**giac [A]** time = 0.18, size = 33, normalized size = 0.77

$$-\frac{1}{4}\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}(3x + 1)\right) - \frac{1}{4}\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}(3x - 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2+1)/(-9\*x^4-2\*x^2-1), x, algorithm="giac")

[Out] -1/4\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(3\*x + 1)) - 1/4\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(3\*x - 1))

**maple [A]** time = 0.00, size = 34, normalized size = 0.79

$$-\frac{\sqrt{2} \arctan\left(\frac{(6x-2)\sqrt{2}}{4}\right)}{4} - \frac{\sqrt{2} \arctan\left(\frac{(6x+2)\sqrt{2}}{4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x^2+1)/(-9\*x^4-2\*x^2-1), x)

[Out] -1/4\*2^(1/2)\*arctan(1/4\*(6\*x-2)\*2^(1/2))-1/4\*2^(1/2)\*arctan(1/4\*(6\*x+2)\*2^(1/2))

**maxima [A]** time = 2.49, size = 33, normalized size = 0.77

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+1)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2+1)/(-9\*x^4-2\*x^2-1),x, algorithm="maxima")

[Out] -1/4\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(3\*x + 1)) - 1/4\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(3\*x - 1))

**mupad [B]** time = 0.00, size = 29, normalized size = 0.67

$$\frac{\sqrt{2}\left(\operatorname{atan}\left(\frac{9\sqrt{2}x^3}{4}+\frac{5\sqrt{2}x}{4}\right)+\operatorname{atan}\left(\frac{3\sqrt{2}x}{4}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(3\*x^2 + 1)/(2\*x^2 + 9\*x^4 + 1),x)

[Out] -(2^(1/2)\*(atan((5\*2^(1/2)\*x)/4 + (9\*2^(1/2)\*x^3)/4) + atan((3\*2^(1/2)\*x)/4)))/4

**sympy [A]** time = 0.15, size = 46, normalized size = 1.07

$$\frac{\sqrt{2}\left(2\operatorname{atan}\left(\frac{3\sqrt{2}x}{4}\right)+2\operatorname{atan}\left(\frac{9\sqrt{2}x^3}{4}+\frac{5\sqrt{2}x}{4}\right)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x\*\*2+1)/(-9\*x\*\*4-2\*x\*\*2-1),x)

[Out] -sqrt(2)\*(2\*atan(3\*sqrt(2)\*x/4) + 2\*atan(9\*sqrt(2)\*x\*\*3/4 + 5\*sqrt(2)\*x/4))/8

$$3.82 \quad \int \frac{3+2x^2}{1-2x^2+x^4} dx$$

Optimal. Leaf size=21

$$\frac{5x}{2(1-x^2)} + \frac{1}{2} \tanh^{-1}(x)$$

**Rubi [A]** time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {28, 385, 207}

$$\frac{5x}{2(1-x^2)} + \frac{1}{2} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(3 + 2\*x^2)/(1 - 2\*x^2 + x^4), x]

[Out] (5\*x)/(2\*(1 - x^2)) + ArcTanh[x]/2

Rule 28

Int[(u\_.)\*((a\_.) + (c\_.)\*(x\_)^(n2\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] :> Dist[1/c^p, Int[u\*(b/2 + c\*x^n)^(2\*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2\*n] && EqQ[b^2 - 4\*a\*c, 0] && IntegerQ[p]

Rule 207

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> -Simp[(b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1)/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rubi steps

$$\begin{aligned} \int \frac{3+2x^2}{1-2x^2+x^4} dx &= \int \frac{3+2x^2}{(-1+x^2)^2} dx \\ &= \frac{5x}{2(1-x^2)} - \frac{1}{2} \int \frac{1}{-1+x^2} dx \\ &= \frac{5x}{2(1-x^2)} + \frac{1}{2} \tanh^{-1}(x) \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 27, normalized size = 1.29

$$\frac{1}{4} \left( -\frac{10x}{x^2-1} - \log(1-x) + \log(x+1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 2\*x^2)/(1 - 2\*x^2 + x^4), x]

[Out] ((-10\*x)/(-1 + x^2) - Log[1 - x] + Log[1 + x])/4

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3 + 2x^2}{1 - 2x^2 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 + 2\*x^2)/(1 - 2\*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(3 + 2\*x^2)/(1 - 2\*x^2 + x^4), x]

**fricas** [B] time = 1.03, size = 34, normalized size = 1.62

$$\frac{(x^2 - 1) \log(x + 1) - (x^2 - 1) \log(x - 1) - 10x}{4(x^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+3)/(x^4-2\*x^2+1), x, algorithm="fricas")

[Out] 1/4\*((x^2 - 1)\*log(x + 1) - (x^2 - 1)\*log(x - 1) - 10\*x)/(x^2 - 1)

**giac** [A] time = 0.17, size = 25, normalized size = 1.19

$$-\frac{5x}{2(x^2 - 1)} + \frac{1}{4} \log(|x + 1|) - \frac{1}{4} \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+3)/(x^4-2\*x^2+1), x, algorithm="giac")

[Out] -5/2\*x/(x^2 - 1) + 1/4\*log(abs(x + 1)) - 1/4\*log(abs(x - 1))

**maple** [A] time = 0.01, size = 28, normalized size = 1.33

$$\frac{\ln(x + 1)}{4} - \frac{\ln(x - 1)}{4} - \frac{5}{4(x + 1)} - \frac{5}{4(x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2+3)/(x^4-2\*x^2+1), x)

[Out] -5/4/(x+1)+1/4\*ln(x+1)-5/4/(x-1)-1/4\*ln(x-1)

**maxima** [A] time = 1.10, size = 23, normalized size = 1.10

$$-\frac{5x}{2(x^2 - 1)} + \frac{1}{4} \log(x + 1) - \frac{1}{4} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2+3)/(x^4-2\*x^2+1), x, algorithm="maxima")

[Out] -5/2\*x/(x^2 - 1) + 1/4\*log(x + 1) - 1/4\*log(x - 1)

**mupad** [B] time = 0.03, size = 17, normalized size = 0.81

$$\frac{\operatorname{atanh}(x)}{2} - \frac{5x}{2(x^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 + 3)/(x^4 - 2*x^2 + 1), x)`

[Out] `atanh(x)/2 - (5*x)/(2*(x^2 - 1))`

**sympy** [A] time = 0.13, size = 22, normalized size = 1.05

$$-\frac{5x}{2x^2 - 2} - \frac{\log(x - 1)}{4} + \frac{\log(x + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2+3)/(x**4-2*x**2+1), x)`

[Out] `-5*x/(2*x**2 - 2) - log(x - 1)/4 + log(x + 1)/4`

$$3.83 \quad \int \frac{2+3x^2}{5-8x^2+3x^4} dx$$

Optimal. Leaf size=28

$$\frac{5}{2} \tanh^{-1}(x) - \frac{7}{2} \sqrt{\frac{3}{5}} \tanh^{-1}\left(\sqrt{\frac{3}{5}} x\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {1166, 207}

$$\frac{5}{2} \tanh^{-1}(x) - \frac{7}{2} \sqrt{\frac{3}{5}} \tanh^{-1}\left(\sqrt{\frac{3}{5}} x\right)$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x^2)/(5 - 8\*x^2 + 3\*x^4), x]

[Out] (5\*ArcTanh[x])/2 - (7\*Sqrt[3/5]\*ArcTanh[Sqrt[3/5]\*x])/2

Rule 207

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[e/2 + (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 - q/2 + c\*x^2), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - a\*e^2, 0] && PosQ[b^2 - 4\*a\*c]

Rubi steps

$$\begin{aligned} \int \frac{2+3x^2}{5-8x^2+3x^4} dx &= -\left(\frac{15}{2} \int \frac{1}{-3+3x^2} dx\right) + \frac{21}{2} \int \frac{1}{-5+3x^2} dx \\ &= \frac{5}{2} \tanh^{-1}(x) - \frac{7}{2} \sqrt{\frac{3}{5}} \tanh^{-1}\left(\sqrt{\frac{3}{5}} x\right) \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 53, normalized size = 1.89

$$\frac{1}{20} (7\sqrt{15} \log(\sqrt{15} - 3x) - 25 \log(1 - x) + 25 \log(x + 1) - 7\sqrt{15} \log(3x + \sqrt{15}))$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x^2)/(5 - 8\*x^2 + 3\*x^4), x]

[Out] (7\*Sqrt[15]\*Log[Sqrt[15] - 3\*x] - 25\*Log[1 - x] + 25\*Log[1 + x] - 7\*Sqrt[15]\*Log[Sqrt[15] + 3\*x])/20

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2+3x^2}{5-8x^2+3x^4} dx$$



Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + 3\*x^2)/(5 - 8\*x^2 + 3\*x^4), x]

[Out] IntegrateAlgebraic[(2 + 3\*x^2)/(5 - 8\*x^2 + 3\*x^4), x]

**fricas** [B] time = 1.01, size = 49, normalized size = 1.75

$$\frac{7}{20} \sqrt{5} \sqrt{3} \log\left(-\frac{2\sqrt{5}\sqrt{3}x - 3x^2 - 5}{3x^2 - 5}\right) + \frac{5}{4} \log(x + 1) - \frac{5}{4} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2+2)/(3\*x^4-8\*x^2+5), x, algorithm="fricas")

[Out] 7/20\*sqrt(5)\*sqrt(3)\*log(-(2\*sqrt(5)\*sqrt(3)\*x - 3\*x^2 - 5)/(3\*x^2 - 5)) + 5/4\*log(x + 1) - 5/4\*log(x - 1)

**giac** [B] time = 0.17, size = 44, normalized size = 1.57

$$\frac{7}{20} \sqrt{15} \log\left(\frac{|6x - 2\sqrt{15}|}{|6x + 2\sqrt{15}|}\right) + \frac{5}{4} \log(|x + 1|) - \frac{5}{4} \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2+2)/(3\*x^4-8\*x^2+5), x, algorithm="giac")

[Out] 7/20\*sqrt(15)\*log(abs(6\*x - 2\*sqrt(15))/abs(6\*x + 2\*sqrt(15))) + 5/4\*log(abs(x + 1)) - 5/4\*log(abs(x - 1))

**maple** [A] time = 0.01, size = 26, normalized size = 0.93

$$-\frac{7\sqrt{15} \operatorname{arctanh}\left(\frac{\sqrt{15}x}{5}\right)}{10} + \frac{5 \ln(x + 1)}{4} - \frac{5 \ln(x - 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x^2+2)/(3\*x^4-8\*x^2+5), x)

[Out] -7/10\*arctanh(1/5\*x\*15^(1/2))\*15^(1/2)+5/4\*ln(x+1)-5/4\*ln(x-1)

**maxima** [B] time = 2.36, size = 38, normalized size = 1.36

$$\frac{7}{20} \sqrt{15} \log\left(\frac{3x - \sqrt{15}}{3x + \sqrt{15}}\right) + \frac{5}{4} \log(x + 1) - \frac{5}{4} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2+2)/(3\*x^4-8\*x^2+5), x, algorithm="maxima")

[Out] 7/20\*sqrt(15)\*log((3\*x - sqrt(15))/(3\*x + sqrt(15))) + 5/4\*log(x + 1) - 5/4\*log(x - 1)

**mupad** [B] time = 4.39, size = 17, normalized size = 0.61

$$\frac{5 \operatorname{atanh}(x)}{2} - \frac{7\sqrt{15} \operatorname{atanh}\left(\frac{\sqrt{15}x}{5}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x^2 + 2)/(3\*x^4 - 8\*x^2 + 5), x)

[Out]  $(5*\operatorname{atanh}(x))/2 - (7*15^{(1/2)}*\operatorname{atanh}((15^{(1/2)}*x)/5))/10$

**sympy [B]** time = 0.61, size = 53, normalized size = 1.89

$$-\frac{5 \log(x-1)}{4} + \frac{5 \log(x+1)}{4} + \frac{7\sqrt{15} \log\left(x - \frac{\sqrt{15}}{3}\right)}{20} - \frac{7\sqrt{15} \log\left(x + \frac{\sqrt{15}}{3}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x**2+2)/(3*x**4-8*x**2+5),x)`

[Out]  $-5*\log(x - 1)/4 + 5*\log(x + 1)/4 + 7*\sqrt{15}*\log(x - \sqrt{15}/3)/20 - 7*\sqrt{15}*\log(x + \sqrt{15}/3)/20$

$$3.84 \quad \int \frac{d+ex^2}{5-8x^2+3x^4} dx$$

Optimal. Leaf size=36

$$\frac{1}{2}(d+e) \tanh^{-1}(x) - \frac{(3d+5e) \tanh^{-1}\left(\sqrt{\frac{3}{5}}x\right)}{2\sqrt{15}}$$

**Rubi [A]** time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {1166, 207}

$$\frac{1}{2}(d+e) \tanh^{-1}(x) - \frac{(3d+5e) \tanh^{-1}\left(\sqrt{\frac{3}{5}}x\right)}{2\sqrt{15}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)/(5 - 8\*x^2 + 3\*x^4), x]

[Out] ((d + e)\*ArcTanh[x])/2 - ((3\*d + 5\*e)\*ArcTanh[Sqrt[3/5]\*x])/(2\*Sqrt[15])

Rule 207

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[e/2 + (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 - q/2 + c\*x^2), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - a\*e^2, 0] && PosQ[b^2 - 4\*a\*c]

Rubi steps

$$\begin{aligned} \int \frac{d+ex^2}{5-8x^2+3x^4} dx &= -\left(\frac{1}{2}(3(d+e)) \int \frac{1}{-3+3x^2} dx\right) + \frac{1}{2}(3d+5e) \int \frac{1}{-5+3x^2} dx \\ &= \frac{1}{2}(d+e) \tanh^{-1}(x) - \frac{(3d+5e) \tanh^{-1}\left(\sqrt{\frac{3}{5}}x\right)}{2\sqrt{15}} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 72, normalized size = 2.00

$$\frac{1}{60} \left( \sqrt{15} (3d+5e) \log(\sqrt{15}-3x) - 15(d+e) \log(1-x) + 15(d+e) \log(x+1) - \sqrt{15} (3d+5e) \log(3x+\sqrt{15}) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)/(5 - 8\*x^2 + 3\*x^4), x]

[Out] (Sqrt[15]\*(3\*d + 5\*e)\*Log[Sqrt[15] - 3\*x] - 15\*(d + e)\*Log[1 - x] + 15\*(d + e)\*Log[1 + x] - Sqrt[15]\*(3\*d + 5\*e)\*Log[Sqrt[15] + 3\*x])/60

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^2}{5-8x^2+3x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)/(5 - 8\*x^2 + 3\*x^4), x]

[Out] IntegrateAlgebraic[(d + e\*x^2)/(5 - 8\*x^2 + 3\*x^4), x]

**fricas** [B] time = 1.00, size = 55, normalized size = 1.53

$$\frac{1}{60} \sqrt{15} (3d + 5e) \log\left(\frac{3x^2 - 2\sqrt{15}x + 5}{3x^2 - 5}\right) + \frac{1}{4} (d + e) \log(x + 1) - \frac{1}{4} (d + e) \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(3\*x^4-8\*x^2+5),x, algorithm="fricas")

[Out] 1/60\*sqrt(15)\*(3\*d + 5\*e)\*log((3\*x^2 - 2\*sqrt(15)\*x + 5)/(3\*x^2 - 5)) + 1/4\*(d + e)\*log(x + 1) - 1/4\*(d + e)\*log(x - 1)

**giac** [B] time = 0.16, size = 60, normalized size = 1.67

$$\frac{1}{60} \sqrt{15} (3d + 5e) \log\left(\frac{|6x - 2\sqrt{15}|}{|6x + 2\sqrt{15}|}\right) + \frac{1}{4} (d + e) \log(|x + 1|) - \frac{1}{4} (d + e) \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(3\*x^4-8\*x^2+5),x, algorithm="giac")

[Out] 1/60\*sqrt(15)\*(3\*d + 5\*e)\*log(abs(6\*x - 2\*sqrt(15))/abs(6\*x + 2\*sqrt(15))) + 1/4\*(d + e)\*log(abs(x + 1)) - 1/4\*(d + e)\*log(abs(x - 1))

**maple** [B] time = 0.01, size = 56, normalized size = 1.56

$$-\frac{\sqrt{15} d \operatorname{arctanh}\left(\frac{\sqrt{15} x}{5}\right)}{10} + \frac{d \ln(x + 1)}{4} - \frac{d \ln(x - 1)}{4} - \frac{\sqrt{15} e \operatorname{arctanh}\left(\frac{\sqrt{15} x}{5}\right)}{6} + \frac{e \ln(x + 1)}{4} - \frac{e \ln(x - 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)/(3\*x^4-8\*x^2+5), x)

[Out] -1/10\*15^(1/2)\*arctanh(1/5\*15^(1/2)\*x)\*d-1/6\*15^(1/2)\*arctanh(1/5\*15^(1/2)\*x)\*e+1/4\*ln(x+1)\*d+1/4\*ln(x+1)\*e-1/4\*ln(x-1)\*d-1/4\*ln(x-1)\*e

**maxima** [A] time = 2.41, size = 51, normalized size = 1.42

$$\frac{1}{60} \sqrt{15} (3d + 5e) \log\left(\frac{3x - \sqrt{15}}{3x + \sqrt{15}}\right) + \frac{1}{4} (d + e) \log(x + 1) - \frac{1}{4} (d + e) \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(3\*x^4-8\*x^2+5),x, algorithm="maxima")

[Out] 1/60\*sqrt(15)\*(3\*d + 5\*e)\*log((3\*x - sqrt(15))/(3\*x + sqrt(15))) + 1/4\*(d + e)\*log(x + 1) - 1/4\*(d + e)\*log(x - 1)

**mupad** [B] time = 4.39, size = 290, normalized size = 8.06

$$\frac{\sqrt{15} \operatorname{atanh}\left(\frac{54\sqrt{15}dx}{25\left(\frac{9d^2-18d^2e+18d^2e+30e^2}{30}\right)} - \frac{6\sqrt{15}d^2x}{-3d^2-18d^2e+18d^2e+30e^2} - \frac{18\sqrt{15}d^2x}{5\left(\frac{9d^2-18d^2e+18d^2e+30e^2}{30}\right)} + \frac{18\sqrt{15}d^2x}{5\left(\frac{9d^2-18d^2e+18d^2e+30e^2}{30}\right)}\right)(3d+5e)}{-\operatorname{atanh}\left(\frac{18d^2x}{-18d^3-18d^2e+30d^2e+30e^3} - \frac{30e^2x}{-18d^3-18d^2e+30d^2e+30e^3} - \frac{30d^2x}{-18d^3-18d^2e+30d^2e+30e^3} + \frac{18d^2ex}{-18d^3-18d^2e+30d^2e+30e^3}\right)\left(\frac{d}{2} + \frac{e}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)/(3\*x^4 - 8\*x^2 + 5), x)

```
[Out] (15^(1/2)*atanh((54*15^(1/2)*d^3*x)/(25*(18*d*e^2 - 18*d^2*e - (54*d^3)/5 + 30*e^3)) - (6*15^(1/2)*e^3*x)/(18*d*e^2 - 18*d^2*e - (54*d^3)/5 + 30*e^3) - (18*15^(1/2)*d*e^2*x)/(5*(18*d*e^2 - 18*d^2*e - (54*d^3)/5 + 30*e^3)) + (18*15^(1/2)*d^2*e*x)/(5*(18*d*e^2 - 18*d^2*e - (54*d^3)/5 + 30*e^3)))*(3*d + 5*e))/30 - atanh((18*d^3*x)/(30*d*e^2 - 18*d^2*e - 18*d^3 + 30*e^3) - (30*e^3*x)/(30*d*e^2 - 18*d^2*e - 18*d^3 + 30*e^3) - (30*d*e^2*x)/(30*d*e^2 - 18*d^2*e - 18*d^3 + 30*e^3) + (18*d^2*e*x)/(30*d*e^2 - 18*d^2*e - 18*d^3 + 30*e^3))*(d/2 + e/2)
```

**sympy [B]** time = 1.50, size = 474, normalized size = 13.17

$$\frac{(d+e)\log\left(x + \frac{54\sqrt{15}d^3 + 180d^2e + 225d^2e^2 + 30d^2e^3 - 100d^2e^4 + 75d^2e^5}{9d^4 + 24d^3e - 40d^2e^3 - 25e^4}\right)}{4} - \frac{(d+e)\log\left(x + \frac{54\sqrt{15}d^3 + 180d^2e + 225d^2e^2 + 30d^2e^3 - 100d^2e^4 + 75d^2e^5}{9d^4 + 24d^3e - 40d^2e^3 - 25e^4}\right)}{4} + \frac{\sqrt{15}(3d+5e)\log\left(x + \frac{12\sqrt{15}d^3 + 12\sqrt{15}d^2e + 12\sqrt{15}d^2e^2 + 12\sqrt{15}d^2e^3 - 12\sqrt{15}d^2e^4 + 12\sqrt{15}d^2e^5}{9d^4 + 24d^3e - 40d^2e^3 - 25e^4}\right)}{60} - \frac{\sqrt{15}(3d+5e)\log\left(x + \frac{12\sqrt{15}d^3 + 12\sqrt{15}d^2e + 12\sqrt{15}d^2e^2 + 12\sqrt{15}d^2e^3 - 12\sqrt{15}d^2e^4 + 12\sqrt{15}d^2e^5}{9d^4 + 24d^3e - 40d^2e^3 - 25e^4}\right)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(3*x**4-8*x**2+5), x)
```

```
[Out] (d + e)*log(x + (-51*d**3*(d + e) - 180*d**2*e*(d + e) - 225*d*e**2*(d + e) + 60*d*(d + e)**3 - 100*e**3*(d + e) + 75*e*(d + e)**3)/(9*d**4 + 24*d**3*e - 40*d*e**3 - 25*e**4))/4 - (d + e)*log(x + (51*d**3*(d + e) + 180*d**2*e*(d + e) + 225*d*e**2*(d + e) - 60*d*(d + e)**3 + 100*e**3*(d + e) - 75*e*(d + e)**3)/(9*d**4 + 24*d**3*e - 40*d*e**3 - 25*e**4))/4 + sqrt(15)*(3*d + 5*e)*log(x + (-17*sqrt(15)*d**3*(3*d + 5*e)/5 - 12*sqrt(15)*d**2*e*(3*d + 5*e) - 15*sqrt(15)*d*e**2*(3*d + 5*e) + 4*sqrt(15)*d*(3*d + 5*e)**3/15 - 20*sqrt(15)*e**3*(3*d + 5*e)/3 + sqrt(15)*e*(3*d + 5*e)**3/3)/(9*d**4 + 24*d**3*e - 40*d*e**3 - 25*e**4))/60 - sqrt(15)*(3*d + 5*e)*log(x + (17*sqrt(15)*d**3*(3*d + 5*e)/5 + 12*sqrt(15)*d**2*e*(3*d + 5*e) + 15*sqrt(15)*d*e**2*(3*d + 5*e) - 4*sqrt(15)*d*(3*d + 5*e)**3/15 + 20*sqrt(15)*e**3*(3*d + 5*e)/3 - sqrt(15)*e*(3*d + 5*e)**3/3)/(9*d**4 + 24*d**3*e - 40*d*e**3 - 25*e**4))/60
```

$$3.85 \quad \int \frac{3+x^2}{1+3x^2+x^4} dx$$

**Optimal.** Leaf size=74

$$\frac{(3 + \sqrt{5})^{3/2} \tan^{-1}\left(\sqrt{\frac{1}{2}(3 + \sqrt{5})}x\right)}{2\sqrt{10}} - \frac{1}{10}\sqrt{180 - 80\sqrt{5}} \tan^{-1}\left(\sqrt{\frac{2}{3 + \sqrt{5}}}x\right)$$

**Rubi [A]** time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {1166, 203}

$$\frac{(3 + \sqrt{5})^{3/2} \tan^{-1}\left(\sqrt{\frac{1}{2}(3 + \sqrt{5})}x\right)}{2\sqrt{10}} - \frac{1}{10}\sqrt{180 - 80\sqrt{5}} \tan^{-1}\left(\sqrt{\frac{2}{3 + \sqrt{5}}}x\right)$$

Antiderivative was successfully verified.

[In] Int[(3 + x^2)/(1 + 3\*x^2 + x^4), x]

[Out] -(Sqrt[180 - 80\*Sqrt[5]]\*ArcTan[Sqrt[2/(3 + Sqrt[5]])\*x])/10 + ((3 + Sqrt[5])^(3/2)\*ArcTan[Sqrt[(3 + Sqrt[5])/2]\*x])/(2\*Sqrt[10])

**Rule 203**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

**Rule 1166**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[e/2 + (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 - q/2 + c\*x^2), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - a\*e^2, 0] && PosQ[b^2 - 4\*a\*c]

**Rubi steps**

$$\begin{aligned} \int \frac{3+x^2}{1+3x^2+x^4} dx &= \frac{1}{10}(5-3\sqrt{5}) \int \frac{1}{\frac{3}{2} + \frac{\sqrt{5}}{2} + x^2} dx + \frac{1}{10}(5+3\sqrt{5}) \int \frac{1}{\frac{3}{2} - \frac{\sqrt{5}}{2} + x^2} dx \\ &= -\frac{1}{5}\sqrt{45-20\sqrt{5}} \tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right) + \frac{(3+\sqrt{5})^{3/2} \tan^{-1}\left(\sqrt{\frac{1}{2}(3+\sqrt{5})}x\right)}{2\sqrt{10}} \end{aligned}$$

**Mathematica [A]** time = 0.10, size = 73, normalized size = 0.99

$$\frac{(3 + \sqrt{5})^{3/2} \tan^{-1}\left(\sqrt{\frac{1}{2}(3 + \sqrt{5})}x\right) - (3 - \sqrt{5})^{3/2} \tan^{-1}\left(\sqrt{\frac{2}{3 + \sqrt{5}}}x\right)}{2\sqrt{10}}$$

Antiderivative was successfully verified.

[In] Integrate[(3 + x^2)/(1 + 3\*x^2 + x^4), x]

[Out]  $(-(3 - \sqrt{5})^{3/2} \operatorname{ArcTan}[\sqrt{2/(3 + \sqrt{5})}] * x) + (3 + \sqrt{5})^{3/2} \operatorname{ArcTan}[\sqrt{(3 + \sqrt{5})/2}] * x) / (2 * \sqrt{10})$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3 + x^2}{1 + 3x^2 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 + x^2)/(1 + 3\*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(3 + x^2)/(1 + 3\*x^2 + x^4), x]

**fricas** [B] time = 1.05, size = 137, normalized size = 1.85

$\frac{2}{5} \sqrt{5} \sqrt{-4\sqrt{5} + 9} \arctan\left(\frac{1}{4} \sqrt{2x^2 + \sqrt{5} + 3} (\sqrt{5}\sqrt{2} + 3\sqrt{2}) \sqrt{-4\sqrt{5} + 9} - \frac{1}{2} (\sqrt{5}x + 3x) \sqrt{-4\sqrt{5} + 9}\right) + \frac{2}{5} \sqrt{5} \sqrt{4\sqrt{5} + 9} \arctan\left(\frac{1}{4} (\sqrt{2x^2 - \sqrt{5} + 3} (\sqrt{5}\sqrt{2} - 3\sqrt{2}) - 2\sqrt{5}x + 6x) \sqrt{4\sqrt{5} + 9}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+3)/(x^4+3\*x^2+1), x, algorithm="fricas")

[Out]  $\frac{2}{5} \sqrt{5} \sqrt{-4\sqrt{5} + 9} \arctan(1/4 \sqrt{2x^2 + \sqrt{5} + 3} (\sqrt{5}\sqrt{2} + 3\sqrt{2}) \sqrt{-4\sqrt{5} + 9} - 1/2 (\sqrt{5}x + 3x) \sqrt{-4\sqrt{5} + 9}) + \frac{2}{5} \sqrt{5} \sqrt{4\sqrt{5} + 9} \arctan(1/4 (\sqrt{2x^2 - \sqrt{5} + 3} (\sqrt{5}\sqrt{2} - 3\sqrt{2}) - 2\sqrt{5}x + 6x) \sqrt{4\sqrt{5} + 9})$

**giac** [A] time = 0.16, size = 41, normalized size = 0.55

$$\frac{1}{5} (2\sqrt{5} - 5) \arctan\left(\frac{2x}{\sqrt{5} + 1}\right) + \frac{1}{5} (2\sqrt{5} + 5) \arctan\left(\frac{2x}{\sqrt{5} - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+3)/(x^4+3\*x^2+1), x, algorithm="giac")

[Out]  $1/5 (2\sqrt{5} - 5) \arctan(2x/(\sqrt{5} + 1)) + 1/5 (2\sqrt{5} + 5) \arctan(2x/(\sqrt{5} - 1))$

**maple** [B] time = 0.02, size = 104, normalized size = 1.41

$$\frac{2 \arctan\left(\frac{4x}{2\sqrt{5}-2}\right)}{2\sqrt{5}-2} + \frac{6\sqrt{5} \arctan\left(\frac{4x}{2\sqrt{5}-2}\right)}{5(2\sqrt{5}-2)} + \frac{2 \arctan\left(\frac{4x}{2\sqrt{5}+2}\right)}{2\sqrt{5}+2} - \frac{6\sqrt{5} \arctan\left(\frac{4x}{2\sqrt{5}+2}\right)}{5(2\sqrt{5}+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+3)/(x^4+3\*x^2+1), x)

[Out]  $2/(2*5^{(1/2)+2}) \arctan(4/(2*5^{(1/2)+2}) * x) - 6/5 * 5^{(1/2)} / (2*5^{(1/2)+2}) \arctan(4/(2*5^{(1/2)+2}) * x) + 2/(2*5^{(1/2)-2}) \arctan(4/(2*5^{(1/2)-2}) * x) + 6/5 * 5^{(1/2)} / (2*5^{(1/2)-2}) \arctan(4/(2*5^{(1/2)-2}) * x)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + 3}{x^4 + 3x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+3)/(x^4+3\*x^2+1), x, algorithm="maxima")

[Out] integrate((x^2 + 3)/(x^4 + 3\*x^2 + 1), x)

**mupad [B]** time = 0.11, size = 117, normalized size = 1.58

$$2 \operatorname{atanh} \left( \frac{80x \sqrt{\frac{\sqrt{5}}{5} - \frac{9}{20}} - \frac{48\sqrt{5}x \sqrt{\frac{\sqrt{5}}{5} - \frac{9}{20}}}{24\sqrt{5} - 56}} \right) \sqrt{\frac{\sqrt{5}}{5} - \frac{9}{20}} - 2 \operatorname{atanh} \left( \frac{80x \sqrt{-\frac{\sqrt{5}}{5} - \frac{9}{20}} + \frac{48\sqrt{5}x \sqrt{-\frac{\sqrt{5}}{5} - \frac{9}{20}}}{24\sqrt{5} + 56}} \right) \sqrt{-\frac{\sqrt{5}}{5} - \frac{9}{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + 3)/(3*x^2 + x^4 + 1), x)`

[Out] `2*atanh((80*x*(5^(1/2)/5 - 9/20)^(1/2))/(24*5^(1/2) - 56) - (48*5^(1/2)*x*(5^(1/2)/5 - 9/20)^(1/2))/(24*5^(1/2) - 56))*(5^(1/2)/5 - 9/20)^(1/2) - 2*atanh((80*x*(-5^(1/2)/5 - 9/20)^(1/2))/(24*5^(1/2) + 56) + (48*5^(1/2)*x*(-5^(1/2)/5 - 9/20)^(1/2))/(24*5^(1/2) + 56))*(-5^(1/2)/5 - 9/20)^(1/2)`

**sympy [A]** time = 0.21, size = 46, normalized size = 0.62

$$2 \left( \frac{\sqrt{5}}{5} + \frac{1}{2} \right) \operatorname{atan} \left( \frac{2x}{-1 + \sqrt{5}} \right) - 2 \left( \frac{1}{2} - \frac{\sqrt{5}}{5} \right) \operatorname{atan} \left( \frac{2x}{1 + \sqrt{5}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+3)/(x**4+3*x**2+1), x)`

[Out] `2*(sqrt(5)/5 + 1/2)*atan(2*x/(-1 + sqrt(5))) - 2*(1/2 - sqrt(5)/5)*atan(2*x/(1 + sqrt(5)))`



$$3.86 \quad \int \frac{a+bx^2}{1+x^2+x^4} dx$$

**Optimal.** Leaf size=83

$$-\frac{1}{4}(a-b) \log(x^2 - x + 1) + \frac{1}{4}(a-b) \log(x^2 + x + 1) - \frac{(a+b) \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{(a+b) \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

**Rubi [A]** time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {1169, 634, 618, 204, 628}

$$-\frac{1}{4}(a-b) \log(x^2 - x + 1) + \frac{1}{4}(a-b) \log(x^2 + x + 1) - \frac{(a+b) \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{(a+b) \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(1 + x^2 + x^4), x]

[Out] -((a + b)\*ArcTan[(1 - 2\*x)/Sqrt[3]])/(2\*Sqrt[3]) + ((a + b)\*ArcTan[(1 + 2\*x)/Sqrt[3]])/(2\*Sqrt[3]) - ((a - b)\*Log[1 - x + x^2])/4 + ((a - b)\*Log[1 + x + x^2])/4

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 1169

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2\*q - b/c, 2]}, Dist[1/(2\*c\*q\*r), Int[(d\*r - (d - e\*q)\*x)/(q - r\*x + x^2), x], x] + Dist[1/(2\*c\*q\*r), Int[(d\*r + (d - e\*q)\*x)/(q + r\*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NegQ[b^2 - 4\*a\*c]

#### Rubi steps

$$\begin{aligned}
\int \frac{a + bx^2}{1 + x^2 + x^4} dx &= \frac{1}{2} \int \frac{a - (a - b)x}{1 - x + x^2} dx + \frac{1}{2} \int \frac{a + (a - b)x}{1 + x + x^2} dx \\
&= \frac{1}{4}(a - b) \int \frac{1 + 2x}{1 + x + x^2} dx + \frac{1}{4}(-a + b) \int \frac{-1 + 2x}{1 - x + x^2} dx + \frac{1}{4}(a + b) \int \frac{1}{1 - x + x^2} dx + \frac{1}{4}(a + b) \int \frac{1}{1 + x + x^2} dx \\
&= -\frac{1}{4}(a - b) \log(1 - x + x^2) + \frac{1}{4}(a - b) \log(1 + x + x^2) + \frac{1}{2}(-a - b) \text{Subst} \left( \int \frac{1}{-3 - x^2} dx, x, \frac{1 - 2x}{\sqrt{3}} \right) \\
&= -\frac{(a + b) \tan^{-1} \left( \frac{1 - 2x}{\sqrt{3}} \right)}{2\sqrt{3}} + \frac{(a + b) \tan^{-1} \left( \frac{1 + 2x}{\sqrt{3}} \right)}{2\sqrt{3}} - \frac{1}{4}(a - b) \log(1 - x + x^2) + \frac{1}{4}(a - b) \log(1 + x + x^2)
\end{aligned}$$

**Mathematica [C]** time = 0.13, size = 97, normalized size = 1.17

$$\frac{(2ia + (\sqrt{3} - i)b) \tan^{-1} \left( \frac{1}{2}(\sqrt{3} - i)x \right)}{\sqrt{6 + 6i\sqrt{3}}} + \frac{((\sqrt{3} + i)b - 2ia) \tan^{-1} \left( \frac{1}{2}(\sqrt{3} + i)x \right)}{\sqrt{6 - 6i\sqrt{3}}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^2)/(1 + x^2 + x^4), x]

[Out] (((2\*I)\*a + (-I + Sqrt[3])\*b)\*ArcTan[(-I + Sqrt[3])\*x/2])/Sqrt[6 + (6\*I)\*Sqrt[3]] + (((-2\*I)\*a + (I + Sqrt[3])\*b)\*ArcTan[(I + Sqrt[3])\*x/2])/Sqrt[6 - (6\*I)\*Sqrt[3]]

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2}{1 + x^2 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b\*x^2)/(1 + x^2 + x^4), x]

[Out] IntegrateAlgebraic[(a + b\*x^2)/(1 + x^2 + x^4), x]

**fricas [A]** time = 1.04, size = 69, normalized size = 0.83

$$\frac{1}{6}\sqrt{3}(a + b) \arctan\left(\frac{1}{3}\sqrt{3}(2x + 1)\right) + \frac{1}{6}\sqrt{3}(a + b) \arctan\left(\frac{1}{3}\sqrt{3}(2x - 1)\right) + \frac{1}{4}(a - b) \log(x^2 + x + 1) - \frac{1}{4}(a - b) \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/(x^4+x^2+1), x, algorithm="fricas")

[Out] 1/6\*sqrt(3)\*(a + b)\*arctan(1/3\*sqrt(3)\*(2\*x + 1)) + 1/6\*sqrt(3)\*(a + b)\*arctan(1/3\*sqrt(3)\*(2\*x - 1)) + 1/4\*(a - b)\*log(x^2 + x + 1) - 1/4\*(a - b)\*log(x^2 - x + 1)

**giac [A]** time = 0.15, size = 69, normalized size = 0.83

$$\frac{1}{6}\sqrt{3}(a + b) \arctan\left(\frac{1}{3}\sqrt{3}(2x + 1)\right) + \frac{1}{6}\sqrt{3}(a + b) \arctan\left(\frac{1}{3}\sqrt{3}(2x - 1)\right) + \frac{1}{4}(a - b) \log(x^2 + x + 1) - \frac{1}{4}(a - b) \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/(x^4+x^2+1), x, algorithm="giac")

[Out] 1/6\*sqrt(3)\*(a + b)\*arctan(1/3\*sqrt(3)\*(2\*x + 1)) + 1/6\*sqrt(3)\*(a + b)\*arctan(1/3\*sqrt(3)\*(2\*x - 1)) + 1/4\*(a - b)\*log(x^2 + x + 1) - 1/4\*(a - b)\*log(x^2 - x + 1)

**maple [A]** time = 0.00, size = 114, normalized size = 1.37

$$\frac{\sqrt{3} a \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6} + \frac{\sqrt{3} a \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} - \frac{a \ln(x^2 - x + 1)}{4} + \frac{a \ln(x^2 + x + 1)}{4} + \frac{\sqrt{3} b \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6} + \frac{\sqrt{3} b \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} + \frac{b \ln(x^2 - x + 1)}{4} - \frac{b \ln(x^2 + x + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)/(x^4+x^2+1), x)

[Out] 1/4\*ln(x^2+x+1)\*a-1/4\*ln(x^2+x+1)\*b+1/6\*3^(1/2)\*arctan(1/3\*(2\*x+1)\*3^(1/2))\*a+1/6\*3^(1/2)\*arctan(1/3\*(2\*x+1)\*3^(1/2))\*b-1/4\*ln(x^2-x+1)\*a+1/4\*ln(x^2-x+1)\*b+1/6\*3^(1/2)\*arctan(1/3\*(2\*x-1)\*3^(1/2))\*a+1/6\*3^(1/2)\*arctan(1/3\*(2\*x-1)\*3^(1/2))\*b

**maxima [A]** time = 2.43, size = 69, normalized size = 0.83

$$\frac{1}{6} \sqrt{3} (a + b) \arctan\left(\frac{1}{3} \sqrt{3} (2x + 1)\right) + \frac{1}{6} \sqrt{3} (a + b) \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) + \frac{1}{4} (a - b) \log(x^2 + x + 1) - \frac{1}{4} (a - b) \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/(x^4+x^2+1), x, algorithm="maxima")

[Out] 1/6\*sqrt(3)\*(a + b)\*arctan(1/3\*sqrt(3)\*(2\*x + 1)) + 1/6\*sqrt(3)\*(a + b)\*arctan(1/3\*sqrt(3)\*(2\*x - 1)) + 1/4\*(a - b)\*log(x^2 + x + 1) - 1/4\*(a - b)\*log(x^2 - x + 1)

**mupad [B]** time = 4.50, size = 827, normalized size = 9.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^2)/(x^2 + x^4 + 1), x)

[Out] - atan(((x\*(4\*a\*b - 4\*a^2 + 2\*b^2) + (12\*a + 24\*x\*(b/4 - a/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*(b/4 - a/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*((b/4 - a/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12)\*1i + (x\*(4\*a\*b - 4\*a^2 + 2\*b^2) - (12\*a - 24\*x\*(b/4 - a/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*((b/4 - a/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*1i)/((x\*(4\*a\*b - 4\*a^2 + 2\*b^2) + (12\*a + 24\*x\*(b/4 - a/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*((b/4 - a/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*((b/4 - a/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12)) - (x\*(4\*a\*b - 4\*a^2 + 2\*b^2) - (12\*a - 24\*x\*(b/4 - a/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*((b/4 - a/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*1i + (x\*(4\*a\*b - 4\*a^2 + 2\*b^2) - (12\*a - 24\*x\*(a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*((a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*1i + (x\*(4\*a\*b - 4\*a^2 + 2\*b^2) - (12\*a - 24\*x\*(a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*((a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*1i)/((x\*(4\*a\*b - 4\*a^2 + 2\*b^2) + (12\*a + 24\*x\*(a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*((a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*((a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12)) - (x\*(4\*a\*b - 4\*a^2 + 2\*b^2) - (12\*a - 24\*x\*(a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*((a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*1i + (x\*(4\*a\*b - 4\*a^2 + 2\*b^2) - (12\*a - 24\*x\*(a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*((a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*1i)/((x\*(4\*a\*b - 4\*a^2 + 2\*b^2) + (12\*a + 24\*x\*(a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*((a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*((a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12)) - 2\*a\*b^2 + 2\*a^2\*b + 2\*b^3))\*((a\*1i)/2 - (b\*1i)/2 + (3^(1/2)\*a)/6 + (3^(1/2)\*b)/6) - atan(((x\*(4\*a\*b - 4\*a^2 + 2\*b^2) + (12\*a + 24\*x\*(a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*(a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*((a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*1i + (x\*(4\*a\*b - 4\*a^2 + 2\*b^2) - (12\*a - 24\*x\*(a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*((a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*1i + (x\*(4\*a\*b - 4\*a^2 + 2\*b^2) - (12\*a - 24\*x\*(a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*((a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*1i)/((x\*(4\*a\*b - 4\*a^2 + 2\*b^2) + (12\*a + 24\*x\*(a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*((a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*((a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12)) - (x\*(4\*a\*b - 4\*a^2 + 2\*b^2) - (12\*a - 24\*x\*(a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*((a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*1i + (x\*(4\*a\*b - 4\*a^2 + 2\*b^2) - (12\*a - 24\*x\*(a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*((a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*1i)/((x\*(4\*a\*b - 4\*a^2 + 2\*b^2) + (12\*a + 24\*x\*(a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*((a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12))\*((a/4 - b/4 + (3^(1/2)\*a\*1i)/12 + (3^(1/2)\*b\*1i)/12)) - 2\*a\*b^2 + 2\*a^2\*b + 2\*b^3))\*((b\*1i)/2 - (a\*1i)/2 + (3^(1/2)\*a)/6 + (3^(1/2)\*b)/6)

**sympy [C]** time = 1.26, size = 740, normalized size = 8.92

[[[...]]]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)/(x\*\*4+x\*\*2+1),x)

[Out]  $(-a/4 + b/4 - \sqrt{3}I(a + b)/12) \log(x + (2a^{**3}(-a/4 + b/4 - \sqrt{3}I(a + b)/12) + 6a^{**2}b(-a/4 + b/4 - \sqrt{3}I(a + b)/12) - 12ab^{**2}(-a/4 + b/4 - \sqrt{3}I(a + b)/12) + 24a(-a/4 + b/4 - \sqrt{3}I(a + b)/12)^{**3} + 2b^{**3}(-a/4 + b/4 - \sqrt{3}I(a + b)/12) - 48b(-a/4 + b/4 - \sqrt{3}I(a + b)/12)^{**3}) / (a^{**4} - a^{**3}b + ab^{**3} - b^{**4})) + (-a/4 + b/4 + \sqrt{3}I(a + b)/12) \log(x + (2a^{**3}(-a/4 + b/4 + \sqrt{3}I(a + b)/12) + 6a^{**2}b(-a/4 + b/4 + \sqrt{3}I(a + b)/12) - 12ab^{**2}(-a/4 + b/4 + \sqrt{3}I(a + b)/12) + 24a(-a/4 + b/4 + \sqrt{3}I(a + b)/12)^{**3} + 2b^{**3}(-a/4 + b/4 + \sqrt{3}I(a + b)/12) - 48b(-a/4 + b/4 + \sqrt{3}I(a + b)/12)^{**3}) / (a^{**4} - a^{**3}b + ab^{**3} - b^{**4})) + (a/4 - b/4 - \sqrt{3}I(a + b)/12) \log(x + (2a^{**3}(a/4 - b/4 - \sqrt{3}I(a + b)/12) + 6a^{**2}b(a/4 - b/4 - \sqrt{3}I(a + b)/12) - 12ab^{**2}(a/4 - b/4 - \sqrt{3}I(a + b)/12) + 24a(a/4 - b/4 - \sqrt{3}I(a + b)/12)^{**3} + 2b^{**3}(a/4 - b/4 - \sqrt{3}I(a + b)/12) - 48b(a/4 - b/4 - \sqrt{3}I(a + b)/12)^{**3}) / (a^{**4} - a^{**3}b + ab^{**3} - b^{**4})) + (a/4 - b/4 + \sqrt{3}I(a + b)/12) \log(x + (2a^{**3}(a/4 - b/4 + \sqrt{3}I(a + b)/12) + 6a^{**2}b(a/4 - b/4 + \sqrt{3}I(a + b)/12) - 12ab^{**2}(a/4 - b/4 + \sqrt{3}I(a + b)/12) + 24a(a/4 - b/4 + \sqrt{3}I(a + b)/12)^{**3} + 2b^{**3}(a/4 - b/4 + \sqrt{3}I(a + b)/12) - 48b(a/4 - b/4 + \sqrt{3}I(a + b)/12)^{**3}) / (a^{**4} - a^{**3}b + ab^{**3} - b^{**4}))$

$$3.87 \quad \int \frac{a+bx^2}{(1+x^2+x^4)^2} dx$$

**Optimal.** Leaf size=119

$$-\frac{1}{8}(2a-b)\log(x^2-x+1)+\frac{1}{8}(2a-b)\log(x^2+x+1)+\frac{x(-(x^2(a-2b))+a+b)}{6(x^4+x^2+1)}-\frac{(4a+b)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(4a+b)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{12\sqrt{3}}$$

**Rubi [A]** time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {1178, 1169, 634, 618, 204, 628}

$$\frac{x(x^2-(a-2b))+a+b}{6(x^4+x^2+1)}-\frac{1}{8}(2a-b)\log(x^2-x+1)+\frac{1}{8}(2a-b)\log(x^2+x+1)-\frac{(4a+b)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(4a+b)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{12\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(1 + x^2 + x^4)^2, x]

[Out] (x\*(a + b - (a - 2\*b)\*x^2))/(6\*(1 + x^2 + x^4)) - ((4\*a + b)\*ArcTan[(1 - 2\*x)/Sqrt[3]])/(12\*Sqrt[3]) + ((4\*a + b)\*ArcTan[(1 + 2\*x)/Sqrt[3]])/(12\*Sqrt[3]) - ((2\*a - b)\*Log[1 - x + x^2])/8 + ((2\*a - b)\*Log[1 + x + x^2])/8

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 1169

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2\*q - b/c, 2]}, Dist[1/(2\*c\*q\*r), Int[(d\*r - (d - e\*q)\*x)/(q - r\*x + x^2), x], x] + Dist[1/(2\*c\*q\*r), Int[(d\*r + (d - e\*q)\*x)/(q + r\*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NegQ[b^2 - 4\*a\*c]

#### Rule 1178

```
Int[((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol]
:= Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned} \int \frac{a + bx^2}{(1 + x^2 + x^4)^2} dx &= \frac{x(a + b - (a - 2b)x^2)}{6(1 + x^2 + x^4)} + \frac{1}{6} \int \frac{5a - b + (-a + 2b)x^2}{1 + x^2 + x^4} dx \\ &= \frac{x(a + b - (a - 2b)x^2)}{6(1 + x^2 + x^4)} + \frac{1}{12} \int \frac{5a - b - (6a - 3b)x}{1 - x + x^2} dx + \frac{1}{12} \int \frac{5a - b + (6a - 3b)x}{1 + x + x^2} dx \\ &= \frac{x(a + b - (a - 2b)x^2)}{6(1 + x^2 + x^4)} + \frac{1}{8}(2a - b) \int \frac{1 + 2x}{1 + x + x^2} dx + \frac{1}{8}(-2a + b) \int \frac{-1 + 2x}{1 - x + x^2} dx + \frac{1}{24}(-4a + b) \int \frac{1}{1 + x + x^2} dx \\ &= \frac{x(a + b - (a - 2b)x^2)}{6(1 + x^2 + x^4)} - \frac{1}{8}(2a - b) \log(1 - x + x^2) + \frac{1}{8}(2a - b) \log(1 + x + x^2) + \frac{1}{12}(-4a + b) \log(1 + x + x^2) \\ &= \frac{x(a + b - (a - 2b)x^2)}{6(1 + x^2 + x^4)} - \frac{(4a + b) \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}} + \frac{(4a + b) \tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{12\sqrt{3}} - \frac{1}{8}(2a - b) \log(1 + x + x^2) \end{aligned}$$

**Mathematica [C]** time = 0.25, size = 147, normalized size = 1.24

$$\frac{x(-ax^2 + a + 2bx^2 + b)}{6(x^4 + x^2 + 1)} - \frac{((\sqrt{3} - 11i)a - 2(\sqrt{3} - 2i)b) \tan^{-1}\left(\frac{1}{2}(\sqrt{3} - i)x\right)}{6\sqrt{6 + 6i\sqrt{3}}} - \frac{((\sqrt{3} + 11i)a - 2(\sqrt{3} + 2i)b) \tan^{-1}\left(\frac{1}{2}(\sqrt{3} + i)x\right)}{6\sqrt{6 - 6i\sqrt{3}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)/(1 + x^2 + x^4)^2, x]
```

```
[Out] (x*(a + b - a*x^2 + 2*b*x^2))/(6*(1 + x^2 + x^4)) - (((-11*I + Sqrt[3])*a - 2*(-2*I + Sqrt[3])*b)*ArcTan[(-I + Sqrt[3])*x/2])/(6*Sqrt[6 + (6*I)*Sqrt[3]]) - (((11*I + Sqrt[3])*a - 2*(2*I + Sqrt[3])*b)*ArcTan[(I + Sqrt[3])*x/2])/(6*Sqrt[6 - (6*I)*Sqrt[3]])
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2}{(1 + x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)/(1 + x^2 + x^4)^2, x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)/(1 + x^2 + x^4)^2, x]
```

**fricas [A]** time = 0.80, size = 185, normalized size = 1.55

$$\frac{12(a - 2b)x^3 - 2\sqrt{3}((4a + b)x^4 + (4a + b)x^2 + 4a + b) \arctan\left(\frac{1}{3}\sqrt{3}(2x + 1)\right) - 2\sqrt{3}((4a + b)x^4 + (4a + b)x^2 + 4a + b) \arctan\left(\frac{1}{3}\sqrt{3}(2x - 1)\right) - 12(a + b)x - 9((2a - b)x^4 + (2a - b)x^2 + 2a - b) \log(x^2 + x + 1) + 9((2a - b)x^4 + (2a - b)x^2 + 2a - b) \log(x^2 - x + 1)}{72(x^4 + x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/(x^4+x^2+1)^2,x, algorithm="fricas")

[Out] 
$$-1/72*(12*(a - 2*b)*x^3 - 2*\sqrt{3}*((4*a + b)*x^4 + (4*a + b)*x^2 + 4*a + b)*\arctan(1/3*\sqrt{3}*(2*x + 1)) - 2*\sqrt{3}*((4*a + b)*x^4 + (4*a + b)*x^2 + 4*a + b)*\arctan(1/3*\sqrt{3}*(2*x - 1)) - 12*(a + b)*x - 9*((2*a - b)*x^4 + (2*a - b)*x^2 + 2*a - b)*\log(x^2 + x + 1) + 9*((2*a - b)*x^4 + (2*a - b)*x^2 + 2*a - b)*\log(x^2 - x + 1))/(x^4 + x^2 + 1)$$

**giac** [A] time = 0.16, size = 109, normalized size = 0.92

$$\frac{1}{36}\sqrt{3}(4a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{36}\sqrt{3}(4a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{8}(2a-b)\log(x^2+x+1) - \frac{1}{8}(2a-b)\log(x^2-x+1) - \frac{ax^3-2bx^3-ax-bx}{6(x^4+x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/(x^4+x^2+1)^2,x, algorithm="giac")

[Out] 
$$1/36*\sqrt{3}*(4*a + b)*\arctan(1/3*\sqrt{3}*(2*x + 1)) + 1/36*\sqrt{3}*(4*a + b)*\arctan(1/3*\sqrt{3}*(2*x - 1)) + 1/8*(2*a - b)*\log(x^2 + x + 1) - 1/8*(2*a - b)*\log(x^2 - x + 1) - 1/6*(a*x^3 - 2*b*x^3 - a*x - b*x)/(x^4 + x^2 + 1)$$

**maple** [A] time = 0.01, size = 168, normalized size = 1.41

$$\frac{\sqrt{3} a \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right) + \sqrt{3} a \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right) - a \ln(x^2-x+1) + a \ln(x^2+x+1) + \sqrt{3} b \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right) + \sqrt{3} b \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right) + b \ln(x^2-x+1) - b \ln(x^2+x+1) + \frac{-2a}{3} + \frac{b}{3} + \left(\frac{-a}{3} + \frac{2b}{3}\right)x - \frac{2a}{3} + \frac{b}{3} + \left(\frac{a}{3} - \frac{2b}{3}\right)x}{9} + \frac{a \ln(x^2-x+1) + a \ln(x^2+x+1)}{4} + \frac{\sqrt{3} b \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right) + \sqrt{3} b \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right) + b \ln(x^2-x+1) - b \ln(x^2+x+1)}{8} + \frac{-2a}{3} + \frac{b}{3} + \left(\frac{-a}{3} + \frac{2b}{3}\right)x - \frac{2a}{3} + \frac{b}{3} + \left(\frac{a}{3} - \frac{2b}{3}\right)x}{4(x^2-x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)/(x^4+x^2+1)^2,x)

[Out] 
$$1/4*((-1/3*a+2/3*b)*x-2/3*a+1/3*b)/(x^2+x+1)+1/4*a*\ln(x^2+x+1)-1/8*b*\ln(x^2+x+1)+1/9*3^{(1/2)}*a*\arctan(1/3*(2*x+1)*3^{(1/2)})+1/36*3^{(1/2)}*b*\arctan(1/3*(2*x+1)*3^{(1/2)})-1/4*((1/3*a-2/3*b)*x-2/3*a+1/3*b)/(x^2-x+1)-1/4*a*\ln(x^2-x+1)+1/8*b*\ln(x^2-x+1)+1/9*3^{(1/2)}*a*\arctan(1/3*(2*x-1)*3^{(1/2)})+1/36*3^{(1/2)}*b*\arctan(1/3*(2*x-1)*3^{(1/2)})$$

**maxima** [A] time = 2.35, size = 105, normalized size = 0.88

$$\frac{1}{36}\sqrt{3}(4a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{36}\sqrt{3}(4a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{8}(2a-b)\log(x^2+x+1) - \frac{1}{8}(2a-b)\log(x^2-x+1) - \frac{(a-2b)x^3-(a+b)x}{6(x^4+x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/(x^4+x^2+1)^2,x, algorithm="maxima")

[Out] 
$$1/36*\sqrt{3}*(4*a + b)*\arctan(1/3*\sqrt{3}*(2*x + 1)) + 1/36*\sqrt{3}*(4*a + b)*\arctan(1/3*\sqrt{3}*(2*x - 1)) + 1/8*(2*a - b)*\log(x^2 + x + 1) - 1/8*(2*a - b)*\log(x^2 - x + 1) - 1/6*((a - 2*b)*x^3 - (a + b)*x)/(x^4 + x^2 + 1)$$

**mupad** [B] time = 4.49, size = 897, normalized size = 7.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^2)/(x^2 + x^4 + 1)^2,x)

[Out] 
$$\operatorname{atan}\left(\left(\left(\left(2*b - 10*a + 24*x*(b/8 - a/4 + (3^{(1/2)}*a*1i)/18 + (3^{(1/2)}*b*1i)/72)\right)*(b/8 - a/4 + (3^{(1/2)}*a*1i)/18 + (3^{(1/2)}*b*1i)/72) - x*((59*a^2)/18 - (19*a*b)/9 + b^2/9)\right)*(b/8 - a/4 + (3^{(1/2)}*a*1i)/18 + (3^{(1/2)}*b*1i)/72)*1i + ((10*a - 2*b + 24*x*(b/8 - a/4 + (3^{(1/2)}*a*1i)/18 + (3^{(1/2)}*b*1i)/72))*(b/8 - a/4 + (3^{(1/2)}*a*1i)/18 + (3^{(1/2)}*b*1i)/72) - x*((59*a^2)/18 - (19*a*b)/9 + b^2/9)\right)*(b/8 - a/4 + (3^{(1/2)}*a*1i)/18 + (3^{(1/2)}*b*1i)/72)*1i\right)/\left(\left(19*a*b^2\right)/36 - \left(29*a^2*b\right)/36 + \left(31*a^3\right)/108 - \left(7*b^3\right)/54 + \left(\left(2*b - 10*a + 24*x*(b/8 - a/4 + (3^{(1/2)}*a*1i)/18 + (3^{(1/2)}*b*1i)/72)\right)*(b/8 - a/4 + (3^{(1/2)}*a*1i)/18 + (3^{(1/2)}*b*1i)/72) - x*((59*a^2)/18 - (19*a*b)/9 + b^2/9)\right)*\right)$$

$$\begin{aligned} & \left( \frac{b}{8} - \frac{a}{4} + \frac{3^{1/2} a^{1i}}{18} + \frac{3^{1/2} b^{1i}}{72} \right) - \left( (10a - 2b + 24x \left( \frac{b}{8} - \frac{a}{4} + \frac{3^{1/2} a^{1i}}{18} + \frac{3^{1/2} b^{1i}}{72} \right)) \right. \\ & \left. \left( \frac{b}{8} - \frac{a}{4} + \frac{3^{1/2} a^{1i}}{18} + \frac{3^{1/2} b^{1i}}{72} \right) - x \left( \frac{59a^2}{18} - \frac{19ab}{9} + \frac{b^2}{9} \right) \right) \left( \frac{b}{8} - \frac{a}{4} + \frac{3^{1/2} a^{1i}}{18} + \frac{3^{1/2} b^{1i}}{72} \right) \\ & \left. \left( \frac{a^{1i}}{2} - \frac{b^{1i}}{4} + \frac{3^{1/2} a}{9} + \frac{3^{1/2} b}{36} \right) + \operatorname{atan} \left( \frac{(2b - 10a + 24x \left( \frac{a}{4} - \frac{b}{8} + \frac{3^{1/2} a^{1i}}{18} + \frac{3^{1/2} b^{1i}}{72} \right)) \left( \frac{a}{4} - \frac{b}{8} + \frac{3^{1/2} a^{1i}}{18} + \frac{3^{1/2} b^{1i}}{72} \right) - x \left( \frac{59a^2}{18} - \frac{19ab}{9} + \frac{b^2}{9} \right)}{(19ab^2)/36 - (29a^2b)/36 + (31a^3)/108 - (7b^3)/54} \right) \right. \\ & \left. \left( \frac{a}{4} - \frac{b}{8} + \frac{3^{1/2} a^{1i}}{18} + \frac{3^{1/2} b^{1i}}{72} \right) \right) \left( \frac{a}{4} - \frac{b}{8} + \frac{3^{1/2} a^{1i}}{18} + \frac{3^{1/2} b^{1i}}{72} \right) - x \left( \frac{59a^2}{18} - \frac{19ab}{9} + \frac{b^2}{9} \right) \left( \frac{a}{4} - \frac{b}{8} + \frac{3^{1/2} a^{1i}}{18} + \frac{3^{1/2} b^{1i}}{72} \right) \\ & \left. \left( \frac{a^{1i}}{2} - \frac{b^{1i}}{4} + \frac{3^{1/2} a}{9} + \frac{3^{1/2} b}{36} \right) - \left( (10a - 2b + 24x \left( \frac{a}{4} - \frac{b}{8} + \frac{3^{1/2} a^{1i}}{18} + \frac{3^{1/2} b^{1i}}{72} \right)) \left( \frac{a}{4} - \frac{b}{8} + \frac{3^{1/2} a^{1i}}{18} + \frac{3^{1/2} b^{1i}}{72} \right) - x \left( \frac{59a^2}{18} - \frac{19ab}{9} + \frac{b^2}{9} \right) \right) \right. \\ & \left. \left( \frac{a}{4} - \frac{b}{8} + \frac{3^{1/2} a^{1i}}{18} + \frac{3^{1/2} b^{1i}}{72} \right) \right) \left( \frac{b^{1i}}{4} - \frac{a^{1i}}{2} + \frac{3^{1/2} a}{9} + \frac{3^{1/2} b}{36} \right) - \left( x^3 \left( \frac{a}{6} - \frac{b}{3} \right) - x \left( \frac{a}{6} + \frac{b}{6} \right) \right) / \left( x^2 + x^4 + 1 \right) \end{aligned}$$

**sympy [C]** time = 1.89, size = 874, normalized size = 7.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)/(x\*\*4+x\*\*2+1)\*\*2,x)

[Out]  $(x^3(-a + 2b) + x(a + b)) / (6x^4 + 6x^2 + 6) + (-a/4 + b/8 - \sqrt{3}) * I * (4a + b) / 72 * \log(x + (76a^3(-a/4 + b/8 - \sqrt{3}) * I * (4a + b) / 72) + 948a^2b(-a/4 + b/8 - \sqrt{3}) * I * (4a + b) / 72 - 816ab^2(-a/4 + b/8 - \sqrt{3}) * I * (4a + b) / 72 + 12096a(-a/4 + b/8 - \sqrt{3}) * I * (4a + b) / 72 ** 3 + 148b^3(-a/4 + b/8 - \sqrt{3}) * I * (4a + b) / 72) - 8640b(-a/4 + b/8 - \sqrt{3}) * I * (4a + b) / 72 ** 3) / (248a^4 - 262a^3b + 75a^2b^2 + 11ab^3 - 7b^4) + (-a/4 + b/8 + \sqrt{3}) * I * (4a + b) / 72 * \log(x + (76a^3(-a/4 + b/8 + \sqrt{3}) * I * (4a + b) / 72) + 948a^2b(-a/4 + b/8 + \sqrt{3}) * I * (4a + b) / 72 - 816ab^2(-a/4 + b/8 + \sqrt{3}) * I * (4a + b) / 72 + 12096a(-a/4 + b/8 + \sqrt{3}) * I * (4a + b) / 72 ** 3 + 148b^3(-a/4 + b/8 + \sqrt{3}) * I * (4a + b) / 72) - 8640b(-a/4 + b/8 + \sqrt{3}) * I * (4a + b) / 72 ** 3) / (248a^4 - 262a^3b + 75a^2b^2 + 11ab^3 - 7b^4) + (a/4 - b/8 - \sqrt{3}) * I * (4a + b) / 72 * \log(x + (76a^3(a/4 - b/8 - \sqrt{3}) * I * (4a + b) / 72) + 948a^2b(a/4 - b/8 - \sqrt{3}) * I * (4a + b) / 72 - 816ab^2(a/4 - b/8 - \sqrt{3}) * I * (4a + b) / 72 + 12096a(a/4 - b/8 - \sqrt{3}) * I * (4a + b) / 72 ** 3 + 148b^3(a/4 - b/8 - \sqrt{3}) * I * (4a + b) / 72) - 8640b(a/4 - b/8 - \sqrt{3}) * I * (4a + b) / 72 ** 3) / (248a^4 - 262a^3b + 75a^2b^2 + 11ab^3 - 7b^4) + (a/4 - b/8 + \sqrt{3}) * I * (4a + b) / 72 * \log(x + (76a^3(a/4 - b/8 + \sqrt{3}) * I * (4a + b) / 72) + 948a^2b(a/4 - b/8 + \sqrt{3}) * I * (4a + b) / 72 - 816ab^2(a/4 - b/8 + \sqrt{3}) * I * (4a + b) / 72 + 12096a(a/4 - b/8 + \sqrt{3}) * I * (4a + b) / 72 ** 3 + 148b^3(a/4 - b/8 + \sqrt{3}) * I * (4a + b) / 72) - 8640b(a/4 - b/8 + \sqrt{3}) * I * (4a + b) / 72 ** 3) / (248a^4 - 262a^3b + 75a^2b^2 + 11ab^3 - 7b^4)$



$$3.88 \quad \int \frac{a+bx^2}{2+x^2+x^4} dx$$

**Optimal.** Leaf size=234

$$\frac{(a - \sqrt{2}b) \log\left(x^2 - \sqrt{2\sqrt{2} - 1}x + \sqrt{2}\right)}{4\sqrt{2}(2\sqrt{2} - 1)} + \frac{(a - \sqrt{2}b) \log\left(x^2 + \sqrt{2\sqrt{2} - 1}x + \sqrt{2}\right)}{4\sqrt{2}(2\sqrt{2} - 1)} - \frac{1}{2} \sqrt{\frac{1}{14}(2\sqrt{2} - 1)} \left(a + \sqrt{2}b\right) \tan^{-1}\left(\frac{\sqrt{2\sqrt{2} - 1} - 2x}{\sqrt{1 + 2\sqrt{2}}}\right)$$

**Rubi [A]** time = 0.23, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {1169, 634, 618, 204, 628}

$$\frac{(a - \sqrt{2}b) \log\left(x^2 - \sqrt{2\sqrt{2} - 1}x + \sqrt{2}\right)}{4\sqrt{2}(2\sqrt{2} - 1)} + \frac{(a - \sqrt{2}b) \log\left(x^2 + \sqrt{2\sqrt{2} - 1}x + \sqrt{2}\right)}{4\sqrt{2}(2\sqrt{2} - 1)} - \frac{1}{2} \sqrt{\frac{1}{14}(2\sqrt{2} - 1)} \left(a + \sqrt{2}b\right) \tan^{-1}\left(\frac{\sqrt{2\sqrt{2} - 1} - 2x}{\sqrt{1 + 2\sqrt{2}}}\right) + \frac{1}{2} \sqrt{\frac{1}{14}(2\sqrt{2} - 1)} \left(a + \sqrt{2}b\right) \tan^{-1}\left(\frac{2x + \sqrt{2\sqrt{2} - 1}}{\sqrt{1 + 2\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(2 + x^2 + x^4), x]

[Out] -(Sqrt[(-1 + 2\*Sqrt[2])/14]\*(a + Sqrt[2]\*b)\*ArcTan[(Sqrt[-1 + 2\*Sqrt[2]] - 2\*x)/Sqrt[1 + 2\*Sqrt[2]])/2 + (Sqrt[(-1 + 2\*Sqrt[2])/14]\*(a + Sqrt[2]\*b)\*ArcTan[(Sqrt[-1 + 2\*Sqrt[2]] + 2\*x)/Sqrt[1 + 2\*Sqrt[2]])/2 - ((a - Sqrt[2]\*b)\*Log[Sqrt[2] - Sqrt[-1 + 2\*Sqrt[2]]\*x + x^2])/(4\*Sqrt[2\*(-1 + 2\*Sqrt[2])]) + ((a - Sqrt[2]\*b)\*Log[Sqrt[2] + Sqrt[-1 + 2\*Sqrt[2]]\*x + x^2])/(4\*Sqrt[2\*(-1 + 2\*Sqrt[2])])

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

Rule 1169

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2\*q - b/c, 2]}, Dist[1/(2\*c\*q\*r), Int[(d\*r - (d - e\*q)\*x)/(q - r\*x + x^2), x], x] + Dist[1/(2\*c\*q\*r), Int[(d\*r + (d - e\*q)\*x)/(q + r\*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NegQ[b^2 - 4\*a\*c]

Rubi steps

$$\int \frac{a + bx^2}{2 + x^2 + x^4} dx = \frac{\int \frac{\sqrt{-1+2\sqrt{2}} a - (a - \sqrt{2}b)x}{\sqrt{2} - \sqrt{-1+2\sqrt{2}} x + x^2} dx}{2\sqrt{2}(-1 + 2\sqrt{2})} + \frac{\int \frac{\sqrt{-1+2\sqrt{2}} a + (a - \sqrt{2}b)x}{\sqrt{2} + \sqrt{-1+2\sqrt{2}} x + x^2} dx}{2\sqrt{2}(-1 + 2\sqrt{2})}$$

$$= \frac{1}{8}(\sqrt{2}a + 2b) \int \frac{1}{\sqrt{2} - \sqrt{-1 + 2\sqrt{2}} x + x^2} dx + \frac{1}{8}(\sqrt{2}a + 2b) \int \frac{1}{\sqrt{2} + \sqrt{-1 + 2\sqrt{2}} x + x^2} dx$$

$$= -\frac{(a - \sqrt{2}b) \log\left(\sqrt{2} - \sqrt{-1 + 2\sqrt{2}} x + x^2\right)}{4\sqrt{2}(-1 + 2\sqrt{2})} + \frac{(a - \sqrt{2}b) \log\left(\sqrt{2} + \sqrt{-1 + 2\sqrt{2}} x + x^2\right)}{4\sqrt{2}(-1 + 2\sqrt{2})}$$

$$= -\frac{(a + \sqrt{2}b) \tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}} - 2x}{\sqrt{1+2\sqrt{2}}}\right)}{2\sqrt{2}(1 + 2\sqrt{2})} + \frac{(a + \sqrt{2}b) \tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}} + 2x}{\sqrt{1+2\sqrt{2}}}\right)}{2\sqrt{2}(1 + 2\sqrt{2})} - \frac{(a - \sqrt{2}b) \log\left(\sqrt{2} - \sqrt{-1 + 2\sqrt{2}} x + x^2\right)}{4\sqrt{2}(-1 + 2\sqrt{2})}$$

**Mathematica [C]** time = 0.12, size = 111, normalized size = 0.47

$$\frac{((\sqrt{7} + i)b - 2ia) \tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}(1-i\sqrt{7})}}\right)}{\sqrt{14 - 14i\sqrt{7}}} + \frac{(2ia + (\sqrt{7} - i)b) \tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}(1+i\sqrt{7})}}\right)}{\sqrt{14 + 14i\sqrt{7}}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(2 + x^2 + x^4), x]

[Out] (((-2\*I)\*a + (I + Sqrt[7])\*b)\*ArcTan[x/Sqrt[(1 - I\*Sqrt[7])/2]])/Sqrt[14 - (14\*I)\*Sqrt[7]] + (((2\*I)\*a + (-I + Sqrt[7])\*b)\*ArcTan[x/Sqrt[(1 + I\*Sqrt[7])/2]])/Sqrt[14 + (14\*I)\*Sqrt[7]]

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2}{2 + x^2 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b\*x^2)/(2 + x^2 + x^4), x]

[Out] IntegrateAlgebraic[(a + b\*x^2)/(2 + x^2 + x^4), x]

**fricas [B]** time = 1.28, size = 3406, normalized size = 14.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/(x^4+x^2+2), x, algorithm="fricas")

[Out] 1/112\*(28\*sqrt(2)\*sqrt(1/7)\*(8\*a^4 - 16\*a^3\*b + 40\*a^2\*b^2 - 32\*a\*b^3 + 32\*b^4)^(1/4)\*sqrt(a^4 - 2\*a^3\*b + 5\*a^2\*b^2 - 4\*a\*b^3 + 4\*b^4)\*sqrt(a^4 - 4\*a^2\*b^2 + 4\*b^4)\*sqrt((4\*a^4 - 8\*a^3\*b + 20\*a^2\*b^2 - 16\*a\*b^3 + 16\*b^4 - sqrt(2)\*sqrt(a^4 - 2\*a^3\*b + 5\*a^2\*b^2 - 4\*a\*b^3 + 4\*b^4)\*(a^2 - 8\*a\*b + 2\*b^2)))/(a^4 - 4\*a^2\*b^2 + 4\*b^4)\*arctan(-1/28\*(7\*sqrt(1/2)\*sqrt(1/7)\*(8\*a^4 -

$$\begin{aligned}
& 16a^3b + 40a^2b^2 - 32a^2b^3 + 32b^4)^{3/4} * (\sqrt{2} * \sqrt{a^4 - 2a^3b} \\
& * b + 5a^2b^2 - 4a^2b^3 + 4b^4) * \sqrt{a^4 - 4a^2b^2 + 4b^4} * a - 2 * \sqrt{a^4 - 4a^2b^2 + 4b^4} * (a^2b - ab^2 + 2b^3) * \sqrt{(4a^4 - 8a^3b + 20a^2b^2 - 16a^2b^3 + 16b^4 - \sqrt{2} * \sqrt{a^4 - 2a^3b + 5a^2b^2 - 4a^2b^3 + 4b^4} * (a^2 - 8ab + 2b^2)) / (a^4 - 4a^2b^2 + 4b^4)} * \sqrt{(2(a^4 - 2a^3b + 5a^2b^2 - 4a^2b^3 + 4b^4) * x^2 + \sqrt{1/7} * (8a^4 - 16a^3b + 40a^2b^2 - 32a^2b^3 + 32b^4))^{1/4} * (\sqrt{7} * \sqrt{2} * \sqrt{a^4 - 2a^3b + 5a^2b^2 - 4a^2b^3 + 4b^4}) * b * x - \sqrt{7} * (a^3 - a^2b + 2a^2b^2) * x) * \sqrt{(4a^4 - 8a^3b + 20a^2b^2 - 16a^2b^3 + 16b^4 - \sqrt{2} * \sqrt{a^4 - 2a^3b + 5a^2b^2 - 4a^2b^3 + 4b^4} * (a^2 - 8ab + 2b^2)) / (a^4 - 4a^2b^2 + 4b^4)} + 2 * \sqrt{2} * \sqrt{a^4 - 2a^3b + 5a^2b^2 - 4a^2b^3 + 4b^4} * (a^2 - ab + 2b^2) / (a^4 - 2a^3b + 5a^2b^2 - 4a^2b^3 + 4b^4) + 8 * \sqrt{7} * \sqrt{2} * (a^4 - 2a^3b + 5a^2b^2 - 4a^2b^3 + 4b^4)^{3/2} * \sqrt{a^4 - 4a^2b^2 + 4b^4} - 7 * \sqrt{1/7} * (8a^4 - 16a^3b + 40a^2b^2 - 32a^2b^3 + 32b^4)^{3/4} * (\sqrt{2} * \sqrt{a^4 - 2a^3b + 5a^2b^2 - 4a^2b^3 + 4b^4}) * \sqrt{a^4 - 4a^2b^2 + 4b^4} * a * x - 2 * \sqrt{a^4 - 4a^2b^2 + 4b^4} * (a^2b - ab^2 + 2b^3) * x) * \sqrt{(4a^4 - 8a^3b + 20a^2b^2 - 16a^2b^3 + 16b^4 - \sqrt{2} * \sqrt{a^4 - 2a^3b + 5a^2b^2 - 4a^2b^3 + 4b^4} * (a^2 - 8ab + 2b^2)) / (a^4 - 4a^2b^2 + 4b^4)} - 4 * \sqrt{7} * (a^6 - 3a^5b + 9a^4b^2 - 13a^3b^3 + 18a^2b^4 - 12a^2b^5 + 8b^6) * \sqrt{a^4 - 4a^2b^2 + 4b^4} / (a^8 - 3a^7b + 7a^6b^2 - 7a^5b^3 + 14a^3b^5 - 28a^2b^6 + 24a^2b^7 - 16b^8) + 28 * \sqrt{2} * \sqrt{1/7} * (8a^4 - 16a^3b + 40a^2b^2 - 32a^2b^3 + 32b^4)^{1/4} * \sqrt{a^4 - 2a^3b + 5a^2b^2 - 4a^2b^3 + 4b^4} * \sqrt{a^4 - 4a^2b^2 + 4b^4} * \sqrt{(4a^4 - 8a^3b + 20a^2b^2 - 16a^2b^3 + 16b^4 - \sqrt{2} * \sqrt{a^4 - 2a^3b + 5a^2b^2 - 4a^2b^3 + 4b^4} * (a^2 - 8ab + 2b^2)) / (a^4 - 4a^2b^2 + 4b^4)} * \arctan(-1/28 * (7 * \sqrt{1/2}) * \sqrt{1/7} * (8a^4 - 16a^3b + 40a^2b^2 - 32a^2b^3 + 32b^4)^{3/4} * (\sqrt{2} * \sqrt{a^4 - 2a^3b + 5a^2b^2 - 4a^2b^3 + 4b^4}) * \sqrt{a^4 - 4a^2b^2 + 4b^4}) * a - 2 * \sqrt{a^4 - 4a^2b^2 + 4b^4} * (a^2b - ab^2 + 2b^3) * \sqrt{(4a^4 - 8a^3b + 20a^2b^2 - 16a^2b^3 + 16b^4 - \sqrt{2} * \sqrt{a^4 - 2a^3b + 5a^2b^2 - 4a^2b^3 + 4b^4} * (a^2 - 8ab + 2b^2)) / (a^4 - 4a^2b^2 + 4b^4)} * \sqrt{(2(a^4 - 2a^3b + 5a^2b^2 - 4a^2b^3 + 4b^4) * x^2 - \sqrt{1/7} * (8a^4 - 16a^3b + 40a^2b^2 - 32a^2b^3 + 32b^4))^{1/4} * (\sqrt{7} * \sqrt{2} * \sqrt{a^4 - 2a^3b + 5a^2b^2 - 4a^2b^3 + 4b^4}) * b * x - \sqrt{7} * (a^3 - a^2b + 2a^2b^2) * x) * \sqrt{(4a^4 - 8a^3b + 20a^2b^2 - 16a^2b^3 + 16b^4 - \sqrt{2} * \sqrt{a^4 - 2a^3b + 5a^2b^2 - 4a^2b^3 + 4b^4} * (a^2 - 8ab + 2b^2)) / (a^4 - 4a^2b^2 + 4b^4)} + 2 * \sqrt{2} * \sqrt{a^4 - 2a^3b + 5a^2b^2 - 4a^2b^3 + 4b^4} * (a^2 - ab + 2b^2) / (a^4 - 2a^3b + 5a^2b^2 - 4a^2b^3 + 4b^4) - 8 * \sqrt{7} * \sqrt{2} * (a^4 - 2a^3b + 5a^2b^2 - 4a^2b^3 + 4b^4)^{3/2} * \sqrt{a^4 - 4a^2b^2 + 4b^4} - 7 * \sqrt{1/7} * (8a^4 - 16a^3b + 40a^2b^2 - 32a^2b^3 + 32b^4)^{3/4} * (\sqrt{2} * \sqrt{a^4 - 2a^3b + 5a^2b^2 - 4a^2b^3 + 4b^4}) * \sqrt{a^4 - 4a^2b^2 + 4b^4} * a * x - 2 * \sqrt{a^4 - 4a^2b^2 + 4b^4} * (a^2b - ab^2 + 2b^3) * x) * \sqrt{(4a^4 - 8a^3b + 20a^2b^2 - 16a^2b^3 + 16b^4 - \sqrt{2} * \sqrt{a^4 - 2a^3b + 5a^2b^2 - 4a^2b^3 + 4b^4} * (a^2 - 8ab + 2b^2)) / (a^4 - 4a^2b^2 + 4b^4)} + 4 * \sqrt{7} * (a^6 - 3a^5b + 9a^4b^2 - 13a^3b^3 + 18a^2b^4 - 12a^2b^5 + 8b^6) * \sqrt{a^4 - 4a^2b^2 + 4b^4} / (a^8 - 3a^7b + 7a^6b^2 - 7a^5b^3 + 14a^3b^5 - 28a^2b^6 + 24a^2b^7 - 16b^8) - \sqrt{1/7} * (8a^4 - 16a^3b + 40a^2b^2 - 32a^2b^3 + 32b^4)^{1/4} * (\sqrt{7} * \sqrt{2} * \sqrt{a^4 - 2a^3b + 5a^2b^2 - 4a^2b^3 + 4b^4}) * (a^2 - 8ab + 2b^2) + 4 * \sqrt{7} * (a^4 - 2a^3b + 5a^2b^2 - 4a^2b^3 + 4b^4) * \sqrt{(4a^4 - 8a^3b + 20a^2b^2 - 16a^2b^3 + 16b^4 - \sqrt{2} * \sqrt{a^4 - 2a^3b + 5a^2b^2 - 4a^2b^3 + 4b^4} * (a^2 - 8ab + 2b^2)) / (a^4 - 4a^2b^2 + 4b^4)} * \log(8 * (a^4 - 2a^3b + 5a^2b^2 - 4a^2b^3 + 4b^4) * x^2 + 4 * \sqrt{1/7} * (8a^4 - 16a^3b + 40a^2b^2 - 32a^2b^3 + 32b^4)^{1/4} * (\sqrt{7} * \sqrt{2} * \sqrt{a^4 - 2a^3b + 5a^2b^2 - 4a^2b^3 + 4b^4}) * b * x - \sqrt{7} * (a^3 - a^2b + 2a^2b^2) * x) * \sqrt{(4a^4 - 8a^3b + 20a^2b^2 - 16a^2b^3 + 16b^4 - \sqrt{2} * \sqrt{a^4 - 2a^3b + 5a^2b^2 - 4a^2b^3 + 4b^4} * (a^2 - 8ab + 2b^2)) / (a^4 - 4a^2b^2 + 4b^4)} + 8 * \sqrt{2} * \sqrt{a^4 - 2a^3b + 5a^2b^2 - 4a^2b^3 + 4b^4} * (a^2 - ab + 2b^2) + sq
\end{aligned}$$

```

rt(1/7)*(8*a^4 - 16*a^3*b + 40*a^2*b^2 - 32*a*b^3 + 32*b^4)^(1/4)*(sqrt(7)*
sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*
b^2) + 4*sqrt(7)*(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4))*sqrt((4*a^4
- 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 + 16*b^4 - sqrt(2)*sqrt(a^4 - 2*a^3*b +
5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2)))/(a^4 - 4*a^2*b^2 + 4*b^
4))*log(8*(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*x^2 - 4*sqrt(1/7)*(
8*a^4 - 16*a^3*b + 40*a^2*b^2 - 32*a*b^3 + 32*b^4)^(1/4)*(sqrt(7)*sqrt(2)*s
qrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*b*x - sqrt(7)*(a^3 - a^2*b
+ 2*a*b^2)*x)*sqrt((4*a^4 - 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 + 16*b^4 - sqr
t(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2
)))/(a^4 - 4*a^2*b^2 + 4*b^4)) + 8*sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 -
4*a*b^3 + 4*b^4)*(a^2 - a*b + 2*b^2)))/(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3
+ 4*b^4)

```

**giac [B]** time = 0.88, size = 544, normalized size = 2.32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+2),x, algorithm="giac")
```

```

[Out] -1/14336*sqrt(7)*(32*sqrt(7)*2^(1/4)*b*(sqrt(2) + 4)^(3/2) + 96*sqrt(7)*2^(
1/4)*b*sqrt(sqrt(2) + 4)*(sqrt(2) - 4) - 24*2^(3/4)*b*(sqrt(2) + 4)*sqrt(-8
*sqrt(2) + 32) + 2^(3/4)*b*(-8*sqrt(2) + 32)^(3/2) - 128*sqrt(7)*2^(3/4)*a*
sqrt(sqrt(2) + 4) + 64*2^(1/4)*a*sqrt(-8*sqrt(2) + 32))*arctan(2*2^(3/4)*sq
rt(1/2)*(x + 2^(1/4)*sqrt(-1/8*sqrt(2) + 1/2))/sqrt(sqrt(2) + 4)) - 1/14336
*sqrt(7)*(32*sqrt(7)*2^(1/4)*b*(sqrt(2) + 4)^(3/2) + 96*sqrt(7)*2^(1/4)*b*s
qrt(sqrt(2) + 4)*(sqrt(2) - 4) - 24*2^(3/4)*b*(sqrt(2) + 4)*sqrt(-8*sqrt(2)
+ 32) + 2^(3/4)*b*(-8*sqrt(2) + 32)^(3/2) - 128*sqrt(7)*2^(3/4)*a*sqrt(sqr
t(2) + 4) + 64*2^(1/4)*a*sqrt(-8*sqrt(2) + 32))*arctan(2*2^(3/4)*sqrt(1/2)*
(x - 2^(1/4)*sqrt(-1/8*sqrt(2) + 1/2))/sqrt(sqrt(2) + 4)) - 1/28672*sqrt(7)
*(24*sqrt(7)*2^(3/4)*b*(sqrt(2) + 4)*sqrt(-8*sqrt(2) + 32) - sqrt(7)*2^(3/4
)*b*(-8*sqrt(2) + 32)^(3/2) + 32*2^(1/4)*b*(sqrt(2) + 4)^(3/2) + 96*2^(1/4)
*b*sqrt(sqrt(2) + 4)*(sqrt(2) - 4) - 128*2^(3/4)*a*sqrt(sqrt(2) + 4) - 64*s
qrt(7)*2^(1/4)*a*sqrt(-8*sqrt(2) + 32))*log(x^2 + 2*2^(1/4)*x*sqrt(-1/8*sqr
t(2) + 1/2) + sqrt(2)) + 1/28672*sqrt(7)*(24*sqrt(7)*2^(3/4)*b*(sqrt(2) + 4
)*sqrt(-8*sqrt(2) + 32) - sqrt(7)*2^(3/4)*b*(-8*sqrt(2) + 32)^(3/2) + 32*2^
(1/4)*b*(sqrt(2) + 4)^(3/2) + 96*2^(1/4)*b*sqrt(sqrt(2) + 4)*(sqrt(2) - 4)
- 128*2^(3/4)*a*sqrt(sqrt(2) + 4) - 64*sqrt(7)*2^(1/4)*a*sqrt(-8*sqrt(2) +
32))*log(x^2 - 2*2^(1/4)*x*sqrt(-1/8*sqrt(2) + 1/2) + sqrt(2))

```

**maple [B]** time = 0.10, size = 710, normalized size = 3.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(x^4+x^2+2),x)
```

```

[Out] 1/56*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))^(1/2)*2^(1/2)*a-
1/14*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))^(1/2)*2^(1/2)*b+
1/14*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))^(1/2)*a-1/28*ln(
x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))^(1/2)*b-1/28/(1+2*2^(1/2
))^(1/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-1+2*2^(1/
2))*2^(1/2)*a+1/7/(1+2*2^(1/2))^(1/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+
2*2^(1/2))^(1/2))*(-1+2*2^(1/2))*2^(1/2)*b-1/7/(1+2*2^(1/2))^(1/2)*arctan((
2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))*a+1/14/(1+2*2
^(1/2))^(1/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-1+2*
2^(1/2))*b+1/2/(1+2*2^(1/2))^(1/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2
^(1/2))^(1/2))*2^(1/2)*a-1/56*ln(x^2+2^(1/2)-x*(-1+2*2^(1/2))^(1/2))*(-1+2*

```

$$2^{(1/2)} \int \frac{(1/2) \cdot 2^{(1/2)} \cdot a + 1/14 \cdot \ln(x^2 + 2^{(1/2)} - x \cdot (-1 + 2 \cdot 2^{(1/2)}))^{(1/2)} \cdot (-1 + 2 \cdot 2^{(1/2)})^{(1/2)} \cdot 2^{(1/2)} \cdot b - 1/14 \cdot \ln(x^2 + 2^{(1/2)} - x \cdot (-1 + 2 \cdot 2^{(1/2)}))^{(1/2)} \cdot (-1 + 2 \cdot 2^{(1/2)})^{(1/2)} \cdot a + 1/28 \cdot \ln(x^2 + 2^{(1/2)} - x \cdot (-1 + 2 \cdot 2^{(1/2)}))^{(1/2)} \cdot (-1 + 2 \cdot 2^{(1/2)})^{(1/2)} \cdot b - 1/28 / (1 + 2 \cdot 2^{(1/2)})^{(1/2)} \cdot \arctan((2 \cdot x - (-1 + 2 \cdot 2^{(1/2)})^{(1/2)}) / (1 + 2 \cdot 2^{(1/2)})^{(1/2)}) \cdot (-1 + 2 \cdot 2^{(1/2)})^{(1/2)} \cdot a + 1/7 / (1 + 2 \cdot 2^{(1/2)})^{(1/2)} \cdot \arctan((2 \cdot x - (-1 + 2 \cdot 2^{(1/2)})^{(1/2)}) / (1 + 2 \cdot 2^{(1/2)})^{(1/2)}) \cdot (-1 + 2 \cdot 2^{(1/2)})^{(1/2)} \cdot b - 1/7 / (1 + 2 \cdot 2^{(1/2)})^{(1/2)} \cdot \arctan((2 \cdot x - (-1 + 2 \cdot 2^{(1/2)})^{(1/2)}) / (1 + 2 \cdot 2^{(1/2)})^{(1/2)}) \cdot (-1 + 2 \cdot 2^{(1/2)})^{(1/2)} \cdot a + 1/14 / (1 + 2 \cdot 2^{(1/2)})^{(1/2)} \cdot \arctan((2 \cdot x - (-1 + 2 \cdot 2^{(1/2)})^{(1/2)}) / (1 + 2 \cdot 2^{(1/2)})^{(1/2)}) \cdot (-1 + 2 \cdot 2^{(1/2)})^{(1/2)} \cdot b + 1/2 / (1 + 2 \cdot 2^{(1/2)})^{(1/2)} \cdot \arctan((2 \cdot x - (-1 + 2 \cdot 2^{(1/2)})^{(1/2)}) / (1 + 2 \cdot 2^{(1/2)})^{(1/2)}) \cdot 2^{(1/2)} \cdot a$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^2 + a}{x^4 + x^2 + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/(x^4+x^2+2),x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)/(x^4 + x^2 + 2), x)

**mupad** [B] time = 4.49, size = 771, normalized size = 3.29

$$\frac{\sqrt{\frac{a^2 \sqrt{\frac{a^2 + b^2}{a^2 + b^2}} + \frac{a^2 \sqrt{\frac{a^2 + b^2}{a^2 + b^2}}}{\sqrt{a^2 + b^2}}}{\sqrt{a^2 + b^2}} + \frac{a^2 \sqrt{\frac{a^2 + b^2}{a^2 + b^2}}}{\sqrt{a^2 + b^2}} + \frac{a^2 \sqrt{\frac{a^2 + b^2}{a^2 + b^2}}}{\sqrt{a^2 + b^2}} + \frac{a^2 \sqrt{\frac{a^2 + b^2}{a^2 + b^2}}}{\sqrt{a^2 + b^2}}}{\sqrt{a^2 + b^2}} + \frac{a^2 \sqrt{\frac{a^2 + b^2}{a^2 + b^2}}}{\sqrt{a^2 + b^2}} + \frac{a^2 \sqrt{\frac{a^2 + b^2}{a^2 + b^2}}}{\sqrt{a^2 + b^2}} + \frac{a^2 \sqrt{\frac{a^2 + b^2}{a^2 + b^2}}}{\sqrt{a^2 + b^2}}}{\sqrt{a^2 + b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^2)/(x^2 + x^4 + 2),x)

[Out] 
$$-\operatorname{atan}\left(\frac{a^2 x \sqrt{\frac{7}{2} a^2 + 1}}{112} - \frac{a b}{14} - \frac{\sqrt{\frac{7}{2} b^2 + 1}}{56} + \frac{a^2}{112} + \frac{b^2}{56}\right)^{(1/2)} \frac{7 i}{((7^{(1/2)} a^3 + 1) / 2 - a b^2 - 2 a^2 b + a^3 / 2 + 4 b^3 - 7^{(1/2)} a b^2 + 1)} - \left(\frac{b^2 x \sqrt{\frac{7}{2} a^2 + 1}}{112} - \frac{a b}{14} - \frac{\sqrt{\frac{7}{2} b^2 + 1}}{56} + \frac{a^2}{112} + \frac{b^2}{56}\right)^{(1/2)} \frac{14 i}{((7^{(1/2)} a^3 + 1) / 2 - a b^2 - 2 a^2 b + a^3 / 2 + 4 b^3 - 7^{(1/2)} a b^2 + 1)} + \frac{7^{(1/2)} a^2 x \sqrt{\frac{7}{2} a^2 + 1}}{112} - \frac{a b}{14} - \frac{\sqrt{\frac{7}{2} b^2 + 1}}{56} + \frac{a^2}{112} + \frac{b^2}{56} \left(\frac{7^{(1/2)} a^3 + 1}{2} - a b^2 - 2 a^2 b + a^3 / 2 + 4 b^3 - 7^{(1/2)} a b^2 + 1\right) \frac{2 i}{\left(\frac{7^{(1/2)} a^3 + 1}{2} - a b^2 - 2 a^2 b + a^3 / 2 + 4 b^3 - 7^{(1/2)} a b^2 + 1\right)} - 2 \operatorname{atanh}\left(\frac{7 a^2 x \sqrt{\frac{7}{2} b^2 + 1}}{56} - \frac{\sqrt{\frac{7}{2} a^2 + 1}}{112} - \frac{a b}{14} + \frac{a^2}{112} + \frac{b^2}{56}\right)^{(1/2)} \frac{2 i}{\left(\frac{7^{(1/2)} a^3 + 1}{2} + a b^2 + 2 a^2 b - a^3 / 2 - 4 b^3 - 7^{(1/2)} a b^2 + 1\right)} - \left(\frac{14 b^2 x \sqrt{\frac{7}{2} b^2 + 1}}{56} - \frac{\sqrt{\frac{7}{2} b^2 + 1}}{112} - \frac{a b}{14} + \frac{a^2}{112} + \frac{b^2}{56}\right)^{(1/2)} \frac{2 i}{\left(\frac{7^{(1/2)} a^3 + 1}{2} + a b^2 + 2 a^2 b - a^3 / 2 - 4 b^3 - 7^{(1/2)} a b^2 + 1\right)} - \frac{7^{(1/2)} a b^2 x \sqrt{\frac{7}{2} b^2 + 1}}{112} - \frac{a b}{14} + \frac{a^2}{112} + \frac{b^2}{56} \frac{2 i}{\left(\frac{7^{(1/2)} a^3 + 1}{2} + a b^2 + 2 a^2 b - a^3 / 2 - 4 b^3 - 7^{(1/2)} a b^2 + 1\right)} - \left(\frac{7^{(1/2)} b^2 x \sqrt{\frac{7}{2} a^2 + 1}}{56} - \frac{\sqrt{\frac{7}{2} a^2 + 1}}{112} - \frac{a b}{14} + \frac{a^2}{112} + \frac{b^2}{56}\right)^{(1/2)} \frac{2 i}{\left(\frac{7^{(1/2)} a^3 + 1}{2} + a b^2 + 2 a^2 b - a^3 / 2 - 4 b^3 - 7^{(1/2)} a b^2 + 1\right)}$$

**sympy** [A] time = 1.32, size = 122, normalized size = 0.52

$$\operatorname{RootSum}\left(1568t^4 + t^2(-28a^2 + 224ab - 56b^2) + a^4 - 2a^3b + 5a^2b^2 - 4ab^3 + 4b^4, \left(t \mapsto t \log\left(x + \frac{112t^3a - 448t^3b + 6ta^3 + 12ta^2b - 48tab^2 + 8tb^3}{a^4 - a^3b + 2ab^3 - 4b^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)/(x\*\*4+x\*\*2+2),x)

[Out] 
$$\operatorname{RootSum}(1568\_t^{**4} + \_t^{**2}(-28a^{**2} + 224a*b - 56b^{**2}) + a^{**4} - 2a^{**3}*b + 5a^{**2}*b^{**2} - 4a*b^{**3} + 4b^{**4}, \operatorname{Lambda}(\_t, \_t \cdot \log(x + (112\_t^{**3}*a - 448\_t^{**3}*b + 6\_t*a^{**3} + 12\_t*a^{**2}*b - 48\_t*a*b^{**2} + 8\_t*b^{**3}) / (a^{**4} - a^{**3}*b + 2a*b^{**3} - 4b^{**4}))))$$

$$3.89 \quad \int \frac{a+bx^2}{(2+x^2+x^4)^2} dx$$

**Optimal.** Leaf size=316

$$\frac{(\sqrt{2}(a-4b)+11a-2b)\log\left(x^2-\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{112\sqrt{2}(2\sqrt{2}-1)} + \frac{((11+\sqrt{2})a-2(2\sqrt{2}b+b))\log\left(x^2+\sqrt{2\sqrt{2}-1}x\right)}{112\sqrt{2}(2\sqrt{2}-1)}$$

**Rubi [A]** time = 0.29, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 18, number of rules / integrand size = 0.333, Rules used = {1178, 1169, 634, 618, 204, 628}

$$\frac{x(x^2-(a-4b)+3a+2b)}{28(x^2+x^2+2)} - \frac{(\sqrt{2}(a-4b)+11a-2b)\log(x^2-\sqrt{2\sqrt{2}-1}x+\sqrt{2})}{112\sqrt{2}(2\sqrt{2}-1)} + \frac{((11+\sqrt{2})a-2(2\sqrt{2}b+b))\log(x^2+\sqrt{2\sqrt{2}-1}x+\sqrt{2})}{112\sqrt{2}(2\sqrt{2}-1)} - \frac{1}{56\sqrt{14}(2\sqrt{2}-1)}((11-\sqrt{2})a-(2-4\sqrt{2})b)\tan^{-1}\left(\frac{\sqrt{2\sqrt{2}-1}-2x}{\sqrt{1+2\sqrt{2}}}\right) + \frac{1}{56\sqrt{14}(2\sqrt{2}-1)}((11-\sqrt{2})a-(2-4\sqrt{2})b)\tan^{-1}\left(\frac{2x+\sqrt{2\sqrt{2}-1}}{\sqrt{1+2\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(2 + x^2 + x^4)^2,x]

[Out] (x\*(3\*a + 2\*b - (a - 4\*b)\*x^2))/(28\*(2 + x^2 + x^4)) - (Sqrt[(-1 + 2\*Sqrt[2])/14]\*((11 - Sqrt[2])\*a - (2 - 4\*Sqrt[2])\*b)\*ArcTan[(Sqrt[-1 + 2\*Sqrt[2]] - 2\*x)/Sqrt[1 + 2\*Sqrt[2]])]/56 + (Sqrt[(-1 + 2\*Sqrt[2])/14]\*((11 - Sqrt[2])\*a - (2 - 4\*Sqrt[2])\*b)\*ArcTan[(Sqrt[-1 + 2\*Sqrt[2]] + 2\*x)/Sqrt[1 + 2\*Sqrt[2]])]/56 - (((11\*a + Sqrt[2]\*(a - 4\*b) - 2\*b)\*Log[Sqrt[2] - Sqrt[-1 + 2\*Sqrt[2]]\*x + x^2])/(112\*Sqrt[2\*(-1 + 2\*Sqrt[2])]) + (((11 + Sqrt[2])\*a - 2\*(b + 2\*Sqrt[2]\*b))\*Log[Sqrt[2] + Sqrt[-1 + 2\*Sqrt[2]]\*x + x^2])/(112\*Sqrt[2\*(-1 + 2\*Sqrt[2])]))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 1169

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2\*q - b/c, 2]}, Dist[1/(2\*c\*q\*r), Int[(d\*r - (d - e\*q)\*x)/(q - r\*x + x^2), x], x] + Dist[1/(2\*c\*q\*r), Int[(d\*r + (d - e\*q)\*x)/(q + r\*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ

$[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \ \text{NegQ}[b^2 - 4ac]$

### Rule 1178

$\text{Int}[(d_ + (e_)*(x_)^2)*((a_ + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_)), x\_Symbol] :> \text{Simp}[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + \text{Dist}[1/(2*a*(p + 1)*(b^2 - 4*a*c)), \text{Int}[\text{Simp}[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /;$   $\text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{IntegerQ}[2*p]$

### Rubi steps

$$\begin{aligned} \int \frac{a + bx^2}{(2 + x^2 + x^4)^2} dx &= \frac{x(3a + 2b - (a - 4b)x^2)}{28(2 + x^2 + x^4)} + \frac{1}{28} \int \frac{11a - 2b + (-a + 4b)x^2}{2 + x^2 + x^4} dx \\ &= \frac{x(3a + 2b - (a - 4b)x^2)}{28(2 + x^2 + x^4)} + \frac{\int \frac{\sqrt{-1+2\sqrt{2}}(11a-2b) - (11a-2b-\sqrt{2}(-a+4b))x}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2} dx}{56\sqrt{2}(-1+2\sqrt{2})} + \frac{\int \frac{\sqrt{-1+2\sqrt{2}}(11a-2b) + (11a-2b+\sqrt{2}(-a+4b))x}{\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x^2} dx}{56\sqrt{2}(-1+2\sqrt{2})} \\ &= \frac{x(3a + 2b - (a - 4b)x^2)}{28(2 + x^2 + x^4)} - \frac{(11a + \sqrt{2}(a - 4b) - 2b) \int \frac{-\sqrt{-1+2\sqrt{2}} + 2x}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2} dx}{112\sqrt{2}(-1+2\sqrt{2})} + \frac{(11a - \sqrt{2}(a - 4b) + 2b) \int \frac{\sqrt{-1+2\sqrt{2}} - 2x}{\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x^2} dx}{112\sqrt{2}(-1+2\sqrt{2})} \\ &= \frac{x(3a + 2b - (a - 4b)x^2)}{28(2 + x^2 + x^4)} - \frac{(11a + \sqrt{2}(a - 4b) - 2b) \log\left(\sqrt{2} - \sqrt{-1+2\sqrt{2}}x + x^2\right)}{112\sqrt{2}(-1+2\sqrt{2})} + \frac{(11a - \sqrt{2}(a - 4b) + 2b) \log\left(\sqrt{2} + \sqrt{-1+2\sqrt{2}}x + x^2\right)}{112\sqrt{2}(-1+2\sqrt{2})} \\ &= \frac{x(3a + 2b - (a - 4b)x^2)}{28(2 + x^2 + x^4)} - \frac{((11 - \sqrt{2})a - (2 - 4\sqrt{2})b) \tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}} - 2x}{\sqrt{1+2\sqrt{2}}}\right)}{56\sqrt{2}(1+2\sqrt{2})} + \frac{((11 + \sqrt{2})a - (2 + 4\sqrt{2})b) \tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}} + 2x}{\sqrt{1+2\sqrt{2}}}\right)}{56\sqrt{2}(1+2\sqrt{2})} \end{aligned}$$

**Mathematica [C]** time = 0.22, size = 165, normalized size = 0.52

$$\frac{2b(2x^3 + x) - ax(x^2 - 3)}{28(x^4 + x^2 + 2)} - \frac{((\sqrt{7} + 23i)a - 4(\sqrt{7} + 2i)b) \tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}(1-i\sqrt{7})}}\right)}{28\sqrt{14 - 14i\sqrt{7}}} - \frac{((\sqrt{7} - 23i)a - 4(\sqrt{7} - 2i)b) \tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}(1+i\sqrt{7})}}\right)}{28\sqrt{14 + 14i\sqrt{7}}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(2 + x^2 + x^4)^2, x]

[Out]  $(- (a*x*(-3 + x^2)) + 2*b*(x + 2*x^3))/(28*(2 + x^2 + x^4)) - (((23*I + \text{Sqrt}[7])*a - 4*(2*I + \text{Sqrt}[7])*b)*\text{ArcTan}[x/\text{Sqrt}[(1 - I*\text{Sqrt}[7])/2]])/(28*\text{Sqrt}[14 - (14*I)*\text{Sqrt}[7]]) - (((-23*I + \text{Sqrt}[7])*a - 4*(-2*I + \text{Sqrt}[7])*b)*\text{ArcTan}[x/\text{Sqrt}[(1 + I*\text{Sqrt}[7])/2]])/(28*\text{Sqrt}[14 + (14*I)*\text{Sqrt}[7]])$

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2}{(2 + x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b\*x^2)/(2 + x^2 + x^4)^2,x]

[Out] IntegrateAlgebraic[(a + b\*x^2)/(2 + x^2 + x^4)^2, x]

**fricas** [B] time = 1.15, size = 4346, normalized size = 13.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/(x^4+x^2+2)^2,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/21952*(196*2^{(3/4)}*\sqrt{2/7}*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^{(3/4)}*\sqrt{289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4} \\ & *(x^4 + x^2 + 2)*\sqrt{(35912*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 18656*a*b^3 + 3872*b^4 - \sqrt{2}*\sqrt{4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4})} \\ & *(211*a^2 - 428*a*b + 100*b^2))/(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4) \\ & *\arctan(1/14*(2^{(3/4)}*\sqrt{2/7}*\sqrt{1/14}*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^{(3/4)} \\ & *(sqrt(2)*\sqrt{4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4}*\sqrt{289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4} \\ & *(11*a - 2*b) + 2*\sqrt{289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4}*(67*a^3 - 321*a^2*b + 234*a*b^2 - 88*b^3)) \\ & *\sqrt{(35912*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 18656*a*b^3 + 3872*b^4 - \sqrt{2}*\sqrt{4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4})} \\ & *(211*a^2 - 428*a*b + 100*b^2))/(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4) \\ & *\sqrt{(14*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*x^2 + 2^{(1/4)}*\sqrt{2/7}*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^{(1/4)} \\ & *(sqrt(7)*\sqrt{2}*\sqrt{4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4}*(a - 4*b)*x + \sqrt{7}*(737*a^3 - 717*a^2*b + 348*a*b^2 - 44*b^3)*x)} \\ & *\sqrt{(35912*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 18656*a*b^3 + 3872*b^4 - \sqrt{2}*\sqrt{4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4})} \\ & *(211*a^2 - 428*a*b + 100*b^2))/(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4) \\ & + 14*\sqrt{2}*\sqrt{4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4}*(67*a^2 - 53*a*b + 22*b^2) \\ & )/(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4) - 2^{(3/4)}*\sqrt{2/7}*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^{(3/4)} \\ & *(sqrt(2)*\sqrt{4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4}*\sqrt{289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4} \\ & *(11*a - 2*b)*x + 2*\sqrt{289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4}*(67*a^3 - 321*a^2*b + 234*a*b^2 - 88*b^3)*x) \\ & *\sqrt{(35912*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 18656*a*b^3 + 3872*b^4 - \sqrt{2}*\sqrt{4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4})} \\ & *(211*a^2 - 428*a*b + 100*b^2))/(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4) - 4*\sqrt{7}*\sqrt{2} \\ & *(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^{(3/2)}*\sqrt{289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4} \\ & + 2*\sqrt{7}*(300763*a^6 - 713751*a^5*b + 860883*a^4*b^2 - 617609*a^3*b^3 + 282678*a^2*b^4 - 76956*a*b^5 + 10648*b^6) \\ & *\sqrt{289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4}))/((5112971*a^8 - 13336819*a^7*b + 16286963*a^6*b^2 - 11087881*a^5*b^3 + 3832430*a^4*b^4 + 31472*a^3*b^5 - 641872*a^2*b^6 + 265232*a*b^7 - 42592*b^8)) \\ & + 196*2^{(3/4)}*\sqrt{2/7}*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^{(3/4)}*\sqrt{289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4} \\ & *(x^4 + x^2 + 2)*\sqrt{(35912*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 18656*a*b^3 + 3872*b^4 - \sqrt{2}*\sqrt{4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4})} \\ & *(211*a^2 - 428*a*b + 100*b^2))/(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4) \\ & *\arctan(1/14*(2^{(3/4)}*\sqrt{2/7}*\sqrt{1/14}*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^{(3/4)} \\ & *(sqrt(2)*\sqrt{4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4}*\sqrt{289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4} \\ & *(11*a - 2*b) + 2*\sqrt{289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4}*(67*a^3 - 321*a^2*b + 234*a*b^2 - 88*b^3)) \\ & *\sqrt{(35912*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 18656*a*b^3 + 3872*b^4 - \sqrt{2}*\sqrt{4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4})} \\ & *(211*a^2 - 428*a*b + 100*b^2))/(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4) \end{aligned}$$



$$\begin{aligned}
& - 18656*a*b^3 + 3872*b^4 - \sqrt{2}*\sqrt{4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4}*(211*a^2 - 428*a*b + 100*b^2))/(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4))*\sqrt{(14*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*x^2 - 2^{(1/4)}*\sqrt{2/7}*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4))^{(1/4)}*(\sqrt{7}*\sqrt{2}*\sqrt{4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4}*(a - 4*b)*x + \sqrt{7}*(737*a^3 - 717*a^2*b + 348*a*b^2 - 44*b^3)*x)*\sqrt{(35912*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 18656*a*b^3 + 3872*b^4 - \sqrt{2}*\sqrt{4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4}*(211*a^2 - 428*a*b + 100*b^2)))/(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)) + 14*\sqrt{2}*\sqrt{4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4}*(67*a^2 - 53*a*b + 22*b^2))/(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4) - 2^{(3/4)}*\sqrt{2/7}*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^{(3/4)}*(\sqrt{2}*\sqrt{4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4}*\sqrt{289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4}*(11*a - 2*b)*x + 2*\sqrt{289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4}*(67*a^3 - 321*a^2*b + 234*a*b^2 - 88*b^3)*x)*\sqrt{(35912*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 18656*a*b^3 + 3872*b^4 - \sqrt{2}*\sqrt{4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4}*(211*a^2 - 428*a*b + 100*b^2)))/(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)) + 4*\sqrt{7}*\sqrt{2}*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^{(3/2)}*\sqrt{289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4} - 2*\sqrt{7}*(300763*a^6 - 713751*a^5*b + 860883*a^4*b^2 - 617609*a^3*b^3 + 282678*a^2*b^4 - 76956*a*b^5 + 10648*b^6)*\sqrt{289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4}))/((5112971*a^8 - 13336819*a^7*b + 16286963*a^6*b^2 - 11087881*a^5*b^3 + 3832430*a^4*b^4 + 31472*a^3*b^5 - 641872*a^2*b^6 + 265232*a*b^7 - 42592*b^8)) + 784*(4489*a^5 - 25058*a^4*b + 34165*a^3*b^2 - 25360*a^2*b^3 + 9812*a*b^4 - 1936*b^5)*x^3 - 2^{(1/4)}*\sqrt{2/7}*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^{(1/4)}*(\sqrt{7}*\sqrt{2}*((211*a^2 - 428*a*b + 100*b^2)*x^4 + (211*a^2 - 428*a*b + 100*b^2)*x^2 + 422*a^2 - 856*a*b + 200*b^2)*\sqrt{4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4} + 8*\sqrt{7}*((4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*x^4 + 8978*a^4 - 14204*a^3*b + 11514*a^2*b^2 - 4664*a*b^3 + 968*b^4 + (4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*x^2))*\sqrt{(35912*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 18656*a*b^3 + 3872*b^4 - \sqrt{2}*\sqrt{4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4}*(211*a^2 - 428*a*b + 100*b^2)))/(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)) * \log(32*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*x^2 + 16/7*2^{(1/4)}*\sqrt{2/7}*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^{(1/4)}*(\sqrt{7}*\sqrt{2}*\sqrt{4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4}*(a - 4*b)*x + \sqrt{7}*(737*a^3 - 717*a^2*b + 348*a*b^2 - 44*b^3)*x)*\sqrt{(35912*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 18656*a*b^3 + 3872*b^4 - \sqrt{2}*\sqrt{4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4}*(211*a^2 - 428*a*b + 100*b^2)))/(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)) + 32*\sqrt{2}*\sqrt{4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4}*(67*a^2 - 53*a*b + 22*b^2)) + 2^{(1/4)}*\sqrt{2/7}*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^{(1/4)}*(\sqrt{7}*\sqrt{2}*((211*a^2 - 428*a*b + 100*b^2)*x^4 + (211*a^2 - 428*a*b + 100*b^2)*x^2 + 422*a^2 - 856*a*b + 200*b^2)*\sqrt{4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4} + 8*\sqrt{7}*((4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*x^4 + 8978*a^4 - 14204*a^3*b + 11514*a^2*b^2 - 4664*a*b^3 + 968*b^4 + (4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*x^2))*\sqrt{(35912*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 18656*a*b^3 + 3872*b^4 - \sqrt{2}*\sqrt{4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4}*(211*a^2 - 428*a*b + 100*b^2)))/(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4))*\log(32*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*x^2 - 16/7*2^{(1/4)}*\sqrt{2/7}*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^{(1/4)}*(\sqrt{7}*\sqrt{2}*\sqrt{4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4}*(a - 4*b)*x + \sqrt{7}
\end{aligned}$$

)\*(737\*a^3 - 717\*a^2\*b + 348\*a\*b^2 - 44\*b^3)\*x)\*sqrt((35912\*a^4 - 56816\*a^3\*b + 46056\*a^2\*b^2 - 18656\*a\*b^3 + 3872\*b^4 - sqrt(2)\*sqrt(4489\*a^4 - 7102\*a^3\*b + 5757\*a^2\*b^2 - 2332\*a\*b^3 + 484\*b^4))\*(211\*a^2 - 428\*a\*b + 100\*b^2))/(289\*a^4 - 136\*a^3\*b - 120\*a^2\*b^2 + 32\*a\*b^3 + 16\*b^4) + 32\*sqrt(2)\*sqrt(4489\*a^4 - 7102\*a^3\*b + 5757\*a^2\*b^2 - 2332\*a\*b^3 + 484\*b^4)\*(67\*a^2 - 53\*a\*b + 22\*b^2)) - 784\*(13467\*a^5 - 12328\*a^4\*b + 3067\*a^3\*b^2 + 4518\*a^2\*b^3 - 3212\*a\*b^4 + 968\*b^5)\*x)/((4489\*a^4 - 7102\*a^3\*b + 5757\*a^2\*b^2 - 2332\*a\*b^3 + 484\*b^4)\*x^4 + 8978\*a^4 - 14204\*a^3\*b + 11514\*a^2\*b^2 - 4664\*a\*b^3 + 968\*b^4 + (4489\*a^4 - 7102\*a^3\*b + 5757\*a^2\*b^2 - 2332\*a\*b^3 + 484\*b^4)\*x^2)

**giac [B]** time = 0.95, size = 988, normalized size = 3.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/(x^4+x^2+2)^2,x, algorithm="giac")

[Out] 1/401408\*sqrt(7)\*(32\*sqrt(7)\*2^(1/4)\*a\*(sqrt(2) + 4)^(3/2) - 128\*sqrt(7)\*2^(1/4)\*b\*(sqrt(2) + 4)^(3/2) + 96\*sqrt(7)\*2^(1/4)\*a\*sqrt(sqrt(2) + 4)\*(sqrt(2) - 4) - 384\*sqrt(7)\*2^(1/4)\*b\*sqrt(sqrt(2) + 4)\*(sqrt(2) - 4) - 24\*2^(3/4)\*a\*(sqrt(2) + 4)\*sqrt(-8\*sqrt(2) + 32) + 96\*2^(3/4)\*b\*(sqrt(2) + 4)\*sqrt(-8\*sqrt(2) + 32) + 2^(3/4)\*a\*(-8\*sqrt(2) + 32)^(3/2) - 4\*2^(3/4)\*b\*(-8\*sqrt(2) + 32)^(3/2) + 1408\*sqrt(7)\*2^(3/4)\*a\*sqrt(sqrt(2) + 4) - 256\*sqrt(7)\*2^(3/4)\*b\*sqrt(sqrt(2) + 4) - 704\*2^(1/4)\*a\*sqrt(-8\*sqrt(2) + 32) + 128\*2^(1/4)\*b\*sqrt(-8\*sqrt(2) + 32))\*arctan(2\*2^(3/4)\*sqrt(1/2)\*(x + 2^(1/4)\*sqrt(-1/8\*sqrt(2) + 1/2))/sqrt(sqrt(2) + 4)) + 1/401408\*sqrt(7)\*(32\*sqrt(7)\*2^(1/4)\*a\*(sqrt(2) + 4)^(3/2) - 128\*sqrt(7)\*2^(1/4)\*b\*(sqrt(2) + 4)^(3/2) + 96\*sqrt(7)\*2^(1/4)\*a\*sqrt(sqrt(2) + 4)\*(sqrt(2) - 4) - 384\*sqrt(7)\*2^(1/4)\*b\*sqrt(sqrt(2) + 4)\*(sqrt(2) - 4) - 24\*2^(3/4)\*a\*(sqrt(2) + 4)\*sqrt(-8\*sqrt(2) + 32) + 96\*2^(3/4)\*b\*(sqrt(2) + 4)\*sqrt(-8\*sqrt(2) + 32) + 2^(3/4)\*a\*(-8\*sqrt(2) + 32)^(3/2) - 4\*2^(3/4)\*b\*(-8\*sqrt(2) + 32)^(3/2) + 1408\*sqrt(7)\*2^(3/4)\*a\*sqrt(sqrt(2) + 4) - 256\*sqrt(7)\*2^(3/4)\*b\*sqrt(sqrt(2) + 4) - 704\*2^(1/4)\*a\*sqrt(-8\*sqrt(2) + 32) + 128\*2^(1/4)\*b\*sqrt(-8\*sqrt(2) + 32))\*arctan(2\*2^(3/4)\*sqrt(1/2)\*(x - 2^(1/4)\*sqrt(-1/8\*sqrt(2) + 1/2))/sqrt(sqrt(2) + 4)) + 1/802816\*sqrt(7)\*(24\*sqrt(7)\*2^(3/4)\*a\*(sqrt(2) + 4)\*sqrt(-8\*sqrt(2) + 32) - 96\*sqrt(7)\*2^(3/4)\*b\*(sqrt(2) + 4)\*sqrt(-8\*sqrt(2) + 32) - sqrt(7)\*2^(3/4)\*a\*(-8\*sqrt(2) + 32)^(3/2) + 4\*sqrt(7)\*2^(3/4)\*b\*(-8\*sqrt(2) + 32)^(3/2) + 32\*2^(1/4)\*a\*(sqrt(2) + 4)^(3/2) - 128\*2^(1/4)\*b\*(sqrt(2) + 4)^(3/2) + 96\*2^(1/4)\*a\*sqrt(sqrt(2) + 4)\*(sqrt(2) - 4) - 384\*2^(1/4)\*b\*sqrt(sqrt(2) + 4)\*(sqrt(2) - 4) + 1408\*2^(3/4)\*a\*sqrt(sqrt(2) + 4) - 256\*2^(3/4)\*b\*sqrt(sqrt(2) + 4) + 704\*sqrt(7)\*2^(1/4)\*a\*sqrt(-8\*sqrt(2) + 32) - 128\*sqrt(7)\*2^(1/4)\*b\*sqrt(-8\*sqrt(2) + 32))\*log(x^2 + 2\*2^(1/4)\*x\*sqrt(-1/8\*sqrt(2) + 1/2) + sqrt(2)) - 1/802816\*sqrt(7)\*(24\*sqrt(7)\*2^(3/4)\*a\*(sqrt(2) + 4)\*sqrt(-8\*sqrt(2) + 32) - 96\*sqrt(7)\*2^(3/4)\*b\*(sqrt(2) + 4)\*sqrt(-8\*sqrt(2) + 32) - sqrt(7)\*2^(3/4)\*a\*(-8\*sqrt(2) + 32)^(3/2) + 4\*sqrt(7)\*2^(3/4)\*b\*(-8\*sqrt(2) + 32)^(3/2) + 32\*2^(1/4)\*a\*(sqrt(2) + 4)^(3/2) - 128\*2^(1/4)\*b\*(sqrt(2) + 4)^(3/2) + 96\*2^(1/4)\*a\*sqrt(sqrt(2) + 4)\*(sqrt(2) - 4) - 384\*2^(1/4)\*b\*sqrt(sqrt(2) + 4)\*(sqrt(2) - 4) + 1408\*2^(3/4)\*a\*sqrt(sqrt(2) + 4) - 256\*2^(3/4)\*b\*sqrt(sqrt(2) + 4) + 704\*sqrt(7)\*2^(1/4)\*a\*sqrt(-8\*sqrt(2) + 32) - 128\*sqrt(7)\*2^(1/4)\*b\*sqrt(-8\*sqrt(2) + 32))\*log(x^2 - 2\*2^(1/4)\*x\*sqrt(-1/8\*sqrt(2) + 1/2) + sqrt(2)) - 1/28\*(a\*x^3 - 4\*b\*x^3 - 3\*a\*x - 2\*b\*x)/(x^4 + x^2 + 2)

**maple [B]** time = 0.31, size = 756, normalized size = 2.39



Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)/(x^4+x^2+2)^2,x)

```
[Out] 1/784*((-14*a-28*2^(1/2)*a+112*b*2^(1/2)+56*b)/(1+2*2^(1/2))*x+1/(1+2*2^(1/2))
*(-1+2*2^(1/2))^(1/2)*(-70*a-42*2^(1/2)*a+56*b*2^(1/2)+28*b))/(x^2+(-1+2
*2^(1/2))^(1/2)*x+2^(1/2))+107/1568/(1+2*2^(1/2))*ln(x^2+(-1+2*2^(1/2))^(1/2)
*x+2^(1/2))*(-1+2*2^(1/2))^(1/2)*2^(1/2)*a-25/784/(1+2*2^(1/2))*ln(x^2+(-
1+2*2^(1/2))^(1/2)*x+2^(1/2))*(-1+2*2^(1/2))^(1/2)*2^(1/2)*b+53/784/(1+2*2^
(1/2))*ln(x^2+(-1+2*2^(1/2))^(1/2)*x+2^(1/2))*(-1+2*2^(1/2))^(1/2)*a-11/196
/(1+2*2^(1/2))*ln(x^2+(-1+2*2^(1/2))^(1/2)*x+2^(1/2))*(-1+2*2^(1/2))^(1/2)*
b+1/16/(1+2*2^(1/2))^(3/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(
1/2))*2^(1/2)*a+3/8/(1+2*2^(1/2))^(3/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2))/
(1+2*2^(1/2))^(1/2))*a+1/8/(1+2*2^(1/2))^(3/2)*arctan((2*x+(-1+2*2^(1/2))^(
1/2))/(1+2*2^(1/2))^(1/2))*b*2^(1/2)-1/784*(-(-14*a-28*2^(1/2)*a+112*b*2^(1
/2)+56*b)/(1+2*2^(1/2))*x+1/(1+2*2^(1/2))*(-1+2*2^(1/2))^(1/2)*(-70*a-42*2^
(1/2)*a+56*b*2^(1/2)+28*b))/(x^2-(-1+2*2^(1/2))^(1/2)*x+2^(1/2))-107/1568/(
1+2*2^(1/2))*ln(x^2-(-1+2*2^(1/2))^(1/2)*x+2^(1/2))*(-1+2*2^(1/2))^(1/2)*2^
(1/2)*a+25/784/(1+2*2^(1/2))*ln(x^2-(-1+2*2^(1/2))^(1/2)*x+2^(1/2))*(-1+2*2
^(1/2))^(1/2)*2^(1/2)*b-53/784/(1+2*2^(1/2))*ln(x^2-(-1+2*2^(1/2))^(1/2)*x+
2^(1/2))*(-1+2*2^(1/2))^(1/2)*a+11/196/(1+2*2^(1/2))*ln(x^2-(-1+2*2^(1/2))^(
1/2)*x+2^(1/2))*(-1+2*2^(1/2))^(1/2)*b+1/16/(1+2*2^(1/2))^(3/2)*arctan((2*
x-(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*2^(1/2)*a+3/8/(1+2*2^(1/2))^(3
/2)*arctan((2*x-(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*a+1/8/(1+2*2^(1/
2))^(3/2)*arctan((2*x-(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*b*2^(1/2)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{(a-4b)x^3-(3a+2b)x}{28(x^4+x^2+2)} + \frac{1}{28} \int -\frac{(a-4b)x^2-11a+2b}{x^4+x^2+2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+2)^2,x, algorithm="maxima")
```

```
[Out] -1/28*((a-4*b)*x^3-(3*a+2*b)*x)/(x^4+x^2+2)+1/28*integrate(-((a-4*b)*x^2-11*a+2*b)/(x^4+x^2+2),x)
```

**mupad** [B] time = 4.50, size = 1491, normalized size = 4.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^2)/(x^2+x^4+2)^2,x)
```

```
[Out] atan((b^2*x*((7^(1/2)*a^2*17i)/12544-(107*a*b)/21952-(7^(1/2)*b^2*1i)/3136+(211*a^2)/87808+(25*b^2)/21952-(7^(1/2)*a*b*1i)/3136)^(1/2)*1i)/(4*((7^(1/2)*a^3*187i)/6272+(7^(1/2)*b^3*1i)/784+(3*a*b^2)/1568-(183*a^2*b)/3136+(255*a^3)/6272+(9*b^3)/784-(7^(1/2)*a*b^2*9i)/1568-(7^(1/2)*a^2*b*39i)/3136))-(a^2*x*((7^(1/2)*a^2*17i)/12544-(107*a*b)/21952-(7^(1/2)*b^2*1i)/3136+(211*a^2)/87808+(25*b^2)/21952-(7^(1/2)*a*b*1i)/3136)^(1/2)*17i)/(16*((7^(1/2)*a^3*187i)/6272+(7^(1/2)*b^3*1i)/784+(3*a*b^2)/1568-(183*a^2*b)/3136+(255*a^3)/6272+(9*b^3)/784-(7^(1/2)*a*b^2*9i)/1568-(7^(1/2)*a^2*b*39i)/3136))+(a*b*x*((7^(1/2)*a^2*17i)/12544-(107*a*b)/21952-(7^(1/2)*b^2*1i)/3136+(211*a^2)/87808+(25*b^2)/21952-(7^(1/2)*a*b*1i)/3136)^(1/2)*1i)/(4*((7^(1/2)*a^3*187i)/6272+(7^(1/2)*b^3*1i)/784+(3*a*b^2)/1568-(183*a^2*b)/3136+(255*a^3)/6272+(9*b^3)/784-(7^(1/2)*a*b^2*9i)/1568-(7^(1/2)*a^2*b*39i)/3136))-(17*7^(1/2)*a^2*x*((7^(1/2)*a^2*17i)/12544-(107*a*b)/21952-(7^(1/2)*b^2*1i)/3136+(211*a^2)/87808+(25*b^2)/21952-(7^(1/2)*a*b*1i)/3136)^(1/2))/(112*((7^(1/2)*a^3*187i)/6272+(7^(1/2)*b^3*1i)/784+(3*a*b^2)/1568-(183*a^2*b)/3136+(255*a^3)/6272+(9*b^3)/784-(7^(1/2)*a*b^2*9i)/1568-(7^(1/2)*a^2*b*39i)/3136))+(7^(1/2)*b^2*x*((7^(1/2)*a^2*17i)/12544-(107*a*b)/21952-(7^(1/2)*b^2*1i)/3136+(211*a^2)/87808+(25*b^2)/21952-(7^(1/2)*a*b*1i)/3136)^(1/2))/(28*((7^(1/2)*a^3*187i)/6272+(7^(1/2)*b^3*1i)/784+(3*a
```

$$\begin{aligned} & *b^2)/1568 - (183*a^2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 - (7^{(1/2)}*a*b \\ & ^2*9i)/1568 - (7^{(1/2)}*a^2*b*39i)/3136)) + (7^{(1/2)}*a*b*x*((7^{(1/2)}*a^2*17i \\ & )/12544 - (107*a*b)/21952 - (7^{(1/2)}*b^2*1i)/3136 + (211*a^2)/87808 + (25*b \\ & ^2)/21952 - (7^{(1/2)}*a*b*1i)/3136)^{(1/2)})/(28*((7^{(1/2)}*a^3*187i)/6272 + (7 \\ & ^{(1/2)}*b^3*1i)/784 + (3*a*b^2)/1568 - (183*a^2*b)/3136 + (255*a^3)/6272 + ( \\ & 9*b^3)/784 - (7^{(1/2)}*a*b^2*9i)/1568 - (7^{(1/2)}*a^2*b*39i)/3136)))*((7^{(1/2)} \\ & )*a^2*17i)/12544 - (107*a*b)/21952 - (7^{(1/2)}*b^2*1i)/3136 + (211*a^2)/8780 \\ & 8 + (25*b^2)/21952 - (7^{(1/2)}*a*b*1i)/3136)^{(1/2)}*2i - \operatorname{atan}((a^2*x*((7^{(1/2)} \\ & )*b^2*1i)/3136 - (7^{(1/2)}*a^2*17i)/12544 - (107*a*b)/21952 + (211*a^2)/8780 \\ & 8 + (25*b^2)/21952 + (7^{(1/2)}*a*b*1i)/3136)^{(1/2)}*17i)/(16*((3*a*b^2)/1568 \\ & - (7^{(1/2)}*b^3*1i)/784 - (7^{(1/2)}*a^3*187i)/6272 - (183*a^2*b)/3136 + (255*a \\ & ^3)/6272 + (9*b^3)/784 + (7^{(1/2)}*a*b^2*9i)/1568 + (7^{(1/2)}*a^2*b*39i)/313 \\ & 6)) - (b^2*x*((7^{(1/2)}*b^2*1i)/3136 - (7^{(1/2)}*a^2*17i)/12544 - (107*a*b)/2 \\ & 1952 + (211*a^2)/87808 + (25*b^2)/21952 + (7^{(1/2)}*a*b*1i)/3136)^{(1/2)}*1i)/ \\ & (4*((3*a*b^2)/1568 - (7^{(1/2)}*b^3*1i)/784 - (7^{(1/2)}*a^3*187i)/6272 - (183*a \\ & ^2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 + (7^{(1/2)}*a*b^2*9i)/1568 + (7^{( \\ & 1/2)}*a^2*b*39i)/3136)) - (a*b*x*((7^{(1/2)}*b^2*1i)/3136 - (7^{(1/2)}*a^2*17i)/ \\ & 12544 - (107*a*b)/21952 + (211*a^2)/87808 + (25*b^2)/21952 + (7^{(1/2)}*a*b*1 \\ & i)/3136)^{(1/2)}*1i)/(4*((3*a*b^2)/1568 - (7^{(1/2)}*b^3*1i)/784 - (7^{(1/2)}*a^3 \\ & *187i)/6272 - (183*a^2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 + (7^{(1/2)}*a* \\ & b^2*9i)/1568 + (7^{(1/2)}*a^2*b*39i)/3136)) - (17*7^{(1/2)}*a^2*x*((7^{(1/2)}*b^2 \\ & *1i)/3136 - (7^{(1/2)}*a^2*17i)/12544 - (107*a*b)/21952 + (211*a^2)/87808 + ( \\ & 25*b^2)/21952 + (7^{(1/2)}*a*b*1i)/3136)^{(1/2)})/(112*((3*a*b^2)/1568 - (7^{(1/ \\ & 2)}*b^3*1i)/784 - (7^{(1/2)}*a^3*187i)/6272 - (183*a^2*b)/3136 + (255*a^3)/627 \\ & 2 + (9*b^3)/784 + (7^{(1/2)}*a*b^2*9i)/1568 + (7^{(1/2)}*a^2*b*39i)/3136)) + (7 \\ & ^{(1/2)}*b^2*x*((7^{(1/2)}*b^2*1i)/3136 - (7^{(1/2)}*a^2*17i)/12544 - (107*a*b)/2 \\ & 1952 + (211*a^2)/87808 + (25*b^2)/21952 + (7^{(1/2)}*a*b*1i)/3136)^{(1/2)})/(28 \\ & *((3*a*b^2)/1568 - (7^{(1/2)}*b^3*1i)/784 - (7^{(1/2)}*a^3*187i)/6272 - (183*a^ \\ & 2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 + (7^{(1/2)}*a*b^2*9i)/1568 + (7^{(1/ \\ & 2)}*a^2*b*39i)/3136)) + (7^{(1/2)}*a*b*x*((7^{(1/2)}*b^2*1i)/3136 - (7^{(1/2)}*a^2 \\ & *17i)/12544 - (107*a*b)/21952 + (211*a^2)/87808 + (25*b^2)/21952 + (7^{(1/2)} \\ & )*a*b*1i)/3136)^{(1/2)})/(28*((3*a*b^2)/1568 - (7^{(1/2)}*b^3*1i)/784 - (7^{(1/2)} \\ & )*a^3*187i)/6272 - (183*a^2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 + (7^{(1/2)} \\ & )*a*b^2*9i)/1568 + (7^{(1/2)}*a^2*b*39i)/3136)))*((7^{(1/2)}*b^2*1i)/3136 - (7^{ \\ & (1/2)}*a^2*17i)/12544 - (107*a*b)/21952 + (211*a^2)/87808 + (25*b^2)/21952 + \\ & (7^{(1/2)}*a*b*1i)/3136)^{(1/2)}*2i - (x^3*(a/28 - b/7) - x*((3*a)/28 + b/14)) \\ & / (x^2 + x^4 + 2) \end{aligned}$$

**sympy [A]** time = 1.80, size = 165, normalized size = 0.52

$$\frac{x^3(-a+4b)+x(3a+2b)}{28x^4+28x^2+56} + \operatorname{RootSum}\left(240945152t^4 + t^2(-1157968a^2 + 2348864ab - 548800b^2) + 4489a^4 - 7102a^3b + 5757a^2b^2 - 2332ab^3 + 484b^4, \left(t \mapsto t \log\left(x + \frac{2634240t^3a - 3161088t^3b + 11996ta^3 + 12792ta^2b - 21936tab^2 + 4384tb^3}{1139a^4 - 1169a^3b + 318a^2b^2 + 124ab^3 - 88b^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)/(x\*\*4+x\*\*2+2)\*\*2,x)

[Out] (x\*\*3\*(-a + 4\*b) + x\*(3\*a + 2\*b))/(28\*x\*\*4 + 28\*x\*\*2 + 56) + RootSum(240945 152\*\_t\*\*4 + \_t\*\*2\*(-1157968\*a\*\*2 + 2348864\*a\*b - 548800\*b\*\*2) + 4489\*a\*\*4 - 7102\*a\*\*3\*b + 5757\*a\*\*2\*b\*\*2 - 2332\*a\*b\*\*3 + 484\*b\*\*4, Lambda(\_t, \_t\*log(x + (2634240\*\_t\*\*3\*a - 3161088\*\_t\*\*3\*b + 11996\*\_t\*a\*\*3 + 12792\*\_t\*a\*\*2\*b - 21936\*\_t\*a\*b\*\*2 + 4384\*\_t\*b\*\*3)/(1139\*a\*\*4 - 1169\*a\*\*3\*b + 318\*a\*\*2\*b\*\*2 + 124\*a\*b\*\*3 - 88\*b\*\*4))))

$$3.90 \quad \int \frac{\sqrt{2-x^2}}{1-\sqrt{2}x^2+x^4} dx$$

**Optimal.** Leaf size=160

$$-\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(x^2-\sqrt{2+\sqrt{2}}x+1\right)+\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(x^2+\sqrt{2+\sqrt{2}}x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}$$

**Rubi [A]** time = 0.15, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$ , Rules used = {1169, 634, 618, 204, 628}

$$-\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(x^2-\sqrt{2+\sqrt{2}}x+1\right)+\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(x^2+\sqrt{2+\sqrt{2}}x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[2] - x^2)/(1 - Sqrt[2]\*x^2 + x^4), x]

[Out] -ArcTan[(Sqrt[2 + Sqrt[2]] - 2\*x)/Sqrt[2 - Sqrt[2]]]/(2\*Sqrt[2 + Sqrt[2]]) + ArcTan[(Sqrt[2 + Sqrt[2]] + 2\*x)/Sqrt[2 - Sqrt[2]]]/(2\*Sqrt[2 + Sqrt[2]]) - (Sqrt[1 + 1/Sqrt[2]]\*Log[1 - Sqrt[2 + Sqrt[2]]\*x + x^2])/4 + (Sqrt[1 + 1/Sqrt[2]]\*Log[1 + Sqrt[2 + Sqrt[2]]\*x + x^2])/4

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 1169

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2\*q - b/c, 2]}, Dist[1/(2\*c\*q\*r), Int[(d\*r - (d - e\*q)\*x)/(q - r\*x + x^2), x], x] + Dist[1/(2\*c\*q\*r), Int[(d\*r + (d - e\*q)\*x)/(q + r\*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NegQ[b^2 - 4\*a\*c]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{2} - x^2}{1 - \sqrt{2}x^2 + x^4} dx &= \frac{\int \frac{\sqrt{2(2+\sqrt{2})} - (1+\sqrt{2})x}{1 - \sqrt{2+\sqrt{2}}x + x^2} dx}{2\sqrt{2+\sqrt{2}}} + \frac{\int \frac{\sqrt{2(2+\sqrt{2})} + (1+\sqrt{2})x}{1 + \sqrt{2+\sqrt{2}}x + x^2} dx}{2\sqrt{2+\sqrt{2}}} \\
&= \frac{1}{4}\sqrt{3-2\sqrt{2}} \int \frac{1}{1 - \sqrt{2+\sqrt{2}}x + x^2} dx + \frac{1}{4}\sqrt{3-2\sqrt{2}} \int \frac{1}{1 + \sqrt{2+\sqrt{2}}x + x^2} dx + \dots \\
&= -\frac{1}{4}\sqrt{1 + \frac{1}{\sqrt{2}}} \log\left(1 - \sqrt{2+\sqrt{2}}x + x^2\right) + \frac{1}{4}\sqrt{1 + \frac{1}{\sqrt{2}}} \log\left(1 + \sqrt{2+\sqrt{2}}x + x^2\right) - \frac{1}{2}\sqrt{\dots} \\
&= -\frac{1}{2}\sqrt{\frac{1}{2}(2-\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}} - 2x}{\sqrt{2-\sqrt{2}}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}(2-\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}} + 2x}{\sqrt{2-\sqrt{2}}}\right) - \dots
\end{aligned}$$

**Mathematica [C]** time = 0.04, size = 53, normalized size = 0.33

$$\frac{\sqrt{-1-i} \tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{-1-i}}\right) + \sqrt{-1+i} \tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{-1+i}}\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[2] - x^2)/(1 - Sqrt[2]\*x^2 + x^4), x]

[Out] (Sqrt[-1 - I]\*ArcTan[(2^(1/4)\*x)/Sqrt[-1 - I]] + Sqrt[-1 + I]\*ArcTan[(2^(1/4)\*x)/Sqrt[-1 + I]])/2^(3/4)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2} - x^2}{1 - \sqrt{2}x^2 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(Sqrt[2] - x^2)/(1 - Sqrt[2]\*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(Sqrt[2] - x^2)/(1 - Sqrt[2]\*x^2 + x^4), x]

**fricas [C]** time = 1.01, size = 97, normalized size = 0.61

$$\frac{1}{4}\sqrt{(i+1)\sqrt{2}} \log\left(x + \frac{1}{2}\sqrt{2}\sqrt{(i+1)\sqrt{2}}\right) - \frac{1}{4}\sqrt{(i+1)\sqrt{2}} \log\left(x - \frac{1}{2}\sqrt{2}\sqrt{(i+1)\sqrt{2}}\right) + \frac{1}{4}\sqrt{-(i-1)\sqrt{2}} \log\left(x + \frac{1}{2}\sqrt{2}\sqrt{-(i-1)\sqrt{2}}\right) - \frac{1}{4}\sqrt{-(i-1)\sqrt{2}} \log\left(x - \frac{1}{2}\sqrt{2}\sqrt{-(i-1)\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2^(1/2))/(1+x^4-x^2\*2^(1/2)), x, algorithm="fricas")

[Out] 1/4\*sqrt((I + 1)\*sqrt(2))\*log(x + 1/2\*sqrt(2)\*sqrt((I + 1)\*sqrt(2))) - 1/4\*sqrt((I + 1)\*sqrt(2))\*log(x - 1/2\*sqrt(2)\*sqrt((I + 1)\*sqrt(2))) + 1/4\*sqrt(-(I - 1)\*sqrt(2))\*log(x + 1/2\*sqrt(2)\*sqrt(-(I - 1)\*sqrt(2))) - 1/4\*sqrt(-(I - 1)\*sqrt(2))\*log(x - 1/2\*sqrt(2)\*sqrt(-(I - 1)\*sqrt(2)))

**giac [A]** time = 0.38, size = 122, normalized size = 0.76

$$\frac{1}{4}\sqrt{-2\sqrt{2}+4} \arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right) + \frac{1}{4}\sqrt{-2\sqrt{2}+4} \arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right) + \frac{1}{8}\sqrt{2\sqrt{2}+4} \log\left(x^2+x\sqrt{\sqrt{2}+2}+1\right) - \frac{1}{8}\sqrt{2\sqrt{2}+4} \log\left(x^2-x\sqrt{\sqrt{2}+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2^(1/2))/(1+x^4-x^2\*2^(1/2)),x, algorithm="giac")

[Out] 1/4\*sqrt(-2\*sqrt(2) + 4)\*arctan((2\*x + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) + 1/4\*sqrt(-2\*sqrt(2) + 4)\*arctan((2\*x - sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) + 1/8\*sqrt(2\*sqrt(2) + 4)\*log(x^2 + x\*sqrt(sqrt(2) + 2) + 1) - 1/8\*sqrt(2\*sqrt(2) + 4)\*log(x^2 - x\*sqrt(sqrt(2) + 2) + 1)

**maple [A]** time = 0.09, size = 199, normalized size = 1.24

$$\frac{\sqrt{2} \arctan\left(\frac{2x-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right) - \arctan\left(\frac{2x-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right) + \sqrt{2} \arctan\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right) - \arctan\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right) - \frac{\sqrt{2} \sqrt{2+\sqrt{2}} \ln\left(x^2 - \sqrt{2+\sqrt{2}} x + 1\right) + \sqrt{2} \sqrt{2+\sqrt{2}} \ln\left(x^2 + \sqrt{2+\sqrt{2}} x + 1\right)}{8}}{2\sqrt{2-\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+2^(1/2))/(1+x^4-x^2\*2^(1/2)),x)

[Out] 1/8\*2^(1/2)\*(2+2^(1/2))^(1/2)\*ln(1+x^2+x\*(2+2^(1/2))^(1/2))+1/2/(2-2^(1/2))^(1/2)\*arctan((2\*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))\*2^(1/2)-1/2/(2-2^(1/2))^(1/2)\*arctan((2\*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))-1/8\*2^(1/2)\*(2+2^(1/2))^(1/2)\*ln(1+x^2-x\*(2+2^(1/2))^(1/2))+1/2/(2-2^(1/2))^(1/2)\*arctan((2\*x-(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))\*2^(1/2)-1/2/(2-2^(1/2))^(1/2)\*arctan((2\*x-(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 - \sqrt{2}}{x^4 - \sqrt{2}x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2^(1/2))/(1+x^4-x^2\*2^(1/2)),x, algorithm="maxima")

[Out] -integrate((x^2 - sqrt(2))/(x^4 - sqrt(2)\*x^2 + 1), x)

**mupad [B]** time = 4.96, size = 121, normalized size = 0.76

$$-\operatorname{atan}\left(x\sqrt{\frac{\sqrt{2}-\sqrt{8}1i}{16}-\frac{\sqrt{8}1i}{32}}2i-\frac{\sqrt{2}\sqrt{8}x\sqrt{\frac{\sqrt{2}-\sqrt{8}1i}{16}-\frac{\sqrt{8}1i}{32}}}{2}\right)\sqrt{\frac{\sqrt{2}-\sqrt{8}1i}{16}-\frac{\sqrt{8}1i}{32}}2i-\operatorname{atan}\left(x\sqrt{\frac{\sqrt{2}+\sqrt{8}1i}{16}+\frac{\sqrt{8}1i}{32}}2i+\frac{\sqrt{2}\sqrt{8}x\sqrt{\frac{\sqrt{2}+\sqrt{8}1i}{16}+\frac{\sqrt{8}1i}{32}}}{2}\right)\sqrt{\frac{\sqrt{2}+\sqrt{8}1i}{16}+\frac{\sqrt{8}1i}{32}}2i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2^(1/2) - x^2)/(x^4 - 2^(1/2)\*x^2 + 1),x)

[Out] -atan(x\*(2^(1/2)/16 - (8^(1/2)\*1i)/32)^(1/2)\*2i - (2^(1/2)\*8^(1/2)\*x\*(2^(1/2)/16 - (8^(1/2)\*1i)/32)^(1/2))/2\*(2^(1/2)/16 - (8^(1/2)\*1i)/32)^(1/2)\*2i - atan(x\*(2^(1/2)/16 + (8^(1/2)\*1i)/32)^(1/2)\*2i + (2^(1/2)\*8^(1/2)\*x\*(2^(1/2)/16 + (8^(1/2)\*1i)/32)^(1/2))/2\*(2^(1/2)/16 + (8^(1/2)\*1i)/32)^(1/2)\*2i

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*2+2\*\*(1/2))/(1+x\*\*4-x\*\*2\*2\*\*(1/2)),x)

[Out] Exception raised: PolynomialError

$$3.91 \quad \int \frac{\sqrt{2+x^2}}{1+\sqrt{2}x^2+x^4} dx$$

**Optimal.** Leaf size=172

$$-\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(x^2-\sqrt{2-\sqrt{2}}x+1\right)+\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(x^2+\sqrt{2-\sqrt{2}}x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{2x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}$$

**Rubi [A]** time = 0.14, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$ , Rules used = {1169, 634, 618, 204, 628}

$$-\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(x^2-\sqrt{2-\sqrt{2}}x+1\right)+\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(x^2+\sqrt{2-\sqrt{2}}x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{2x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2] + x^2)/(1 + Sqrt[2]*x^2 + x^4), x]
```

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2 - Sqrt[2]])
+ ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2 - Sqrt[2]])
- (Sqrt[1 - 1/Sqrt[2]]*Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2])/4 + (Sqrt[1 - 1
/Sqrt[2]]*Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/4
```

#### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

#### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

#### Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
(d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```



Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{2} + x^2}{1 + \sqrt{2}x^2 + x^4} dx &= \frac{\int \frac{\sqrt{2(2-\sqrt{2})} - (-1+\sqrt{2})x}{1-\sqrt{2-\sqrt{2}}x+x^2} dx}{2\sqrt{2-\sqrt{2}}} + \frac{\int \frac{\sqrt{2(2-\sqrt{2})} + (-1+\sqrt{2})x}{1+\sqrt{2-\sqrt{2}}x+x^2} dx}{2\sqrt{2-\sqrt{2}}} \\
&= \frac{(1-\sqrt{2}) \int \frac{-\sqrt{2-\sqrt{2}}+2x}{1-\sqrt{2-\sqrt{2}}x+x^2} dx}{4\sqrt{2-\sqrt{2}}} + \frac{(-1+\sqrt{2}) \int \frac{\sqrt{2-\sqrt{2}}+2x}{1+\sqrt{2-\sqrt{2}}x+x^2} dx}{4\sqrt{2-\sqrt{2}}} + \frac{1}{4}\sqrt{3+2\sqrt{2}} \int \frac{1}{1-\sqrt{2-\sqrt{2}}x+x^2} dx \\
&= -\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}} \log\left(1-\sqrt{2-\sqrt{2}}x+x^2\right) + \frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}} \log\left(1+\sqrt{2-\sqrt{2}}x+x^2\right) - \frac{1}{4}\sqrt{3+2\sqrt{2}} \log\left(1-\sqrt{2-\sqrt{2}}x+x^2\right) \\
&= -\frac{1}{2}\sqrt{\frac{1}{2}(2+\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}(2+\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}+2x}{\sqrt{2+\sqrt{2}}}\right)
\end{aligned}$$

**Mathematica [C]** time = 0.04, size = 53, normalized size = 0.31

$$\frac{\sqrt{1-i} \tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1-i}}\right) + \sqrt{1+i} \tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+i}}\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[2] + x^2)/(1 + Sqrt[2]\*x^2 + x^4), x]

[Out] (Sqrt[1 - I]\*ArcTan[(2^(1/4)\*x)/Sqrt[1 - I]] + Sqrt[1 + I]\*ArcTan[(2^(1/4)\*x)/Sqrt[1 + I]])/2^(3/4)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2} + x^2}{1 + \sqrt{2}x^2 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(Sqrt[2] + x^2)/(1 + Sqrt[2]\*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(Sqrt[2] + x^2)/(1 + Sqrt[2]\*x^2 + x^4), x]

**fricas [C]** time = 1.13, size = 97, normalized size = 0.56

$$\frac{1}{4}\sqrt{(i-1)\sqrt{2}} \log\left(x + \frac{1}{2}\sqrt{2}\sqrt{(i-1)\sqrt{2}}\right) - \frac{1}{4}\sqrt{(i-1)\sqrt{2}} \log\left(x - \frac{1}{2}\sqrt{2}\sqrt{(i-1)\sqrt{2}}\right) + \frac{1}{4}\sqrt{-(i+1)\sqrt{2}} \log\left(x + \frac{1}{2}\sqrt{2}\sqrt{-(i+1)\sqrt{2}}\right) - \frac{1}{4}\sqrt{-(i+1)\sqrt{2}} \log\left(x - \frac{1}{2}\sqrt{2}\sqrt{-(i+1)\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2^(1/2))/(1+x^4+x^2\*2^(1/2)),x, algorithm="fricas")

[Out] 1/4\*sqrt((I - 1)\*sqrt(2))\*log(x + 1/2\*sqrt(2)\*sqrt((I - 1)\*sqrt(2))) - 1/4\*sqrt((I - 1)\*sqrt(2))\*log(x - 1/2\*sqrt(2)\*sqrt((I - 1)\*sqrt(2))) + 1/4\*sqrt(-(I + 1)\*sqrt(2))\*log(x + 1/2\*sqrt(2)\*sqrt(-(I + 1)\*sqrt(2))) - 1/4\*sqrt(-(I + 1)\*sqrt(2))\*log(x - 1/2\*sqrt(2)\*sqrt(-(I + 1)\*sqrt(2)))

**giac [A]** time = 0.33, size = 126, normalized size = 0.73

$$\frac{1}{4}\sqrt{2\sqrt{2}+4} \arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right) + \frac{1}{4}\sqrt{2\sqrt{2}+4} \arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right) + \frac{1}{8}\sqrt{-2\sqrt{2}+4} \log\left(x^2+x\sqrt{-\sqrt{2}+2}+1\right) - \frac{1}{8}\sqrt{-2\sqrt{2}+4} \log\left(x^2-x\sqrt{-\sqrt{2}+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2^(1/2))/(1+x^4+x^2\*2^(1/2)),x, algorithm="giac")

[Out] 1/4\*sqrt(2\*sqrt(2) + 4)\*arctan((2\*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) + 1/4\*sqrt(2\*sqrt(2) + 4)\*arctan((2\*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) + 1/8\*sqrt(-2\*sqrt(2) + 4)\*log(x^2 + x\*sqrt(-sqrt(2) + 2) + 1) - 1/8\*sqrt(-2\*sqrt(2) + 4)\*log(x^2 - x\*sqrt(-sqrt(2) + 2) + 1)

**maple [A]** time = 0.09, size = 199, normalized size = 1.16

$$\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}} + \frac{\sqrt{2}\arctan\left(\frac{2x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}} + \frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}} + \frac{\sqrt{2}\arctan\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}} - \frac{\sqrt{2}\sqrt{2-\sqrt{2}}\ln\left(x^2-\sqrt{2-\sqrt{2}}x+1\right)}{8} + \frac{\sqrt{2}\sqrt{2-\sqrt{2}}\ln\left(x^2+\sqrt{2-\sqrt{2}}x+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+2^(1/2))/(1+x^4+x^2\*2^(1/2)),x)

[Out] 1/8\*2^(1/2)\*(2-2^(1/2))^(1/2)\*ln(1+x^2+x\*(2-2^(1/2))^(1/2))+1/2/(2+2^(1/2))^(1/2)\*arctan((2\*x+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))+1/2/(2+2^(1/2))^(1/2)\*arctan((2\*x+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))\*2^(1/2)-1/8\*2^(1/2)\*(2-2^(1/2))^(1/2)\*ln(1+x^2-x\*(2-2^(1/2))^(1/2))+1/2/(2+2^(1/2))^(1/2)\*arctan((2\*x-(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))+1/2/(2+2^(1/2))^(1/2)\*arctan((2\*x-(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))\*2^(1/2)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + \sqrt{2}}{x^4 + \sqrt{2}x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2^(1/2))/(1+x^4+x^2\*2^(1/2)),x, algorithm="maxima")

[Out] integrate((x^2 + sqrt(2))/(x^4 + sqrt(2)\*x^2 + 1), x)

**mupad [B]** time = 4.95, size = 121, normalized size = 0.70

$$\operatorname{atan}\left(x\sqrt{-\frac{\sqrt{2}}{16}-\frac{\sqrt{8}1i}{32}}\right)^{2i} + \frac{\sqrt{2}\sqrt{8}x\sqrt{-\frac{\sqrt{2}}{16}-\frac{\sqrt{8}1i}{32}}}{2}\sqrt{-\frac{\sqrt{2}}{16}-\frac{\sqrt{8}1i}{32}}\right)^{2i} + \operatorname{atan}\left(x\sqrt{-\frac{\sqrt{2}}{16}+\frac{\sqrt{8}1i}{32}}\right)^{2i} - \frac{\sqrt{2}\sqrt{8}x\sqrt{-\frac{\sqrt{2}}{16}+\frac{\sqrt{8}1i}{32}}}{2}\sqrt{-\frac{\sqrt{2}}{16}+\frac{\sqrt{8}1i}{32}}\right)^{2i}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2^(1/2) + x^2)/(2^(1/2)\*x^2 + x^4 + 1),x)

[Out] atan(x\*(-2^(1/2)/16 - (8^(1/2)\*1i)/32)^(1/2)\*2i + (2^(1/2)\*8^(1/2)\*x\*(-2^(1/2)/16 - (8^(1/2)\*1i)/32)^(1/2))/2\*(-2^(1/2)/16 - (8^(1/2)\*1i)/32)^(1/2)\*2i + atan(x\*((8^(1/2)\*1i)/32 - 2^(1/2)/16)^(1/2)\*2i - (2^(1/2)\*8^(1/2)\*x\*((8^(1/2)\*1i)/32 - 2^(1/2)/16)^(1/2))/2\*((8^(1/2)\*1i)/32 - 2^(1/2)/16)^(1/2)\*2i

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*2+2\*\*(1/2))/(1+x\*\*4+x\*\*2\*2\*\*(1/2)),x)

[Out] Exception raised: PolynomialError

$$3.92 \quad \int \frac{\sqrt{2-x^2}}{1+bx^2+x^4} dx$$

**Optimal.** Leaf size=160

$$\frac{(1+\sqrt{2})\log(-\sqrt{2-b}x+x^2+1)}{4\sqrt{2-b}} + \frac{(1+\sqrt{2})\log(\sqrt{2-b}x+x^2+1)}{4\sqrt{2-b}} + \frac{(1-\sqrt{2})\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}} - \frac{(1-\sqrt{2})\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}$$

**Rubi [A]** time = 0.12, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$ , Rules used = {1169, 634, 618, 204, 628}

$$\frac{(1+\sqrt{2})\log(-\sqrt{2-b}x+x^2+1)}{4\sqrt{2-b}} + \frac{(1+\sqrt{2})\log(\sqrt{2-b}x+x^2+1)}{4\sqrt{2-b}} + \frac{(1-\sqrt{2})\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}} - \frac{(1-\sqrt{2})\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[2] - x^2)/(1 + b\*x^2 + x^4), x]

[Out] ((1 - Sqrt[2])\*ArcTan[(Sqrt[2 - b] - 2\*x)/Sqrt[2 + b]]/(2\*Sqrt[2 + b]) - (1 - Sqrt[2])\*ArcTan[(Sqrt[2 - b] + 2\*x)/Sqrt[2 + b]]/(2\*Sqrt[2 + b]) - ((1 + Sqrt[2])\*Log[1 - Sqrt[2 - b]\*x + x^2])/(4\*Sqrt[2 - b]) + ((1 + Sqrt[2])\*Log[1 + Sqrt[2 - b]\*x + x^2])/(4\*Sqrt[2 - b]))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 1169

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2\*q - b/c, 2]}, Dist[1/(2\*c\*q\*r), Int[(d\*r - (d - e\*q)\*x)/(q - r\*x + x^2), x], x] + Dist[1/(2\*c\*q\*r), Int[(d\*r + (d - e\*q)\*x)/(q + r\*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NegQ[b^2 - 4\*a\*c]

#### Rubi steps

$$\int \frac{\sqrt{2} - x^2}{1 + bx^2 + x^4} dx = \frac{\int \frac{\sqrt{2}\sqrt{2-b} - (1+\sqrt{2})x}{1-\sqrt{2-b}x+x^2} dx}{2\sqrt{2-b}} + \frac{\int \frac{\sqrt{2}\sqrt{2-b} + (1+\sqrt{2})x}{1+\sqrt{2-b}x+x^2} dx}{2\sqrt{2-b}}$$

$$= \frac{1}{4}(-1 + \sqrt{2}) \int \frac{1}{1 - \sqrt{2-b}x + x^2} dx + \frac{1}{4}(-1 + \sqrt{2}) \int \frac{1}{1 + \sqrt{2-b}x + x^2} dx - \frac{(1 + \sqrt{2})}{4}$$

$$= -\frac{(1 + \sqrt{2}) \log(1 - \sqrt{2-b}x + x^2)}{4\sqrt{2-b}} + \frac{(1 + \sqrt{2}) \log(1 + \sqrt{2-b}x + x^2)}{4\sqrt{2-b}} + \frac{1}{2}(1 - \sqrt{2}) \operatorname{Subst}$$

$$= \frac{(1 - \sqrt{2}) \tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}} - \frac{(1 - \sqrt{2}) \tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}} - \frac{(1 + \sqrt{2}) \log(1 - \sqrt{2-b}x + x^2)}{4\sqrt{2-b}}$$

**Mathematica [A]** time = 0.09, size = 137, normalized size = 0.86

$$\frac{(-\sqrt{b^2-4}+b+2\sqrt{2}) \tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b-\sqrt{b^2-4}}}\right) - (\sqrt{b^2-4}+b+2\sqrt{2}) \tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{b^2-4}+b}}\right)}{\sqrt{b-\sqrt{b^2-4}} \sqrt{\sqrt{b^2-4}+b}}$$

$$\frac{\hspace{10em}}{\sqrt{2}\sqrt{b^2-4}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[2] - x^2)/(1 + b\*x^2 + x^4), x]

[Out] (((2\*Sqrt[2] + b - Sqrt[-4 + b^2])\*ArcTan[(Sqrt[2]\*x)/Sqrt[b - Sqrt[-4 + b^2]]])/Sqrt[b - Sqrt[-4 + b^2]] - ((2\*Sqrt[2] + b + Sqrt[-4 + b^2])\*ArcTan[(Sqrt[2]\*x)/Sqrt[b + Sqrt[-4 + b^2]]])/Sqrt[b + Sqrt[-4 + b^2]])/(Sqrt[2]\*Sqrt[-4 + b^2])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2} - x^2}{1 + bx^2 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(Sqrt[2] - x^2)/(1 + b\*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(Sqrt[2] - x^2)/(1 + b\*x^2 + x^4), x]

**fricas [B]** time = 1.22, size = 451, normalized size = 2.82

$$\frac{1}{2}\sqrt{\frac{2+\sqrt{2}\sqrt{b^2-4}}{b^2-4}} \operatorname{arctan}\left(\frac{\sqrt{2}\sqrt{b^2-4}}{\sqrt{b^2-4}+b}\right) - \frac{1}{2}\sqrt{\frac{2-\sqrt{2}\sqrt{b^2-4}}{b^2-4}} \operatorname{arctan}\left(\frac{\sqrt{2}\sqrt{b^2-4}}{\sqrt{b^2-4}-b}\right) - \frac{1}{2}\sqrt{\frac{2+\sqrt{2}\sqrt{b^2-4}}{b^2-4}} \operatorname{arctan}\left(\frac{\sqrt{2}\sqrt{b^2-4}}{\sqrt{b^2-4}+b}\right) - \frac{1}{2}\sqrt{\frac{2-\sqrt{2}\sqrt{b^2-4}}{b^2-4}} \operatorname{arctan}\left(\frac{\sqrt{2}\sqrt{b^2-4}}{\sqrt{b^2-4}-b}\right) - \frac{1}{2}\sqrt{\frac{2+\sqrt{2}\sqrt{b^2-4}}{b^2-4}} \operatorname{arctan}\left(\frac{\sqrt{2}\sqrt{b^2-4}}{\sqrt{b^2-4}+b}\right) - \frac{1}{2}\sqrt{\frac{2-\sqrt{2}\sqrt{b^2-4}}{b^2-4}} \operatorname{arctan}\left(\frac{\sqrt{2}\sqrt{b^2-4}}{\sqrt{b^2-4}-b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2^(1/2))/(x^4+b\*x^2+1), x, algorithm="fricas")

[Out] -1/2\*sqrt(1/2)\*sqrt(-(3\*b + 4\*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))\*log(1/2\*(2\*b + 3\*sqrt(2))\*x + 1/2\*sqrt(1/2)\*(b^2 - (b^3 + sqrt(2)\*b^2 - 4\*b - 4\*sqrt(2)))/sqrt(b^2 - 4) - 4)\*sqrt(-(3\*b + 4\*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4)) + 1/2\*sqrt(1/2)\*sqrt(-(3\*b + 4\*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))\*log(1/2\*(2\*b + 3\*sqrt(2))\*x - 1/2\*sqrt(1/2)\*(b^2 - (b^3 + sqrt(2)\*b^2 - 4\*b - 4\*sqrt(2)))/sqrt(b^2 - 4) - 4)\*sqrt(-(3\*b + 4\*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4)) - 1/2\*sqrt(1/2)\*sqrt(-(3\*b + 4\*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))\*log(1/2\*(2\*b + 3\*sqrt(2))\*x + 1/2\*sqrt(1/2)\*(b^2 + (b^3 + sqrt(2)\*b^2 - 4\*b - 4\*sqrt(2)))/sqrt(b^2 - 4) - 4)\*sqrt(-(3\*b + 4\*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))

4))) + 1/2\*sqrt(1/2)\*sqrt(-(3\*b + 4\*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))\*log(1/2\*(2\*b + 3\*sqrt(2))\*x - 1/2\*sqrt(1/2)\*(b^2 + (b^3 + sqrt(2)\*b^2 - 4\*b - 4\*sqrt(2))/sqrt(b^2 - 4) - 4)\*sqrt(-(3\*b + 4\*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4)))

**giac [B]** time = 0.32, size = 1501, normalized size = 9.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2^(1/2))/(x^4+b\*x^2+1),x, algorithm="giac")

[Out] 1/4\*(sqrt(2)\*sqrt(b + 2)\*b^4 + sqrt(2)\*sqrt(b - 2)\*b^4 - sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b + 2)\*b^3 - sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b - 2)\*b^3 - sqrt(2)\*sqrt(b + 2)\*sqrt(b - 2)\*b^3 - 3\*sqrt(2)\*b^4 + 3\*sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b + 2)\*sqrt(b - 2)\*b^2 + sqrt(2)\*sqrt(b^2 - 4)\*b^3 - sqrt(2)\*sqrt(b + 2)\*b^3 - sqrt(2)\*sqrt(b - 2)\*b^3 + sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b + 2)\*b^2 + sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b - 2)\*b^2 + sqrt(2)\*sqrt(b + 2)\*sqrt(b - 2)\*b^2 + 3\*sqrt(2)\*b^3 - 3\*sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b + 2)\*sqrt(b - 2)\*b - sqrt(2)\*sqrt(b^2 - 4)\*b^2 - 10\*sqrt(2)\*sqrt(b + 2)\*b^2 - 2\*sqrt(b^2 - 4)\*sqrt(b + 2)\*b^2 - 6\*sqrt(2)\*sqrt(b - 2)\*b^2 - 2\*sqrt(b^2 - 4)\*sqrt(b - 2)\*b^2 - 2\*sqrt(b + 2)\*sqrt(b - 2)\*b^2 + 4\*sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b + 2)\*b + 4\*sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b - 2)\*b + 4\*sqrt(2)\*sqrt(b + 2)\*sqrt(b - 2)\*b + 24\*sqrt(2)\*b^2 + 2\*sqrt(b^2 - 4)\*b^2 - 12\*sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b + 2)\*sqrt(b - 2) - 4\*sqrt(2)\*sqrt(b^2 - 4)\*b + 6\*sqrt(2)\*sqrt(b + 2)\*b + 4\*sqrt(b^2 - 4)\*sqrt(b + 2)\*b + 2\*sqrt(2)\*sqrt(b - 2)\*b + 4\*sqrt(b^2 - 4)\*sqrt(b - 2)\*b + 4\*sqrt(b + 2)\*sqrt(b - 2)\*b + 6\*b^2 + 4\*sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b + 2) + 4\*sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b - 2) + 4\*sqrt(2)\*sqrt(b + 2)\*sqrt(b - 2) - 6\*sqrt(b^2 - 4)\*sqrt(b + 2)\*sqrt(b - 2) - 12\*sqrt(2)\*b - 4\*sqrt(b^2 - 4)\*b - 2\*sqrt(b + 2)\*b - 2\*sqrt(b - 2)\*b - 4\*sqrt(2)\*sqrt(b^2 - 4) + 20\*sqrt(2)\*sqrt(b + 2) + 8\*sqrt(b^2 - 4)\*sqrt(b + 2) + 4\*sqrt(2)\*sqrt(b - 2) + 8\*sqrt(b^2 - 4)\*sqrt(b - 2) + 8\*sqrt(b + 2)\*sqrt(b - 2) - 48\*sqrt(2) - 8\*sqrt(b^2 - 4) + 4\*sqrt(b + 2) - 4\*sqrt(b - 2) - 24)\*arctan(x/sqrt(1/2\*b + 1/2\*sqrt(b^2 - 4)))/(b^4 - 2\*b^3 - 7\*b^2 + 8\*b + 12) + 1/4\*(sqrt(2)\*sqrt(b + 2)\*b^4 - sqrt(2)\*sqrt(b - 2)\*b^4 + sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b + 2)\*b^3 - sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b - 2)\*b^3 - sqrt(2)\*sqrt(b + 2)\*sqrt(b - 2)\*b^3 + 3\*sqrt(2)\*b^4 - 3\*sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b + 2)\*sqrt(b - 2)\*b^2 + sqrt(2)\*sqrt(b^2 - 4)\*b^3 - sqrt(2)\*sqrt(b + 2)\*b^3 + sqrt(2)\*sqrt(b - 2)\*b^3 - sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b + 2)\*b^2 + sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b - 2)\*b^2 + sqrt(2)\*sqrt(b + 2)\*sqrt(b - 2)\*b^2 - 3\*sqrt(2)\*b^3 + 3\*sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b + 2)\*sqrt(b - 2)\*b - sqrt(2)\*sqrt(b^2 - 4)\*b^2 - 10\*sqrt(2)\*sqrt(b + 2)\*b^2 + 2\*sqrt(b^2 - 4)\*sqrt(b + 2)\*b^2 + 6\*sqrt(2)\*sqrt(b - 2)\*b^2 - 2\*sqrt(b^2 - 4)\*sqrt(b - 2)\*b^2 - 4\*sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b + 2)\*b + 4\*sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b - 2)\*b + 4\*sqrt(2)\*sqrt(b + 2)\*sqrt(b - 2)\*b - 24\*sqrt(2)\*b^2 + 2\*sqrt(b^2 - 4)\*b^2 + 12\*sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b + 2)\*sqrt(b - 2) - 4\*sqrt(2)\*sqrt(b^2 - 4)\*b + 6\*sqrt(2)\*sqrt(b + 2)\*b - 4\*sqrt(b^2 - 4)\*sqrt(b + 2)\*b - 2\*sqrt(2)\*sqrt(b - 2)\*b + 4\*sqrt(b^2 - 4)\*sqrt(b - 2)\*b + 4\*sqrt(b + 2)\*sqrt(b - 2)\*b - 6\*b^2 - 4\*sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b + 2) + 4\*sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b - 2) + 4\*sqrt(2)\*sqrt(b + 2)\*sqrt(b - 2) + 6\*sqrt(b^2 - 4)\*sqrt(b + 2)\*sqrt(b - 2) + 12\*sqrt(2)\*b - 4\*sqrt(b^2 - 4)\*b - 2\*sqrt(b + 2)\*b + 2\*sqrt(b - 2)\*b - 4\*sqrt(2)\*sqrt(b^2 - 4) + 20\*sqrt(2)\*sqrt(b + 2) - 8\*sqrt(b^2 - 4)\*sqrt(b + 2) - 4\*sqrt(2)\*sqrt(b - 2) + 8\*sqrt(b^2 - 4)\*sqrt(b - 2) + 8\*sqrt(b + 2)\*sqrt(b - 2) + 48\*sqrt(2) - 8\*sqrt(b^2 - 4) + 4\*sqrt(b + 2) + 4\*sqrt(b - 2) + 24)\*arctan(x/sqrt(1/2\*b - 1/2\*sqrt(b^2 - 4)))/(b^4 - 2\*b^3 - 7\*b^2 + 8\*b + 12)

**maple [B]** time = 0.02, size = 285, normalized size = 1.78

$$\frac{b \arctan\left(\frac{2x}{\sqrt{2b-2\sqrt{(b-2)(b+2)}}}\right)}{\sqrt{(b-2)(b+2)}\sqrt{2b-2\sqrt{(b-2)(b+2)}}} - \frac{b \arctan\left(\frac{2x}{\sqrt{2b+2\sqrt{(b-2)(b+2)}}}\right)}{\sqrt{(b-2)(b+2)}\sqrt{2b+2\sqrt{(b-2)(b+2)}}} - \frac{\arctan\left(\frac{2x}{\sqrt{2b-2\sqrt{(b-2)(b+2)}}}\right)}{\sqrt{2b-2\sqrt{(b-2)(b+2)}}} + \frac{2\sqrt{2} \arctan\left(\frac{2x}{\sqrt{2b-2\sqrt{(b-2)(b+2)}}}\right)}{\sqrt{(b-2)(b+2)}\sqrt{2b-2\sqrt{(b-2)(b+2)}}} - \frac{\arctan\left(\frac{2x}{\sqrt{2b+2\sqrt{(b-2)(b+2)}}}\right)}{\sqrt{2b+2\sqrt{(b-2)(b+2)}}} - \frac{2\sqrt{2} \arctan\left(\frac{2x}{\sqrt{2b+2\sqrt{(b-2)(b+2)}}}\right)}{\sqrt{(b-2)(b+2)}\sqrt{2b+2\sqrt{(b-2)(b+2)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-x^2+2^(1/2))/(x^4+b*x^2+1),x)`

[Out] 
$$-1/(2*b+2*((b-2)*(b+2))^(1/2))^(1/2)*\arctan(2/(2*b+2*((b-2)*(b+2))^(1/2))^(1/2)*x)-1/((b-2)*(b+2))^(1/2)/(2*b+2*((b-2)*(b+2))^(1/2))^(1/2)*b*\arctan(2/(2*b+2*((b-2)*(b+2))^(1/2))^(1/2)*x)-2/((b-2)*(b+2))^(1/2)/(2*b+2*((b-2)*(b+2))^(1/2))^(1/2)*\arctan(2/(2*b+2*((b-2)*(b+2))^(1/2))^(1/2)*x)*2^(1/2)-1/(2*b-2*((b-2)*(b+2))^(1/2))^(1/2)*\arctan(2/(2*b-2*((b-2)*(b+2))^(1/2))^(1/2)*x)+1/((b-2)*(b+2))^(1/2)/(2*b-2*((b-2)*(b+2))^(1/2))^(1/2)*b*\arctan(2/(2*b-2*((b-2)*(b+2))^(1/2))^(1/2)*x)+2/((b-2)*(b+2))^(1/2)/(2*b-2*((b-2)*(b+2))^(1/2))^(1/2)*\arctan(2/(2*b-2*((b-2)*(b+2))^(1/2))^(1/2)*x)*2^(1/2)$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 - \sqrt{2}}{x^4 + bx^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="maxima")`

[Out] `-integrate((x^2 - sqrt(2))/(x^4 + b*x^2 + 1), x)`

**mupad** [B] time = 1.07, size = 1227, normalized size = 7.67

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2^(1/2) - x^2)/(b*x^2 + x^4 + 1),x)`

[Out] 
$$\begin{aligned} & \operatorname{atan}\left(\frac{x*(-(4*2^{1/2})b^2 - 16*2^{1/2} - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^{1/2})}{(8*b^4 - 64*b^2 + 128)^{1/2}}\right)*32i - b*x*\left(\frac{-(4*2^{1/2})b^2 - 16*2^{1/2} - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^{1/2}}{(8*b^4 - 64*b^2 + 128)^{1/2}}\right)*256i + b^2*x*\left(\frac{-(4*2^{1/2})b^2 - 16*2^{1/2} - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^{1/2}}{(8*b^4 - 64*b^2 + 128)^{1/2}}\right)*8i - b^4*x*\left(\frac{-(4*2^{1/2})b^2 - 16*2^{1/2} - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^{1/2}}{(8*b^4 - 64*b^2 + 128)^{1/2}}\right)*4i + b^3*x*\left(\frac{-(4*2^{1/2})b^2 - 16*2^{1/2} - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^{1/2}}{(8*b^4 - 64*b^2 + 128)^{1/2}}\right)*128i - b^5*x*\left(\frac{-(4*2^{1/2})b^2 - 16*2^{1/2} - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^{1/2}}{(8*b^4 - 64*b^2 + 128)^{1/2}}\right)*16i + 2^{1/2}*b*x*\left(\frac{-(4*2^{1/2})b^2 - 16*2^{1/2} - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^{1/2}}{(8*b^4 - 64*b^2 + 128)^{1/2}}\right)*32i - 2^{1/2}*b^3*x*\left(\frac{-(4*2^{1/2})b^2 - 16*2^{1/2} - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^{1/2}}{(8*b^4 - 64*b^2 + 128)^{1/2}}\right)*8i/(2^{1/2}*b^3 - 4*2^{1/2}*b + 2^{1/2}*(48*b^2 - 12*b^4 + b^6 - 64)^{1/2} + 2*b^2 - 8))*\left(\frac{-(4*2^{1/2})b^2 - 16*2^{1/2} - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^{1/2}}{(8*b^4 - 64*b^2 + 128)^{1/2}}\right)*2i - \operatorname{atan}\left(\frac{x*((12*b + 16*2^{1/2}) - 4*2^{1/2}*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^{1/2})}{(8*b^4 - 64*b^2 + 128)^{1/2}}\right)*32i - b*x*\left(\frac{(12*b + 16*2^{1/2}) - 4*2^{1/2}*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^{1/2}}{(8*b^4 - 64*b^2 + 128)^{1/2}}\right)*256i + b^2*x*\left(\frac{(12*b + 16*2^{1/2}) - 4*2^{1/2}*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^{1/2}}{(8*b^4 - 64*b^2 + 128)^{1/2}}\right)*8i - b^4*x*\left(\frac{(12*b + 16*2^{1/2}) - 4*2^{1/2}*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^{1/2}}{(8*b^4 - 64*b^2 + 128)^{1/2}}\right)*4i + b^3*x*\left(\frac{(12*b + 16*2^{1/2}) - 4*2^{1/2}*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^{1/2}}{(8*b^4 - 64*b^2 + 128)^{1/2}}\right)*128i - b^5*x*\left(\frac{(12*b + 16*2^{1/2}) - 4*2^{1/2}*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^{1/2}}{(8*b^4 - 64*b^2 + 128)^{1/2}}\right)*16i + 2^{1/2}*b*x*\left(\frac{(12*b + 16*2^{1/2}) - 4*2^{1/2}*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^{1/2}}{(8*b^4 - 64*b^2 + 128)^{1/2}}\right)*32i - 2^{1/2}*b^3*x*\left(\frac{(12*b + 16*2^{1/2}) - 4*2^{1/2}*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^{1/2}}{(8*b^4 - 64*b^2 + 128)^{1/2}}\right)*8i/(4*2^{1/2}*b - 2^{1/2}*b^3 + 2 \end{aligned}$$

$$\sqrt{\frac{1}{2}} \cdot (48b^2 - 12b^4 + b^6 - 64)^{\frac{1}{2}} - 2b^2 + 8) \cdot ((12b + 16 \cdot 2^{\frac{1}{2}} - 4 \cdot 2^{\frac{1}{2}} \cdot b^2 - 3b^3 + (48b^2 - 12b^4 + b^6 - 64)^{\frac{1}{2}}) / (8b^4 - 64b^2 + 128))^{\frac{1}{2}} \cdot 2i$$

**sympy [B]** time = 2.86, size = 1469, normalized size = 9.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*2+2\*\*(1/2))/(x\*\*4+b\*x\*\*2+1),x)

[Out] -RootSum(\_t\*\*4\*(16\*b\*\*4 - 128\*b\*\*2 + 256) + \_t\*\*2\*(12\*b\*\*3 + 16\*sqrt(2)\*b\*\*2 - 48\*b - 64\*sqrt(2)) + 2\*b\*\*2 + 6\*sqrt(2)\*b + 9, Lambda(\_t, \_t\*log(\_t\*\*3\*(64\*b\*\*12/(8\*b\*\*10 + 88\*sqrt(2)\*b\*\*9 + 828\*b\*\*8 + 2144\*sqrt(2)\*b\*\*7 + 6470\*b\*\*6 + 5310\*sqrt(2)\*b\*\*5 + 2781\*b\*\*4 - 2322\*sqrt(2)\*b\*\*3 - 3402\*b\*\*2 + 729) + 672\*sqrt(2)\*b\*\*11/(8\*b\*\*10 + 88\*sqrt(2)\*b\*\*9 + 828\*b\*\*8 + 2144\*sqrt(2)\*b\*\*7 + 6470\*b\*\*6 + 5310\*sqrt(2)\*b\*\*5 + 2781\*b\*\*4 - 2322\*sqrt(2)\*b\*\*3 - 3402\*b\*\*2 + 729) + 5760\*b\*\*10/(8\*b\*\*10 + 88\*sqrt(2)\*b\*\*9 + 828\*b\*\*8 + 2144\*sqrt(2)\*b\*\*7 + 6470\*b\*\*6 + 5310\*sqrt(2)\*b\*\*5 + 2781\*b\*\*4 - 2322\*sqrt(2)\*b\*\*3 - 3402\*b\*\*2 + 729) + 12064\*sqrt(2)\*b\*\*9/(8\*b\*\*10 + 88\*sqrt(2)\*b\*\*9 + 828\*b\*\*8 + 2144\*sqrt(2)\*b\*\*7 + 6470\*b\*\*6 + 5310\*sqrt(2)\*b\*\*5 + 2781\*b\*\*4 - 2322\*sqrt(2)\*b\*\*3 - 3402\*b\*\*2 + 729) + 17744\*b\*\*8/(8\*b\*\*10 + 88\*sqrt(2)\*b\*\*9 + 828\*b\*\*8 + 2144\*sqrt(2)\*b\*\*7 + 6470\*b\*\*6 + 5310\*sqrt(2)\*b\*\*5 + 2781\*b\*\*4 - 2322\*sqrt(2)\*b\*\*3 - 3402\*b\*\*2 + 729) - 27480\*sqrt(2)\*b\*\*7/(8\*b\*\*10 + 88\*sqrt(2)\*b\*\*9 + 828\*b\*\*8 + 2144\*sqrt(2)\*b\*\*7 + 6470\*b\*\*6 + 5310\*sqrt(2)\*b\*\*5 + 2781\*b\*\*4 - 2322\*sqrt(2)\*b\*\*3 - 3402\*b\*\*2 + 729) - 154608\*b\*\*6/(8\*b\*\*10 + 88\*sqrt(2)\*b\*\*9 + 828\*b\*\*8 + 2144\*sqrt(2)\*b\*\*7 + 6470\*b\*\*6 + 5310\*sqrt(2)\*b\*\*5 + 2781\*b\*\*4 - 2322\*sqrt(2)\*b\*\*3 - 3402\*b\*\*2 + 729) - 141376\*sqrt(2)\*b\*\*5/(8\*b\*\*10 + 88\*sqrt(2)\*b\*\*9 + 828\*b\*\*8 + 2144\*sqrt(2)\*b\*\*7 + 6470\*b\*\*6 + 5310\*sqrt(2)\*b\*\*5 + 2781\*b\*\*4 - 2322\*sqrt(2)\*b\*\*3 - 3402\*b\*\*2 + 729) - 69072\*b\*\*4/(8\*b\*\*10 + 88\*sqrt(2)\*b\*\*9 + 828\*b\*\*8 + 2144\*sqrt(2)\*b\*\*7 + 6470\*b\*\*6 + 5310\*sqrt(2)\*b\*\*5 + 2781\*b\*\*4 - 2322\*sqrt(2)\*b\*\*3 - 3402\*b\*\*2 + 729) + 61704\*sqrt(2)\*b\*\*3/(8\*b\*\*10 + 88\*sqrt(2)\*b\*\*9 + 828\*b\*\*8 + 2144\*sqrt(2)\*b\*\*7 + 6470\*b\*\*6 + 5310\*sqrt(2)\*b\*\*5 + 2781\*b\*\*4 - 2322\*sqrt(2)\*b\*\*3 - 3402\*b\*\*2 + 729) + 78192\*b\*\*2/(8\*b\*\*10 + 88\*sqrt(2)\*b\*\*9 + 828\*b\*\*8 + 2144\*sqrt(2)\*b\*\*7 + 6470\*b\*\*6 + 5310\*sqrt(2)\*b\*\*5 + 2781\*b\*\*4 - 2322\*sqrt(2)\*b\*\*3 - 3402\*b\*\*2 + 729) - 2592\*sqrt(2)\*b/(8\*b\*\*10 + 88\*sqrt(2)\*b\*\*9 + 828\*b\*\*8 + 2144\*sqrt(2)\*b\*\*7 + 6470\*b\*\*6 + 5310\*sqrt(2)\*b\*\*5 + 2781\*b\*\*4 - 2322\*sqrt(2)\*b\*\*3 - 3402\*b\*\*2 + 729) - 15552/(8\*b\*\*10 + 88\*sqrt(2)\*b\*\*9 + 828\*b\*\*8 + 2144\*sqrt(2)\*b\*\*7 + 6470\*b\*\*6 + 5310\*sqrt(2)\*b\*\*5 + 2781\*b\*\*4 - 2322\*sqrt(2)\*b\*\*3 - 3402\*b\*\*2 + 729)) + \_t\*(16\*b\*\*7/(4\*b\*\*6 + 28\*sqrt(2)\*b\*\*5 + 152\*b\*\*4 + 192\*sqrt(2)\*b\*\*3 + 189\*b\*\*2 - 27\*sqrt(2)\*b - 81) + 116\*sqrt(2)\*b\*\*6/(4\*b\*\*6 + 28\*sqrt(2)\*b\*\*5 + 152\*b\*\*4 + 192\*sqrt(2)\*b\*\*3 + 189\*b\*\*2 - 27\*sqrt(2)\*b - 81) + 668\*b\*\*5/(4\*b\*\*6 + 28\*sqrt(2)\*b\*\*5 + 152\*b\*\*4 + 192\*sqrt(2)\*b\*\*3 + 189\*b\*\*2 - 27\*sqrt(2)\*b - 81) + 942\*sqrt(2)\*b\*\*4/(4\*b\*\*6 + 28\*sqrt(2)\*b\*\*5 + 152\*b\*\*4 + 192\*sqrt(2)\*b\*\*3 + 189\*b\*\*2 - 27\*sqrt(2)\*b - 81) + 1226\*b\*\*3/(4\*b\*\*6 + 28\*sqrt(2)\*b\*\*5 + 152\*b\*\*4 + 192\*sqrt(2)\*b\*\*3 + 189\*b\*\*2 - 27\*sqrt(2)\*b - 81) + 144\*sqrt(2)\*b\*\*2/(4\*b\*\*6 + 28\*sqrt(2)\*b\*\*5 + 152\*b\*\*4 + 192\*sqrt(2)\*b\*\*3 + 189\*b\*\*2 - 27\*sqrt(2)\*b - 81) - 378\*b/(4\*b\*\*6 + 28\*sqrt(2)\*b\*\*5 + 152\*b\*\*4 + 192\*sqrt(2)\*b\*\*3 + 189\*b\*\*2 - 27\*sqrt(2)\*b - 81) - 108\*sqrt(2)/(4\*b\*\*6 + 28\*sqrt(2)\*b\*\*5 + 152\*b\*\*4 + 192\*sqrt(2)\*b\*\*3 + 189\*b\*\*2 - 27\*sqrt(2)\*b - 81)) + x))

$$3.93 \quad \int \frac{\sqrt{2+x^2}}{1+bx^2+x^4} dx$$

**Optimal.** Leaf size=160

$$\frac{(1-\sqrt{2})\log(-\sqrt{2-b}x+x^2+1)}{4\sqrt{2-b}} - \frac{(1-\sqrt{2})\log(\sqrt{2-b}x+x^2+1)}{4\sqrt{2-b}} - \frac{(1+\sqrt{2})\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}} + \frac{(1+\sqrt{2})\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}$$

**Rubi [A]** time = 0.10, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$ , Rules used = {1169, 634, 618, 204, 628}

$$\frac{(1-\sqrt{2})\log(-\sqrt{2-b}x+x^2+1)}{4\sqrt{2-b}} - \frac{(1-\sqrt{2})\log(\sqrt{2-b}x+x^2+1)}{4\sqrt{2-b}} - \frac{(1+\sqrt{2})\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}} + \frac{(1+\sqrt{2})\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[2] + x^2)/(1 + b\*x^2 + x^4), x]

[Out] -((1 + Sqrt[2])\*ArcTan[(Sqrt[2 - b] - 2\*x)/Sqrt[2 + b]]/(2\*Sqrt[2 + b]) + ((1 + Sqrt[2])\*ArcTan[(Sqrt[2 - b] + 2\*x)/Sqrt[2 + b]]/(2\*Sqrt[2 + b]) + (1 - Sqrt[2])\*Log[1 - Sqrt[2 - b]\*x + x^2])/(4\*Sqrt[2 - b]) - ((1 - Sqrt[2])\*Log[1 + Sqrt[2 - b]\*x + x^2])/(4\*Sqrt[2 - b])

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 1169

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2\*q - b/c, 2]}, Dist[1/(2\*c\*q\*r), Int[(d\*r - (d - e\*q)\*x)/(q - r\*x + x^2), x], x] + Dist[1/(2\*c\*q\*r), Int[(d\*r + (d - e\*q)\*x)/(q + r\*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NegQ[b^2 - 4\*a\*c]

#### Rubi steps



$$\begin{aligned}
\int \frac{\sqrt{2} + x^2}{1 + bx^2 + x^4} dx &= \frac{\int \frac{\sqrt{2}\sqrt{2-b} - (-1+\sqrt{2})x}{1-\sqrt{2-b}x+x^2} dx}{2\sqrt{2-b}} + \frac{\int \frac{\sqrt{2}\sqrt{2-b} + (-1+\sqrt{2})x}{1+\sqrt{2-b}x+x^2} dx}{2\sqrt{2-b}} \\
&= \frac{1}{4} (1 + \sqrt{2}) \int \frac{1}{1 - \sqrt{2-b}x + x^2} dx + \frac{1}{4} (1 + \sqrt{2}) \int \frac{1}{1 + \sqrt{2-b}x + x^2} dx + \frac{(1 - \sqrt{2})}{4} \int \frac{1}{1 + \sqrt{2-b}x + x^2} dx \\
&= \frac{(1 - \sqrt{2}) \log(1 - \sqrt{2-b}x + x^2)}{4\sqrt{2-b}} - \frac{(1 - \sqrt{2}) \log(1 + \sqrt{2-b}x + x^2)}{4\sqrt{2-b}} + \frac{1}{2} (-1 - \sqrt{2}) \operatorname{Su} \\
&= -\frac{(1 + \sqrt{2}) \tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}} + \frac{(1 + \sqrt{2}) \tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}} + \frac{(1 - \sqrt{2}) \log(1 - \sqrt{2-b}x + x^2)}{4\sqrt{2-b}}
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 136, normalized size = 0.85

$$\frac{\left(\sqrt{b^2-4}-b+2\sqrt{2}\right) \tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b-\sqrt{b^2-4}}}\right) + \left(\sqrt{b^2-4}+b-2\sqrt{2}\right) \tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{b^2-4}+b}}\right)}{\sqrt{b-\sqrt{b^2-4}} + \sqrt{\sqrt{b^2-4}+b}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[2] + x^2)/(1 + b\*x^2 + x^4), x]

[Out] (((2\*Sqrt[2] - b + Sqrt[-4 + b^2])\*ArcTan[(Sqrt[2]\*x)/Sqrt[b - Sqrt[-4 + b^2]]])/Sqrt[b - Sqrt[-4 + b^2]] + ((-2\*Sqrt[2] + b + Sqrt[-4 + b^2])\*ArcTan[(Sqrt[2]\*x)/Sqrt[b + Sqrt[-4 + b^2]]])/Sqrt[b + Sqrt[-4 + b^2]])/(Sqrt[2]\*Sqrt[-4 + b^2])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2} + x^2}{1 + bx^2 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(Sqrt[2] + x^2)/(1 + b\*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(Sqrt[2] + x^2)/(1 + b\*x^2 + x^4), x]

**fricas [B]** time = 1.10, size = 455, normalized size = 2.84

$$\frac{1}{2} \sqrt{\frac{3b+\sqrt{b^2-4}}{b^2-4}} \operatorname{arctan}\left(\frac{1}{\sqrt{b^2-4}} \sqrt{\frac{3b+\sqrt{b^2-4}}{b^2-4}}\right) - \frac{1}{2} \sqrt{\frac{3b-\sqrt{b^2-4}}{b^2-4}} \operatorname{arctan}\left(\frac{1}{\sqrt{b^2-4}} \sqrt{\frac{3b-\sqrt{b^2-4}}{b^2-4}}\right) + \frac{1}{2} \sqrt{\frac{3b+\sqrt{b^2-4}}{b^2-4}} \operatorname{arctan}\left(\frac{1}{\sqrt{b^2-4}} \sqrt{\frac{3b+\sqrt{b^2-4}}{b^2-4}}\right) - \frac{1}{2} \sqrt{\frac{3b-\sqrt{b^2-4}}{b^2-4}} \operatorname{arctan}\left(\frac{1}{\sqrt{b^2-4}} \sqrt{\frac{3b-\sqrt{b^2-4}}{b^2-4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2^(1/2))/(x^4+b\*x^2+1), x, algorithm="fricas")

[Out] 1/2\*sqrt(1/2)\*sqrt(-(3\*b - 4\*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))\*log(1/2\*(2\*b - 3\*sqrt(2))\*x + 1/2\*sqrt(1/2)\*(b^2 - (b^3 - sqrt(2)\*b^2 - 4\*b + 4\*sqrt(2))/sqrt(b^2 - 4) - 4)\*sqrt(-(3\*b - 4\*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))) - 1/2\*sqrt(1/2)\*sqrt(-(3\*b - 4\*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))\*log(1/2\*(2\*b - 3\*sqrt(2))\*x - 1/2\*sqrt(1/2)\*(b^2 - (b^3 - sqrt(2)\*b^2 - 4\*b + 4\*sqrt(2))/sqrt(b^2 - 4) - 4)\*sqrt(-(3\*b - 4\*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))) + 1/2\*sqrt(1/2)\*sqrt(-(3\*b - 4\*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))\*log(1/2\*(2\*b - 3\*sqrt(2))\*x + 1/2\*sqrt(1/2)\*(b^2 + (b^3 - sqrt(2)\*b^2 - 4\*b + 4\*sqrt(2))/sqrt(b^2 - 4) - 4)\*sqrt(-(3\*b - 4\*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4)))

))) - 1/2\*sqrt(1/2)\*sqrt(-(3\*b - 4\*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))\*log(1/2\*(2\*b - 3\*sqrt(2))\*x - 1/2\*sqrt(1/2)\*(b^2 + (b^3 - sqrt(2)\*b^2 - 4\*b + 4\*sqrt(2))/sqrt(b^2 - 4) - 4)\*sqrt(-(3\*b - 4\*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4)))

**giac [B]** time = 0.35, size = 1501, normalized size = 9.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2^(1/2))/(x^4+b\*x^2+1),x, algorithm="giac")

[Out] 1/4\*(sqrt(2)\*sqrt(b + 2)\*b^4 + sqrt(2)\*sqrt(b - 2)\*b^4 - sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b + 2)\*b^3 - sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b - 2)\*b^3 - sqrt(2)\*sqrt(b + 2)\*sqrt(b - 2)\*b^3 - 3\*sqrt(2)\*b^4 + 3\*sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b + 2)\*sqrt(b - 2)\*b^2 + sqrt(2)\*sqrt(b^2 - 4)\*b^3 - sqrt(2)\*sqrt(b + 2)\*b^3 - sqrt(2)\*sqrt(b - 2)\*b^3 + sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b + 2)\*b^2 + sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b - 2)\*b^2 + 3\*sqrt(2)\*b^3 - 3\*sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b + 2)\*sqrt(b - 2)\*b - sqrt(2)\*sqrt(b^2 - 4)\*b^2 - 10\*sqrt(2)\*sqrt(b + 2)\*b^2 + 2\*sqrt(b^2 - 4)\*sqrt(b + 2)\*b^2 - 6\*sqrt(2)\*sqrt(b - 2)\*b^2 + 2\*sqrt(b^2 - 4)\*sqrt(b - 2)\*b^2 + 2\*sqrt(b + 2)\*sqrt(b - 2)\*b^2 + 4\*sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b + 2)\*b + 4\*sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b - 2)\*b + 4\*sqrt(2)\*sqrt(b + 2)\*sqrt(b - 2)\*b + 24\*sqrt(2)\*b^2 - 2\*sqrt(b^2 - 4)\*b^2 - 12\*sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b + 2)\*sqrt(b - 2) - 4\*sqrt(2)\*sqrt(b^2 - 4)\*b + 6\*sqrt(2)\*sqrt(b + 2)\*b - 4\*sqrt(b^2 - 4)\*sqrt(b + 2)\*b + 2\*sqrt(2)\*sqrt(b - 2)\*b - 4\*sqrt(b^2 - 4)\*sqrt(b - 2)\*b - 4\*sqrt(b + 2)\*sqrt(b - 2)\*b - 6\*b^2 + 4\*sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b + 2) + 4\*sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b - 2) + 4\*sqrt(2)\*sqrt(b + 2)\*sqrt(b - 2) + 6\*sqrt(b^2 - 4)\*sqrt(b + 2)\*sqrt(b - 2) - 12\*sqrt(2)\*b + 4\*sqrt(b^2 - 4)\*b + 2\*sqrt(b + 2)\*b + 2\*sqrt(b - 2)\*b - 4\*sqrt(2)\*sqrt(b^2 - 4) + 20\*sqrt(2)\*sqrt(b + 2) - 8\*sqrt(b^2 - 4)\*sqrt(b + 2) + 4\*sqrt(2)\*sqrt(b - 2) - 8\*sqrt(b^2 - 4)\*sqrt(b - 2) - 8\*sqrt(b + 2)\*sqrt(b - 2) - 48\*sqrt(2) + 8\*sqrt(b^2 - 4) - 4\*sqrt(b + 2) + 4\*sqrt(b - 2) + 24)\*arctan(x/sqrt(1/2\*b + 1/2\*sqrt(b^2 - 4)))/(b^4 - 2\*b^3 - 7\*b^2 + 8\*b + 12) + 1/4\*(sqrt(2)\*sqrt(b + 2)\*b^4 - sqrt(2)\*sqrt(b - 2)\*b^4 + sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b + 2)\*b^3 - sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b - 2)\*b^3 - sqrt(2)\*sqrt(b + 2)\*sqrt(b - 2)\*b^3 + 3\*sqrt(2)\*b^4 - 3\*sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b + 2)\*sqrt(b - 2)\*b^2 + sqrt(2)\*sqrt(b^2 - 4)\*b^3 - sqrt(2)\*sqrt(b + 2)\*b^3 + sqrt(2)\*sqrt(b - 2)\*b^3 - sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b + 2)\*b^2 + sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b - 2)\*b^2 + sqrt(2)\*sqrt(b + 2)\*sqrt(b - 2)\*b^2 - 3\*sqrt(2)\*b^3 + 3\*sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b + 2)\*sqrt(b - 2)\*b - sqrt(2)\*sqrt(b^2 - 4)\*b^2 - 10\*sqrt(2)\*sqrt(b + 2)\*b^2 - 2\*sqrt(b^2 - 4)\*sqrt(b + 2)\*b^2 + 6\*sqrt(2)\*sqrt(b - 2)\*b^2 + 2\*sqrt(b^2 - 4)\*sqrt(b - 2)\*b^2 + 2\*sqrt(b + 2)\*sqrt(b - 2)\*b^2 - 4\*sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b + 2)\*b + 4\*sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b - 2)\*b + 4\*sqrt(2)\*sqrt(b + 2)\*sqrt(b - 2)\*b - 24\*sqrt(2)\*b^2 - 2\*sqrt(b^2 - 4)\*b^2 + 12\*sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b + 2)\*sqrt(b - 2) - 4\*sqrt(2)\*sqrt(b^2 - 4)\*b + 6\*sqrt(2)\*sqrt(b + 2)\*b + 4\*sqrt(b^2 - 4)\*sqrt(b + 2)\*b - 2\*sqrt(2)\*sqrt(b - 2)\*b - 4\*sqrt(b^2 - 4)\*sqrt(b - 2)\*b - 4\*sqrt(b + 2)\*sqrt(b - 2)\*b + 6\*b^2 - 4\*sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b + 2) + 4\*sqrt(2)\*sqrt(b^2 - 4)\*sqrt(b - 2) + 4\*sqrt(2)\*sqrt(b + 2)\*sqrt(b - 2) - 6\*sqrt(b^2 - 4)\*sqrt(b + 2)\*sqrt(b - 2) + 12\*sqrt(2)\*b + 4\*sqrt(b^2 - 4)\*b + 2\*sqrt(b + 2)\*b - 2\*sqrt(b - 2)\*b - 4\*sqrt(2)\*sqrt(b^2 - 4) + 20\*sqrt(2)\*sqrt(b + 2) + 8\*sqrt(b^2 - 4)\*sqrt(b + 2) - 4\*sqrt(2)\*sqrt(b - 2) - 8\*sqrt(b^2 - 4)\*sqrt(b - 2) - 8\*sqrt(b + 2)\*sqrt(b - 2) + 48\*sqrt(2) + 8\*sqrt(b^2 - 4) - 4\*sqrt(b + 2) - 4\*sqrt(b - 2) - 24)\*arctan(x/sqrt(1/2\*b - 1/2\*sqrt(b^2 - 4)))/(b^4 - 2\*b^3 - 7\*b^2 + 8\*b + 12)

**maple [B]** time = 0.02, size = 283, normalized size = 1.77

$$\frac{b \arctan\left(\frac{2x}{\sqrt{2b-2\sqrt{(b-2)(b+2)}}}\right)}{\sqrt{(b-2)(b+2)}\sqrt{2b-2\sqrt{(b-2)(b+2)}}} + \frac{b \arctan\left(\frac{2x}{\sqrt{2b+2\sqrt{(b-2)(b+2)}}}\right)}{\sqrt{(b-2)(b+2)}\sqrt{2b+2\sqrt{(b-2)(b+2)}}} + \frac{\arctan\left(\frac{2x}{\sqrt{2b-2\sqrt{(b-2)(b+2)}}}\right)}{\sqrt{2b-2\sqrt{(b-2)(b+2)}}} + \frac{2\sqrt{2} \arctan\left(\frac{2x}{\sqrt{2b-2\sqrt{(b-2)(b+2)}}}\right)}{\sqrt{(b-2)(b+2)}\sqrt{2b-2\sqrt{(b-2)(b+2)}}} + \frac{\arctan\left(\frac{2x}{\sqrt{2b+2\sqrt{(b-2)(b+2)}}}\right)}{\sqrt{2b+2\sqrt{(b-2)(b+2)}}} - \frac{2\sqrt{2} \arctan\left(\frac{2x}{\sqrt{2b+2\sqrt{(b-2)(b+2)}}}\right)}{\sqrt{(b-2)(b+2)}\sqrt{2b+2\sqrt{(b-2)(b+2)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+2^(1/2))/(x^4+b*x^2+1),x)`

[Out]  $\frac{1}{(2b+2((b-2)(b+2))^{1/2})^{1/2}} \arctan\left(\frac{2}{(2b+2((b-2)(b+2))^{1/2})^{1/2}} x\right) + \frac{1}{((b-2)(b+2))^{1/2}} \frac{1}{(2b+2((b-2)(b+2))^{1/2})^{1/2}} b \arctan\left(\frac{2}{(2b+2((b-2)(b+2))^{1/2})^{1/2}} x\right) - \frac{2}{((b-2)(b+2))^{1/2}} \frac{1}{(2b+2((b-2)(b+2))^{1/2})^{1/2}} 2^{1/2} \arctan\left(\frac{2}{(2b+2((b-2)(b+2))^{1/2})^{1/2}} x\right) + \frac{1}{(2b-2((b-2)(b+2))^{1/2})^{1/2}} \arctan\left(\frac{2}{(2b-2((b-2)(b+2))^{1/2})^{1/2}} x\right) - \frac{1}{((b-2)(b+2))^{1/2}} \frac{1}{(2b-2((b-2)(b+2))^{1/2})^{1/2}} b \arctan\left(\frac{2}{(2b-2((b-2)(b+2))^{1/2})^{1/2}} x\right) + \frac{2}{((b-2)(b+2))^{1/2}} \frac{1}{(2b-2((b-2)(b+2))^{1/2})^{1/2}} 2^{1/2} \arctan\left(\frac{2}{(2b-2((b-2)(b+2))^{1/2})^{1/2}} x\right)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + \sqrt{2}}{x^4 + bx^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="maxima")`

[Out] `integrate((x^2 + sqrt(2))/(x^4 + b*x^2 + 1), x)`

**mupad** [B] time = 5.25, size = 1227, normalized size = 7.67

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2^(1/2) + x^2)/(b*x^2 + x^4 + 1),x)`

[Out]  $\operatorname{atan}\left(\frac{x(-16\sqrt{2} - 12b - 4\sqrt{2}b^2 + 3b^3 + (48b^2 - 12b^4 + b^6 - 64)^{1/2})}{(8b^4 - 64b^2 + 128)^{1/2}}\right) 32i - b x \operatorname{atan}\left(\frac{x(-16\sqrt{2} - 12b - 4\sqrt{2}b^2 + 3b^3 + (48b^2 - 12b^4 + b^6 - 64)^{1/2})}{(8b^4 - 64b^2 + 128)^{1/2}}\right) 256i + b^2 x \operatorname{atan}\left(\frac{x(-16\sqrt{2} - 12b - 4\sqrt{2}b^2 + 3b^3 + (48b^2 - 12b^4 + b^6 - 64)^{1/2})}{(8b^4 - 64b^2 + 128)^{1/2}}\right) 8i - b^4 x \operatorname{atan}\left(\frac{x(-16\sqrt{2} - 12b - 4\sqrt{2}b^2 + 3b^3 + (48b^2 - 12b^4 + b^6 - 64)^{1/2})}{(8b^4 - 64b^2 + 128)^{1/2}}\right) 4i + b^3 x \operatorname{atan}\left(\frac{x(-16\sqrt{2} - 12b - 4\sqrt{2}b^2 + 3b^3 + (48b^2 - 12b^4 + b^6 - 64)^{1/2})}{(8b^4 - 64b^2 + 128)^{1/2}}\right) 128i - b^5 x \operatorname{atan}\left(\frac{x(-16\sqrt{2} - 12b - 4\sqrt{2}b^2 + 3b^3 + (48b^2 - 12b^4 + b^6 - 64)^{1/2})}{(8b^4 - 64b^2 + 128)^{1/2}}\right) 16i - 2^{1/2} b x \operatorname{atan}\left(\frac{x(-16\sqrt{2} - 12b - 4\sqrt{2}b^2 + 3b^3 + (48b^2 - 12b^4 + b^6 - 64)^{1/2})}{(8b^4 - 64b^2 + 128)^{1/2}}\right) 32i + 2^{1/2} b^3 x \operatorname{atan}\left(\frac{x(-16\sqrt{2} - 12b - 4\sqrt{2}b^2 + 3b^3 + (48b^2 - 12b^4 + b^6 - 64)^{1/2})}{(8b^4 - 64b^2 + 128)^{1/2}}\right) 8i / (2^{1/2} b^3 - 4\sqrt{2} b + 2^{1/2} (48b^2 - 12b^4 + b^6 - 64)^{1/2} - 2b^2 + 8) * (-16\sqrt{2} - 12b - 4\sqrt{2}b^2 + 3b^3 + (48b^2 - 12b^4 + b^6 - 64)^{1/2}) / (8b^4 - 64b^2 + 128)^{1/2} * 2i - \operatorname{atan}\left(\frac{x((12b - 16\sqrt{2} + 4\sqrt{2}b^2 - 3b^3 + (48b^2 - 12b^4 + b^6 - 64)^{1/2})}{(8b^4 - 64b^2 + 128)^{1/2}}\right) 32i - b x \operatorname{atan}\left(\frac{x((12b - 16\sqrt{2} + 4\sqrt{2}b^2 - 3b^3 + (48b^2 - 12b^4 + b^6 - 64)^{1/2})}{(8b^4 - 64b^2 + 128)^{1/2}}\right) 256i + b^2 x \operatorname{atan}\left(\frac{x((12b - 16\sqrt{2} + 4\sqrt{2}b^2 - 3b^3 + (48b^2 - 12b^4 + b^6 - 64)^{1/2})}{(8b^4 - 64b^2 + 128)^{1/2}}\right) 8i - b^4 x \operatorname{atan}\left(\frac{x((12b - 16\sqrt{2} + 4\sqrt{2}b^2 - 3b^3 + (48b^2 - 12b^4 + b^6 - 64)^{1/2})}{(8b^4 - 64b^2 + 128)^{1/2}}\right) 4i + b^3 x \operatorname{atan}\left(\frac{x((12b - 16\sqrt{2} + 4\sqrt{2}b^2 - 3b^3 + (48b^2 - 12b^4 + b^6 - 64)^{1/2})}{(8b^4 - 64b^2 + 128)^{1/2}}\right) 128i - b^5 x \operatorname{atan}\left(\frac{x((12b - 16\sqrt{2} + 4\sqrt{2}b^2 - 3b^3 + (48b^2 - 12b^4 + b^6 - 64)^{1/2})}{(8b^4 - 64b^2 + 128)^{1/2}}\right) 16i - 2^{1/2} b x \operatorname{atan}\left(\frac{x((12b - 16\sqrt{2} + 4\sqrt{2}b^2 - 3b^3 + (48b^2 - 12b^4 + b^6 - 64)^{1/2})}{(8b^4 - 64b^2 + 128)^{1/2}}\right) 32i + 2^{1/2} b^3 x \operatorname{atan}\left(\frac{x((12b - 16\sqrt{2} + 4\sqrt{2}b^2 - 3b^3 + (48b^2 - 12b^4 + b^6 - 64)^{1/2})}{(8b^4 - 64b^2 + 128)^{1/2}}\right) 8i / (4\sqrt{2} b - 2^{1/2} b^3 + 2)$

$$\sqrt{\frac{1}{2}} \cdot (48b^2 - 12b^4 + b^6 - 64)^{\frac{1}{2}} + 2b^2 - 8) \cdot ((12b - 16 \cdot 2^{\frac{1}{2}} + 4 \cdot 2^{\frac{1}{2}})b^2 - 3b^3 + (48b^2 - 12b^4 + b^6 - 64)^{\frac{1}{2}}) / (8b^4 - 64b^2 + 128)^{\frac{1}{2}} \cdot 2i$$

**sympy [B]** time = 2.73, size = 1467, normalized size = 9.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*2+2\*\*(1/2))/(x\*\*4+b\*x\*\*2+1),x)

[Out] RootSum(\_t\*\*4\*(16\*b\*\*4 - 128\*b\*\*2 + 256) + \_t\*\*2\*(12\*b\*\*3 - 16\*sqrt(2)\*b\*\*2 - 48\*b + 64\*sqrt(2)) + 2\*b\*\*2 - 6\*sqrt(2)\*b + 9, Lambda(\_t, \_t\*log(\_t\*\*3\*(64\*b\*\*12/(8\*b\*\*10 - 88\*sqrt(2)\*b\*\*9 + 828\*b\*\*8 - 2144\*sqrt(2)\*b\*\*7 + 6470\*b\*\*6 - 5310\*sqrt(2)\*b\*\*5 + 2781\*b\*\*4 + 2322\*sqrt(2)\*b\*\*3 - 3402\*b\*\*2 + 729) - 672\*sqrt(2)\*b\*\*11/(8\*b\*\*10 - 88\*sqrt(2)\*b\*\*9 + 828\*b\*\*8 - 2144\*sqrt(2)\*b\*\*7 + 6470\*b\*\*6 - 5310\*sqrt(2)\*b\*\*5 + 2781\*b\*\*4 + 2322\*sqrt(2)\*b\*\*3 - 3402\*b\*\*2 + 729) + 5760\*b\*\*10/(8\*b\*\*10 - 88\*sqrt(2)\*b\*\*9 + 828\*b\*\*8 - 2144\*sqrt(2)\*b\*\*7 + 6470\*b\*\*6 - 5310\*sqrt(2)\*b\*\*5 + 2781\*b\*\*4 + 2322\*sqrt(2)\*b\*\*3 - 3402\*b\*\*2 + 729) - 12064\*sqrt(2)\*b\*\*9/(8\*b\*\*10 - 88\*sqrt(2)\*b\*\*9 + 828\*b\*\*8 - 2144\*sqrt(2)\*b\*\*7 + 6470\*b\*\*6 - 5310\*sqrt(2)\*b\*\*5 + 2781\*b\*\*4 + 2322\*sqrt(2)\*b\*\*3 - 3402\*b\*\*2 + 729) + 17744\*b\*\*8/(8\*b\*\*10 - 88\*sqrt(2)\*b\*\*9 + 828\*b\*\*8 - 2144\*sqrt(2)\*b\*\*7 + 6470\*b\*\*6 - 5310\*sqrt(2)\*b\*\*5 + 2781\*b\*\*4 + 2322\*sqrt(2)\*b\*\*3 - 3402\*b\*\*2 + 729) + 27480\*sqrt(2)\*b\*\*7/(8\*b\*\*10 - 88\*sqrt(2)\*b\*\*9 + 828\*b\*\*8 - 2144\*sqrt(2)\*b\*\*7 + 6470\*b\*\*6 - 5310\*sqrt(2)\*b\*\*5 + 2781\*b\*\*4 + 2322\*sqrt(2)\*b\*\*3 - 3402\*b\*\*2 + 729) - 154608\*b\*\*6/(8\*b\*\*10 - 88\*sqrt(2)\*b\*\*9 + 828\*b\*\*8 - 2144\*sqrt(2)\*b\*\*7 + 6470\*b\*\*6 - 5310\*sqrt(2)\*b\*\*5 + 2781\*b\*\*4 + 2322\*sqrt(2)\*b\*\*3 - 3402\*b\*\*2 + 729) + 141376\*sqrt(2)\*b\*\*5/(8\*b\*\*10 - 88\*sqrt(2)\*b\*\*9 + 828\*b\*\*8 - 2144\*sqrt(2)\*b\*\*7 + 6470\*b\*\*6 - 5310\*sqrt(2)\*b\*\*5 + 2781\*b\*\*4 + 2322\*sqrt(2)\*b\*\*3 - 3402\*b\*\*2 + 729) - 69072\*b\*\*4/(8\*b\*\*10 - 88\*sqrt(2)\*b\*\*9 + 828\*b\*\*8 - 2144\*sqrt(2)\*b\*\*7 + 6470\*b\*\*6 - 5310\*sqrt(2)\*b\*\*5 + 2781\*b\*\*4 + 2322\*sqrt(2)\*b\*\*3 - 3402\*b\*\*2 + 729) - 61704\*sqrt(2)\*b\*\*3/(8\*b\*\*10 - 88\*sqrt(2)\*b\*\*9 + 828\*b\*\*8 - 2144\*sqrt(2)\*b\*\*7 + 6470\*b\*\*6 - 5310\*sqrt(2)\*b\*\*5 + 2781\*b\*\*4 + 2322\*sqrt(2)\*b\*\*3 - 3402\*b\*\*2 + 729) + 78192\*b\*\*2/(8\*b\*\*10 - 88\*sqrt(2)\*b\*\*9 + 828\*b\*\*8 - 2144\*sqrt(2)\*b\*\*7 + 6470\*b\*\*6 - 5310\*sqrt(2)\*b\*\*5 + 2781\*b\*\*4 + 2322\*sqrt(2)\*b\*\*3 - 3402\*b\*\*2 + 729) + 2592\*sqrt(2)\*b/(8\*b\*\*10 - 88\*sqrt(2)\*b\*\*9 + 828\*b\*\*8 - 2144\*sqrt(2)\*b\*\*7 + 6470\*b\*\*6 - 5310\*sqrt(2)\*b\*\*5 + 2781\*b\*\*4 + 2322\*sqrt(2)\*b\*\*3 - 3402\*b\*\*2 + 729) - 15552/(8\*b\*\*10 - 88\*sqrt(2)\*b\*\*9 + 828\*b\*\*8 - 2144\*sqrt(2)\*b\*\*7 + 6470\*b\*\*6 - 5310\*sqrt(2)\*b\*\*5 + 2781\*b\*\*4 + 2322\*sqrt(2)\*b\*\*3 - 3402\*b\*\*2 + 729)) + \_t\*(16\*b\*\*7/(4\*b\*\*6 - 28\*sqrt(2)\*b\*\*5 + 152\*b\*\*4 - 192\*sqrt(2)\*b\*\*3 + 189\*b\*\*2 + 27\*sqrt(2)\*b - 81) - 116\*sqrt(2)\*b\*\*6/(4\*b\*\*6 - 28\*sqrt(2)\*b\*\*5 + 152\*b\*\*4 - 192\*sqrt(2)\*b\*\*3 + 189\*b\*\*2 + 27\*sqrt(2)\*b - 81) + 668\*b\*\*5/(4\*b\*\*6 - 28\*sqrt(2)\*b\*\*5 + 152\*b\*\*4 - 192\*sqrt(2)\*b\*\*3 + 189\*b\*\*2 + 27\*sqrt(2)\*b - 81) - 942\*sqrt(2)\*b\*\*4/(4\*b\*\*6 - 28\*sqrt(2)\*b\*\*5 + 152\*b\*\*4 - 192\*sqrt(2)\*b\*\*3 + 189\*b\*\*2 + 27\*sqrt(2)\*b - 81) + 1226\*b\*\*3/(4\*b\*\*6 - 28\*sqrt(2)\*b\*\*5 + 152\*b\*\*4 - 192\*sqrt(2)\*b\*\*3 + 189\*b\*\*2 + 27\*sqrt(2)\*b - 81) - 144\*sqrt(2)\*b\*\*2/(4\*b\*\*6 - 28\*sqrt(2)\*b\*\*5 + 152\*b\*\*4 - 192\*sqrt(2)\*b\*\*3 + 189\*b\*\*2 + 27\*sqrt(2)\*b - 81) - 378\*b/(4\*b\*\*6 - 28\*sqrt(2)\*b\*\*5 + 152\*b\*\*4 - 192\*sqrt(2)\*b\*\*3 + 189\*b\*\*2 + 27\*sqrt(2)\*b - 81) + 108\*sqrt(2)/(4\*b\*\*6 - 28\*sqrt(2)\*b\*\*5 + 152\*b\*\*4 - 192\*sqrt(2)\*b\*\*3 + 189\*b\*\*2 + 27\*sqrt(2)\*b - 81)) + x)))

$$3.94 \quad \int \frac{2a-x^2}{a^2-ax^2+x^4} dx$$

**Optimal.** Leaf size=114

$$-\frac{\sqrt{3} \log(-\sqrt{3} \sqrt{a} x + a + x^2)}{4\sqrt{a}} + \frac{\sqrt{3} \log(\sqrt{3} \sqrt{a} x + a + x^2)}{4\sqrt{a}} - \frac{\tan^{-1}\left(\sqrt{3} - \frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}} + \frac{\tan^{-1}\left(\frac{2x}{\sqrt{a}} + \sqrt{3}\right)}{2\sqrt{a}}$$

**Rubi [A]** time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1169, 634, 617, 204, 628}

$$-\frac{\sqrt{3} \log(-\sqrt{3} \sqrt{a} x + a + x^2)}{4\sqrt{a}} + \frac{\sqrt{3} \log(\sqrt{3} \sqrt{a} x + a + x^2)}{4\sqrt{a}} - \frac{\tan^{-1}\left(\sqrt{3} - \frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}} + \frac{\tan^{-1}\left(\frac{2x}{\sqrt{a}} + \sqrt{3}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[(2\*a - x^2)/(a^2 - a\*x^2 + x^4), x]

[Out] -ArcTan[Sqrt[3] - (2\*x)/Sqrt[a]]/(2\*Sqrt[a]) + ArcTan[Sqrt[3] + (2\*x)/Sqrt[a]]/(2\*Sqrt[a]) - (Sqrt[3]\*Log[a - Sqrt[3]\*Sqrt[a]\*x + x^2])/(4\*Sqrt[a]) + (Sqrt[3]\*Log[a + Sqrt[3]\*Sqrt[a]\*x + x^2])/(4\*Sqrt[a])

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 1169

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2\*q - b/c, 2]}, Dist[1/(2\*c\*q\*r), Int[(d\*r - (d - e\*q)\*x)/(q - r\*x + x^2), x], x] + Dist[1/(2\*c\*q\*r), Int[(d\*r + (d - e\*q)\*x)/(q + r\*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NegQ[b^2 - 4\*a\*c]

#### Rubi steps

$$\begin{aligned}
\int \frac{2a - x^2}{a^2 - ax^2 + x^4} dx &= \frac{\int \frac{2\sqrt{3}a^{3/2} - 3ax}{a - \sqrt{3}\sqrt{a}x + x^2} dx}{2\sqrt{3}a^{3/2}} + \frac{\int \frac{2\sqrt{3}a^{3/2} + 3ax}{a + \sqrt{3}\sqrt{a}x + x^2} dx}{2\sqrt{3}a^{3/2}} \\
&= \frac{1}{4} \int \frac{1}{a - \sqrt{3}\sqrt{a}x + x^2} dx + \frac{1}{4} \int \frac{1}{a + \sqrt{3}\sqrt{a}x + x^2} dx - \frac{\sqrt{3}}{4\sqrt{a}} \int \frac{-\sqrt{3}\sqrt{a} + 2x}{a - \sqrt{3}\sqrt{a}x + x^2} dx + \frac{\sqrt{3}}{4} \int \frac{1}{a + \sqrt{3}\sqrt{a}x + x^2} dx \\
&= -\frac{\sqrt{3} \log(a - \sqrt{3}\sqrt{a}x + x^2)}{4\sqrt{a}} + \frac{\sqrt{3} \log(a + \sqrt{3}\sqrt{a}x + x^2)}{4\sqrt{a}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{3} - x^2} dx, x, 1 - \frac{x}{\sqrt{3}}\right)}{2\sqrt{3}\sqrt{a}} \\
&= -\frac{\tan^{-1}\left(\sqrt{3} - \frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}} + \frac{\tan^{-1}\left(\sqrt{3} + \frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt{3} \log(a - \sqrt{3}\sqrt{a}x + x^2)}{4\sqrt{a}} + \frac{\sqrt{3} \log(a + \sqrt{3}\sqrt{a}x + x^2)}{4\sqrt{a}}
\end{aligned}$$

**Mathematica [C]** time = 0.17, size = 115, normalized size = 1.01

$$\frac{\sqrt[4]{-1} \left( \sqrt{\sqrt{3} - i} (\sqrt{3} - 3i) \tanh^{-1} \left( \frac{(1+i)x}{\sqrt{\sqrt{3} + i\sqrt{a}}} \right) - \sqrt{\sqrt{3} + i} (\sqrt{3} + 3i) \tan^{-1} \left( \frac{(1+i)x}{\sqrt{\sqrt{3} - i\sqrt{a}}} \right) \right)}{2\sqrt{6}\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[(2\*a - x^2)/(a^2 - a\*x^2 + x^4), x]

[Out] ((-1)^(1/4)\*(-(Sqrt[I + Sqrt[3]]\*(3\*I + Sqrt[3])\*ArcTan[((1 + I)\*x)/(Sqrt[-I + Sqrt[3]]\*Sqrt[a]])) + Sqrt[-I + Sqrt[3]]\*(-3\*I + Sqrt[3])\*ArcTanh[((1 + I)\*x)/(Sqrt[I + Sqrt[3]]\*Sqrt[a]])))/(2\*Sqrt[6]\*Sqrt[a])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2a - x^2}{a^2 - ax^2 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2\*a - x^2)/(a^2 - a\*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(2\*a - x^2)/(a^2 - a\*x^2 + x^4), x]

**fricas [B]** time = 0.85, size = 517, normalized size = 4.54

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2\*a)/(x^4-a\*x^2+a^2), x, algorithm="fricas")

[Out] 1/24\*(sqrt(3)\*a\*sqrt(a^(-2)) + 2\*sqrt(3))\*sqrt(-4\*a\*sqrt(a^(-2)) + 8)\*(a^(-2))^(1/4)\*log(6\*a^2\*sqrt(a^(-2)) + 6\*x^2 + (sqrt(3)\*a^2\*sqrt(a^(-2)))\*x + 2\*sqrt(3)\*a\*x)\*sqrt(-4\*a\*sqrt(a^(-2)) + 8)\*(a^(-2))^(1/4)) - 1/24\*(sqrt(3)\*a\*sqrt(a^(-2)) + 2\*sqrt(3))\*sqrt(-4\*a\*sqrt(a^(-2)) + 8)\*(a^(-2))^(1/4)\*log(6\*a^2\*sqrt(a^(-2)) + 6\*x^2 - (sqrt(3)\*a^2\*sqrt(a^(-2)))\*x + 2\*sqrt(3)\*a\*x)\*sqrt(-4\*a\*sqrt(a^(-2)) + 8)\*(a^(-2))^(1/4)) - 1/2\*sqrt(-4\*a\*sqrt(a^(-2)) + 8)\*(a^(-2))^(1/4)\*arctan(1/18\*(sqrt(6)\*a^2\*sqrt(a^(-2)) + 2\*sqrt(6)\*a)\*sqrt(6\*a^2\*sqrt(a^(-2)) + 6\*x^2 + (sqrt(3)\*a^2\*sqrt(a^(-2)))\*x + 2\*sqrt(3)\*a\*x)\*sqrt(-4\*a\*sqrt(a^(-2)) + 8)\*(a^(-2))^(1/4))\*sqrt(-4\*a\*sqrt(a^(-2)) + 8)\*(a^(-2))^(3/4) - 1/3\*(a^2\*sqrt(a^(-2))\*x + 2\*a\*x)\*sqrt(-4\*a\*sqrt(a^(-2)) + 8)\*(a^(-2))^(3/4) - 1/3\*sqrt(3)\*a\*sqrt(a^(-2)) - 2/3\*sqrt(3)) - 1/2\*sqrt(-4\*a\*sqrt(a^(-2)) + 8)\*(a^(-2))^(1/4)\*sqrt(-4\*a\*sqrt(a^(-2)) + 8)









) / 2) / sqrt(abs(a)))

**maple** [A] time = 0.04, size = 92, normalized size = 0.81

$$\frac{\arctan\left(\frac{2x+\sqrt{3}\sqrt{a}}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\arctan\left(\frac{-2x+\sqrt{3}\sqrt{a}}{\sqrt{a}}\right)}{2\sqrt{a}} + \frac{\sqrt{3}\ln(x^2+\sqrt{3}\sqrt{a}x+a)}{4\sqrt{a}} - \frac{\sqrt{3}\ln(-x^2+\sqrt{3}\sqrt{a}x-a)}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+2\*a)/(x^4-a\*x^2+a^2),x)

[Out] -1/4/a^(1/2)\*3^(1/2)\*ln(x\*3^(1/2)\*a^(1/2)-x^2-a)-1/2/a^(1/2)\*arctan((3^(1/2)\*a^(1/2)-2\*x)/a^(1/2))+1/4\*ln(a+x^2+x\*3^(1/2)\*a^(1/2))\*3^(1/2)/a^(1/2)+1/2/a^(1/2)\*arctan((2\*x+3^(1/2)\*a^(1/2))/a^(1/2))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 - 2a}{x^4 - ax^2 + a^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2\*a)/(x^4-a\*x^2+a^2),x, algorithm="maxima")

[Out] -integrate((x^2 - 2\*a)/(x^4 - a\*x^2 + a^2), x)

**mupad** [B] time = 4.48, size = 133, normalized size = 1.17

$$\frac{\sqrt{8} \operatorname{atan}\left(x\sqrt{\frac{1}{8a} + \frac{\sqrt{3}i}{8a}} + \sqrt{3}x\sqrt{\frac{1}{8a} + \frac{\sqrt{3}i}{8a}}\right)\sqrt{\frac{1+\sqrt{3}i}{a}} + \sqrt{8} \operatorname{atan}\left(x\sqrt{\frac{1}{8a} - \frac{\sqrt{3}i}{8a}} - \sqrt{3}x\sqrt{\frac{1}{8a} - \frac{\sqrt{3}i}{8a}}\right)\sqrt{\frac{-1+\sqrt{3}i}{a}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*a - x^2)/(a^2 - a\*x^2 + x^4),x)

[Out] -(8^(1/2)\*atan(x\*((3^(1/2)\*1i)/(8\*a) + 1/(8\*a))^(1/2)\*1i + 3^(1/2)\*x\*((3^(1/2)\*1i)/(8\*a) + 1/(8\*a))^(1/2))\*((3^(1/2)\*1i + 1)/a)^(1/2)\*1i)/4 - (8^(1/2)\*atan(x\*(1/(8\*a) - (3^(1/2)\*1i)/(8\*a))^(1/2)\*1i - 3^(1/2)\*x\*(1/(8\*a) - (3^(1/2)\*1i)/(8\*a))^(1/2))\*(-(3^(1/2)\*1i - 1)/a)^(1/2)\*1i)/4

**sympy** [A] time = 0.25, size = 27, normalized size = 0.24

$$-\operatorname{RootSum}\left(16t^4a^2 - 4t^2a + 1, (t \mapsto t \log(-2ta + x))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*2+2\*a)/(x\*\*4-a\*x\*\*2+a\*\*2),x)

[Out] -RootSum(16\*\_t\*\*4\*a\*\*2 - 4\*\_t\*\*2\*a + 1, Lambda(\_t, \_t\*log(-2\*\_t\*a + x)))

$$3.95 \quad \int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}x^2+x^4} dx$$

**Optimal.** Leaf size=122

$$\frac{\sqrt{3} \log\left(-\sqrt{3} \sqrt[4]{a} x + \sqrt{a} + x^2\right)}{4\sqrt[4]{a}} + \frac{\sqrt{3} \log\left(\sqrt{3} \sqrt[4]{a} x + \sqrt{a} + x^2\right)}{4\sqrt[4]{a}} - \frac{\tan^{-1}\left(\sqrt{3} - \frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}} + \frac{\tan^{-1}\left(\frac{2x}{\sqrt[4]{a}} + \sqrt{3}\right)}{2\sqrt[4]{a}}$$

**Rubi [A]** time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {1169, 634, 617, 204, 628}

$$\frac{\sqrt{3} \log\left(-\sqrt{3} \sqrt[4]{a} x + \sqrt{a} + x^2\right)}{4\sqrt[4]{a}} + \frac{\sqrt{3} \log\left(\sqrt{3} \sqrt[4]{a} x + \sqrt{a} + x^2\right)}{4\sqrt[4]{a}} - \frac{\tan^{-1}\left(\sqrt{3} - \frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}} + \frac{\tan^{-1}\left(\frac{2x}{\sqrt[4]{a}} + \sqrt{3}\right)}{2\sqrt[4]{a}}$$

Antiderivative was successfully verified.

[In] Int[(2\*Sqrt[a] - x^2)/(a - Sqrt[a]\*x^2 + x^4), x]

[Out] -ArcTan[Sqrt[3] - (2\*x)/a^(1/4)]/(2\*a^(1/4)) + ArcTan[Sqrt[3] + (2\*x)/a^(1/4)]/(2\*a^(1/4)) - (Sqrt[3]\*Log[Sqrt[a] - Sqrt[3]\*a^(1/4)\*x + x^2])/(4\*a^(1/4)) + (Sqrt[3]\*Log[Sqrt[a] + Sqrt[3]\*a^(1/4)\*x + x^2])/(4\*a^(1/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 1169

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2\*q - b/c, 2]}, Dist[1/(2\*c\*q\*r), Int[(d\*r - (d - e\*q)\*x)/(q - r\*x + x^2), x], x] + Dist[1/(2\*c\*q\*r), Int[(d\*r + (d - e\*q)\*x)/(q + r\*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NegQ[b^2 - 4\*a\*c]

Rubi steps

$$\begin{aligned}
\int \frac{2\sqrt{a} - x^2}{a - \sqrt{a}x^2 + x^4} dx &= \frac{\int \frac{2\sqrt{3}a^{3/4} - 3\sqrt{a}x}{\sqrt{a} - \sqrt{3}\sqrt[4]{a}x + x^2} dx}{2\sqrt{3}a^{3/4}} + \frac{\int \frac{2\sqrt{3}a^{3/4} + 3\sqrt{a}x}{\sqrt{a} + \sqrt{3}\sqrt[4]{a}x + x^2} dx}{2\sqrt{3}a^{3/4}} \\
&= \frac{1}{4} \int \frac{1}{\sqrt{a} - \sqrt{3}\sqrt[4]{a}x + x^2} dx + \frac{1}{4} \int \frac{1}{\sqrt{a} + \sqrt{3}\sqrt[4]{a}x + x^2} dx - \frac{\sqrt{3}}{4\sqrt[4]{a}} \int \frac{-\sqrt{3}\sqrt[4]{a} + 2x}{\sqrt{a} - \sqrt{3}\sqrt[4]{a}x + x^2} dx + \frac{\sqrt{3}}{4\sqrt[4]{a}} \int \frac{\sqrt{3}\sqrt[4]{a} + 2x}{\sqrt{a} + \sqrt{3}\sqrt[4]{a}x + x^2} dx \\
&= -\frac{\sqrt{3} \log(\sqrt{a} - \sqrt{3}\sqrt[4]{a}x + x^2)}{4\sqrt[4]{a}} + \frac{\sqrt{3} \log(\sqrt{a} + \sqrt{3}\sqrt[4]{a}x + x^2)}{4\sqrt[4]{a}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2} dx, x, \frac{\sqrt{3}\sqrt[4]{a} + 2x}{\sqrt{a} - \sqrt{3}\sqrt[4]{a}x + x^2}\right)}{2\sqrt{3}\sqrt[4]{a}} \\
&= -\frac{\tan^{-1}\left(\sqrt{3} - \frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}} + \frac{\tan^{-1}\left(\sqrt{3} + \frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}} - \frac{\sqrt{3} \log(\sqrt{a} - \sqrt{3}\sqrt[4]{a}x + x^2)}{4\sqrt[4]{a}} + \frac{\sqrt{3} \log(\sqrt{a} + \sqrt{3}\sqrt[4]{a}x + x^2)}{4\sqrt[4]{a}}
\end{aligned}$$

**Mathematica** [C] time = 0.15, size = 115, normalized size = 0.94

$$\frac{\sqrt[4]{-1} \left( \sqrt{\sqrt{3} - i} (\sqrt{3} - 3i) \tanh^{-1} \left( \frac{(1+i)x}{\sqrt{\sqrt{3} + i} \sqrt[4]{a}} \right) - \sqrt{\sqrt{3} + i} (\sqrt{3} + 3i) \tan^{-1} \left( \frac{(1+i)x}{\sqrt{\sqrt{3} - i} \sqrt[4]{a}} \right) \right)}{2\sqrt{6} \sqrt[4]{a}}$$

Antiderivative was successfully verified.

[In] Integrate[(2\*Sqrt[a] - x^2)/(a - Sqrt[a]\*x^2 + x^4), x]

[Out] ((-1)^(1/4)\*(-(Sqrt[I + Sqrt[3]]\*(3\*I + Sqrt[3])\*ArcTan[((1 + I)\*x)/(Sqrt[-I + Sqrt[3]]\*a^(1/4))]) + Sqrt[-I + Sqrt[3]]\*(-3\*I + Sqrt[3])\*ArcTanh[((1 + I)\*x)/(Sqrt[I + Sqrt[3]]\*a^(1/4))]))/(2\*Sqrt[6]\*a^(1/4))

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\sqrt{a} - x^2}{a - \sqrt{a}x^2 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2\*Sqrt[a] - x^2)/(a - Sqrt[a]\*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(2\*Sqrt[a] - x^2)/(a - Sqrt[a]\*x^2 + x^4), x]

**fricas** [B] time = 1.07, size = 251, normalized size = 2.06

$$\frac{1}{2} \sqrt{\frac{1}{2}} \sqrt{\frac{\sqrt{3}a\sqrt{-1} + \sqrt{a}}{a}} \log\left(\sqrt{\frac{1}{2}} \sqrt{a} \sqrt{\frac{\sqrt{3}a\sqrt{-1} + \sqrt{a}}{a}} + x\right) + \frac{1}{2} \sqrt{\frac{1}{2}} \sqrt{\frac{\sqrt{3}a\sqrt{1} + \sqrt{a}}{a}} \log\left(\sqrt{\frac{1}{2}} \sqrt{a} \sqrt{\frac{\sqrt{3}a\sqrt{1} + \sqrt{a}}{a}} + x\right) - \frac{1}{2} \sqrt{\frac{1}{2}} \sqrt{\frac{\sqrt{3}a\sqrt{-1} - \sqrt{a}}{a}} \log\left(\sqrt{\frac{1}{2}} \sqrt{a} \sqrt{\frac{\sqrt{3}a\sqrt{-1} - \sqrt{a}}{a}} + x\right) - \frac{1}{2} \sqrt{\frac{1}{2}} \sqrt{\frac{\sqrt{3}a\sqrt{1} - \sqrt{a}}{a}} \log\left(\sqrt{\frac{1}{2}} \sqrt{a} \sqrt{\frac{\sqrt{3}a\sqrt{1} - \sqrt{a}}{a}} + x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2\*a^(1/2))/(a+x^4-x^2\*a^(1/2)), x, algorithm="fricas")

[Out] 1/2\*sqrt(1/2)\*sqrt((sqrt(3)\*a\*sqrt(-1/a) + sqrt(a))/a)\*log(sqrt(1/2)\*sqrt(a)\*sqrt((sqrt(3)\*a\*sqrt(-1/a) + sqrt(a))/a) + x) - 1/2\*sqrt(1/2)\*sqrt((sqrt(3)\*a\*sqrt(-1/a) + sqrt(a))/a)\*log(-sqrt(1/2)\*sqrt(a)\*sqrt((sqrt(3)\*a\*sqrt(-1/a) + sqrt(a))/a) + x) + 1/2\*sqrt(1/2)\*sqrt(-(sqrt(3)\*a\*sqrt(-1/a) - sqrt(a))/a)\*log(sqrt(1/2)\*sqrt(a)\*sqrt(-(sqrt(3)\*a\*sqrt(-1/a) - sqrt(a))/a) + x) - 1/2\*sqrt(1/2)\*sqrt(-(sqrt(3)\*a\*sqrt(-1/a) - sqrt(a))/a)\*log(-sqrt(1/2)\*sqrt(a)\*sqrt(-(sqrt(3)\*a\*sqrt(-1/a) - sqrt(a))/a) + x)

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2\*a^(1/2))/(a+x^4-x^2\*a^(1/2)),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index\_m &  
i,const vecteur & l) Error: Bad Argument Value

**maple [A]** time = 0.05, size = 96, normalized size = 0.79

$$\frac{\arctan\left(\frac{2x+\sqrt{3}a^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{2a^{\frac{1}{4}}} - \frac{\arctan\left(\frac{-2x+\sqrt{3}a^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{2a^{\frac{1}{4}}} + \frac{\sqrt{3} \ln\left(x^2 + \sqrt{3}a^{\frac{1}{4}}x + \sqrt{a}\right)}{4a^{\frac{1}{4}}} - \frac{\sqrt{3} \ln\left(-x^2 + \sqrt{3}a^{\frac{1}{4}}x - \sqrt{a}\right)}{4a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+2\*a^(1/2))/(a+x^4-a^(1/2)\*x^2),x)

[Out] 1/4\*ln(x^2+a^(1/4)\*x\*3^(1/2)+a^(1/2))\*3^(1/2)/a^(1/4)+1/2/a^(1/4)\*arctan((2  
\*x+3^(1/2)\*a^(1/4))/a^(1/4))-1/4/a^(1/4)\*3^(1/2)\*ln(a^(1/4)\*x\*3^(1/2)-x^2-a  
^(1/2))-1/2/a^(1/4)\*arctan((3^(1/2)\*a^(1/4)-2\*x)/a^(1/4))

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{x^2 - 2\sqrt{a}}{x^4 - \sqrt{a}x^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2\*a^(1/2))/(a+x^4-x^2\*a^(1/2)),x, algorithm="maxima")

[Out] -integrate((x^2 - 2\*sqrt(a))/(x^4 - sqrt(a)\*x^2 + a), x)

**mupad [B]** time = 5.06, size = 159, normalized size = 1.30

$$2 \operatorname{atanh} \left( x \sqrt{\frac{1}{8\sqrt{a}} - \frac{\sqrt{-27a^3}}{24a^2}} - \frac{9a^{3/2}x \sqrt{\frac{1}{8\sqrt{a}} - \frac{\sqrt{-27a^3}}{24a^2}}}{\sqrt{-27a^3}} \right) \sqrt{\frac{1}{8\sqrt{a}} - \frac{\sqrt{-27a^3}}{24a^2}} + 2 \operatorname{atanh} \left( x \sqrt{\frac{\sqrt{-27a^3}}{24a^2} + \frac{1}{8\sqrt{a}}} + \frac{9a^{3/2}x \sqrt{\frac{\sqrt{-27a^3}}{24a^2} + \frac{1}{8\sqrt{a}}}}{\sqrt{-27a^3}} \right) \sqrt{\frac{\sqrt{-27a^3}}{24a^2} + \frac{1}{8\sqrt{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*a^(1/2) - x^2)/(a + x^4 - a^(1/2)\*x^2),x)

[Out] 2\*atanh(x\*(1/(8\*a^(1/2)) - (-27\*a^3)^(1/2)/(24\*a^2))^(1/2) - (9\*a^(3/2)\*x\*(  
1/(8\*a^(1/2)) - (-27\*a^3)^(1/2)/(24\*a^2))^(1/2))/(-27\*a^3)^(1/2))\*1/(8\*a^(  
1/2)) - (-27\*a^3)^(1/2)/(24\*a^2))^(1/2) + 2\*atanh(x\*((-27\*a^3)^(1/2)/(24\*a^  
2) + 1/(8\*a^(1/2)))^(1/2) + (9\*a^(3/2)\*x\*((-27\*a^3)^(1/2)/(24\*a^2) + 1/(8\*a  
^(1/2)))^(1/2))/(-27\*a^3)^(1/2))\*((-27\*a^3)^(1/2)/(24\*a^2) + 1/(8\*a^(1/2)))  
^(1/2)

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*2+2\*a\*\*(1/2))/(a+x\*\*4-x\*\*2\*a\*\*(1/2)),x)

[Out] Exception raised: PolynomialError

$$3.96 \quad \int \frac{2b^{2/3} + x^2}{b^{4/3} + b^{2/3}x^2 + x^4} dx$$

**Optimal.** Leaf size=124

$$-\frac{\log(b^{2/3} - \sqrt[3]{b}x + x^2)}{4\sqrt[3]{b}} + \frac{\log(b^{2/3} + \sqrt[3]{b}x + x^2)}{4\sqrt[3]{b}} - \frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{b}-2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{b}+2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}$$

**Rubi [A]** time = 0.08, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {1169, 634, 617, 204, 628}

$$-\frac{\log(b^{2/3} - \sqrt[3]{b}x + x^2)}{4\sqrt[3]{b}} + \frac{\log(b^{2/3} + \sqrt[3]{b}x + x^2)}{4\sqrt[3]{b}} - \frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{b}-2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{b}+2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[(2\*b^(2/3) + x^2)/(b^(4/3) + b^(2/3)\*x^2 + x^4), x]

[Out] -(Sqrt[3]\*ArcTan[(b^(1/3) - 2\*x)/(Sqrt[3]\*b^(1/3))]/(2\*b^(1/3)) + (Sqrt[3]\*ArcTan[(b^(1/3) + 2\*x)/(Sqrt[3]\*b^(1/3))]/(2\*b^(1/3)) - Log[b^(2/3) - b^(1/3)\*x + x^2]/(4\*b^(1/3)) + Log[b^(2/3) + b^(1/3)\*x + x^2]/(4\*b^(1/3)))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 1169

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2\*q - b/c, 2]}, Dist[1/(2\*c\*q\*r), Int[(d\*r - (d - e\*q)\*x)/(q - r\*x + x^2), x], x] + Dist[1/(2\*c\*q\*r), Int[(d\*r + (d - e\*q)\*x)/(q + r\*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NegQ[b^2 - 4\*a\*c]

#### Rubi steps

$$\begin{aligned}
\int \frac{2b^{2/3} + x^2}{b^{4/3} + b^{2/3}x^2 + x^4} dx &= \frac{\int \frac{2b - b^{2/3}x}{b^{2/3} - \sqrt[3]{b}x + x^2} dx}{2b} + \frac{\int \frac{2b + b^{2/3}x}{b^{2/3} + \sqrt[3]{b}x + x^2} dx}{2b} \\
&= \frac{3}{4} \int \frac{1}{b^{2/3} - \sqrt[3]{b}x + x^2} dx + \frac{3}{4} \int \frac{1}{b^{2/3} + \sqrt[3]{b}x + x^2} dx - \frac{\int \frac{-\sqrt[3]{b} + 2x}{b^{2/3} - \sqrt[3]{b}x + x^2} dx}{4\sqrt[3]{b}} + \frac{\int \frac{\sqrt[3]{b} + 2x}{b^{2/3} + \sqrt[3]{b}x + x^2} dx}{4\sqrt[3]{b}} \\
&= -\frac{\log(b^{2/3} - \sqrt[3]{b}x + x^2)}{4\sqrt[3]{b}} + \frac{\log(b^{2/3} + \sqrt[3]{b}x + x^2)}{4\sqrt[3]{b}} + \frac{3 \operatorname{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 - \frac{2x}{\sqrt[3]{b}}\right)}{2\sqrt[3]{b}} \\
&= -\frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{b}-2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{b}+2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}} - \frac{\log(b^{2/3} - \sqrt[3]{b}x + x^2)}{4\sqrt[3]{b}} + \frac{\log(b^{2/3} + \sqrt[3]{b}x + x^2)}{4\sqrt[3]{b}}
\end{aligned}$$

**Mathematica [C]** time = 0.13, size = 115, normalized size = 0.93

$$\frac{\sqrt[4]{-1} \left( \sqrt{\sqrt{3} - i} (\sqrt{3} - 3i) \tan^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3} + i} \sqrt[3]{b}}\right) - \sqrt{\sqrt{3} + i} (\sqrt{3} + 3i) \tanh^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3} - i} \sqrt[3]{b}}\right) \right)}{2\sqrt{6} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(2\*b^(2/3) + x^2)/(b^(4/3) + b^(2/3)\*x^2 + x^4), x]

[Out] ((-1)^(1/4)\*(Sqrt[-I + Sqrt[3]]\*(-3\*I + Sqrt[3])\*ArcTan[((1 + I)\*x)/(Sqrt[I + Sqrt[3]]\*b^(1/3))]) - Sqrt[I + Sqrt[3]]\*(3\*I + Sqrt[3])\*ArcTanh[((1 + I)\*x)/(Sqrt[-I + Sqrt[3]]\*b^(1/3)))]/(2\*Sqrt[6]\*b^(1/3))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2b^{2/3} + x^2}{b^{4/3} + b^{2/3}x^2 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2\*b^(2/3) + x^2)/(b^(4/3) + b^(2/3)\*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(2\*b^(2/3) + x^2)/(b^(4/3) + b^(2/3)\*x^2 + x^4), x]

**fricas [A]** time = 1.29, size = 264, normalized size = 2.13

$$\frac{\sqrt{3}b \sqrt{\frac{1}{b^3}} \log\left(\frac{2x^2 + \sqrt{3}\sqrt{2b^3x^2 + bx + b^3}}{x^2 + b}\right) + \sqrt{3}b \sqrt{\frac{1}{b^3}} \log\left(\frac{2x^2 + \sqrt{3}\sqrt{2b^3x^2 - bx - b^3}}{x^2 - b}\right) + b^{\frac{2}{3}} \log(x^2 + b^{\frac{1}{3}}x + b^{\frac{2}{3}}) - b^{\frac{2}{3}} \log(x^2 - b^{\frac{1}{3}}x + b^{\frac{2}{3}})}{4b} - \frac{2\sqrt{3}b^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\sqrt{2x^2 + b^{\frac{1}{3}}}}{3b^{\frac{1}{3}}}\right) - 2\sqrt{3}b^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\sqrt{2x^2 - b^{\frac{1}{3}}}}{3b^{\frac{1}{3}}}\right) + b^{\frac{2}{3}} \log(x^2 + b^{\frac{1}{3}}x + b^{\frac{2}{3}}) - b^{\frac{2}{3}} \log(x^2 - b^{\frac{1}{3}}x + b^{\frac{2}{3}})}{4b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)\*x^2+x^4), x, algorithm="fricas")

[Out] [1/4\*(sqrt(3)\*b\*sqrt(-1/b^(2/3))\*log((2\*x^3 + sqrt(3)\*(2\*b^(2/3)\*x^2 + b\*x - b^(4/3))\*sqrt(-1/b^(2/3)) - 3\*b^(2/3)\*x - b)/(x^3 + b)) + sqrt(3)\*b\*sqrt(-1/b^(2/3))\*log((2\*x^3 + sqrt(3)\*(2\*b^(2/3)\*x^2 - b\*x - b^(4/3))\*sqrt(-1/b^(2/3)) - 3\*b^(2/3)\*x + b)/(x^3 - b)) + b^(2/3)\*log(x^2 + b^(1/3)\*x + b^(2/3)) - b^(2/3)\*log(x^2 - b^(1/3)\*x + b^(2/3)))/b, 1/4\*(2\*sqrt(3)\*b^(2/3)\*arctan(1/3\*sqrt(3)\*(2\*x + b^(1/3))/b^(1/3)) - 2\*sqrt(3)\*b^(2/3)\*arctan(-1/3\*sqrt(3)\*(2\*x - b^(1/3))/b^(1/3)) + b^(2/3)\*log(x^2 + b^(1/3)\*x + b^(2/3)) - b^(2/3)\*log(x^2 - b^(1/3)\*x + b^(2/3)))/b]

**giac** [A] time = 0.18, size = 92, normalized size = 0.74

$$\frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2x+b^{\frac{1}{3}}\right)}{3|b|^{\frac{1}{3}}}\right)}{2|b|^{\frac{1}{3}}} + \frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2x-b^{\frac{1}{3}}\right)}{3|b|^{\frac{1}{3}}}\right)}{2|b|^{\frac{1}{3}}} + \frac{\log\left(x^2 + b^{\frac{1}{3}}x + b^{\frac{2}{3}}\right)}{4b^{\frac{1}{3}}} - \frac{\log\left(x^2 - b^{\frac{1}{3}}x + b^{\frac{2}{3}}\right)}{4b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)\*x^2+x^4),x, algorithm="giac")

[Out] 1/2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*x + b^(1/3))/abs(b)^(1/3))/abs(b)^(1/3) + 1/2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*x - b^(1/3))/abs(b)^(1/3))/abs(b)^(1/3) + 1/4\*log(x^2 + b^(1/3)\*x + b^(2/3))/b^(1/3) - 1/4\*log(x^2 - b^(1/3)\*x + b^(2/3))/b^(1/3)

**maple** [A] time = 0.03, size = 89, normalized size = 0.72

$$\frac{\sqrt{3} \arctan\left(\frac{\left(2x-b^{\frac{1}{3}}\right)\sqrt{3}}{3b^{\frac{1}{3}}}\right)}{2b^{\frac{1}{3}}} + \frac{\sqrt{3} \arctan\left(\frac{\left(2x+b^{\frac{1}{3}}\right)\sqrt{3}}{3b^{\frac{1}{3}}}\right)}{2b^{\frac{1}{3}}} - \frac{\ln\left(x^2 - b^{\frac{1}{3}}x + b^{\frac{2}{3}}\right)}{4b^{\frac{1}{3}}} + \frac{\ln\left(x^2 + b^{\frac{1}{3}}x + b^{\frac{2}{3}}\right)}{4b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)\*x^2+x^4),x)

[Out] -1/4\*ln(b^(2/3)-b^(1/3)\*x+x^2)/b^(1/3)+1/2\*3^(1/2)/b^(1/3)\*arctan(1/3\*(-b^(1/3)+2\*x)\*3^(1/2)/b^(1/3))+1/4\*ln(b^(2/3)+b^(1/3)\*x+x^2)/b^(1/3)+1/2\*arctan(1/3\*(b^(1/3)+2\*x)/b^(1/3)\*3^(1/2))\*3^(1/2)/b^(1/3)

**maxima** [A] time = 2.29, size = 88, normalized size = 0.71

$$\frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2x+b^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}}\right)}{2b^{\frac{1}{3}}} + \frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2x-b^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}}\right)}{2b^{\frac{1}{3}}} + \frac{\log\left(x^2 + b^{\frac{1}{3}}x + b^{\frac{2}{3}}\right)}{4b^{\frac{1}{3}}} - \frac{\log\left(x^2 - b^{\frac{1}{3}}x + b^{\frac{2}{3}}\right)}{4b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)\*x^2+x^4),x, algorithm="maxima")

[Out] 1/2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*x + b^(1/3))/b^(1/3))/b^(1/3) + 1/2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*x - b^(1/3))/b^(1/3))/b^(1/3) + 1/4\*log(x^2 + b^(1/3)\*x + b^(2/3))/b^(1/3) - 1/4\*log(x^2 - b^(1/3)\*x + b^(2/3))/b^(1/3)

**mupad** [B] time = 0.24, size = 133, normalized size = 1.07

$$\frac{\sqrt{8} \operatorname{atan}\left(x \sqrt{\frac{1}{8b^{2/3}} - \frac{\sqrt{3} 1i}{8b^{2/3}}} + \sqrt{3} x \sqrt{\frac{1}{8b^{2/3}} - \frac{\sqrt{3} 1i}{8b^{2/3}}}\right) \sqrt{\frac{-1+\sqrt{3} 1i}{b^{2/3}}} 1i}{4} + \frac{\sqrt{8} \operatorname{atan}\left(x \sqrt{\frac{1}{8b^{2/3}} + \frac{\sqrt{3} 1i}{8b^{2/3}}} 1i - \sqrt{3} x \sqrt{\frac{1}{8b^{2/3}} + \frac{\sqrt{3} 1i}{8b^{2/3}}}\right) \sqrt{\frac{-1+\sqrt{3} 1i}{b^{2/3}}} 1i}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*b^(2/3) + x^2)/(b^(4/3) + x^4 + b^(2/3)\*x^2),x)

[Out] (8^(1/2)\*atan(x\*(-3^(1/2)\*1i)/(8\*b^(2/3)) - 1/(8\*b^(2/3)))^(1/2)\*1i + 3^(1/2)\*x\*(-3^(1/2)\*1i)/(8\*b^(2/3)) - 1/(8\*b^(2/3)))^(1/2)\*(-3^(1/2)\*1i + 1)/b^(2/3))^(1/2)\*1i)/4 + (8^(1/2)\*atan(x\*((3^(1/2)\*1i)/(8\*b^(2/3)) - 1/(8\*b^(2/3)))^(1/2)\*1i - 3^(1/2)\*x\*((3^(1/2)\*1i)/(8\*b^(2/3)) - 1/(8\*b^(2/3)))^(1/2))\*((3^(1/2)\*1i - 1)/b^(2/3))^(1/2)\*1i)/4



**sympy [C]** time = 0.31, size = 143, normalized size = 1.15

$$\frac{\left(-\frac{1}{4} - \frac{\sqrt{3}i}{4}\right) \log\left(2\sqrt[3]{b}\left(-\frac{1}{4} - \frac{\sqrt{3}i}{4}\right) + x\right) + \left(-\frac{1}{4} + \frac{\sqrt{3}i}{4}\right) \log\left(2\sqrt[3]{b}\left(-\frac{1}{4} + \frac{\sqrt{3}i}{4}\right) + x\right) + \left(\frac{1}{4} - \frac{\sqrt{3}i}{4}\right) \log\left(2\sqrt[3]{b}\left(\frac{1}{4} - \frac{\sqrt{3}i}{4}\right) + x\right) + \left(\frac{1}{4} + \frac{\sqrt{3}i}{4}\right) \log\left(2\sqrt[3]{b}\left(\frac{1}{4} + \frac{\sqrt{3}i}{4}\right) + x\right)}{\sqrt[3]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*b\*\*(2/3)+x\*\*2)/(b\*\*(4/3)+b\*\*(2/3)\*x\*\*2+x\*\*4),x)

[Out] ((-1/4 - sqrt(3)\*I/4)\*log(2\*b\*\*(1/3)\*(-1/4 - sqrt(3)\*I/4) + x) + (-1/4 + sqrt(3)\*I/4)\*log(2\*b\*\*(1/3)\*(-1/4 + sqrt(3)\*I/4) + x) + (1/4 - sqrt(3)\*I/4)\*log(2\*b\*\*(1/3)\*(1/4 - sqrt(3)\*I/4) + x) + (1/4 + sqrt(3)\*I/4)\*log(2\*b\*\*(1/3)\*(1/4 + sqrt(3)\*I/4) + x))/b\*\*(1/3)

$$3.97 \quad \int \frac{A+Bx^2}{a^2-ax^2+x^4} dx$$

**Optimal.** Leaf size=136

$$\frac{(A - aB) \log(-\sqrt{3} \sqrt{a} x + a + x^2)}{4\sqrt{3} a^{3/2}} + \frac{(A - aB) \log(\sqrt{3} \sqrt{a} x + a + x^2)}{4\sqrt{3} a^{3/2}} - \frac{(aB + A) \tan^{-1}\left(\sqrt{3} - \frac{2x}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{(aB + A) \tan^{-1}\left(\frac{2x}{\sqrt{a}} + \sqrt{3}\right)}{2a^{3/2}}$$

**Rubi [A]** time = 0.10, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$ , Rules used = {1169, 634, 617, 204, 628}

$$\frac{(A - aB) \log(-\sqrt{3} \sqrt{a} x + a + x^2)}{4\sqrt{3} a^{3/2}} + \frac{(A - aB) \log(\sqrt{3} \sqrt{a} x + a + x^2)}{4\sqrt{3} a^{3/2}} - \frac{(aB + A) \tan^{-1}\left(\sqrt{3} - \frac{2x}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{(aB + A) \tan^{-1}\left(\frac{2x}{\sqrt{a}} + \sqrt{3}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(a^2 - a\*x^2 + x^4), x]

[Out] -((A + a\*B)\*ArcTan[Sqrt[3] - (2\*x)/Sqrt[a]]/(2\*a^(3/2)) + ((A + a\*B)\*ArcTan[Sqrt[3] + (2\*x)/Sqrt[a]]/(2\*a^(3/2)) - ((A - a\*B)\*Log[a - Sqrt[3]\*Sqrt[a]\*x + x^2])/(4\*Sqrt[3]\*a^(3/2)) + ((A - a\*B)\*Log[a + Sqrt[3]\*Sqrt[a]\*x + x^2])/(4\*Sqrt[3]\*a^(3/2))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 1169

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2\*q - b/c, 2]}, Dist[1/(2\*c\*q\*r), Int[(d\*r - (d - e\*q)\*x)/(q - r\*x + x^2), x], x] + Dist[1/(2\*c\*q\*r), Int[(d\*r + (d - e\*q)\*x)/(q + r\*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NegQ[b^2 - 4\*a\*c]

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^2}{a^2 - ax^2 + x^4} dx &= \frac{\int \frac{\sqrt{3}\sqrt{a}A - (A-aB)x}{a - \sqrt{3}\sqrt{a}x + x^2} dx}{2\sqrt{3}a^{3/2}} + \frac{\int \frac{\sqrt{3}\sqrt{a}A + (A-aB)x}{a + \sqrt{3}\sqrt{a}x + x^2} dx}{2\sqrt{3}a^{3/2}} \\
&= -\frac{(A-aB) \int \frac{-\sqrt{3}\sqrt{a} + 2x}{a - \sqrt{3}\sqrt{a}x + x^2} dx}{4\sqrt{3}a^{3/2}} + \frac{(A-aB) \int \frac{\sqrt{3}\sqrt{a} + 2x}{a + \sqrt{3}\sqrt{a}x + x^2} dx}{4\sqrt{3}a^{3/2}} + \frac{(A+aB) \int \frac{1}{a - \sqrt{3}\sqrt{a}x + x^2} dx}{4a} \\
&= -\frac{(A-aB) \log(a - \sqrt{3}\sqrt{a}x + x^2)}{4\sqrt{3}a^{3/2}} + \frac{(A-aB) \log(a + \sqrt{3}\sqrt{a}x + x^2)}{4\sqrt{3}a^{3/2}} + \frac{(A+aB) \operatorname{Subst}\left(\int \frac{1}{u} du, a - \sqrt{3}\sqrt{a}x + x^2\right)}{4a} \\
&= -\frac{(A+aB) \tan^{-1}\left(\sqrt{3} - \frac{2x}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{(A+aB) \tan^{-1}\left(\sqrt{3} + \frac{2x}{\sqrt{a}}\right)}{2a^{3/2}} - \frac{(A-aB) \log(a - \sqrt{3}\sqrt{a}x + x^2)}{4\sqrt{3}a^{3/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.15, size = 130, normalized size = 0.96

$$\frac{\sqrt[4]{-1} \left( \frac{((\sqrt{3}-i)aB-2iA) \tan^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}-i}\sqrt{a}}\right)}{\sqrt{\sqrt{3}-i}} - \frac{((\sqrt{3}+i)aB+2iA) \operatorname{tanh}^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}+i}\sqrt{a}}\right)}{\sqrt{\sqrt{3}+i}} \right)}{\sqrt{6}a^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(a^2 - a\*x^2 + x^4), x]

[Out] ((-1)^(1/4)\*((( (-2\*I)\*A + (-I + Sqrt[3])\*a\*B)\*ArcTan[((1 + I)\*x)/(Sqrt[-I + Sqrt[3]]\*Sqrt[a]])/Sqrt[-I + Sqrt[3]] - ((2\*I)\*A + (I + Sqrt[3])\*a\*B)\*ArcTanh[((1 + I)\*x)/(Sqrt[I + Sqrt[3]]\*Sqrt[a]])/Sqrt[I + Sqrt[3]]))/Sqrt[6]\*a^(3/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{a^2 - ax^2 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B\*x^2)/(a^2 - a\*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(A + B\*x^2)/(a^2 - a\*x^2 + x^4), x]

**fricas [B]** time = 2.41, size = 4551, normalized size = 33.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(x^4-a\*x^2+a^2), x, algorithm="fricas")

[Out] 1/4\*(4\*(1/9)^(1/4)\*a^6\*sqrt((2\*B^4\*a^4 + 4\*A\*B^3\*a^3 + 6\*A^2\*B^2\*a^2 + 4\*A^3\*B\*a + 2\*A^4 + (B^2\*a^5 + 4\*A\*B\*a^4 + A^2\*a^3)\*sqrt((B^4\*a^4 + 2\*A\*B^3\*a^3 + 3\*A^2\*B^2\*a^2 + 2\*A^3\*B\*a + A^4)/a^6)))/(B^4\*a^4 - 2\*A^2\*B^2\*a^2 + A^4))\*((B^4\*a^4 + 2\*A\*B^3\*a^3 + 3\*A^2\*B^2\*a^2 + 2\*A^3\*B\*a + A^4)/a^6)^(3/4)\*sqrt((B^4\*a^4 - 2\*A^2\*B^2\*a^2 + A^4)/a^6)\*arctan((18\*sqrt(1/3)\*(1/9)^(3/4)\*(sqrt(1/3)\*A\*a^10\*sqrt((B^4\*a^4 + 2\*A\*B^3\*a^3 + 3\*A^2\*B^2\*a^2 + 2\*A^3\*B\*a + A^4)/a^6)\*sqrt((B^4\*a^4 - 2\*A^2\*B^2\*a^2 + A^4)/a^6) - sqrt(1/3)\*(B^3\*a^10 + A\*B^2\*a^9 + A^2\*B\*a^8)\*sqrt((B^4\*a^4 - 2\*A^2\*B^2\*a^2 + A^4)/a^6))\*sqrt((2\*B^4\*a^4 + 4\*A\*B^3\*a^3 + 6\*A^2\*B^2\*a^2 + 4\*A^3\*B\*a + 2\*A^4 + (B^2\*a^5 + 4\*A\*B\*a^4 + A^2\*a^3)\*sqrt((B^4\*a^4 + 2\*A\*B^3\*a^3 + 3\*A^2\*B^2\*a^2 + 2\*A^3\*B\*a + A^4)/a^6)))/(B^4\*a^4 - 2\*A^2\*B^2\*a^2 + A^4))

$$\begin{aligned}
& 4 + A^2 a^3) \sqrt{(B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) / a^6)} / (B^4 a^4 - 2 A^2 B^2 a^2 + A^4) \sqrt{((B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) x^2 + 3 \sqrt{1/3} (1/9)^{1/4} (B a^6 x \sqrt{(B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) / a^6}) - (A B^2 a^4 + A^2 B a^3 + A^3 a^2) x) \sqrt{(2 B^4 a^4 + 4 A B^3 a^3 + 6 A^2 B^2 a^2 + 4 A^3 B a + 2 A^4 + (B^2 a^5 + 4 A B a^4 + A^2 a^3) \sqrt{(B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) / a^6)}) / (B^4 a^4 - 2 A^2 B^2 a^2 + A^4)} \\
& ) * ((B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) / a^6)^{1/4} + (B^2 a^6 + A B a^5 + A^2 a^4) \sqrt{(B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) / a^6)} / (B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) * ((B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) / a^6)^{3/4} \\
& ) - 18 \sqrt{1/3} (1/9)^{3/4} (\sqrt{1/3} A a^{10} x \sqrt{(B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) / a^6}) \sqrt{(B^4 a^4 - 2 A^2 B^2 a^2 + A^4) / a^6} - \sqrt{1/3} (B^3 a^{10} + A B^2 a^9 + A^2 B a^8) x \sqrt{(B^4 a^4 - 2 A^2 B^2 a^2 + A^4) / a^6)} \\
& ) \sqrt{(2 B^4 a^4 + 4 A B^3 a^3 + 6 A^2 B^2 a^2 + 4 A^3 B a + 2 A^4 + (B^2 a^5 + 4 A B a^4 + A^2 a^3) \sqrt{(B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) / a^6)}) / (B^4 a^4 - 2 A^2 B^2 a^2 + A^4)} \\
& ) * ((B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) / a^6)^{3/4} + 2 \sqrt{1/3} (B^4 a^{10} + 2 A B^3 a^9 + 3 A^2 B^2 a^8 + 2 A^3 B a^7 + A^4 a^6) \sqrt{(B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) / a^6} \sqrt{(B^4 a^4 - 2 A^2 B^2 a^2 + A^4) / a^6} \\
& + \sqrt{1/3} (B^6 a^9 + 3 A B^5 a^8 + 6 A^2 B^4 a^7 + 7 A^3 B^3 a^6 + 6 A^4 B^2 a^5 + 3 A^5 B a^4 + A^6 a^3) \sqrt{(B^4 a^4 - 2 A^2 B^2 a^2 + A^4) / a^6)} / (B^8 a^8 + 3 A B^7 a^7 + 5 A^2 B^6 a^6 + 4 A^3 B^5 a^5 - 4 A^5 B^3 a^3 - 5 A^6 B^2 a^2 - 3 A^7 B a - A^8)) + 4 * (1/9)^{1/4} a^6 \sqrt{(2 B^4 a^4 + 4 A B^3 a^3 + 6 A^2 B^2 a^2 + 4 A^3 B a + 2 A^4 + (B^2 a^5 + 4 A B a^4 + A^2 a^3) \sqrt{(B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) / a^6)}) / (B^4 a^4 - 2 A^2 B^2 a^2 + A^4)} \\
& ) * ((B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) / a^6)^{3/4} \sqrt{(B^4 a^4 - 2 A^2 B^2 a^2 + A^4) / a^6} \arctan((18 \sqrt{1/3} (1/9)^{3/4} (\sqrt{1/3} A a^{10} \sqrt{(B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) / a^6}) \sqrt{(B^4 a^4 - 2 A^2 B^2 a^2 + A^4) / a^6} - \sqrt{1/3} (B^3 a^{10} + A B^2 a^9 + A^2 B a^8) \sqrt{(B^4 a^4 - 2 A^2 B^2 a^2 + A^4) / a^6)}) \sqrt{(2 B^4 a^4 + 4 A B^3 a^3 + 6 A^2 B^2 a^2 + 4 A^3 B a + 2 A^4 + (B^2 a^5 + 4 A B a^4 + A^2 a^3) \sqrt{(B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) / a^6)}) / (B^4 a^4 - 2 A^2 B^2 a^2 + A^4)} \\
& ) \sqrt{((B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) x^2 - 3 \sqrt{1/3} (1/9)^{1/4} (B a^6 x \sqrt{(B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) / a^6}) - (A B^2 a^4 + A^2 B a^3 + A^3 a^2) x) \sqrt{(2 B^4 a^4 + 4 A B^3 a^3 + 6 A^2 B^2 a^2 + 4 A^3 B a + 2 A^4 + (B^2 a^5 + 4 A B a^4 + A^2 a^3) \sqrt{(B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) / a^6)}) / (B^4 a^4 - 2 A^2 B^2 a^2 + A^4)} \\
& ) * ((B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) / a^6)^{1/4} + (B^2 a^6 + A B a^5 + A^2 a^4) \sqrt{(B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) / a^6)} / (B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) * ((B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) / a^6)^{3/4} - 18 \sqrt{1/3} (1/9)^{3/4} (\sqrt{1/3} A a^{10} x \sqrt{(B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) / a^6}) \sqrt{(B^4 a^4 - 2 A^2 B^2 a^2 + A^4) / a^6} \\
& - \sqrt{1/3} (B^3 a^{10} + A B^2 a^9 + A^2 B a^8) x \sqrt{(B^4 a^4 - 2 A^2 B^2 a^2 + A^4) / a^6)} \sqrt{(2 B^4 a^4 + 4 A B^3 a^3 + 6 A^2 B^2 a^2 + 4 A^3 B a + 2 A^4 + (B^2 a^5 + 4 A B a^4 + A^2 a^3) \sqrt{(B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) / a^6)}) / (B^4 a^4 - 2 A^2 B^2 a^2 + A^4)} \\
& ) * ((B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) / a^6)^{3/4} - 2 \sqrt{1/3} (B^4 a^{10} + 2 A B^3 a^9 + 3 A^2 B^2 a^8 + 2 A^3 B a^7 + A^4 a^6) \sqrt{(B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) / a^6} \sqrt{(B^4 a^4 - 2 A^2 B^2 a^2 + A^4) / a^6} - \sqrt{1/3} (B^6 a^9 + 3 A B^5 a^8 + 6 A^2 B^4 a^7 + 7 A^3 B^3 a^6 + 6 A^4 B^2 a^5 + 3 A^5 B a^4 + A^6 a^3) \sqrt{(B^4 a^4 - 2 A^2 B^2 a^2 + A^4) / a^6)} / (B^8 a^8 + 3 A B^7 a^7 + 5 A^2 B^6 a^6 + 4 A^3 B^5 a^5 - 4 A^5 B^3 a^3 - 5 A^6 B^2 a^2 - 3 A^7 B a - A^8)) - \sqrt{1/3} (1/9)^{1/4} (2 B^4 a^4 + 4 A B^3 a^3 + 6 A^2 B^2 a^2 + 4 A^3 B a + 2 A^4 - (B^2 a^5 + 4 A B a^4 + A^2 a^3) \sqrt{(B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) / a^6}) \sqrt{(B^4 a^4 + 2 A B^3 a^3 + 3 A^2 B^2 a^2 + 2 A^3 B a + A^4) / a^6}
\end{aligned}$$

$$\begin{aligned}
& + 2A^3B^2a + A^4)/a^6))\sqrt{(2B^4a^4 + 4AB^3a^3 + 6A^2B^2a^2 + 4A^3B^2a + 2A^4 + (B^2a^5 + 4AB^2a^4 + A^2a^3)\sqrt{(B^4a^4 + 2AB^3a^3 + 3A^2B^2a^2 + 2A^3B^2a + A^4)/a^6)))/(B^4a^4 - 2A^2B^2a^2 + A^4)} \\
& )*((B^4a^4 + 2AB^3a^3 + 3A^2B^2a^2 + 2A^3B^2a + A^4)/a^6)^{(1/4)}\log(2(B^4a^4 + 2AB^3a^3 + 3A^2B^2a^2 + 2A^3B^2a + A^4)x^2 + 6\sqrt{(1/3)}(1/9)^{(1/4)}(B^4a^4 + 2AB^3a^3 + 3A^2B^2a^2 + 2A^3B^2a + A^4)/a^6) - (AB^2a^4 + A^2B^2a^3 + A^3a^2)x)\sqrt{(2B^4a^4 + 4AB^3a^3 + 6A^2B^2a^2 + 4A^3B^2a + 2A^4 + (B^2a^5 + 4AB^2a^4 + A^2a^3)\sqrt{(B^4a^4 + 2AB^3a^3 + 3A^2B^2a^2 + 2A^3B^2a + A^4)/a^6)))/(B^4a^4 - 2A^2B^2a^2 + A^4)}*((B^4a^4 + 2AB^3a^3 + 3A^2B^2a^2 + 2A^3B^2a + A^4)/a^6)^{(1/4)} + 2(B^2a^6 + AB^2a^5 + A^2a^4)\sqrt{(B^4a^4 + 2AB^3a^3 + 3A^2B^2a^2 + 2A^3B^2a + A^4)/a^6)} + \sqrt{(1/3)}(1/9)^{(1/4)}(2B^4a^4 + 4AB^3a^3 + 6A^2B^2a^2 + 4A^3B^2a + 2A^4 - (B^2a^5 + 4AB^2a^4 + A^2a^3)\sqrt{(B^4a^4 + 2AB^3a^3 + 3A^2B^2a^2 + 2A^3B^2a + A^4)/a^6))\sqrt{(2B^4a^4 + 4AB^3a^3 + 6A^2B^2a^2 + 4A^3B^2a + 2A^4 + (B^2a^5 + 4AB^2a^4 + A^2a^3)\sqrt{(B^4a^4 + 2AB^3a^3 + 3A^2B^2a^2 + 2A^3B^2a + A^4)/a^6)))/(B^4a^4 - 2A^2B^2a^2 + A^4)}*((B^4a^4 + 2AB^3a^3 + 3A^2B^2a^2 + 2A^3B^2a + A^4)/a^6)^{(1/4)}\log(2(B^4a^4 + 2AB^3a^3 + 3A^2B^2a^2 + 2A^3B^2a + A^4)x^2 - 6\sqrt{(1/3)}(1/9)^{(1/4)}(B^4a^4 + 2AB^3a^3 + 3A^2B^2a^2 + 2A^3B^2a + A^4)/a^6) - (AB^2a^4 + A^2B^2a^3 + A^3a^2)x)\sqrt{(2B^4a^4 + 4AB^3a^3 + 6A^2B^2a^2 + 4A^3B^2a + 2A^4 + (B^2a^5 + 4AB^2a^4 + A^2a^3)\sqrt{(B^4a^4 + 2AB^3a^3 + 3A^2B^2a^2 + 2A^3B^2a + A^4)/a^6)))/(B^4a^4 - 2A^2B^2a^2 + A^4)}*((B^4a^4 + 2AB^3a^3 + 3A^2B^2a^2 + 2A^3B^2a + A^4)/a^6)^{(1/4)} + 2(B^2a^6 + AB^2a^5 + A^2a^4)\sqrt{(B^4a^4 + 2AB^3a^3 + 3A^2B^2a^2 + 2A^3B^2a + A^4)/a^6)})))/(B^4a^4 + 2AB^3a^3 + 3A^2B^2a^2 + 2A^3B^2a + A^4)
\end{aligned}$$

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(x^4-a\*x^2+a^2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0,%%{-16,[2,0]%%}+%%{-4,[0,1]%%},0,%%{64,[4,0]%%}+%%{8,[2,2]%%}+%%{16,[2,1]%%}+%%{6,[0,2]%%},0,%%{-64,[4,2]%%}+%%{-128,[4,1]%%}+%%{48,[2,3]%%}+%%{16,[2,2]%%}+%%{-4,[0,3]%%},0,%%{16,[4,4]%%}+%%{-64,[4,3]%%}+%%{64,[4,2]%%}+%%{8,[2,4]%%}+%%{-16,[2,3]%%}+%%{1,[0,4]%%}] at parameters values [71,-96]Warning, choosing root of [1,0,%%{-16,[2,0]%%}+%%{-4,[0,1]%%},0,%%{64,[4,0]%%}+%%{8,[2,2]%%}+%%{16,[2,1]%%}+%%{6,[0,2]%%},0,%%{-64,[4,2]%%}+%%{-128,[4,1]%%}+%%{48,[2,3]%%}+%%{16,[2,2]%%}+%%{-4,[0,3]%%},0,%%{16,[4,4]%%}+%%{-64,[4,3]%%}+%%{64,[4,2]%%}+%%{8,[2,4]%%}+%%{-16,[2,3]%%}+%%{1,[0,4]%%}] at parameters values [72,-72]((64\*a^3\*sqrt(abs(a))\*abs(a)+32\*sqrt(3)\*a^4\*sqrt(abs(a))+32\*a^4\*sqrt(abs(a)))\*A\*im(sign(cos(acos(a/2/abs(a))/2))))+(64\*sqrt(3)\*a^5+192\*abs(a)\*a^4-128\*sqrt(3)\*a^4\*abs(a))\*1/2/sqrt(abs(a))\*A\*im(sign(sin(acos(a/2/abs(a))/2)))+(-64\*sqrt(3)\*a^5+192\*abs(a)\*a^4-128\*sqrt(3)\*a^4\*abs(a))\*1/2/sqrt(abs(a))\*A\*re(sign(cos(acos(a/2/abs(a))/2)))+(32\*sqrt(3)\*a^5-64\*a^5+32\*a^4\*abs(a))/sqrt(abs(a))\*A\*re(sign(sin(acos(a/2/abs(a))/2)))+(-32\*a^6-40\*a^5\*abs(a)+8\*sqrt(3)\*a^5\*sqrt(5\*a^2+4\*a\*abs(a)))/sqrt(abs(a))\*B\*im(sign(cos(acos(a/2/abs(a))/2)))^3-1/12\*(-864\*sqrt(3)\*a^6+864\*a^5\*sqrt(5\*a^2+4\*a\*abs(a)))/sqrt(abs(a))\*B\*im(sign(cos(acos(a/2/abs(a))/2)))^2\*im(sign(sin(acos(a/2/abs(a))/2)))-1/24\*(-2880\*sqrt(3)\*a^6+1728\*abs(a)\*a^4\*sqrt(5\*a^2+4\*a\*abs(a))-2304\*sqrt(3)\*a^5\*abs(a))/sqrt(abs(a))\*B\*im(sign(cos(acos(a/2/abs(a))/2)))^2\*re(sign(cos(acos(a/2/abs(a))/2)))-(-72\*a^5\*abs(a)+24\*sqrt(3)\*a^5\*sqrt(5\*a^2+4\*a\*abs(a)))/sqrt(abs(a))\*B\*im(sign(cos(acos(a/2/abs(a))/2)))^2\*re(sign(sin(acos(a/2/abs(a))/2)))-(-72\*a^5\*abs(a)+24\*sqrt(3)\*a^5\*sqrt(5\*a^2+4\*a\*abs(a)))/sqrt(abs(a))\*B\*im(sign(cos(acos(a/2/abs(a))/2)))\*im



$$\begin{aligned}
& n(\sin(\cos(a/2/\text{abs}(a))/2))^{2-} - (32*a^6 - 40*a^5*\text{abs}(a) + 8*\sqrt{3})*a^5*\sqrt{5*a^2 - 4*a*\text{abs}(a)}/\sqrt{\text{abs}(a)} * B * \text{im}(\text{sign}(\sin(\cos(a/2/\text{abs}(a))/2)))^3 - (-72*a^5 * \\
& \text{abs}(a) + 24*\sqrt{3})*a^5*\sqrt{5*a^2 - 4*a*\text{abs}(a)}/\sqrt{\text{abs}(a)} * B * \text{im}(\text{sign}(\sin(\cos(a/2/\text{abs}(a))/2)))^2 * \text{re}(\text{sign}(\cos(\cos(a/2/\text{abs}(a))/2)))) + 1/24 * (-2880*\sqrt{3} \\
& *a^6 + 1728*\text{abs}(a)*a^4*\sqrt{5*a^2 - 4*a*\text{abs}(a)} + 2304*\sqrt{3})*a^5*\text{abs}(a)/\sqrt{\text{abs}(a)} * B * \text{im}(\text{sign}(\sin(\cos(a/2/\text{abs}(a))/2)))^2 * \text{re}(\text{sign}(\sin(\cos(a/2/\text{abs}(a))/2))) \\
& ) - (-72*a^5*\text{abs}(a) + 24*\sqrt{3})*a^5*\sqrt{5*a^2 + 4*a*\text{abs}(a)}/\sqrt{\text{abs}(a)} * B * \text{im}(\text{sign}(\sin(\cos(a/2/\text{abs}(a))/2))) * \text{re}(\text{sign}(\cos(\cos(a/2/\text{abs}(a))/2))))^2 + 1/24 * ( \\
& -3456*\sqrt{3})*a^6 + 3456*a^5*\sqrt{5*a^2 - 4*a*\text{abs}(a)}/\sqrt{\text{abs}(a)} * B * \text{im}(\text{sign}(\sin(\cos(a/2/\text{abs}(a))/2))) * \text{re}(\text{sign}(\cos(\cos(a/2/\text{abs}(a))/2)))) * \text{re}(\text{sign}(\sin(\cos \\
& (a/2/\text{abs}(a))/2))) + (96*a^6 - 120*a^5*\text{abs}(a) + 24*\sqrt{3})*a^5*\sqrt{5*a^2 - 4*a*\text{abs}(a)}/\sqrt{\text{abs}(a)} * B * \text{im}(\text{sign}(\sin(\cos(a/2/\text{abs}(a))/2))) * \text{re}(\text{sign}(\sin(\cos(a/2/ \\
& \text{abs}(a))/2)))^2 - (-32*a^6 - 40*a^5*\text{abs}(a) + 8*\sqrt{3})*a^5*\sqrt{5*a^2 + 4*a*\text{abs}(a)}/\sqrt{\text{abs}(a)} * B * \text{re}(\text{sign}(\cos(\cos(a/2/\text{abs}(a))/2))))^3 + 1/12 * (-864*\sqrt{3})*a^6 + \\
& 864*a^5*\sqrt{5*a^2 + 4*a*\text{abs}(a)}/\sqrt{\text{abs}(a)} * B * \text{re}(\text{sign}(\cos(\cos(a/2/\text{abs}(a))/2))))^2 * \text{re}(\text{sign}(\sin(\cos(a/2/\text{abs}(a))/2)))) + (-72*a^5*\text{abs}(a) + 24*\sqrt{3})*a^5*\sqrt{5*a^2 - 4*a*\text{abs}(a)}/\sqrt{\text{abs}(a)} * B * \text{re}(\text{sign}(\cos(\cos(a/2/\text{abs}(a))/2)))) * \text{re}(\text{sign}(\sin(\cos(a/2/\text{abs}(a))/2)))^2 - 1/8 * (-320*\sqrt{3})*a^6 + 192*\text{abs}(a)*a^4*\sqrt{5 \\
& *a^2 - 4*a*\text{abs}(a)} + 256*\sqrt{3})*a^5*\text{abs}(a)/\sqrt{\text{abs}(a)} * B * \text{re}(\text{sign}(\sin(\cos(a/2/\text{abs}(a))/2))))^3 / ((128*a^4*\sqrt{2*a^2 + a*\text{abs}(a)})*\sqrt{3}*\text{abs}(a) - 128*a^4*\sqrt{2*a^2 - a*\text{abs}(a)})*\sqrt{3}*\text{abs}(a)) * \text{atan}((x - \text{sign}(\cos(\cos(a*1/2/\text{abs}(a))/2)))*\sqrt{3} * \sqrt{(1 + a*1/2/\text{abs}(a))/2} * \sqrt{\text{abs}(a)}) / \text{sign}(\sin(\cos(a*1/2/\text{abs}(a))/2)) / \sqrt{(1 - a*1/2/\text{abs}(a))/2} * \sqrt{\text{abs}(a)}) + (-\text{abs}(a) * \sqrt{\text{abs}(a)} * A * a * \cosh(\text{im}(\cos(a/2/\text{abs}(a))/2) * \sin(\text{re}(\cos(a/2/\text{abs}(a))/2) + \text{abs}(a) * \sqrt{\text{abs}(a)} * A * a * \sin(\text{re}(\cos(a/2/\text{abs}(a))/2) * \sinh(\text{im}(\cos(a/2/\text{abs}(a))/2) + \sqrt{3})*a^2*\sqrt{\text{abs}(a)} * A * \cos(\text{re}(\cos(a/2/\text{abs}(a))/2) * \cosh(\text{im}(\cos(a/2/\text{abs}(a))/2) - \sqrt{3})*a^2*\sqrt{\text{abs}(a)} * A * \cos(\text{re}(\cos(a/2/\text{abs}(a))/2) * \sinh(\text{im}(\cos(a/2/\text{abs}(a))/2) - 3*a^2*\sqrt{\text{abs}(a)} * B * a * \cos(\text{re}(\cos(a/2/\text{abs}(a))/2))^2 * \cosh(\text{im}(\cos(a/2/\text{abs}(a))/2))^3 * \sin(\text{re}(\cos(a/2/\text{abs}(a))/2) + 9*a^2*\sqrt{\text{abs}(a)} * B * a * \cos(\text{re}(\cos(a/2/\text{abs}(a))/2))^2 * \cosh(\text{im}(\cos(a/2/\text{abs}(a))/2))^2 * \sin(\text{re}(\cos(a/2/\text{abs}(a))/2) * \sinh(\text{im}(\cos(a/2/\text{abs}(a))/2) - 9*a^2*\sqrt{\text{abs}(a)} * B * a * \cos(\text{re}(\cos(a/2/\text{abs}(a))/2))^2 * \cosh(\text{im}(\cos(a/2/\text{abs}(a))/2))^3 * \sin(\text{re}(\cos(a/2/\text{abs}(a))/2) * \sinh(\text{im}(\cos(a/2/\text{abs}(a))/2))^2 + 3*a^2*\sqrt{\text{abs}(a)} * B * a * \cos(\text{re}(\cos(a/2/\text{abs}(a))/2))^2 * \sin(\text{re}(\cos(a/2/\text{abs}(a))/2) * \sinh(\text{im}(\cos(a/2/\text{abs}(a))/2))^3 + a^2*\sqrt{\text{abs}(a)} * B * a * \cosh(\text{im}(\cos(a/2/\text{abs}(a))/2))^3 * \sin(\text{re}(\cos(a/2/\text{abs}(a))/2) * \sinh(\text{im}(\cos(a/2/\text{abs}(a))/2))^3 * \sin(\text{re}(\cos(a/2/\text{abs}(a))/2) * \sinh(\text{im}(\cos(a/2/\text{abs}(a))/2))^2 - a^2*\sqrt{\text{abs}(a)} * B * a * \sin(\text{re}(\cos(a/2/\text{abs}(a))/2) * \sinh(\text{im}(\cos(a/2/\text{abs}(a))/2))^3 + \sqrt{3})*\text{abs}(a) * a^2*\sqrt{\text{abs}(a)} * B * \cos(\text{re}(\cos(a/2/\text{abs}(a))/2))^3 * \cosh(\text{im}(\cos(a/2/\text{abs}(a))/2))^3 * \sqrt{3})*\text{abs}(a) * a^2*\sqrt{\text{abs}(a)} * B * \cos(\text{re}(\cos(a/2/\text{abs}(a))/2))^3 * \cosh(\text{im}(\cos(a/2/\text{abs}(a))/2))^2 * \sinh(\text{im}(\cos(a/2/\text{abs}(a))/2) + 3*\sqrt{3})*\text{abs}(a) * a^2*\sqrt{\text{abs}(a)} * B * \cos(\text{re}(\cos(a/2/\text{abs}(a))/2))^3 * \cosh(\text{im}(\cos(a/2/\text{abs}(a))/2))^2 * \sinh(\text{im}(\cos(a/2/\text{abs}(a))/2) - \sqrt{3})*\text{abs}(a) * a^2*\sqrt{\text{abs}(a)} * B * \cos(\text{re}(\cos(a/2/\text{abs}(a))/2))^3 * \sinh(\text{im}(\cos(a/2/\text{abs}(a))/2))^3 - 3*\sqrt{3})*\text{abs}(a) * a^2*\sqrt{\text{abs}(a)} * B * \cos(\text{re}(\cos(a/2/\text{abs}(a))/2)) * \cosh(\text{im}(\cos(a/2/\text{abs}(a))/2))^3 * \sin(\text{re}(\cos(a/2/\text{abs}(a))/2) * \sinh(\text{im}(\cos(a/2/\text{abs}(a))/2))^2 + 9*\sqrt{3})*\text{abs}(a) * a^2*\sqrt{\text{abs}(a)} * B * \cos(\text{re}(\cos(a/2/\text{abs}(a))/2) * \cosh(\text{im}(\cos(a/2/\text{abs}(a))/2))^2 * \sin(\text{re}(\cos(a/2/\text{abs}(a))/2) * \sinh(\text{im}(\cos(a/2/\text{abs}(a))/2) - 9*\sqrt{3})*\text{abs}(a) * a^2*\sqrt{\text{abs}(a)} * B * \cos(\text{re}(\cos(a/2/\text{abs}(a))/2) * \cosh(\text{im}(\cos(a/2/\text{abs}(a))/2) * \sin(\text{re}(\cos(a/2/\text{abs}(a))/2) * \sinh(\text{im}(\cos(a/2/\text{abs}(a))/2))^2 + 3*\sqrt{3})*\text{abs}(a) * a^2*\sqrt{\text{abs}(a)} * B * \cos(\text{re}(\cos(a/2/\text{abs}(a))/2) * \sin(\text{re}(\cos(a/2/\text{abs}(a))/2) * \sinh(\text{im}(\cos(a/2/\text{abs}(a))/2))^2 * \sinh(\text{im}(\cos(a/2/\text{abs}(a))/2))^3) * 1/4/\sqrt{3}/a^4 * \ln(x^2 + 2*\sqrt{\text{abs}(a)} * \cos(\cos(a*1/2/\text{abs}(a))/2) * x + \sqrt{\text{abs}(a)} * \sqrt{\text{abs}(a)}) - (-\text{abs}(a) * \sqrt{\text{abs}(a)} * A * a * \cos(\text{re}(\cos(a/2/\text{abs}(a))/2) * \cosh(\text{im}(\cos(a/2/\text{abs}(a))/2) + \text{abs}(a) * \sqrt{\text{abs}(a)} * A * a * \cos(\text{re}(\cos(a/2/\text{abs}(a))/2) * \sinh(\text{im}(\cos(a/2/\text{abs}(a))/2) - \sqrt{3})*a^2*\sqrt{\text{abs}(a)} * A * \cosh(\text{im}(\cos(a/2/\text{abs}(a))/2) * \sin(\text{re}(\cos(a/2/\text{abs}(a))/2) + \sqrt{3})*a^2*\sqrt{\text{abs}(a)} * A * \sin(\text{re}(\cos(a/2/\text{abs}(a))/2) * \sinh(\text{im}(\cos(a/2/\text{abs}(a))/2) - a^2*\sqrt{\text{abs}(a)} * B * a * \cos(\text{re}(\cos(a/2/\text{abs}(a))/2))^3 * \cosh(\text{im}(\cos(a/2/\text{abs}(a))/2))^3 + 3*a^2*
\end{aligned}$$

```

sqrt(abs(a))*B*a*cos(re(acos(a/2/abs(a))))/2)^3*cosh(im(acos(a/2/abs(a))))/2
^2*sinh(im(acos(a/2/abs(a))))/2-3*a^2*sqrt(abs(a))*B*a*cos(re(acos(a/2/abs(
a))))/2)^3*cosh(im(acos(a/2/abs(a))))/2)*sinh(im(acos(a/2/abs(a))))/2^2+a^2*s
qrt(abs(a))*B*a*cos(re(acos(a/2/abs(a))))/2)^3*sinh(im(acos(a/2/abs(a))))/2^
3+3*a^2*sqrt(abs(a))*B*a*cos(re(acos(a/2/abs(a))))/2)*cosh(im(acos(a/2/abs(a
))))/2)^3*sin(re(acos(a/2/abs(a))))/2^2-9*a^2*sqrt(abs(a))*B*a*cos(re(acos(a
/2/abs(a))))/2)*cosh(im(acos(a/2/abs(a))))/2^2*sin(re(acos(a/2/abs(a))))/2^2
*sinh(im(acos(a/2/abs(a))))/2+9*a^2*sqrt(abs(a))*B*a*cos(re(acos(a/2/abs(a)
))))/2)*cosh(im(acos(a/2/abs(a))))/2)*sin(re(acos(a/2/abs(a))))/2^2*sinh(im(ac
os(a/2/abs(a))))/2^2-3*a^2*sqrt(abs(a))*B*a*cos(re(acos(a/2/abs(a))))/2)*sin
(re(acos(a/2/abs(a))))/2^2*sinh(im(acos(a/2/abs(a))))/2^3-3*sqrt(3)*abs(a)*
a^2*sqrt(abs(a))*B*cos(re(acos(a/2/abs(a))))/2)^2*cosh(im(acos(a/2/abs(a))))/
2)^3*sin(re(acos(a/2/abs(a))))/2+9*sqrt(3)*abs(a)*a^2*sqrt(abs(a))*B*cos(re
(acos(a/2/abs(a))))/2)^2*cosh(im(acos(a/2/abs(a))))/2)^2*sin(re(acos(a/2/abs(
a))))/2)*sinh(im(acos(a/2/abs(a))))/2-9*sqrt(3)*abs(a)*a^2*sqrt(abs(a))*B*co
s(re(acos(a/2/abs(a))))/2)^2*cosh(im(acos(a/2/abs(a))))/2)*sin(re(acos(a/2/ab
s(a))))/2)*sinh(im(acos(a/2/abs(a))))/2^2+3*sqrt(3)*abs(a)*a^2*sqrt(abs(a))*
B*cos(re(acos(a/2/abs(a))))/2)^2*sin(re(acos(a/2/abs(a))))/2)*sinh(im(acos(a/
2/abs(a))))/2)^3+sqrt(3)*abs(a)*a^2*sqrt(abs(a))*B*cosh(im(acos(a/2/abs(a)))
/2)^3*sin(re(acos(a/2/abs(a))))/2)^3-3*sqrt(3)*abs(a)*a^2*sqrt(abs(a))*B*cos
h(im(acos(a/2/abs(a))))/2)^2*sin(re(acos(a/2/abs(a))))/2)^3*sinh(im(acos(a/2/
abs(a))))/2)+3*sqrt(3)*abs(a)*a^2*sqrt(abs(a))*B*cosh(im(acos(a/2/abs(a))))/2
)*sin(re(acos(a/2/abs(a))))/2)^3*sinh(im(acos(a/2/abs(a))))/2^2-sqrt(3)*abs(
a)*a^2*sqrt(abs(a))*B*sin(re(acos(a/2/abs(a))))/2)^3*sinh(im(acos(a/2/abs(a)
))))/2)^3)*1/2/sqrt(3)/a^4*atan((x+cos(acos(a*1/2/abs(a)))/2)*sqrt(abs(a)))/si
n(acos(a*1/2/abs(a)))/2)/sqrt(abs(a))

```

**maple [A]** time = 0.03, size = 190, normalized size = 1.40

$$\frac{B \arctan\left(\frac{2x+\sqrt{3}\sqrt{a}}{\sqrt{a}}\right) - B \arctan\left(\frac{-2x+\sqrt{3}\sqrt{a}}{\sqrt{a}}\right) - \sqrt{3} B \ln(x^2 + \sqrt{3}\sqrt{a}x + a)}{2\sqrt{a}} + \frac{\sqrt{3} B \ln(x^2 + \sqrt{3}\sqrt{a}x - a)}{12\sqrt{a}} + \frac{A \arctan\left(\frac{2x+\sqrt{3}\sqrt{a}}{\sqrt{a}}\right) - A \arctan\left(\frac{-2x+\sqrt{3}\sqrt{a}}{\sqrt{a}}\right) + \sqrt{3} A \ln(x^2 + \sqrt{3}\sqrt{a}x + a) - \sqrt{3} A \ln(x^2 + \sqrt{3}\sqrt{a}x - a)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] int((B*x^2+A)/(x^4-a*x^2+a^2),x)
[Out] 1/12/a^(1/2)*ln(-x^2+3^(1/2)*a^(1/2)*x-a)*B*3^(1/2)-1/12/a^(3/2)*ln(-x^2+3^(
1/2)*a^(1/2)*x-a)*A*3^(1/2)-1/2/a^(1/2)*arctan((-2*x+3^(1/2)*a^(1/2))/a^(1
/2))*B-1/2/a^(3/2)*arctan((-2*x+3^(1/2)*a^(1/2))/a^(1/2))*A-1/12/a^(1/2)*ln
(x^2+3^(1/2)*a^(1/2)*x+a)*B*3^(1/2)+1/12/a^(3/2)*ln(x^2+3^(1/2)*a^(1/2)*x+a
)*A*3^(1/2)+1/2/a^(1/2)*arctan((2*x+3^(1/2)*a^(1/2))/a^(1/2))*B+1/2/a^(3/2)
*arctan((2*x+3^(1/2)*a^(1/2))/a^(1/2))*A

```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{x^4 - ax^2 + a^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((B*x^2+A)/(x^4-a*x^2+a^2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)/(x^4 - a*x^2 + a^2), x)

```

**mupad [B]** time = 4.59, size = 1007, normalized size = 7.40

Verification of antiderivative is not currently implemented for this CAS.

```

[In] int((A + B*x^2)/(a^2 - a*x^2 + x^4),x)
[Out] atan((A^2*x*((3^(1/2)*B^2*1i)/(24*a) - B^2/(24*a) - (3^(1/2)*A^2*1i)/(24*a^
3) - A^2/(24*a^3) - (A*B)/(6*a^2))^(1/2)*6i)/(2*A^2*B + A^3/a - 2*B^3*a^2 +

```



$$\begin{aligned} & (3^{(1/2)}A^3*1i)/a - A*B^2*a - 3^{(1/2)}*A*B^2*a*1i) + (2*3^{(1/2)}*A^2*x*((3^{(1/2)}*B^2*1i)/(24*a) - B^2/(24*a) - (3^{(1/2)}*A^2*1i)/(24*a^3) - A^2/(24*a^3) \\ & ) - (A*B)/(6*a^2))^{(1/2)})/(2*A^2*B + A^3/a - 2*B^3*a^2 + (3^{(1/2)}*A^3*1i)/a - A*B^2*a - 3^{(1/2)}*A*B^2*a*1i) - (B^2*a^2*x*((3^{(1/2)}*B^2*1i)/(24*a) - B^2/(24*a) - (3^{(1/2)}*A^2*1i)/(24*a^3) - A^2/(24*a^3) - (A*B)/(6*a^2))^{(1/2)}*6i)/(2*A^2*B + A^3/a - 2*B^3*a^2 + (3^{(1/2)}*A^3*1i)/a - A*B^2*a - 3^{(1/2)}*A*B^2*a*1i) - (2*3^{(1/2)}*B^2*a^2*x*((3^{(1/2)}*B^2*1i)/(24*a) - B^2/(24*a) - (3^{(1/2)}*A^2*1i)/(24*a^3) - A^2/(24*a^3) - (A*B)/(6*a^2))^{(1/2)})/(2*A^2*B + A^3/a - 2*B^3*a^2 + (3^{(1/2)}*A^3*1i)/a - A*B^2*a - 3^{(1/2)}*A*B^2*a*1i))*(-(3^{(1/2)}*A^2*1i + A^2 + B^2*a^2 - 3^{(1/2)}*B^2*a^2*1i + 4*A*B*a)/(24*a^3))^{(1/2)}*2i + atan((A^2*x*((3^{(1/2)}*A^2*1i)/(24*a^3) - B^2/(24*a) - A^2/(24*a^3) - (3^{(1/2)}*B^2*1i)/(24*a) - (A*B)/(6*a^2))^{(1/2)}*6i)/(2*A^2*B + A^3/a - 2*B^3*a^2 - (3^{(1/2)}*A^3*1i)/a - A*B^2*a + 3^{(1/2)}*A*B^2*a*1i) - (2*3^{(1/2)}*A^2*x*((3^{(1/2)}*A^2*1i)/(24*a^3) - B^2/(24*a) - A^2/(24*a^3) - (3^{(1/2)}*B^2*1i)/(24*a) - (A*B)/(6*a^2))^{(1/2)})/(2*A^2*B + A^3/a - 2*B^3*a^2 - (3^{(1/2)}*A^3*1i)/a - A*B^2*a + 3^{(1/2)}*A*B^2*a*1i) - (B^2*a^2*x*((3^{(1/2)}*A^2*1i)/(24*a^3) - B^2/(24*a) - A^2/(24*a^3) - (3^{(1/2)}*B^2*1i)/(24*a) - (A*B)/(6*a^2))^{(1/2)}*6i)/(2*A^2*B + A^3/a - 2*B^3*a^2 - (3^{(1/2)}*A^3*1i)/a - A*B^2*a + 3^{(1/2)}*A*B^2*a*1i) + (2*3^{(1/2)}*B^2*a^2*x*((3^{(1/2)}*A^2*1i)/(24*a^3) - B^2/(24*a) - A^2/(24*a^3) - (3^{(1/2)}*B^2*1i)/(24*a) - (A*B)/(6*a^2))^{(1/2)})/(2*A^2*B + A^3/a - 2*B^3*a^2 - (3^{(1/2)}*A^3*1i)/a - A*B^2*a + 3^{(1/2)}*A*B^2*a*1i))*(-(A^2 - 3^{(1/2)}*A^2*1i + B^2*a^2 + 3^{(1/2)}*B^2*a^2*1i + 4*A*B*a)/(24*a^3))^{(1/2)}*2i \end{aligned}$$

**sympy [A]** time = 1.91, size = 172, normalized size = 1.26

$$\text{RootSum}\left(144t^4a^6 + t^2(12A^2a^3 + 48ABa^4 + 12B^2a^5) + A^4 + 2A^3Ba + 3A^2B^2a^2 + 2AB^3a^3 + B^4a^4, \left(t \mapsto t \log\left(x + \frac{24t^3Aa^5 + 48t^3Ba^6 - 2tA^3a^2 + 6tA^2Ba^3 + 12tAB^2a^4 + 2tB^3a^5}{-A^4 - A^3Ba + AB^3a^3 + B^4a^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)/(x\*\*4-a\*x\*\*2+a\*\*2),x)

[Out] RootSum(144\*\_t\*\*4\*a\*\*6 + \_t\*\*2\*(12\*A\*\*2\*a\*\*3 + 48\*A\*B\*a\*\*4 + 12\*B\*\*2\*a\*\*5) + A\*\*4 + 2\*A\*\*3\*B\*a + 3\*A\*\*2\*B\*\*2\*a\*\*2 + 2\*A\*B\*\*3\*a\*\*3 + B\*\*4\*a\*\*4, Lambda(\_t, \_t\*log(x + (24\*\_t\*\*3\*A\*a\*\*5 + 48\*\_t\*\*3\*B\*a\*\*6 - 2\*\_t\*A\*\*3\*a\*\*2 + 6\*\_t\*A\*\*2\*B\*a\*\*3 + 12\*\_t\*A\*B\*\*2\*a\*\*4 + 2\*\_t\*B\*\*3\*a\*\*5)/(-A\*\*4 - A\*\*3\*B\*a + A\*B\*\*3\*a\*\*3 + B\*\*4\*a\*\*4))))

$$3.98 \quad \int \frac{A+Bx^2}{a-\sqrt{a}x^2+x^4} dx$$

**Optimal.** Leaf size=160

$$\frac{(A-\sqrt{a}B)\log(-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2)}{4\sqrt{3}a^{3/4}} + \frac{(A-\sqrt{a}B)\log(\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2)}{4\sqrt{3}a^{3/4}} - \frac{(\sqrt{a}B+A)\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}}$$

**Rubi [A]** time = 0.12, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1169, 634, 617, 204, 628}

$$\frac{(A-\sqrt{a}B)\log(-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2)}{4\sqrt{3}a^{3/4}} + \frac{(A-\sqrt{a}B)\log(\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2)}{4\sqrt{3}a^{3/4}} - \frac{(\sqrt{a}B+A)\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}} + \frac{(\sqrt{a}B+A)\tan^{-1}\left(\frac{2x}{\sqrt[4]{a}}+\sqrt{3}\right)}{2a^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(a - Sqrt[a]\*x^2 + x^4), x]

[Out] -((A + Sqrt[a]\*B)\*ArcTan[Sqrt[3] - (2\*x)/a^(1/4)]/(2\*a^(3/4)) + ((A + Sqrt[a]\*B)\*ArcTan[Sqrt[3] + (2\*x)/a^(1/4)]/(2\*a^(3/4)) - ((A - Sqrt[a]\*B)\*Log[Sqrt[a] - Sqrt[3]\*a^(1/4)\*x + x^2])/(4\*Sqrt[3]\*a^(3/4)) + ((A - Sqrt[a]\*B)\*Log[Sqrt[a] + Sqrt[3]\*a^(1/4)\*x + x^2])/(4\*Sqrt[3]\*a^(3/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 1169

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2\*q - b/c, 2]}, Dist[1/(2\*c\*q\*r), Int[(d\*r - (d - e\*q)\*x)/(q - r\*x + x^2), x], x] + Dist[1/(2\*c\*q\*r), Int[(d\*r + (d - e\*q)\*x)/(q + r\*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NegQ[b^2 - 4\*a\*c]

#### Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^2}{a - \sqrt{a}x^2 + x^4} dx &= \frac{\int \frac{\sqrt{3} \sqrt[4]{a} A - (A - \sqrt{a} B)x}{\sqrt{a} - \sqrt{3} \sqrt[4]{a} x + x^2} dx}{2\sqrt{3} a^{3/4}} + \frac{\int \frac{\sqrt{3} \sqrt[4]{a} A + (A - \sqrt{a} B)x}{\sqrt{a} + \sqrt{3} \sqrt[4]{a} x + x^2} dx}{2\sqrt{3} a^{3/4}} \\
&= \frac{1}{4} \left( \frac{A}{\sqrt{a}} + B \right) \int \frac{1}{\sqrt{a} - \sqrt{3} \sqrt[4]{a} x + x^2} dx + \frac{1}{4} \left( \frac{A}{\sqrt{a}} + B \right) \int \frac{1}{\sqrt{a} + \sqrt{3} \sqrt[4]{a} x + x^2} dx - \frac{(A - \sqrt{a} B) \log(\sqrt{a} - \sqrt{3} \sqrt[4]{a} x + x^2)}{4\sqrt{3} a^{3/4}} + \frac{(A - \sqrt{a} B) \log(\sqrt{a} + \sqrt{3} \sqrt[4]{a} x + x^2)}{4\sqrt{3} a^{3/4}} \\
&= -\frac{(A + \sqrt{a} B) \tan^{-1}\left(\sqrt{3} - \frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}} + \frac{(A + \sqrt{a} B) \tan^{-1}\left(\sqrt{3} + \frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}} - \frac{(A - \sqrt{a} B) \log(\sqrt{a} - \sqrt{3} \sqrt[4]{a} x + x^2)}{4\sqrt{3} a^{3/4}} + \frac{(A - \sqrt{a} B) \log(\sqrt{a} + \sqrt{3} \sqrt[4]{a} x + x^2)}{4\sqrt{3} a^{3/4}}
\end{aligned}$$

**Mathematica [C]** time = 0.13, size = 138, normalized size = 0.86

$$\frac{\sqrt[4]{-1} \left( \frac{((\sqrt{3}-i)\sqrt{a}B-2iA) \tan^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}-i}\sqrt[4]{a}}\right)}{\sqrt{\sqrt{3}-i}} - \frac{((\sqrt{3}+i)\sqrt{a}B+2iA) \tanh^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}+i}\sqrt[4]{a}}\right)}{\sqrt{\sqrt{3}+i}} \right)}{\sqrt{6} a^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(a - Sqrt[a]\*x^2 + x^4), x]

[Out]  $((-1)^{(1/4)} * ((((-2*I)*A + (-I + \text{Sqrt}[3])*\text{Sqrt}[a]*B)*\text{ArcTan}[(1 + I)*x]/(\text{Sqrt}[-I + \text{Sqrt}[3]]*a^{(1/4)})))/\text{Sqrt}[-I + \text{Sqrt}[3]] - (((2*I)*A + (I + \text{Sqrt}[3])*S\text{qrt}[a]*B)*\text{ArcTanh}[(1 + I)*x]/(\text{Sqrt}[I + \text{Sqrt}[3]]*a^{(1/4)})))/\text{Sqrt}[I + \text{Sqrt}[3]])))/(\text{Sqrt}[6]*a^{(3/4)})$

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{a - \sqrt{a}x^2 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B\*x^2)/(a - Sqrt[a]\*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(A + B\*x^2)/(a - Sqrt[a]\*x^2 + x^4), x]

**fricas [B]** time = 1.68, size = 1141, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(a+x^4-x^2\*a^(1/2)),x, algorithm="fricas")

[Out]  $1/2*\text{sqrt}(1/6)*\text{sqrt}(-4*A*B*a + 3*\text{sqrt}(1/3)*a^2*\text{sqrt}(-B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*\text{sqrt}(a)/a^2*\log(2*(B^6*a^3 - A^6)*x + 3*\text{sqrt}(1/6)*(A*B^4*a^3 - A^5*a - \text{sqrt}(1/3)*(2*B^3*a^4 + A^2*B*a^3)*\text{sqrt}(-B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) - (A^2*B^3*a^2 - A^4*B*a - \text{sqrt}(1/3)*(A*B^2*a^3 - A^3*a^2)*\text{sqrt}(-B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3))*\text{sqrt}(a))*\text{sqrt}(-4*A*B*a + 3*\text{sqrt}(1/3)*a^2*\text{sqrt}(-B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*\text{sqrt}(a)/a^2) - 1/2*\text{sqrt}(1/6)*\text{sqrt}(-4*A*B*a + 3*\text{sqrt}(1/3)*a^2*\text{sqrt}(-B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*\text{sqrt}(a)/a^2*\log(2*(B^6*a^3 - A^6)*x + 3*\text{sqrt}(1/6)*(A*B^4*a^3 - A^5*a - \text{sqrt}(1/3)*(2*B^3*a^4 + A^2*B*a^3)*\text{sqrt}(-B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) - (A^2*B^3*a^2 - A^4*B*a - \text{sqrt}(1/3)*(A*B^2*a^3 - A^3*a^2)*\text{sqrt}(-B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3))*\text{sqrt}(a))*\text{sqrt}(-4*A*B*a + 3*\text{sqrt}(1/3)*a^2*\text{sqrt}(-B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*\text{sqrt}(a)/a^2)$

```

3 - A^6)*x - 3*sqrt(1/6)*(A*B^4*a^3 - A^5*a - sqrt(1/3)*(2*B^3*a^4 + A^2*B*
a^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) - (A^2*B^3*a^2 - A^4*B*a - sq
rt(1/3)*(A*B^2*a^3 - A^3*a^2)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3))*sq
rt(a))*sqrt(-(4*A*B*a + 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/
a^3) + (B^2*a + A^2)*sqrt(a))/a^2)) + 1/2*sqrt(1/6)*sqrt(-(4*A*B*a - 3*sqrt
(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*sqrt(a)
/a^2)*log(2*(B^6*a^3 - A^6)*x + 3*sqrt(1/6)*(A*B^4*a^3 - A^5*a + sqrt(1/3)*
(2*B^3*a^4 + A^2*B*a^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) - (A^2*B^3
*a^2 - A^4*B*a + sqrt(1/3)*(A*B^2*a^3 - A^3*a^2)*sqrt(-(B^4*a^2 - 2*A^2*B^2
*a + A^4)/a^3))*sqrt(a))*sqrt(-(4*A*B*a - 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 -
2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))/a^2)) - 1/2*sqrt(1/6)*sqrt
(-(4*A*B*a - 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B
^2*a + A^2)*sqrt(a))/a^2)*log(2*(B^6*a^3 - A^6)*x - 3*sqrt(1/6)*(A*B^4*a^3 -
A^5*a + sqrt(1/3)*(2*B^3*a^4 + A^2*B*a^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A
^4)/a^3) - (A^2*B^3*a^2 - A^4*B*a + sqrt(1/3)*(A*B^2*a^3 - A^3*a^2)*sqrt(-
(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3))*sqrt(a))*sqrt(-(4*A*B*a - 3*sqrt(1/3)*a
^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))/a^2))

```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(a+x^4-x^2\*a^(1/2)),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
UT:sage2:=int(sage0,x);OUTPUT:sym2poly/r2sym(const gen & e,const index\_m &  
i,const vecteur & l) Error: Bad Argument Value

**maple** [A] time = 0.04, size = 198, normalized size = 1.24

$$\frac{B \arctan\left(\frac{2x+\sqrt{3}a^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{2a^{\frac{3}{4}}} - \frac{B \arctan\left(\frac{-2x+\sqrt{3}a^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{2a^{\frac{3}{4}}} - \frac{\sqrt{3} B \ln\left(x^2 + \sqrt{3}a^{\frac{1}{4}}x + \sqrt{a}\right)}{12a^{\frac{3}{4}}} + \frac{\sqrt{3} B \ln\left(-x^2 + \sqrt{3}a^{\frac{1}{4}}x - \sqrt{a}\right)}{12a^{\frac{3}{4}}} + \frac{A \arctan\left(\frac{2x+\sqrt{3}a^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{2a^{\frac{3}{4}}} - \frac{A \arctan\left(\frac{-2x+\sqrt{3}a^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{2a^{\frac{3}{4}}} + \frac{\sqrt{3} A \ln\left(x^2 + \sqrt{3}a^{\frac{1}{4}}x + \sqrt{a}\right)}{12a^{\frac{3}{4}}} - \frac{\sqrt{3} A \ln\left(-x^2 + \sqrt{3}a^{\frac{1}{4}}x - \sqrt{a}\right)}{12a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)/(a+x^4-a^(1/2)\*x^2),x)

[Out] 1/12/a^(3/4)\*ln(x^2+3^(1/2)\*a^(1/4)\*x+a^(1/2))\*A\*3^(1/2)-1/12/a^(1/4)\*ln(x^2+3^(1/2)\*a^(1/4)\*x+a^(1/2))\*B\*3^(1/2)+1/2/a^(3/4)\*arctan((2\*x+3^(1/2)\*a^(1/4))/a^(1/4))/a^(1/4))\*A+1/2/a^(1/4)\*arctan((2\*x+3^(1/2)\*a^(1/4))/a^(1/4))\*B-1/12/a^(3/4)\*ln(-x^2+3^(1/2)\*a^(1/4)\*x-a^(1/2))\*A\*3^(1/2)+1/12/a^(1/4)\*ln(-x^2+3^(1/2)\*a^(1/4)\*x-a^(1/2))\*B\*3^(1/2)-1/2/a^(3/4)\*arctan((-2\*x+3^(1/2)\*a^(1/4))/a^(1/4))\*A-1/2/a^(1/4)\*arctan((-2\*x+3^(1/2)\*a^(1/4))/a^(1/4))\*B

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

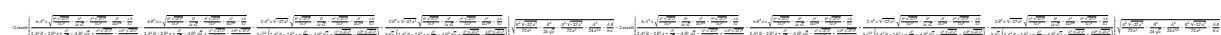
$$\int \frac{Bx^2 + A}{x^4 - \sqrt{a}x^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(a+x^4-x^2\*a^(1/2)),x, algorithm="maxima")

[Out] integrate((B\*x^2 + A)/(x^4 - sqrt(a)\*x^2 + a), x)

**mupad** [B] time = 4.99, size = 1155, normalized size = 7.22



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + x^4 - a^(1/2)*x^2),x)
```

```
[Out] - 2*atanh((6*A^2*x*((B^2*(-27*a^3)^(1/2))/(72*a^2) - B^2/(24*a^(1/2)) - (A^2*(-27*a^3)^(1/2))/(72*a^3) - A^2/(24*a^(3/2)) - (A*B)/(6*a))^(1/2))/(2*A^2*B - 2*B^3*a + A^3/a^(1/2) - A*B^2*a^(1/2) + (A^3*(-27*a^3)^(1/2))/(3*a^2) - (A*B^2*(-27*a^3)^(1/2))/(3*a)) - (6*B^2*a*x*((B^2*(-27*a^3)^(1/2))/(72*a^2) - B^2/(24*a^(1/2)) - (A^2*(-27*a^3)^(1/2))/(72*a^3) - A^2/(24*a^(3/2)) - (A*B)/(6*a))^(1/2))/(2*A^2*B - 2*B^3*a + A^3/a^(1/2) - A*B^2*a^(1/2) + (A^3*(-27*a^3)^(1/2))/(3*a^2) - (A*B^2*(-27*a^3)^(1/2))/(3*a)) - (2*A^2*x*((-27*a^3)^(1/2)*((B^2*(-27*a^3)^(1/2))/(72*a^2) - B^2/(24*a^(1/2)) - (A^2*(-27*a^3)^(1/2))/(72*a^3) - A^2/(24*a^(3/2)) - (A*B)/(6*a))^(1/2))/(3*a^(3/2)*(2*A^2*B - 2*B^3*a + A^3/a^(1/2) - A*B^2*a^(1/2) + (A^3*(-27*a^3)^(1/2))/(3*a^2) - (A*B^2*(-27*a^3)^(1/2))/(3*a))) + (2*B^2*x*((-27*a^3)^(1/2)*((B^2*(-27*a^3)^(1/2))/(72*a^2) - B^2/(24*a^(1/2)) - (A^2*(-27*a^3)^(1/2))/(72*a^3) - A^2/(24*a^(3/2)) - (A*B)/(6*a))^(1/2))/(3*a^(1/2)*(2*A^2*B - 2*B^3*a + A^3/a^(1/2) - A*B^2*a^(1/2) + (A^3*(-27*a^3)^(1/2))/(3*a^2) - (A*B^2*(-27*a^3)^(1/2))/(3*a))))*((B^2*(-27*a^3)^(1/2))/(72*a^2) - B^2/(24*a^(1/2)) - (A^2*(-27*a^3)^(1/2))/(72*a^3) - A^2/(24*a^(3/2)) - (A*B)/(6*a))^(1/2) - 2*atanh((6*A^2*x*((A^2*(-27*a^3)^(1/2))/(72*a^3) - B^2/(24*a^(1/2)) - A^2/(24*a^(3/2)) - (B^2*(-27*a^3)^(1/2))/(72*a^2) - (A*B)/(6*a))^(1/2))/(2*A^2*B - 2*B^3*a + A^3/a^(1/2) - A*B^2*a^(1/2) - (A^3*(-27*a^3)^(1/2))/(3*a^2) + (A*B^2*(-27*a^3)^(1/2))/(3*a)) - (6*B^2*a*x*((A^2*(-27*a^3)^(1/2))/(72*a^3) - B^2/(24*a^(1/2)) - A^2/(24*a^(3/2)) - (B^2*(-27*a^3)^(1/2))/(72*a^2) - (A*B)/(6*a))^(1/2))/(2*A^2*B - 2*B^3*a + A^3/a^(1/2) - A*B^2*a^(1/2) - (A^3*(-27*a^3)^(1/2))/(3*a^2) + (A*B^2*(-27*a^3)^(1/2))/(3*a)) + (2*A^2*x*((-27*a^3)^(1/2)*((A^2*(-27*a^3)^(1/2))/(72*a^3) - B^2/(24*a^(1/2)) - A^2/(24*a^(3/2)) - (B^2*(-27*a^3)^(1/2))/(72*a^2) - (A*B)/(6*a))^(1/2))/(3*a^(3/2)*(2*A^2*B - 2*B^3*a + A^3/a^(1/2) - A*B^2*a^(1/2) - (A^3*(-27*a^3)^(1/2))/(3*a^2) + (A*B^2*(-27*a^3)^(1/2))/(3*a))) - (2*B^2*x*((-27*a^3)^(1/2)*((A^2*(-27*a^3)^(1/2))/(72*a^3) - B^2/(24*a^(1/2)) - A^2/(24*a^(3/2)) - (B^2*(-27*a^3)^(1/2))/(72*a^2) - (A*B)/(6*a))^(1/2))/(3*a^(1/2)*(2*A^2*B - 2*B^3*a + A^3/a^(1/2) - A*B^2*a^(1/2) - (A^3*(-27*a^3)^(1/2))/(3*a^2) + (A*B^2*(-27*a^3)^(1/2))/(3*a))))*((A^2*(-27*a^3)^(1/2))/(72*a^3) - B^2/(24*a^(1/2)) - A^2/(24*a^(3/2)) - (B^2*(-27*a^3)^(1/2))/(72*a^2) - (A*B)/(6*a))^(1/2)
```

```
sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(a+x**4-x**2*a**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError
```

$$3.99 \quad \int \frac{A+Bx^2}{a-\sqrt{ac}x^2+cx^4} dx$$

**Optimal.** Leaf size=414

$$\frac{\left(A - \frac{\sqrt{a}B}{\sqrt{c}}\right) \log\left(-x\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}} + \sqrt{a} + \sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}} + \frac{\left(A - \frac{\sqrt{a}B}{\sqrt{c}}\right) \log\left(x\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}} + \sqrt{a} + \sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}}$$

**Rubi [A]** time = 0.45, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 29, number of rules / integrand size = 0.172, Rules used = {1169, 634, 618, 204, 628}

$$\frac{\left(A - \frac{\sqrt{a}B}{\sqrt{c}}\right) \log\left(-x\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}} + \sqrt{a} + \sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}} + \frac{\left(A - \frac{\sqrt{a}B}{\sqrt{c}}\right) \log\left(x\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}} + \sqrt{a} + \sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}} - \frac{(\sqrt{a}B + A\sqrt{c}) \tan^{-1}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}} - 2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c} - \sqrt{ac}}}\right)}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c} - \sqrt{ac}}} + \frac{(\sqrt{a}B + A\sqrt{c}) \tan^{-1}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}} + 2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c} - \sqrt{ac}}}\right)}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c} - \sqrt{ac}}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(a - Sqrt[a\*c]\*x^2 + c\*x^4), x]

[Out] -((Sqrt[a]\*B + A\*Sqrt[c])\*ArcTan[(Sqrt[2\*Sqrt[a]\*Sqrt[c] + Sqrt[a\*c]] - 2\*Sqrt[c]\*x)/Sqrt[2\*Sqrt[a]\*Sqrt[c] - Sqrt[a\*c]])/(2\*Sqrt[a]\*Sqrt[c]\*Sqrt[2\*Sqrt[a]\*Sqrt[c] - Sqrt[a\*c]]) + ((Sqrt[a]\*B + A\*Sqrt[c])\*ArcTan[(Sqrt[2\*Sqrt[a]\*Sqrt[c] + Sqrt[a\*c]] + 2\*Sqrt[c]\*x)/Sqrt[2\*Sqrt[a]\*Sqrt[c] - Sqrt[a\*c]])/(2\*Sqrt[a]\*Sqrt[c]\*Sqrt[2\*Sqrt[a]\*Sqrt[c] - Sqrt[a\*c]]) - ((A - (Sqrt[a]\*B)/Sqrt[c])\*Log[Sqrt[a] - Sqrt[2\*Sqrt[a]\*Sqrt[c] + Sqrt[a\*c]]\*x + Sqrt[c]\*x^2])/(4\*Sqrt[a]\*Sqrt[2\*Sqrt[a]\*Sqrt[c] + Sqrt[a\*c]]) + ((A - (Sqrt[a]\*B)/Sqrt[c])\*Log[Sqrt[a] + Sqrt[2\*Sqrt[a]\*Sqrt[c] + Sqrt[a\*c]]\*x + Sqrt[c]\*x^2])/(4\*Sqrt[a]\*Sqrt[2\*Sqrt[a]\*Sqrt[c] + Sqrt[a\*c]])

#### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 1169

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2\*q - b/c, 2]}, Dist[1/(2\*c\*q\*r), Int

$[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + \text{Dist}[1/(2*c*q*r), \text{Int}[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{NegQ}[b^2 - 4*a*c]$

Rubi steps

$$\int \frac{A + Bx^2}{a - \sqrt{ac}x^2 + cx^4} dx = \int \frac{\frac{A\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}}{\sqrt{c}} - \left(A - \frac{\sqrt{a}B}{\sqrt{c}}\right)x}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}x}{\sqrt{c}} + x^2} dx + \int \frac{\frac{A\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}}{\sqrt{c}} + \left(A - \frac{\sqrt{a}B}{\sqrt{c}}\right)x}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}x}{\sqrt{c}} + x^2} dx$$

$$= \frac{\left(B + \frac{A\sqrt{c}}{\sqrt{a}}\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}x}{\sqrt{c}} + x^2} dx}{4c} + \frac{\left(B + \frac{A\sqrt{c}}{\sqrt{a}}\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}x}{\sqrt{c}} + x^2} dx}{4c} - \frac{\left(A - \frac{\sqrt{a}B}{\sqrt{c}}\right) \log\left(\sqrt{a} - \sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}x + \sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}} + \frac{\left(A - \frac{\sqrt{a}B}{\sqrt{c}}\right) \log\left(\sqrt{a} + \sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}x + \sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}}}$$

$$= -\frac{\left(B + \frac{A\sqrt{c}}{\sqrt{a}}\right) \tan^{-1}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}} - 2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c} - \sqrt{ac}}}\right)}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c} - \sqrt{ac}}} + \frac{\left(B + \frac{A\sqrt{c}}{\sqrt{a}}\right) \tan^{-1}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c} + \sqrt{ac}} + 2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c} - \sqrt{ac}}}\right)}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c} - \sqrt{ac}}}$$

**Mathematica [C]** time = 0.20, size = 247, normalized size = 0.60

$$\frac{(\sqrt{3}\sqrt{a}B\sqrt{c} - i(B\sqrt{ac} + 2Ac)) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-\sqrt{ac} - i\sqrt{3}\sqrt{a}\sqrt{c}}}\right)}{\sqrt{-\sqrt{ac} - i\sqrt{3}\sqrt{a}\sqrt{c}}} + \frac{(\sqrt{3}\sqrt{a}B\sqrt{c} + i(B\sqrt{ac} + 2Ac)) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-\sqrt{ac} + i\sqrt{3}\sqrt{a}\sqrt{c}}}\right)}{\sqrt{-\sqrt{ac} + i\sqrt{3}\sqrt{a}\sqrt{c}}}$$

$$\sqrt{6}\sqrt{a}c$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(a - Sqrt[a\*c]\*x^2 + c\*x^4), x]

[Out] (((Sqrt[3]\*Sqrt[a]\*B\*Sqrt[c] - I\*(2\*A\*c + B\*Sqrt[a\*c]))\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[(-I)\*Sqrt[3]\*Sqrt[a]\*Sqrt[c] - Sqrt[a\*c]]])/Sqrt[(-I)\*Sqrt[3]\*Sqrt[a]\*Sqrt[c] - Sqrt[a\*c]] + ((Sqrt[3]\*Sqrt[a]\*B\*Sqrt[c] + I\*(2\*A\*c + B\*Sqrt[a\*c]))\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[I\*Sqrt[3]\*Sqrt[a]\*Sqrt[c] - Sqrt[a\*c]]])/Sqrt[I\*Sqrt[3]\*Sqrt[a]\*Sqrt[c] - Sqrt[a\*c]])/(Sqrt[6]\*Sqrt[a]\*c)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{a - \sqrt{ac}x^2 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B\*x^2)/(a - Sqrt[a\*c]\*x^2 + c\*x^4), x]

[Out] IntegrateAlgebraic[(A + B\*x^2)/(a - Sqrt[a\*c]\*x^2 + c\*x^4), x]

**fricas [B]** time = 1.51, size = 1457, normalized size = 3.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+c*x^4-x^2*(a*c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/6)*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))*log(-2*(B^6*a^3 - A^6*c^3)*x + 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 - sqrt(1/3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - (A^2*B^3*a^2*c - A^4*B*a*c^2 - sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a*c))*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))) + 1/2*sqrt(1/6)*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))*log(-2*(B^6*a^3 - A^6*c^3)*x - 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 - sqrt(1/3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - (A^2*B^3*a^2*c - A^4*B*a*c^2 - sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a*c))*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))) - 1/2*sqrt(1/6)*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))*log(-2*(B^6*a^3 - A^6*c^3)*x + 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 + sqrt(1/3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - (A^2*B^3*a^2*c - A^4*B*a*c^2 + sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a*c))*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))) + 1/2*sqrt(1/6)*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))*log(-2*(B^6*a^3 - A^6*c^3)*x - 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 + sqrt(1/3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - (A^2*B^3*a^2*c - A^4*B*a*c^2 + sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a*c))*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2)))
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+c*x^4-x^2*(a*c)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument Value
```

```
maple [A] time = 0.06, size = 404, normalized size = 0.98
```

$$\frac{\operatorname{Arctan}\left(\frac{2\sqrt{c}\sqrt{ax^2+3\sqrt{ac}}}{\sqrt{4\sqrt{c}\sqrt{c-3\sqrt{ac}}}}\right)}{2\sqrt{4\sqrt{c}\sqrt{c-3\sqrt{ac}}}} + \frac{\operatorname{Arctan}\left(\frac{2\sqrt{c}\sqrt{ax^2+3\sqrt{ac}}}{\sqrt{4\sqrt{c}\sqrt{c-3\sqrt{ac}}}}\right)}{2\sqrt{4\sqrt{c}\sqrt{c-3\sqrt{ac}}}} + \frac{\operatorname{Arctan}\left(\frac{2\sqrt{c}\sqrt{ax^2+3\sqrt{ac}}}{\sqrt{4\sqrt{c}\sqrt{c-3\sqrt{ac}}}}\right)}{2\sqrt{4\sqrt{c}\sqrt{c-3\sqrt{ac}}}} + \frac{\operatorname{Arctan}\left(\frac{2\sqrt{c}\sqrt{ax^2+3\sqrt{ac}}}{\sqrt{4\sqrt{c}\sqrt{c-3\sqrt{ac}}}}\right)}{2\sqrt{4\sqrt{c}\sqrt{c-3\sqrt{ac}}}} + \frac{\sqrt{3}\operatorname{arctan}\left(\frac{2\sqrt{c}\sqrt{ax^2+3\sqrt{ac}}}{\sqrt{4\sqrt{c}\sqrt{c-3\sqrt{ac}}}}\right)}{12a^2c} + \frac{\sqrt{3}\operatorname{arctan}\left(\frac{2\sqrt{c}\sqrt{ax^2+3\sqrt{ac}}}{\sqrt{4\sqrt{c}\sqrt{c-3\sqrt{ac}}}}\right)}{12a^2c} + \frac{\sqrt{3}\operatorname{arctan}\left(\frac{2\sqrt{c}\sqrt{ax^2+3\sqrt{ac}}}{\sqrt{4\sqrt{c}\sqrt{c-3\sqrt{ac}}}}\right)}{12a^2c} + \frac{\sqrt{3}\operatorname{arctan}\left(\frac{2\sqrt{c}\sqrt{ax^2+3\sqrt{ac}}}{\sqrt{4\sqrt{c}\sqrt{c-3\sqrt{ac}}}}\right)}{12a^2c} + \frac{\sqrt{3}\operatorname{arctan}\left(\frac{2\sqrt{c}\sqrt{ax^2+3\sqrt{ac}}}{\sqrt{4\sqrt{c}\sqrt{c-3\sqrt{ac}}}}\right)}{12a^2c} + \frac{\sqrt{3}\operatorname{arctan}\left(\frac{2\sqrt{c}\sqrt{ax^2+3\sqrt{ac}}}{\sqrt{4\sqrt{c}\sqrt{c-3\sqrt{ac}}}}\right)}{12a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(a+c*x^4-x^2*(a*c)^(1/2)),x)
```

```
[Out] 1/12/a/c^(3/2)*ln(x*3^(1/2)*(a*c)^(1/4)-c^(1/2)*x^2-a^(1/2))*B*3^(1/2)*(a*c)^(3/4)-1/12/a^(3/2)/c*ln(x*3^(1/2)*(a*c)^(1/4)-c^(1/2)*x^2-a^(1/2))*A*3^(1/2)*(a*c)^(3/4)-1/2/a^(1/2)/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2)*arctan(
```



$$\begin{aligned} & (3^{1/2} * (a*c)^{1/4} - 2*c^{1/2} * x) / (4*a^{1/2} * c^{1/2} - 3*(a*c)^{1/2})^{1/2} * \\ & A - 1/2/c^{1/2} / (4*a^{1/2} * c^{1/2} - 3*(a*c)^{1/2})^{1/2} * \arctan((3^{1/2} * (a*c)^{1/4} - 2*c^{1/2} * x) / (4*a^{1/2} * c^{1/2} - 3*(a*c)^{1/2})^{1/2}) * B - 1/12/a/c^{3/2} * \\ & \ln(c^{1/2} * x^2 + x * 3^{1/2} * (a*c)^{1/4} + a^{1/2}) * B * 3^{1/2} * (a*c)^{3/4} + 1/12/a^{3/2} / c * \ln(c^{1/2} * x^2 + x * 3^{1/2} * (a*c)^{1/4} + a^{1/2}) * A * 3^{1/2} * (a*c)^{3/4} + \\ & 1/2/a^{1/2} / (4*a^{1/2} * c^{1/2} - 3*(a*c)^{1/2})^{1/2} * \arctan((2*c^{1/2} * x + 3^{1/2} * (a*c)^{1/4}) / (4*a^{1/2} * c^{1/2} - 3*(a*c)^{1/2})^{1/2}) * A + 1/2/c^{1/2} / (4*a^{1/2} * c^{1/2} - 3*(a*c)^{1/2})^{1/2} * \arctan((2*c^{1/2} * x + 3^{1/2} * (a*c)^{1/4}) / (4*a^{1/2} * c^{1/2} - 3*(a*c)^{1/2})^{1/2}) * B \end{aligned}$$

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{cx^4 - \sqrt{ac}x^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(a+c\*x^4-x^2\*(a\*c)^(1/2)),x, algorithm="maxima")

[Out] integrate((B\*x^2 + A)/(c\*x^4 - sqrt(a\*c)\*x^2 + a), x)

**mupad [B]** time = 5.22, size = 3285, normalized size = 7.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)/(a + c\*x^4 - x^2\*(a\*c)^(1/2)),x)

[Out] 
$$\begin{aligned} & - \operatorname{atan}\left(\frac{\left(\frac{12Aa}{c^2} - (2x(4c(a^3)^{3/2}) - 16a^2c^2(a^3)^{1/2})\right) * \left(- (B^2a^3c^3)^{1/2} - A^2c^2(-27a^3c^3)^{1/2} - B^2a^2(a^3)^{3/2} - A^2c^2(a^3)^{3/2} + 12ABa^2c^2 + 4A^2a^2c^2(a^3)^{1/2} + 4B^2a^2c^2(a^3)^{1/2}\right)}{(72a^3c^3)^{1/2}}\right)}{c^4} * \left(- (B^2a^3c^3)^{1/2} - A^2c^2(-27a^3c^3)^{1/2} - B^2a^2(a^3)^{3/2} - A^2c^2(a^3)^{3/2} + 12ABa^2c^2 + 4A^2a^2c^2(a^3)^{1/2} + 4B^2a^2c^2(a^3)^{1/2}\right) / (72a^3c^3)^{1/2} \\ & + \frac{(2x(2A^2c^2 - B^2a^2c + 2ABc^2(a^3)^{1/2}))}{c^4} * \left(- (B^2a^3c^3)^{1/2} - A^2c^2(-27a^3c^3)^{1/2} - B^2a^2(a^3)^{3/2} - A^2c^2(a^3)^{3/2} + 12ABa^2c^2 + 4A^2a^2c^2(a^3)^{1/2} + 4B^2a^2c^2(a^3)^{1/2}\right) / (72a^3c^3)^{1/2} \\ & + \frac{\left(\frac{12Aa}{c^2} + (2x(4c(a^3)^{3/2}) - 16a^2c^2(a^3)^{1/2})\right) * \left(- (B^2a^3c^3)^{1/2} - A^2c^2(-27a^3c^3)^{1/2} - B^2a^2(a^3)^{3/2} - A^2c^2(a^3)^{3/2} + 12ABa^2c^2 + 4A^2a^2c^2(a^3)^{1/2} + 4B^2a^2c^2(a^3)^{1/2}\right)}{(72a^3c^3)^{1/2}} * \operatorname{atan}\left(\frac{\left(\frac{12Aa}{c^2} + (2x(4c(a^3)^{3/2}) - 16a^2c^2(a^3)^{1/2})\right) * \left(- (B^2a^3c^3)^{1/2} - A^2c^2(-27a^3c^3)^{1/2} - B^2a^2(a^3)^{3/2} - A^2c^2(a^3)^{3/2} + 12ABa^2c^2 + 4A^2a^2c^2(a^3)^{1/2} + 4B^2a^2c^2(a^3)^{1/2}\right)}{(72a^3c^3)^{1/2}}\right)}{c^4} * \left(- (B^2a^3c^3)^{1/2} - A^2c^2(-27a^3c^3)^{1/2} - B^2a^2(a^3)^{3/2} - A^2c^2(a^3)^{3/2} + 12ABa^2c^2 + 4A^2a^2c^2(a^3)^{1/2} + 4B^2a^2c^2(a^3)^{1/2}\right) / (72a^3c^3)^{1/2} \\ & - \frac{(2x(2A^2c^2 - B^2a^2c + 2ABc^2(a^3)^{1/2}))}{c^4} * \left(- (B^2a^3c^3)^{1/2} - A^2c^2(-27a^3c^3)^{1/2} - B^2a^2(a^3)^{3/2} - A^2c^2(a^3)^{3/2} + 12ABa^2c^2 + 4A^2a^2c^2(a^3)^{1/2} + 4B^2a^2c^2(a^3)^{1/2}\right) / (72a^3c^3)^{1/2} \\ & + \frac{\left(\frac{12Aa}{c^2} - (2x(4c(a^3)^{3/2}) - 16a^2c^2(a^3)^{1/2})\right) * \left(- (B^2a^3c^3)^{1/2} - A^2c^2(-27a^3c^3)^{1/2} - B^2a^2(a^3)^{3/2} - A^2c^2(a^3)^{3/2} + 12ABa^2c^2 + 4A^2a^2c^2(a^3)^{1/2} + 4B^2a^2c^2(a^3)^{1/2}\right)}{(72a^3c^3)^{1/2}} * \operatorname{atan}\left(\frac{\left(\frac{12Aa}{c^2} - (2x(4c(a^3)^{3/2}) - 16a^2c^2(a^3)^{1/2})\right) * \left(- (B^2a^3c^3)^{1/2} - A^2c^2(-27a^3c^3)^{1/2} - B^2a^2(a^3)^{3/2} - A^2c^2(a^3)^{3/2} + 12ABa^2c^2 + 4A^2a^2c^2(a^3)^{1/2} + 4B^2a^2c^2(a^3)^{1/2}\right)}{(72a^3c^3)^{1/2}}\right)}{c^4} * \left(- (B^2a^3c^3)^{1/2} - A^2c^2(-27a^3c^3)^{1/2} - B^2a^2(a^3)^{3/2} - A^2c^2(a^3)^{3/2} + 12ABa^2c^2 + 4A^2a^2c^2(a^3)^{1/2} + 4B^2a^2c^2(a^3)^{1/2}\right) / (72a^3c^3)^{1/2} \\ & + \frac{(2x(2A^2c^2 - B^2a^2c + 2ABc^2(a^3)^{1/2}))}{c^4} * \left(- (B^2a^3c^3)^{1/2} - A^2c^2(-27a^3c^3)^{1/2} - B^2a^2(a^3)^{3/2} - A^2c^2(a^3)^{3/2} + 12ABa^2c^2 + 4A^2a^2c^2(a^3)^{1/2} + 4B^2a^2c^2(a^3)^{1/2}\right) / (72a^3c^3)^{1/2} \\ & + \frac{\left(\frac{12Aa}{c^2} + (2x(4c(a^3)^{3/2}) - 16a^2c^2(a^3)^{1/2})\right) * \left(- (B^2a^3c^3)^{1/2} - A^2c^2(-27a^3c^3)^{1/2} - B^2a^2(a^3)^{3/2} - A^2c^2(a^3)^{3/2} + 12ABa^2c^2 + 4A^2a^2c^2(a^3)^{1/2} + 4B^2a^2c^2(a^3)^{1/2}\right)}{(72a^3c^3)^{1/2}} * \operatorname{atan}\left(\frac{\left(\frac{12Aa}{c^2} + (2x(4c(a^3)^{3/2}) - 16a^2c^2(a^3)^{1/2})\right) * \left(- (B^2a^3c^3)^{1/2} - A^2c^2(-27a^3c^3)^{1/2} - B^2a^2(a^3)^{3/2} - A^2c^2(a^3)^{3/2} + 12ABa^2c^2 + 4A^2a^2c^2(a^3)^{1/2} + 4B^2a^2c^2(a^3)^{1/2}\right)}{(72a^3c^3)^{1/2}}\right)}{c^4} * \left(- (B^2a^3c^3)^{1/2} - A^2c^2(-27a^3c^3)^{1/2} - B^2a^2(a^3)^{3/2} - A^2c^2(a^3)^{3/2} + 12ABa^2c^2 + 4A^2a^2c^2(a^3)^{1/2} + 4B^2a^2c^2(a^3)^{1/2}\right) / (72a^3c^3)^{1/2} \\ & - \frac{(2x(2A^2c^2 - B^2a^2c + 2ABc^2(a^3)^{1/2}))}{c^4} * \left(- (B^2a^3c^3)^{1/2} - A^2c^2(-27a^3c^3)^{1/2} - B^2a^2(a^3)^{3/2} - A^2c^2(a^3)^{3/2} + 12ABa^2c^2 + 4A^2a^2c^2(a^3)^{1/2} + 4B^2a^2c^2(a^3)^{1/2}\right) / (72a^3c^3)^{1/2} \\ & + \frac{\left(\frac{12Aa}{c^2} - (2x(4c(a^3)^{3/2}) - 16a^2c^2(a^3)^{1/2})\right) * \left(- (B^2a^3c^3)^{1/2} - A^2c^2(-27a^3c^3)^{1/2} - B^2a^2(a^3)^{3/2} - A^2c^2(a^3)^{3/2} + 12ABa^2c^2 + 4A^2a^2c^2(a^3)^{1/2} + 4B^2a^2c^2(a^3)^{1/2}\right)}{(72a^3c^3)^{1/2}} * \operatorname{atan}\left(\frac{\left(\frac{12Aa}{c^2} - (2x(4c(a^3)^{3/2}) - 16a^2c^2(a^3)^{1/2})\right) * \left(- (B^2a^3c^3)^{1/2} - A^2c^2(-27a^3c^3)^{1/2} - B^2a^2(a^3)^{3/2} - A^2c^2(a^3)^{3/2} + 12ABa^2c^2 + 4A^2a^2c^2(a^3)^{1/2} + 4B^2a^2c^2(a^3)^{1/2}\right)}{(72a^3c^3)^{1/2}}\right)}{c^4} * \left(- (B^2a^3c^3)^{1/2} - A^2c^2(-27a^3c^3)^{1/2} - B^2a^2(a^3)^{3/2} - A^2c^2(a^3)^{3/2} + 12ABa^2c^2 + 4A^2a^2c^2(a^3)^{1/2} + 4B^2a^2c^2(a^3)^{1/2}\right) / (72a^3c^3)^{1/2} \end{aligned}$$

```

) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2) - (2*x*(2*A^2*c^2 - B^2*a*
c + 2*A*B*c*(a*c)^(1/2)))/c^4)*(-(B^2*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^
3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A
^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2) + (2*(B
^3*a + A^2*B*c + A*B^2*(a*c)^(1/2)))/c^4)*(-(B^2*a*(-27*a^3*c^3)^(1/2) - A
^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a
^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(
1/2)*2i - atan((((12*A*a)/c^2 - (2*x*(4*c*(a*c)^(3/2) - 16*a*c^2*(a*c)^(1/
2)))*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(
3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^
2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2))/c^4)*(-(A^2*c*(-27*a^3*c^3)^(1/2)
- B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A
*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3
))^(1/2) + (2*x*(2*A^2*c^2 - B^2*a*c + 2*A*B*c*(a*c)^(1/2)))/c^4)*(-(A^2*c*
(-27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c
*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)
^(1/2))/(72*a^3*c^3))^(1/2)*1i - (((12*A*a)/c^2 + (2*x*(4*c*(a*c)^(3/2) - 1
6*a*c^2*(a*c)^(1/2)))*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)^(1/
2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(
a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2))/c^4)*(-(A^2*c*(-
27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(
a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(
1/2))/(72*a^3*c^3))^(1/2) - (2*x*(2*A^2*c^2 - B^2*a*c + 2*A*B*c*(a*c)^(1/2)
))/c^4)*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a
*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) +
4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2)*1i)/((((12*A*a)/c^2 - (2*x*(4*
c*(a*c)^(3/2) - 16*a*c^2*(a*c)^(1/2)))*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*
(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^
2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2)
)/c^4)*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c
)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*
B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2) + (2*x*(2*A^2*c^2 - B^2*a*c + 2*
A*B*c*(a*c)^(1/2)))/c^4)*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)
^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c
^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2) + (((12*A*a)/
c^2 + (2*x*(4*c*(a*c)^(3/2) - 16*a*c^2*(a*c)^(1/2)))*(-(A^2*c*(-27*a^3*c^3)^(
1/2) - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) +
12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^
3*c^3))^(1/2))/c^4)*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)^(1/2)
- B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a
*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2) - (2*x*(2*A^2*c^2
- B^2*a*c + 2*A*B*c*(a*c)^(1/2)))/c^4)*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a
*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c
^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2)
+ (2*(B^3*a + A^2*B*c + A*B^2*(a*c)^(1/2)))/c^4)*(-(A^2*c*(-27*a^3*c^3)^(
1/2) - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) +
12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3
*c^3))^(1/2)*2i

```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)/(a+c\*x\*\*4-x\*\*2\*(a\*c)\*\*(1/2)),x)

[Out] Exception raised: PolynomialError

$$3.100 \quad \int \frac{A+Bx^2}{a-\sqrt{a}\sqrt{c}x^2+cx^4} dx$$

**Optimal.** Leaf size=234

$$\frac{(\sqrt{a}B + A\sqrt{c}) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[4]{c}x}{\sqrt{a}}\right)}{2a^{3/4}c^{3/4}} + \frac{(\sqrt{a}B + A\sqrt{c}) \tan^{-1}\left(\frac{2\sqrt[4]{c}x}{\sqrt{a}} + \sqrt{3}\right)}{2a^{3/4}c^{3/4}} - \frac{\left(A - \frac{\sqrt{a}B}{\sqrt{c}}\right) \log\left(-\sqrt{3}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a}\sqrt{c}\right)}{4\sqrt{3}a^{3/4}\sqrt[4]{c}}$$

**Rubi [A]** time = 0.17, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {1169, 634, 617, 204, 628}

$$\frac{(\sqrt{a}B + A\sqrt{c}) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[4]{c}x}{\sqrt{a}}\right)}{2a^{3/4}c^{3/4}} + \frac{(\sqrt{a}B + A\sqrt{c}) \tan^{-1}\left(\frac{2\sqrt[4]{c}x}{\sqrt{a}} + \sqrt{3}\right)}{2a^{3/4}c^{3/4}} - \frac{\left(A - \frac{\sqrt{a}B}{\sqrt{c}}\right) \log\left(-\sqrt{3}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a}\sqrt{c}\right)}{4\sqrt{3}a^{3/4}\sqrt[4]{c}} + \frac{\left(A - \frac{\sqrt{a}B}{\sqrt{c}}\right) \log\left(\sqrt{3}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a}\sqrt{c}\right)}{4\sqrt{3}a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x^2)/(a - Sqrt[a]\*Sqrt[c]\*x^2 + c\*x^4), x]

[Out] -((Sqrt[a]\*B + A\*Sqrt[c])\*ArcTan[Sqrt[3] - (2\*c^(1/4)\*x)/a^(1/4)]/(2\*a^(3/4)\*c^(3/4)) + ((Sqrt[a]\*B + A\*Sqrt[c])\*ArcTan[Sqrt[3] + (2\*c^(1/4)\*x)/a^(1/4)]/(2\*a^(3/4)\*c^(3/4)) - ((A - (Sqrt[a]\*B)/Sqrt[c])\*Log[Sqrt[a] - Sqrt[3]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/(4\*Sqrt[3]\*a^(3/4)\*c^(1/4)) + ((A - (Sqrt[a]\*B)/Sqrt[c])\*Log[Sqrt[a] + Sqrt[3]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/(4\*Sqrt[3]\*a^(3/4)\*c^(1/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 1169

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2\*q - b/c, 2]}, Dist[1/(2\*c\*q\*r), Int[(d\*r - (d - e\*q)\*x)/(q - r\*x + x^2), x], x] + Dist[1/(2\*c\*q\*r), Int[(d\*r + (d - e\*q)\*x)/(q + r\*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NegQ[b^2 - 4\*a\*c]

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^2}{a - \sqrt{a}\sqrt{c}x^2 + cx^4} dx &= \int \frac{\frac{\sqrt{3}\sqrt[4]{a}A}{\sqrt[4]{c}} - \left(A - \frac{\sqrt{a}B}{\sqrt{c}}\right)x}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx + \int \frac{\frac{\sqrt{3}\sqrt[4]{a}A}{\sqrt[4]{c}} + \left(A - \frac{\sqrt{a}B}{\sqrt{c}}\right)x}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx \\
&= \frac{\left(B + \frac{A\sqrt{c}}{\sqrt{a}}\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{4c} + \frac{\left(B + \frac{A\sqrt{c}}{\sqrt{a}}\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{4c} + \frac{(\sqrt{a}B - A\sqrt{c}) \int \frac{1}{\sqrt{a} - \sqrt{3}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2} dx}{4\sqrt{3}a^{3/4}\sqrt[4]{c}} \\
&= \frac{(\sqrt{a}B - A\sqrt{c}) \log(\sqrt{a} - \sqrt{3}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{4\sqrt{3}a^{3/4}c^{3/4}} + \frac{\left(A - \frac{\sqrt{a}B}{\sqrt{c}}\right) \log(\sqrt{a} + \sqrt{3}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{4\sqrt{3}a^{3/4}\sqrt[4]{c}} \\
&= -\frac{(\sqrt{a}B + A\sqrt{c}) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{3/4}} + \frac{(\sqrt{a}B + A\sqrt{c}) \tan^{-1}\left(\sqrt{3} + \frac{2\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{3/4}} + \frac{(\sqrt{a}B - A\sqrt{c}) \log(\sqrt{a} - \sqrt{3}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{4\sqrt{3}a^{3/4}\sqrt[4]{c}}
\end{aligned}$$

**Mathematica** [C] time = 0.19, size = 163, normalized size = 0.70

$$\frac{\sqrt[4]{-1} \left( \frac{((\sqrt{3}-i)\sqrt{a}B-2iA\sqrt{c}) \tan^{-1}\left(\frac{(1+i)\sqrt[4]{c}x}{\sqrt{\sqrt{3}-i}\sqrt[4]{a}}\right)}{\sqrt{\sqrt{3}-i}} - \frac{((\sqrt{3}+i)\sqrt{a}B+2iA\sqrt{c}) \tanh^{-1}\left(\frac{(1+i)\sqrt[4]{c}x}{\sqrt{\sqrt{3}+i}\sqrt[4]{a}}\right)}{\sqrt{\sqrt{3}+i}} \right)}{\sqrt{6}a^{3/4}c^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x^2)/(a - Sqrt[a]\*Sqrt[c]\*x^2 + c\*x^4), x]

[Out]  $((-1)^{(1/4)} * ((((-I + \text{Sqrt}[3]) * \text{Sqrt}[a] * B - (2 * I) * A * \text{Sqrt}[c]) * \text{ArcTan}[\frac{(1 + I) * c^{(1/4)} * x}{(\text{Sqrt}[-I + \text{Sqrt}[3]] * a^{(1/4)})})] / \text{Sqrt}[-I + \text{Sqrt}[3]] - ((I + \text{Sqrt}[3]) * \text{Sqrt}[a] * B + (2 * I) * A * \text{Sqrt}[c]) * \text{ArcTanh}[\frac{(1 + I) * c^{(1/4)} * x}{(\text{Sqrt}[I + \text{Sqrt}[3]] * a^{(1/4)})})] / \text{Sqrt}[I + \text{Sqrt}[3]])) / (\text{Sqrt}[6] * a^{(3/4)} * c^{(3/4)})$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{a - \sqrt{a}\sqrt{c}x^2 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B\*x^2)/(a - Sqrt[a]\*Sqrt[c]\*x^2 + c\*x^4), x]

[Out] IntegrateAlgebraic[(A + B\*x^2)/(a - Sqrt[a]\*Sqrt[c]\*x^2 + c\*x^4), x]

**fricas** [B] time = 2.81, size = 1469, normalized size = 6.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(a+c\*x^4-x^2\*a^(1/2)\*c^(1/2)), x, algorithm="fricas")

[Out]  $-1/2 * \text{sqrt}(1/6) * \text{sqrt}(- (3 * \text{sqrt}(1/3) * a^2 * c^2 * \text{sqrt}(- (B^4 * a^2 - 2 * A^2 * B^2 * a * c + A^4 * c^2) / (a^3 * c^3))) + 4 * A * B * a * c + (B^2 * a + A^2 * c) * \text{sqrt}(a) * \text{sqrt}(c)) / (a^2 * c^2) * \log(-2 * (B^6 * a^3 - A^6 * c^3) * x + 3 * \text{sqrt}(1/6) * (A * B^4 * a^3 * c - A^5 * a * c^3 - (A$

$$\begin{aligned} & ^2*B^3*a^2*c - A^4*B*a*c^2 - \sqrt{1/3}*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*\sqrt{(-} \\ & (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*\sqrt{a}*\sqrt{c} - \sqrt{1/3} \\ & *(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*\sqrt{-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/} \\ & (a^3*c^3)))*\sqrt{-(3*\sqrt{1/3}*a^2*c^2*\sqrt{-(B^4*a^2 - 2*A^2*B^2*a*c + A^4} \\ & *c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*\sqrt{a}*\sqrt{c))/(a^2*c^2)} \\ & + 1/2*\sqrt{1/6}*\sqrt{-(3*\sqrt{1/3}*a^2*c^2*\sqrt{-(B^4*a^2 - 2*A^2*B^2*a*c} \\ & + A^4*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*\sqrt{a}*\sqrt{c))/(a^2*c} \\ & ^2)}*\log(-2*(B^6*a^3 - A^6*c^3)*x - 3*\sqrt{1/6}*(A*B^4*a^3*c - A^5*a*c^3 - \\ & (A^2*B^3*a^2*c - A^4*B*a*c^2 - \sqrt{1/3}*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*\sqrt{(-} \\ & (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*\sqrt{a}*\sqrt{c} - \sqrt{1/} \\ & 3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*\sqrt{-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2} \\ & )/(a^3*c^3)))*\sqrt{-(3*\sqrt{1/3}*a^2*c^2*\sqrt{-(B^4*a^2 - 2*A^2*B^2*a*c + A^4} \\ & *c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*\sqrt{a}*\sqrt{c))/(a^2*c^2)} \\ & )) - 1/2*\sqrt{1/6}*\sqrt{(3*\sqrt{1/3}*a^2*c^2*\sqrt{-(B^4*a^2 - 2*A^2*B^2*a*c} \\ & + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*\sqrt{a}*\sqrt{c))/(a^2*c} \\ & ^2)}*\log(-2*(B^6*a^3 - A^6*c^3)*x + 3*\sqrt{1/6}*(A*B^4*a^3*c - A^5*a*c^3 - \\ & (A^2*B^3*a^2*c - A^4*B*a*c^2 + \sqrt{1/3}*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*\sqrt{(-} \\ & (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*\sqrt{a}*\sqrt{c} + \sqrt{1/} \\ & 3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*\sqrt{-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2} \\ & )/(a^3*c^3)))*\sqrt{(3*\sqrt{1/3}*a^2*c^2*\sqrt{-(B^4*a^2 - 2*A^2*B^2*a*c + A^4} \\ & *c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*\sqrt{a}*\sqrt{c))/(a^2*c^2)} \\ & )) + 1/2*\sqrt{1/6}*\sqrt{(3*\sqrt{1/3}*a^2*c^2*\sqrt{-(B^4*a^2 - 2*A^2*B^2*a*c} \\ & + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*\sqrt{a}*\sqrt{c))/(a^2*c} \\ & ^2)}*\log(-2*(B^6*a^3 - A^6*c^3)*x - 3*\sqrt{1/6}*(A*B^4*a^3*c - A^5*a*c^3 - \\ & (A^2*B^3*a^2*c - A^4*B*a*c^2 + \sqrt{1/3}*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*\sqrt{(-} \\ & (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*\sqrt{a}*\sqrt{c} + \sqrt{1/} \\ & 3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*\sqrt{-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2} \\ & )/(a^3*c^3)))*\sqrt{(3*\sqrt{1/3}*a^2*c^2*\sqrt{-(B^4*a^2 - 2*A^2*B^2*a*c + A^4} \\ & *c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*\sqrt{a}*\sqrt{c))/(a^2*c^2)} \\ & )) \end{aligned}$$

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(a+c\*x^4-x^2\*a^(1/2)\*c^(1/2)),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index\_m &  
i,const vecteur & l) Error: Bad Argument Value

**maple** [A] time = 0.07, size = 320, normalized size = 1.37

$$\frac{A \arctan\left(\frac{2\sqrt{c}\sqrt{a}\sqrt{a^2+c^2}}{\sqrt{a^2+c^2}}\right)}{2\sqrt{a}\sqrt{c}\sqrt{a}} - \frac{A \arctan\left(\frac{-2\sqrt{c}\sqrt{a}\sqrt{a^2+c^2}}{\sqrt{a^2+c^2}}\right)}{2\sqrt{a}\sqrt{c}\sqrt{a}} + \frac{B \arctan\left(\frac{2\sqrt{c}\sqrt{a}\sqrt{a^2+c^2}}{\sqrt{a^2+c^2}}\right)}{2\sqrt{a}\sqrt{c}\sqrt{c}} - \frac{B \arctan\left(\frac{-2\sqrt{c}\sqrt{a}\sqrt{a^2+c^2}}{\sqrt{a^2+c^2}}\right)}{2\sqrt{a}\sqrt{c}\sqrt{c}} + \frac{\sqrt{3} A \ln\left(\sqrt{c}x^2 + \sqrt{3} a^{\frac{1}{2}} c^{\frac{1}{2}} x + \sqrt{a}\right)}{12a^{\frac{3}{2}}c^{\frac{1}{2}}} - \frac{\sqrt{3} A \ln\left(-\sqrt{c}x^2 + \sqrt{3} a^{\frac{1}{2}} c^{\frac{1}{2}} x - \sqrt{a}\right)}{12a^{\frac{3}{2}}c^{\frac{1}{2}}} - \frac{\sqrt{3} B \ln\left(\sqrt{c}x^2 + \sqrt{3} a^{\frac{1}{2}} c^{\frac{1}{2}} x + \sqrt{a}\right)}{12a^{\frac{3}{2}}c^{\frac{1}{2}}} + \frac{\sqrt{3} B \ln\left(-\sqrt{c}x^2 + \sqrt{3} a^{\frac{1}{2}} c^{\frac{1}{2}} x - \sqrt{a}\right)}{12a^{\frac{3}{2}}c^{\frac{1}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*x^2+A)/(a+c\*x^4-x^2\*a^(1/2)\*c^(1/2)),x)

[Out]  $-1/12/c^{1/4}/a^{3/4}*\ln(a^{1/4}*c^{1/4}*x^3^{1/2}-c^{1/2}*x^2-a^{1/2})*A*3^{1/2}+1/12/c^{3/4}/a^{1/4}*\ln(a^{1/4}*c^{1/4}*x^3^{1/2}-c^{1/2}*x^2-a^{1/2})*B*3^{1/2}-1/2/a^{1/2}/(a^{1/2}*c^{1/2})^{1/2}*\arctan((3^{1/2}*c^{1/4})a^{1/4}-2*c^{1/2}*x)/(a^{1/2}*c^{1/2})^{1/2})*A-1/2/c^{1/2}/(a^{1/2}*c^{1/2})^{1/2}*\arctan((3^{1/2}*c^{1/4})a^{1/4}-2*c^{1/2}*x)/(a^{1/2}*c^{1/2})^{1/2})*B+1/12/c^{1/4}/a^{3/4}*\ln(a^{1/4}*c^{1/4}*x^3^{1/2}+a^{1/2}+c^{1/2}*x^2)*A*3^{1/2}-1/12/c^{3/4}/a^{1/4}*\ln(a^{1/4}*c^{1/4}*x^3^{1/2}+a^{1/2}+c^{1/2}*x^2)*B*3^{1/2}+1/2/a^{1/2}/(a^{1/2}*c^{1/2})^{1/2}*\arctan((2*c^{1/2}*x+3^{1/2}*c^{1/4})a^{1/4})/(a^{1/2}*c^{1/2})^{1/2})*A+1/2/c^{1/2}/(a^{1/2}*c^{1/2})^{1/2}$

$2))^{(1/2)} * \arctan((2*c^{(1/2)}*x+3^{(1/2)}*c^{(1/4)}*a^{(1/4)})/(a^{(1/2)}*c^{(1/2)})^{(1/2)}) * B$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{cx^4 - \sqrt{a}\sqrt{c}x^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x^2+A)/(a+c\*x^4-x^2\*a^(1/2)\*c^(1/2)),x, algorithm="maxima")

[Out] integrate((B\*x^2 + A)/(c\*x^4 - sqrt(a)\*sqrt(c)\*x^2 + a), x)

**mupad** [B] time = 5.29, size = 1575, normalized size = 6.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x^2)/(a + c\*x^4 - a^(1/2)\*c^(1/2)\*x^2),x)

[Out]  $-2 * \operatorname{atanh}((6 * A^2 * x * ((B^2 * (-27 * a^3 * c^3)^{(1/2)}) / (72 * a^2 * c^3) - B^2 / (24 * a^{(1/2)} * c^{(3/2)})) * c^{(3/2)}) - (A * B) / (6 * a * c) - (A^2 * (-27 * a^3 * c^3)^{(1/2)}) / (72 * a^3 * c^2) - A^2 / (24 * a^{(3/2)} * c^{(1/2)})^{(1/2)}) / ((2 * A^2 * B) / c - (2 * B^3 * a) / c^2 + A^3 / (a^{(1/2)} * c^{(1/2)})) + (A^3 * (-27 * a^3 * c^3)^{(1/2)}) / (3 * a^2 * c^2) - (A * B^2 * a^{(1/2)}) / c^{(3/2)} - (A * B^2 * (-27 * a^3 * c^3)^{(1/2)}) / (3 * a * c^3) - (6 * B^2 * a * x * ((B^2 * (-27 * a^3 * c^3)^{(1/2)}) / (72 * a^2 * c^3) - B^2 / (24 * a^{(1/2)} * c^{(3/2)})) - (A * B) / (6 * a * c) - (A^2 * (-27 * a^3 * c^3)^{(1/2)}) / (72 * a^3 * c^2) - A^2 / (24 * a^{(3/2)} * c^{(1/2)})^{(1/2)}) / (2 * A^2 * B - (2 * B^3 * a) / c + (A^3 * c^{(1/2)}) / a^{(1/2)} + (A^3 * (-27 * a^3 * c^3)^{(1/2)}) / (3 * a^2 * c) - (A * B^2 * a^{(1/2)}) / c^{(1/2)} - (A * B^2 * (-27 * a^3 * c^3)^{(1/2)}) / (3 * a * c^2)) - (2 * A^2 * x * (-27 * a^3 * c^3)^{(1/2)} * ((B^2 * (-27 * a^3 * c^3)^{(1/2)}) / (72 * a^2 * c^3) - B^2 / (24 * a^{(1/2)} * c^{(3/2)})) - (A * B) / (6 * a * c) - (A^2 * (-27 * a^3 * c^3)^{(1/2)}) / (72 * a^3 * c^2) - A^2 / (24 * a^{(3/2)} * c^{(1/2)})^{(1/2)}) / (3 * a^{(3/2)} * c^{(7/2)} * ((2 * A^2 * B) / c^3 - (2 * B^3 * a) / c^4 + A^3 / (a^{(1/2)} * c^{(5/2)})) + (A^3 * (-27 * a^3 * c^3)^{(1/2)}) / (3 * a^2 * c^4) - (A * B^2 * a^{(1/2)}) / c^{(7/2)} - (A * B^2 * (-27 * a^3 * c^3)^{(1/2)}) / (3 * a * c^5))) + (2 * B^2 * x * (-27 * a^3 * c^3)^{(1/2)} * ((B^2 * (-27 * a^3 * c^3)^{(1/2)}) / (72 * a^2 * c^3) - B^2 / (24 * a^{(1/2)} * c^{(3/2)})) - (A * B) / (6 * a * c) - (A^2 * (-27 * a^3 * c^3)^{(1/2)}) / (72 * a^3 * c^2) - A^2 / (24 * a^{(3/2)} * c^{(1/2)})^{(1/2)}) / (3 * a^{(1/2)} * c^{(9/2)} * ((2 * A^2 * B) / c^3 - (2 * B^3 * a) / c^4 + A^3 / (a^{(1/2)} * c^{(5/2)})) + (A^3 * (-27 * a^3 * c^3)^{(1/2)}) / (3 * a^2 * c^4) - (A * B^2 * a^{(1/2)}) / c^{(7/2)} - (A * B^2 * (-27 * a^3 * c^3)^{(1/2)}) / (3 * a * c^5))) * ((B^2 * (-27 * a^3 * c^3)^{(1/2)}) / (72 * a^2 * c^3) - B^2 / (24 * a^{(1/2)} * c^{(3/2)})) - (A * B) / (6 * a * c) - (A^2 * (-27 * a^3 * c^3)^{(1/2)}) / (72 * a^3 * c^2) - A^2 / (24 * a^{(3/2)} * c^{(1/2)})^{(1/2)} - 2 * \operatorname{atanh}((6 * A^2 * x * ((A^2 * (-27 * a^3 * c^3)^{(1/2)}) / (72 * a^3 * c^2) - B^2 / (24 * a^{(1/2)} * c^{(3/2)})) - (A * B) / (6 * a * c) - A^2 / (24 * a^{(3/2)} * c^{(1/2)}) - (B^2 * (-27 * a^3 * c^3)^{(1/2)}) / (72 * a^2 * c^3))^{(1/2)}) / ((2 * A^2 * B) / c - (2 * B^3 * a) / c^2 + A^3 / (a^{(1/2)} * c^{(1/2)})) - (A^3 * (-27 * a^3 * c^3)^{(1/2)}) / (3 * a^2 * c^2) - (A * B^2 * a^{(1/2)}) / c^{(3/2)} + (A * B^2 * (-27 * a^3 * c^3)^{(1/2)}) / (3 * a * c^3) - (6 * B^2 * a * x * ((A^2 * (-27 * a^3 * c^3)^{(1/2)}) / (72 * a^3 * c^2) - B^2 / (24 * a^{(1/2)} * c^{(3/2)})) - (A * B) / (6 * a * c) - A^2 / (24 * a^{(3/2)} * c^{(1/2)}) - (B^2 * (-27 * a^3 * c^3)^{(1/2)}) / (72 * a^2 * c^3))^{(1/2)}) / (2 * A^2 * B - (2 * B^3 * a) / c + (A^3 * c^{(1/2)}) / a^{(1/2)} - (A^3 * (-27 * a^3 * c^3)^{(1/2)}) / (3 * a^2 * c) - (A * B^2 * a^{(1/2)}) / c^{(1/2)} + (A * B^2 * (-27 * a^3 * c^3)^{(1/2)}) / (3 * a * c^2)) + (2 * A^2 * x * (-27 * a^3 * c^3)^{(1/2)} * ((A^2 * (-27 * a^3 * c^3)^{(1/2)}) / (72 * a^3 * c^2) - B^2 / (24 * a^{(1/2)} * c^{(3/2)})) - (A * B) / (6 * a * c) - A^2 / (24 * a^{(3/2)} * c^{(1/2)}) - (B^2 * (-27 * a^3 * c^3)^{(1/2)}) / (72 * a^2 * c^3))^{(1/2)}) / (3 * a^{(3/2)} * c^{(7/2)} * ((2 * A^2 * B) / c^3 - (2 * B^3 * a) / c^4 + A^3 / (a^{(1/2)} * c^{(5/2)})) - (A^3 * (-27 * a^3 * c^3)^{(1/2)}) / (3 * a^2 * c^4) - (A * B^2 * a^{(1/2)}) / c^{(7/2)} + (A * B^2 * (-27 * a^3 * c^3)^{(1/2)}) / (3 * a * c^5))) - (2 * B^2 * x * (-27 * a^3 * c^3)^{(1/2)} * ((A^2 * (-27 * a^3 * c^3)^{(1/2)}) / (72 * a^3 * c^2) - B^2 / (24 * a^{(1/2)} * c^{(3/2)})) - (A * B) / (6 * a * c) - A^2 / (24 * a^{(3/2)} * c^{(1/2)}) - (B^2 * (-27 * a^3 * c^3)^{(1/2)}) / (72 * a^2 * c^3))^{(1/2)}) / (3 * a^{(1/2)} * c^{(9/2)} * ((2 * A^2 * B) / c^3 - (2 * B^3 * a) / c^4 + A^3 / (a^{(1/2)} * c^{(5/2)})) - (A^3 * (-27 * a^3 * c^3)^{(1/2)}) / (3 * a^2 * c^4) - (A * B^2 * a^{(1/2)}) / c^{(7/2)} + (A * B^2 * (-27 * a^3 * c^3)^{(1/2)}) / (3 * a * c^5))) * ((A^2 * (-27 * a^3 * c^3)^{(1/2)}) / (72 * a^3 * c^2) - B^2 / (24 * a^{(1/2)} * c^{(3/2)})) - (A * B) / (6 * a * c) - A^2 / (24 * a^{(3/2)} * c^{(1/2)}) - (B^2 * (-27 * a^3 * c^3)^{(1/2)}) / (72 * a^2 * c^3))^{(1/2)}$

$$72*a^3*c^2) - B^2/(24*a^{(1/2)}*c^{(3/2)}) - (A*B)/(6*a*c) - A^2/(24*a^{(3/2)}*c^{(1/2)}) - (B^2*(-27*a^3*c^3)^{(1/2)})/(72*a^2*c^3)^{(1/2)}$$

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B\*x\*\*2+A)/(a+c\*x\*\*4-x\*\*2\*a\*\*(1/2)\*c\*\*(1/2)),x)

[Out] Exception raised: PolynomialError

### 3.101 $\int (d + ex^2)^4 (a + cx^4) dx$

**Optimal.** Leaf size=106

$$\frac{1}{9}e^2x^9 (ae^2 + 6cd^2) + \frac{4}{7}dex^7 (ae^2 + cd^2) + \frac{1}{5}d^2x^5 (6ae^2 + cd^2) + ad^4x + \frac{4}{3}ad^3ex^3 + \frac{4}{11}cde^3x^{11} + \frac{1}{13}ce^4x^{13}$$

**Rubi [A]** time = 0.08, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {1154}

$$\frac{1}{9}e^2x^9 (ae^2 + 6cd^2) + \frac{4}{7}dex^7 (ae^2 + cd^2) + \frac{1}{5}d^2x^5 (6ae^2 + cd^2) + \frac{4}{3}ad^3ex^3 + ad^4x + \frac{4}{11}cde^3x^{11} + \frac{1}{13}ce^4x^{13}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^4\*(a + c\*x^4), x]

[Out] a\*d^4\*x + (4\*a\*d^3\*e\*x^3)/3 + (d^2\*(c\*d^2 + 6\*a\*e^2)\*x^5)/5 + (4\*d\*e\*(c\*d^2 + a\*e^2)\*x^7)/7 + (e^2\*(6\*c\*d^2 + a\*e^2)\*x^9)/9 + (4\*c\*d\*e^3\*x^11)/11 + (c\*e^4\*x^13)/13

Rule 1154

Int[((d\_) + (e\_)\*(x\_)^2)^(q\_)\*((a\_) + (c\_)\*(x\_)^4)^(p\_), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^2)^q\*(a + c\*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int (d + ex^2)^4 (a + cx^4) dx &= \int (ad^4 + 4ad^3ex^2 + d^2(cd^2 + 6ae^2)x^4 + 4de(cd^2 + ae^2)x^6 + e^2(6cd^2 + ae^2)x^8 + 4ad^4x + \frac{4}{3}ad^3ex^3 + \frac{1}{5}d^2(cd^2 + 6ae^2)x^5 + \frac{4}{7}de(cd^2 + ae^2)x^7 + \frac{1}{9}e^2(6cd^2 + ae^2)x^9 + \dots) dx \\ &= ad^4x + \frac{4}{3}ad^3ex^3 + \frac{1}{5}d^2(cd^2 + 6ae^2)x^5 + \frac{4}{7}de(cd^2 + ae^2)x^7 + \frac{1}{9}e^2(6cd^2 + ae^2)x^9 + \dots \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 106, normalized size = 1.00

$$\frac{1}{9}e^2x^9 (ae^2 + 6cd^2) + \frac{4}{7}dex^7 (ae^2 + cd^2) + \frac{1}{5}d^2x^5 (6ae^2 + cd^2) + ad^4x + \frac{4}{3}ad^3ex^3 + \frac{4}{11}cde^3x^{11} + \frac{1}{13}ce^4x^{13}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^4\*(a + c\*x^4), x]

[Out] a\*d^4\*x + (4\*a\*d^3\*e\*x^3)/3 + (d^2\*(c\*d^2 + 6\*a\*e^2)\*x^5)/5 + (4\*d\*e\*(c\*d^2 + a\*e^2)\*x^7)/7 + (e^2\*(6\*c\*d^2 + a\*e^2)\*x^9)/9 + (4\*c\*d\*e^3\*x^11)/11 + (c\*e^4\*x^13)/13

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex^2)^4 (a + cx^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)^4\*(a + c\*x^4), x]

[Out] IntegrateAlgebraic[(d + e\*x^2)^4\*(a + c\*x^4), x]



**fricas** [A] time = 0.87, size = 98, normalized size = 0.92

$$\frac{1}{13}x^{13}e^4c + \frac{4}{11}x^{11}e^3dc + \frac{2}{3}x^9e^2d^2c + \frac{1}{9}x^9e^4a + \frac{4}{7}x^7ed^3c + \frac{4}{7}x^7e^3da + \frac{1}{5}x^5d^4c + \frac{6}{5}x^5e^2d^2a + \frac{4}{3}x^3ed^3a + xd^4a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^4\*(c\*x^4+a),x, algorithm="fricas")

[Out] 1/13\*x^13\*e^4\*c + 4/11\*x^11\*e^3\*d\*c + 2/3\*x^9\*e^2\*d^2\*c + 1/9\*x^9\*e^4\*a + 4/7\*x^7\*e\*d^3\*c + 4/7\*x^7\*e^3\*d\*a + 1/5\*x^5\*d^4\*c + 6/5\*x^5\*e^2\*d^2\*a + 4/3\*x^3\*e\*d^3\*a + x\*d^4\*a

**giac** [A] time = 0.15, size = 94, normalized size = 0.89

$$\frac{1}{13}cx^{13}e^4 + \frac{4}{11}cdx^{11}e^3 + \frac{2}{3}cd^2x^9e^2 + \frac{4}{7}cd^3x^7e + \frac{1}{9}ax^9e^4 + \frac{1}{5}cd^4x^5 + \frac{4}{7}adx^7e^3 + \frac{6}{5}ad^2x^5e^2 + \frac{4}{3}ad^3x^3e + ad^4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^4\*(c\*x^4+a),x, algorithm="giac")

[Out] 1/13\*c\*x^13\*e^4 + 4/11\*c\*d\*x^11\*e^3 + 2/3\*c\*d^2\*x^9\*e^2 + 4/7\*c\*d^3\*x^7\*e + 1/9\*a\*x^9\*e^4 + 1/5\*c\*d^4\*x^5 + 4/7\*a\*d\*x^7\*e^3 + 6/5\*a\*d^2\*x^5\*e^2 + 4/3\*a\*d^3\*x^3\*e + a\*d^4\*x

**maple** [A] time = 0.00, size = 97, normalized size = 0.92

$$\frac{ce^4x^{13}}{13} + \frac{4cde^3x^{11}}{11} + \frac{(e^4a + 6d^2e^2c)x^9}{9} + \frac{4ad^3ex^3}{3} + \frac{(4de^3a + 4d^3ec)x^7}{7} + ad^4x + \frac{(6d^2e^2a + d^4c)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^4\*(c\*x^4+a),x)

[Out] 1/13\*c\*e^4\*x^13+4/11\*c\*d\*e^3\*x^11+1/9\*(a\*e^4+6\*c\*d^2\*e^2)\*x^9+1/7\*(4\*a\*d\*e^3+4\*c\*d^3\*e)\*x^7+1/5\*(6\*a\*d^2\*e^2+c\*d^4)\*x^5+4/3\*a\*d^3\*e\*x^3+a\*d^4\*x

**maxima** [A] time = 1.05, size = 94, normalized size = 0.89

$$\frac{1}{13}ce^4x^{13} + \frac{4}{11}cde^3x^{11} + \frac{1}{9}(6cd^2e^2 + ae^4)x^9 + \frac{4}{3}ad^3ex^3 + \frac{4}{7}(cd^3e + ade^3)x^7 + ad^4x + \frac{1}{5}(cd^4 + 6ad^2e^2)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^4\*(c\*x^4+a),x, algorithm="maxima")

[Out] 1/13\*c\*e^4\*x^13 + 4/11\*c\*d\*e^3\*x^11 + 1/9\*(6\*c\*d^2\*e^2 + a\*e^4)\*x^9 + 4/3\*a\*d^3\*e\*x^3 + 4/7\*(c\*d^3\*e + a\*d\*e^3)\*x^7 + a\*d^4\*x + 1/5\*(c\*d^4 + 6\*a\*d^2\*e^2)\*x^5

**mupad** [B] time = 4.35, size = 95, normalized size = 0.90

$$x^5 \left( \frac{cd^4}{5} + \frac{6ad^2e^2}{5} \right) + x^9 \left( \frac{2cd^2e^2}{3} + \frac{ae^4}{9} \right) + x^7 \left( \frac{4cd^3e}{7} + \frac{4ade^3}{7} \right) + \frac{ce^4x^{13}}{13} + ad^4x + \frac{4ad^3ex^3}{3} + \frac{4cde^3x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c\*x^4)\*(d + e\*x^2)^4,x)

[Out] x^5\*((c\*d^4)/5 + (6\*a\*d^2\*e^2)/5) + x^9\*((a\*e^4)/9 + (2\*c\*d^2\*e^2)/3) + x^7\*((4\*a\*d\*e^3)/7 + (4\*c\*d^3\*e)/7) + (c\*e^4\*x^13)/13 + a\*d^4\*x + (4\*a\*d^3\*e\*x^3)/3 + (4\*c\*d\*e^3\*x^11)/11

sympy [A] time = 0.09, size = 110, normalized size = 1.04

$$ad^4x + \frac{4ad^3ex^3}{3} + \frac{4cde^3x^{11}}{11} + \frac{ce^4x^{13}}{13} + x^9 \left( \frac{ae^4}{9} + \frac{2cd^2e^2}{3} \right) + x^7 \left( \frac{4ade^3}{7} + \frac{4cd^3e}{7} \right) + x^5 \left( \frac{6ad^2e^2}{5} + \frac{cd^4}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*4\*(c\*x\*\*4+a),x)

[Out] a\*d\*\*4\*x + 4\*a\*d\*\*3\*e\*x\*\*3/3 + 4\*c\*d\*e\*\*3\*x\*\*11/11 + c\*e\*\*4\*x\*\*13/13 + x\*\*9\*(a\*e\*\*4/9 + 2\*c\*d\*\*2\*e\*\*2/3) + x\*\*7\*(4\*a\*d\*e\*\*3/7 + 4\*c\*d\*\*3\*e/7) + x\*\*5\*(6\*a\*d\*\*2\*e\*\*2/5 + c\*d\*\*4/5)

### 3.102 $\int (d + ex^2)^3 (a + cx^4) dx$

**Optimal.** Leaf size=79

$$\frac{1}{7}ex^7 (ae^2 + 3cd^2) + \frac{1}{5}dx^5 (3ae^2 + cd^2) + ad^3x + ad^2ex^3 + \frac{1}{3}cde^2x^9 + \frac{1}{11}ce^3x^{11}$$

**Rubi [A]** time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {1154}

$$\frac{1}{7}ex^7 (ae^2 + 3cd^2) + \frac{1}{5}dx^5 (3ae^2 + cd^2) + ad^2ex^3 + ad^3x + \frac{1}{3}cde^2x^9 + \frac{1}{11}ce^3x^{11}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^3\*(a + c\*x^4), x]

[Out] a\*d^3\*x + a\*d^2\*e\*x^3 + (d\*(c\*d^2 + 3\*a\*e^2)\*x^5)/5 + (e\*(3\*c\*d^2 + a\*e^2)\*x^7)/7 + (c\*d\*e^2\*x^9)/3 + (c\*e^3\*x^11)/11

**Rule 1154**

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^2)^q\*(a + c\*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

**Rubi steps**

$$\begin{aligned} \int (d + ex^2)^3 (a + cx^4) dx &= \int (ad^3 + 3ad^2ex^2 + d(cd^2 + 3ae^2)x^4 + e(3cd^2 + ae^2)x^6 + 3cde^2x^8 + ce^3x^{10}) dx \\ &= ad^3x + ad^2ex^3 + \frac{1}{5}d(cd^2 + 3ae^2)x^5 + \frac{1}{7}e(3cd^2 + ae^2)x^7 + \frac{1}{3}cde^2x^9 + \frac{1}{11}ce^3x^{11} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 79, normalized size = 1.00

$$\frac{1}{7}ex^7 (ae^2 + 3cd^2) + \frac{1}{5}dx^5 (3ae^2 + cd^2) + ad^3x + ad^2ex^3 + \frac{1}{3}cde^2x^9 + \frac{1}{11}ce^3x^{11}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^3\*(a + c\*x^4), x]

[Out] a\*d^3\*x + a\*d^2\*e\*x^3 + (d\*(c\*d^2 + 3\*a\*e^2)\*x^5)/5 + (e\*(3\*c\*d^2 + a\*e^2)\*x^7)/7 + (c\*d\*e^2\*x^9)/3 + (c\*e^3\*x^11)/11

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex^2)^3 (a + cx^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)^3\*(a + c\*x^4), x]

[Out] IntegrateAlgebraic[(d + e\*x^2)^3\*(a + c\*x^4), x]

**fricas [A]** time = 0.87, size = 73, normalized size = 0.92

$$\frac{1}{11}x^{11}e^3c + \frac{1}{3}x^9e^2dc + \frac{3}{7}x^7ed^2c + \frac{1}{7}x^7e^3a + \frac{1}{5}x^5d^3c + \frac{3}{5}x^5e^2da + x^3ed^2a + xd^3a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^3\*(c\*x^4+a),x, algorithm="fricas")

[Out]  $\frac{1}{11}x^{11}e^3c + \frac{1}{3}x^9e^2dc + \frac{3}{7}x^7e^2d^2c + \frac{1}{7}x^7e^3a + \frac{1}{5}x^5d^3c + \frac{3}{5}x^5e^2da + x^3e^2d^2a + x^d^3a$

**giac** [A] time = 0.15, size = 71, normalized size = 0.90

$$\frac{1}{11}cx^{11}e^3 + \frac{1}{3}cdx^9e^2 + \frac{3}{7}cd^2x^7e + \frac{1}{5}cd^3x^5 + \frac{1}{7}ax^7e^3 + \frac{3}{5}adx^5e^2 + ad^2x^3e + ad^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^3\*(c\*x^4+a),x, algorithm="giac")

[Out]  $\frac{1}{11}c*x^{11}*e^3 + \frac{1}{3}c*d*x^9*e^2 + \frac{3}{7}c*d^2*x^7*e + \frac{1}{5}c*d^3*x^5 + \frac{1}{7}a*x^7*e^3 + \frac{3}{5}a*d*x^5*e^2 + a*d^2*x^3*e + a*d^3*x$

**maple** [A] time = 0.00, size = 72, normalized size = 0.91

$$\frac{ce^3x^{11}}{11} + \frac{cde^2x^9}{3} + ad^2ex^3 + \frac{(ae^3 + 3cd^2e)x^7}{7} + ad^3x + \frac{(3de^2a + d^3c)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^3\*(c\*x^4+a),x)

[Out]  $\frac{1}{11}c*e^3*x^{11} + \frac{1}{3}c*d*e^2*x^9 + \frac{1}{7}*(a*e^3 + 3*c*d^2*e)*x^7 + \frac{1}{5}*(3*a*d*e^2 + c*d^3)*x^5 + a*d^2*e*x^3 + a*d^3*x$

**maxima** [A] time = 1.04, size = 71, normalized size = 0.90

$$\frac{1}{11}ce^3x^{11} + \frac{1}{3}cde^2x^9 + \frac{1}{7}(3cd^2e + ae^3)x^7 + ad^2ex^3 + \frac{1}{5}(cd^3 + 3ade^2)x^5 + ad^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^3\*(c\*x^4+a),x, algorithm="maxima")

[Out]  $\frac{1}{11}c*e^3*x^{11} + \frac{1}{3}c*d*e^2*x^9 + \frac{1}{7}*(3*c*d^2*e + a*e^3)*x^7 + a*d^2*e*x^3 + \frac{1}{5}*(c*d^3 + 3*a*d*e^2)*x^5 + a*d^3*x$

**mupad** [B] time = 0.03, size = 71, normalized size = 0.90

$$x^5 \left( \frac{cd^3}{5} + \frac{3ade^2}{5} \right) + x^7 \left( \frac{3cd^2e}{7} + \frac{ae^3}{7} \right) + \frac{ce^3x^{11}}{11} + ad^3x + ad^2ex^3 + \frac{cde^2x^9}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c\*x^4)\*(d + e\*x^2)^3,x)

[Out]  $x^5*((c*d^3)/5 + (3*a*d*e^2)/5) + x^7*((a*e^3)/7 + (3*c*d^2*e)/7) + (c*e^3*x^{11})/11 + a*d^3*x + a*d^2*e*x^3 + (c*d*e^2*x^9)/3$

**sympy** [A] time = 0.09, size = 78, normalized size = 0.99

$$ad^3x + ad^2ex^3 + \frac{cde^2x^9}{3} + \frac{ce^3x^{11}}{11} + x^7 \left( \frac{ae^3}{7} + \frac{3cd^2e}{7} \right) + x^5 \left( \frac{3ade^2}{5} + \frac{cd^3}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*3\*(c\*x\*\*4+a),x)

[Out]  $a*d**3*x + a*d**2*e*x**3 + c*d*e**2*x**9/3 + c*e**3*x**11/11 + x**7*(a*e**3/7 + 3*c*d**2*e/7) + x**5*(3*a*d*e**2/5 + c*d**3/5)$

### 3.103 $\int (d + ex^2)^2 (a + cx^4) dx$

**Optimal.** Leaf size=56

$$\frac{1}{5}x^5 (ae^2 + cd^2) + ad^2x + \frac{2}{3}adex^3 + \frac{2}{7}cdex^7 + \frac{1}{9}ce^2x^9$$

**Rubi [A]** time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {1154}

$$\frac{1}{5}x^5 (ae^2 + cd^2) + ad^2x + \frac{2}{3}adex^3 + \frac{2}{7}cdex^7 + \frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^2\*(a + c\*x^4), x]

[Out] a\*d^2\*x + (2\*a\*d\*e\*x^3)/3 + ((c\*d^2 + a\*e^2)\*x^5)/5 + (2\*c\*d\*e\*x^7)/7 + (c\*e^2\*x^9)/9

**Rule 1154**

Int[((d\_) + (e\_)\*(x\_)^2)^(q\_)\*((a\_) + (c\_)\*(x\_)^4)^(p\_), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^2)^q\*(a + c\*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

**Rubi steps**

$$\begin{aligned} \int (d + ex^2)^2 (a + cx^4) dx &= \int (ad^2 + 2adex^2 + (cd^2 + ae^2)x^4 + 2cdex^6 + ce^2x^8) dx \\ &= ad^2x + \frac{2}{3}adex^3 + \frac{1}{5}(cd^2 + ae^2)x^5 + \frac{2}{7}cdex^7 + \frac{1}{9}ce^2x^9 \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 56, normalized size = 1.00

$$\frac{1}{5}x^5 (ae^2 + cd^2) + ad^2x + \frac{2}{3}adex^3 + \frac{2}{7}cdex^7 + \frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^2\*(a + c\*x^4), x]

[Out] a\*d^2\*x + (2\*a\*d\*e\*x^3)/3 + ((c\*d^2 + a\*e^2)\*x^5)/5 + (2\*c\*d\*e\*x^7)/7 + (c\*e^2\*x^9)/9

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex^2)^2 (a + cx^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)^2\*(a + c\*x^4), x]

[Out] IntegrateAlgebraic[(d + e\*x^2)^2\*(a + c\*x^4), x]

**fricas [A]** time = 0.57, size = 50, normalized size = 0.89

$$\frac{1}{9}x^9e^2c + \frac{2}{7}x^7edc + \frac{1}{5}x^5d^2c + \frac{1}{5}x^5e^2a + \frac{2}{3}x^3eda + xd^2a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(c\*x^4+a),x, algorithm="fricas")

[Out] 1/9\*x^9\*e^2\*c + 2/7\*x^7\*e\*d\*c + 1/5\*x^5\*d^2\*c + 1/5\*x^5\*e^2\*a + 2/3\*x^3\*e\*d\*a + x\*d^2\*a

**giac** [A] time = 0.20, size = 50, normalized size = 0.89

$$\frac{1}{9}cx^9e^2 + \frac{2}{7}cdx^7e + \frac{1}{5}cd^2x^5 + \frac{1}{5}ax^5e^2 + \frac{2}{3}adx^3e + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(c\*x^4+a),x, algorithm="giac")

[Out] 1/9\*c\*x^9\*e^2 + 2/7\*c\*d\*x^7\*e + 1/5\*c\*d^2\*x^5 + 1/5\*a\*x^5\*e^2 + 2/3\*a\*d\*x^3\*e + a\*d^2\*x

**maple** [A] time = 0.00, size = 49, normalized size = 0.88

$$\frac{ce^2x^9}{9} + \frac{2cde x^7}{7} + \frac{2ade x^3}{3} + \frac{(ae^2 + cd^2)x^5}{5} + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^2\*(c\*x^4+a),x)

[Out] a\*d^2\*x+2/3\*a\*d\*e\*x^3+1/5\*(a\*e^2+c\*d^2)\*x^5+2/7\*c\*d\*e\*x^7+1/9\*c\*e^2\*x^9

**maxima** [A] time = 0.97, size = 48, normalized size = 0.86

$$\frac{1}{9}ce^2x^9 + \frac{2}{7}cdex^7 + \frac{2}{3}adex^3 + \frac{1}{5}(cd^2 + ae^2)x^5 + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(c\*x^4+a),x, algorithm="maxima")

[Out] 1/9\*c\*e^2\*x^9 + 2/7\*c\*d\*e\*x^7 + 2/3\*a\*d\*e\*x^3 + 1/5\*(c\*d^2 + a\*e^2)\*x^5 + a\*d^2\*x

**mupad** [B] time = 0.02, size = 49, normalized size = 0.88

$$x^5 \left( \frac{cd^2}{5} + \frac{ae^2}{5} \right) + \frac{ce^2x^9}{9} + ad^2x + \frac{2adex^3}{3} + \frac{2cdex^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c\*x^4)\*(d + e\*x^2)^2,x)

[Out] x^5\*((a\*e^2)/5 + (c\*d^2)/5) + (c\*e^2\*x^9)/9 + a\*d^2\*x + (2\*a\*d\*e\*x^3)/3 + (2\*c\*d\*e\*x^7)/7

**sympy** [A] time = 0.08, size = 56, normalized size = 1.00

$$ad^2x + \frac{2adex^3}{3} + \frac{2cdex^7}{7} + \frac{ce^2x^9}{9} + x^5 \left( \frac{ae^2}{5} + \frac{cd^2}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*2\*(c\*x\*\*4+a),x)

[Out] a\*d\*\*2\*x + 2\*a\*d\*e\*x\*\*3/3 + 2\*c\*d\*e\*x\*\*7/7 + c\*e\*\*2\*x\*\*9/9 + x\*\*5\*(a\*e\*\*2/5 + c\*d\*\*2/5)

### 3.104 $\int (d + ex^2)(a + cx^4) dx$

**Optimal.** Leaf size=32

$$adx + \frac{1}{3}aex^3 + \frac{1}{5}cdx^5 + \frac{1}{7}cex^7$$

**Rubi [A]** time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {1154}

$$adx + \frac{1}{3}aex^3 + \frac{1}{5}cdx^5 + \frac{1}{7}cex^7$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)\*(a + c\*x^4), x]

[Out] a\*d\*x + (a\*e\*x^3)/3 + (c\*d\*x^5)/5 + (c\*e\*x^7)/7

**Rule 1154**

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^2)^q\*(a + c\*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

**Rubi steps**

$$\begin{aligned} \int (d + ex^2)(a + cx^4) dx &= \int (ad + aex^2 + cdx^4 + cex^6) dx \\ &= adx + \frac{1}{3}aex^3 + \frac{1}{5}cdx^5 + \frac{1}{7}cex^7 \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 32, normalized size = 1.00

$$adx + \frac{1}{3}aex^3 + \frac{1}{5}cdx^5 + \frac{1}{7}cex^7$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)\*(a + c\*x^4), x]

[Out] a\*d\*x + (a\*e\*x^3)/3 + (c\*d\*x^5)/5 + (c\*e\*x^7)/7

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex^2)(a + cx^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)\*(a + c\*x^4), x]

[Out] IntegrateAlgebraic[(d + e\*x^2)\*(a + c\*x^4), x]

**fricas [A]** time = 0.81, size = 26, normalized size = 0.81

$$\frac{1}{7}x^7ec + \frac{1}{5}x^5dc + \frac{1}{3}x^3ea + xda$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(c\*x^4+a),x, algorithm="fricas")

[Out] 1/7\*x^7\*e\*c + 1/5\*x^5\*d\*c + 1/3\*x^3\*e\*a + x\*d\*a

**giac** [A] time = 0.18, size = 28, normalized size = 0.88

$$\frac{1}{7}cx^7e + \frac{1}{5}cdx^5 + \frac{1}{3}ax^3e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(c\*x^4+a),x, algorithm="giac")

[Out] 1/7\*c\*x^7\*e + 1/5\*c\*d\*x^5 + 1/3\*a\*x^3\*e + a\*d\*x

**maple** [A] time = 0.00, size = 27, normalized size = 0.84

$$\frac{1}{7}ce x^7 + \frac{1}{5}cd x^5 + \frac{1}{3}ae x^3 + adx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)\*(c\*x^4+a),x)

[Out] a\*d\*x+1/3\*a\*e\*x^3+1/5\*c\*d\*x^5+1/7\*c\*e\*x^7

**maxima** [A] time = 1.06, size = 26, normalized size = 0.81

$$\frac{1}{7}cex^7 + \frac{1}{5}cdx^5 + \frac{1}{3}aex^3 + adx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(c\*x^4+a),x, algorithm="maxima")

[Out] 1/7\*c\*e\*x^7 + 1/5\*c\*d\*x^5 + 1/3\*a\*e\*x^3 + a\*d\*x

**mupad** [B] time = 0.04, size = 26, normalized size = 0.81

$$\frac{cex^7}{7} + \frac{cdx^5}{5} + \frac{aex^3}{3} + adx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c\*x^4)\*(d + e\*x^2),x)

[Out] a\*d\*x + (a\*e\*x^3)/3 + (c\*d\*x^5)/5 + (c\*e\*x^7)/7

**sympy** [A] time = 0.08, size = 29, normalized size = 0.91

$$adx + \frac{aex^3}{3} + \frac{cdx^5}{5} + \frac{cex^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*(c\*x\*\*4+a),x)

[Out] a\*d\*x + a\*e\*x\*\*3/3 + c\*d\*x\*\*5/5 + c\*e\*x\*\*7/7



$$3.105 \quad \int \frac{a+cx^4}{d+ex^2} dx$$

Optimal. Leaf size=55

$$\frac{(ae^2 + cd^2) \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}} - \frac{cdx}{e^2} + \frac{cx^3}{3e}$$

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {1154, 205}

$$\frac{(ae^2 + cd^2) \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}} - \frac{cdx}{e^2} + \frac{cx^3}{3e}$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^4)/(d + e\*x^2), x]

[Out] -((c\*d\*x)/e^2) + (c\*x^3)/(3\*e) + ((c\*d^2 + a\*e^2)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(Sqrt[d]\*e^(5/2))

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1154

Int(((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^2)^q\*(a + c\*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int \frac{a+cx^4}{d+ex^2} dx &= \int \left( -\frac{cd}{e^2} + \frac{cx^2}{e} + \frac{cd^2+ae^2}{e^2(d+ex^2)} \right) dx \\ &= -\frac{cdx}{e^2} + \frac{cx^3}{3e} + \left( a + \frac{cd^2}{e^2} \right) \int \frac{1}{d+ex^2} dx \\ &= -\frac{cdx}{e^2} + \frac{cx^3}{3e} + \frac{(cd^2+ae^2) \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 1.00

$$\frac{(ae^2 + cd^2) \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}} - \frac{cdx}{e^2} + \frac{cx^3}{3e}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c\*x^4)/(d + e\*x^2), x]

[Out] -((c\*d\*x)/e^2) + (c\*x^3)/(3\*e) + ((c\*d^2 + a\*e^2)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(Sqrt[d]\*e^(5/2))

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + cx^4}{d + ex^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + c\*x^4)/(d + e\*x^2), x]

[Out] IntegrateAlgebraic[(a + c\*x^4)/(d + e\*x^2), x]

**fricas** [A] time = 1.11, size = 131, normalized size = 2.38

$$\left[ \frac{2cde^2x^3 - 6cd^2ex - 3(cd^2 + ae^2)\sqrt{-de} \log\left(\frac{ex^2 - 2\sqrt{-de}x - d}{ex^2 + d}\right)}{6de^3}, \frac{cde^2x^3 - 3cd^2ex + 3(cd^2 + ae^2)\sqrt{de} \arctan\left(\frac{\sqrt{de}x}{d}\right)}{3de^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+a)/(e\*x^2+d), x, algorithm="fricas")

[Out] [1/6\*(2\*c\*d\*e^2\*x^3 - 6\*c\*d^2\*e\*x - 3\*(c\*d^2 + a\*e^2)\*sqrt(-d\*e)\*log((e\*x^2 - 2\*sqrt(-d\*e)\*x - d)/(e\*x^2 + d)))/(d\*e^3), 1/3\*(c\*d\*e^2\*x^3 - 3\*c\*d^2\*e\*x + 3\*(c\*d^2 + a\*e^2)\*sqrt(d\*e)\*arctan(sqrt(d\*e)\*x/d))/(d\*e^3)]

**giac** [A] time = 0.17, size = 44, normalized size = 0.80

$$\frac{(cd^2 + ae^2) \arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{5}{2}\right)}}{\sqrt{d}} + \frac{1}{3} (cx^3e^2 - 3cdxe)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+a)/(e\*x^2+d), x, algorithm="giac")

[Out] (c\*d^2 + a\*e^2)\*arctan(x\*e^(1/2)/sqrt(d))\*e^(-5/2)/sqrt(d) + 1/3\*(c\*x^3\*e^2 - 3\*c\*d\*x\*e)\*e^(-3)

**maple** [A] time = 0.01, size = 57, normalized size = 1.04

$$\frac{cx^3}{3e} + \frac{a \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} + \frac{cd^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de} e^2} - \frac{cdx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^4+a)/(e\*x^2+d), x)

[Out] 1/3\*c\*x^3/e - c\*d\*x/e^2 + 1/(d\*e)^(1/2)\*arctan(x\*e/(d\*e)^(1/2))\*a + 1/e^2/(d\*e)^(1/2)\*arctan(x\*e/(d\*e)^(1/2))\*c\*d^2

**maxima** [A] time = 2.55, size = 47, normalized size = 0.85

$$\frac{(cd^2 + ae^2) \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de} e^2} + \frac{cex^3 - 3cdx}{3e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+a)/(e\*x^2+d), x, algorithm="maxima")

[Out] (c\*d^2 + a\*e^2)\*arctan(e\*x/sqrt(d\*e))/(sqrt(d\*e)\*e^2) + 1/3\*(c\*e\*x^3 - 3\*c\*d\*x)/e^2

**mupad [B]** time = 0.07, size = 45, normalized size = 0.82

$$\frac{c x^3}{3 e} + \frac{\operatorname{atan}\left(\frac{\sqrt{e} x}{\sqrt{d}}\right) (c d^2 + a e^2)}{\sqrt{d} e^{5/2}} - \frac{c d x}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c\*x^4)/(d + e\*x^2), x)

[Out] (c\*x^3)/(3\*e) + (atan((e^(1/2)\*x)/d^(1/2))\*(a\*e^2 + c\*d^2))/(d^(1/2)\*e^(5/2)) - (c\*d\*x)/e^2

**sympy [B]** time = 0.32, size = 104, normalized size = 1.89

$$-\frac{cdx}{e^2} + \frac{cx^3}{3e} - \frac{\sqrt{-\frac{1}{de^5}} (ae^2 + cd^2) \log\left(-de^2\sqrt{-\frac{1}{de^5}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{de^5}} (ae^2 + cd^2) \log\left(de^2\sqrt{-\frac{1}{de^5}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*4+a)/(e\*x\*\*2+d), x)

[Out] -c\*d\*x/e\*\*2 + c\*x\*\*3/(3\*e) - sqrt(-1/(d\*e\*\*5))\*(a\*e\*\*2 + c\*d\*\*2)\*log(-d\*e\*\*2\*sqrt(-1/(d\*e\*\*5)) + x)/2 + sqrt(-1/(d\*e\*\*5))\*(a\*e\*\*2 + c\*d\*\*2)\*log(d\*e\*\*2\*sqrt(-1/(d\*e\*\*5)) + x)/2

$$3.106 \quad \int \frac{a+cx^4}{(d+ex^2)^2} dx$$

Optimal. Leaf size=74

$$\frac{x\left(a + \frac{cd^2}{e^2}\right)}{2d(d+ex^2)} - \frac{(3cd^2 - ae^2) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}} + \frac{cx}{e^2}$$

**Rubi [A]** time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {1158, 388, 205}

$$\frac{x\left(a + \frac{cd^2}{e^2}\right)}{2d(d+ex^2)} - \frac{(3cd^2 - ae^2) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}} + \frac{cx}{e^2}$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^4)/(d + e\*x^2)^2, x]

[Out] (c\*x)/e^2 + ((a + (c\*d^2)/e^2)\*x)/(2\*d\*(d + e\*x^2)) - ((3\*c\*d^2 - a\*e^2)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(2\*d^(3/2)\*e^(5/2))

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(d\*x\*(a + b\*x^n)^(p+1)/(b\*(n\*(p+1)+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1)+1))/(b\*(n\*(p+1)+1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p+1)+1, 0]

#### Rule 1158

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> With[{Qx = PolynomialQuotient[(a + c\*x^4)^p, d + e\*x^2, x], R = Coeff[PolynomialRemainder[(a + c\*x^4)^p, d + e\*x^2, x], x, 0]}, -Simp[(R\*x\*(d + e\*x^2)^(q+1))/(2\*d\*(q+1)), x] + Dist[1/(2\*d\*(q+1)), Int[(d + e\*x^2)^(q+1)\*ExpandToSum[2\*d\*(q+1)\*Qx + R\*(2\*q+3), x], x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{a+cx^4}{(d+ex^2)^2} dx &= \frac{\left(a + \frac{cd^2}{e^2}\right)x}{2d(d+ex^2)} - \int \frac{-a + \frac{cd^2}{e^2} - \frac{2cdx^2}{e}}{d+ex^2} dx \\ &= \frac{cx}{e^2} + \frac{\left(a + \frac{cd^2}{e^2}\right)x}{2d(d+ex^2)} + \frac{\left(a - \frac{3cd^2}{e^2}\right) \int \frac{1}{d+ex^2} dx}{2d} \\ &= \frac{cx}{e^2} + \frac{\left(a + \frac{cd^2}{e^2}\right)x}{2d(d+ex^2)} + \frac{\left(a - \frac{3cd^2}{e^2}\right) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 78, normalized size = 1.05

$$\frac{x(ae^2 + cd^2)}{2de^2(d + ex^2)} - \frac{(3cd^2 - ae^2) \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}} + \frac{cx}{e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c\*x^4)/(d + e\*x^2)^2,x]

[Out] (c\*x)/e^2 + ((c\*d^2 + a\*e^2)\*x)/(2\*d\*e^2\*(d + e\*x^2)) - ((3\*c\*d^2 - a\*e^2)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(2\*d^(3/2)\*e^(5/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + cx^4}{(d + ex^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + c\*x^4)/(d + e\*x^2)^2,x]

[Out] IntegrateAlgebraic[(a + c\*x^4)/(d + e\*x^2)^2, x]

**fricas [A]** time = 1.11, size = 222, normalized size = 3.00

$$\left[ \frac{4cd^2e^2x^3 + (3cd^3 - ade^2 + (3cd^2e - ae^3)x^2)\sqrt{-de} \log\left(\frac{ex^2 - 2\sqrt{-de}x - d}{ex^2 + d}\right) + 2(3cd^3e + ade^3)x}{4(d^2e^4x^2 + d^3e^3)}, \frac{2cd^2e^2x^3 - (3cd^3 - ade^2 + (3cd^2e - ae^3)x^2)\sqrt{de} \arctan\left(\frac{\sqrt{de}x}{d}\right) + (3cd^3e + ade^3)x}{2(d^2e^4x^2 + d^3e^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+a)/(e\*x^2+d)^2,x, algorithm="fricas")

[Out] [1/4\*(4\*c\*d^2\*e^2\*x^3 + (3\*c\*d^3 - a\*d\*e^2 + (3\*c\*d^2\*e - a\*e^3)\*x^2)\*sqrt(-d\*e)\*log((e\*x^2 - 2\*sqrt(-d\*e)\*x - d)/(e\*x^2 + d)) + 2\*(3\*c\*d^3\*e + a\*d\*e^3)\*x)/(d^2\*e^4\*x^2 + d^3\*e^3), 1/2\*(2\*c\*d^2\*e^2\*x^3 - (3\*c\*d^3 - a\*d\*e^2 + (3\*c\*d^2\*e - a\*e^3)\*x^2)\*sqrt(d\*e)\*arctan(sqrt(d\*e)\*x/d) + (3\*c\*d^3\*e + a\*d\*e^3)\*x)/(d^2\*e^4\*x^2 + d^3\*e^3)]

**giac [A]** time = 0.16, size = 62, normalized size = 0.84

$$cxe^{(-2)} - \frac{(3cd^2 - ae^2) \arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{5}{2}\right)}}{2d^{\frac{3}{2}}} + \frac{(cd^2x + axe^2)e^{(-2)}}{2(x^2e + d)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+a)/(e\*x^2+d)^2,x, algorithm="giac")

[Out] c\*x\*e^(-2) - 1/2\*(3\*c\*d^2 - a\*e^2)\*arctan(x\*e^(1/2)/sqrt(d))\*e^(-5/2)/d^(3/2) + 1/2\*(c\*d^2\*x + a\*x\*e^2)\*e^(-2)/((x^2\*e + d)\*d)

**maple [A]** time = 0.01, size = 82, normalized size = 1.11

$$\frac{ax}{2(e x^2 + d) d} + \frac{a \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de} d} + \frac{cdx}{2(e x^2 + d) e^2} - \frac{3cd \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de} e^2} + \frac{cx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^4+a)/(e\*x^2+d)^2,x)

[Out]  $c*x/e^{2+1/2/d*x/(e*x^2+d)*a+1/2/e^{2*d*x/(e*x^2+d)*c+1/2/d/(d*e)^{1/2}*arctan(1/(d*e)^{1/2}*e*x)*a-3/2/e^{2*d/(d*e)^{1/2}*arctan(1/(d*e)^{1/2}*e*x)*c}$

**maxima** [A] time = 2.24, size = 74, normalized size = 1.00

$$\frac{(cd^2 + ae^2)x}{2(de^3x^2 + d^2e^2)} + \frac{cx}{e^2} - \frac{(3cd^2 - ae^2) \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+a)/(e\*x^2+d)^2,x, algorithm="maxima")

[Out]  $1/2*(c*d^2 + a*e^2)*x/(d*e^3*x^2 + d^2*e^2) + c*x/e^2 - 1/2*(3*c*d^2 - a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d*e^2)$

**mupad** [B] time = 4.44, size = 68, normalized size = 0.92

$$\frac{cx}{e^2} + \frac{\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(ae^2 - 3cd^2)}{2d^{3/2}e^{5/2}} + \frac{x(cd^2 + ae^2)}{2d(e^3x^2 + de^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c\*x^4)/(d + e\*x^2)^2,x)

[Out]  $(c*x)/e^2 + (\operatorname{atan}((e^{1/2}*x)/d^{1/2})*(a*e^2 - 3*c*d^2))/(2*d^{3/2}*e^{5/2}) + (x*(a*e^2 + c*d^2))/(2*d*(d*e^2 + e^3*x^2))$

**sympy** [B] time = 0.51, size = 138, normalized size = 1.86

$$\frac{cx}{e^2} + \frac{x(ae^2 + cd^2)}{2d^2e^2 + 2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2 - 3cd^2) \log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2 - 3cd^2) \log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*4+a)/(e\*x\*\*2+d)\*\*2,x)

[Out]  $c*x/e^{**2} + x*(a*e^{**2} + c*d^{**2})/(2*d^{**2}*e^{**2} + 2*d*e^{**3}*x^{**2}) - \operatorname{sqrt}(-1/(d^{**3}*e^{**5}))*(a*e^{**2} - 3*c*d^{**2})*\log(-d^{**2}*e^{**2}*\operatorname{sqrt}(-1/(d^{**3}*e^{**5})) + x)/4 + \operatorname{sqrt}(-1/(d^{**3}*e^{**5}))*(a*e^{**2} - 3*c*d^{**2})*\log(d^{**2}*e^{**2}*\operatorname{sqrt}(-1/(d^{**3}*e^{**5})) + x)/4$

$$3.107 \quad \int \frac{a+cx^4}{(d+ex^2)^3} dx$$

Optimal. Leaf size=93

$$\frac{x\left(\frac{3a}{d^2} - \frac{5c}{e^2}\right)}{8(d+ex^2)} + \frac{x\left(a + \frac{cd^2}{e^2}\right)}{4d(d+ex^2)^2} + \frac{3(ae^2 + cd^2)\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

**Rubi [A]** time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {1158, 385, 205}

$$\frac{x\left(\frac{3a}{d^2} - \frac{5c}{e^2}\right)}{8(d+ex^2)} + \frac{x\left(a + \frac{cd^2}{e^2}\right)}{4d(d+ex^2)^2} + \frac{3(ae^2 + cd^2)\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^4)/(d + e\*x^2)^3,x]

[Out] ((a + (c\*d^2)/e^2)\*x)/(4\*d\*(d + e\*x^2)^2) + (((3\*a)/d^2 - (5\*c)/e^2)\*x)/(8\*(d + e\*x^2)) + (3\*(c\*d^2 + a\*e^2)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(8\*d^(5/2)\*e^(5/2))

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[(b\*c - a\*d)\*x\*(a + b\*x^n)^(p+1)/(a\*b\*n\*(p+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1)+1))/(a\*b\*n\*(p+1)), Int[(a + b\*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 1158

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] := With[{Qx = PolynomialQuotient[(a + c\*x^4)^p, d + e\*x^2, x], R = Coeff[PolynomialRemainder[(a + c\*x^4)^p, d + e\*x^2, x], x, 0]}, -Simp[(R\*x\*(d + e\*x^2)^(q+1))/(2\*d\*(q+1)), x] + Dist[1/(2\*d\*(q+1)), Int[(d + e\*x^2)^(q+1)\*ExpandToSum[2\*d\*(q+1)\*Qx + R\*(2\*q+3), x], x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rubi steps

$$\begin{aligned} \int \frac{a + cx^4}{(d + ex^2)^3} dx &= \frac{\left(a + \frac{cd^2}{e^2}\right)x}{4d(d + ex^2)^2} - \frac{\int \frac{-3a + \frac{cd^2}{e^2} - \frac{4cdx^2}{e}}{(d+ex^2)^2} dx}{4d} \\ &= \frac{\left(a + \frac{cd^2}{e^2}\right)x}{4d(d + ex^2)^2} + \frac{\left(\frac{3a}{d^2} - \frac{5c}{e^2}\right)x}{8(d + ex^2)} + \frac{1}{8} \left(3\left(\frac{a}{d^2} + \frac{c}{e^2}\right)\right) \int \frac{1}{d + ex^2} dx \\ &= \frac{\left(a + \frac{cd^2}{e^2}\right)x}{4d(d + ex^2)^2} + \frac{\left(\frac{3a}{d^2} - \frac{5c}{e^2}\right)x}{8(d + ex^2)} + \frac{3(cd^2 + ae^2) \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}} \end{aligned}$$

**Mathematica [A]** time = 0.06, size = 92, normalized size = 0.99

$$\frac{ae^2x(5d + 3ex^2) - cd^2x(3d + 5ex^2)}{8d^2e^2(d + ex^2)^2} + \frac{3(ae^2 + cd^2) \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c\*x^4)/(d + e\*x^2)^3,x]

[Out] (a\*e^2\*x\*(5\*d + 3\*e\*x^2) - c\*d^2\*x\*(3\*d + 5\*e\*x^2))/(8\*d^2\*e^2\*(d + e\*x^2)^2) + (3\*(c\*d^2 + a\*e^2)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(8\*d^(5/2)\*e^(5/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + cx^4}{(d + ex^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + c\*x^4)/(d + e\*x^2)^3,x]

[Out] IntegrateAlgebraic[(a + c\*x^4)/(d + e\*x^2)^3, x]

**fricas [A]** time = 0.89, size = 306, normalized size = 3.29

$$\left[ \frac{2(5cd^3e^2 - 3ade^4)x^3 + 3(cd^4 + ae^4) + (cd^2e^2 + ae^4)x^4 + 2(cd^3e + ade^3)x^2 \sqrt{-de} \log\left(\frac{x^2 - 2\sqrt{-de}x - d}{x^2 + d}\right) + 2(3cd^4e - 5ad^2e^3)x}{16(d^3e^5x^4 + 2d^4e^4x^2 + d^5e^3)}, \frac{(5cd^3e^2 - 3ade^4)x^3 - 3(cd^4 + ad^2e^2 + (cd^2e^2 + ae^4)x^4 + 2(cd^3e + ade^3)x^2) \sqrt{de} \arctan\left(\frac{\sqrt{de}x}{d}\right) + (3cd^4e - 5ad^2e^3)x}{8(d^3e^5x^4 + 2d^4e^4x^2 + d^5e^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+a)/(e\*x^2+d)^3,x, algorithm="fricas")

[Out] [-1/16\*(2\*(5\*c\*d^3\*e^2 - 3\*a\*d\*e^4)\*x^3 + 3\*(c\*d^4 + a\*d^2\*e^2 + (c\*d^2\*e^2 + a\*e^4)\*x^4 + 2\*(c\*d^3\*e + a\*d\*e^3)\*x^2)\*sqrt(-d\*e)\*log((e\*x^2 - 2\*sqrt(-d\*e)\*x - d)/(e\*x^2 + d)) + 2\*(3\*c\*d^4\*e - 5\*a\*d^2\*e^3)\*x)/(d^3\*e^5\*x^4 + 2\*d^4\*e^4\*x^2 + d^5\*e^3), -1/8\*((5\*c\*d^3\*e^2 - 3\*a\*d\*e^4)\*x^3 - 3\*(c\*d^4 + a\*d^2\*e^2 + (c\*d^2\*e^2 + a\*e^4)\*x^4 + 2\*(c\*d^3\*e + a\*d\*e^3)\*x^2)\*sqrt(d\*e)\*arctan(sqrt(d\*e)\*x/d) + (3\*c\*d^4\*e - 5\*a\*d^2\*e^3)\*x)/(d^3\*e^5\*x^4 + 2\*d^4\*e^4\*x^2 + d^5\*e^3)]

**giac [A]** time = 0.16, size = 77, normalized size = 0.83

$$\frac{3(cd^2 + ae^2) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right) e^{\left(-\frac{5}{2}\right)}}{8d^{\frac{5}{2}}} - \frac{(5cd^2x^3e + 3cd^3x - 3ax^3e^3 - 5adxe^2)e^{(-2)}}{8(x^2e + d)^2d^2}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+a)/(e\*x^2+d)^3,x, algorithm="giac")

[Out] 3/8\*(c\*d^2 + a\*e^2)\*arctan(x\*e^(1/2)/sqrt(d))\*e^(-5/2)/d^(5/2) - 1/8\*(5\*c\*d^2\*x^3\*e + 3\*c\*d^3\*x - 3\*a\*x^3\*e^3 - 5\*a\*d\*x\*e^2)\*e^(-2)/((x^2\*e + d)^2\*d^2)

**maple** [A] time = 0.01, size = 99, normalized size = 1.06

$$\frac{3a \arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de} d^2} + \frac{3c \arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de} e^2} + \frac{\frac{(3ae^2-5cd^2)x^3}{8d^2e} + \frac{(5ae^2-3cd^2)x}{8de^2}}{(ex^2+d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^4+a)/(e\*x^2+d)^3,x)

[Out] (1/8\*(3\*a\*e^2-5\*c\*d^2)/d^2/e\*x^3+1/8\*(5\*a\*e^2-3\*c\*d^2)/d/e^2\*x)/(e\*x^2+d)^2 +3/8/d^2/(d\*e)^(1/2)\*arctan(1/(d\*e)^(1/2)\*e\*x)\*a+3/8/e^2/(d\*e)^(1/2)\*arctan(1/(d\*e)^(1/2)\*e\*x)\*c

**maxima** [A] time = 2.56, size = 102, normalized size = 1.10

$$-\frac{(5cd^2e - 3ae^3)x^3 + (3cd^3 - 5ade^2)x}{8(d^2e^4x^4 + 2d^3e^3x^2 + d^4e^2)} + \frac{3(cd^2 + ae^2) \arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de} d^2 e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+a)/(e\*x^2+d)^3,x, algorithm="maxima")

[Out] -1/8\*((5\*c\*d^2\*e - 3\*a\*e^3)\*x^3 + (3\*c\*d^3 - 5\*a\*d\*e^2)\*x)/(d^2\*e^4\*x^4 + 2\*d^3\*e^3\*x^2 + d^4\*e^2) + 3/8\*(c\*d^2 + a\*e^2)\*arctan(e\*x/sqrt(d\*e))/(sqrt(d\*e)\*d^2\*e^2)

**mupad** [B] time = 4.48, size = 97, normalized size = 1.04

$$\frac{\frac{x^3(3ae^2-5cd^2)}{8d^2e} + \frac{x(5ae^2-3cd^2)}{8de^2}}{d^2 + 2dex^2 + e^2x^4} + \frac{3 \operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) (cd^2 + ae^2)}{8d^{5/2}e^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c\*x^4)/(d + e\*x^2)^3,x)

[Out] ((x^3\*(3\*a\*e^2 - 5\*c\*d^2))/(8\*d^2\*e) + (x\*(5\*a\*e^2 - 3\*c\*d^2))/(8\*d\*e^2))/(d^2 + e^2\*x^4 + 2\*d\*e\*x^2) + (3\*atan((e^(1/2)\*x)/d^(1/2))\*(a\*e^2 + c\*d^2))/(8\*d^(5/2)\*e^(5/2))

**sympy** [B] time = 0.75, size = 219, normalized size = 2.35

$$-\frac{3\sqrt{-\frac{1}{d^5e^5}}(ae^2 + cd^2) \log\left(-\frac{3d^3e^2\sqrt{-\frac{1}{d^5e^5}}(ae^2 + cd^2)}{3ae^2 + 3cd^2} + x\right)}{16} + \frac{3\sqrt{-\frac{1}{d^5e^5}}(ae^2 + cd^2) \log\left(\frac{3d^3e^2\sqrt{-\frac{1}{d^5e^5}}(ae^2 + cd^2)}{3ae^2 + 3cd^2} + x\right)}{16} + \frac{x^3(3ae^3 - 5cd^2e) + x(5ade^2 - 3cd^3)}{8d^4e^2 + 16d^3e^3x^2 + 8d^2e^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*4+a)/(e\*x\*\*2+d)\*\*3,x)

[Out] -3\*sqrt(-1/(d\*\*5\*e\*\*5))\*(a\*e\*\*2 + c\*d\*\*2)\*log(-3\*d\*\*3\*e\*\*2\*sqrt(-1/(d\*\*5\*e\*\*5))\*(a\*e\*\*2 + c\*d\*\*2)/(3\*a\*e\*\*2 + 3\*c\*d\*\*2) + x)/16 + 3\*sqrt(-1/(d\*\*5\*e\*\*5))\*(a\*e\*\*2 + c\*d\*\*2)\*log(3\*d\*\*3\*e\*\*2\*sqrt(-1/(d\*\*5\*e\*\*5))\*(a\*e\*\*2 + c\*d\*\*2)/(3\*a\*e\*\*2 + 3\*c\*d\*\*2) + x)/16 + (x\*\*3\*(3\*a\*e\*\*3 - 5\*c\*d\*\*2\*e) + x\*(5\*a\*d\*e\*\*2 - 3\*c\*d\*\*3))/(8\*d\*\*4\*e\*\*2 + 16\*d\*\*3\*e\*\*3\*x\*\*2 + 8\*d\*\*2\*e\*\*4\*x\*\*4)

$$3.108 \quad \int \frac{a+cx^4}{(d+ex^2)^4} dx$$

**Optimal.** Leaf size=123

$$\frac{x \left( \frac{5a}{d^2} + \frac{c}{e^2} \right)}{16d(d+ex^2)} + \frac{x \left( \frac{5a}{d^2} - \frac{7c}{e^2} \right)}{24(d+ex^2)^2} + \frac{x \left( a + \frac{cd^2}{e^2} \right)}{6d(d+ex^2)^3} + \frac{(5ae^2 + cd^2) \tan^{-1} \left( \frac{\sqrt{ex}}{\sqrt{d}} \right)}{16d^{7/2}e^{5/2}}$$

**Rubi [A]** time = 0.11, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$ , Rules used = {1158, 385, 199, 205}

$$\frac{x \left( \frac{5a}{d^2} + \frac{c}{e^2} \right)}{16d(d+ex^2)} + \frac{x \left( \frac{5a}{d^2} - \frac{7c}{e^2} \right)}{24(d+ex^2)^2} + \frac{x \left( a + \frac{cd^2}{e^2} \right)}{6d(d+ex^2)^3} + \frac{(5ae^2 + cd^2) \tan^{-1} \left( \frac{\sqrt{ex}}{\sqrt{d}} \right)}{16d^{7/2}e^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^4)/(d + e\*x^2)^4,x]

[Out] ((a + (c\*d^2)/e^2)\*x)/(6\*d\*(d + e\*x^2)^3) + (((5\*a)/d^2 - (7\*c)/e^2)\*x)/(24\*(d + e\*x^2)^2) + (((5\*a)/d^2 + c/e^2)\*x)/(16\*d\*(d + e\*x^2)) + ((c\*d^2 + 5\*a\*e^2)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(16\*d^(7/2)\*e^(5/2))

Rule 199

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2\*p] || (n == 2 && IntegerQ[4\*p]) || (n == 2 && IntegerQ[3\*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 1158

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] := With[{Qx = PolynomialQuotient[(a + c\*x^4)^p, d + e\*x^2, x], R = Coeff[PolynomialRemainder[(a + c\*x^4)^p, d + e\*x^2, x], x, 0]}, -Simp[(R\*x\*(d + e\*x^2)^(q + 1))/(2\*d\*(q + 1)), x] + Dist[1/(2\*d\*(q + 1)), Int[(d + e\*x^2)^(q + 1)\*ExpandToSum[2\*d\*(q + 1)\*Qx + R\*(2\*q + 3), x], x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rubi steps



**giac** [A] time = 0.15, size = 100, normalized size = 0.81

$$\frac{(cd^2 + 5ae^2) \arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{5}{2}\right)}}{16d^{\frac{7}{2}}} + \frac{(3cd^2x^5e^2 - 8cd^3x^3e + 15ax^5e^4 - 3cd^4x + 40adx^3e^3 + 33ad^2xe^2)e^{(-2)}}{48(x^2e + d)^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+a)/(e\*x^2+d)^4,x, algorithm="giac")

[Out] 1/16\*(c\*d^2 + 5\*a\*e^2)\*arctan(x\*e^(1/2)/sqrt(d))\*e^(-5/2)/d^(7/2) + 1/48\*(3\*c\*d^2\*x^5\*e^2 - 8\*c\*d^3\*x^3\*e + 15\*a\*x^5\*e^4 - 3\*c\*d^4\*x + 40\*a\*d\*x^3\*e^3 + 33\*a\*d^2\*x\*e^2)\*e^(-2)/((x^2\*e + d)^3\*d^3)

**maple** [A] time = 0.01, size = 122, normalized size = 0.99

$$\frac{5a \arctan\left(\frac{ex}{\sqrt{de}}\right)}{16\sqrt{de} d^3} + \frac{c \arctan\left(\frac{ex}{\sqrt{de}}\right)}{16\sqrt{de} d e^2} + \frac{(5ae^2+cd^2)x^5}{16d^3} + \frac{(5ae^2-cd^2)x^3}{6d^2e} + \frac{(11ae^2-cd^2)x}{16de^2} + \frac{(ex^2+d)^3}{(ex^2+d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^4+a)/(e\*x^2+d)^4,x)

[Out] (1/16\*(5\*a\*e^2+c\*d^2)/d^3\*x^5+1/6\*(5\*a\*e^2-c\*d^2)/d^2/e\*x^3+1/16\*(11\*a\*e^2-c\*d^2)/d/e^2\*x)/(e\*x^2+d)^3+5/16/d^3/(d\*e)^(1/2)\*arctan(1/(d\*e)^(1/2)\*e\*x)\*a+1/16/d/e^2/(d\*e)^(1/2)\*arctan(1/(d\*e)^(1/2)\*e\*x)\*c

**maxima** [A] time = 2.36, size = 137, normalized size = 1.11

$$\frac{3(cd^2e^2 + 5ae^4)x^5 - 8(cd^3e - 5ade^3)x^3 - 3(cd^4 - 11ad^2e^2)x}{48(d^3e^5x^6 + 3d^4e^4x^4 + 3d^5e^3x^2 + d^6e^2)} + \frac{(cd^2 + 5ae^2) \arctan\left(\frac{ex}{\sqrt{de}}\right)}{16\sqrt{de} d^3 e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+a)/(e\*x^2+d)^4,x, algorithm="maxima")

[Out] 1/48\*(3\*(c\*d^2\*e^2 + 5\*a\*e^4)\*x^5 - 8\*(c\*d^3\*e - 5\*a\*d\*e^3)\*x^3 - 3\*(c\*d^4 - 11\*a\*d^2\*e^2)\*x)/(d^3\*e^5\*x^6 + 3\*d^4\*e^4\*x^4 + 3\*d^5\*e^3\*x^2 + d^6\*e^2) + 1/16\*(c\*d^2 + 5\*a\*e^2)\*arctan(e\*x/sqrt(d\*e))/(sqrt(d\*e)\*d^3\*e^2)

**mupad** [B] time = 4.48, size = 129, normalized size = 1.05

$$\frac{\frac{x^5(cd^2+5ae^2)}{16d^3} + \frac{x^3(5ae^2-cd^2)}{6d^2e} + \frac{x(11ae^2-cd^2)}{16de^2}}{d^3 + 3d^2ex^2 + 3de^2x^4 + e^3x^6} + \frac{\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(cd^2 + 5ae^2)}{16d^{7/2}e^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c\*x^4)/(d + e\*x^2)^4,x)

[Out] ((x^5\*(5\*a\*e^2 + c\*d^2))/(16\*d^3) + (x^3\*(5\*a\*e^2 - c\*d^2))/(6\*d^2\*e) + (x\*(11\*a\*e^2 - c\*d^2))/(16\*d\*e^2))/(d^3 + e^3\*x^6 + 3\*d^2\*e\*x^2 + 3\*d\*e^2\*x^4) + (atan((e^(1/2)\*x)/d^(1/2))\*(5\*a\*e^2 + c\*d^2))/(16\*d^(7/2)\*e^(5/2))

**sympy** [A] time = 0.95, size = 204, normalized size = 1.66

$$\frac{\sqrt{-\frac{1}{d^7e^5}}(5ae^2 + cd^2) \log\left(-d^4e^2\sqrt{-\frac{1}{d^7e^5}} + x\right)}{32} + \frac{\sqrt{-\frac{1}{d^7e^5}}(5ae^2 + cd^2) \log\left(d^4e^2\sqrt{-\frac{1}{d^7e^5}} + x\right)}{32} + \frac{x^5(15ae^4 + 3cd^2e^2) + x^3(40ade^3 - 8cd^3e) + x(33ad^2e^2 - 3cd^4)}{48d^6e^2 + 144d^5e^3x^2 + 144d^4e^4x^4 + 48d^3e^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*4+a)/(e\*x\*\*2+d)\*\*4,x)

[Out]  $-\sqrt{-1/(d^7e^5)}(5ae^2 + cd^2)\log(-d^4e^2\sqrt{-1/(d^7e^5)} + x)/32 + \sqrt{-1/(d^7e^5)}(5ae^2 + cd^2)\log(d^4e^2\sqrt{-1/(d^7e^5)} + x)/32 + (x^5(15ae^4 + 3cd^2e^2) + x^3(40ad e^3 - 8cd^3e) + x(33ad^2e^2 - 3cd^4))/(48d^6e^2 + 144d^5e^3x^2 + 144d^4e^4x^4 + 48d^3e^5x^6)$

$$3.109 \quad \int (d + ex^2)^3 (a + cx^4)^2 dx$$

**Optimal.** Leaf size=133

$$a^2d^3x + a^2d^2ex^3 + \frac{1}{11}cex^{11}(2ae^2 + 3cd^2) + \frac{1}{9}cdx^9(6ae^2 + cd^2) + \frac{1}{7}aex^7(ae^2 + 6cd^2) + \frac{1}{5}adx^5(3ae^2 + 2cd^2) + \frac{3}{13}c^2de^2x^{13} + \frac{1}{15}c^2e^3x^{15}$$

**Rubi [A]** time = 0.11, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {1154}

$$a^2d^2ex^3 + a^2d^3x + \frac{1}{11}cex^{11}(2ae^2 + 3cd^2) + \frac{1}{9}cdx^9(6ae^2 + cd^2) + \frac{1}{7}aex^7(ae^2 + 6cd^2) + \frac{1}{5}adx^5(3ae^2 + 2cd^2) + \frac{3}{13}c^2de^2x^{13} + \frac{1}{15}c^2e^3x^{15}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^3\*(a + c\*x^4)^2,x]

[Out] a^2\*d^3\*x + a^2\*d^2\*e\*x^3 + (a\*d\*(2\*c\*d^2 + 3\*a\*e^2)\*x^5)/5 + (a\*e\*(6\*c\*d^2 + a\*e^2)\*x^7)/7 + (c\*d\*(c\*d^2 + 6\*a\*e^2)\*x^9)/9 + (c\*e\*(3\*c\*d^2 + 2\*a\*e^2)\*x^11)/11 + (3\*c^2\*d\*e^2\*x^13)/13 + (c^2\*e^3\*x^15)/15

**Rule 1154**

Int[((d\_) + (e\_)\*(x\_)^2)^(q\_)\*((a\_) + (c\_)\*(x\_)^4)^(p\_), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^2)^q\*(a + c\*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

**Rubi steps**

$$\begin{aligned} \int (d + ex^2)^3 (a + cx^4)^2 dx &= \int (a^2d^3 + 3a^2d^2ex^2 + ad(2cd^2 + 3ae^2)x^4 + ae(6cd^2 + ae^2)x^6 + cd(cd^2 + 6ae^2)x^8) dx \\ &= a^2d^3x + a^2d^2ex^3 + \frac{1}{5}ad(2cd^2 + 3ae^2)x^5 + \frac{1}{7}ae(6cd^2 + ae^2)x^7 + \frac{1}{9}cd(cd^2 + 6ae^2)x^9 \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 133, normalized size = 1.00

$$a^2d^3x + a^2d^2ex^3 + \frac{1}{11}cex^{11}(2ae^2 + 3cd^2) + \frac{1}{9}cdx^9(6ae^2 + cd^2) + \frac{1}{7}aex^7(ae^2 + 6cd^2) + \frac{1}{5}adx^5(3ae^2 + 2cd^2) + \frac{3}{13}c^2de^2x^{13} + \frac{1}{15}c^2e^3x^{15}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^3\*(a + c\*x^4)^2,x]

[Out] a^2\*d^3\*x + a^2\*d^2\*e\*x^3 + (a\*d\*(2\*c\*d^2 + 3\*a\*e^2)\*x^5)/5 + (a\*e\*(6\*c\*d^2 + a\*e^2)\*x^7)/7 + (c\*d\*(c\*d^2 + 6\*a\*e^2)\*x^9)/9 + (c\*e\*(3\*c\*d^2 + 2\*a\*e^2)\*x^11)/11 + (3\*c^2\*d\*e^2\*x^13)/13 + (c^2\*e^3\*x^15)/15

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex^2)^3 (a + cx^4)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)^3\*(a + c\*x^4)^2,x]

[Out] IntegrateAlgebraic[(d + e\*x^2)^3\*(a + c\*x^4)^2, x]

**fricas [A]** time = 0.79, size = 131, normalized size = 0.98

$$\frac{1}{15}x^{15}e^3c^2 + \frac{3}{13}x^{13}e^2dc^2 + \frac{3}{11}x^{11}ed^2c^2 + \frac{2}{11}x^{11}e^3ca + \frac{1}{9}x^9d^3c^2 + \frac{2}{3}x^9e^2dca + \frac{6}{7}x^7ed^2ca + \frac{1}{7}x^7e^3a^2 + \frac{2}{5}x^5d^3ca + \frac{3}{5}x^5e^2da^2 + x^3ed^2a^2 + xd^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^3\*(c\*x^4+a)^2,x, algorithm="fricas")

[Out]  $1/15*x^{15}*e^3*c^2 + 3/13*x^{13}*e^2*d*c^2 + 3/11*x^{11}*e*d^2*c^2 + 2/11*x^{11}*e^3*c*a + 1/9*x^9*d^3*c^2 + 2/3*x^9*e^2*d*c*a + 6/7*x^7*e*d^2*c*a + 1/7*x^7*e^3*a^2 + 2/5*x^5*d^3*c*a + 3/5*x^5*e^2*d*a^2 + x^3*e*d^2*a^2 + x*d^3*a^2$

**giac** [A] time = 0.16, size = 128, normalized size = 0.96

$$\frac{1}{15}c^2x^{15}e^3 + \frac{3}{13}c^2dx^{13}e^2 + \frac{3}{11}c^2d^2x^{11}e + \frac{1}{9}c^2d^3x^9 + \frac{2}{11}acx^{11}e^3 + \frac{2}{3}acdx^9e^2 + \frac{6}{7}acd^2x^7e + \frac{2}{5}acd^3x^5 + \frac{1}{7}a^2x^7e^3 + \frac{3}{5}a^2dx^5e^2 + a^2d^2x^3e + a^2d^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^3\*(c\*x^4+a)^2,x, algorithm="giac")

[Out]  $1/15*c^2*x^{15}*e^3 + 3/13*c^2*d*x^{13}*e^2 + 3/11*c^2*d^2*x^{11}*e + 1/9*c^2*d^3*x^9 + 2/11*a*c*x^{11}*e^3 + 2/3*a*c*d*x^9*e^2 + 6/7*a*c*d^2*x^7*e + 2/5*a*c*d^3*x^5 + 1/7*a^2*x^7*e^3 + 3/5*a^2*d*x^5*e^2 + a^2*d^2*x^3*e + a^2*d^3*x$

**maple** [A] time = 0.00, size = 130, normalized size = 0.98

$$\frac{c^2e^3x^{15}}{15} + \frac{3c^2de^2x^{13}}{13} + \frac{(2e^3ac + 3d^2e^2c^2)x^{11}}{11} + \frac{(6acd^2e^2 + c^2d^3)x^9}{9} + a^2d^2ex^3 + \frac{(e^3a^2 + 6d^2eac)x^7}{7} + a^2d^3x + \frac{(3de^2a^2 + 2d^3ac)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^3\*(c\*x^4+a)^2,x)

[Out]  $1/15*c^2*e^3*x^{15} + 3/13*c^2*d*e^2*x^{13} + 1/11*(2*a*c*e^3 + 3*c^2*d^2*e)*x^{11} + 1/9*(6*a*c*d*e^2 + c^2*d^3)*x^9 + 1/7*(a^2*e^3 + 6*a*c*d^2*e)*x^7 + 1/5*(3*a^2*d*e^2 + 2*a*c*d^3)*x^5 + a^2*d^2*e*x^3 + a^2*d^3*x$

**maxima** [A] time = 1.07, size = 129, normalized size = 0.97

$$\frac{1}{15}c^2e^3x^{15} + \frac{3}{13}c^2de^2x^{13} + \frac{1}{11}(3c^2d^2e + 2ace^3)x^{11} + \frac{1}{9}(c^2d^3 + 6acde^2)x^9 + a^2d^2ex^3 + \frac{1}{7}(6acd^2e + a^2e^3)x^7 + a^2d^3x + \frac{1}{5}(2acd^3 + 3a^2de^2)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^3\*(c\*x^4+a)^2,x, algorithm="maxima")

[Out]  $1/15*c^2*e^3*x^{15} + 3/13*c^2*d*e^2*x^{13} + 1/11*(3*c^2*d^2*e + 2*a*c*e^3)*x^{11} + 1/9*(c^2*d^3 + 6*a*c*d*e^2)*x^9 + a^2*d^2*e*x^3 + 1/7*(6*a*c*d^2*e + a^2*e^3)*x^7 + a^2*d^3*x + 1/5*(2*a*c*d^3 + 3*a^2*d*e^2)*x^5$

**mupad** [B] time = 0.06, size = 127, normalized size = 0.95

$$x^5 \left( \frac{3a^2de^2}{5} + \frac{2cad^3}{5} \right) + x^7 \left( \frac{a^2e^3}{7} + \frac{6cad^2e}{7} \right) + x^9 \left( \frac{c^2d^3}{9} + \frac{2acd^2e}{3} \right) + x^{11} \left( \frac{3c^2d^2e}{11} + \frac{2ace^3}{11} \right) + a^2d^3x + \frac{c^2e^3x^{15}}{15} + a^2d^2ex^3 + \frac{3c^2de^2x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c\*x^4)^2\*(d + e\*x^2)^3,x)

[Out]  $x^5*((3*a^2*d*e^2)/5 + (2*a*c*d^3)/5) + x^7*((a^2*e^3)/7 + (6*a*c*d^2*e)/7) + x^9*((c^2*d^3)/9 + (2*a*c*d*e^2)/3) + x^{11}*((3*c^2*d^2*e)/11 + (2*a*c*e^3)/11) + a^2*d^3*x + (c^2*e^3*x^{15})/15 + a^2*d^2*e*x^3 + (3*c^2*d*e^2*x^{13})/13$

**sympy** [A] time = 0.09, size = 144, normalized size = 1.08

$$a^2d^3x + a^2d^2ex^3 + \frac{3c^2de^2x^{13}}{13} + \frac{c^2e^3x^{15}}{15} + x^{11} \left( \frac{2ace^3}{11} + \frac{3c^2d^2e}{11} \right) + x^9 \left( \frac{2acde^2}{3} + \frac{c^2d^3}{9} \right) + x^7 \left( \frac{a^2e^3}{7} + \frac{6acd^2e}{7} \right) + x^5 \left( \frac{3a^2de^2}{5} + \frac{2acd^3}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(c*x**4+a)**2,x)
```

```
[Out] a**2*d**3*x + a**2*d**2*e*x**3 + 3*c**2*d*e**2*x**13/13 + c**2*e**3*x**15/15 + x**11*(2*a*c*e**3/11 + 3*c**2*d**2*e/11) + x**9*(2*a*c*d*e**2/3 + c**2*d**3/9) + x**7*(a**2*e**3/7 + 6*a*c*d**2*e/7) + x**5*(3*a**2*d*e**2/5 + 2*a*c*d**3/5)
```



$$3.110 \quad \int (d + ex^2)^2 (a + cx^4)^2 dx$$

**Optimal.** Leaf size=97

$$a^2d^2x + \frac{2}{3}a^2dex^3 + \frac{1}{9}cx^9(2ae^2 + cd^2) + \frac{1}{5}ax^5(ae^2 + 2cd^2) + \frac{4}{7}acdex^7 + \frac{2}{11}c^2dex^{11} + \frac{1}{13}c^2e^2x^{13}$$

**Rubi [A]** time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {1154}

$$a^2d^2x + \frac{2}{3}a^2dex^3 + \frac{1}{9}cx^9(2ae^2 + cd^2) + \frac{1}{5}ax^5(ae^2 + 2cd^2) + \frac{4}{7}acdex^7 + \frac{2}{11}c^2dex^{11} + \frac{1}{13}c^2e^2x^{13}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^2\*(a + c\*x^4)^2,x]

[Out] a^2\*d^2\*x + (2\*a^2\*d\*e\*x^3)/3 + (a\*(2\*c\*d^2 + a\*e^2)\*x^5)/5 + (4\*a\*c\*d\*e\*x^7)/7 + (c\*(c\*d^2 + 2\*a\*e^2)\*x^9)/9 + (2\*c^2\*d\*e\*x^11)/11 + (c^2\*e^2\*x^13)/13

**Rule 1154**

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^2)^q\*(a + c\*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

**Rubi steps**

$$\begin{aligned} \int (d + ex^2)^2 (a + cx^4)^2 dx &= \int (a^2d^2 + 2a^2dex^2 + a(2cd^2 + ae^2)x^4 + 4acdex^6 + c(cd^2 + 2ae^2)x^8 + 2c^2dex^{10} \\ &\quad + a^2d^2x + \frac{2}{3}a^2dex^3 + \frac{1}{5}a(2cd^2 + ae^2)x^5 + \frac{4}{7}acdex^7 + \frac{1}{9}c(cd^2 + 2ae^2)x^9 + \frac{2}{11}c^2dex^{11} + \frac{1}{13}c^2e^2x^{13}) dx \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 97, normalized size = 1.00

$$a^2d^2x + \frac{2}{3}a^2dex^3 + \frac{1}{9}cx^9(2ae^2 + cd^2) + \frac{1}{5}ax^5(ae^2 + 2cd^2) + \frac{4}{7}acdex^7 + \frac{2}{11}c^2dex^{11} + \frac{1}{13}c^2e^2x^{13}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^2\*(a + c\*x^4)^2,x]

[Out] a^2\*d^2\*x + (2\*a^2\*d\*e\*x^3)/3 + (a\*(2\*c\*d^2 + a\*e^2)\*x^5)/5 + (4\*a\*c\*d\*e\*x^7)/7 + (c\*(c\*d^2 + 2\*a\*e^2)\*x^9)/9 + (2\*c^2\*d\*e\*x^11)/11 + (c^2\*e^2\*x^13)/13

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex^2)^2 (a + cx^4)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)^2\*(a + c\*x^4)^2,x]

[Out] IntegrateAlgebraic[(d + e\*x^2)^2\*(a + c\*x^4)^2, x]

**fricas [A]** time = 0.75, size = 91, normalized size = 0.94

$$\frac{1}{13}x^{13}e^2c^2 + \frac{2}{11}x^{11}edc^2 + \frac{1}{9}x^9d^2c^2 + \frac{2}{9}x^9e^2ca + \frac{4}{7}x^7edca + \frac{2}{5}x^5d^2ca + \frac{1}{5}x^5e^2a^2 + \frac{2}{3}x^3eda^2 + xd^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(c\*x^4+a)^2,x, algorithm="fricas")

[Out] 1/13\*x^13\*e^2\*c^2 + 2/11\*x^11\*e\*d\*c^2 + 1/9\*x^9\*d^2\*c^2 + 2/9\*x^9\*e^2\*c\*a + 4/7\*x^7\*e\*d\*c\*a + 2/5\*x^5\*d^2\*c\*a + 1/5\*x^5\*e^2\*a^2 + 2/3\*x^3\*e\*d\*a^2 + x\*d^2\*a^2

**giac [A]** time = 0.15, size = 91, normalized size = 0.94

$$\frac{1}{13}c^2x^{13}e^2 + \frac{2}{11}c^2dx^{11}e + \frac{1}{9}c^2d^2x^9 + \frac{2}{9}acx^9e^2 + \frac{4}{7}acdx^7e + \frac{2}{5}acd^2x^5 + \frac{1}{5}a^2x^5e^2 + \frac{2}{3}a^2dx^3e + a^2d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(c\*x^4+a)^2,x, algorithm="giac")

[Out] 1/13\*c^2\*x^13\*e^2 + 2/11\*c^2\*d\*x^11\*e + 1/9\*c^2\*d^2\*x^9 + 2/9\*a\*c\*x^9\*e^2 + 4/7\*a\*c\*d\*x^7\*e + 2/5\*a\*c\*d^2\*x^5 + 1/5\*a^2\*x^5\*e^2 + 2/3\*a^2\*d\*x^3\*e + a^2\*d^2\*x

**maple [A]** time = 0.00, size = 90, normalized size = 0.93

$$\frac{c^2e^2x^{13}}{13} + \frac{2c^2dex^{11}}{11} + \frac{4acdex^7}{7} + \frac{(2e^2ac + c^2d^2)x^9}{9} + \frac{2a^2dex^3}{3} + a^2d^2x + \frac{(e^2a^2 + 2d^2ac)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^2\*(c\*x^4+a)^2,x)

[Out] 1/13\*c^2\*e^2\*x^13+2/11\*c^2\*d\*e\*x^11+1/9\*(2\*a\*c\*e^2+c^2\*d^2)\*x^9+4/7\*a\*c\*d\*e\*x^7+1/5\*(a^2\*e^2+2\*a\*c\*d^2)\*x^5+2/3\*a^2\*d\*e\*x^3+a^2\*d^2\*x

**maxima [A]** time = 1.03, size = 89, normalized size = 0.92

$$\frac{1}{13}c^2e^2x^{13} + \frac{2}{11}c^2dex^{11} + \frac{4}{7}acdex^7 + \frac{1}{9}(c^2d^2 + 2ace^2)x^9 + \frac{2}{3}a^2dex^3 + \frac{1}{5}(2acd^2 + a^2e^2)x^5 + a^2d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(c\*x^4+a)^2,x, algorithm="maxima")

[Out] 1/13\*c^2\*e^2\*x^13 + 2/11\*c^2\*d\*e\*x^11 + 4/7\*a\*c\*d\*e\*x^7 + 1/9\*(c^2\*d^2 + 2\*a\*c\*e^2)\*x^9 + 2/3\*a^2\*d\*e\*x^3 + 1/5\*(2\*a\*c\*d^2 + a^2\*e^2)\*x^5 + a^2\*d^2\*x

**mupad [B]** time = 0.05, size = 89, normalized size = 0.92

$$x^5 \left( \frac{a^2e^2}{5} + \frac{2ca^2d^2}{5} \right) + x^9 \left( \frac{c^2d^2}{9} + \frac{2ac^2e^2}{9} \right) + a^2d^2x + \frac{c^2e^2x^{13}}{13} + \frac{2a^2dex^3}{3} + \frac{2c^2dex^{11}}{11} + \frac{4acdex^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c\*x^4)^2\*(d + e\*x^2)^2,x)

[Out] x^5\*((a^2\*e^2)/5 + (2\*a\*c\*d^2)/5) + x^9\*((c^2\*d^2)/9 + (2\*a\*c\*e^2)/9) + a^2\*d^2\*x + (c^2\*e^2\*x^13)/13 + (2\*a^2\*d\*e\*x^3)/3 + (2\*c^2\*d\*e\*x^11)/11 + (4\*a\*c\*d\*e\*x^7)/7

**sympy [A]** time = 0.09, size = 104, normalized size = 1.07

$$a^2 d^2 x + \frac{2a^2 d e x^3}{3} + \frac{4a c d e x^7}{7} + \frac{2c^2 d e x^{11}}{11} + \frac{c^2 e^2 x^{13}}{13} + x^9 \left( \frac{2a c e^2}{9} + \frac{c^2 d^2}{9} \right) + x^5 \left( \frac{a^2 e^2}{5} + \frac{2a c d^2}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*2\*(c\*x\*\*4+a)\*\*2,x)

[Out] a\*\*2\*d\*\*2\*x + 2\*a\*\*2\*d\*e\*x\*\*3/3 + 4\*a\*c\*d\*e\*x\*\*7/7 + 2\*c\*\*2\*d\*e\*x\*\*11/11 + c\*\*2\*e\*\*2\*x\*\*13/13 + x\*\*9\*(2\*a\*c\*e\*\*2/9 + c\*\*2\*d\*\*2/9) + x\*\*5\*(a\*\*2\*e\*\*2/5 + 2\*a\*c\*d\*\*2/5)

$$3.111 \quad \int (d + ex^2)(a + cx^4)^2 dx$$

**Optimal.** Leaf size=60

$$a^2dx + \frac{1}{3}a^2ex^3 + \frac{2}{5}acdx^5 + \frac{2}{7}acex^7 + \frac{1}{9}c^2dx^9 + \frac{1}{11}c^2ex^{11}$$

**Rubi [A]** time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {1154}

$$a^2dx + \frac{1}{3}a^2ex^3 + \frac{2}{5}acdx^5 + \frac{2}{7}acex^7 + \frac{1}{9}c^2dx^9 + \frac{1}{11}c^2ex^{11}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)\*(a + c\*x^4)^2,x]

[Out] a^2\*d\*x + (a^2\*e\*x^3)/3 + (2\*a\*c\*d\*x^5)/5 + (2\*a\*c\*e\*x^7)/7 + (c^2\*d\*x^9)/9 + (c^2\*e\*x^11)/11

Rule 1154

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^2)^q\*(a + c\*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int (d + ex^2)(a + cx^4)^2 dx &= \int (a^2d + a^2ex^2 + 2acdx^4 + 2acex^6 + c^2dx^8 + c^2ex^{10}) dx \\ &= a^2dx + \frac{1}{3}a^2ex^3 + \frac{2}{5}acdx^5 + \frac{2}{7}acex^7 + \frac{1}{9}c^2dx^9 + \frac{1}{11}c^2ex^{11} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 60, normalized size = 1.00

$$a^2dx + \frac{1}{3}a^2ex^3 + \frac{2}{5}acdx^5 + \frac{2}{7}acex^7 + \frac{1}{9}c^2dx^9 + \frac{1}{11}c^2ex^{11}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)\*(a + c\*x^4)^2,x]

[Out] a^2\*d\*x + (a^2\*e\*x^3)/3 + (2\*a\*c\*d\*x^5)/5 + (2\*a\*c\*e\*x^7)/7 + (c^2\*d\*x^9)/9 + (c^2\*e\*x^11)/11

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex^2)(a + cx^4)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)\*(a + c\*x^4)^2,x]

[Out] IntegrateAlgebraic[(d + e\*x^2)\*(a + c\*x^4)^2, x]

**fricas [A]** time = 1.27, size = 50, normalized size = 0.83

$$\frac{1}{11}x^{11}ec^2 + \frac{1}{9}x^9dc^2 + \frac{2}{7}x^7eca + \frac{2}{5}x^5dca + \frac{1}{3}x^3ea^2 + xda^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(c\*x^4+a)^2,x, algorithm="fricas")

[Out]  $1/11*x^{11}*e*c^2 + 1/9*x^9*d*c^2 + 2/7*x^7*e*c*a + 2/5*x^5*d*c*a + 1/3*x^3*e*a^2 + x*d*a^2$

**giac** [A] time = 0.15, size = 53, normalized size = 0.88

$$\frac{1}{11}c^2x^{11}e + \frac{1}{9}c^2dx^9 + \frac{2}{7}acx^7e + \frac{2}{5}acdx^5 + \frac{1}{3}a^2x^3e + a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(c\*x^4+a)^2,x, algorithm="giac")

[Out]  $1/11*c^2*x^{11}*e + 1/9*c^2*d*x^9 + 2/7*a*c*x^7*e + 2/5*a*c*d*x^5 + 1/3*a^2*x^3*e + a^2*d*x$

**maple** [A] time = 0.00, size = 51, normalized size = 0.85

$$\frac{1}{11}c^2ex^{11} + \frac{1}{9}c^2dx^9 + \frac{2}{7}acex^7 + \frac{2}{5}acdx^5 + \frac{1}{3}a^2ex^3 + a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)\*(c\*x^4+a)^2,x)

[Out]  $a^2*d*x + 1/3*a^2*e*x^3 + 2/5*a*c*d*x^5 + 2/7*a*c*e*x^7 + 1/9*c^2*d*x^9 + 1/11*c^2*e*x^{11}$

**maxima** [A] time = 1.04, size = 50, normalized size = 0.83

$$\frac{1}{11}c^2ex^{11} + \frac{1}{9}c^2dx^9 + \frac{2}{7}acex^7 + \frac{2}{5}acdx^5 + \frac{1}{3}a^2ex^3 + a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(c\*x^4+a)^2,x, algorithm="maxima")

[Out]  $1/11*c^2*e*x^{11} + 1/9*c^2*d*x^9 + 2/7*a*c*e*x^7 + 2/5*a*c*d*x^5 + 1/3*a^2*e*x^3 + a^2*d*x$

**mupad** [B] time = 0.03, size = 50, normalized size = 0.83

$$\frac{ea^2x^3}{3} + da^2x + \frac{2eacx^7}{7} + \frac{2dacx^5}{5} + \frac{ec^2x^{11}}{11} + \frac{dc^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c\*x^4)^2\*(d + e\*x^2),x)

[Out]  $(a^2*e*x^3)/3 + (c^2*d*x^9)/9 + (c^2*e*x^{11})/11 + a^2*d*x + (2*a*c*d*x^5)/5 + (2*a*c*e*x^7)/7$

**sympy** [A] time = 0.08, size = 60, normalized size = 1.00

$$a^2dx + \frac{a^2ex^3}{3} + \frac{2acdx^5}{5} + \frac{2acex^7}{7} + \frac{c^2dx^9}{9} + \frac{c^2ex^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*(c\*x\*\*4+a)\*\*2,x)

[Out]  $a**2*d*x + a**2*e*x**3/3 + 2*a*c*d*x**5/5 + 2*a*c*e*x**7/7 + c**2*d*x**9/9 + c**2*e*x**11/11$

$$3.112 \quad \int (a + cx^4)^2 dx$$

**Optimal.** Leaf size=25

$$a^2x + \frac{2}{5}acx^5 + \frac{c^2x^9}{9}$$

**Rubi [A]** time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {194}

$$a^2x + \frac{2}{5}acx^5 + \frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^4)^2,x]

[Out] a^2\*x + (2\*a\*c\*x^5)/5 + (c^2\*x^9)/9

Rule 194

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (a + cx^4)^2 dx &= \int (a^2 + 2acx^4 + c^2x^8) dx \\ &= a^2x + \frac{2}{5}acx^5 + \frac{c^2x^9}{9} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 25, normalized size = 1.00

$$a^2x + \frac{2}{5}acx^5 + \frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c\*x^4)^2,x]

[Out] a^2\*x + (2\*a\*c\*x^5)/5 + (c^2\*x^9)/9

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a + cx^4)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + c\*x^4)^2,x]

[Out] IntegrateAlgebraic[(a + c\*x^4)^2, x]

**fricas [A]** time = 0.37, size = 21, normalized size = 0.84

$$\frac{1}{9}x^9c^2 + \frac{2}{5}x^5ca + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+a)^2,x, algorithm="fricas")

[Out] 1/9\*x^9\*c^2 + 2/5\*x^5\*c\*a + x\*a^2

**giac** [A] time = 0.17, size = 21, normalized size = 0.84

$$\frac{1}{9}c^2x^9 + \frac{2}{5}acx^5 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+a)^2,x, algorithm="giac")

[Out] 1/9\*c^2\*x^9 + 2/5\*a\*c\*x^5 + a^2\*x

**maple** [A] time = 0.00, size = 22, normalized size = 0.88

$$\frac{1}{9}c^2x^9 + \frac{2}{5}acx^5 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^4+a)^2,x)

[Out] a^2\*x+2/5\*a\*c\*x^5+1/9\*c^2\*x^9

**maxima** [A] time = 1.00, size = 21, normalized size = 0.84

$$\frac{1}{9}c^2x^9 + \frac{2}{5}acx^5 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+a)^2,x, algorithm="maxima")

[Out] 1/9\*c^2\*x^9 + 2/5\*a\*c\*x^5 + a^2\*x

**mupad** [B] time = 0.03, size = 21, normalized size = 0.84

$$a^2x + \frac{2acx^5}{5} + \frac{c^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c\*x^4)^2,x)

[Out] a^2\*x + (c^2\*x^9)/9 + (2\*a\*c\*x^5)/5

**sympy** [A] time = 0.07, size = 22, normalized size = 0.88

$$a^2x + \frac{2acx^5}{5} + \frac{c^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*4+a)\*\*2,x)

[Out] a\*\*2\*x + 2\*a\*c\*x\*\*5/5 + c\*\*2\*x\*\*9/9

$$3.113 \quad \int \frac{(a+cx^4)^2}{d+ex^2} dx$$

**Optimal.** Leaf size=108

$$\frac{(ae^2 + cd^2)^2 \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{d} e^{9/2}} - \frac{cdx(2ae^2 + cd^2)}{e^4} + \frac{cx^3(2ae^2 + cd^2)}{3e^3} - \frac{c^2 dx^5}{5e^2} + \frac{c^2 x^7}{7e}$$

**Rubi [A]** time = 0.08, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {1154, 205}

$$\frac{cx^3(2ae^2 + cd^2)}{3e^3} - \frac{cdx(2ae^2 + cd^2)}{e^4} + \frac{(ae^2 + cd^2)^2 \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{d} e^{9/2}} - \frac{c^2 dx^5}{5e^2} + \frac{c^2 x^7}{7e}$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^4)^2/(d + e\*x^2),x]

[Out] -((c\*d\*(c\*d^2 + 2\*a\*e^2)\*x)/e^4) + (c\*(c\*d^2 + 2\*a\*e^2)\*x^3)/(3\*e^3) - (c^2\*d\*x^5)/(5\*e^2) + (c^2\*x^7)/(7\*e) + ((c\*d^2 + a\*e^2)^2\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(Sqrt[d]\*e^(9/2))

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1154

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^2)^q\*(a + c\*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int \frac{(a+cx^4)^2}{d+ex^2} dx &= \int \left( -\frac{cd(cd^2 + 2ae^2)}{e^4} + \frac{c(cd^2 + 2ae^2)x^2}{e^3} - \frac{c^2 dx^4}{e^2} + \frac{c^2 x^6}{e} + \frac{c^2 d^4 + 2acd^2 e^2 + a^2 e^4}{e^4(d+ex^2)} \right) dx \\ &= -\frac{cd(cd^2 + 2ae^2)x}{e^4} + \frac{c(cd^2 + 2ae^2)x^3}{3e^3} - \frac{c^2 dx^5}{5e^2} + \frac{c^2 x^7}{7e} + \frac{(cd^2 + ae^2)^2 \int \frac{1}{d+ex^2} dx}{e^4} \\ &= -\frac{cd(cd^2 + 2ae^2)x}{e^4} + \frac{c(cd^2 + 2ae^2)x^3}{3e^3} - \frac{c^2 dx^5}{5e^2} + \frac{c^2 x^7}{7e} + \frac{(cd^2 + ae^2)^2 \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{d} e^{9/2}} \end{aligned}$$

**Mathematica [A]** time = 0.08, size = 97, normalized size = 0.90

$$\frac{(ae^2 + cd^2)^2 \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{d} e^{9/2}} + \frac{cx(70ae^2(ex^2 - 3d) + c(-105d^3 + 35d^2ex^2 - 21de^2x^4 + 15e^3x^6))}{105e^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c\*x^4)^2/(d + e\*x^2),x]



[Out]  $(c*x*(70*a*e^2*(-3*d + e*x^2) + c*(-105*d^3 + 35*d^2*e*x^2 - 21*d*e^2*x^4 + 15*e^3*x^6)))/(105*e^4) + ((c*d^2 + a*e^2)^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^{(9/2)})$

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + cx^4)^2}{d + ex^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + c\*x^4)^2/(d + e\*x^2), x]

[Out] IntegrateAlgebraic[(a + c\*x^4)^2/(d + e\*x^2), x]

**fricas [A]** time = 1.27, size = 268, normalized size = 2.48

$$\frac{30c^2de^4x^7 - 42c^2d^2e^3x^5 + 70(c^2d^4 + 2acde^2 + a^2e^4)\sqrt{de} \log\left(\frac{x^2 - 2\sqrt{de}x - d}{e^2 + d}\right) - 210(c^2d^4e + 2acde^2)x + 15c^2de^4x^7 - 21c^2d^2e^3x^5 + 35(c^2d^4 + 2acde^2)x^3 + 105(c^2d^4 + 2acde^2 + a^2e^4)\sqrt{de} \arctan\left(\frac{\sqrt{de}x}{d}\right) - 105(c^2d^4 + 2acde^2)x}{210d^5} + \frac{15c^2de^4x^7 - 21c^2d^2e^3x^5 + 35(c^2d^4 + 2acde^2)x^3 + 105(c^2d^4 + 2acde^2 + a^2e^4)\sqrt{de} \arctan\left(\frac{\sqrt{de}x}{d}\right) - 105(c^2d^4 + 2acde^2)x}{105de^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+a)^2/(e\*x^2+d), x, algorithm="fricas")

[Out]  $[1/210*(30*c^2*d*e^4*x^7 - 42*c^2*d^2*e^3*x^5 + 70*(c^2*d^4 + 2*a*c*d*e^2)*x^3 - 105*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 210*(c^2*d^4*e + 2*a*c*d^2*e^3)*x)/(d*e^5), 1/105*(15*c^2*d*e^4*x^7 - 21*c^2*d^2*e^3*x^5 + 35*(c^2*d^4 + 2*a*c*d*e^2)*x^3 + 105*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 105*(c^2*d^4*e + 2*a*c*d^2*e^3)*x)/(d*e^5)]$

**giac [A]** time = 0.16, size = 105, normalized size = 0.97

$$\frac{(c^2d^4 + 2acd^2e^2 + a^2e^4) \arctan\left(\frac{x\sqrt{d}}{\sqrt{de}}\right) e^{(-\frac{9}{2})}}{\sqrt{d}} + \frac{1}{105} (15c^2x^7e^6 - 21c^2dx^5e^5 + 35c^2d^2x^3e^4 - 105c^2d^3xe^3 + 70acx^3e^6 - 210acdxe^5)e^{(-7)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+a)^2/(e\*x^2+d), x, algorithm="giac")

[Out]  $(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*arctan(x*e^{(1/2)}/sqrt(d))*e^{(-9/2)}/sqrt(d) + 1/105*(15*c^2*x^7*e^6 - 21*c^2*d*x^5*e^5 + 35*c^2*d^2*x^3*e^4 - 105*c^2*d^3*x*e^3 + 70*a*c*x^3*e^6 - 210*a*c*d*x*e^5)*e^{(-7)}$

**maple [A]** time = 0.00, size = 136, normalized size = 1.26

$$\frac{c^2x^7}{7e} - \frac{c^2dx^5}{5e^2} + \frac{2acx^3}{3e} + \frac{c^2d^2x^3}{3e^3} + \frac{a^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} + \frac{2acd^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}e^2} + \frac{c^2d^4 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}e^4} - \frac{2acdx}{e^2} - \frac{c^2d^3x}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^4+a)^2/(e\*x^2+d), x)

[Out]  $1/7*c^2*x^7/e - 1/5*c^2*d*x^5/e^2 + 2/3*c/e*x^3*a + 1/3*c^2/e^3*x^3*d^2 - 2*c/e^2*d*a*x - c^2/e^4*d^3*x + 1/(d*e)^{(1/2)}*arctan(1/(d*e)^{(1/2)}*e*x)*a^2 + 2/e^2/(d*e)^{(1/2)}*arctan(1/(d*e)^{(1/2)}*e*x)*a*c*d^2 + 1/e^4/(d*e)^{(1/2)}*arctan(1/(d*e)^{(1/2)}*e*x)*c^2*d^4$

**maxima [A]** time = 2.45, size = 113, normalized size = 1.05

$$\frac{(c^2d^4 + 2acd^2e^2 + a^2e^4) \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}e^4} + \frac{15c^2e^3x^7 - 21c^2de^2x^5 + 35(c^2d^2e + 2ace^3)x^3 - 105(c^2d^3 + 2acde^2)x}{105e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+a)^2/(e\*x^2+d),x, algorithm="maxima")

[Out] (c^2\*d^4 + 2\*a\*c\*d^2\*e^2 + a^2\*e^4)\*arctan(e\*x/sqrt(d\*e))/(sqrt(d\*e)\*e^4) + 1/105\*(15\*c^2\*e^3\*x^7 - 21\*c^2\*d\*e^2\*x^5 + 35\*(c^2\*d^2\*e + 2\*a\*c\*e^3)\*x^3 - 105\*(c^2\*d^3 + 2\*a\*c\*d\*e^2)\*x)/e^4

**mupad [B]** time = 4.39, size = 141, normalized size = 1.31

$$x^3 \left( \frac{c^2 d^2}{3e^3} + \frac{2ac}{3e} \right) + \frac{c^2 x^7}{7e} - \frac{c^2 d x^5}{5e^2} + \frac{\operatorname{atan} \left( \frac{\sqrt{e} x (c d^2 + a e^2)^2}{\sqrt{d} (a^2 e^4 + 2 a c d^2 e^2 + c^2 d^4)} \right) (c d^2 + a e^2)^2}{\sqrt{d} e^{9/2}} - \frac{d x \left( \frac{c^2 d^2}{e^3} + \frac{2ac}{e} \right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c\*x^4)^2/(d + e\*x^2),x)

[Out] x^3\*((c^2\*d^2)/(3\*e^3) + (2\*a\*c)/(3\*e)) + (c^2\*x^7)/(7\*e) - (c^2\*d\*x^5)/(5\*e^2) + (atan((e^(1/2)\*x\*(a\*e^2 + c\*d^2)^2)/(d^(1/2)\*(a^2\*e^4 + c^2\*d^4 + 2\*a\*c\*d^2\*e^2)))\*(a\*e^2 + c\*d^2)^2)/(d^(1/2)\*e^(9/2)) - (d\*x\*((c^2\*d^2)/e^3 + (2\*a\*c)/e))/e

**sympy [B]** time = 0.50, size = 236, normalized size = 2.19

$$-\frac{c^2 d x^5}{5e^2} + \frac{c^2 x^7}{7e} + x^3 \left( \frac{2ac}{3e} + \frac{c^2 d^2}{3e^3} \right) + x \left( -\frac{2acd}{e^2} - \frac{c^2 d^3}{e^4} \right) - \frac{\sqrt{-\frac{1}{de^9}} (ae^2 + cd^2)^2 \log \left( -\frac{de^4 \sqrt{-\frac{1}{de^9}} (ae^2 + cd^2)^2}{a^2 e^4 + 2acd^2 e^2 + c^2 d^4} + x \right)}{2} + \frac{\sqrt{-\frac{1}{de^9}} (ae^2 + cd^2)^2 \log \left( \frac{de^4 \sqrt{-\frac{1}{de^9}} (ae^2 + cd^2)^2}{a^2 e^4 + 2acd^2 e^2 + c^2 d^4} + x \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*4+a)\*\*2/(e\*x\*\*2+d),x)

[Out] -c\*\*2\*d\*x\*\*5/(5\*e\*\*2) + c\*\*2\*x\*\*7/(7\*e) + x\*\*3\*(2\*a\*c/(3\*e) + c\*\*2\*d\*\*2/(3\*e\*\*3)) + x\*(-2\*a\*c\*d/e\*\*2 - c\*\*2\*d\*\*3/e\*\*4) - sqrt(-1/(d\*e\*\*9))\*(a\*e\*\*2 + c\*d\*\*2)\*\*2\*log(-d\*e\*\*4\*sqrt(-1/(d\*e\*\*9))\*(a\*e\*\*2 + c\*d\*\*2)\*\*2/(a\*\*2\*e\*\*4 + 2\*a\*c\*d\*\*2\*e\*\*2 + c\*\*2\*d\*\*4) + x)/2 + sqrt(-1/(d\*e\*\*9))\*(a\*e\*\*2 + c\*d\*\*2)\*\*2\*log(d\*e\*\*4\*sqrt(-1/(d\*e\*\*9))\*(a\*e\*\*2 + c\*d\*\*2)\*\*2/(a\*\*2\*e\*\*4 + 2\*a\*c\*d\*\*2\*e\*\*2 + c\*\*2\*d\*\*4) + x)/2

$$3.114 \quad \int \frac{(a+cx^4)^2}{(d+ex^2)^2} dx$$

**Optimal.** Leaf size=131

$$\frac{x(ae^2 + cd^2)^2}{2de^4(d + ex^2)} + \frac{cx(2ae^2 + 3cd^2)}{e^4} - \frac{(7cd^2 - ae^2)(ae^2 + cd^2) \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}} - \frac{2c^2dx^3}{3e^3} + \frac{c^2x^5}{5e^2}$$

**Rubi [A]** time = 0.19, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {1158, 1810, 205}

$$\frac{x(ae^2 + cd^2)^2}{2de^4(d + ex^2)} + \frac{cx(2ae^2 + 3cd^2)}{e^4} - \frac{(7cd^2 - ae^2)(ae^2 + cd^2) \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}} - \frac{2c^2dx^3}{3e^3} + \frac{c^2x^5}{5e^2}$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^4)^2/(d + e\*x^2)^2,x]

[Out] (c\*(3\*c\*d^2 + 2\*a\*e^2)\*x)/e^4 - (2\*c^2\*d\*x^3)/(3\*e^3) + (c^2\*x^5)/(5\*e^2) + ((c\*d^2 + a\*e^2)^2\*x)/(2\*d\*e^4\*(d + e\*x^2)) - ((7\*c\*d^2 - a\*e^2)\*(c\*d^2 + a\*e^2)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(2\*d^(3/2)\*e^(9/2))

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 1158

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] := With[{Qx = PolynomialQuotient[(a + c\*x^4)^p, d + e\*x^2, x], R = Coeff[PolynomialRemainder[(a + c\*x^4)^p, d + e\*x^2, x], x, 0]}, -Simp[(R\*x\*(d + e\*x^2)^(q + 1))/(2\*d\*(q + 1)), x] + Dist[1/(2\*d\*(q + 1)), Int[(d + e\*x^2)^(q + 1)\*ExpandToSum[2\*d\*(q + 1)\*Qx + R\*(2\*q + 3), x], x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

#### Rule 1810

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[ExpandIntegrand[Pq\*(a + b\*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

#### Rubi steps

$$\begin{aligned}
\int \frac{(a + cx^4)^2}{(d + ex^2)^2} dx &= \frac{(cd^2 + ae^2)^2 x}{2de^4 (d + ex^2)} - \frac{\int \frac{-a^2 + \frac{c^2 d^4}{e^4} + \frac{2acd^2}{e^2} - \frac{2cd(cd^2 + 2ae^2)x^2}{e^3} + \frac{2c^2 d^2 x^4}{e^2} - \frac{2c^2 dx^6}{e}}{d + ex^2} dx}{2d} \\
&= \frac{(cd^2 + ae^2)^2 x}{2de^4 (d + ex^2)} - \frac{\int \left( -\frac{2cd(3cd^2 + 2ae^2)}{e^4} + \frac{4c^2 d^2 x^2}{e^3} - \frac{2c^2 dx^4}{e^2} + \frac{7c^2 d^4 + 6acd^2 e^2 - a^2 e^4}{e^4 (d + ex^2)} \right) dx}{2d} \\
&= \frac{c(3cd^2 + 2ae^2)x}{e^4} - \frac{2c^2 dx^3}{3e^3} + \frac{c^2 x^5}{5e^2} + \frac{(cd^2 + ae^2)^2 x}{2de^4 (d + ex^2)} - \frac{((7cd^2 - ae^2)(cd^2 + ae^2)) \int \frac{1}{d + ex^2} dx}{2de^4} \\
&= \frac{c(3cd^2 + 2ae^2)x}{e^4} - \frac{2c^2 dx^3}{3e^3} + \frac{c^2 x^5}{5e^2} + \frac{(cd^2 + ae^2)^2 x}{2de^4 (d + ex^2)} - \frac{(7cd^2 - ae^2)(cd^2 + ae^2) \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 134, normalized size = 1.02

$$-\frac{(-a^2 e^4 + 6acd^2 e^2 + 7c^2 d^4) \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}} + \frac{x(ae^2 + cd^2)^2}{2de^4(d + ex^2)} + \frac{cx(2ae^2 + 3cd^2)}{e^4} - \frac{2c^2 dx^3}{3e^3} + \frac{c^2 x^5}{5e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c\*x^4)^2/(d + e\*x^2)^2,x]

[Out] (c\*(3\*c\*d^2 + 2\*a\*e^2)\*x)/e^4 - (2\*c^2\*d\*x^3)/(3\*e^3) + (c^2\*x^5)/(5\*e^2) + ((c\*d^2 + a\*e^2)^2\*x)/(2\*d\*e^4\*(d + e\*x^2)) - ((7\*c^2\*d^4 + 6\*a\*c\*d^2\*e^2 - a^2\*e^4)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(2\*d^(3/2)\*e^(9/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + cx^4)^2}{(d + ex^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + c\*x^4)^2/(d + e\*x^2)^2,x]

[Out] IntegrateAlgebraic[(a + c\*x^4)^2/(d + e\*x^2)^2, x]

**fricas [A]** time = 1.60, size = 394, normalized size = 3.01

$$\frac{12c^2d^4e^7 - 28c^2d^3e^5 + 20(7c^2d^4e^2 + 6acd^2e^2)^2 + 15(7c^2d^4 + 6acd^2 - a^2e^4)\sqrt{d}\log\left(\frac{e^2x^2 + d}{2d}\right) + 30(7c^2d^4 + 6acd^2 + a^2e^4)x + 6c^2d^4e^2 - 14c^2d^3e^3 + 10(7c^2d^4 + 6acd^2)^2 - 15(7c^2d^4 + 6acd^2 - a^2e^4)\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) + 15(7c^2d^4 + 6acd^2 + a^2e^4)}{60(d^2e^2 + d^2e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+a)^2/(e\*x^2+d)^2,x, algorithm="fricas")

[Out] [1/60\*(12\*c^2\*d^2\*e^4\*x^7 - 28\*c^2\*d^3\*e^3\*x^5 + 20\*(7\*c^2\*d^4\*e^2 + 6\*a\*c\*d^2\*e^4)\*x^3 + 15\*(7\*c^2\*d^4 + 6\*a\*c\*d^3\*e^2 - a^2\*d\*e^4 + (7\*c^2\*d^4\*e + 6\*a\*c\*d^2\*e^3 - a^2\*e^5)\*x^2)\*sqrt(-d\*e)\*log((e\*x^2 - 2\*sqrt(-d\*e)\*x - d)/(e\*x^2 + d)) + 30\*(7\*c^2\*d^5\*e + 6\*a\*c\*d^3\*e^3 + a^2\*d\*e^5)\*x)/(d^2\*e^6\*x^2 + d^3\*e^5), 1/30\*(6\*c^2\*d^2\*e^4\*x^7 - 14\*c^2\*d^3\*e^3\*x^5 + 10\*(7\*c^2\*d^4\*e^2 + 6\*a\*c\*d^2\*e^4)\*x^3 - 15\*(7\*c^2\*d^4 + 6\*a\*c\*d^3\*e^2 - a^2\*d\*e^4 + (7\*c^2\*d^4\*e + 6\*a\*c\*d^2\*e^3 - a^2\*e^5)\*x^2)\*sqrt(d\*e)\*arctan(sqrt(d\*e)\*x/d) + 15\*(7\*c^2\*d^5\*e + 6\*a\*c\*d^3\*e^3 + a^2\*d\*e^5)\*x)/(d^2\*e^6\*x^2 + d^3\*e^5)]

**giac [A]** time = 0.17, size = 128, normalized size = 0.98

$$\frac{1}{15} (3c^2x^5e^8 - 10c^2dx^3e^7 + 45c^2d^2xe^6 + 30acxe^8)e^{(-10)} - \frac{(7c^2d^4 + 6acd^2e^2 - a^2e^4) \arctan\left(\frac{1}{\sqrt{d}}\right) e^{\left(-\frac{9}{2}\right)}}{2d^{\frac{3}{2}}} + \frac{(c^2d^4x + 2acd^2xe^2 + a^2xe^4)e^{(-4)}}{2(x^2e + d)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+a)^2/(e\*x^2+d)^2,x, algorithm="giac")

[Out] 1/15\*(3\*c^2\*x^5\*e^8 - 10\*c^2\*d\*x^3\*e^7 + 45\*c^2\*d^2\*x\*e^6 + 30\*a\*c\*x\*e^8)\*e^(-10) - 1/2\*(7\*c^2\*d^4 + 6\*a\*c\*d^2\*e^2 - a^2\*e^4)\*arctan(x\*e^(1/2)/sqrt(d))\*e^(-9/2)/d^(3/2) + 1/2\*(c^2\*d^4\*x + 2\*a\*c\*d^2\*x\*e^2 + a^2\*x\*e^4)\*e^(-4)/((x^2\*e + d)\*d)

**maple [A]** time = 0.01, size = 170, normalized size = 1.30

$$\frac{c^2x^5}{5e^2} - \frac{2c^2dx^3}{3e^3} + \frac{a^2x}{2(e^2x^2+d)} + \frac{a^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}d} + \frac{acdx}{(ex^2+d)e^2} - \frac{3acd \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}e^2} + \frac{c^2d^3x}{2(e^2x^2+d)e^4} - \frac{7c^2d^3 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}e^4} + \frac{2acx}{e^2} + \frac{3c^2d^2x}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^4+a)^2/(e\*x^2+d)^2,x)

[Out] 1/5\*c^2\*x^5/e^2-2/3\*c^2\*d\*x^3/e^3+2\*c/e^2\*a\*x+3\*c^2/e^4\*d^2\*x+1/2/d\*x/(e\*x^2+d)\*a^2+1/e^2\*d\*x/(e\*x^2+d)\*a\*c+1/2/e^4\*d^3\*x/(e\*x^2+d)\*c^2+1/2/d/(d\*e)^(1/2)\*arctan(1/(d\*e)^(1/2)\*e\*x)\*a^2-3/e^2\*d/(d\*e)^(1/2)\*arctan(1/(d\*e)^(1/2)\*e\*x)\*a\*c-7/2/e^4\*d^3/(d\*e)^(1/2)\*arctan(1/(d\*e)^(1/2)\*e\*x)\*c^2

**maxima [A]** time = 2.28, size = 142, normalized size = 1.08

$$\frac{(c^2d^4 + 2acd^2e^2 + a^2e^4)x}{2(de^5x^2 + d^2e^4)} + \frac{3c^2e^2x^5 - 10c^2dex^3 + 15(3c^2d^2 + 2ace^2)x}{15e^4} - \frac{(7c^2d^4 + 6acd^2e^2 - a^2e^4) \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}de^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+a)^2/(e\*x^2+d)^2,x, algorithm="maxima")

[Out] 1/2\*(c^2\*d^4 + 2\*a\*c\*d^2\*e^2 + a^2\*e^4)\*x/(d\*e^5\*x^2 + d^2\*e^4) + 1/15\*(3\*c^2\*e^2\*x^5 - 10\*c^2\*d\*e\*x^3 + 15\*(3\*c^2\*d^2 + 2\*a\*c\*e^2)\*x)/e^4 - 1/2\*(7\*c^2\*d^4 + 6\*a\*c\*d^2\*e^2 - a^2\*e^4)\*arctan(e\*x/sqrt(d\*e))/(sqrt(d\*e)\*d\*e^4)

**mupad [B]** time = 4.40, size = 183, normalized size = 1.40

$$x \left( \frac{3c^2d^2}{e^4} + \frac{2ac}{e^2} \right) + \frac{c^2x^5}{5e^2} - \frac{2c^2dx^3}{3e^3} + \frac{x(a^2e^4 + 2ac d^2e^2 + c^2d^4)}{2d(e^5x^2 + de^4)} - \frac{\operatorname{atan}\left(\frac{\sqrt{e}x(c d^2 + a e^2)(a e^2 - 7 c d^2)}{\sqrt{d}(-a^2 e^4 + 6 a c d^2 e^2 + 7 c^2 d^4)}\right) (c d^2 + a e^2) (a e^2 - 7 c d^2)}{2 d^{3/2} e^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c\*x^4)^2/(d + e\*x^2)^2,x)

[Out] x\*((3\*c^2\*d^2)/e^4 + (2\*a\*c)/e^2) + (c^2\*x^5)/(5\*e^2) - (2\*c^2\*d\*x^3)/(3\*e^3) + (x\*(a^2\*e^4 + c^2\*d^4 + 2\*a\*c\*d^2\*e^2))/(2\*d\*(d\*e^4 + e^5\*x^2)) - (atan((e^(1/2)\*x\*(a\*e^2 + c\*d^2)\*(a\*e^2 - 7\*c\*d^2))/(d^(1/2)\*(7\*c^2\*d^4 - a^2\*e^4 + 6\*a\*c\*d^2\*e^2)))\*(a\*e^2 + c\*d^2)\*(a\*e^2 - 7\*c\*d^2))/(2\*d^(3/2)\*e^(9/2))

**sympy [B]** time = 0.93, size = 314, normalized size = 2.40

$$-\frac{2c^2dx^3}{3e^3} + \frac{c^2x^5}{5e^2} + x \left( \frac{2ac}{e^2} + \frac{3c^2d^2}{e^4} \right) + \frac{x(a^2e^4 + 2acd^2e^2 + c^2d^4)}{2d^2e^4 + 2de^5x^2} - \frac{\sqrt{\frac{1}{\beta\beta}}(ae^2 - 7cd^2)(ae^2 + cd^2) \log\left(\frac{d^2e^4\sqrt{\frac{1}{\beta\beta}}(ae^2 - 7cd^2)(ae^2 + cd^2)}{x^2e^4 - 6acd^2e^2 - 7c^2d^4} + x\right)}{4} + \frac{\sqrt{\frac{1}{\beta\beta}}(ae^2 - 7cd^2)(ae^2 + cd^2) \log\left(\frac{d^2e^4\sqrt{\frac{1}{\beta\beta}}(ae^2 - 7cd^2)(ae^2 + cd^2)}{x^2e^4 - 6acd^2e^2 - 7c^2d^4} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*4+a)\*\*2/(e\*x\*\*2+d)\*\*2,x)

[Out] 
$$-2*c**2*d*x**3/(3*e**3) + c**2*x**5/(5*e**2) + x*(2*a*c/e**2 + 3*c**2*d**2/e**4) + x*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4)/(2*d**2*e**4 + 2*d*e**5*x**2) - \sqrt{-1/(d**3*e**9)}*(a*e**2 - 7*c*d**2)*(a*e**2 + c*d**2)*\log(-d**2*e**4*\sqrt{-1/(d**3*e**9)}*(a*e**2 - 7*c*d**2)*(a*e**2 + c*d**2)/(a**2*e**4 - 6*a*c*d**2*e**2 - 7*c**2*d**4) + x)/4 + \sqrt{-1/(d**3*e**9)}*(a*e**2 - 7*c*d**2)*(a*e**2 + c*d**2)*\log(d**2*e**4*\sqrt{-1/(d**3*e**9)}*(a*e**2 - 7*c*d**2)*(a*e**2 + c*d**2)/(a**2*e**4 - 6*a*c*d**2*e**2 - 7*c**2*d**4) + x)/4$$

$$3.115 \quad \int \frac{(a+cx^4)^2}{(d+ex^2)^3} dx$$

**Optimal.** Leaf size=155

$$\frac{x \left( 3a^2 - \frac{10acd^2}{e^2} - \frac{13c^2d^4}{e^4} \right)}{8d^2(d+ex^2)} + \frac{(3a^2e^4 + 6acd^2e^2 + 35c^2d^4) \tan^{-1} \left( \frac{\sqrt{e}x}{\sqrt{d}} \right)}{8d^{5/2}e^{9/2}} + \frac{x(ae^2 + cd^2)^2}{4de^4(d+ex^2)^2} - \frac{3c^2dx}{e^4} + \frac{c^2x^3}{3e^3}$$

**Rubi [A]** time = 0.25, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$ , Rules used = {1158, 1814, 1153, 205}

$$\frac{x \left( 3a^2 - \frac{10acd^2}{e^2} - \frac{13c^2d^4}{e^4} \right)}{8d^2(d+ex^2)} + \frac{(3a^2e^4 + 6acd^2e^2 + 35c^2d^4) \tan^{-1} \left( \frac{\sqrt{e}x}{\sqrt{d}} \right)}{8d^{5/2}e^{9/2}} + \frac{x(ae^2 + cd^2)^2}{4de^4(d+ex^2)^2} - \frac{3c^2dx}{e^4} + \frac{c^2x^3}{3e^3}$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^4)^2/(d + e\*x^2)^3,x]

[Out] (-3\*c^2\*d\*x)/e^4 + (c^2\*x^3)/(3\*e^3) + ((c\*d^2 + a\*e^2)^2\*x)/(4\*d\*e^4\*(d + e\*x^2)^2) + ((3\*a^2 - (13\*c^2\*d^4)/e^4 - (10\*a\*c\*d^2)/e^2)\*x)/(8\*d^2\*(d + e\*x^2)) + ((35\*c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + 3\*a^2\*e^4)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(8\*d^(5/2)\*e^(9/2))

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 1153

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x^2)^q\*(a + b\*x^2 + c\*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

#### Rule 1158

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] := With[{Qx = PolynomialQuotient[(a + c\*x^4)^p, d + e\*x^2, x], R = Coeff[PolynomialRemainder[(a + c\*x^4)^p, d + e\*x^2, x], x, 0]}, -Simp[(R\*x\*(d + e\*x^2)^(q + 1))/(2\*d\*(q + 1)), x] + Dist[1/(2\*d\*(q + 1)), Int[(d + e\*x^2)^(q + 1)\*ExpandToSum[2\*d\*(q + 1)\*Qx + R\*(2\*q + 3), x], x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

#### Rule 1814

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 1]}, Simp[((a\*g - b\*f\*x)\*(a + b\*x^2)^(p + 1))/(2\*a\*b\*(p + 1)), x] + Dist[1/(2\*a\*(p + 1)), Int[(a + b\*x^2)^(p + 1)\*ExpandToSum[2\*a\*(p + 1)\*Q + f\*(2\*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

#### Rubi steps





$a*c*d^3*e^3 + 3*a^2*d*e^5)*x^2)*\sqrt{-d*e}*\log((e*x^2 - 2*\sqrt{-d*e})*x - d)/(e*x^2 + d) - 6*(35*c^2*d^6*e + 6*a*c*d^4*e^3 - 5*a^2*d^2*e^5)*x)/(d^3*e^7*x^4 + 2*d^4*e^6*x^2 + d^5*e^5), 1/24*(8*c^2*d^3*e^4*x^7 - 56*c^2*d^4*e^3*x^5 - (175*c^2*d^5*e^2 + 30*a*c*d^3*e^4 - 9*a^2*d*e^6)*x^3 + 3*(35*c^2*d^6 + 6*a*c*d^4*e^2 + 3*a^2*d^2*e^4 + (35*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + 3*a^2*e^6)*x^4 + 2*(35*c^2*d^5*e + 6*a*c*d^3*e^3 + 3*a^2*d*e^5)*x^2)*\sqrt{d*e}*\arctan(\sqrt{d*e}*x/d) - 3*(35*c^2*d^6*e + 6*a*c*d^4*e^3 - 5*a^2*d^2*e^5)*x)/(d^3*e^7*x^4 + 2*d^4*e^6*x^2 + d^5*e^5)]$

**giac** [A] time = 0.17, size = 145, normalized size = 0.94

$$\frac{1}{3}(c^2x^3e^6 - 9c^2dxe^5)e^{(-9)} + \frac{(35c^2d^4 + 6acd^2e^2 + 3a^2e^4)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{8d^{\frac{5}{2}}} - \frac{(13c^2d^4x^3e + 11c^2d^5x + 10acd^2x^3e^3 + 6acd^3xe^2 - 3a^2x^3e^5 - 5a^2dxe^4)e^{(-4)}}{8(x^2e + d)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+a)^2/(e\*x^2+d)^3,x, algorithm="giac")

[Out]  $1/3*(c^2*x^3*e^6 - 9*c^2*d*x*e^5)*e^{(-9)} + 1/8*(35*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*\arctan(x*e^{(1/2)}/\sqrt{d})*e^{(-9/2)}/d^{(5/2)} - 1/8*(13*c^2*d^4*x^3*e + 11*c^2*d^5*x + 10*a*c*d^2*x^3*e^3 + 6*a*c*d^3*x*e^2 - 3*a^2*x^3*e^5 - 5*a^2*d*x*e^4)*e^{(-4)}/((x^2*e + d)^2*d^2)$

**maple** [A] time = 0.01, size = 211, normalized size = 1.36

$$\frac{3a^2ex^3}{8(e^2x+d)^2d^2} - \frac{5acx^3}{4(e^2x+d)^2e} - \frac{13c^2d^2x^3}{8(e^2x+d)^2e^3} + \frac{5a^2x}{8(e^2x+d)^2d} - \frac{3acdx}{4(e^2x+d)^2e^2} - \frac{11c^2d^3x}{8(e^2x+d)^2e^4} + \frac{c^2x^3}{3e^3} + \frac{3a^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}d^2} + \frac{3ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{4\sqrt{de}e^2} + \frac{35c^2d^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}e^4} - \frac{3c^2dx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^4+a)^2/(e\*x^2+d)^3,x)

[Out]  $1/3*c^2*x^3/e^3 - 3*c^2*d*x/e^4 + 3/8*e/(e*x^2+d)^2/d^2*x^3*a^2 - 5/4/e/(e*x^2+d)^2*x^3*a*c - 13/8/e^3/(e*x^2+d)^2*d^2*x^3*c^2 + 5/8/(e*x^2+d)^2/d*x*a^2 - 3/4/e^2/(e*x^2+d)^2*d*x*a*c - 11/8/e^4/(e*x^2+d)^2*d^3*x*c^2 + 3/8/d^2/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)}*e*x)*a^2 + 3/4/e^2/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)}*e*x)*a*c + 35/8/e^4*d^2/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)}*e*x)*c^2$

**maxima** [A] time = 2.31, size = 167, normalized size = 1.08

$$\frac{(13c^2d^4e + 10acd^2e^3 - 3a^2e^5)x^3 + (11c^2d^5 + 6acd^3e^2 - 5a^2de^4)x}{8(d^2e^6x^4 + 2d^3e^5x^2 + d^4e^4)} + \frac{c^2ex^3 - 9c^2dx}{3e^4} + \frac{(35c^2d^4 + 6acd^2e^2 + 3a^2e^4)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}d^2e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+a)^2/(e\*x^2+d)^3,x, algorithm="maxima")

[Out]  $-1/8*((13*c^2*d^4*e + 10*a*c*d^2*e^3 - 3*a^2*e^5)*x^3 + (11*c^2*d^5 + 6*a*c*d^3*e^2 - 5*a^2*d*e^4)*x)/(d^2*e^6*x^4 + 2*d^3*e^5*x^2 + d^4*e^4) + 1/3*(c^2*e*x^3 - 9*c^2*d*x)/e^4 + 1/8*(35*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*\arctan(e*x/\sqrt{d*e})/(\sqrt{d*e}*d^2*e^4)$

**mupad** [B] time = 4.41, size = 164, normalized size = 1.06

$$\frac{c^2x^3}{3e^3} - \frac{x^3(-3a^2e^5 + 10acd^2e^3 + 13c^2d^4e)}{8d^2} + \frac{x(-5a^2e^4 + 6acd^2e^2 + 11c^2d^4)}{8d} + \frac{\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(3a^2e^4 + 6acd^2e^2 + 35c^2d^4)}{8d^{5/2}e^{9/2}} - \frac{3c^2dx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c\*x^4)^2/(d + e\*x^2)^3,x)

[Out]  $(c^2*x^3)/(3*e^3) - ((x^3*(13*c^2*d^4*e - 3*a^2*e^5 + 10*a*c*d^2*e^3))/(8*d^2) + (x*(11*c^2*d^4 - 5*a^2*e^4 + 6*a*c*d^2*e^2))/(8*d))/((d^2*e^4 + e^6*x^2))$

$$4 + 2*d*e^5*x^2) + (\text{atan}((e^{(1/2)*x})/d^{(1/2)}))*(3*a^2*e^4 + 35*c^2*d^4 + 6*a*c*d^2*e^2))/(8*d^{(5/2)*e^{(9/2)}}) - (3*c^2*d*x)/e^4$$

**sympy [A]** time = 1.71, size = 257, normalized size = 1.66

$$-\frac{3c^2dx}{e^4} + \frac{c^2x^3}{3e^3} - \frac{\sqrt{-\frac{1}{d^5\beta}}(3a^2e^4 + 6acd^2e^2 + 35c^2d^4)\log\left(-d^3e^4\sqrt{-\frac{1}{d^5\beta}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{d^5\beta}}(3a^2e^4 + 6acd^2e^2 + 35c^2d^4)\log\left(d^3e^4\sqrt{-\frac{1}{d^5\beta}} + x\right)}{16} + \frac{x^3(3a^2e^5 - 10acd^2e^3 - 13c^2d^4e) + x(5a^2de^4 - 6acd^3e^2 - 11c^2d^5)}{8d^4e^4 + 16d^3e^5x^2 + 8d^2e^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*4+a)\*\*2/(e\*x\*\*2+d)\*\*3,x)

[Out]  $-3*c**2*d*x/e**4 + c**2*x**3/(3*e**3) - \text{sqrt}(-1/(d**5*e**9))*(3*a**2*e**4 + 6*a*c*d**2*e**2 + 35*c**2*d**4)*\log(-d**3*e**4*\text{sqrt}(-1/(d**5*e**9)) + x)/16 + \text{sqrt}(-1/(d**5*e**9))*(3*a**2*e**4 + 6*a*c*d**2*e**2 + 35*c**2*d**4)*\log(d**3*e**4*\text{sqrt}(-1/(d**5*e**9)) + x)/16 + (x**3*(3*a**2*e**5 - 10*a*c*d**2*e**3 - 13*c**2*d**4*e) + x*(5*a**2*d*e**4 - 6*a*c*d**3*e**2 - 11*c**2*d**5))/(8*d**4*e**4 + 16*d**3*e**5*x**2 + 8*d**2*e**6*x**4)$

$$3.116 \quad \int \frac{(a+cx^4)^2}{(d+ex^2)^4} dx$$

**Optimal.** Leaf size=184

$$\frac{x \left( 5a^2 - \frac{14acd^2}{e^2} - \frac{19c^2d^4}{e^4} \right) (-5a^2e^4 - 2acd^2e^2 + 35c^2d^4) \tan^{-1} \left( \frac{\sqrt{e}x}{\sqrt{d}} \right)}{24d^2 (d+ex^2)^2} + \frac{x \left( 5a^2 + \frac{2acd^2}{e^2} + \frac{29c^2d^4}{e^4} \right)}{16d^{7/2}e^{9/2}} + \frac{x (ae^2 + cd^2)^2}{16d^3 (d+ex^2)} + \frac{x (ae^2 + cd^2)^2}{6de^4 (d+ex^2)^3}$$

**Rubi [A]** time = 0.30, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {1158, 1814, 1157, 388, 205}

$$\frac{x \left( 5a^2 + \frac{2acd^2}{e^2} + \frac{29c^2d^4}{e^4} \right)}{16d^3 (d+ex^2)} + \frac{x \left( 5a^2 - \frac{14acd^2}{e^2} - \frac{19c^2d^4}{e^4} \right) (-5a^2e^4 - 2acd^2e^2 + 35c^2d^4) \tan^{-1} \left( \frac{\sqrt{e}x}{\sqrt{d}} \right)}{24d^2 (d+ex^2)^2} + \frac{x (ae^2 + cd^2)^2}{16d^{7/2}e^{9/2}} + \frac{x (ae^2 + cd^2)^2}{6de^4 (d+ex^2)^3} + \frac{c^2x}{e^4}$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^4)^2/(d + e\*x^2)^4,x]

[Out] (c^2\*x)/e^4 + ((c\*d^2 + a\*e^2)^2\*x)/(6\*d\*e^4\*(d + e\*x^2)^3) + ((5\*a^2 - (19\*c^2\*d^4)/e^4 - (14\*a\*c\*d^2)/e^2)\*x)/(24\*d^2\*(d + e\*x^2)^2) + ((5\*a^2 + (29\*c^2\*d^4)/e^4 + (2\*a\*c\*d^2)/e^2)\*x)/(16\*d^3\*(d + e\*x^2)) - ((35\*c^2\*d^4 - 2\*a\*c\*d^2\*e^2 - 5\*a^2\*e^4)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(16\*d^(7/2)\*e^(9/2))

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p+1)/(b\*(n\*(p+1)+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1)+1))/(b\*(n\*(p+1)+1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p+1)+1, 0]

#### Rule 1157

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] := With[{Qx = PolynomialQuotient[(a + b\*x^2 + c\*x^4)^p, d + e\*x^2, x], R = Coeff[PolynomialRemainder[(a + b\*x^2 + c\*x^4)^p, d + e\*x^2, x], x, 0]}, -Simp[(R\*x\*(d + e\*x^2)^(q+1))/(2\*d\*(q+1)), x] + Dist[1/(2\*d\*(q+1)), Int[(d + e\*x^2)^(q+1)\*ExpandToSum[2\*d\*(q+1)\*Qx + R\*(2\*q+3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

#### Rule 1158

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] := With[{Qx = PolynomialQuotient[(a + c\*x^4)^p, d + e\*x^2, x], R = Coeff[PolynomialRemainder[(a + c\*x^4)^p, d + e\*x^2, x], x, 0]}, -Simp[(R\*x\*(d + e\*x^2)^(q+1))/(2\*d\*(q+1)), x] + Dist[1/(2\*d\*(q+1)), Int[(d + e\*x^2)^(q+1)\*ExpandToSum[2\*d\*(q+1)\*Qx + R\*(2\*q+3), x], x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

#### Rule 1814

```
Int[(Pq_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] / ; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(a + cx^4)^2}{(d + ex^2)^4} dx = \frac{(cd^2 + ae^2)^2 x}{6de^4 (d + ex^2)^3} - \frac{\int \frac{-5a^2 + \frac{c^2 d^4}{e^4} + \frac{2acd^2}{e^2} - \frac{6cd(cd^2 + 2ae^2)x^2}{e^3} + \frac{6c^2 d^2 x^4}{e^2} - \frac{6c^2 dx^6}{e}}{(d + ex^2)^3} dx}{6d}$$

$$= \frac{(cd^2 + ae^2)^2 x}{6de^4 (d + ex^2)^3} + \frac{\left(5a^2 - \frac{19c^2 d^4}{e^4} - \frac{14acd^2}{e^2}\right) x}{24d^2 (d + ex^2)^2} + \frac{\int \frac{3\left(5a^2 + \frac{5c^2 d^4}{e^4} + \frac{2acd^2}{e^2}\right) - \frac{48c^2 d^3 x^2}{e^3} + \frac{24c^2 d^2 x^4}{e^2}}{(d + ex^2)^2} dx}{24d^2}$$

$$= \frac{(cd^2 + ae^2)^2 x}{6de^4 (d + ex^2)^3} + \frac{\left(5a^2 - \frac{19c^2 d^4}{e^4} - \frac{14acd^2}{e^2}\right) x}{24d^2 (d + ex^2)^2} + \frac{\left(5a^2 + \frac{29c^2 d^4}{e^4} + \frac{2acd^2}{e^2}\right) x}{16d^3 (d + ex^2)} - \frac{\int \frac{-3\left(5a^2 - \frac{19c^2 d^4}{e^4} + \frac{2acd^2}{e^2}\right)}{d + ex^2} dx}{48d^3}$$

$$= \frac{c^2 x}{e^4} + \frac{(cd^2 + ae^2)^2 x}{6de^4 (d + ex^2)^3} + \frac{\left(5a^2 - \frac{19c^2 d^4}{e^4} - \frac{14acd^2}{e^2}\right) x}{24d^2 (d + ex^2)^2} + \frac{\left(5a^2 + \frac{29c^2 d^4}{e^4} + \frac{2acd^2}{e^2}\right) x}{16d^3 (d + ex^2)} - \frac{(35c^2 d^4 - 2acd^2)}{48d^3}$$

$$= \frac{c^2 x}{e^4} + \frac{(cd^2 + ae^2)^2 x}{6de^4 (d + ex^2)^3} + \frac{\left(5a^2 - \frac{19c^2 d^4}{e^4} - \frac{14acd^2}{e^2}\right) x}{24d^2 (d + ex^2)^2} + \frac{\left(5a^2 + \frac{29c^2 d^4}{e^4} + \frac{2acd^2}{e^2}\right) x}{16d^3 (d + ex^2)} - \frac{(35c^2 d^4 - 2acd^2)}{48d^3}$$

**Mathematica [A]** time = 0.14, size = 174, normalized size = 0.95

$$\frac{x(a^2 e^4 (33d^2 + 40dex^2 + 15e^2 x^4) - 2acd^2 e^2 (3d^2 + 8dex^2 - 3e^2 x^4) + c^2 d^3 (105d^3 + 280d^2 ex^2 + 231de^2 x^4 + 48e^3 x^6))}{48d^3 e^4 (d + ex^2)^3} - \frac{(-5a^2 e^4 - 2acd^2 e^2 + 35c^2 d^4) \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2} e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^2/(d + e*x^2)^4,x]
```

```
[Out] (x*(-2*a*c*d^2*e^2*(3*d^2 + 8*d*e*x^2 - 3*e^2*x^4) + a^2*e^4*(33*d^2 + 40*d*e*x^2 + 15*e^2*x^4) + c^2*d^3*(105*d^3 + 280*d^2*e*x^2 + 231*d*e^2*x^4 + 48*e^3*x^6)))/(48*d^3*e^4*(d + e*x^2)^3) - ((35*c^2*d^4 - 2*a*c*d^2*e^2 - 5*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(9/2))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + cx^4)^2}{(d + ex^2)^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + c*x^4)^2/(d + e*x^2)^4,x]
```

```
[Out] IntegrateAlgebraic[(a + c*x^4)^2/(d + e*x^2)^4, x]
```

**fricas [A]** time = 0.87, size = 662, normalized size = 3.60

$\frac{c^2 x}{e^4} + \frac{(cd^2 + ae^2)^2 x}{6de^4 (d + ex^2)^3} + \frac{\left(5a^2 - \frac{19c^2 d^4}{e^4} - \frac{14acd^2}{e^2}\right) x}{24d^2 (d + ex^2)^2} + \frac{\left(5a^2 + \frac{29c^2 d^4}{e^4} + \frac{2acd^2}{e^2}\right) x}{16d^3 (d + ex^2)} - \frac{(35c^2 d^4 - 2acd^2)}{48d^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+a)^2/(e\*x^2+d)^4,x, algorithm="fricas")

[Out] [1/96\*(96\*c^2\*d^4\*e^4\*x^7 + 6\*(77\*c^2\*d^5\*e^3 + 2\*a\*c\*d^3\*e^5 + 5\*a^2\*d\*e^7)\*x^5 + 16\*(35\*c^2\*d^6\*e^2 - 2\*a\*c\*d^4\*e^4 + 5\*a^2\*d^2\*e^6)\*x^3 + 3\*(35\*c^2\*d^7 - 2\*a\*c\*d^5\*e^2 - 5\*a^2\*d^3\*e^4 + (35\*c^2\*d^4\*e^3 - 2\*a\*c\*d^2\*e^5 - 5\*a^2\*e^7)\*x^6 + 3\*(35\*c^2\*d^5\*e^2 - 2\*a\*c\*d^3\*e^4 - 5\*a^2\*d\*e^6)\*x^4 + 3\*(35\*c^2\*d^6\*e - 2\*a\*c\*d^4\*e^3 - 5\*a^2\*d^2\*e^5)\*x^2)\*sqrt(-d\*e)\*log((e\*x^2 - 2\*sqrt(-d\*e)\*x - d)/(e\*x^2 + d)) + 6\*(35\*c^2\*d^7\*e - 2\*a\*c\*d^5\*e^3 + 11\*a^2\*d^3\*e^5)\*x)/(d^4\*e^8\*x^6 + 3\*d^5\*e^7\*x^4 + 3\*d^6\*e^6\*x^2 + d^7\*e^5), 1/48\*(48\*c^2\*d^4\*e^4\*x^7 + 3\*(77\*c^2\*d^5\*e^3 + 2\*a\*c\*d^3\*e^5 + 5\*a^2\*d\*e^7)\*x^5 + 8\*(35\*c^2\*d^6\*e^2 - 2\*a\*c\*d^4\*e^4 + 5\*a^2\*d^2\*e^6)\*x^3 - 3\*(35\*c^2\*d^7 - 2\*a\*c\*d^5\*e^2 - 5\*a^2\*d^3\*e^4 + (35\*c^2\*d^4\*e^3 - 2\*a\*c\*d^2\*e^5 - 5\*a^2\*e^7)\*x^6 + 3\*(35\*c^2\*d^5\*e^2 - 2\*a\*c\*d^3\*e^4 - 5\*a^2\*d\*e^6)\*x^4 + 3\*(35\*c^2\*d^6\*e - 2\*a\*c\*d^4\*e^3 - 5\*a^2\*d^2\*e^5)\*x^2)\*sqrt(d\*e)\*arctan(sqrt(d\*e)\*x/d) + 3\*(35\*c^2\*d^7\*e - 2\*a\*c\*d^5\*e^3 + 11\*a^2\*d^3\*e^5)\*x)/(d^4\*e^8\*x^6 + 3\*d^5\*e^7\*x^4 + 3\*d^6\*e^6\*x^2 + d^7\*e^5)]

**giac** [A] time = 0.16, size = 167, normalized size = 0.91

$$c^2 x e^{(-4)} - \frac{(35 c^2 d^4 - 2 a c d^2 e^2 - 5 a^2 e^4) \arctan\left(\frac{x \sqrt{d}}{\sqrt{d}}\right) e^{(-\frac{9}{2})}}{16 d^2} + \frac{(87 c^2 d^4 x^5 e^2 + 136 c^2 d^5 x^3 e + 6 a c d^2 x^5 e^4 + 57 c^2 d^6 x - 16 a c d^3 x^3 e^3 + 15 a^2 x^5 e^6 - 6 a c d^4 x e^2 + 40 a^2 d x^3 e^5 + 33 a^2 d^2 x e^4) e^{(-4)}}{48 (x^2 e + d)^3 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+a)^2/(e\*x^2+d)^4,x, algorithm="giac")

[Out] c^2\*x\*e^(-4) - 1/16\*(35\*c^2\*d^4 - 2\*a\*c\*d^2\*e^2 - 5\*a^2\*e^4)\*arctan(x\*e^(1/2)/sqrt(d))\*e^(-9/2)/d^(7/2) + 1/48\*(87\*c^2\*d^4\*x^5\*e^2 + 136\*c^2\*d^5\*x^3\*e + 6\*a\*c\*d^2\*x^5\*e^4 + 57\*c^2\*d^6\*x - 16\*a\*c\*d^3\*x^3\*e^3 + 15\*a^2\*x^5\*e^6 - 6\*a\*c\*d^4\*x\*e^2 + 40\*a^2\*d\*x^3\*e^5 + 33\*a^2\*d^2\*x\*e^4)\*e^(-4)/((x^2\*e + d)^3\*d^3)

**maple** [A] time = 0.01, size = 262, normalized size = 1.42

$$\frac{5 a^2 e^2 x^5}{16 (e x^2 + d)^3 d^3} + \frac{a c x^5}{8 (e x^2 + d)^3 d} + \frac{29 c^2 d x^5}{16 (e x^2 + d)^3 d^2} + \frac{5 a^2 e x^3}{6 (e x^2 + d)^3 d^2} - \frac{a c x^3}{3 (e x^2 + d)^3 e} + \frac{17 c^2 d^2 x^3}{6 (e x^2 + d)^3 e^3} + \frac{11 a^2 x}{16 (e x^2 + d)^3 d} - \frac{a c d x}{8 (e x^2 + d)^3 e^2} + \frac{19 c^2 d^3 x}{16 (e x^2 + d)^3 e^4} + \frac{5 a^2 \arctan\left(\frac{x}{\sqrt{d e}}\right)}{16 \sqrt{d e} d^3} + \frac{a c \arctan\left(\frac{x}{\sqrt{d e}}\right)}{8 \sqrt{d e} d^2} - \frac{35 c^2 d \arctan\left(\frac{x}{\sqrt{d e}}\right)}{16 \sqrt{d e} e^4} + \frac{c^2 x}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^4+a)^2/(e\*x^2+d)^4,x)

[Out] c^2\*x/e^4+5/16\*e^2/(e\*x^2+d)^3/d^3\*x^5\*a^2+1/8/(e\*x^2+d)^3/d\*x^5\*a\*c+29/16/e^2/(e\*x^2+d)^3\*d\*x^5\*c^2+5/6\*e/(e\*x^2+d)^3/d^2\*x^3\*a^2-1/3/e/(e\*x^2+d)^3\*x^3\*a\*c+17/6/e^3/(e\*x^2+d)^3\*d^2\*x^3\*c^2+11/16/(e\*x^2+d)^3/d\*x\*a^2-1/8/e^2/(e\*x^2+d)^3\*d\*x\*a\*c+19/16/e^4/(e\*x^2+d)^3\*d^3\*x\*c^2+5/16/d^3/(d\*e)^(1/2)\*arctan(1/(d\*e)^(1/2)\*e\*x)\*a^2+1/8/e^2/d/(d\*e)^(1/2)\*arctan(1/(d\*e)^(1/2)\*e\*x)\*a\*c-35/16/e^4\*d/(d\*e)^(1/2)\*arctan(1/(d\*e)^(1/2)\*e\*x)\*c^2

**maxima** [A] time = 2.39, size = 205, normalized size = 1.11

$$\frac{3(29 c^2 d^4 e^2 + 2 a c d^2 e^4 + 5 a^2 e^6) x^5 + 8(17 c^2 d^5 e - 2 a c d^3 e^3 + 5 a^2 d e^5) x^3 + 3(19 c^2 d^6 - 2 a c d^4 e^2 + 11 a^2 d^2 e^4) x}{48 (d^3 e^7 x^6 + 3 d^4 e^6 x^4 + 3 d^5 e^5 x^2 + d^6 e^4)} + \frac{c^2 x}{e^4} - \frac{(35 c^2 d^4 - 2 a c d^2 e^2 - 5 a^2 e^4) \arctan\left(\frac{e x}{\sqrt{d e}}\right)}{16 \sqrt{d e} d^3 e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+a)^2/(e\*x^2+d)^4,x, algorithm="maxima")

[Out] 1/48\*(3\*(29\*c^2\*d^4\*e^2 + 2\*a\*c\*d^2\*e^4 + 5\*a^2\*e^6)\*x^5 + 8\*(17\*c^2\*d^5\*e - 2\*a\*c\*d^3\*e^3 + 5\*a^2\*d\*e^5)\*x^3 + 3\*(19\*c^2\*d^6 - 2\*a\*c\*d^4\*e^2 + 11\*a^2\*d^2\*e^4)\*x)/(d^3\*e^7\*x^6 + 3\*d^4\*e^6\*x^4 + 3\*d^5\*e^5\*x^2 + d^6\*e^4) + c^2\*x/e^4 - 1/16\*(35\*c^2\*d^4 - 2\*a\*c\*d^2\*e^2 - 5\*a^2\*e^4)\*arctan(e\*x/sqrt(d\*e))/(sqrt(d\*e)\*d^3\*e^4)

**mupad [B]** time = 4.49, size = 199, normalized size = 1.08

$$\frac{x^3(5a^2e^5 - 2acd^2e^3 + 17c^2d^4e)}{6d^2} + \frac{x(11a^2e^4 - 2acd^2e^2 + 19c^2d^4)}{16d} + \frac{x^5(5a^2e^6 + 2acd^2e^4 + 29c^2d^4e^2)}{16d^3} + \frac{c^2x}{e^4} + \frac{\operatorname{atan}\left(\frac{\sqrt{6}x}{\sqrt{d}}\right)(5a^2e^4 + 2acd^2e^2 - 35c^2d^4)}{16d^{7/2}e^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + c*x^4)^2/(d + e*x^2)^4,x)`

[Out]  $((x^3(5a^2e^5 + 17c^2d^4e - 2a*c*d^2*e^3))/(6*d^2) + (x*(11a^2e^4 + 19c^2d^4 - 2a*c*d^2*e^2))/(16*d) + (x^5(5a^2e^6 + 29c^2d^4e^2 + 2a*c*d^2*e^4))/(16*d^3))/(d^3*e^4 + e^7*x^6 + 3*d*e^6*x^4 + 3*d^2*e^5*x^2) + (c^2*x)/e^4 + (\operatorname{atan}((e^{1/2}*x)/d^{1/2})*(5a^2e^4 - 35c^2d^4 + 2a*c*d^2*e^2))/(16*d^{7/2}*e^{9/2})$

**sympy [A]** time = 2.61, size = 292, normalized size = 1.59

$$\frac{c^2x}{e^4} - \frac{\sqrt{-\frac{1}{d^2e}}(5a^2e^4 + 2acd^2e^2 - 35c^2d^4)\log\left(-d^4e^4\sqrt{-\frac{1}{d^2e}} + x\right)}{32} + \frac{\sqrt{-\frac{1}{d^2e}}(5a^2e^4 + 2acd^2e^2 - 35c^2d^4)\log\left(d^4e^4\sqrt{-\frac{1}{d^2e}} + x\right)}{32} + \frac{x^5(15a^2e^6 + 6acd^2e^4 + 87c^2d^4e^2) + x^3(40a^2de^5 - 16acd^3e^3 + 136c^2d^2e) + x(33a^2d^2e^4 - 6acd^4e^2 + 57c^2d^6)}{48d^6e^4 + 144d^5e^5x^2 + 144d^4e^6x^4 + 48d^3e^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**4+a)**2/(e*x**2+d)**4,x)`

[Out]  $c**2*x/e**4 - \operatorname{sqrt}(-1/(d**7*e**9))*(5a**2*e**4 + 2a*c*d**2*e**2 - 35c**2*d**4)*\log(-d**4*e**4*\operatorname{sqrt}(-1/(d**7*e**9)) + x)/32 + \operatorname{sqrt}(-1/(d**7*e**9))*(5a**2*e**4 + 2a*c*d**2*e**2 - 35c**2*d**4)*\log(d**4*e**4*\operatorname{sqrt}(-1/(d**7*e**9)) + x)/32 + (x**5*(15a**2*e**6 + 6a*c*d**2*e**4 + 87*c**2*d**4*e**2) + x**3*(40a**2*d*e**5 - 16a*c*d**3*e**3 + 136*c**2*d**5*e) + x*(33a**2*d**2*e**4 - 6a*c*d**4*e**2 + 57*c**2*d**6))/(48*d**6*e**4 + 144*d**5*e**5*x**2 + 144*d**4*e**6*x**4 + 48*d**3*e**7*x**6)$

$$3.117 \quad \int \frac{(a+cx^4)^2}{(d+ex^2)^5} dx$$

**Optimal.** Leaf size=223

$$\frac{x(-35a^2e^4 - 6acd^2e^2 + 93c^2d^4)}{128d^4e^4(d+ex^2)} + \frac{x\left(7a^2 - \frac{18acd^2}{e^2} - \frac{25c^2d^4}{e^4}\right)}{48d^2(d+ex^2)^3} + \frac{(35a^2e^4 + 6acd^2e^2 + 35c^2d^4)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{128d^{9/2}e^{9/2}} + \frac{x(35a^2e^4 + 6acd^2e^2 + 93c^2d^4)}{128d^4e^4(d+ex^2)}$$

**Rubi [A]** time = 0.34, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {1158, 1814, 1157, 385, 205}

$$\frac{x(-35a^2e^4 - 6acd^2e^2 + 93c^2d^4)}{128d^4e^4(d+ex^2)} + \frac{x\left(35a^2 + \frac{6acd^2}{e^2} + \frac{163c^2d^4}{e^4}\right)}{192d^3(d+ex^2)^2} + \frac{x\left(7a^2 - \frac{18acd^2}{e^2} - \frac{25c^2d^4}{e^4}\right)}{48d^2(d+ex^2)^3} + \frac{(35a^2e^4 + 6acd^2e^2 + 35c^2d^4)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{128d^{9/2}e^{9/2}} + \frac{x(ae^2 + cd^2)^2}{8de^4(d+ex^2)^4}$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^4)^2/(d + e\*x^2)^5, x]

[Out] ((c\*d^2 + a\*e^2)^2\*x)/(8\*d\*e^4\*(d + e\*x^2)^4) + ((7\*a^2 - (25\*c^2\*d^4)/e^4 - (18\*a\*c\*d^2)/e^2)\*x)/(48\*d^2\*(d + e\*x^2)^3) + ((35\*a^2 + (163\*c^2\*d^4)/e^4 + (6\*a\*c\*d^2)/e^2)\*x)/(192\*d^3\*(d + e\*x^2)^2) - ((93\*c^2\*d^4 - 6\*a\*c\*d^2\*e^2 - 35\*a^2\*e^4)\*x)/(128\*d^4\*e^4\*(d + e\*x^2)) + ((35\*c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + 35\*a^2\*e^4)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(128\*d^(9/2)\*e^(9/2))

**Rule 205**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 385**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p+1))/(a\*b\*n\*(p+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1)+1))/(a\*b\*n\*(p+1)), Int[(a + b\*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

**Rule 1157**

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> With[{Qx = PolynomialQuotient[(a + b\*x^2 + c\*x^4)^p, d + e\*x^2, x], R = Coeff[PolynomialRemainder[(a + b\*x^2 + c\*x^4)^p, d + e\*x^2, x], x, 0]}, -Simp[(R\*x\*(d + e\*x^2)^(q+1))/(2\*d\*(q+1)), x] + Dist[1/(2\*d\*(q+1)), Int[(d + e\*x^2)^(q+1)\*ExpandToSum[2\*d\*(q+1)\*Qx + R\*(2\*q+3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

**Rule 1158**

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> With[{Qx = PolynomialQuotient[(a + c\*x^4)^p, d + e\*x^2, x], R = Coeff[PolynomialRemainder[(a + c\*x^4)^p, d + e\*x^2, x], x, 0]}, -Simp[(R\*x\*(d + e\*x^2)^(q+1))/(2\*d\*(q+1)), x] + Dist[1/(2\*d\*(q+1)), Int[(d + e\*x^2)^(q+1)\*ExpandToSum[2\*d\*(q+1)\*Qx + R\*(2\*q+3), x], x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

**Rule 1814**

```
Int[(Pq_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] / ; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(a + cx^4)^2}{(d + ex^2)^5} dx = \frac{(cd^2 + ae^2)^2 x}{8de^4 (d + ex^2)^4} - \frac{\int \frac{-7a^2 + \frac{c^2 d^4}{e^4} + \frac{2acd^2}{e^2} - \frac{8cd(cd^2 + 2ae^2)x^2}{e^3} + \frac{8c^2 d^2 x^4}{e^2} - \frac{8c^2 dx^6}{e}}{(d + ex^2)^4} dx}{8d}$$

$$= \frac{(cd^2 + ae^2)^2 x}{8de^4 (d + ex^2)^4} + \frac{\left(7a^2 - \frac{25c^2 d^4}{e^4} - \frac{18acd^2}{e^2}\right) x}{48d^2 (d + ex^2)^3} + \frac{\int \frac{35a^2 + \frac{19c^2 d^4}{e^4} + \frac{6acd^2}{e^2} - \frac{96c^2 d^3 x^2}{e^3} + \frac{48c^2 d^2 x^4}{e^2}}{(d + ex^2)^3} dx}{48d^2}$$

$$= \frac{(cd^2 + ae^2)^2 x}{8de^4 (d + ex^2)^4} + \frac{\left(7a^2 - \frac{25c^2 d^4}{e^4} - \frac{18acd^2}{e^2}\right) x}{48d^2 (d + ex^2)^3} + \frac{\left(35a^2 + \frac{163c^2 d^4}{e^4} + \frac{6acd^2}{e^2}\right) x}{192d^3 (d + ex^2)^2} - \frac{\int \frac{-3\left(35a^2 - \frac{29c^2 d^4}{e^4} + \frac{6acd^2}{e^2}\right)}{(d + ex^2)^2} dx}{192d^3}$$

$$= \frac{(cd^2 + ae^2)^2 x}{8de^4 (d + ex^2)^4} + \frac{\left(7a^2 - \frac{25c^2 d^4}{e^4} - \frac{18acd^2}{e^2}\right) x}{48d^2 (d + ex^2)^3} + \frac{\left(35a^2 + \frac{163c^2 d^4}{e^4} + \frac{6acd^2}{e^2}\right) x}{192d^3 (d + ex^2)^2} - \frac{(93c^2 d^4 - 6acd^2 e^2)}{128d^4 e^4 (d + ex^2)}$$

$$= \frac{(cd^2 + ae^2)^2 x}{8de^4 (d + ex^2)^4} + \frac{\left(7a^2 - \frac{25c^2 d^4}{e^4} - \frac{18acd^2}{e^2}\right) x}{48d^2 (d + ex^2)^3} + \frac{\left(35a^2 + \frac{163c^2 d^4}{e^4} + \frac{6acd^2}{e^2}\right) x}{192d^3 (d + ex^2)^2} - \frac{(93c^2 d^4 - 6acd^2 e^2)}{128d^4 e^4 (d + ex^2)}$$

**Mathematica [A]** time = 0.19, size = 200, normalized size = 0.90

$$\frac{3(35a^2 e^4 + 6acd^2 e^2 + 35c^2 d^4) \tan^{-1}\left(\frac{\sqrt{e} x}{\sqrt{d}}\right) + \frac{\sqrt{d} \sqrt{e} x (a^2 e^4 (279d^3 + 511d^2 e x^2 + 385d e^2 x^4 + 105e^3 x^6) - 6acd^2 e^2 (3d^3 + 11d^2 e x^2 - 11d e^2 x^4 - 3e^3 x^6) - c^2 d^4 (105d^3 + 385d^2 e x^2 + 511d e^2 x^4 + 279e^3 x^6))}{(d + ex^2)^4}}{384d^{9/2} e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^2/(d + e*x^2)^5,x]
```

```
[Out] ((Sqrt[d]*Sqrt[e]*x*(-6*a*c*d^2*e^2*(3*d^3 + 11*d^2*e*x^2 - 11*d*e^2*x^4 - 3*e^3*x^6) + a^2*e^4*(279*d^3 + 511*d^2*e*x^2 + 385*d*e^2*x^4 + 105*e^3*x^6) - c^2*d^4*(105*d^3 + 385*d^2*e*x^2 + 511*d*e^2*x^4 + 279*e^3*x^6)))/(d + e*x^2)^4 + 3*(35*c^2*d^4 + 6*a*c*d^2*e^2 + 35*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(384*d^(9/2)*e^(9/2))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + cx^4)^2}{(d + ex^2)^5} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + c*x^4)^2/(d + e*x^2)^5,x]
```

```
[Out] IntegrateAlgebraic[(a + c*x^4)^2/(d + e*x^2)^5, x]
```



**fricas** [A] time = 0.96, size = 806, normalized size = 3.61

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+a)^2/(e\*x^2+d)^5,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/768*(6*(93*c^2*d^5*e^4 - 6*a*c*d^3*e^6 - 35*a^2*d*e^8)*x^7 + 2*(511*c^2*d^6*e^3 - 66*a*c*d^4*e^5 - 385*a^2*d^2*e^7)*x^5 + 2*(385*c^2*d^7*e^2 + 66*a*c*d^5*e^4 - 511*a^2*d^3*e^6)*x^3 + 3*(35*c^2*d^8 + 6*a*c*d^6*e^2 + 35*a^2*d^4*e^4 + (35*c^2*d^4*e^4 + 6*a*c*d^2*e^6 + 35*a^2*e^8)*x^8 + 4*(35*c^2*d^5*e^3 + 6*a*c*d^3*e^5 + 35*a^2*d*e^7)*x^6 + 6*(35*c^2*d^6*e^2 + 6*a*c*d^4*e^4 + 35*a^2*d^2*e^6)*x^4 + 4*(35*c^2*d^7*e + 6*a*c*d^5*e^3 + 35*a^2*d^3*e^5)*x^2)*\sqrt{-d*e}*\log((e*x^2 - 2*\sqrt{-d*e}*x - d)/(e*x^2 + d)) + 6*(35*c^2*d^8*e + 6*a*c*d^6*e^3 - 93*a^2*d^4*e^5)*x)/(d^5*e^9*x^8 + 4*d^6*e^8*x^6 + 6*d^7*e^7*x^4 + 4*d^8*e^6*x^2 + d^9*e^5), \\ & -1/384*(3*(93*c^2*d^5*e^4 - 6*a*c*d^3*e^6 - 35*a^2*d*e^8)*x^7 + (511*c^2*d^6*e^3 - 66*a*c*d^4*e^5 - 385*a^2*d^2*e^7)*x^5 + (385*c^2*d^7*e^2 + 66*a*c*d^5*e^4 - 511*a^2*d^3*e^6)*x^3 - 3*(35*c^2*d^8 + 6*a*c*d^6*e^2 + 35*a^2*d^4*e^4 + (35*c^2*d^4*e^4 + 6*a*c*d^2*e^6 + 35*a^2*e^8)*x^8 + 4*(35*c^2*d^5*e^3 + 6*a*c*d^3*e^5 + 35*a^2*d*e^7)*x^6 + 6*(35*c^2*d^6*e^2 + 6*a*c*d^4*e^4 + 35*a^2*d^2*e^6)*x^4 + 4*(35*c^2*d^7*e + 6*a*c*d^5*e^3 + 35*a^2*d^3*e^5)*x^2)*\sqrt{d*e}*\arctan(\sqrt{d*e}*x/d) + 3*(35*c^2*d^8*e + 6*a*c*d^6*e^3 - 93*a^2*d^4*e^5)*x)/(d^5*e^9*x^8 + 4*d^6*e^8*x^6 + 6*d^7*e^7*x^4 + 4*d^8*e^6*x^2 + d^9*e^5)] \end{aligned}$$

**giac** [A] time = 0.25, size = 198, normalized size = 0.89

$$\frac{(35c^2d^4 + 6acd^2e^2 + 35a^2e^4) \arctan\left(\frac{\sqrt{x^2+d}}{\sqrt{d}}\right) e^{\frac{1}{2}}}{128d^{\frac{9}{2}}} - \frac{(279c^2d^4x^7e^3 + 511c^2d^5x^5e^2 - 18a^2c^2d^2x^7e^5 + 385c^2d^6x^3e - 66a^2c^2d^3x^5e^4 + 105c^2d^7x - 105a^2x^7e^7 + 66a^2c^2d^4x^3e^3 - 385a^2d^5x^5e^6 + 18a^2c^2d^5x^7e^2 - 511a^2d^2x^3e^5 - 279a^2d^3x^5e^4)*e^{-4}}{384(x^2+d)^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+a)^2/(e\*x^2+d)^5,x, algorithm="giac")

[Out] 
$$\begin{aligned} & 1/128*(35*c^2*d^4 + 6*a*c*d^2*e^2 + 35*a^2*e^4)*\arctan(x*e^{(1/2)}/\sqrt{d})*e^{(-9/2)}/d^{(9/2)} - 1/384*(279*c^2*d^4*x^7*e^3 + 511*c^2*d^5*x^5*e^2 - 18*a^2*c^2*d^2*x^7*e^5 + 385*c^2*d^6*x^3*e - 66*a^2*c^2*d^3*x^5*e^4 + 105*c^2*d^7*x - 105*a^2*x^7*e^7 + 66*a^2*c^2*d^4*x^3*e^3 - 385*a^2*d^5*x^5*e^6 + 18*a^2*c^2*d^5*x^7*e^2 - 511*a^2*d^2*x^3*e^5 - 279*a^2*d^3*x^5*e^4)*e^{-4}/((x^2*e + d)^4*d^4) \end{aligned}$$

**maple** [A] time = 0.01, size = 231, normalized size = 1.04

$$\frac{35a^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{128\sqrt{de} d^4} + \frac{3ac \arctan\left(\frac{ex}{\sqrt{de}}\right)}{64\sqrt{de} d^2 e^2} + \frac{35c^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{128\sqrt{de} e^4} + \frac{(35a^2e^4 + 6acd^2e^2 - 93c^2d^4)x^7}{128d^4e} + \frac{(385a^2e^4 + 66acd^2e^2 - 511c^2d^4)x^5}{384d^3e^2} + \frac{(511a^2e^4 - 66acd^2e^2 - 385c^2d^4)x^3}{384d^2e^3} + \frac{(93a^2e^4 - 6acd^2e^2 - 35c^2d^4)x}{128d e^4} \frac{1}{(ex^2 + d)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^4+a)^2/(e\*x^2+d)^5,x)

[Out] 
$$\begin{aligned} & (1/128*(35*a^2*e^4 + 6*a*c*d^2*e^2 - 93*c^2*d^4)/d^4/e*x^7 + 1/384*(385*a^2*e^4 + 66*a*c*d^2*e^2 - 511*c^2*d^4)/d^3/e^2*x^5 + 1/384*(511*a^2*e^4 - 66*a*c*d^2*e^2 - 385*c^2*d^4)/d^2/e^3*x^3 + 1/128*(93*a^2*e^4 - 6*a*c*d^2*e^2 - 35*c^2*d^4)/d/e^4*x) / (e*x^2+d)^4 + 35/128/d^4/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)}*e*x)*a^2 + 3/64/d^2/e^2/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)}*e*x)*a*c + 35/128/e^4/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)}*e*x)*c^2 \end{aligned}$$

**maxima** [A] time = 2.41, size = 244, normalized size = 1.09

$$\frac{3(93c^2d^4e^3 - 6acd^2e^5 - 35a^2e^7)x^7 + (511c^2d^5e^2 - 66acd^3e^4 - 385a^2d^6e^3)x^5 + (385c^2d^7e + 66acd^4e^3 - 511a^2d^8e^2)x^3 + 3(35c^2d^7 + 6acd^5e^2 - 93a^2d^6e^4)x}{384(d^4e^3x^8 + 4d^5e^4x^6 + 6d^6e^5x^4 + 4d^7e^6x^2 + d^8e^7)} + \frac{(35c^2d^4 + 6acd^2e^2 + 35a^2e^4) \arctan\left(\frac{ex}{\sqrt{de}}\right)}{128\sqrt{de} d^4 e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+a)^2/(e\*x^2+d)^5,x, algorithm="maxima")

[Out] 
$$-1/384*(3*(93*c^2*d^4*e^3 - 6*a*c*d^2*e^5 - 35*a^2*e^7)*x^7 + (511*c^2*d^5*e^2 - 66*a*c*d^3*e^4 - 385*a^2*d*e^6)*x^5 + (385*c^2*d^6*e + 66*a*c*d^4*e^3 - 511*a^2*d^2*e^5)*x^3 + 3*(35*c^2*d^7 + 6*a*c*d^5*e^2 - 93*a^2*d^3*e^4)*x) / (d^4*e^8*x^8 + 4*d^5*e^7*x^6 + 6*d^6*e^6*x^4 + 4*d^7*e^5*x^2 + d^8*e^4) + 1/128*(35*c^2*d^4 + 6*a*c*d^2*e^2 + 35*a^2*e^4)*\arctan(e*x/\sqrt{d*e}) / (\sqrt{d*e}*d^4*e^4)$$

**mupad [B]** time = 4.49, size = 240, normalized size = 1.08

$$\frac{\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(35a^2e^4 + 6ac d^2e^2 + 35c^2d^4)}{128d^{9/2}e^{9/2}} - \frac{x(-93a^2e^4 + 6ac d^2e^2 + 35c^2d^4)}{128de^4} - \frac{x^7(35a^2e^4 + 6ac d^2e^2 - 93c^2d^4)}{128d^4e} + \frac{x^3(-511a^2e^4 + 66ac d^2e^2 + 385c^2d^4)}{384d^2e^3} - \frac{x^5(385a^2e^4 + 66ac d^2e^2 - 511c^2d^4)}{384d^3e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c\*x^4)^2/(d + e\*x^2)^5,x)

[Out] 
$$\left(\operatorname{atan}\left(\frac{e^{1/2}x}{d^{1/2}}\right)*(35a^2e^4 + 35c^2d^4 + 6a*c*d^2*e^2)\right) / (128*d^{9/2}*e^{9/2}) - \left(\frac{x*(35c^2*d^4 - 93a^2*e^4 + 6a*c*d^2*e^2)}{(128*d*e^4)} - \frac{x^7*(35a^2*e^4 - 93c^2*d^4 + 6a*c*d^2*e^2)}{(128*d^4*e)} + \frac{x^3*(385c^2*d^4 - 511a^2*e^4 + 66a*c*d^2*e^2)}{(384*d^2*e^3)} - \frac{x^5*(385a^2*e^4 - 511c^2*d^4 + 66a*c*d^2*e^2)}{(384*d^3*e^2)}\right) / (d^4 + e^4*x^8 + 4*d^3*e*x^2 + 4*d^2*e^3*x^6 + 6*d^2*e^2*x^4)$$

**sympy [A]** time = 4.11, size = 335, normalized size = 1.50

$$\frac{\sqrt{\frac{1}{2d}}(35a^2e^4 + 6ac d^2e^2 + 35c^2d^4) \log\left(-\frac{d^2e^4}{\sqrt{2d}} + x\right)}{256} + \frac{\sqrt{\frac{1}{2d}}(35a^2e^4 + 6ac d^2e^2 + 35c^2d^4) \log\left(\frac{d^2e^4}{\sqrt{2d}} + x\right)}{256} + \frac{x^7(105a^2e^7 + 18ac d^2e^5 - 279c^2d^4e^3) + x^5(385a^2d^6e^6 + 66ac d^4e^4 - 511c^2d^2e^2) + x^3(511a^2d^5e^5 - 66ac d^3e^3 - 385c^2d^2e) + x(279a^2d^4e^4 - 18ac d^2e^2 - 105c^2d^2)}{384d^8e^4 + 1536d^7e^3x^2 + 2304d^6e^2x^4 + 1536d^5e^2x^6 + 384d^4e^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*4+a)\*\*2/(e\*x\*\*2+d)\*\*5,x)

[Out] 
$$-\sqrt{-1/(d**9*e**9)}*(35*a**2*e**4 + 6*a*c*d**2*e**2 + 35*c**2*d**4)*\log(-d**5*e**4*\sqrt{-1/(d**9*e**9)} + x)/256 + \sqrt{-1/(d**9*e**9)}*(35*a**2*e**4 + 6*a*c*d**2*e**2 + 35*c**2*d**4)*\log(d**5*e**4*\sqrt{-1/(d**9*e**9)} + x)/256 + (x**7*(105*a**2*e**7 + 18*a*c*d**2*e**5 - 279*c**2*d**4*e**3) + x**5*(385*a**2*d**6*e**6 + 66*a*c*d**3*e**4 - 511*c**2*d**5*e**2) + x**3*(511*a**2*d**2*e**5 - 66*a*c*d**4*e**3 - 385*c**2*d**6*e) + x*(279*a**2*d**3*e**4 - 18*a*c*d**5*e**2 - 105*c**2*d**7))/(384*d**8*e**4 + 1536*d**7*e**5*x**2 + 2304*d**6*e**6*x**4 + 1536*d**5*e**7*x**6 + 384*d**4*e**8*x**8)$$

$$3.118 \quad \int \frac{(d+ex^2)^4}{a+cx^4} dx$$

**Optimal.** Leaf size=437

$$\frac{(a^2e^4 - 6acd^2e^2 - 4\sqrt{a}\sqrt{c}de(cd^2 - ae^2) + c^2d^4) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}c^{9/4}} + \frac{(a^2e^4 - 6acd^2e^2 - 4\sqrt{a}\sqrt{c}de(cd^2 - ae^2) + c^2d^4) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}c^{9/4}}$$

**Rubi [A]** time = 0.45, antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, number of rules / integrand size = 0.368, Rules used = {1171, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(d^4 - 6acd^2e^2 - 4\sqrt{a}\sqrt{c}de(cd^2 - ae^2) + c^2d^4) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}c^{9/4}} + \frac{(d^4 - 6acd^2e^2 - 4\sqrt{a}\sqrt{c}de(cd^2 - ae^2) + c^2d^4) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}c^{9/4}} + \frac{(d^4 - 6acd^2e^2 + 4\sqrt{a}\sqrt{c}de(cd^2 - ae^2) + c^2d^4) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a} + \sqrt{c}x^2}\right)}{2\sqrt{2}a^{3/4}c^{9/4}} + \frac{(d^4 - 6acd^2e^2 + 4\sqrt{a}\sqrt{c}de(cd^2 - ae^2) + c^2d^4) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a} + \sqrt{c}x^2} + 1\right)}{2\sqrt{2}a^{3/4}c^{9/4}} + \frac{d^4 \arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a} + \sqrt{c}x^2}\right)}{c^2} + \frac{d^4 \arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a} + \sqrt{c}x^2} + 1\right)}{3c^2} + \frac{c^2d^4}{5c^2}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^4/(a + c\*x^4), x]

[Out] (e^2\*(6\*c\*d^2 - a\*e^2)\*x)/c^2 + (4\*d\*e^3\*x^3)/(3\*c) + (e^4\*x^5)/(5\*c) - ((c^2\*d^4 - 6\*a\*c\*d^2\*e^2 + a^2\*e^4 + 4\*sqrt[a]\*sqrt[c]\*d\*e\*(c\*d^2 - a\*e^2))\*ArcTan[1 - (sqrt[2]\*c^(1/4)\*x)/a^(1/4)]/(2\*sqrt[2]\*a^(3/4)\*c^(9/4)) + ((c^2\*d^4 - 6\*a\*c\*d^2\*e^2 + a^2\*e^4 + 4\*sqrt[a]\*sqrt[c]\*d\*e\*(c\*d^2 - a\*e^2))\*ArcTan[1 + (sqrt[2]\*c^(1/4)\*x)/a^(1/4)]/(2\*sqrt[2]\*a^(3/4)\*c^(9/4)) - ((c^2\*d^4 - 6\*a\*c\*d^2\*e^2 + a^2\*e^4 - 4\*sqrt[a]\*sqrt[c]\*d\*e\*(c\*d^2 - a\*e^2))\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/(4\*sqrt[2]\*a^(3/4)\*c^(9/4)) + ((c^2\*d^4 - 6\*a\*c\*d^2\*e^2 + a^2\*e^4 - 4\*sqrt[a]\*sqrt[c]\*d\*e\*(c\*d^2 - a\*e^2))\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/(4\*sqrt[2]\*a^(3/4)\*c^(9/4))

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 617**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])]; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

**Rule 628**

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

**Rule 1162**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

**Rule 1165**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x],

x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

Rule 1168

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[-(a\*c)]

Rule 1171

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x^2)^q/(a + c\*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[q]

Rubi steps

$$\int \frac{(d + ex^2)^4}{a + cx^4} dx = \int \left( \frac{e^2(6cd^2 - ae^2)}{c^2} + \frac{4de^3x^2}{c} + \frac{e^4x^4}{c} + \frac{c^2d^4 - 6acd^2e^2 + a^2e^4 + 4cde(cd^2 - ae^2)x^2}{c^2(a + cx^4)} \right) dx$$

$$= \frac{e^2(6cd^2 - ae^2)x}{c^2} + \frac{4de^3x^3}{3c} + \frac{e^4x^5}{5c} + \frac{\int \frac{c^2d^4 - 6acd^2e^2 + a^2e^4 + 4cde(cd^2 - ae^2)x^2}{a + cx^4} dx}{c^2}$$

$$= \frac{e^2(6cd^2 - ae^2)x}{c^2} + \frac{4de^3x^3}{3c} + \frac{e^4x^5}{5c} - \frac{\left(4cd^3e - 4ade^3 - \frac{c^2d^4 - 6acd^2e^2 + a^2e^4}{\sqrt{a}\sqrt{c}}\right) \int \frac{\sqrt{a}\sqrt{c} - cx^2}{a + cx^4} dx}{2c^2} + \frac{(4cd^3e - 4ade^3 - \frac{c^2d^4 - 6acd^2e^2 + a^2e^4}{\sqrt{a}\sqrt{c}}) \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt{c}} + 2x}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt{c}} - x^2} dx}{4\sqrt{2}\sqrt[4]{a}c^{7/4}}$$

$$= \frac{e^2(6cd^2 - ae^2)x}{c^2} + \frac{4de^3x^3}{3c} + \frac{e^4x^5}{5c} + \frac{\left(4cd^3e - 4ade^3 - \frac{c^2d^4 - 6acd^2e^2 + a^2e^4}{\sqrt{a}\sqrt{c}}\right) \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\right)}{4\sqrt{2}\sqrt[4]{a}c^{7/4}}$$

$$= \frac{e^2(6cd^2 - ae^2)x}{c^2} + \frac{4de^3x^3}{3c} + \frac{e^4x^5}{5c} - \frac{\left(4cd^3e - 4ade^3 + \frac{c^2d^4 - 6acd^2e^2 + a^2e^4}{\sqrt{a}\sqrt{c}}\right) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}c^{7/4}} +$$

**Mathematica [A]** time = 0.34, size = 444, normalized size = 1.02

160\*a^(3/4)\*c^(1/4)\*e^2\*(-6\*c\*d^2 + a\*e^2)\*x + 160\*a^(3/4)\*c^(5/4)\*d\*e^3\*x^3 + 24\*a^(3/4)\*c^(5/4)\*e^4\*x^5 - 30\*Sqrt[2]\*(c^2\*d^4 + 4\*Sqrt[a]\*c^(3/2)\*d^3\*e - 6\*a\*c\*d^2\*e^2 - 4\*a^(3/2)\*Sqrt[c]\*d\*e^3 + a^2\*e^4)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)] + 30\*Sqrt[2]\*(c^2\*d^4 + 4\*Sqrt[a]\*c^(3/2)\*d^3\*e - 6\*a\*c\*d^2\*e^2 - 4\*a^(3/2)\*Sqrt[c]\*d\*e^3 + a^2\*e^4)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)] - 15\*Sqrt[2]\*(c^2\*d^4 - 4\*Sqrt[a]\*c^(3/2)\*d^3\*e - 6\*a\*c\*d^2\*e^2 + 4\*a^(3/2)\*Sqrt[c]\*d\*e^3 + a^2\*e^4)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2] + 15\*Sqrt[2]\*(c^2\*d^4 - 4\*Sqrt[a]\*c^(3/2)\*d^3\*e - 6\*a\*c\*d^2\*e^2 + 4\*a^(3/2)\*Sqrt[c]\*d\*e^3 + a^2\*e^4)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2]

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^4/(a + c\*x^4), x]

[Out] (-120\*a^(3/4)\*c^(1/4)\*e^2\*(-6\*c\*d^2 + a\*e^2)\*x + 160\*a^(3/4)\*c^(5/4)\*d\*e^3\*x^3 + 24\*a^(3/4)\*c^(5/4)\*e^4\*x^5 - 30\*Sqrt[2]\*(c^2\*d^4 + 4\*Sqrt[a]\*c^(3/2)\*d^3\*e - 6\*a\*c\*d^2\*e^2 - 4\*a^(3/2)\*Sqrt[c]\*d\*e^3 + a^2\*e^4)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)] + 30\*Sqrt[2]\*(c^2\*d^4 + 4\*Sqrt[a]\*c^(3/2)\*d^3\*e - 6\*a\*c\*d^2\*e^2 - 4\*a^(3/2)\*Sqrt[c]\*d\*e^3 + a^2\*e^4)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)] - 15\*Sqrt[2]\*(c^2\*d^4 - 4\*Sqrt[a]\*c^(3/2)\*d^3\*e - 6\*a\*c\*d^2\*e^2 + 4\*a^(3/2)\*Sqrt[c]\*d\*e^3 + a^2\*e^4)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2] + 15\*Sqrt[2]\*(c^2\*d^4 - 4\*Sqrt[a]\*c^(3/2)\*d^3\*e - 6\*a\*c\*d^2\*e^2 + 4\*a^(3/2)\*Sqrt[c]\*d\*e^3 + a^2\*e^4)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2]

$c*d^2*e^2 + 4*a^{(3/2)*\text{Sqrt}[c]*d*e^3 + a^2*e^4)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)*x + \text{Sqrt}[c]*x^2}]/(120*a^{(3/4)*c^{(9/4)})}$

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^2)^4}{a + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)^4/(a + c\*x^4),x]

[Out] IntegrateAlgebraic[(d + e\*x^2)^4/(a + c\*x^4), x]

**fricas [B]** time = 11.04, size = 2878, normalized size = 6.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^4/(c\*x^4+a),x, algorithm="fricas")

[Out]  $\frac{1}{60} * (12*c*e^4*x^5 + 80*c*d*e^3*x^3 + 15*c^2*\text{sqrt}(-(8*c^3*d^7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 + a*c^4*\text{sqrt}(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9))))/(a*c^4)) * \text{log}((c^8*d^16 - 24*a*c^7*d^14*e^2 - 36*a^2*c^6*d^12*e^4 + 88*a^3*c^5*d^10*e^6 + 198*a^4*c^4*d^8*e^8 + 88*a^5*c^3*d^6*e^10 - 36*a^6*c^2*d^4*e^12 - 24*a^7*c*d^2*e^14 + a^8*e^16)*x + (a*c^8*d^12 - 34*a^2*c^7*d^10*e^2 + 239*a^3*c^6*d^8*e^4 - 476*a^4*c^5*d^6*e^6 + 239*a^5*c^4*d^4*e^8 - 34*a^6*c^3*d^2*e^10 + a^7*c^2*e^12 + 4*(a^3*c^8*d^3*e - a^4*c^7*d*e^3)*\text{sqrt}(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))) * \text{sqrt}(-(8*c^3*d^7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 + a*c^4*\text{sqrt}(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9))))/(a*c^4)) - 15*c^2*\text{sqrt}(-(8*c^3*d^7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 + a*c^4*\text{sqrt}(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9))))/(a*c^4)) + 15*c^2*\text{sqrt}(-(8*c^3*d^7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 - a*c^4*\text{sqrt}(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9))))/(a*c^4)) * \text{log}((c^8*d^16 - 24*a*c^7*d^14*e^2 - 36*a^2*c^6*d^12*e^4 + 88*a^3*c^5*d^10*e^6 + 198*a^4*c^4*d^8*e^8 + 88*a^5*c^3*d^6*e^10 - 36*a^6*c^2*d^4*e^12 - 24*a^7*c*d^2*e^14 + a^8*e^16)*x + (a*c^8*d^12 - 34*a^2*c^7*d^10*e^2 + 239*a^3*c^6*d^8*e^4 - 476*a^4*c^5*d^6*e^6 + 239*a^5*c^4*d^4*e^8 - 34*a^6*c^3*d^2*e^10 + a^7*c^2*e^12 - 4*(a^3*c^8*d^3*e - a^4*c^7*d*e^3)*\text{sqrt}(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9))))/(a*c^4)) + 15*c^2*\text{sqrt}(-(8*c^3*d^7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 + a*c^4*\text{sqrt}(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9))))/(a*c^4)) + 15*c^2*\text{sqrt}(-(8*c^3*d^7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 + a*c^4*\text{sqrt}(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9))))/(a*c^4))$

```

4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 +
924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))*sqrt(-(8*c
^3*d^7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 - a*c^4*sqrt(-
(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^
6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 5
6*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c^4))) - 15*c^2*sqrt(-(8*c^3*d^
7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 - a*c^4*sqrt(-(c^8*
d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6
470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7
*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c^4))*log((c^8*d^16 - 24*a*c^7*d^14*
e^2 - 36*a^2*c^6*d^12*e^4 + 88*a^3*c^5*d^10*e^6 + 198*a^4*c^4*d^8*e^8 + 88*
a^5*c^3*d^6*e^10 - 36*a^6*c^2*d^4*e^12 - 24*a^7*c*d^2*e^14 + a^8*e^16)*x -
(a*c^8*d^12 - 34*a^2*c^7*d^10*e^2 + 239*a^3*c^6*d^8*e^4 - 476*a^4*c^5*d^6*e
^6 + 239*a^5*c^4*d^4*e^8 - 34*a^6*c^3*d^2*e^10 + a^7*c^2*e^12 - 4*(a^3*c^8*
d^3*e - a^4*c^7*d*e^3)*sqrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^
12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^
10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))*sqrt(-
(8*c^3*d^7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 - a*c^4*s
qrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^
10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^1
2 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c^4))) + 60*(6*c*d^2*e^2 -
a*e^4)*x)/c^2

```

**giac [A]** time = 0.19, size = 498, normalized size = 1.14

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^4/(c\*x^4+a),x, algorithm="giac")

```

[Out] 1/4*sqrt(2)*((a*c^3)^(1/4)*c^3*d^4 - 6*(a*c^3)^(1/4)*a*c^2*d^2*e^2 + 4*(a*c
^3)^(3/4)*c*d^3*e + (a*c^3)^(1/4)*a^2*c*e^4 - 4*(a*c^3)^(3/4)*a*d*e^3)*arct
an(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 1/4*sqrt(
2)*((a*c^3)^(1/4)*c^3*d^4 - 6*(a*c^3)^(1/4)*a*c^2*d^2*e^2 + 4*(a*c^3)^(3/4)
*c*d^3*e + (a*c^3)^(1/4)*a^2*c*e^4 - 4*(a*c^3)^(3/4)*a*d*e^3)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 1/8*sqrt(2)*((a*c^
3)^(1/4)*c^3*d^4 - 6*(a*c^3)^(1/4)*a*c^2*d^2*e^2 - 4*(a*c^3)^(3/4)*c*d^3*e
+ (a*c^3)^(1/4)*a^2*c*e^4 + 4*(a*c^3)^(3/4)*a*d*e^3)*log(x^2 + sqrt(2)*x*(a
/c)^(1/4) + sqrt(a/c))/(a*c^4) - 1/8*sqrt(2)*((a*c^3)^(1/4)*c^3*d^4 - 6*(a*
c^3)^(1/4)*a*c^2*d^2*e^2 - 4*(a*c^3)^(3/4)*c*d^3*e + (a*c^3)^(1/4)*a^2*c*e^
4 + 4*(a*c^3)^(3/4)*a*d*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(
a*c^4) + 1/15*(3*c^4*x^5*e^4 + 20*c^4*d*x^3*e^3 + 90*c^4*d^2*x*e^2 - 15*a*c
^3*x*e^4)/c^5

```

**maple [B]** time = 0.01, size = 741, normalized size = 1.70

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^4/(c\*x^4+a),x)

```

[Out] 1/5*e^4*x^5/c+4/3*d*e^3*x^3/c-e^4/c^2*a*x+6*e^2/c*d^2*x+1/4/c^2*(a/c)^(1/4)
*a^2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*e^4-3/2/c*(a/c)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^2*e^2+1/4*(a/c)^(1/4)/a^2^(1/2)*arctan(2^(1
/2)/(a/c)^(1/4)*x-1)*d^4+1/8/c^2*(a/c)^(1/4)*a^2^(1/2)*ln((x^2+(a/c)^(1/4)*
x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*e^4-3/4/c*(
a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1
/4)*x*2^(1/2)+(a/c)^(1/2)))*d^2*e^2+1/8*(a/c)^(1/4)/a^2^(1/2)*ln((x^2+(a/c)
^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^4+

```

$$\frac{1}{4}c^{-2}(a/c)^{1/4}a^2e^{1/2}\arctan(2^{1/2}/(a/c)^{1/4}x+1)e^{-4-3/2}c(a/c)^{1/4}2^{1/2}\arctan(2^{1/2}/(a/c)^{1/4}x+1)d^2e^{2+1/4}(a/c)^{1/4}/a^22^{1/2}\arctan(2^{1/2}/(a/c)^{1/4}x+1)d^4-1/2c^2/(a/c)^{1/4}2^{1/2}\ln((x^2-(a/c)^{1/4}x^2+(a/c)^{1/2})/(x^2+(a/c)^{1/4}x^2+(a/c)^{1/2}))a^2d^3e^{3+1/2}c/(a/c)^{1/4}2^{1/2}\ln((x^2-(a/c)^{1/4}x^2+(a/c)^{1/2})/(x^2+(a/c)^{1/4}x^2+(a/c)^{1/2}))d^3e^{-1}c^2/(a/c)^{1/4}2^{1/2}\arctan(2^{1/2}/(a/c)^{1/4}x-1)a^2d^3e^{3+1}c/(a/c)^{1/4}2^{1/2}\arctan(2^{1/2}/(a/c)^{1/4}x-1)d^3e^{-1}c^2/(a/c)^{1/4}2^{1/2}\arctan(2^{1/2}/(a/c)^{1/4}x+1)a^2d^3e^{3+1}c/(a/c)^{1/4}2^{1/2}\arctan(2^{1/2}/(a/c)^{1/4}x+1)d^3e$$

**maxima** [A] time = 2.45, size = 432, normalized size = 0.99

$$\frac{3c^2x^5 + 20cd^2x^3 + 15(6a^2d^2 - ad^4)x}{15c^2} + \frac{2\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{a}\sqrt{c^2d^3e - 6a^3c^{3/2}d^2e^2 - 4a^4c^{3/2}d^2e^3 + a^2\sqrt{c}e^4}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{a}\sqrt{c^2d^3e - 6a^3c^{3/2}d^2e^2 - 4a^4c^{3/2}d^2e^3 + a^2\sqrt{c}e^4}}{2\sqrt{c^2d^3e - 6a^3c^{3/2}d^2e^2 - 4a^4c^{3/2}d^2e^3 + a^2\sqrt{c}e^4}}\right)}{\sqrt{c}\sqrt{d}\sqrt{a}\sqrt{c^2d^3e - 6a^3c^{3/2}d^2e^2 - 4a^4c^{3/2}d^2e^3 + a^2\sqrt{c}e^4}} + \frac{2\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{a}\sqrt{c^2d^3e - 6a^3c^{3/2}d^2e^2 - 4a^4c^{3/2}d^2e^3 + a^2\sqrt{c}e^4}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{a}\sqrt{c^2d^3e - 6a^3c^{3/2}d^2e^2 - 4a^4c^{3/2}d^2e^3 + a^2\sqrt{c}e^4}}{2\sqrt{c^2d^3e - 6a^3c^{3/2}d^2e^2 - 4a^4c^{3/2}d^2e^3 + a^2\sqrt{c}e^4}}\right)}{\sqrt{c}\sqrt{d}\sqrt{a}\sqrt{c^2d^3e - 6a^3c^{3/2}d^2e^2 - 4a^4c^{3/2}d^2e^3 + a^2\sqrt{c}e^4}} + \frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{a}\sqrt{c^2d^3e - 6a^3c^{3/2}d^2e^2 - 4a^4c^{3/2}d^2e^3 + a^2\sqrt{c}e^4}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{a}\sqrt{c^2d^3e - 6a^3c^{3/2}d^2e^2 - 4a^4c^{3/2}d^2e^3 + a^2\sqrt{c}e^4}}{2\sqrt{c^2d^3e - 6a^3c^{3/2}d^2e^2 - 4a^4c^{3/2}d^2e^3 + a^2\sqrt{c}e^4}}\right)}{\sqrt{c}\sqrt{d}\sqrt{a}\sqrt{c^2d^3e - 6a^3c^{3/2}d^2e^2 - 4a^4c^{3/2}d^2e^3 + a^2\sqrt{c}e^4}} - \frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{a}\sqrt{c^2d^3e - 6a^3c^{3/2}d^2e^2 - 4a^4c^{3/2}d^2e^3 + a^2\sqrt{c}e^4}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{a}\sqrt{c^2d^3e - 6a^3c^{3/2}d^2e^2 - 4a^4c^{3/2}d^2e^3 + a^2\sqrt{c}e^4}}{2\sqrt{c^2d^3e - 6a^3c^{3/2}d^2e^2 - 4a^4c^{3/2}d^2e^3 + a^2\sqrt{c}e^4}}\right)}{\sqrt{c}\sqrt{d}\sqrt{a}\sqrt{c^2d^3e - 6a^3c^{3/2}d^2e^2 - 4a^4c^{3/2}d^2e^3 + a^2\sqrt{c}e^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+a), x, algorithm="maxima")
```

```
[Out] 1/15*(3*c*e^4*x^5 + 20*c*d*e^3*x^3 + 15*(6*c*d^2*e^2 - a*e^4)*x)/c^2 + 1/8*(2*sqrt(2)*(c^(5/2)*d^4 + 4*sqrt(a)*c^2*d^3*e - 6*a*c^(3/2)*d^2*e^2 - 4*a^(3/2)*c*d*e^3 + a^2*sqrt(c)*e^4)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c) + 2*sqrt(2)*(c^(5/2)*d^4 + 4*sqrt(a)*c^2*d^3*e - 6*a*c^(3/2)*d^2*e^2 - 4*a^(3/2)*c*d*e^3 + a^2*sqrt(c)*e^4)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c) + sqrt(2)*(c^(5/2)*d^4 - 4*sqrt(a)*c^2*d^3*e - 6*a*c^(3/2)*d^2*e^2 + 4*a^(3/2)*c*d*e^3 + a^2*sqrt(c)*e^4)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(c^(5/2)*d^4 - 4*sqrt(a)*c^2*d^3*e - 6*a*c^(3/2)*d^2*e^2 + 4*a^(3/2)*c*d*e^3 + a^2*sqrt(c)*e^4)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4))/c^2
```

**mupad** [B] time = 5.08, size = 4022, normalized size = 9.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^4/(a + c*x^4), x)
```

```
[Out] atan((((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c*d^2*e^6 + 70*a^2*c^2*d^4*e^4))/c - (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^2*c^5*d^2*e^2)*(a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8*a^5*c^5*d*e^7 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))/c^3)*((a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8*a^5*c^5*d*e^7 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))*i1 + ((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c*d^2*e^6 + 70*a^2*c^2*d^4*e^4))/c + (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^2*c^5*d^2*e^2)*(a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8*a^5*c^5*d*e^7 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))/c^3)*((a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8*a^5*c^5*d*e^7 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))*i1)/((((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c*d^2*e^6 + 70*a^2*c^2*d^4*e^4))/c - (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^2*c^5*d^2*e^2)*(a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8*a^5*c^5*d*e^7 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))/c^3)*((a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8*a^5*c^5*d*e^7 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))*i1 + ((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c*d^2*e^6 + 70*a^2*c^2*d^4*e^4))/c + (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^2*c^5*d^2*e^2)*(a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8*a^5*c^5*d*e^7 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))/c^3)*((a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8*a^5*c^5*d*e^7 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))*i1)/c^2
```





$$56*a^3*c^7*d^5*e^3 + 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^{(1/2)} - 28*a^3*c*d^2*e^6*(-a^3*c^9)^{(1/2)} + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^{(1/2)} / ((16*a^3*c^9)^{(1/2)}*2i + (e^4*x^5)/(5*c) + (4*d*e^3*x^3)/(3*c))$$

**sympy [A]** time = 3.75, size = 500, normalized size = 1.14

$$\left( \frac{e^4}{5c} + \frac{4d^3e^3}{3c} \right) + \text{RootSum} \left( 256t^4a^3c^9 + t^2(-256a^5c^5d^7e^7 + 1792a^4c^6d^3e^5 - 1792a^3c^7d^5e^3 + 256a^2c^8d^7e) + a^8e^{16} + 8a^7c^2d^2e^{14} + 28a^6c^3d^4e^{12} + 56a^5c^5d^6e^{10} + 70a^4c^4d^8e^8 + 56a^3c^5d^{10}e^6 + 28a^2c^6d^{12}e^4 + 8ac^7d^{14}e^2 + c^8d^{16} \right), \text{Lambda}(t, t \log(x + \frac{256t^3a^4c^7d^3e^3 - 256t^3a^3c^8d^3e + 4t^2a^7c^2e^{12} - 264t^2a^6c^3d^2e^{10} + 1980t^2a^5c^4d^4e^8 - 3696t^2a^4c^5d^6e^6 + 1980t^2a^3c^6d^8e^4 - 264t^2a^2c^7d^{10}e^2 + 4t^2a^c^8d^{12}}{a^8e^{16} - 24a^7c^2d^2e^{14} - 36a^6c^3d^4e^{12} + 88a^5c^3d^6e^{10} + 198a^4c^4d^8e^8 + 88a^3c^5d^{10}e^6 - 36a^2c^6d^{12}e^4 - 24ac^7d^{14}e^2 + c^8d^{16}})) + 4d^3e^3x^3/(3c) + e^4x^5/(5c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*4/(c\*x\*\*4+a), x)

[Out] x\*(-a\*e\*\*4/c\*\*2 + 6\*d\*\*2\*e\*\*2/c) + RootSum(256\*\_t\*\*4\*a\*\*3\*c\*\*9 + \_t\*\*2\*(-256\*a\*\*5\*c\*\*5\*d\*\*7\*e\*\*7 + 1792\*a\*\*4\*c\*\*6\*d\*\*3\*e\*\*5 - 1792\*a\*\*3\*c\*\*7\*d\*\*5\*e\*\*3 + 256\*a\*\*2\*c\*\*8\*d\*\*7\*e) + a\*\*8\*e\*\*16 + 8\*a\*\*7\*c\*\*2\*d\*\*2\*e\*\*14 + 28\*a\*\*6\*c\*\*3\*d\*\*4\*e\*\*12 + 56\*a\*\*5\*c\*\*5\*d\*\*6\*e\*\*10 + 70\*a\*\*4\*c\*\*4\*d\*\*8\*e\*\*8 + 56\*a\*\*3\*c\*\*5\*d\*\*10\*e\*\*6 + 28\*a\*\*2\*c\*\*6\*d\*\*12\*e\*\*4 + 8\*a\*c\*\*7\*d\*\*14\*e\*\*2 + c\*\*8\*d\*\*16, Lambda(\_t, \_t\*log(x + (256\*\_t\*\*3\*a\*\*4\*c\*\*7\*d\*\*3\*e\*\*3 - 256\*\_t\*\*3\*a\*\*3\*c\*\*8\*d\*\*3\*e + 4\*\_t\*a\*\*7\*c\*\*2\*e\*\*12 - 264\*\_t\*a\*\*6\*c\*\*3\*d\*\*2\*e\*\*10 + 1980\*\_t\*a\*\*5\*c\*\*4\*d\*\*4\*e\*\*8 - 3696\*\_t\*a\*\*4\*c\*\*5\*d\*\*6\*e\*\*6 + 1980\*\_t\*a\*\*3\*c\*\*6\*d\*\*8\*e\*\*4 - 264\*\_t\*a\*\*2\*c\*\*7\*d\*\*10\*e\*\*2 + 4\*\_t\*a\*\*c\*\*8\*d\*\*12)/(a\*\*8\*e\*\*16 - 24\*a\*\*7\*c\*\*2\*d\*\*2\*e\*\*14 - 36\*a\*\*6\*c\*\*3\*d\*\*4\*e\*\*12 + 88\*a\*\*5\*c\*\*3\*d\*\*6\*e\*\*10 + 198\*a\*\*4\*c\*\*4\*d\*\*8\*e\*\*8 + 88\*a\*\*3\*c\*\*5\*d\*\*10\*e\*\*6 - 36\*a\*\*2\*c\*\*6\*d\*\*12\*e\*\*4 - 24\*a\*c\*\*7\*d\*\*14\*e\*\*2 + c\*\*8\*d\*\*16)))) + 4\*d\*e\*\*3\*x\*\*3/(3\*c) + e\*\*4\*x\*\*5/(5\*c)

$$3.119 \quad \int \frac{(d+ex^2)^3}{a+cx^4} dx$$

**Optimal.** Leaf size=370

$$\frac{(\sqrt{c}d(cd^2 - 3ae^2) - \sqrt{a}e(3cd^2 - ae^2)) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}c^{7/4}} + \frac{(\sqrt{c}d(cd^2 - 3ae^2) - \sqrt{a}e(3cd^2 - ae^2)) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}c^{7/4}}$$

**Rubi [A]** time = 0.50, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, number of rules / integrand size = 0.368, Rules used = {1171, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(\sqrt{c}d(cd^2 - 3ae^2) - \sqrt{a}e(3cd^2 - ae^2)) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}c^{7/4}} + \frac{(\sqrt{c}d(cd^2 - 3ae^2) - \sqrt{a}e(3cd^2 - ae^2)) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}c^{7/4}} - \frac{(\sqrt{c}d(cd^2 - 3ae^2) + \sqrt{a}e(3cd^2 - ae^2)) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{a}}{\sqrt[4]{c}}\right)}{2\sqrt{2}a^{3/4}c^{7/4}} + \frac{(\sqrt{c}d(cd^2 - 3ae^2) + \sqrt{a}e(3cd^2 - ae^2)) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{a}}{\sqrt[4]{c}} + 1\right)}{2\sqrt{2}a^{3/4}c^{7/4}} + \frac{3de^2x}{c} + \frac{e^3x^3}{3c}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^3/(a + c\*x^4), x]

[Out] (3\*d\*e^2\*x)/c + (e^3\*x^3)/(3\*c) - ((Sqrt[c]\*d\*(c\*d^2 - 3\*a\*e^2) + Sqrt[a]\*e\*(3\*c\*d^2 - a\*e^2))\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)]/(2\*Sqrt[2]\*a^(3/4)\*c^(7/4)) + ((Sqrt[c]\*d\*(c\*d^2 - 3\*a\*e^2) + Sqrt[a]\*e\*(3\*c\*d^2 - a\*e^2))\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)]/(2\*Sqrt[2]\*a^(3/4)\*c^(7/4)) - ((Sqrt[c]\*d\*(c\*d^2 - 3\*a\*e^2) - Sqrt[a]\*e\*(3\*c\*d^2 - a\*e^2))\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/(4\*Sqrt[2]\*a^(3/4)\*c^(7/4)) + ((Sqrt[c]\*d\*(c\*d^2 - 3\*a\*e^2) - Sqrt[a]\*e\*(3\*c\*d^2 - a\*e^2))\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/(4\*Sqrt[2]\*a^(3/4)\*c^(7/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 1162

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

#### Rule 1165

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

Rule 1168

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[-(a\*c)]

Rule 1171

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x^2)^q/(a + c\*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[q]

Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)^3}{a + cx^4} dx &= \int \left( \frac{3de^2}{c} + \frac{e^3x^2}{c} + \frac{cd^3 - 3ade^2 + e(3cd^2 - ae^2)x^2}{c(a + cx^4)} \right) dx \\ &= \frac{3de^2x}{c} + \frac{e^3x^3}{3c} + \frac{\int \frac{cd^3 - 3ade^2 + e(3cd^2 - ae^2)x^2}{a + cx^4} dx}{c} \\ &= \frac{3de^2x}{c} + \frac{e^3x^3}{3c} - \frac{\left(3cd^2e - ae^3 - \frac{\sqrt{c}d(cd^2 - 3ae^2)}{\sqrt{a}}\right) \int \frac{\sqrt{a}\sqrt{c} - cx^2}{a + cx^4} dx}{2c^2} + \frac{\left(3cd^2e - ae^3 + \frac{\sqrt{c}d(cd^2 - 3ae^2)}{\sqrt{a}}\right)}{2c^2} \\ &= \frac{3de^2x}{c} + \frac{e^3x^3}{3c} + \frac{\left(3cd^2e - ae^3 - \frac{\sqrt{c}d(cd^2 - 3ae^2)}{\sqrt{a}}\right) \int \frac{\frac{\sqrt{2}\sqrt[4]{a} + 2x}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt{c}} - x^2}{a + cx^4} dx}{4\sqrt{2}\sqrt[4]{a}c^{7/4}} + \frac{\left(3cd^2e - ae^3 - \frac{\sqrt{c}d(cd^2 - 3ae^2)}{\sqrt{a}}\right)}{4\sqrt{2}\sqrt[4]{a}c^{7/4}} \\ &= \frac{3de^2x}{c} + \frac{e^3x^3}{3c} + \frac{\left(3cd^2e - ae^3 - \frac{\sqrt{c}d(cd^2 - 3ae^2)}{\sqrt{a}}\right) \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}c^{7/4}} - \frac{\left(3cd^2e - ae^3 + \frac{\sqrt{c}d(cd^2 - 3ae^2)}{\sqrt{a}}\right) \log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}c^{7/4}} \\ &= \frac{3de^2x}{c} + \frac{e^3x^3}{3c} - \frac{\left(3cd^2e - ae^3 + \frac{\sqrt{c}d(cd^2 - 3ae^2)}{\sqrt{a}}\right) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}c^{7/4}} + \frac{\left(3cd^2e - ae^3 + \frac{\sqrt{c}d(cd^2 - 3ae^2)}{\sqrt{a}}\right) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}c^{7/4}} \end{aligned}$$

**Mathematica [A]** time = 0.28, size = 360, normalized size = 0.97

$-3\sqrt{2}(a^2e^2 - 3\sqrt{a}cd^2e - 3a\sqrt{c}de^2 + e^2d^2)\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2) + 3\sqrt{2}(a^2e^2 - 3\sqrt{a}cd^2e - 3a\sqrt{c}de^2 + e^2d^2)\log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2) + 6\sqrt{2}(a^2e^2 - 3\sqrt{a}cd^2e + 3a\sqrt{c}de^2 - e^2d^2)\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) + 6\sqrt{2}(-a^2e^2 + 3\sqrt{a}cd^2e - 3a\sqrt{c}de^2 + e^2d^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right) + 72a^{3/4}\sqrt[4]{a}d^2e + 8a^{3/4}\sqrt[4]{a}e^2d^2$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^3/(a + c\*x^4), x]

[Out] (72\*a^(3/4)\*c^(3/4)\*d\*e^2\*x + 8\*a^(3/4)\*c^(3/4)\*e^3\*x^3 + 6\*Sqrt[2]\*(-(c^(3/2)\*d^3) - 3\*Sqrt[a]\*c\*d^2\*e + 3\*a\*Sqrt[c]\*d\*e^2 + a^(3/2)\*e^3)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)] + 6\*Sqrt[2]\*(c^(3/2)\*d^3 + 3\*Sqrt[a]\*c\*d^2\*e - 3\*a\*Sqrt[c]\*d\*e^2 - a^(3/2)\*e^3)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)] - 3\*Sqrt[2]\*(c^(3/2)\*d^3 - 3\*Sqrt[a]\*c\*d^2\*e - 3\*a\*Sqrt[c]\*d\*e^2 + a^(3/2)\*e^3)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2] + 3\*Sqrt[2]\*(c^(3/2)\*d^3 - 3\*Sqrt[a]\*c\*d^2\*e - 3\*a\*Sqrt[c]\*d\*e^2 + a^(3/2)\*e^3)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2]/(24\*a^(3/4)\*c^(7/4))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^2)^3}{a + cx^4} dx$$



**giac** [A] time = 0.21, size = 405, normalized size = 1.09

$$\frac{c^2 d^2 e^3 + 9 d^2 e^2 x}{3c} + \frac{\sqrt{2} \left( (ac)^{\frac{3}{4}} c^2 d^2 e^3 - 3 (ac)^{\frac{3}{4}} ac^2 d^2 e^2 + 3 (ac)^{\frac{3}{4}} ac^2 d e^2 - (ac)^{\frac{3}{4}} ac^2 \right) \arctan\left(\frac{\sqrt{2} \sqrt{c} \sqrt{d} \sqrt{e}}{2 \sqrt{a}}\right)}{4ac^4} + \frac{\sqrt{2} \left( (ac)^{\frac{3}{4}} c^2 d^2 e^3 - 3 (ac)^{\frac{3}{4}} ac^2 d^2 e^2 + 3 (ac)^{\frac{3}{4}} ac^2 d e^2 - (ac)^{\frac{3}{4}} ac^2 \right) \arctan\left(\frac{\sqrt{2} \sqrt{c} \sqrt{d} \sqrt{e}}{2 \sqrt{a}}\right)}{4ac^4} + \frac{\sqrt{2} \left( (ac)^{\frac{3}{4}} c^2 d^2 e^3 - 3 (ac)^{\frac{3}{4}} ac^2 d^2 e^2 + 3 (ac)^{\frac{3}{4}} ac^2 d e^2 - (ac)^{\frac{3}{4}} ac^2 \right) \log\left(\sqrt{c^2 + \sqrt{2} a} \sqrt{d^2 + \sqrt{2} a} + \sqrt{e}\right)}{8ac^4} + \frac{\sqrt{2} \left( (ac)^{\frac{3}{4}} c^2 d^2 e^3 - 3 (ac)^{\frac{3}{4}} ac^2 d^2 e^2 + 3 (ac)^{\frac{3}{4}} ac^2 d e^2 - (ac)^{\frac{3}{4}} ac^2 \right) \log\left(\sqrt{c^2 - \sqrt{2} a} \sqrt{d^2 + \sqrt{2} a} + \sqrt{e}\right)}{8ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^3/(c\*x^4+a),x, algorithm="giac")

[Out] 1/3\*(c^2\*x^3\*e^3 + 9\*c^2\*d\*x\*e^2)/c^3 + 1/4\*sqrt(2)\*((a\*c^3)^(1/4)\*c^3\*d^3 - 3\*(a\*c^3)^(1/4)\*a\*c^2\*d\*e^2 + 3\*(a\*c^3)^(3/4)\*c\*d^2\*e - (a\*c^3)^(3/4)\*a\*e^3)\*arctan(1/2\*sqrt(2)\*(2\*x + sqrt(2)\*(a/c)^(1/4))/(a/c)^(1/4))/(a\*c^4) + 1/4\*sqrt(2)\*((a\*c^3)^(1/4)\*c^3\*d^3 - 3\*(a\*c^3)^(1/4)\*a\*c^2\*d\*e^2 + 3\*(a\*c^3)^(3/4)\*c\*d^2\*e - (a\*c^3)^(3/4)\*a\*e^3)\*arctan(1/2\*sqrt(2)\*(2\*x - sqrt(2)\*(a/c)^(1/4))/(a/c)^(1/4))/(a\*c^4) + 1/8\*sqrt(2)\*((a\*c^3)^(1/4)\*c^3\*d^3 - 3\*(a\*c^3)^(1/4)\*a\*c^2\*d\*e^2 - 3\*(a\*c^3)^(3/4)\*c\*d^2\*e + (a\*c^3)^(3/4)\*a\*e^3)\*log(x^2 + sqrt(2)\*x\*(a/c)^(1/4) + sqrt(a/c))/(a\*c^4) - 1/8\*sqrt(2)\*((a\*c^3)^(1/4)\*c^3\*d^3 - 3\*(a\*c^3)^(1/4)\*a\*c^2\*d\*e^2 - 3\*(a\*c^3)^(3/4)\*c\*d^2\*e + (a\*c^3)^(3/4)\*a\*e^3)\*log(x^2 - sqrt(2)\*x\*(a/c)^(1/4) + sqrt(a/c))/(a\*c^4)

**maple** [A] time = 0.00, size = 572, normalized size = 1.55

$$\frac{c^2 d^2 e^3 + 9 d^2 e^2 x}{3c} + \frac{\sqrt{2} c^2 d^2 e^3 \arctan\left(\frac{\sqrt{2} c}{2 \sqrt{a}}\right)}{4 (c^2)^{\frac{3}{2}}} + \frac{\sqrt{2} c^2 d^2 e^2 \arctan\left(\frac{\sqrt{2} c}{2 \sqrt{a}}\right)}{4 (c^2)^{\frac{3}{2}}} + \frac{\sqrt{2} c^2 d^2 e \arctan\left(\frac{\sqrt{2} c}{2 \sqrt{a}}\right)}{8 (c^2)^{\frac{3}{2}}} + \frac{(c^2)^{\frac{3}{2}} \sqrt{2} d^2 e^3 \arctan\left(\frac{\sqrt{2} c}{2 \sqrt{a}}\right)}{4c} + \frac{(c^2)^{\frac{3}{2}} \sqrt{2} d^2 e^2 \arctan\left(\frac{\sqrt{2} c}{2 \sqrt{a}}\right)}{4c} + \frac{(c^2)^{\frac{3}{2}} \sqrt{2} d^2 e \arctan\left(\frac{\sqrt{2} c}{2 \sqrt{a}}\right)}{8c} + \frac{3 \sqrt{2} c^2 d^2 e^3 \arctan\left(\frac{\sqrt{2} c}{2 \sqrt{a}}\right)}{4 (c^2)^{\frac{3}{2}}} + \frac{3 \sqrt{2} c^2 d^2 e^2 \arctan\left(\frac{\sqrt{2} c}{2 \sqrt{a}}\right)}{4 (c^2)^{\frac{3}{2}}} + \frac{3 \sqrt{2} c^2 d^2 e \arctan\left(\frac{\sqrt{2} c}{2 \sqrt{a}}\right)}{8 (c^2)^{\frac{3}{2}}} + \frac{3 (c^2)^{\frac{3}{2}} \sqrt{2} d^2 e^3 \arctan\left(\frac{\sqrt{2} c}{2 \sqrt{a}}\right)}{4c} + \frac{3 (c^2)^{\frac{3}{2}} \sqrt{2} d^2 e^2 \arctan\left(\frac{\sqrt{2} c}{2 \sqrt{a}}\right)}{4c} + \frac{3 (c^2)^{\frac{3}{2}} \sqrt{2} d^2 e \arctan\left(\frac{\sqrt{2} c}{2 \sqrt{a}}\right)}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^3/(c\*x^4+a),x)

[Out] 1/3\*e^3\*x^3/c+3\*d\*e^2\*x/c-3/4/c\*(a/c)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/c)^(1/4)\*x-1)\*d\*e^2+1/4\*(a/c)^(1/4)/a\*2^(1/2)\*arctan(2^(1/2)/(a/c)^(1/4)\*x-1)\*d^3-3/8/c\*(a/c)^(1/4)\*2^(1/2)\*ln((x^2+(a/c)^(1/4)\*2^(1/2)\*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)\*2^(1/2)\*x+(a/c)^(1/2)))\*d\*e^2+1/8\*(a/c)^(1/4)/a\*2^(1/2)\*ln((x^2+(a/c)^(1/4)\*2^(1/2)\*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)\*2^(1/2)\*x+(a/c)^(1/2)))\*d^3-3/4/c\*(a/c)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/c)^(1/4)\*x+1)\*d\*e^2+1/4\*(a/c)^(1/4)/a\*2^(1/2)\*arctan(2^(1/2)/(a/c)^(1/4)\*x+1)\*d^3-1/8/c^2/(a/c)^(1/4)\*2^(1/2)\*ln((x^2-(a/c)^(1/4)\*2^(1/2)\*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)\*2^(1/2)\*x+(a/c)^(1/2)))\*a\*e^3+3/8/c/(a/c)^(1/4)\*2^(1/2)\*ln((x^2-(a/c)^(1/4)\*2^(1/2)\*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)\*2^(1/2)\*x+(a/c)^(1/2)))\*d^2\*e-1/4/c^2/(a/c)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/c)^(1/4)\*x-1)\*a\*e^3+3/4/c/(a/c)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/c)^(1/4)\*x-1)\*d^2\*e-1/4/c^2/(a/c)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/c)^(1/4)\*x+1)\*a\*e^3+3/4/c/(a/c)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/c)^(1/4)\*x+1)\*d^2\*e

**maxima** [A] time = 2.48, size = 342, normalized size = 0.92

$$\frac{e^3 x^3 + 9 d e^2 x}{3c} + \frac{2 \sqrt{2} \left( \frac{3}{2} d^2 e^3 + 3 \sqrt{a} c d^2 e^2 - 3 a \sqrt{c} d e^2 + a^{\frac{3}{2}} e^3 \right) \arctan\left(\frac{\sqrt{2} \sqrt{c} \sqrt{d} \sqrt{e}}{2 \sqrt{a}}\right)}{\sqrt{a} \sqrt{c} \sqrt{d} \sqrt{e}} + \frac{2 \sqrt{2} \left( \frac{3}{2} d^2 e^3 + 3 \sqrt{a} c d^2 e^2 - 3 a \sqrt{c} d e^2 + a^{\frac{3}{2}} e^3 \right) \arctan\left(\frac{\sqrt{2} \sqrt{c} \sqrt{d} \sqrt{e}}{2 \sqrt{a}}\right)}{\sqrt{a} \sqrt{c} \sqrt{d} \sqrt{e}} + \frac{\sqrt{2} \left( \frac{3}{2} d^2 e^3 - 3 \sqrt{a} c d^2 e^2 - 3 a \sqrt{c} d e^2 + a^{\frac{3}{2}} e^3 \right) \log\left(\sqrt{c^2 + \sqrt{2} a} \sqrt{d^2 + \sqrt{2} a} + \sqrt{e}\right)}{a^{\frac{3}{2}} \sqrt{c}} + \frac{\sqrt{2} \left( \frac{3}{2} d^2 e^3 - 3 \sqrt{a} c d^2 e^2 - 3 a \sqrt{c} d e^2 + a^{\frac{3}{2}} e^3 \right) \log\left(\sqrt{c^2 - \sqrt{2} a} \sqrt{d^2 + \sqrt{2} a} + \sqrt{e}\right)}{a^{\frac{3}{2}} \sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^3/(c\*x^4+a),x, algorithm="maxima")

[Out] 1/3\*(e^3\*x^3 + 9\*d\*e^2\*x)/c + 1/8\*(2\*sqrt(2)\*(c^(3/2)\*d^3 + 3\*sqrt(a)\*c\*d^2\*e - 3\*a\*sqrt(c)\*d\*e^2 - a^(3/2)\*e^3)\*arctan(1/2\*sqrt(2)\*(2\*sqrt(c)\*x + sqrt(2)\*a^(1/4)\*c^(1/4))/sqrt(sqrt(a)\*sqrt(c)))/sqrt(a)\*sqrt(sqrt(a)\*sqrt(c))\*sqrt(c) + 2\*sqrt(2)\*(c^(3/2)\*d^3 + 3\*sqrt(a)\*c\*d^2\*e - 3\*a\*sqrt(c)\*d\*e^2 - a^(3/2)\*e^3)\*arctan(1/2\*sqrt(2)\*(2\*sqrt(c)\*x - sqrt(2)\*a^(1/4)\*c^(1/4))/sqrt(sqrt(a)\*sqrt(c)))/sqrt(a)\*sqrt(sqrt(a)\*sqrt(c))\*sqrt(c) + sqrt(2)\*(c^(3/2)\*d^3 - 3\*sqrt(a)\*c\*d^2\*e - 3\*a\*sqrt(c)\*d\*e^2 + a^(3/2)\*e^3)\*log(sqrt(c)\*x^2 + sqrt(2)\*a^(1/4)\*c^(1/4)\*x + sqrt(a))/(a^(3/4)\*c^(3/4)) - sqrt(2)\*(c^(3/2)\*d^3 - 3\*sqrt(a)\*c\*d^2\*e - 3\*a\*sqrt(c)\*d\*e^2 + a^(3/2)\*e^3)\*log(sqrt(c)\*x^2 - sqrt(2)\*a^(1/4)\*c^(1/4)\*x + sqrt(a))/(a^(3/4)\*c^(3/4))/c



$$\begin{aligned} & )) / (16*c^7) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) + (d^6*(-a^3*c^7)^{(1/2)}) / (16*a^3*c^4) + (15*d^2*e^4*(-a^3*c^7)^{(1/2)}) / (16*a*c^6) - (15*d^4*e^2*(-a^3*c^7)^{(1/2)}) / (16*a^2*c^5))^{(1/2)} * 120i / (6*c^2*d^8*e + (2*a^4*e^9)/c^2 + 120*a^2*d^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 - (2*d^9*(-a^3*c^7)^{(1/2)}) / (a^2*c) - (120*d^5*e^4*(-a^3*c^7)^{(1/2)}) / c^3 + (92*a*d^3*e^6*(-a^3*c^7)^{(1/2)}) / c^4 - (6*a^2*d*e^8*(-a^3*c^7)^{(1/2)}) / c^5 + (36*d^7*e^2*(-a^3*c^7)^{(1/2)}) / (a*c^2)) * (-a^3*e^6*(-a^3*c^7)^{(1/2)} - c^3*d^6*(-a^3*c^7)^{(1/2)} + 6*a^2*c^6*d^5*e + 6*a^4*c^4*d*e^5 - 20*a^3*c^5*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^7)^{(1/2)} - 15*a^2*c*d^2*e^4*(-a^3*c^7)^{(1/2)}) / (16*a^3*c^7))^{(1/2)} * 2i + (3*d*e^2*x)/c \end{aligned}$$

**sympy [A]** time = 2.27, size = 350, normalized size = 0.95

RootSum  $\left( 256t^4a^3c^7 + t^2(192a^4c^4d^5e - 640a^3c^5d^3e^3 + 192a^2c^6d^2e^5) + a^6e^{12} + 6a^5c^2d^2e^{10} + 15a^4c^3d^3e^8 + 20a^3c^4d^4e^6 + 15a^2c^5d^5e^4 + 6a^2c^6d^6e^2 + c^6d^{12} \left( t + t \log \left( x + \frac{-64t^3a^3c^3e^3 + 192t^2a^4c^4d^2e - 36t^2c^2d^2e^3 + 336t^2c^3d^3e^5 - 504t^2c^4d^4e^7 + 144t^2c^5d^5e^9 - 4t^2c^6d^6e^{11}}{d^6e^{12} - 12a^5c^2d^2e^{10} - 27a^4c^3d^3e^8 + 27a^3c^4d^4e^6 + 12a^2c^5d^5e^4 - c^6d^{12}} \right) \right) \right) + \frac{3d^2x}{c} + \frac{c^3}{3c}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*3/(c\*x\*\*4+a),x)

[Out] RootSum(256\*\_t\*\*4\*a\*\*3\*c\*\*7 + \_t\*\*2\*(192\*a\*\*4\*c\*\*4\*d\*\*5\*e - 640\*a\*\*3\*c\*\*5\*d\*\*3\*e\*\*3 + 192\*a\*\*2\*c\*\*6\*d\*\*5\*e) + a\*\*6\*e\*\*12 + 6\*a\*\*5\*c\*d\*\*2\*e\*\*10 + 15\*a\*\*4\*c\*\*2\*d\*\*4\*e\*\*8 + 20\*a\*\*3\*c\*\*3\*d\*\*6\*e\*\*6 + 15\*a\*\*2\*c\*\*4\*d\*\*8\*e\*\*4 + 6\*a\*c\*\*5\*d\*\*10\*e\*\*2 + c\*\*6\*d\*\*12, Lambda(\_t, \_t\*log(x + (-64\*\_t\*\*3\*a\*\*4\*c\*\*5\*e\*\*3 + 192\*\_t\*\*3\*a\*\*3\*c\*\*6\*d\*\*2\*e - 36\*\_t\*a\*\*5\*c\*\*2\*d\*\*e\*\*8 + 336\*\_t\*a\*\*4\*c\*\*3\*d\*\*3\*e\*\*6 - 504\*\_t\*a\*\*3\*c\*\*4\*d\*\*5\*e\*\*4 + 144\*\_t\*a\*\*2\*c\*\*5\*d\*\*7\*e\*\*2 - 4\*\_t\*a\*c\*\*6\*d\*\*9)/(a\*\*6\*e\*\*12 - 12\*a\*\*5\*c\*d\*\*2\*e\*\*10 - 27\*a\*\*4\*c\*\*2\*d\*\*4\*e\*\*8 + 27\*a\*\*2\*c\*\*4\*d\*\*8\*e\*\*4 + 12\*a\*c\*\*5\*d\*\*10\*e\*\*2 - c\*\*6\*d\*\*12)))) + 3\*d\*e\*\*2\*x/c + e\*\*3\*x\*\*3/(3\*c)

$$3.120 \quad \int \frac{(d+ex^2)^2}{a+cx^4} dx$$

**Optimal.** Leaf size=297

$$\frac{(-2\sqrt{a}\sqrt{c}de - ae^2 + cd^2) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}c^{5/4}} + \frac{(-2\sqrt{a}\sqrt{c}de - ae^2 + cd^2) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a})}{4\sqrt{2}a^{3/4}c^{5/4}}$$

**Rubi [A]** time = 0.29, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {1171, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(-2\sqrt{a}\sqrt{c}de - ae^2 + cd^2) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}c^{5/4}} + \frac{(-2\sqrt{a}\sqrt{c}de - ae^2 + cd^2) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a})}{4\sqrt{2}a^{3/4}c^{5/4}} - \frac{(2\sqrt{a}\sqrt{c}de - ae^2 + cd^2) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{5/4}} + \frac{(2\sqrt{a}\sqrt{c}de - ae^2 + cd^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2}a^{3/4}c^{5/4}} + \frac{e^2x}{c}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^2/(a + c\*x^4), x]

[Out] (e^2\*x)/c - ((c\*d^2 + 2\*Sqrt[a]\*Sqrt[c]\*d\*e - a\*e^2)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)]/(2\*Sqrt[2]\*a^(3/4)\*c^(5/4)) + ((c\*d^2 + 2\*Sqrt[a]\*Sqrt[c]\*d\*e - a\*e^2)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)]/(2\*Sqrt[2]\*a^(3/4)\*c^(5/4)) - ((c\*d^2 - 2\*Sqrt[a]\*Sqrt[c]\*d\*e - a\*e^2)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/(4\*Sqrt[2]\*a^(3/4)\*c^(5/4)) + ((c\*d^2 - 2\*Sqrt[a]\*Sqrt[c]\*d\*e - a\*e^2)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/(4\*Sqrt[2]\*a^(3/4)\*c^(5/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 1162

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

#### Rule 1165

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]



Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] := Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)^2}{a + cx^4} dx &= \int \left( \frac{e^2}{c} + \frac{cd^2 - ae^2 + 2cdex^2}{c(a + cx^4)} \right) dx \\ &= \frac{e^2 x}{c} + \frac{\int \frac{cd^2 - ae^2 + 2cdex^2}{a + cx^4} dx}{c} \\ &= \frac{e^2 x}{c} + \frac{(cd^2 - 2\sqrt{a}\sqrt{c}de - ae^2) \int \frac{\sqrt{a}\sqrt{c} - cx^2}{a + cx^4} dx}{2\sqrt{a}c^{3/2}} + \frac{(cd^2 + 2\sqrt{a}\sqrt{c}de - ae^2) \int \frac{\sqrt{a}\sqrt{c} + cx^2}{a + cx^4} dx}{2\sqrt{a}c^{3/2}} \\ &= \frac{e^2 x}{c} - \frac{(cd^2 - 2\sqrt{a}\sqrt{c}de - ae^2) \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt{c}} + 2x}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt{c}} - x^2} dx}{4\sqrt{2}a^{3/4}c^{5/4}} - \frac{(cd^2 - 2\sqrt{a}\sqrt{c}de - ae^2) \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt{c}} - 2x}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt{c}} - x^2} dx}{4\sqrt{2}a^{3/4}c^{5/4}} \\ &= \frac{e^2 x}{c} - \frac{(cd^2 - 2\sqrt{a}\sqrt{c}de - ae^2) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}c^{5/4}} + \frac{(cd^2 - 2\sqrt{a}\sqrt{c}de - ae^2)}{4\sqrt{2}a^{3/4}c^{5/4}} \\ &= \frac{e^2 x}{c} - \frac{(cd^2 + 2\sqrt{a}\sqrt{c}de - ae^2) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}}\right)}{2\sqrt{2}a^{3/4}c^{5/4}} + \frac{(cd^2 + 2\sqrt{a}\sqrt{c}de - ae^2) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}}\right)}{2\sqrt{2}a^{3/4}c^{5/4}} \end{aligned}$$

**Mathematica [A]** time = 0.26, size = 269, normalized size = 0.91

$$\frac{8a^{3/4}\sqrt[4]{c}e^2x + \sqrt{2}(2\sqrt{a}\sqrt{c}de + ae^2 - cd^2) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2) + \sqrt{2}(-2\sqrt{a}\sqrt{c}de - ae^2 + cd^2) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2) - 2\sqrt{2}(2\sqrt{a}\sqrt{c}de - ae^2 + cd^2) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}}\right) + 2\sqrt{2}(2\sqrt{a}\sqrt{c}de - ae^2 + cd^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}} + 1\right)}{8a^{3/4}c^{5/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^2/(a + c\*x^4), x]

[Out] (8\*a^(3/4)\*c^(1/4)\*e^2\*x - 2\*Sqrt[2]\*(c\*d^2 + 2\*Sqrt[a]\*Sqrt[c]\*d\*e - a\*e^2)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)] + 2\*Sqrt[2]\*(c\*d^2 + 2\*Sqrt[a]\*Sqrt[c]\*d\*e - a\*e^2)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)] + Sqrt[2]\*(-(c\*d^2) + 2\*Sqrt[a]\*Sqrt[c]\*d\*e + a\*e^2)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2] + Sqrt[2]\*(c\*d^2 - 2\*Sqrt[a]\*Sqrt[c]\*d\*e - a\*e^2)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2]/(8\*a^(3/4)\*c^(5/4))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^2)^2}{a + cx^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^2)^2/(a + c*x^4),x]
[Out] IntegrateAlgebraic[(d + e*x^2)^2/(a + c*x^4), x]
fricas [B]   time = 1.36, size = 1480, normalized size = 4.98
result too large to display
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a),x, algorithm="fricas")
[Out] 1/4*(4*e^2*x + c*sqrt(-(4*c*d^3*e - 4*a*d*e^3 + a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5))))/(a*c^2))*log((c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*x + (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 + 2*a^3*c^4*d*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))*sqrt(-(4*c*d^3*e - 4*a*d*e^3 + a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))) - c*sqrt(-(4*c*d^3*e - 4*a*d*e^3 + a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))*log((c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*x - (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 + 2*a^3*c^4*d*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))*sqrt(-(4*c*d^3*e - 4*a*d*e^3 - a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))) + c*sqrt(-(4*c*d^3*e - 4*a*d*e^3 - a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))*log((c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*x + (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 - 2*a^3*c^4*d*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))*sqrt(-(4*c*d^3*e - 4*a*d*e^3 - a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))) - c*sqrt(-(4*c*d^3*e - 4*a*d*e^3 - a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))*log((c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*x - (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 - 2*a^3*c^4*d*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))*sqrt(-(4*c*d^3*e - 4*a*d*e^3 - a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2)))/c
```

```
giac [A]   time = 0.18, size = 318, normalized size = 1.07
```

$$\frac{\sqrt{2} \left( (ac)^{\frac{1}{2}} c^{\frac{1}{2}} d^{\frac{1}{2}} - (ac)^{\frac{1}{2}} ac^{\frac{1}{2}} + 2 (ac)^{\frac{1}{2}} d^{\frac{1}{2}} \right) \arctan \left( \frac{\sqrt{2} \sqrt{2 + \sqrt{2}} \left( \frac{d}{c} \right)^{\frac{1}{2}}}{2 \left( \frac{d}{c} \right)^{\frac{1}{2}}} \right) + \sqrt{2} \left( (ac)^{\frac{1}{2}} c^{\frac{1}{2}} d^{\frac{1}{2}} - (ac)^{\frac{1}{2}} ac^{\frac{1}{2}} + 2 (ac)^{\frac{1}{2}} d^{\frac{1}{2}} \right) \arctan \left( \frac{\sqrt{2} \sqrt{2 + \sqrt{2}} \left( \frac{d}{c} \right)^{\frac{1}{2}}}{2 \left( \frac{d}{c} \right)^{\frac{1}{2}}} \right) + \sqrt{2} \left( (ac)^{\frac{1}{2}} c^{\frac{1}{2}} d^{\frac{1}{2}} - (ac)^{\frac{1}{2}} ac^{\frac{1}{2}} - 2 (ac)^{\frac{1}{2}} d^{\frac{1}{2}} \right) \log \left( x^2 + \sqrt{2} x \left( \frac{d}{c} \right)^{\frac{1}{2}} + \sqrt{\frac{d}{c}} \right) - \sqrt{2} \left( (ac)^{\frac{1}{2}} c^{\frac{1}{2}} d^{\frac{1}{2}} - (ac)^{\frac{1}{2}} ac^{\frac{1}{2}} - 2 (ac)^{\frac{1}{2}} d^{\frac{1}{2}} \right) \log \left( x^2 - \sqrt{2} x \left( \frac{d}{c} \right)^{\frac{1}{2}} + \sqrt{\frac{d}{c}} \right)}{4 ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a),x, algorithm="giac")
[Out] x*e^2/c + 1/4*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 + 2*(a*c^3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/4*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 + 2*(a*c^3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 - 2*(a*c^3)^(3/4)*d*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 - 2*(a*c^3)^(3/4)*d*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3)
```

**maple [A]** time = 0.00, size = 412, normalized size = 1.39

$$\frac{\binom{2}{2}^{\frac{1}{2}} \sqrt{2} d^2 \arctan\left(\frac{\sqrt{2x-1}}{\binom{2}{2}^{\frac{1}{2}}}\right) + \binom{2}{2}^{\frac{1}{2}} \sqrt{2} d^2 \arctan\left(\frac{\sqrt{2x+1}}{\binom{2}{2}^{\frac{1}{2}}}\right) + \binom{2}{2}^{\frac{1}{2}} \sqrt{2} d^2 \ln\left(\frac{x^2 \binom{2}{2}^{\frac{1}{2}} \sqrt{2x+1} \sqrt{2x-1}}{x^2 \binom{2}{2}^{\frac{1}{2}} \sqrt{2x+1} \sqrt{2x-1}}\right) + \frac{\sqrt{2} d \arctan\left(\frac{\sqrt{2x-1}}{\binom{2}{2}^{\frac{1}{2}}}\right) + \sqrt{2} d \arctan\left(\frac{\sqrt{2x+1}}{\binom{2}{2}^{\frac{1}{2}}}\right) + \sqrt{2} d \ln\left(\frac{x^2 \binom{2}{2}^{\frac{1}{2}} \sqrt{2x+1} \sqrt{2x-1}}{x^2 \binom{2}{2}^{\frac{1}{2}} \sqrt{2x+1} \sqrt{2x-1}}\right) + \frac{c^2 x}{c} - \frac{\binom{2}{2}^{\frac{1}{2}} \sqrt{2} d^2 \arctan\left(\frac{\sqrt{2x-1}}{\binom{2}{2}^{\frac{1}{2}}}\right) + \binom{2}{2}^{\frac{1}{2}} \sqrt{2} d^2 \arctan\left(\frac{\sqrt{2x+1}}{\binom{2}{2}^{\frac{1}{2}}}\right) + \binom{2}{2}^{\frac{1}{2}} \sqrt{2} d^2 \ln\left(\frac{x^2 \binom{2}{2}^{\frac{1}{2}} \sqrt{2x+1} \sqrt{2x-1}}{x^2 \binom{2}{2}^{\frac{1}{2}} \sqrt{2x+1} \sqrt{2x-1}}\right)}{8c} - \frac{\binom{2}{2}^{\frac{1}{2}} \sqrt{2} d^2 \arctan\left(\frac{\sqrt{2x-1}}{\binom{2}{2}^{\frac{1}{2}}}\right) + \binom{2}{2}^{\frac{1}{2}} \sqrt{2} d^2 \arctan\left(\frac{\sqrt{2x+1}}{\binom{2}{2}^{\frac{1}{2}}}\right) + \binom{2}{2}^{\frac{1}{2}} \sqrt{2} d^2 \ln\left(\frac{x^2 \binom{2}{2}^{\frac{1}{2}} \sqrt{2x+1} \sqrt{2x-1}}{x^2 \binom{2}{2}^{\frac{1}{2}} \sqrt{2x+1} \sqrt{2x-1}}\right)}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^2/(c\*x^4+a), x)

[Out]  $e^{2x}/c - 1/4/c * (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)}/(a/c)^{(1/4)} * x - 1) * e^{2+1/4} * (a/c)^{(1/4)} / a * 2^{(1/2)} * \arctan(2^{(1/2)}/(a/c)^{(1/4)} * x - 1) * d^{2-1/8} / c * (a/c)^{(1/4)} * 2^{(1/2)} * \ln((x^2 + (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)}) / (x^2 - (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)})) * e^{2+1/8} * (a/c)^{(1/4)} / a * 2^{(1/2)} * \ln((x^2 + (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)}) / (x^2 - (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)})) * d^{2-1/4} / c * (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)}/(a/c)^{(1/4)} * x + 1) * e^{2+1/4} * (a/c)^{(1/4)} / a * 2^{(1/2)} * \arctan(2^{(1/2)}/(a/c)^{(1/4)} * x + 1) * d^{2+1/4} / c * d * e / (a/c)^{(1/4)} * 2^{(1/2)} * \ln((x^2 - (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)}) / (x^2 + (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)})) + 1/2 / c * d * e / (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)}/(a/c)^{(1/4)} * x + 1) + 1/2 / c * d * e / (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)}/(a/c)^{(1/4)} * x - 1)$

**maxima [A]** time = 2.36, size = 288, normalized size = 0.97

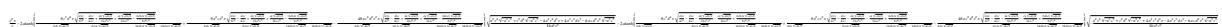
$$\frac{e^2 x}{c} + \frac{2 \sqrt{2} \left( c^{\frac{3}{2}} d^2 + 2 \sqrt{a} c d e - a \sqrt{c} e^2 \right) \arctan\left(\frac{\sqrt{2} \left( 2 \sqrt{c x + \sqrt{2} a^{\frac{1}{4}} \frac{1}{4}} \right)}{2 \sqrt{a} \sqrt{c}}\right) + 2 \sqrt{2} \left( c^{\frac{3}{2}} d^2 + 2 \sqrt{a} c d e - a \sqrt{c} e^2 \right) \arctan\left(\frac{\sqrt{2} \left( 2 \sqrt{c x - \sqrt{2} a^{\frac{1}{4}} \frac{1}{4}} \right)}{2 \sqrt{a} \sqrt{c}}\right) + \frac{\sqrt{2} \left( c^{\frac{3}{2}} d^2 - 2 \sqrt{a} c d e - a \sqrt{c} e^2 \right) \log\left(\sqrt{c} x^2 + \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x + \sqrt{a}\right) - \sqrt{2} \left( c^{\frac{3}{2}} d^2 - 2 \sqrt{a} c d e - a \sqrt{c} e^2 \right) \log\left(\sqrt{c} x^2 - \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x + \sqrt{a}\right)}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2/(c\*x^4+a), x, algorithm="maxima")

[Out]  $e^{2x}/c + 1/8 * (2 * \sqrt{2}) * (c^{(3/2)} * d^2 + 2 * \sqrt{a} * c * d * e - a * \sqrt{c} * e^2) * \arctan(1/2 * \sqrt{2} * (2 * \sqrt{2} * \sqrt{c} * x + \sqrt{2} * a^{(1/4)} * c^{(1/4)}) / \sqrt{a} * \sqrt{c}) / (\sqrt{a} * \sqrt{c}) + 2 * \sqrt{2} * (c^{(3/2)} * d^2 + 2 * \sqrt{a} * c * d * e - a * \sqrt{c} * e^2) * \arctan(1/2 * \sqrt{2} * (2 * \sqrt{2} * \sqrt{c} * x - \sqrt{2} * a^{(1/4)} * c^{(1/4)}) / \sqrt{a} * \sqrt{c}) / (\sqrt{a} * \sqrt{c}) + \sqrt{2} * (c^{(3/2)} * d^2 - 2 * \sqrt{a} * c * d * e - a * \sqrt{c} * e^2) * \log(\sqrt{c} * x^2 + \sqrt{2} * a^{(1/4)} * c^{(1/4)} * x + \sqrt{a}) / (a^{(3/4)} * c^{(3/4)}) - \sqrt{2} * (c^{(3/2)} * d^2 - 2 * \sqrt{a} * c * d * e - a * \sqrt{c} * e^2) * \log(\sqrt{c} * x^2 - \sqrt{2} * a^{(1/4)} * c^{(1/4)} * x + \sqrt{a}) / (a^{(3/4)} * c^{(3/4)}) / c$

**mupad [B]** time = 4.79, size = 1479, normalized size = 4.98



Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)^2/(a + c\*x^4), x)

[Out]  $(e^{2x})/c - 2 * \operatorname{atanh}\left(\frac{8 * c^3 * d^4 * x * ((d * e^3)/(4 * c^2) - (d^3 * e)/(4 * a * c)) + (d^4 * (-a^3 * c^5)^{(1/2)})}{(16 * a^3 * c^3) + (e^4 * (-a^3 * c^5)^{(1/2)}) / (16 * a * c^5) - (3 * d^2 * e^2 * (-a^3 * c^5)^{(1/2)}) / (8 * a^2 * c^4)}\right) / (4 * a^2 * d * e^5 - (2 * d^6 * (-a^3 * c^5)^{(1/2)}) / a^2 + 4 * c^2 * d^5 * e + (2 * a * e^6 * (-a^3 * c^5)^{(1/2)}) / c^3 - 24 * a * c * d^3 * e^3 - (14 * d^2 * e^4 * (-a^3 * c^5)^{(1/2)}) / c^2 + (14 * d^4 * e^2 * (-a^3 * c^5)^{(1/2)}) / (a * c)) + (8 * a^2 * c * e^4 * x * ((d * e^3)/(4 * c^2) - (d^3 * e)/(4 * a * c)) + (d^4 * (-a^3 * c^5)^{(1/2)}) / (16 * a^3 * c^3) + (e^4 * (-a^3 * c^5)^{(1/2)}) / (16 * a * c^5) - (3 * d^2 * e^2 * (-a^3 * c^5)^{(1/2)}) / (8 * a^2 * c^4)}\right) / (4 * a^2 * d * e^5 - (2 * d^6 * (-a^3 * c^5)^{(1/2)}) / a^2 + 4 * c^2 * d^5 * e + (2 * a * e^6 * (-a^3 * c^5)^{(1/2)}) / c^3 - 24 * a * c * d^3 * e^3 - (14 * d^2 * e^4 * (-a^3 * c^5)^{(1/2)}) / c^2 + (14 * d^4 * e^2 * (-a^3 * c^5)^{(1/2)}) / (a * c)) - (48 * a * c^2 * d^2 * e^2 * x * ((d * e^3)/(4 * c^2) - (d^3 * e)/(4 * a * c)) + (d^4 * (-a^3 * c^5)^{(1/2)}) / (16 * a^3 * c^3) + (e^4 * (-a^3 * c^5)^{(1/2)}) / (16 * a * c^5) - (3 * d^2 * e^2 * (-a^3 * c^5)^{(1/2)}) / (8 * a^2 * c^4)}\right) / (4 * a^2 * d * e^5 - (2 * d^6 * (-a^3 * c^5)^{(1/2)}) / a^2 + 4 * c^2 * d^5 * e + (2 * a * e^6 * (-a^3 * c^5)^{(1/2)}) / c^3 - 24 * a * c * d^3 * e^3 - (14 * d^2 * e^4 * (-a^3 * c^5)^{(1/2)}) / c^2 + (14 * d^4 * e^2 * (-a^3 * c^5)^{(1/2)}) / (a * c))$

$$\begin{aligned} & \frac{1}{c^2} + (14d^4e^2(-a^3c^5)^{1/2})/(ac)) * ((a^2e^4(-a^3c^5)^{1/2} + c^2d^4(-a^3c^5)^{1/2} - 4a^2c^4d^3e + 4a^3c^3d^2e^3 - 6a^2c^2d^2e^2(-a^3c^5)^{1/2})/(16a^3c^5)^{1/2} - 2 \operatorname{atanh}((8c^3d^4x((d^3e^3)/(4c^2) - (d^3e)/(4ac) - (d^4(-a^3c^5)^{1/2})/(16a^3c^3) - (e^4(-a^3c^5)^{1/2})/(16ac^5) + (3d^2e^2(-a^3c^5)^{1/2})/(8a^2c^4))^{1/2})/((2d^6(-a^3c^5)^{1/2})/a^2 + 4a^2d^5e + 4c^2d^5e - (2ae^6(-a^3c^5)^{1/2})/c^3 - 24acd^3e^3 + (14d^2e^4(-a^3c^5)^{1/2})/c^2 - (14d^4e^2(-a^3c^5)^{1/2})/(ac)) + (8a^2c^4e^4x((d^3e^3)/(4c^2) - (d^3e)/(4ac) - (d^4(-a^3c^5)^{1/2})/(16a^3c^3) - (e^4(-a^3c^5)^{1/2})/(16ac^5) + (3d^2e^2(-a^3c^5)^{1/2})/(8a^2c^4))^{1/2})/((2d^6(-a^3c^5)^{1/2})/a^2 + 4a^2d^5e + 4c^2d^5e - (2ae^6(-a^3c^5)^{1/2})/c^3 - 24acd^3e^3 + (14d^2e^4(-a^3c^5)^{1/2})/c^2 - (14d^4e^2(-a^3c^5)^{1/2})/(ac)) - (48a^2c^2d^2e^2x((d^3e^3)/(4c^2) - (d^3e)/(4ac) - (d^4(-a^3c^5)^{1/2})/(16a^3c^3) - (e^4(-a^3c^5)^{1/2})/(16ac^5) + (3d^2e^2(-a^3c^5)^{1/2})/(8a^2c^4))^{1/2})/((2d^6(-a^3c^5)^{1/2})/a^2 + 4a^2d^5e + 4c^2d^5e - (2ae^6(-a^3c^5)^{1/2})/c^3 - 24acd^3e^3 + (14d^2e^4(-a^3c^5)^{1/2})/c^2 - (14d^4e^2(-a^3c^5)^{1/2})/(ac))) * (-a^2e^4(-a^3c^5)^{1/2} + c^2d^4(-a^3c^5)^{1/2} + 4a^2c^4d^3e - 4a^3c^3d^2e^3 - 6a^2c^2d^2e^2(-a^3c^5)^{1/2})/(16a^3c^5)^{1/2} \end{aligned}$$

**sympy [A]** time = 1.48, size = 238, normalized size = 0.80

$$\operatorname{RootSum}\left(256t^4a^3c^5 + t^2(-128a^3c^3de^3 + 128a^2c^4d^3e) + a^4e^8 + 4a^3cd^2e^6 + 6a^2c^2d^4e^4 + 4ac^3d^6e^2 + c^4d^8, \left(t \mapsto t \log\left(x + \frac{-128t^3a^3c^4de - 4ta^4ce^6 + 60ta^3c^2d^2e^4 - 60ta^2c^3d^4e^2 + 4ta^4d^6}{a^4e^8 - 4a^3cd^2e^6 - 10a^2c^2d^4e^4 - 4ac^3d^6e^2 + c^4d^8}\right)\right) + \frac{e^2x}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*2/(c\*x\*\*4+a),x)

[Out] RootSum(256\*\_t\*\*4\*a\*\*3\*c\*\*5 + \_t\*\*2\*(-128\*a\*\*3\*c\*\*3\*d\*e\*\*3 + 128\*a\*\*2\*c\*\*4\*d\*\*3\*e) + a\*\*4\*e\*\*8 + 4\*a\*\*3\*c\*d\*\*2\*e\*\*6 + 6\*a\*\*2\*c\*\*2\*d\*\*4\*e\*\*4 + 4\*a\*c\*\*3\*d\*\*6\*e\*\*2 + c\*\*4\*d\*\*8, Lambda(\_t, \_t\*log(x + (-128\*\_t\*\*3\*a\*\*3\*c\*\*4\*d\*e - 4\*\_t\*a\*\*4\*c\*e\*\*6 + 60\*\_t\*a\*\*3\*c\*\*2\*d\*\*2\*e\*\*4 - 60\*\_t\*a\*\*2\*c\*\*3\*d\*\*4\*e\*\*2 + 4\*\_t\*a\*c\*\*4\*d\*\*6)/(a\*\*4\*e\*\*8 - 4\*a\*\*3\*c\*d\*\*2\*e\*\*6 - 10\*a\*\*2\*c\*\*2\*d\*\*4\*e\*\*4 - 4\*a\*c\*\*3\*d\*\*6\*e\*\*2 + c\*\*4\*d\*\*8)))) + e\*\*2\*x/c

$$3.121 \quad \int \frac{d+ex^2}{a+cx^4} dx$$

**Optimal.** Leaf size=247

$$\frac{(\sqrt{c}d - \sqrt{a}e) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} a^{3/4} c^{3/4}} + \frac{(\sqrt{c}d - \sqrt{a}e) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} a^{3/4} c^{3/4}} - \frac{(\sqrt{a}e + \sqrt{c}d) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2} a^{3/4} c^{3/4}} + \frac{(\sqrt{a}e + \sqrt{c}d) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4} c^{3/4}}$$

**Rubi [A]** time = 0.15, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$ , Rules used = {1168, 1162, 617, 204, 1165, 628}

$$\frac{(\sqrt{c}d - \sqrt{a}e) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} a^{3/4} c^{3/4}} + \frac{(\sqrt{c}d - \sqrt{a}e) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} a^{3/4} c^{3/4}} - \frac{(\sqrt{a}e + \sqrt{c}d) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4} c^{3/4}} + \frac{(\sqrt{a}e + \sqrt{c}d) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2} a^{3/4} c^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)/(a + c\*x^4), x]

[Out] -((Sqrt[c]\*d + Sqrt[a]\*e)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)]/(2\*Sqrt[2]\*a^(3/4)\*c^(3/4)) + ((Sqrt[c]\*d + Sqrt[a]\*e)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)]/(2\*Sqrt[2]\*a^(3/4)\*c^(3/4)) - ((Sqrt[c]\*d - Sqrt[a]\*e)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/(4\*Sqrt[2]\*a^(3/4)\*c^(3/4)) + ((Sqrt[c]\*d - Sqrt[a]\*e)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/(4\*Sqrt[2]\*a^(3/4)\*c^(3/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 1162

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

#### Rule 1165

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

#### Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

### Rubi steps

$$\begin{aligned} \int \frac{d + ex^2}{a + cx^4} dx &= \frac{\left(\frac{\sqrt{c}d}{\sqrt{a}} - e\right) \int \frac{\sqrt{a}\sqrt{c-cx^2}}{a+cx^4} dx}{2c} + \frac{\left(\frac{\sqrt{c}d}{\sqrt{a}} + e\right) \int \frac{\sqrt{a}\sqrt{c+cx^2}}{a+cx^4} dx}{2c} \\ &= \frac{\left(\frac{\sqrt{c}d}{\sqrt{a}} + e\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{4c} + \frac{\left(\frac{\sqrt{c}d}{\sqrt{a}} + e\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{4c} - \frac{(\sqrt{c}d - \sqrt{a}e) \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}} + 2x}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} - x^2} dx}{4\sqrt{2}a^{3/4}c^{3/4}} \\ &= -\frac{(\sqrt{c}d - \sqrt{a}e) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{(\sqrt{c}d - \sqrt{a}e) \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}c^{3/4}} \\ &= -\frac{(\sqrt{c}d + \sqrt{a}e) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} + \frac{(\sqrt{c}d + \sqrt{a}e) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} - \frac{(\sqrt{c}d - \sqrt{a}e) \log(\sqrt{a}}{4\sqrt{2}} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 183, normalized size = 0.74

$$\frac{-(\sqrt{c}d - \sqrt{a}e) \left( \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2) - \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2) \right) - 2(\sqrt{a}e + \sqrt{c}d) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) + 2(\sqrt{a}e + \sqrt{c}d) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right)}{4\sqrt{2}a^{3/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(a + c*x^4), x]
```

```
[Out] (-2*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - (Sqrt[c]*d - Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]))/(4*Sqrt[2]*a^(3/4)*c^(3/4))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{a + cx^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^2)/(a + c*x^4), x]
```

```
[Out] IntegrateAlgebraic[(d + e*x^2)/(a + c*x^4), x]
```

**fricas [B]** time = 0.98, size = 767, normalized size = 3.11

```
-----
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-(a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) + 2*d*e)/(a*c))*log(-(c^2*d^4 - a^2*e^4)*x + (a^3*c^2*e*sqrt(-(c^2*d^4 - 2*a*c*d^2
```

$$2e^2 + a^2e^4)/(a^3c^3)) + ac^2d^3 - a^2cde^2) \sqrt{-(ac \sqrt{-(c^2d^4 - 2ac^2d^2e^2 + a^2e^4)/(a^3c^3)} + 2de)/(ac))} + 1/4 \sqrt{-(ac \sqrt{-(c^2d^4 - 2ac^2d^2e^2 + a^2e^4)/(a^3c^3)} + 2de)/(ac))} \log(-(c^2d^4 - a^2e^4)x - (a^3c^2e \sqrt{-(c^2d^4 - 2ac^2d^2e^2 + a^2e^4)/(a^3c^3)} + ac^2d^3 - a^2cde^2) \sqrt{-(ac \sqrt{-(c^2d^4 - 2ac^2d^2e^2 + a^2e^4)/(a^3c^3)} + 2de)/(ac))} + 1/4 \sqrt{(ac \sqrt{-(c^2d^4 - 2ac^2d^2e^2 + a^2e^4)/(a^3c^3)} - 2de)/(ac))} \log(-(c^2d^4 - a^2e^4)x + (a^3c^2e \sqrt{-(c^2d^4 - 2ac^2d^2e^2 + a^2e^4)/(a^3c^3)} - ac^2d^3 + a^2cde^2) \sqrt{(ac \sqrt{-(c^2d^4 - 2ac^2d^2e^2 + a^2e^4)/(a^3c^3)} - 2de)/(ac))} - 1/4 \sqrt{(ac \sqrt{-(c^2d^4 - 2ac^2d^2e^2 + a^2e^4)/(a^3c^3)} - 2de)/(ac))} \log(-(c^2d^4 - a^2e^4)x - (a^3c^2e \sqrt{-(c^2d^4 - 2ac^2d^2e^2 + a^2e^4)/(a^3c^3)} - ac^2d^3 + a^2cde^2) \sqrt{(ac \sqrt{-(c^2d^4 - 2ac^2d^2e^2 + a^2e^4)/(a^3c^3)} - 2de)/(ac))} - 2de)/(ac))$$

**giac** [A] time = 0.18, size = 245, normalized size = 0.99

$$\frac{\sqrt{2} \left( (ac^3)^{\frac{1}{4}} c^2 d + (ac^3)^{\frac{3}{4}} e \right) \arctan \left( \frac{\sqrt{2} \left( 2x + \sqrt{2} \left( \frac{e}{c} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{e}{c} \right)^{\frac{1}{4}}} \right)}{4ac^3} + \frac{\sqrt{2} \left( (ac^3)^{\frac{1}{4}} c^2 d + (ac^3)^{\frac{3}{4}} e \right) \arctan \left( \frac{\sqrt{2} \left( 2x - \sqrt{2} \left( \frac{e}{c} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{e}{c} \right)^{\frac{1}{4}}} \right)}{4ac^3} + \frac{\sqrt{2} \left( (ac^3)^{\frac{1}{4}} c^2 d - (ac^3)^{\frac{3}{4}} e \right) \log \left( x^2 + \sqrt{2} x \left( \frac{e}{c} \right)^{\frac{1}{4}} + \sqrt{\frac{e}{c}} \right)}{8ac^3} - \frac{\sqrt{2} \left( (ac^3)^{\frac{1}{4}} c^2 d - (ac^3)^{\frac{3}{4}} e \right) \log \left( x^2 - \sqrt{2} x \left( \frac{e}{c} \right)^{\frac{1}{4}} + \sqrt{\frac{e}{c}} \right)}{8ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(c\*x^4+a),x, algorithm="giac")

[Out]  $1/4 \sqrt{2} \left( (ac^3)^{\frac{1}{4}} c^2 d + (ac^3)^{\frac{3}{4}} e \right) \arctan \left( \frac{1/2 \sqrt{2} \left( (ac^3)^{\frac{1}{4}} c^2 d + (ac^3)^{\frac{3}{4}} e \right)}{\sqrt{2} \left( \frac{e}{c} \right)^{\frac{1}{4}}} \right) / \left( \frac{e}{c} \right)^{\frac{1}{4}} + 1/4 \sqrt{2} \left( (ac^3)^{\frac{1}{4}} c^2 d + (ac^3)^{\frac{3}{4}} e \right) \arctan \left( \frac{1/2 \sqrt{2} \left( (ac^3)^{\frac{1}{4}} c^2 d - (ac^3)^{\frac{3}{4}} e \right)}{\sqrt{2} \left( \frac{e}{c} \right)^{\frac{1}{4}}} \right) / \left( \frac{e}{c} \right)^{\frac{1}{4}} + 1/8 \sqrt{2} \left( (ac^3)^{\frac{1}{4}} c^2 d - (ac^3)^{\frac{3}{4}} e \right) \log \left( x^2 + \sqrt{2} x \left( \frac{e}{c} \right)^{\frac{1}{4}} + \sqrt{\frac{e}{c}} \right) / \left( \frac{e}{c} \right)^{\frac{1}{4}} - 1/8 \sqrt{2} \left( (ac^3)^{\frac{1}{4}} c^2 d - (ac^3)^{\frac{3}{4}} e \right) \log \left( x^2 - \sqrt{2} x \left( \frac{e}{c} \right)^{\frac{1}{4}} + \sqrt{\frac{e}{c}} \right) / \left( \frac{e}{c} \right)^{\frac{1}{4}}$

**maple** [A] time = 0.00, size = 260, normalized size = 1.05

$$\frac{\left( \frac{e}{c} \right)^{\frac{1}{4}} \sqrt{2} d \arctan \left( \frac{\sqrt{2} x - 1}{\left( \frac{e}{c} \right)^{\frac{1}{4}}} \right)}{4a} + \frac{\left( \frac{e}{c} \right)^{\frac{1}{4}} \sqrt{2} d \arctan \left( \frac{\sqrt{2} x + 1}{\left( \frac{e}{c} \right)^{\frac{1}{4}}} \right)}{4a} + \frac{\left( \frac{e}{c} \right)^{\frac{1}{4}} \sqrt{2} d \ln \left( \frac{x^2 + \left( \frac{e}{c} \right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{e}{c}}}{x^2 - \left( \frac{e}{c} \right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{e}{c}}} \right)}{8a} + \frac{\sqrt{2} e \arctan \left( \frac{\sqrt{2} x - 1}{\left( \frac{e}{c} \right)^{\frac{1}{4}}} \right)}{4 \left( \frac{e}{c} \right)^{\frac{1}{4}} c} + \frac{\sqrt{2} e \arctan \left( \frac{\sqrt{2} x + 1}{\left( \frac{e}{c} \right)^{\frac{1}{4}}} \right)}{4 \left( \frac{e}{c} \right)^{\frac{1}{4}} c} + \frac{\sqrt{2} e \ln \left( \frac{x^2 - \left( \frac{e}{c} \right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{e}{c}}}{x^2 + \left( \frac{e}{c} \right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{e}{c}}} \right)}{8 \left( \frac{e}{c} \right)^{\frac{1}{4}} c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)/(c\*x^4+a),x)

[Out]  $1/8 d \left( \frac{e}{c} \right)^{\frac{1}{4}} / a^2 \ln \left( \left( x^2 + \left( \frac{e}{c} \right)^{\frac{1}{4}} \sqrt{2} x + \left( \frac{e}{c} \right)^{\frac{1}{2}} \right) / \left( x^2 - \left( \frac{e}{c} \right)^{\frac{1}{4}} \sqrt{2} x + \left( \frac{e}{c} \right)^{\frac{1}{2}} \right) \right) + 1/4 d \left( \frac{e}{c} \right)^{\frac{1}{4}} / a^2 \arctan \left( \frac{2 \sqrt{2} \left( \frac{e}{c} \right)^{\frac{1}{4}} x + 1}{\left( \frac{e}{c} \right)^{\frac{1}{4}}} \right) + 1/4 d \left( \frac{e}{c} \right)^{\frac{1}{4}} / a^2 \arctan \left( \frac{2 \sqrt{2} \left( \frac{e}{c} \right)^{\frac{1}{4}} x - 1}{\left( \frac{e}{c} \right)^{\frac{1}{4}}} \right) + 1/8 e/c \left( \frac{e}{c} \right)^{\frac{1}{4}} \ln \left( \left( x^2 + \left( \frac{e}{c} \right)^{\frac{1}{4}} \sqrt{2} x + \left( \frac{e}{c} \right)^{\frac{1}{2}} \right) / \left( x^2 - \left( \frac{e}{c} \right)^{\frac{1}{4}} \sqrt{2} x + \left( \frac{e}{c} \right)^{\frac{1}{2}} \right) \right) + 1/4 e/c \left( \frac{e}{c} \right)^{\frac{1}{4}} \arctan \left( \frac{2 \sqrt{2} \left( \frac{e}{c} \right)^{\frac{1}{4}} x + 1}{\left( \frac{e}{c} \right)^{\frac{1}{4}}} \right) + 1/4 e/c \left( \frac{e}{c} \right)^{\frac{1}{4}} \arctan \left( \frac{2 \sqrt{2} \left( \frac{e}{c} \right)^{\frac{1}{4}} x - 1}{\left( \frac{e}{c} \right)^{\frac{1}{4}}} \right)$

**maxima** [A] time = 2.53, size = 221, normalized size = 0.89

$$\frac{\sqrt{2} (\sqrt{c} d + \sqrt{a} e) \arctan \left( \frac{\sqrt{2} (2\sqrt{c} x + \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}})}{2\sqrt{a}\sqrt{c}} \right)}{4\sqrt{a}\sqrt{a}\sqrt{c}\sqrt{c}} + \frac{\sqrt{2} (\sqrt{c} d + \sqrt{a} e) \arctan \left( \frac{\sqrt{2} (2\sqrt{c} x - \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}})}{2\sqrt{a}\sqrt{c}} \right)}{4\sqrt{a}\sqrt{a}\sqrt{c}\sqrt{c}} + \frac{\sqrt{2} (\sqrt{c} d - \sqrt{a} e) \log \left( \sqrt{c} x^2 + \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x + \sqrt{a} \right)}{8a^{\frac{3}{4}} c^{\frac{3}{4}}} - \frac{\sqrt{2} (\sqrt{c} d - \sqrt{a} e) \log \left( \sqrt{c} x^2 - \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x + \sqrt{a} \right)}{8a^{\frac{3}{4}} c^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(c\*x^4+a),x, algorithm="maxima")

[Out]  $1/4 \sqrt{2} \left( \sqrt{c} d + \sqrt{a} e \right) \arctan \left( \frac{1/2 \sqrt{2} \left( \sqrt{c} d + \sqrt{a} e \right)}{\sqrt{a} \sqrt{c}} \right) / \left( \sqrt{a} \sqrt{c} \right) + 1/4 \sqrt{2} \left( \sqrt{c} d + \sqrt{a} e \right) \arctan \left( \frac{1/2 \sqrt{2} \left( \sqrt{c} d - \sqrt{a} e \right)}{\sqrt{a} \sqrt{c}} \right) / \left( \sqrt{a} \sqrt{c} \right) + 1/8 \left( \sqrt{c} d - \sqrt{a} e \right) \log \left( \sqrt{c} x^2 + \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x + \sqrt{a} \right) / \left( a^{\frac{3}{4}} c^{\frac{3}{4}} \right) - 1/8 \left( \sqrt{c} d - \sqrt{a} e \right) \log \left( \sqrt{c} x^2 - \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x + \sqrt{a} \right) / \left( a^{\frac{3}{4}} c^{\frac{3}{4}} \right)$

\*x - sqrt(2)\*a^(1/4)\*c^(1/4))/sqrt(sqrt(a)\*sqrt(c)))/(sqrt(a)\*sqrt(sqrt(a)\*sqrt(c))\*sqrt(c)) + 1/8\*sqrt(2)\*(sqrt(c)\*d - sqrt(a)\*e)\*log(sqrt(c)\*x^2 + sqrt(2)\*a^(1/4)\*c^(1/4)\*x + sqrt(a))/(a^(3/4)\*c^(3/4)) - 1/8\*sqrt(2)\*(sqrt(c)\*d - sqrt(a)\*e)\*log(sqrt(c)\*x^2 - sqrt(2)\*a^(1/4)\*c^(1/4)\*x + sqrt(a))/(a^(3/4)\*c^(3/4))

**mupad [B]** time = 4.68, size = 599, normalized size = 2.43

$$-2 \operatorname{atanh} \left( \frac{8c^3 d^2 x \sqrt{\frac{a^2 \sqrt{c^3}}{16a^3 c^3}} - \frac{d^2 \sqrt{c^3}}{16a^3 c^3} - \frac{d^2}{8ac}}{2c^2 d^2 e - 2ac^2 e + \frac{2d^2 \sqrt{c^3}}{a} - \frac{2d^2 \sqrt{c^3}}{a}} \right) \sqrt{\frac{cd^2 \sqrt{-a^3 c^3 - ac^2 \sqrt{-a^3 c^3} + 2d^2 c^2 de}}{16a^3 c^3}} - 2 \operatorname{atanh} \left( \frac{8c^3 d^2 x \sqrt{\frac{a^2 \sqrt{c^3}}{16a^3 c^3}} - \frac{d^2 \sqrt{c^3}}{16a^3 c^3} - \frac{d^2}{8ac}}{2c^2 d^2 e - 2ac^2 e - \frac{2d^2 \sqrt{c^3}}{a} + \frac{2d^2 \sqrt{c^3}}{a}} \right) \sqrt{\frac{a^2 \sqrt{-a^3 c^3 - cd^2 \sqrt{-a^3 c^3} + 2d^2 c^2 de}}{16a^3 c^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)/(a + c\*x^4), x)

[Out] - 2\*atanh((8\*c^3\*d^2\*x\*((e^2\*(-a^3\*c^3)^(1/2))/(16\*a^2\*c^3) - (d^2\*(-a^3\*c^3)^(1/2))/(16\*a^3\*c^2) - (d\*e)/(8\*a\*c))^(1/2))/(2\*c^2\*d^2\*e - 2\*a\*c\*e^3 + (2\*c\*d^3\*(-a^3\*c^3)^(1/2))/a^2 - (2\*d\*e^2\*(-a^3\*c^3)^(1/2))/a) - (8\*a\*c^2\*e^2\*x\*((e^2\*(-a^3\*c^3)^(1/2))/(16\*a^2\*c^3) - (d^2\*(-a^3\*c^3)^(1/2))/(16\*a^3\*c^2) - (d\*e)/(8\*a\*c))^(1/2))/(2\*c^2\*d^2\*e - 2\*a\*c\*e^3 + (2\*c\*d^3\*(-a^3\*c^3)^(1/2))/a^2 - (2\*d\*e^2\*(-a^3\*c^3)^(1/2))/a))\*(-(c\*d^2\*(-a^3\*c^3)^(1/2) - a\*e^2\*(-a^3\*c^3)^(1/2) + 2\*a^2\*c^2\*d\*e)/(16\*a^3\*c^3))^(1/2) - 2\*atanh((8\*c^3\*d^2\*x\*((d^2\*(-a^3\*c^3)^(1/2))/(16\*a^3\*c^2) - (d\*e)/(8\*a\*c) - (e^2\*(-a^3\*c^3)^(1/2))/(16\*a^2\*c^3))^(1/2))/(2\*c^2\*d^2\*e - 2\*a\*c\*e^3 - (2\*c\*d^3\*(-a^3\*c^3)^(1/2))/a^2 + (2\*d\*e^2\*(-a^3\*c^3)^(1/2))/a) - (8\*a\*c^2\*e^2\*x\*((d^2\*(-a^3\*c^3)^(1/2))/(16\*a^3\*c^2) - (d\*e)/(8\*a\*c) - (e^2\*(-a^3\*c^3)^(1/2))/(16\*a^2\*c^3))^(1/2))/(2\*c^2\*d^2\*e - 2\*a\*c\*e^3 - (2\*c\*d^3\*(-a^3\*c^3)^(1/2))/a^2 + (2\*d\*e^2\*(-a^3\*c^3)^(1/2))/a))\*(-(a\*e^2\*(-a^3\*c^3)^(1/2) - c\*d^2\*(-a^3\*c^3)^(1/2) + 2\*a^2\*c^2\*d\*e)/(16\*a^3\*c^3))^(1/2)

**sympy [A]** time = 0.68, size = 109, normalized size = 0.44

$$\operatorname{RootSum} \left( 256t^4 a^3 c^3 + 64t^2 a^2 c^2 d e + a^2 e^4 + 2acd^2 e^2 + c^2 d^4, \left( t \mapsto t \log \left( x + \frac{64t^3 a^3 c^2 e + 12ta^2 c d e^2 - 4tac^2 d^3}{a^2 e^4 - c^2 d^4} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)/(c\*x\*\*4+a), x)

[Out] RootSum(256\*\_t\*\*4\*a\*\*3\*c\*\*3 + 64\*\_t\*\*2\*a\*\*2\*c\*\*2\*d\*e + a\*\*2\*e\*\*4 + 2\*a\*c\*d\*\*2\*e\*\*2 + c\*\*2\*d\*\*4, Lambda(\_t, \_t\*log(x + (64\*\_t\*\*3\*a\*\*3\*c\*\*2\*e + 12\*\_t\*a\*\*2\*c\*d\*e\*\*2 - 4\*\_t\*a\*c\*\*2\*d\*\*3)/(a\*\*2\*e\*\*4 - c\*\*2\*d\*\*4))))



$$3.122 \quad \int \frac{1}{a+cx^4} dx$$

**Optimal.** Leaf size=185

$$\frac{\log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{c}} + \frac{\log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{c}} - \frac{\tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4} \sqrt[4]{c}} + \frac{\tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2} a^{3/4} \sqrt[4]{c}}$$

**Rubi [A]** time = 0.11, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {211, 1165, 628, 1162, 617, 204}

$$\frac{\log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{c}} + \frac{\log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{c}} - \frac{\tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4} \sqrt[4]{c}} + \frac{\tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2} a^{3/4} \sqrt[4]{c}}$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^4)^(-1), x]

[Out] -ArcTan[1 - (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)]/(2\*Sqrt[2]\*a^(3/4)\*c^(1/4)) + ArcTan[1 + (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)]/(2\*Sqrt[2]\*a^(3/4)\*c^(1/4)) - Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2]/(4\*Sqrt[2]\*a^(3/4)\*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2]/(4\*Sqrt[2]\*a^(3/4)\*c^(1/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 1162

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

## Rule 1165

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

## Rubi steps

$$\begin{aligned} \int \frac{1}{a+cx^4} dx &= \frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4} dx}{2\sqrt{a}} + \frac{\int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4} dx}{2\sqrt{a}} \\ &= \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{4\sqrt{a}\sqrt{c}} + \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{4\sqrt{a}\sqrt{c}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}} + 2x}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} - x^2} dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}} - 2x}{-\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} - x^2} dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} \\ &= -\frac{\log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} \\ &= -\frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 134, normalized size = 0.72

$$\frac{-\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2) + \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2) - 2\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) + 2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c\*x^4)^(-1), x]

[Out] (-2\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)] + 2\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2] + Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/(4\*Sqrt[2]\*a^(3/4)\*c^(1/4))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + c\*x^4)^(-1), x]

[Out] IntegrateAlgebraic[(a + c\*x^4)^(-1), x]

**fricas [A]** time = 1.76, size = 121, normalized size = 0.65

$$\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} \arctan\left(-a^2cx\left(-\frac{1}{a^3c}\right)^{\frac{3}{4}} + \sqrt{a^2\sqrt{-\frac{1}{a^3c}} + x^2}a^2c\left(-\frac{1}{a^3c}\right)^{\frac{3}{4}}\right) + \frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} \log\left(a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} + x\right) - \frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} \log\left(-a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} + x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^4+a),x, algorithm="fricas")

[Out]  $(-1/(a^3c))^{1/4} \arctan(-a^2cx(-1/(a^3c))^{3/4} + \sqrt{a^2\sqrt{-1/(a^3c)} + x^2})a^2c(-1/(a^3c))^{3/4} + 1/4(-1/(a^3c))^{1/4} \log(a(-1/(a^3c))^{1/4} + x) - 1/4(-1/(a^3c))^{1/4} \log(-a(-1/(a^3c))^{1/4} + x)$

**giac** [A] time = 0.18, size = 179, normalized size = 0.97

$$\frac{\sqrt{2} (ac^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac} + \frac{\sqrt{2} (ac^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac} + \frac{\sqrt{2} (ac^3)^{\frac{1}{4}} \log\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8ac} - \frac{\sqrt{2} (ac^3)^{\frac{1}{4}} \log\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^4+a),x, algorithm="giac")

[Out]  $1/4\sqrt{2}*(a*c^3)^{1/4}*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(a/c)^{1/4}))/((a/c)^{1/4})/(a*c) + 1/4*\sqrt{2}*(a*c^3)^{1/4}*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/c)^{1/4}))/((a/c)^{1/4})/(a*c) + 1/8*\sqrt{2}*(a*c^3)^{1/4}*\log(x^2 + \sqrt{2}*x*(a/c)^{1/4} + \sqrt{a/c})/(a*c) - 1/8*\sqrt{2}*(a*c^3)^{1/4}*\log(x^2 - \sqrt{2}*x*(a/c)^{1/4} + \sqrt{a/c})/(a*c)$

**maple** [A] time = 0.00, size = 128, normalized size = 0.69

$$\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}} - 1\right)}{4a} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}} + 1\right)}{4a} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{x^2 + \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}x + \sqrt{\frac{a}{c}}}{x^2 - \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}x + \sqrt{\frac{a}{c}}}\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x^4+a),x)

[Out]  $1/8*(a/c)^{1/4}/a*2^{1/2}*ln((x^2+(a/c)^{1/4}*2^{1/2}*x+(a/c)^{1/2}))/((x^2-(a/c)^{1/4}*2^{1/2}*x+(a/c)^{1/2}))+1/4*(a/c)^{1/4}/a*2^{1/2}*\arctan(2^{1/2}/(a/c)^{1/4}*x+1)+1/4*(a/c)^{1/4}/a*2^{1/2}*\arctan(2^{1/2}/(a/c)^{1/4}*x-1)$

**maxima** [A] time = 2.44, size = 169, normalized size = 0.91

$$\frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{a}\sqrt{c}}\right)}{4\sqrt{a}\sqrt{a}\sqrt{c}} + \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{a}\sqrt{c}}\right)}{4\sqrt{a}\sqrt{a}\sqrt{c}} + \frac{\sqrt{2} \log\left(\sqrt{c}x^2 + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{\sqrt{2} \log\left(\sqrt{c}x^2 - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^4+a),x, algorithm="maxima")

[Out]  $1/4*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*\sqrt{c}*x + \sqrt{2}*a^{1/4}*c^{1/4}))/\sqrt{c}(\sqrt{a}*\sqrt{c}))/(\sqrt{a}*\sqrt{c}(\sqrt{a}*\sqrt{c})) + 1/4*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*\sqrt{c}*x - \sqrt{2}*a^{1/4}*c^{1/4}))/\sqrt{c}(\sqrt{a}*\sqrt{c}))/(\sqrt{a}*\sqrt{c}(\sqrt{a}*\sqrt{c})) + 1/8*\sqrt{2}*\log(\sqrt{c}*x^2 + \sqrt{2}*a^{1/4}*c^{1/4}*x + \sqrt{a})/(a^{3/4}*c^{1/4}) - 1/8*\sqrt{2}*\log(\sqrt{c}*x^2 - \sqrt{2}*a^{1/4}*c^{1/4}*x + \sqrt{a})/(a^{3/4}*c^{1/4})$

**mupad** [B] time = 4.41, size = 33, normalized size = 0.18

$$\frac{\operatorname{atan}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right) + \operatorname{atanh}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)}{2(-a)^{3/4}c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + c\*x^4),x)

[Out]  $-(\operatorname{atan}((c^{1/4}x)/(-a)^{1/4}) + \operatorname{atanh}((c^{1/4}x)/(-a)^{1/4}))/2(-a)^{3/4}c^{1/4}$

sympy [A] time = 0.17, size = 20, normalized size = 0.11

$$\operatorname{RootSum}(256t^4a^3c + 1, (t \mapsto t \log(4ta + x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x**4+a), x)`

[Out] `RootSum(256*_t**4*a**3*c + 1, Lambda(_t, _t*log(4*_t*a + x)))`

$$3.123 \quad \int \frac{1}{(d+ex^2)(a+cx^4)} dx$$

**Optimal.** Leaf size=336

$$\frac{\sqrt[4]{c} (\sqrt{a} e + \sqrt{c} d) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} a^{3/4} (ae^2 + cd^2)} + \frac{\sqrt[4]{c} (\sqrt{a} e + \sqrt{c} d) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} a^{3/4} (ae^2 + cd^2)}$$

**Rubi [A]** time = 0.27, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {1171, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{c} (\sqrt{a} e + \sqrt{c} d) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} a^{3/4} (ae^2 + cd^2)} + \frac{\sqrt[4]{c} (\sqrt{a} e + \sqrt{c} d) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} a^{3/4} (ae^2 + cd^2)} - \frac{\sqrt[4]{c} (\sqrt{c} d - \sqrt{a} e) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4} (ae^2 + cd^2)} + \frac{\sqrt[4]{c} (\sqrt{c} d - \sqrt{a} e) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2} a^{3/4} (ae^2 + cd^2)} + \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{c} x}{\sqrt{a}}\right)}{\sqrt{d} (ae^2 + cd^2)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x^2)\*(a + c\*x^4)), x]

[Out] (e^(3/2)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]]/(Sqrt[d]\*(c\*d^2 + a\*e^2)) - (c^(1/4)\*(Sqrt[c]\*d - Sqrt[a]\*e)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)]/(2\*Sqrt[2]\*a^(3/4)\*(c\*d^2 + a\*e^2)) + (c^(1/4)\*(Sqrt[c]\*d - Sqrt[a]\*e)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)]/(2\*Sqrt[2]\*a^(3/4)\*(c\*d^2 + a\*e^2)) - (c^(1/4)\*(Sqrt[c]\*d + Sqrt[a]\*e)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2]/(4\*Sqrt[2]\*a^(3/4)\*(c\*d^2 + a\*e^2)) + (c^(1/4)\*(Sqrt[c]\*d + Sqrt[a]\*e)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2]/(4\*Sqrt[2]\*a^(3/4)\*(c\*d^2 + a\*e^2))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 1162

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

#### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] := Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{1}{(d + ex^2)(a + cx^4)} dx = \int \left( \frac{e^2}{(cd^2 + ae^2)(d + ex^2)} + \frac{c(d - ex^2)}{(cd^2 + ae^2)(a + cx^4)} \right) dx$$

$$= \frac{c \int \frac{d-ex^2}{a+cx^4} dx}{cd^2 + ae^2} + \frac{e^2 \int \frac{1}{d+ex^2} dx}{cd^2 + ae^2}$$

$$= \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(cd^2 + ae^2)} + \frac{\left(\frac{\sqrt{c}d}{\sqrt{a}} - e\right) \int \frac{\sqrt{a}\sqrt{c+cx^2}}{a+cx^4} dx}{2(cd^2 + ae^2)} + \frac{\left(\frac{\sqrt{c}d}{\sqrt{a}} + e\right) \int \frac{\sqrt{a}\sqrt{c-cx^2}}{a+cx^4} dx}{2(cd^2 + ae^2)}$$

$$= \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(cd^2 + ae^2)} + \frac{\left(\frac{\sqrt{c}d}{\sqrt{a}} - e\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt{c}} + x^2} dx}{4(cd^2 + ae^2)} + \frac{\left(\frac{\sqrt{c}d}{\sqrt{a}} + e\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt{c}} + x^2} dx}{4(cd^2 + ae^2)}$$

$$= \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(cd^2 + ae^2)} - \frac{4\sqrt{c}(\sqrt{c}d + \sqrt{a}e) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}(cd^2 + ae^2)} + \frac{4\sqrt{c}(\sqrt{c}d + \sqrt{a}e) \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}(cd^2 + ae^2)}$$

$$= \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(cd^2 + ae^2)} - \frac{4\sqrt{c}(\sqrt{c}d - \sqrt{a}e) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2 + ae^2)} + \frac{4\sqrt{c}(\sqrt{c}d - \sqrt{a}e) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2 + ae^2)}$$

**Mathematica [A]** time = 0.15, size = 234, normalized size = 0.70

$$\frac{8a^{3/4}e^{3/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) + \sqrt{2}\sqrt{c}\sqrt{d} \left( -(\sqrt{a}e + \sqrt{c}d) \left( \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2) - \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2) \right) + (2\sqrt{a}e - 2\sqrt{c}d) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}}\right) + 2(\sqrt{c}d - \sqrt{a}e) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}} + 1\right) \right)}{8a^{3/4}\sqrt{d}(ae^2 + cd^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(a + c*x^4)), x]
```

```
[Out] (8*a^(3/4)*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*c^(1/4)*Sqrt[d]*((
-2*Sqrt[c]*d + 2*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sq
rt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - (Sqrt[c]*d +
Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]))/(8*a^(3/4)*Sqrt[d]*(c*
d^2 + a*e^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex^2)(a + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e\*x^2)\*(a + c\*x^4)),x]

[Out] IntegrateAlgebraic[1/((d + e\*x^2)\*(a + c\*x^4)), x]

fricas [B] time = 2.56, size = 4084, normalized size = 12.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)/(c\*x^4+a),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/4*((c*d^2 + a*e^2)*\sqrt{(2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*\sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)}})/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*\log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*\sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)})))*\sqrt{(2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*\sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)}})/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)) - (c*d^2 + a*e^2)*\sqrt{(2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*\sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)}})/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*\log(-(c^2*d^2 - a*c*e^2)*x - (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*\sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)})))*\sqrt{(2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*\sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)}})/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)) + (c*d^2 + a*e^2)*\sqrt{(2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*\sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)}})/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*\log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*\sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)})))*\sqrt{(2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*\sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)}})/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)) - (c*d^2 + a*e^2)*\sqrt{(2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*\sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)}})/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*\log(-(c^2*d^2 - a*c*e^2)*x - (a*c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*\sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)})))*\sqrt{(2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*\sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)}})/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)) - 2*e*\sqrt{-e/d)*\log((e*x^2 + 2*d*x*\sqrt{-e/d} - d)/(e*x^2 + d)))/(c*d^2 + a*e^2), 1/4*(4*e*\sqrt{e/d)*\arctan(x*\sqrt{e/d}) - (c*d^2 + a*e^2)*\sqrt{(2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*\sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)}})/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*\log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 -$$

a^2\*c\*d\*e^2 + (a^3\*c^2\*d^4\*e + 2\*a^4\*c\*d^2\*e^3 + a^5\*e^5)\*sqrt(-(c^3\*d^4 - 2\*a\*c^2\*d^2\*e^2 + a^2\*c\*e^4)/(a^3\*c^4\*d^8 + 4\*a^4\*c^3\*d^6\*e^2 + 6\*a^5\*c^2\*d^4\*e^4 + 4\*a^6\*c\*d^2\*e^6 + a^7\*e^8))\*sqrt((2\*c\*d\*e + (a\*c^2\*d^4 + 2\*a^2\*c\*d^2\*e^2 + a^3\*e^4)\*sqrt(-(c^3\*d^4 - 2\*a\*c^2\*d^2\*e^2 + a^2\*c\*e^4)/(a^3\*c^4\*d^8 + 4\*a^4\*c^3\*d^6\*e^2 + 6\*a^5\*c^2\*d^4\*e^4 + 4\*a^6\*c\*d^2\*e^6 + a^7\*e^8)))/(a\*c^2\*d^4 + 2\*a^2\*c\*d^2\*e^2 + a^3\*e^4)) + (c\*d^2 + a\*e^2)\*sqrt((2\*c\*d\*e + (a\*c^2\*d^4 + 2\*a^2\*c\*d^2\*e^2 + a^3\*e^4)\*sqrt(-(c^3\*d^4 - 2\*a\*c^2\*d^2\*e^2 + a^2\*c\*e^4)/(a^3\*c^4\*d^8 + 4\*a^4\*c^3\*d^6\*e^2 + 6\*a^5\*c^2\*d^4\*e^4 + 4\*a^6\*c\*d^2\*e^6 + a^7\*e^8)))/(a\*c^2\*d^4 + 2\*a^2\*c\*d^2\*e^2 + a^3\*e^4))\*log(-(c^2\*d^2 - a\*c\*e^2)\*x - (a\*c^2\*d^3 - a^2\*c\*d\*e^2 + (a^3\*c^2\*d^4\*e + 2\*a^4\*c\*d^2\*e^3 + a^5\*e^5)\*sqrt(-(c^3\*d^4 - 2\*a\*c^2\*d^2\*e^2 + a^2\*c\*e^4)/(a^3\*c^4\*d^8 + 4\*a^4\*c^3\*d^6\*e^2 + 6\*a^5\*c^2\*d^4\*e^4 + 4\*a^6\*c\*d^2\*e^6 + a^7\*e^8)))\*sqrt((2\*c\*d\*e + (a\*c^2\*d^4 + 2\*a^2\*c\*d^2\*e^2 + a^3\*e^4)\*sqrt(-(c^3\*d^4 - 2\*a\*c^2\*d^2\*e^2 + a^2\*c\*e^4)/(a^3\*c^4\*d^8 + 4\*a^4\*c^3\*d^6\*e^2 + 6\*a^5\*c^2\*d^4\*e^4 + 4\*a^6\*c\*d^2\*e^6 + a^7\*e^8)))/(a\*c^2\*d^4 + 2\*a^2\*c\*d^2\*e^2 + a^3\*e^4)) - (c\*d^2 + a\*e^2)\*sqrt((2\*c\*d\*e - (a\*c^2\*d^4 + 2\*a^2\*c\*d^2\*e^2 + a^3\*e^4)\*sqrt(-(c^3\*d^4 - 2\*a\*c^2\*d^2\*e^2 + a^2\*c\*e^4)/(a^3\*c^4\*d^8 + 4\*a^4\*c^3\*d^6\*e^2 + 6\*a^5\*c^2\*d^4\*e^4 + 4\*a^6\*c\*d^2\*e^6 + a^7\*e^8)))/(a\*c^2\*d^4 + 2\*a^2\*c\*d^2\*e^2 + a^3\*e^4))\*log(-(c^2\*d^2 - a\*c\*e^2)\*x + (a\*c^2\*d^3 - a^2\*c\*d\*e^2 - (a^3\*c^2\*d^4\*e + 2\*a^4\*c\*d^2\*e^3 + a^5\*e^5)\*sqrt(-(c^3\*d^4 - 2\*a\*c^2\*d^2\*e^2 + a^2\*c\*e^4)/(a^3\*c^4\*d^8 + 4\*a^4\*c^3\*d^6\*e^2 + 6\*a^5\*c^2\*d^4\*e^4 + 4\*a^6\*c\*d^2\*e^6 + a^7\*e^8)))\*sqrt((2\*c\*d\*e - (a\*c^2\*d^4 + 2\*a^2\*c\*d^2\*e^2 + a^3\*e^4)\*sqrt(-(c^3\*d^4 - 2\*a\*c^2\*d^2\*e^2 + a^2\*c\*e^4)/(a^3\*c^4\*d^8 + 4\*a^4\*c^3\*d^6\*e^2 + 6\*a^5\*c^2\*d^4\*e^4 + 4\*a^6\*c\*d^2\*e^6 + a^7\*e^8)))/(a\*c^2\*d^4 + 2\*a^2\*c\*d^2\*e^2 + a^3\*e^4)) + (c\*d^2 + a\*e^2)\*sqrt((2\*c\*d\*e - (a\*c^2\*d^4 + 2\*a^2\*c\*d^2\*e^2 + a^3\*e^4)\*sqrt(-(c^3\*d^4 - 2\*a\*c^2\*d^2\*e^2 + a^2\*c\*e^4)/(a^3\*c^4\*d^8 + 4\*a^4\*c^3\*d^6\*e^2 + 6\*a^5\*c^2\*d^4\*e^4 + 4\*a^6\*c\*d^2\*e^6 + a^7\*e^8)))/(a\*c^2\*d^4 + 2\*a^2\*c\*d^2\*e^2 + a^3\*e^4))\*log(-(c^2\*d^2 - a\*c\*e^2)\*x - (a\*c^2\*d^3 - a^2\*c\*d\*e^2 - (a^3\*c^2\*d^4\*e + 2\*a^4\*c\*d^2\*e^3 + a^5\*e^5)\*sqrt(-(c^3\*d^4 - 2\*a\*c^2\*d^2\*e^2 + a^2\*c\*e^4)/(a^3\*c^4\*d^8 + 4\*a^4\*c^3\*d^6\*e^2 + 6\*a^5\*c^2\*d^4\*e^4 + 4\*a^6\*c\*d^2\*e^6 + a^7\*e^8)))\*sqrt((2\*c\*d\*e + (a\*c^2\*d^4 + 2\*a^2\*c\*d^2\*e^2 + a^3\*e^4)\*sqrt(-(c^3\*d^4 - 2\*a\*c^2\*d^2\*e^2 + a^2\*c\*e^4)/(a^3\*c^4\*d^8 + 4\*a^4\*c^3\*d^6\*e^2 + 6\*a^5\*c^2\*d^4\*e^4 + 4\*a^6\*c\*d^2\*e^6 + a^7\*e^8)))/(a\*c^2\*d^4 + 2\*a^2\*c\*d^2\*e^2 + a^3\*e^4)))/(c\*d^2 + a\*e^2)]

**giac** [A] time = 0.21, size = 339, normalized size = 1.01

$$\frac{\left((ac^3)^{\frac{1}{4}}c^2d - (ac^3)^{\frac{3}{4}}e\right) \arctan\left(\frac{\sqrt{2x+\sqrt{2}}\left(\frac{x}{c}\right)^{\frac{1}{4}}}{2\left(\frac{x}{c}\right)^{\frac{1}{4}}}\right)}{2(\sqrt{2}ac^3d^2 + \sqrt{2}a^2c^2e^2)} + \frac{\left((ac^3)^{\frac{1}{4}}c^2d - (ac^3)^{\frac{3}{4}}e\right) \arctan\left(\frac{\sqrt{2x-\sqrt{2}}\left(\frac{x}{c}\right)^{\frac{1}{4}}}{2\left(\frac{x}{c}\right)^{\frac{1}{4}}}\right)}{2(\sqrt{2}ac^3d^2 + \sqrt{2}a^2c^2e^2)} + \frac{\left((ac^3)^{\frac{1}{4}}c^2d + (ac^3)^{\frac{3}{4}}e\right) \log\left(x^2 + \sqrt{2}x\left(\frac{x}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{4(\sqrt{2}ac^3d^2 + \sqrt{2}a^2c^2e^2)} - \frac{\left((ac^3)^{\frac{1}{4}}c^2d + (ac^3)^{\frac{3}{4}}e\right) \log\left(x^2 - \sqrt{2}x\left(\frac{x}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{4(\sqrt{2}ac^3d^2 + \sqrt{2}a^2c^2e^2)} + \frac{\arctan\left(\frac{x^2}{\sqrt{d}}\right)e^{\frac{3}{2}}}{(cd^2 + ae^2)\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
[Out] 1/2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/4*((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) - 1/4*((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + arctan(x*e^(1/2)/sqrt(d))*e^(3/2)/((c*d^2 + a*e^2)*sqrt(d))
```

**maple** [A] time = 0.01, size = 363, normalized size = 1.08

$$\frac{e^2 \arctan\left(\frac{cx}{\sqrt{de}}\right)}{(ae^2 + cd^2)\sqrt{de}} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}cd \arctan\left(\frac{\sqrt{2}x}{\left(\frac{x}{c}\right)^{\frac{1}{4}}}\right)}{4(ae^2 + cd^2)a} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}cd \arctan\left(\frac{\sqrt{2}x}{\left(\frac{x}{c}\right)^{\frac{1}{4}}}\right)}{4(ae^2 + cd^2)a} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}cd \ln\left(\frac{x^2+\left(\frac{x}{c}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{x}{c}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{c}}}\right)}{8(ae^2 + cd^2)a} - \frac{\sqrt{2}e \arctan\left(\frac{\sqrt{2}x}{\left(\frac{x}{c}\right)^{\frac{1}{4}}}\right)}{4(ae^2 + cd^2)\left(\frac{a}{c}\right)^{\frac{1}{4}}} - \frac{\sqrt{2}e \arctan\left(\frac{\sqrt{2}x}{\left(\frac{x}{c}\right)^{\frac{1}{4}}}\right)}{4(ae^2 + cd^2)\left(\frac{a}{c}\right)^{\frac{1}{4}}} - \frac{\sqrt{2}e \ln\left(\frac{x^2-\left(\frac{x}{c}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{x}{c}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{c}}}\right)}{8(ae^2 + cd^2)\left(\frac{a}{c}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] int(1/(e\*x^2+d)/(c\*x^4+a),x)

[Out]  $e^2/(a*e^2+c*d^2)/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)}*e*x)+1/8*c/(a*e^2+c*d^2)*d*(a/c)^{(1/4)}/a^2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))+1/4*c/(a*e^2+c*d^2)*d*(a/c)^{(1/4)}/a^2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)+1/4*c/(a*e^2+c*d^2)*d*(a/c)^{(1/4)}/a^2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)-1/8/(a*e^2+c*d^2)*e/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))-1/4/(a*e^2+c*d^2)*e/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)-1/4/(a*e^2+c*d^2)*e/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)$

**maxima** [A] time = 2.38, size = 268, normalized size = 0.80

$$\frac{e^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{(cd^2 + ae^2)\sqrt{de}} + \frac{c \left( \frac{2\sqrt{2}(\sqrt{c}d - \sqrt{ae}) \arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x + \sqrt{2a^{\frac{1}{4}}c^{\frac{1}{4}}}\right)}{2\sqrt{a}\sqrt{c}}\right)}{\sqrt{a}\sqrt{a}\sqrt{c}}\right) + \frac{2\sqrt{2}(\sqrt{c}d - \sqrt{ae}) \arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x - \sqrt{2a^{\frac{1}{4}}c^{\frac{1}{4}}}\right)}{2\sqrt{a}\sqrt{c}}\right)}{\sqrt{a}\sqrt{a}\sqrt{c}} + \frac{\sqrt{2}(\sqrt{c}d + \sqrt{ae}) \log\left(\frac{\sqrt{c}x^2 + \sqrt{2a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}}}{a^{\frac{3}{4}}c^{\frac{3}{4}}}\right) - \frac{\sqrt{2}(\sqrt{c}d + \sqrt{ae}) \log\left(\frac{\sqrt{c}x^2 - \sqrt{2a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}}}{a^{\frac{3}{4}}c^{\frac{3}{4}}}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}}{8(cd^2 + ae^2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)/(c\*x^4+a),x, algorithm="maxima")

[Out]  $e^2*\arctan(e*x/\sqrt{d*e})/((c*d^2 + a*e^2)*\sqrt{d*e}) + 1/8*c*(2*\sqrt{2}*(\sqrt{c}*d - \sqrt{a}*e)*\arctan(1/2*\sqrt{2}*(2*\sqrt{c}*x + \sqrt{2})*a^{(1/4)}*c^{(1/4)}/\sqrt{c}*\sqrt{a}*\sqrt{c}))/(\sqrt{a}*\sqrt{c}*\sqrt{a}*\sqrt{c})*\sqrt{c} + 2*\sqrt{2}*(\sqrt{c}*d - \sqrt{a}*e)*\arctan(1/2*\sqrt{2}*(2*\sqrt{c}*x - \sqrt{2})*a^{(1/4)}*c^{(1/4)}/\sqrt{c}*\sqrt{a}*\sqrt{c}))/(\sqrt{a}*\sqrt{c}*\sqrt{a}*\sqrt{c})*\sqrt{c} + \sqrt{2}*(\sqrt{c}*d + \sqrt{a}*e)*\log(\sqrt{c}*x^2 + \sqrt{2})*a^{(1/4)}*c^{(1/4)}*x + \sqrt{a}))/(\sqrt{a}^{(3/4)}*c^{(3/4)}) - \sqrt{2}*(\sqrt{c}*d + \sqrt{a}*e)*\log(\sqrt{c}*x^2 - \sqrt{2})*a^{(1/4)}*c^{(1/4)}*x + \sqrt{a}))/(\sqrt{a}^{(3/4)}*c^{(3/4)})/(c*d^2 + a*e^2)$

**mupad** [B] time = 5.71, size = 4802, normalized size = 14.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + c\*x^4)\*(d + e\*x^2)),x)

[Out]  $\operatorname{atan}\left(\frac{((a^5e^2*(-a^3c)^{(1/2)} - c*d^2*(-a^3c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5e^4 + a^3c^2*d^4 + 2*a^4*c*d^2*e^2))^{(1/2)}*(4*c^6*d^3*e^3 - ((a^5e^2*(-a^3c)^{(1/2)} - c*d^2*(-a^3c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5e^4 + a^3c^2*d^4 + 2*a^4*c*d^2*e^2))^{(1/2)}*(256*a^4*c^4*e^8 + x*((a^5e^2*(-a^3c)^{(1/2)} - c*d^2*(-a^3c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5e^4 + a^3c^2*d^4 + 2*a^4*c*d^2*e^2))^{(1/2)}*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) + x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))}{((a^5e^2*(-a^3c)^{(1/2)} - c*d^2*(-a^3c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5e^4 + a^3c^2*d^4 + 2*a^4*c*d^2*e^2))^{(1/2)} + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((a^5e^2*(-a^3c)^{(1/2)} - c*d^2*(-a^3c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5e^4 + a^3c^2*d^4 + 2*a^4*c*d^2*e^2))^{(1/2)}*i - ((a^5e^2*(-a^3c)^{(1/2)} - c*d^2*(-a^3c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5e^4 + a^3c^2*d^4 + 2*a^4*c*d^2*e^2))^{(1/2)}*(4*c^6*d^3*e^3 - ((a^5e^2*(-a^3c)^{(1/2)} - c*d^2*(-a^3c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5e^4 + a^3c^2*d^4 + 2*a^4*c*d^2*e^2))^{(1/2)}*(256*a^4*c^4*e^8 - x*((a^5e^2*(-a^3c)^{(1/2)} - c*d^2*(-a^3c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5e^4 + a^3c^2*d^4 + 2*a^4*c*d^2*e^2))^{(1/2)}*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))}{((a^5e^2*(-a^3c)^{(1/2)} - c*d^2*(-a^3c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5e^4 + a^3c^2*d^4 + 2*a^4*c*d^2*e^2))^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5e^4 + a^3c^2*d^4 + 2*a^4*c*d^2*e^2))^{(1/2)}}\right)$



$$\begin{aligned}
&^{(1/2)} + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((c*d^2*(-a^3*c)^{(1/2)} - a*e^2*(-a^3*c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^{(1/2)} + (((c*d^2*(-a^3*c)^{(1/2)} - a*e^2*(-a^3*c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^{(1/2)}*(4*c^6*d^3*e^3 - (((c*d^2*(-a^3*c)^{(1/2)} - a*e^2*(-a^3*c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^{(1/2)}*(256*a^4*c^4*e^8 - x*((c*d^2*(-a^3*c)^{(1/2)} - a*e^2*(-a^3*c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^{(1/2)}*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6)) * ((c*d^2*(-a^3*c)^{(1/2)} - a*e^2*(-a^3*c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^{(1/2)} + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*((c*d^2*(-a^3*c)^{(1/2)} - a*e^2*(-a^3*c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^{(1/2)})))*((c*d^2*(-a^3*c)^{(1/2)} - a*e^2*(-a^3*c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^{(1/2)}*2i - (log(16*a^2*e^2*(-d*e^3)^{(3/2)} + c^2*d^5*e^3*x - c^2*d^5*e*(-d*e^3)^{(1/2)} + 16*a^2*d*e^7*x + a*c*d^2*(-d*e^3)^{(3/2)} + a*c*d^3*e^5*x)*(-d*e^3)^{(1/2)})/(2*(c*d^3 + a*d*e^2)) + (log(c^2*d^5*e^3*x - 16*a^2*e^2*(-d*e^3)^{(3/2)} + c^2*d^5*e*(-d*e^3)^{(1/2)} + 16*a^2*d*e^7*x + 4*a*c*d^2*(-d*e^3)^{(3/2)} + a*c*d^3*e^5*x + 5*a*c*d^3*e^3*(-d*e^3)^{(1/2)})*(-d*e^3)^{(1/2)})/(2*c*d^3 + 2*a*d*e^2)
\end{aligned}$$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x\*\*2+d)/(c\*x\*\*4+a), x)

[Out] Timed out

$$3.124 \quad \int \frac{1}{(d+ex^2)^2(a+cx^4)} dx$$

**Optimal.** Leaf size=453

$$\frac{c^{3/4} (2\sqrt{a}\sqrt{c}de - ae^2 + cd^2) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}(ae^2 + cd^2)^2} + \frac{c^{3/4} (2\sqrt{a}\sqrt{c}de - ae^2 + cd^2) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}(ae^2 + cd^2)^2}$$

**Rubi [A]** time = 0.38, antiderivative size = 453, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$ , Rules used = {1171, 199, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{c^{3/4} (2\sqrt{a}\sqrt{c}de - ae^2 + cd^2) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}(ae^2 + cd^2)^2} + \frac{c^{3/4} (2\sqrt{a}\sqrt{c}de - ae^2 + cd^2) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}(ae^2 + cd^2)^2} - \frac{c^{3/4} (-2\sqrt{a}\sqrt{c}de - ae^2 + cd^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}}\right)}{2\sqrt{2}a^{3/4}(ae^2 + cd^2)^2} + \frac{c^{3/4} (-2\sqrt{a}\sqrt{c}de - ae^2 + cd^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}} + 1\right)}{2\sqrt{2}a^{3/4}(ae^2 + cd^2)^2} + \frac{e^2 x}{2d(d+ex^2)(ae^2 + cd^2)} + \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2d^{3/2}(ae^2 + cd^2)} + \frac{2c\sqrt{d}e^{3/2} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{(ae^2 + cd^2)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x^2)^2\*(a + c\*x^4)),x]

[Out] (e^2\*x)/(2\*d\*(c\*d^2 + a\*e^2)\*(d + e\*x^2)) + (2\*c\*Sqrt[d]\*e^(3/2)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]]/(c\*d^2 + a\*e^2)^2 + (e^(3/2)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(2\*d^(3/2)\*(c\*d^2 + a\*e^2)) - (c^(3/4)\*(c\*d^2 - 2\*Sqrt[a]\*Sqrt[c]\*d\*e - a\*e^2)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)]/(2\*Sqrt[2]\*a^(3/4)\*(c\*d^2 + a\*e^2)^2) + (c^(3/4)\*(c\*d^2 - 2\*Sqrt[a]\*Sqrt[c]\*d\*e - a\*e^2)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)]/(2\*Sqrt[2]\*a^(3/4)\*(c\*d^2 + a\*e^2)^2) - (c^(3/4)\*(c\*d^2 + 2\*Sqrt[a]\*Sqrt[c]\*d\*e - a\*e^2)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/(4\*Sqrt[2]\*a^(3/4)\*(c\*d^2 + a\*e^2)^2) + (c^(3/4)\*(c\*d^2 + 2\*Sqrt[a]\*Sqrt[c]\*d\*e - a\*e^2)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/(4\*Sqrt[2]\*a^(3/4)\*(c\*d^2 + a\*e^2)^2)

#### Rule 199

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2\*p] || (n == 2 && IntegerQ[4\*p]) || (n == 2 && IntegerQ[3\*p]) || Denominator[p + 1/n] < Denominator[p])

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,

e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1162

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1165

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1168

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[-(a\*c)]

### Rule 1171

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x^2)^q/(a + c\*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[q]

### Rubi steps

$$\begin{aligned} \int \frac{1}{(d+ex^2)^2(a+cx^4)} dx &= \int \left( \frac{e^2}{(cd^2+ae^2)(d+ex^2)^2} + \frac{2cde^2}{(cd^2+ae^2)^2(d+ex^2)} + \frac{c(cd^2-ae^2-2cdex^2)}{(cd^2+ae^2)^2(a+cx^4)} \right) dx \\ &= \frac{c \int \frac{cd^2-ae^2-2cdex^2}{a+cx^4} dx}{(cd^2+ae^2)^2} + \frac{(2cde^2) \int \frac{1}{d+ex^2} dx}{(cd^2+ae^2)^2} + \frac{e^2 \int \frac{1}{(d+ex^2)^2} dx}{cd^2+ae^2} \\ &= \frac{e^2 x}{2d(cd^2+ae^2)(d+ex^2)} + \frac{2c\sqrt{d}e^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{(\sqrt{c}(cd^2-2\sqrt{a}\sqrt{c}de-ae^2))}{2\sqrt{a}(cd^2+ae^2)} \\ &= \frac{e^2 x}{2d(cd^2+ae^2)(d+ex^2)} + \frac{2c\sqrt{d}e^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}(cd^2+ae^2)} + \frac{(\sqrt{c}(cd^2-2\sqrt{a}\sqrt{c}de-ae^2))}{2\sqrt{a}(cd^2+ae^2)} \\ &= \frac{e^2 x}{2d(cd^2+ae^2)(d+ex^2)} + \frac{2c\sqrt{d}e^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}(cd^2+ae^2)} - \frac{c^{3/4}(cd^2-2\sqrt{a}\sqrt{c}de-ae^2)}{2\sqrt{a}(cd^2+ae^2)} \\ &= \frac{e^2 x}{2d(cd^2+ae^2)(d+ex^2)} + \frac{2c\sqrt{d}e^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}(cd^2+ae^2)} - \frac{c^{3/4}(cd^2-2\sqrt{a}\sqrt{c}de-ae^2)}{2\sqrt{a}(cd^2+ae^2)} \end{aligned}$$

**Mathematica [A]** time = 0.47, size = 362, normalized size = 0.80

$$\frac{\sqrt{2}c^{3/4}(-2\sqrt{a}\sqrt{cd+ae^2})\log(-\sqrt{2}\sqrt[4]{c}\sqrt{a+\sqrt{c}x^2}) + \sqrt{2}c^{3/4}(2\sqrt{a}\sqrt{cd-ae^2+cd^2})\log(\sqrt{2}\sqrt[4]{c}\sqrt{a+\sqrt{c}x^2}) + \frac{2\sqrt{2}c^{3/4}(2\sqrt{a}\sqrt{cd+ae^2-cd^2})\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{c}}\right) - 2\sqrt{2}c^{3/4}(2\sqrt{a}\sqrt{cd+ae^2-cd^2})\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{c}}+1\right) + \frac{4c^2x(ae^2+cd^2)}{d(d+cx^2)} + \frac{4c^{3/2}(a^2+5cd^2)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{d^{3/2}}}{8(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x^2)^2\*(a + c\*x^4)),x]

[Out] ((4\*e^2\*(c\*d^2 + a\*e^2)\*x)/(d\*(d + e\*x^2)) + (4\*e^(3/2)\*(5\*c\*d^2 + a\*e^2)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/d^(3/2) + (2\*Sqrt[2]\*c^(3/4)\*(-(c\*d^2) + 2\*Sqrt[a]\*Sqrt[c]\*d\*e + a\*e^2)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)]/a^(3/4) - (2\*Sqrt[2]\*c^(3/4)\*(-(c\*d^2) + 2\*Sqrt[a]\*Sqrt[c]\*d\*e + a\*e^2)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)]/a^(3/4) + (Sqrt[2]\*c^(3/4)\*(-(c\*d^2) - 2\*Sqrt[a]\*Sqrt[c]\*d\*e + a\*e^2)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/a^(3/4) + (Sqrt[2]\*c^(3/4)\*(c\*d^2 + 2\*Sqrt[a]\*Sqrt[c]\*d\*e - a\*e^2)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/a^(3/4))/(8\*(c\*d^2 + a\*e^2)^2)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex^2)^2 (a + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e\*x^2)^2\*(a + c\*x^4)),x]

[Out] IntegrateAlgebraic[1/((d + e\*x^2)^2\*(a + c\*x^4)), x]

**fricas [B]** time = 42.20, size = 8409, normalized size = 18.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)^2/(c\*x^4+a),x, algorithm="fricas")

[Out] [1/4\*((c^2\*d^6 + 2\*a\*c\*d^4\*e^2 + a^2\*d^2\*e^4 + (c^2\*d^5\*e + 2\*a\*c\*d^3\*e^3 + a^2\*d\*e^5)\*x^2)\*sqrt((4\*c^3\*d^3\*e - 4\*a\*c^2\*d\*e^3 + (a\*c^4\*d^8 + 4\*a^2\*c^3\*d^6\*e^2 + 6\*a^3\*c^2\*d^4\*e^4 + 4\*a^4\*c\*d^2\*e^6 + a^5\*e^8)\*sqrt(-(c^7\*d^8 - 12\*a\*c^6\*d^6\*e^2 + 38\*a^2\*c^5\*d^4\*e^4 - 12\*a^3\*c^4\*d^2\*e^6 + a^4\*c^3\*e^8)/(a^3\*c^8\*d^16 + 8\*a^4\*c^7\*d^14\*e^2 + 28\*a^5\*c^6\*d^12\*e^4 + 56\*a^6\*c^5\*d^10\*e^6 + 70\*a^7\*c^4\*d^8\*e^8 + 56\*a^8\*c^3\*d^6\*e^10 + 28\*a^9\*c^2\*d^4\*e^12 + 8\*a^10\*c\*d^2\*e^14 + a^11\*e^16)))/(a\*c^4\*d^8 + 4\*a^2\*c^3\*d^6\*e^2 + 6\*a^3\*c^2\*d^4\*e^4 + 4\*a^4\*c\*d^2\*e^6 + a^5\*e^8))\*log((c^4\*d^4 - 6\*a\*c^3\*d^2\*e^2 + a^2\*c^2\*e^4)\*x + (a\*c^4\*d^6 - 7\*a^2\*c^3\*d^4\*e^2 + 7\*a^3\*c^2\*d^2\*e^4 - a^4\*c\*e^6 + 2\*(a^3\*c^4\*d^9\*e + 4\*a^4\*c^3\*d^7\*e^3 + 6\*a^5\*c^2\*d^5\*e^5 + 4\*a^6\*c\*d^3\*e^7 + a^7\*d\*e^9)\*sqrt(-(c^7\*d^8 - 12\*a\*c^6\*d^6\*e^2 + 38\*a^2\*c^5\*d^4\*e^4 - 12\*a^3\*c^4\*d^2\*e^6 + a^4\*c^3\*e^8)/(a^3\*c^8\*d^16 + 8\*a^4\*c^7\*d^14\*e^2 + 28\*a^5\*c^6\*d^12\*e^4 + 56\*a^6\*c^5\*d^10\*e^6 + 70\*a^7\*c^4\*d^8\*e^8 + 56\*a^8\*c^3\*d^6\*e^10 + 28\*a^9\*c^2\*d^4\*e^12 + 8\*a^10\*c\*d^2\*e^14 + a^11\*e^16)))\*sqrt((4\*c^3\*d^3\*e - 4\*a\*c^2\*d\*e^3 + (a\*c^4\*d^8 + 4\*a^2\*c^3\*d^6\*e^2 + 6\*a^3\*c^2\*d^4\*e^4 + 4\*a^4\*c\*d^2\*e^6 + a^5\*e^8)\*sqrt(-(c^7\*d^8 - 12\*a\*c^6\*d^6\*e^2 + 38\*a^2\*c^5\*d^4\*e^4 - 12\*a^3\*c^4\*d^2\*e^6 + a^4\*c^3\*e^8)/(a^3\*c^8\*d^16 + 8\*a^4\*c^7\*d^14\*e^2 + 28\*a^5\*c^6\*d^12\*e^4 + 56\*a^6\*c^5\*d^10\*e^6 + 70\*a^7\*c^4\*d^8\*e^8 + 56\*a^8\*c^3\*d^6\*e^10 + 28\*a^9\*c^2\*d^4\*e^12 + 8\*a^10\*c\*d^2\*e^14 + a^11\*e^16)))/(a\*c^4\*d^8 + 4\*a^2\*c^3\*d^6\*e^2 + 6\*a^3\*c^2\*d^4\*e^4 + 4\*a^4\*c\*d^2\*e^6 + a^5\*e^8)) - (c^2\*d^6 + 2\*a\*c\*d^4\*e^2 + a^2\*d^2\*e^4 + (c^2\*d^5\*e + 2\*a\*c\*d^3\*e^3 + a^2\*d\*e^5)\*x^2)\*sqrt((4\*c^3\*d^3\*e - 4\*a\*c^2\*d\*e^3 + (a\*c^4\*d^8 + 4\*a^2\*c^3\*d^6\*e^2 + 6\*a^3\*c^2\*d^4\*e^4 + 4\*a^4\*c\*d^2\*e^6 + a^5\*e^8)\*sqrt(-(c^7\*d^8 - 12\*a\*c^6\*d^6\*e^2 + 38\*a^2\*c^5\*d^4\*e^4 - 12\*a^3\*c^4\*d^2\*e^6 + a^4\*c^3\*e^8)/(a^3\*c^8\*d^16 + 8\*a^4\*c^7\*d^14\*e^2 + 28\*a^5\*c^6\*d^12\*e^4 + 56\*a^6\*c^5\*d^10\*e^6 + 70\*a^7\*c^4\*d^8\*e^8 + 56\*a^8\*c^3\*d^6\*e^10 + 28\*a^9\*c^2\*d^4\*e^12 + 8\*a^10\*c\*d^2\*e^14 + a^11\*e^16)))/(a^3\*c^6\*d^6\*e^2 + 38\*a^2\*c^5\*d^4\*e^4 - 12\*a^3\*c^4\*d^2\*e^6 + a^4\*c^3\*e^8)/(a^3\*c^8\*d^16 + 8\*a^4\*c^7\*d^14\*e^2 + 28\*a^5\*c^6\*d^12\*e^4 + 56\*a^6\*c^5\*d^10\*e^6 + 70\*a^7\*c^4\*d^8\*e^8 + 56\*a^8\*c^3\*d^6\*e^10 + 28\*a^9\*c^2\*d^4\*e^12 + 8\*a^10\*c\*d^2\*e^14 + a^11\*e^16)))]

$$\begin{aligned}
& c^8 d^{16} + 8 a^4 c^7 d^{14} e^2 + 28 a^5 c^6 d^{12} e^4 + 56 a^6 c^5 d^{10} e^6 + \\
& 70 a^7 c^4 d^8 e^8 + 56 a^8 c^3 d^6 e^{10} + 28 a^9 c^2 d^4 e^{12} + 8 a^{10} c d^2 e^{14} + a^{11} e^{16} \Big) \Big/ \Big( a^4 c^4 d^8 + 4 a^2 c^3 d^6 e^2 + 6 a^3 c^2 d^4 e^4 + \\
& 4 a^4 c d^2 e^6 + a^5 e^8 \Big) \log \Big( (c^4 d^4 - 6 a c^3 d^2 e^2 + a^2 c^2 e^4) * x - \\
& (a^4 c^4 d^6 - 7 a^2 c^3 d^4 e^2 + 7 a^3 c^2 d^2 e^4 - a^4 c e^6 + 2 (a^3 c^4 d^9 e + \\
& 4 a^4 c^3 d^7 e^3 + 6 a^5 c^2 d^5 e^5 + 4 a^6 c d^3 e^7 + a^7 d e^9) * \sqrt{-(c^7 d^8 - 12 a c^6 d^6 e^2 + 38 a^2 c^5 d^4 e^4 - 12 a^3 c^4 d^2 e^6 + a^4 c^3 e^8)} \Big/ \\
& (a^3 c^8 d^{16} + 8 a^4 c^7 d^{14} e^2 + 28 a^5 c^6 d^{12} e^4 + 56 a^6 c^5 d^{10} e^6 + 70 a^7 c^4 d^8 e^8 + 56 a^8 c^3 d^6 e^{10} + 28 a^9 c^2 d^4 e^{12} + 8 a^{10} c d^2 e^{14} + a^{11} e^{16})) * \sqrt{((4 c^3 d^3 e - 4 a c^2 d e^3 + \\
& (a^4 c^4 d^8 + 4 a^2 c^3 d^6 e^2 + 6 a^3 c^2 d^4 e^4 + 4 a^4 c d^2 e^6 + a^5 e^8) * \sqrt{-(c^7 d^8 - 12 a c^6 d^6 e^2 + 38 a^2 c^5 d^4 e^4 - 12 a^3 c^4 d^2 e^6 + a^4 c^3 e^8)} \Big/ \\
& (a^3 c^8 d^{16} + 8 a^4 c^7 d^{14} e^2 + 28 a^5 c^6 d^{12} e^4 + 56 a^6 c^5 d^{10} e^6 + 70 a^7 c^4 d^8 e^8 + 56 a^8 c^3 d^6 e^{10} + 28 a^9 c^2 d^4 e^{12} + 8 a^{10} c d^2 e^{14} + a^{11} e^{16})) \Big) \Big/ \Big( a^4 c^4 d^8 + 4 a^2 c^3 d^6 e^2 + 6 a^3 c^2 d^4 e^4 + 4 a^4 c d^2 e^6 + a^5 e^8 \Big) \Big) + \\
& (c^2 d^6 + 2 a c d^4 e^2 + a^2 d^2 e^4 + (c^2 d^5 e + 2 a c d^3 e^3 + a^2 d e^5) * x^2) * \sqrt{((4 c^3 d^3 e - 4 a c^2 d e^3 - (a^4 c^4 d^8 + 4 a^2 c^3 d^6 e^2 + 6 a^3 c^2 d^4 e^4 + 4 a^4 c d^2 e^6 + a^5 e^8) * \sqrt{-(c^7 d^8 - 12 a c^6 d^6 e^2 + 38 a^2 c^5 d^4 e^4 - 12 a^3 c^4 d^2 e^6 + a^4 c^3 e^8)} \Big/ \\
& (a^3 c^8 d^{16} + 8 a^4 c^7 d^{14} e^2 + 28 a^5 c^6 d^{12} e^4 + 56 a^6 c^5 d^{10} e^6 + 70 a^7 c^4 d^8 e^8 + 56 a^8 c^3 d^6 e^{10} + 28 a^9 c^2 d^4 e^{12} + 8 a^{10} c d^2 e^{14} + a^{11} e^{16})) \Big) \Big/ \Big( a^4 c^4 d^8 + 4 a^2 c^3 d^6 e^2 + 6 a^3 c^2 d^4 e^4 + 4 a^4 c d^2 e^6 + a^5 e^8 \Big) \Big) * \log \Big( (c^4 d^4 - 6 a c^3 d^2 e^2 + a^2 c^2 e^4) * x + \\
& (a^4 c^4 d^6 - 7 a^2 c^3 d^4 e^2 + 7 a^3 c^2 d^2 e^4 - a^4 c e^6 - 2 (a^3 c^4 d^9 e + 4 a^4 c^3 d^7 e^3 + 6 a^5 c^2 d^5 e^5 + 4 a^6 c d^3 e^7 + a^7 d e^9) * \sqrt{-(c^7 d^8 - 12 a c^6 d^6 e^2 + 38 a^2 c^5 d^4 e^4 - 12 a^3 c^4 d^2 e^6 + a^4 c^3 e^8)} \Big/ \\
& (a^3 c^8 d^{16} + 8 a^4 c^7 d^{14} e^2 + 28 a^5 c^6 d^{12} e^4 + 56 a^6 c^5 d^{10} e^6 + 70 a^7 c^4 d^8 e^8 + 56 a^8 c^3 d^6 e^{10} + 28 a^9 c^2 d^4 e^{12} + 8 a^{10} c d^2 e^{14} + a^{11} e^{16})) * \sqrt{((4 c^3 d^3 e - 4 a c^2 d e^3 - (a^4 c^4 d^8 + 4 a^2 c^3 d^6 e^2 + 6 a^3 c^2 d^4 e^4 + 4 a^4 c d^2 e^6 + a^5 e^8) * \sqrt{-(c^7 d^8 - 12 a c^6 d^6 e^2 + 38 a^2 c^5 d^4 e^4 - 12 a^3 c^4 d^2 e^6 + a^4 c^3 e^8)} \Big/ \\
& (a^3 c^8 d^{16} + 8 a^4 c^7 d^{14} e^2 + 28 a^5 c^6 d^{12} e^4 + 56 a^6 c^5 d^{10} e^6 + 70 a^7 c^4 d^8 e^8 + 56 a^8 c^3 d^6 e^{10} + 28 a^9 c^2 d^4 e^{12} + 8 a^{10} c d^2 e^{14} + a^{11} e^{16})) \Big) \Big/ \Big( a^4 c^4 d^8 + 4 a^2 c^3 d^6 e^2 + 6 a^3 c^2 d^4 e^4 + 4 a^4 c d^2 e^6 + a^5 e^8 \Big) \Big) - \\
& (c^2 d^6 + 2 a c d^4 e^2 + a^2 d^2 e^4 + (c^2 d^5 e + 2 a c d^3 e^3 + a^2 d e^5) * x^2) * \sqrt{((4 c^3 d^3 e - 4 a c^2 d e^3 - (a^4 c^4 d^8 + 4 a^2 c^3 d^6 e^2 + 6 a^3 c^2 d^4 e^4 + 4 a^4 c d^2 e^6 + a^5 e^8) * \sqrt{-(c^7 d^8 - 12 a c^6 d^6 e^2 + 38 a^2 c^5 d^4 e^4 - 12 a^3 c^4 d^2 e^6 + a^4 c^3 e^8)} \Big/ \\
& (a^3 c^8 d^{16} + 8 a^4 c^7 d^{14} e^2 + 28 a^5 c^6 d^{12} e^4 + 56 a^6 c^5 d^{10} e^6 + 70 a^7 c^4 d^8 e^8 + 56 a^8 c^3 d^6 e^{10} + 28 a^9 c^2 d^4 e^{12} + 8 a^{10} c d^2 e^{14} + a^{11} e^{16})) \Big) \Big/ \Big( a^4 c^4 d^8 + 4 a^2 c^3 d^6 e^2 + 6 a^3 c^2 d^4 e^4 + 4 a^4 c d^2 e^6 + a^5 e^8 \Big) \Big) + \\
& (5 c^3 d^3 e + a d e^3 + (5 c^2 d^2 e^2 + a e^4) * x^2) * \sqrt{-e/d} * \log \Big( (e x^2 + 2 d x * \sqrt{-e/d} - e/d) - d \Big) \Big/ (e x^2 + d) + 2 * (c^2 d^2 e^2 + a e^4) * x \Big/ (c^2 d^6 + 2 a c d^4 e^2 + a^2 d^2 e^4 + (c^2 d^5 e + 2 a c d^3 e^3 + a^2 d e^5) * x^2), 1/4 * (2 * (5 c^3
\end{aligned}$$





$$\begin{aligned}
& (28*a^9*c^2*d^4*e^{12} + 8*a^{10}*c*d^2*e^{14} + a^{11}*e^{16}))/((a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)) - (c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x^2) \\
& *sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 - (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^{10}*c*d^2*e^{14} + a^{11}*e^{16}))))/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^{10}*c*d^2*e^{14} + a^{11}*e^{16}))) \\
& *log((c^4*d^4 - 6*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x - (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 - 2*(a^3*c^4*d^9*e + 4*a^4*c^3*d^7*e^3 + 6*a^5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 + a^7*d*e^9)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^{10}*c*d^2*e^{14} + a^{11}*e^{16}))))*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 - (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^{10}*c*d^2*e^{14} + a^{11}*e^{16}))))/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^{10}*c*d^2*e^{14} + a^{11}*e^{16}))) \\
& + 2*(c*d^2*e^2 + a*e^4)*x)/(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x^2)]
\end{aligned}$$

**giac** [A] time = 0.25, size = 517, normalized size = 1.14

$$\frac{(5c^2d^2 + ad^2) \arctan\left(\frac{x}{\sqrt{d}}\right) + \frac{(ac)^2 c^2 d^2 - (ac)^2 ac^2 - 2(ac)^2 ad}{2(d^2)} \arctan\left(\frac{\sqrt{2}c^2 d^2 + \sqrt{2}c^2 d^2 + \sqrt{2}c^2 d^2}{2(d^2)}\right) + \frac{(ac)^2 c^2 d^2 - (ac)^2 ac^2 - 2(ac)^2 ad}{2(d^2)} \arctan\left(\frac{\sqrt{2}c^2 d^2 + \sqrt{2}c^2 d^2 + \sqrt{2}c^2 d^2}{2(d^2)}\right) + \frac{(\sqrt{2}c^2 d^2 - \sqrt{2}c^2 d^2 + 2\sqrt{2}c^2 d^2) \log\left(\frac{x + \sqrt{2}c^2 d^2 + \sqrt{2}c^2 d^2}{\sqrt{2}c^2 d^2}\right) + (\sqrt{2}c^2 d^2 - \sqrt{2}c^2 d^2 + 2\sqrt{2}c^2 d^2) \log\left(\frac{x - \sqrt{2}c^2 d^2 + \sqrt{2}c^2 d^2}{\sqrt{2}c^2 d^2}\right)}{8(ac^2 d^2 + 2ac^2 d^2 + ad^2)}}{2(ac^2 d^2 + 2ac^2 d^2 + ad^2)\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)^2/(c\*x^4+a),x, algorithm="giac")

[Out] 1/2\*(5\*c\*d^2\*e^2 + a\*e^4)\*arctan(x\*e^(1/2)/sqrt(d))\*e^(-1/2)/((c^2\*d^5 + 2\*a\*c\*d^3\*e^2 + a^2\*d\*e^4)\*sqrt(d)) + 1/2\*((a\*c^3)^(1/4)\*c^2\*d^2 - (a\*c^3)^(1/4)\*a\*c\*e^2 - 2\*(a\*c^3)^(3/4)\*d\*e)\*arctan(1/2\*sqrt(2)\*(2\*x + sqrt(2))\*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)\*a\*c^3\*d^4 + 2\*sqrt(2)\*a^2\*c^2\*d^2\*e^2 + sqrt(2)\*a^3\*c\*e^4) + 1/2\*((a\*c^3)^(1/4)\*c^2\*d^2 - (a\*c^3)^(1/4)\*a\*c\*e^2 - 2\*(a\*c^3)^(3/4)\*d\*e)\*arctan(1/2\*sqrt(2)\*(2\*x - sqrt(2))\*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)\*a\*c^3\*d^4 + 2\*sqrt(2)\*a^2\*c^2\*d^2\*e^2 + sqrt(2)\*a^3\*c\*e^4) + 1/8\*(sqrt(2)\*(a\*c^3)^(1/4)\*c^2\*d^2 - sqrt(2)\*(a\*c^3)^(1/4)\*a\*c\*e^2 + 2\*sqrt(2)\*(a\*c^3)^(3/4)\*d\*e)\*log(x^2 + sqrt(2)\*x\*(a/c)^(1/4) + sqrt(a/c))/(a\*c^3\*d^4 + 2\*a^2\*c^2\*d^2\*e^2 + a^3\*c\*e^4) - 1/8\*(sqrt(2)\*(a\*c^3)^(1/4)\*c^2\*d^2 - sqrt(2)\*(a\*c^3)^(1/4)\*a\*c\*e^2 + 2\*sqrt(2)\*(a\*c^3)^(3/4)\*d\*e)\*log(x^2 - sqrt(2)\*x\*(a/c)^(1/4) + sqrt(a/c))/(a\*c^3\*d^4 + 2\*a^2\*c^2\*d^2\*e^2 + a^3\*c\*e^4) + 1/2\*x\*e^2/((c\*d^3 + a\*d\*e^2)\*(x^2\*e + d))

**maple** [A] time = 0.01, size = 650, normalized size = 1.43

$$\frac{ax}{2(ac^2 + c^2d^2)(c^2 + d)} + \frac{a^2 \arctan\left(\frac{x}{\sqrt{d}}\right)}{2(ac^2 + c^2d^2)\sqrt{d}} + \frac{cdx}{2(ac^2 + c^2d^2)(c^2 + d)} + \frac{cd \arctan\left(\frac{x}{\sqrt{d}}\right)}{2(ac^2 + c^2d^2)\sqrt{d}} + \frac{(c^2) \sqrt{2} c^2 d^2 \arctan\left(\frac{\sqrt{2}x}{(c^2) \sqrt{2} c^2 d^2 + \sqrt{2} c^2 d^2}\right)}{4(ac^2 + c^2d^2)d} + \frac{(c^2) \sqrt{2} c^2 d^2 \arctan\left(\frac{\sqrt{2}x}{(c^2) \sqrt{2} c^2 d^2 + \sqrt{2} c^2 d^2}\right)}{4(ac^2 + c^2d^2)d} + \frac{(c^2) \sqrt{2} c^2 d^2 \arctan\left(\frac{\sqrt{2}x}{(c^2) \sqrt{2} c^2 d^2 + \sqrt{2} c^2 d^2}\right)}{8(ac^2 + c^2d^2)d} + \frac{\sqrt{2} cd \arctan\left(\frac{\sqrt{2}x}{(c^2) \sqrt{2} c^2 d^2 + \sqrt{2} c^2 d^2}\right)}{2(ac^2 + c^2d^2)(c^2)} + \frac{\sqrt{2} cd \arctan\left(\frac{\sqrt{2}x}{(c^2) \sqrt{2} c^2 d^2 + \sqrt{2} c^2 d^2}\right)}{2(ac^2 + c^2d^2)(c^2)} + \frac{\sqrt{2} cd \ln\left(\frac{(c^2) \sqrt{2} c^2 d^2 + \sqrt{2} c^2 d^2}{(c^2) \sqrt{2} c^2 d^2 + \sqrt{2} c^2 d^2}\right)}{4(ac^2 + c^2d^2)(c^2)} + \frac{\sqrt{2} cd \ln\left(\frac{(c^2) \sqrt{2} c^2 d^2 + \sqrt{2} c^2 d^2}{(c^2) \sqrt{2} c^2 d^2 + \sqrt{2} c^2 d^2}\right)}{4(ac^2 + c^2d^2)(c^2)} + \frac{(c^2) \sqrt{2} c^2 d^2 \arctan\left(\frac{\sqrt{2}x}{(c^2) \sqrt{2} c^2 d^2 + \sqrt{2} c^2 d^2}\right)}{4(ac^2 + c^2d^2)} + \frac{(c^2) \sqrt{2} c^2 d^2 \arctan\left(\frac{\sqrt{2}x}{(c^2) \sqrt{2} c^2 d^2 + \sqrt{2} c^2 d^2}\right)}{4(ac^2 + c^2d^2)} + \frac{(c^2) \sqrt{2} c^2 d^2 \arctan\left(\frac{\sqrt{2}x}{(c^2) \sqrt{2} c^2 d^2 + \sqrt{2} c^2 d^2}\right)}{8(ac^2 + c^2d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x^2+d)^2/(c\*x^4+a),x)

[Out] 1/2\*e^4/(a\*e^2+c\*d^2)^2/d\*x/(e\*x^2+d)\*a+1/2\*e^2/(a\*e^2+c\*d^2)^2\*d\*x/(e\*x^2+d)\*c+1/2\*e^4/(a\*e^2+c\*d^2)^2/d/(d\*e)^(1/2)\*arctan(1/(d\*e)^(1/2)\*e\*x)\*a+5/2\*e^2/(a\*e^2+c\*d^2)^2\*d/(d\*e)^(1/2)\*arctan(1/(d\*e)^(1/2)\*e\*x)\*c-1/8\*c/(a\*e^2+c\*d^2)^2\*(a/c)^(1/4)\*2^(1/2)\*ln((x^2+(a/c)^(1/4)\*2^(1/2)\*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)\*2^(1/2)\*x+(a/c)^(1/2)))\*e^2+1/8\*c^2/(a\*e^2+c\*d^2)^2\*(a/c)^(1/4)/a\*2^(1/2)\*ln((x^2+(a/c)^(1/4)\*2^(1/2)\*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)\*2^(1/2)\*x+(a/c)^(1/2)))\*e^2

$(1/2)*x+(a/c)^{(1/2)}) * d^{-1/4} * c / (a * e^{2+c*d^2})^{2} * (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x - 1) * e^{2+1/4*c^2} / (a * e^{2+c*d^2})^{2} * (a/c)^{(1/4)} / a * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x - 1) * d^{-1/4} * c / (a * e^{2+c*d^2})^{2} * (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x + 1) * e^{2+1/4*c^2} / (a * e^{2+c*d^2})^{2} * (a/c)^{(1/4)} / a * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x + 1) * d^{-1/4} * c / (a * e^{2+c*d^2})^{2} * d * e / (a/c)^{(1/4)} * 2^{(1/2)} * \ln((x^2 - (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)}) / (x^2 + (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)})) - 1/2 * c / (a * e^{2+c*d^2})^{2} * d * e / (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x + 1) - 1/2 * c / (a * e^{2+c*d^2})^{2} * d * e / (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x - 1)$

**maxima [A]** time = 2.45, size = 403, normalized size = 0.89

$$\frac{e^{2x}}{2(ad^4 + ad^2e^2 + (cd^3e + ade^3)x^2)^2} + \frac{\left( \frac{2\sqrt{d}\left(\frac{3}{2}\mu^2 - 2\sqrt{d}cd - \sqrt{c^2}\right) \arctan\left(\frac{\sqrt{d}\left(2\sqrt{c^2} + \sqrt{d}\frac{1}{2}\right)}{2\sqrt{d}\sqrt{c}}\right)}{\sqrt{d}\sqrt{c}\sqrt{c}} + \frac{2\sqrt{d}\left(\frac{3}{2}\mu^2 - 2\sqrt{d}cd - \sqrt{c^2}\right) \arctan\left(\frac{\sqrt{d}\left(2\sqrt{c^2} - \sqrt{d}\frac{1}{2}\right)}{2\sqrt{d}\sqrt{c}}\right)}{\sqrt{d}\sqrt{c}\sqrt{c}} + \frac{\sqrt{d}\left(\frac{3}{2}\mu^2 + 2\sqrt{d}cd - \sqrt{c^2}\right) \log\left(\sqrt{c^2} + \sqrt{d}\frac{1}{2} + \sqrt{d}\right) - \sqrt{d}\left(\frac{3}{2}\mu^2 + 2\sqrt{d}cd - \sqrt{c^2}\right) \log\left(\sqrt{c^2} - \sqrt{d}\frac{1}{2} + \sqrt{d}\right)}{d^{\frac{3}{2}}\frac{1}{2}} \right)}{8(c^2d^4 + 2acd^2e^2 + a^2e^4)} + \frac{(5cd^2e^2 + ad^4) \arctan\left(\frac{cx}{\sqrt{de}}\right)}{2(c^2d^3 + 2acd^2e^2 + a^2de^4)\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)^2/(c\*x^4+a),x, algorithm="maxima")

[Out]  $1/2 * e^{2x} / (c * d^4 + a * d^2 * e^2 + (c * d^3 * e + a * d * e^3) * x^2) + 1/8 * c * (2 * \sqrt{2}) * (c^{(3/2)} * d^2 - 2 * \sqrt{a} * c * d * e - a * \sqrt{c} * e^2) * \arctan(1/2 * \sqrt{2} * (2 * \sqrt{2} * (c * x + \sqrt{2}) * a^{(1/4)} * c^{(1/4)}) / \sqrt{a} * \sqrt{c})) / (\sqrt{a} * \sqrt{c}) + 2 * \sqrt{2} * (c^{(3/2)} * d^2 - 2 * \sqrt{a} * c * d * e - a * \sqrt{c} * e^2) * \arctan(1/2 * \sqrt{2} * (2 * \sqrt{2} * (c * x - \sqrt{2}) * a^{(1/4)} * c^{(1/4)}) / \sqrt{a} * \sqrt{c})) / (\sqrt{a} * \sqrt{c}) + \sqrt{2} * (c^{(3/2)} * d^2 + 2 * \sqrt{a} * c * d * e - a * \sqrt{c} * e^2) * \log(\sqrt{c} * x^2 + \sqrt{2} * a^{(1/4)} * c^{(1/4)} * x + \sqrt{a})) / (a^{(3/4)} * c^{(3/4)}) - \sqrt{2} * (c^{(3/2)} * d^2 + 2 * \sqrt{a} * c * d * e - a * \sqrt{c} * e^2) * \log(\sqrt{c} * x^2 - \sqrt{2} * a^{(1/4)} * c^{(1/4)} * x + \sqrt{a})) / (a^{(3/4)} * c^{(3/4)}) / (c^2 * d^4 + 2 * a * c * d^2 * e^2 + a^2 * e^4) + 1/2 * (5 * c * d^2 * e^2 + a * e^4) * \arctan(e * x / \sqrt{d * e}) / ((c^2 * d^5 + 2 * a * c * d^3 * e^2 + a^2 * d * e^4) * \sqrt{d * e})$

**mupad [B]** time = 6.55, size = 16369, normalized size = 36.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + c\*x^4)\*(d + e\*x^2)^2),x)

[Out]  $(e^{2x}) / (2 * d * (d + e * x^2) * (a * e^2 + c * d^2)) - \operatorname{atan}\left(\frac{\left(\left(\left(\left(\left(256 * a^8 * c^4 * d * e^{16} - 128 * a * c^{11} * d^{15} * e^2 + 256 * a^2 * c^{10} * d^{13} * e^4 + 3456 * a^3 * c^9 * d^{11} * e^6 + 8960 * a^4 * c^8 * d^9 * e^8 + 10880 * a^5 * c^7 * d^7 * e^{10} + 6912 * a^6 * c^6 * d^5 * e^{12} + 2176 * a^7 * c^5 * d^3 * e^{14}\right) / \left(2 * (c^4 * d^{10} + a^4 * d^2 * e^8 + 4 * a * c^3 * d^8 * e^2 + 4 * a^3 * c * d^4 * e^6 + 6 * a^2 * c^2 * d^6 * e^4)\right) + (x * ((a^2 * e^4 * (-a^3 * c^3)^{(1/2)} + c^2 * d^4 * (-a^3 * c^3)^{(1/2)} + 4 * a^2 * c^3 * d^3 * e - 4 * a^3 * c^2 * d * e^3 - 6 * a * c * d^2 * e^2 * (-a^3 * c^3)^{(1/2)})) / (16 * (a^7 * e^8 + a^3 * c^4 * d^8 + 4 * a^6 * c * d^2 * e^6 + 4 * a^4 * c^3 * d^6 * e^2 + 6 * a^5 * c^2 * d^4 * e^4))\right)^{(1/2)} * (512 * a^2 * c^{11} * d^{16} * e^3 + 2560 * a^3 * c^{10} * d^{14} * e^5 + 4608 * a^4 * c^9 * d^{12} * e^7 + 2560 * a^5 * c^8 * d^{10} * e^9 - 2560 * a^6 * c^7 * d^8 * e^{11} - 4608 * a^7 * c^6 * d^6 * e^{13} - 2560 * a^8 * c^5 * d^4 * e^{15} - 512 * a^9 * c^4 * d^2 * e^{17}) / (c^4 * d^{10} + a^4 * d^2 * e^8 + 4 * a * c^3 * d^8 * e^2 + 4 * a^3 * c * d^4 * e^6 + 6 * a^2 * c^2 * d^6 * e^4)) * ((a^2 * e^4 * (-a^3 * c^3)^{(1/2)} + c^2 * d^4 * (-a^3 * c^3)^{(1/2)} + 4 * a^2 * c^3 * d^3 * e - 4 * a^3 * c^2 * d * e^3 - 6 * a * c * d^2 * e^2 * (-a^3 * c^3)^{(1/2)}) / (16 * (a^7 * e^8 + a^3 * c^4 * d^8 + 4 * a^6 * c * d^2 * e^6 + 4 * a^4 * c^3 * d^6 * e^2 + 6 * a^5 * c^2 * d^4 * e^4))\right)^{(1/2)} + (x * (32 * a^6 * c^5 * d * e^{14} - 48 * a * c^{10} * d^{11} * e^4 - 16 * c^{11} * d^{13} * e^2 + 1024 * a^2 * c^9 * d^9 * e^6 + 2208 * a^3 * c^8 * d^7 * e^8 + 1264 * a^4 * c^7 * d^5 * e^{10} + 144 * a^5 * c^6 * d^3 * e^{12})) / (c^4 * d^{10} + a^4 * d^2 * e^8 + 4 * a * c^3 * d^8 * e^2 + 4 * a^3 * c * d^4 * e^6 + 6 * a^2 * c^2 * d^6 * e^4)) * ((a^2 * e^4 * (-a^3 * c^3)^{(1/2)} + c^2 * d^4 * (-a^3 * c^3)^{(1/2)} + 4 * a^2 * c^3 * d^3 * e - 4 * a^3 * c^2 * d * e^3 - 6 * a * c * d^2 * e^2 * (-a^3 * c^3)^{(1/2)}) / (16 * (a^7 * e^8 + a^3 * c^4 * d^8 + 4 * a^6 * c * d^2 * e^6 + 4 * a^4 * c^3 * d^6 * e^2 + 6 * a^5 * c^2 * d^4 * e^4))\right)^{(1/2)} + (480 * a^2 * c^8 * d^6 * e^7 - 200 * a * c^9 * d^8 * e^5 - 8 * a^5 * c^5 * e^{13} + 784 * a^3 * c^7 * d^4 * e^9 + 96 * a^4 * c^6 * d^2 * e^{11}) / (2 * (c^4 * d^{10} + a^4 * d^2 * e^8 + 4 * a * c^3 * d^8 * e^2$

$$\begin{aligned}
& + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)) * ((a^2*e^4*(-a^3*c^3)^{(1/2)} + c^2*d^4*(-a^3*c^3)^{(1/2)} + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^{(1/2)}) / (16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^{(1/2)} + (x*(a^3*c^6*e^{11} - 27*c^9*d^6*e^5 + 11*a*c^8*d^4*e^7 + 7*a^2*c^7*d^2*e^9)) / (c^4*d^{10} + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)) * ((a^2*e^4*(-a^3*c^3)^{(1/2)} + c^2*d^4*(-a^3*c^3)^{(1/2)} + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^{(1/2)}) / (16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^{(1/2)} * i - (((((256*a^8*c^4*d*e^{16} - 128*a*c^{11}*d^{15}*e^2 + 256*a^2*c^{10}*d^{13}*e^4 + 3456*a^3*c^9*d^{11}*e^6 + 8960*a^4*c^8*d^9*e^8 + 10880*a^5*c^7*d^7*e^{10} + 6912*a^6*c^6*d^5*e^{12} + 2176*a^7*c^5*d^3*e^{14}) / (2*(c^4*d^{10} + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)) - (x*((a^2*e^4*(-a^3*c^3)^{(1/2)} + c^2*d^4*(-a^3*c^3)^{(1/2)} + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^{(1/2)}) / (16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^{(1/2)} * (512*a^2*c^{11}*d^{16}*e^3 + 2560*a^3*c^{10}*d^{14}*e^5 + 4608*a^4*c^9*d^{12}*e^7 + 2560*a^5*c^8*d^{10}*e^9 - 2560*a^6*c^7*d^8*e^{11} - 4608*a^7*c^6*d^6*e^{13} - 2560*a^8*c^5*d^4*e^{15} - 512*a^9*c^4*d^2*e^{17})) / (c^4*d^{10} + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)) * ((a^2*e^4*(-a^3*c^3)^{(1/2)} + c^2*d^4*(-a^3*c^3)^{(1/2)} + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^{(1/2)}) / (16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^{(1/2)} - (x*(32*a^6*c^5*d*e^{14} - 48*a*c^{10}*d^{11}*e^4 - 16*c^{11}*d^{13}*e^2 + 1024*a^2*c^9*d^9*e^6 + 2208*a^3*c^8*d^7*e^8 + 1264*a^4*c^7*d^5*e^{10} + 144*a^5*c^6*d^3*e^{12})) / (c^4*d^{10} + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)) * ((a^2*e^4*(-a^3*c^3)^{(1/2)} + c^2*d^4*(-a^3*c^3)^{(1/2)} + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^{(1/2)}) / (16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^{(1/2)} + (480*a^2*c^8*d^6*e^7 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^{13} + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^{11}) / (2*(c^4*d^{10} + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)) * ((a^2*e^4*(-a^3*c^3)^{(1/2)} + c^2*d^4*(-a^3*c^3)^{(1/2)} + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^{(1/2)}) / (16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^{(1/2)} - (x*(a^3*c^6*e^{11} - 27*c^9*d^6*e^5 + 11*a*c^8*d^4*e^7 + 7*a^2*c^7*d^2*e^9)) / (c^4*d^{10} + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)) * ((a^2*e^4*(-a^3*c^3)^{(1/2)} + c^2*d^4*(-a^3*c^3)^{(1/2)} + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^{(1/2)}) / (16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^{(1/2)} * i) / (((((256*a^8*c^4*d*e^{16} - 128*a*c^{11}*d^{15}*e^2 + 256*a^2*c^{10}*d^{13}*e^4 + 3456*a^3*c^9*d^{11}*e^6 + 8960*a^4*c^8*d^9*e^8 + 10880*a^5*c^7*d^7*e^{10} + 6912*a^6*c^6*d^5*e^{12} + 2176*a^7*c^5*d^3*e^{14}) / (2*(c^4*d^{10} + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)) + (x*((a^2*e^4*(-a^3*c^3)^{(1/2)} + c^2*d^4*(-a^3*c^3)^{(1/2)} + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^{(1/2)}) / (16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^{(1/2)} + (480*a^2*c^8*d^6*e^7 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^{13} + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^{11}) / (2*(c^4*d^{10} + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)) * ((a^2*e^4*(-a^3*c^3)^{(1/2)} + c^2*d^4*(-a^3*c^3)^{(1/2)} + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^{(1/2)}) / (16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^{(1/2)} - (x*(a^3*c^6*e^{11} - 27*c^9*d^6*e^5 + 11*a*c^8*d^4*e^7 + 7*a^2*c^7*d^2*e^9)) / (c^4*d^{10} + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)) * ((a^2*e^4*(-a^3*c^3)^{(1/2)} + c^2*d^4*(-a^3*c^3)^{(1/2)} + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^{(1/2)}) / (16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^{(1/2)} + (x*(32*a^6*c^5*d*e^{14} - 48*a*c^{10}*d^{11}*e^4 - 16*c^{11}*d^{13}*e^2 + 1024*a^2*c^9*d^9*e^6 + 2208*a^3*c^8*d^7*e^8 + 1264*a^4*c^7*d^5*e^{10} + 144*a^5*c^6*d^3*e^{12})) / (c^4*d^{10} + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)) * ((a^2*e^4*(-a^3*c^3)^{(1/2)} + c^2*d^4*(-a^3*c^3)^{(1/2)} + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^{(1/2)}) / (16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^{(1/2)} + (480*a^2*c^8*d^6*e^7 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^{13} + 784*a^3*c^7*d^4*e^9 + 96*a^4
\end{aligned}$$

$$\begin{aligned}
& *c^6*d^2*e^{11})/(2*(c^4*d^{10} + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)) * ((a^2*e^4*(-a^3*c^3)^{(1/2)} + c^2*d^4*(-a^3*c^3)^{(1/2)} + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^{(1/2)}) / \\
& (16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^{(1/2)} + (x*(a^3*c^6*e^{11} - 27*c^9*d^6*e^5 + 11*a*c^8*d^4*e^7 + 7*a^2*c^7*d^2*e^9)) / (c^4*d^{10} + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)) * ((a^2*e^4*(-a^3*c^3)^{(1/2)} + c^2*d^4*(-a^3*c^3)^{(1/2)} + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^{(1/2)}) / (16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^{(1/2)} + (((((256*a^8*c^4*d*e^{16} - 128*a*c^{11}*d^{15}*e^2 + 256*a^2*c^{10}*d^{13}*e^4 + 3456*a^3*c^9*d^{11}*e^6 + 8960*a^4*c^8*d^9*e^8 + 10880*a^5*c^7*d^7*e^{10} + 6912*a^6*c^6*d^5*e^{12} + 2176*a^7*c^5*d^3*e^{14}) / (2*(c^4*d^{10} + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)) - (x*((a^2*e^4*(-a^3*c^3)^{(1/2)} + c^2*d^4*(-a^3*c^3)^{(1/2)} + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^{(1/2)}) / (16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^{(1/2)} * (512*a^2*c^{11}*d^{16}*e^3 + 2560*a^3*c^{10}*d^{14}*e^5 + 4608*a^4*c^9*d^{12}*e^7 + 2560*a^5*c^8*d^{10}*e^9 - 2560*a^6*c^7*d^8*e^{11} - 4608*a^7*c^6*d^6*e^{13} - 2560*a^8*c^5*d^4*e^{15} - 512*a^9*c^4*d^2*e^{17})) / (c^4*d^{10} + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)) * ((a^2*e^4*(-a^3*c^3)^{(1/2)} + c^2*d^4*(-a^3*c^3)^{(1/2)} + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^{(1/2)}) / (16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^{(1/2)} - (x*(32*a^6*c^5*d*e^{14} - 48*a*c^{10}*d^{11}*e^4 - 16*c^{11}*d^{13}*e^2 + 1024*a^2*c^9*d^9*e^6 + 2208*a^3*c^8*d^7*e^8 + 1264*a^4*c^7*d^5*e^{10} + 144*a^5*c^6*d^3*e^{12})) / (c^4*d^{10} + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)) * ((a^2*e^4*(-a^3*c^3)^{(1/2)} + c^2*d^4*(-a^3*c^3)^{(1/2)} + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^{(1/2)}) / (16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^{(1/2)} + (480*a^2*c^8*d^6*e^7 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^{13} + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^{11}) / (2*(c^4*d^{10} + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)) * ((a^2*e^4*(-a^3*c^3)^{(1/2)} + c^2*d^4*(-a^3*c^3)^{(1/2)} + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^{(1/2)}) / (16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^{(1/2)} - (x*(a^3*c^6*e^{11} - 27*c^9*d^6*e^5 + 11*a*c^8*d^4*e^7 + 7*a^2*c^7*d^2*e^9)) / (c^4*d^{10} + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)) * ((a^2*e^4*(-a^3*c^3)^{(1/2)} + c^2*d^4*(-a^3*c^3)^{(1/2)} + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^{(1/2)}) / (16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^{(1/2)} + (5*c^8*d^3*e^6 + a*c^7*d*e^8) / (c^4*d^{10} + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)) * ((a^2*e^4*(-a^3*c^3)^{(1/2)} + c^2*d^4*(-a^3*c^3)^{(1/2)} + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^{(1/2)}) / (16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^{(1/2)} * 2i - (atan((((x*(a^3*c^6*e^{11} - 27*c^9*d^6*e^5 + 11*a*c^8*d^4*e^7 + 7*a^2*c^7*d^2*e^9)) / (c^4*d^{10} + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4) - (((240*a^2*c^8*d^6*e^7 - 100*a*c^9*d^8*e^5 - 4*a^5*c^5*e^{13} + 392*a^3*c^7*d^4*e^9 + 48*a^4*c^6*d^2*e^{11}) / (c^4*d^{10} + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4) - (((x*(32*a^6*c^5*d*e^{14} - 48*a*c^{10}*d^{11}*e^4 - 16*c^{11}*d^{13}*e^2 + 1024*a^2*c^9*d^9*e^6 + 2208*a^3*c^8*d^7*e^8 + 1264*a^4*c^7*d^5*e^{10} + 144*a^5*c^6*d^3*e^{12})) / (c^4*d^{10} + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4) - ((a^e^2 + 5*c*d^2) * ((128*a^8*c^4*d*e^{16} - 64*a*c^{11}*d^{15}*e^2 + 128*a^2*c^{10}*d^{13}*e^4 + 1728*a^3*c^9*d^{11}*e^6 + 4480*a^4*c^8*d^9*e^8 + 5440*a^5*c^7*d^7*e^{10} + 3456*a^6*c^6*d^5*e^{12} + 1088*a^7*c^5*d^3*e^{14}) / (c^4*d^{10} + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4) - (x*(a^e^2 + 5*c*d^2) * (-d^3*e^3)^{(1/2)} * (512*a^2*c^{11}*d^{16}*e^3 + 2560*a^3*c^{10}*d^{14}*e^5 + 4608*a^4*c^9*d^{12}*e^7 + 2560*a^5*c^8*d^{10}*e^9 - 2560*a^6*c^7*d^8*e^{11} - 4608*a^7*c^6*d^6*e^{13} - 2560*a^8*c^5*d^4*e^{15} - 512*a^9*c^4*d^2*e^{17})) / (4*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^
\end{aligned}$$





$$\begin{aligned}
& \left( a^8 + a^3c^4d^8 + 4a^6c^2d^2e^6 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 \right)^{1/2} + (480a^2c^8d^6e^7 - 200a^3c^9d^8e^5 - 8a^5c^5e^{13} + 784a^3c^7d^4e^9 + 96a^4c^6d^2e^{11}) / (2(c^4d^{10} + a^4d^2e^8 + 4a^3c^3d^8e^2 + 4a^3c^3d^4e^6 + 6a^2c^2d^6e^4)) * (-a^2e^4(-a^3c^3)^{1/2} + c^2d^4(-a^3c^3)^{1/2} - 4a^2c^3d^3e + 4a^3c^2d^2e^3 - 6a^3c^2d^2e^2(-a^3c^3)^{1/2}) / (16(a^7e^8 + a^3c^4d^8 + 4a^6c^2d^2e^6 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4))^{1/2} - (x(a^3c^6e^{11} - 27c^9d^6e^5 + 11a^3c^8d^4e^7 + 7a^2c^7d^2e^9)) / (c^4d^{10} + a^4d^2e^8 + 4a^3c^3d^8e^2 + 4a^3c^3d^4e^6 + 6a^2c^2d^6e^4) * (-a^2e^4(-a^3c^3)^{1/2} + c^2d^4(-a^3c^3)^{1/2} - 4a^2c^3d^3e + 4a^3c^2d^2e^3 - 6a^3c^2d^2e^2(-a^3c^3)^{1/2}) / (16(a^7e^8 + a^3c^4d^8 + 4a^6c^2d^2e^6 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4))^{1/2} * i) / (((((256a^8c^4d^2e^{16} - 128a^3c^{11}d^{15}e^2 + 256a^2c^{10}d^{13}e^4 + 3456a^3c^9d^{11}e^6 + 8960a^4c^8d^9e^8 + 10880a^5c^7d^7e^{10} + 6912a^6c^6d^5e^{12} + 2176a^7c^5d^3e^{14}) / (2(c^4d^{10} + a^4d^2e^8 + 4a^3c^3d^8e^2 + 4a^3c^3d^4e^6 + 6a^2c^2d^6e^4)) + (x(-a^2e^4(-a^3c^3)^{1/2} + c^2d^4(-a^3c^3)^{1/2} - 4a^2c^3d^3e + 4a^3c^2d^2e^3 - 6a^3c^2d^2e^2(-a^3c^3)^{1/2}) / (16(a^7e^8 + a^3c^4d^8 + 4a^6c^2d^2e^6 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4))^{1/2} * (512a^2c^{11}d^{16}e^3 + 2560a^3c^{10}d^{14}e^5 + 4608a^4c^9d^{12}e^7 + 2560a^5c^8d^{10}e^9 - 2560a^6c^7d^8e^{11} - 4608a^7c^6d^6e^{13} - 2560a^8c^5d^4e^{15} - 512a^9c^4d^2e^{17})) / (c^4d^{10} + a^4d^2e^8 + 4a^3c^3d^8e^2 + 4a^3c^3d^4e^6 + 6a^2c^2d^6e^4) * (-a^2e^4(-a^3c^3)^{1/2} + c^2d^4(-a^3c^3)^{1/2} - 4a^2c^3d^3e + 4a^3c^2d^2e^3 - 6a^3c^2d^2e^2(-a^3c^3)^{1/2}) / (16(a^7e^8 + a^3c^4d^8 + 4a^6c^2d^2e^6 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4))^{1/2} + (x(32a^6c^5d^2e^{14} - 48a^3c^{10}d^{11}e^4 - 16c^{11}d^{13}e^2 + 1024a^2c^9d^9e^6 + 2208a^3c^8d^7e^8 + 1264a^4c^7d^5e^{10} + 144a^5c^6d^3e^{12})) / (c^4d^{10} + a^4d^2e^8 + 4a^3c^3d^8e^2 + 4a^3c^3d^4e^6 + 6a^2c^2d^6e^4) * (-a^2e^4(-a^3c^3)^{1/2} + c^2d^4(-a^3c^3)^{1/2} - 4a^2c^3d^3e + 4a^3c^2d^2e^3 - 6a^3c^2d^2e^2(-a^3c^3)^{1/2}) / (16(a^7e^8 + a^3c^4d^8 + 4a^6c^2d^2e^6 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4))^{1/2} + (480a^2c^8d^6e^7 - 200a^3c^9d^8e^5 - 8a^5c^5e^{13} + 784a^3c^7d^4e^9 + 96a^4c^6d^2e^{11}) / (2(c^4d^{10} + a^4d^2e^8 + 4a^3c^3d^8e^2 + 4a^3c^3d^4e^6 + 6a^2c^2d^6e^4)) * (-a^2e^4(-a^3c^3)^{1/2} + c^2d^4(-a^3c^3)^{1/2} - 4a^2c^3d^3e + 4a^3c^2d^2e^3 - 6a^3c^2d^2e^2(-a^3c^3)^{1/2}) / (16(a^7e^8 + a^3c^4d^8 + 4a^6c^2d^2e^6 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4))^{1/2} + (x(a^3c^6e^{11} - 27c^9d^6e^5 + 11a^3c^8d^4e^7 + 7a^2c^7d^2e^9)) / (c^4d^{10} + a^4d^2e^8 + 4a^3c^3d^8e^2 + 4a^3c^3d^4e^6 + 6a^2c^2d^6e^4) * (-a^2e^4(-a^3c^3)^{1/2} + c^2d^4(-a^3c^3)^{1/2} - 4a^2c^3d^3e + 4a^3c^2d^2e^3 - 6a^3c^2d^2e^2(-a^3c^3)^{1/2}) / (16(a^7e^8 + a^3c^4d^8 + 4a^6c^2d^2e^6 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4))^{1/2} + (((((256a^8c^4d^2e^{16} - 128a^3c^{11}d^{15}e^2 + 256a^2c^{10}d^{13}e^4 + 3456a^3c^9d^{11}e^6 + 8960a^4c^8d^9e^8 + 10880a^5c^7d^7e^{10} + 6912a^6c^6d^5e^{12} + 2176a^7c^5d^3e^{14}) / (2(c^4d^{10} + a^4d^2e^8 + 4a^3c^3d^8e^2 + 4a^3c^3d^4e^6 + 6a^2c^2d^6e^4)) - (x(-a^2e^4(-a^3c^3)^{1/2} + c^2d^4(-a^3c^3)^{1/2} - 4a^2c^3d^3e + 4a^3c^2d^2e^3 - 6a^3c^2d^2e^2(-a^3c^3)^{1/2}) / (16(a^7e^8 + a^3c^4d^8 + 4a^6c^2d^2e^6 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4))^{1/2} * (512a^2c^{11}d^{16}e^3 + 2560a^3c^{10}d^{14}e^5 + 4608a^4c^9d^{12}e^7 + 2560a^5c^8d^{10}e^9 - 2560a^6c^7d^8e^{11} - 4608a^7c^6d^6e^{13} - 2560a^8c^5d^4e^{15} - 512a^9c^4d^2e^{17})) / (c^4d^{10} + a^4d^2e^8 + 4a^3c^3d^8e^2 + 4a^3c^3d^4e^6 + 6a^2c^2d^6e^4) * (-a^2e^4(-a^3c^3)^{1/2} + c^2d^4(-a^3c^3)^{1/2} - 4a^2c^3d^3e + 4a^3c^2d^2e^3 - 6a^3c^2d^2e^2(-a^3c^3)^{1/2}) / (16(a^7e^8 + a^3c^4d^8 + 4a^6c^2d^2e^6 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4))^{1/2} - (x(32a^6c^5d^2e^{14} - 48a^3c^{10}d^{11}e^4 - 16c^{11}d^{13}e^2 + 1024a^2c^9d^9e^6 + 2208a^3c^8d^7e^8 + 1264a^4c^7d^5e^{10} + 144a^5c^6d^3e^{12})) / (c^4d^{10} + a^4d^2e^8 + 4a^3c^3d^8e^2 + 4a^3c^3d^4e^6 + 6a^2c^2d^6e^4) * (-a^2e^4(-a^3c^3)^{1/2} + c^2d^4(-a^3c^3)^{1/2} - 4a^2c^3d^3e + 4a^3c^2d^2e^3 - 6a^3c^2d^2e^2(-a^3c^3)^{1/2}) / (16(a^7e^8 + a^3c^4d^8 + 4a^6c^2d^2e^6 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4))^{1/2}
\end{aligned}$$

$$\begin{aligned}
& 3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^{(1/2)} / (16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4))^{(1/2)} \\
& + (480*a^2*c^8*d^6*e^7 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^{13} + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^{11}) / (2*(c^4*d^{10} + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)) * (-a^2*e^4*(-a^3*c^3)^{(1/2)} + c^2*d^4*(-a^3*c^3)^{(1/2)} - 4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^{(1/2)}) / (16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4))^{(1/2)} - (x*(a^3*c^6*e^{11} - 27*c^9*d^6*e^5 + 11*a*c^8*d^4*e^7 + 7*a^2*c^7*d^2*e^9)) / (c^4*d^{10} + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)) * (-a^2*e^4*(-a^3*c^3)^{(1/2)} + c^2*d^4*(-a^3*c^3)^{(1/2)} - 4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^{(1/2)}) / (16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4))^{(1/2)} + (5*c^8*d^3*e^6 + a*c^7*d*e^8) / (c^4*d^{10} + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)) * (-a^2*e^4*(-a^3*c^3)^{(1/2)} + c^2*d^4*(-a^3*c^3)^{(1/2)} - 4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^{(1/2)}) / (16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4))^{(1/2)} * 2i
\end{aligned}$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x\*\*2+d)\*\*2/(c\*x\*\*4+a),x)

[Out] Timed out



$$3.125 \quad \int \frac{(d+ex^2)^3}{(a+cx^4)^2} dx$$

**Optimal.** Leaf size=363

$$\frac{3(\sqrt{c}d - \sqrt{a}e)(ae^2 + cd^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2}a^{7/4}c^{7/4}} + \frac{3(\sqrt{c}d - \sqrt{a}e)(ae^2 + cd^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x)}{16\sqrt{2}a^{7/4}c^{7/4}}$$

**Rubi [A]** time = 0.41, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {1207, 1858, 1168, 1162, 617, 204, 1165, 628}

$$\frac{3(\sqrt{c}d - \sqrt{a}e)(ae^2 + cd^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2}a^{7/4}c^{7/4}} + \frac{3(\sqrt{c}d - \sqrt{a}e)(ae^2 + cd^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2}a^{7/4}c^{7/4}} - \frac{3(\sqrt{a}e + \sqrt{c}d)(ae^2 + cd^2) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{c}}{\sqrt{a}}\right)}{8\sqrt{2}a^{7/4}c^{7/4}} + \frac{3(\sqrt{a}e + \sqrt{c}d)(ae^2 + cd^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{a}} + 1\right)}{8\sqrt{2}a^{7/4}c^{7/4}} + \frac{x(3cx^2(ae^2 + cd^2) + d(cd^2 - 3ae^2))}{4ac(a + cx^4)} - \frac{e^2x^3}{c(a + cx^4)}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^3/(a + c\*x^4)^2,x]

[Out] -((e^3\*x^3)/(c\*(a + c\*x^4))) + (x\*(d\*(c\*d^2 - 3\*a\*e^2) + 3\*e\*(c\*d^2 + a\*e^2)\*x^2))/(4\*a\*c\*(a + c\*x^4)) - (3\*(Sqrt[c]\*d + Sqrt[a]\*e)\*(c\*d^2 + a\*e^2)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)])/(8\*Sqrt[2]\*a^(7/4)\*c^(7/4)) + (3\*(Sqrt[c]\*d + Sqrt[a]\*e)\*(c\*d^2 + a\*e^2)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)])/(8\*Sqrt[2]\*a^(7/4)\*c^(7/4)) - (3\*(Sqrt[c]\*d - Sqrt[a]\*e)\*(c\*d^2 + a\*e^2)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/(16\*Sqrt[2]\*a^(7/4)\*c^(7/4)) + (3\*(Sqrt[c]\*d - Sqrt[a]\*e)\*(c\*d^2 + a\*e^2)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/(16\*Sqrt[2]\*a^(7/4)\*c^(7/4))

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 617**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

**Rule 628**

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

**Rule 1162**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

**Rule 1165**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; Fre

EqQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

Rule 1168

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[-(a\*c)]

Rule 1207

Int[((d\_) + (e\_)\*(x\_)^2)^(q\_)\*((a\_) + (c\_)\*(x\_)^4)^(p\_), x\_Symbol] := Simp[(e^q\*x^(2\*q - 3)\*(a + c\*x^4)^(p + 1))/(c\*(4\*p + 2\*q + 1)), x] + Dist[1/(c\*(4\*p + 2\*q + 1)), Int[(a + c\*x^4)^p\*ExpandToSum[c\*(4\*p + 2\*q + 1)\*(d + e\*x^2)^q - a\*(2\*q - 3)\*e^q\*x^(2\*q - 4) - c\*(4\*p + 2\*q + 1)\*e^q\*x^(2\*q), x], x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IGtQ[q, 1]

Rule 1858

Int[(Pq\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)\*Pq, a + b\*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)\*Pq, a + b\*x^n, x]}, Dist[1/(a\*n\*(p + 1)\*b^(Floor[(q - 1)/n] + 1)), Int[(a + b\*x^n)^(p + 1)\*ExpandToSum[a\*n\*(p + 1)\*Q + n\*(p + 1)\*R + D[x\*R, x], x], x] - Simp[(x\*R\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)\*b^(Floor[(q - 1)/n] + 1)), x] /; GeQ[q, n] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]

Rubi steps

$$\int \frac{(d + ex^2)^3}{(a + cx^4)^2} dx = -\frac{e^3 x^3}{c(a + cx^4)} - \frac{\int \frac{-cd^3 - 3e(cd^2 + ae^2)x^2 - 3cde^2 x^4}{(a + cx^4)^2} dx}{c}$$

$$= -\frac{e^3 x^3}{c(a + cx^4)} + \frac{x(d(cd^2 - 3ae^2) + 3e(cd^2 + ae^2)x^2)}{4ac(a + cx^4)} + \frac{\int \frac{3cd(cd^2 + ae^2) + 3ce(cd^2 + ae^2)x^2}{a + cx^4} dx}{4ac^2}$$

$$= -\frac{e^3 x^3}{c(a + cx^4)} + \frac{x(d(cd^2 - 3ae^2) + 3e(cd^2 + ae^2)x^2)}{4ac(a + cx^4)} + \frac{(3(\sqrt{c}d - \sqrt{a}e)(cd^2 + ae^2)) \int \frac{\sqrt{a}\sqrt{c}}{a + cx^4} dx}{8a^{3/2}c^2}$$

$$= -\frac{e^3 x^3}{c(a + cx^4)} + \frac{x(d(cd^2 - 3ae^2) + 3e(cd^2 + ae^2)x^2)}{4ac(a + cx^4)} - \frac{(3(\sqrt{c}d - \sqrt{a}e)(cd^2 + ae^2)) \int \frac{\sqrt{2}\sqrt{c}}{\sqrt{a} + \sqrt{c}x^2} dx}{16\sqrt{2}a^{7/4}c^{7/4}}$$

$$= -\frac{e^3 x^3}{c(a + cx^4)} + \frac{x(d(cd^2 - 3ae^2) + 3e(cd^2 + ae^2)x^2)}{4ac(a + cx^4)} - \frac{3(\sqrt{c}d - \sqrt{a}e)(cd^2 + ae^2) \log(\sqrt{a} - \sqrt{c}x^2)}{16\sqrt{2}a^{7/4}c^{7/4}}$$

$$= -\frac{e^3 x^3}{c(a + cx^4)} + \frac{x(d(cd^2 - 3ae^2) + 3e(cd^2 + ae^2)x^2)}{4ac(a + cx^4)} - \frac{3(\sqrt{c}d + \sqrt{a}e)(cd^2 + ae^2) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{c}}{\sqrt{a} + \sqrt{c}x^2}\right)}{8\sqrt{2}a^{7/4}c^{7/4}}$$

**Mathematica [A]** time = 0.26, size = 371, normalized size = 1.02

$$\frac{3e^3 x^3 (a^2 c^2 (3a + c^2) + 3\sqrt{2} (a^{3/2} c^2 + \sqrt{a} c d^2 e - a\sqrt{c} d e^2 - c^{3/2} d^2) \log(-\sqrt{2}\sqrt{a}\sqrt{c}x + \sqrt{a} + \sqrt{c}x^2) + 3\sqrt{2} (-a^{3/2} c^2 - \sqrt{a} c d^2 e + a\sqrt{c} d e^2 + c^{3/2} d^2) \log(\sqrt{2}\sqrt{a}\sqrt{c}x + \sqrt{a} + \sqrt{c}x^2) - 6\sqrt{2} (a^{3/2} c^2 + \sqrt{a} c d^2 e + a\sqrt{c} d e^2 + c^{3/2} d^2) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{c}}{\sqrt{a} + \sqrt{c}x^2}\right) + 6\sqrt{2} (a^{3/2} c^2 + \sqrt{a} c d^2 e + a\sqrt{c} d e^2 + c^{3/2} d^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{a} + \sqrt{c}x^2} + 1\right)}{32a^{7/4}c^{7/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^3/(a + c\*x^4)^2,x]

[Out] 
$$\frac{((-8a^{3/4}c^{3/4}(ae^2x(3d + ex^2) - cd^2x(d + 3ex^2)))/(a + cx^4) - 6\sqrt{2}(c^{3/2}d^3 + \sqrt{a}cd^2e + a\sqrt{c}d^2e^2 + a^{3/2}e^3)\text{ArcTan}[1 - (\sqrt{2}c^{1/4}x)/a^{1/4}] + 6\sqrt{2}(c^{3/2}d^3 + \sqrt{a}cd^2e + a\sqrt{c}d^2e^2 + a^{3/2}e^3)\text{ArcTan}[1 + (\sqrt{2}c^{1/4}x)/a^{1/4}] + 3\sqrt{2}(-(c^{3/2}d^3) + \sqrt{a}cd^2e - a\sqrt{c}d^2e^2 + a^{3/2}e^3)\text{Log}[\sqrt{a} - \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2] + 3\sqrt{2}(c^{3/2}d^3 - \sqrt{a}cd^2e + a\sqrt{c}d^2e^2 - a^{3/2}e^3)\text{Log}[\sqrt{a} + \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2])}{(32a^{7/4}c^{7/4})}$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^2)^3}{(a + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)^3/(a + c\*x^4)^2,x]

[Out] IntegrateAlgebraic[(d + e\*x^2)^3/(a + c\*x^4)^2, x]

fricas [B] time = 1.13, size = 2116, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^3/(c\*x^4+a)^2,x, algorithm="fricas")

[Out] 
$$\frac{1}{16} \cdot \frac{(4(3cd^2e - ae^3)x^3 - 3(ac^2x^4 + a^2c)\sqrt{-(2c^2d^5e + 4acd^3e^3 + 2a^2d^5e + a^3c^3)\sqrt{-(c^6d^{12} + 2ac^5d^{10}e^2 - a^2c^4d^8e^4 - 4a^3c^3d^6e^6 - a^4c^2d^4e^8 + 2a^5cd^2e^{10} + a^6e^{12})}}/(a^7c^7)))/(a^3c^3)) \cdot \log(-27(c^5d^{10} + 3ac^4d^8e^2 + 2a^2c^3d^6e^4 - 2a^3c^2d^4e^6 - 3a^4cd^2e^8 - a^5e^{10})x + 27(a^2c^5d^7 + a^3c^4d^5e^2 - a^4c^3d^3e^4 - a^5c^2d^2e^6 + a^6c^5e\sqrt{-(c^6d^{12} + 2ac^5d^{10}e^2 - a^2c^4d^8e^4 - 4a^3c^3d^6e^6 - a^4c^2d^4e^8 + 2a^5cd^2e^{10} + a^6e^{12})}}/(a^7c^7)))\sqrt{-(2c^2d^5e + 4acd^3e^3 + 2a^2d^5e + a^3c^3)\sqrt{-(c^6d^{12} + 2ac^5d^{10}e^2 - a^2c^4d^8e^4 - 4a^3c^3d^6e^6 - a^4c^2d^4e^8 + 2a^5cd^2e^{10} + a^6e^{12})}}/(a^7c^7)))/(a^3c^3)) + 3(ac^2x^4 + a^2c)\sqrt{-(2c^2d^5e + 4acd^3e^3 + 2a^2d^5e + a^3c^3)\sqrt{-(c^6d^{12} + 2ac^5d^{10}e^2 - a^2c^4d^8e^4 - 4a^3c^3d^6e^6 - a^4c^2d^4e^8 + 2a^5cd^2e^{10} + a^6e^{12})}}/(a^7c^7)))/(a^3c^3)) \cdot \log(-27(c^5d^{10} + 3ac^4d^8e^2 + 2a^2c^3d^6e^4 - 2a^3c^2d^4e^6 - 3a^4cd^2e^8 - a^5e^{10})x - 27(a^2c^5d^7 + a^3c^4d^5e^2 - a^4c^3d^3e^4 - a^5c^2d^2e^6 + a^6c^5e\sqrt{-(c^6d^{12} + 2ac^5d^{10}e^2 - a^2c^4d^8e^4 - 4a^3c^3d^6e^6 - a^4c^2d^4e^8 + 2a^5cd^2e^{10} + a^6e^{12})}}/(a^7c^7)))\sqrt{-(2c^2d^5e + 4acd^3e^3 + 2a^2d^5e + a^3c^3)\sqrt{-(c^6d^{12} + 2ac^5d^{10}e^2 - a^2c^4d^8e^4 - 4a^3c^3d^6e^6 - a^4c^2d^4e^8 + 2a^5cd^2e^{10} + a^6e^{12})}}/(a^7c^7)))/(a^3c^3)) - 3(ac^2x^4 + a^2c)\sqrt{-(2c^2d^5e + 4acd^3e^3 + 2a^2d^5e - a^3c^3)\sqrt{-(c^6d^{12} + 2ac^5d^{10}e^2 - a^2c^4d^8e^4 - 4a^3c^3d^6e^6 - a^4c^2d^4e^8 + 2a^5cd^2e^{10} + a^6e^{12})}}/(a^7c^7)))/(a^3c^3)) \cdot \log(-27(c^5d^{10} + 3ac^4d^8e^2 + 2a^2c^3d^6e^4 - 2a^3c^2d^4e^6 - 3a^4cd^2e^8 - a^5e^{10})x + 27(a^2c^5d^7 + a^3c^4d^5e^2 - a^4c^3d^3e^4 - a^5c^2d^2e^6 - a^6c^5e\sqrt{-(c^6d^{12} + 2ac^5d^{10}e^2 - a^2c^4d^8e^4 - 4a^3c^3d^6e^6 - a^4c^2d^4e^8 + 2a^5cd^2e^{10} + a^6e^{12})}}/(a^7c^7)))\sqrt{-(2c^2d^5e + 4acd^3e^3 + 2a^2d^5e - a^3c^3)\sqrt{-(c^6d^{12} + 2ac^5d^{10}e^2 - a^2c^4d^8e^4 - 4a^3c^3d^6e^6 - a^4c^2d^4e^8 + 2a^5cd^2e^{10} + a^6e^{12})}}/(a^7c^7)))/(a^3c^3))$$

$2*a^5*c*d^2*e^{10} + a^6*e^{12})/(a^7*c^7)))/(a^3*c^3))) + 3*(a*c^2*x^4 + a^2*c)*sqrt(-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 - a^3*c^3*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^{10} + a^6*e^{12})/(a^7*c^7)))/(a^3*c^3)))*log(-27*(c^5*d^{10} + 3*a*c^4*d^8*e^2 + 2*a^2*c^3*d^6*e^4 - 2*a^3*c^2*d^4*e^6 - 3*a^4*c*d^2*e^8 - a^5*e^{10})*x - 27*(a^2*c^5*d^7 + a^3*c^4*d^5*e^2 - a^4*c^3*d^3*e^4 - a^5*c^2*d*e^6 - a^6*c^5*e*sqrt(-(c^6*d^{12} + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^{10} + a^6*e^{12})/(a^7*c^7)))*sqrt(-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 - a^3*c^3*sqrt(-(c^6*d^{12} + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^{10} + a^6*e^{12})/(a^7*c^7)))/(a^3*c^3))) + 4*(c*d^3 - 3*a*d*e^2)*x)/(a*c^2*x^4 + a^2*c)$

**giac [A]** time = 0.19, size = 425, normalized size = 1.17

$$\frac{3\sqrt{2}e^{\frac{1}{2} \arctan\left(\frac{\sqrt{2}x}{1+\sqrt{2}x}\right)} + (ac)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}x}{1+\sqrt{2}x}\right) + (ac)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{1+\sqrt{2}x}\right)}{16\sqrt{2}ac} + \frac{3\sqrt{2}e^{\frac{1}{2} \arctan\left(\frac{\sqrt{2}x}{1+\sqrt{2}x}\right)} + (ac)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}x}{1+\sqrt{2}x}\right) + (ac)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{1+\sqrt{2}x}\right)}{16\sqrt{2}ac} + \frac{3\sqrt{2}e^{\frac{1}{2} \arctan\left(\frac{\sqrt{2}x}{1+\sqrt{2}x}\right)} + (ac)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}x}{1+\sqrt{2}x}\right) + (ac)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{1+\sqrt{2}x}\right)}{32\sqrt{2}ac} + \frac{3\sqrt{2}e^{\frac{1}{2} \arctan\left(\frac{\sqrt{2}x}{1+\sqrt{2}x}\right)} + (ac)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}x}{1+\sqrt{2}x}\right) + (ac)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{1+\sqrt{2}x}\right)}{32\sqrt{2}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^3/(c\*x^4+a)^2,x, algorithm="giac")

[Out]  $\frac{1}{4}*(3*c*d^2*x^3*e + c*d^3*x - a*x^3*e^3 - 3*a*d*x*e^2)/((c*x^4 + a)*a*c) + \frac{3}{16}*sqrt(2)*((a*c^3)^{(1/4)}*c^3*d^3 + (a*c^3)^{(1/4)}*a*c^2*d*e^2 + (a*c^3)^{(3/4)}*c*d^2*e + (a*c^3)^{(3/4)}*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^{(1/4)})/(a/c)^{(1/4)})/(a^2*c^4) + \frac{3}{16}*sqrt(2)*((a*c^3)^{(1/4)}*c^3*d^3 + (a*c^3)^{(1/4)}*a*c^2*d*e^2 + (a*c^3)^{(3/4)}*c*d^2*e + (a*c^3)^{(3/4)}*a*e^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^{(1/4)})/(a/c)^{(1/4)})/(a^2*c^4) + \frac{3}{32}*sqrt(2)*((a*c^3)^{(1/4)}*c^3*d^3 + (a*c^3)^{(1/4)}*a*c^2*d*e^2 - (a*c^3)^{(3/4)}*c*d^2*e - (a*c^3)^{(3/4)}*a*e^3)*log(x^2 + sqrt(2)*x*(a/c)^{(1/4)} + sqrt(a/c))/(a^2*c^4) - \frac{3}{32}*sqrt(2)*((a*c^3)^{(1/4)}*c^3*d^3 + (a*c^3)^{(1/4)}*a*c^2*d*e^2 - (a*c^3)^{(3/4)}*c*d^2*e - (a*c^3)^{(3/4)}*a*e^3)*log(x^2 - sqrt(2)*x*(a/c)^{(1/4)} + sqrt(a/c))/(a^2*c^4)$

**maple [B]** time = 0.01, size = 624, normalized size = 1.72

$$\frac{3\sqrt{2}e^{\frac{1}{2} \arctan\left(\frac{\sqrt{2}x}{1+\sqrt{2}x}\right)} + (ac)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}x}{1+\sqrt{2}x}\right) + (ac)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{1+\sqrt{2}x}\right)}{16\sqrt{2}ac} + \frac{3\sqrt{2}e^{\frac{1}{2} \arctan\left(\frac{\sqrt{2}x}{1+\sqrt{2}x}\right)} + (ac)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}x}{1+\sqrt{2}x}\right) + (ac)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{1+\sqrt{2}x}\right)}{16\sqrt{2}ac} + \frac{3\sqrt{2}e^{\frac{1}{2} \arctan\left(\frac{\sqrt{2}x}{1+\sqrt{2}x}\right)} + (ac)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}x}{1+\sqrt{2}x}\right) + (ac)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{1+\sqrt{2}x}\right)}{32\sqrt{2}ac} + \frac{3\sqrt{2}e^{\frac{1}{2} \arctan\left(\frac{\sqrt{2}x}{1+\sqrt{2}x}\right)} + (ac)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}x}{1+\sqrt{2}x}\right) + (ac)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{1+\sqrt{2}x}\right)}{32\sqrt{2}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^3/(c\*x^4+a)^2,x)

[Out]  $(-1/4*e*(a*e^2-3*c*d^2)/a/c*x^3-1/4*d*(3*a*e^2-c*d^2)/a/c*x)/(c*x^4+a)+3/32/a/c*d*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))e^2+3/32/a^2*d^3*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})))+3/16/a/c*d*(a/c)^{(1/4)}*2^{(1/2)}*arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*e^2+3/16/a^2*d^3*(a/c)^{(1/4)}*2^{(1/2)}*arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)+3/16/a/c*d*(a/c)^{(1/4)}*2^{(1/2)}*arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*e^2+3/16/a^2*d^3*(a/c)^{(1/4)}*2^{(1/2)}*arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)+3/32/c^2*e^3/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})))+3/32/a/c*e/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})))*d^2+3/16/c^2*e^3/(a/c)^{(1/4)}*2^{(1/2)}*arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)+3/16/a/c*e/(a/c)^{(1/4)}*2^{(1/2)}*arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)+3/16/a/c*e/(a/c)^{(1/4)}*2^{(1/2)}*arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^2$

**maxima [A]** time = 2.36, size = 292, normalized size = 0.80

$$\frac{(3cd^2e - ae^3)x^3 + (cd^3 - 3ade^2)x}{4(ac^2x^4 + a^2c)} + \frac{2\sqrt{2}(\sqrt{cd+\sqrt{ae}})\arctan\left(\frac{\sqrt{2}\sqrt{cd+\sqrt{ae}}}{2\sqrt{\sqrt{a}\sqrt{c}}}\right) + 2\sqrt{2}(\sqrt{cd+\sqrt{ae}})\arctan\left(\frac{\sqrt{2}\sqrt{cd+\sqrt{ae}}}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{\sqrt{2}(\sqrt{cd-\sqrt{ae}})\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right) - \sqrt{2}(\sqrt{cd-\sqrt{ae}})\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^3/(c\*x^4+a)^2,x, algorithm="maxima")

[Out]  $\frac{1}{4} * ((3 * c * d^2 * e - a * e^3) * x^3 + (c * d^3 - 3 * a * d * e^2) * x) / (a * c^2 * x^4 + a^2 * c) + \frac{3}{32} * (c * d^2 + a * e^2) * (2 * \sqrt{2}) * (\sqrt{c} * d + \sqrt{a} * e) * \arctan(1/2 * \sqrt{2}) * (2 * \sqrt{c} * x + \sqrt{2}) * a^{1/4} * c^{1/4} / \sqrt{(\sqrt{a} * \sqrt{c})} / (\sqrt{a} * \sqrt{(\sqrt{a} * \sqrt{c}) * \sqrt{c}}) + 2 * \sqrt{2} * (\sqrt{c} * d + \sqrt{a} * e) * \arctan(1/2 * \sqrt{2}) * (2 * \sqrt{c} * x - \sqrt{2}) * a^{1/4} * c^{1/4} / \sqrt{(\sqrt{a} * \sqrt{c})} / (\sqrt{a} * \sqrt{(\sqrt{a} * \sqrt{c}) * \sqrt{c}}) + \sqrt{2} * (\sqrt{c} * d - \sqrt{a} * e) * \log(\sqrt{c} * x^2 + \sqrt{2}) * a^{1/4} * c^{1/4} * x + \sqrt{a}) / (a^{3/4} * c^{3/4}) - \sqrt{2} * (\sqrt{c} * d - \sqrt{a} * e) * \log(\sqrt{c} * x^2 - \sqrt{2}) * a^{1/4} * c^{1/4} * x + \sqrt{a}) / (a^{3/4} * c^{3/4}) / (a * c)$

**mupad [B]** time = 4.94, size = 2560, normalized size = 7.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)^3/(a + c\*x^4)^2,x)

[Out]  $-\frac{(d * x * (3 * a * e^2 - c * d^2))}{(4 * a * c)} + \frac{(e * x^3 * (a * e^2 - 3 * c * d^2))}{(4 * a * c)} / (a + c * x^4) - 2 * \operatorname{atanh}\left(\frac{(9 * c^3 * d^6 * x * ((9 * e^6 * (-a^7 * c^7)^{1/2})) / (256 * a^4 * c^7) - (9 * d^5 * e) / (128 * a^3 * c) - (9 * d^3 * e^3) / (64 * a^2 * c^2) - (9 * d^6 * (-a^7 * c^7)^{1/2})) / (256 * a^7 * c^4) - (9 * d * e^5) / (128 * a * c^3) + (9 * d^2 * e^4 * (-a^7 * c^7)^{1/2}) / (256 * a^5 * c^6) - (9 * d^4 * e^2 * (-a^7 * c^7)^{1/2}) / (256 * a^6 * c^5))^{1/2}}{(2 * ((27 * c * d^6 * e^3) / 16 - (27 * a^3 * e^9) / (32 * c^2) + (27 * c^2 * d^8 * e) / (32 * a) - (27 * a^2 * d^2 * e^7) / (16 * c) + (27 * d^9 * (-a^7 * c^7)^{1/2}) / (32 * a^5 * c) - (27 * d * e^8 * (-a^7 * c^7)^{1/2}) / (32 * a * c^5) - (27 * d^3 * e^6 * (-a^7 * c^7)^{1/2}) / (16 * a^2 * c^4) + (27 * d^7 * e^2 * (-a^7 * c^7)^{1/2}) / (16 * a^4 * c^2))}\right) + \frac{(9 * a * e^6 * x * ((9 * e^6 * (-a^7 * c^7)^{1/2})) / (256 * a^4 * c^7) - (9 * d^5 * e) / (128 * a^3 * c) - (9 * d^3 * e^3) / (64 * a^2 * c^2) - (9 * d^6 * (-a^7 * c^7)^{1/2}) / (256 * a^7 * c^4) - (9 * d * e^5) / (128 * a * c^3) + (9 * d^2 * e^4 * (-a^7 * c^7)^{1/2}) / (256 * a^5 * c^6) - (9 * d^4 * e^2 * (-a^7 * c^7)^{1/2}) / (256 * a^6 * c^5))^{1/2}}{(2 * ((27 * a * e^9) / (32 * c^2) + (27 * d^2 * e^7) / (16 * c) - (27 * c * d^6 * e^3) / (16 * a^2) - (27 * c^2 * d^8 * e) / (32 * a^3) - (27 * d^9 * (-a^7 * c^7)^{1/2}) / (32 * a^7 * c) + (27 * d * e^8 * (-a^7 * c^7)^{1/2}) / (32 * a^3 * c^5) + (27 * d^3 * e^6 * (-a^7 * c^7)^{1/2}) / (16 * a^4 * c^4) - (27 * d^7 * e^2 * (-a^7 * c^7)^{1/2}) / (16 * a^6 * c^2))}\right) + \frac{(9 * c * d^2 * e^4 * x * ((9 * e^6 * (-a^7 * c^7)^{1/2})) / (256 * a^4 * c^7) - (9 * d^5 * e) / (128 * a^3 * c) - (9 * d^3 * e^3) / (64 * a^2 * c^2) - (9 * d^6 * (-a^7 * c^7)^{1/2}) / (256 * a^7 * c^4) - (9 * d * e^5) / (128 * a * c^3) + (9 * d^2 * e^4 * (-a^7 * c^7)^{1/2}) / (256 * a^5 * c^6) - (9 * d^4 * e^2 * (-a^7 * c^7)^{1/2}) / (256 * a^6 * c^5))^{1/2}}{(2 * ((27 * a * e^9) / (32 * c^2) + (27 * d^2 * e^7) / (16 * c) - (27 * c * d^6 * e^3) / (16 * a^2) - (27 * c^2 * d^8 * e) / (32 * a^3) - (27 * d^9 * (-a^7 * c^7)^{1/2}) / (32 * a^7 * c) + (27 * d * e^8 * (-a^7 * c^7)^{1/2}) / (32 * a^3 * c^5) + (27 * d^3 * e^6 * (-a^7 * c^7)^{1/2}) / (16 * a^4 * c^4) - (27 * d^7 * e^2 * (-a^7 * c^7)^{1/2}) / (16 * a^6 * c^2))}\right) - \frac{(9 * c^2 * d^4 * e^2 * x * ((9 * e^6 * (-a^7 * c^7)^{1/2})) / (256 * a^4 * c^7) - (9 * d^5 * e) / (128 * a^3 * c) - (9 * d^3 * e^3) / (64 * a^2 * c^2) - (9 * d^6 * (-a^7 * c^7)^{1/2}) / (256 * a^7 * c^4) - (9 * d * e^5) / (128 * a * c^3) + (9 * d^2 * e^4 * (-a^7 * c^7)^{1/2}) / (256 * a^5 * c^6) - (9 * d^4 * e^2 * (-a^7 * c^7)^{1/2}) / (256 * a^6 * c^5))^{1/2}}{(2 * ((27 * a * e^9) / (32 * c^2) + (27 * d^2 * e^7) / (16 * c) - (27 * c * d^6 * e^3) / (16 * a^2) - (27 * c^2 * d^8 * e) / (32 * a^3) - (27 * d^9 * (-a^7 * c^7)^{1/2}) / (32 * a^7 * c) + (27 * d * e^8 * (-a^7 * c^7)^{1/2}) / (32 * a^3 * c^5) + (27 * d^3 * e^6 * (-a^7 * c^7)^{1/2}) / (16 * a^4 * c^4) - (27 * d^7 * e^2 * (-a^7 * c^7)^{1/2}) / (16 * a^6 * c^2))}\right) * (-\frac{(9 * (c^3 * d^6 * (-a^7 * c^7)^{1/2}) - a^3 * e^6 * (-a^7 * c^7)^{1/2} + 2 * a^4 * c^6 * d^5 * e + 2 * a^6 * c^4 * d * e^5 + 4 * a^5 * c^5 * d^3 * e^3 + a * c^2 * d^4 * e^2 * (-a^7 * c^7)^{1/2} - a^2 * c * d^2 * e^4 * (-a^7 * c^7)^{1/2})}{(256 * a^7 * c^7))^{1/2}} - 2 * \operatorname{atanh}\left(\frac{(9 * c^3 * d^6 * x * ((9 * d^6 * (-a^7 * c^7)^{1/2})) / (256 * a^7 * c^4) - (9 * d^5 * e) / (128 * a^3 * c) - (9 * d^3 * e^3) / (64 * a^2 * c^2) - (9 * d * e^5) / (128 * a * c^3) - (9 * e^6 * (-a^7 * c^7)^{1/2})) / (256 * a^4 * c^7) - (9 * d^2 * e^4 * (-a^7 * c^7)^{1/2}) / (256 * a^5 * c^6) + (9 * d^4 * e^2 * (-a^7 * c^7)^{1/2}) / (256 * a^6 * c^5))^{1/2}}{(2 * ((27 * c * d^6 * e^3) / 16 - (27 * a^3 * e^9) / (32 * c^2) + (27 * c^2 * d^8 * e) / (32 * a) - (27 * a^2 * d^2 * e^7) / (16 * c) - (27 * d^9 * (-a^7 * c^7)^{1/2}) / (32 * a^5 * c) + (27 * d * e^8 * (-a^7 * c^7)^{1/2}) / (32 * a * c^5) + (27 * d^3 * e^6 * (-a^7 * c^7)^{1/2}) / (16 * a^2 * c^4) - (27 * d^7 * e^2 * (-a^7 * c^7)^{1/2}) / (16 * a^4 * c^2))}\right)$

$c^2))) + (9*a*e^6*x*((9*d^6*(-a^7*c^7)^{(1/2)})/(256*a^7*c^4) - (9*d^5*e)/(128*a^3*c) - (9*d^3*e^3)/(64*a^2*c^2) - (9*d*e^5)/(128*a*c^3) - (9*e^6*(-a^7*c^7)^{(1/2)})/(256*a^4*c^7) - (9*d^2*e^4*(-a^7*c^7)^{(1/2)})/(256*a^5*c^6) + (9*d^4*e^2*(-a^7*c^7)^{(1/2)})/(256*a^6*c^5))^{(1/2)})/(2*((27*a*e^9)/(32*c^2) + (27*d^2*e^7)/(16*c) - (27*c*d^6*e^3)/(16*a^2) - (27*c^2*d^8*e)/(32*a^3) + (27*d^9*(-a^7*c^7)^{(1/2)})/(32*a^7*c) - (27*d*e^8*(-a^7*c^7)^{(1/2)})/(32*a^3*c^5) - (27*d^3*e^6*(-a^7*c^7)^{(1/2)})/(16*a^4*c^4) + (27*d^7*e^2*(-a^7*c^7)^{(1/2)})/(16*a^6*c^2))) + (9*c*d^2*e^4*x*((9*d^6*(-a^7*c^7)^{(1/2)})/(256*a^7*c^4) - (9*d^5*e)/(128*a^3*c) - (9*d^3*e^3)/(64*a^2*c^2) - (9*d*e^5)/(128*a*c^3) - (9*e^6*(-a^7*c^7)^{(1/2)})/(256*a^4*c^7) - (9*d^2*e^4*(-a^7*c^7)^{(1/2)})/(256*a^5*c^6) + (9*d^4*e^2*(-a^7*c^7)^{(1/2)})/(256*a^6*c^5))^{(1/2)})/(2*((27*a^2*e^9)/(32*c^2) + (27*a*d^2*e^7)/(16*c) - (27*c*d^6*e^3)/(16*a) - (27*c^2*d^8*e)/(32*a^2) + (27*d^9*(-a^7*c^7)^{(1/2)})/(32*a^6*c) - (27*d*e^8*(-a^7*c^7)^{(1/2)})/(32*a^2*c^5) - (27*d^3*e^6*(-a^7*c^7)^{(1/2)})/(16*a^3*c^4) + (27*d^7*e^2*(-a^7*c^7)^{(1/2)})/(16*a^5*c^2))))*(-(9*(a^3*e^6*(-a^7*c^7)^{(1/2)} - c^3*d^6*(-a^7*c^7)^{(1/2)} + 2*a^4*c^6*d^5*e + 2*a^6*c^4*d*e^5 + 4*a^5*c^5*d^3*e^3 - a*c^2*d^4*e^2*(-a^7*c^7)^{(1/2)} + a^2*c*d^2*e^4*(-a^7*c^7)^{(1/2)}))/(256*a^7*c^7))^{(1/2)}$

**sympy [A]** time = 3.37, size = 352, normalized size = 0.97

$$\text{RootSum}\left(65536t^7z^2 + t^9(9216a^6c^4d^5 + 18432a^7c^2d^3 + 9216a^8c^2d^2) + 81a^{10}e^{12} + 486a^9c^5e^{10} + 1215a^8c^4d^2e^8 + 1620a^7c^3d^2e^6 + 1215a^6c^2d^4e^4 + 486a^5c^4d^6 + 81a^4d^{12}\left(1 + t \log\left(x + \frac{4096a^6d^5c + 432a^5c^2d^6 + 720a^4c^3d^4e + 144a^3c^4d^2e^2 - 144a^2c^5d^2e^4}{27a^5e^{10} + 81a^4c^4d^6 + 54a^3c^3d^4e^2 - 54a^2c^2d^3e^4 - 81ac^4d^2e^6 - 27a^6d^{10}}\right)\right) + \frac{x^3(-ae^3 + 3cd^2) + x(-3ad^2 + cd^3)}{4a^2c + 4ac^2d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*3/(c\*x\*\*4+a)\*\*2,x)

[Out] RootSum(65536\*\_t\*\*4\*a\*\*7\*c\*\*7 + \_t\*\*2\*(9216\*a\*\*6\*c\*\*4\*d\*e\*\*5 + 18432\*a\*\*5\*c\*\*5\*d\*\*3\*e\*\*3 + 9216\*a\*\*4\*c\*\*6\*d\*\*5\*e) + 81\*a\*\*6\*e\*\*12 + 486\*a\*\*5\*c\*d\*\*2\*e\*\*10 + 1215\*a\*\*4\*c\*\*2\*d\*\*4\*e\*\*8 + 1620\*a\*\*3\*c\*\*3\*d\*\*6\*e\*\*6 + 1215\*a\*\*2\*c\*\*4\*d\*\*8\*e\*\*4 + 486\*a\*c\*\*5\*d\*\*10\*e\*\*2 + 81\*c\*\*6\*d\*\*12, Lambda(\_t, \_t\*log(x + (4096\*\_t\*\*3\*a\*\*6\*c\*\*5\*e + 432\*\_t\*a\*\*5\*c\*\*2\*d\*e\*\*6 + 720\*\_t\*a\*\*4\*c\*\*3\*d\*\*3\*e\*\*4 + 144\*\_t\*a\*\*3\*c\*\*4\*d\*\*5\*e\*\*2 - 144\*\_t\*a\*\*2\*c\*\*5\*d\*\*7)/(27\*a\*\*5\*e\*\*10 + 81\*a\*\*4\*c\*d\*\*2\*e\*\*8 + 54\*a\*\*3\*c\*\*2\*d\*\*4\*e\*\*6 - 54\*a\*\*2\*c\*\*3\*d\*\*6\*e\*\*4 - 81\*a\*c\*\*4\*d\*\*8\*e\*\*2 - 27\*c\*\*5\*d\*\*10)))) + (x\*\*3\*(-a\*e\*\*3 + 3\*c\*d\*\*2\*e) + x\*(-3\*a\*d\*e\*\*2 + c\*d\*\*3))/(4\*a\*\*2\*c + 4\*a\*c\*\*2\*x\*\*4)

$$3.126 \quad \int \frac{(d+ex^2)^2}{(a+cx^4)^2} dx$$

**Optimal.** Leaf size=349

$$\frac{(-2\sqrt{a}\sqrt{c}de + ae^2 + 3cd^2) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2}a^{7/4}c^{5/4}} + \frac{(-2\sqrt{a}\sqrt{c}de + ae^2 + 3cd^2) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x)}{16\sqrt{2}a^{7/4}c^{5/4}}$$

**Rubi [A]** time = 0.31, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {1207, 1179, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(-2\sqrt{a}\sqrt{c}de + ae^2 + 3cd^2) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2}a^{7/4}c^{5/4}} + \frac{(-2\sqrt{a}\sqrt{c}de + ae^2 + 3cd^2) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2}a^{7/4}c^{5/4}} - \frac{(2\sqrt{a}\sqrt{c}de + ae^2 + 3cd^2) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}}\right)}{8\sqrt{2}a^{7/4}c^{5/4}} + \frac{(2\sqrt{a}\sqrt{c}de + ae^2 + 3cd^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}} + 1\right)}{8\sqrt{2}a^{7/4}c^{5/4}} + \frac{x(ae^2 + 3cd^2 + 6dex^2)}{12ac(a+cx^4)} - \frac{e^2x}{3c(a+cx^4)}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^2/(a + c\*x^4)^2,x]

[Out]  $-(e^2x)/(3c(a + cx^4)) + (x(3cd^2 + ae^2 + 6cdex^2))/(12ac(a + cx^4)) - ((3cd^2 + 2\sqrt{a}\sqrt{c}de + ae^2)\text{ArcTan}[1 - (\sqrt{2}\sqrt[4]{c}x/a^{1/4})]/(8\sqrt{2}a^{7/4}c^{5/4}) + ((3cd^2 + 2\sqrt{a}\sqrt{c}de + ae^2)\text{ArcTan}[1 + (\sqrt{2}\sqrt[4]{c}x/a^{1/4})]/(8\sqrt{2}a^{7/4}c^{5/4}) - ((3cd^2 - 2\sqrt{a}\sqrt{c}de + ae^2)\text{Log}[\sqrt{a} - \sqrt{2}\sqrt[4]{c}x/a^{1/4}]/(16\sqrt{2}a^{7/4}c^{5/4}) + ((3cd^2 - 2\sqrt{a}\sqrt{c}de + ae^2)\text{Log}[\sqrt{a} + \sqrt{2}\sqrt[4]{c}x/a^{1/4}]/(16\sqrt{2}a^{7/4}c^{5/4}))$

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 1162

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

#### Rule 1165

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; Fre

$eQ[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

### Rule 1168

$\text{Int}[\{(d\_)+(e\_)*(x\_)^2\}/\{(a\_)+(c\_)*(x\_)^4\}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[-(a*c)]$

### Rule 1179

$\text{Int}[\{(d\_)+(e\_)*(x\_)^2\}*\{(a\_)+(c\_)*(x\_)^4\}^{(p\_)}, x\_Symbol] \ :> \ -\text{Simp}[(x*(d + e*x^2)*(a + c*x^4)^{(p + 1)})/(4*a*(p + 1)), x] + \text{Dist}[1/(4*a*(p + 1)), \text{Int}[\text{Simp}[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^{(p + 1)}, x], x] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{IntegerQ}[2*p]$

### Rule 1207

$\text{Int}[\{(d\_)+(e\_)*(x\_)^2\}^{(q\_)}*\{(a\_)+(c\_)*(x\_)^4\}^{(p\_)}, x\_Symbol] \ :> \ \text{Simp}[(e^q*x^{(2*q - 3)}*(a + c*x^4)^{(p + 1)})/(c*(4*p + 2*q + 1)), x] + \text{Dist}[1/(c*(4*p + 2*q + 1)), \text{Int}[(a + c*x^4)^p*\text{ExpandToSum}[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^{(2*q - 4)} - c*(4*p + 2*q + 1)*e^q*x^{(2*q)}, x], x] \ /; \ \text{FreeQ}[\{a, c, d, e, p\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{IGtQ}[q, 1]$

### Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)^2}{(a + cx^4)^2} dx &= -\frac{e^2 x}{3c(a + cx^4)} - \frac{\int \frac{-3cd^2 - ae^2 - 6cdex^2}{(a + cx^4)^2} dx}{3c} \\ &= -\frac{e^2 x}{3c(a + cx^4)} + \frac{x(3cd^2 + ae^2 + 6cdex^2)}{12ac(a + cx^4)} + \frac{\int \frac{3(3cd^2 + ae^2) + 6cdex^2}{a + cx^4} dx}{12ac} \\ &= -\frac{e^2 x}{3c(a + cx^4)} + \frac{x(3cd^2 + ae^2 + 6cdex^2)}{12ac(a + cx^4)} + \frac{(3cd^2 - 2\sqrt{a}\sqrt{c}de + ae^2) \int \frac{\sqrt{a}\sqrt{c} - cx^2}{a + cx^4} dx}{8a^{3/2}c^{3/2}} + \frac{(3cd^2 - 2\sqrt{a}\sqrt{c}de + ae^2) \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt{c}} + 2x}{\sqrt{c} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt{c}} - x^2} dx}{16\sqrt{2}a^{7/4}c^{5/4}}}{16\sqrt{2}a^{7/4}c^{5/4}} \\ &= -\frac{e^2 x}{3c(a + cx^4)} + \frac{x(3cd^2 + ae^2 + 6cdex^2)}{12ac(a + cx^4)} - \frac{(3cd^2 - 2\sqrt{a}\sqrt{c}de + ae^2) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx})}{16\sqrt{2}a^{7/4}c^{5/4}} \\ &= -\frac{e^2 x}{3c(a + cx^4)} + \frac{x(3cd^2 + ae^2 + 6cdex^2)}{12ac(a + cx^4)} - \frac{(3cd^2 + 2\sqrt{a}\sqrt{c}de + ae^2) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt{a}}\right)}{8\sqrt{2}a^{7/4}c^{5/4}} + \frac{(3cd^2 - 2\sqrt{a}\sqrt{c}de + ae^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt{a}}\right)}{8\sqrt{2}a^{7/4}c^{5/4}} \end{aligned}$$

**Mathematica [A]** time = 0.17, size = 295, normalized size = 0.85

$$\frac{8e^{2q}\sqrt{c}(a^2x - a(d + 2ex^2))}{a^{2+2q}} - \sqrt{2}(-2\sqrt{a}\sqrt{c}de + ae^2 + 3cd^2) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{c}x^2) + \sqrt{2}(-2\sqrt{a}\sqrt{c}de + ae^2 + 3cd^2) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{c}x^2) - 2\sqrt{2}(2\sqrt{a}\sqrt{c}de + ae^2 + 3cd^2) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt{a}}\right) + 2\sqrt{2}(2\sqrt{a}\sqrt{c}de + ae^2 + 3cd^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt{a}}\right) + \frac{32e^{2q}x^{2q+4}}{32e^{2q}x^{2q+4}}$$

Antiderivative was successfully verified.



[In] Integrate[(d + e\*x^2)^2/(a + c\*x^4)^2,x]

[Out] 
$$\frac{((-8*a^{3/4}*c^{1/4}*(a*e^2*x - c*d*x*(d + 2*e*x^2)))/(a + c*x^4) - 2*\text{Sqrt}[2]*(3*c*d^2 + 2*\text{Sqrt}[a]*\text{Sqrt}[c]*d*e + a*e^2)*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{1/4}*x)/a^{1/4}] + 2*\text{Sqrt}[2]*(3*c*d^2 + 2*\text{Sqrt}[a]*\text{Sqrt}[c]*d*e + a*e^2)*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{1/4}*x)/a^{1/4}] - \text{Sqrt}[2]*(3*c*d^2 - 2*\text{Sqrt}[a]*\text{Sqrt}[c]*d*e + a*e^2)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{1/4}*c^{1/4}*x + \text{Sqrt}[c]*x^2] + \text{Sqrt}[2]*(3*c*d^2 - 2*\text{Sqrt}[a]*\text{Sqrt}[c]*d*e + a*e^2)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{1/4}*c^{1/4}*x + \text{Sqrt}[c]*x^2])/(32*a^{7/4}*c^{5/4})$$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^2)^2}{(a + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)^2/(a + c\*x^4)^2,x]

[Out] IntegrateAlgebraic[(d + e\*x^2)^2/(a + c\*x^4)^2, x]

**fricas** [B] time = 1.77, size = 1596, normalized size = 4.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2/(c\*x^4+a)^2,x, algorithm="fricas")

[Out] 
$$\frac{1}{16}*(8*c*d*e*x^3 + (a*c^2*x^4 + a^2*c)*\text{sqrt}(-(a^3*c^2*\text{sqrt}(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 12*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))*\text{log}((81*c^4*d^8 + 108*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)*x + (2*a^6*c^4*d*e*\text{sqrt}(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 27*a^2*c^4*d^6 + 15*a^3*c^3*d^4*e^2 + 5*a^4*c^2*d^2*e^4 + a^5*c*e^6)*\text{sqrt}(-(a^3*c^2*\text{sqrt}(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 12*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))) - (a*c^2*x^4 + a^2*c)*\text{sqrt}(-(a^3*c^2*\text{sqrt}(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 12*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))*\text{log}((81*c^4*d^8 + 108*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)*x - (2*a^6*c^4*d*e*\text{sqrt}(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 27*a^2*c^4*d^6 + 15*a^3*c^3*d^4*e^2 + 5*a^4*c^2*d^2*e^4 + a^5*c*e^6)*\text{sqrt}(-(a^3*c^2*\text{sqrt}(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 12*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))) - (a*c^2*x^4 + a^2*c)*\text{sqrt}((a^3*c^2*\text{sqrt}(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 12*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))*\text{log}((81*c^4*d^8 + 108*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)*x + (2*a^6*c^4*d*e*\text{sqrt}(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 27*a^2*c^4*d^6 - 15*a^3*c^3*d^4*e^2 - 5*a^4*c^2*d^2*e^4 - a^5*c*e^6)*\text{sqrt}((a^3*c^2*\text{sqrt}(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 12*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))) + (a*c^2*x^4 + a^2*c)*\text{sqrt}((a^3*c^2*\text{sqrt}(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 12*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))*\text{log}((81*c^4*d^8 + 108*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)*x - (2*a^6*c^4*d*e*\text{sqrt}(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 27*a^2*c^4*d^6 - 15*a^3*c^3*d^4*e^2 - 5*a^4*c^2*d^2*e^4 - a^5*c*e^6)*\text{sqrt}((a^3*c^2*\text{sqrt}(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 12*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))) + 4*(c*d^2 - a*e^2)*x)/(a*c^2*x^4 + a^2*c)$$

**giac** [A] time = 0.19, size = 350, normalized size = 1.00

$$\frac{2cdex^3 + ad^2x - ad^2}{4(c^2x^4 + a)ac} + \frac{\sqrt{2} \left( 3(ac)^{\frac{1}{2}} c^{\frac{1}{2}} d^2 + (ac)^{\frac{1}{2}} ac^2 + 2(ac)^{\frac{1}{2}} d \right) \arctan\left(\frac{\sqrt{2} \sqrt{2+3} \sqrt{2}}{2 \sqrt{2}}\right)}{16a^2c^{\frac{3}{2}}} + \frac{\sqrt{2} \left( 3(ac)^{\frac{1}{2}} c^{\frac{1}{2}} d^2 + (ac)^{\frac{1}{2}} ac^2 + 2(ac)^{\frac{1}{2}} d \right) \arctan\left(\frac{\sqrt{2} \sqrt{2-3} \sqrt{2}}{2 \sqrt{2}}\right)}{16a^2c^{\frac{3}{2}}} + \frac{\sqrt{2} \left( (ac)^{\frac{1}{2}} c^{\frac{1}{2}} d^2 + (ac)^{\frac{1}{2}} ac^2 - 2(ac)^{\frac{1}{2}} d \right) \log\left(x^2 + \sqrt{2}x\left(\frac{2}{3}\right) + \sqrt{\frac{2}{3}}\right)}{32a^2c^{\frac{3}{2}}} + \frac{\sqrt{2} \left( (ac)^{\frac{1}{2}} c^{\frac{1}{2}} d^2 + (ac)^{\frac{1}{2}} ac^2 - 2(ac)^{\frac{1}{2}} d \right) \log\left(x^2 - \sqrt{2}x\left(\frac{2}{3}\right) + \sqrt{\frac{2}{3}}\right)}{32a^2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2/(c\*x^4+a)^2,x, algorithm="giac")

[Out] 1/4\*(2\*c\*d\*x^3\*e + c\*d^2\*x - a\*x\*e^2)/((c\*x^4 + a)\*a\*c) + 1/16\*sqrt(2)\*(3\*(a\*c^3)^(1/4)\*c^2\*d^2 + (a\*c^3)^(1/4)\*a\*c\*e^2 + 2\*(a\*c^3)^(3/4)\*d\*e)\*arctan(1/2\*sqrt(2)\*(2\*x + sqrt(2)\*(a/c)^(1/4))/(a/c)^(1/4))/(a^2\*c^3) + 1/16\*sqrt(2)\*(3\*(a\*c^3)^(1/4)\*c^2\*d^2 + (a\*c^3)^(1/4)\*a\*c\*e^2 + 2\*(a\*c^3)^(3/4)\*d\*e)\*arctan(1/2\*sqrt(2)\*(2\*x - sqrt(2)\*(a/c)^(1/4))/(a/c)^(1/4))/(a^2\*c^3) + 1/32\*sqrt(2)\*(3\*(a\*c^3)^(1/4)\*c^2\*d^2 + (a\*c^3)^(1/4)\*a\*c\*e^2 - 2\*(a\*c^3)^(3/4)\*d\*e)\*log(x^2 + sqrt(2)\*x\*(a/c)^(1/4) + sqrt(a/c))/(a^2\*c^3) - 1/32\*sqrt(2)\*(3\*(a\*c^3)^(1/4)\*c^2\*d^2 + (a\*c^3)^(1/4)\*a\*c\*e^2 - 2\*(a\*c^3)^(3/4)\*d\*e)\*log(x^2 - sqrt(2)\*x\*(a/c)^(1/4) + sqrt(a/c))/(a^2\*c^3)

**maple** [A] time = 0.01, size = 464, normalized size = 1.33

$$\frac{\sqrt{2} d e \arctan\left(\frac{\sqrt{2} x}{(c)^{\frac{1}{4}}}\right)}{8(c)^{\frac{3}{4}} a c} + \frac{\sqrt{2} d e \arctan\left(\frac{\sqrt{2} x}{(c)^{\frac{1}{4}}}\right)}{8(c)^{\frac{3}{4}} a c} + \frac{\sqrt{2} d e \ln\left(\frac{x^2 + \sqrt{2} x \sqrt{c}}{x^2 + \sqrt{2} x \sqrt{c}}\right)}{16 a c} + \frac{\left(\frac{2}{3}\right)^{\frac{1}{4}} \sqrt{2} d^2 \arctan\left(\frac{\sqrt{2} x}{(c)^{\frac{1}{4}}}\right)}{16 a c} + \frac{\left(\frac{2}{3}\right)^{\frac{1}{4}} \sqrt{2} d^2 \arctan\left(\frac{\sqrt{2} x}{(c)^{\frac{1}{4}}}\right)}{16 a c} + \frac{\left(\frac{2}{3}\right)^{\frac{1}{4}} \sqrt{2} d^2 \ln\left(\frac{x^2 + \sqrt{2} x \sqrt{c}}{x^2 + \sqrt{2} x \sqrt{c}}\right)}{32 a c} + \frac{3\left(\frac{2}{3}\right)^{\frac{1}{4}} \sqrt{2} d^2 \arctan\left(\frac{\sqrt{2} x}{(c)^{\frac{1}{4}}}\right)}{16 a^2} + \frac{3\left(\frac{2}{3}\right)^{\frac{1}{4}} \sqrt{2} d^2 \arctan\left(\frac{\sqrt{2} x}{(c)^{\frac{1}{4}}}\right)}{16 a^2} + \frac{3\left(\frac{2}{3}\right)^{\frac{1}{4}} \sqrt{2} d^2 \ln\left(\frac{x^2 + \sqrt{2} x \sqrt{c}}{x^2 + \sqrt{2} x \sqrt{c}}\right)}{32 a^2} + \frac{3\left(\frac{2}{3}\right)^{\frac{1}{4}} \sqrt{2} d^2 \ln\left(\frac{x^2 + \sqrt{2} x \sqrt{c}}{x^2 + \sqrt{2} x \sqrt{c}}\right)}{32 a^2} + \frac{d e^2 (a^2 - c^2)}{c^2 x^4 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^2/(c\*x^4+a)^2,x)

[Out] (1/2\*d\*e/a\*x^3-1/4\*(a\*e^2-c\*d^2)/a/c\*x)/(c\*x^4+a)+1/16/a/c\*(a/c)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/c)^(1/4)\*x-1)\*e^2+3/16/a^2\*(a/c)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/c)^(1/4)\*x-1)\*d^2+1/32/a/c\*(a/c)^(1/4)\*2^(1/2)\*ln((x^2+(a/c)^(1/4)\*2^(1/2)\*x+(a/c)^(1/2))/((x^2-(a/c)^(1/4)\*2^(1/2)\*x+(a/c)^(1/2))))\*e^2+3/32/a^2\*(a/c)^(1/4)\*2^(1/2)\*ln((x^2+(a/c)^(1/4)\*2^(1/2)\*x+(a/c)^(1/2))/((x^2-(a/c)^(1/4)\*2^(1/2)\*x+(a/c)^(1/2))))\*d^2+1/16/a/c\*(a/c)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/c)^(1/4)\*x+1)\*e^2+3/16/a^2\*(a/c)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/c)^(1/4)\*x+1)\*d^2+1/16/a/c\*d\*e/(a/c)^(1/4)\*2^(1/2)\*ln((x^2-(a/c)^(1/4)\*2^(1/2)\*x+(a/c)^(1/2))/((x^2+(a/c)^(1/4)\*2^(1/2)\*x+(a/c)^(1/2))))+1/8/a/c\*d\*e/(a/c)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/c)^(1/4)\*x+1)+1/8/a/c\*d\*e/(a/c)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/c)^(1/4)\*x-1)

**maxima** [A] time = 2.59, size = 324, normalized size = 0.93

$$\frac{2cdex^3 + (cd^2 - ad^2)x}{4(a^2x^4 + a^2c)} + \frac{2\sqrt{2} \left( 3c^{\frac{3}{2}}d^2 + 2\sqrt{a}cde + a\sqrt{c}d^2 \right) \arctan\left(\frac{\sqrt{2} \sqrt{c} \sqrt{2+3} \sqrt{2}}{2\sqrt{2}}\right)}{\sqrt{a} \sqrt{a} \sqrt{c} \sqrt{c}} + \frac{2\sqrt{2} \left( 3c^{\frac{3}{2}}d^2 + 2\sqrt{a}cde + a\sqrt{c}d^2 \right) \arctan\left(\frac{\sqrt{2} \sqrt{c} \sqrt{2-3} \sqrt{2}}{2\sqrt{2}}\right)}{\sqrt{a} \sqrt{a} \sqrt{c} \sqrt{c}} + \frac{\sqrt{2} \left( 3c^{\frac{3}{2}}d^2 - 2\sqrt{a}cde + a\sqrt{c}d^2 \right) \log\left(\sqrt{c}x^2 + \sqrt{2}a^{\frac{1}{4}}\sqrt{c}x + \sqrt{a}\right)}{a^{\frac{3}{2}}c^{\frac{3}{2}}} - \frac{\sqrt{2} \left( 3c^{\frac{3}{2}}d^2 - 2\sqrt{a}cde + a\sqrt{c}d^2 \right) \log\left(\sqrt{c}x^2 - \sqrt{2}a^{\frac{1}{4}}\sqrt{c}x + \sqrt{a}\right)}{a^{\frac{3}{2}}c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2/(c\*x^4+a)^2,x, algorithm="maxima")

[Out] 1/4\*(2\*c\*d\*e\*x^3 + (c\*d^2 - a\*e^2)\*x)/(a\*c^2\*x^4 + a^2\*c) + 1/32\*(2\*sqrt(2)\*(3\*c^(3/2)\*d^2 + 2\*sqrt(a)\*c\*d\*e + a\*sqrt(c)\*e^2)\*arctan(1/2\*sqrt(2)\*(2\*sqrt(c)\*x + sqrt(2)\*a^(1/4)\*c^(1/4))/sqrt(sqrt(a)\*sqrt(c)))/sqrt(a)\*sqrt(sqrt(a)\*sqrt(c))\*sqrt(c) + 2\*sqrt(2)\*(3\*c^(3/2)\*d^2 + 2\*sqrt(a)\*c\*d\*e + a\*sqrt(c)\*e^2)\*arctan(1/2\*sqrt(2)\*(2\*sqrt(c)\*x - sqrt(2)\*a^(1/4)\*c^(1/4))/sqrt(sqrt(a)\*sqrt(c)))/sqrt(a)\*sqrt(sqrt(a)\*sqrt(c))\*sqrt(c) + sqrt(2)\*(3\*c^(3/2)\*d^2 - 2\*sqrt(a)\*c\*d\*e + a\*sqrt(c)\*e^2)\*log(sqrt(c)\*x^2 + sqrt(2)\*a^(1/4)\*c^(1/4)\*x + sqrt(a))/(a^(3/4)\*c^(3/4)) - sqrt(2)\*(3\*c^(3/2)\*d^2 - 2\*sqrt(a)\*c\*d\*e + a\*sqrt(c)\*e^2)\*log(sqrt(c)\*x^2 - sqrt(2)\*a^(1/4)\*c^(1/4)\*x + sqrt(a))/(a^(3/4)\*c^(3/4))/(a\*c)

**mupad** [B] time = 4.79, size = 1565, normalized size = 4.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x^2)^2/(a + c*x^4)^2,x)`

[Out]  $2*\operatorname{atanh}\left(\frac{9*c^3*d^4*x*((9*d^4*(-a^7*c^5)^{1/2})/(256*a^7*c^3) - (3*d^3*e)/(64*a^3*c) - (d*e^3)/(64*a^2*c^2) + (e^4*(-a^7*c^5)^{1/2})/(256*a^5*c^5) + (d^2*e^2*(-a^7*c^5)^{1/2})/(128*a^6*c^4))^{1/2}}{2*((27*d^6*(-a^7*c^5)^{1/2})/(32*a^5) - (c*d^3*e^3)/8 - (a*d*e^5)/16 - (9*c^2*d^5*e)/(16*a) + (e^6*(-a^7*c^5)^{1/2})/(32*a^2*c^3) + (5*d^2*e^4*(-a^7*c^5)^{1/2})/(32*a^3*c^2) + (15*d^4*e^2*(-a^7*c^5)^{1/2})/(32*a^4*c))} + (c*e^4*x*((9*d^4*(-a^7*c^5)^{1/2})/(256*a^7*c^3) - (3*d^3*e)/(64*a^3*c) - (d*e^3)/(64*a^2*c^2) + (e^4*(-a^7*c^5)^{1/2})/(256*a^5*c^5) + (d^2*e^2*(-a^7*c^5)^{1/2})/(128*a^6*c^4))^{1/2}}{2*((27*d^6*(-a^7*c^5)^{1/2})/(32*a^7) - (d*e^5)/(16*a) - (c*d^3*e^3)/(8*a^2) - (9*c^2*d^5*e)/(16*a^3) + (e^6*(-a^7*c^5)^{1/2})/(32*a^4*c^3) + (5*d^2*e^4*(-a^7*c^5)^{1/2})/(32*a^5*c^2) + (15*d^4*e^2*(-a^7*c^5)^{1/2})/(32*a^6*c))} + (c^2*d^2*e^2*x*((9*d^4*(-a^7*c^5)^{1/2})/(256*a^7*c^3) - (3*d^3*e)/(64*a^3*c) - (d*e^3)/(64*a^2*c^2) + (e^4*(-a^7*c^5)^{1/2})/(256*a^5*c^5) + (d^2*e^2*(-a^7*c^5)^{1/2})/(128*a^6*c^4))^{1/2}}{(27*d^6*(-a^7*c^5)^{1/2})/(32*a^6) - (d*e^5)/16 - (c*d^3*e^3)/(8*a) - (9*c^2*d^5*e)/(16*a^2) + (e^6*(-a^7*c^5)^{1/2})/(32*a^3*c^3) + (5*d^2*e^4*(-a^7*c^5)^{1/2})/(32*a^4*c^2) + (15*d^4*e^2*(-a^7*c^5)^{1/2})/(32*a^5*c))} * ((a^2*e^4*(-a^7*c^5)^{1/2} + 9*c^2*d^4*(-a^7*c^5)^{1/2} - 12*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 + 2*a*c*d^2*e^2*(-a^7*c^5)^{1/2})/(256*a^7*c^5))^{1/2} - 2*\operatorname{atanh}\left(\frac{9*c^3*d^4*x*(-(d*e^3)/(64*a^2*c^2) - (3*d^3*e)/(64*a^3*c) - (9*d^4*(-a^7*c^5)^{1/2})/(256*a^7*c^3) - (e^4*(-a^7*c^5)^{1/2})/(256*a^5*c^5) - (d^2*e^2*(-a^7*c^5)^{1/2})/(128*a^6*c^4))^{1/2}}{2*((27*d^6*(-a^7*c^5)^{1/2})/(32*a^5) + (c*d^3*e^3)/8 + (a*d*e^5)/16 + (9*c^2*d^5*e)/(16*a) + (e^6*(-a^7*c^5)^{1/2})/(32*a^2*c^3) + (5*d^2*e^4*(-a^7*c^5)^{1/2})/(32*a^3*c^2) + (15*d^4*e^2*(-a^7*c^5)^{1/2})/(32*a^4*c))} + (c*e^4*x*(-(d*e^3)/(64*a^2*c^2) - (3*d^3*e)/(64*a^3*c) - (9*d^4*(-a^7*c^5)^{1/2})/(256*a^7*c^3) - (e^4*(-a^7*c^5)^{1/2})/(256*a^5*c^5) - (d^2*e^2*(-a^7*c^5)^{1/2})/(128*a^6*c^4))^{1/2}}{2*((27*d^6*(-a^7*c^5)^{1/2})/(32*a^7) + (d*e^5)/(16*a) + (c*d^3*e^3)/(8*a^2) + (9*c^2*d^5*e)/(16*a^3) + (e^6*(-a^7*c^5)^{1/2})/(32*a^4*c^3) + (5*d^2*e^4*(-a^7*c^5)^{1/2})/(32*a^5*c^2) + (15*d^4*e^2*(-a^7*c^5)^{1/2})/(32*a^6*c))} + (c^2*d^2*e^2*x*(-(d*e^3)/(64*a^2*c^2) - (3*d^3*e)/(64*a^3*c) - (9*d^4*(-a^7*c^5)^{1/2})/(256*a^7*c^3) - (e^4*(-a^7*c^5)^{1/2})/(256*a^5*c^5) - (d^2*e^2*(-a^7*c^5)^{1/2})/(128*a^6*c^4))^{1/2}}{(d*e^5)/16 + (27*d^6*(-a^7*c^5)^{1/2})/(32*a^6) + (c*d^3*e^3)/(8*a) + (9*c^2*d^5*e)/(16*a^2) + (e^6*(-a^7*c^5)^{1/2})/(32*a^3*c^3) + (5*d^2*e^4*(-a^7*c^5)^{1/2})/(32*a^4*c^2) + (15*d^4*e^2*(-a^7*c^5)^{1/2})/(32*a^5*c))} * (-a^2*e^4*(-a^7*c^5)^{1/2} + 9*c^2*d^4*(-a^7*c^5)^{1/2} + 12*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^3 + 2*a*c*d^2*e^2*(-a^7*c^5)^{1/2})/(256*a^7*c^5))^{1/2} + ((d*e*x^3)/(2*a) - (x*(a*e^2 - c*d^2))/(4*a*c))/(a + c*x^4)$

**sympy [A]** time = 2.07, size = 275, normalized size = 0.79

$\operatorname{RootSum}\left(65536*a^7*c^5 + t^2(2048*a^5*c^3*d^3*e^3 + 6144*a^4*c^4*d^3*e) + a^4*e^8 + 20*a^3*c*d^2*e^6 + 118*a^2*c^2*d^4*e^4 + 180*a*c^3*d^6*e^2 + 81*c^4*d^8\right)\left(t \mapsto t \log\left(x + \frac{-8192t^3a^6c^4de + 16ta^5c^6 - 48ta^4c^2d^2e^4 - 144ta^3c^3d^4e^2 + 432ta^2c^4d^6}{a^4d^8 + 12a^3c^2d^6 + 38a^2c^2d^4e^4 + 108ac^3d^6e^2 + 81c^4d^8}\right)\right) + \frac{2cdex^3 + x(-ae^2 + cd^2)}{4a^2c + 4ac^2x^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**2/(c*x**4+a)**2,x)`

[Out]  $\operatorname{RootSum}(65536*_t**4*a**7*c**5 + _t**2*(2048*a**5*c**3*d**3*e**3 + 6144*a**4*c**4*d**3*e) + a**4*e**8 + 20*a**3*c*d**2*e**6 + 118*a**2*c**2*d**4*e**4 + 180*a*c**3*d**6*e**2 + 81*c**4*d**8, \operatorname{Lambda}(_t, _t*\log(x + (-8192*_t**3*a**6*c**4*d*e + 16*_t*a**5*c**e**6 - 48*_t*a**4*c**2*d**2*e**4 - 144*_t*a**3*c**3*d**4*e**2 + 432*_t*a**2*c**4*d**6)/(a**4*e**8 + 12*a**3*c*d**2*e**6 + 38*a**2*c**2*d**4*e**4 + 108*a*c**3*d**6*e**2 + 81*c**4*d**8))) + (2*c*d*e*x**3 + x*(-a*e**2 + c*d**2))/(4*a**2*c + 4*a*c**2*x**4)$

$$3.127 \quad \int \frac{d+ex^2}{(a+cx^4)^2} dx$$

**Optimal.** Leaf size=275

$$\frac{(3\sqrt{c}d - \sqrt{a}e) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2} a^{7/4} c^{3/4}} + \frac{(3\sqrt{c}d - \sqrt{a}e) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2} a^{7/4} c^{3/4}} - \frac{(\sqrt{a}e + 3\sqrt{c}d) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2} a^{7/4} c^{3/4}} + \frac{(\sqrt{a}e + 3\sqrt{c}d) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right)}{8\sqrt{2} a^{7/4} c^{3/4}} + \frac{x(d+ex^2)}{4a(a+cx^4)}$$

**Rubi [A]** time = 0.20, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$ , Rules used = {1179, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(3\sqrt{c}d - \sqrt{a}e) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2} a^{7/4} c^{3/4}} + \frac{(3\sqrt{c}d - \sqrt{a}e) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2} a^{7/4} c^{3/4}} - \frac{(\sqrt{a}e + 3\sqrt{c}d) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2} a^{7/4} c^{3/4}} + \frac{(\sqrt{a}e + 3\sqrt{c}d) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right)}{8\sqrt{2} a^{7/4} c^{3/4}} + \frac{x(d+ex^2)}{4a(a+cx^4)}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)/(a + c\*x^4)^2, x]

[Out] (x\*(d + e\*x^2))/(4\*a\*(a + c\*x^4)) - ((3\*sqrt[c]\*d + sqrt[a]\*e)\*ArcTan[1 - (sqrt[2]\*c^(1/4)\*x)/a^(1/4)]/(8\*sqrt[2]\*a^(7/4)\*c^(3/4)) + ((3\*sqrt[c]\*d + sqrt[a]\*e)\*ArcTan[1 + (sqrt[2]\*c^(1/4)\*x)/a^(1/4)]/(8\*sqrt[2]\*a^(7/4)\*c^(3/4)) - ((3\*sqrt[c]\*d - sqrt[a]\*e)\*Log[sqrt[a] - sqrt[2]\*a^(1/4)\*c^(1/4)\*x + sqrt[c]\*x^2])/(16\*sqrt[2]\*a^(7/4)\*c^(3/4)) + ((3\*sqrt[c]\*d - sqrt[a]\*e)\*Log[sqrt[a] + sqrt[2]\*a^(1/4)\*c^(1/4)\*x + sqrt[c]\*x^2])/(16\*sqrt[2]\*a^(7/4)\*c^(3/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 1162

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

#### Rule 1165

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\begin{aligned} \int \frac{d + ex^2}{(a + cx^4)^2} dx &= \frac{x(d + ex^2)}{4a(a + cx^4)} - \frac{\int \frac{-3d - ex^2}{a + cx^4} dx}{4a} \\ &= \frac{x(d + ex^2)}{4a(a + cx^4)} + \frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}} - e\right) \int \frac{\sqrt{a}\sqrt{c} - cx^2}{a + cx^4} dx}{8ac} + \frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}} + e\right) \int \frac{\sqrt{a}\sqrt{c} + cx^2}{a + cx^4} dx}{8ac} \\ &= \frac{x(d + ex^2)}{4a(a + cx^4)} + \frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}} + e\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt{c}} + x^2} dx}{16ac} + \frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}} + e\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt{c}} + x^2} dx}{16ac} - \frac{(3\sqrt{c}d - \sqrt{a}e) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{16\sqrt{2}a^{7/4}c^{3/4}} + \frac{(3\sqrt{c}d - \sqrt{a}e) \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{16\sqrt{2}a^{7/4}c^{3/4}} \\ &= \frac{x(d + ex^2)}{4a(a + cx^4)} - \frac{(3\sqrt{c}d + \sqrt{a}e) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}} + \frac{(3\sqrt{c}d + \sqrt{a}e) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}} - \end{aligned}$$

**Mathematica [A]** time = 0.27, size = 267, normalized size = 0.97

$$\frac{\sqrt{2}(a^{3/4}e - 3\sqrt[4]{a}\sqrt{c}d) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{c^{3/4}} + \frac{\sqrt{2}(3\sqrt[4]{a}\sqrt{c}d - a^{3/4}e) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{c^{3/4}} - \frac{2\sqrt{2}\sqrt[4]{a}(\sqrt{a}e + 3\sqrt{c}d) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}}\right)}{c^{3/4}} + \frac{2\sqrt{2}\sqrt[4]{a}(\sqrt{a}e + 3\sqrt{c}d) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}} + 1\right)}{c^{3/4}} + \frac{8ax(d + ex^2)}{a + cx^4}$$

32a<sup>2</sup>

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)/(a + c\*x^4)^2, x]

[Out] ((8\*a\*x\*(d + e\*x^2))/(a + c\*x^4) - (2\*Sqrt[2]\*a^(1/4)\*(3\*Sqrt[c]\*d + Sqrt[a]\*e)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)])/c^(3/4) + (2\*Sqrt[2]\*a^(1/4)\*(3\*Sqrt[c]\*d + Sqrt[a]\*e)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)])/c^(3/4) + (Sqrt[2]\*(-3\*a^(1/4)\*Sqrt[c]\*d + a^(3/4)\*e)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/c^(3/4) + (Sqrt[2]\*(3\*a^(1/4)\*Sqrt[c]\*d - a^(3/4)\*e)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/c^(3/4))/(32\*a^2)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{(a + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)/(a + c\*x^4)^2,x]

[Out] IntegrateAlgebraic[(d + e\*x^2)/(a + c\*x^4)^2, x]

**fricas** [B] time = 1.10, size = 873, normalized size = 3.17

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(c\*x^4+a)^2,x, algorithm="fricas")

[Out]  $\frac{1}{16} \cdot (4 \cdot e \cdot x^3 - (a \cdot c \cdot x^4 + a^2) \cdot \sqrt{-\left(a^3 \cdot c \cdot \sqrt{-(81 \cdot c^2 \cdot d^4 - 18 \cdot a \cdot c \cdot d^2 \cdot e^2 + a^2 \cdot e^4)}\right)} + 6 \cdot d \cdot e) / (a^7 \cdot c^3) + \frac{6 \cdot d \cdot e}{a^3 \cdot c} \cdot \log\left(-\frac{81 \cdot c^2 \cdot d^4 - a^2 \cdot e^4}{a^7 \cdot c^3} \cdot x + \frac{a^6 \cdot c^2 \cdot e \cdot \sqrt{-(81 \cdot c^2 \cdot d^4 - 18 \cdot a \cdot c \cdot d^2 \cdot e^2 + a^2 \cdot e^4)}}{a^7 \cdot c^3} + 27 \cdot a^2 \cdot c^2 \cdot d^3 - 3 \cdot a^3 \cdot c \cdot d \cdot e^2\right) \cdot \sqrt{-\left(a^3 \cdot c \cdot \sqrt{-(81 \cdot c^2 \cdot d^4 - 18 \cdot a \cdot c \cdot d^2 \cdot e^2 + a^2 \cdot e^4)}\right)} + \frac{6 \cdot d \cdot e}{a^3 \cdot c} + (a \cdot c \cdot x^4 + a^2) \cdot \sqrt{-\left(a^3 \cdot c \cdot \sqrt{-(81 \cdot c^2 \cdot d^4 - 18 \cdot a \cdot c \cdot d^2 \cdot e^2 + a^2 \cdot e^4)}\right)} + 6 \cdot d \cdot e) / (a^3 \cdot c) \cdot \log\left(-\frac{81 \cdot c^2 \cdot d^4 - a^2 \cdot e^4}{a^7 \cdot c^3} \cdot x - \frac{a^6 \cdot c^2 \cdot e \cdot \sqrt{-(81 \cdot c^2 \cdot d^4 - 18 \cdot a \cdot c \cdot d^2 \cdot e^2 + a^2 \cdot e^4)}}{a^7 \cdot c^3} + 27 \cdot a^2 \cdot c^2 \cdot d^3 - 3 \cdot a^3 \cdot c \cdot d \cdot e^2\right) \cdot \sqrt{-\left(a^3 \cdot c \cdot \sqrt{-(81 \cdot c^2 \cdot d^4 - 18 \cdot a \cdot c \cdot d^2 \cdot e^2 + a^2 \cdot e^4)}\right)} + \frac{6 \cdot d \cdot e}{a^3 \cdot c} + (a \cdot c \cdot x^4 + a^2) \cdot \sqrt{\left(a^3 \cdot c \cdot \sqrt{-(81 \cdot c^2 \cdot d^4 - 18 \cdot a \cdot c \cdot d^2 \cdot e^2 + a^2 \cdot e^4)}\right)} / (a^7 \cdot c^3) - \frac{6 \cdot d \cdot e}{a^3 \cdot c} \cdot \log\left(-\frac{81 \cdot c^2 \cdot d^4 - a^2 \cdot e^4}{a^7 \cdot c^3} \cdot x + \frac{a^6 \cdot c^2 \cdot e \cdot \sqrt{-(81 \cdot c^2 \cdot d^4 - 18 \cdot a \cdot c \cdot d^2 \cdot e^2 + a^2 \cdot e^4)}}{a^7 \cdot c^3} - 27 \cdot a^2 \cdot c^2 \cdot d^3 + 3 \cdot a^3 \cdot c \cdot d \cdot e^2\right) \cdot \sqrt{\left(a^3 \cdot c \cdot \sqrt{-(81 \cdot c^2 \cdot d^4 - 18 \cdot a \cdot c \cdot d^2 \cdot e^2 + a^2 \cdot e^4)}\right)} / (a^7 \cdot c^3) - \frac{6 \cdot d \cdot e}{a^3 \cdot c} + (a \cdot c \cdot x^4 + a^2) \cdot \sqrt{\left(a^3 \cdot c \cdot \sqrt{-(81 \cdot c^2 \cdot d^4 - 18 \cdot a \cdot c \cdot d^2 \cdot e^2 + a^2 \cdot e^4)}\right)} / (a^7 \cdot c^3) - \frac{6 \cdot d \cdot e}{a^3 \cdot c} \cdot \log\left(-\frac{81 \cdot c^2 \cdot d^4 - a^2 \cdot e^4}{a^7 \cdot c^3} \cdot x - \frac{a^6 \cdot c^2 \cdot e \cdot \sqrt{-(81 \cdot c^2 \cdot d^4 - 18 \cdot a \cdot c \cdot d^2 \cdot e^2 + a^2 \cdot e^4)}}{a^7 \cdot c^3} - 27 \cdot a^2 \cdot c^2 \cdot d^3 + 3 \cdot a^3 \cdot c \cdot d \cdot e^2\right) \cdot \sqrt{\left(a^3 \cdot c \cdot \sqrt{-(81 \cdot c^2 \cdot d^4 - 18 \cdot a \cdot c \cdot d^2 \cdot e^2 + a^2 \cdot e^4)}\right)} / (a^7 \cdot c^3) - \frac{6 \cdot d \cdot e}{a^3 \cdot c} + 4 \cdot d \cdot x) / (a \cdot c \cdot x^4 + a^2)$

**giac** [A] time = 0.44, size = 273, normalized size = 0.99

$$\frac{x^3 e + dx}{4(c x^4 + a)a} + \frac{\sqrt{2} \left(3(a c^3)^{\frac{1}{4}} c^2 d + (a c^3)^{\frac{3}{4}} e\right) \arctan\left(\frac{\sqrt{2} \left(2x + \sqrt{2} \left(\frac{c}{a}\right)^{\frac{1}{4}}\right)}{\left(\frac{c}{a}\right)^{\frac{1}{4}}}\right)}{16 a^2 c^3} + \frac{\sqrt{2} \left(3(a c^3)^{\frac{1}{4}} c^2 d + (a c^3)^{\frac{3}{4}} e\right) \arctan\left(\frac{\sqrt{2} \left(2x - \sqrt{2} \left(\frac{c}{a}\right)^{\frac{1}{4}}\right)}{\left(\frac{c}{a}\right)^{\frac{1}{4}}}\right)}{16 a^2 c^3} + \frac{\sqrt{2} \left(3(a c^3)^{\frac{1}{4}} c^2 d - (a c^3)^{\frac{3}{4}} e\right) \log\left(x^2 + \sqrt{2} x \left(\frac{c}{a}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{a}}\right)}{32 a^2 c^3} - \frac{\sqrt{2} \left(3(a c^3)^{\frac{1}{4}} c^2 d - (a c^3)^{\frac{3}{4}} e\right) \log\left(x^2 - \sqrt{2} x \left(\frac{c}{a}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{a}}\right)}{32 a^2 c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(c\*x^4+a)^2,x, algorithm="giac")

[Out]  $\frac{1}{4} \cdot (x^3 \cdot e + d \cdot x) / ((c \cdot x^4 + a) \cdot a) + \frac{1}{16} \cdot \sqrt{2} \cdot (3 \cdot (a \cdot c^3)^{\frac{1}{4}} \cdot c^2 \cdot d + (a \cdot c^3)^{\frac{3}{4}} \cdot e) \cdot \arctan\left(\frac{1}{2} \cdot \sqrt{2} \cdot (2 \cdot x + \sqrt{2} \cdot (a/c)^{\frac{1}{4}}) / (a/c)^{\frac{1}{4}}\right) / (a^2 \cdot c^3) + \frac{1}{16} \cdot \sqrt{2} \cdot (3 \cdot (a \cdot c^3)^{\frac{1}{4}} \cdot c^2 \cdot d + (a \cdot c^3)^{\frac{3}{4}} \cdot e) \cdot \arctan\left(\frac{1}{2} \cdot \sqrt{2} \cdot (2 \cdot x - \sqrt{2} \cdot (a/c)^{\frac{1}{4}}) / (a/c)^{\frac{1}{4}}\right) / (a^2 \cdot c^3) + \frac{1}{32} \cdot \sqrt{2} \cdot (3 \cdot (a \cdot c^3)^{\frac{1}{4}} \cdot c^2 \cdot d - (a \cdot c^3)^{\frac{3}{4}} \cdot e) \cdot \log\left(x^2 + \sqrt{2} \cdot x \cdot (a/c)^{\frac{1}{4}} + \sqrt{a/c}\right) / (a^2 \cdot c^3) - \frac{1}{32} \cdot \sqrt{2} \cdot (3 \cdot (a \cdot c^3)^{\frac{1}{4}} \cdot c^2 \cdot d - (a \cdot c^3)^{\frac{3}{4}} \cdot e) \cdot \log\left(x^2 - \sqrt{2} \cdot x \cdot (a/c)^{\frac{1}{4}} + \sqrt{a/c}\right) / (a^2 \cdot c^3)$

**maple** [A] time = 0.01, size = 303, normalized size = 1.10

$$\frac{e x^3}{4(c x^4 + a)a} + \frac{d x}{4(c x^4 + a)a} + \frac{\sqrt{2} e \arctan\left(\frac{\sqrt{2} x - 1}{\left(\frac{c}{a}\right)^{\frac{1}{4}}}\right)}{16 \left(\frac{c}{a}\right)^{\frac{1}{4}} a c} + \frac{\sqrt{2} e \arctan\left(\frac{\sqrt{2} x + 1}{\left(\frac{c}{a}\right)^{\frac{1}{4}}}\right)}{16 \left(\frac{c}{a}\right)^{\frac{1}{4}} a c} + \frac{\sqrt{2} e \ln\left(\frac{x^2 - \left(\frac{c}{a}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{c}{a}}}{x^2 + \left(\frac{c}{a}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{c}{a}}}\right)}{32 \left(\frac{c}{a}\right)^{\frac{1}{4}} a c} + \frac{3 \left(\frac{c}{a}\right)^{\frac{1}{4}} \sqrt{2} d \arctan\left(\frac{\sqrt{2} x - 1}{\left(\frac{c}{a}\right)^{\frac{1}{4}}}\right)}{16 a^2} + \frac{3 \left(\frac{c}{a}\right)^{\frac{1}{4}} \sqrt{2} d \arctan\left(\frac{\sqrt{2} x + 1}{\left(\frac{c}{a}\right)^{\frac{1}{4}}}\right)}{16 a^2} + \frac{3 \left(\frac{c}{a}\right)^{\frac{1}{4}} \sqrt{2} d \ln\left(\frac{x^2 + \left(\frac{c}{a}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{c}{a}}}{x^2 - \left(\frac{c}{a}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{c}{a}}}\right)}{32 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)/(c\*x^4+a)^2,x)

[Out]  $\frac{1}{4} \cdot d \cdot x / a / (c \cdot x^4 + a) + \frac{3}{32} \cdot d / a^2 \cdot (a/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot \ln\left(\frac{(x^2 + (a/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot x + (a/c)^{\frac{1}{2}}) / (x^2 - (a/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot x + (a/c)^{\frac{1}{2}})}{2}\right) + \frac{3}{16} \cdot d / a^2 \cdot (a/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot \arctan\left(\frac{2^{\frac{1}{2}}}{(a/c)^{\frac{1}{4}} \cdot x + 1}\right) + \frac{3}{16} \cdot d / a^2 \cdot (a/c)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot \arctan\left(\frac{2^{\frac{1}{2}}}{(a/c)^{\frac{1}{4}} \cdot x - 1}\right)$

$/2) * \arctan(2^{1/2} / (a/c)^{1/4} * x - 1) + 1/4 * e * x^3 / a / (c * x^4 + a) + 1/32 * e / a / c / (a/c)^{1/4} * 2^{1/2} * \ln((x^2 - (a/c)^{1/4}) * 2^{1/2} * x + (a/c)^{1/2}) / (x^2 + (a/c)^{1/4}) * 2^{1/2} * x + (a/c)^{1/2})) + 1/16 * e / a / c / (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x + 1) + 1/16 * e / a / c / (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x - 1)$

**maxima** [A] time = 2.27, size = 253, normalized size = 0.92

$$\frac{e x^3 + d x}{4(a c x^4 + a^2)} + \frac{2 \sqrt{2}(3 \sqrt{c} d + \sqrt{a} e) \arctan\left(\frac{\sqrt{2}\left(2 \sqrt{c} x + \sqrt{2 a^{\frac{1}{4}} c^{\frac{1}{4}}}\right)}{2 \sqrt{a} \sqrt{c}}\right)}{\sqrt{a} \sqrt{a} \sqrt{c} \sqrt{c}} + \frac{2 \sqrt{2}(3 \sqrt{c} d + \sqrt{a} e) \arctan\left(\frac{\sqrt{2}\left(2 \sqrt{c} x - \sqrt{2 a^{\frac{1}{4}} c^{\frac{1}{4}}}\right)}{2 \sqrt{a} \sqrt{c}}\right)}{\sqrt{a} \sqrt{a} \sqrt{c} \sqrt{c}} + \frac{\sqrt{2}(3 \sqrt{c} d - \sqrt{a} e) \log\left(\sqrt{c} x^2 + \sqrt{2 a^{\frac{1}{4}} c^{\frac{1}{4}} x} + \sqrt{a}\right)}{\frac{3}{a^{\frac{3}{4}} c^{\frac{3}{4}}}} - \frac{\sqrt{2}(3 \sqrt{c} d - \sqrt{a} e) \log\left(\sqrt{c} x^2 - \sqrt{2 a^{\frac{1}{4}} c^{\frac{1}{4}} x} + \sqrt{a}\right)}{\frac{3}{a^{\frac{3}{4}} c^{\frac{3}{4}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(c\*x^4+a)^2,x, algorithm="maxima")

[Out]  $1/4 * (e * x^3 + d * x) / (a * c * x^4 + a^2) + 1/32 * (2 * \text{sqrt}(2) * (3 * \text{sqrt}(c) * d + \text{sqrt}(a) * e) * \arctan(1/2 * \text{sqrt}(2) * (2 * \text{sqrt}(c) * x + \text{sqrt}(2) * a^{1/4} * c^{1/4})) / \text{sqrt}(\text{sqrt}(a) * \text{sqrt}(c))) / (\text{sqrt}(a) * \text{sqrt}(\text{sqrt}(a) * \text{sqrt}(c)) * \text{sqrt}(c)) + 2 * \text{sqrt}(2) * (3 * \text{sqrt}(c) * d + \text{sqrt}(a) * e) * \arctan(1/2 * \text{sqrt}(2) * (2 * \text{sqrt}(c) * x - \text{sqrt}(2) * a^{1/4} * c^{1/4})) / \text{sqrt}(\text{sqrt}(a) * \text{sqrt}(c))) / (\text{sqrt}(a) * \text{sqrt}(\text{sqrt}(a) * \text{sqrt}(c)) * \text{sqrt}(c)) + \text{sqrt}(2) * (3 * \text{sqrt}(c) * d - \text{sqrt}(a) * e) * \log(\text{sqrt}(c) * x^2 + \text{sqrt}(2) * a^{1/4} * c^{1/4} * x + \text{sqrt}(a)) / (a^{3/4} * c^{3/4}) - \text{sqrt}(2) * (3 * \text{sqrt}(c) * d - \text{sqrt}(a) * e) * \log(\text{sqrt}(c) * x^2 - \text{sqrt}(2) * a^{1/4} * c^{1/4} * x + \text{sqrt}(a)) / (a^{3/4} * c^{3/4})) / a$

**mupad** [B] time = 0.40, size = 637, normalized size = 2.32

$$\frac{\frac{e^2}{4a} + \frac{d}{4a} - 2 \operatorname{atanh}\left(\frac{c^2 d^2 x \sqrt{\frac{c^2 \sqrt{c^2 d^2} - 9 d^2 \sqrt{c^2 d^2} - 3 d e}}{2 \sqrt{c^2 d^2} - 9 d^2 \sqrt{c^2 d^2} - 3 d e}}{2 \sqrt{\frac{c^2}{32 a} - \frac{9 d^2 e}{32 a} + \frac{3 d^2 \sqrt{c^2 d^2}}{32 a}}}\right) - \frac{9 c^2 d^2 x \sqrt{\frac{c^2 \sqrt{c^2 d^2} - 9 d^2 \sqrt{c^2 d^2} - 3 d e}}{2 \sqrt{c^2 d^2} - 9 d^2 \sqrt{c^2 d^2} - 3 d e}}{\sqrt{\frac{c^2}{32 a} - \frac{9 d^2 e}{32 a} + \frac{3 d^2 \sqrt{c^2 d^2}}{32 a}}}}{\sqrt{\frac{9 c^2 \sqrt{c^2 d^2} - 9 d^2 \sqrt{c^2 d^2} - 6 d^2 d e}}{256 a^6 c^3}} - 2 \operatorname{atanh}\left(\frac{c^2 d^2 x \sqrt{\frac{c^2 \sqrt{c^2 d^2} - 9 d^2 \sqrt{c^2 d^2} - 3 d e}}{2 \sqrt{c^2 d^2} - 9 d^2 \sqrt{c^2 d^2} - 3 d e}}{2 \sqrt{\frac{c^2}{32 a} - \frac{9 d^2 e}{32 a} + \frac{3 d^2 \sqrt{c^2 d^2}}{32 a}}}\right) - \frac{9 c^2 d^2 x \sqrt{\frac{c^2 \sqrt{c^2 d^2} - 9 d^2 \sqrt{c^2 d^2} - 3 d e}}{2 \sqrt{c^2 d^2} - 9 d^2 \sqrt{c^2 d^2} - 3 d e}}{\sqrt{\frac{c^2}{32 a} - \frac{9 d^2 e}{32 a} + \frac{3 d^2 \sqrt{c^2 d^2}}{32 a}}}}{\sqrt{\frac{9 c^2 \sqrt{c^2 d^2} - 9 d^2 \sqrt{c^2 d^2} - 6 d^2 d e}}{256 a^6 c^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)/(a + c\*x^4)^2,x)

[Out]  $((e * x^3) / (4 * a) + (d * x) / (4 * a)) / (a + c * x^4) - 2 * \operatorname{atanh}((c^2 * e^2 * x * ((e^2 * (-a^7 * c^3)^{1/2}) / (256 * a^6 * c^3) - (9 * d^2 * (-a^7 * c^3)^{1/2}) / (256 * a^7 * c^2) - (3 * d * e) / (128 * a^3 * c))^{1/2}) / (2 * ((c * e^3) / (32 * a) - (9 * c^2 * d^2 * e) / (32 * a^2) - (27 * c * d^3 * (-a^7 * c^3)^{1/2}) / (32 * a^6) + (3 * d * e^2 * (-a^7 * c^3)^{1/2}) / (32 * a^5))) - (9 * c^3 * d^2 * x * ((e^2 * (-a^7 * c^3)^{1/2}) / (256 * a^6 * c^3) - (9 * d^2 * (-a^7 * c^3)^{1/2}) / (256 * a^7 * c^2) - (3 * d * e) / (128 * a^3 * c))^{1/2}) / (2 * ((c * e^3) / 32 - (9 * c^2 * d^2 * e) / (32 * a) - (27 * c * d^3 * (-a^7 * c^3)^{1/2}) / (32 * a^5) + (3 * d * e^2 * (-a^7 * c^3)^{1/2}) / (32 * a^4))) * (- (9 * c * d^2 * (-a^7 * c^3)^{1/2}) - a * e^2 * (-a^7 * c^3)^{1/2} + 6 * a^4 * c^2 * d * e) / (256 * a^7 * c^3))^{1/2} - 2 * \operatorname{atanh}((c^2 * e^2 * x * ((9 * d^2 * (-a^7 * c^3)^{1/2}) / (256 * a^7 * c^2) - (3 * d * e) / (128 * a^3 * c) - (e^2 * (-a^7 * c^3)^{1/2}) / (256 * a^6 * c^3))^{1/2}) / (2 * ((c * e^3) / (32 * a) - (9 * c^2 * d^2 * e) / (32 * a^2) + (27 * c * d^3 * (-a^7 * c^3)^{1/2}) / (32 * a^6) - (3 * d * e^2 * (-a^7 * c^3)^{1/2}) / (32 * a^5))) - (9 * c^3 * d^2 * x * ((9 * d^2 * (-a^7 * c^3)^{1/2}) / (256 * a^7 * c^2) - (3 * d * e) / (128 * a^3 * c) - (e^2 * (-a^7 * c^3)^{1/2}) / (256 * a^6 * c^3))^{1/2}) / (2 * ((c * e^3) / 32 - (9 * c^2 * d^2 * e) / (32 * a) + (27 * c * d^3 * (-a^7 * c^3)^{1/2}) / (32 * a^5) - (3 * d * e^2 * (-a^7 * c^3)^{1/2}) / (32 * a^4))) * (- (a * e^2 * (-a^7 * c^3)^{1/2}) - 9 * c * d^2 * (-a^7 * c^3)^{1/2} + 6 * a^4 * c^2 * d * e) / (256 * a^7 * c^3))^{1/2}$

**sympy** [A] time = 1.03, size = 136, normalized size = 0.49

$$\operatorname{RootSum}\left(65536 t^4 a^7 c^3 + 3072 t^2 a^4 c^2 d e + a^2 e^4 + 18 a c d^2 e^2 + 81 c^2 d^4, \left(t \mapsto t \log\left(x + \frac{4096 t^3 a^6 c^2 e + 144 t a^3 c d e^2 - 432 t a^2 c^2 d^3}{a^2 e^4 - 81 c^2 d^4}\right)\right)\right) + \frac{d x + e x^3}{4 a^2 + 4 a c x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)/(c\*x\*\*4+a)\*\*2,x)

[Out]  $\operatorname{RootSum}(65536 * t ** 4 * a ** 7 * c ** 3 + 3072 * t ** 2 * a ** 4 * c ** 2 * d * e + a ** 2 * e ** 4 + 18 * a * c * d ** 2 * e ** 2 + 81 * c ** 2 * d ** 4, \operatorname{Lambda}(t, t * \log(x + (4096 * t ** 3 * a ** 6 * c ** 2 * e + 144 * t * a ** 3 * c * d * e ** 2 - 432 * t * a ** 2 * c ** 2 * d ** 3) / (a ** 2 * e ** 4 - 81 * c ** 2 * d ** 4)))) + (d * x + e * x ** 3) / (4 * a ** 2 + 4 * a * c * x ** 4)$

$$3.128 \quad \int \frac{1}{(a+cx^4)^2} dx$$

**Optimal.** Leaf size=202

$$\frac{3 \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{16\sqrt{2} a^{7/4} \sqrt[4]{c}} + \frac{3 \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{16\sqrt{2} a^{7/4} \sqrt[4]{c}} - \frac{3 \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{8\sqrt{2} a^{7/4} \sqrt[4]{c}} + \frac{3 \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{8\sqrt{2} a^{7/4} \sqrt[4]{c}}$$

**Rubi [A]** time = 0.13, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$ , Rules used = {199, 211, 1165, 628, 1162, 617, 204}

$$\frac{3 \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{16\sqrt{2} a^{7/4} \sqrt[4]{c}} + \frac{3 \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{16\sqrt{2} a^{7/4} \sqrt[4]{c}} - \frac{3 \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{8\sqrt{2} a^{7/4} \sqrt[4]{c}} + \frac{3 \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}} + 1\right)}{8\sqrt{2} a^{7/4} \sqrt[4]{c}} + \frac{x}{4a(a+cx^4)}$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^4)^(-2), x]

[Out] x/(4\*a\*(a + c\*x^4)) - (3\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)])/(8\*Sqrt[2]\*a^(7/4)\*c^(1/4)) + (3\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)])/(8\*Sqrt[2]\*a^(7/4)\*c^(1/4)) - (3\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/(16\*Sqrt[2]\*a^(7/4)\*c^(1/4)) + (3\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/(16\*Sqrt[2]\*a^(7/4)\*c^(1/4))

#### Rule 199

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2\*p] || (n == 2 && IntegerQ[4\*p]) || (n == 2 && IntegerQ[3\*p]) || Denominator[p + 1/n] < Denominator[p])

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,



e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1162

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1165

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rubi steps

$$\begin{aligned} \int \frac{1}{(a+cx^4)^2} dx &= \frac{x}{4a(a+cx^4)} + \frac{3 \int \frac{1}{a+cx^4} dx}{4a} \\ &= \frac{x}{4a(a+cx^4)} + \frac{3 \int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4} dx}{8a^{3/2}} + \frac{3 \int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4} dx}{8a^{3/2}} \\ &= \frac{x}{4a(a+cx^4)} + \frac{3 \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{16a^{3/2}\sqrt{c}} + \frac{3 \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{16a^{3/2}\sqrt{c}} - \frac{3 \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}} + 2x}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} - x^2} dx}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} - \frac{3 \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}} - 2x}{-\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} - x^2} dx}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} \\ &= \frac{x}{4a(a+cx^4)} - \frac{3 \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3 \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3 \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} - \frac{3 \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} \end{aligned}$$

**Mathematica [A]** time = 0.12, size = 183, normalized size = 0.91

$$\frac{\frac{8a^{3/4}x}{a+cx^4} - \frac{3\sqrt{2} \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{\sqrt[4]{c}} + \frac{3\sqrt{2} \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{\sqrt[4]{c}} - \frac{6\sqrt{2} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} + \frac{6\sqrt{2} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right)}{\sqrt[4]{c}}}{32a^{7/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c\*x^4)^(-2), x]

[Out] ((8\*a^(3/4)\*x)/(a + c\*x^4) - (6\*Sqrt[2]\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)])/c^(1/4) + (6\*Sqrt[2]\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)])/c^(1/4) - (3\*Sqrt[2]\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/c^(1/4) + (3\*Sqrt[2]\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/c^(1/4))/(32\*a^(7/4))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + c\*x^4)^(-2), x]

[Out] IntegrateAlgebraic[(a + c\*x^4)^(-2), x]

**fricas** [A] time = 1.09, size = 173, normalized size = 0.86

$$\frac{12(acx^4 + a^2)\left(-\frac{1}{a^5c}\right)^{\frac{1}{4}} \arctan\left(-a^5cx\left(-\frac{1}{a^5c}\right)^{\frac{3}{4}} + \sqrt{a^4\sqrt{-\frac{1}{a^5c}} + x^2}a^5c\left(-\frac{1}{a^5c}\right)^{\frac{3}{4}}\right) + 3(acx^4 + a^2)\left(-\frac{1}{a^5c}\right)^{\frac{1}{4}} \log\left(a^2\left(-\frac{1}{a^5c}\right)^{\frac{1}{4}} + x\right) - 3(acx^4 + a^2)\left(-\frac{1}{a^5c}\right)^{\frac{1}{4}} \log\left(-a^2\left(-\frac{1}{a^5c}\right)^{\frac{1}{4}} + x\right) + 4x}{16(acx^4 + a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^4+a)^2,x, algorithm="fricas")

[Out] 1/16\*(12\*(a\*c\*x^4 + a^2)\*(-1/(a^7\*c))^(1/4)\*arctan(-a^5\*c\*x\*(-1/(a^7\*c))^(3/4) + sqrt(a^4\*sqrt(-1/(a^7\*c)) + x^2)\*a^5\*c\*(-1/(a^7\*c))^(3/4)) + 3\*(a\*c\*x^4 + a^2)\*(-1/(a^7\*c))^(1/4)\*log(a^2\*(-1/(a^7\*c))^(1/4) + x) - 3\*(a\*c\*x^4 + a^2)\*(-1/(a^7\*c))^(1/4)\*log(-a^2\*(-1/(a^7\*c))^(1/4) + x) + 4\*x)/(a\*c\*x^4 + a^2)

**giac** [A] time = 0.18, size = 194, normalized size = 0.96

$$\frac{x}{4(cx^4 + a)a} + \frac{3\sqrt{2}(ac^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^2c} + \frac{3\sqrt{2}(ac^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^2c} + \frac{3\sqrt{2}(ac^3)^{\frac{1}{4}} \log\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{32a^2c} - \frac{3\sqrt{2}(ac^3)^{\frac{1}{4}} \log\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{32a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^4+a)^2,x, algorithm="giac")

[Out] 1/4\*x/((c\*x^4 + a)\*a) + 3/16\*sqrt(2)\*(a\*c^3)^(1/4)\*arctan(1/2\*sqrt(2)\*(2\*x + sqrt(2)\*(a/c)^(1/4))/(a/c)^(1/4))/(a^2\*c) + 3/16\*sqrt(2)\*(a\*c^3)^(1/4)\*arctan(1/2\*sqrt(2)\*(2\*x - sqrt(2)\*(a/c)^(1/4))/(a/c)^(1/4))/(a^2\*c) + 3/32\*sqrt(2)\*(a\*c^3)^(1/4)\*log(x^2 + sqrt(2)\*x\*(a/c)^(1/4) + sqrt(a/c))/(a^2\*c) - 3/32\*sqrt(2)\*(a\*c^3)^(1/4)\*log(x^2 - sqrt(2)\*x\*(a/c)^(1/4) + sqrt(a/c))/(a^2\*c)

**maple** [A] time = 0.00, size = 143, normalized size = 0.71

$$\frac{x}{4(cx^4 + a)a} + \frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}} - 1\right)}{16a^2} + \frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}} + 1\right)}{16a^2} + \frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{x^2 + \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}x + \sqrt{\frac{a}{c}}}{x^2 - \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}x + \sqrt{\frac{a}{c}}}\right)}{32a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x^4+a)^2,x)

[Out] 1/4\*x/a/(c\*x^4+a)+3/32/a^2\*(a/c)^(1/4)\*2^(1/2)\*ln((x^2+(a/c)^(1/4)\*2^(1/2)\*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)\*2^(1/2)\*x+(a/c)^(1/2)))+3/16/a^2\*(a/c)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/c)^(1/4)\*x+1)+3/16/a^2\*(a/c)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/c)^(1/4)\*x-1)

**maxima** [A] time = 2.43, size = 189, normalized size = 0.94

$$\frac{x}{4(acx^4 + a^2)} + \frac{3 \left( \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(2\sqrt{cx} + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{a}\sqrt{c}}\right)}{\sqrt{a}\sqrt{a}\sqrt{c}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(2\sqrt{cx} - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{a}\sqrt{c}}\right)}{\sqrt{a}\sqrt{a}\sqrt{c}} + \frac{\sqrt{2} \log\left(\sqrt{c}x^2 + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{\sqrt{2} \log\left(\sqrt{c}x^2 - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}} \right)}{32a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^4+a)^2,x, algorithm="maxima")

[Out]  $\frac{1}{4}x/(acx^4 + a^2) + \frac{3}{32}(2\sqrt{2})\arctan\left(\frac{1}{2}\sqrt{2}(2\sqrt{c}x + \sqrt{2}a^{1/4}c^{1/4})/\sqrt{\sqrt{a}\sqrt{c}}\right)/(\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}) + 2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2\sqrt{c}x - \sqrt{2}a^{1/4}c^{1/4})/\sqrt{\sqrt{a}\sqrt{c}}\right)/(\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}) + \sqrt{2}\log(\sqrt{c}x^2 + \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{a})/(a^{3/4}c^{1/4}) - \sqrt{2}\log(\sqrt{c}x^2 - \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{a})/(a^{3/4}c^{1/4})/a$

**mupad [B]** time = 0.08, size = 58, normalized size = 0.29

$$\frac{x}{4a(cx^4 + a)} + \frac{3 \operatorname{atan}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)}{8(-a)^{7/4}c^{1/4}} + \frac{3 \operatorname{atanh}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)}{8(-a)^{7/4}c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + c\*x^4)^2,x)

[Out]  $x/(4a(a + cx^4)) + (3\operatorname{atan}((c^{1/4}x)/(-a)^{1/4}))/((8(-a)^{7/4})c^{1/4}) + (3\operatorname{atanh}((c^{1/4}x)/(-a)^{1/4}))/((8(-a)^{7/4})c^{1/4})$

**sympy [A]** time = 0.35, size = 39, normalized size = 0.19

$$\frac{x}{4a^2 + 4acx^4} + \operatorname{RootSum}\left(65536t^4a^7c + 81, \left(t \mapsto t \log\left(\frac{16ta^2}{3} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x\*\*4+a)\*\*2,x)

[Out]  $x/(4a**2 + 4a*c*x**4) + \operatorname{RootSum}(65536*_t**4*a**7*c + 81, \operatorname{Lambda}(_t, _t*\log(16*_t*a**2/3 + x)))$

**3.129**  $\int \frac{1}{(d+ex^2)(a+cx^4)^2} dx$

**Optimal.** Leaf size=689

$$\frac{\sqrt[4]{c} e^2 (\sqrt{a} e + \sqrt{c} d) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} a^{3/4} (ae^2 + cd^2)^2} + \frac{\sqrt[4]{c} e^2 (\sqrt{a} e + \sqrt{c} d) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} a^{3/4} (ae^2 + cd^2)^2}$$

**Rubi [A]** time = 0.62, antiderivative size = 689, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 19, number of rules / integrand size = 0.474, Rules used = {1239, 205, 1179, 1168, 1162, 617, 204, 1165, 628}

$\frac{\sqrt[4]{c} (\sqrt{a} + \sqrt{c} d) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} a^{3/4} (ae^2 + cd^2)^2}, \frac{\sqrt[4]{c} (\sqrt{a} + \sqrt{c} d) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} a^{3/4} (ae^2 + cd^2)^2}, \frac{\sqrt[4]{c} (\sqrt{a} + \sqrt{c} d) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{16\sqrt{2} a^{3/4} (ae^2 + cd^2)^2}, \frac{\sqrt[4]{c} (\sqrt{a} + \sqrt{c} d) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{16\sqrt{2} a^{3/4} (ae^2 + cd^2)^2}, \frac{\sqrt[4]{c} (\sqrt{a} + \sqrt{c} d) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{2\sqrt{2} a^{3/4} (ae^2 + cd^2)^2}, \frac{\sqrt[4]{c} (\sqrt{a} + \sqrt{c} d) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{2\sqrt{2} a^{3/4} (ae^2 + cd^2)^2}, \frac{\sqrt[4]{c} (\sqrt{a} + \sqrt{c} d) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{8\sqrt{2} a^{3/4} (ae^2 + cd^2)^2}, \frac{\sqrt[4]{c} (\sqrt{a} + \sqrt{c} d) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{8\sqrt{2} a^{3/4} (ae^2 + cd^2)^2}, \frac{c \sqrt{a} - c^2}{4(a + c^2)(ae^2 + cd^2)}, \frac{c^2 \sqrt{a} - \sqrt{c} d}{4(a + c^2)(ae^2 + cd^2)}$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(a + c*x^4)^2),x]
[Out] (c*x*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)^2) - (c^(1/4)*e^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c^(1/4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2))
```

**Rule 204**

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

**Rule 205**

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

**Rule 617**

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

**Rule 628**

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

**Rule 1162**

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

#### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

#### Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

#### Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

#### Rule 1239

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := Int
[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e,
p, q}, x] && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0])
```

#### Rubi steps

$$\int \frac{1}{(d + ex^2)(a + cx^4)^2} dx = \int \left( \frac{e^4}{(cd^2 + ae^2)^2 (d + ex^2)} + \frac{c(d - ex^2)}{(cd^2 + ae^2)(a + cx^4)^2} - \frac{ce^2(-d + ex^2)}{(cd^2 + ae^2)^2 (a + cx^4)} \right) dx$$

$$= -\frac{(ce^2) \int \frac{-d+ex^2}{a+cx^4} dx}{(cd^2 + ae^2)^2} + \frac{e^4 \int \frac{1}{d+ex^2} dx}{(cd^2 + ae^2)^2} + \frac{c \int \frac{d-ex^2}{(a+cx^4)^2} dx}{cd^2 + ae^2}$$

$$= \frac{cx(d - ex^2)}{4a(cd^2 + ae^2)(a + cx^4)} + \frac{e^{7/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(cd^2 + ae^2)^2} + \frac{\left(\left(\frac{\sqrt{c}d}{\sqrt{a}} - e\right)e^2\right) \int \frac{\sqrt{a}\sqrt{c+cx^2}}{a+cx^4} dx}{2(cd^2 + ae^2)^2} + \frac{e^2 \int \frac{1}{a+cx^4} dx}{2(cd^2 + ae^2)^2}$$

$$= \frac{cx(d - ex^2)}{4a(cd^2 + ae^2)(a + cx^4)} + \frac{e^{7/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(cd^2 + ae^2)^2} + \frac{\left(\left(\frac{\sqrt{c}d}{\sqrt{a}} - e\right)e^2\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt{c}} + x^2} dx}{4(cd^2 + ae^2)^2} + \frac{e^2 \int \frac{1}{a+cx^4} dx}{4(cd^2 + ae^2)^2}$$

$$= \frac{cx(d - ex^2)}{4a(cd^2 + ae^2)(a + cx^4)} + \frac{e^{7/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(cd^2 + ae^2)^2} - \frac{\sqrt[4]{c}e^2(\sqrt{c}d + \sqrt{a}e) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}x)}{4\sqrt{2}a^{3/4}(cd^2 + ae^2)^2} + \frac{e^2 \int \frac{1}{a+cx^4} dx}{4(cd^2 + ae^2)^2}$$

$$= \frac{cx(d - ex^2)}{4a(cd^2 + ae^2)(a + cx^4)} + \frac{e^{7/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(cd^2 + ae^2)^2} - \frac{\sqrt[4]{c}e^2(\sqrt{c}d - \sqrt{a}e) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2 + ae^2)^2} + \frac{e^2 \int \frac{1}{a+cx^4} dx}{4(cd^2 + ae^2)^2}$$

$$= \frac{cx(d - ex^2)}{4a(cd^2 + ae^2)(a + cx^4)} + \frac{e^{7/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(cd^2 + ae^2)^2} - \frac{\sqrt[4]{c}e^2(\sqrt{c}d - \sqrt{a}e) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2 + ae^2)^2} + \frac{e^2 \int \frac{1}{a+cx^4} dx}{4(cd^2 + ae^2)^2}$$

**Mathematica [A]** time = 0.30, size = 429, normalized size = 0.62

$$\frac{\sqrt{2}\sqrt[4]{c}(\sqrt{a^2d^2 + \sqrt{c}cd^2 + 7e\sqrt{c}d^2 + 3e^2d^2}) \log(-\sqrt{2}\sqrt[4]{c}\sqrt{c+e} + \sqrt{c+e}) + \sqrt{2}\sqrt[4]{c}(\sqrt{a^2d^2 + \sqrt{c}cd^2 + 7e\sqrt{c}d^2 + 3e^2d^2}) \log(\sqrt{2}\sqrt[4]{c}\sqrt{c+e} + \sqrt{c+e}) + \frac{2\sqrt{2}\sqrt[4]{c}(\sqrt{a^2d^2 + \sqrt{c}cd^2 - 7e\sqrt{c}d^2 - 3e^2d^2}) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}}\right) - 2\sqrt{2}\sqrt[4]{c}(\sqrt{a^2d^2 + \sqrt{c}cd^2 - 7e\sqrt{c}d^2 - 3e^2d^2}) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}} + 1\right) + \frac{8c(d-c^2)(a^2+cd)}{d(a+cx^2)} + \frac{32e^2 \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}}{32(a^2 + cd)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x^2)\*(a + c\*x^4)^2), x]

[Out] ((8\*c\*(c\*d^2 + a\*e^2)\*x\*(d - e\*x^2))/(a\*(a + c\*x^4)) + (32\*e^(7/2)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]]/Sqrt[d] + (2\*Sqrt[2]\*c^(1/4)\*(-3\*c^(3/2)\*d^3 + Sqrt[a]\*c\*d^2\*e - 7\*a\*Sqrt[c]\*d\*e^2 + 5\*a^(3/2)\*e^3)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)]/a^(7/4) - (2\*Sqrt[2]\*c^(1/4)\*(-3\*c^(3/2)\*d^3 + Sqrt[a]\*c\*d^2\*e - 7\*a\*Sqrt[c]\*d\*e^2 + 5\*a^(3/2)\*e^3)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)]/a^(7/4) - (Sqrt[2]\*c^(1/4)\*(3\*c^(3/2)\*d^3 + Sqrt[a]\*c\*d^2\*e + 7\*a\*Sqrt[c]\*d\*e^2 + 5\*a^(3/2)\*e^3)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/a^(7/4) + (Sqrt[2]\*c^(1/4)\*(3\*c^(3/2)\*d^3 + Sqrt[a]\*c\*d^2\*e + 7\*a\*Sqrt[c]\*d\*e^2 + 5\*a^(3/2)\*e^3)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/a^(7/4))/(32\*(c\*d^2 + a\*e^2)^2)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex^2)(a + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e\*x^2)\*(a + c\*x^4)^2), x]



$$\begin{aligned}
& *c^4 * x^4) * \sqrt{(6c^3 d^5 e + 44a^2 c^2 d^3 e^3 + 70a^2 c d^2 e^5 - (a^3 c^4 d^8 + 4a^4 c^3 d^6 e^2 + 6a^5 c^2 d^4 e^4 + 4a^6 c d^2 e^6 + a^7 e^8))} \\
& * \sqrt{-(81c^7 d^{12} + 738a^2 c^6 d^{10} e^2 + 2383a^2 c^5 d^8 e^4 + 2748a^3 c^4 d^6 e^6 - 529a^4 c^3 d^4 e^8 - 1950a^5 c^2 d^2 e^{10} + 625a^6 c e^{12})} \\
& / (a^7 c^8 d^{16} + 8a^8 c^7 d^{14} e^2 + 28a^9 c^6 d^{12} e^4 + 56a^{10} c^5 d^{10} e^6 + 70a^{11} c^4 d^8 e^8 + 56a^{12} c^3 d^6 e^{10} + 28a^{13} c^2 d^4 e^{12} + \\
& 8a^{14} c d^2 e^{14} + a^{15} e^{16})) / (a^3 c^4 d^8 + 4a^4 c^3 d^6 e^2 + 6a^5 c^2 d^4 e^4 + 4a^6 c d^2 e^6 + a^7 e^8)) * \log(-(81c^5 d^8 + 594a^2 c^4 d^6 e^2 + \\
& 1376a^2 c^3 d^4 e^4 + 750a^3 c^2 d^2 e^6 - 625a^4 c e^8) * x + (27a^2 c^5 d^9 + 186a^3 c^4 d^7 e^2 + 404a^4 c^3 d^5 e^4 + 198a^5 c^2 d^3 e^6 - \\
& 175a^6 c d e^8 - (a^6 c^5 d^{10} e + 9a^7 c^4 d^8 e^3 + 26a^8 c^3 d^6 e^5 + 34a^9 c^2 d^4 e^7 + 21a^{10} c d^2 e^9 + 5a^{11} e^{11})) * \sqrt{-(81c^7 d^{12} + \\
& 738a^2 c^6 d^{10} e^2 + 2383a^2 c^5 d^8 e^4 + 2748a^3 c^4 d^6 e^6 - 529a^4 c^3 d^4 e^8 - 1950a^5 c^2 d^2 e^{10} + 625a^6 c e^{12})} / (a^7 c^8 d^{16} + \\
& 8a^8 c^7 d^{14} e^2 + 28a^9 c^6 d^{12} e^4 + 56a^{10} c^5 d^{10} e^6 + 70a^{11} c^4 d^8 e^8 + 56a^{12} c^3 d^6 e^{10} + 28a^{13} c^2 d^4 e^{12} + 8a^{14} c d^2 e^{14} + \\
& a^{15} e^{16})) * \sqrt{(6c^3 d^5 e + 44a^2 c^2 d^3 e^3 + 70a^2 c d^2 e^5 - (a^3 c^4 d^8 + 4a^4 c^3 d^6 e^2 + 6a^5 c^2 d^4 e^4 + 4a^6 c d^2 e^6 + a^7 e^8))} \\
& * \sqrt{-(81c^7 d^{12} + 738a^2 c^6 d^{10} e^2 + 2383a^2 c^5 d^8 e^4 + 2748a^3 c^4 d^6 e^6 - 529a^4 c^3 d^4 e^8 - 1950a^5 c^2 d^2 e^{10} + 625a^6 c e^{12})} / (a^7 c^8 d^{16} + \\
& 8a^8 c^7 d^{14} e^2 + 28a^9 c^6 d^{12} e^4 + 56a^{10} c^5 d^{10} e^6 + 70a^{11} c^4 d^8 e^8 + 56a^{12} c^3 d^6 e^{10} + 28a^{13} c^2 d^4 e^{12} + 8a^{14} c d^2 e^{14} + \\
& a^{15} e^{16})) / (a^3 c^4 d^8 + 4a^4 c^3 d^6 e^2 + 6a^5 c^2 d^4 e^4 + 4a^6 c d^2 e^6 + a^7 e^8)) - (a^2 c^2 d^4 + 2a^3 c d^2 e^2 + a^4 e^4 + (a^3 c^3 d^4 + 2a^2 c^2 d^2 e^2 + a^3 c e^4) * x^4) * \sqrt{(6c^3 d^5 e + \\
& 44a^2 c^2 d^3 e^3 + 70a^2 c d^2 e^5 - (a^3 c^4 d^8 + 4a^4 c^3 d^6 e^2 + 6a^5 c^2 d^4 e^4 + 4a^6 c d^2 e^6 + a^7 e^8))} * \sqrt{-(81c^7 d^{12} + \\
& 738a^2 c^6 d^{10} e^2 + 2383a^2 c^5 d^8 e^4 + 2748a^3 c^4 d^6 e^6 - 529a^4 c^3 d^4 e^8 - 1950a^5 c^2 d^2 e^{10} + 625a^6 c e^{12})} / (a^7 c^8 d^{16} + 8a^8 c^7 d^{14} e^2 + \\
& 28a^9 c^6 d^{12} e^4 + 56a^{10} c^5 d^{10} e^6 + 70a^{11} c^4 d^8 e^8 + 56a^{12} c^3 d^6 e^{10} + 28a^{13} c^2 d^4 e^{12} + 8a^{14} c d^2 e^{14} + a^{15} e^{16})) / (a^3 c^4 d^8 + 4a^4 c^3 d^6 e^2 + 6a^5 c^2 d^4 e^4 + 4a^6 c d^2 e^6 + a^7 e^8)) * \log(-(81c^5 d^8 + 594a^2 c^4 d^6 e^2 + 1376a^2 c^3 d^4 e^4 + \\
& 750a^3 c^2 d^2 e^6 - 625a^4 c e^8) * x - (27a^2 c^5 d^9 + 186a^3 c^4 d^7 e^2 + 404a^4 c^3 d^5 e^4 + 198a^5 c^2 d^3 e^6 - 175a^6 c d e^8 - \\
& (a^6 c^5 d^{10} e + 9a^7 c^4 d^8 e^3 + 26a^8 c^3 d^6 e^5 + 34a^9 c^2 d^4 e^7 + 21a^{10} c d^2 e^9 + 5a^{11} e^{11})) * \sqrt{-(81c^7 d^{12} + 738a^2 c^6 d^{10} e^2 + \\
& 2383a^2 c^5 d^8 e^4 + 2748a^3 c^4 d^6 e^6 - 529a^4 c^3 d^4 e^8 - 1950a^5 c^2 d^2 e^{10} + 625a^6 c e^{12})} / (a^7 c^8 d^{16} + 8a^8 c^7 d^{14} e^2 + \\
& 28a^9 c^6 d^{12} e^4 + 56a^{10} c^5 d^{10} e^6 + 70a^{11} c^4 d^8 e^8 + 56a^{12} c^3 d^6 e^{10} + 28a^{13} c^2 d^4 e^{12} + 8a^{14} c d^2 e^{14} + a^{15} e^{16})) * \sqrt{(6c^3 d^5 e + 44a^2 c^2 d^3 e^3 + 70a^2 c d^2 e^5 - (a^3 c^4 d^8 + 4a^4 c^3 d^6 e^2 + 6a^5 c^2 d^4 e^4 + 4a^6 c d^2 e^6 + a^7 e^8))} \\
& * \sqrt{-(81c^7 d^{12} + 738a^2 c^6 d^{10} e^2 + 2383a^2 c^5 d^8 e^4 + 2748a^3 c^4 d^6 e^6 - 529a^4 c^3 d^4 e^8 - 1950a^5 c^2 d^2 e^{10} + 625a^6 c e^{12})} / (a^7 c^8 d^{16} + 8a^8 c^7 d^{14} e^2 + \\
& 28a^9 c^6 d^{12} e^4 + 56a^{10} c^5 d^{10} e^6 + 70a^{11} c^4 d^8 e^8 + 56a^{12} c^3 d^6 e^{10} + 28a^{13} c^2 d^4 e^{12} + 8a^{14} c d^2 e^{14} + a^{15} e^{16})) / (a^3 c^4 d^8 + 4a^4 c^3 d^6 e^2 + 6a^5 c^2 d^4 e^4 + 4a^6 c d^2 e^6 + a^7 e^8)) - 8(a^2 c^2 d^4 + 2a^3 c d^2 e^2 + a^4 e^4) * \sqrt{-e/d} * \log((e * x^2 + 2 * d * x * \sqrt{-e/d} - d) / (e * x^2 + d)) - 4(c^2 d^3 + a^2 c d e^2) * x / (a^2 c^2 d^4 + 2a^3 c d^2 e^2 + a^4 e^4 + (a^3 c^3 d^4 + 2a^2 c^2 d^2 e^2 + a^3 c e^4) * x^4), -1/16(4(c^2 d^2 e + a^2 c e^3) * x^3 - 16(a^2 c e^3 * x^4 + a^2 e^3) * \sqrt{e/d} * \arctan(x * \sqrt{e/d})) + (a^2 c^2 d^4 + 2a^3 c d^2 e^2 + a^4 e^4 + (a^3 c^3 d^4 + 2a^2 c^2 d^2 e^2 + a^3 c e^4) * x^4) * \sqrt{(6c^3 d^5 e + 44a^2 c^2 d^3 e^3 + 70a^2 c d^2 e^5 + (a^3 c^4 d^8 + 4a^4 c^3 d^6 e^2 + 6a^5 c^2 d^4 e^4 + 4a^6 c d^2 e^6 + a^7 e^8))} * \sqrt{-(81c^7 d^{12} + 738a^2 c^6 d^{10} e^2 + 2383a^2 c^5 d^8 e^4 + 2748a^3 c^4 d^6 e^6 - 529a^4 c^3 d^4 e^8 - 1950a^5 c^2 d^2 e^{10} + 625a^6 c e^{12})} / (a^7 c^8 d^{16} + 8a^8 c^7 d^{14} e^2 + 28a^9 c^6 d^{12} e^4 + 56a^{10} c^5 d^{10} e^6 + 70a^{11} c^4 d^8 e^8 + 56a^{12} c^3 d^6 e^{10} + 28a^{13} c^2 d^4 e^{12} + 8a^{14} c d^2 e^{14} + a^{15} e^{16})) / (a^3 c^4 d^8 + 4a^4 c^3 d^6 e^2 + 6a^5 c^2 d^4 e^4 + 4a^6 c d^2 e^6 + a^7 e^8))
\end{aligned}$$





$$8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}c^2d^2e^{14} + a^{15}e^{16}))\sqrt{(6c^3d^5e + 44a^2c^2d^3e^3 + 70a^2c^2d^5e^5 - (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c^2d^2e^6 + a^7e^8))\sqrt{-(81c^7d^{12} + 738a^2c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^2e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}c^2d^2e^{14} + a^{15}e^{16})))/(a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c^2d^2e^6 + a^7e^8)) - (a^2c^2d^4 + 2a^3c^2d^2e^2 + a^4e^4 + (a^3c^3d^4 + 2a^2c^2d^2e^2 + a^3c^2e^4)*x^4)\sqrt{(6c^3d^5e + 44a^2c^2d^3e^3 + 70a^2c^2d^5e^5 - (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c^2d^2e^6 + a^7e^8))\sqrt{-(81c^7d^{12} + 738a^2c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^2e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}c^2d^2e^{14} + a^{15}e^{16})))/(a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c^2d^2e^6 + a^7e^8))*\log(-(81c^5d^8 + 594a^2c^4d^6e^2 + 1376a^2c^3d^4e^4 + 750a^3c^2d^2e^6 - 625a^4c^2e^8)*x - (27a^2c^5d^9 + 186a^3c^4d^7e^2 + 404a^4c^3d^5e^4 + 198a^5c^2d^3e^6 - 175a^6c^2d^2e^8 - (a^6c^5d^{10}e + 9a^7c^4d^8e^3 + 26a^8c^3d^6e^5 + 34a^9c^2d^4e^7 + 21a^{10}c^2d^2e^9 + 5a^{11}e^{11})*\sqrt{-(81c^7d^{12} + 738a^2c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^2e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}c^2d^2e^{14} + a^{15}e^{16})))*\sqrt{(6c^3d^5e + 44a^2c^2d^3e^3 + 70a^2c^2d^5e^5 - (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c^2d^2e^6 + a^7e^8))\sqrt{-(81c^7d^{12} + 738a^2c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^2e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}c^2d^2e^{14} + a^{15}e^{16})))/(a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c^2d^2e^6 + a^7e^8)) - 4*(c^2d^3 + a^2c^2d^2e^2)*x)/(a^2c^2d^4 + 2a^3c^2d^2e^2 + a^4e^4 + (a^3c^3d^4 + 2a^2c^2d^2e^2 + a^3c^2e^4)*x^4)]$$

**giac [A]** time = 0.21, size = 603, normalized size = 0.88

$$\frac{\left(\frac{d}{c}\right)^{\frac{1}{4}} \sqrt{d} \sqrt{c^2 d^2 + a^2} \sqrt{c^2 d^2 + a^2} \sqrt{c^2 d^2 + a^2} \arctan\left(\frac{\sqrt{c^2 d^2 + a^2}}{\sqrt{d}}\right)}{4 \sqrt{d} \sqrt{c^2 d^2 + a^2} \sqrt{c^2 d^2 + a^2} \sqrt{c^2 d^2 + a^2}} + \frac{\left(\frac{d}{c}\right)^{\frac{1}{4}} \sqrt{d} \sqrt{c^2 d^2 + a^2} \sqrt{c^2 d^2 + a^2} \sqrt{c^2 d^2 + a^2} \arctan\left(\frac{\sqrt{c^2 d^2 + a^2}}{\sqrt{d}}\right)}{4 \sqrt{d} \sqrt{c^2 d^2 + a^2} \sqrt{c^2 d^2 + a^2} \sqrt{c^2 d^2 + a^2}} + \frac{\left(\frac{d}{c}\right)^{\frac{1}{4}} \sqrt{d} \sqrt{c^2 d^2 + a^2} \sqrt{c^2 d^2 + a^2} \sqrt{c^2 d^2 + a^2} \arctan\left(\frac{\sqrt{c^2 d^2 + a^2}}{\sqrt{d}}\right)}{4 \sqrt{d} \sqrt{c^2 d^2 + a^2} \sqrt{c^2 d^2 + a^2} \sqrt{c^2 d^2 + a^2}} + \frac{\left(\frac{d}{c}\right)^{\frac{1}{4}} \sqrt{d} \sqrt{c^2 d^2 + a^2} \sqrt{c^2 d^2 + a^2} \sqrt{c^2 d^2 + a^2} \arctan\left(\frac{\sqrt{c^2 d^2 + a^2}}{\sqrt{d}}\right)}{4 \sqrt{d} \sqrt{c^2 d^2 + a^2} \sqrt{c^2 d^2 + a^2} \sqrt{c^2 d^2 + a^2}} + \frac{\left(\frac{d}{c}\right)^{\frac{1}{4}} \sqrt{d} \sqrt{c^2 d^2 + a^2} \sqrt{c^2 d^2 + a^2} \sqrt{c^2 d^2 + a^2} \arctan\left(\frac{\sqrt{c^2 d^2 + a^2}}{\sqrt{d}}\right)}{4 \sqrt{d} \sqrt{c^2 d^2 + a^2} \sqrt{c^2 d^2 + a^2} \sqrt{c^2 d^2 + a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)/(c\*x^4+a)^2,x, algorithm="giac")

[Out] 1/8\*(3\*(a\*c^3)^(1/4)\*c^3\*d^3 + 7\*(a\*c^3)^(1/4)\*a\*c^2\*d\*e^2 - (a\*c^3)^(3/4)\*c\*d^2\*e - 5\*(a\*c^3)^(3/4)\*a\*e^3)\*arctan(1/2\*sqrt(2)\*(2\*x + sqrt(2))\*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)\*a^2\*c^4\*d^4 + 2\*sqrt(2)\*a^3\*c^3\*d^2\*e^2 + sqrt(2)\*a^4\*c^2\*e^4) + 1/8\*(3\*(a\*c^3)^(1/4)\*c^3\*d^3 + 7\*(a\*c^3)^(1/4)\*a\*c^2\*d\*e^2 - (a\*c^3)^(3/4)\*c\*d^2\*e - 5\*(a\*c^3)^(3/4)\*a\*e^3)\*arctan(1/2\*sqrt(2)\*(2\*x - sqrt(2))\*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)\*a^2\*c^4\*d^4 + 2\*sqrt(2)\*a^3\*c^3\*d^2\*e^2 + sqrt(2)\*a^4\*c^2\*e^4) + 1/16\*(3\*(a\*c^3)^(1/4)\*c^3\*d^3 + 7\*(a\*c^3)^(1/4)\*a\*c^2\*d\*e^2 + (a\*c^3)^(3/4)\*c\*d^2\*e + 5\*(a\*c^3)^(3/4)\*a\*e^3)\*log(x^2 + sqrt(2)\*x\*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)\*a^2\*c^4\*d^4 + 2\*sqrt(2)\*a^3\*c^3\*d^2\*e^2 + sqrt(2)\*a^4\*c^2\*e^4) - 1/16\*(3\*(a\*c^3)^(1/4)\*c^3\*d^3 + 7\*(a\*c^3)^(1/4)\*a\*c^2\*d\*e^2 + (a\*c^3)^(3/4)\*c\*d^2\*e + 5\*(a\*c^3)^(3/4)\*a\*e^3)\*log(x^2 - sqrt(2)\*x\*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)\*a^2\*c^4\*d^4 + 2\*sqrt(2)\*a^3\*c^3\*d^2\*e^2 + sqrt(2)\*a^4\*c^2\*e^4) + arctan(x\*e^(1/2)/sqrt(d))\*e^(7/2)/((c^2\*d^4 + 2\*a\*c\*d^2\*e^2 + a^2\*e^4)\*sqrt(d)) - 1/4\*(c\*x^3\*e - c\*d\*x)/((c\*x^4 + a)\*(a\*c\*d^2 + a^2\*e^2))

**maple [A]** time = 0.02, size = 873, normalized size = 1.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x^2+d)/(c\*x^4+a)^2,x)

[Out] 
$$e^4/(a*e^2+c*d^2)^2/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)}*e*x)-1/4/(a*e^2+c*d^2)^2*c/(c*x^4+a)*e^3*x^3-1/4/(a*e^2+c*d^2)^2*c^2/(c*x^4+a)*e/a*x^3*d^2+1/4/(a*e^2+c*d^2)^2*c/(c*x^4+a)*d*x*e^2+1/4/(a*e^2+c*d^2)^2*c^2/(c*x^4+a)*d^3/a*x+7/16/(a*e^2+c*d^2)^2*c/a*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d*e^2+3/16/(a*e^2+c*d^2)^2*c^2/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^3+7/32/(a*e^2+c*d^2)^2*c/a*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d*e^2+3/32/(a*e^2+c*d^2)^2*c^2/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d^3+7/16/(a*e^2+c*d^2)^2*c/a*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d*e^2+3/16/(a*e^2+c*d^2)^2*c^2/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^3-5/32/(a*e^2+c*d^2)^2/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*e^3-1/32/(a*e^2+c*d^2)^2*c/a*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d^2*e-5/16/(a*e^2+c*d^2)^2/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*e^3-1/16/(a*e^2+c*d^2)^2*c/a*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^2*e-5/16/(a*e^2+c*d^2)^2/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*e^3-1/16/(a*e^2+c*d^2)^2*c/a*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^2*e$$

**maxima** [A] time = 2.45, size = 506, normalized size = 0.73

$$\frac{e^4 \arctan\left(\frac{x}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)/(c\*x^4+a)^2,x, algorithm="maxima")

[Out] 
$$e^4*\arctan(e*x/\sqrt{d*e})/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*\sqrt{d*e}) + 1/32*c*(2*\sqrt{2}*(3*c^{(3/2)}*d^3 - \sqrt{a}*c*d^2*e + 7*a*\sqrt{c}*d*e^2 - 5*a^{(3/2)}*e^3)*\arctan(1/2*\sqrt{2}*(2*\sqrt{c}*x + \sqrt{2})*a^{(1/4)}*c^{(1/4)})/\sqrt{a*\sqrt{c}})/(\sqrt{a}*\sqrt{\sqrt{a}*\sqrt{c}}*\sqrt{c}) + 2*\sqrt{2}*(3*c^{(3/2)}*d^3 - \sqrt{a}*c*d^2*e + 7*a*\sqrt{c}*d*e^2 - 5*a^{(3/2)}*e^3)*\arctan(1/2*\sqrt{2}*(2*\sqrt{c}*x - \sqrt{2})*a^{(1/4)}*c^{(1/4)})/\sqrt{a*\sqrt{c}})/(\sqrt{a}*\sqrt{\sqrt{a}*\sqrt{c}}*\sqrt{c}) + \sqrt{2}*(3*c^{(3/2)}*d^3 + \sqrt{a}*c*d^2*e + 7*a*\sqrt{c}*d*e^2 + 5*a^{(3/2)}*e^3)*\log(\sqrt{c}*x^2 + \sqrt{2})*a^{(1/4)}*c^{(1/4)}*x + \sqrt{a})/(a^{(3/4)}*c^{(3/4)}) - \sqrt{2}*(3*c^{(3/2)}*d^3 + \sqrt{a}*c*d^2*e + 7*a*\sqrt{c}*d*e^2 + 5*a^{(3/2)}*e^3)*\log(\sqrt{c}*x^2 - \sqrt{2})*a^{(1/4)}*c^{(1/4)}*x + \sqrt{a})/(a^{(3/4)}*c^{(3/4)})/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4) - 1/4*((c^2*d^2*e + a*c*e^3)*x^3 - (c^2*d^3 + a*c*d*e^2)*x)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)$$

**mupad** [B] time = 6.78, size = 17945, normalized size = 26.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + c\*x^4)^2\*(d + e\*x^2)),x)

[Out] 
$$((c*d*x)/(4*a*(a*e^2 + c*d^2)) - (c*e*x^3)/(4*a*(a*e^2 + c*d^2)))/(a + c*x^4) - \operatorname{atan}\left(\frac{((65536*a^{11}*c^4*e^{16} - 12288*a^4*c^{11}*d^{14}*e^2 - 57344*a^5*c^{10}*d^{12}*e^4 - 36864*a^6*c^9*d^{10}*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^{10} + 663552*a^9*c^6*d^4*e^{12} + 331776*a^{10}*c^5*d^2*e^{14})/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4))}{(c*d*x)/(4*a*(a*e^2 + c*d^2)) - (c*e*x^3)/(4*a*(a*e^2 + c*d^2))}\right)$$





$$\begin{aligned}
& 5e + 44a^5c^2d^3e^3 + 70a^6c^2de^5 + 41a^2c^2d^4e^2(-a^7c)^{(1/2)} \\
& + 39a^2c^2d^2e^4(-a^7c)^{(1/2)} / (256(a^{11}e^8 + a^7c^4d^8 + 4a^{10}c \\
& *d^2e^6 + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4))^{(1/2)} + (x(1152a^2c^ \\
& 11d^{13}e^2 - 49024a^8c^5d^7e^8 + 7936a^3c^{10}d^{11}e^4 + 20352a^4c^9 \\
& *d^9e^6 + 8704a^5c^8d^7e^8 - 66688a^6c^7d^5e^{10} - 110848a^7c^6d \\
& ^3e^{12})) / (128(a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 \\
& + 6a^6c^2d^4e^4)) * ((9c^3d^6(-a^7c)^{(1/2)} - 25a^3e^6(-a^7c)^{(1/2)} \\
& + 6a^4c^3d^5e + 44a^5c^2d^3e^3 + 70a^6c^2de^5 + 41a^2c^2d^4e^2(-a^7c)^{(1/2)} \\
& + 39a^2c^2d^2e^4(-a^7c)^{(1/2)}) / (256(a^{11}e^8 + a^7c^4d^8 + 4a^{10}c \\
& ^4d^8 + 4a^{10}c^2d^2e^6 + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4))^{(1/2)} \\
& - (720a^2c^{10}d^{11}e^3 + 20432a^6c^5d^7e^{13} + 4880a^2c^9d^9e^5 + 123 \\
& 20a^3c^8d^7e^7 + 21024a^4c^7d^5e^9 + 33296a^5c^6d^3e^{11}) / (256(a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) * ((9c^3d^6(-a^7c)^{(1/2)} - 25a^3e^6(-a^7c)^{(1/2)} + 6a^4c^3d^5e + 44a^5c^2d^3e^3 + 70a^6c^2de^5 + 41a^2c^2d^4e^2(-a^7c)^{(1/2)} + 39a^2c^2d^2e^4(-a^7c)^{(1/2)}) / (256(a^{11}e^8 + a^7c^4d^8 + 4a^{10}c^2d^2e^6 + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4))^{(1/2)} + (x(1425a^4c^5e^{13} + 81c^9d^8e^5 + 612a^2c^8d^6e^7 + 1894a^2c^7d^4e^9 + 2532a^3c^6d^2e^{11})) / (128(a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) * ((9c^3d^6(-a^7c)^{(1/2)} - 25a^3e^6(-a^7c)^{(1/2)} + 6a^4c^3d^5e + 44a^5c^2d^3e^3 + 70a^6c^2de^5 + 41a^2c^2d^4e^2(-a^7c)^{(1/2)} + 39a^2c^2d^2e^4(-a^7c)^{(1/2)}) / (256(a^{11}e^8 + a^7c^4d^8 + 4a^{10}c^2d^2e^6 + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4))^{(1/2)})) * ((((((65536a^{11}c^4e^{16} - 12288a^4c^{11}d^{14}e^2 - 57344a^5c^{10}d^{12}e^4 - 36864a^6c^9d^{10}e^6 + 245760a^7c^8d^8e^8 + 634880a^8c^7d^6e^{10} + 663552a^9c^6d^4e^{12} + 331776a^{10}c^5d^2e^{14}) / (256(a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) - (x((25a^3e^6(-a^7c)^{(1/2)} - 9c^3d^6(-a^7c)^{(1/2)} + 6a^4c^3d^5e + 44a^5c^2d^3e^3 + 70a^6c^2de^5 - 41a^2c^2d^4e^2(-a^7c)^{(1/2)} - 39a^2c^2d^2e^4(-a^7c)^{(1/2)}) / (256(a^{11}e^8 + a^7c^4d^8 + 4a^{10}c^2d^2e^6 + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4))^{(1/2)} * (65536a^{13}c^4e^{17} - 65536a^6c^{11}d^{14}e^3 - 327680a^7c^{10}d^{12}e^5 - 589824a^8c^9d^{10}e^7 - 327680a^9c^8d^8e^9 + 327680a^{10}c^7d^6e^{11} + 589824a^{11}c^6d^4e^{13} + 327680a^{12}c^5d^2e^{15})) / (128(a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) * ((25a^3e^6(-a^7c)^{(1/2)} - 9c^3d^6(-a^7c)^{(1/2)} + 6a^4c^3d^5e + 44a^5c^2d^3e^3 + 70a^6c^2de^5 - 41a^2c^2d^4e^2(-a^7c)^{(1/2)} - 39a^2c^2d^2e^4(-a^7c)^{(1/2)}) / (256(a^{11}e^8 + a^7c^4d^8 + 4a^{10}c^2d^2e^6 + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4))^{(1/2)} - (x(1152a^2c^{11}d^{13}e^2 - 49024a^8c^5d^7e^8 + 7936a^3c^{10}d^{11}e^4 + 20352a^4c^9d^9e^6 + 8704a^5c^8d^7e^8 - 66688a^6c^7d^5e^{10} - 110848a^7c^6d^3e^{12})) / (128(a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) * ((25a^3e^6(-a^7c)^{(1/2)} - 9c^3d^6(-a^7c)^{(1/2)} + 6a^4c^3d^5e + 44a^5c^2d^3e^3 + 70a^6c^2de^5 - 41a^2c^2d^4e^2(-a^7c)^{(1/2)} - 39a^2c^2d^2e^4(-a^7c)^{(1/2)}) / (256(a^{11}e^8 + a^7c^4d^8 + 4a^{10}c^2d^2e^6 + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4))^{(1/2)} - (720a^2c^{10}d^{11}e^3 + 20432a^6c^5d^7e^{13} + 4880a^2c^9d^9e^5 + 12320a^3c^8d^7e^7 + 21024a^4c^7d^5e^9 + 33296a^5c^6d^3e^{11}) / (256(a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) * ((25a^3e^6(-a^7c)^{(1/2)} - 9c^3d^6(-a^7c)^{(1/2)} + 6a^4c^3d^5e + 44a^5c^2d^3e^3 + 70a^6c^2de^5 - 41a^2c^2d^4e^2(-a^7c)^{(1/2)} - 39a^2c^2d^2e^4(-a^7c)^{(1/2)}) / (256(a^{11}e^8 + a^7c^4d^8 + 4a^{10}c^2d^2e^6 + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4))^{(1/2)} - (x(1425a^4c^5e^{13} + 81c^9d^8e^5 + 612a^2c^8d^6e^7 + 1894a^2c^7d^4e^9 + 2532a^3c^6d^2e^{11})) / (128(a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4))
\end{aligned}$$

$$\begin{aligned}
& )) * ((25*a^3*e^6*(-a^7*c)^{(1/2)} - 9*c^3*d^6*(-a^7*c)^{(1/2)} + 6*a^4*c^3*d^5*e \\
& + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^{(1/2)} - \\
& 39*a^2*c*d^2*e^4*(-a^7*c)^{(1/2)}) / (256*(a^{11}*e^8 + a^7*c^4*d^8 + 4*a^{10}*c*d^2 \\
& *e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4))^{(1/2)} * i - (((((65536*a^{11} \\
& *c^4*e^{16} - 12288*a^4*c^{11}*d^{14}*e^2 - 57344*a^5*c^{10}*d^{12}*e^4 - 36864*a^6*c^9*d^{10}*e^6 \\
& + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^{10} + 663552*a^9 \\
& *c^6*d^4*e^{12} + 331776*a^{10}*c^5*d^2*e^{14}) / (256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 \\
& + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (x*((25*a^3*e^6*(-a^7*c)^{(1/2)} - 9*c^3*d^6*(-a^7*c)^{(1/2)} \\
& + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^7*c)^{(1/2)}) / (256*(a^{11}*e^8 + a^7*c^4*d^8 + 4*a^{10}*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4))^{(1/2)} * (65536*a^{13}*c^4*e^{17} - 65536*a^6*c^{11}*d^{14}*e^3 - 327680*a^7*c^{10}*d^{12}*e^5 - 589824*a^8*c^9*d^{10}*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^{10}*c^7*d^6*e^{11} + 589824*a^{11}*c^6*d^4*e^{13} + 327680*a^{12}*c^5*d^2*e^{15}) / (128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4))) * ((25*a^3*e^6*(-a^7*c)^{(1/2)} - 9*c^3*d^6*(-a^7*c)^{(1/2)} + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^7*c)^{(1/2)}) / (256*(a^{11}*e^8 + a^7*c^4*d^8 + 4*a^{10}*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4))^{(1/2)} + (x*(1152*a^2*c^{11}*d^{13}*e^2 - 49024*a^8*c^5*d*e^{14} + 7936*a^3*c^{10}*d^{11}*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^{10} - 110848*a^7*c^6*d^3*e^{12}) / (128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4))) * ((25*a^3*e^6*(-a^7*c)^{(1/2)} - 9*c^3*d^6*(-a^7*c)^{(1/2)} + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^7*c)^{(1/2)}) / (256*(a^{11}*e^8 + a^7*c^4*d^8 + 4*a^{10}*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4))^{(1/2)} - (720*a*c^{10}*d^{11}*e^3 + 20432*a^6*c^5*d*e^{13} + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^{11}) / (256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4))) * ((25*a^3*e^6*(-a^7*c)^{(1/2)} - 9*c^3*d^6*(-a^7*c)^{(1/2)} + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^7*c)^{(1/2)}) / (256*(a^{11}*e^8 + a^7*c^4*d^8 + 4*a^{10}*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4))^{(1/2)} + (x*(1425*a^4*c^5*e^{13} + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^{11}) / (128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4))) * ((25*a^3*e^6*(-a^7*c)^{(1/2)} - 9*c^3*d^6*(-a^7*c)^{(1/2)} + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^7*c)^{(1/2)}) / (256*(a^{11}*e^8 + a^7*c^4*d^8 + 4*a^{10}*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4))^{(1/2)} * i) / (((125*a^2*c^5*e^{12} + 81*c^7*d^4*e^8 + 270*a*c^6*d^2*e^{10}) / (128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (((((65536*a^{11}*c^4*e^{16} - 12288*a^4*c^{11}*d^{14}*e^2 - 57344*a^5*c^{10}*d^{12}*e^4 - 36864*a^6*c^9*d^{10}*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^{10} + 663552*a^9*c^6*d^4*e^{12} + 331776*a^{10}*c^5*d^2*e^{14}) / (256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (x*((25*a^3*e^6*(-a^7*c)^{(1/2)} - 9*c^3*d^6*(-a^7*c)^{(1/2)} + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^7*c)^{(1/2)}) / (256*(a^{11}*e^8 + a^7*c^4*d^8 + 4*a^{10}*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4))^{(1/2)} * (65536*a^{13}*c^4*e^{17} - 65536*a^6*c^{11}*d^{14}*e^3 - 327680*a^7*c^{10}*d^{12}*e^5 - 589824*a^8*c^9*d^{10}*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^{10}*c^7*d^6*e^{11} + 589824*a^{11}*c^6*d^4*e^{13} + 327680*a^{12}*c^5*d^2*e^{15}) / (128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4))) * ((25*a^3*e^6*(-a^7*c)^{(1/2)} - 9*c^3*d^6*(-a^7*c)^{(1/2)} + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^7*c)^{(1/2)}) / (256*(a^{11}*e^8 + a^7*c^4*d^8 + 4*a^{10}*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4))^{(1/2)} - (x*(1152*a^2*c^{11}*d^{13}*e^2 - 49024*a^8*c^5*d*e^{14} + 7936*a^3*c^{10}*d^{11}*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^{10} - 110848*
\end{aligned}$$

$$\begin{aligned}
& a^7 c^6 d^3 e^{12}) / (128 (a^8 e^8 + a^4 c^4 d^8 + 4 a^7 c^3 d^2 e^6 + 4 a^5 c^3 d^6 e^2 + 6 a^6 c^2 d^4 e^4)) * ((25 a^3 e^6 (-a^7 c)^{1/2} - 9 c^3 d^6 (-a^7 c)^{1/2} + 6 a^4 c^3 d^5 e + 44 a^5 c^2 d^3 e^3 + 70 a^6 c^2 d^4 e^2 - 41 a^6 c^2 d^4 e^2 (-a^7 c)^{1/2} - 39 a^2 c^2 d^2 e^4 (-a^7 c)^{1/2}) / (256 (a^{11} e^8 + a^7 c^4 d^8 + 4 a^{10} c^2 d^2 e^6 + 4 a^8 c^3 d^6 e^2 + 6 a^9 c^2 d^4 e^4)))^{1/2} - (720 a^2 c^10 d^11 e^3 + 20432 a^6 c^5 d^5 e^{13} + 4880 a^2 c^9 d^9 e^5 + 12320 a^3 c^8 d^7 e^7 + 21024 a^4 c^7 d^5 e^9 + 33296 a^5 c^6 d^3 e^{11}) / (256 (a^8 e^8 + a^4 c^4 d^8 + 4 a^7 c^3 d^2 e^6 + 4 a^5 c^3 d^6 e^2 + 6 a^6 c^2 d^4 e^4))) * ((25 a^3 e^6 (-a^7 c)^{1/2} - 9 c^3 d^6 (-a^7 c)^{1/2} + 6 a^4 c^3 d^5 e + 44 a^5 c^2 d^3 e^3 + 70 a^6 c^2 d^4 e^2 - 41 a^6 c^2 d^4 e^2 (-a^7 c)^{1/2} - 39 a^2 c^2 d^2 e^4 (-a^7 c)^{1/2}) / (256 (a^{11} e^8 + a^7 c^4 d^8 + 4 a^{10} c^2 d^2 e^6 + 4 a^8 c^3 d^6 e^2 + 6 a^9 c^2 d^4 e^4)))^{1/2} - (x * (1425 a^4 c^5 e^{13} + 81 c^9 d^8 e^5 + 612 a^2 c^8 d^6 e^7 + 1894 a^2 c^7 d^4 e^9 + 2532 a^3 c^6 d^2 e^{11})) / (128 (a^8 e^8 + a^4 c^4 d^8 + 4 a^7 c^3 d^2 e^6 + 4 a^5 c^3 d^6 e^2 + 6 a^6 c^2 d^4 e^4))) * ((25 a^3 e^6 (-a^7 c)^{1/2} - 9 c^3 d^6 (-a^7 c)^{1/2} + 6 a^4 c^3 d^5 e + 44 a^5 c^2 d^3 e^3 + 70 a^6 c^2 d^4 e^2 - 41 a^6 c^2 d^4 e^2 (-a^7 c)^{1/2} - 39 a^2 c^2 d^2 e^4 (-a^7 c)^{1/2}) / (256 (a^{11} e^8 + a^7 c^4 d^8 + 4 a^{10} c^2 d^2 e^6 + 4 a^8 c^3 d^6 e^2 + 6 a^9 c^2 d^4 e^4)))^{1/2} + (((((65536 a^{11} c^4 e^{16} - 12288 a^4 c^{11} d^{14} e^2 - 57344 a^5 c^{10} d^{12} e^4 - 36864 a^6 c^9 d^{10} e^6 + 245760 a^7 c^8 d^8 e^8 + 634880 a^8 c^7 d^6 e^{10} + 663552 a^9 c^6 d^4 e^{12} + 331776 a^{10} c^5 d^2 e^{14}) / (256 (a^8 e^8 + a^4 c^4 d^8 + 4 a^7 c^3 d^2 e^6 + 4 a^5 c^3 d^6 e^2 + 6 a^6 c^2 d^4 e^4)) + (x * ((25 a^3 e^6 (-a^7 c)^{1/2} - 9 c^3 d^6 (-a^7 c)^{1/2} + 6 a^4 c^3 d^5 e + 44 a^5 c^2 d^3 e^3 + 70 a^6 c^2 d^4 e^2 - 41 a^6 c^2 d^4 e^2 (-a^7 c)^{1/2} - 39 a^2 c^2 d^2 e^4 (-a^7 c)^{1/2}) / (256 (a^{11} e^8 + a^7 c^4 d^8 + 4 a^{10} c^2 d^2 e^6 + 4 a^8 c^3 d^6 e^2 + 6 a^9 c^2 d^4 e^4)))^{1/2} * (65536 a^{13} c^4 e^{17} - 65536 a^6 c^{11} d^{14} e^3 - 327680 a^7 c^{10} d^{12} e^5 - 589824 a^8 c^9 d^{10} e^7 - 327680 a^9 c^8 d^8 e^9 + 327680 a^{10} c^7 d^6 e^{11} + 589824 a^{11} c^6 d^4 e^{13} + 327680 a^{12} c^5 d^2 e^{15})) / (128 (a^8 e^8 + a^4 c^4 d^8 + 4 a^7 c^3 d^2 e^6 + 4 a^5 c^3 d^6 e^2 + 6 a^6 c^2 d^4 e^4))) * ((25 a^3 e^6 (-a^7 c)^{1/2} - 9 c^3 d^6 (-a^7 c)^{1/2} + 6 a^4 c^3 d^5 e + 44 a^5 c^2 d^3 e^3 + 70 a^6 c^2 d^4 e^2 - 41 a^6 c^2 d^4 e^2 (-a^7 c)^{1/2} - 39 a^2 c^2 d^2 e^4 (-a^7 c)^{1/2}) / (256 (a^{11} e^8 + a^7 c^4 d^8 + 4 a^{10} c^2 d^2 e^6 + 4 a^8 c^3 d^6 e^2 + 6 a^9 c^2 d^4 e^4)))^{1/2} + (x * (1152 a^2 c^{11} d^{13} e^2 - 49024 a^8 c^5 d^5 e^{14} + 7936 a^3 c^{10} d^{11} e^4 + 20352 a^4 c^9 d^9 e^6 + 8704 a^5 c^8 d^7 e^8 - 66688 a^6 c^7 d^5 e^{10} - 110848 a^7 c^6 d^3 e^{12})) / (128 (a^8 e^8 + a^4 c^4 d^8 + 4 a^7 c^3 d^2 e^6 + 4 a^5 c^3 d^6 e^2 + 6 a^6 c^2 d^4 e^4))) * ((25 a^3 e^6 (-a^7 c)^{1/2} - 9 c^3 d^6 (-a^7 c)^{1/2} + 6 a^4 c^3 d^5 e + 44 a^5 c^2 d^3 e^3 + 70 a^6 c^2 d^4 e^2 - 41 a^6 c^2 d^4 e^2 (-a^7 c)^{1/2} - 39 a^2 c^2 d^2 e^4 (-a^7 c)^{1/2}) / (256 (a^{11} e^8 + a^7 c^4 d^8 + 4 a^{10} c^2 d^2 e^6 + 4 a^8 c^3 d^6 e^2 + 6 a^9 c^2 d^4 e^4)))^{1/2} - (720 a^2 c^{10} d^{11} e^3 + 20432 a^6 c^5 d^5 e^{13} + 4880 a^2 c^9 d^9 e^5 + 12320 a^3 c^8 d^7 e^7 + 21024 a^4 c^7 d^5 e^9 + 33296 a^5 c^6 d^3 e^{11}) / (256 (a^8 e^8 + a^4 c^4 d^8 + 4 a^7 c^3 d^2 e^6 + 4 a^5 c^3 d^6 e^2 + 6 a^6 c^2 d^4 e^4))) * ((25 a^3 e^6 (-a^7 c)^{1/2} - 9 c^3 d^6 (-a^7 c)^{1/2} + 6 a^4 c^3 d^5 e + 44 a^5 c^2 d^3 e^3 + 70 a^6 c^2 d^4 e^2 - 41 a^6 c^2 d^4 e^2 (-a^7 c)^{1/2} - 39 a^2 c^2 d^2 e^4 (-a^7 c)^{1/2}) / (256 (a^{11} e^8 + a^7 c^4 d^8 + 4 a^{10} c^2 d^2 e^6 + 4 a^8 c^3 d^6 e^2 + 6 a^9 c^2 d^4 e^4)))^{1/2} + (x * (1425 a^4 c^5 e^{13} + 81 c^9 d^8 e^5 + 612 a^2 c^8 d^6 e^7 + 1894 a^2 c^7 d^4 e^9 + 2532 a^3 c^6 d^2 e^{11})) / (128 (a^8 e^8 + a^4 c^4 d^8 + 4 a^7 c^3 d^2 e^6 + 4 a^5 c^3 d^6 e^2 + 6 a^6 c^2 d^4 e^4))) * ((25 a^3 e^6 (-a^7 c)^{1/2} - 9 c^3 d^6 (-a^7 c)^{1/2} + 6 a^4 c^3 d^5 e + 44 a^5 c^2 d^3 e^3 + 70 a^6 c^2 d^4 e^2 - 41 a^6 c^2 d^4 e^2 (-a^7 c)^{1/2} - 39 a^2 c^2 d^2 e^4 (-a^7 c)^{1/2}) / (256 (a^{11} e^8 + a^7 c^4 d^8 + 4 a^{10} c^2 d^2 e^6 + 4 a^8 c^3 d^6 e^2 + 6 a^9 c^2 d^4 e^4)))^{1/2} * 2i + (\operatorname{atan}(-((((((45 a^5 c^{10} d^{11} e^3) / 16 + (1277 a^6 c^5 d^5 e^{13}) / 16 + (305 a^2 c^9 d^9 e^5) / 16
\end{aligned}$$



$$\begin{aligned}
& + (385*a^3*c^8*d^7*e^7)/8 + (657*a^4*c^7*d^5*e^9)/8 + (2081*a^5*c^6*d^3*e^11)/16) / (2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (((((256*a^11*c^4*e^16 - 48*a^4*c^11*d^14*e^2 - 224*a^5*c^10*d^12*e^4 - 144*a^6*c^9*d^10*e^6 + 960*a^7*c^8*d^8*e^8 + 2480*a^8*c^7*d^6*e^10 + 2592*a^9*c^6*d^4*e^12 + 1296*a^10*c^5*d^2*e^14) / (2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (x*(-d*e^7)^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15)) / (512*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) * (a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4))) * (-d*e^7)^(1/2)) / (2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) - (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12)) / (256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4))) * (-d*e^7)^(1/2)) / (2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) + (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11)) / (256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4))) * (-d*e^7)^(1/2)*i) / (c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2) - ((((((45*a*c^10*d^11*e^3)/16 + (1277*a^6*c^5*d*e^13)/16 + (305*a^2*c^9*d^9*e^5)/16 + (385*a^3*c^8*d^7*e^7)/8 + (657*a^4*c^7*d^5*e^9)/8 + (2081*a^5*c^6*d^3*e^11)/16) / (2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (((((256*a^11*c^4*e^16 - 48*a^4*c^11*d^14*e^2 - 224*a^5*c^10*d^12*e^4 - 144*a^6*c^9*d^10*e^6 + 960*a^7*c^8*d^8*e^8 + 2480*a^8*c^7*d^6*e^10 + 2592*a^9*c^6*d^4*e^12 + 1296*a^10*c^5*d^2*e^14) / (2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (x*(-d*e^7)^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15)) / (512*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) * (a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4))) * (-d*e^7)^(1/2)) / (2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) + (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12)) / (256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4))) * (-d*e^7)^(1/2)) / (2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) - (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11)) / (256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4))) * (-d*e^7)^(1/2)*i) / (c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2) / ((((((45*a*c^10*d^11*e^3)/16 + (1277*a^6*c^5*d*e^13)/16 + (305*a^2*c^9*d^9*e^5)/16 + (385*a^3*c^8*d^7*e^7)/8 + (657*a^4*c^7*d^5*e^9)/8 + (2081*a^5*c^6*d^3*e^11)/16) / (2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (((((256*a^11*c^4*e^16 - 48*a^4*c^11*d^14*e^2 - 224*a^5*c^10*d^12*e^4 - 144*a^6*c^9*d^10*e^6 + 960*a^7*c^8*d^8*e^8 + 2480*a^8*c^7*d^6*e^10 + 2592*a^9*c^6*d^4*e^12 + 1296*a^10*c^5*d^2*e^14) / (2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (x*(-d*e^7)^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15)) / (512*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) * (a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4))) * (-d*e^7)^(1/2)) / (2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) - (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12)) / (256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4))) * (-d*e^7)^(1/2)) / (2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) + (x*(1425*a^4*c^5*
\end{aligned}$$

$$\begin{aligned}
& e^{13} + 81c^9d^8e^5 + 612a^2c^8d^6e^7 + 1894a^2c^7d^4e^9 + 2532a^3 \\
& *c^6d^2e^{11}) / (256(a^8e^8 + a^4c^4d^8 + 4a^7c^3d^2e^6 + 4a^5c^3d \\
& ^6e^2 + 6a^6c^2d^4e^4)) * (-d^7)^{(1/2)} / (c^2d^5 + a^2d^4e^4 + 2a^2c^3d \\
& ^3e^2) - (((125a^2c^5e^{12}) / 128 + (81c^7d^4e^8) / 128 + (135a^2c^6d^2e \\
& ^{10}) / 64) / (a^8e^8 + a^4c^4d^8 + 4a^7c^3d^2e^6 + 4a^5c^3d^6e^2 + 6 \\
& a^6c^2d^4e^4) + ((((((45a^2c^10d^11e^3) / 16 + (1277a^6c^5d^5e^{13}) / 16 \\
& + (305a^2c^9d^9e^5) / 16 + (385a^3c^8d^7e^7) / 8 + (657a^4c^7d^5e^9 \\
& ) / 8 + (2081a^5c^6d^3e^{11}) / 16) / (2(a^8e^8 + a^4c^4d^8 + 4a^7c^3d^2e^6 \\
& + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) - ((((((256a^{11}c^4e^{16} - 48 \\
& a^4c^{11}d^{14}e^2 - 224a^5c^{10}d^{12}e^4 - 144a^6c^9d^{10}e^6 + 960a^7c^8d^8e^8 \\
& + 2480a^8c^7d^6e^{10} + 2592a^9c^6d^4e^{12} + 1296a^{10}c^5 \\
& *d^2e^{14}) / (2(a^8e^8 + a^4c^4d^8 + 4a^7c^3d^2e^6 + 4a^5c^3d^6e^2 \\
& + 6a^6c^2d^4e^4)) + (x * (-d^7)^{(1/2)} * (65536a^{13}c^4e^{17} - 65536a^6c \\
& ^{11}d^{14}e^3 - 327680a^7c^{10}d^{12}e^5 - 589824a^8c^9d^{10}e^7 - 327680 \\
& *a^9c^8d^8e^9 + 327680a^{10}c^7d^6e^{11} + 589824a^{11}c^6d^4e^{13} + 32 \\
& 7680a^{12}c^5d^2e^{15})) / (512(c^2d^5 + a^2d^4e^4 + 2a^2c^3d^3e^2)) * (a^8e^8 \\
& + a^4c^4d^8 + 4a^7c^3d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) \\
& ) * (-d^7)^{(1/2)} / (2(c^2d^5 + a^2d^4e^4 + 2a^2c^3d^3e^2)) + (x * (1152a^2c \\
& ^{11}d^{13}e^2 - 49024a^8c^5d^5e^{14} + 7936a^3c^{10}d^{11}e^4 + 20352a^4c^9d^9e^6 \\
& + 8704a^5c^8d^7e^8 - 66688a^6c^7d^5e^{10} - 110848a^7c^6 \\
& *d^3e^{12})) / (256(a^8e^8 + a^4c^4d^8 + 4a^7c^3d^2e^6 + 4a^5c^3d^6e^2 \\
& + 6a^6c^2d^4e^4)) * (-d^7)^{(1/2)} / (2(c^2d^5 + a^2d^4e^4 + 2a^2c^3d^3e^2)) \\
& )) * (-d^7)^{(1/2)} / (2(c^2d^5 + a^2d^4e^4 + 2a^2c^3d^3e^2)) - (x * (1 \\
& 425a^4c^5e^{13} + 81c^9d^8e^5 + 612a^2c^8d^6e^7 + 1894a^2c^7d^4e^9 + 2532a^3 \\
& *c^6d^2e^{11})) / (256(a^8e^8 + a^4c^4d^8 + 4a^7c^3d^2e^6 + 4a^5c^3d^6e^2 \\
& + 6a^6c^2d^4e^4)) * (-d^7)^{(1/2)} / (c^2d^5 + a^2d^4e^4 + 2a^2c^3d^3e^2)) \\
& ) * (-d^7)^{(1/2)} * i) / (c^2d^5 + a^2d^4e^4 + 2a^2c^3d^3e^2)
\end{aligned}$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x\*\*2+d)/(c\*x\*\*4+a)\*\*2,x)

[Out] Timed out

$$3.130 \quad \int \frac{1}{(d+ex^2)^2(a+cx^4)^2} dx$$

**Optimal.** Leaf size=864

$$\frac{xe^4}{2d(cd^2 + ae^2)^2(ex^2 + d)} + \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)e^{7/2}}{2d^{3/2}(cd^2 + ae^2)^2} + \frac{4c\sqrt{d}\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)e^{7/2}}{(cd^2 + ae^2)^3} - \frac{c^{3/4}(3cd^2 - 4\sqrt{a}\sqrt{c}ed - ae^2)\tan^{-1}\left(1 - \frac{\sqrt{ex}}{\sqrt{d}}\right)}{2\sqrt{2}a^{3/4}(cd^2 + ae^2)^3}$$

**Rubi [A]** time = 0.91, antiderivative size = 864, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$ , Rules used = {1239, 199, 205, 1179, 1168, 1162, 617, 204, 1165, 628}

Antiderivative was successfully verified.

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x^2)^2\*(a + c\*x^4)^2), x]

[Out] (e^4\*x)/(2\*d\*(c\*d^2 + a\*e^2)^2\*(d + e\*x^2)) + (c\*x\*(c\*d^2 - a\*e^2 - 2\*c\*d\*e\*x^2))/(4\*a\*(c\*d^2 + a\*e^2)^2\*(a + c\*x^4)) + (4\*c\*Sqrt[d]\*e^(7/2)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(c\*d^2 + a\*e^2)^3 + (e^(7/2)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(2\*d^(3/2)\*(c\*d^2 + a\*e^2)^2) - (c^(3/4)\*e^2\*(3\*c\*d^2 - 4\*Sqrt[a]\*Sqrt[c]\*d\*e - a\*e^2)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)])/(2\*Sqrt[2]\*a^(3/4)\*(c\*d^2 + a\*e^2)^3) - (c^(3/4)\*(3\*c\*d^2 - 2\*Sqrt[a]\*Sqrt[c]\*d\*e - 3\*a\*e^2)\*ArcTan[1 - (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)])/(8\*Sqrt[2]\*a^(7/4)\*(c\*d^2 + a\*e^2)^2) + (c^(3/4)\*e^2\*(3\*c\*d^2 - 4\*Sqrt[a]\*Sqrt[c]\*d\*e - a\*e^2)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)])/(2\*Sqrt[2]\*a^(3/4)\*(c\*d^2 + a\*e^2)^3) + (c^(3/4)\*(3\*c\*d^2 - 2\*Sqrt[a]\*Sqrt[c]\*d\*e - 3\*a\*e^2)\*ArcTan[1 + (Sqrt[2]\*c^(1/4)\*x)/a^(1/4)])/(8\*Sqrt[2]\*a^(7/4)\*(c\*d^2 + a\*e^2)^2) - (c^(3/4)\*e^2\*(3\*c\*d^2 + 4\*Sqrt[a]\*Sqrt[c]\*d\*e - a\*e^2)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/(4\*Sqrt[2]\*a^(3/4)\*(c\*d^2 + a\*e^2)^3) - (c^(3/4)\*(3\*c\*d^2 + 2\*Sqrt[a]\*Sqrt[c]\*d\*e - 3\*a\*e^2)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/(16\*Sqrt[2]\*a^(7/4)\*(c\*d^2 + a\*e^2)^2) + (c^(3/4)\*e^2\*(3\*c\*d^2 + 4\*Sqrt[a]\*Sqrt[c]\*d\*e - a\*e^2)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/(4\*Sqrt[2]\*a^(3/4)\*(c\*d^2 + a\*e^2)^3) + (c^(3/4)\*(3\*c\*d^2 + 2\*Sqrt[a]\*Sqrt[c]\*d\*e - 3\*a\*e^2)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2])/(16\*Sqrt[2]\*a^(7/4)\*(c\*d^2 + a\*e^2)^2)

**Rule 199**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2\*p] || (n == 2 && IntegerQ[4\*p]) || (n == 2 && IntegerQ[3\*p]) || Denominator[p + 1/n] < Denominator[p])

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 205**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 617**

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]
```

### Rule 1179

```
Int[((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := -Simp[(x*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

### Rule 1239

```
Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e, p, q}, x] && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0])
```

### Rubi steps

$$\int \frac{1}{(d + ex^2)^2 (a + cx^4)^2} dx = \int \left( \frac{e^4}{(cd^2 + ae^2)^2 (d + ex^2)^2} + \frac{4cde^4}{(cd^2 + ae^2)^3 (d + ex^2)} + \frac{c(cd^2 - ae^2 - 2cdex^2)}{(cd^2 + ae^2)^2 (a + cx^4)^2} \right) dx$$

$$= -\frac{(ce^2) \int \frac{-3cd^2 + ae^2 + 4cdex^2}{a + cx^4} dx}{(cd^2 + ae^2)^3} + \frac{(4cde^4) \int \frac{1}{d + ex^2} dx}{(cd^2 + ae^2)^3} + \frac{c \int \frac{cd^2 - ae^2 - 2cdex^2}{(a + cx^4)^2} dx}{(cd^2 + ae^2)^2} + \frac{e^4 \int \frac{1}{(d + ex^2)^2} dx}{(cd^2 + ae^2)^2}$$

$$= \frac{e^4 x}{2d (cd^2 + ae^2)^2 (d + ex^2)} + \frac{cx (cd^2 - ae^2 - 2cdex^2)}{4a (cd^2 + ae^2)^2 (a + cx^4)} + \frac{4c\sqrt{d} e^{7/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{(cd^2 + ae^2)^3}$$

$$= \frac{e^4 x}{2d (cd^2 + ae^2)^2 (d + ex^2)} + \frac{cx (cd^2 - ae^2 - 2cdex^2)}{4a (cd^2 + ae^2)^2 (a + cx^4)} + \frac{4c\sqrt{d} e^{7/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{(cd^2 + ae^2)^3}$$

$$= \frac{e^4 x}{2d (cd^2 + ae^2)^2 (d + ex^2)} + \frac{cx (cd^2 - ae^2 - 2cdex^2)}{4a (cd^2 + ae^2)^2 (a + cx^4)} + \frac{4c\sqrt{d} e^{7/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{(cd^2 + ae^2)^3}$$

$$= \frac{e^4 x}{2d (cd^2 + ae^2)^2 (d + ex^2)} + \frac{cx (cd^2 - ae^2 - 2cdex^2)}{4a (cd^2 + ae^2)^2 (a + cx^4)} + \frac{4c\sqrt{d} e^{7/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{(cd^2 + ae^2)^3}$$

**Mathematica [A]** time = 0.58, size = 540, normalized size = 0.62

$$\frac{\sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \left(16 c^2 d^2 + a e^2\right) x + \left(8 c^2 \left(c d^2 + a e^2\right) x \left(-\left(a e^2 + c d \left(d - 2 e x^2\right)\right)\right) / \left(a \left(a + c x^4\right)\right) + \left(16 e^{7/2}\right) \left(9 c d^2 + a e^2\right) \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] / d^{3/2} + \left(2 \sqrt{2}\right) c^{3/4} \left(-3 c^2 d^4 + 2 \sqrt{2} \sqrt{a} c^{3/2} d^3 e - 12 a c d^2 e^2 + 18 a^{3/2} \sqrt{c} d e^3 + 7 a^2 e^4\right) \operatorname{ArcTan}\left[1 - \left(\sqrt{2} c^{1/4} x\right) / a^{1/4}\right] / a^{7/4} - \left(2 \sqrt{2}\right) c^{3/4} \left(-3 c^2 d^4 + 2 \sqrt{2} \sqrt{a} c^{3/2} d^3 e - 12 a c d^2 e^2 + 18 a^{3/2} \sqrt{c} d e^3 + 7 a^2 e^4\right) \operatorname{ArcTan}\left[1 + \left(\sqrt{2} c^{1/4} x\right) / a^{1/4}\right] / a^{7/4} - \left(\sqrt{2}\right) c^{3/4} \left(3 c^2 d^4 + 2 \sqrt{2} \sqrt{a} c^{3/2} d^3 e + 12 a c d^2 e^2 + 18 a^{3/2} \sqrt{c} d e^3 - 7 a^2 e^4\right) \operatorname{Log}\left[\sqrt{a} - \sqrt{2} a^{1/4} c^{1/4} x + \sqrt{c} x^2\right] / a^{7/4} + \left(\sqrt{2}\right) c^{3/4} \left(3 c^2 d^4 + 2 \sqrt{2} \sqrt{a} c^{3/2} d^3 e + 12 a c d^2 e^2 + 18 a^{3/2} \sqrt{c} d e^3 - 7 a^2 e^4\right) \operatorname{Log}\left[\sqrt{a} + \sqrt{2} a^{1/4} c^{1/4} x + \sqrt{c} x^2\right] / a^{7/4}}{\left(32 \left(c d^2 + a e^2\right)^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*(a + c*x^4)^2), x]
[Out] ((16*e^4*(c*d^2 + a*e^2)*x)/(d*(d + e*x^2)) + (8*c*(c*d^2 + a*e^2)*x*(-(a*e^2 + c*d*(d - 2*e*x^2)))/(a*(a + c*x^4)) + (16*e^(7/2)*(9*c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + (2*Sqrt[2]*c^(3/4)*(-3*c^2*d^4 + 2*Sqrt[a]*c^(3/2)*d^3*e - 12*a*c*d^2*e^2 + 18*a^(3/2)*Sqrt[c]*d*e^3 + 7*a^2*e^4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/a^(7/4) - (2*Sqrt[2]*c^(3/4)*(-3*c^2*d^4 + 2*Sqrt[a]*c^(3/2)*d^3*e - 12*a*c*d^2*e^2 + 18*a^(3/2)*Sqrt[c]*d*e^3 + 7*a^2*e^4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/a^(7/4) - (Sqrt[2]*c^(3/4)*(3*c^2*d^4 + 2*Sqrt[a]*c^(3/2)*d^3*e + 12*a*c*d^2*e^2 + 18*a^(3/2)*Sqrt[c]*d*e^3 - 7*a^2*e^4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) + (Sqrt[2]*c^(3/4)*(3*c^2*d^4 + 2*Sqrt[a]*c^(3/2)*d^3*e + 12*a*c*d^2*e^2 + 18*a^(3/2)*Sqrt[c]*d*e^3 - 7*a^2*e^4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4))/(32*(c*d^2 + a*e^2)^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex^2)^2 (a + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e\*x^2)^2\*(a + c\*x^4)^2),x]

[Out] IntegrateAlgebraic[1/((d + e\*x^2)^2\*(a + c\*x^4)^2), x]

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)^2/(c\*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out

**giac** [A] time = 0.25, size = 855, normalized size = 0.99

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)^2/(c\*x^4+a)^2,x, algorithm="giac")

[Out]  $\frac{1}{2} \cdot (9 \cdot c \cdot d^2 \cdot e^4 + a \cdot e^6) \cdot \arctan\left(\frac{x \cdot e^{1/2}}{\sqrt{d}}\right) \cdot e^{-1/2} / \left( (c^3 \cdot d^7 + 3 \cdot a \cdot c^2 \cdot d^5 \cdot e^2 + 3 \cdot a^2 \cdot c \cdot d^3 \cdot e^4 + a^3 \cdot d \cdot e^6) \cdot \sqrt{d} \right) + \frac{1}{8} \cdot (3 \cdot (a \cdot c^3)^{1/4} \cdot c^3 \cdot d^4 + 12 \cdot (a \cdot c^3)^{1/4} \cdot a \cdot c^2 \cdot d^2 \cdot e^2 - 2 \cdot (a \cdot c^3)^{3/4} \cdot c \cdot d^3 \cdot e - 7 \cdot (a \cdot c^3)^{1/4} \cdot a^2 \cdot c \cdot e^4 - 18 \cdot (a \cdot c^3)^{3/4} \cdot a \cdot d \cdot e^3) \cdot \arctan\left(\frac{1}{2} \cdot \sqrt{2} \cdot (2 \cdot x + \sqrt{2}) \cdot (a/c)^{1/4}\right) / (a/c)^{1/4} / \left( \sqrt{2} \cdot a^2 \cdot c^4 \cdot d^6 + 3 \cdot \sqrt{2} \cdot a^3 \cdot c^3 \cdot d^4 \cdot e^2 + 3 \cdot \sqrt{2} \cdot a^4 \cdot c^2 \cdot d^2 \cdot e^4 + \sqrt{2} \cdot a^5 \cdot c \cdot e^6 \right) + \frac{1}{8} \cdot (3 \cdot (a \cdot c^3)^{1/4} \cdot c^3 \cdot d^4 + 12 \cdot (a \cdot c^3)^{1/4} \cdot a \cdot c^2 \cdot d^2 \cdot e^2 - 2 \cdot (a \cdot c^3)^{3/4} \cdot c \cdot d^3 \cdot e - 7 \cdot (a \cdot c^3)^{1/4} \cdot a^2 \cdot c \cdot e^4 - 18 \cdot (a \cdot c^3)^{3/4} \cdot a \cdot d \cdot e^3) \cdot \arctan\left(\frac{1}{2} \cdot \sqrt{2} \cdot (2 \cdot x - \sqrt{2}) \cdot (a/c)^{1/4}\right) / (a/c)^{1/4} / \left( \sqrt{2} \cdot a^2 \cdot c^4 \cdot d^6 + 3 \cdot \sqrt{2} \cdot a^3 \cdot c^3 \cdot d^4 \cdot e^2 + 3 \cdot \sqrt{2} \cdot a^4 \cdot c^2 \cdot d^2 \cdot e^4 + \sqrt{2} \cdot a^5 \cdot c \cdot e^6 \right) + \frac{1}{32} \cdot (3 \cdot \sqrt{2} \cdot (a \cdot c^3)^{1/4} \cdot c^3 \cdot d^4 + 12 \cdot \sqrt{2} \cdot (a \cdot c^3)^{1/4} \cdot a \cdot c^2 \cdot d^2 \cdot e^2 + 2 \cdot \sqrt{2} \cdot (a \cdot c^3)^{3/4} \cdot c \cdot d^3 \cdot e - 7 \cdot \sqrt{2} \cdot (a \cdot c^3)^{1/4} \cdot a^2 \cdot c \cdot e^4 + 18 \cdot \sqrt{2} \cdot (a \cdot c^3)^{3/4} \cdot a \cdot d \cdot e^3) \cdot \log(x^2 + \sqrt{2} \cdot x \cdot (a/c)^{1/4} + \sqrt{a/c}) / (a^2 \cdot c^4 \cdot d^6 + 3 \cdot a^3 \cdot c^3 \cdot d^4 \cdot e^2 + 3 \cdot a^4 \cdot c^2 \cdot d^2 \cdot e^4 + a^5 \cdot c \cdot e^6) - \frac{1}{32} \cdot (3 \cdot \sqrt{2} \cdot (a \cdot c^3)^{1/4} \cdot c^3 \cdot d^4 + 12 \cdot \sqrt{2} \cdot (a \cdot c^3)^{1/4} \cdot a \cdot c^2 \cdot d^2 \cdot e^2 + 2 \cdot \sqrt{2} \cdot (a \cdot c^3)^{3/4} \cdot c \cdot d^3 \cdot e - 7 \cdot \sqrt{2} \cdot (a \cdot c^3)^{1/4} \cdot a^2 \cdot c \cdot e^4 + 18 \cdot \sqrt{2} \cdot (a \cdot c^3)^{3/4} \cdot a \cdot d \cdot e^3) \cdot \log(x^2 - \sqrt{2} \cdot x \cdot (a/c)^{1/4} + \sqrt{a/c}) / (a^2 \cdot c^4 \cdot d^6 + 3 \cdot a^3 \cdot c^3 \cdot d^4 \cdot e^2 + 3 \cdot a^4 \cdot c^2 \cdot d^2 \cdot e^4 + a^5 \cdot c \cdot e^6) - \frac{1}{4} \cdot (2 \cdot c^2 \cdot d^2 \cdot x^5 \cdot e^2 + c^2 \cdot d^3 \cdot x^3 \cdot e - 2 \cdot a \cdot c \cdot x^5 \cdot e^4 - c^2 \cdot d^4 \cdot x + a \cdot c \cdot d \cdot x^3 \cdot e^3 + a \cdot c \cdot d^2 \cdot x \cdot e^2 - 2 \cdot a^2 \cdot x \cdot e^4) / \left( (a \cdot c^2 \cdot d^5 + 2 \cdot a^2 \cdot c \cdot d^3 \cdot e^2 + a^3 \cdot d \cdot e^4) \cdot (c \cdot x^6 \cdot e + c \cdot d \cdot x^4 + a \cdot x^2 \cdot e + a \cdot d) \right)$

**maple** [A] time = 0.02, size = 1169, normalized size = 1.35

result too large to display

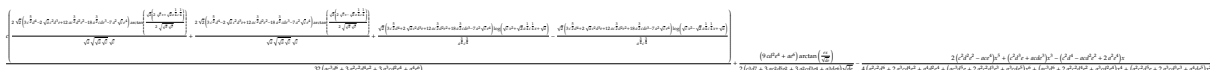
Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x^2+d)^2/(c\*x^4+a)^2,x)

[Out]  $\frac{1}{2} \cdot e^6 / (a \cdot e^2 + c \cdot d^2)^3 \cdot d \cdot x / (e \cdot x^2 + d) \cdot a + \frac{1}{2} \cdot e^4 / (a \cdot e^2 + c \cdot d^2)^3 \cdot d \cdot x / (e \cdot x^2 + d) \cdot c + \frac{1}{2} \cdot e^6 / (a \cdot e^2 + c \cdot d^2)^3 \cdot d / (d \cdot e)^{1/2} \cdot \arctan\left(\frac{1}{(d \cdot e)^{1/2}} \cdot e \cdot x\right) \cdot a + \frac{9}{2} \cdot e^4 / (a \cdot e^2 + c \cdot d^2)^3 \cdot d / (d \cdot e)^{1/2} \cdot \arctan\left(\frac{1}{(d \cdot e)^{1/2}} \cdot e \cdot x\right) \cdot c - \frac{1}{2} \cdot c^2 / (a \cdot e^2 + c \cdot d^2)^3 \cdot (c \cdot x^4 + a) \cdot d \cdot e^3 \cdot x^3 - \frac{1}{2} \cdot c^3 / (a \cdot e^2 + c \cdot d^2)^3 \cdot (c \cdot x^4 + a) \cdot d^3 \cdot e / a \cdot x^3 - \frac{1}{4} \cdot c / (a \cdot e^2 + c \cdot d^2)^3 \cdot (c \cdot x^4 + a) \cdot a \cdot x \cdot e^4 + \frac{1}{4} \cdot c^3 / (a \cdot e^2 + c \cdot d^2)^3 \cdot (c \cdot x^4 + a) / a \cdot x \cdot d^4 - \frac{7}{16} \cdot c / (a \cdot e^2 + c \cdot d^2)^3 \cdot (a/c)^{1/4} \cdot 2^{1/2} \cdot \arctan\left(2^{1/2} / (a/c)^{1/4} \cdot x + 1\right) \cdot e^4 + \frac{3}{4} \cdot c^2 / (a \cdot e^2 + c \cdot d^2)^3 \cdot a \cdot (a/c)^{1/4} \cdot 2^{1/2} \cdot \arctan\left(2^{1/2} / (a/c)^{1/4} \cdot x + 1\right) \cdot d^2 \cdot e^2 + \frac{3}{16} \cdot c^3 / (a \cdot e^2 + c \cdot d^2)^3 \cdot a^2 \cdot (a/c)^{1/4} \cdot 2^{1/2} \cdot \arctan\left(2^{1/2} / (a/c)^{1/4} \cdot x + 1\right) \cdot d^4 - \frac{7}{16} \cdot c / (a \cdot e^2 + c \cdot d^2)^3 \cdot (a/c)^{1/4} \cdot 2^{1/2} \cdot \arctan\left(2^{1/2} / (a/c)^{1/4} \cdot x - 1\right) \cdot e^4 + \frac{3}{4} \cdot c^2 / (a \cdot e^2 + c \cdot d^2)^3 \cdot a \cdot (a/c)^{1/4} \cdot 2^{1/2} \cdot \arctan\left(2^{1/2} / (a/c)^{1/4} \cdot x - 1\right) \cdot d^2 \cdot e^2 + \frac{3}{16} \cdot c^3 / (a \cdot e^2 + c \cdot d^2)^3 \cdot a^2 \cdot (a/c)^{1/4} \cdot 2^{1/2} \cdot \arctan\left(2^{1/2} / (a/c)^{1/4} \cdot x - 1\right) \cdot d^4 - \frac{7}{32} \cdot c / (a \cdot e^2 + c$

$$d^2)^3*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})))*e^4+3/8*c^2/(a*e^2+c*d^2)^3/a*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})))*d^2*e^2+3/32*c^3/(a*e^2+c*d^2)^3/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})))*d^4-9/16*c/(a*e^2+c*d^2)^3/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})))*d*e^3-1/16*c^2/(a*e^2+c*d^2)^3/a/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})))*d^3*e-9/8*c/(a*e^2+c*d^2)^3/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d*e^3-1/8*c^2/(a*e^2+c*d^2)^3/a/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^3*e-9/8*c/(a*e^2+c*d^2)^3/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d*e^3-1/8*c^2/(a*e^2+c*d^2)^3/a/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^3*e$$

**maxima** [A] time = 2.61, size = 732, normalized size = 0.85



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)^2/(c\*x^4+a)^2,x, algorithm="maxima")

[Out]  $\frac{1}{32}c*(2*\sqrt{2})*(3*c^{(5/2)}*d^4 - 2*\sqrt{a}*c^2*d^3*e + 12*a*c^{(3/2)}*d^2*e^2 - 18*a^{(3/2)}*c*d*e^3 - 7*a^2*\sqrt{c}*e^4)*\arctan(1/2*\sqrt{2}*(2*\sqrt{c}*x + \sqrt{2})*a^{(1/4)}*c^{(1/4)})/\sqrt{\sqrt{a}*\sqrt{c}})/(\sqrt{a}*\sqrt{\sqrt{a}*\sqrt{c}})*\sqrt{c} + 2*\sqrt{2}*(3*c^{(5/2)}*d^4 - 2*\sqrt{a}*c^2*d^3*e + 12*a*c^{(3/2)}*d^2*e^2 - 18*a^{(3/2)}*c*d*e^3 - 7*a^2*\sqrt{c}*e^4)*\arctan(1/2*\sqrt{2}*(2*\sqrt{c}*x - \sqrt{2})*a^{(1/4)}*c^{(1/4)})/\sqrt{\sqrt{a}*\sqrt{c}})/(\sqrt{a}*\sqrt{\sqrt{a}*\sqrt{c}})*\sqrt{c} + \sqrt{2}*(3*c^{(5/2)}*d^4 + 2*\sqrt{a}*c^2*d^3*e + 12*a*c^{(3/2)}*d^2*e^2 + 18*a^{(3/2)}*c*d*e^3 - 7*a^2*\sqrt{c}*e^4)*\log(\sqrt{c}*x^2 + \sqrt{2})*a^{(1/4)}*c^{(1/4)}*x + \sqrt{a})/(a^{(3/4)}*c^{(3/4)}) - \sqrt{2}*(3*c^{(5/2)}*d^4 + 2*\sqrt{a}*c^2*d^3*e + 12*a*c^{(3/2)}*d^2*e^2 + 18*a^{(3/2)}*c*d*e^3 - 7*a^2*\sqrt{c}*e^4)*\log(\sqrt{c}*x^2 - \sqrt{2})*a^{(1/4)}*c^{(1/4)}*x + \sqrt{a})/(a^{(3/4)}*c^{(3/4)})/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6) + 1/2*(9*c*d^2*e^4 + a*e^6)*\arctan(e*x/\sqrt{d*e})/((c^3*d^7 + 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d*e^6)*\sqrt{d*e}) - 1/4*(2*(c^2*d^2*e^2 - a*c*e^4)*x^5 + (c^2*d^3*e + a*c*d*e^3)*x^3 - (c^2*d^4 - a*c*d^2*e^2 + 2*a^2*e^4)*x)/(a^2*c^2*d^6 + 2*a^3*c*d^4*e^2 + a^4*d^2*e^4 + (a*c^3*d^5*e + 2*a^2*c^2*d^3*e^3 + a^3*c*d*e^5)*x^6 + (a*c^3*d^6 + 2*a^2*c^2*d^4*e^2 + a^3*c*d^2*e^4)*x^4 + (a^2*c^2*d^5*e + 2*a^3*c*d^3*e^3 + a^4*d*e^5)*x^2)$

**mupad** [B] time = 8.33, size = 28923, normalized size = 33.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + c\*x^4)^2\*(d + e\*x^2)^2),x)

[Out]  $\frac{(x*(2*a^2*e^4 + c^2*d^4 - a*c*d^2*e^2))/(4*a*d*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) - (c*e*x^3)/(4*a*(a*e^2 + c*d^2)) + (c*e^2*x^5*(a*e^2 - c*d^2))/(2*a*d*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))/(a*d + a*e*x^2 + c*d*x^4 + c*e*x^6) + \operatorname{atan}\left(\frac{(3584*a^{10}*c^5*e^{21} + 1152*a*c^{14}*d^{18}*e^3 + 13184*a^2*c^{13}*d^{16}*e^5 + 54912*a^3*c^{12}*d^{14}*e^7 + 296832*a^4*c^{11}*d^{12}*e^9 + 1282432*a^5*c^{10}*d^{10}*e^{11} + 769152*a^6*c^9*d^8*e^{13} - 1421440*a^7*c^8*d^6*e^{15} - 1254784*a^8*c^7*d^4*e^{17} - 89088*a^9*c^6*d^2*e^{19})}{(512*(a^4*c^8*d^{18} + a^{12}*d^2*e^{16} + 8*a^{11}*c*d^4*e^{14} + 8*a^5*c^7*d^{16}*e^2 + 28*a^6*c^6*d^{14}*e^4 + 56*a^7*c^5*d^{12}*e^6 + 70*a^8*c^4*d^{10}*e^8 + 56*a^9*c^3*d^8*e^{10} + 28*a^{10}*c^2*d^6*e^{12})} - ((65536*a^{15}*c^4*d*e^{24} - 24576*a^4*c^{15}*d^{23}*e^2 - 212992*a^5*c^{14}*d^{21}*e^4 - 352256*a^6*c^{13}*d^{19}*e^6 + 1966080*a^7*c^{12}*d^{17}*e^8 + 10960896*a^8*c^{11}*d^{15}*e^{10} + 25460736*a^9*c^{10}*d^{13}*e^{12} + 34750464*a^{10}*c^9$

$$\begin{aligned}
& *d^{11}e^{14} + 30081024a^{11}c^8d^9e^{16} + 16588800a^{12}c^7d^7e^{18} + 5554 \\
& 176a^{13}c^6d^5e^{20} + 991232a^{14}c^5d^3e^{22}) / (512(a^4c^8d^{18} + a^{12} \\
& *d^2e^{16} + 8a^{11}c*d^4e^{14} + 8a^5c^7d^{16}e^2 + 28a^6c^6d^{14}e^4 + \\
& 56a^7c^5d^{12}e^6 + 70a^8c^4d^{10}e^8 + 56a^9c^3d^8e^{10} + 28a^{10}c^2 \\
& *d^6e^{12})) - (x*(-(49a^4e^8*(-a^7c^3)^{(1/2)} + 9c^4d^8*(-a^7c^3)^{(1/2)} \\
& - 12a^4c^5d^7e + 252a^7c^2d^7e^7 - 156a^5c^4d^5e^3 - 404a^6c^3 \\
& *d^3e^5 + 68a*c^3d^6e^2*(-a^7c^3)^{(1/2)} - 492a^3c*d^2e^6*(-a^7c^3)^{(1/2)} \\
& + 30a^2c^2d^4e^4*(-a^7c^3)^{(1/2)}) / (256(a^{13}e^{12} + a^7c^6d^{12} \\
& + 6a^{12}c*d^2e^{10} + 6a^8c^5d^{10}e^2 + 15a^9c^4d^8e^4 + 20a^1 \\
& 0c^3d^6e^6 + 15a^{11}c^2d^4e^8)))^{(1/2)} * (65536a^6c^{15}d^{24}e^3 + 589 \\
& 824a^7c^{14}d^{22}e^5 + 2293760a^8c^{13}d^{20}e^7 + 4915200a^9c^{12}d^{18}e^9 \\
& + 5898240a^{10}c^{11}d^{16}e^{11} + 2752512a^{11}c^{10}d^{14}e^{13} - 2752512a^{12} \\
& *c^9d^{12}e^{15} - 5898240a^{13}c^8d^{10}e^{17} - 4915200a^{14}c^7d^8e^{19} - \\
& 2293760a^{15}c^6d^6e^{21} - 589824a^{16}c^5d^4e^{23} - 65536a^{17}c^4d^2e^{25}) \\
& ) / (128(a^4c^8d^{18} + a^{12}d^2e^{16} + 8a^{11}c*d^4e^{14} + 8a^5c^7d^{16}e^2 + 28 \\
& *a^6c^6d^{14}e^4 + 56a^7c^5d^{12}e^6 + 70a^8c^4d^{10}e^8 + 56a^9c^3d^8e^{10} \\
& + 28a^{10}c^2d^6e^{12})) * (-(49a^4e^8*(-a^7c^3)^{(1/2)} + 9c^4d^8*(-a^7c^3)^{(1/2)} \\
& - 12a^4c^5d^7e + 252a^7c^2d^7e^7 - 156a^5c^4d^5e^3 - 404a^6c^3d^3e^5 \\
& + 68a*c^3d^6e^2*(-a^7c^3)^{(1/2)} - 492a^3c*d^2e^6*(-a^7c^3)^{(1/2)} \\
& + 30a^2c^2d^4e^4*(-a^7c^3)^{(1/2)}) / (256(a^{13}e^{12} + a^7c^6d^{12} + 6a^{12}c*d^2e^{10} \\
& + 6a^8c^5d^{10}e^2 + 15a^9c^4d^8e^4 + 20a^{10}c^3d^6e^6 + 15a^{11}c^2d^4e^8)))^{(1/2)} - \\
& (x*(4096a^{12}c^5d^5e^{22} - 1152a^2c^{15}d^{21}e^2 - 15232a^3c^{14}d^{19}e^4 \\
& - 78336a^4c^{13}d^{17}e^6 - 140800a^5c^{12}d^{15}e^8 + 489728a^6c^{11}d^{13}e^{10} \\
& + 2219776a^7c^{10}d^{11}e^{12} + 3155456a^8c^9d^9e^{14} + 1901056a^9c^8d^7e^{16} \\
& + 362368a^{10}c^7d^5e^{18} - 32640a^{11}c^6d^3e^{20})) / (128(a^4c^8d^{18} + a^{12}d^2e^{16} \\
& + 8a^{11}c*d^4e^{14} + 8a^5c^7d^{16}e^2 + 28a^6c^6d^{14}e^4 + 56a^7c^5d^{12}e^6 \\
& + 70a^8c^4d^{10}e^8 + 56a^9c^3d^8e^{10} + 28a^{10}c^2d^6e^{12})) * (-(49a^4e^8*(-a^7c^3)^{(1/2)} \\
& + 9c^4d^8*(-a^7c^3)^{(1/2)} - 12a^4c^5d^7e + 252a^7c^2d^7e^7 - 156a^5c^4d^5e^3 \\
& - 404a^6c^3d^3e^5 + 68a*c^3d^6e^2*(-a^7c^3)^{(1/2)} - 492a^3c*d^2e^6*(-a^7c^3)^{(1/2)} \\
& + 30a^2c^2d^4e^4*(-a^7c^3)^{(1/2)}) / (256(a^{13}e^{12} + a^7c^6d^{12} + 6a^{12}c*d^2e^{10} \\
& + 6a^8c^5d^{10}e^2 + 15a^9c^4d^8e^4 + 20a^{10}c^3d^6e^6 + 15a^{11}c^2d^4e^8)))^{(1/2)} * (-(49a^4e^8 \\
& *(-a^7c^3)^{(1/2)} + 9c^4d^8*(-a^7c^3)^{(1/2)} - 12a^4c^5d^7e + 252a^7 \\
& *c^2d^7e^7 - 156a^5c^4d^5e^3 - 404a^6c^3d^3e^5 + 68a*c^3d^6e^2*(-a^7c^3)^{(1/2)} \\
& - 492a^3c*d^2e^6*(-a^7c^3)^{(1/2)} + 30a^2c^2d^4e^4*(-a^7c^3)^{(1/2)}) / (256(a^{13}e^{12} \\
& + a^7c^6d^{12} + 6a^{12}c*d^2e^{10} + 6a^8c^5d^{10}e^2 + 15a^9c^4d^8e^4 + 20a^{10}c^3d^6e^6 \\
& + 15a^{11}c^2d^4e^8)))^{(1/2)} - (x*(81c^{13}d^{14}e^5 - 392a^7c^6e^{19} + 1206a*c^{12}d^{12}e^7 \\
& + 12247a^2c^{11}d^{10}e^9 + 58636a^3c^{10}d^8e^{11} + 114927a^4c^9d^6e^{13} - 1306a^5c^8d^4e^{15} \\
& - 3575a^6c^7d^2e^{17})) / (128(a^4c^8d^{18} + a^{12}d^2e^{16} + 8a^{11}c*d^4e^{14} + 8a^5c^7d^{16}e^2 \\
& + 28a^6c^6d^{14}e^4 + 56a^7c^5d^{12}e^6 + 70a^8c^4d^{10}e^8 + 56a^9c^3d^8e^{10} + 28a^{10}c^2d^6e^{12})) \\
& ) * (-(49a^4e^8*(-a^7c^3)^{(1/2)} + 9c^4d^8*(-a^7c^3)^{(1/2)} - 12a^4c^5d^7e + 252a^7 \\
& *c^2d^7e^7 - 156a^5c^4d^5e^3 - 404a^6c^3d^3e^5 + 68a*c^3d^6e^2*(-a^7c^3)^{(1/2)} - 492a^3c \\
& *d^2e^6*(-a^7c^3)^{(1/2)} + 30a^2c^2d^4e^4*(-a^7c^3)^{(1/2)}) / (256(a^{13}e^{12} + a^7c^6d^{12} \\
& + 6a^{12}c*d^2e^{10} + 6a^8c^5d^{10}e^2 + 15a^9c^4d^8e^4 + 20a^{10}c^3d^6e^6 + 15a^{11}c^2d^4e^8)))^{(1/2)} * 1i - (((3584a^{10}c^5e^{21} + \\
& 1152a*c^{14}d^{18}e^3 + 13184a^2c^{13}d^{16}e^5 + 54912a^3c^{12}d^{14}e^7 + 296832a^4c^{11}d^{12}e^9 \\
& + 1282432a^5c^{10}d^{10}e^{11} + 769152a^6c^9d^8e^{13} - 1421440a^7c^8d^6e^{15} - 1254784a^8c^7d^4e^{17} \\
& - 89088a^9c^6d^2e^{19}) / (512(a^4c^8d^{18} + a^{12}d^2e^{16} + 8a^{11}c*d^4e^{14} + 8a^5c^7d^{16}e^2 \\
& + 28a^6c^6d^{14}e^4 + 56a^7c^5d^{12}e^6 + 70a^8c^4d^{10}e^8 + 56a^9c^3d^8e^{10} + 28a^{10}c^2d^6e^{12})) \\
& - (((65536a^{15}c^4d^4e^24 - 24576a^4c^{15}d^{23}e^2 - 212992a^5c^{14}d^{21}e^4 - 352256a^6c^{13}d^{19}e^6 \\
& + 1966080a^7c^{12}d^{17}e^8 + 10960896a^8c^{11}d^{15}e^{10} + 25460736a^9c^{10}d^{13}e^{12} + 34750464a^{10}c^9d^{11}e^{14} \\
& + 30081024a^{11}c^8d^9e^{16}
\end{aligned}$$



$$\begin{aligned}
& ^{16} + 16588800a^{12}c^7d^7e^{18} + 5554176a^{13}c^6d^5e^{20} + 991232a^{14}c^5d^3e^{22}) / (512(a^4c^8d^{18} + a^{12}d^2e^{16} + 8a^{11}c^4d^4e^{14} + 8a^5c^7d^{16}e^2 + 28a^6c^6d^{14}e^4 + 56a^7c^5d^{12}e^6 + 70a^8c^4d^{10}e^8 + 56a^9c^3d^8e^{10} + 28a^{10}c^2d^6e^{12})) + (x(-(49a^4e^8(-a^7c^3)^{1/2} + 9c^4d^8(-a^7c^3)^{1/2} - 12a^4c^5d^7e + 252a^7c^2d^2e^7 - 156a^5c^4d^5e^3 - 404a^6c^3d^3e^5 + 68a^3c^3d^6e^2(-a^7c^3)^{1/2} - 492a^3c^3d^2e^6(-a^7c^3)^{1/2} + 30a^2c^2d^4e^4(-a^7c^3)^{1/2})) / (256(a^{13}e^{12} + a^7c^6d^{12} + 6a^{12}c^2d^2e^{10} + 6a^8c^5d^{10}e^2 + 15a^9c^4d^8e^4 + 20a^{10}c^3d^6e^6 + 15a^{11}c^2d^4e^8)))^{1/2} * (65536a^6c^{15}d^{24}e^3 + 589824a^7c^{14}d^{22}e^5 + 2293760a^8c^{13}d^{20}e^7 + 4915200a^9c^{12}d^{18}e^9 + 5898240a^{10}c^{11}d^{16}e^{11} + 2752512a^{11}c^{10}d^{14}e^{13} - 2752512a^{12}c^9d^{12}e^{15} - 5898240a^{13}c^8d^{10}e^{17} - 4915200a^{14}c^7d^8e^{19} - 2293760a^{15}c^6d^6e^{21} - 589824a^{16}c^5d^4e^{23} - 65536a^{17}c^4d^2e^{25})) / (128(a^4c^8d^{18} + a^{12}d^2e^{16} + 8a^{11}c^4d^4e^{14} + 8a^5c^7d^{16}e^2 + 28a^6c^6d^{14}e^4 + 56a^7c^5d^{12}e^6 + 70a^8c^4d^{10}e^8 + 56a^9c^3d^8e^{10} + 28a^{10}c^2d^6e^{12})) * (-(49a^4e^8(-a^7c^3)^{1/2} + 9c^4d^8(-a^7c^3)^{1/2} - 12a^4c^5d^7e + 252a^7c^2d^2e^7 - 156a^5c^4d^5e^3 - 404a^6c^3d^3e^5 + 68a^3c^3d^6e^2(-a^7c^3)^{1/2} - 492a^3c^3d^2e^6(-a^7c^3)^{1/2} + 30a^2c^2d^4e^4(-a^7c^3)^{1/2})) / (256(a^{13}e^{12} + a^7c^6d^{12} + 6a^{12}c^2d^2e^{10} + 6a^8c^5d^{10}e^2 + 15a^9c^4d^8e^4 + 20a^{10}c^3d^6e^6 + 15a^{11}c^2d^4e^8)))^{1/2} + (x*(4096a^{12}c^5d^2e^{22} - 1152a^2c^{15}d^{21}e^2 - 15232a^3c^{14}d^{19}e^4 - 78336a^4c^{13}d^{17}e^6 - 140800a^5c^{12}d^{15}e^8 + 489728a^6c^{11}d^{13}e^{10} + 2219776a^7c^{10}d^{11}e^{12} + 3155456a^8c^9d^9e^{14} + 1901056a^9c^8d^7e^{16} + 362368a^{10}c^7d^5e^{18} - 32640a^{11}c^6d^3e^{20})) / (128(a^4c^8d^{18} + a^{12}d^2e^{16} + 8a^{11}c^4d^4e^{14} + 8a^5c^7d^{16}e^2 + 28a^6c^6d^{14}e^4 + 56a^7c^5d^{12}e^6 + 70a^8c^4d^{10}e^8 + 56a^9c^3d^8e^{10} + 28a^{10}c^2d^6e^{12})) * (-(49a^4e^8(-a^7c^3)^{1/2} + 9c^4d^8(-a^7c^3)^{1/2} - 12a^4c^5d^7e + 252a^7c^2d^2e^7 - 156a^5c^4d^5e^3 - 404a^6c^3d^3e^5 + 68a^3c^3d^6e^2(-a^7c^3)^{1/2} - 492a^3c^3d^2e^6(-a^7c^3)^{1/2} + 30a^2c^2d^4e^4(-a^7c^3)^{1/2})) / (256(a^{13}e^{12} + a^7c^6d^{12} + 6a^{12}c^2d^2e^{10} + 6a^8c^5d^{10}e^2 + 15a^9c^4d^8e^4 + 20a^{10}c^3d^6e^6 + 15a^{11}c^2d^4e^8)))^{1/2} * (-(49a^4e^8(-a^7c^3)^{1/2} + 9c^4d^8(-a^7c^3)^{1/2} - 12a^4c^5d^7e + 252a^7c^2d^2e^7 - 156a^5c^4d^5e^3 - 404a^6c^3d^3e^5 + 68a^3c^3d^6e^2(-a^7c^3)^{1/2} - 492a^3c^3d^2e^6(-a^7c^3)^{1/2} + 30a^2c^2d^4e^4(-a^7c^3)^{1/2})) / (256(a^{13}e^{12} + a^7c^6d^{12} + 6a^{12}c^2d^2e^{10} + 6a^8c^5d^{10}e^2 + 15a^9c^4d^8e^4 + 20a^{10}c^3d^6e^6 + 15a^{11}c^2d^4e^8)))^{1/2} + (x*(81c^{13}d^{14}e^5 - 392a^7c^6e^{19} + 1206a^3c^{12}d^{12}e^7 + 12247a^2c^{11}d^{10}e^9 + 58636a^3c^{10}d^8e^{11} + 114927a^4c^9d^6e^{13} - 1306a^5c^8d^4e^{15} - 3575a^6c^7d^2e^{17})) / (128(a^4c^8d^{18} + a^{12}d^2e^{16} + 8a^{11}c^4d^4e^{14} + 8a^5c^7d^{16}e^2 + 28a^6c^6d^{14}e^4 + 56a^7c^5d^{12}e^6 + 70a^8c^4d^{10}e^8 + 56a^9c^3d^8e^{10} + 28a^{10}c^2d^6e^{12})) * (-(49a^4e^8(-a^7c^3)^{1/2} + 9c^4d^8(-a^7c^3)^{1/2} - 12a^4c^5d^7e + 252a^7c^2d^2e^7 - 156a^5c^4d^5e^3 - 404a^6c^3d^3e^5 + 68a^3c^3d^6e^2(-a^7c^3)^{1/2} - 492a^3c^3d^2e^6(-a^7c^3)^{1/2} + 30a^2c^2d^4e^4(-a^7c^3)^{1/2})) / (256(a^{13}e^{12} + a^7c^6d^{12} + 6a^{12}c^2d^2e^{10} + 6a^8c^5d^{10}e^2 + 15a^9c^4d^8e^4 + 20a^{10}c^3d^6e^6 + 15a^{11}c^2d^4e^8)))^{1/2} * i) / (((3584a^{10}c^5e^{21} + 1152a^3c^{14}d^{18}e^3 + 13184a^2c^{13}d^{16}e^5 + 54912a^3c^{12}d^{14}e^7 + 296832a^4c^{11}d^{12}e^9 + 1282432a^5c^{10}d^{10}e^{11} + 769152a^6c^9d^8e^{13} - 1421440a^7c^8d^6e^{15} - 1254784a^8c^7d^4e^{17} - 89088a^9c^6d^2e^{19}) / (512(a^4c^8d^{18} + a^{12}d^2e^{16} + 8a^{11}c^4d^4e^{14} + 8a^5c^7d^{16}e^2 + 28a^6c^6d^{14}e^4 + 56a^7c^5d^{12}e^6 + 70a^8c^4d^{10}e^8 + 56a^9c^3d^8e^{10} + 28a^{10}c^2d^6e^{12})) - (((65536a^{15}c^4d^2e^{24} - 24576a^4c^{15}d^{23}e^2 - 212992a^5c^{14}d^{21}e^4 - 352256a^6c^{13}d^{19}e^6 + 1966080a^7c^{12}d^{17}e^8 + 10960896a^8c^{11}d^{15}e^{10} + 25460736a^9c^{10}d^{13}e^{12} + 34750464a^{10}c^9d^{11}e^{14} + 30081024a^{11}c^8d^9e^{16} + 16588800a^{12}c^7d^7e^{18} + 5
\end{aligned}$$

$$\begin{aligned}
& 554176*a^{13}*c^6*d^5*e^{20} + 991232*a^{14}*c^5*d^3*e^{22})/(512*(a^4*c^8*d^{18} + a^{12}*d^2*e^{16} + 8*a^{11}*c*d^4*e^{14} + 8*a^5*c^7*d^{16}*e^2 + 28*a^6*c^6*d^{14}*e^4 + 56*a^7*c^5*d^{12}*e^6 + 70*a^8*c^4*d^{10}*e^8 + 56*a^9*c^3*d^8*e^{10} + 28*a^{10}*c^2*d^6*e^{12})) - (x*(-(49*a^4*e^8*(-a^7*c^3)^{(1/2)} + 9*c^4*d^8*(-a^7*c^3)^{(1/2)} - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^{(1/2)} - 492*a^3*c*d^2*e^6*(-a^7*c^3)^{(1/2)} + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^{(1/2)}))/(256*(a^{13}*e^{12} + a^7*c^6*d^{12} + 6*a^{12}*c*d^2*e^{10} + 6*a^8*c^5*d^{10}*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^{10}*c^3*d^6*e^6 + 15*a^{11}*c^2*d^4*e^8)))^{(1/2)}*(65536*a^6*c^{15}*d^{24}*e^3 + 589824*a^7*c^{14}*d^{22}*e^5 + 2293760*a^8*c^{13}*d^{20}*e^7 + 4915200*a^9*c^{12}*d^{18}*e^9 + 5898240*a^{10}*c^{11}*d^{16}*e^{11} + 2752512*a^{11}*c^{10}*d^{14}*e^{13} - 2752512*a^{12}*c^9*d^{12}*e^{15} - 5898240*a^{13}*c^8*d^{10}*e^{17} - 4915200*a^{14}*c^7*d^8*e^{19} - 2293760*a^{15}*c^6*d^6*e^{21} - 589824*a^{16}*c^5*d^4*e^{23} - 65536*a^{17}*c^4*d^2*e^{25}))/((128*(a^4*c^8*d^{18} + a^{12}*d^2*e^{16} + 8*a^{11}*c*d^4*e^{14} + 8*a^5*c^7*d^{16}*e^2 + 28*a^6*c^6*d^{14}*e^4 + 56*a^7*c^5*d^{12}*e^6 + 70*a^8*c^4*d^{10}*e^8 + 56*a^9*c^3*d^8*e^{10} + 28*a^{10}*c^2*d^6*e^{12})))*(-(49*a^4*e^8*(-a^7*c^3)^{(1/2)} + 9*c^4*d^8*(-a^7*c^3)^{(1/2)} - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^{(1/2)} - 492*a^3*c*d^2*e^6*(-a^7*c^3)^{(1/2)} + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^{(1/2)}))/(256*(a^{13}*e^{12} + a^7*c^6*d^{12} + 6*a^{12}*c*d^2*e^{10} + 6*a^8*c^5*d^{10}*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^{10}*c^3*d^6*e^6 + 15*a^{11}*c^2*d^4*e^8)))^{(1/2)} - (x*(4096*a^{12}*c^5*d*e^{22} - 1152*a^2*c^{15}*d^{21}*e^2 - 15232*a^3*c^{14}*d^{19}*e^4 - 78336*a^4*c^{13}*d^{17}*e^6 - 140800*a^5*c^{12}*d^{15}*e^8 + 489728*a^6*c^{11}*d^{13}*e^{10} + 2219776*a^7*c^{10}*d^{11}*e^{12} + 3155456*a^8*c^9*d^9*e^{14} + 1901056*a^9*c^8*d^7*e^{16} + 362368*a^{10}*c^7*d^5*e^{18} - 32640*a^{11}*c^6*d^3*e^{20}))/((128*(a^4*c^8*d^{18} + a^{12}*d^2*e^{16} + 8*a^{11}*c*d^4*e^{14} + 8*a^5*c^7*d^{16}*e^2 + 28*a^6*c^6*d^{14}*e^4 + 56*a^7*c^5*d^{12}*e^6 + 70*a^8*c^4*d^{10}*e^8 + 56*a^9*c^3*d^8*e^{10} + 28*a^{10}*c^2*d^6*e^{12})))*(-(49*a^4*e^8*(-a^7*c^3)^{(1/2)} + 9*c^4*d^8*(-a^7*c^3)^{(1/2)} - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^{(1/2)} - 492*a^3*c*d^2*e^6*(-a^7*c^3)^{(1/2)} + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^{(1/2)}))/(256*(a^{13}*e^{12} + a^7*c^6*d^{12} + 6*a^{12}*c*d^2*e^{10} + 6*a^8*c^5*d^{10}*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^{10}*c^3*d^6*e^6 + 15*a^{11}*c^2*d^4*e^8)))^{(1/2)} - (x*(81*c^{13}*d^{14}*e^5 - 392*a^7*c^6*e^{19} + 1206*a*c^{12}*d^{12}*e^7 + 12247*a^2*c^{11}*d^{10}*e^9 + 58636*a^3*c^{10}*d^8*e^{11} + 114927*a^4*c^9*d^6*e^{13} - 1306*a^5*c^8*d^4*e^{15} - 3575*a^6*c^7*d^2*e^{17}))/((128*(a^4*c^8*d^{18} + a^{12}*d^2*e^{16} + 8*a^{11}*c*d^4*e^{14} + 8*a^5*c^7*d^{16}*e^2 + 28*a^6*c^6*d^{14}*e^4 + 56*a^7*c^5*d^{12}*e^6 + 70*a^8*c^4*d^{10}*e^8 + 56*a^9*c^3*d^8*e^{10} + 28*a^{10}*c^2*d^6*e^{12})))*(-(49*a^4*e^8*(-a^7*c^3)^{(1/2)} + 9*c^4*d^8*(-a^7*c^3)^{(1/2)} - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^{(1/2)} - 492*a^3*c*d^2*e^6*(-a^7*c^3)^{(1/2)} + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^{(1/2)}))/(256*(a^{13}*e^{12} + a^7*c^6*d^{12} + 6*a^{12}*c*d^2*e^{10} + 6*a^8*c^5*d^{10}*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^{10}*c^3*d^6*e^6 + 15*a^{11}*c^2*d^4*e^8)))^{(1/2)} + (((3584*a^{10}*c^5*e^{21} + 1152*a*c^{14}*d^{18}*e^3 + 13184*a^2*c^{13}*d^{16}*e^5 + 54912*a^3*c^{12}*d^{14}*e^7 + 296832*a^4*c^{11}*d^{12}*e^9 + 1282432*a^5*c^{10}*d^{10}*e^{11} + 769152*a^6*c^9*d^8*e^{13} - 1421440*a^7*c^8*d^6*e^{15} - 1254784*a^8*c^7*d^4*e^{17} - 89088*a^9*c^6*d^2*e^{19}))/((512*(a^4*c^8*d^{18} + a^{12}*d^2*e^{16} + 8*a^{11}*c*d^4*e^{14} + 8*a^5*c^7*d^{16}*e^2 + 28*a^6*c^6*d^{14}*e^4 + 56*a^7*c^5*d^{12}*e^6 + 70*a^8*c^4*d^{10}*e^8 + 56*a^9*c^3*d^8*e^{10} + 28*a^{10}*c^2*d^6*e^{12})) - (((65536*a^{15}*c^4*d*e^24 - 24576*a^4*c^{15}*d^{23}*e^2 - 212992*a^5*c^{14}*d^{21}*e^4 - 352256*a^6*c^{13}*d^{19}*e^6 + 1966080*a^7*c^{12}*d^{17}*e^8 + 10960896*a^8*c^{11}*d^{15}*e^{10} + 25460736*a^9*c^{10}*d^{13}*e^{12} + 34750464*a^{10}*c^9*d^{11}*e^{14} + 30081024*a^{11}*c^8*d^9*e^{16} + 16588800*a^{12}*c^7*d^7*e^{18} + 5554176*a^{13}*c^6*d^5*e^{20} + 991232*a^{14}
\end{aligned}$$

$$\begin{aligned}
& c^5 d^3 e^{22} / (512 (a^4 c^8 d^{18} + a^{12} d^2 e^{16} + 8 a^{11} c d^4 e^{14} + 8 a^5 c^7 d^{16} e^2 + 28 a^6 c^6 d^{14} e^4 + 56 a^7 c^5 d^{12} e^6 + 70 a^8 c^4 d^{10} e^8 + 56 a^9 c^3 d^8 e^{10} + 28 a^{10} c^2 d^6 e^{12})) + (x * (- (49 a^4 e^8 (-a^7 c^3)^{(1/2)} + 9 c^4 d^8 (-a^7 c^3)^{(1/2)} - 12 a^4 c^5 d^7 e + 252 a^7 c^2 d e^7 - 156 a^5 c^4 d^5 e^3 - 404 a^6 c^3 d^3 e^5 + 68 a^8 c^3 d^6 e^2 (-a^7 c^3)^{(1/2)} - 492 a^3 c^2 d^2 e^6 (-a^7 c^3)^{(1/2)} + 30 a^2 c^2 d^4 e^4 (-a^7 c^3)^{(1/2)})) / (256 (a^{13} e^{12} + a^7 c^6 d^{12} + 6 a^{12} c d^2 e^{10} + 6 a^8 c^5 d^{10} e^2 + 15 a^9 c^4 d^8 e^4 + 20 a^{10} c^3 d^6 e^6 + 15 a^{11} c^2 d^4 e^8)))^{(1/2)} * (65536 a^6 c^{15} d^{24} e^3 + 589824 a^7 c^{14} d^{22} e^5 + 2293760 a^8 c^{13} d^{20} e^7 + 4915200 a^9 c^{12} d^{18} e^9 + 5898240 a^{10} c^{11} d^{16} e^{11} + 2752512 a^{11} c^{10} d^{14} e^{13} - 2752512 a^{12} c^9 d^{12} e^{15} - 5898240 a^{13} c^8 d^{10} e^{17} - 4915200 a^{14} c^7 d^8 e^{19} - 2293760 a^{15} c^6 d^6 e^{21} - 589824 a^{16} c^5 d^4 e^{23} - 65536 a^{17} c^4 d^2 e^{25})) / (128 (a^4 c^8 d^{18} + a^{12} d^2 e^{16} + 8 a^{11} c d^4 e^{14} + 8 a^5 c^7 d^{16} e^2 + 28 a^6 c^6 d^{14} e^4 + 56 a^7 c^5 d^{12} e^6 + 70 a^8 c^4 d^{10} e^8 + 56 a^9 c^3 d^8 e^{10} + 28 a^{10} c^2 d^6 e^{12})) * (- (49 a^4 e^8 (-a^7 c^3)^{(1/2)} + 9 c^4 d^8 (-a^7 c^3)^{(1/2)} - 12 a^4 c^5 d^7 e + 252 a^7 c^2 d e^7 - 156 a^5 c^4 d^5 e^3 - 404 a^6 c^3 d^3 e^5 + 68 a^8 c^3 d^6 e^2 (-a^7 c^3)^{(1/2)} - 492 a^3 c^2 d^2 e^6 (-a^7 c^3)^{(1/2)} + 30 a^2 c^2 d^4 e^4 (-a^7 c^3)^{(1/2)})) / (256 (a^{13} e^{12} + a^7 c^6 d^{12} + 6 a^{12} c d^2 e^{10} + 6 a^8 c^5 d^{10} e^2 + 15 a^9 c^4 d^8 e^4 + 20 a^{10} c^3 d^6 e^6 + 15 a^{11} c^2 d^4 e^8)))^{(1/2)} + (x * (4096 a^{12} c^5 d e^{22} - 1152 a^2 c^{15} d^{21} e^2 - 15232 a^3 c^{14} d^{19} e^4 - 78336 a^4 c^{13} d^{17} e^6 - 140800 a^5 c^{12} d^{15} e^8 + 489728 a^6 c^{11} d^{13} e^{10} + 2219776 a^7 c^{10} d^{11} e^{12} + 3155456 a^8 c^9 d^9 e^{14} + 1901056 a^9 c^8 d^7 e^{16} + 362368 a^{10} c^7 d^5 e^{18} - 32640 a^{11} c^6 d^3 e^{20})) / (128 (a^4 c^8 d^{18} + a^{12} d^2 e^{16} + 8 a^{11} c d^4 e^{14} + 8 a^5 c^7 d^{16} e^2 + 28 a^6 c^6 d^{14} e^4 + 56 a^7 c^5 d^{12} e^6 + 70 a^8 c^4 d^{10} e^8 + 56 a^9 c^3 d^8 e^{10} + 28 a^{10} c^2 d^6 e^{12})) * (- (49 a^4 e^8 (-a^7 c^3)^{(1/2)} + 9 c^4 d^8 (-a^7 c^3)^{(1/2)} - 12 a^4 c^5 d^7 e + 252 a^7 c^2 d e^7 - 156 a^5 c^4 d^5 e^3 - 404 a^6 c^3 d^3 e^5 + 68 a^8 c^3 d^6 e^2 (-a^7 c^3)^{(1/2)} - 492 a^3 c^2 d^2 e^6 (-a^7 c^3)^{(1/2)} + 30 a^2 c^2 d^4 e^4 (-a^7 c^3)^{(1/2)})) / (256 (a^{13} e^{12} + a^7 c^6 d^{12} + 6 a^{12} c d^2 e^{10} + 6 a^8 c^5 d^{10} e^2 + 15 a^9 c^4 d^8 e^4 + 20 a^{10} c^3 d^6 e^6 + 15 a^{11} c^2 d^4 e^8)))^{(1/2)} * (- (49 a^4 e^8 (-a^7 c^3)^{(1/2)} + 9 c^4 d^8 (-a^7 c^3)^{(1/2)} - 12 a^4 c^5 d^7 e + 252 a^7 c^2 d e^7 - 156 a^5 c^4 d^5 e^3 - 404 a^6 c^3 d^3 e^5 + 68 a^8 c^3 d^6 e^2 (-a^7 c^3)^{(1/2)} - 492 a^3 c^2 d^2 e^6 (-a^7 c^3)^{(1/2)} + 30 a^2 c^2 d^4 e^4 (-a^7 c^3)^{(1/2)})) / (256 (a^{13} e^{12} + a^7 c^6 d^{12} + 6 a^{12} c d^2 e^{10} + 6 a^8 c^5 d^{10} e^2 + 15 a^9 c^4 d^8 e^4 + 20 a^{10} c^3 d^6 e^6 + 15 a^{11} c^2 d^4 e^8)))^{(1/2)} + (x * (81 c^{13} d^{14} e^5 - 392 a^7 c^6 e^{19} + 1206 a^8 c^{12} d^{12} e^7 + 12247 a^2 c^{11} d^{10} e^9 + 58636 a^3 c^{10} d^8 e^{11} + 114927 a^4 c^9 d^6 e^{13} - 1306 a^5 c^8 d^4 e^{15} - 3575 a^6 c^7 d^2 e^{17})) / (128 (a^4 c^8 d^{18} + a^{12} d^2 e^{16} + 8 a^{11} c d^4 e^{14} + 8 a^5 c^7 d^{16} e^2 + 28 a^6 c^6 d^{14} e^4 + 56 a^7 c^5 d^{12} e^6 + 70 a^8 c^4 d^{10} e^8 + 56 a^9 c^3 d^8 e^{10} + 28 a^{10} c^2 d^6 e^{12})) * (- (49 a^4 e^8 (-a^7 c^3)^{(1/2)} + 9 c^4 d^8 (-a^7 c^3)^{(1/2)} - 12 a^4 c^5 d^7 e + 252 a^7 c^2 d e^7 - 156 a^5 c^4 d^5 e^3 - 404 a^6 c^3 d^3 e^5 + 68 a^8 c^3 d^6 e^2 (-a^7 c^3)^{(1/2)} - 492 a^3 c^2 d^2 e^6 (-a^7 c^3)^{(1/2)} + 30 a^2 c^2 d^4 e^4 (-a^7 c^3)^{(1/2)})) / (256 (a^{13} e^{12} + a^7 c^6 d^{12} + 6 a^{12} c d^2 e^{10} + 6 a^8 c^5 d^{10} e^2 + 15 a^9 c^4 d^8 e^4 + 20 a^{10} c^3 d^6 e^6 + 15 a^{11} c^2 d^4 e^8)))^{(1/2)} - (729 c^{11} d^9 e^8 + 2916 a^2 c^{10} d^7 e^{10} + 2009 a^4 c^7 d e^{16} - 2538 a^2 c^9 d^5 e^{12} + 17764 a^3 c^8 d^3 e^{14}) / (256 (a^4 c^8 d^{18} + a^{12} d^2 e^{16} + 8 a^{11} c d^4 e^{14} + 8 a^5 c^7 d^{16} e^2 + 28 a^6 c^6 d^{14} e^4 + 56 a^7 c^5 d^{12} e^6 + 70 a^8 c^4 d^{10} e^8 + 56 a^9 c^3 d^8 e^{10} + 28 a^{10} c^2 d^6 e^{12})) * (- (49 a^4 e^8 (-a^7 c^3)^{(1/2)} + 9 c^4 d^8 (-a^7 c^3)^{(1/2)} - 12 a^4 c^5 d^7 e + 252 a^7 c^2 d e^7 - 156 a^5 c^4 d^5 e^3 - 404 a^6 c^3 d^3 e^5 + 68 a^8 c^3 d^6 e^2 (-a^7 c^3)^{(1/2)} - 492 a^3 c^2 d^2 e^6 (-a^7 c^3)^{(1/2)} + 30 a^2 c^2 d^4 e^4 (-a^7 c^3)^{(1/2)})) / (256 (a^{13} e^{12} + a^7 c^6 d^{12} + 6 a^{12} c d^2 e^{10} + 6 a^8 c^5 d^{10} e^2 + 15 a^9 c^4 d^8 e^4 + 20 a^{10} c^3 d^6 e^6 + 15 a^{11} c^2 d^4 e^8)))^{(1/2)} * 2i + \operatorname{atan}((((3584 a^{10} c^5 e^{21} + 1152 a^2 c^{14} d^{18} e^3 + 13184 a^2 c^{13} d^{16} e^5 + 54912 a^3 c^{12} d^{14} e^7
\end{aligned}$$

$$\begin{aligned}
& + 296832*a^4*c^11*d^12*e^9 + 1282432*a^5*c^10*d^10*e^11 + 769152*a^6*c^9*d^8*e^13 - 1421440*a^7*c^8*d^6*e^15 - 1254784*a^8*c^7*d^4*e^17 - 89088*a^9*c^6*d^2*e^19)/(512*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)) - (((65536*a^15*c^4*d*e^24 - 24576*a^4*c^15*d^23*e^2 - 212992*a^5*c^14*d^21*e^4 - 352256*a^6*c^13*d^19*e^6 + 1966080*a^7*c^12*d^17*e^8 + 10960896*a^8*c^11*d^15*e^10 + 25460736*a^9*c^10*d^13*e^12 + 34750464*a^10*c^9*d^11*e^14 + 30081024*a^11*c^8*d^9*e^16 + 16588800*a^12*c^7*d^7*e^18 + 5554176*a^13*c^6*d^5*e^20 + 991232*a^14*c^5*d^3*e^22)/(512*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)) - (x*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2)))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2)*(65536*a^6*c^15*d^24*e^3 + 589824*a^7*c^14*d^22*e^5 + 2293760*a^8*c^13*d^20*e^7 + 4915200*a^9*c^12*d^18*e^9 + 5898240*a^10*c^11*d^16*e^11 + 2752512*a^11*c^10*d^14*e^13 - 2752512*a^12*c^9*d^12*e^15 - 5898240*a^13*c^8*d^10*e^17 - 4915200*a^14*c^7*d^8*e^19 - 2293760*a^15*c^6*d^6*e^21 - 589824*a^16*c^5*d^4*e^23 - 65536*a^17*c^4*d^2*e^25))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12))*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2)))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) - (x*(4096*a^12*c^5*d*e^22 - 1152*a^2*c^15*d^21*e^2 - 15232*a^3*c^14*d^19*e^4 - 78336*a^4*c^13*d^17*e^6 - 140800*a^5*c^12*d^15*e^8 + 489728*a^6*c^11*d^13*e^10 + 2219776*a^7*c^10*d^11*e^12 + 3155456*a^8*c^9*d^9*e^14 + 1901056*a^9*c^8*d^7*e^16 + 362368*a^10*c^7*d^5*e^18 - 32640*a^11*c^6*d^3*e^20))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12))*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2)))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) - (x*(81*c^13*d^14*e^5 - 392*a^7*c^6*e^19 + 1206*a*c^12*d^12*e^7 + 12247*a^2*c^11*d^10*e^9 + 58636*a^3*c^10*d^8*e^11 + 114927*a^4*c^9*d^6*e^13 - 1306*a^5*c^8*d^4*e^15 - 3575*a^6*c^7*d^2*e^17))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12))*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2)))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2)*i - (((3584*a^10*c^5*e^21 + 1152*a*c^14*d^18*e^3 + 13184*a^2*c^13*d^16*e^5 + 54912*a^3*c^12*d^14*e^7 + 296832*a^4*c^11*d^12*e^9 + 1282432*a^5
\end{aligned}$$

$$\begin{aligned}
& *c^{10}d^{10}e^{11} + 769152*a^6*c^9*d^8*e^{13} - 1421440*a^7*c^8*d^6*e^{15} - 1254 \\
& 784*a^8*c^7*d^4*e^{17} - 89088*a^9*c^6*d^2*e^{19})/(512*(a^4*c^8*d^{18} + a^{12}*d^2 \\
& *e^{16} + 8*a^{11}*c*d^4*e^{14} + 8*a^5*c^7*d^{16}*e^2 + 28*a^6*c^6*d^{14}*e^4 + 56* \\
& a^7*c^5*d^{12}*e^6 + 70*a^8*c^4*d^{10}*e^8 + 56*a^9*c^3*d^8*e^{10} + 28*a^{10}*c^2* \\
& d^6*e^{12})) - (((65536*a^{15}*c^4*d*e^{24} - 24576*a^4*c^{15}*d^{23}*e^2 - 212992*a^5 \\
& *c^{14}*d^{21}*e^4 - 352256*a^6*c^{13}*d^{19}*e^6 + 1966080*a^7*c^{12}*d^{17}*e^8 + 10 \\
& 960896*a^8*c^{11}*d^{15}*e^{10} + 25460736*a^9*c^{10}*d^{13}*e^{12} + 34750464*a^{10}*c^9 \\
& *d^{11}*e^{14} + 30081024*a^{11}*c^8*d^9*e^{16} + 16588800*a^{12}*c^7*d^7*e^{18} + 5554 \\
& 176*a^{13}*c^6*d^5*e^{20} + 991232*a^{14}*c^5*d^3*e^{22})/(512*(a^4*c^8*d^{18} + a^{12} \\
& *d^2*e^{16} + 8*a^{11}*c*d^4*e^{14} + 8*a^5*c^7*d^{16}*e^2 + 28*a^6*c^6*d^{14}*e^4 + \\
& 56*a^7*c^5*d^{12}*e^6 + 70*a^8*c^4*d^{10}*e^8 + 56*a^9*c^3*d^8*e^{10} + 28*a^{10}*c^2 \\
& *d^6*e^{12})) + (x*((49*a^4*e^8*(-a^7*c^3)^{(1/2)} + 9*c^4*d^8*(-a^7*c^3)^{(1/2)} \\
& + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 \\
& + 68*a*c^3*d^6*e^2*(-a^7*c^3)^{(1/2)} - 492*a^3*c*d^2*e^6*(-a^7*c^3)^{(1/2)} + 30*a^2*c^2*d^4*e^4 \\
& *(-a^7*c^3)^{(1/2)})/(256*(a^{13}*e^{12} + a^7*c^6*d^{12} + 6*a^{12}*c*d^2*e^{10} + 6*a^8*c^5*d^{10}*e^2 \\
& + 15*a^9*c^4*d^8*e^4 + 20*a^{10}*c^3*d^6*e^6 + 15*a^{11}*c^2*d^4*e^8)))^{(1/2)}*(65536*a^6*c^{15}*d^{24}*e^3 + 5898 \\
& 24*a^7*c^{14}*d^{22}*e^5 + 2293760*a^8*c^{13}*d^{20}*e^7 + 4915200*a^9*c^{12}*d^{18}*e^9 \\
& + 5898240*a^{10}*c^{11}*d^{16}*e^{11} + 2752512*a^{11}*c^{10}*d^{14}*e^{13} - 2752512*a^{12} \\
& *c^9*d^{12}*e^{15} - 5898240*a^{13}*c^8*d^{10}*e^{17} - 4915200*a^{14}*c^7*d^8*e^{19} - \\
& 2293760*a^{15}*c^6*d^6*e^{21} - 589824*a^{16}*c^5*d^4*e^{23} - 65536*a^{17}*c^4*d^2*e^{25}))/ \\
& (128*(a^4*c^8*d^{18} + a^{12}*d^2*e^{16} + 8*a^{11}*c*d^4*e^{14} + 8*a^5*c^7*d^{16}*e^2 + 28*a^6*c^6*d^{14}*e^4 \\
& + 56*a^7*c^5*d^{12}*e^6 + 70*a^8*c^4*d^{10}*e^8 + 56*a^9*c^3*d^8*e^{10} + 28*a^{10}*c^2*d^6*e^{12}))) * \\
& ((49*a^4*e^8*(-a^7*c^3)^{(1/2)} + 9*c^4*d^8*(-a^7*c^3)^{(1/2)} + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 \\
& + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^{(1/2)} - 492*a^3*c*d^2*e^6 \\
& *(-a^7*c^3)^{(1/2)} + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^{(1/2)})/(256*(a^{13}*e^{12} + a^7*c^6*d^{12} + 6*a^{12}*c*d^2*e^{10} \\
& + 6*a^8*c^5*d^{10}*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^{10}*c^3*d^6*e^6 + 15*a^{11}*c^2*d^4*e^8)))^{(1/2)} + (x \\
& *(4096*a^{12}*c^5*d*e^{22} - 1152*a^2*c^{15}*d^{21}*e^2 - 15232*a^3*c^{14}*d^{19}*e^4 - 78336*a^4*c^{13}*d^{17}*e^6 \\
& - 140800*a^5*c^{12}*d^{15}*e^8 + 489728*a^6*c^{11}*d^{13}*e^{10} + 2219776*a^7*c^{10}*d^{11}*e^{12} + 3155456*a^8*c^9*d^9*e^{14} \\
& + 1901056*a^9*c^8*d^7*e^{16} + 362368*a^{10}*c^7*d^5*e^{18} - 32640*a^{11}*c^6*d^3*e^{20}))/ \\
& (128*(a^4*c^8*d^{18} + a^{12}*d^2*e^{16} + 8*a^{11}*c*d^4*e^{14} + 8*a^5*c^7*d^{16}*e^2 + 28*a^6*c^6*d^{14}*e^4 \\
& + 56*a^7*c^5*d^{12}*e^6 + 70*a^8*c^4*d^{10}*e^8 + 56*a^9*c^3*d^8*e^{10} + 28*a^{10}*c^2*d^6*e^{12}))) * \\
& ((49*a^4*e^8*(-a^7*c^3)^{(1/2)} + 9*c^4*d^8*(-a^7*c^3)^{(1/2)} + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 \\
& + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^{(1/2)} - 492*a^3*c*d^2*e^6 \\
& *(-a^7*c^3)^{(1/2)} + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^{(1/2)})/(256*(a^{13}*e^{12} + a^7*c^6*d^{12} + 6*a^{12}*c*d^2*e^{10} \\
& + 6*a^8*c^5*d^{10}*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^{10}*c^3*d^6*e^6 + 15*a^{11}*c^2*d^4*e^8)))^{(1/2)} + (x \\
& *(81*c^{13}*d^{14}*e^5 - 392*a^7*c^6*e^{19} + 1206*a*c^{12}*d^{12}*e^7 + 12247*a^2*c^{11}*d^{10}*e^9 \\
& + 58636*a^3*c^{10}*d^8*e^{11} + 114927*a^4*c^9*d^6*e^{13} - 1306*a^5*c^8*d^4*e^{15} - 3575*a^6*c^7*d^2*e^{17}))/ \\
& (128*(a^4*c^8*d^{18} + a^{12}*d^2*e^{16} + 8*a^{11}*c*d^4*e^{14} + 8*a^5*c^7*d^{16}*e^2 + 28*a^6*c^6*d^{14}*e^4 \\
& + 56*a^7*c^5*d^{12}*e^6 + 70*a^8*c^4*d^{10}*e^8 + 56*a^9*c^3*d^8*e^{10} + 28*a^{10}*c^2*d^6*e^{12}))) * \\
& ((49*a^4*e^8*(-a^7*c^3)^{(1/2)} + 9*c^4*d^8*(-a^7*c^3)^{(1/2)} + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 \\
& + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^{(1/2)} - 492*a^3*c*d^2*e^6 \\
& *(-a^7*c^3)^{(1/2)} + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^{(1/2)})/(256*(a^{13}*e^{12} + a^7*c^6*d^{12} + 6*a^{12}*c*d^2*e^{10} \\
& + 6*a^8*c^5*d^{10}*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^{10}*c^3*d^6*e^6 + 15*a^{11}*c^2*d^4*e^8)))^{(1/2)} * i)/ \\
& (((3584*a^{10}*c^5*e^{21} + 1152*a*c^{14}*d^{18}*e^3 + 13184*a^2*c^{13}*d^{16}*e^5 + 54912*a^3*c^{12}*d^{14}*e^7 + 2968 \\
& 32*a^4*c^{11}*d^{12}*e^9 + 1282432*a^5*c^{10}*d^{10}*e^{11} + 769152*a^6*c^9*d^8*e^{13}
\end{aligned}$$

$$\begin{aligned}
& - 1421440*a^7*c^8*d^6*e^15 - 1254784*a^8*c^7*d^4*e^17 - 89088*a^9*c^6*d^2* \\
& e^19)/(512*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^ \\
& 16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + \\
& 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)) - (((65536*a^15*c^4*d*e^24 - 2 \\
& 4576*a^4*c^15*d^23*e^2 - 212992*a^5*c^14*d^21*e^4 - 352256*a^6*c^13*d^19*e^ \\
& 6 + 1966080*a^7*c^12*d^17*e^8 + 10960896*a^8*c^11*d^15*e^10 + 25460736*a^9* \\
& c^10*d^13*e^12 + 34750464*a^10*c^9*d^11*e^14 + 30081024*a^11*c^8*d^9*e^16 + \\
& 16588800*a^12*c^7*d^7*e^18 + 5554176*a^13*c^6*d^5*e^20 + 991232*a^14*c^5*d \\
& ^3*e^22))/(512*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7 \\
& *d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 \\
& + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)) - (x*((49*a^4*e^8*(-a^7*c^3 \\
& )^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 \\
& + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^( \\
& 1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^( \\
& 1/2)))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10* \\
& e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/ \\
& 2)*(65536*a^6*c^15*d^24*e^3 + 589824*a^7*c^14*d^22*e^5 + 2293760*a^8*c^13*d \\
& ^20*e^7 + 4915200*a^9*c^12*d^18*e^9 + 5898240*a^10*c^11*d^16*e^11 + 2752512 \\
& *a^11*c^10*d^14*e^13 - 2752512*a^12*c^9*d^12*e^15 - 5898240*a^13*c^8*d^10*e \\
& ^17 - 4915200*a^14*c^7*d^8*e^19 - 2293760*a^15*c^6*d^6*e^21 - 589824*a^16*c \\
& ^5*d^4*e^23 - 65536*a^17*c^4*d^2*e^25))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 \\
& + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5 \\
& *d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^1 \\
& 2)))*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^ \\
& 5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 6 \\
& 8*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30* \\
& a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c \\
& *d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + \\
& 15*a^11*c^2*d^4*e^8)))^(1/2) - (x*(4096*a^12*c^5*d*e^22 - 1152*a^2*c^15*d^ \\
& 21*e^2 - 15232*a^3*c^14*d^19*e^4 - 78336*a^4*c^13*d^17*e^6 - 140800*a^5*c^1 \\
& 2*d^15*e^8 + 489728*a^6*c^11*d^13*e^10 + 2219776*a^7*c^10*d^11*e^12 + 31554 \\
& 56*a^8*c^9*d^9*e^14 + 1901056*a^9*c^8*d^7*e^16 + 362368*a^10*c^7*d^5*e^18 - \\
& 32640*a^11*c^6*d^3*e^20))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^ \\
& 4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 7 \\
& 0*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*((49*a^4 \\
& *e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252 \\
& *a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e \\
& ^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e \\
& ^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6 \\
& *a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2* \\
& d^4*e^8)))^(1/2)*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2 \\
& ) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^ \\
& 3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3 \\
& )^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^ \\
& 12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10* \\
& c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) - (x*(81*c^13*d^14*e^5 - 392*a^7 \\
& *c^6*e^19 + 1206*a*c^12*d^12*e^7 + 12247*a^2*c^11*d^10*e^9 + 58636*a^3*c^10 \\
& *d^8*e^11 + 114927*a^4*c^9*d^6*e^13 - 1306*a^5*c^8*d^4*e^15 - 3575*a^6*c^7* \\
& d^2*e^17))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c \\
& ^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e \\
& ^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*((49*a^4*e^8*(-a^7*c^3)^( \\
& 1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + \\
& 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1 \\
& /2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1 \\
& /2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^ \\
& 2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) \\
& + (((3584*a^10*c^5*e^21 + 1152*a*c^14*d^18*e^3 + 13184*a^2*c^13*d^16*e^5 + \\
& 54912*a^3*c^12*d^14*e^7 + 296832*a^4*c^11*d^12*e^9 + 1282432*a^5*c^10*d^10 \\
& *e^11 + 769152*a^6*c^9*d^8*e^13 - 1421440*a^7*c^8*d^6*e^15 - 1254784*a^8*c^
\end{aligned}$$

$$\begin{aligned}
& 7*d^4*e^{17} - 89088*a^9*c^6*d^2*e^{19})/(512*(a^4*c^8*d^{18} + a^{12}*d^2*e^{16} + 8 \\
& *a^{11}*c*d^4*e^{14} + 8*a^5*c^7*d^{16}*e^2 + 28*a^6*c^6*d^{14}*e^4 + 56*a^7*c^5*d^{12} \\
& *e^6 + 70*a^8*c^4*d^{10}*e^8 + 56*a^9*c^3*d^8*e^{10} + 28*a^{10}*c^2*d^6*e^{12})) \\
& - (((65536*a^{15}*c^4*d*e^{24} - 24576*a^4*c^{15}*d^23*e^2 - 212992*a^5*c^{14}*d^2 \\
& 1*e^4 - 352256*a^6*c^{13}*d^{19}*e^6 + 1966080*a^7*c^{12}*d^{17}*e^8 + 10960896*a^8 \\
& *c^{11}*d^{15}*e^{10} + 25460736*a^9*c^{10}*d^{13}*e^{12} + 34750464*a^{10}*c^9*d^{11}*e^{14} \\
& + 30081024*a^{11}*c^8*d^9*e^{16} + 16588800*a^{12}*c^7*d^7*e^{18} + 5554176*a^{13}*c \\
& ^6*d^5*e^{20} + 991232*a^{14}*c^5*d^3*e^{22})/(512*(a^4*c^8*d^{18} + a^{12}*d^2*e^{16} \\
& + 8*a^{11}*c*d^4*e^{14} + 8*a^5*c^7*d^{16}*e^2 + 28*a^6*c^6*d^{14}*e^4 + 56*a^7*c^5 \\
& *d^{12}*e^6 + 70*a^8*c^4*d^{10}*e^8 + 56*a^9*c^3*d^8*e^{10} + 28*a^{10}*c^2*d^6*e^{12} \\
& )) + (x*((49*a^4*e^8*(-a^7*c^3)^{(1/2)} + 9*c^4*d^8*(-a^7*c^3)^{(1/2)} + 12*a^4 \\
& *c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 \\
& + 68*a*c^3*d^6*e^2*(-a^7*c^3)^{(1/2)} - 492*a^3*c*d^2*e^6*(-a^7*c^3)^{(1/2)} + \\
& 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^{(1/2)})/(256*(a^{13}*e^{12} + a^7*c^6*d^{12} + 6*a^{12} \\
& *c*d^2*e^{10} + 6*a^8*c^5*d^{10}*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^{10}*c^3*d^6*e^6 \\
& + 15*a^{11}*c^2*d^4*e^8)))^{(1/2)}*(65536*a^6*c^{15}*d^{24}*e^3 + 589824*a^7*c^{14} \\
& *d^{22}*e^5 + 2293760*a^8*c^{13}*d^{20}*e^7 + 4915200*a^9*c^{12}*d^{18}*e^9 + 589824 \\
& 0*a^{10}*c^{11}*d^{16}*e^{11} + 2752512*a^{11}*c^{10}*d^{14}*e^{13} - 2752512*a^{12}*c^9*d^{12} \\
& *e^{15} - 5898240*a^{13}*c^8*d^{10}*e^{17} - 4915200*a^{14}*c^7*d^8*e^{19} - 2293760*a^{15} \\
& *c^6*d^6*e^{21} - 589824*a^{16}*c^5*d^4*e^{23} - 65536*a^{17}*c^4*d^2*e^{25}))/((128 \\
& *(a^4*c^8*d^{18} + a^{12}*d^2*e^{16} + 8*a^{11}*c*d^4*e^{14} + 8*a^5*c^7*d^{16}*e^2 + 2 \\
& 8*a^6*c^6*d^{14}*e^4 + 56*a^7*c^5*d^{12}*e^6 + 70*a^8*c^4*d^{10}*e^8 + 56*a^9*c^3 \\
& *d^8*e^{10} + 28*a^{10}*c^2*d^6*e^{12}))*((49*a^4*e^8*(-a^7*c^3)^{(1/2)} + 9*c^4*d^8 \\
& *(-a^7*c^3)^{(1/2)} + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5 \\
& *e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^{(1/2)} - 492*a^3*c \\
& *d^2*e^6*(-a^7*c^3)^{(1/2)} + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^{(1/2)})/(256*(a^{13} \\
& *e^{12} + a^7*c^6*d^{12} + 6*a^{12}*c*d^2*e^{10} + 6*a^8*c^5*d^{10}*e^2 + 15*a^9*c^4* \\
& d^8*e^4 + 20*a^{10}*c^3*d^6*e^6 + 15*a^{11}*c^2*d^4*e^8)))^{(1/2)} + (x*(4096*a^{12} \\
& *c^5*d*e^{22} - 1152*a^2*c^{15}*d^{21}*e^2 - 15232*a^3*c^{14}*d^{19}*e^4 - 78336*a^4 \\
& *c^{13}*d^{17}*e^6 - 140800*a^5*c^{12}*d^{15}*e^8 + 489728*a^6*c^{11}*d^{13}*e^{10} + 221 \\
& 9776*a^7*c^{10}*d^{11}*e^{12} + 3155456*a^8*c^9*d^9*e^{14} + 1901056*a^9*c^8*d^7*e^{16} \\
& + 362368*a^{10}*c^7*d^5*e^{18} - 32640*a^{11}*c^6*d^3*e^{20}))/((128*(a^4*c^8*d^{18} \\
& + a^{12}*d^2*e^{16} + 8*a^{11}*c*d^4*e^{14} + 8*a^5*c^7*d^{16}*e^2 + 28*a^6*c^6*d^{14} \\
& *e^4 + 56*a^7*c^5*d^{12}*e^6 + 70*a^8*c^4*d^{10}*e^8 + 56*a^9*c^3*d^8*e^{10} + 2 \\
& 8*a^{10}*c^2*d^6*e^{12}))*((49*a^4*e^8*(-a^7*c^3)^{(1/2)} + 9*c^4*d^8*(-a^7*c^3) \\
& ^{(1/2)} + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6 \\
& *c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^{(1/2)} - 492*a^3*c*d^2*e^6*(-a^7 \\
& *c^3)^{(1/2)} + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^{(1/2)})/(256*(a^{13}*e^{12} + a^7*c^6 \\
& *d^{12} + 6*a^{12}*c*d^2*e^{10} + 6*a^8*c^5*d^{10}*e^2 + 15*a^9*c^4*d^8*e^4 + 20* \\
& a^{10}*c^3*d^6*e^6 + 15*a^{11}*c^2*d^4*e^8)))^{(1/2)}*((49*a^4*e^8*(-a^7*c^3)^{(1/2)} \\
& + 9*c^4*d^8*(-a^7*c^3)^{(1/2)} + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 1 \\
& 56*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^{(1/2)} \\
& ) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^{(1/2)} + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^{(1/2)} \\
& ))/(256*(a^{13}*e^{12} + a^7*c^6*d^{12} + 6*a^{12}*c*d^2*e^{10} + 6*a^8*c^5*d^{10}*e^2 \\
& + 15*a^9*c^4*d^8*e^4 + 20*a^{10}*c^3*d^6*e^6 + 15*a^{11}*c^2*d^4*e^8)))^{(1/2)} + \\
& (x*(81*c^{13}*d^{14}*e^5 - 392*a^7*c^6*e^{19} + 1206*a*c^{12}*d^{12}*e^7 + 12247*a^2 \\
& *c^{11}*d^{10}*e^9 + 58636*a^3*c^{10}*d^8*e^{11} + 114927*a^4*c^9*d^6*e^{13} - 1306*a^5 \\
& *c^8*d^4*e^{15} - 3575*a^6*c^7*d^2*e^{17}))/((128*(a^4*c^8*d^{18} + a^{12}*d^2*e^{16} \\
& + 8*a^{11}*c*d^4*e^{14} + 8*a^5*c^7*d^{16}*e^2 + 28*a^6*c^6*d^{14}*e^4 + 56*a^7*c^5 \\
& *d^{12}*e^6 + 70*a^8*c^4*d^{10}*e^8 + 56*a^9*c^3*d^8*e^{10} + 28*a^{10}*c^2*d^6*e^{12} \\
& ))*((49*a^4*e^8*(-a^7*c^3)^{(1/2)} + 9*c^4*d^8*(-a^7*c^3)^{(1/2)} + 12*a^4*c^5 \\
& *d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + \\
& 68*a*c^3*d^6*e^2*(-a^7*c^3)^{(1/2)} - 492*a^3*c*d^2*e^6*(-a^7*c^3)^{(1/2)} + 3 \\
& 0*a^2*c^2*d^4*e^4*(-a^7*c^3)^{(1/2)})/(256*(a^{13}*e^{12} + a^7*c^6*d^{12} + 6*a^{12} \\
& *c*d^2*e^{10} + 6*a^8*c^5*d^{10}*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^{10}*c^3*d^6*e^6 \\
& + 15*a^{11}*c^2*d^4*e^8)))^{(1/2)} - (729*c^{11}*d^9*e^8 + 2916*a*c^{10}*d^7*e^{10} \\
& + 2009*a^4*c^7*d^5*e^{16} - 2538*a^2*c^9*d^5*e^{12} + 17764*a^3*c^8*d^3*e^{14}))/((25 \\
& 6*(a^4*c^8*d^{18} + a^{12}*d^2*e^{16} + 8*a^{11}*c*d^4*e^{14} + 8*a^5*c^7*d^{16}*e^2 + \\
& 28*a^6*c^6*d^{14}*e^4 + 56*a^7*c^5*d^{12}*e^6 + 70*a^8*c^4*d^{10}*e^8 + 56*a^9*c^
\end{aligned}$$

$$\begin{aligned}
& 3*d^8*e^{10} + 28*a^{10}*c^2*d^6*e^{12}))) * ((49*a^4*e^8*(-a^7*c^3)^{(1/2)} + 9*c^4 \\
& *d^8*(-a^7*c^3)^{(1/2)} + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4* \\
& d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^{(1/2)} - 492*a^3 \\
& *c*d^2*e^6*(-a^7*c^3)^{(1/2)} + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^{(1/2)}) / (256*(a^ \\
& 13*e^{12} + a^7*c^6*d^{12} + 6*a^{12}*c*d^2*e^{10} + 6*a^8*c^5*d^{10}*e^2 + 15*a^9*c^ \\
& 4*d^8*e^4 + 20*a^{10}*c^3*d^6*e^6 + 15*a^{11}*c^2*d^4*e^8)))^{(1/2)} * 2i + (\operatorname{atan}(( \\
& ((x*(81*c^{13}*d^{14}*e^5 - 392*a^7*c^6*e^{19} + 1206*a*c^{12}*d^{12}*e^7 + 12247*a^ \\
& 2*c^{11}*d^{10}*e^9 + 58636*a^3*c^{10}*d^8*e^{11} + 114927*a^4*c^9*d^6*e^{13} - 1306* \\
& a^5*c^8*d^4*e^{15} - 3575*a^6*c^7*d^2*e^{17}))/ (128*(a^4*c^8*d^{18} + a^{12}*d^2*e^ \\
& 16 + 8*a^{11}*c*d^4*e^{14} + 8*a^5*c^7*d^{16}*e^2 + 28*a^6*c^6*d^{14}*e^4 + 56*a^7* \\
& c^5*d^{12}*e^6 + 70*a^8*c^4*d^{10}*e^8 + 56*a^9*c^3*d^8*e^{10} + 28*a^{10}*c^2*d^6* \\
& e^{12})) - (((7*a^{10}*c^5*e^{21} + (9*a*c^{14}*d^{18}*e^3)/4 + (103*a^2*c^{13}*d^{16}*e^ \\
& 5)/4 + (429*a^3*c^{12}*d^{14}*e^7)/4 + (2319*a^4*c^{11}*d^{12}*e^9)/4 + (10019*a^5* \\
& c^{10}*d^{10}*e^{11})/4 + (6009*a^6*c^9*d^8*e^{13})/4 - (11105*a^7*c^8*d^6*e^{15})/4 \\
& - (9803*a^8*c^7*d^4*e^{17})/4 - 174*a^9*c^6*d^2*e^{19}) / (a^4*c^8*d^{18} + a^{12}*d^ \\
& 2*e^{16} + 8*a^{11}*c*d^4*e^{14} + 8*a^5*c^7*d^{16}*e^2 + 28*a^6*c^6*d^{14}*e^4 + 56* \\
& a^7*c^5*d^{12}*e^6 + 70*a^8*c^4*d^{10}*e^8 + 56*a^9*c^3*d^8*e^{10} + 28*a^{10}*c^2* \\
& d^6*e^{12}) + ((a*e^2 + 9*c*d^2)*(-d^3*e^7)^{(1/2)}) * ((x*(4096*a^{12}*c^5*d*e^{22} - \\
& 1152*a^2*c^{15}*d^{21}*e^2 - 15232*a^3*c^{14}*d^{19}*e^4 - 78336*a^4*c^{13}*d^{17}*e^6 \\
& - 140800*a^5*c^{12}*d^{15}*e^8 + 489728*a^6*c^{11}*d^{13}*e^{10} + 2219776*a^7*c^{10}* \\
& d^{11}*e^{12} + 3155456*a^8*c^9*d^9*e^{14} + 1901056*a^9*c^8*d^7*e^{16} + 362368*a^ \\
& 10*c^7*d^5*e^{18} - 32640*a^{11}*c^6*d^3*e^{20}))/ (128*(a^4*c^8*d^{18} + a^{12}*d^2*e^ \\
& 16 + 8*a^{11}*c*d^4*e^{14} + 8*a^5*c^7*d^{16}*e^2 + 28*a^6*c^6*d^{14}*e^4 + 56*a^7* \\
& c^5*d^{12}*e^6 + 70*a^8*c^4*d^{10}*e^8 + 56*a^9*c^3*d^8*e^{10} + 28*a^{10}*c^2*d^6* \\
& e^{12})) - (((128*a^{15}*c^4*d*e^{24} - 48*a^4*c^{15}*d^{23}*e^2 - 416*a^5*c^{14}*d^{21} \\
& *e^4 - 688*a^6*c^{13}*d^{19}*e^6 + 3840*a^7*c^{12}*d^{17}*e^8 + 21408*a^8*c^{11}*d^{15} \\
& *e^{10} + 49728*a^9*c^{10}*d^{13}*e^{12} + 67872*a^{10}*c^9*d^{11}*e^{14} + 58752*a^{11}*c^ \\
& 8*d^9*e^{16} + 32400*a^{12}*c^7*d^7*e^{18} + 10848*a^{13}*c^6*d^5*e^{20} + 1936*a^{14}* \\
& c^5*d^3*e^{22}) / (a^4*c^8*d^{18} + a^{12}*d^2*e^{16} + 8*a^{11}*c*d^4*e^{14} + 8*a^5*c^7* \\
& d^{16}*e^2 + 28*a^6*c^6*d^{14}*e^4 + 56*a^7*c^5*d^{12}*e^6 + 70*a^8*c^4*d^{10}*e^8 \\
& + 56*a^9*c^3*d^8*e^{10} + 28*a^{10}*c^2*d^6*e^{12}) - (x*(a*e^2 + 9*c*d^2)*(-d^3 \\
& *e^7)^{(1/2)}) * (65536*a^6*c^{15}*d^{24}*e^3 + 589824*a^7*c^{14}*d^{22}*e^5 + 2293760*a^ \\
& 8*c^{13}*d^{20}*e^7 + 4915200*a^9*c^{12}*d^{18}*e^9 + 5898240*a^{10}*c^{11}*d^{16}*e^{11} \\
& + 2752512*a^{11}*c^{10}*d^{14}*e^{13} - 2752512*a^{12}*c^9*d^{12}*e^{15} - 5898240*a^{13}*c^ \\
& 8*d^{10}*e^{17} - 4915200*a^{14}*c^7*d^8*e^{19} - 2293760*a^{15}*c^6*d^6*e^{21} - 5898 \\
& 24*a^{16}*c^5*d^4*e^{23} - 65536*a^{17}*c^4*d^2*e^{25}))/ (512*(c^3*d^9 + a^3*d^3*e^ \\
& 6 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4)*(a^4*c^8*d^{18} + a^{12}*d^2*e^{16} + 8*a^ \\
& 11*c*d^4*e^{14} + 8*a^5*c^7*d^{16}*e^2 + 28*a^6*c^6*d^{14}*e^4 + 56*a^7*c^5*d^{12}* \\
& e^6 + 70*a^8*c^4*d^{10}*e^8 + 56*a^9*c^3*d^8*e^{10} + 28*a^{10}*c^2*d^6*e^{12}))) * ( \\
& a*e^2 + 9*c*d^2)*(-d^3*e^7)^{(1/2)}) / (4*(c^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7* \\
& e^2 + 3*a^2*c*d^5*e^4))) / (4*(c^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7*e^2 + 3*a^ \\
& 2*c*d^5*e^4))) * (a*e^2 + 9*c*d^2)*(-d^3*e^7)^{(1/2)}) / (4*(c^3*d^9 + a^3*d^3*e^ \\
& 6 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4))) * (a*e^2 + 9*c*d^2)*(-d^3*e^7)^{(1/2)} \\
& ) * 1i) / (4*(c^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4)) + ((( \\
& x*(81*c^{13}*d^{14}*e^5 - 392*a^7*c^6*e^{19} + 1206*a*c^{12}*d^{12}*e^7 + 12247*a^2*c^ \\
& 11*d^{10}*e^9 + 58636*a^3*c^{10}*d^8*e^{11} + 114927*a^4*c^9*d^6*e^{13} - 1306*a^5* \\
& c^8*d^4*e^{15} - 3575*a^6*c^7*d^2*e^{17}))/ (128*(a^4*c^8*d^{18} + a^{12}*d^2*e^{16} \\
& + 8*a^{11}*c*d^4*e^{14} + 8*a^5*c^7*d^{16}*e^2 + 28*a^6*c^6*d^{14}*e^4 + 56*a^7*c^5* \\
& d^{12}*e^6 + 70*a^8*c^4*d^{10}*e^8 + 56*a^9*c^3*d^8*e^{10} + 28*a^{10}*c^2*d^6*e^{1 \\
& 2})) + (((7*a^{10}*c^5*e^{21} + (9*a*c^{14}*d^{18}*e^3)/4 + (103*a^2*c^{13}*d^{16}*e^5)/ \\
& 4 + (429*a^3*c^{12}*d^{14}*e^7)/4 + (2319*a^4*c^{11}*d^{12}*e^9)/4 + (10019*a^5*c^1 \\
& 0*d^{10}*e^{11})/4 + (6009*a^6*c^9*d^8*e^{13})/4 - (11105*a^7*c^8*d^6*e^{15})/4 - ( \\
& 9803*a^8*c^7*d^4*e^{17})/4 - 174*a^9*c^6*d^2*e^{19}) / (a^4*c^8*d^{18} + a^{12}*d^2*e^ \\
& 16 + 8*a^{11}*c*d^4*e^{14} + 8*a^5*c^7*d^{16}*e^2 + 28*a^6*c^6*d^{14}*e^4 + 56*a^7* \\
& c^5*d^{12}*e^6 + 70*a^8*c^4*d^{10}*e^8 + 56*a^9*c^3*d^8*e^{10} + 28*a^{10}*c^2*d^6* \\
& e^{12}) - ((a*e^2 + 9*c*d^2)*(-d^3*e^7)^{(1/2)}) * ((x*(4096*a^{12}*c^5*d*e^{22} - 11 \\
& 52*a^2*c^{15}*d^{21}*e^2 - 15232*a^3*c^{14}*d^{19}*e^4 - 78336*a^4*c^{13}*d^{17}*e^6 - \\
& 140800*a^5*c^{12}*d^{15}*e^8 + 489728*a^6*c^{11}*d^{13}*e^{10} + 2219776*a^7*c^{10}*d^{1 \\
& 1}*e^{12} + 3155456*a^8*c^9*d^9*e^{14} + 1901056*a^9*c^8*d^7*e^{16} + 362368*a^{10}*
\end{aligned}$$



$$\begin{aligned}
& c^7 d^5 e^{18} - 32640 a^{11} c^6 d^3 e^{20}) / (128 (a^4 c^8 d^{18} + a^{12} d^2 e^{16} \\
& + 8 a^{11} c d^4 e^{14} + 8 a^5 c^7 d^{16} e^2 + 28 a^6 c^6 d^{14} e^4 + 56 a^7 c^5 d^{12} e^6 + 70 a^8 c^4 d^{10} e^8 + 56 a^9 c^3 d^8 e^{10} + 28 a^{10} c^2 d^6 e^{12})) + (((128 a^{15} c^4 d^2 e^{24} - 48 a^4 c^{15} d^{23} e^2 - 416 a^5 c^{14} d^{21} e^4 - 688 a^6 c^{13} d^{19} e^6 + 3840 a^7 c^{12} d^{17} e^8 + 21408 a^8 c^{11} d^{15} e^{10} + 49728 a^9 c^{10} d^{13} e^{12} + 67872 a^{10} c^9 d^{11} e^{14} + 58752 a^{11} c^8 d^9 e^{16} + 32400 a^{12} c^7 d^7 e^{18} + 10848 a^{13} c^6 d^5 e^{20} + 1936 a^{14} c^5 d^3 e^{22}) / (a^4 c^8 d^{18} + a^{12} d^2 e^{16} + 8 a^{11} c d^4 e^{14} + 8 a^5 c^7 d^{16} e^2 + 28 a^6 c^6 d^{14} e^4 + 56 a^7 c^5 d^{12} e^6 + 70 a^8 c^4 d^{10} e^8 + 56 a^9 c^3 d^8 e^{10} + 28 a^{10} c^2 d^6 e^{12}) + (x (a e^2 + 9 c d^2) (-d^3 e^7)^{1/2}) * (65536 a^6 c^{15} d^{24} e^3 + 589824 a^7 c^{14} d^{22} e^5 + 2293760 a^8 c^{13} d^{20} e^7 + 4915200 a^9 c^{12} d^{18} e^9 + 5898240 a^{10} c^{11} d^{16} e^{11} + 2752512 a^{11} c^{10} d^{14} e^{13} - 2752512 a^{12} c^9 d^{12} e^{15} - 5898240 a^{13} c^8 d^{10} e^{17} - 4915200 a^{14} c^7 d^8 e^{19} - 2293760 a^{15} c^6 d^6 e^{21} - 589824 a^{16} c^5 d^4 e^{23} - 65536 a^{17} c^4 d^2 e^{25})) / (512 (c^3 d^9 + a^3 d^3 e^6 + 3 a^2 c^2 d^7 e^2 + 3 a^2 c^2 d^5 e^4)) * (a^4 c^8 d^{18} + a^{12} d^2 e^{16} + 8 a^{11} c d^4 e^{14} + 8 a^5 c^7 d^{16} e^2 + 28 a^6 c^6 d^{14} e^4 + 56 a^7 c^5 d^{12} e^6 + 70 a^8 c^4 d^{10} e^8 + 56 a^9 c^3 d^8 e^{10} + 28 a^{10} c^2 d^6 e^{12})) * (a e^2 + 9 c d^2) (-d^3 e^7)^{1/2} / (4 (c^3 d^9 + a^3 d^3 e^6 + 3 a^2 c^2 d^7 e^2 + 3 a^2 c^2 d^5 e^4)) / (4 (c^3 d^9 + a^3 d^3 e^6 + 3 a^2 c^2 d^7 e^2 + 3 a^2 c^2 d^5 e^4)) * (a e^2 + 9 c d^2) (-d^3 e^7)^{1/2} / (4 (c^3 d^9 + a^3 d^3 e^6 + 3 a^2 c^2 d^7 e^2 + 3 a^2 c^2 d^5 e^4)) * (a e^2 + 9 c d^2) (-d^3 e^7)^{1/2} * i / (4 (c^3 d^9 + a^3 d^3 e^6 + 3 a^2 c^2 d^7 e^2 + 3 a^2 c^2 d^5 e^4)) / (((729 c^{11} d^9 e^8) / 256 + (729 a^2 c^{10} d^7 e^{10}) / 64 + (2009 a^4 c^7 d^5 e^{16}) / 256 - (1269 a^2 c^9 d^5 e^{12}) / 128 + (4441 a^3 c^8 d^3 e^{14}) / 64) / (a^4 c^8 d^{18} + a^{12} d^2 e^{16} + 8 a^{11} c d^4 e^{14} + 8 a^5 c^7 d^{16} e^2 + 28 a^6 c^6 d^{14} e^4 + 56 a^7 c^5 d^{12} e^6 + 70 a^8 c^4 d^{10} e^8 + 56 a^9 c^3 d^8 e^{10} + 28 a^{10} c^2 d^6 e^{12}) + (((x (81 c^{13} d^{14} e^5 - 392 a^7 c^6 e^{19} + 1206 a^2 c^{12} d^{12} e^7 + 12247 a^2 c^{11} d^{10} e^9 + 58636 a^3 c^{10} d^8 e^{11} + 114927 a^4 c^9 d^6 e^{13} - 1306 a^5 c^8 d^4 e^{15} - 3575 a^6 c^7 d^2 e^{17})) / (128 (a^4 c^8 d^{18} + a^{12} d^2 e^{16} + 8 a^{11} c d^4 e^{14} + 8 a^5 c^7 d^{16} e^2 + 28 a^6 c^6 d^{14} e^4 + 56 a^7 c^5 d^{12} e^6 + 70 a^8 c^4 d^{10} e^8 + 56 a^9 c^3 d^8 e^{10} + 28 a^{10} c^2 d^6 e^{12})) - (((7 a^{10} c^5 e^{21} + (9 a^2 c^{14} d^{18} e^3) / 4 + (10 3 a^2 c^{13} d^{16} e^5) / 4 + (429 a^3 c^{12} d^{14} e^7) / 4 + (2319 a^4 c^{11} d^{12} e^9) / 4 + (10019 a^5 c^{10} d^{10} e^{11}) / 4 + (6009 a^6 c^9 d^8 e^{13}) / 4 - (11105 a^7 c^8 d^6 e^{15}) / 4 - (9803 a^8 c^7 d^4 e^{17}) / 4 - 174 a^9 c^6 d^2 e^{19}) / (a^4 c^8 d^{18} + a^{12} d^2 e^{16} + 8 a^{11} c d^4 e^{14} + 8 a^5 c^7 d^{16} e^2 + 28 a^6 c^6 d^{14} e^4 + 56 a^7 c^5 d^{12} e^6 + 70 a^8 c^4 d^{10} e^8 + 56 a^9 c^3 d^8 e^{10} + 28 a^{10} c^2 d^6 e^{12}) + ((a e^2 + 9 c d^2) (-d^3 e^7)^{1/2}) * ((x (4096 a^{12} c^5 d^2 e^{22} - 1152 a^2 c^{15} d^{21} e^2 - 15232 a^3 c^{14} d^{19} e^4 - 78336 a^4 c^{13} d^{17} e^6 - 140800 a^5 c^{12} d^{15} e^8 + 489728 a^6 c^{11} d^{13} e^{10} + 2219776 a^7 c^{10} d^{11} e^{12} + 3155456 a^8 c^9 d^9 e^{14} + 1901056 a^9 c^8 d^7 e^{16} + 362368 a^{10} c^7 d^5 e^{18} - 32640 a^{11} c^6 d^3 e^{20})) / (128 (a^4 c^8 d^{18} + a^{12} d^2 e^{16} + 8 a^{11} c d^4 e^{14} + 8 a^5 c^7 d^{16} e^2 + 28 a^6 c^6 d^{14} e^4 + 56 a^7 c^5 d^{12} e^6 + 70 a^8 c^4 d^{10} e^8 + 56 a^9 c^3 d^8 e^{10} + 28 a^{10} c^2 d^6 e^{12})) - (((128 a^{15} c^4 d^2 e^{24} - 48 a^4 c^{15} d^{23} e^2 - 416 a^5 c^{14} d^{21} e^4 - 688 a^6 c^{13} d^{19} e^6 + 3840 a^7 c^{12} d^{17} e^8 + 21408 a^8 c^{11} d^{15} e^{10} + 49728 a^9 c^{10} d^{13} e^{12} + 67872 a^{10} c^9 d^{11} e^{14} + 58752 a^{11} c^8 d^9 e^{16} + 32400 a^{12} c^7 d^7 e^{18} + 10848 a^{13} c^6 d^5 e^{20} + 1936 a^{14} c^5 d^3 e^{22}) / (a^4 c^8 d^{18} + a^{12} d^2 e^{16} + 8 a^{11} c d^4 e^{14} + 8 a^5 c^7 d^{16} e^2 + 28 a^6 c^6 d^{14} e^4 + 56 a^7 c^5 d^{12} e^6 + 70 a^8 c^4 d^{10} e^8 + 56 a^9 c^3 d^8 e^{10} + 28 a^{10} c^2 d^6 e^{12})) - (x (a e^2 + 9 c d^2) (-d^3 e^7)^{1/2}) * (65536 a^6 c^{15} d^{24} e^3 + 589824 a^7 c^{14} d^{22} e^5 + 2293760 a^8 c^{13} d^{20} e^7 + 4915200 a^9 c^{12} d^{18} e^9 + 5898240 a^{10} c^{11} d^{16} e^{11} + 2752512 a^{11} c^{10} d^{14} e^{13} - 2752512 a^{12} c^9 d^{12} e^{15} - 5898240 a^{13} c^8 d^{10} e^{17} - 4915200 a^{14} c^7 d^8 e^{19} - 2293760 a^{15} c^6 d^6 e^{21} - 589824 a^{16} c^5 d^4 e^{23} - 65536 a^{17} c^4 d^2 e^{25})) / (512 (c^3 d^9 + a^3 d^3 e^6 + 3 a^2 c^2 d^7 e^2 + 3 a^2 c^2 d^5 e^4)) * (a^4 c^8 d^{18} + a^{12} d^2 e^{16} + 8 a^{11} c d^4 e^{14} + 8 a^5 c^7 d^{16} e^2 + 28 a^6 c^6 d^{14} e^4
\end{aligned}$$



$$3.131 \quad \int \frac{(d+ex^2)^4}{d^2-e^2x^4} dx$$

Optimal. Leaf size=51

$$\frac{8d^{5/2} \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{e}} - 7d^2x - \frac{4}{3}dex^3 - \frac{1}{5}e^2x^5$$

Rubi [A] time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1150, 390, 208}

$$\frac{8d^{5/2} \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{e}} - 7d^2x - \frac{4}{3}dex^3 - \frac{1}{5}e^2x^5$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^4/(d^2 - e^2\*x^4), x]

[Out] -7\*d^2\*x - (4\*d\*e\*x^3)/3 - (e^2\*x^5)/5 + (8\*d^(5/2)\*ArcTanh[(Sqrt[e]\*x)/Sqrt[d]])/Sqrt[e]

Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 390

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Int[PolynomialDivide[(a + b\*x^n)^p, (c + d\*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 1150

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[(d + e\*x^2)^(p+q)\*(a/d + (c\*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x] && EqQ[c\*d^2 + a\*e^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{(d+ex^2)^4}{d^2-e^2x^4} dx &= \int \frac{(d+ex^2)^3}{d-ex^2} dx \\ &= \int \left( -7d^2 - 4dex^2 - e^2x^4 + \frac{8d^3}{d-ex^2} \right) dx \\ &= -7d^2x - \frac{4}{3}dex^3 - \frac{e^2x^5}{5} + (8d^3) \int \frac{1}{d-ex^2} dx \\ &= -7d^2x - \frac{4}{3}dex^3 - \frac{e^2x^5}{5} + \frac{8d^{5/2} \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{e}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.00

$$\frac{8d^{5/2} \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{e}} - 7d^2x - \frac{4}{3}dex^3 - \frac{1}{5}e^2x^5$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^4/(d^2 - e^2\*x^4), x]

[Out]  $-7*d^2*x - (4*d*e*x^3)/3 - (e^2*x^5)/5 + (8*d^{(5/2)}*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^2)^4}{d^2 - e^2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)^4/(d^2 - e^2\*x^4), x]

[Out] IntegrateAlgebraic[(d + e\*x^2)^4/(d^2 - e^2\*x^4), x]

**fricas** [A] time = 0.83, size = 116, normalized size = 2.27

$$\left[ -\frac{1}{5}e^2x^5 - \frac{4}{3}dex^3 + 4d^2\sqrt{\frac{d}{e}} \log\left(\frac{ex^2 + 2ex\sqrt{\frac{d}{e}} + d}{ex^2 - d}\right) - 7d^2x, -\frac{1}{5}e^2x^5 - \frac{4}{3}dex^3 - 8d^2\sqrt{-\frac{d}{e}} \arctan\left(\frac{ex\sqrt{-\frac{d}{e}}}{d}\right) - 7d^2x \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^4/(-e^2\*x^4+d^2), x, algorithm="fricas")

[Out]  $[-1/5*e^2*x^5 - 4/3*d*e*x^3 + 4*d^2*\sqrt{d/e}*\log((e*x^2 + 2*e*x*\sqrt{d/e} + d)/(e*x^2 - d)) - 7*d^2*x, -1/5*e^2*x^5 - 4/3*d*e*x^3 - 8*d^2*\sqrt{-d/e}*\arctan(e*x*\sqrt{-d/e}/d) - 7*d^2*x]$

**giac** [B] time = 0.21, size = 144, normalized size = 2.82

$$4\left(\left(\frac{1}{d^2}\right)^{\frac{1}{2}}d^2e^{\frac{11}{2}} - \left(\frac{1}{d^2}\right)^{\frac{1}{2}}d\sqrt{d}e^{\frac{11}{2}}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\left(\frac{1}{d^2}\right)^{\frac{1}{2}}}\right)e^{(-6)} + 2\left(\left(\frac{1}{d^2}\right)^{\frac{1}{2}}d^2e^{\frac{15}{2}} + \left(\frac{1}{d^2}\right)^{\frac{3}{2}}d^2e^{\frac{15}{2}}\right)e^{(-8)}\log\left(\left(\frac{1}{d^2}\right)^{\frac{1}{2}}e^{\left(-\frac{1}{2}\right)} + x\right) - 2\left(\left(\frac{1}{d^2}\right)^{\frac{1}{2}}d^2e^{\frac{11}{2}} + \left(\frac{1}{d^2}\right)^{\frac{1}{2}}d\sqrt{d}e^{\frac{11}{2}}\right)e^{(-6)}\log\left(-\left(\frac{1}{d^2}\right)^{\frac{1}{2}}e^{\left(-\frac{1}{2}\right)} + x\right) - \frac{1}{15}(3x^5e^{12} + 20dx^3e^{11} + 105d^2xe^{10})e^{(-10)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^4/(-e^2\*x^4+d^2), x, algorithm="giac")

[Out]  $4*((d^2)^{(1/4)}*d^2*e^{(11/2)} - (d^2)^{(1/4)}*d*abs(d)*e^{(11/2)})*\arctan(x*e^{(1/2)}/(d^2)^{(1/4)})*e^{(-6)} + 2*((d^2)^{(1/4)}*d^2*e^{(15/2)} + (d^2)^{(3/4)}*d*e^{(15/2)})*e^{(-8)}*\log(abs((d^2)^{(1/4)}*e^{(-1/2)} + x)) - 2*((d^2)^{(1/4)}*d^2*e^{(11/2)} + (d^2)^{(1/4)}*d*abs(d)*e^{(11/2)})*e^{(-6)}*\log(abs(-(d^2)^{(1/4)}*e^{(-1/2)} + x)) - 1/15*(3*x^5*e^12 + 20*d*x^3*e^11 + 105*d^2*x*e^10)*e^{(-10)}$

**maple** [A] time = 0.00, size = 42, normalized size = 0.82

$$-\frac{e^2x^5}{5} - \frac{4dex^3}{3} + \frac{8d^3 \operatorname{arctanh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} - 7d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^4/(-e^2\*x^4+d^2), x)

[Out]  $-1/5*e^2*x^5 - 4/3*d*e*x^3 - 7*d^2*x + 8*d^3/(d*e)^{(1/2)}*\operatorname{arctanh}(1/(d*e)^{(1/2)}*e*x)$

**maxima** [A] time = 2.25, size = 56, normalized size = 1.10

$$-\frac{1}{5}e^2x^5 - \frac{4}{3}dex^3 - \frac{4d^3 \log\left(\frac{ex - \sqrt{de}}{ex + \sqrt{de}}\right)}{\sqrt{de}} - 7d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^4/(-e^2\*x^4+d^2),x, algorithm="maxima")

[Out]  $-1/5*e^2*x^5 - 4/3*d*e*x^3 - 4*d^3*\log((e*x - \sqrt{d*e})/(e*x + \sqrt{d*e}))/\sqrt{d*e} - 7*d^2*x$

mupad [B] time = 0.09, size = 42, normalized size = 0.82

$$-7d^2x - \frac{e^2x^5}{5} - \frac{4dex^3}{3} - \frac{d^{5/2} \operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) 8i}{\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)^4/(d^2 - e^2\*x^4),x)

[Out]  $-7*d^2*x - (e^2*x^5)/5 - (d^{(5/2)}*\operatorname{atan}((e^{(1/2)}*x)/d^{(1/2)}))*8i)/e^{(1/2)} - (4*d*e*x^3)/3$

sympy [A] time = 0.24, size = 75, normalized size = 1.47

$$-7d^2x - \frac{4dex^3}{3} - \frac{e^2x^5}{5} - 4\sqrt{\frac{d^5}{e}} \log\left(x - \frac{\sqrt{\frac{d^5}{e}}}{d^2}\right) + 4\sqrt{\frac{d^5}{e}} \log\left(x + \frac{\sqrt{\frac{d^5}{e}}}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*4/(-e\*\*2\*x\*\*4+d\*\*2),x)

[Out]  $-7*d**2*x - 4*d*e*x**3/3 - e**2*x**5/5 - 4*\sqrt{d**5/e}*\log(x - \sqrt{d**5/e}/d**2) + 4*\sqrt{d**5/e}*\log(x + \sqrt{d**5/e}/d**2)$

$$3.132 \quad \int \frac{(d+ex^2)^3}{d^2-e^2x^4} dx$$

**Optimal.** Leaf size=38

$$\frac{4d^{3/2} \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - 3dx - \frac{ex^3}{3}$$

**Rubi [A]** time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1150, 390, 208}

$$\frac{4d^{3/2} \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - 3dx - \frac{ex^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^3/(d^2 - e^2\*x^4), x]

[Out] -3\*d\*x - (e\*x^3)/3 + (4\*d^(3/2)\*ArcTanh[(Sqrt[e]\*x)/Sqrt[d]])/Sqrt[e]

**Rule 208**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

**Rule 390**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Int[PolynomialDivide[(a + b\*x^n)^p, (c + d\*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

**Rule 1150**

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[(d + e\*x^2)^(p+q)\*(a/d + (c\*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x] && EqQ[c\*d^2 + a\*e^2, 0] && IntegerQ[p]

**Rubi steps**

$$\begin{aligned} \int \frac{(d+ex^2)^3}{d^2-e^2x^4} dx &= \int \frac{(d+ex^2)^2}{d-ex^2} dx \\ &= \int \left( -3d - ex^2 + \frac{4d^2}{d-ex^2} \right) dx \\ &= -3dx - \frac{ex^3}{3} + (4d^2) \int \frac{1}{d-ex^2} dx \\ &= -3dx - \frac{ex^3}{3} + \frac{4d^{3/2} \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 38, normalized size = 1.00

$$\frac{4d^{3/2} \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - 3dx - \frac{ex^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^3/(d^2 - e^2\*x^4), x]

[Out] -3\*d\*x - (e\*x^3)/3 + (4\*d^(3/2)\*ArcTanh[(Sqrt[e]\*x)/Sqrt[d]])/Sqrt[e]

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^2)^3}{d^2 - e^2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)^3/(d^2 - e^2\*x^4), x]

[Out] IntegrateAlgebraic[(d + e\*x^2)^3/(d^2 - e^2\*x^4), x]

**fricas** [A] time = 0.87, size = 90, normalized size = 2.37

$$\left[ -\frac{1}{3}ex^3 + 2d\sqrt{\frac{d}{e}} \log\left(\frac{ex^2 + 2ex\sqrt{\frac{d}{e}} + d}{ex^2 - d}\right) - 3dx, -\frac{1}{3}ex^3 - 4d\sqrt{-\frac{d}{e}} \arctan\left(\frac{ex\sqrt{-\frac{d}{e}}}{d}\right) - 3dx \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^3/(-e^2\*x^4+d^2), x, algorithm="fricas")

[Out] [-1/3\*e\*x^3 + 2\*d\*sqrt(d/e)\*log((e\*x^2 + 2\*e\*x\*sqrt(d/e) + d)/(e\*x^2 - d)) - 3\*d\*x, -1/3\*e\*x^3 - 4\*d\*sqrt(-d/e)\*arctan(e\*x\*sqrt(-d/e)/d) - 3\*d\*x]

**giac** [B] time = 0.23, size = 123, normalized size = 3.24

$$2\left((d^2)^{\frac{1}{4}}de^{\frac{11}{2}} - (d^2)^{\frac{1}{4}}|d|e^{\frac{11}{2}}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{(d^2)^{\frac{1}{4}}}\right)e^{(-6)} + \left((d^2)^{\frac{1}{4}}de^{\frac{15}{2}} + (d^2)^{\frac{3}{4}}e^{\frac{15}{2}}\right)e^{(-8)}\log\left(\left((d^2)^{\frac{1}{4}}e^{(-\frac{1}{2})} + x\right)\right) - \left((d^2)^{\frac{1}{4}}de^{\frac{11}{2}} + (d^2)^{\frac{1}{4}}|d|e^{\frac{11}{2}}\right)e^{(-6)}\log\left(\left(- (d^2)^{\frac{1}{4}}e^{(\frac{1}{2})} + x\right)\right) - \frac{1}{3}(x^3e^7 + 9dxe^6)e^{(-6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^3/(-e^2\*x^4+d^2), x, algorithm="giac")

[Out] 2\*((d^2)^(1/4)\*d\*e^(11/2) - (d^2)^(1/4)\*abs(d)\*e^(11/2))\*arctan(x\*e^(1/2)/((d^2)^(1/4))\*e^(-6) + ((d^2)^(1/4)\*d\*e^(15/2) + (d^2)^(3/4)\*e^(15/2))\*e^(-8)\*log(abs((d^2)^(1/4)\*e^(-1/2) + x)) - ((d^2)^(1/4)\*d\*e^(11/2) + (d^2)^(1/4)\*abs(d)\*e^(11/2))\*e^(-6)\*log(abs(-(d^2)^(1/4)\*e^(-1/2) + x)) - 1/3\*(x^3\*e^7 + 9\*d\*x\*e^6)\*e^(-6)

**maple** [A] time = 0.00, size = 31, normalized size = 0.82

$$-\frac{e x^3}{3} + \frac{4d^2 \operatorname{arctanh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} - 3dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^3/(-e^2\*x^4+d^2), x)

[Out] -1/3\*e\*x^3-3\*d\*x+4\*d^2/(d\*e)^(1/2)\*arctanh(1/(d\*e)^(1/2)\*e\*x)

**maxima** [A] time = 2.45, size = 45, normalized size = 1.18

$$-\frac{1}{3}ex^3 - \frac{2d^2 \log\left(\frac{ex - \sqrt{de}}{ex + \sqrt{de}}\right)}{\sqrt{de}} - 3dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^3/(-e^2\*x^4+d^2),x, algorithm="maxima")

[Out] -1/3\*e\*x^3 - 2\*d^2\*log((e\*x - sqrt(d\*e))/(e\*x + sqrt(d\*e)))/sqrt(d\*e) - 3\*d\*x

**mupad [B]** time = 0.05, size = 28, normalized size = 0.74

$$\frac{4d^{3/2} \operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{ex^3}{3} - 3dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)^3/(d^2 - e^2\*x^4),x)

[Out] (4\*d^(3/2)\*atanh((e^(1/2)\*x)/d^(1/2)))/e^(1/2) - (e\*x^3)/3 - 3\*d\*x

**sympy [A]** time = 0.20, size = 58, normalized size = 1.53

$$-3dx - \frac{ex^3}{3} - 2\sqrt{\frac{d^3}{e}} \log\left(x - \frac{\sqrt{\frac{d^3}{e}}}{d}\right) + 2\sqrt{\frac{d^3}{e}} \log\left(x + \frac{\sqrt{\frac{d^3}{e}}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*3/(-e\*\*2\*x\*\*4+d\*\*2),x)

[Out] -3\*d\*x - e\*x\*\*3/3 - 2\*sqrt(d\*\*3/e)\*log(x - sqrt(d\*\*3/e)/d) + 2\*sqrt(d\*\*3/e)\*log(x + sqrt(d\*\*3/e)/d)



$$3.133 \quad \int \frac{(d+ex^2)^2}{d^2-e^2x^4} dx$$

Optimal. Leaf size=29

$$\frac{2\sqrt{d} \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - x$$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1150, 388, 208}

$$\frac{2\sqrt{d} \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - x$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^2/(d^2 - e^2\*x^4), x]

[Out] -x + (2\*sqrt[d]\*ArcTanh[(sqrt[e]\*x)/sqrt[d]])/sqrt[e]

Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(d\*x\*(a + b\*x^n)^(p + 1)/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

Rule 1150

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[(d + e\*x^2)^(p + q)\*(a/d + (c\*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x] && EqQ[c\*d^2 + a\*e^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{(d+ex^2)^2}{d^2-e^2x^4} dx &= \int \frac{d+ex^2}{d-ex^2} dx \\ &= -x + (2d) \int \frac{1}{d-ex^2} dx \\ &= -x + \frac{2\sqrt{d} \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{2\sqrt{d} \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - x$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^2/(d^2 - e^2\*x^4), x]

[Out] -x + (2\*sqrt[d]\*ArcTanh[(sqrt[e]\*x)/sqrt[d]])/sqrt[e]

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^2)^2}{d^2 - e^2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)^2/(d^2 - e^2\*x^4), x]

[Out] IntegrateAlgebraic[(d + e\*x^2)^2/(d^2 - e^2\*x^4), x]

**fricas [A]** time = 1.68, size = 73, normalized size = 2.52

$$\left[ \sqrt{\frac{d}{e}} \log\left(\frac{ex^2 + 2ex\sqrt{\frac{d}{e}} + d}{ex^2 - d}\right) - x, -2\sqrt{-\frac{d}{e}} \arctan\left(\frac{ex\sqrt{-\frac{d}{e}}}{d}\right) - x \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2/(-e^2\*x^4+d^2), x, algorithm="fricas")

[Out] [sqrt(d/e)\*log((e\*x^2 + 2\*e\*x\*sqrt(d/e) + d)/(e\*x^2 - d)) - x, -2\*sqrt(-d/e)\*arctan(e\*x\*sqrt(-d/e)/d) - x]

**giac [B]** time = 0.21, size = 118, normalized size = 4.07

$$\frac{\left((d^2)^{\frac{1}{4}}de^{\frac{7}{2}} - (d^2)^{\frac{1}{4}}|d|e^{\frac{7}{2}}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{(d^2)^{\frac{1}{4}}}\right)e^{(-4)}}{d} + \frac{\left((d^2)^{\frac{1}{4}}de^{\frac{11}{2}} + (d^2)^{\frac{3}{4}}e^{\frac{11}{2}}\right)e^{(-6)}\log\left(\left|(d^2)^{\frac{1}{4}}e^{\left(\frac{1}{2}\right)} + x\right|\right)}{2d} - \frac{\left((d^2)^{\frac{1}{4}}de^{\frac{7}{2}} + (d^2)^{\frac{1}{4}}|d|e^{\frac{7}{2}}\right)e^{(-4)}\log\left(\left|-(d^2)^{\frac{1}{4}}e^{\left(\frac{1}{2}\right)} + x\right|\right)}{2d} - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2/(-e^2\*x^4+d^2), x, algorithm="giac")

[Out] ((d^2)^(1/4)\*d\*e^(7/2) - (d^2)^(1/4)\*abs(d)\*e^(7/2))\*arctan(x\*e^(1/2)/(d^2)^(1/4))\*e^(-4)/d + 1/2\*((d^2)^(1/4)\*d\*e^(11/2) + (d^2)^(3/4)\*e^(11/2))\*e^(-6)\*log(abs((d^2)^(1/4)\*e^(-1/2) + x))/d - 1/2\*((d^2)^(1/4)\*d\*e^(7/2) + (d^2)^(1/4)\*abs(d)\*e^(7/2))\*e^(-4)\*log(abs(-(d^2)^(1/4)\*e^(-1/2) + x))/d - x

**maple [A]** time = 0.00, size = 22, normalized size = 0.76

$$\frac{2d \operatorname{arctanh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^2/(-e^2\*x^4+d^2), x)

[Out] -x+2\*d/(d\*e)^(1/2)\*arctanh(1/(d\*e)^(1/2)\*e\*x)

**maxima [A]** time = 2.45, size = 36, normalized size = 1.24

$$-\frac{d \log\left(\frac{ex - \sqrt{de}}{ex + \sqrt{de}}\right)}{\sqrt{de}} - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2/(-e^2\*x^4+d^2), x, algorithm="maxima")

[Out]  $-d \cdot \log\left(\frac{e \cdot x - \sqrt{d \cdot e}}{e \cdot x + \sqrt{d \cdot e}}\right) / \sqrt{d \cdot e} - x$

**mupad [B]** time = 4.43, size = 21, normalized size = 0.72

$$\frac{2 \sqrt{d} \operatorname{atanh}\left(\frac{\sqrt{e} x}{\sqrt{d}}\right)}{\sqrt{e}} - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x^2)^2/(d^2 - e^2*x^4), x)`

[Out]  $(2 \cdot d^{1/2} \cdot \operatorname{atanh}(e^{1/2} \cdot x / d^{1/2})) / e^{1/2} - x$

**sympy [A]** time = 0.18, size = 34, normalized size = 1.17

$$-x - \sqrt{\frac{d}{e}} \log\left(x - \sqrt{\frac{d}{e}}\right) + \sqrt{\frac{d}{e}} \log\left(x + \sqrt{\frac{d}{e}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**2/(-e**2*x**4+d**2), x)`

[Out]  $-x - \sqrt{d/e} \cdot \log(x - \sqrt{d/e}) + \sqrt{d/e} \cdot \log(x + \sqrt{d/e})$

$$3.134 \quad \int \frac{d+ex^2}{d^2-e^2x^4} dx$$

**Optimal.** Leaf size=24

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

**Rubi [A]** time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {1150, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)/(d^2 - e^2\*x^4), x]

[Out] ArcTanh[(Sqrt[e]\*x)/Sqrt[d]]/(Sqrt[d]\*Sqrt[e])

**Rule 208**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

**Rule 1150**

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[(d + e\*x^2)^(p+q)\*(a/d + (c\*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x] && EqQ[c\*d^2 + a\*e^2, 0] && IntegerQ[p]

**Rubi steps**

$$\begin{aligned} \int \frac{d+ex^2}{d^2-e^2x^4} dx &= \int \frac{1}{d-ex^2} dx \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 24, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)/(d^2 - e^2\*x^4), x]

[Out] ArcTanh[(Sqrt[e]\*x)/Sqrt[d]]/(Sqrt[d]\*Sqrt[e])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^2}{d^2-e^2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)/(d^2 - e^2\*x^4), x]

[Out] IntegrateAlgebraic[(d + e\*x^2)/(d^2 - e^2\*x^4), x]

**fricas** [A] time = 1.09, size = 68, normalized size = 2.83

$$\left[ \frac{\sqrt{de} \log\left(\frac{ex^2 + 2\sqrt{de}x + d}{ex^2 - d}\right)}{2de}, -\frac{\sqrt{-de} \arctan\left(\frac{\sqrt{-de}x}{d}\right)}{de} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(-e^2\*x^4+d^2), x, algorithm="fricas")

[Out] [1/2\*sqrt(d\*e)\*log((e\*x^2 + 2\*sqrt(d\*e)\*x + d)/(e\*x^2 - d))/(d\*e), -sqrt(-d\*e)\*arctan(sqrt(-d\*e)\*x/d)/(d\*e)]

**giac** [B] time = 0.29, size = 116, normalized size = 4.83

$$\frac{\left((d^2)^{\frac{1}{4}}de^{\frac{7}{2}} - (d^2)^{\frac{1}{4}}|d|e^{\frac{7}{2}}\right) \arctan\left(\frac{xe^{\frac{1}{2}}}{(d^2)^{\frac{1}{4}}}\right) e^{(-4)}}{2d^2} + \frac{\left((d^2)^{\frac{1}{4}}de^{\frac{11}{2}} + (d^2)^{\frac{3}{4}}e^{\frac{11}{2}}\right) e^{(-6)} \log\left(\left|(d^2)^{\frac{1}{4}}e^{\left(\frac{1}{2}\right)} + x\right|\right)}{4d^2} - \frac{\left((d^2)^{\frac{1}{4}}de^{\frac{7}{2}} + (d^2)^{\frac{1}{4}}|d|e^{\frac{7}{2}}\right) e^{(-4)} \log\left(\left|-(d^2)^{\frac{1}{4}}e^{\left(\frac{1}{2}\right)} + x\right|\right)}{4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(-e^2\*x^4+d^2), x, algorithm="giac")

[Out] 1/2\*((d^2)^(1/4)\*d\*e^(7/2) - (d^2)^(1/4)\*abs(d)\*e^(7/2))\*arctan(x\*e^(1/2)/(d^2)^(1/4))\*e^(-4)/d^2 + 1/4\*((d^2)^(1/4)\*d\*e^(11/2) + (d^2)^(3/4)\*e^(11/2))\*e^(-6)\*log(abs((d^2)^(1/4)\*e^(-1/2) + x))/d^2 - 1/4\*((d^2)^(1/4)\*d\*e^(7/2) + (d^2)^(1/4)\*abs(d)\*e^(7/2))\*e^(-4)\*log(abs(-(d^2)^(1/4)\*e^(-1/2) + x))/d^2

**maple** [A] time = 0.00, size = 16, normalized size = 0.67

$$\frac{\operatorname{arctanh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)/(-e^2\*x^4+d^2), x)

[Out] 1/(d\*e)^(1/2)\*arctanh(1/(d\*e)^(1/2)\*e\*x)

**maxima** [A] time = 2.35, size = 31, normalized size = 1.29

$$\frac{\log\left(\frac{ex - \sqrt{de}}{ex + \sqrt{de}}\right)}{2\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(-e^2\*x^4+d^2), x, algorithm="maxima")

[Out] -1/2\*log((e\*x - sqrt(d\*e))/(e\*x + sqrt(d\*e)))/sqrt(d\*e)

**mupad** [B] time = 0.06, size = 16, normalized size = 0.67

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x^2)/(d^2 - e^2*x^4),x)`

[Out] `atanh((e^(1/2)*x)/d^(1/2))/(d^(1/2)*e^(1/2))`

**sympy [B]** time = 0.15, size = 46, normalized size = 1.92

$$-\frac{\sqrt{\frac{1}{de}} \log\left(-d\sqrt{\frac{1}{de}} + x\right)}{2} + \frac{\sqrt{\frac{1}{de}} \log\left(d\sqrt{\frac{1}{de}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)/(-e**2*x**4+d**2),x)`

[Out] `-sqrt(1/(d*e))*log(-d*sqrt(1/(d*e)) + x)/2 + sqrt(1/(d*e))*log(d*sqrt(1/(d*e)) + x)/2`

$$3.135 \quad \int \frac{1}{(d+ex^2)(d^2-e^2x^4)} dx$$

Optimal. Leaf size=72

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{e}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{4d^{5/2}\sqrt{e}} + \frac{x}{4d^2(d+ex^2)}$$

Rubi [A] time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$ , Rules used = {1150, 414, 522, 208, 205}

$$\frac{x}{4d^2(d+ex^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{e}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{4d^{5/2}\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x^2)\*(d^2 - e^2\*x^4)),x]

[Out] x/(4\*d^2\*(d + e\*x^2)) + ArcTan[(Sqrt[e]\*x)/Sqrt[d]]/(2\*d^(5/2)\*Sqrt[e]) + ArcTanh[(Sqrt[e]\*x)/Sqrt[d]]/(4\*d^(5/2)\*Sqrt[e])

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 414

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> -Simp[(b\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(p+1)\*(b\*c - a\*d)), x] + Dist[1/(a\*n\*(p+1)\*(b\*c - a\*d)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q\*Simp[b\*c + n\*(p+1)\*(b\*c - a\*d) + d\*b\*(n\*(p+q+2)+1)\*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e\_) + (f\_.)\*(x\_)^(n\_))/((a\_) + (b\_.)\*(x\_)^(n\_))\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Dist[(b\*e - a\*f)/(b\*c - a\*d), Int[1/(a + b\*x^n), x], x] - Dist[(d\*e - c\*f)/(b\*c - a\*d), Int[1/(c + d\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 1150

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[(d + e\*x^2)^(p+q)\*(a/d + (c\*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x] && EqQ[c\*d^2 + a\*e^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex^2)(d^2-e^2x^4)} dx &= \int \frac{1}{(d-ex^2)(d+ex^2)^2} dx \\
&= \frac{x}{4d^2(d+ex^2)} - \frac{\int \frac{-3de+e^2x^2}{(d-ex^2)(d+ex^2)} dx}{4d^2e} \\
&= \frac{x}{4d^2(d+ex^2)} + \frac{\int \frac{1}{d-ex^2} dx}{4d^2} + \frac{\int \frac{1}{d+ex^2} dx}{2d^2} \\
&= \frac{x}{4d^2(d+ex^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{e}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{4d^{5/2}\sqrt{e}}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 65, normalized size = 0.90

$$\frac{\frac{\sqrt{d}x}{d+ex^2} + \frac{2 \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}}{4d^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x^2)\*(d^2 - e^2\*x^4)), x]

[Out] ((Sqrt[d]\*x)/(d + e\*x^2) + (2\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/Sqrt[e] + ArcTan h[(Sqrt[e]\*x)/Sqrt[d]]/Sqrt[e])/(4\*d^(5/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e\*x^2)\*(d^2 - e^2\*x^4)), x]

[Out] IntegrateAlgebraic[1/((d + e\*x^2)\*(d^2 - e^2\*x^4)), x]

**fricas [A]** time = 0.88, size = 189, normalized size = 2.62

$$\left[ \frac{2dex + 4(ex^2 + d)\sqrt{de} \arctan\left(\frac{\sqrt{de}x}{d}\right) + (ex^2 + d)\sqrt{de} \log\left(\frac{ex^2 + 2\sqrt{de}x + d}{ex^2 - d}\right)}{8(d^3e^2x^2 + d^4e)}, \frac{dex - (ex^2 + d)\sqrt{-de} \arctan\left(\frac{\sqrt{-de}x}{d}\right) - (ex^2 + d)\sqrt{-de} \log\left(\frac{ex^2 - 2\sqrt{-de}x - d}{ex^2 + d}\right)}{4(d^3e^2x^2 + d^4e)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)/(-e^2\*x^4+d^2), x, algorithm="fricas")

[Out] [1/8\*(2\*d\*e\*x + 4\*(e\*x^2 + d)\*sqrt(d\*e)\*arctan(sqrt(d\*e)\*x/d) + (e\*x^2 + d)\*sqrt(d\*e)\*log((e\*x^2 + 2\*sqrt(d\*e)\*x + d)/(e\*x^2 - d)))/(d^3\*e^2\*x^2 + d^4\*e), 1/4\*(d\*e\*x - (e\*x^2 + d)\*sqrt(-d\*e)\*arctan(sqrt(-d\*e)\*x/d) - (e\*x^2 + d)\*sqrt(-d\*e)\*log((e\*x^2 - 2\*sqrt(-d\*e)\*x - d)/(e\*x^2 + d)))/(d^3\*e^2\*x^2 + d^4\*e)]

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(1/(e\*x^2+d)/(-e^2\*x^4+d^2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output:  $-\left(\left(d^2 \exp(2)^3\right)^{1/4} \operatorname{abs}(d) \exp(1)^2 - d \exp(2) \left(d^2 \exp(2)^3\right)^{1/4}\right) / \left(4 d^4 \exp(2) \exp(1)^2 - 4 d^4 \exp(2)^2\right) \ln(\operatorname{abs}(x - (d^2 / \exp(2))^{1/4})) + \left(\left(d^2 \exp(2)^3\right)^{1/4}\right)^3 / \left(4 d^4 \exp(2)^2 \exp(1) - 4 d^4 \exp(1) \exp(2)^2\right) \ln(\operatorname{abs}(x + (d^2 / \exp(2))^{1/4})) - \left(\left(d^2 \exp(2)^3\right)^{1/4} \operatorname{abs}(d) \exp(1)^2 + d \exp(2) \left(d^2 \exp(2)^3\right)^{1/4}\right) / \left(2 d^4 \exp(2) \exp(1)^2 - 2 d^4 \exp(2)^2\right) \operatorname{atan}(x / (d^2 / \exp(2))^{1/4}) - 2 \exp(1)^2 * 1/2 / (\exp(2) * d^2 - d^2 \exp(1)^2) / \sqrt{d \exp(1)} * \operatorname{atan}(x \exp(1) / \sqrt{d \exp(1)})$

**maple [A]** time = 0.01, size = 55, normalized size = 0.76

$$\frac{x}{4(e x^2 + d) d^2} + \frac{\operatorname{arctanh}\left(\frac{e x}{\sqrt{d e}}\right)}{4 \sqrt{d e} d^2} + \frac{\operatorname{arctan}\left(\frac{e x}{\sqrt{d e}}\right)}{2 \sqrt{d e} d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x^2+d)/(-e^2\*x^4+d^2),x)

[Out]  $1/4 * x / d^2 / (e * x^2 + d) + 1/2 / d^2 / (d * e)^{1/2} * \operatorname{arctan}(1 / (d * e)^{1/2} * e * x) + 1/4 / d^2 / (d * e)^{1/2} * \operatorname{arctanh}(1 / (d * e)^{1/2} * e * x)$

**maxima [A]** time = 2.44, size = 71, normalized size = 0.99

$$\frac{x}{4(d^2 e x^2 + d^3)} + \frac{\operatorname{arctan}\left(\frac{e x}{\sqrt{d e}}\right)}{2 \sqrt{d e} d^2} - \frac{\log\left(\frac{e x - \sqrt{d e}}{e x + \sqrt{d e}}\right)}{8 \sqrt{d e} d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)/(-e^2\*x^4+d^2),x, algorithm="maxima")

[Out]  $1/4 * x / (d^2 * e * x^2 + d^3) + 1/2 * \operatorname{arctan}(e * x / \sqrt{d * e}) / (\sqrt{d * e} * d^2) - 1/8 * \log((e * x - \sqrt{d * e}) / (e * x + \sqrt{d * e})) / (\sqrt{d * e} * d^2)$

**mupad [B]** time = 0.16, size = 74, normalized size = 1.03

$$\frac{x}{4 d^2 (e x^2 + d)} + \frac{\operatorname{atanh}\left(\frac{x \sqrt{d^5} e}{d^3}\right) \sqrt{d^5} e}{4 d^5 e} - \frac{\operatorname{atanh}\left(\frac{x \sqrt{-d^5} e}{d^3}\right) \sqrt{-d^5} e}{2 d^5 e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d^2 - e^2\*x^4)\*(d + e\*x^2)),x)

[Out]  $x / (4 * d^2 * (d + e * x^2)) + (\operatorname{atanh}((x * (d^5 * e)^{1/2}) / d^3) * (d^5 * e)^{1/2}) / (4 * d^5 * e) - (\operatorname{atanh}((x * (-d^5 * e)^{1/2}) / d^3) * (-d^5 * e)^{1/2}) / (2 * d^5 * e)$

**sympy [B]** time = 0.45, size = 226, normalized size = 3.14

$$\frac{x}{4 d^5 + 4 d^2 e x^2} - \frac{\sqrt{\frac{1}{d^5 e}} \log\left(-\frac{d^6 e \left(\frac{1}{d^5 e}\right)^{\frac{3}{2}}}{10} - \frac{9 d^3 \sqrt{\frac{1}{d^5 e}}}{10} + x\right)}{8} + \frac{\sqrt{\frac{1}{d^5 e}} \log\left(\frac{d^6 e \left(\frac{1}{d^5 e}\right)^{\frac{3}{2}}}{10} + \frac{9 d^3 \sqrt{\frac{1}{d^5 e}}}{10} + x\right)}{8} - \frac{\sqrt{-\frac{1}{d^5 e}} \log\left(-\frac{4 d^6 e \left(\frac{1}{d^5 e}\right)^{\frac{3}{2}}}{5} - \frac{9 d^3 \sqrt{\frac{1}{d^5 e}}}{5} + x\right)}{4} + \frac{\sqrt{-\frac{1}{d^5 e}} \log\left(\frac{4 d^6 e \left(\frac{1}{d^5 e}\right)^{\frac{3}{2}}}{5} + \frac{9 d^3 \sqrt{\frac{1}{d^5 e}}}{5} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x\*\*2+d)/(-e\*\*2\*x\*\*4+d\*\*2),x)

[Out]  $x / (4 * d ** 3 + 4 * d ** 2 * e * x ** 2) - \sqrt{1 / (d ** 5 * e)} * \log(-d ** 8 * e * (1 / (d ** 5 * e)) ** (3 / 2) / 10 - 9 * d ** 3 * \sqrt{1 / (d ** 5 * e)} / 10 + x) / 8 + \sqrt{1 / (d ** 5 * e)} * \log(d ** 8 * e * (1 / (d ** 5 * e)) ** (3 / 2) / 10 + 9 * d ** 3 * \sqrt{1 / (d ** 5 * e)} / 10 + x) / 8 - \sqrt{-1 / (d ** 5 * e)} * \log(-4 * d ** 8 * e * (-1 / (d ** 5 * e)) ** (3 / 2) / 5 - 9 * d ** 3 * \sqrt{-1 / (d ** 5 * e)} / 5 + x) / 4 + \sqrt{-1 / (d ** 5 * e)} * \log(4 * d ** 8 * e * (-1 / (d ** 5 * e)) ** (3 / 2) / 5 + 9 * d ** 3 * \sqrt{-1 / (d ** 5 * e)} / 5 + x) / 4$

$$3.136 \quad \int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)} dx$$

**Optimal.** Leaf size=89

$$\frac{7 \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}\sqrt{e}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{7/2}\sqrt{e}} + \frac{5x}{16d^3(d+ex^2)} + \frac{x}{8d^2(d+ex^2)^2}$$

**Rubi [A]** time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {1150, 414, 527, 522, 208, 205}

$$\frac{5x}{16d^3(d+ex^2)} + \frac{x}{8d^2(d+ex^2)^2} + \frac{7 \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}\sqrt{e}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{7/2}\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x^2)^2\*(d^2 - e^2\*x^4)),x]

[Out] x/(8\*d^2\*(d + e\*x^2)^2) + (5\*x)/(16\*d^3\*(d + e\*x^2)) + (7\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]]/(16\*d^(7/2)\*Sqrt[e]) + ArcTanh[(Sqrt[e]\*x)/Sqrt[d]]/(8\*d^(7/2)\*Sqrt[e]))

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 414

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(b\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(p+1)\*(b\*c - a\*d)), x] + Dist[1/(a\*n\*(p+1)\*(b\*c - a\*d)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q\*Simp[b\*c + n\*(p+1)\*(b\*c - a\*d) + d\*b\*(n\*(p+q+2) + 1)\*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e\_) + (f\_.)\*(x\_)^(n\_))/((a\_) + (b\_.)\*(x\_)^(n\_))\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Dist[(b\*e - a\*f)/(b\*c - a\*d), Int[1/(a + b\*x^n), x], x] - Dist[(d\*e - c\*f)/(b\*c - a\*d), Int[1/(c + d\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_)\*((e\_) + (f\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*e - a\*f)\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(b\*c - a\*d)\*(p+1)), x] + Dist[1/(a\*n\*(b\*c - a\*d)\*(p+1)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q\*Simp[c\*(b\*e - a\*f) + e\*n\*(b\*c - a\*d)\*(p+1) + d\*(b\*e - a\*f)\*(n\*(p+q+2) + 1)\*x^n, x], x], x] /; FreeQ

[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

### Rule 1150

Int[((d\_) + (e\_)\*(x\_)^2)^(q\_)\*((a\_) + (c\_)\*(x\_)^4)^(p\_), x\_Symbol] := Int[(d + e\*x^2)^(p + q)\*(a/d + (c\*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x] && EqQ[c\*d^2 + a\*e^2, 0] && IntegerQ[p]

### Rubi steps

$$\begin{aligned} \int \frac{1}{(d + ex^2)^2 (d^2 - e^2 x^4)} dx &= \int \frac{1}{(d - ex^2)(d + ex^2)^3} dx \\ &= \frac{x}{8d^2 (d + ex^2)^2} - \frac{\int \frac{-7de + 3e^2 x^2}{(d - ex^2)(d + ex^2)^2} dx}{8d^2 e} \\ &= \frac{x}{8d^2 (d + ex^2)^2} + \frac{5x}{16d^3 (d + ex^2)} + \frac{\int \frac{18d^2 e^2 - 10de^3 x^2}{(d - ex^2)(d + ex^2)} dx}{32d^4 e^2} \\ &= \frac{x}{8d^2 (d + ex^2)^2} + \frac{5x}{16d^3 (d + ex^2)} + \frac{\int \frac{1}{d - ex^2} dx}{8d^3} + \frac{7 \int \frac{1}{d + ex^2} dx}{16d^3} \\ &= \frac{x}{8d^2 (d + ex^2)^2} + \frac{5x}{16d^3 (d + ex^2)} + \frac{7 \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}\sqrt{e}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{7/2}\sqrt{e}} \end{aligned}$$

**Mathematica [A]** time = 0.06, size = 76, normalized size = 0.85

$$\frac{\frac{\sqrt{d}x(7d+5ex^2)}{(d+ex^2)^2} + \frac{7 \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + \frac{2 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}}{16d^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x^2)^2\*(d^2 - e^2\*x^4)), x]

[Out] ((Sqrt[d]\*x\*(7\*d + 5\*e\*x^2))/(d + e\*x^2)^2 + (7\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/Sqrt[e] + (2\*ArcTanh[(Sqrt[e]\*x)/Sqrt[d]])/Sqrt[e])/(16\*d^(7/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex^2)^2 (d^2 - e^2 x^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e\*x^2)^2\*(d^2 - e^2\*x^4)), x]

[Out] IntegrateAlgebraic[1/((d + e\*x^2)^2\*(d^2 - e^2\*x^4)), x]

**fricas [B]** time = 1.24, size = 278, normalized size = 3.12

$$\frac{5de^2x^3 + 7d^2ex + 7(e^2x^4 + 2dex^2 + d^2)\sqrt{de} \arctan\left(\frac{\sqrt{de}x}{d}\right) + (e^2x^4 + 2dex^2 + d^2)\sqrt{de} \log\left(\frac{e^2x^2 + 2dex + d}{e^2x^2 - d}\right) + 10de^2x^3 + 14d^2ex - 4(e^2x^4 + 2dex^2 + d^2)\sqrt{-de} \arctan\left(\frac{\sqrt{-de}x}{d}\right) - 7(e^2x^4 + 2dex^2 + d^2)\sqrt{-de} \log\left(\frac{e^2x^2 - 2\sqrt{-de}x - d}{e^2x^2 + d}\right)}{16(d^4e^3x^4 + 2d^5e^2x^2 + d^6e)} \frac{10de^2x^3 + 14d^2ex - 4(e^2x^4 + 2dex^2 + d^2)\sqrt{-de} \arctan\left(\frac{\sqrt{-de}x}{d}\right) - 7(e^2x^4 + 2dex^2 + d^2)\sqrt{-de} \log\left(\frac{e^2x^2 - 2\sqrt{-de}x - d}{e^2x^2 + d}\right)}{32(d^4e^3x^4 + 2d^5e^2x^2 + d^6e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)^2/(-e^2\*x^4+d^2),x, algorithm="fricas")

[Out] [1/16\*(5\*d\*e^2\*x^3 + 7\*d^2\*e\*x + 7\*(e^2\*x^4 + 2\*d\*e\*x^2 + d^2)\*sqrt(d\*e)\*arctan(sqrt(d\*e)\*x/d) + (e^2\*x^4 + 2\*d\*e\*x^2 + d^2)\*sqrt(d\*e)\*log((e\*x^2 + 2\*sqrt(d\*e)\*x + d)/(e\*x^2 - d)))/(d^4\*e^3\*x^4 + 2\*d^5\*e^2\*x^2 + d^6\*e), 1/32\*(10\*d\*e^2\*x^3 + 14\*d^2\*e\*x - 4\*(e^2\*x^4 + 2\*d\*e\*x^2 + d^2)\*sqrt(-d\*e)\*arctan(sqrt(-d\*e)\*x/d) - 7\*(e^2\*x^4 + 2\*d\*e\*x^2 + d^2)\*sqrt(-d\*e)\*log((e\*x^2 - 2\*sqrt(-d\*e)\*x - d)/(e\*x^2 + d)))/(d^4\*e^3\*x^4 + 2\*d^5\*e^2\*x^2 + d^6\*e)]

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)^2/(-e^2\*x^4+d^2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output:  $-( -2*(d^2*\exp(2)^3)^{1/4}*abs(d)*\exp(1)^2+d*(d^2*\exp(2)^3)^{1/4}*\exp(1)^2+d*\exp(2)*(d^2*\exp(2)^3)^{1/4})/(4*d^5*\exp(1)^4-8*d^5*\exp(2)*\exp(1)^2+4*d^5*\exp(2)^2)*\ln(abs(x-(d^2/\exp(2))^{1/4}))+\exp(2)*(d^2*\exp(2)^3)^{1/4}/(4*d^4*\exp(2)*\exp(1)^2-8*d^4*\exp(1)*\exp(2)*\exp(1)+4*d^4*\exp(2)^2)*\ln(abs(x+(d^2/\exp(2))^{1/4}))-(-2*(d^2*\exp(2)^3)^{1/4}*abs(d)*\exp(1)^2-d*(d^2*\exp(2)^3)^{1/4}*\exp(1)^2-d*\exp(2)*(d^2*\exp(2)^3)^{1/4})/(2*d^5*\exp(1)^4-4*d^5*\exp(2)*\exp(1)^2+2*d^5*\exp(2)^2)*\operatorname{atan}(x/(d^2/\exp(2))^{1/4})-(-5*\exp(2)*\exp(1)^2+\exp(1)^4)*1/2/(-\exp(2)^2*d^3+2*\exp(2)*d^3*\exp(1)^2-d^3*\exp(1)^4)/\sqrt{d*\exp(1)}*\operatorname{atan}(x*\exp(1)/\sqrt{d*\exp(1)})+x*\exp(1)^2/(-2*\exp(2)*d^3+2*d^3*\exp(1)^2)/(x^2*\exp(1)+d)$

**maple** [A] time = 0.01, size = 73, normalized size = 0.82

$$\frac{5ex^3}{16(e x^2 + d)^2 d^3} + \frac{7x}{16(e x^2 + d)^2 d^2} + \frac{\operatorname{arctanh}\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de} d^3} + \frac{7 \operatorname{arctan}\left(\frac{ex}{\sqrt{de}}\right)}{16\sqrt{de} d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x^2+d)^2/(-e^2\*x^4+d^2),x)

[Out]  $5/16/d^3/(e*x^2+d)^2*x^3*e+7/16*x/d^2/(e*x^2+d)^2+7/16/d^3/(d*e)^{1/2}*\operatorname{arctan}(1/(d*e)^{1/2}*e*x)+1/8/d^3/(d*e)^{1/2}*\operatorname{arctanh}(1/(d*e)^{1/2}*e*x)$

**maxima** [A] time = 2.49, size = 92, normalized size = 1.03

$$\frac{5ex^3 + 7dx}{16(d^3e^2x^4 + 2d^4ex^2 + d^5)} + \frac{7 \operatorname{arctan}\left(\frac{ex}{\sqrt{de}}\right)}{16\sqrt{de} d^3} - \frac{\log\left(\frac{ex-\sqrt{de}}{ex+\sqrt{de}}\right)}{16\sqrt{de} d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)^2/(-e^2\*x^4+d^2),x, algorithm="maxima")

[Out]  $1/16*(5*e*x^3 + 7*d*x)/(d^3*e^2*x^4 + 2*d^4*e*x^2 + d^5) + 7/16*\operatorname{arctan}(e*x/\sqrt{d*e})/(\sqrt{d*e}*d^3) - 1/16*\log((e*x - \sqrt{d*e})/(e*x + \sqrt{d*e}))/(\sqrt{d*e}*d^3)$

**mapad** [B] time = 0.16, size = 96, normalized size = 1.08

$$\frac{\frac{7x}{16d^2} + \frac{5ex^3}{16d^3}}{d^2 + 2dex^2 + e^2x^4} + \frac{\operatorname{atanh}\left(\frac{x\sqrt{d^7e}}{d^4}\right)\sqrt{d^7e}}{8d^7e} - \frac{7 \operatorname{atanh}\left(\frac{x\sqrt{-d^7e}}{d^4}\right)\sqrt{-d^7e}}{16d^7e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d^2 - e^2\*x^4)\*(d + e\*x^2)^2),x)

[Out] ((7\*x)/(16\*d^2) + (5\*e\*x^3)/(16\*d^3))/(d^2 + e^2\*x^4 + 2\*d\*e\*x^2) + (atanh(x\*(d^7\*e)^(1/2))/d^4)\*(d^7\*e)^(1/2)/(8\*d^7\*e) - (7\*atanh(x\*(-d^7\*e)^(1/2))/d^4)\*(-d^7\*e)^(1/2)/(16\*d^7\*e)

**sympy [B]** time = 0.72, size = 257, normalized size = 2.89

$$\frac{\sqrt{\frac{1}{d^7 e}} \log\left(\frac{20d^{11}\left(\frac{1}{d^7 e}\right)^{\frac{3}{2}} - 351d^4\sqrt{\frac{1}{d^7 e}}}{371} + x\right)}{16} + \frac{\sqrt{\frac{1}{d^7 e}} \log\left(\frac{20d^{11}\left(\frac{1}{d^7 e}\right)^{\frac{3}{2}} + 351d^4\sqrt{\frac{1}{d^7 e}}}{371} + x\right)}{16} - \frac{7\sqrt{\frac{1}{d^7 e}} \log\left(\frac{245d^{11}\left(-\frac{1}{d^7 e}\right)^{\frac{3}{2}} - 351d^4\sqrt{\frac{1}{d^7 e}}}{106} + x\right)}{32} + \frac{7\sqrt{\frac{1}{d^7 e}} \log\left(\frac{245d^{11}\left(-\frac{1}{d^7 e}\right)^{\frac{3}{2}} + 351d^4\sqrt{\frac{1}{d^7 e}}}{106} + x\right)}{32} - \frac{-7dx - 5ex^3}{16d^5 + 32d^4ex^2 + 16d^3e^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x\*\*2+d)\*\*2/(-e\*\*2\*x\*\*4+d\*\*2),x)

[Out] -sqrt(1/(d\*\*7\*e))\*log(-20\*d\*\*11\*e\*(1/(d\*\*7\*e))\*\*(3/2)/371 - 351\*d\*\*4\*sqrt(1/(d\*\*7\*e))/371 + x)/16 + sqrt(1/(d\*\*7\*e))\*log(20\*d\*\*11\*e\*(1/(d\*\*7\*e))\*\*(3/2)/371 + 351\*d\*\*4\*sqrt(1/(d\*\*7\*e))/371 + x)/16 - 7\*sqrt(-1/(d\*\*7\*e))\*log(-245\*d\*\*11\*e\*(-1/(d\*\*7\*e))\*\*(3/2)/106 - 351\*d\*\*4\*sqrt(-1/(d\*\*7\*e))/106 + x)/32 + 7\*sqrt(-1/(d\*\*7\*e))\*log(245\*d\*\*11\*e\*(-1/(d\*\*7\*e))\*\*(3/2)/106 + 351\*d\*\*4\*sqrt(-1/(d\*\*7\*e))/106 + x)/32 - (-7\*d\*x - 5\*e\*x\*\*3)/(16\*d\*\*5 + 32\*d\*\*4\*e\*x\*\*2 + 16\*d\*\*3\*e\*\*2\*x\*\*4)

$$3.137 \quad \int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4} dx$$

Optimal. Leaf size=62

$$\frac{\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{\sqrt{e}} - \frac{\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}$$

**Rubi [A]** time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {1150, 402, 217, 206, 377, 208}

$$\frac{\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{\sqrt{e}} - \frac{\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^(3/2)/(d^2 - e^2\*x^4), x]

[Out] -(ArcTanh[(Sqrt[e]\*x)/Sqrt[d + e\*x^2]]/Sqrt[e]) + (Sqrt[2]\*ArcTanh[(Sqrt[2]\*Sqrt[e]\*x)/Sqrt[d + e\*x^2]])/Sqrt[e]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

#### Rule 402

Int[((a\_) + (b\_.)\*(x\_)^2)^(p\_)/((c\_) + (d\_.)\*(x\_)^2), x\_Symbol] := Dist[b/d, Int[(a + b\*x^2)^(p - 1), x], x] - Dist[(b\*c - a\*d)/d, Int[(a + b\*x^2)^(p - 1)/(c + d\*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])

#### Rule 1150

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] := Int[(d + e\*x^2)^(p + q)\*(a/d + (c\*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x] && EqQ[c\*d^2 + a\*e^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4} dx &= \int \frac{\sqrt{d+ex^2}}{d-ex^2} dx \\
&= (2d) \int \frac{1}{(d-ex^2)\sqrt{d+ex^2}} dx - \int \frac{1}{\sqrt{d+ex^2}} dx \\
&= (2d) \operatorname{Subst}\left(\int \frac{1}{d-2dex^2} dx, x, \frac{x}{\sqrt{d+ex^2}}\right) - \operatorname{Subst}\left(\int \frac{1}{1-ex^2} dx, x, \frac{x}{\sqrt{d+ex^2}}\right) \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{\sqrt{e}} + \frac{\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 61, normalized size = 0.98

$$\frac{\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d+ex^2}}\right) - \log\left(\sqrt{e}\sqrt{d+ex^2} + ex\right)}{\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^(3/2)/(d^2 - e^2\*x^4), x]

[Out] (Sqrt[2]\*ArcTanh[(Sqrt[2]\*Sqrt[e]\*x)/Sqrt[d + e\*x^2]] - Log[e\*x + Sqrt[e]\*Sqrt[d + e\*x^2]])/Sqrt[e]

**IntegrateAlgebraic [A]** time = 0.12, size = 86, normalized size = 1.39

$$\frac{\log\left(\sqrt{d+ex^2} - \sqrt{ex}\right)}{\sqrt{e}} + \frac{\sqrt{2} \tanh^{-1}\left(-\frac{ex^2}{\sqrt{2}d} + \frac{\sqrt{ex}\sqrt{d+ex^2}}{\sqrt{2}d} + \frac{1}{\sqrt{2}}\right)}{\sqrt{e}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e\*x^2)^(3/2)/(d^2 - e^2\*x^4), x]

[Out] (Sqrt[2]\*ArcTanh[1/Sqrt[2] - (e\*x^2)/(Sqrt[2]\*d) + (Sqrt[e]\*x\*Sqrt[d + e\*x^2])/(Sqrt[2]\*d)]/Sqrt[e] + Log[-(Sqrt[e]\*x) + Sqrt[d + e\*x^2]]/Sqrt[e])

**fricas [A]** time = 0.85, size = 199, normalized size = 3.21

$$\left[ \frac{\sqrt{2}\sqrt{e} \log\left(\frac{17e^2x^4 + 14dex^2 + d^2 + \frac{4\sqrt{2}(3e^2x^3 + dex)\sqrt{ex^2+d}}{\sqrt{e}}}{e^2x^4 - 2dex^2 + d^2}\right) + 2\sqrt{e} \log(-2ex^2 + 2\sqrt{ex^2+d}\sqrt{ex-d})}{4e}, -\frac{\sqrt{2}e\sqrt{-\frac{1}{e}} \arctan\left(\frac{\sqrt{2}(3ex^2+d)\sqrt{ex^2+d}\sqrt{-\frac{1}{e}}}{4(ex^3+dx)}\right) - 2\sqrt{-e} \arctan\left(\frac{\sqrt{-ex}}{\sqrt{ex^2+d}}\right)}{2e} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)/(-e^2\*x^4+d^2), x, algorithm="fricas")

[Out] [1/4\*(sqrt(2)\*sqrt(e)\*log((17\*e^2\*x^4 + 14\*d\*e\*x^2 + d^2 + 4\*sqrt(2)\*(3\*e^2\*x^3 + d\*e\*x)\*sqrt(e\*x^2 + d)/sqrt(e))/(e^2\*x^4 - 2\*d\*e\*x^2 + d^2)) + 2\*sqrt(e)\*log(-2\*e\*x^2 + 2\*sqrt(e\*x^2 + d)\*sqrt(e)\*x - d))/e, -1/2\*(sqrt(2)\*e\*sqrt(-1/e)\*arctan(1/4\*sqrt(2)\*(3\*e\*x^2 + d)\*sqrt(e\*x^2 + d)\*sqrt(-1/e)/(e\*x^3 + d\*x)) - 2\*sqrt(-e)\*arctan(sqrt(-e)\*x/sqrt(e\*x^2 + d)))/e]

**giac [A]** time = 0.25, size = 24, normalized size = 0.39

$$\frac{1}{2} e^{\left(-\frac{1}{2}\right)} \log\left(\left(xe^{\frac{1}{2}} - \sqrt{x^2e + d}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)/(-e^2\*x^4+d^2),x, algorithm="giac")

[Out] 1/2\*e^(-1/2)\*log((x\*e^(1/2) - sqrt(x^2\*e + d))^2)

**maple [B]** time = 0.06, size = 1442, normalized size = 23.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^(3/2)/(-e^2\*x^4+d^2),x)

[Out] 1/6\*e/((-d\*e)^(1/2)+(d\*e)^(1/2))/(-(-d\*e)^(1/2)+(d\*e)^(1/2))/(-d\*e)^(1/2)\*((x-1/e\*(-d\*e)^(1/2))^2\*e+2\*(-d\*e)^(1/2)\*(x-1/e\*(-d\*e)^(1/2)))^(3/2)+1/4\*e/((-d\*e)^(1/2)+(d\*e)^(1/2))/(-(-d\*e)^(1/2)+(d\*e)^(1/2))\*((x-1/e\*(-d\*e)^(1/2))^2\*e+2\*(-d\*e)^(1/2)\*(x-1/e\*(-d\*e)^(1/2)))^(1/2)\*x+1/4\*e^(1/2)/((-d\*e)^(1/2)+(d\*e)^(1/2))/(-(-d\*e)^(1/2)+(d\*e)^(1/2))\*d\*ln(((x-1/e\*(-d\*e)^(1/2))\*e+(-d\*e)^(1/2))/e^(1/2)+((x-1/e\*(-d\*e)^(1/2))^2\*e+2\*(-d\*e)^(1/2)\*(x-1/e\*(-d\*e)^(1/2)))^(1/2))-1/6\*e/(d\*e)^(1/2)/((-d\*e)^(1/2)+(d\*e)^(1/2))/(-(-d\*e)^(1/2)+(d\*e)^(1/2))\*((x-(d\*e)^(1/2)/e)^2\*e+2\*(d\*e)^(1/2)\*(x-(d\*e)^(1/2)/e)+2\*d)^(3/2)-1/4\*e/((-d\*e)^(1/2)+(d\*e)^(1/2))/(-(-d\*e)^(1/2)+(d\*e)^(1/2))\*((x-(d\*e)^(1/2)/e)^2\*e+2\*(d\*e)^(1/2)\*(x-(d\*e)^(1/2)/e)+2\*d)^(1/2)\*x-5/4\*e^(1/2)/((-d\*e)^(1/2)+(d\*e)^(1/2))/(-(-d\*e)^(1/2)+(d\*e)^(1/2))\*d\*ln(((x-(d\*e)^(1/2)/e)\*e+(d\*e)^(1/2))/e^(1/2)+((x-(d\*e)^(1/2)/e)^2\*e+2\*(d\*e)^(1/2)\*(x-(d\*e)^(1/2)/e)+2\*d)^(1/2))-e/(d\*e)^(1/2)/((-d\*e)^(1/2)+(d\*e)^(1/2))/(-(-d\*e)^(1/2)+(d\*e)^(1/2))\*d\*((x-(d\*e)^(1/2)/e)^2\*e+2\*(d\*e)^(1/2)\*(x-(d\*e)^(1/2)/e)+2\*d)^(1/2)+e/(d\*e)^(1/2)/((-d\*e)^(1/2)+(d\*e)^(1/2))/(-(-d\*e)^(1/2)+(d\*e)^(1/2))\*d^(3/2)\*2^(1/2)\*ln((4\*d+2\*(d\*e)^(1/2)\*(x-(d\*e)^(1/2)/e)+2\*2^(1/2)\*d^(1/2))\*((x-(d\*e)^(1/2)/e)^2\*e+2\*(d\*e)^(1/2)\*(x-(d\*e)^(1/2)/e)+2\*d)^(1/2)/(x-(d\*e)^(1/2)/e))-1/6\*e/((-d\*e)^(1/2)+(d\*e)^(1/2))/(-(-d\*e)^(1/2)+(d\*e)^(1/2))/(-d\*e)^(1/2))\*((x+1/e\*(-d\*e)^(1/2))^2\*e-2\*(-d\*e)^(1/2)\*(x+1/e\*(-d\*e)^(1/2)))^(3/2)+1/4\*e/((-d\*e)^(1/2)+(d\*e)^(1/2))/(-(-d\*e)^(1/2)+(d\*e)^(1/2))\*((x+1/e\*(-d\*e)^(1/2))^2\*e-2\*(-d\*e)^(1/2)\*(x+1/e\*(-d\*e)^(1/2)))^(1/2)\*x+1/4\*e^(1/2)/((-d\*e)^(1/2)+(d\*e)^(1/2))/(-(-d\*e)^(1/2)+(d\*e)^(1/2))\*d\*ln(((x+1/e\*(-d\*e)^(1/2))\*e-(-d\*e)^(1/2))/e^(1/2)+((x+1/e\*(-d\*e)^(1/2))^2\*e-2\*(-d\*e)^(1/2)\*(x+1/e\*(-d\*e)^(1/2)))^(1/2))+1/6\*e/(d\*e)^(1/2)/((-d\*e)^(1/2)+(d\*e)^(1/2))/(-(-d\*e)^(1/2)+(d\*e)^(1/2))\*((x+(d\*e)^(1/2)/e)^2\*e-2\*(d\*e)^(1/2)\*(x+(d\*e)^(1/2)/e)+2\*d)^(3/2)-1/4\*e/((-d\*e)^(1/2)+(d\*e)^(1/2))/(-(-d\*e)^(1/2)+(d\*e)^(1/2))\*((x+(d\*e)^(1/2)/e)^2\*e-2\*(d\*e)^(1/2)\*(x+(d\*e)^(1/2)/e)+2\*d)^(1/2)\*x-5/4\*e^(1/2)/((-d\*e)^(1/2)+(d\*e)^(1/2))/(-(-d\*e)^(1/2)+(d\*e)^(1/2))\*d\*ln(((x+(d\*e)^(1/2)/e)\*e-(d\*e)^(1/2))/e^(1/2)+((x+(d\*e)^(1/2)/e)^2\*e-2\*(d\*e)^(1/2)\*(x+(d\*e)^(1/2)/e)+2\*d)^(1/2))+e/(d\*e)^(1/2)/((-d\*e)^(1/2)+(d\*e)^(1/2))/(-(-d\*e)^(1/2)+(d\*e)^(1/2))\*d\*((x+(d\*e)^(1/2)/e)^2\*e-2\*(d\*e)^(1/2)\*(x+(d\*e)^(1/2)/e)+2\*d)^(1/2))-e/(d\*e)^(1/2)/((-d\*e)^(1/2)+(d\*e)^(1/2))/(-(-d\*e)^(1/2)+(d\*e)^(1/2))\*d^(3/2)\*2^(1/2)\*ln((4\*d-2\*(d\*e)^(1/2)\*(x+(d\*e)^(1/2)/e)+2\*2^(1/2)\*d^(1/2))\*((x+(d\*e)^(1/2)/e)^2\*e-2\*(d\*e)^(1/2)\*(x+(d\*e)^(1/2)/e)+2\*d)^(1/2)/(x+(d\*e)^(1/2)/e))

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ex^2 + d)^{\frac{3}{2}}}{e^2x^4 - d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)/(-e^2\*x^4+d^2),x, algorithm="maxima")

[Out] -integrate((e\*x^2 + d)^(3/2)/(e^2\*x^4 - d^2), x)



**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(ex^2 + d)^{3/2}}{d^2 - e^2x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)^(3/2)/(d^2 - e^2\*x^4), x)

[Out] int((d + e\*x^2)^(3/2)/(d^2 - e^2\*x^4), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{d + ex^2}}{-d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*(3/2)/(-e\*\*2\*x\*\*4+d\*\*2), x)

[Out] -Integral(sqrt(d + e\*x\*\*2)/(-d + e\*x\*\*2), x)

$$3.138 \quad \int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4} dx$$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{\sqrt{2}d\sqrt{e}}$$

**Rubi [A]** time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {1150, 377, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{\sqrt{2}d\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e\*x^2]/(d^2 - e^2\*x^4),x]

[Out] ArcTanh[(Sqrt[2]\*Sqrt[e]\*x)/Sqrt[d + e\*x^2]]/(Sqrt[2]\*d\*Sqrt[e])

Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

Rule 1150

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[(d + e\*x^2)^(p+q)\*(a/d + (c\*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x] && EqQ[c\*d^2 + a\*e^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4} dx &= \int \frac{1}{(d-ex^2)\sqrt{d+ex^2}} dx \\ &= \text{Subst}\left(\int \frac{1}{d-2dex^2} dx, x, \frac{x}{\sqrt{d+ex^2}}\right) \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{\sqrt{2}d\sqrt{e}} \end{aligned}$$

**Mathematica [A]** time = 0.15, size = 38, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{\sqrt{2}d\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e\*x^2]/(d^2 - e^2\*x^4), x]

[Out] ArcTanh[(Sqrt[2]\*Sqrt[e]\*x)/Sqrt[d + e\*x^2]]/(Sqrt[2]\*d\*Sqrt[e])

**IntegrateAlgebraic [A]** time = 0.07, size = 61, normalized size = 1.61

$$\frac{\tanh^{-1}\left(-\frac{ex^2}{\sqrt{2}d} + \frac{\sqrt{ex\sqrt{d+ex^2}}}{\sqrt{2}d} + \frac{1}{\sqrt{2}}\right)}{\sqrt{2}d\sqrt{e}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[d + e\*x^2]/(d^2 - e^2\*x^4), x]

[Out] ArcTanh[1/Sqrt[2] - (e\*x^2)/(Sqrt[2]\*d) + (Sqrt[e]\*x\*Sqrt[d + e\*x^2])/(Sqrt[2]\*d)]/(Sqrt[2]\*d\*Sqrt[e])

**fricas [A]** time = 0.56, size = 138, normalized size = 3.63

$$\left[ \frac{\sqrt{2} \log\left(\frac{17e^2x^4 + 14dex^2 + 4\sqrt{2}(3ex^3 + dx)\sqrt{ex^2 + d}\sqrt{e + d^2}}{e^2x^4 - 2dex^2 + d^2}\right)}{8d\sqrt{e}}, -\frac{\sqrt{2}\sqrt{-e} \arctan\left(\frac{\sqrt{2}(3ex^2 + d)\sqrt{ex^2 + d}\sqrt{-e}}{4(e^2x^3 + dex)}\right)}{4de} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(1/2)/(-e^2\*x^4+d^2), x, algorithm="fricas")

[Out] [1/8\*sqrt(2)\*log((17\*e^2\*x^4 + 14\*d\*e\*x^2 + 4\*sqrt(2)\*(3\*e\*x^3 + d\*x)\*sqrt(e\*x^2 + d)\*sqrt(e) + d^2)/(e^2\*x^4 - 2\*d\*e\*x^2 + d^2))/(d\*sqrt(e)), -1/4\*sqrt(2)\*sqrt(-e)\*arctan(1/4\*sqrt(2)\*(3\*e\*x^2 + d)\*sqrt(e\*x^2 + d)\*sqrt(-e)/(e^2\*x^3 + d\*e\*x))/(d\*e)]

**giac [B]** time = 0.53, size = 131, normalized size = 3.45

$$\frac{\left(\sqrt{2}i \arctan\left(\frac{e^{\frac{1}{2}}}{\sqrt{\frac{de + \sqrt{d^2}e}{d}}}\right)e^{\frac{1}{2}} - \sqrt{2}i \arctan\left(\frac{e^{\frac{1}{2}}}{\sqrt{\frac{de - \sqrt{d^2}e}{d}}}\right)e^{\frac{1}{2}}\right)e^{(-1)\operatorname{sgn}(x)} + \sqrt{2}i \arctan\left(\frac{\sqrt{\frac{d}{x^2} + e}}{\sqrt{\frac{desgn(x) + \sqrt{d^2}e}{d\operatorname{sgn}(x)}}}\right)e^{\left(-\frac{1}{2}\right)}}{4|d|} + \frac{\sqrt{2}i \arctan\left(\frac{\sqrt{\frac{d}{x^2} + e}}{\sqrt{\frac{desgn(x) + \sqrt{d^2}e}{d\operatorname{sgn}(x)}}}\right)e^{\left(-\frac{1}{2}\right)}}{2|d|\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(1/2)/(-e^2\*x^4+d^2), x, algorithm="giac")

[Out] -1/4\*(sqrt(2)\*i\*arctan(e^(1/2)/sqrt(-(d\*e + sqrt(d^2)\*e)/d))\*e^(1/2) - sqrt(2)\*i\*arctan(e^(1/2)/sqrt(-(d\*e - sqrt(d^2)\*e)/d))\*e^(1/2))\*e^(-1)\*sgn(x)/abs(d) + 1/2\*sqrt(2)\*i\*arctan(sqrt(d/x^2 + e)/sqrt(-(d\*e\*sgn(x) + sqrt(d^2)\*e)/(d\*sgn(x))))\*e^(-1/2)/(abs(d)\*abs(sgn(x)))

**maple [B]** time = 0.02, size = 986, normalized size = 25.95

[In] int((e\*x^2+d)^(1/2)/(-e^2\*x^4+d^2), x)

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^(1/2)/(-e^2\*x^4+d^2), x)

[Out] -1/2\*e/((-d\*e)^(1/2)+(d\*e)^(1/2))/((-d\*e)^(1/2)-(d\*e)^(1/2))/(-d\*e)^(1/2)\*(x-(-d\*e)^(1/2)/e)^2\*e+2\*(-d\*e)^(1/2)\*(x-(-d\*e)^(1/2)/e)^(1/2)-1/2\*e^(1/2)/((-d\*e)^(1/2)+(d\*e)^(1/2))/((-d\*e)^(1/2)-(d\*e)^(1/2))\*ln(((x-(-d\*e)^(1/2)/e)\*e+(-d\*e)^(1/2))/e^(1/2)+((x-(-d\*e)^(1/2)/e)^2\*e+2\*(-d\*e)^(1/2)\*(x-(-d\*e)^(1/2)/e)^(1/2))+1/2\*e/(d\*e)^(1/2))/((-d\*e)^(1/2)+(d\*e)^(1/2))/((-d\*e)^(1/2)

$$\begin{aligned}
 & -(d*e)^{(1/2)} * (2*d + (x - (d*e)^{(1/2)}/e)^2 * e + 2*(d*e)^{(1/2)} * (x - (d*e)^{(1/2)}/e))^{(1/2)} \\
 & + 1/2 * e^{(1/2)} / ((-d*e)^{(1/2)} + (d*e)^{(1/2)}) / ((-d*e)^{(1/2)} - (d*e)^{(1/2)}) * \ln( \\
 & ((x - (d*e)^{(1/2)}/e) * e + (d*e)^{(1/2)}) / e^{(1/2)} + (2*d + (x - (d*e)^{(1/2)}/e)^2 * e + 2*(d*e)^{(1/2)} * (x - (d*e)^{(1/2)}/e))^{(1/2)} \\
 & - 1/2 * e / (d*e)^{(1/2)} / ((-d*e)^{(1/2)} + (d*e)^{(1/2)}) / ((-d*e)^{(1/2)} - (d*e)^{(1/2)}) * d^{(1/2)} * 2^{(1/2)} * \ln( \\
 & (4*d + 2*2^{(1/2)} * (2*d + (x - (d*e)^{(1/2)}/e)^2 * e + 2*(d*e)^{(1/2)} * (x - (d*e)^{(1/2)}/e))^{(1/2)} * d^{(1/2)} + 2*(d*e)^{(1/2)} * (x - (d*e)^{(1/2)}/e)) / (x - (d*e)^{(1/2)}/e) \\
 & + 1/2 * e / ((-d*e)^{(1/2)} + (d*e)^{(1/2)}) / ((-d*e)^{(1/2)} - (d*e)^{(1/2)}) / (-d*e)^{(1/2)} * ((x + (-d*e)^{(1/2)}/e)^2 * e - 2*(-d*e)^{(1/2)} * (x + (-d*e)^{(1/2)}/e))^{(1/2)} \\
 & - 1/2 * e^{(1/2)} / ((-d*e)^{(1/2)} + (d*e)^{(1/2)}) / ((-d*e)^{(1/2)} - (d*e)^{(1/2)}) * \ln( \\
 & ((x + (-d*e)^{(1/2)}/e) * e - (-d*e)^{(1/2)}) / e^{(1/2)} + ((x + (-d*e)^{(1/2)}/e)^2 * e - 2*(-d*e)^{(1/2)} * (x + (-d*e)^{(1/2)}/e))^{(1/2)} \\
 & - 1/2 * e / (d*e)^{(1/2)} / ((-d*e)^{(1/2)} + (d*e)^{(1/2)}) / ((-d*e)^{(1/2)} - (d*e)^{(1/2)}) * (2*d + (x + (d*e)^{(1/2)}/e)^2 * e - 2*(d*e)^{(1/2)} * (x + (d*e)^{(1/2)}/e))^{(1/2)} \\
 & + 1/2 * e^{(1/2)} / ((-d*e)^{(1/2)} + (d*e)^{(1/2)}) / ((-d*e)^{(1/2)} - (d*e)^{(1/2)}) * \ln( \\
 & ((x + (d*e)^{(1/2)}/e) * e - (d*e)^{(1/2)}) / e^{(1/2)} + (2*d + (x + (d*e)^{(1/2)}/e)^2 * e - 2*(d*e)^{(1/2)} * (x + (d*e)^{(1/2)}/e))^{(1/2)} \\
 & + 1/2 * e / (d*e)^{(1/2)} / ((-d*e)^{(1/2)} + (d*e)^{(1/2)}) / ((-d*e)^{(1/2)} - (d*e)^{(1/2)}) * d^{(1/2)} * 2^{(1/2)} * \ln( \\
 & (4*d + 2*2^{(1/2)} * (2*d + (x + (d*e)^{(1/2)}/e)^2 * e - 2*(d*e)^{(1/2)} * (x + (d*e)^{(1/2)}/e))^{(1/2)} * d^{(1/2)} - 2*(d*e)^{(1/2)} * (x + (d*e)^{(1/2)}/e)) / (x + (d*e)^{(1/2)}/e)
 \end{aligned}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{\sqrt{ex^2 + d}}{e^2x^4 - d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(1/2)/(-e^2\*x^4+d^2),x, algorithm="maxima")

[Out] -integrate(sqrt(e\*x^2 + d)/(e^2\*x^4 - d^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{ex^2 + d}}{d^2 - e^2x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)^(1/2)/(d^2 - e^2\*x^4),x)

[Out] int((d + e\*x^2)^(1/2)/(d^2 - e^2\*x^4), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{1}{-d\sqrt{d + ex^2} + ex^2\sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*(1/2)/(-e\*\*2\*x\*\*4+d\*\*2),x)

[Out] -Integral(1/(-d\*sqrt(d + e\*x\*\*2) + e\*x\*\*2\*sqrt(d + e\*x\*\*2)), x)

$$3.139 \quad \int \frac{1}{\sqrt{d+ex^2}(d^2-e^2x^4)} dx$$

**Optimal.** Leaf size=61

$$\frac{x}{2d^2\sqrt{d+ex^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}d^2\sqrt{e}}$$

**Rubi [A]** time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {1150, 382, 377, 208}

$$\frac{x}{2d^2\sqrt{d+ex^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}d^2\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[d + e\*x^2]\*(d^2 - e^2\*x^4)),x]

[Out] x/(2\*d^2\*Sqrt[d + e\*x^2]) + ArcTanh[(Sqrt[2]\*Sqrt[e]\*x)/Sqrt[d + e\*x^2]]/(2\*Sqrt[2]\*d^2\*Sqrt[e])

**Rule 208**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

**Rule 377**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

**Rule 382**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(b\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(p+1)\*(b\*c - a\*d)), x] + Dist[(b\*c + n\*(p+1)\*(b\*c - a\*d))/(a\*n\*(p+1)\*(b\*c - a\*d)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p+q+2)+1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]

**Rule 1150**

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] := Int[(d + e\*x^2)^(p+q)\*(a/d + (c\*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x] && EqQ[c\*d^2 + a\*e^2, 0] && IntegerQ[p]

**Rubi steps**

$$\begin{aligned}
\int \frac{1}{\sqrt{d+ex^2}(d^2-e^2x^4)} dx &= \int \frac{1}{(d-ex^2)(d+ex^2)^{3/2}} dx \\
&= \frac{x}{2d^2\sqrt{d+ex^2}} + \frac{\int \frac{1}{(d-ex^2)\sqrt{d+ex^2}} dx}{2d} \\
&= \frac{x}{2d^2\sqrt{d+ex^2}} + \frac{\text{Subst}\left(\int \frac{1}{d-2dex^2} dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2d} \\
&= \frac{x}{2d^2\sqrt{d+ex^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}d^2\sqrt{e}}
\end{aligned}$$

**Mathematica [A]** time = 0.13, size = 108, normalized size = 1.77

$$\frac{\frac{4x}{\sqrt{d+ex^2}} - \frac{\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{d}-\sqrt{ex}}{\sqrt{2}\sqrt{d+ex^2}}\right)}{\sqrt{e}} + \frac{\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{ex}}{\sqrt{2}\sqrt{d+ex^2}}\right)}{\sqrt{e}}}{8d^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[d + e\*x^2]\*(d^2 - e^2\*x^4)), x]

[Out] ((4\*x)/Sqrt[d + e\*x^2] - (Sqrt[2]\*ArcTanh[(Sqrt[d] - Sqrt[e]\*x)/(Sqrt[2]\*Sqrt[d + e\*x^2])])/Sqrt[e] + (Sqrt[2]\*ArcTanh[(Sqrt[d] + Sqrt[e]\*x)/(Sqrt[2]\*Sqrt[d + e\*x^2])])/Sqrt[e])/(8\*d^2)

**IntegrateAlgebraic [A]** time = 0.15, size = 84, normalized size = 1.38

$$\frac{x}{2d^2\sqrt{d+ex^2}} + \frac{\tanh^{-1}\left(-\frac{ex^2}{\sqrt{2}d} + \frac{\sqrt{ex}\sqrt{d+ex^2}}{\sqrt{2}d} + \frac{1}{\sqrt{2}}\right)}{2\sqrt{2}d^2\sqrt{e}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[d + e\*x^2]\*(d^2 - e^2\*x^4)), x]

[Out] x/(2\*d^2\*Sqrt[d + e\*x^2]) + ArcTanh[1/Sqrt[2] - (e\*x^2)/(Sqrt[2]\*d) + (Sqrt[e]\*x\*Sqrt[d + e\*x^2])/(Sqrt[2]\*d)]/(2\*Sqrt[2]\*d^2\*Sqrt[e])

**fricas [B]** time = 1.08, size = 209, normalized size = 3.43

$$\left[ \frac{\sqrt{2}(ex^2+d)\sqrt{e} \log\left(\frac{17e^2x^4+14dex^2+4\sqrt{2}(3ex^3+dx)\sqrt{ex^2+d}\sqrt{e+d^2}}{e^2x^4-2dex^2+d^2}\right) + 8\sqrt{ex^2+d}ex - \sqrt{2}(ex^2+d)\sqrt{e} \arctan\left(\frac{\sqrt{2}(3ex^2+d)\sqrt{ex^2+d}\sqrt{-e}}{4(e^2x^3+dex)}\right) - 4\sqrt{ex^2+d}ex}{16(d^2e^2x^2+d^3e)}, -\frac{\sqrt{2}(ex^2+d)\sqrt{e} \arctan\left(\frac{\sqrt{2}(3ex^2+d)\sqrt{ex^2+d}\sqrt{-e}}{4(e^2x^3+dex)}\right) - 4\sqrt{ex^2+d}ex}{8(d^2e^2x^2+d^3e)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)^(1/2)/(-e^2\*x^4+d^2), x, algorithm="fricas")

[Out] [1/16\*(sqrt(2)\*(e\*x^2 + d)\*sqrt(e)\*log((17\*e^2\*x^4 + 14\*d\*e\*x^2 + 4\*sqrt(2)\*(3\*e\*x^3 + d\*x)\*sqrt(e\*x^2 + d)\*sqrt(e) + d^2)/(e^2\*x^4 - 2\*d\*e\*x^2 + d^2)) + 8\*sqrt(e\*x^2 + d)\*e\*x)/(d^2\*e^2\*x^2 + d^3\*e), -1/8\*(sqrt(2)\*(e\*x^2 + d)\*sqrt(-e)\*arctan(1/4\*sqrt(2)\*(3\*e\*x^2 + d)\*sqrt(e\*x^2 + d)\*sqrt(-e)/(e^2\*x^3 + d\*e\*x)) - 4\*sqrt(e\*x^2 + d)\*e\*x)/(d^2\*e^2\*x^2 + d^3\*e)]

**giac [A]** time = 0.33, size = 1, normalized size = 0.02

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)^(1/2)/(-e^2\*x^4+d^2),x, algorithm="giac")

[Out] +Infinity

**maple [B]** time = 0.02, size = 441, normalized size = 7.23

$$\frac{\sqrt{2} e \ln \left( \frac{4d+2\sqrt{2} \sqrt{2d+\left(x-\frac{\sqrt{de}}{e}\right)^2} e+2\sqrt{de} \left(x-\frac{\sqrt{de}}{e}\right) \sqrt{d+2\sqrt{de} \left(x-\frac{\sqrt{de}}{e}\right)}}{x-\frac{\sqrt{de}}{e}} \right)}{4\sqrt{de} (\sqrt{-de} + \sqrt{de})(\sqrt{-de} - \sqrt{de}) \sqrt{d}} + \frac{\sqrt{2} e \ln \left( \frac{4d+2\sqrt{2} \sqrt{2d+\left(x+\frac{\sqrt{de}}{e}\right)^2} e-2\sqrt{de} \left(x+\frac{\sqrt{de}}{e}\right) \sqrt{d-2\sqrt{de} \left(x+\frac{\sqrt{de}}{e}\right)}}{x+\frac{\sqrt{de}}{e}} \right)}{4\sqrt{de} (\sqrt{-de} + \sqrt{de})(\sqrt{-de} - \sqrt{de}) \sqrt{d}} - \frac{\sqrt{\left(x-\frac{\sqrt{de}}{e}\right)^2 e+2\sqrt{de} \left(x-\frac{\sqrt{de}}{e}\right)}}{2(\sqrt{-de} + \sqrt{de})(\sqrt{-de} - \sqrt{de}) \left(x-\frac{\sqrt{de}}{e}\right) d} - \frac{\sqrt{\left(x+\frac{\sqrt{de}}{e}\right)^2 e-2\sqrt{de} \left(x+\frac{\sqrt{de}}{e}\right)}}{2(\sqrt{-de} + \sqrt{de})(\sqrt{-de} - \sqrt{de}) \left(x+\frac{\sqrt{de}}{e}\right) d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x^2+d)^(1/2)/(-e^2\*x^4+d^2),x)

[Out] -1/2/((-d\*e)^(1/2)+(d\*e)^(1/2))/((-d\*e)^(1/2)-(d\*e)^(1/2))/d/(x-(-d\*e)^(1/2)/e)\*((x-(-d\*e)^(1/2)/e)^2\*e+2\*(-d\*e)^(1/2)\*(x-(-d\*e)^(1/2)/e))^(1/2)-1/4\*e/(d\*e)^(1/2)/((-d\*e)^(1/2)+(d\*e)^(1/2))/((-d\*e)^(1/2)-(d\*e)^(1/2))\*2^(1/2)/d^(1/2)\*ln((4\*d+2\*2^(1/2)\*(2\*d+(x-(d\*e)^(1/2)/e)^2\*e+2\*(d\*e)^(1/2)\*(x-(d\*e)^(1/2)/e))^(1/2)\*d^(1/2)+2\*(d\*e)^(1/2)\*(x-(d\*e)^(1/2)/e))/(x-(d\*e)^(1/2)/e)-1/2/((-d\*e)^(1/2)+(d\*e)^(1/2))/((-d\*e)^(1/2)-(d\*e)^(1/2))/d/(x+(d\*e)^(1/2)/e)\*((x+(d\*e)^(1/2)/e)^2\*e-2\*(-d\*e)^(1/2)\*(x+(d\*e)^(1/2)/e))^(1/2)+1/4\*e/(d\*e)^(1/2)/((-d\*e)^(1/2)+(d\*e)^(1/2))/((-d\*e)^(1/2)-(d\*e)^(1/2))\*2^(1/2)/d^(1/2)\*ln((4\*d+2\*2^(1/2)\*(2\*d+(x+(d\*e)^(1/2)/e)^2\*e-2\*(d\*e)^(1/2)\*(x+(d\*e)^(1/2)/e))^(1/2)\*d^(1/2)-2\*(d\*e)^(1/2)\*(x+(d\*e)^(1/2)/e))/(x+(d\*e)^(1/2)/e))

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{(e^2x^4 - d^2)\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)^(1/2)/(-e^2\*x^4+d^2),x, algorithm="maxima")

[Out] -integrate(1/((e^2\*x^4 - d^2)\*sqrt(e\*x^2 + d)), x)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(d^2 - e^2 x^4) \sqrt{e x^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d^2 - e^2\*x^4)\*(d + e\*x^2)^(1/2)),x)

[Out] int(1/((d^2 - e^2\*x^4)\*(d + e\*x^2)^(1/2)), x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-d^2\sqrt{d + ex^2} + e^2x^4\sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x\*\*2+d)\*\*(1/2)/(-e\*\*2\*x\*\*4+d\*\*2),x)

[Out] -Integral(1/(-d\*\*2\*sqrt(d + e\*x\*\*2) + e\*\*2\*x\*\*4\*sqrt(d + e\*x\*\*2)), x)

$$3.140 \quad \int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)} dx$$

Optimal. Leaf size=80

$$\frac{7x}{12d^3\sqrt{d+ex^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{4\sqrt{2}d^3\sqrt{e}} + \frac{x}{6d^2(d+ex^2)^{3/2}}$$

**Rubi [A]** time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {1150, 414, 527, 12, 377, 208}

$$\frac{7x}{12d^3\sqrt{d+ex^2}} + \frac{x}{6d^2(d+ex^2)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{4\sqrt{2}d^3\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x^2)^(3/2)\*(d^2 - e^2\*x^4)),x]

[Out] x/(6\*d^2\*(d + e\*x^2)^(3/2)) + (7\*x)/(12\*d^3\*Sqrt[d + e\*x^2]) + ArcTanh[(Sqrt[2]\*Sqrt[e]\*x)/Sqrt[d + e\*x^2]]/(4\*Sqrt[2]\*d^3\*Sqrt[e])

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

#### Rule 414

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(b\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(p+1)\*(b\*c - a\*d)), x] + Dist[1/(a\*n\*(p+1)\*(b\*c - a\*d)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q\*Simp[b\*c + n\*(p+1)\*(b\*c - a\*d) + d\*b\*(n\*(p+q+2)+1)\*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

#### Rule 527

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.)\*((e\_) + (f\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*e - a\*f)\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(b\*c - a\*d)\*(p+1)), x] + Dist[1/(a\*n\*(b\*c - a\*d)\*(p+1)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q\*Simp[c\*(b\*e - a\*f) + e\*n\*(b\*c - a\*d)\*(p+1) + d\*(b\*e - a\*f)\*(n\*(p+q+2)+1)\*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]



## Rule 1150

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] := Int[(d + e\*x^2)^(p + q)\*(a/d + (c\*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x] && EqQ[c\*d^2 + a\*e^2, 0] && IntegerQ[p]

## Rubi steps

$$\begin{aligned}
 \int \frac{1}{(d + ex^2)^{3/2} (d^2 - e^2x^4)} dx &= \int \frac{1}{(d - ex^2) (d + ex^2)^{5/2}} dx \\
 &= \frac{x}{6d^2 (d + ex^2)^{3/2}} - \frac{\int \frac{-5de + 2e^2x^2}{(d - ex^2)(d + ex^2)^{3/2}} dx}{6d^2e} \\
 &= \frac{x}{6d^2 (d + ex^2)^{3/2}} + \frac{7x}{12d^3 \sqrt{d + ex^2}} + \frac{\int \frac{3d^2e^2}{(d - ex^2)\sqrt{d + ex^2}} dx}{12d^4e^2} \\
 &= \frac{x}{6d^2 (d + ex^2)^{3/2}} + \frac{7x}{12d^3 \sqrt{d + ex^2}} + \frac{\int \frac{1}{(d - ex^2)\sqrt{d + ex^2}} dx}{4d^2} \\
 &= \frac{x}{6d^2 (d + ex^2)^{3/2}} + \frac{7x}{12d^3 \sqrt{d + ex^2}} + \frac{\text{Subst}\left(\int \frac{1}{d - 2dex^2} dx, x, \frac{x}{\sqrt{d + ex^2}}\right)}{4d^2} \\
 &= \frac{x}{6d^2 (d + ex^2)^{3/2}} + \frac{7x}{12d^3 \sqrt{d + ex^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d + ex^2}}\right)}{4\sqrt{2}d^3\sqrt{e}}
 \end{aligned}$$

**Mathematica [C]** time = 3.34, size = 345, normalized size = 4.31

$$\frac{\frac{384e^4x^8(d+ex^2)^2 {}_3F_2\left(2, 2, 2; 1, \frac{9}{2}; -\frac{2ex^2}{d-ex^2}\right)}{ex^2-d} + \frac{384e^4x^8(4d^2+7dex^2+3e^2x^4) {}_2F_1\left(2, 2; \frac{9}{2}; -\frac{2ex^2}{d-ex^2}\right)}{ex^2-d} + \frac{35\sqrt{2}\sqrt{\frac{ex^2}{ex^2-d}}(-15d^3-5d^2ex^2+12de^2x^4+8e^3x^6)\left(\sqrt{2}\sqrt{\frac{ex^2}{ex^2-d}}\sqrt{\frac{d+ex^2}{d-ex^2}}(-3d^2-2dex^2+5e^2x^4)+3(d+ex^2)^2\sin^{-1}\left(\sqrt{2}\sqrt{\frac{ex^2}{ex^2-d}}\right)\right)}{\sqrt{\frac{d+ex^2}{d-ex^2}}}}{2520d^5e^3x^5\sqrt{d+ex^2}\left(1-\frac{e^2x^4}{d^2}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((d + e\*x^2)^(3/2)\*(d^2 - e^2\*x^4)), x]

[Out] ((35\*Sqrt[2]\*Sqrt[(e\*x^2)/(-d + e\*x^2)]\*(-15\*d^3 - 5\*d^2\*e\*x^2 + 12\*d\*e^2\*x^4 + 8\*e^3\*x^6)\*(Sqrt[2]\*Sqrt[(e\*x^2)/(-d + e\*x^2)]\*Sqrt[(d + e\*x^2)/(d - e\*x^2)]\*(-3\*d^2 - 2\*d\*e\*x^2 + 5\*e^2\*x^4) + 3\*(d + e\*x^2)^2\*ArcSin[Sqrt[2]\*Sqrt[(e\*x^2)/(-d + e\*x^2)]])/Sqrt[(d + e\*x^2)/(d - e\*x^2)] + (384\*e^4\*x^8\*(4\*d^2 + 7\*d\*e\*x^2 + 3\*e^2\*x^4)\*Hypergeometric2F1[2, 2, 9/2, (-2\*e\*x^2)/(d - e\*x^2)]/(-d + e\*x^2) + (384\*e^4\*x^8\*(d + e\*x^2)^2\*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, (-2\*e\*x^2)/(d - e\*x^2)]/(-d + e\*x^2))/(2520\*d^5\*e^3\*x^5\*Sqrt[d + e\*x^2]\*(1 - (e^2\*x^4)/d^2)))

**IntegrateAlgebraic [A]** time = 0.20, size = 94, normalized size = 1.18

$$\frac{\tanh^{-1}\left(-\frac{ex^2}{\sqrt{2}d} + \frac{\sqrt{ex}\sqrt{d+ex^2}}{\sqrt{2}d} + \frac{1}{\sqrt{2}}\right)}{4\sqrt{2}d^3\sqrt{e}} + \frac{9dx + 7ex^3}{12d^3(d + ex^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((d + e\*x^2)^(3/2)\*(d^2 - e^2\*x^4)),x]

[Out] (9\*d\*x + 7\*e\*x^3)/(12\*d^3\*(d + e\*x^2)^(3/2)) + ArcTanh[1/Sqrt[2] - (e\*x^2)/(Sqrt[2]\*d) + (Sqrt[e]\*x\*Sqrt[d + e\*x^2])/(Sqrt[2]\*d)]/(4\*Sqrt[2]\*d^3\*Sqrt[e])

**fricas** [B] time = 1.74, size = 279, normalized size = 3.49

$$\left[ \frac{3\sqrt{2}(e^2x^4 + 2dex^2 + d^2)\sqrt{e} \log\left(\frac{17e^2x^4 + 14dex^2 + 4\sqrt{2}(3ex^3 + dx)\sqrt{ex^2 + d}\sqrt{e} + d^2}{e^2x^4 - 2dex^2 + d^2}\right) + 8(7e^2x^3 + 9dex)\sqrt{ex^2 + d}}{96(d^3e^3x^4 + 2d^4e^2x^2 + d^5e)}, -\frac{3\sqrt{2}(e^2x^4 + 2dex^2 + d^2)\sqrt{-e} \arctan\left(\frac{\sqrt{2}(3ex^2 + d)\sqrt{ex^2 + d}\sqrt{-e}}{4(e^2x^3 + dex)}\right) - 4(7e^2x^3 + 9dex)\sqrt{ex^2 + d}}{48(d^3e^3x^4 + 2d^4e^2x^2 + d^5e)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)^(3/2)/(-e^2\*x^4+d^2),x, algorithm="fricas")

[Out] [1/96\*(3\*sqrt(2)\*(e^2\*x^4 + 2\*d\*e\*x^2 + d^2)\*sqrt(e)\*log((17\*e^2\*x^4 + 14\*d\*e\*x^2 + 4\*sqrt(2)\*(3\*e\*x^3 + d\*x)\*sqrt(e\*x^2 + d)\*sqrt(e) + d^2)/(e^2\*x^4 - 2\*d\*e\*x^2 + d^2)) + 8\*(7\*e^2\*x^3 + 9\*d\*e\*x)\*sqrt(e\*x^2 + d))/(d^3\*e^3\*x^4 + 2\*d^4\*e^2\*x^2 + d^5\*e), -1/48\*(3\*sqrt(2)\*(e^2\*x^4 + 2\*d\*e\*x^2 + d^2)\*sqrt(-e)\*arctan(1/4\*sqrt(2)\*(3\*e\*x^2 + d)\*sqrt(e\*x^2 + d)\*sqrt(-e)/(e^2\*x^3 + d\*e\*x)) - 4\*(7\*e^2\*x^3 + 9\*d\*e\*x)\*sqrt(e\*x^2 + d))/(d^3\*e^3\*x^4 + 2\*d^4\*e^2\*x^2 + d^5\*e)]

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)^(3/2)/(-e^2\*x^4+d^2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Unable to transpose Error: Bad Argument Value

**maple** [B] time = 0.03, size = 911, normalized size = 11.39



Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x^2+d)^(3/2)/(-e^2\*x^4+d^2),x)

[Out] -1/6/((-d\*e)^(1/2)+(d\*e)^(1/2))/((-d\*e)^(1/2)-(d\*e)^(1/2))/d/(x-(-d\*e)^(1/2)/e)/((x-(-d\*e)^(1/2)/e)^2\*e+2\*(-d\*e)^(1/2)\*(x-(-d\*e)^(1/2)/e)^(1/2)-1/3\*e/((-d\*e)^(1/2)+(d\*e)^(1/2))/((-d\*e)^(1/2)-(d\*e)^(1/2))/d^2/((x-(-d\*e)^(1/2)/e)^2\*e+2\*(-d\*e)^(1/2)\*(x-(-d\*e)^(1/2)/e)^(1/2)\*x+1/4\*e/(d\*e)^(1/2))/((-d\*e)^(1/2)+(d\*e)^(1/2))/((-d\*e)^(1/2)-(d\*e)^(1/2))/d/(2\*d+(x-(d\*e)^(1/2)/e)^2\*e+2\*(d\*e)^(1/2)\*(x-(d\*e)^(1/2)/e)^(1/2)-1/4\*e/((-d\*e)^(1/2)+(d\*e)^(1/2))/((-d\*e)^(1/2)-(d\*e)^(1/2))/d^2/(2\*d+(x-(d\*e)^(1/2)/e)^2\*e+2\*(d\*e)^(1/2)\*(x-(d\*e)^(1/2)/e)^(1/2)\*x-1/8\*e/(d\*e)^(1/2))/((-d\*e)^(1/2)+(d\*e)^(1/2))/((-d\*e)^(1/2)-(d\*e)^(1/2))/d^(3/2)\*2^(1/2)\*ln((4\*d+2\*2^(1/2)\*(2\*d+(x-(d\*e)^(1/2)/e)^2\*e+2\*(d\*e)^(1/2)\*(x-(d\*e)^(1/2)/e)^(1/2)\*d^(1/2)+2\*(d\*e)^(1/2)\*(x-(d\*e)^(1/2)/e))/((x-(d\*e)^(1/2)/e))-1/6/((-d\*e)^(1/2)+(d\*e)^(1/2))/((-d\*e)^(1/2)-(d\*e)^(1/2))/d/(x+(-d\*e)^(1/2)/e)/((x+(-d\*e)^(1/2)/e)^2\*e-2\*(-d\*e)^(1/2)\*(x+(-d\*e)^(1/2)/e)^(1/2)-1/3\*e/((-d\*e)^(1/2)+(d\*e)^(1/2))/((-d\*e)^(1/2)-(d\*e)^(1/2))/d^2/((x+(-d\*e)^(1/2)/e)^2\*e-2\*(-d\*e)^(1/2)\*(x+(-d\*e)^(1/2)/e)^(1/2)\*x-1/4\*e/(d\*e)^(1/2))/((-d\*e)^(1/2)+(d\*e)^(1/2))/((-d\*e)^(1/2)-(d\*e)^(1/2))/d/(2\*d+(x+(d\*e)^(1/2)/e)^2\*e-2\*(d\*e)^(1/2)\*(x+(d\*e)^(1/2)/e)^(1/2)-1/4\*e/((-d\*e)^(1/2)+(d\*e)^(1/2))/((-d\*e)^(1/2)-(d\*e)^(1/2))/d^2/(2\*d+(x+(d\*e)^(1/2)/e)^2\*e-2\*(d\*e)^(1/2)\*(x+(d\*e)^(1/2)/e)^(1/2)\*x+1/8\*e/(d\*e)^(1/2))/((-d\*e)^(1/2)+(d\*e)^(1/2))/((-d\*e)^(1/2)-(d\*e)^(1/2))/d^(3/2)\*2^(1/2)\*ln((4\*d+2\*

$2^{1/2} * (2*d + (x + (d*e)^{1/2}/e)^{2*e} - 2*(d*e)^{1/2} * (x + (d*e)^{1/2}/e))^{1/2} * d^{1/2} - 2*(d*e)^{1/2} * (x + (d*e)^{1/2}/e) / (x + (d*e)^{1/2}/e)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{1}{(e^2 x^4 - d^2)(e x^2 + d)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)^(3/2)/(-e^2\*x^4+d^2),x, algorithm="maxima")

[Out] -integrate(1/((e^2\*x^4 - d^2)\*(e\*x^2 + d)^(3/2)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(d^2 - e^2 x^4)(e x^2 + d)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d^2 - e^2\*x^4)\*(d + e\*x^2)^(3/2)),x)

[Out] int(1/((d^2 - e^2\*x^4)\*(d + e\*x^2)^(3/2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{1}{-d^3 \sqrt{d + e x^2} - d^2 e x^2 \sqrt{d + e x^2} + d e^2 x^4 \sqrt{d + e x^2} + e^3 x^6 \sqrt{d + e x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x\*\*2+d)\*\*(3/2)/(-e\*\*2\*x\*\*4+d\*\*2),x)

[Out] -Integral(1/(-d\*\*3\*sqrt(d + e\*x\*\*2) - d\*\*2\*e\*x\*\*2\*sqrt(d + e\*x\*\*2) + d\*e\*\*2\*x\*\*4\*sqrt(d + e\*x\*\*2) + e\*\*3\*x\*\*6\*sqrt(d + e\*x\*\*2)), x)

$$3.141 \quad \int \frac{(a+bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}} dx$$

**Optimal.** Leaf size=153

$$\frac{9ax(a-bx^2)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)(a+bx^2)^{3/2}}{4\sqrt{a^2-b^2x^4}} + \frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}$$

**Rubi [A]** time = 0.05, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$ , Rules used = {1152, 416, 388, 217, 203}

$$\frac{9ax(a-bx^2)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)(a+bx^2)^{3/2}}{4\sqrt{a^2-b^2x^4}} + \frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(5/2)/Sqrt[a^2 - b^2\*x^4], x]

[Out] (-9\*a\*x\*(a - b\*x^2)\*Sqrt[a + b\*x^2])/(8\*Sqrt[a^2 - b^2\*x^4]) - (x\*(a - b\*x^2)\*(a + b\*x^2)^(3/2))/(4\*Sqrt[a^2 - b^2\*x^4]) + (19\*a^2\*Sqrt[a - b\*x^2]\*Sqrt[a + b\*x^2]\*ArcTan[(Sqrt[b]\*x)/Sqrt[a - b\*x^2]])/(8\*Sqrt[b]\*Sqrt[a^2 - b^2\*x^4])

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rule 416

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(b\*(n\*(p + q) + 1)), x] + Dist[1/(b\*(n\*(p + q) + 1)), Int[(a + b\*x^n)^p\*(c + d\*x^n)^(q - 2)\*Simp[c\*(b\*c\*(n\*(p + q) + 1) - a\*d] + d\*(b\*c\*(n\*(p + 2\*q - 1) + 1) - a\*d\*(n\*(q - 1) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && GtQ[q, 1] && NeQ[n\*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

#### Rule 1152

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] := Dist[(a + c\*x^4)^FracPart[p]/((d + e\*x^2)^FracPart[p]\*(a/d + (c\*x^2)/e)^FracPart[p]), Int[(d + e\*x^2)^(p + q)\*(a/d + (c\*x^2)/e)^p, x], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}} dx &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right) \int \frac{(a+bx^2)^2}{\sqrt{a-bx^2}} dx}{\sqrt{a^2-b^2x^4}} \\
&= -\frac{x(a-bx^2)(a+bx^2)^{3/2}}{4\sqrt{a^2-b^2x^4}} - \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right) \int \frac{-5a^2b-9ab^2x^2}{\sqrt{a-bx^2}} dx}{4b\sqrt{a^2-b^2x^4}} \\
&= -\frac{9ax(a-bx^2)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)(a+bx^2)^{3/2}}{4\sqrt{a^2-b^2x^4}} + \frac{\left(19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\right) \int \frac{1}{\sqrt{a-bx^2}}}{8\sqrt{a^2-b^2x^4}} \\
&= -\frac{9ax(a-bx^2)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)(a+bx^2)^{3/2}}{4\sqrt{a^2-b^2x^4}} + \frac{\left(19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\right) \text{Subst}\left(\frac{1}{\sqrt{a-bx^2}}\right)}{8\sqrt{a^2-b^2x^4}} \\
&= -\frac{9ax(a-bx^2)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)(a+bx^2)^{3/2}}{4\sqrt{a^2-b^2x^4}} + \frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2} \tan^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}
\end{aligned}$$

**Mathematica [C]** time = 0.17, size = 98, normalized size = 0.64

$$-\frac{(11ax+2bx^3)\sqrt{a^2-b^2x^4}}{8\sqrt{a+bx^2}} + \frac{19ia^2 \log\left(\frac{2\sqrt{a^2-b^2x^4}}{\sqrt{a+bx^2}} - 2i\sqrt{b}x\right)}{8\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(5/2)/Sqrt[a^2 - b^2\*x^4], x]

[Out] -1/8\*((11\*a\*x + 2\*b\*x^3)\*Sqrt[a^2 - b^2\*x^4])/Sqrt[a + b\*x^2] + (((19\*I)/8)\*a^2\*Log[(-2\*I)\*Sqrt[b]\*x + (2\*Sqrt[a^2 - b^2\*x^4])/Sqrt[a + b\*x^2]])/Sqrt[b]

**IntegrateAlgebraic [F]** time = 3.03, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b\*x^2)^(5/2)/Sqrt[a^2 - b^2\*x^4], x]

[Out] Defer[IntegrateAlgebraic] [(a + b\*x^2)^(5/2)/Sqrt[a^2 - b^2\*x^4], x]

**fricas [A]** time = 1.13, size = 251, normalized size = 1.64

$$\left[ \frac{19(a^2bx^2+a^3)\sqrt{-b} \log\left(\frac{2b^2x^4+abx^2-2\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{-bx-a^2}}{bx^2+a}\right) + 2\sqrt{-b^2x^4+a^2}(2b^2x^3+11abx)\sqrt{bx^2+a}}{16(b^2x^2+ab)}, \frac{19(a^2bx^2+a^3)\sqrt{b} \arctan\left(\frac{\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{b}}{b^2x^3+abx}\right) + \sqrt{-b^2x^4+a^2}(2b^2x^3+11abx)\sqrt{bx^2+a}}{8(b^2x^2+ab)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^(5/2)/(-b^2\*x^4+a^2)^(1/2), x, algorithm="fricas")

[Out] [-1/16\*(19\*(a^2\*b\*x^2 + a^3)\*sqrt(-b)\*log(-(2\*b^2\*x^4 + a\*b\*x^2 - 2\*sqrt(-b^2\*x^4 + a^2)\*sqrt(b\*x^2 + a)\*sqrt(-b)\*x - a^2)/(b\*x^2 + a)) + 2\*sqrt(-b^2\*x^4 + a^2)\*(2\*b^2\*x^3 + 11\*a\*b\*x)\*sqrt(b\*x^2 + a))/(b^2\*x^2 + a\*b), -1/8\*(19\*(a^2\*b\*x^2 + a^3)\*sqrt(b)\*arctan(sqrt(-b^2\*x^4 + a^2)\*sqrt(b\*x^2 + a)\*sqrt

$t(b)/(b^2x^3 + a*b*x) + \text{sqrt}(-b^2x^4 + a^2)*(2*b^2x^3 + 11*a*b*x)*\text{sqrt}(b*x^2 + a)/(b^2x^2 + a*b]$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^2 + a)^{\frac{5}{2}}}{\sqrt{-b^2x^4 + a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^(5/2)/(-b^2\*x^4+a^2)^(1/2),x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(5/2)/sqrt(-b^2\*x^4 + a^2), x)

**maple** [A] time = 0.07, size = 132, normalized size = 0.86

$$\frac{\sqrt{-b^2x^4 + a^2} \left( 2\sqrt{-bx^2 + a} b^{\frac{3}{2}}x^3 - 32a^2 \arctan\left(\frac{\sqrt{b}x}{\sqrt{\frac{(-bx + \sqrt{ab})(bx + \sqrt{ab})}{b}}}\right) + 13a^2 \arctan\left(\frac{\sqrt{b}x}{\sqrt{-bx^2 + a}}\right) + 11\sqrt{-bx^2 + a} a\sqrt{b}x \right)}{8\sqrt{bx^2 + a} \sqrt{-bx^2 + a} \sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(5/2)/(-b^2\*x^4+a^2)^(1/2),x)

[Out]  $-1/8*(-b^2*x^4+a^2)^{(1/2)}*(2*x^3*b^{(3/2)}*(-b*x^2+a)^{(1/2)}+11*(-b*x^2+a)^{(1/2)}*b^{(1/2)}*x*a+13*\arctan(1/(-b*x^2+a)^{(1/2)}*b^{(1/2)}*x)*a^2-32*\arctan(b^{(1/2)}*x/((-b*x+(a*b)^{(1/2)})/b*(b*x+(a*b)^{(1/2)}))^{(1/2)})*a^2)/(b*x^2+a)^{(1/2)}/(-b*x^2+a)^{(1/2)}/b^{(1/2)}$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^2 + a)^{\frac{5}{2}}}{\sqrt{-b^2x^4 + a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^(5/2)/(-b^2\*x^4+a^2)^(1/2),x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(5/2)/sqrt(-b^2\*x^4 + a^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(bx^2 + a)^{\frac{5}{2}}}{\sqrt{a^2 - b^2x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^2)^(5/2)/(a^2 - b^2\*x^4)^(1/2),x)

[Out] int((a + b\*x^2)^(5/2)/(a^2 - b^2\*x^4)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)^{\frac{5}{2}}}{\sqrt{-(-a + bx^2)(a + bx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(5/2)/(-b\*\*2\*x\*\*4+a\*\*2)\*\*(1/2),x)

[Out] Integral((a + b\*x\*\*2)\*\*(5/2)/sqrt(-(-a + b\*x\*\*2)\*(a + b\*x\*\*2)), x)

$$3.142 \quad \int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}} dx$$

Optimal. Leaf size=110

$$\frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}}$$

Rubi [A] time = 0.03, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {1152, 388, 217, 203}

$$\frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(3/2)/Sqrt[a^2 - b^2\*x^4], x]

[Out] -(x\*(a - b\*x^2)\*Sqrt[a + b\*x^2])/(2\*Sqrt[a^2 - b^2\*x^4]) + (3\*a\*Sqrt[a - b\*x^2]\*Sqrt[a + b\*x^2]\*ArcTan[(Sqrt[b]\*x)/Sqrt[a - b\*x^2]])/(2\*Sqrt[b]\*Sqrt[a^2 - b^2\*x^4])

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rule 1152

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] := Dist[(a + c\*x^4)^FracPart[p]/((d + e\*x^2)^FracPart[p]\*(a/d + (c\*x^2)/e)^FracPart[p]), Int[(d + e\*x^2)^(p + q)\*(a/d + (c\*x^2)/e)^p, x], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[p]

#### Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}} dx &= \frac{(\sqrt{a-bx^2}\sqrt{a+bx^2}) \int \frac{a+bx^2}{\sqrt{a-bx^2}} dx}{\sqrt{a^2-b^2x^4}} \\
&= -\frac{x(a-bx^2)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}} + \frac{(3a\sqrt{a-bx^2}\sqrt{a+bx^2}) \int \frac{1}{\sqrt{a-bx^2}} dx}{2\sqrt{a^2-b^2x^4}} \\
&= -\frac{x(a-bx^2)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}} + \frac{(3a\sqrt{a-bx^2}\sqrt{a+bx^2}) \text{Subst}\left(\int \frac{1}{1+bx^2} dx, x, \frac{x}{\sqrt{a-bx^2}}\right)}{2\sqrt{a^2-b^2x^4}} \\
&= -\frac{x(a-bx^2)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}} + \frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2} \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}}
\end{aligned}$$

**Mathematica [C]** time = 0.07, size = 86, normalized size = 0.78

$$-\frac{x\sqrt{a^2-b^2x^4}}{2\sqrt{a+bx^2}} + \frac{3ia \log\left(\frac{2\sqrt{a^2-b^2x^4}}{\sqrt{a+bx^2}} - 2i\sqrt{b}x\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(3/2)/Sqrt[a^2 - b^2\*x^4], x]

[Out] -1/2\*(x\*Sqrt[a^2 - b^2\*x^4])/Sqrt[a + b\*x^2] + (((3\*I)/2)\*a\*Log[(-2\*I)\*Sqrt[b]\*x + (2\*Sqrt[a^2 - b^2\*x^4])/Sqrt[a + b\*x^2]])/Sqrt[b]

**IntegrateAlgebraic [F]** time = 2.55, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b\*x^2)^(3/2)/Sqrt[a^2 - b^2\*x^4], x]

[Out] Defer[IntegrateAlgebraic] [(a + b\*x^2)^(3/2)/Sqrt[a^2 - b^2\*x^4], x]

**fricas [A]** time = 1.09, size = 223, normalized size = 2.03

$$\left[ \frac{2\sqrt{-b^2x^4+a^2}\sqrt{bx^2+abx+3(abx^2+a^2)\sqrt{-b}} \log\left(-\frac{2b^2x^4+abx^2-2\sqrt{-b^2x^4+a^2}\sqrt{bx^2+ab}\sqrt{-bx-a^2}}{bx^2+a}\right), \frac{\sqrt{-b^2x^4+a^2}\sqrt{bx^2+abx+3(abx^2+a^2)\sqrt{b}} \arctan\left(\frac{\sqrt{-b^2x^4+a^2}\sqrt{bx^2+ab}\sqrt{b}}{b^2x^3+abx}\right)}{2(b^2x^2+ab)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^(3/2)/(-b^2\*x^4+a^2)^(1/2), x, algorithm="fricas")

[Out] [-1/4\*(2\*sqrt(-b^2\*x^4 + a^2)\*sqrt(b\*x^2 + a)\*b\*x + 3\*(a\*b\*x^2 + a^2)\*sqrt(-b)\*log((-2\*b^2\*x^4 + a\*b\*x^2 - 2\*sqrt(-b^2\*x^4 + a^2)\*sqrt(b\*x^2 + a)\*sqrt(-b)\*x - a^2)/(b\*x^2 + a)))/(b^2\*x^2 + a\*b), -1/2\*(sqrt(-b^2\*x^4 + a^2)\*sqrt(b\*x^2 + a)\*b\*x + 3\*(a\*b\*x^2 + a^2)\*sqrt(b)\*arctan(sqrt(-b^2\*x^4 + a^2)\*sqrt(b\*x^2 + a)\*sqrt(b)/(b^2\*x^3 + a\*b\*x)))/(b^2\*x^2 + a\*b)]

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^2+a)^{3/2}}{\sqrt{-b^2x^4+a^2}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^(3/2)/(-b^2\*x^4+a^2)^(1/2),x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(3/2)/sqrt(-b^2\*x^4 + a^2), x)

**maple** [A] time = 0.02, size = 107, normalized size = 0.97

$$\frac{\sqrt{-b^2x^4 + a^2} \left( -4a \arctan\left(\frac{\sqrt{b}x}{\sqrt{\frac{(-bx + \sqrt{ab})(bx + \sqrt{ab})}{b}}}\right) + a \arctan\left(\frac{\sqrt{b}x}{\sqrt{-bx^2 + a}}\right) + \sqrt{-bx^2 + a} \sqrt{b}x \right)}{2\sqrt{bx^2 + a} \sqrt{-bx^2 + a} \sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(3/2)/(-b^2\*x^4+a^2)^(1/2),x)

[Out] -1/2/(b\*x^2+a)^(1/2)\*(-b^2\*x^4+a^2)^(1/2)\*(x\*(-b\*x^2+a)^(1/2)\*b^(1/2)+a\*arctan(1/(-b\*x^2+a)^(1/2)\*b^(1/2)\*x)-4\*arctan(1/((-b\*x+(a\*b)^(1/2))\*(b\*x+(a\*b)^(1/2)))/b)^(1/2)\*b^(1/2)\*x)\*a)/(-b\*x^2+a)^(1/2)/b^(1/2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^2 + a)^{\frac{3}{2}}}{\sqrt{-b^2x^4 + a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^(3/2)/(-b^2\*x^4+a^2)^(1/2),x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(3/2)/sqrt(-b^2\*x^4 + a^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(bx^2 + a)^{3/2}}{\sqrt{a^2 - b^2x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^2)^(3/2)/(a^2 - b^2\*x^4)^(1/2),x)

[Out] int((a + b\*x^2)^(3/2)/(a^2 - b^2\*x^4)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)^{\frac{3}{2}}}{\sqrt{-(-a + bx^2)(a + bx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(3/2)/(-b\*\*2\*x\*\*4+a\*\*2)\*\*(1/2),x)

[Out] Integral((a + b\*x\*\*2)\*\*(3/2)/sqrt(-(-a + b\*x\*\*2)\*(a + b\*x\*\*2)), x)

$$3.143 \quad \int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}} dx$$

**Optimal.** Leaf size=65

$$\frac{\sqrt{a-bx^2} \sqrt{a+bx^2} \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{b} \sqrt{a^2-b^2x^4}}$$

**Rubi [A]** time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$ , Rules used = {1152, 217, 203}

$$\frac{\sqrt{a-bx^2} \sqrt{a+bx^2} \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{b} \sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x^2]/Sqrt[a^2 - b^2\*x^4], x]

[Out] (Sqrt[a - b\*x^2]\*Sqrt[a + b\*x^2]\*ArcTan[(Sqrt[b]\*x)/Sqrt[a - b\*x^2]])/(Sqrt[b]\*Sqrt[a^2 - b^2\*x^4])

Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 1152

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] :> Dist[(a + c\*x^4)^FracPart[p]/((d + e\*x^2)^FracPart[p]\*(a/d + (c\*x^2)/e)^FracPart[p]), Int[(d + e\*x^2)^(p + q)\*(a/d + (c\*x^2)/e)^p, x], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}} dx &= \frac{\left(\sqrt{a-bx^2} \sqrt{a+bx^2}\right) \int \frac{1}{\sqrt{a-bx^2}} dx}{\sqrt{a^2-b^2x^4}} \\ &= \frac{\left(\sqrt{a-bx^2} \sqrt{a+bx^2}\right) \text{Subst}\left(\int \frac{1}{1+bx^2} dx, x, \frac{x}{\sqrt{a-bx^2}}\right)}{\sqrt{a^2-b^2x^4}} \\ &= \frac{\sqrt{a-bx^2} \sqrt{a+bx^2} \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{b} \sqrt{a^2-b^2x^4}} \end{aligned}$$

**Mathematica [C]** time = 0.04, size = 50, normalized size = 0.77

$$\frac{i \log\left(\frac{2\sqrt{a^2-b^2x^4}}{\sqrt{a+bx^2}} - 2i\sqrt{b}x\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b\*x^2]/Sqrt[a^2 - b^2\*x^4], x]

[Out] (I\*Log[(-2\*I)\*Sqrt[b]\*x + (2\*Sqrt[a^2 - b^2\*x^4])/Sqrt[a + b\*x^2]])/Sqrt[b]

**IntegrateAlgebraic** [F] time = 1.33, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^2}}{\sqrt{a^2 - b^2x^4}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[Sqrt[a + b\*x^2]/Sqrt[a^2 - b^2\*x^4], x]

[Out] Defer[IntegrateAlgebraic][Sqrt[a + b\*x^2]/Sqrt[a^2 - b^2\*x^4], x]

**fricas** [A] time = 1.02, size = 121, normalized size = 1.86

$$\left[ \frac{\sqrt{-b} \log\left(-\frac{2b^2x^4 + abx^2 - 2\sqrt{-b^2x^4 + a^2}\sqrt{bx^2 + a}\sqrt{-bx - a^2}}{bx^2 + a}\right)}{2b}, \frac{\arctan\left(\frac{\sqrt{-b^2x^4 + a^2}\sqrt{bx^2 + a}\sqrt{b}}{b^2x^3 + abx}\right)}{\sqrt{b}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^(1/2)/(-b^2\*x^4+a^2)^(1/2), x, algorithm="fricas")

[Out] [-1/2\*sqrt(-b)\*log(-(2\*b^2\*x^4 + a\*b\*x^2 - 2\*sqrt(-b^2\*x^4 + a^2)\*sqrt(b\*x^2 + a)\*sqrt(-b)\*x - a^2)/(b\*x^2 + a))/b, -arctan(sqrt(-b^2\*x^4 + a^2)\*sqrt(b\*x^2 + a)\*sqrt(b)/(b^2\*x^3 + a\*b\*x))/sqrt(b)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2 + a}}{\sqrt{-b^2x^4 + a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^(1/2)/(-b^2\*x^4+a^2)^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(b\*x^2 + a)/sqrt(-b^2\*x^4 + a^2), x)

**maple** [A] time = 0.02, size = 69, normalized size = 1.06

$$\frac{\sqrt{-b^2x^4 + a^2} \arctan\left(\frac{\sqrt{b} x}{\sqrt{\frac{(-bx + \sqrt{ab})(bx + \sqrt{ab})}{b}}}\right)}{\sqrt{bx^2 + a} \sqrt{-bx^2 + a} \sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(1/2)/(-b^2\*x^4+a^2)^(1/2), x)

[Out] 1/(b\*x^2+a)^(1/2)\*(-b^2\*x^4+a^2)^(1/2)\*arctan(1/((-b\*x+(a\*b)^(1/2))\*(b\*x+(a\*b)^(1/2))/b)^(1/2)\*b^(1/2)\*x)/(-b\*x^2+a)^(1/2)/b^(1/2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2 + a}}{\sqrt{-b^2x^4 + a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^(1/2)/(-b^2\*x^4+a^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b\*x^2 + a)/sqrt(-b^2\*x^4 + a^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{bx^2 + a}}{\sqrt{a^2 - b^2x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^2)^(1/2)/(a^2 - b^2\*x^4)^(1/2),x)

[Out] int((a + b\*x^2)^(1/2)/(a^2 - b^2\*x^4)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^2}}{\sqrt{-(-a + bx^2)(a + bx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(1/2)/(-b\*\*2\*x\*\*4+a\*\*2)\*\*(1/2),x)

[Out] Integral(sqrt(a + b\*x\*\*2)/sqrt(-(-a + b\*x\*\*2)\*(a + b\*x\*\*2)), x)

$$3.144 \quad \int \frac{1}{\sqrt{a+bx^2} \sqrt{a^2-b^2x^4}} dx$$

**Optimal.** Leaf size=78

$$\frac{\sqrt{a-bx^2} \sqrt{a+bx^2} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{\sqrt{2}a\sqrt{b} \sqrt{a^2-b^2x^4}}$$

**Rubi [A]** time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$ , Rules used = {1152, 377, 205}

$$\frac{\sqrt{a-bx^2} \sqrt{a+bx^2} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{\sqrt{2}a\sqrt{b} \sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b\*x^2]\*Sqrt[a^2 - b^2\*x^4]), x]

[Out] (Sqrt[a - b\*x^2]\*Sqrt[a + b\*x^2]\*ArcTan[(Sqrt[2]\*Sqrt[b]\*x)/Sqrt[a - b\*x^2]])/(Sqrt[2]\*a\*Sqrt[b]\*Sqrt[a^2 - b^2\*x^4])

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

Rule 1152

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] := Dist[(a + c\*x^4)^FracPart[p]/((d + e\*x^2)^FracPart[p]\*(a/d + (c\*x^2)/e)^FracPart[p]), Int[(d + e\*x^2)^(p+q)\*(a/d + (c\*x^2)/e)^p, x], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a+bx^2} \sqrt{a^2-b^2x^4}} dx &= \frac{\left(\sqrt{a-bx^2} \sqrt{a+bx^2}\right) \int \frac{1}{\sqrt{a-bx^2} (a+bx^2)} dx}{\sqrt{a^2-b^2x^4}} \\ &= \frac{\left(\sqrt{a-bx^2} \sqrt{a+bx^2}\right) \text{Subst}\left(\int \frac{1}{a+2abx^2} dx, x, \frac{x}{\sqrt{a-bx^2}}\right)}{\sqrt{a^2-b^2x^4}} \\ &= \frac{\sqrt{a-bx^2} \sqrt{a+bx^2} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{\sqrt{2}a\sqrt{b} \sqrt{a^2-b^2x^4}} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 78, normalized size = 1.00

$$\frac{\sqrt{a^2-b^2x^4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{\sqrt{2}a\sqrt{b} \sqrt{a-bx^2} \sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + b\*x^2]\*Sqrt[a^2 - b^2\*x^4]), x]

[Out] (Sqrt[a^2 - b^2\*x^4]\*ArcTan[(Sqrt[2]\*Sqrt[b]\*x)/Sqrt[a - b\*x^2]])/(Sqrt[2]\*a\*Sqrt[b]\*Sqrt[a - b\*x^2]\*Sqrt[a + b\*x^2])

**IntegrateAlgebraic** [F] time = 1.62, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bx^2} \sqrt{a^2 - b^2x^4}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(Sqrt[a + b\*x^2]\*Sqrt[a^2 - b^2\*x^4]), x]

[Out] Defer[IntegrateAlgebraic][1/(Sqrt[a + b\*x^2]\*Sqrt[a^2 - b^2\*x^4]), x]

**fricas** [A] time = 0.85, size = 152, normalized size = 1.95

$$\left[ -\frac{\sqrt{2} \sqrt{-b} \log\left(-\frac{3b^2x^4 + 2abx^2 - 2\sqrt{2}\sqrt{-b^2x^4 + a^2}\sqrt{bx^2 + a}\sqrt{-bx - a^2}}{b^2x^4 + 2abx^2 + a^2}\right)}{4ab}, -\frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}\sqrt{-b^2x^4 + a^2}\sqrt{bx^2 + a}\sqrt{b}}{2(b^2x^3 + abx)}\right)}{2a\sqrt{b}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^2+a)^(1/2)/(-b^2\*x^4+a^2)^(1/2), x, algorithm="fricas")

[Out] [-1/4\*sqrt(2)\*sqrt(-b)\*log(-(3\*b^2\*x^4 + 2\*a\*b\*x^2 - 2\*sqrt(2)\*sqrt(-b^2\*x^4 + a^2)\*sqrt(b\*x^2 + a)\*sqrt(-b)\*x - a^2)/(b^2\*x^4 + 2\*a\*b\*x^2 + a^2))/(a\*b), -1/2\*sqrt(2)\*arctan(1/2\*sqrt(2)\*sqrt(-b^2\*x^4 + a^2)\*sqrt(b\*x^2 + a)\*sqrt(b)/(b^2\*x^3 + a\*b\*x))/(a\*sqrt(b))]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2} \sqrt{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^2+a)^(1/2)/(-b^2\*x^4+a^2)^(1/2), x, algorithm="giac")

[Out] integrate(1/(sqrt(-b^2\*x^4 + a^2)\*sqrt(b\*x^2 + a)), x)

**maple** [B] time = 0.06, size = 249, normalized size = 3.19

$$\frac{\sqrt{-b^2x^4 + a^2} \left( \sqrt{2} \sqrt{a} \sqrt{b} \ln\left(\frac{2(a - \sqrt{-ab}x + \sqrt{2}\sqrt{-b^2x^2 + a}\sqrt{a})b}{bx - \sqrt{-ab}}\right) - \sqrt{2} \sqrt{a} \sqrt{b} \ln\left(\frac{2(a + \sqrt{-ab}x + \sqrt{2}\sqrt{-b^2x^2 + a}\sqrt{a})b}{bx + \sqrt{-ab}}\right) - 2\sqrt{-ab} \arctan\left(\frac{\sqrt{b}x}{\sqrt{(-bx + \sqrt{ab})(bx + \sqrt{ab})}}\right) + 2\sqrt{-ab} \arctan\left(\frac{\sqrt{b}x}{\sqrt{-b^2x^2 + a}}\right) \right) \sqrt{b}}{2\sqrt{bx^2 + a} \sqrt{-b^2x^2 + a} (\sqrt{-ab} + \sqrt{ab})(\sqrt{-ab} - \sqrt{ab}) \sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^2+a)^(1/2)/(-b^2\*x^4+a^2)^(1/2), x)

[Out] 1/2\*(-b^2\*x^4+a^2)^(1/2)\*b^(1/2)\*(a^(1/2)\*2^(1/2)\*ln(2\*b\*(2^(1/2)\*a^(1/2)\*(-b\*x^2+a)^(1/2)-(-a\*b)^(1/2)\*x+a)/(b\*x-(-a\*b)^(1/2))) \* b^(1/2)-a^(1/2)\*2^(1/2)\*ln(2\*b\*(2^(1/2)\*a^(1/2)\*(-b\*x^2+a)^(1/2)+(-a\*b)^(1/2)\*x+a)/(b\*x+(-a\*b)^(1/2))) \* b^(1/2)-2\*(-a\*b)^(1/2)\*arctan(1/((-b\*x+(a\*b)^(1/2))\*(b\*x+(a\*b)^(1/2)))/b)^(1/2)\*b^(1/2)\*x+2\*(-a\*b)^(1/2)\*arctan(1/(-b\*x^2+a)^(1/2)\*b^(1/2)\*x))/(b\*x^2+a)^(1/2)/(-b\*x^2+a)^(1/2)/((-a\*b)^(1/2)+(a\*b)^(1/2))/((-a\*b)^(1/2)-(a\*b)^(1/2))/(-a\*b)^(1/2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2} \sqrt{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^2+a)^(1/2)/(-b^2\*x^4+a^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-b^2\*x^4 + a^2)\*sqrt(b\*x^2 + a)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a^2 - b^2 x^4} \sqrt{b x^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a^2 - b^2\*x^4)^(1/2)\*(a + b\*x^2)^(1/2)),x)

[Out] int(1/((a^2 - b^2\*x^4)^(1/2)\*(a + b\*x^2)^(1/2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(-a + bx^2)(a + bx^2)} \sqrt{a + bx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*2+a)\*\*(1/2)/(-b\*\*2\*x\*\*4+a\*\*2)\*\*(1/2),x)

[Out] Integral(1/(sqrt(-(-a + b\*x\*\*2)\*(a + b\*x\*\*2))\*sqrt(a + b\*x\*\*2)), x)

$$3.145 \quad \int \frac{1}{(a+bx^2)^{3/2} \sqrt{a^2-b^2x^4}} dx$$

Optimal. Leaf size=125

$$\frac{x(a-bx^2)}{4a^2\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{3\sqrt{a+bx^2}\sqrt{a-bx^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}}$$

**Rubi [A]** time = 0.05, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {1152, 382, 377, 205}

$$\frac{x(a-bx^2)}{4a^2\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{3\sqrt{a+bx^2}\sqrt{a-bx^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^2)^(3/2)\*Sqrt[a^2 - b^2\*x^4]), x]

[Out] (x\*(a - b\*x^2))/(4\*a^2\*Sqrt[a + b\*x^2]\*Sqrt[a^2 - b^2\*x^4]) + (3\*Sqrt[a - b\*x^2]\*Sqrt[a + b\*x^2]\*ArcTan[(Sqrt[2]\*Sqrt[b]\*x)/Sqrt[a - b\*x^2]])/(4\*Sqrt[2]\*a^2\*Sqrt[b]\*Sqrt[a^2 - b^2\*x^4])

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

#### Rule 382

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(b\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(p+1)\*(b\*c - a\*d)), x] + Dist[(b\*c + n\*(p+1)\*(b\*c - a\*d))/(a\*n\*(p+1)\*(b\*c - a\*d)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p+q+2)+1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]

#### Rule 1152

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] := Dist[(a + c\*x^4)^FracPart[p]/((d + e\*x^2)^FracPart[p]\*(a/d + (c\*x^2)/e)^FracPart[p]), Int[(d + e\*x^2)^(p+q)\*(a/d + (c\*x^2)/e)^p, x], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[p]

#### Rubi steps



$$\begin{aligned}
\int \frac{1}{(a+bx^2)^{3/2} \sqrt{a^2-b^2x^4}} dx &= \frac{\left(\sqrt{a-bx^2} \sqrt{a+bx^2}\right) \int \frac{1}{\sqrt{a-bx^2}(a+bx^2)^2} dx}{\sqrt{a^2-b^2x^4}} \\
&= \frac{x(a-bx^2)}{4a^2\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(3\sqrt{a-bx^2}\sqrt{a+bx^2}\right) \int \frac{1}{\sqrt{a-bx^2}(a+bx^2)} dx}{4a\sqrt{a^2-b^2x^4}} \\
&= \frac{x(a-bx^2)}{4a^2\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(3\sqrt{a-bx^2}\sqrt{a+bx^2}\right) \text{Subst}\left(\int \frac{1}{a+2abx^2} dx, x, \frac{\sqrt{a-bx^2}}{\sqrt{a+bx^2}}\right)}{4a\sqrt{a^2-b^2x^4}} \\
&= \frac{x(a-bx^2)}{4a^2\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{3\sqrt{a-bx^2}\sqrt{a+bx^2} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}}
\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 111, normalized size = 0.89

$$\frac{\sqrt{a^2-b^2x^4} \left(2\sqrt{b}x\sqrt{a-bx^2} + 3\sqrt{2}(a+bx^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a-bx^2}}\right)\right)}{8a^2\sqrt{b}\sqrt{a-bx^2}(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^2)^(3/2)\*Sqrt[a^2 - b^2\*x^4]), x]

[Out] (Sqrt[a^2 - b^2\*x^4]\*(2\*Sqrt[b]\*x\*Sqrt[a - b\*x^2] + 3\*Sqrt[2]\*(a + b\*x^2)\*ArcTan[(Sqrt[2]\*Sqrt[b]\*x)/Sqrt[a - b\*x^2]])/(8\*a^2\*Sqrt[b]\*Sqrt[a - b\*x^2]\*(a + b\*x^2)^(3/2))

**IntegrateAlgebraic [F]** time = 2.62, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^2)^{3/2} \sqrt{a^2-b^2x^4}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((a + b\*x^2)^(3/2)\*Sqrt[a^2 - b^2\*x^4]), x]

[Out] Defer[IntegrateAlgebraic][1/((a + b\*x^2)^(3/2)\*Sqrt[a^2 - b^2\*x^4]), x]

**fricas [A]** time = 0.89, size = 297, normalized size = 2.38

$$\left[ \frac{4\sqrt{-b^2x^4+a^2}\sqrt{bx^2+ax}-3\sqrt{2}(b^2x^4+2abx^2+a^2)\sqrt{-b}\log\left(-\frac{3i^2x^4+2abx^2-2\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{bx^2+ax}\sqrt{-bx-a^2}}{b^2x^4+2abx^2+a^2}\right)}{16(a^2b^3x^4+2a^3b^2x^2+a^4b)}, \frac{2\sqrt{-b^2x^4+a^2}\sqrt{bx^2+ax}-3\sqrt{2}(b^2x^4+2abx^2+a^2)\sqrt{b}\arctan\left(\frac{\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{bx^2+ax}\sqrt{b}}{2(b^2x^3+abx)}\right)}{8(a^2b^3x^4+2a^3b^2x^2+a^4b)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^2+a)^(3/2)/(-b^2\*x^4+a^2)^(1/2), x, algorithm="fricas")

[Out] [1/16\*(4\*sqrt(-b^2\*x^4 + a^2)\*sqrt(b\*x^2 + a)\*b\*x - 3\*sqrt(2)\*(b^2\*x^4 + 2\*a\*b\*x^2 + a^2)\*sqrt(-b)\*log(-(3\*b^2\*x^4 + 2\*a\*b\*x^2 - 2\*sqrt(2)\*sqrt(-b^2\*x^4 + a^2)\*sqrt(b\*x^2 + a)\*sqrt(-b)\*x - a^2)/(b^2\*x^4 + 2\*a\*b\*x^2 + a^2)))/(a^2\*b^3\*x^4 + 2\*a^3\*b^2\*x^2 + a^4\*b), 1/8\*(2\*sqrt(-b^2\*x^4 + a^2)\*sqrt(b\*x^2 + a)\*b\*x - 3\*sqrt(2)\*(b^2\*x^4 + 2\*a\*b\*x^2 + a^2)\*sqrt(b)\*arctan(1/2\*sqrt(2)\*sqrt(-b^2\*x^4 + a^2)\*sqrt(b\*x^2 + a)\*sqrt(b)/(b^2\*x^3 + a\*b\*x)))/(a^2\*b^3\*x^4 + 2\*a^3\*b^2\*x^2 + a^4\*b)]



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a + b*x**2)**(3/2)), x)
```

$$3.146 \quad \int \frac{1}{(a+bx^2)^{5/2} \sqrt{a^2-b^2x^4}} dx$$

**Optimal.** Leaf size=168

$$\frac{x(a-bx^2)}{8a^2(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x(a-bx^2)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{19\sqrt{a+bx^2}\sqrt{a-bx^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}}$$

**Rubi [A]** time = 0.09, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {1152, 414, 527, 12, 377, 205}

$$\frac{9x(a-bx^2)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{x(a-bx^2)}{8a^2(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{19\sqrt{a+bx^2}\sqrt{a-bx^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a-bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^2)^(5/2)\*Sqrt[a^2 - b^2\*x^4]), x]

[Out] (x\*(a - b\*x^2))/(8\*a^2\*(a + b\*x^2)^(3/2)\*Sqrt[a^2 - b^2\*x^4]) + (9\*x\*(a - b\*x^2))/(32\*a^3\*Sqrt[a + b\*x^2]\*Sqrt[a^2 - b^2\*x^4]) + (19\*Sqrt[a - b\*x^2]\*Sqrt[a + b\*x^2]\*ArcTan[(Sqrt[2]\*Sqrt[b]\*x)/Sqrt[a - b\*x^2]])/(32\*Sqrt[2]\*a^3\*Sqrt[b]\*Sqrt[a^2 - b^2\*x^4])

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

#### Rule 414

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(b\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(p+1)\*(b\*c - a\*d)), x] + Dist[1/(a\*n\*(p+1)\*(b\*c - a\*d)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q\*Simp[b\*c + n\*(p+1)\*(b\*c - a\*d) + d\*b\*(n\*(p+q+2) + 1)\*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

#### Rule 527

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_)\*((e\_) + (f\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*e - a\*f)\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(b\*c - a\*d)\*(p+1)), x] + Dist[1/(a\*n\*(b\*c - a\*d)\*(p+1)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q\*Simp[c\*(b\*e - a\*f) + e\*n\*(b\*c - a\*d)\*(p+1) + d\*(b\*e - a\*f)\*(n\*(p+q+2) + 1)\*x^n, x], x] /; FreeQ

[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

### Rule 1152

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] := Dis  
t[(a + c\*x^4)^FracPart[p]/((d + e\*x^2)^FracPart[p]\*(a/d + (c\*x^2)/e)^FracPa  
rt[p]), Int[(d + e\*x^2)^(p + q)\*(a/d + (c\*x^2)/e)^p, x], x] /; FreeQ[{a, c,  
d, e, p, q}, x] && EqQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[p]

### Rubi steps

$$\begin{aligned} \int \frac{1}{(a + bx^2)^{5/2} \sqrt{a^2 - b^2x^4}} dx &= \frac{\left(\sqrt{a - bx^2} \sqrt{a + bx^2}\right) \int \frac{1}{\sqrt{a - bx^2} (a + bx^2)^3} dx}{\sqrt{a^2 - b^2x^4}} \\ &= \frac{x(a - bx^2)}{8a^2(a + bx^2)^{3/2} \sqrt{a^2 - b^2x^4}} - \frac{\left(\sqrt{a - bx^2} \sqrt{a + bx^2}\right) \int \frac{-7ab + 2b^2x^2}{\sqrt{a - bx^2} (a + bx^2)^2} dx}{8a^2b\sqrt{a^2 - b^2x^4}} \\ &= \frac{x(a - bx^2)}{8a^2(a + bx^2)^{3/2} \sqrt{a^2 - b^2x^4}} + \frac{9x(a - bx^2)}{32a^3\sqrt{a + bx^2} \sqrt{a^2 - b^2x^4}} + \frac{\left(\sqrt{a - bx^2} \sqrt{a + bx^2}\right) \int \frac{19\sqrt{a - bx^2} \sqrt{a + bx^2}}{(a + bx^2)^2} dx}{32a^4b} \\ &= \frac{x(a - bx^2)}{8a^2(a + bx^2)^{3/2} \sqrt{a^2 - b^2x^4}} + \frac{9x(a - bx^2)}{32a^3\sqrt{a + bx^2} \sqrt{a^2 - b^2x^4}} + \frac{\left(19\sqrt{a - bx^2} \sqrt{a + bx^2}\right) \int \frac{1}{(a + bx^2)^2} dx}{32a^4b} \\ &= \frac{x(a - bx^2)}{8a^2(a + bx^2)^{3/2} \sqrt{a^2 - b^2x^4}} + \frac{9x(a - bx^2)}{32a^3\sqrt{a + bx^2} \sqrt{a^2 - b^2x^4}} + \frac{\left(19\sqrt{a - bx^2} \sqrt{a + bx^2}\right) \int \frac{1}{(a + bx^2)^2} dx}{32a^4b} \\ &= \frac{x(a - bx^2)}{8a^2(a + bx^2)^{3/2} \sqrt{a^2 - b^2x^4}} + \frac{9x(a - bx^2)}{32a^3\sqrt{a + bx^2} \sqrt{a^2 - b^2x^4}} + \frac{19\sqrt{a - bx^2} \sqrt{a + bx^2}}{32\sqrt{2} a^3 \sqrt{a^2 - b^2x^4}} \end{aligned}$$

**Mathematica [A]** time = 0.11, size = 123, normalized size = 0.73

$$\frac{\sqrt{a^2 - b^2x^4} \left( 2\sqrt{b} x \sqrt{a - bx^2} (13a + 9bx^2) + 19\sqrt{2} (a + bx^2)^2 \tan^{-1} \left( \frac{\sqrt{2} \sqrt{b} x}{\sqrt{a - bx^2}} \right) \right)}{64a^3 \sqrt{b} \sqrt{a - bx^2} (a + bx^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^2)^(5/2)\*Sqrt[a^2 - b^2\*x^4]), x]

[Out] (Sqrt[a^2 - b^2\*x^4]\*(2\*Sqrt[b]\*x\*Sqrt[a - b\*x^2]\*(13\*a + 9\*b\*x^2) + 19\*Sqr  
t[2]\*(a + b\*x^2)^2\*ArcTan[(Sqrt[2]\*Sqrt[b]\*x)/Sqrt[a - b\*x^2]]))/(64\*a^3\*Sq  
rt[b]\*Sqrt[a - b\*x^2]\*(a + b\*x^2)^(5/2))

**IntegrateAlgebraic [F]** time = 2.96, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^2)^{5/2} \sqrt{a^2 - b^2x^4}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((a + b\*x^2)^(5/2)\*Sqrt[a^2 - b^2\*x^4]), x]

[Out] Defer[IntegrateAlgebraic][1/((a + b\*x^2)^(5/2)\*sqrt[a^2 - b^2\*x^4]), x]

**fricas** [A] time = 0.74, size = 365, normalized size = 2.17

$$\frac{19\sqrt{2}(b^3x^6 + 3ab^2x^4 + 3a^2bx^2 + a^3)\sqrt{-b} \log\left(\frac{3b^2x^4 + 2abx^2 - 2\sqrt{-b^2x^4 + a^2}\sqrt{bx^2 + a}}{b^2x^4 + 2abx^2 + a^2}\right) - 4\sqrt{-b^2x^4 + a^2}(9b^2x^3 + 13abx)\sqrt{bx^2 + a} - 19\sqrt{2}(b^3x^6 + 3ab^2x^4 + 3a^2bx^2 + a^3)\sqrt{b} \arctan\left(\frac{\sqrt{2}\sqrt{-b^2x^4 + a^2}\sqrt{bx^2 + a}}{2(b^2x^2 + ab)}\right) - 2\sqrt{-b^2x^4 + a^2}(9b^2x^3 + 13abx)\sqrt{bx^2 + a}}{128(a^3b^4x^6 + 3a^4b^3x^4 + 3a^5b^2x^2 + a^6b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^2+a)^(5/2)/(-b^2\*x^4+a^2)^(1/2), x, algorithm="fricas")

[Out] [-1/128\*(19\*sqrt(2)\*(b^3\*x^6 + 3\*a\*b^2\*x^4 + 3\*a^2\*b\*x^2 + a^3)\*sqrt(-b)\*log(-(3\*b^2\*x^4 + 2\*a\*b\*x^2 - 2\*sqrt(2)\*sqrt(-b^2\*x^4 + a^2)\*sqrt(b\*x^2 + a)\*sqrt(-b)\*x - a^2)/(b^2\*x^4 + 2\*a\*b\*x^2 + a^2)) - 4\*sqrt(-b^2\*x^4 + a^2)\*(9\*b^2\*x^3 + 13\*a\*b\*x)\*sqrt(b\*x^2 + a)/(a^3\*b^4\*x^6 + 3\*a^4\*b^3\*x^4 + 3\*a^5\*b^2\*x^2 + a^6\*b), -1/64\*(19\*sqrt(2)\*(b^3\*x^6 + 3\*a\*b^2\*x^4 + 3\*a^2\*b\*x^2 + a^3)\*sqrt(b)\*arctan(1/2\*sqrt(2)\*sqrt(-b^2\*x^4 + a^2)\*sqrt(b\*x^2 + a)\*sqrt(b)/(b^2\*x^3 + a\*b\*x)) - 2\*sqrt(-b^2\*x^4 + a^2)\*(9\*b^2\*x^3 + 13\*a\*b\*x)\*sqrt(b\*x^2 + a)/(a^3\*b^4\*x^6 + 3\*a^4\*b^3\*x^4 + 3\*a^5\*b^2\*x^2 + a^6\*b)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2} (bx^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^2+a)^(5/2)/(-b^2\*x^4+a^2)^(1/2), x, algorithm="giac")

[Out] integrate(1/(sqrt(-b^2\*x^4 + a^2)\*(b\*x^2 + a)^(5/2)), x)

**maple** [B] time = 0.06, size = 711, normalized size = 4.23

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2} (bx^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^2+a)^(5/2)/(-b^2\*x^4+a^2)^(1/2), x)

[Out] -1/16\*(-b^2\*x^4+a^2)^(1/2)\*b^(9/2)\*(19\*2^(1/2)\*ln(2\*(a-(-a\*b)^(1/2)\*x+2^(1/2))\*(-b\*x^2+a)^(1/2)\*a^(1/2))/(b\*x-(-a\*b)^(1/2))\*b\*x^4\*b^(5/2)\*a^(1/2)-19\*2^(1/2)\*ln(2\*(a+(-a\*b)^(1/2)\*x+2^(1/2))\*(-b\*x^2+a)^(1/2)\*a^(1/2))/(b\*x+(-a\*b)^(1/2))\*b\*x^4\*b^(5/2)\*a^(1/2)+38\*2^(1/2)\*ln(2\*(a-(-a\*b)^(1/2)\*x+2^(1/2))\*(-b\*x^2+a)^(1/2)\*a^(1/2))/(b\*x-(-a\*b)^(1/2))\*b\*x^2\*a^(3/2)\*b^(3/2)-38\*2^(1/2)\*ln(2\*(a+(-a\*b)^(1/2)\*x+2^(1/2))\*(-b\*x^2+a)^(1/2)\*a^(1/2))/(b\*x+(-a\*b)^(1/2))\*b\*x^2\*a^(3/2)\*b^(3/2)+16\*arctan(1/(-b\*x^2+a)^(1/2)\*b^(1/2)\*x)\*x^4\*b^2\*(-a\*b)^(1/2)-16\*arctan(1/((-b\*x+(a\*b)^(1/2))\*(b\*x+(a\*b)^(1/2))/b)^(1/2)\*b^(1/2)\*x)\*x^4\*b^2\*(-a\*b)^(1/2)-36\*b^(3/2)\*(-a\*b)^(1/2)\*(-b\*x^2+a)^(1/2)\*x^3+19\*2^(1/2)\*ln(2\*(a-(-a\*b)^(1/2)\*x+2^(1/2))\*(-b\*x^2+a)^(1/2)\*a^(1/2))/(b\*x-(-a\*b)^(1/2))\*b\*a^(5/2)\*b^(1/2)-19\*2^(1/2)\*ln(2\*(a+(-a\*b)^(1/2)\*x+2^(1/2))\*(-b\*x^2+a)^(1/2)\*a^(1/2))/(b\*x+(-a\*b)^(1/2))\*b\*a^(5/2)\*b^(1/2)+32\*arctan(1/(-b\*x^2+a)^(1/2)\*b^(1/2)\*x)\*x^2\*a\*b\*(-a\*b)^(1/2)-32\*arctan(1/((-b\*x+(a\*b)^(1/2))\*(b\*x+(a\*b)^(1/2))/b)^(1/2)\*b^(1/2)\*x)\*x^2\*a\*b\*(-a\*b)^(1/2)-52\*a\*(-a\*b)^(1/2)\*b^(1/2)\*(-b\*x^2+a)^(1/2)\*x+16\*arctan(1/(-b\*x^2+a)^(1/2)\*b^(1/2)\*x)\*a^2\*(-a\*b)^(1/2)-16\*arctan(1/((-b\*x+(a\*b)^(1/2))\*(b\*x+(a\*b)^(1/2))/b)^(1/2)\*b^(1/2)\*x)\*a^2\*(-a\*b)^(1/2))/(b\*x^2+a)^(1/2)/(-b\*x^2+a)^(1/2)/(-a\*b)^(1/2)/((-a\*b)^(1/2)+(a\*b)^(1/2))^3/(-(-a\*b)^(1/2)+(a\*b)^(1/2))^3/(b\*x+(-a\*b)^(1/2))^2/2/(b\*x-(-a\*b)^(1/2))^2

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2} (bx^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^2+a)^(5/2)/(-b^2\*x^4+a^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-b^2\*x^4 + a^2)\*(b\*x^2 + a)^(5/2)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a^2 - b^2 x^4} (b x^2 + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a^2 - b^2\*x^4)^(1/2)\*(a + b\*x^2)^(5/2)),x)

[Out] int(1/((a^2 - b^2\*x^4)^(1/2)\*(a + b\*x^2)^(5/2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(-a + bx^2)(a + bx^2)} (a + bx^2)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*2+a)\*\*(5/2)/(-b\*\*2\*x\*\*4+a\*\*2)\*\*(1/2),x)

[Out] Integral(1/(sqrt(-(-a + b\*x\*\*2)\*(a + b\*x\*\*2))\*(a + b\*x\*\*2)\*\*(5/2)), x)

$$3.147 \quad \int \frac{(a-bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}} dx$$

**Optimal.** Leaf size=152

$$\frac{9ax\sqrt{a-bx^2}(a+bx^2)}{8\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)^{3/2}(a+bx^2)}{4\sqrt{a^2-b^2x^4}} + \frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}$$

**Rubi [A]** time = 0.05, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$ , Rules used = {1152, 416, 388, 217, 206}

$$\frac{9ax\sqrt{a-bx^2}(a+bx^2)}{8\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)^{3/2}(a+bx^2)}{4\sqrt{a^2-b^2x^4}} + \frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(5/2)/Sqrt[a^2 - b^2\*x^4], x]

[Out] (-9\*a\*x\*Sqrt[a - b\*x^2]\*(a + b\*x^2))/(8\*Sqrt[a^2 - b^2\*x^4]) - (x\*(a - b\*x^2)^(3/2)\*(a + b\*x^2))/(4\*Sqrt[a^2 - b^2\*x^4]) + (19\*a^2\*Sqrt[a - b\*x^2]\*Sqrt[a + b\*x^2]\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/(8\*Sqrt[b]\*Sqrt[a^2 - b^2\*x^4])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rule 416

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(b\*(n\*(p + q) + 1)), x] + Dist[1/(b\*(n\*(p + q) + 1)), Int[(a + b\*x^n)^p\*(c + d\*x^n)^(q - 2)\*Simp[c\*(b\*c\*(n\*(p + q) + 1) - a\*d] + d\*(b\*c\*(n\*(p + 2\*q - 1) + 1) - a\*d\*(n\*(q - 1) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && GtQ[q, 1] && NeQ[n\*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

#### Rule 1152

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] := Dist[(a + c\*x^4)^FracPart[p]/((d + e\*x^2)^FracPart[p]\*(a/d + (c\*x^2)/e)^FracPart[p]), Int[(d + e\*x^2)^(p + q)\*(a/d + (c\*x^2)/e)^p, x], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[p]



Rubi steps

$$\begin{aligned}
\int \frac{(a - bx^2)^{5/2}}{\sqrt{a^2 - b^2x^4}} dx &= \frac{\left(\sqrt{a - bx^2} \sqrt{a + bx^2}\right) \int \frac{(a - bx^2)^2}{\sqrt{a + bx^2}} dx}{\sqrt{a^2 - b^2x^4}} \\
&= -\frac{x(a - bx^2)^{3/2}(a + bx^2)}{4\sqrt{a^2 - b^2x^4}} + \frac{\left(\sqrt{a - bx^2} \sqrt{a + bx^2}\right) \int \frac{5a^2b - 9ab^2x^2}{\sqrt{a + bx^2}} dx}{4b\sqrt{a^2 - b^2x^4}} \\
&= -\frac{9ax\sqrt{a - bx^2}(a + bx^2)}{8\sqrt{a^2 - b^2x^4}} - \frac{x(a - bx^2)^{3/2}(a + bx^2)}{4\sqrt{a^2 - b^2x^4}} + \frac{\left(19a^2\sqrt{a - bx^2} \sqrt{a + bx^2}\right) \int \frac{1}{\sqrt{a + bx^2}} dx}{8\sqrt{a^2 - b^2x^4}} \\
&= -\frac{9ax\sqrt{a - bx^2}(a + bx^2)}{8\sqrt{a^2 - b^2x^4}} - \frac{x(a - bx^2)^{3/2}(a + bx^2)}{4\sqrt{a^2 - b^2x^4}} + \frac{\left(19a^2\sqrt{a - bx^2} \sqrt{a + bx^2}\right) \text{Subst}\left(\frac{1}{\sqrt{u}}, u, a + bx^2\right)}{8\sqrt{a^2 - b^2x^4}} \\
&= -\frac{9ax\sqrt{a - bx^2}(a + bx^2)}{8\sqrt{a^2 - b^2x^4}} - \frac{x(a - bx^2)^{3/2}(a + bx^2)}{4\sqrt{a^2 - b^2x^4}} + \frac{19a^2\sqrt{a - bx^2} \sqrt{a + bx^2} \tanh^{-1}\left(\frac{\sqrt{a - bx^2}}{\sqrt{a + bx^2}}\right)}{8\sqrt{b} \sqrt{a^2 - b^2x^4}}
\end{aligned}$$

**Mathematica [A]** time = 0.20, size = 123, normalized size = 0.81

$$\frac{1}{8} \left( \frac{x(2bx^2 - 11a)\sqrt{a^2 - b^2x^4}}{\sqrt{a - bx^2}} + \frac{19a^2 \log\left(\sqrt{b} \sqrt{a - bx^2} \sqrt{a^2 - b^2x^4} + abx - b^2x^3\right)}{\sqrt{b}} - \frac{19a^2 \log(bx^2 - a)}{\sqrt{b}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(5/2)/Sqrt[a^2 - b^2\*x^4], x]

[Out] ((x\*(-11\*a + 2\*b\*x^2)\*Sqrt[a^2 - b^2\*x^4])/Sqrt[a - b\*x^2] - (19\*a^2\*Log[-a + b\*x^2])/Sqrt[b] + (19\*a^2\*Log[a\*b\*x - b^2\*x^3 + Sqrt[b]\*Sqrt[a - b\*x^2]\*Sqrt[a^2 - b^2\*x^4]))/Sqrt[b])/8

**IntegrateAlgebraic [F]** time = 3.07, size = 0, normalized size = 0.00

$$\int \frac{(a - bx^2)^{5/2}}{\sqrt{a^2 - b^2x^4}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a - b\*x^2)^(5/2)/Sqrt[a^2 - b^2\*x^4], x]

[Out] Defer[IntegrateAlgebraic] [(a - b\*x^2)^(5/2)/Sqrt[a^2 - b^2\*x^4], x]

**fricas [A]** time = 1.32, size = 265, normalized size = 1.74

$$\left[ \frac{19(a^2bx^2 - a^3)\sqrt{b} \log\left(\frac{2b^2x^4 - abx^2 - 2\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + a}}{bx^2 - a}\right) - 2\sqrt{-b^2x^4 + a^2}(2b^2x^3 - 11abx)\sqrt{-bx^2 + a}}{16(b^2x^2 - ab)}, \frac{19(a^2bx^2 - a^3)\sqrt{-b} \arctan\left(\frac{\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + a}}{b^2x^3 - abx}\right) - \sqrt{-b^2x^4 + a^2}(2b^2x^3 - 11abx)\sqrt{-bx^2 + a}}{8(b^2x^2 - ab)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2+a)^(5/2)/(-b^2\*x^4+a^2)^(1/2), x, algorithm="fricas")

[Out] [1/16\*(19\*(a^2\*b\*x^2 - a^3)\*sqrt(b)\*log((2\*b^2\*x^4 - a\*b\*x^2 - 2\*sqrt(-b^2\*x^4 + a^2)\*sqrt(-b\*x^2 + a)\*sqrt(b)\*x - a^2)/(b\*x^2 - a) - 2\*sqrt(-b^2\*x^4 + a^2)\*(2\*b^2\*x^3 - 11\*a\*b\*x)\*sqrt(-b\*x^2 + a))/(b^2\*x^2 - a\*b), 1/8\*(19\*(a^2\*b\*x^2 - a^3)\*sqrt(-b)\*arctan(sqrt(-b^2\*x^4 + a^2)\*sqrt(-b\*x^2 + a)\*sqrt(-b)/(b^2\*x^3 - a\*b\*x)) - sqrt(-b^2\*x^4 + a^2)\*(2\*b^2\*x^3 - 11\*a\*b\*x)\*sqrt(-b\*x^2 + a))/(b^2\*x^2 - a\*b)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-bx^2 + a)^{\frac{5}{2}}}{\sqrt{-b^2x^4 + a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2+a)^(5/2)/(-b^2\*x^4+a^2)^(1/2),x, algorithm="giac")

[Out] integrate((-b\*x^2 + a)^(5/2)/sqrt(-b^2\*x^4 + a^2), x)

**maple** [A] time = 0.02, size = 105, normalized size = 0.69

$$\frac{\sqrt{-bx^2 + a} \sqrt{-b^2x^4 + a^2} \left( 2\sqrt{bx^2 + a} b^{\frac{3}{2}}x^3 + 19a^2 \ln\left(\sqrt{b} x + \sqrt{bx^2 + a}\right) - 11\sqrt{bx^2 + a} a\sqrt{b} x \right)}{8(bx^2 - a)\sqrt{bx^2 + a}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^2+a)^(5/2)/(-b^2\*x^4+a^2)^(1/2),x)

[Out] -1/8\*(-b\*x^2+a)^(1/2)\*(-b^2\*x^4+a^2)^(1/2)\*(2\*x^3\*b^(3/2)\*(b\*x^2+a)^(1/2)-11\*x\*a\*b^(1/2)\*(b\*x^2+a)^(1/2)+19\*ln(b^(1/2)\*x+(b\*x^2+a)^(1/2))\*a^2)/(b\*x^2-a)/(b\*x^2+a)^(1/2)/b^(1/2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-bx^2 + a)^{\frac{5}{2}}}{\sqrt{-b^2x^4 + a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2+a)^(5/2)/(-b^2\*x^4+a^2)^(1/2),x, algorithm="maxima")

[Out] integrate((-b\*x^2 + a)^(5/2)/sqrt(-b^2\*x^4 + a^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a - bx^2)^{\frac{5}{2}}}{\sqrt{a^2 - b^2x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - b\*x^2)^(5/2)/(a^2 - b^2\*x^4)^(1/2),x)

[Out] int((a - b\*x^2)^(5/2)/(a^2 - b^2\*x^4)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a - bx^2)^{\frac{5}{2}}}{\sqrt{-(-a + bx^2)(a + bx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*2+a)\*\*(5/2)/(-b\*\*2\*x\*\*4+a\*\*2)\*\*(1/2),x)

[Out] Integral((a - b\*x\*\*2)\*\*(5/2)/sqrt(-(-a + b\*x\*\*2)\*(a + b\*x\*\*2)), x)

$$3.148 \quad \int \frac{(a-bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}} dx$$

Optimal. Leaf size=109

$$\frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}} - \frac{x\sqrt{a-bx^2}(a+bx^2)}{2\sqrt{a^2-b^2x^4}}$$

Rubi [A] time = 0.04, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {1152, 388, 217, 206}

$$\frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}} - \frac{x\sqrt{a-bx^2}(a+bx^2)}{2\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(3/2)/Sqrt[a^2 - b^2\*x^4], x]

[Out] -(x\*Sqrt[a - b\*x^2]\*(a + b\*x^2))/(2\*Sqrt[a^2 - b^2\*x^4]) + (3\*a\*Sqrt[a - b\*x^2]\*Sqrt[a + b\*x^2]\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/(2\*Sqrt[b]\*Sqrt[a^2 - b^2\*x^4])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rule 1152

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] := Dist[(a + c\*x^4)^FracPart[p]/((d + e\*x^2)^FracPart[p]\*(a/d + (c\*x^2)/e)^FracPart[p]), Int[(d + e\*x^2)^(p + q)\*(a/d + (c\*x^2)/e)^p, x], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[p]

#### Rubi steps

$$\begin{aligned}
\int \frac{(a - bx^2)^{3/2}}{\sqrt{a^2 - b^2x^4}} dx &= \frac{(\sqrt{a - bx^2} \sqrt{a + bx^2}) \int \frac{a - bx^2}{\sqrt{a + bx^2}} dx}{\sqrt{a^2 - b^2x^4}} \\
&= -\frac{x\sqrt{a - bx^2} (a + bx^2)}{2\sqrt{a^2 - b^2x^4}} + \frac{(3a\sqrt{a - bx^2} \sqrt{a + bx^2}) \int \frac{1}{\sqrt{a + bx^2}} dx}{2\sqrt{a^2 - b^2x^4}} \\
&= -\frac{x\sqrt{a - bx^2} (a + bx^2)}{2\sqrt{a^2 - b^2x^4}} + \frac{(3a\sqrt{a - bx^2} \sqrt{a + bx^2}) \operatorname{Subst}\left(\int \frac{1}{1 - bx^2} dx, x, \frac{x}{\sqrt{a + bx^2}}\right)}{2\sqrt{a^2 - b^2x^4}} \\
&= -\frac{x\sqrt{a - bx^2} (a + bx^2)}{2\sqrt{a^2 - b^2x^4}} + \frac{3a\sqrt{a - bx^2} \sqrt{a + bx^2} \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a + bx^2}}\right)}{2\sqrt{b} \sqrt{a^2 - b^2x^4}}
\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 110, normalized size = 1.01

$$\frac{1}{2} \left( -\frac{x\sqrt{a^2 - b^2x^4}}{\sqrt{a - bx^2}} + \frac{3a \log\left(\sqrt{b} \sqrt{a - bx^2} \sqrt{a^2 - b^2x^4} + abx - b^2x^3\right)}{\sqrt{b}} - \frac{3a \log(bx^2 - a)}{\sqrt{b}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(3/2)/Sqrt[a^2 - b^2\*x^4], x]

[Out] (-((x\*Sqrt[a^2 - b^2\*x^4])/Sqrt[a - b\*x^2]) - (3\*a\*Log[-a + b\*x^2])/Sqrt[b] + (3\*a\*Log[a\*b\*x - b^2\*x^3 + Sqrt[b]\*Sqrt[a - b\*x^2]\*Sqrt[a^2 - b^2\*x^4]])/Sqrt[b])/2

**IntegrateAlgebraic [F]** time = 2.58, size = 0, normalized size = 0.00

$$\int \frac{(a - bx^2)^{3/2}}{\sqrt{a^2 - b^2x^4}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a - b\*x^2)^(3/2)/Sqrt[a^2 - b^2\*x^4], x]

[Out] Defer[IntegrateAlgebraic] [(a - b\*x^2)^(3/2)/Sqrt[a^2 - b^2\*x^4], x]

**fricas [A]** time = 1.04, size = 236, normalized size = 2.17

$$\left[ \frac{2\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + a}bx + 3(abx^2 - a^2)\sqrt{b} \log\left(\frac{2b^2x^4 - abx^2 - 2\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + a}\sqrt{bx - a^2}}{bx^2 - a}\right)}{4(b^2x^2 - ab)}, \frac{\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + a}bx + 3(abx^2 - a^2)\sqrt{-b} \arctan\left(\frac{\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + a}\sqrt{-b}}{b^2x^3 - abx}\right)}{2(b^2x^2 - ab)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2+a)^(3/2)/(-b^2\*x^4+a^2)^(1/2), x, algorithm="fricas")

[Out] [1/4\*(2\*sqrt(-b^2\*x^4 + a^2)\*sqrt(-b\*x^2 + a)\*b\*x + 3\*(a\*b\*x^2 - a^2)\*sqrt(b)\*log((2\*b^2\*x^4 - a\*b\*x^2 - 2\*sqrt(-b^2\*x^4 + a^2)\*sqrt(-b\*x^2 + a)\*sqrt(b)\*x - a^2)/(b\*x^2 - a)))/(b^2\*x^2 - a\*b), 1/2\*(sqrt(-b^2\*x^4 + a^2)\*sqrt(-b\*x^2 + a)\*b\*x + 3\*(a\*b\*x^2 - a^2)\*sqrt(-b)\*arctan(sqrt(-b^2\*x^4 + a^2)\*sqrt(-b\*x^2 + a)\*sqrt(-b)/(b^2\*x^3 - a\*b\*x)))/(b^2\*x^2 - a\*b)]

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-bx^2 + a)^{3/2}}{\sqrt{-b^2x^4 + a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2+a)^(3/2)/(-b^2\*x^4+a^2)^(1/2),x, algorithm="giac")

[Out] integrate((-b\*x^2 + a)^(3/2)/sqrt(-b^2\*x^4 + a^2), x)

**maple** [A] time = 0.01, size = 85, normalized size = 0.78

$$-\frac{\sqrt{-bx^2+a}\sqrt{-b^2x^4+a^2}\left(3a\ln\left(\sqrt{b}x+\sqrt{bx^2+a}\right)-\sqrt{bx^2+a}\sqrt{b}x\right)}{2(bx^2-a)\sqrt{bx^2+a}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^2+a)^(3/2)/(-b^2\*x^4+a^2)^(1/2),x)

[Out] -1/2\*(-b\*x^2+a)^(1/2)\*(-b^2\*x^4+a^2)^(1/2)\*(-x\*(b\*x^2+a)^(1/2)\*b^(1/2)+3\*ln(b^(1/2)\*x+(b\*x^2+a)^(1/2))\*a)/(b\*x^2-a)/(b\*x^2+a)^(1/2)/b^(1/2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-bx^2+a)^{\frac{3}{2}}}{\sqrt{-b^2x^4+a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2+a)^(3/2)/(-b^2\*x^4+a^2)^(1/2),x, algorithm="maxima")

[Out] integrate((-b\*x^2 + a)^(3/2)/sqrt(-b^2\*x^4 + a^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - b\*x^2)^(3/2)/(a^2 - b^2\*x^4)^(1/2),x)

[Out] int((a - b\*x^2)^(3/2)/(a^2 - b^2\*x^4)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a-bx^2)^{\frac{3}{2}}}{\sqrt{-(-a+bx^2)(a+bx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*2+a)\*\*(3/2)/(-b\*\*2\*x\*\*4+a\*\*2)\*\*(1/2),x)

[Out] Integral((a - b\*x\*\*2)\*\*(3/2)/sqrt(-(-a + b\*x\*\*2)\*(a + b\*x\*\*2)), x)

$$3.149 \quad \int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}} dx$$

**Optimal.** Leaf size=64

$$\frac{\sqrt{a-bx^2} \sqrt{a+bx^2} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{\sqrt{b} \sqrt{a^2-b^2x^4}}$$

**Rubi [A]** time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {1152, 217, 206}

$$\frac{\sqrt{a-bx^2} \sqrt{a+bx^2} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{\sqrt{b} \sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a - b\*x^2]/Sqrt[a^2 - b^2\*x^4], x]

[Out] (Sqrt[a - b\*x^2]\*Sqrt[a + b\*x^2]\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/(Sqrt[b]\*Sqrt[a^2 - b^2\*x^4])

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 1152

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] := Dist[(a + c\*x^4)^FracPart[p]/((d + e\*x^2)^FracPart[p]\*(a/d + (c\*x^2)/e)^FracPart[p]), Int[(d + e\*x^2)^(p + q)\*(a/d + (c\*x^2)/e)^p, x], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}} dx &= \frac{\left(\sqrt{a-bx^2} \sqrt{a+bx^2}\right) \int \frac{1}{\sqrt{a+bx^2}} dx}{\sqrt{a^2-b^2x^4}} \\ &= \frac{\left(\sqrt{a-bx^2} \sqrt{a+bx^2}\right) \text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{\sqrt{a^2-b^2x^4}} \\ &= \frac{\sqrt{a-bx^2} \sqrt{a+bx^2} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{\sqrt{b} \sqrt{a^2-b^2x^4}} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 67, normalized size = 1.05

$$\frac{\log\left(\sqrt{b} \sqrt{a-bx^2} \sqrt{a^2-b^2x^4} + abx - b^2x^3\right) - \log(bx^2 - a)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a - b\*x^2]/Sqrt[a^2 - b^2\*x^4], x]

[Out] (-Log[-a + b\*x^2] + Log[a\*b\*x - b^2\*x^3 + Sqrt[b]\*Sqrt[a - b\*x^2]\*Sqrt[a^2 - b^2\*x^4]])/Sqrt[b]

**IntegrateAlgebraic** [F] time = 1.36, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a - bx^2}}{\sqrt{a^2 - b^2x^4}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[Sqrt[a - b\*x^2]/Sqrt[a^2 - b^2\*x^4], x]

[Out] Defer[IntegrateAlgebraic][Sqrt[a - b\*x^2]/Sqrt[a^2 - b^2\*x^4], x]

**fricas** [A] time = 1.07, size = 125, normalized size = 1.95

$$\left[ \frac{\log\left(\frac{2b^2x^4 - abx^2 - 2\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + a}\sqrt{bx - a^2}}{bx^2 - a}\right)}{2\sqrt{b}}, \frac{\sqrt{-b} \arctan\left(\frac{\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + a}\sqrt{-b}}{b^2x^3 - abx}\right)}{b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2+a)^(1/2)/(-b^2\*x^4+a^2)^(1/2), x, algorithm="fricas")

[Out] [1/2\*log((2\*b^2\*x^4 - a\*b\*x^2 - 2\*sqrt(-b^2\*x^4 + a^2)\*sqrt(-b\*x^2 + a)\*sqrt(b)\*x - a^2)/(b\*x^2 - a))/sqrt(b), sqrt(-b)\*arctan(sqrt(-b^2\*x^4 + a^2)\*sqrt(-b\*x^2 + a)\*sqrt(-b)/(b^2\*x^3 - a\*b\*x))/b]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-bx^2 + a}}{\sqrt{-b^2x^4 + a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2+a)^(1/2)/(-b^2\*x^4+a^2)^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(-b\*x^2 + a)/sqrt(-b^2\*x^4 + a^2), x)

**maple** [A] time = 0.01, size = 54, normalized size = 0.84

$$\frac{\sqrt{-b^2x^4 + a^2} \ln\left(\sqrt{b}x + \sqrt{bx^2 + a}\right)}{\sqrt{-bx^2 + a} \sqrt{bx^2 + a} \sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^2+a)^(1/2)/(-b^2\*x^4+a^2)^(1/2), x)

[Out] 1/(-b\*x^2+a)^(1/2)/(b\*x^2+a)^(1/2)/b^(1/2)\*(-b^2\*x^4+a^2)^(1/2)\*ln(b^(1/2)\*x+(b\*x^2+a)^(1/2))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-bx^2 + a}}{\sqrt{-b^2x^4 + a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2+a)^(1/2)/(-b^2\*x^4+a^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-b\*x^2 + a)/sqrt(-b^2\*x^4 + a^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a - bx^2}}{\sqrt{a^2 - b^2x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - b\*x^2)^(1/2)/(a^2 - b^2\*x^4)^(1/2),x)

[Out] int((a - b\*x^2)^(1/2)/(a^2 - b^2\*x^4)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a - bx^2}}{\sqrt{-(-a + bx^2)(a + bx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*2+a)\*\*(1/2)/(-b\*\*2\*x\*\*4+a\*\*2)\*\*(1/2),x)

[Out] Integral(sqrt(a - b\*x\*\*2)/sqrt(-(-a + b\*x\*\*2)\*(a + b\*x\*\*2)), x)



$$3.150 \quad \int \frac{1}{\sqrt{a-bx^2} \sqrt{a^2-b^2x^4}} dx$$

**Optimal.** Leaf size=77

$$\frac{\sqrt{a-bx^2} \sqrt{a+bx^2} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a^2-b^2x^4}}$$

**Rubi [A]** time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {1152, 377, 208}

$$\frac{\sqrt{a-bx^2} \sqrt{a+bx^2} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a - b\*x^2]\*Sqrt[a^2 - b^2\*x^4]),x]

[Out] (Sqrt[a - b\*x^2]\*Sqrt[a + b\*x^2]\*ArcTanh[(Sqrt[2]\*Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/(Sqrt[2]\*a\*Sqrt[b]\*Sqrt[a^2 - b^2\*x^4])

Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

Rule 1152

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] := Dist[(a + c\*x^4)^FracPart[p]/((d + e\*x^2)^FracPart[p]\*(a/d + (c\*x^2)/e)^FracPart[p]), Int[(d + e\*x^2)^(p+q)\*(a/d + (c\*x^2)/e)^p, x], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a-bx^2} \sqrt{a^2-b^2x^4}} dx &= \frac{\left(\sqrt{a-bx^2} \sqrt{a+bx^2}\right) \int \frac{1}{(a-bx^2)\sqrt{a+bx^2}} dx}{\sqrt{a^2-b^2x^4}} \\ &= \frac{\left(\sqrt{a-bx^2} \sqrt{a+bx^2}\right) \text{Subst}\left(\int \frac{1}{a-2abx^2} dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{\sqrt{a^2-b^2x^4}} \\ &= \frac{\sqrt{a-bx^2} \sqrt{a+bx^2} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a^2-b^2x^4}} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 77, normalized size = 1.00

$$\frac{\sqrt{a^2-b^2x^4} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a-bx^2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a - b\*x^2]\*Sqrt[a^2 - b^2\*x^4]), x]

[Out] (Sqrt[a^2 - b^2\*x^4]\*ArcTanh[(Sqrt[2]\*Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/(Sqrt[2]\*a\*Sqrt[b]\*Sqrt[a - b\*x^2]\*Sqrt[a + b\*x^2])

**IntegrateAlgebraic** [F] time = 1.59, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a - bx^2} \sqrt{a^2 - b^2x^4}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(Sqrt[a - b\*x^2]\*Sqrt[a^2 - b^2\*x^4]), x]

[Out] Defer[IntegrateAlgebraic][1/(Sqrt[a - b\*x^2]\*Sqrt[a^2 - b^2\*x^4]), x]

**fricas** [A] time = 1.08, size = 155, normalized size = 2.01

$$\left[ \frac{\sqrt{2} \log\left(-\frac{3b^2x^4 - 2abx^2 - 2\sqrt{2}\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + a}\sqrt{bx - a^2}}{b^2x^4 - 2abx^2 + a^2}\right)}{4a\sqrt{b}}, \frac{\sqrt{2}\sqrt{-b} \arctan\left(\frac{\sqrt{2}\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + a}\sqrt{-b}}{2(b^2x^3 - abx)}\right)}{2ab} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x^2+a)^(1/2)/(-b^2\*x^4+a^2)^(1/2), x, algorithm="fricas")

[Out] [1/4\*sqrt(2)\*log(-(3\*b^2\*x^4 - 2\*a\*b\*x^2 - 2\*sqrt(2)\*sqrt(-b^2\*x^4 + a^2))\*sqrt(-b\*x^2 + a)\*sqrt(b)\*x - a^2)/(b^2\*x^4 - 2\*a\*b\*x^2 + a^2))/(a\*sqrt(b)), 1/2\*sqrt(2)\*sqrt(-b)\*arctan(1/2\*sqrt(2)\*sqrt(-b^2\*x^4 + a^2)\*sqrt(-b\*x^2 + a)\*sqrt(-b)/(b^2\*x^3 - a\*b\*x))/(a\*b)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2} \sqrt{-bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x^2+a)^(1/2)/(-b^2\*x^4+a^2)^(1/2), x, algorithm="giac")

[Out] integrate(1/(sqrt(-b^2\*x^4 + a^2)\*sqrt(-b\*x^2 + a)), x)

**maple** [B] time = 0.06, size = 267, normalized size = 3.47

$$\frac{\sqrt{-bx^2 + a} \sqrt{-b^2x^4 + a^2} \left( \sqrt{2} \sqrt{a} \sqrt{b} \ln\left(\frac{2(a - \sqrt{ab}x + \sqrt{2}\sqrt{bx^2 + a}\sqrt{a})b}{bx + \sqrt{ab}}\right) - \sqrt{2} \sqrt{a} \sqrt{b} \ln\left(\frac{2(a + \sqrt{ab}x + \sqrt{2}\sqrt{bx^2 + a}\sqrt{a})b}{bx - \sqrt{ab}}\right) - 2\sqrt{ab} \ln\left(\frac{bx + \sqrt{\frac{(bx + \sqrt{ab})(-bx + \sqrt{ab})}{b}} \sqrt{b}}{\sqrt{b}}\right) + 2\sqrt{ab} \ln\left(\frac{bx + \sqrt{bx^2 + a} \sqrt{b}}{\sqrt{b}}\right) \right) \sqrt{b}}{2(bx^2 - a) \sqrt{bx^2 + a} (\sqrt{-ab} + \sqrt{ab}) (-\sqrt{-ab} + \sqrt{ab}) \sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b\*x^2+a)^(1/2)/(-b^2\*x^4+a^2)^(1/2), x)

[Out] 1/2\*(-b\*x^2+a)^(1/2)\*(-b^2\*x^4+a^2)^(1/2)\*b^(1/2)\*(a^(1/2)\*2^(1/2)\*ln(2\*b\*(2^(1/2)\*a^(1/2)\*(b\*x^2+a)^(1/2)-(a\*b)^(1/2)\*x+a)/(b\*x+(a\*b)^(1/2))))\*b^(1/2)-a^(1/2)\*2^(1/2)\*ln(2\*b\*(2^(1/2)\*a^(1/2)\*(b\*x^2+a)^(1/2)+(a\*b)^(1/2)\*x+a)/(b\*x-(a\*b)^(1/2))))\*b^(1/2)-2\*(a\*b)^(1/2)\*ln((b^(1/2)\*(-b\*x+(-a\*b)^(1/2)))/b\*(-b\*x+(-a\*b)^(1/2)))^(1/2)+b\*x)/b^(1/2))+2\*(a\*b)^(1/2)\*ln((b^(1/2)\*(b\*x^2+a)^(1/2)+b\*x)/b^(1/2)))/(b\*x^2-a)/(b\*x^2+a)^(1/2)/((-a\*b)^(1/2)+(a\*b)^(1/2))/((-a\*b)^(1/2)+(a\*b)^(1/2))/(a\*b)^(1/2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2} \sqrt{-bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x^2+a)^(1/2)/(-b^2\*x^4+a^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-b^2\*x^4 + a^2)\*sqrt(-b\*x^2 + a)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a^2 - b^2 x^4} \sqrt{a - b x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a^2 - b^2\*x^4)^(1/2)\*(a - b\*x^2)^(1/2)),x)

[Out] int(1/((a^2 - b^2\*x^4)^(1/2)\*(a - b\*x^2)^(1/2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(-a + bx^2)(a + bx^2)} \sqrt{a - bx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x\*\*2+a)\*\*(1/2)/(-b\*\*2\*x\*\*4+a\*\*2)\*\*(1/2),x)

[Out] Integral(1/(sqrt(-(-a + b\*x\*\*2)\*(a + b\*x\*\*2))\*sqrt(a - b\*x\*\*2)), x)

$$3.151 \quad \int \frac{1}{(a-bx^2)^{3/2} \sqrt{a^2-b^2x^4}} dx$$

Optimal. Leaf size=124

$$\frac{x(a+bx^2)}{4a^2\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}} + \frac{3\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}}$$

**Rubi [A]** time = 0.05, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {1152, 382, 377, 208}

$$\frac{x(a+bx^2)}{4a^2\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}} + \frac{3\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[1/((a - b\*x^2)^(3/2)\*Sqrt[a^2 - b^2\*x^4]),x]

[Out] (x\*(a + b\*x^2))/(4\*a^2\*Sqrt[a - b\*x^2]\*Sqrt[a^2 - b^2\*x^4]) + (3\*Sqrt[a - b\*x^2]\*Sqrt[a + b\*x^2]\*ArcTanh[(Sqrt[2]\*Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/(4\*Sqrt[2]\*a^2\*Sqrt[b]\*Sqrt[a^2 - b^2\*x^4])

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

#### Rule 382

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(b\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(p+1)\*(b\*c - a\*d)), x] + Dist[(b\*c + n\*(p+1)\*(b\*c - a\*d))/(a\*n\*(p+1)\*(b\*c - a\*d)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p+q+2)+1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]

#### Rule 1152

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] := Dist[(a + c\*x^4)^FracPart[p]/((d + e\*x^2)^FracPart[p]\*(a/d + (c\*x^2)/e)^FracPart[p]), Int[(d + e\*x^2)^(p+q)\*(a/d + (c\*x^2)/e)^p, x], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[p]

#### Rubi steps

$$\int \frac{1}{(a-bx^2)^{3/2} \sqrt{a^2-b^2x^4}} dx = \frac{\left(\sqrt{a-bx^2} \sqrt{a+bx^2}\right) \int \frac{1}{(a-bx^2)^2 \sqrt{a+bx^2}} dx}{\sqrt{a^2-b^2x^4}}$$

$$= \frac{x(a+bx^2)}{4a^2 \sqrt{a-bx^2} \sqrt{a^2-b^2x^4}} + \frac{\left(3\sqrt{a-bx^2} \sqrt{a+bx^2}\right) \int \frac{1}{(a-bx^2) \sqrt{a+bx^2}} dx}{4a \sqrt{a^2-b^2x^4}}$$

$$= \frac{x(a+bx^2)}{4a^2 \sqrt{a-bx^2} \sqrt{a^2-b^2x^4}} + \frac{\left(3\sqrt{a-bx^2} \sqrt{a+bx^2}\right) \text{Subst}\left(\int \frac{1}{a-2abx^2} dx, x, \frac{\sqrt{a-bx^2}}{\sqrt{a+bx^2}}\right)}{4a \sqrt{a^2-b^2x^4}}$$

$$= \frac{x(a+bx^2)}{4a^2 \sqrt{a-bx^2} \sqrt{a^2-b^2x^4}} + \frac{3\sqrt{a-bx^2} \sqrt{a+bx^2} \tanh^{-1}\left(\frac{\sqrt{2} \sqrt{bx}}{\sqrt{a+bx^2}}\right)}{4\sqrt{2} a^2 \sqrt{b} \sqrt{a^2-b^2x^4}}$$

**Mathematica [A]** time = 0.08, size = 110, normalized size = 0.89

$$\frac{\sqrt{a^2-b^2x^4} \left(2\sqrt{b} x \sqrt{a+bx^2} + 3\sqrt{2} (a-bx^2) \tanh^{-1}\left(\frac{\sqrt{2} \sqrt{bx}}{\sqrt{a+bx^2}}\right)\right)}{8a^2 \sqrt{b} (a-bx^2)^{3/2} \sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a - b\*x^2)^(3/2)\*Sqrt[a^2 - b^2\*x^4]), x]

[Out] (Sqrt[a^2 - b^2\*x^4]\*(2\*Sqrt[b]\*x\*Sqrt[a + b\*x^2] + 3\*Sqrt[2]\*(a - b\*x^2)\*ArcTanh[(Sqrt[2]\*Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/(8\*a^2\*Sqrt[b]\*(a - b\*x^2)^(3/2)\*Sqrt[a + b\*x^2])

**IntegrateAlgebraic [F]** time = 2.68, size = 0, normalized size = 0.00

$$\int \frac{1}{(a-bx^2)^{3/2} \sqrt{a^2-b^2x^4}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((a - b\*x^2)^(3/2)\*Sqrt[a^2 - b^2\*x^4]), x]

[Out] Defer[IntegrateAlgebraic][1/((a - b\*x^2)^(3/2)\*Sqrt[a^2 - b^2\*x^4]), x]

**fricas [A]** time = 1.14, size = 302, normalized size = 2.44

$$\frac{4\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+ax}+3\sqrt{2}(b^2x^4-2abx^2+a^2)\sqrt{b}\log\left(\frac{-3b^2x^4-2abx^2-2\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+ax}\sqrt{bx-a^2}}{b^2x^4-2abx^2+a^2}\right)+2\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+ax}+3\sqrt{2}(b^2x^4-2abx^2+a^2)\sqrt{-b}\arctan\left(\frac{\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+ax}\sqrt{-b}}{2(b^2x^4-2abx^2+a^2)}\right)}{16(a^2b^3x^4-2a^3b^2x^2+a^4b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x^2+a)^(3/2)/(-b^2\*x^4+a^2)^(1/2), x, algorithm="fricas")

[Out] [1/16\*(4\*sqrt(-b^2\*x^4 + a^2)\*sqrt(-b\*x^2 + a)\*b\*x + 3\*sqrt(2)\*(b^2\*x^4 - 2\*a\*b\*x^2 + a^2)\*sqrt(b)\*log(-(3\*b^2\*x^4 - 2\*a\*b\*x^2 - 2\*sqrt(2)\*sqrt(-b^2\*x^4 + a^2)\*sqrt(-b\*x^2 + a)\*sqrt(b)\*x - a^2)/(b^2\*x^4 - 2\*a\*b\*x^2 + a^2)))/(a^2\*b^3\*x^4 - 2\*a^3\*b^2\*x^2 + a^4\*b), 1/8\*(2\*sqrt(-b^2\*x^4 + a^2)\*sqrt(-b\*x^2 + a)\*b\*x + 3\*sqrt(2)\*(b^2\*x^4 - 2\*a\*b\*x^2 + a^2)\*sqrt(-b)\*arctan(1/2\*sqrt(2)\*sqrt(-b^2\*x^4 + a^2)\*sqrt(-b\*x^2 + a)\*sqrt(-b)/(b^2\*x^4 - a\*b\*x)))/(a^2\*b^3\*x^4 - 2\*a^3\*b^2\*x^2 + a^4\*b)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2} (-bx^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x^2+a)^(3/2)/(-b^2\*x^4+a^2)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(-b^2\*x^4 + a^2)\*(-b\*x^2 + a)^(3/2)), x)

**maple** [B] time = 0.04, size = 510, normalized size = 4.11

$$\frac{\sqrt{-3x^2+a}\sqrt{-3bx^2+a^2}\left(3\sqrt{2}\sqrt{b^2x^2}\ln\left(\frac{b(-\sqrt{3}\sqrt{bx^2+a^2}+\sqrt{3bx^2+a^2})}{bx-\sqrt{3a}}\right)-3\sqrt{2}\sqrt{b^2x^2}\ln\left(\frac{b(-\sqrt{3}\sqrt{bx^2+a^2}+\sqrt{3bx^2+a^2})}{bx-\sqrt{3a}}\right)-4\sqrt{3}bx^2\ln\left(\frac{bx-\sqrt{3}\sqrt{bx^2+a^2}}{a}\right)+4\sqrt{3}bx^2\ln\left(\frac{bx-\sqrt{3}\sqrt{bx^2+a^2}}{a}\right)-3\sqrt{2}x^2\sqrt{b}\ln\left(\frac{b(-\sqrt{3}\sqrt{bx^2+a^2}+\sqrt{3bx^2+a^2})}{bx-\sqrt{3a}}\right)+3\sqrt{2}x^2\sqrt{b}\ln\left(\frac{b(-\sqrt{3}\sqrt{bx^2+a^2}+\sqrt{3bx^2+a^2})}{bx-\sqrt{3a}}\right)+4\sqrt{3}bx\ln\left(\frac{bx-\sqrt{3}\sqrt{bx^2+a^2}}{a}\right)-4\sqrt{3}bx\ln\left(\frac{bx-\sqrt{3}\sqrt{bx^2+a^2}}{a}\right)+4\sqrt{3}\sqrt{bx^2+a^2}\sqrt{b}x\right)}{4(bx^2-a)\sqrt{bx^2+a^2}(\sqrt{3a}+\sqrt{3b})^2(\sqrt{3a}-\sqrt{3b})^2\sqrt{3a}(bx-\sqrt{3a})(bx+\sqrt{3a})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b\*x^2+a)^(3/2)/(-b^2\*x^4+a^2)^(1/2),x)

[Out] 1/4\*(-b\*x^2+a)^(1/2)\*(-b^2\*x^4+a^2)^(1/2)\*b^(5/2)\*(3\*2^(1/2)\*ln(2\*(a-(a\*b)^(1/2)\*x+2^(1/2)\*(b\*x^2+a)^(1/2)\*a^(1/2)))/(b\*x+(a\*b)^(1/2))\*b)\*x^2\*b^(3/2)\*a^(1/2)-3\*2^(1/2)\*ln(2\*(a+(a\*b)^(1/2)\*x+2^(1/2)\*(b\*x^2+a)^(1/2)\*a^(1/2)))/(b\*x-(a\*b)^(1/2))\*b)\*x^2\*b^(3/2)\*a^(1/2)-3\*2^(1/2)\*ln(2\*(a-(a\*b)^(1/2)\*x+2^(1/2)\*(b\*x^2+a)^(1/2)\*a^(1/2)))/(b\*x+(a\*b)^(1/2))\*b)\*a^(3/2)\*b^(1/2)+3\*2^(1/2)\*ln(2\*(a+(a\*b)^(1/2)\*x+2^(1/2)\*(b\*x^2+a)^(1/2)\*a^(1/2)))/(b\*x-(a\*b)^(1/2))\*b)\*a^(3/2)\*b^(1/2)+4\*ln((b\*x+(b\*x^2+a)^(1/2)\*b^(1/2))/b^(1/2))\*x^2\*b\*(a\*b)^(1/2)-4\*ln((b\*x+(-(b\*x+(-a\*b)^(1/2))\*(-b\*x+(-a\*b)^(1/2)))/b^(1/2))\*b^(1/2))/b^(1/2))\*x^2\*b\*(a\*b)^(1/2)+4\*b^(1/2)\*(a\*b)^(1/2)\*(b\*x^2+a)^(1/2)\*x-4\*ln((b\*x+(b\*x^2+a)^(1/2)\*b^(1/2))/b^(1/2))\*a\*(a\*b)^(1/2)+4\*ln((b\*x+(-(b\*x+(-a\*b)^(1/2))\*(-b\*x+(-a\*b)^(1/2)))/b^(1/2))\*b^(1/2))/b^(1/2))\*a\*(a\*b)^(1/2)/(b\*x^2-a)/(b\*x^2+a)^(1/2)/((-a\*b)^(1/2)+(a\*b)^(1/2))^2/((-a\*b)^(1/2)-(a\*b)^(1/2))^2/(a\*b)^(1/2)/(b\*x-(a\*b)^(1/2))/(b\*x+(a\*b)^(1/2))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2} (-bx^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x^2+a)^(3/2)/(-b^2\*x^4+a^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-b^2\*x^4 + a^2)\*(-b\*x^2 + a)^(3/2)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a^2 - b^2x^4} (a - bx^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a^2 - b^2\*x^4)^(1/2)\*(a - b\*x^2)^(3/2)),x)

[Out] int(1/((a^2 - b^2\*x^4)^(1/2)\*(a - b\*x^2)^(3/2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(-a + bx^2)(a + bx^2)} (a - bx^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a - b*x**2)**(3/2)), x)
```

$$3.152 \quad \int \frac{1}{(a-bx^2)^{5/2} \sqrt{a^2-b^2x^4}} dx$$

**Optimal.** Leaf size=167

$$\frac{x(a+bx^2)}{8a^2(a-bx^2)^{3/2} \sqrt{a^2-b^2x^4}} + \frac{9x(a+bx^2)}{32a^3 \sqrt{a-bx^2} \sqrt{a^2-b^2x^4}} + \frac{19\sqrt{a-bx^2} \sqrt{a+bx^2} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b} \sqrt{a^2-b^2x^4}}$$

**Rubi [A]** time = 0.09, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$ , Rules used = {1152, 414, 527, 12, 377, 208}

$$\frac{9x(a+bx^2)}{32a^3 \sqrt{a-bx^2} \sqrt{a^2-b^2x^4}} + \frac{x(a+bx^2)}{8a^2(a-bx^2)^{3/2} \sqrt{a^2-b^2x^4}} + \frac{19\sqrt{a-bx^2} \sqrt{a+bx^2} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b} \sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[1/((a - b\*x^2)^(5/2)\*Sqrt[a^2 - b^2\*x^4]), x]

[Out] (x\*(a + b\*x^2))/(8\*a^2\*(a - b\*x^2)^(3/2)\*Sqrt[a^2 - b^2\*x^4]) + (9\*x\*(a + b\*x^2))/(32\*a^3\*Sqrt[a - b\*x^2]\*Sqrt[a^2 - b^2\*x^4]) + (19\*Sqrt[a - b\*x^2]\*Sqrt[a + b\*x^2]\*ArcTanh[(Sqrt[2]\*Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/(32\*Sqrt[2]\*a^3\*Sqrt[b]\*Sqrt[a^2 - b^2\*x^4])

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

#### Rule 414

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(b\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(p+1)\*(b\*c - a\*d)), x] + Dist[1/(a\*n\*(p+1)\*(b\*c - a\*d)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q\*Simp[b\*c + n\*(p+1)\*(b\*c - a\*d) + d\*b\*(n\*(p+q+2) + 1)\*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

#### Rule 527

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_)\*((e\_) + (f\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*e - a\*f)\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(b\*c - a\*d)\*(p+1)), x] + Dist[1/(a\*n\*(b\*c - a\*d)\*(p+1)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q\*Simp[c\*(b\*e - a\*f) + e\*n\*(b\*c - a\*d)\*(p+1) + d\*(b\*e - a\*f)\*(n\*(p+q+2) + 1)\*x^n, x], x] /; FreeQ



[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

### Rule 1152

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] := Dis  
t[(a + c\*x^4)^FracPart[p]/((d + e\*x^2)^FracPart[p]\*(a/d + (c\*x^2)/e)^FracPa  
rt[p]), Int[(d + e\*x^2)^(p + q)\*(a/d + (c\*x^2)/e)^p, x], x] /; FreeQ[{a, c,  
d, e, p, q}, x] && EqQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[p]

### Rubi steps

$$\begin{aligned} \int \frac{1}{(a-bx^2)^{5/2} \sqrt{a^2-b^2x^4}} dx &= \frac{\left(\sqrt{a-bx^2} \sqrt{a+bx^2}\right) \int \frac{1}{(a-bx^2)^3 \sqrt{a+bx^2}} dx}{\sqrt{a^2-b^2x^4}} \\ &= \frac{x(a+bx^2)}{8a^2(a-bx^2)^{3/2} \sqrt{a^2-b^2x^4}} + \frac{\left(\sqrt{a-bx^2} \sqrt{a+bx^2}\right) \int \frac{7ab+2b^2x^2}{(a-bx^2)^2 \sqrt{a+bx^2}} dx}{8a^2b \sqrt{a^2-b^2x^4}} \\ &= \frac{x(a+bx^2)}{8a^2(a-bx^2)^{3/2} \sqrt{a^2-b^2x^4}} + \frac{9x(a+bx^2)}{32a^3 \sqrt{a-bx^2} \sqrt{a^2-b^2x^4}} + \frac{\left(\sqrt{a-bx^2} \sqrt{a+bx^2}\right) \int \frac{19\sqrt{a-bx^2} \sqrt{a+bx^2}}{32a^3 \sqrt{a+bx^2}} dx}{32a^4b} \\ &= \frac{x(a+bx^2)}{8a^2(a-bx^2)^{3/2} \sqrt{a^2-b^2x^4}} + \frac{9x(a+bx^2)}{32a^3 \sqrt{a-bx^2} \sqrt{a^2-b^2x^4}} + \frac{\left(19\sqrt{a-bx^2} \sqrt{a+bx^2}\right) \int \frac{19\sqrt{a-bx^2} \sqrt{a+bx^2}}{32a^3 \sqrt{a+bx^2}} dx}{32a^4b} \\ &= \frac{x(a+bx^2)}{8a^2(a-bx^2)^{3/2} \sqrt{a^2-b^2x^4}} + \frac{9x(a+bx^2)}{32a^3 \sqrt{a-bx^2} \sqrt{a^2-b^2x^4}} + \frac{\left(19\sqrt{a-bx^2} \sqrt{a+bx^2}\right) \int \frac{19\sqrt{a-bx^2} \sqrt{a+bx^2}}{32a^3 \sqrt{a+bx^2}} dx}{32a^4b} \\ &= \frac{x(a+bx^2)}{8a^2(a-bx^2)^{3/2} \sqrt{a^2-b^2x^4}} + \frac{9x(a+bx^2)}{32a^3 \sqrt{a-bx^2} \sqrt{a^2-b^2x^4}} + \frac{19\sqrt{a-bx^2} \sqrt{a+bx^2}}{32\sqrt{2} a^3 \sqrt{a^2-b^2x^4}} \end{aligned}$$

**Mathematica [A]** time = 0.11, size = 122, normalized size = 0.73

$$\frac{\sqrt{a^2-b^2x^4} \left(2\sqrt{b} x (13a-9bx^2) \sqrt{a+bx^2} + 19\sqrt{2} (a-bx^2)^2 \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{bx^2}}{\sqrt{a+bx^2}}\right)\right)}{64a^3\sqrt{b} (a-bx^2)^{5/2} \sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a - b\*x^2)^(5/2)\*Sqrt[a^2 - b^2\*x^4]), x]

[Out] (Sqrt[a^2 - b^2\*x^4]\*(2\*Sqrt[b]\*x\*(13\*a - 9\*b\*x^2)\*Sqrt[a + b\*x^2] + 19\*Sqr  
t[2]\*(a - b\*x^2)^2\*ArcTanh[(Sqrt[2]\*Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/(64\*a^3\*S  
qrt[b]\*(a - b\*x^2)^(5/2)\*Sqrt[a + b\*x^2])

**IntegrateAlgebraic [F]** time = 2.94, size = 0, normalized size = 0.00

$$\int \frac{1}{(a-bx^2)^{5/2} \sqrt{a^2-b^2x^4}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((a - b\*x^2)^(5/2)\*Sqrt[a^2 - b^2\*x^4]), x]

[Out] Defer[IntegrateAlgebraic][1/((a - b\*x^2)^(5/2)\*Sqrt[a^2 - b^2\*x^4]), x]

**fricas** [A] time = 1.16, size = 376, normalized size = 2.25

$$\frac{19\sqrt{2}(b^3x^6 - 3ab^2x^4 + 3a^2bx^2 - a^3)\sqrt{b}\log\left(\frac{3b^2x^4 - 2ab^2x^2 - a^2\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + a}}{b^2x^4 - 2ab^2x^2 + a^2}\right) + 4\sqrt{-b^2x^4 + a^2}(9b^2x^3 - 13abx)\sqrt{-bx^2 + a} - 19\sqrt{2}(b^3x^6 - 3ab^2x^4 + 3a^2bx^2 - a^3)\sqrt{-b}\arctan\left(\frac{\sqrt{2}\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + a}}{2(b^2x^4 - ab^2x^2 + a^2)}\right) + 2\sqrt{-b^2x^4 + a^2}(9b^2x^3 - 13abx)\sqrt{-bx^2 + a}}{128(a^3b^4x^6 - 3a^4b^3x^4 + 3a^5b^2x^2 - a^6b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x^2+a)^(5/2)/(-b^2\*x^4+a^2)^(1/2),x, algorithm="fricas")

[Out] [1/128\*(19\*sqrt(2)\*(b^3\*x^6 - 3\*a\*b^2\*x^4 + 3\*a^2\*b\*x^2 - a^3)\*sqrt(b)\*log(- (3\*b^2\*x^4 - 2\*a\*b\*x^2 - 2\*sqrt(2)\*sqrt(-b^2\*x^4 + a^2)\*sqrt(-b\*x^2 + a)\*sqrt(b)\*x - a^2)/(b^2\*x^4 - 2\*a\*b\*x^2 + a^2)) + 4\*sqrt(-b^2\*x^4 + a^2)\*(9\*b^2\*x^3 - 13\*a\*b\*x)\*sqrt(-b\*x^2 + a))/(a^3\*b^4\*x^6 - 3\*a^4\*b^3\*x^4 + 3\*a^5\*b^2\*x^2 - a^6\*b), 1/64\*(19\*sqrt(2)\*(b^3\*x^6 - 3\*a\*b^2\*x^4 + 3\*a^2\*b\*x^2 - a^3)\*sqrt(-b)\*arctan(1/2\*sqrt(2)\*sqrt(-b^2\*x^4 + a^2)\*sqrt(-b\*x^2 + a)\*sqrt(-b)/(b^2\*x^3 - a\*b\*x)) + 2\*sqrt(-b^2\*x^4 + a^2)\*(9\*b^2\*x^3 - 13\*a\*b\*x)\*sqrt(-b\*x^2 + a))/(a^3\*b^4\*x^6 - 3\*a^4\*b^3\*x^4 + 3\*a^5\*b^2\*x^2 - a^6\*b)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2}(-bx^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x^2+a)^(5/2)/(-b^2\*x^4+a^2)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(-b^2\*x^4 + a^2)\*(-b\*x^2 + a)^(5/2)), x)

**maple** [B] time = 0.05, size = 739, normalized size = 4.43

$$\frac{1}{\sqrt{-b^2x^4 + a^2}(-bx^2 + a)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b\*x^2+a)^(5/2)/(-b^2\*x^4+a^2)^(1/2),x)

[Out] 1/16\*(-b\*x^2+a)^(1/2)\*(-b^2\*x^4+a^2)^(1/2)\*b^(9/2)\*(19\*ln(2\*(a-(a\*b)^(1/2)\*x+2^(1/2)\*(b\*x^2+a)^(1/2)\*a^(1/2))/(b\*x+(a\*b)^(1/2))\*b)\*2^(1/2)\*x^4\*b^(5/2)\*a^(1/2)-19\*ln(2\*(a+(a\*b)^(1/2)\*x+2^(1/2)\*(b\*x^2+a)^(1/2)\*a^(1/2))/(b\*x-(a\*b)^(1/2))\*b)\*2^(1/2)\*x^4\*b^(5/2)\*a^(1/2)+16\*ln((b\*x+(b\*x^2+a)^(1/2)\*b^(1/2))/b^(1/2))\*x^4\*b^2\*(a\*b)^(1/2)-16\*ln((b\*x+(-(b\*x+(-a\*b)^(1/2))\*(-b\*x+(-a\*b)^(1/2))/b)^(1/2)\*b^(1/2))/b^(1/2))\*x^4\*b^2\*(a\*b)^(1/2)-38\*ln(2\*(a-(a\*b)^(1/2)\*x+2^(1/2)\*(b\*x^2+a)^(1/2)\*a^(1/2))/(b\*x-(a\*b)^(1/2))\*b)\*2^(1/2)\*x^2\*a^(3/2)\*b^(3/2)+36\*b^(3/2)\*(a\*b)^(1/2)\*(b\*x^2+a)^(1/2)\*x^3-32\*ln((b\*x+(b\*x^2+a)^(1/2)\*b^(1/2))/b^(1/2))\*x^2\*a\*b\*(a\*b)^(1/2)+32\*ln((b\*x+(-(b\*x+(-a\*b)^(1/2))\*(-b\*x+(-a\*b)^(1/2))/b)^(1/2)\*b^(1/2))/b^(1/2))\*x^2\*a\*b\*(a\*b)^(1/2)+19\*ln(2\*(a-(a\*b)^(1/2)\*x+2^(1/2)\*(b\*x^2+a)^(1/2)\*a^(1/2))/(b\*x+(a\*b)^(1/2))\*b)\*2^(1/2)\*a^(5/2)\*b^(1/2)-19\*ln(2\*(a+(a\*b)^(1/2)\*x+2^(1/2)\*(b\*x^2+a)^(1/2)\*a^(1/2))/(b\*x-(a\*b)^(1/2))\*b)\*2^(1/2)\*a^(5/2)\*b^(1/2)-52\*a\*(a\*b)^(1/2)\*(b\*x^2+a)^(1/2)\*b^(1/2)\*x+16\*ln((b\*x+(b\*x^2+a)^(1/2)\*b^(1/2))/b^(1/2))\*a^2\*(a\*b)^(1/2)-16\*ln((b\*x+(-(b\*x+(-a\*b)^(1/2))\*(-b\*x+(-a\*b)^(1/2))/b)^(1/2)\*b^(1/2))/b^(1/2))\*a^2\*(a\*b)^(1/2)/(b\*x^2-a)/(b\*x^2+a)^(1/2)/((-a\*b)^(1/2)+(a\*b)^(1/2))^3/(-(-a\*b)^(1/2)+(a\*b)^(1/2))^3/(a\*b)^(1/2)/(b\*x-(a\*b)^(1/2))^2/(b\*x+(a\*b)^(1/2))^2

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2}(-bx^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x^2+a)^(5/2)/(-b^2\*x^4+a^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-b^2\*x^4 + a^2)\*(-b\*x^2 + a)^(5/2)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a^2 - b^2 x^4} (a - b x^2)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a^2 - b^2\*x^4)^(1/2)\*(a - b\*x^2)^(5/2)),x)

[Out] int(1/((a^2 - b^2\*x^4)^(1/2)\*(a - b\*x^2)^(5/2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(-a + bx^2)(a + bx^2)} (a - bx^2)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x\*\*2+a)\*\*(5/2)/(-b\*\*2\*x\*\*4+a\*\*2)\*\*(1/2),x)

[Out] Integral(1/(sqrt(-(-a + b\*x\*\*2)\*(a + b\*x\*\*2))\*(a - b\*x\*\*2)\*\*(5/2)), x)

$$3.153 \quad \int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}} dx$$

**Optimal.** Leaf size=30

$$\frac{\sqrt{x^2-1} \sqrt{x^2+1} \sinh^{-1}(x)}{\sqrt{x^4-1}}$$

**Rubi [A]** time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {1152, 215}

$$\frac{\sqrt{x^2-1} \sqrt{x^2+1} \sinh^{-1}(x)}{\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-1 + x^2]/Sqrt[-1 + x^4], x]

[Out] (Sqrt[-1 + x^2]\*Sqrt[1 + x^2]\*ArcSinh[x])/Sqrt[-1 + x^4]

**Rule 215**

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

**Rule 1152**

Int[((d\_) + (e\_)\*(x\_)^2)^(q\_)\*((a\_) + (c\_)\*(x\_)^4)^(p\_), x\_Symbol] := Dist[(a + c\*x^4)^FracPart[p]/((d + e\*x^2)^FracPart[p]\*(a/d + (c\*x^2)/e)^FracPart[p]), Int[(d + e\*x^2)^(p + q)\*(a/d + (c\*x^2)/e)^p, x], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[p]

**Rubi steps**

$$\begin{aligned} \int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}} dx &= \frac{\left(\sqrt{-1+x^2} \sqrt{1+x^2}\right) \int \frac{1}{\sqrt{1+x^2}} dx}{\sqrt{-1+x^4}} \\ &= \frac{\sqrt{-1+x^2} \sqrt{1+x^2} \sinh^{-1}(x)}{\sqrt{-1+x^4}} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 38, normalized size = 1.27

$$\log\left(x^3 + \sqrt{x^2-1} \sqrt{x^4-1} - x\right) - \log(1-x^2)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-1 + x^2]/Sqrt[-1 + x^4], x]

[Out] -Log[1 - x^2] + Log[-x + x^3 + Sqrt[-1 + x^2]\*Sqrt[-1 + x^4]]

**IntegrateAlgebraic [F]** time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[Sqrt[-1 + x^2]/Sqrt[-1 + x^4], x]

[Out] Defer[IntegrateAlgebraic][Sqrt[-1 + x^2]/Sqrt[-1 + x^4], x]

**fricas** [B] time = 0.69, size = 73, normalized size = 2.43

$$\frac{1}{2} \log\left(\frac{x^3 + \sqrt{x^4 - 1} \sqrt{x^2 - 1} - x}{x^3 - x}\right) - \frac{1}{2} \log\left(-\frac{x^3 - \sqrt{x^4 - 1} \sqrt{x^2 - 1} - x}{x^3 - x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)^(1/2)/(x^4-1)^(1/2), x, algorithm="fricas")

[Out] 1/2\*log((x^3 + sqrt(x^4 - 1)\*sqrt(x^2 - 1) - x)/(x^3 - x)) - 1/2\*log(-(x^3 - sqrt(x^4 - 1)\*sqrt(x^2 - 1) - x)/(x^3 - x))

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 - 1}}{\sqrt{x^4 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)^(1/2)/(x^4-1)^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(x^2 - 1)/sqrt(x^4 - 1), x)

**maple** [A] time = 0.01, size = 25, normalized size = 0.83

$$\frac{\sqrt{x^4 - 1} \operatorname{arcsinh}(x)}{\sqrt{x^2 - 1} \sqrt{x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-1)^(1/2)/(x^4-1)^(1/2), x)

[Out] 1/(x^2-1)^(1/2)\*(x^4-1)^(1/2)/(x^2+1)^(1/2)\*arcsinh(x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 - 1}}{\sqrt{x^4 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)^(1/2)/(x^4-1)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(x^2 - 1)/sqrt(x^4 - 1), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{x^2 - 1}}{\sqrt{x^4 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 - 1)^(1/2)/(x^4 - 1)^(1/2), x)

[Out] int((x^2 - 1)^(1/2)/(x^4 - 1)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x-1)(x+1)}}{\sqrt{(x-1)(x+1)(x^2+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**(1/2)/(x**4-1)**(1/2),x)
```

```
[Out] Integral(sqrt((x - 1)*(x + 1))/sqrt((x - 1)*(x + 1)*(x**2 + 1)), x)
```

$$3.154 \quad \int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}} dx$$

**Optimal.** Leaf size=24

$$-\frac{\sqrt{x^4-1} \sin^{-1}(x)}{\sqrt{1-x^4}}$$

**Rubi [A]** time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.67, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {1152, 217, 206}

$$\frac{\sqrt{x^2-1} \sqrt{x^2+1} \tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)}{\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + x^2]/Sqrt[-1 + x^4], x]

[Out] (Sqrt[-1 + x^2]\*Sqrt[1 + x^2]\*ArcTanh[x/Sqrt[-1 + x^2]])/Sqrt[-1 + x^4]

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 217**

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

**Rule 1152**

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] :> Dist[(a + c\*x^4)^FracPart[p]/((d + e\*x^2)^FracPart[p]\*(a/d + (c\*x^2)/e)^FracPart[p]), Int[(d + e\*x^2)^(p + q)\*(a/d + (c\*x^2)/e)^p, x], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[p]

**Rubi steps**

$$\begin{aligned} \int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}} dx &= \frac{\left(\sqrt{-1+x^2} \sqrt{1+x^2}\right) \int \frac{1}{\sqrt{-1+x^2}} dx}{\sqrt{-1+x^4}} \\ &= \frac{\left(\sqrt{-1+x^2} \sqrt{1+x^2}\right) \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^4}} \\ &= \frac{\sqrt{-1+x^2} \sqrt{1+x^2} \tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^4}} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 34, normalized size = 1.42

$$\log\left(x^3 + \sqrt{x^2+1} \sqrt{x^4-1} + x\right) - \log(x^2+1)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + x^2]/Sqrt[-1 + x^4], x]

[Out] -Log[1 + x^2] + Log[x + x^3 + Sqrt[1 + x^2]\*Sqrt[-1 + x^4]]

**IntegrateAlgebraic** [F] time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[Sqrt[1 + x^2]/Sqrt[-1 + x^4], x]

[Out] Defer[IntegrateAlgebraic][Sqrt[1 + x^2]/Sqrt[-1 + x^4], x]

**fricas** [B] time = 1.03, size = 65, normalized size = 2.71

$$\frac{1}{2} \log\left(\frac{x^3 + \sqrt{x^4-1}\sqrt{x^2+1} + x}{x^3 + x}\right) - \frac{1}{2} \log\left(-\frac{x^3 - \sqrt{x^4-1}\sqrt{x^2+1} + x}{x^3 + x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^(1/2)/(x^4-1)^(1/2), x, algorithm="fricas")

[Out] 1/2\*log((x^3 + sqrt(x^4 - 1)\*sqrt(x^2 + 1) + x)/(x^3 + x)) - 1/2\*log(-(x^3 - sqrt(x^4 - 1)\*sqrt(x^2 + 1) + x)/(x^3 + x))

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}}{\sqrt{x^4-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^(1/2)/(x^4-1)^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(x^2 + 1)/sqrt(x^4 - 1), x)

**maple** [A] time = 0.01, size = 33, normalized size = 1.38

$$\frac{\sqrt{x^4-1} \ln(x + \sqrt{x^2-1})}{\sqrt{x^2+1} \sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)^(1/2)/(x^4-1)^(1/2), x)

[Out] 1/(x^2+1)^(1/2)\*(x^4-1)^(1/2)/(x^2-1)^(1/2)\*ln(x+(x^2-1)^(1/2))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}}{\sqrt{x^4-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^(1/2)/(x^4-1)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(x^2 + 1)/sqrt(x^4 - 1), x)



**mupad** [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{x^2 + 1}}{\sqrt{x^4 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 1)^(1/2)/(x^4 - 1)^(1/2), x)

[Out] int((x^2 + 1)^(1/2)/(x^4 - 1)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 + 1}}{\sqrt{(x - 1)(x + 1)(x^2 + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*2+1)\*\*(1/2)/(x\*\*4-1)\*\*(1/2), x)

[Out] Integral(sqrt(x\*\*2 + 1)/sqrt((x - 1)\*(x + 1)\*(x\*\*2 + 1)), x)

$$3.155 \quad \int \frac{-\sqrt{-1+x^2} + \sqrt{1+x^2}}{\sqrt{-1+x^4}} dx$$

**Optimal.** Leaf size=73

$$\frac{\sqrt{x^2-1} \sqrt{x^4-1} \sinh^{-1}(x)}{(1-x^2) \sqrt{x^2+1}} - \frac{\sqrt{x^4-1} \sin^{-1}(x)}{\sqrt{1-x^2} \sqrt{x^2+1}}$$

**Rubi [A]** time = 0.12, antiderivative size = 72, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {6742, 1152, 215, 217, 206}

$$\frac{\sqrt{x^2-1} \sqrt{x^2+1} \tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)}{\sqrt{x^4-1}} - \frac{\sqrt{x^2-1} \sqrt{x^2+1} \sinh^{-1}(x)}{\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

[In] Int[(-Sqrt[-1 + x^2] + Sqrt[1 + x^2])/Sqrt[-1 + x^4], x]

[Out] -((Sqrt[-1 + x^2]\*Sqrt[1 + x^2]\*ArcSinh[x])/Sqrt[-1 + x^4]) + (Sqrt[-1 + x^2]\*Sqrt[1 + x^2]\*ArcTanh[x/Sqrt[-1 + x^2]])/Sqrt[-1 + x^4]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 1152

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] := Dist[(a + c\*x^4)^FracPart[p]/((d + e\*x^2)^FracPart[p]\*(a/d + (c\*x^2)/e)^FracPart[p]), Int[(d + e\*x^2)^(p + q)\*(a/d + (c\*x^2)/e)^p, x], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[p]

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rubi steps

$$\begin{aligned}
\int \frac{-\sqrt{-1+x^2} + \sqrt{1+x^2}}{\sqrt{-1+x^4}} dx &= \int \left( \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}} + \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}} \right) dx \\
&= -\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}} dx + \int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}} dx \\
&= \frac{(\sqrt{-1+x^2} \sqrt{1+x^2}) \int \frac{1}{\sqrt{-1+x^2}} dx}{\sqrt{-1+x^4}} - \frac{(\sqrt{-1+x^2} \sqrt{1+x^2}) \int \frac{1}{\sqrt{1+x^2}} dx}{\sqrt{-1+x^4}} \\
&= -\frac{\sqrt{-1+x^2} \sqrt{1+x^2} \sinh^{-1}(x)}{\sqrt{-1+x^4}} + \frac{(\sqrt{-1+x^2} \sqrt{1+x^2}) \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^4}} \\
&= -\frac{\sqrt{-1+x^2} \sqrt{1+x^2} \sinh^{-1}(x)}{\sqrt{-1+x^4}} + \frac{\sqrt{-1+x^2} \sqrt{1+x^2} \tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^4}}
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 71, normalized size = 0.97

$$\log(1-x^2) - \log(x^2+1) - \log(x^3 + \sqrt{x^2-1} \sqrt{x^4-1} - x) + \log(x^3 + \sqrt{x^2+1} \sqrt{x^4-1} + x)$$

Antiderivative was successfully verified.

[In] Integrate[(-Sqrt[-1 + x^2] + Sqrt[1 + x^2])/Sqrt[-1 + x^4], x]

[Out] Log[1 - x^2] - Log[1 + x^2] - Log[-x + x^3 + Sqrt[-1 + x^2]\*Sqrt[-1 + x^4]] + Log[x + x^3 + Sqrt[1 + x^2]\*Sqrt[-1 + x^4]]

**IntegrateAlgebraic [F]** time = 4.67, size = 0, normalized size = 0.00

$$\int \frac{-\sqrt{-1+x^2} + \sqrt{1+x^2}}{\sqrt{-1+x^4}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-Sqrt[-1 + x^2] + Sqrt[1 + x^2])/Sqrt[-1 + x^4], x]

[Out] Defer[IntegrateAlgebraic][(-Sqrt[-1 + x^2] + Sqrt[1 + x^2])/Sqrt[-1 + x^4], x]

**fricas [B]** time = 1.42, size = 137, normalized size = 1.88

$$\frac{1}{2} \log\left(\frac{x^3 + \sqrt{x^4-1} \sqrt{x^2+1} + x}{x^3+x}\right) - \frac{1}{2} \log\left(-\frac{x^3 - \sqrt{x^4-1} \sqrt{x^2+1} + x}{x^3+x}\right) - \frac{1}{2} \log\left(\frac{x^3 + \sqrt{x^4-1} \sqrt{x^2-1} - x}{x^3-x}\right) + \frac{1}{2} \log\left(-\frac{x^3 - \sqrt{x^4-1} \sqrt{x^2-1} - x}{x^3-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((- (x^2-1)^(1/2) + (x^2+1)^(1/2)) / ((x^4-1)^(1/2)), x, algorithm="fricas")

[Out] 1/2\*log((x^3 + sqrt(x^4 - 1)\*sqrt(x^2 + 1) + x)/(x^3 + x)) - 1/2\*log(-(x^3 - sqrt(x^4 - 1)\*sqrt(x^2 + 1) + x)/(x^3 + x)) - 1/2\*log((x^3 + sqrt(x^4 - 1)\*sqrt(x^2 - 1) - x)/(x^3 - x)) + 1/2\*log(-(x^3 - sqrt(x^4 - 1)\*sqrt(x^2 - 1) - x)/(x^3 - x))

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1} - \sqrt{x^2-1}}{\sqrt{x^4-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x, algorithm="giac")

[Out] integrate((sqrt(x^2 + 1) - sqrt(x^2 - 1))/sqrt(x^4 - 1), x)

maple [A] time = 0.00, size = 59, normalized size = 0.81

$$-\frac{\sqrt{x^4-1} \operatorname{arcsinh}(x)}{\sqrt{x^2-1} \sqrt{x^2+1}} + \frac{\sqrt{x^4-1} \ln\left(x + \sqrt{x^2-1}\right)}{\sqrt{x^2+1} \sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x)

[Out] -1/(x^2-1)^(1/2)\*(x^4-1)^(1/2)/(x^2+1)^(1/2)\*arcsinh(x)+1/(x^2+1)^(1/2)\*(x^4-1)^(1/2)/(x^2-1)^(1/2)\*ln(x+(x^2-1)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1} - \sqrt{x^2-1}}{\sqrt{x^4-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x, algorithm="maxima")

[Out] integrate((sqrt(x^2 + 1) - sqrt(x^2 - 1))/sqrt(x^4 - 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{x^2-1} - \sqrt{x^2+1}}{\sqrt{x^4-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-((x^2 - 1)^(1/2) - (x^2 + 1)^(1/2))/(x^4 - 1)^(1/2),x)

[Out] int(-((x^2 - 1)^(1/2) - (x^2 + 1)^(1/2))/(x^4 - 1)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-\sqrt{x^2-1} + \sqrt{x^2+1}}{\sqrt{(x-1)(x+1)(x^2+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*2-1)\*\*(1/2)+(x\*\*2+1)\*\*(1/2))/(x\*\*4-1)\*\*(1/2),x)

[Out] Integral((-sqrt(x\*\*2 - 1) + sqrt(x\*\*2 + 1))/sqrt((x - 1)\*(x + 1)\*(x\*\*2 + 1)), x)

$$3.156 \quad \int \frac{(d+ex^2)^4}{-cd^2+bde+be^2x^2+ce^2x^4} dx$$

**Optimal.** Leaf size=121

$$\frac{x(b^2e^2 - 5bcde + 7c^2d^2)}{c^3} - \frac{(2cd - be)^3 \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{cd-be}}\right)}{c^{7/2}\sqrt{e}\sqrt{cd-be}} + \frac{ex^3(4cd - be)}{3c^2} + \frac{e^2x^5}{5c}$$

**Rubi [A]** time = 0.16, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {1149, 390, 208}

$$\frac{x(b^2e^2 - 5bcde + 7c^2d^2)}{c^3} + \frac{ex^3(4cd - be)}{3c^2} - \frac{(2cd - be)^3 \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{cd-be}}\right)}{c^{7/2}\sqrt{e}\sqrt{cd-be}} + \frac{e^2x^5}{5c}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^4/(-(c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4), x]

[Out] ((7\*c^2\*d^2 - 5\*b\*c\*d\*e + b^2\*e^2)\*x)/c^3 + (e\*(4\*c\*d - b\*e)\*x^3)/(3\*c^2) + (e^2\*x^5)/(5\*c) - ((2\*c\*d - b\*e)^3\*ArcTanh[(Sqrt[c]\*Sqrt[e]\*x)/Sqrt[c\*d - b\*e]]/(c^(7/2)\*Sqrt[e]\*Sqrt[c\*d - b\*e])

Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 390

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Int[PolynomialDivide[(a + b\*x^n)^p, (c + d\*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 1149

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[(d + e\*x^2)^(p+q)\*(a/d + (c\*x^2)/e)^p, x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{(d+ex^2)^4}{-cd^2+bde+be^2x^2+ce^2x^4} dx &= \int \frac{(d+ex^2)^3}{\frac{-cd^2+bde}{d} + cex^2} dx \\ &= \int \left( \frac{7c^2d^2 - 5bcde + b^2e^2}{c^3} + \frac{e(4cd - be)x^2}{c^2} + \frac{e^2x^4}{c} + \frac{8c^3d^3 - 12bc^2d^2e + 6c^2d^2e^2 - 4bcde^2 + b^2e^3}{c^3(-cd + be + cex^2)} \right) dx \\ &= \frac{(7c^2d^2 - 5bcde + b^2e^2)x}{c^3} + \frac{e(4cd - be)x^3}{3c^2} + \frac{e^2x^5}{5c} + \frac{(2cd - be)^3 \int \frac{1}{-cd + be + cex^2}}{c^3} \\ &= \frac{(7c^2d^2 - 5bcde + b^2e^2)x}{c^3} + \frac{e(4cd - be)x^3}{3c^2} + \frac{e^2x^5}{5c} - \frac{(2cd - be)^3 \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{cd-be}}\right)}{c^{7/2}\sqrt{e}\sqrt{cd-be}} \end{aligned}$$













Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)^4/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)`

[Out]  $\frac{1}{5}e^2x^5/c - \frac{1}{3}c^2x^3b^2e^2 + \frac{4}{3}cx^3d^2e + \frac{1}{c^3}b^2e^2x - \frac{5}{c^2}b^2d^2e^2x + \frac{7}{c}d^2x - \frac{1}{c^3}((b^2e - c^2d) * c^2e)^{(1/2)} * \arctan(c^2e^2x / ((b^2e - c^2d) * c^2e)^{(1/2)}) * b^3e^3 + \frac{6}{c^2}((b^2e - c^2d) * c^2e)^{(1/2)} * \arctan(c^2e^2x / ((b^2e - c^2d) * c^2e)^{(1/2)}) * b^2d^2e^2 - \frac{12}{c}((b^2e - c^2d) * c^2e)^{(1/2)} * \arctan(c^2e^2x / ((b^2e - c^2d) * c^2e)^{(1/2)}) * b^2d^2e^2 + \frac{8}{((b^2e - c^2d) * c^2e)^{(1/2)} * \arctan(c^2e^2x / ((b^2e - c^2d) * c^2e)^{(1/2)})} * d^3$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^4/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(b\*e-c\*d>0)', see 'assume?' for more details) Is b\*e-c\*d positive or negative?

**mupad** [B] time = 4.53, size = 182, normalized size = 1.50

$$x \left( \frac{3d^2}{c} + \frac{\left( \frac{e(b^2e - c^2d)}{c^2} - \frac{3de}{c} \right) (be - cd)}{ce} \right) - x^3 \left( \frac{e(b^2e - c^2d)}{3c^2} - \frac{de}{c} \right) + \frac{e^2x^5}{5c} - \frac{\operatorname{atan}\left( \frac{\sqrt{c}ex(b^2e - c^2d)^3}{\sqrt{be^2 - c^2d}e(b^3e^3 - 6b^2cd^2 + 12b^2d^2e - 8c^3d^3)} \right) (be - cd)^3}{c^{7/2}\sqrt{be^2 - c^2d}e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x^2)^4/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)`

[Out]  $x \left( \frac{3d^2}{c} + \frac{((e(b^2e - c^2d))/c^2 - (3d^2e)/c) * (b^2e - c^2d)}{(c^2e)} \right) - x^3 * \left( \frac{e(b^2e - c^2d)}{3c^2} - \frac{de}{c} \right) + \frac{e^2x^5}{5c} - \frac{\operatorname{atan}\left( \frac{c^{(1/2)} * e * x * (b^2e - 2c^2d)^3}{(b^2e^2 - c^2d^2)^{(1/2)} * (b^3e^3 - 8c^3d^3 + 12b^2c^2d^2 * 2e - 6b^2 * 2c^2d^2 * e^2)} \right) * (b^2e - 2c^2d)^3}{c^{(7/2)} * (b^2e^2 - c^2d^2)^{(1/2)}}$

**sympy** [B] time = 1.00, size = 345, normalized size = 2.85

$$x^3 \left( -\frac{be^2}{3c^2} + \frac{4de}{3c} \right) + x \left( \frac{b^2e^2}{c^3} - \frac{5bde}{c^2} + \frac{7d^2}{c} \right) + \frac{\sqrt{-\frac{1}{c^2e(b^2e - c^2d)}} (be - 2cd)^3 \log \left( x + \frac{-b^2c^2e \sqrt{-\frac{1}{c^2e(b^2e - c^2d)}} (be - 2cd)^3 + c^2d \sqrt{-\frac{1}{c^2e(b^2e - c^2d)}} (be - 2cd)^3}{b^3e^3 - 6b^2cd^2 + 12b^2d^2e - 8c^3d^3} \right) - \frac{\sqrt{-\frac{1}{c^2e(b^2e - c^2d)}} (be - 2cd)^3 \log \left( x + \frac{bc^2e \sqrt{-\frac{1}{c^2e(b^2e - c^2d)}} (be - 2cd)^3 - c^2d \sqrt{-\frac{1}{c^2e(b^2e - c^2d)}} (be - 2cd)^3}{b^3e^3 - 6b^2cd^2 + 12b^2d^2e - 8c^3d^3} \right)}{2} + \frac{e^2x^5}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**4/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)`

[Out]  $x^3 * \left( -\frac{b^2e^2}{3c^2} + \frac{4d^2e}{3c} \right) + x * \left( \frac{b^2e^2}{c^3} - \frac{5b^2d^2e}{c^2} + \frac{7d^2e^2}{c} \right) + \sqrt{-1/(c^2e^2(b^2e - c^2d))} * (b^2e - 2c^2d)^3 * \log(x + (-b^2c^2e^2 * e * \sqrt{-1/(c^2e^2(b^2e - c^2d))}) * (b^2e - 2c^2d)^3 + c^2d^2 * \sqrt{-1/(c^2e^2(b^2e - c^2d))} * (b^2e - c^2d)) * (b^2e - 2c^2d)^3 / (b^3e^3 - 6b^2c^2d^2e^2 + 12b^2c^2d^2 * 2e - 8c^3d^3) / 2 - \sqrt{-1/(c^2e^2(b^2e - c^2d))} * (b^2e - 2c^2d)^3 * \log(x + (b^2c^2e^2 * e * \sqrt{-1/(c^2e^2(b^2e - c^2d))}) * (b^2e - 2c^2d)^3 - c^2d^2 * \sqrt{-1/(c^2e^2(b^2e - c^2d))} * (b^2e - c^2d)) * (b^2e - 2c^2d)^3 / (b^3e^3 - 6b^2c^2d^2e^2 + 12b^2c^2d^2 * 2e - 8c^3d^3) / 2 + e^2x^5 / (5c)$

$$3.157 \quad \int \frac{(d+ex^2)^3}{-cd^2+bde+be^2x^2+ce^2x^4} dx$$

**Optimal.** Leaf size=86

$$-\frac{(2cd-be)^2 \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{cd-be}}\right)}{c^{5/2}\sqrt{e}\sqrt{cd-be}} + \frac{x(3cd-be)}{c^2} + \frac{ex^3}{3c}$$

**Rubi [A]** time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {1149, 390, 208}

$$\frac{x(3cd-be)}{c^2} - \frac{(2cd-be)^2 \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{cd-be}}\right)}{c^{5/2}\sqrt{e}\sqrt{cd-be}} + \frac{ex^3}{3c}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^3/(-(c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4), x]

[Out] ((3\*c\*d - b\*e)\*x)/c^2 + (e\*x^3)/(3\*c) - ((2\*c\*d - b\*e)^2\*ArcTanh[(Sqrt[c]\*Sqrt[e]\*x)/Sqrt[c\*d - b\*e]]/(c^(5/2)\*Sqrt[e]\*Sqrt[c\*d - b\*e])

Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 390

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Int[PolynomialDivide[(a + b\*x^n)^p, (c + d\*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 1149

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[(d + e\*x^2)^(p+q)\*(a/d + (c\*x^2)/e)^p, x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{(d+ex^2)^3}{-cd^2+bde+be^2x^2+ce^2x^4} dx &= \int \frac{(d+ex^2)^2}{\frac{-cd^2+bde}{d}+cex^2} dx \\ &= \int \left( \frac{3cd-be}{c^2} + \frac{ex^2}{c} + \frac{4c^2d^2-4bcde+b^2e^2}{c^2(-cd+be+cex^2)} \right) dx \\ &= \frac{(3cd-be)x}{c^2} + \frac{ex^3}{3c} + \frac{(2cd-be)^2 \int \frac{1}{-cd+be+cex^2} dx}{c^2} \\ &= \frac{(3cd-be)x}{c^2} + \frac{ex^3}{3c} - \frac{(2cd-be)^2 \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{cd-be}}\right)}{c^{5/2}\sqrt{e}\sqrt{cd-be}} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 84, normalized size = 0.98

$$\frac{(be - 2cd)^2 \tan^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{be - cd}}\right)}{c^{5/2}\sqrt{e}\sqrt{be - cd}} - \frac{x(be - 3cd)}{c^2} + \frac{ex^3}{3c}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^3/(-(c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4), x]

[Out] -((( -3\*c\*d + b\*e)\*x)/c^2) + (e\*x^3)/(3\*c) + ((-2\*c\*d + b\*e)^2\*ArcTan[(Sqrt[c]\*Sqrt[e]\*x)/Sqrt[-(c\*d) + b\*e]])/(c^(5/2)\*Sqrt[e]\*Sqrt[-(c\*d) + b\*e])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^2)^3}{-cd^2 + bde + be^2x^2 + ce^2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)^3/(-(c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4), x]

[Out] IntegrateAlgebraic[(d + e\*x^2)^3/(-(c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4), x]

**fricas [A]** time = 1.70, size = 311, normalized size = 3.62

$$\frac{2(c^3de^2 - bc^2e^3)x^3 + 3(4c^2d^2 - 4bcde + b^2e^2)\sqrt{c^2de - bce^2} \log\left(\frac{cx^2 + d - be - 2\sqrt{c^2de - bce^2}x}{cx^2 - cd + be}\right) + 6(3c^3d^2e - 4bc^2de^2 + b^2ce^3)x^2 + (c^3de^2 - bc^2e^3)x^3 - 3(4c^2d^2 - 4bcde + b^2e^2)\sqrt{-c^2de + bce^2} \arctan\left(\frac{-\sqrt{c^2de + bce^2}x}{cd - be}\right) + 3(3c^3d^2e - 4bc^2de^2 + b^2ce^3)x}{6(c^4de - bc^3e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^3/(c\*e^2\*x^4+b\*e^2\*x^2+b\*d\*e-c\*d^2), x, algorithm="fricas")

[Out] [1/6\*(2\*(c^3\*d\*e^2 - b\*c^2\*e^3)\*x^3 + 3\*(4\*c^2\*d^2 - 4\*b\*c\*d\*e + b^2\*e^2)\*sqrt(c^2\*d\*e - b\*c\*e^2)\*log((c\*e\*x^2 + c\*d - b\*e - 2\*sqrt(c^2\*d\*e - b\*c\*e^2)\*x)/(c\*e\*x^2 - c\*d + b\*e)) + 6\*(3\*c^3\*d^2\*e - 4\*b\*c^2\*d\*e^2 + b^2\*c\*e^3)\*x)/(c^4\*d\*e - b\*c^3\*e^2), 1/3\*((c^3\*d\*e^2 - b\*c^2\*e^3)\*x^3 - 3\*(4\*c^2\*d^2 - 4\*b\*c\*d\*e + b^2\*e^2)\*sqrt(-c^2\*d\*e + b\*c\*e^2)\*arctan(-sqrt(-c^2\*d\*e + b\*c\*e^2)\*x/(c\*d - b\*e)) + 3\*(3\*c^3\*d^2\*e - 4\*b\*c^2\*d\*e^2 + b^2\*c\*e^3)\*x)/(c^4\*d\*e - b\*c^3\*e^2)]

**giac [B]** time = 5.30, size = 8680, normalized size = 100.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^3/(c\*e^2\*x^4+b\*e^2\*x^2+b\*d\*e-c\*d^2), x, algorithm="giac")

[Out] -1/8\*(64\*b\*c^9\*d^5\*e^8 - 32\*sqrt(2)\*sqrt(4\*c^2\*d^2\*e^2 - 4\*b\*c\*d\*e^3 + b^2\*e^4)\*sqrt(b\*c\*e^4 + sqrt(4\*c^2\*d^2\*e^2 - 4\*b\*c\*d\*e^3 + b^2\*e^4)\*c\*e^2)\*b\*c^7\*d^5\*e^4 - 160\*b^2\*c^8\*d^4\*e^9 + 80\*sqrt(2)\*sqrt(4\*c^2\*d^2\*e^2 - 4\*b\*c\*d\*e^3 + b^2\*e^4)\*sqrt(b\*c\*e^4 + sqrt(4\*c^2\*d^2\*e^2 - 4\*b\*c\*d\*e^3 + b^2\*e^4)\*c\*e^2)\*b^2\*c^6\*d^4\*e^5 + 160\*b^3\*c^7\*d^3\*e^10 - 80\*sqrt(2)\*sqrt(4\*c^2\*d^2\*e^2 - 4\*b\*c\*d\*e^3 + b^2\*e^4)\*sqrt(b\*c\*e^4 + sqrt(4\*c^2\*d^2\*e^2 - 4\*b\*c\*d\*e^3 + b^2\*e^4)\*c\*e^2)\*b^3\*c^5\*d^3\*e^6 + 16\*sqrt(2)\*sqrt(4\*c^2\*d^2\*e^2 - 4\*b\*c\*d\*e^3 + b^2\*e^4)\*sqrt(b\*c\*e^4 + sqrt(4\*c^2\*d^2\*e^2 - 4\*b\*c\*d\*e^3 + b^2\*e^4)\*c\*e^2)\*b^2\*c^6\*d^3\*e^6 - 8\*sqrt(2)\*sqrt(4\*c^2\*d^2\*e^2 - 4\*b\*c\*d\*e^3 + b^2\*e^4)\*sqrt(b\*c\*e^4 + sqrt(4\*c^2\*d^2\*e^2 - 4\*b\*c\*d\*e^3 + b^2\*e^4)\*c\*e^2)\*b\*c^7\*

$$\begin{aligned}
& d^3e^6 - 80b^4c^6d^2e^{11} - 16(4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4) * \\
& b^7c^7d^3e^6 + 40\sqrt{2}\sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}\sqrt{ \\
& (b^2c^2e^4 + \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4})c^2e^2} * b^4c^4d^2e^7 - 24\sqrt{2}\sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}\sqrt{ \\
& (b^2c^2e^4 + \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4})c^2e^2} * b^3c^5d^2e^7 + 12\sqrt{2}\sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}\sqrt{ \\
& (b^2c^2e^4 + \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4})c^2e^2} * b^2c^6d^2e^7 + 20b^5c^5d^2e^{12} \\
& + 24(4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4) * b^2c^6d^2e^7 - 10\sqrt{2}\sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}\sqrt{ \\
& (b^2c^2e^4 + \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4})c^2e^2} * b^5c^3d^2e^8 + 12\sqrt{2}\sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}\sqrt{ \\
& (b^2c^2e^4 + \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4})c^2e^2} * b^4c^4d^2e^8 - 6\sqrt{2}\sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}\sqrt{ \\
& (b^2c^2e^4 + \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4})c^2e^2} * b^3c^5d^2e^8 - 2b^6c^4e^{13} - 12(4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4) * \\
& b^3c^5d^2e^8 + \sqrt{2}\sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}\sqrt{ \\
& (b^2c^2e^4 + \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4})c^2e^2} * b^6c^2e^9 - 2\sqrt{2}\sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}\sqrt{ \\
& (b^2c^2e^4 + \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4})c^2e^2} * b^5c^3e^9 + \sqrt{2}\sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}\sqrt{ \\
& (b^2c^2e^4 + \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4})c^2e^2} * b^4c^4e^9 + 2(4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4) * \\
& b^4c^4e^9 + (128c^8d^6e^7 - 64\sqrt{2}\sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}\sqrt{ \\
& (b^2c^2e^4 + \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4})c^2e^2} * c^6d^6e^3 - 384b^7c^7d^5e^8 + 192\sqrt{2}\sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}\sqrt{ \\
& (b^2c^2e^4 + \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4})c^2e^2} * b^5d^5e^4 + 480b^2c^6d^4e^9 - 240\sqrt{2}\sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}\sqrt{ \\
& (b^2c^2e^4 + \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4})c^2e^2} * b^2c^4d^4e^5 + 32\sqrt{2}\sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}\sqrt{ \\
& (b^2c^2e^4 + \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4})c^2e^2} * b^5d^4e^5 - 16\sqrt{2}\sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}\sqrt{ \\
& (b^2c^2e^4 + \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4})c^2e^2} * c^6d^4e^5 - 320b^3c^5d^3e^{10} - 32(4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4) * \\
& c^6d^4e^5 + 160\sqrt{2}\sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}\sqrt{ \\
& (b^2c^2e^4 + \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4})c^2e^2} * b^3c^3d^3e^6 - 64\sqrt{2}\sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}\sqrt{ \\
& (b^2c^2e^4 + \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4})c^2e^2} * b^2c^4d^3e^6 + 32\sqrt{2}\sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}\sqrt{ \\
& (b^2c^2e^4 + \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4})c^2e^2} * b^3c^3d^3e^6 + 120 * \\
& b^4c^4d^2e^{11} + 64(4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4) * b^5d^3e^6 - 60\sqrt{2}\sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}\sqrt{ \\
& (b^2c^2e^4 + \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4})c^2e^2} * b^4c^2d^2e^7 + 48\sqrt{2}\sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}\sqrt{ \\
& (b^2c^2e^4 + \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4})c^2e^2} * b^3c^3d^2e^7 - 24\sqrt{2}\sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}\sqrt{ \\
& (b^2c^2e^4 + \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4})c^2e^2} * b^2c^4d^2e^7 - 24b^5c^3d^2e^{12} - 48(4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4) * \\
& b^2c^4d^2e^7 + 12\sqrt{2}\sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}\sqrt{ \\
& (b^2c^2e^4 + \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4})c^2e^2} * b^5c^3d^2e^8 - 16\sqrt{2}\sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}\sqrt{ \\
& (b^2c^2e^4 + \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4})c^2e^2} * b^4c^2d^2e^8 + 8\sqrt{2}\sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}\sqrt{ \\
& (b^2c^2e^4 + \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4})c^2e^2} * b^3c^3d^2e^8 + 2b^6c^2e^{13} + 16(4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4) * \\
& b^3c^3d^2e^8 - \sqrt{2}\sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}\sqrt{ \\
& (b^2c^2e^4 + \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4})c^2e^2} * b^6e^9 + 2\sqrt{2}\sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}\sqrt{ \\
& (b^2c^2e^4 + \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4})c^2e^2} * b^4c^2e^9 - 2(4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4) * \\
& b^4c^2e^9 - 2(128c^9d^7e^6 - 64\sqrt{2}\sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}\sqrt{ \\
& (b^2c^2e^4 + \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4})c^2e^2} * c^8d^7e^4 - 384b^8c^8d^6e^7 + 192 *
\end{aligned}$$

$$\begin{aligned}
& \text{qrt}(2) * \text{sqrt}(b * c * e^4 + \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * c * e^2) * b * \\
& c^7 * d^6 * e^5 + 480 * b^2 * c^7 * d^5 * e^8 - 240 * \text{sqrt}(2) * \text{sqrt}(b * c * e^4 + \text{sqrt}(4 * c^2 * d^2 * e^2 - \\
& 4 * b * c * d * e^3 + b^2 * e^4) * c * e^2) * b^2 * c^6 * d^5 * e^6 + 32 * \text{sqrt}(2) * \text{sqrt}(b * c * e^4 + \\
& \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * c * e^2) * b * c^7 * d^5 * e^6 - \\
& 16 * \text{sqrt}(2) * \text{sqrt}(b * c * e^4 + \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * c * e^2) \\
& ) * c^8 * d^5 * e^6 - 320 * b^3 * c^6 * d^4 * e^9 + 160 * \text{sqrt}(2) * \text{sqrt}(b * c * e^4 + \text{sqrt}(4 * c^2 * \\
& d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * c * e^2) * b^3 * c^5 * d^4 * e^7 - 64 * \text{sqrt}(2) * \text{sqrt}( \\
& b * c * e^4 + \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * c * e^2) * b^2 * c^6 * d^4 * e^7 \\
& + 32 * \text{sqrt}(2) * \text{sqrt}(b * c * e^4 + \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * c \\
& * e^2) * b * c^7 * d^4 * e^7 - 32 * (4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * c^7 * d^5 * e^4 \\
& + 120 * b^4 * c^5 * d^3 * e^{10} - 60 * \text{sqrt}(2) * \text{sqrt}(b * c * e^4 + \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 \\
& * b * c * d * e^3 + b^2 * e^4) * c * e^2) * b^4 * c^4 * d^3 * e^8 + 48 * \text{sqrt}(2) * \text{sqrt}(b * c * e^4 + \text{sq} \\
& \text{rt}(4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * c * e^2) * b^3 * c^5 * d^3 * e^8 - 24 * \text{sqrt}( \\
& 2) * \text{sqrt}(b * c * e^4 + \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * c * e^2) * b^2 * c^6 \\
& * d^3 * e^8 + 64 * (4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * b * c^6 * d^4 * e^5 - 24 * b \\
& ^5 * c^4 * d^2 * e^{11} + 12 * \text{sqrt}(2) * \text{sqrt}(b * c * e^4 + \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 \\
& + b^2 * e^4) * c * e^2) * b^5 * c^3 * d^2 * e^9 - 16 * \text{sqrt}(2) * \text{sqrt}(b * c * e^4 + \text{sqrt}(4 * c^2 * \\
& d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * c * e^2) * b^4 * c^4 * d^2 * e^9 + 8 * \text{sqrt}(2) * \text{sqrt}(b * \\
& c * e^4 + \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * c * e^2) * b^3 * c^5 * d^2 * e^9 \\
& - 48 * (4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * b^2 * c^5 * d^3 * e^6 + 2 * b^6 * c^3 * d * \\
& e^{12} - \text{sqrt}(2) * \text{sqrt}(b * c * e^4 + \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * c \\
& * e^2) * b^6 * c^2 * d * e^{10} + 2 * \text{sqrt}(2) * \text{sqrt}(b * c * e^4 + \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * \\
& d * e^3 + b^2 * e^4) * c * e^2) * b^5 * c^3 * d * e^{10} - \text{sqrt}(2) * \text{sqrt}(b * c * e^4 + \text{sqrt}(4 * c^2 * \\
& d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * c * e^2) * b^4 * c^4 * d * e^{10} + 16 * (4 * c^2 * d^2 * e^2 \\
& - 4 * b * c * d * e^3 + b^2 * e^4) * b^3 * c^4 * d^2 * e^7 - 2 * (4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + \\
& b^2 * e^4) * b^4 * c^3 * d * e^8) * \text{abs}(c) * \text{arctan}(2 * \text{sqrt}(1/2) * x * e^4 / \text{sqrt}((b * c^3 * e^8 + \\
& \text{sqrt}(b^2 * c^6 * e^{16} + 4 * (c^4 * d^2 * e^6 - b * c^3 * d * e^7) * c^4 * e^8)) / c^4)) / ((16 * c^9 * \\
& d^6 * e^6 - 48 * b * c^8 * d^5 * e^7 + 56 * b^2 * c^7 * d^4 * e^8 - 8 * b * c^8 * d^4 * e^8 + 4 * c^9 * \\
& d^4 * e^8 - 32 * b^3 * c^6 * d^3 * e^9 + 16 * b^2 * c^7 * d^3 * e^9 - 8 * b * c^8 * d^3 * e^9 + 9 * b^4 * \\
& c^5 * d^2 * e^{10} - 10 * b^3 * c^6 * d^2 * e^{10} + 5 * b^2 * c^7 * d^2 * e^{10} - b^5 * c^4 * d * e^{11} + \\
& 2 * b^4 * c^5 * d * e^{11} - b^3 * c^6 * d * e^{11}) * c^2) + 1/8 * (64 * b * c^9 * d^5 * e^8 - 32 * \text{sqrt}( \\
& 2) * \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * \text{sqrt}(b * c * e^4 - \text{sqrt}(4 * c^2 * d^2 * \\
& e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * c * e^2) * b * c^7 * d^5 * e^4 - 160 * b^2 * c^8 * d^4 * e^9 + \\
& 80 * \text{sqrt}(2) * \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * \text{sqrt}(b * c * e^4 - \text{sqrt}( \\
& 4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * c * e^2) * b^2 * c^6 * d^4 * e^5 + 160 * b^3 * c^7 * \\
& d^3 * e^{10} - 80 * \text{sqrt}(2) * \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * \text{sqrt}(b * c * e^4 - \\
& \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * c * e^2) * b^3 * c^5 * d^3 * e^6 + 16 * \text{sqrt}(2) * \\
& \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * \text{sqrt}(b * c * e^4 - \text{sqrt}(4 * c^2 * d^2 * e^2 - \\
& 4 * b * c * d * e^3 + b^2 * e^4) * c * e^2) * b^2 * c^6 * d^3 * e^6 - 8 * \text{sqrt}(2) * \text{sqrt}(4 * c^2 * d^2 * e^2 - \\
& 4 * b * c * d * e^3 + b^2 * e^4) * \text{sqrt}(b * c * e^4 - \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + \\
& b^2 * e^4) * c * e^2) * b * c^7 * d^3 * e^6 - 80 * b^4 * c^6 * d^2 * e^{11} - 16 * (4 * c^2 * d^2 * e^2 - 4 * b * c * \\
& d * e^3 + b^2 * e^4) * b * c^7 * d^3 * e^6 + 40 * \text{sqrt}(2) * \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + \\
& b^2 * e^4) * \text{sqrt}(b * c * e^4 - \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * c * e^2) * b^4 * \\
& c^4 * d^2 * e^7 - 24 * \text{sqrt}(2) * \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * \text{sqrt}(b * c * e^4 - \\
& \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * c * e^2) * b^3 * c^5 * d^2 * e^7 + 12 * \text{sqrt}(2) * \\
& \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * \text{sqrt}(b * c * e^4 - \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * \\
& d * e^3 + b^2 * e^4) * c * e^2) * b^2 * c^6 * d^2 * e^7 + 20 * b^5 * c^5 * d * e^{12} + 24 * (4 * c^2 * d^2 * e^2 - \\
& 4 * b * c * d * e^3 + b^2 * e^4) * b^2 * c^6 * d^2 * e^7 - 10 * \text{sqrt}(2) * \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * \\
& d * e^3 + b^2 * e^4) * \text{sqrt}(b * c * e^4 - \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * c * e^2) \\
& ) * b^5 * c^3 * d * e^8 + 12 * \text{sqrt}(2) * \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * \text{sqrt}( \\
& b * c * e^4 - \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * c * e^2) * b^4 * c^4 * d * e^8 - \\
& 6 * \text{sqrt}(2) * \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * \text{sqrt}(b * c * e^4 - \text{sqrt}(4 * c^2 * \\
& d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * c * e^2) * b^3 * c^5 * d * e^8 - 2 * b^6 * c^4 * e^{13} - 12 * (4 * c^2 * \\
& d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * b^3 * c^5 * d * e^8 + \text{sqrt}(2) * \\
& \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + b^2 * e^4) * \text{sqrt}(b * c * e^4 - \text{sqrt}(4 * c^2 * d^2 * e^2 - \\
& 4 * b * c * d * e^3 + b^2 * e^4) * c * e^2) * b^6 * c^2 * e^9 - 2 * \text{sqrt}(2) * \text{sqrt}(4 * c^2 * d^2 * e^2 - \\
& 4 * b * c * d * e^3 + b^2 * e^4) * \text{sqrt}(b * c * e^4 - \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 + \\
& b^2 * e^4) * c * e^2) * b^5 * c^3 * e^9 + \text{sqrt}(2) * \text{sqrt}(4 * c^2 * d^2 * e^2 - 4 * b * c * d * e^3 +
\end{aligned}$$

$$\begin{aligned}
& b^2e^4) \sqrt{b^2e^4 - \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}} c^2e^2) \\
& * b^4c^4e^9 + 2(4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4) b^4c^4e^9 + (128 \\
& * c^8d^6e^7 - 64\sqrt{2}) \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4} \sqrt{ \\
& b^2e^4 - \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}} c^6d^6e^3 - \\
& 384b^2c^7d^5e^8 + 192\sqrt{2}) \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4} \\
& * \sqrt{b^2e^4 - \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}} c^2e^2) * b^2c^5d^5 \\
& * e^4 + 480b^2c^6d^4e^9 - 240\sqrt{2}) \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4} \\
& * \sqrt{b^2e^4 - \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}} c^2e^2) * b^2c^4d^4e^5 \\
& + 32\sqrt{2}) \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4} * \\
& \sqrt{b^2e^4 - \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}} c^2e^2) * b^2c^5d^4 \\
& * e^5 - 16\sqrt{2}) \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4} \sqrt{b^2e^4 - \\
& - \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}} c^2e^2) * c^6d^4e^5 - 320b^3c^5 \\
& * d^3e^{10} - 32(4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4) * c^6d^4e^5 + 160 \\
& * \sqrt{2}) \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4} \sqrt{b^2e^4 - \sqrt{4c^2d^2e^2 - \\
& 4b^2c^2d^2e^2 + b^2e^4}} c^2e^2) * b^3c^3d^3e^6 - 64\sqrt{2}) \sqrt{ \\
& 4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4} \sqrt{b^2e^4 - \sqrt{4c^2d^2e^2 - \\
& - 4b^2c^2d^2e^2 + b^2e^4}} c^2e^2) * b^2c^4d^3e^6 + 32\sqrt{2}) \sqrt{4c^2d^2e^2 - \\
& 4b^2c^2d^2e^2 + b^2e^4} \sqrt{b^2e^4 - \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}} \\
& * c^2e^2) * b^2c^5d^3e^6 + 120b^4c^4d^2e^{11} + 64(4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4) \\
& * b^2c^5d^3e^6 - 60\sqrt{2}) \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4} \sqrt{b^2e^4 - \\
& - \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}} c^2e^2) * b^4c^2d^2e^7 + 48\sqrt{2}) \sqrt{ \\
& 4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4} \sqrt{b^2e^4 - \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}} \\
& * c^2e^2) * b^3c^3d^2e^7 - 24\sqrt{2}) \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4} \\
& * \sqrt{b^2e^4 - \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}} c^2e^2) * b^2c^4d^2e^7 \\
& - 24b^5c^3d^2e^{12} - 48(4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4) * \\
& b^2c^4d^2e^7 + 12\sqrt{2}) \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4} \sqrt{ \\
& b^2e^4 - \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}} c^2e^2) * b^5c^3d^2e^8 \\
& - 16\sqrt{2}) \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4} \sqrt{b^2e^4 - \sqrt{4c^2d^2e^2 - \\
& 4b^2c^2d^2e^2 + b^2e^4}} c^2e^2) * b^4c^2d^2e^8 + 8\sqrt{2}) \sqrt{4c^2d^2e^2 - \\
& 4b^2c^2d^2e^2 + b^2e^4} \sqrt{b^2e^4 - \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}} \\
& * c^2e^2) * b^3c^3d^2e^8 + 2b^6c^2e^{13} + 16(4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4) \\
& * b^3c^3d^2e^8 - \sqrt{2}) \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4} \sqrt{b^2e^4 - \sqrt{4c^2d^2e^2 - \\
& 4b^2c^2d^2e^2 + b^2e^4}} c^2e^2) * b^6e^9 + 2\sqrt{2}) \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4} \\
& * \sqrt{b^2e^4 - \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}} c^2e^2) * \\
& b^5c^2e^9 - \sqrt{2}) \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4} \sqrt{b^2e^4 - \sqrt{4c^2d^2e^2 - \\
& 4b^2c^2d^2e^2 + b^2e^4}} c^2e^2) * b^4c^2e^9 - 2(4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4) \\
& * b^4c^2e^9) * c^2 - 2(128c^9d^7e^6 + \\
& 64\sqrt{2}) \sqrt{b^2e^4 - \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}} c^2e^2) \\
& * c^8d^7e^4 - 384b^2c^8d^6e^7 - 192\sqrt{2}) \sqrt{b^2e^4 - \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}} \\
& * c^2e^2) * b^2c^7d^6e^5 + 480b^2c^7d^5e^8 \\
& + 240\sqrt{2}) \sqrt{b^2e^4 - \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}} c^2e^2) * \\
& b^2c^6d^5e^6 - 32\sqrt{2}) \sqrt{b^2e^4 - \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}} \\
& * c^2e^2) * b^2c^7d^5e^6 + 16\sqrt{2}) \sqrt{b^2e^4 - \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}} \\
& * c^2e^2) * c^8d^5e^6 - 320b^3c^6d^4e^9 \\
& - 160\sqrt{2}) \sqrt{b^2e^4 - \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}} c^2e^2) * \\
& b^3c^5d^4e^7 + 64\sqrt{2}) \sqrt{b^2e^4 - \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}} \\
& * c^2e^2) * b^2c^6d^4e^7 - 32\sqrt{2}) \sqrt{b^2e^4 - \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}} \\
& * c^2e^2) * b^2c^7d^4e^7 - 32(4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4) * c^7d^5e^4 \\
& + 120b^4c^5d^3e^{10} + 60\sqrt{2}) \sqrt{b^2e^4 - \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}} \\
& * c^2e^2) * b^4c^4d^3e^8 - 48\sqrt{2}) \sqrt{b^2e^4 - \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}} \\
& * c^2e^2) * b^3c^5d^3e^8 + 24\sqrt{2}) \sqrt{b^2e^4 - \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}} \\
& * c^2e^2) * b^2c^6d^3e^8 + 64(4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4) * b^2c^6d^4e^5 \\
& - 24b^5c^4d^2e^{11} - 12\sqrt{2}) \sqrt{b^2e^4 - \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}} \\
& * c^2e^2) * b^5c^3d^2e^9 + 16\sqrt{2}) \sqrt{b^2e^4 - \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}} \\
& * c^2e^2) * b^4c^4d^2e^9 - 8\sqrt{2}) \sqrt{b^2e^4 - \sqrt{4c^2d^2e^2 - 4b^2c^2d^2e^2 + b^2e^4}}
\end{aligned}$$



$$c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^5*d^2*e^9 - 48*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^2*c^5*d^3*e^6 + 2*b^6*c^3*d*e^12 + \sqrt{2}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b^6*c^2*d*e^10 - 2*\sqrt{2}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b^5*c^3*d*e^10 + \sqrt{2}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b^4*c^4*d*e^10 + 16*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^3*c^4*d^2*e^7 - 2*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^4*c^3*d*e^8)*\text{abs}(c))*\arctan(2*\sqrt{1/2}*x*e^4/\sqrt{((b*c^3*e^8 - \sqrt{b^2*c^6*e^16 + 4*(c^4*d^2*e^6 - b*c^3*d*e^7)*c^4*e^8))/c^4}))/((16*c^9*d^6*e^6 - 48*b*c^8*d^5*e^7 + 56*b^2*c^7*d^4*e^8 - 8*b*c^8*d^4*e^8 + 4*c^9*d^4*e^8 - 32*b^3*c^6*d^3*e^9 + 16*b^2*c^7*d^3*e^9 - 8*b*c^8*d^3*e^9 + 9*b^4*c^5*d^2*e^10 - 10*b^3*c^6*d^2*e^10 + 5*b^2*c^7*d^2*e^10 - b^5*c^4*d*e^11 + 2*b^4*c^5*d*e^11 - b^3*c^6*d*e^11)*c^2) + 1/3*(c^2*x^3*e^7 + 9*c^2*d*x*e^6 - 3*b*c*x*e^7)*e^(-6)/c^3$$

**maple** [A] time = 0.00, size = 142, normalized size = 1.65

$$\frac{b^2 e^2 \arctan\left(\frac{cex}{\sqrt{(be-cd)ce}}\right)}{\sqrt{(be-cd)ce} c^2} - \frac{4bde \arctan\left(\frac{cex}{\sqrt{(be-cd)ce}}\right)}{\sqrt{(be-cd)ce} c} + \frac{ex^3}{3c} + \frac{4d^2 \arctan\left(\frac{cex}{\sqrt{(be-cd)ce}}\right)}{\sqrt{(be-cd)ce}} - \frac{bex}{c^2} + \frac{3dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^3/(c\*e^2\*x^4+b\*e^2\*x^2+b\*d\*e-c\*d^2), x)

[Out] 1/3\*e\*x^3/c-1/c^2\*b\*e\*x+3/c\*d\*x+1/c^2/((b\*e-c\*d)\*c\*e)^(1/2)\*arctan(1/((b\*e-c\*d)\*c\*e)^(1/2)\*c\*e\*x)\*b^2\*e^2-4/c/((b\*e-c\*d)\*c\*e)^(1/2)\*arctan(1/((b\*e-c\*d)\*c\*e)^(1/2)\*c\*e\*x)\*b\*d\*e+4/((b\*e-c\*d)\*c\*e)^(1/2)\*arctan(1/((b\*e-c\*d)\*c\*e)^(1/2)\*c\*e\*x)\*d^2

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^3/(c\*e^2\*x^4+b\*e^2\*x^2+b\*d\*e-c\*d^2), x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(b\*e-c\*d>0)', see 'assume?' for more details)Is b\*e-c\*d positive or negative?

**mupad** [B] time = 4.52, size = 113, normalized size = 1.31

$$x \left( \frac{2d}{c} - \frac{be-cd}{c^2} \right) + \frac{ex^3}{3c} + \frac{\text{atan}\left(\frac{\sqrt{c} ex (be-2cd)^2}{\sqrt{be^2-cde} (b^2 e^2-4bcde+4c^2 d^2)}\right) (be-2cd)^2}{c^{5/2} \sqrt{be^2-cde}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)^3/(b\*e^2\*x^2 - c\*d^2 + c\*e^2\*x^4 + b\*d\*e), x)

[Out] x\*((2\*d)/c - (b\*e - c\*d)/c^2) + (e\*x^3)/(3\*c) + (atan((c^(1/2))\*e\*x\*(b\*e - 2\*c\*d)^2)/((b\*e^2 - c\*d\*e)^(1/2)\*(b^2\*e^2 + 4\*c^2\*d^2 - 4\*b\*c\*d\*e)))\*(b\*e - 2\*c\*d)^2/(c^(5/2)\*(b\*e^2 - c\*d\*e)^(1/2))

**sympy** [B] time = 0.72, size = 275, normalized size = 3.20

$$x \left( -\frac{be}{c^2} + \frac{3d}{c} \right) - \frac{\sqrt{\frac{1}{c^5(b-cd)}} (be-2cd)^2 \log\left(x + \frac{-bc^2e\sqrt{\frac{1}{c^5(b-cd)}}(be-2cd)^2+c^3d\sqrt{\frac{1}{c^5(b-cd)}}(be-2cd)^2}{b^2e^2-4bcde+4c^2d^2}\right)}{2} + \frac{\sqrt{\frac{1}{c^5(b-cd)}} (be-2cd)^2 \log\left(x + \frac{bc^2e\sqrt{\frac{1}{c^5(b-cd)}}(be-2cd)^2-c^3d\sqrt{\frac{1}{c^5(b-cd)}}(be-2cd)^2}{b^2e^2-4bcde+4c^2d^2}\right)}{2} + \frac{ex^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**3/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)`

[Out]  $x*(-b*e/c**2 + 3*d/c) - \sqrt{-1/(c**5*e*(b*e - c*d))}*(b*e - 2*c*d)**2*\log(x + (-b*c**2*e*\sqrt{-1/(c**5*e*(b*e - c*d))}*(b*e - 2*c*d)**2 + c**3*d*\sqrt{-1/(c**5*e*(b*e - c*d))}*(b*e - 2*c*d)**2)/(b**2*e**2 - 4*b*c*d*e + 4*c**2*d**2))/2 + \sqrt{-1/(c**5*e*(b*e - c*d))}*(b*e - 2*c*d)**2*\log(x + (b*c**2*e*\sqrt{-1/(c**5*e*(b*e - c*d))}*(b*e - 2*c*d)**2 - c**3*d*\sqrt{-1/(c**5*e*(b*e - c*d))}*(b*e - 2*c*d)**2)/(b**2*e**2 - 4*b*c*d*e + 4*c**2*d**2))/2 + e*x**3/(3*c)$

$$3.158 \quad \int \frac{(d+ex^2)^2}{-cd^2+bde+be^2x^2+ce^2x^4} dx$$

**Optimal.** Leaf size=64

$$\frac{x}{c} - \frac{(2cd - be) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{cd-be}}\right)}{c^{3/2}\sqrt{e}\sqrt{cd-be}}$$

**Rubi [A]** time = 0.08, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {1149, 388, 208}

$$\frac{x}{c} - \frac{(2cd - be) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{cd-be}}\right)}{c^{3/2}\sqrt{e}\sqrt{cd-be}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^2/(-(c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4),x]

[Out] x/c - ((2\*c\*d - b\*e)\*ArcTanh[(Sqrt[c]\*Sqrt[e]\*x)/Sqrt[c\*d - b\*e]]/(c^(3/2)\*Sqrt[e]\*Sqrt[c\*d - b\*e])

**Rule 208**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

**Rule 388**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

**Rule 1149**

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[(d + e\*x^2)^(p + q)\*(a/d + (c\*x^2)/e)^p, x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IntegerQ[p]

**Rubi steps**

$$\begin{aligned} \int \frac{(d+ex^2)^2}{-cd^2+bde+be^2x^2+ce^2x^4} dx &= \int \frac{d+ex^2}{\frac{-cd^2+bde}{d}+ce^2x^2} dx \\ &= \frac{x}{c} - \frac{\left(-cde + \frac{e(-cd^2+bde)}{d}\right) \int \frac{1}{\frac{-cd^2+bde}{d}+ce^2x^2} dx}{ce} \\ &= \frac{x}{c} - \frac{(2cd - be) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{cd-be}}\right)}{c^{3/2}\sqrt{e}\sqrt{cd-be}} \end{aligned}$$

**Mathematica [A]** time = 0.06, size = 63, normalized size = 0.98

$$\frac{x}{c} - \frac{(be - 2cd) \tan^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{be-cd}}\right)}{c^{3/2}\sqrt{e}\sqrt{be-cd}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^2/(-(c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4), x]

[Out] x/c - ((-2\*c\*d + b\*e)\*ArcTan[(Sqrt[c]\*Sqrt[e]\*x)/Sqrt[-(c\*d) + b\*e]])/(c^(3/2)\*Sqrt[e]\*Sqrt[-(c\*d) + b\*e])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^2)^2}{-cd^2 + bde + be^2x^2 + ce^2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)^2/(-(c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4), x]

[Out] IntegrateAlgebraic[(d + e\*x^2)^2/(-(c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4), x]

**fricas [A]** time = 1.11, size = 210, normalized size = 3.28

$$\left[ \frac{\sqrt{c^2de - bce^2} (2cd - be) \log\left(\frac{cex^2 + cd - be + 2\sqrt{c^2de - bce^2}x}{cex^2 - cd + be}\right) - 2(c^2de - bce^2)x}{2(c^3de - bc^2e^2)}, \frac{\sqrt{-c^2de + bce^2} (2cd - be) \arctan\left(\frac{-\sqrt{-c^2de + bce^2}x}{cd - be}\right) - (c^2de - bce^2)x}{c^3de - bc^2e^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2/(c\*e^2\*x^4+b\*e^2\*x^2+b\*d\*e-c\*d^2),x, algorithm="fricas")

[Out] [-1/2\*(sqrt(c^2\*d\*e - b\*c\*e^2)\*(2\*c\*d - b\*e)\*log((c\*e\*x^2 + c\*d - b\*e + 2\*sqrt(c^2\*d\*e - b\*c\*e^2)\*x)/(c\*e\*x^2 - c\*d + b\*e)) - 2\*(c^2\*d\*e - b\*c\*e^2)\*x/(c^3\*d\*e - b\*c^2\*e^2), -(sqrt(-c^2\*d\*e + b\*c\*e^2)\*(2\*c\*d - b\*e)\*arctan(-sqrt(-c^2\*d\*e + b\*c\*e^2)\*x/(c\*d - b\*e)) - (c^2\*d\*e - b\*c\*e^2)\*x/(c^3\*d\*e - b\*c^2\*e^2)]

**giac [B]** time = 4.82, size = 7051, normalized size = 110.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2/(c\*e^2\*x^4+b\*e^2\*x^2+b\*d\*e-c\*d^2),x, algorithm="giac")

[Out] x/c - 1/8\*(32\*b\*c^8\*d^4\*e^8 - 16\*sqrt(2)\*sqrt(4\*c^2\*d^2\*e^2 - 4\*b\*c\*d\*e^3 + b^2\*e^4)\*sqrt(b\*c\*e^4 + sqrt(4\*c^2\*d^2\*e^2 - 4\*b\*c\*d\*e^3 + b^2\*e^4)\*c\*e^2)\*b\*c^6\*d^4\*e^4 - 64\*b^2\*c^7\*d^3\*e^9 + 32\*sqrt(2)\*sqrt(4\*c^2\*d^2\*e^2 - 4\*b\*c\*d\*e^3 + b^2\*e^4)\*sqrt(b\*c\*e^4 + sqrt(4\*c^2\*d^2\*e^2 - 4\*b\*c\*d\*e^3 + b^2\*e^4)\*c\*e^2)\*b^2\*c^5\*d^3\*e^5 + 48\*b^3\*c^6\*d^2\*e^10 - 24\*sqrt(2)\*sqrt(4\*c^2\*d^2\*e^2 - 4\*b\*c\*d\*e^3 + b^2\*e^4)\*sqrt(b\*c\*e^4 + sqrt(4\*c^2\*d^2\*e^2 - 4\*b\*c\*d\*e^3 + b^2\*e^4)\*c\*e^2)\*b^3\*c^4\*d^2\*e^6 + 8\*sqrt(2)\*sqrt(4\*c^2\*d^2\*e^2 - 4\*b\*c\*d\*e^3 + b^2\*e^4)\*sqrt(b\*c\*e^4 + sqrt(4\*c^2\*d^2\*e^2 - 4\*b\*c\*d\*e^3 + b^2\*e^4)\*c\*e^2)\*b^2\*c^5\*d^2\*e^6 - 4\*sqrt(2)\*sqrt(4\*c^2\*d^2\*e^2 - 4\*b\*c\*d\*e^3 + b^2\*e^4)\*sqrt(b\*c\*e^4 + sqrt(4\*c^2\*d^2\*e^2 - 4\*b\*c\*d\*e^3 + b^2\*e^4)\*c\*e^2)\*b\*c^6\*d^2\*e^6 - 16\*b^4\*c^5\*d\*e^11 - 8\*(4\*c^2\*d^2\*e^2 - 4\*b\*c\*d\*e^3 + b^2\*e^4)\*b

$$\begin{aligned}
& *c^6*d^2*e^6 + 8*\sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b^4*c^3*d*e^7 - \\
& 8*\sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b^3*c^4*d*e^7 + 4*\sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b^2*c^5*d*e^7 + 2*b^5*c^4*e^{12} + 8*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^2*c^5*d*e^7 - \sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b^5*c^2*e^8 + 2*\sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b^4*c^3*e^8 - \sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b^3*c^4*e^8 - 2*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^3*c^4*e^8 + (64*c^7*d^5*e^7 - 32*\sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*c^5*d^5*e^3 - 160*b*c^6*d^4*e^8 + 80*\sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b*c^4*d^4*e^4 + 160*b^2*c^5*d^3*e^9 - 80*\sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b^2*c^3*d^3*e^5 + 16*\sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b*c^4*d^3*e^5 - 8*\sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*c^5*d^3*e^5 - 80*b^3*c^4*d^2*e^{10} - 16*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c^5*d^3*e^5 + 40*\sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b^3*c^2*d^2*e^6 - 24*\sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b^2*c^3*d^2*e^6 + 12*\sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b*c^4*d^2*e^6 + 20*b^4*c^3*d*e^{11} + 24*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b*c^4*d^2*e^6 - 10*\sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b^4*c*d*e^7 + 12*\sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b^3*c^2*d*e^7 - 6*\sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b^2*c^3*d*e^7 + \sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b^5*e^8 - 2*\sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b^4*c*e^8 + \sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b^3*c^2*e^8 + 2*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^3*c^2*e^8)*c^2 - 2*(64*c^8*d^6*e^6 - 32*\sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*c^7*d^6*e^4 - 160*b*c^7*d^5*e^7 + 80*\sqrt{2}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b*c^6*d^5*e^5 + 160*b^2*c^6*d^4*e^8 - 80*\sqrt{2}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b^2*c^5*d^4*e^6 + 16*\sqrt{2}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b*c^6*d^4*e^6 - 8*\sqrt{2}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*c^7*d^4*e^6 - 80*b^3*c^5*d^3*e^9 + 40*\sqrt{2}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b^3*c^4*d^3*e^7 - 24*\sqrt{2}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b^2*c^5*d^3*e^7 + 12*\sqrt{2}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b*c^6*d^3*e^7 - 16*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c^6*d^4*e^4 + 20*b^4*c^4*d^2*e^{10} - 10*\sqrt{2}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b^4*c^3*d^2*e^8 + 12*\sqrt{2}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b^3*c^4*d^2*e^8 - 6*\sqrt{2}*\sqrt{(b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*c*e^2)*b^2*c^5*d^2*e^8 + 24*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c^5*d^2*e^8 + 24*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c^5*d^2*e^8
\end{aligned}$$



$$4*b*c*d*e^3 + b^2*e^4)*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*c*e^2)*b^5*e^8 - 2*\sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*c*e^2)*b^4*c*e^8 + \sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4})*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*c*e^2)*b^3*c^2*e^8 + 2*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^3*c^2*e^8)*c^2 - 2*(64*c^8*d^6*e^6 + 32*\sqrt{2}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*c*e^2)*c^7*d^6*e^4 - 160*b*c^7*d^5*e^7 - 80*\sqrt{2}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*c*e^2)*b*c^6*d^5*e^5 + 160*b^2*c^6*d^4*e^8 + 80*\sqrt{2}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*c*e^2)*b^2*c^5*d^4*e^6 - 16*\sqrt{2}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*c*e^2)*b*c^6*d^4*e^6 + 8*\sqrt{2}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*c*e^2)*c^7*d^4*e^6 - 80*b^3*c^5*d^3*e^9 - 40*\sqrt{2}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*c*e^2)*b^3*c^4*d^3*e^7 + 24*\sqrt{2}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*c*e^2)*b^2*c^5*d^3*e^7 - 12*\sqrt{2}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*c*e^2)*b*c^6*d^3*e^7 - 16*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c^6*d^4*e^4 + 20*b^4*c^4*d^2*e^10 + 10*\sqrt{2}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*c*e^2)*b^4*c^3*d^2*e^8 - 12*\sqrt{2}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*c*e^2)*b^3*c^4*d^2*e^8 + 6*\sqrt{2}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*c*e^2)*b^2*c^5*d^2*e^8 + 24*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b*c^5*d^3*e^5 - 2*b^5*c^3*d*e^11 - \sqrt{2}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*c*e^2)*b^5*c^2*d*e^9 + 2*\sqrt{2}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*c*e^2)*b^4*c^3*d*e^9 - \sqrt{2}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*c*e^2)*b^3*c^4*d*e^9 - 12*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^2*c^4*d^2*e^6 + 2*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^3*c^3*d*e^7)*\text{abs}(c))*\arctan(2*\sqrt{1/2}*x*e^2/\sqrt{(b*c*e^4 - \sqrt{b^2*c^2*e^8 + 4*(c^2*d^2*e^2 - b*c*d*e^3)*c^2*e^4}))/c^2))/((16*c^8*d^6*e^6 - 48*b*c^7*d^5*e^7 + 56*b^2*c^6*d^4*e^8 - 8*b*c^7*d^4*e^8 + 4*c^8*d^4*e^8 - 32*b^3*c^5*d^3*e^9 + 16*b^2*c^6*d^3*e^9 - 8*b*c^7*d^3*e^9 + 9*b^4*c^4*d^2*e^10 - 10*b^3*c^5*d^2*e^10 + 5*b^2*c^6*d^2*e^10 - b^5*c^3*d*e^11 + 2*b^4*c^4*d*e^11 - b^3*c^5*d*e^11)*c^2)$$

**maple [A]** time = 0.00, size = 79, normalized size = 1.23

$$-\frac{be \arctan\left(\frac{cex}{\sqrt{(be-cd)ce}}\right)}{\sqrt{(be-cd)ce}c} + \frac{2d \arctan\left(\frac{cex}{\sqrt{(be-cd)ce}}\right)}{\sqrt{(be-cd)ce}} + \frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^2/(c\*e^2\*x^4+b\*e^2\*x^2+b\*d\*e-c\*d^2), x)

[Out] 1/c\*x-1/c/((b\*e-c\*d)\*c\*e)^(1/2)\*arctan(1/((b\*e-c\*d)\*c\*e)^(1/2)\*c\*e\*x)\*b\*e+2/((b\*e-c\*d)\*c\*e)^(1/2)\*arctan(1/((b\*e-c\*d)\*c\*e)^(1/2)\*c\*e\*x)\*d

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2/(c\*e^2\*x^4+b\*e^2\*x^2+b\*d\*e-c\*d^2), x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(b\*e-c\*d>0)', see 'assume?' for more details)Is b\*e-c\*d positive or negative?

**mupad [B]** time = 0.07, size = 52, normalized size = 0.81

$$\frac{x}{c} - \frac{\operatorname{atan}\left(\frac{\sqrt{c} e x}{\sqrt{b e^2 - c d e}}\right) (b e - 2 c d)}{c^{3/2} \sqrt{b e^2 - c d e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x^2)^2/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e), x)`

[Out] `x/c - (atan((c^(1/2)*e*x)/(b*e^2 - c*d*e)^(1/2))*(b*e - 2*c*d))/(c^(3/2)*(b*e^2 - c*d*e)^(1/2))`

**sympy [B]** time = 0.49, size = 212, normalized size = 3.31

$$\frac{\sqrt{-\frac{1}{c^3 e (b e - c d)}} (b e - 2 c d) \log\left(x + \frac{-b c e \sqrt{-\frac{1}{c^3 e (b e - c d)}} (b e - 2 c d) + c^2 d \sqrt{-\frac{1}{c^3 e (b e - c d)}} (b e - 2 c d)}{b e - 2 c d}\right)}{2} - \frac{\sqrt{-\frac{1}{c^3 e (b e - c d)}} (b e - 2 c d) \log\left(x + \frac{b c e \sqrt{-\frac{1}{c^3 e (b e - c d)}} (b e - 2 c d) - c^2 d \sqrt{-\frac{1}{c^3 e (b e - c d)}} (b e - 2 c d)}{b e - 2 c d}\right)}{2} + \frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**2/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2), x)`

[Out] `sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d)*log(x + (-b*c*e*sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d)))/(b*e - 2*c*d) + c**2*d*sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d)/(b*e - 2*c*d)/2 - sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d)*log(x + (b*c*e*sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d) - c**2*d*sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d)))/(b*e - 2*c*d)/2 + x/c`



$$3.159 \quad \int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4} dx$$

Optimal. Leaf size=49

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{c}\sqrt{e}\sqrt{cd-be}}$$

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$ , Rules used = {1149, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{c}\sqrt{e}\sqrt{cd-be}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)/(-(c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4), x]

[Out] -(ArcTanh[(Sqrt[c]\*Sqrt[e]\*x)/Sqrt[c\*d - b\*e]]/(Sqrt[c]\*Sqrt[e]\*Sqrt[c\*d - b\*e]))

Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1149

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[(d + e\*x^2)^(p + q)\*(a/d + (c\*x^2)/e)^p, x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4} dx &= \int \frac{1}{\frac{-cd^2+bde}{d} + cex^2} dx \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{c}\sqrt{e}\sqrt{cd-be}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 0.98

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{be-cd}}\right)}{\sqrt{c}\sqrt{e}\sqrt{be-cd}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)/(-(c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4), x]

[Out] ArcTan[(Sqrt[c]\*Sqrt[e]\*x)/Sqrt[-(c\*d) + b\*e]]/(Sqrt[c]\*Sqrt[e]\*Sqrt[-(c\*d) + b\*e])



$$\begin{aligned}
&^2e^6 + 8*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b*c^2*d*e^3 + 4*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c^3*d*e^3 - \sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}}*c*e^2)*b^3*e^4 + 2*\sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}}*b^2*c*e^4 - \sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{b*c*e^4 + \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}}*b*c^2*e^4 - 2*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^2*c*e^4 - 2*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b*c^2*e^4)*\arctan(2*\sqrt{1/2}*x/e/\sqrt{(b*e^2 + \sqrt{b^2*e^4 + 4*(c*d^2 - b*d*e)*c*e^2})/c})/((16*c^5*d^5*e^4 - 48*b*c^4*d^4*e^5 + 56*b^2*c^3*d^3*e^6 - 8*b*c^4*d^3*e^6 + 4*c^5*d^3*e^6 - 32*b^3*c^2*d^2*e^7 + 16*b^2*c^3*d^2*e^7 - 8*b*c^4*d^2*e^7 + 9*b^4*c*d*e^8 - 10*b^3*c^2*d*e^8 + 5*b^2*c^3*d*e^8 - b^5*e^9 + 2*b^4*c*e^9 - b^3*c^2*e^9)*\text{abs}(c)) - 1/4*(32*c^5*d^4*e^4 + 16*\sqrt{2}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}}*c*e^2)*c^4*d^4*e^2 - 64*b*c^4*d^3*e^5 - 16*c^5*d^3*e^5 - 32*\sqrt{2}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}}*c*e^2)*b*c^3*d^3*e^3 + 8*\sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}}*c^3*d^3*e + 48*b^2*c^3*d^2*e^6 + 24*b*c^4*d^2*e^6 + 24*\sqrt{2}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}}*c*e^2)*b^2*c^2*d^2*e^4 - 8*\sqrt{2}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}}*c*e^2)*b*c^3*d^2*e^4 + 4*\sqrt{2}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}}*c*e^2)*c^4*d^2*e^4 - 12*\sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}}*b*c^2*d^2*e^2 - 16*b^3*c^2*d*e^7 - 12*b^2*c^3*d*e^7 - 8*\sqrt{2}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}}*c*e^2)*b^3*c*d*e^5 + 8*\sqrt{2}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}}*c*e^2)*b^2*c^2*d*e^5 - 4*\sqrt{2}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}}*c*e^2)*b*c^3*d*e^5 - 8*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c^3*d^2*e^2 + 6*\sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}}*c*e^2)*b^2*c*d*e^3 - 4*\sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}}*c*e^2)*b*c^2*d*e^3 + 2*\sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}}*c^3*d*e^3 + 2*b^4*c*e^8 + 2*b^3*c^2*e^8 + \sqrt{2}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}}*c*e^2)*b^4*e^6 - 2*\sqrt{2}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}}*c*e^2)*b^3*c*e^6 + \sqrt{2}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}}*c*e^2)*b^2*c^2*e^6 + 8*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b*c^2*d*e^3 + 4*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c^3*d*e^3 - \sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}}*c*e^2)*b^3*e^4 + 2*\sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}}*b^2*c*e^4 - \sqrt{2}*\sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}*\sqrt{b*c*e^4 - \sqrt{4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4}}*b^2*c*e^4 - 2*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b*c^2*e^4)*\arctan(2*\sqrt{1/2}*x/e/\sqrt{(b*e^2 - \sqrt{b^2*e^4 + 4*(c*d^2 - b*d*e)*c*e^2})/c})/((16*c^5*d^5*e^4 - 48*b*c^4*d^4*e^5 + 56*b^2*c^3*d^3*e^6 - 8*b*c^4*d^3*e^6 + 4*c^5*d^3*e^6 - 32*b^3*c^2*d^2*e^7 + 16*b^2*c^3*d^2*e^7 - 8*b*c^4*d^2*e^7 + 9*b^4*c*d*e^8 - 10*b^3*c^2*d*e^8 + 5*b^2*c^3*d*e^8 - b^5*e^9 + 2*b^4*c*e^9 - b^3*c^2*e^9)*\text{abs}(c))
\end{aligned}$$

**maple [A]** time = 0.00, size = 33, normalized size = 0.67

$$\frac{\arctan\left(\frac{cex}{\sqrt{(be-cd)ce}}\right)}{\sqrt{(be-cd)ce}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)/(c\*e^2\*x^4+b\*e^2\*x^2+b\*d\*e-c\*d^2), x)

[Out]  $1/((b*e-c*d)*c*e)^{(1/2)}*\arctan(1/((b*e-c*d)*c*e)^{(1/2)}*c*e*x)$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(c\*e^2\*x^4+b\*e^2\*x^2+b\*d\*e-c\*d^2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(b\*e-c\*d>0)', see 'assume?' for more details)Is b\*e-c\*d positive or negative?

**mupad** [B] time = 4.49, size = 38, normalized size = 0.78

$$\frac{\operatorname{atan}\left(\frac{cex}{\sqrt{bce^2-c^2de}}\right)}{\sqrt{bce^2-c^2de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)/(b\*e^2\*x^2 - c\*d^2 + c\*e^2\*x^4 + b\*d\*e),x)

[Out]  $\operatorname{atan}\left(\frac{c*e*x}{(b*c*e^2 - c^2*d*e)^{(1/2)}}\right)/(b*c*e^2 - c^2*d*e)^{(1/2)}$

**sympy** [B] time = 0.32, size = 124, normalized size = 2.53

$$-\frac{\sqrt{-\frac{1}{ce(be-cd)}} \log\left(-be\sqrt{-\frac{1}{ce(be-cd)}} + cd\sqrt{-\frac{1}{ce(be-cd)}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{ce(be-cd)}} \log\left(be\sqrt{-\frac{1}{ce(be-cd)}} - cd\sqrt{-\frac{1}{ce(be-cd)}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)/(c\*e\*\*2\*x\*\*4+b\*e\*\*2\*x\*\*2+b\*d\*e-c\*d\*\*2),x)

[Out]  $-\operatorname{sqrt}(-1/(c*e*(b*e - c*d)))*\log(-b*e*\operatorname{sqrt}(-1/(c*e*(b*e - c*d))) + c*d*\operatorname{sqrt}(-1/(c*e*(b*e - c*d))) + x)/2 + \operatorname{sqrt}(-1/(c*e*(b*e - c*d)))*\log(b*e*\operatorname{sqrt}(-1/(c*e*(b*e - c*d))) - c*d*\operatorname{sqrt}(-1/(c*e*(b*e - c*d))) + x)/2$

$$3.160 \quad \int \frac{1}{(d+ex^2)(-cd^2+bde+be^2x^2+ce^2x^4)} dx$$

**Optimal.** Leaf size=136

$$-\frac{c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^2} - \frac{(4cd-be) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}(2cd-be)^2} - \frac{x}{2d(d+ex^2)(2cd-be)}$$

**Rubi [A]** time = 0.18, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$ , Rules used = {1149, 414, 522, 205, 208}

$$-\frac{c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{ex}}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^2} - \frac{(4cd-be) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}(2cd-be)^2} - \frac{x}{2d(d+ex^2)(2cd-be)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x^2)\*(-(c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4)), x]

[Out] -x/(2\*d\*(2\*c\*d - b\*e)\*(d + e\*x^2)) - ((4\*c\*d - b\*e)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(2\*d^(3/2)\*Sqrt[e]\*(2\*c\*d - b\*e)^2) - (c^(3/2)\*ArcTanh[(Sqrt[c]\*Sqrt[e]\*x)/Sqrt[c\*d - b\*e]])/(Sqrt[e]\*Sqrt[c\*d - b\*e]\*(2\*c\*d - b\*e)^2)

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 414

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(b\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(p+1)\*(b\*c - a\*d)), x] + Dist[1/(a\*n\*(p+1)\*(b\*c - a\*d)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q\*Simp[b\*c + n\*(p+1)\*(b\*c - a\*d) + d\*b\*(n\*(p+q+2) + 1)\*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e\_) + (f\_.)\*(x\_)^(n\_))/(((a\_) + (b\_.)\*(x\_)^(n\_))\*((c\_) + (d\_.)\*(x\_)^(n\_))), x\_Symbol] := Dist[(b\*e - a\*f)/(b\*c - a\*d), Int[1/(a + b\*x^n), x], x] - Dist[(d\*e - c\*f)/(b\*c - a\*d), Int[1/(c + d\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 1149

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] := Int[(d + e\*x^2)^(p+q)\*(a/d + (c\*x^2)/e)^p, x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IntegerQ[p]

Rubi steps

$$\int \frac{1}{(d + ex^2)(-cd^2 + bde + be^2x^2 + ce^2x^4)} dx = \int \frac{1}{(d + ex^2)^2 \left( \frac{-cd^2 + bde}{d} + cex^2 \right)} dx$$

$$= -\frac{x}{2d(2cd - be)(d + ex^2)} + \frac{\int \frac{e(3cd - be) - ce^2x^2}{(d + ex^2) \left( \frac{-cd^2 + bde}{d} + cex^2 \right)} dx}{2de(2cd - be)}$$

$$= -\frac{x}{2d(2cd - be)(d + ex^2)} + \frac{c^2 \int \frac{1}{\frac{-cd^2 + bde}{d} + cex^2} dx}{(2cd - be)^2} - \frac{(4cd - be) \int \frac{1}{d + ex^2} dx}{2d(2cd - be)}$$

$$= -\frac{x}{2d(2cd - be)(d + ex^2)} - \frac{(4cd - be) \tan^{-1} \left( \frac{\sqrt{e}x}{\sqrt{d}} \right)}{2d^{3/2} \sqrt{e} (2cd - be)^2} - \frac{c^{3/2} \tanh^{-1} \left( \frac{\sqrt{e}x}{\sqrt{d}} \right)}{\sqrt{e} \sqrt{cd - be}}$$

**Mathematica [A]** time = 0.20, size = 133, normalized size = 0.98

$$\frac{c^{3/2} \tan^{-1} \left( \frac{\sqrt{c} \sqrt{e} x}{\sqrt{be - cd}} \right)}{\sqrt{e} (be - 2cd)^2 \sqrt{be - cd}} + \frac{(be - 4cd) \tan^{-1} \left( \frac{\sqrt{e} x}{\sqrt{d}} \right)}{2d^{3/2} \sqrt{e} (2cd - be)^2} - \frac{x}{2d(d + ex^2)(2cd - be)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x^2)\*(-c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4)),x]

[Out] -1/2\*x/(d\*(2\*c\*d - b\*e)\*(d + e\*x^2)) + ((-4\*c\*d + b\*e)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(2\*d^(3/2)\*Sqrt[e]\*(2\*c\*d - b\*e)^2) + (c^(3/2)\*ArcTan[(Sqrt[c]\*Sqrt[e]\*x)/Sqrt[-(c\*d) + b\*e]])/(Sqrt[e]\*(-2\*c\*d + b\*e)^2\*Sqrt[-(c\*d) + b\*e])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex^2)(-cd^2 + bde + be^2x^2 + ce^2x^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e\*x^2)\*(-c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4)),x]

[Out] IntegrateAlgebraic[1/((d + e\*x^2)\*(-c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4)), x]

**fricas [A]** time = 1.29, size = 895, normalized size = 6.58

$$\frac{c^{3/2} \tan^{-1} \left( \frac{\sqrt{c} \sqrt{e} x}{\sqrt{be - cd}} \right)}{\sqrt{e} (be - 2cd)^2 \sqrt{be - cd}} + \frac{(be - 4cd) \tan^{-1} \left( \frac{\sqrt{e} x}{\sqrt{d}} \right)}{2d^{3/2} \sqrt{e} (2cd - be)^2} - \frac{x}{2d(d + ex^2)(2cd - be)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)/(c\*e^2\*x^4+b\*e^2\*x^2+b\*d\*e-c\*d^2),x, algorithm="fricas")

[Out] [1/4\*(2\*(c\*d^2\*e^2\*x^2 + c\*d^3\*e)\*sqrt(c/(c\*d\*e - b\*e^2))\*log((c\*e\*x^2 - 2\*(c\*d\*e - b\*e^2)\*x\*sqrt(c/(c\*d\*e - b\*e^2)) + c\*d - b\*e)/(c\*e\*x^2 - c\*d + b\*e)) + (4\*c\*d^2 - b\*d\*e + (4\*c\*d\*e - b\*e^2)\*x^2)\*sqrt(-d\*e)\*log((e\*x^2 - 2\*sqrt(-d\*e)\*x - d)/(e\*x^2 + d)) - 2\*(2\*c\*d^2\*e - b\*d\*e^2)\*x/(4\*c^2\*d^5\*e - 4\*b\*c\*d^4\*e^2 + b^2\*d^3\*e^3 + (4\*c^2\*d^4\*e^2 - 4\*b\*c\*d^3\*e^3 + b^2\*d^2\*e^4)\*x^2), -1/2\*((4\*c\*d^2 - b\*d\*e + (4\*c\*d\*e - b\*e^2)\*x^2)\*sqrt(d\*e)\*arctan(sqrt(d\*e)\*x/d) - (c\*d^2\*e^2\*x^2 + c\*d^3\*e)\*sqrt(c/(c\*d\*e - b\*e^2))\*log((c\*e\*x^2

$$- 2*(c*d*e - b*e^2)*x*\sqrt{c/(c*d*e - b*e^2)} + c*d - b*e)/(c*e*x^2 - c*d + b*e) + (2*c*d^2*e - b*d*e^2)*x)/(4*c^2*d^5*e - 4*b*c*d^4*e^2 + b^2*d^3*e^3 + (4*c^2*d^4*e^2 - 4*b*c*d^3*e^3 + b^2*d^2*e^4)*x^2), 1/4*(4*(c*d^2*e^2*x^2 + c*d^3*e)*\sqrt{-c/(c*d*e - b*e^2)}*\arctan(e*x*\sqrt{-c/(c*d*e - b*e^2)})) + (4*c*d^2 - b*d*e + (4*c*d*e - b*e^2)*x^2)*\sqrt{-d*e}*log((e*x^2 - 2*\sqrt{-d*e})*x - d)/(e*x^2 + d)) - 2*(2*c*d^2*e - b*d*e^2)*x)/(4*c^2*d^5*e - 4*b*c*d^4*e^2 + b^2*d^3*e^3 + (4*c^2*d^4*e^2 - 4*b*c*d^3*e^3 + b^2*d^2*e^4)*x^2), 1/2*(2*(c*d^2*e^2*x^2 + c*d^3*e)*\sqrt{-c/(c*d*e - b*e^2)}*\arctan(e*x*\sqrt{-c/(c*d*e - b*e^2)})) - (4*c*d^2 - b*d*e + (4*c*d*e - b*e^2)*x^2)*\sqrt{d*e})*\arctan(\sqrt{d*e}*x/d) - (2*c*d^2*e - b*d*e^2)*x)/(4*c^2*d^5*e - 4*b*c*d^4*e^2 + b^2*d^3*e^3 + (4*c^2*d^4*e^2 - 4*b*c*d^3*e^3 + b^2*d^2*e^4)*x^2)]$$

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)/(c\*e^2\*x^4+b\*e^2\*x^2+b\*d\*e-c\*d^2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
 UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [b,c,d,exp(1),exp(2)]=[-95,-68,60,-66,8]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [b,c,d,exp(1),exp(2)]=[79,32,2,-92,39]sym2poly/r2sym(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 10.77Done

**maple** [A] time = 0.01, size = 155, normalized size = 1.14

$$\frac{c^2 \arctan\left(\frac{cex}{\sqrt{(be-cd)ce}}\right)}{(be-2cd)^2 \sqrt{(be-cd)ce}} + \frac{bex}{2(be-2cd)^2 (ex^2+d)d} + \frac{be \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2(be-2cd)^2 \sqrt{de}d} - \frac{cx}{(be-2cd)^2 (ex^2+d)} - \frac{2c \arctan\left(\frac{ex}{\sqrt{de}}\right)}{(be-2cd)^2 \sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x^2+d)/(c\*e^2\*x^4+b\*e^2\*x^2+b\*d\*e-c\*d^2),x)

[Out] c^2/(b\*e-2\*c\*d)^2/((b\*e-c\*d)\*c\*e)^(1/2)\*arctan(1/((b\*e-c\*d)\*c\*e)^(1/2)\*c\*e\*x)+1/2/(b\*e-2\*c\*d)^2/d\*x/(e\*x^2+d)\*b\*e-1/(b\*e-2\*c\*d)^2\*x/(e\*x^2+d)\*c+1/2/(b\*e-2\*c\*d)^2/d/(d\*e)^(1/2)\*arctan(1/(d\*e)^(1/2)\*e\*x)\*b\*e-2/(b\*e-2\*c\*d)^2/(d\*e)^(1/2)\*arctan(1/(d\*e)^(1/2)\*e\*x)\*c

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)/(c\*e^2\*x^4+b\*e^2\*x^2+b\*d\*e-c\*d^2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(b\*e-c\*d>0)', see `assume?` for more details)Is b\*e-c\*d positive or negative?





$$\frac{4d^3e^9 + 22b^4c^3d^2e^{10}}{(8c^3d^5 - b^3d^2e^3 + 6b^2cd^3e^2 - 12b^2c^2d^4e) + (x(-d^3e)^{1/2})(be - 4cd)(256b^6c^6d^6e^8 - 512b^2c^5d^5e^9 + 384b^3c^4d^4e^{10} - 128b^4c^3d^3e^{11} + 16b^5c^2d^2e^{12})} / ((8(4c^2d^4 + b^2d^2e^2 - 4b^2cd^3e)(4c^2d^5e + b^2d^3e^3 - 4b^2cd^4e^2)))(be - 4cd) / (4(4c^2d^5e + b^2d^3e^3 - 4b^2cd^4e^2)))(-d^3e)^{1/2}(be - 4cd) * 1i / (4(4c^2d^5e + b^2d^3e^3 - 4b^2cd^4e^2)) / (((b^4c^6)/2 - 2c^5d^5e^5) / (8c^3d^5 - b^3d^2e^3 + 6b^2cd^3e^2 - 12b^2c^2d^4e) + (((x(b^2c^3e^8 + 20c^5d^2e^6 - 8b^4d^7e^7)) / (2(4c^2d^4 + b^2d^2e^2 - 4b^2cd^3e)) - ((-d^3e)^{1/2}) * ((96c^7d^6e^6 - 224b^6c^6d^5e^7 - 2b^5c^2d^11e + 208b^2c^5d^4e^8 - 96b^3c^4d^3e^9 + 22b^4c^3d^2e^{10}) / (8c^3d^5 - b^3d^2e^3 + 6b^2cd^3e^2 - 12b^2c^2d^4e) - (x(-d^3e)^{1/2})(be - 4cd) * (256b^6c^6d^6e^8 - 512b^2c^5d^5e^9 + 384b^3c^4d^4e^{10} - 128b^4c^3d^3e^{11} + 16b^5c^2d^2e^{12})) / (8(4c^2d^4 + b^2d^2e^2 - 4b^2cd^3e) * (4c^2d^5e + b^2d^3e^3 - 4b^2cd^4e^2)))(be - 4cd) / (4(4c^2d^5e + b^2d^3e^3 - 4b^2cd^4e^2)))(-d^3e)^{1/2}(be - 4cd) / (4(4c^2d^5e + b^2d^3e^3 - 4b^2cd^4e^2)) - (((x(b^2c^3e^8 + 20c^5d^2e^6 - 8b^4d^7e^7)) / (2(4c^2d^4 + b^2d^2e^2 - 4b^2cd^3e)) + ((-d^3e)^{1/2}) * ((96c^7d^6e^6 - 224b^6c^6d^5e^7 - 2b^5c^2d^11e + 208b^2c^5d^4e^8 - 96b^3c^4d^3e^9 + 22b^4c^3d^2e^{10}) / (8c^3d^5 - b^3d^2e^3 + 6b^2cd^3e^2 - 12b^2c^2d^4e) + (x(-d^3e)^{1/2})(be - 4cd) * (256b^6c^6d^6e^8 - 512b^2c^5d^5e^9 + 384b^3c^4d^4e^{10} - 128b^4c^3d^3e^{11} + 16b^5c^2d^2e^{12})) / (8(4c^2d^4 + b^2d^2e^2 - 4b^2cd^3e) * (4c^2d^5e + b^2d^3e^3 - 4b^2cd^4e^2)))(be - 4cd) / (4(4c^2d^5e + b^2d^3e^3 - 4b^2cd^4e^2)))(-d^3e)^{1/2}(be - 4cd) / (4(4c^2d^5e + b^2d^3e^3 - 4b^2cd^4e^2)))(-d^3e)^{1/2}(be - 4cd) * 1i / (2(4c^2d^5e + b^2d^3e^3 - 4b^2cd^4e^2))$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x\*\*2+d)/(c\*e\*\*2\*x\*\*4+b\*e\*\*2\*x\*\*2+b\*d\*e-c\*d\*\*2),x)

[Out] Timed out

$$3.161 \quad \int \frac{1}{(d+ex^2)^2(-cd^2+bde+be^2x^2+ce^2x^4)} dx$$

**Optimal.** Leaf size=187

$$\frac{(3b^2e^2 - 16bcde + 28c^2d^2) \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) - \frac{c^{5/2} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^3} - \frac{x(10cd-3be)}{8d^2(d+ex^2)(2cd-be)^2} - \frac{x}{4d(d+ex^2)^2(2cd-be)}}{8d^{5/2}\sqrt{e}(2cd-be)^3}$$

**Rubi [A]** time = 0.28, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {1149, 414, 527, 522, 205, 208}

$$\frac{(3b^2e^2 - 16bcde + 28c^2d^2) \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) - \frac{c^{5/2} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^3} - \frac{x(10cd-3be)}{8d^2(d+ex^2)(2cd-be)^2} - \frac{x}{4d(d+ex^2)^2(2cd-be)}}{8d^{5/2}\sqrt{e}(2cd-be)^3}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x^2)^2\*(-(c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4)),x]

[Out] -x/(4\*d\*(2\*c\*d - b\*e)\*(d + e\*x^2)^2) - ((10\*c\*d - 3\*b\*e)\*x)/(8\*d^2\*(2\*c\*d - b\*e)^2\*(d + e\*x^2)) - ((28\*c^2\*d^2 - 16\*b\*c\*d\*e + 3\*b^2\*e^2)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(8\*d^(5/2)\*Sqrt[e]\*(2\*c\*d - b\*e)^3) - (c^(5/2)\*ArcTanh[(Sqrt[c]\*Sqrt[e]\*x)/Sqrt[c\*d - b\*e]])/(Sqrt[e]\*Sqrt[c\*d - b\*e]\*(2\*c\*d - b\*e)^3)

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 414

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(b\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(p+1)\*(b\*c - a\*d)), x] + Dist[1/(a\*n\*(p+1)\*(b\*c - a\*d)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q\*Simp[b\*c + n\*(p+1)\*(b\*c - a\*d) + d\*b\*(n\*(p+q+2) + 1)\*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

#### Rule 522

Int[((e\_) + (f\_.)\*(x\_)^(n\_))/((a\_) + (b\_.)\*(x\_)^(n\_))\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Dist[(b\*e - a\*f)/(b\*c - a\*d), Int[1/(a + b\*x^n), x], x] - Dist[(d\*e - c\*f)/(b\*c - a\*d), Int[1/(c + d\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

#### Rule 527

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.)\*((e\_) + (f\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*e - a\*f)\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(b\*c - a\*d)\*(p+1)), x] + Dist[1/(a\*n\*(b\*c - a\*d)\*(p+1)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q\*Simp[c\*(b\*e - a\*f) + e\*n\*(b\*c - a\*d)\*(p+1) + d\*(b\*e - a\*f)\*(n\*(p+q+2) + 1)\*x^n, x], x] /; FreeQ

[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

### Rule 1149

Int[((d\_) + (e\_)\*(x\_)^2)^(q\_)\*((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4)^(p\_), x\_Symbol] :> Int[(d + e\*x^2)^(p + q)\*(a/d + (c\*x^2)/e)^p, x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IntegerQ[p]

### Rubi steps

$$\begin{aligned} \int \frac{1}{(d + ex^2)^2 (-cd^2 + bde + be^2x^2 + ce^2x^4)} dx &= \int \frac{1}{(d + ex^2)^3 \left( \frac{-cd^2 + bde}{d} + cex^2 \right)} dx \\ &= -\frac{x}{4d(2cd - be)(d + ex^2)^2} + \frac{\int \frac{e(7cd - 3be) - 3ce^2x^2}{(d + ex^2)^2 \left( \frac{-cd^2 + bde}{d} + cex^2 \right)} dx}{4de(2cd - be)} \\ &= -\frac{x}{4d(2cd - be)(d + ex^2)^2} - \frac{(10cd - 3be)x}{8d^2(2cd - be)^2(d + ex^2)} + \frac{\int \frac{e^2(1}{c^3 \int \dots} \\ &= -\frac{x}{4d(2cd - be)(d + ex^2)^2} - \frac{(10cd - 3be)x}{8d^2(2cd - be)^2(d + ex^2)} + \frac{c^3 \int \dots}{(2} \\ &= -\frac{x}{4d(2cd - be)(d + ex^2)^2} - \frac{(10cd - 3be)x}{8d^2(2cd - be)^2(d + ex^2)} - \frac{(28c^2}{ \end{aligned}$$

**Mathematica [A]** time = 0.41, size = 177, normalized size = 0.95

$$\frac{1}{8} \left( \frac{(3b^2e^2 - 16bcde + 28c^2d^2) \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) - \frac{8c^{5/2} \tan^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{be-cd}}\right) + \frac{x(be-2cd)(2cd(7d+5ex^2) - be(5d+3ex^2))}{d^2(d+ex^2)^2}}{d^{5/2}\sqrt{e}(2cd-be)^3} - \frac{\dots}{(be-2cd)^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x^2)^2\*(-(c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4)), x]

[Out] (-(((28\*c^2\*d^2 - 16\*b\*c\*d\*e + 3\*b^2\*e^2)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(d^(5/2)\*Sqrt[e]\*(2\*c\*d - b\*e)^3) - (((-2\*c\*d + b\*e)\*x\*(-(b\*e\*(5\*d + 3\*e\*x^2)) + 2\*c\*d\*(7\*d + 5\*e\*x^2)))/(d^2\*(d + e\*x^2)^2) + (8\*c^(5/2)\*ArcTan[(Sqrt[c]\*Sqrt[e]\*x)/Sqrt[-(c\*d) + b\*e]]/(Sqrt[e]\*Sqrt[-(c\*d) + b\*e]))/(-2\*c\*d + b\*e)^3)/8

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex^2)^2 (-cd^2 + bde + be^2x^2 + ce^2x^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e\*x^2)^2\*(-(c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4)),x]

[Out] IntegrateAlgebraic[1/((d + e\*x^2)^2\*(-(c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4)), x]

**fricas** [B] time = 3.71, size = 1765, normalized size = 9.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)^2/(c\*e^2\*x^4+b\*e^2\*x^2+b\*d\*e-c\*d^2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/16*(2*(20*c^2*d^3*e^2 - 16*b*c*d^2*e^3 + 3*b^2*d*e^4)*x^3 + 8*(c^2*d^3*e^3*x^4 + 2*c^2*d^4*e^2*x^2 + c^2*d^5*e)*\sqrt{c/(c*d*e - b*e^2)}*\log((c*e*x^2 + 2*(c*d*e - b*e^2)*x*\sqrt{c/(c*d*e - b*e^2)} + c*d - b*e)/(c*e*x^2 - c*d + b*e)) - (28*c^2*d^4 - 16*b*c*d^3*e + 3*b^2*d^2*e^2 + (28*c^2*d^2*e^2 - 16*b*c*d*e^3 + 3*b^2*e^4)*x^4 + 2*(28*c^2*d^3*e - 16*b*c*d^2*e^2 + 3*b^2*d*e^3)*x^2)*\sqrt{-d*e}*\log((e*x^2 - 2*\sqrt{-d*e}*x - d)/(e*x^2 + d)) + 2*(28*c^2*d^4*e - 24*b*c*d^3*e^2 + 5*b^2*d^2*e^3)*x)/(8*c^3*d^8*e - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^6*e^3 - b^3*d^5*e^4 + (8*c^3*d^6*e^3 - 12*b*c^2*d^5*e^4 + 6*b^2*c*d^4*e^5 - b^3*d^3*e^6)*x^4 + 2*(8*c^3*d^7*e^2 - 12*b*c^2*d^6*e^3 + 6*b^2*c*d^5*e^4 - b^3*d^4*e^5)*x^2), -1/8*((20*c^2*d^3*e^2 - 16*b*c*d^2*e^3 + 3*b^2*d*e^4)*x^3 + (28*c^2*d^4 - 16*b*c*d^3*e + 3*b^2*d^2*e^2 + (28*c^2*d^2*e^2 - 16*b*c*d*e^3 + 3*b^2*e^4)*x^4 + 2*(28*c^2*d^3*e - 16*b*c*d^2*e^2 + 3*b^2*d*e^3)*x^2)*\sqrt{d*e}*\arctan(\sqrt{d*e}*x/d) + 4*(c^2*d^3*e^3*x^4 + 2*c^2*d^4*e^2*x^2 + c^2*d^5*e)*\sqrt{c/(c*d*e - b*e^2)}*\log((c*e*x^2 + 2*(c*d*e - b*e^2)*x*\sqrt{c/(c*d*e - b*e^2)} + c*d - b*e)/(c*e*x^2 - c*d + b*e)) + (28*c^2*d^4*e - 24*b*c*d^3*e^2 + 5*b^2*d^2*e^3)*x)/(8*c^3*d^8*e - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^6*e^3 - b^3*d^5*e^4 + (8*c^3*d^6*e^3 - 12*b*c^2*d^5*e^4 + 6*b^2*c*d^4*e^5 - b^3*d^3*e^6)*x^4 + 2*(8*c^3*d^7*e^2 - 12*b*c^2*d^6*e^3 + 6*b^2*c*d^5*e^4 - b^3*d^4*e^5)*x^2), -1/16*(2*(20*c^2*d^3*e^2 - 16*b*c*d^2*e^3 + 3*b^2*d*e^4)*x^3 - 16*(c^2*d^3*e^3*x^4 + 2*c^2*d^4*e^2*x^2 + c^2*d^5*e)*\sqrt{-c/(c*d*e - b*e^2)}*\arctan(e*x*\sqrt{-c/(c*d*e - b*e^2)})) - (28*c^2*d^4 - 16*b*c*d^3*e + 3*b^2*d^2*e^2 + (28*c^2*d^2*e^2 - 16*b*c*d*e^3 + 3*b^2*e^4)*x^4 + 2*(28*c^2*d^3*e - 16*b*c*d^2*e^2 + 3*b^2*d*e^3)*x^2)*\sqrt{-d*e}*\log((e*x^2 - 2*\sqrt{-d*e}*x - d)/(e*x^2 + d)) + 2*(28*c^2*d^4*e - 24*b*c*d^3*e^2 + 5*b^2*d^2*e^3)*x)/(8*c^3*d^8*e - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^6*e^3 - b^3*d^5*e^4 + (8*c^3*d^6*e^3 - 12*b*c^2*d^5*e^4 + 6*b^2*c*d^4*e^5 - b^3*d^3*e^6)*x^4 + 2*(8*c^3*d^7*e^2 - 12*b*c^2*d^6*e^3 + 6*b^2*c*d^5*e^4 - b^3*d^4*e^5)*x^2), -1/8*((20*c^2*d^3*e^2 - 16*b*c*d^2*e^3 + 3*b^2*d*e^4)*x^3 - 8*(c^2*d^3*e^3*x^4 + 2*c^2*d^4*e^2*x^2 + c^2*d^5*e)*\sqrt{-c/(c*d*e - b*e^2)}*\arctan(e*x*\sqrt{-c/(c*d*e - b*e^2)})) + (28*c^2*d^4 - 16*b*c*d^3*e + 3*b^2*d^2*e^2 + (28*c^2*d^2*e^2 - 16*b*c*d*e^3 + 3*b^2*e^4)*x^4 + 2*(28*c^2*d^3*e - 16*b*c*d^2*e^2 + 3*b^2*d*e^3)*x^2)*\sqrt{d*e}*\arctan(\sqrt{d*e}*x/d) + (28*c^2*d^4*e - 24*b*c*d^3*e^2 + 5*b^2*d^2*e^3)*x)/(8*c^3*d^8*e - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^6*e^3 - b^3*d^5*e^4 + (8*c^3*d^6*e^3 - 12*b*c^2*d^5*e^4 + 6*b^2*c*d^4*e^5 - b^3*d^3*e^6)*x^4 + 2*(8*c^3*d^7*e^2 - 12*b*c^2*d^6*e^3 + 6*b^2*c*d^5*e^4 - b^3*d^4*e^5)*x^2)] \end{aligned}$$

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)^2/(c\*e^2\*x^4+b\*e^2\*x^2+b\*d\*e-c\*d^2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output:  $(4*b^5*c*\exp(1)*\exp(2)^5+2*b^5*\sqrt{2}*\sqrt{b*c*\exp(2)^2-c*\sqrt{b^2*\exp(2)^2+4*c}}$

$$\begin{aligned}
& ^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2)) * exp(2)) * exp(1) * exp(2)^4-36*b^4*c^2*d*exp(1)^2*exp(2)^4-4*b^4*c^2*d*exp(2)^5-4*b^4*c^2*exp(1) * exp(2)^5-18*b^4*c*d*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(2)) * exp(1)^2*exp(2)^3-2*b^4*c*d*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(2)) * exp(2)) * exp(2)^4-4*b^4*c*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(2)) * exp(1) * exp(2)^4+2*b^4*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(2)) * sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(1) * exp(2)^3+96*b^3*c^3*d^2*exp(1)^3*exp(2)^3+64*b^3*c^3*d^2*exp(1) * exp(2)^4+28*b^3*c^3*d*exp(1)^2*exp(2)^4+4*b^3*c^3*d*exp(2)^5+48*b^3*c^2*d^2*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(2)) * exp(1)^3*exp(2)^2+32*b^3*c^2*d^2*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(2)) * exp(1) * exp(2)^3+20*b^3*c^2*d*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(2)) * exp(2)) * exp(1)^2*exp(2)^3+4*b^3*c^2*d*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(2)) * exp(2)) * exp(2)^4+2*b^3*c^2*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(2)) * exp(1) * exp(2)^4-14*b^3*c*d*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(2)) * sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(1)^2*exp(2)^2-2*b^3*c*d*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(2)) * sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(2)) * sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(1) * exp(2)^3-4*b^3*c*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(2)) * sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(1) * exp(2)^3-4*b^3*c*(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(1) * exp(2)^3-64*b^2*c^4*d^3*exp(1)^4*exp(2)^2-224*b^2*c^4*d^3*exp(1)^2*exp(2)^3-32*b^2*c^4*d^3*exp(2)^4-48*b^2*c^4*d^2*exp(1)^3*exp(2)^3-48*b^2*c^4*d^2*exp(1) * exp(2)^4-32*b^2*c^3*d^3*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(2)) * exp(1)^4*exp(2)-112*b^2*c^3*d^3*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(2)) * exp(2)) * exp(1)^2*exp(2)^2-16*b^2*c^3*d^3*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(2)) * exp(2)) * exp(2)^3-16*b^2*c^3*d^2*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(2)) * sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(2)) * exp(1)^3*exp(2)^2-32*b^2*c^3*d^2*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(2)) * sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(1) * exp(2)^3-10*b^2*c^3*d*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(2)) * exp(2)) * exp(1)^2*exp(2)^3-2*b^2*c^3*d*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(2)) * exp(2)) * exp(2)^4+24*b^2*c^2*d^2*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(2)) * sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(1) * exp(2)^2+12*b^2*c^2*d*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(2)) * sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(1)^2*exp(2)^2+4*b^2*c^2*d*(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(2)^3+2*b^2*c^2*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(2)) * sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(1) * exp(2)^3+4*b^2*c^2*(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(1) * exp(2)^3+128*b*c^5*d^4*exp(1)^3*exp(2)^2+192*b*c^5*d^4*exp(1) * exp(2)^3+112*b*c^5*d^3*exp(1)^2*exp(2)^3+16*b*c^5*d^3*exp(2)^4+64*b*c^4*d^4*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1) * exp(2)) * exp(2)) * exp(1)^3*exp(2)+96*b*c^4*d^4*sqrt(2)*sqrt(b
\end{aligned}$$





$$\begin{aligned}
& *b^2*c^3*d^2*\sqrt{2}*\sqrt{b*c*\exp(2)^2+c*\sqrt{b^2*\exp(2)^2+4*c^2*d^2*\exp(2)} \\
& -4*b*c*d*\exp(1)*\exp(2))*\exp(2))*\exp(1)^3*\exp(2)^2+32*b^2*c^3*d^2*\sqrt{2}*\sqrt{ \\
& \text{rt}(b*c*\exp(2)^2+c*\sqrt{b^2*\exp(2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)*\exp(2)) \\
& *\exp(2))*\exp(1)*\exp(2)^3+10*b^2*c^3*d*\sqrt{2}*\sqrt{b*c*\exp(2)^2+c*\sqrt{b^2* \\
& \exp(2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)*\exp(2))*\exp(2))*\exp(1)^2*\exp(2)^3+ \\
& 2*b^2*c^3*d*\sqrt{2}*\sqrt{b*c*\exp(2)^2+c*\sqrt{b^2*\exp(2)^2+4*c^2*d^2*\exp(2)}- \\
& 4*b*c*d*\exp(1)*\exp(2))*\exp(2))*\exp(2)^4+24*b^2*c^2*d^2*\sqrt{2}*\sqrt{b*c*\exp \\
& (2)^2+c*\sqrt{b^2*\exp(2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)*\exp(2))*\exp(2))*s \\
& \text{qrt}(b^2*\exp(2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)*\exp(2))*\exp(1)^3*\exp(2)+24 \\
& *b^2*c^2*d^2*\sqrt{2}*\sqrt{b*c*\exp(2)^2+c*\sqrt{b^2*\exp(2)^2+4*c^2*d^2*\exp(2)} \\
& -4*b*c*d*\exp(1)*\exp(2))*\exp(2))*\sqrt{b^2*\exp(2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d* \\
& \exp(1)*\exp(2))*\exp(1)*\exp(2)^2+12*b^2*c^2*d*\sqrt{2}*\sqrt{b*c*\exp(2)^2+c*\sqrt{ \\
& \text{t}(b^2*\exp(2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)*\exp(2))*\exp(2))*\sqrt{b^2*\exp \\
& (2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)*\exp(2))*\exp(1)^2*\exp(2)^2+4*b^2*c^2*d \\
& *\sqrt{2}*\sqrt{b*c*\exp(2)^2+c*\sqrt{b^2*\exp(2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp \\
& (1)*\exp(2))*\exp(2))*\sqrt{b^2*\exp(2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)*\exp(2) \\
& ))*\exp(2)^3+20*b^2*c^2*d*(b^2*\exp(2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)*\exp( \\
& 2))*\exp(1)^2*\exp(2)^2+4*b^2*c^2*d*(b^2*\exp(2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp \\
& (1)*\exp(2))*\exp(2)^3+2*b^2*c^2*\sqrt{2}*\sqrt{b*c*\exp(2)^2+c*\sqrt{b^2*\exp(2) \\
& ^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)*\exp(2))*\exp(2))*\sqrt{b^2*\exp(2)^2+4*c^2* \\
& d^2*\exp(2)}-4*b*c*d*\exp(1)*\exp(2))*\exp(1)*\exp(2)^3+4*b^2*c^2*(b^2*\exp(2)^2+4 \\
& *c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)*\exp(2))*\exp(1)*\exp(2)^3+128*b*c^5*d^4*\exp(1) \\
& ^3*\exp(2)^2+192*b*c^5*d^4*\exp(1)*\exp(2)^3+112*b*c^5*d^3*\exp(1)^2*\exp(2)^3+1 \\
& 6*b*c^5*d^3*\exp(2)^4-64*b*c^4*d^4*\sqrt{2}*\sqrt{b*c*\exp(2)^2+c*\sqrt{b^2*\exp( \\
& 2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)*\exp(2))*\exp(2))*\exp(1)^3*\exp(2)-96*b*c \\
& ^4*d^4*\sqrt{2}*\sqrt{b*c*\exp(2)^2+c*\sqrt{b^2*\exp(2)^2+4*c^2*d^2*\exp(2)}-4*b*c \\
& *d*\exp(1)*\exp(2))*\exp(2))*\exp(1)*\exp(2)^2-16*b*c^4*d^3*\sqrt{2}*\sqrt{b*c*\exp \\
& (2)^2+c*\sqrt{b^2*\exp(2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)*\exp(2))*\exp(2))*e \\
& \text{xp}(1)^2*\exp(2)^2-16*b*c^4*d^3*\sqrt{2}*\sqrt{b*c*\exp(2)^2+c*\sqrt{b^2*\exp(2)^2 \\
& +4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)*\exp(2))*\exp(2))*\exp(2)^3-8*b*c^4*d^2*\sqrt{2} \\
& *\sqrt{b*c*\exp(2)^2+c*\sqrt{b^2*\exp(2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)*\exp \\
& (2))*\exp(2))*\exp(1)^3*\exp(2)^2-16*b*c^4*d^2*\sqrt{2}*\sqrt{b*c*\exp(2)^2+c*\sqrt{ \\
& \text{rt}(b^2*\exp(2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)*\exp(2))*\exp(2))*\exp(1)*\exp( \\
& 2)^3-56*b*c^3*d^3*\sqrt{2}*\sqrt{b*c*\exp(2)^2+c*\sqrt{b^2*\exp(2)^2+4*c^2*d^2*\exp \\
& (2)}-4*b*c*d*\exp(1)*\exp(2))*\exp(2))*\sqrt{b^2*\exp(2)^2+4*c^2*d^2*\exp(2)}-4*b \\
& *c*d*\exp(1)*\exp(2))*\exp(1)^2*\exp(2)-8*b*c^3*d^3*\sqrt{2}*\sqrt{b*c*\exp(2)^2+c \\
& *\sqrt{b^2*\exp(2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)*\exp(2))*\exp(2))*\sqrt{b^2 \\
& *\exp(2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)*\exp(2))*\exp(2)^2-16*b*c^3*d^2*\sqrt{2} \\
& *\sqrt{b*c*\exp(2)^2+c*\sqrt{b^2*\exp(2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)* \\
& \exp(2))*\exp(2))*\sqrt{b^2*\exp(2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)*\exp(2))* \\
& \text{xp}(1)*\exp(2)^2-16*b*c^3*d^2*(b^2*\exp(2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)* \\
& \exp(2))*\exp(1)^3*\exp(2)-32*b*c^3*d^2*(b^2*\exp(2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d* \\
& \exp(1)*\exp(2))*\exp(1)*\exp(2)^2-6*b*c^3*d*\sqrt{2}*\sqrt{b*c*\exp(2)^2+c*\sqrt{b \\
& ^2*\exp(2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)*\exp(2))*\exp(2))*\sqrt{b^2*\exp(2) \\
& ^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)*\exp(2))*\exp(1)^2*\exp(2)^2-2*b*c^3*d*\sqrt{2} \\
& *\sqrt{b*c*\exp(2)^2+c*\sqrt{b^2*\exp(2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)* \\
& \exp(2))*\exp(2))*\sqrt{b^2*\exp(2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)*\exp(2))* \\
& \text{xp}(2)^3-12*b*c^3*d*(b^2*\exp(2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)*\exp(2))*\exp \\
& (1)^2*\exp(2)^2-4*b*c^3*d*(b^2*\exp(2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)*\exp( \\
& 2))*\exp(2)^3-64*c^6*d^5*\exp(1)^2*\exp(2)^2-64*c^6*d^5*\exp(2)^3-64*c^6*d^4*\exp \\
& (1)*\exp(2)^3+32*c^5*d^5*\sqrt{2}*\sqrt{b*c*\exp(2)^2+c*\sqrt{b^2*\exp(2)^2+4*c^ \\
& 2*d^2*\exp(2)}-4*b*c*d*\exp(1)*\exp(2))*\exp(2))*\exp(1)^2*\exp(2)+32*c^5*d^5*\sqrt{2} \\
& *\sqrt{b*c*\exp(2)^2+c*\sqrt{b^2*\exp(2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)* \\
& \exp(2))*\exp(2))*\exp(2)^2+8*c^5*d^3*\sqrt{2}*\sqrt{b*c*\exp(2)^2+c*\sqrt{b^2*\exp( \\
& 2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)*\exp(2))*\exp(2))*\exp(1)^2*\exp(2)^2+8*c^ \\
& 5*d^3*\sqrt{2}*\sqrt{b*c*\exp(2)^2+c*\sqrt{b^2*\exp(2)^2+4*c^2*d^2*\exp(2)}-4*b*c* \\
& d*\exp(1)*\exp(2))*\exp(2))*\exp(2)^3+32*c^4*d^4*\sqrt{2}*\sqrt{b*c*\exp(2)^2+c*\sqrt{ \\
& \text{rt}(b^2*\exp(2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)*\exp(2))*\exp(2))*\sqrt{b^2*\exp \\
& (2)^2+4*c^2*d^2*\exp(2)}-4*b*c*d*\exp(1)*\exp(2))*\exp(1)*\exp(2)+16*c^4*d^3*(b^
\end{aligned}$$



$2*\exp(2)^2+4*c^2*d^2*\exp(2)-4*b*c*d*\exp(1)*\exp(2))*\exp(1)^2*\exp(2)+16*c^4*d^3*(b^2*\exp(2)^2+4*c^2*d^2*\exp(2)-4*b*c*d*\exp(1)*\exp(2))*\exp(2)^2+8*c^4*d^2*\sqrt{2}*\sqrt{b*c*\exp(2)^2+c*\sqrt{b^2*\exp(2)^2+4*c^2*d^2*\exp(2)-4*b*c*d*\exp(1)*\exp(2))*\exp(2))*\sqrt{b^2*\exp(2)^2+4*c^2*d^2*\exp(2)-4*b*c*d*\exp(1)*\exp(2)))*\exp(1)*\exp(2)^2+16*c^4*d^2*(b^2*\exp(2)^2+4*c^2*d^2*\exp(2)-4*b*c*d*\exp(1)*\exp(2))*\exp(1)*\exp(2)^2)/(8*b^6*d^3*\exp(1)^6*\exp(2)^3-16*b^6*d^3*\exp(1)^4*\exp(2)^4+8*b^6*d^3*\exp(1)^2*\exp(2)^5-64*b^5*c*d^4*\exp(1)^7*\exp(2)^2+112*b^5*c*d^4*\exp(1)^5*\exp(2)^3-32*b^5*c*d^4*\exp(1)^3*\exp(2)^4-16*b^5*c*d^4*\exp(1)*\exp(2)^5-16*b^5*c*d^3*\exp(1)^6*\exp(2)^3+32*b^5*c*d^3*\exp(1)^4*\exp(2)^4-16*b^5*c*d^3*\exp(1)^2*\exp(2)^5+128*b^4*c^2*d^5*\exp(1)^8*\exp(2)-64*b^4*c^2*d^5*\exp(1)^6*\exp(2)^2-248*b^4*c^2*d^5*\exp(1)^4*\exp(2)^3+176*b^4*c^2*d^5*\exp(1)^2*\exp(2)^4+8*b^4*c^2*d^5*\exp(2)^5+64*b^4*c^2*d^4*\exp(1)^7*\exp(2)^2-96*b^4*c^2*d^4*\exp(1)^5*\exp(2)^3+32*b^4*c^2*d^4*\exp(1)*\exp(2)^5+8*b^4*c^2*d^3*\exp(1)^6*\exp(2)^3-16*b^4*c^2*d^3*\exp(1)^4*\exp(2)^4+8*b^4*c^2*d^3*\exp(1)^2*\exp(2)^5-512*b^3*c^3*d^6*\exp(1)^7*\exp(2)+832*b^3*c^3*d^6*\exp(1)^5*\exp(2)^2-128*b^3*c^3*d^6*\exp(1)^3*\exp(2)^3-192*b^3*c^3*d^6*\exp(1)*\exp(2)^4-192*b^3*c^3*d^5*\exp(1)^6*\exp(2)^2+368*b^3*c^3*d^5*\exp(1)^4*\exp(2)^3-160*b^3*c^3*d^5*\exp(1)^2*\exp(2)^4-16*b^3*c^3*d^5*\exp(2)^5-32*b^3*c^3*d^4*\exp(1)^7*\exp(2)^2+48*b^3*c^3*d^4*\exp(1)^5*\exp(2)^3-16*b^3*c^3*d^4*\exp(1)*\exp(2)^5+768*b^2*c^4*d^7*\exp(1)^6*\exp(2)-1472*b^2*c^4*d^7*\exp(1)^4*\exp(2)^2+640*b^2*c^4*d^7*\exp(1)^2*\exp(2)^3+64*b^2*c^4*d^7*\exp(2)^4+192*b^2*c^4*d^6*\exp(1)^5*\exp(2)^2-384*b^2*c^4*d^6*\exp(1)^3*\exp(2)^3+192*b^2*c^4*d^6*\exp(1)*\exp(2)^4+96*b^2*c^4*d^5*\exp(1)^6*\exp(2)^2-184*b^2*c^4*d^5*\exp(1)^4*\exp(2)^3+80*b^2*c^4*d^5*\exp(1)^2*\exp(2)^4+8*b^2*c^4*d^5*\exp(2)^5-512*b*c^5*d^8*\exp(1)^5*\exp(2)+1024*b*c^5*d^8*\exp(1)^3*\exp(2)^2-512*b*c^5*d^8*\exp(1)*\exp(2)^3-64*b*c^5*d^7*\exp(1)^4*\exp(2)^2+128*b*c^5*d^7*\exp(1)^2*\exp(2)^3-64*b*c^5*d^7*\exp(2)^4-96*b*c^5*d^6*\exp(1)^5*\exp(2)^2+192*b*c^5*d^6*\exp(1)^3*\exp(2)^3-96*b*c^5*d^6*\exp(1)*\exp(2)^4+128*c^6*d^9*\exp(1)^4*\exp(2)-256*c^6*d^9*\exp(1)^2*\exp(2)^2+128*c^6*d^9*\exp(2)^3+32*c^6*d^7*\exp(1)^4*\exp(2)^2-64*c^6*d^7*\exp(1)^2*\exp(2)^3+32*c^6*d^7*\exp(2)^4)/\text{abs}(c)*\text{atan}(x/\sqrt{-(c^2*\exp(2)^3*b*d^4-2*c^2*\exp(2)^2*b*d^4*\exp(1)^2+c^2*\exp(2)*b*d^4*\exp(1)^4-2*c*\exp(2)^3*b^2*d^3*\exp(1)+4*c*\exp(2)^2*b^2*d^3*\exp(1)^3-2*c*\exp(2)*b^2*d^3*\exp(1)^5+\exp(2)^3*b^3*d^2*\exp(1)^2-2*\exp(2)^2*b^3*d^2*\exp(1)^4+\exp(2)*b^3*d^2*\exp(1)^6-\sqrt{(-(c^2*\exp(2)^3*b*d^4+2*c^2*\exp(2)^2*b*d^4*\exp(1)^2-c^2*\exp(2)*b*d^4*\exp(1)^4+2*c*\exp(2)^3*b^2*d^3*\exp(1)-4*c*\exp(2)^2*b^2*d^3*\exp(1)^3+2*c*\exp(2)*b^2*d^3*\exp(1)^5-\exp(2)^3*b^3*d^2*\exp(1)^2+2*\exp(2)^2*b^3*d^2*\exp(1)^4-\exp(2)*b^3*d^2*\exp(1)^6)*(-(c^2*\exp(2)^3*b*d^4+2*c^2*\exp(2)^2*b*d^4*\exp(1)^2-c^2*\exp(2)*b*d^4*\exp(1)^4+2*c*\exp(2)^3*b^2*d^3*\exp(1)-4*c*\exp(2)^2*b^2*d^3*\exp(1)^3+2*c*\exp(2)*b^2*d^3*\exp(1)^5-\exp(2)^3*b^3*d^2*\exp(1)^2+2*\exp(2)^2*b^3*d^2*\exp(1)^4-\exp(2)*b^3*d^2*\exp(1)^6)}-4*(-c^3*\exp(2)^3*d^4+2*c^3*\exp(2)^2*d^4*\exp(1)^2-c^3*\exp(2)*d^4*\exp(1)^4+2*c^2*\exp(2)^3*b*d^3*\exp(1)-4*c^2*\exp(2)^2*b*d^3*\exp(1)^3+2*c^2*\exp(2)*b*d^3*\exp(1)^5-c*\exp(2)^3*b^2*d^2*\exp(1)^2+2*c*\exp(2)^2*b^2*d^2*\exp(1)^4-c*\exp(2)*b^2*d^2*\exp(1)^6)*(c^3*\exp(2)^2*d^6-2*c^3*\exp(2)*d^6*\exp(1)^2+c^3*d^6*\exp(1)^4-3*c^2*\exp(2)^2*b*d^5*\exp(1)+6*c^2*\exp(2)*b*d^5*\exp(1)^3-3*c^2*b*d^5*\exp(1)^5+3*c*\exp(2)^2*b^2*d^4*\exp(1)^2-6*c*\exp(2)*b^2*d^4*\exp(1)^4+3*c*b^2*d^4*\exp(1)^6-\exp(2)^2*b^3*d^3*\exp(1)^3+2*\exp(2)*b^3*d^3*\exp(1)^5-b^3*d^3*\exp(1)^7)))/2/(-(c^3*\exp(2)^3*d^4+2*c^3*\exp(2)^2*d^4*\exp(1)^2-c^3*\exp(2)*d^4*\exp(1)^4+2*c^2*\exp(2)^3*b*d^3*\exp(1)-4*c^2*\exp(2)^2*b*d^3*\exp(1)^3+2*c^2*\exp(2)*b*d^3*\exp(1)^5-c*\exp(2)^3*b^2*d^2*\exp(1)^2+2*c*\exp(2)^2*b^2*d^2*\exp(1)^4-c*\exp(2)*b^2*d^2*\exp(1)^6)))+(-5*c*\exp(2)*d*\exp(1)^2+c*d*\exp(1)^4+3*\exp(2)*b*\exp(1)^3-b*\exp(1)^5)*1/2/(-(c^2*\exp(2)^2*d^4+2*c^2*\exp(2)*d^4*\exp(1)^2-c^2*d^4*\exp(1)^4+2*c*\exp(2)^2*b*d^3*\exp(1)-4*c*\exp(2)*b*d^3*\exp(1)^3+2*c*b*d^3*\exp(1)^5-\exp(2)^2*b^2*d^2*\exp(1)^2+2*\exp(2)*b^2*d^2*\exp(1)^4-b^2*d^2*\exp(1)^6)/\sqrt{d*\exp(1)}*\text{atan}(x*\exp(1)/\sqrt{d*\exp(1)})-x*\exp(1)^2/(-2*c*\exp(2)*d^3+2*c*d^3*\exp(1)^2+2*\exp(2)*b*d^2*\exp(1)-2*b*d^2*\exp(1)^3)/(x^2*\exp(1)+d)$

**maple [A]** time = 0.01, size = 319, normalized size = 1.71

$$\frac{3b^2e^3x^3}{8(be-2cd)^3(e^2+d)^2d^2} - \frac{2bce^2x^3}{(be-2cd)^3(e^2+d)^2d} + \frac{5c^2e^2x^3}{2(be-2cd)^3(e^2+d)^2} + \frac{5b^2e^2x}{8(be-2cd)^3(e^2+d)^2d} - \frac{3bcex}{(be-2cd)^3(e^2+d)^2} - \frac{c^3 \arctan\left(\frac{cx}{\sqrt{be-cd}}\right)}{(be-2cd)^3\sqrt{be-cd}} + \frac{7c^2dx}{2(be-2cd)^3(e^2+d)^2} + \frac{3b^2d^2 \arctan\left(\frac{cx}{\sqrt{be-cd}}\right)}{8(be-2cd)^3\sqrt{be-cd}} - \frac{2bce \arctan\left(\frac{cx}{\sqrt{be-cd}}\right)}{(be-2cd)^3\sqrt{be-cd}} + \frac{7c^2 \arctan\left(\frac{cx}{\sqrt{be-cd}}\right)}{2(be-2cd)^3\sqrt{be-cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(1/(e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2), x)$

[Out]  $-c^3/(b*e-2*c*d)^3/((b*e-c*d)*c*e)^{(1/2)}*\arctan(1/((b*e-c*d)*c*e)^{(1/2)}*c*e*x)+3/8/(b*e-2*c*d)^3/(e*x^2+d)^2*e^3/d^2*x^3*b^2-2/(b*e-2*c*d)^3/(e*x^2+d)^2*e^2/d*x^3*b*c+5/2/(b*e-2*c*d)^3/(e*x^2+d)^2*e*x^3*c^2+5/8/(b*e-2*c*d)^3/(e*x^2+d)^2/d*x*b^2*e^2-3/(b*e-2*c*d)^3/(e*x^2+d)^2*x*b*c*e+7/2/(b*e-2*c*d)^3/(e*x^2+d)^2*d*x*c^2+3/8/(b*e-2*c*d)^3/d^2/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)}*e*x)*b^2*e^2-2/(b*e-2*c*d)^3/d/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)}*e*x)*b*c*e+7/2/(b*e-2*c*d)^3/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)}*e*x)*c^2$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(1/(e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2), x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(b\*e-c\*d>0)', see 'assume?' for more details)Is b\*e-c\*d positive or negative?

**mupad** [B] time = 6.45, size = 6267, normalized size = 33.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(1/((d + e*x^2)^2*(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e)), x)$

[Out]  $((x*(5*b*e - 14*c*d))/(8*d*(b^2*e^2 + 4*c^2*d^2 - 4*b*c*d*e)) + (e*x^3*(3*b*e - 10*c*d))/(8*d^2*(b^2*e^2 + 4*c^2*d^2 - 4*b*c*d*e)))/(d^2 + e^2*x^4 + 2*d*e*x^2) - (\text{atan}(\frac{(x*(9*b^4*c^3*e^{10} + 848*c^7*d^4*e^6 - 896*b*c^6*d^3*e^7 - 96*b^3*c^4*d*e^9 + 424*b^2*c^5*d^2*e^8))}{(64*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)) - ((576*c^{10}*d^{10}*e^6 - 2144*b*c^9*d^9*e^7 + 3504*b^2*c^8*d^8*e^8 - 3288*b^3*c^7*d^7*e^9 + 1940*b^4*c^6*d^6*e^{10} - 738*b^5*c^5*d^5*e^{11} + 177*b^6*c^4*d^4*e^{12} - (49*b^7*c^3*d^3*e^{13})/2 + (3*b^8*c^2*d^2*e^{14})/2)/(2*(64*c^6*d^{10} + b^6*d^4*e^6 - 12*b^5*c*d^5*e^5 + 240*b^2*c^4*d^8*e^2 - 160*b^3*c^3*d^7*e^3 + 60*b^4*c^2*d^6*e^4 - 192*b*c^5*d^9*e)) - (x*(-c^5*e*(b*e - c*d))^{(1/2)}*(16384*b*c^8*d^{10}*e^8 - 49152*b^2*c^7*d^9*e^9 + 61440*b^3*c^6*d^8*e^{10} - 40960*b^4*c^5*d^7*e^{11} + 15360*b^5*c^4*d^6*e^{12} - 3072*b^6*c^3*d^5*e^{13} + 256*b^7*c^2*d^4*e^{14})/(128*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)*(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)))*(-c^5*e*(b*e - c*d))^{(1/2)})/(2*(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)))*(-c^5*e*(b*e - c*d))^{(1/2)}*i)/(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4) + ((x*(9*b^4*c^3*e^{10} + 848*c^7*d^4*e^6 - 896*b*c^6*d^3*e^7 - 96*b^3*c^4*d*e^9 + 424*b^2*c^5*d^2*e^8))/(64*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)) + ((576*c^{10}*d^{10}*e^6 - 2144*b*c^9*d^9*e^7 + 3504*b^2*c^8*d^8*e^8 - 3288*b^3*c^7*d^7*e^9 + 1940*b^4*c^6*d^6*e^{10} - 738*b^5*c^5*d^5*e^{11} + 177*b^6*c^4*d^4*e^{12} - (49*b^7*c^3*d^3*e^{13})/2 + (3*b^8*c^2*d^2*e^{14})/2)/(2*(64*c^6*d^{10} + b^6*d^4*e^6 - 12*b^5*c*d^5*e^5 + 240*b^2*c^4*d^8*e^2 - 160*b^3*c^3*d^7*e^3 + 60*b^4*c^2*d^6*e^4 - 192*b*c^5*d^9*e)) + (x*(-c^5*e*(b*e - c*d))^{(1/2)}*(16384*b*c^8*d^{10}*e^8 - 49152*b^2*c^7*d^9*e^9 + 61440*b^3*c^6*d^8*e^{10} - 40960*b^4*c^5*d^7*e^{11} + 15360*b^5*c^4*d^6*e^{12} - 3072*b^6*c^3*d^5*e^{13} + 256*b^7*c^2*d^4*e^{14})/(128*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)*(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18$

$$\begin{aligned}
& *b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)) * (-c^5*e*(b*e - c*d))^{(1/2)} / (2*(b^4*e^5 \\
& + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)) * ( \\
& -c^5*e*(b*e - c*d))^{(1/2)} * i) / (b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 1 \\
& 8*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)) / (((9*b^3*c^5*e^8)/32 - (35*c^8*d^3*e^5) \\
& /4 + (61*b*c^7*d^2*e^6)/8 - (39*b^2*c^6*d*e^7)/16) / (64*c^6*d^10 + b^6*d^4*e \\
& ^6 - 12*b^5*c*d^5*e^5 + 240*b^2*c^4*d^8*e^2 - 160*b^3*c^3*d^7*e^3 + 60*b^4*c \\
& ^2*d^6*e^4 - 192*b*c^5*d^9*e) + (((x*(9*b^4*c^3*e^10 + 848*c^7*d^4*e^6 - 8 \\
& 96*b*c^6*d^3*e^7 - 96*b^3*c^4*d*e^9 + 424*b^2*c^5*d^2*e^8)) / (64*(16*c^4*d^8 \\
& + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)) - \\
& (((576*c^10*d^10*e^6 - 2144*b*c^9*d^9*e^7 + 3504*b^2*c^8*d^8*e^8 - 3288*b^3 \\
& *c^7*d^7*e^9 + 1940*b^4*c^6*d^6*e^10 - 738*b^5*c^5*d^5*e^11 + 177*b^6*c^4*d \\
& ^4*e^12 - (49*b^7*c^3*d^3*e^13)/2 + (3*b^8*c^2*d^2*e^14)/2) / (2*(64*c^6*d^10 \\
& + b^6*d^4*e^6 - 12*b^5*c*d^5*e^5 + 240*b^2*c^4*d^8*e^2 - 160*b^3*c^3*d^7*e \\
& ^3 + 60*b^4*c^2*d^6*e^4 - 192*b*c^5*d^9*e)) - (x*(-c^5*e*(b*e - c*d))^{(1/2)} \\
& *(16384*b*c^8*d^10*e^8 - 49152*b^2*c^7*d^9*e^9 + 61440*b^3*c^6*d^8*e^10 - 4 \\
& 0960*b^4*c^5*d^7*e^11 + 15360*b^5*c^4*d^6*e^12 - 3072*b^6*c^3*d^5*e^13 + 25 \\
& 6*b^7*c^2*d^4*e^14)) / (128*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24* \\
& b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)) * (b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 \\
& + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)) * (-c^5*e*(b*e - c*d))^{(1/2)} / (2*(b^ \\
& 4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4 \\
& )) * (-c^5*e*(b*e - c*d))^{(1/2)} / (b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + \\
& 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4) - (((x*(9*b^4*c^3*e^10 + 848*c^7*d^4*e^6 \\
& ^6 - 896*b*c^6*d^3*e^7 - 96*b^3*c^4*d*e^9 + 424*b^2*c^5*d^2*e^8)) / (64*(16*c \\
& ^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7* \\
& e)) + (((576*c^10*d^10*e^6 - 2144*b*c^9*d^9*e^7 + 3504*b^2*c^8*d^8*e^8 - 32 \\
& 88*b^3*c^7*d^7*e^9 + 1940*b^4*c^6*d^6*e^10 - 738*b^5*c^5*d^5*e^11 + 177*b^6 \\
& *c^4*d^4*e^12 - (49*b^7*c^3*d^3*e^13)/2 + (3*b^8*c^2*d^2*e^14)/2) / (2*(64*c^ \\
& 6*d^10 + b^6*d^4*e^6 - 12*b^5*c*d^5*e^5 + 240*b^2*c^4*d^8*e^2 - 160*b^3*c^3 \\
& *d^7*e^3 + 60*b^4*c^2*d^6*e^4 - 192*b*c^5*d^9*e)) + (x*(-c^5*e*(b*e - c*d)) \\
& ^{(1/2)} * (16384*b*c^8*d^10*e^8 - 49152*b^2*c^7*d^9*e^9 + 61440*b^3*c^6*d^8*e^ \\
& 10 - 40960*b^4*c^5*d^7*e^11 + 15360*b^5*c^4*d^6*e^12 - 3072*b^6*c^3*d^5*e^1 \\
& 3 + 256*b^7*c^2*d^4*e^14)) / (128*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 \\
& + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)) * (b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d \\
& ^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)) * (-c^5*e*(b*e - c*d))^{(1/2)} / \\
& (2*(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c \\
& *d*e^4)) * (-c^5*e*(b*e - c*d))^{(1/2)} / (b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3 \\
& *e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)) * (-c^5*e*(b*e - c*d))^{(1/2)} * i) \\
& / (b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d \\
& *e^4) - (atan((((x*(9*b^4*c^3*e^10 + 848*c^7*d^4*e^6 - 896*b*c^6*d^3*e^7 - \\
& 96*b^3*c^4*d*e^9 + 424*b^2*c^5*d^2*e^8)) / (32*(16*c^4*d^8 + b^4*d^4*e^4 - 8 \\
& *b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)) - (((576*c^10*d^10*e \\
& ^6 - 2144*b*c^9*d^9*e^7 + 3504*b^2*c^8*d^8*e^8 - 3288*b^3*c^7*d^7*e^9 + 194 \\
& 0*b^4*c^6*d^6*e^10 - 738*b^5*c^5*d^5*e^11 + 177*b^6*c^4*d^4*e^12 - (49*b^7* \\
& c^3*d^3*e^13)/2 + (3*b^8*c^2*d^2*e^14)/2) / (64*c^6*d^10 + b^6*d^4*e^6 - 12*b \\
& ^5*c*d^5*e^5 + 240*b^2*c^4*d^8*e^2 - 160*b^3*c^3*d^7*e^3 + 60*b^4*c^2*d^6*e \\
& ^4 - 192*b*c^5*d^9*e) - (x*(-d^5*e))^{(1/2)} * (3*b^2*e^2 + 28*c^2*d^2 - 16*b*c \\
& *d*e) * (16384*b*c^8*d^10*e^8 - 49152*b^2*c^7*d^9*e^9 + 61440*b^3*c^6*d^8*e^10 \\
& - 40960*b^4*c^5*d^7*e^11 + 15360*b^5*c^4*d^6*e^12 - 3072*b^6*c^3*d^5*e^13 \\
& + 256*b^7*c^2*d^4*e^14)) / (512*(8*c^3*d^8*e - b^3*d^5*e^4 - 12*b*c^2*d^7*e^2 \\
& + 6*b^2*c*d^6*e^3) * (16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^ \\
& 2*d^6*e^2 - 32*b*c^3*d^7*e)) * (-d^5*e)^{(1/2)} * (3*b^2*e^2 + 28*c^2*d^2 - 16*b \\
& *c*d*e)) / (16*(8*c^3*d^8*e - b^3*d^5*e^4 - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^6*e^ \\
& 3))) * (-d^5*e)^{(1/2)} * (3*b^2*e^2 + 28*c^2*d^2 - 16*b*c*d*e) * i) / (16*(8*c^3*d^ \\
& 8*e - b^3*d^5*e^4 - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^6*e^3)) + (((x*(9*b^4*c^3* \\
& e^10 + 848*c^7*d^4*e^6 - 896*b*c^6*d^3*e^7 - 96*b^3*c^4*d*e^9 + 424*b^2*c^5 \\
& *d^2*e^8)) / (32*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6 \\
& *e^2 - 32*b*c^3*d^7*e)) + (((576*c^10*d^10*e^6 - 2144*b*c^9*d^9*e^7 + 3504* \\
& b^2*c^8*d^8*e^8 - 3288*b^3*c^7*d^7*e^9 + 1940*b^4*c^6*d^6*e^10 - 738*b^5*c^ \\
& 5*d^5*e^11 + 177*b^6*c^4*d^4*e^12 - (49*b^7*c^3*d^3*e^13)/2 + (3*b^8*c^2*d^
\end{aligned}$$



$$3.162 \quad \int \frac{(d+ex^2)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4} dx$$

**Optimal.** Leaf size=139

$$-\frac{(2cd-be)^{3/2} \tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{c^2\sqrt{e}\sqrt{cd-be}} + \frac{(5cd-2be) \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}} + \frac{x\sqrt{d+ex^2}}{2c}$$

**Rubi [A]** time = 0.28, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$ , Rules used = {1149, 416, 523, 217, 206, 377, 208}

$$-\frac{(2cd-be)^{3/2} \tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{c^2\sqrt{e}\sqrt{cd-be}} + \frac{(5cd-2be) \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}} + \frac{x\sqrt{d+ex^2}}{2c}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^(5/2)/(-(c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4),x]

[Out] (x\*sqrt[d + e\*x^2])/(2\*c) + ((5\*c\*d - 2\*b\*e)\*ArcTanh[(sqrt[e]\*x)/sqrt[d + e\*x^2]])/(2\*c^2\*sqrt[e]) - ((2\*c\*d - b\*e)^(3/2)\*ArcTanh[(sqrt[e]\*sqrt[2\*c\*d - b\*e]\*x)/(sqrt[c\*d - b\*e]\*sqrt[d + e\*x^2])])/(c^2\*sqrt[e]\*sqrt[c\*d - b\*e])

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

Rule 416

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q-1))/(b\*(n\*(p+q) + 1)), x] + Dist[1/(b\*(n\*(p+q) + 1)), Int[(a + b\*x^n)^p\*(c + d\*x^n)^(q-2)\*Simp[c\*(b\*c\*(n\*(p+q) + 1) - a\*d) + d\*(b\*c\*(n\*(p+2\*q-1) + 1) - a\*d\*(n\*(q-1) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && GtQ[q, 1] && NeQ[n\*(p+q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 523

Int[((e\_) + (f\_.)\*(x\_)^(n\_))/((a\_) + (b\_.)\*(x\_)^(n\_))\*sqrt[(c\_) + (d\_.)\*(x\_)^(n\_)], x\_Symbol] := Dist[f/b, Int[1/Sqrt[c + d\*x^n], x], x] + Dist[(b\*e

- a\*f)/b, Int[1/((a + b\*x^n)\*Sqrt[c + d\*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

### Rule 1149

Int[((d\_) + (e\_)\*(x\_)^2)^(q\_)\*((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4)^(p\_), x\_Symbol] := Int[(d + e\*x^2)^(p + q)\*(a/d + (c\*x^2)/e)^p, x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IntegerQ[p]

### Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)^{5/2}}{-cd^2 + bde + be^2x^2 + ce^2x^4} dx &= \int \frac{(d + ex^2)^{3/2}}{\frac{-cd^2 + bde}{d} + cex^2} dx \\ &= \frac{x\sqrt{d + ex^2}}{2c} + \frac{\int \frac{de(3cd - be) + e^2(5cd - 2be)x^2}{\sqrt{d + ex^2} \left( \frac{-cd^2 + bde}{d} + cex^2 \right)} dx}{2ce} \\ &= \frac{x\sqrt{d + ex^2}}{2c} + \frac{(5cd - 2be) \int \frac{1}{\sqrt{d + ex^2}} dx}{2c^2} + \frac{(2cd - be)^2 \int \frac{1}{\sqrt{d + ex^2} \left( \frac{-cd^2 + bde}{d} + cex^2 \right)} dx}{c^2} \\ &= \frac{x\sqrt{d + ex^2}}{2c} + \frac{(5cd - 2be) \text{Subst} \left( \int \frac{1}{1 - ex^2} dx, x, \frac{x}{\sqrt{d + ex^2}} \right)}{2c^2} + \frac{(2cd - be)^2 \text{Subst} \left( \int \frac{1}{1 - ex^2} dx, x, \frac{x}{\sqrt{d + ex^2}} \right)}{c^2} \\ &= \frac{x\sqrt{d + ex^2}}{2c} + \frac{(5cd - 2be) \tanh^{-1} \left( \frac{\sqrt{e}x}{\sqrt{d + ex^2}} \right)}{2c^2\sqrt{e}} - \frac{(2cd - be)^{3/2} \tanh^{-1} \left( \frac{\sqrt{e}\sqrt{2cd - be}}{\sqrt{cd - be}\sqrt{d + ex^2}} \right)}{c^2\sqrt{e}\sqrt{cd - be}} \end{aligned}$$

**Mathematica [A]** time = 0.26, size = 134, normalized size = 0.96

$$\frac{(2be - 5cd) \log(\sqrt{e}\sqrt{d + ex^2} + ex)}{\sqrt{e}} - \frac{2(be - 2cd)^{3/2} \tanh^{-1} \left( \frac{\sqrt{e}x\sqrt{be - 2cd}}{\sqrt{d + ex^2}\sqrt{be - cd}} \right)}{\sqrt{e}\sqrt{be - cd}} - cx\sqrt{d + ex^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^(5/2)/(-(c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4), x]

[Out] -1/2\*(-(c\*x\*Sqrt[d + e\*x^2]) - (2\*(-2\*c\*d + b\*e)^(3/2)\*ArcTanh[(Sqrt[e]\*Sqrt[-2\*c\*d + b\*e]\*x)/(Sqrt[-(c\*d) + b\*e]\*Sqrt[d + e\*x^2])])/(Sqrt[e]\*Sqrt[-(c\*d) + b\*e]) + ((-5\*c\*d + 2\*b\*e)\*Log[e\*x + Sqrt[e]\*Sqrt[d + e\*x^2]])/Sqrt[e])/c^2

**IntegrateAlgebraic [A]** time = 0.38, size = 179, normalized size = 1.29

$$\frac{(2cd - be)\sqrt{b^2e^2 - 3bcde + 2c^2d^2} \tanh^{-1} \left( \frac{-be + c\sqrt{e}x\sqrt{d + ex^2} + cd - cex^2}{\sqrt{b^2e^2 - 3bcde + 2c^2d^2}} \right)}{c^2\sqrt{e}(cd - be)} + \frac{(2be - 5cd) \log(\sqrt{d + ex^2} - \sqrt{e}x)}{2c^2\sqrt{e}} + \frac{x\sqrt{d + ex^2}}{2c}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e\*x^2)^(5/2)/(-(c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4), x]

```
[Out] (x*Sqrt[d + e*x^2])/(2*c) - ((2*c*d - b*e)*Sqrt[2*c^2*d^2 - 3*b*c*d*e + b^2
*e^2]*ArcTanh[(c*d - b*e - c*e*x^2 + c*Sqrt[e]*x*Sqrt[d + e*x^2])/Sqrt[2*c^
2*d^2 - 3*b*c*d*e + b^2*e^2]])/(c^2*Sqrt[e]*(c*d - b*e)) + ((-5*c*d + 2*b*e
)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(2*c^2*Sqrt[e])
```

**fricas** [A] time = 2.74, size = 1079, normalized size = 7.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="f
ricas")
```

```
[Out] [1/4*(2*sqrt(e*x^2 + d)*c*e*x - (5*c*d - 2*b*e)*sqrt(e)*log(-2*e*x^2 + 2*sq
rt(e*x^2 + d)*sqrt(e)*x - d) - (2*c*d*e - b*e^2)*sqrt((2*c*d - b*e)/(c*d*e
- b*e^2))*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b
*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*
x^2 + 4*((3*c^2*d^2*e^2 - 5*b*c*d*e^3 + 2*b^2*e^4)*x^3 + (c^2*d^3*e - 2*b*c
*d^2*e^2 + b^2*d*e^3)*x)*sqrt(e*x^2 + d)*sqrt((2*c*d - b*e)/(c*d*e - b*e^2
)))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^
2))/(c^2*e), 1/4*(2*sqrt(e*x^2 + d)*c*e*x - 2*(5*c*d - 2*b*e)*sqrt(-e)*arct
an(sqrt(-e)*x/sqrt(e*x^2 + d)) - (2*c*d*e - b*e^2)*sqrt((2*c*d - b*e)/(c*d*
e - b*e^2))*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24
*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3
)*x^2 + 4*((3*c^2*d^2*e^2 - 5*b*c*d*e^3 + 2*b^2*e^4)*x^3 + (c^2*d^3*e - 2*b
*c*d^2*e^2 + b^2*d*e^3)*x)*sqrt(e*x^2 + d)*sqrt((2*c*d - b*e)/(c*d*e - b*e^
2)))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x
^2))/(c^2*e), 1/4*(2*sqrt(e*x^2 + d)*c*e*x + 2*(2*c*d*e - b*e^2)*sqrt(-(2*
c*d - b*e)/(c*d*e - b*e^2))*arctan(1/2*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2)
*x^2)*sqrt(e*x^2 + d)*sqrt(-(2*c*d - b*e)/(c*d*e - b*e^2)))/((2*c*d*e - b*e^
2)*x^3 + (2*c*d^2 - b*d*e)*x)) - (5*c*d - 2*b*e)*sqrt(e)*log(-2*e*x^2 + 2*s
qrt(e*x^2 + d)*sqrt(e)*x - d))/(c^2*e), 1/2*(sqrt(e*x^2 + d)*c*e*x - (5*c*d
- 2*b*e)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + (2*c*d*e - b*e^2)*s
qrt(-(2*c*d - b*e)/(c*d*e - b*e^2))*arctan(1/2*(c*d^2 - b*d*e + (3*c*d*e -
2*b*e^2)*x^2)*sqrt(e*x^2 + d)*sqrt(-(2*c*d - b*e)/(c*d*e - b*e^2)))/((2*c*d*
e - b*e^2)*x^3 + (2*c*d^2 - b*d*e)*x)))/(c^2*e)]
```

**giac** [A] time = 2.39, size = 54, normalized size = 0.39

$$-\frac{(5cd - 2be)e^{\left(-\frac{1}{2}\right)} \log\left(\left(xe^{\frac{1}{2}} - \sqrt{x^2e + d}\right)^2\right)}{4c^2} + \frac{\sqrt{x^2e + d}x}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="g
iac")
```

```
[Out] -1/4*(5*c*d - 2*b*e)*e^(-1/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c^2 + 1/
2*sqrt(x^2*e + d)*x/c
```

**maple** [B] time = 0.06, size = 7043, normalized size = 50.67

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(5/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] result too large to display
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{5}{2}}}{ce^2x^4 + be^2x^2 - cd^2 + bde} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(5/2)/(c\*e^2\*x^4+b\*e^2\*x^2+b\*d\*e-c\*d^2),x, algorithm="maxima")

[Out] integrate((e\*x^2 + d)^(5/2)/(c\*e^2\*x^4 + b\*e^2\*x^2 - c\*d^2 + b\*d\*e), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(ex^2 + d)^{5/2}}{-cd^2 + bde + ce^2x^4 + be^2x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)^(5/2)/(b\*e^2\*x^2 - c\*d^2 + c\*e^2\*x^4 + b\*d\*e),x)

[Out] int((d + e\*x^2)^(5/2)/(b\*e^2\*x^2 - c\*d^2 + c\*e^2\*x^4 + b\*d\*e), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^2)^{\frac{3}{2}}}{be - cd + cex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*(5/2)/(c\*e\*\*2\*x\*\*4+b\*e\*\*2\*x\*\*2+b\*d\*e-c\*d\*\*2),x)

[Out] Integral((d + e\*x\*\*2)\*\*(3/2)/(b\*e - c\*d + c\*e\*x\*\*2), x)



$$3.163 \quad \int \frac{(d+ex^2)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4} dx$$

**Optimal.** Leaf size=108

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}} - \frac{\sqrt{2cd-be} \tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{c\sqrt{e}\sqrt{cd-be}}$$

**Rubi [A]** time = 0.13, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$ , Rules used = {1149, 402, 217, 206, 377, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}} - \frac{\sqrt{2cd-be} \tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{c\sqrt{e}\sqrt{cd-be}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^(3/2)/(-(c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4),x]

[Out] ArcTanh[(Sqrt[e]\*x)/Sqrt[d + e\*x^2]]/(c\*Sqrt[e]) - (Sqrt[2\*c\*d - b\*e]\*ArcTanh[(Sqrt[e]\*Sqrt[2\*c\*d - b\*e]\*x)/(Sqrt[c\*d - b\*e]\*Sqrt[d + e\*x^2])])/(c\*Sqrt[e]\*Sqrt[c\*d - b\*e])

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

Rule 402

Int[((a\_) + (b\_.)\*(x\_)^2)^(p\_)/((c\_) + (d\_.)\*(x\_)^2), x\_Symbol] :> Dist[b/d, Int[(a + b\*x^2)^(p - 1), x], x] - Dist[(b\*c - a\*d)/d, Int[(a + b\*x^2)^(p - 1)/(c + d\*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])

Rule 1149

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[(d + e\*x^2)^(p + q)\*(a/d + (c\*x^2)/e)^p, x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4} dx &= \int \frac{\sqrt{d+ex^2}}{\frac{-cd^2+bde}{d}+ce^2x^2} dx \\
&= \frac{\int \frac{1}{\sqrt{d+ex^2}} dx}{c} - \frac{\left(-cde + \frac{e(-cd^2+bde)}{d}\right) \int \frac{1}{\sqrt{d+ex^2} \left(\frac{-cd^2+bde}{d}+ce^2x^2\right)} dx}{ce} \\
&= \frac{\text{Subst}\left(\int \frac{1}{1-ex^2} dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{c} - \frac{\left(-cde + \frac{e(-cd^2+bde)}{d}\right) \text{Subst}\left(\int \frac{1}{\frac{-cd^2+bde}{d} - \left(-cde + \frac{e(-cd^2+bde)}{d}\right)} dx\right)}{ce} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}} - \frac{\sqrt{2cd-be} \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{2cd-bex}}{\sqrt{cd-be}\sqrt{d+ex^2}}\right)}{c\sqrt{e}\sqrt{cd-be}}
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 103, normalized size = 0.95

$$\frac{\frac{\sqrt{be-2cd} \tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{be-2cd}}{\sqrt{d+ex^2}\sqrt{be-cd}}\right)}{\sqrt{be-cd}} - \log\left(\sqrt{e}\sqrt{d+ex^2} + ex\right)}{c\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^(3/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4), x]
[Out] -(((Sqrt[-2*c*d + b*e]*ArcTanh[(Sqrt[e]*Sqrt[-2*c*d + b*e]*x)/(Sqrt[-(c*d) + b*e]*Sqrt[d + e*x^2])])/Sqrt[-(c*d) + b*e] - Log[e*x + Sqrt[e]*Sqrt[d + e*x^2])]/(c*Sqrt[e]))
```

**IntegrateAlgebraic [B]** time = 0.26, size = 217, normalized size = 2.01

$$\frac{\frac{\sqrt{b^2e^2-3bcde+2c^2d^2} \tanh^{-1}\left(-\frac{ce^2}{\sqrt{b^2e^2-3bcde+2c^2d^2}} + \frac{c\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{b^2e^2-3bcde+2c^2d^2}} + \frac{cd}{\sqrt{b^2e^2-3bcde+2c^2d^2}} - \frac{be}{\sqrt{b^2e^2-3bcde+2c^2d^2}}\right)}{c\sqrt{e}(cd-be)} - \log\left(\sqrt{d+ex^2} - \sqrt{ex}\right)}{c\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x^2)^(3/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4), x]
[Out] -(((Sqrt[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2]*ArcTanh[(c*d)/Sqrt[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2] - (b*e)/Sqrt[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2] - (c*e*x^2)/Sqrt[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2] + (c*Sqrt[e]*x*Sqrt[d + e*x^2])/Sqrt[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2]])/(c*Sqrt[e]*(c*d - b*e))) - Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]]/(c*Sqrt[e]))
```

**fricas [A]** time = 1.04, size = 940, normalized size = 8.70

$$\frac{\sqrt{be-2cd} \tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{be-2cd}}{\sqrt{d+ex^2}\sqrt{be-cd}}\right) - \log\left(\sqrt{e}\sqrt{d+ex^2} + ex\right)}{c\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2), x, algorithm="fricas")
```

```
[Out] [1/4*(e*sqrt((2*c*d - b*e)/(c*d*e - b*e^2))*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*
```

$$e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2 - 4*((3*c^2*d^2*e^2 - 5*b*c*d*e^3 + 2*b^2*e^4)*x^3 + (c^2*d^3*e - 2*b*c*d^2*e^2 + b^2*d*e^3)*x)*\sqrt{e*x^2 + d}*\sqrt{(2*c*d - b*e)/(c*d*e - b*e^2)))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^2)) + 2*\sqrt{e}*\log(-2*e*x^2 - 2*\sqrt{e*x^2 + d}*\sqrt{e}*x - d))/(c*e), 1/4*(e*\sqrt{(2*c*d - b*e)/(c*d*e - b*e^2)})*\log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2 - 4*((3*c^2*d^2*e^2 - 5*b*c*d*e^3 + 2*b^2*e^4)*x^3 + (c^2*d^3*e - 2*b*c*d^2*e^2 + b^2*d*e^3)*x)*\sqrt{e*x^2 + d}*\sqrt{(2*c*d - b*e)/(c*d*e - b*e^2)))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^2)) - 4*\sqrt{-e}*\arctan(\sqrt{-e}*x/\sqrt{e*x^2 + d}))/c), 1/2*(e*\sqrt{-(2*c*d - b*e)/(c*d*e - b*e^2)})*\arctan(1/2*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2)*x^2)*\sqrt{e*x^2 + d}*\sqrt{-(2*c*d - b*e)/(c*d*e - b*e^2)})/((2*c*d*e - b*e^2)*x^3 + (2*c*d^2 - b*d*e)*x)) + \sqrt{e}*\log(-2*e*x^2 - 2*\sqrt{e*x^2 + d}*\sqrt{e}*x - d))/(c*e), 1/2*(e*\sqrt{-(2*c*d - b*e)/(c*d*e - b*e^2)})*\arctan(1/2*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2)*x^2)*\sqrt{e*x^2 + d}*\sqrt{-(2*c*d - b*e)/(c*d*e - b*e^2)})/((2*c*d*e - b*e^2)*x^3 + (2*c*d^2 - b*d*e)*x)) - 2*\sqrt{-e}*\arctan(\sqrt{-e}*x/\sqrt{e*x^2 + d}))/c)]$$

**giac** [A] time = 2.39, size = 27, normalized size = 0.25

$$\frac{e^{\left(-\frac{1}{2}\right)} \log\left(\left(xe^{\frac{1}{2}} - \sqrt{x^2e + d}\right)^2\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)/(c\*e^2\*x^4+b\*e^2\*x^2+b\*d\*e-c\*d^2),x, algorithm="giac")

[Out] -1/2\*e^(-1/2)\*log((x\*e^(1/2) - sqrt(x^2\*e + d))^2)/c

**maple** [B] time = 0.02, size = 4308, normalized size = 39.89

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^(3/2)/(c\*e^2\*x^4+b\*e^2\*x^2+b\*d\*e-c\*d^2),x)

[Out] -1/6\*c^2\*e/((-d\*e)^(1/2)\*c+(-(b\*e-c\*d)\*c\*e)^(1/2))/((-d\*e)^(1/2)\*c+(-(b\*e-c\*d)\*c\*e)^(1/2))/(-(b\*e-c\*d)\*c\*e)^(1/2)\*((x+(-(b\*e-c\*d)\*c\*e)^(1/2)/c/e)^2\*e-2\*(-(b\*e-c\*d)\*c\*e)^(1/2)/c\*(x+(-(b\*e-c\*d)\*c\*e)^(1/2)/c/e)-(b\*e-2\*c\*d)/c)^(3/2)+1/4\*c\*e/((-d\*e)^(1/2)\*c+(-(b\*e-c\*d)\*c\*e)^(1/2))/((-d\*e)^(1/2)\*c+(-(b\*e-c\*d)\*c\*e)^(1/2))\*((x+(-(b\*e-c\*d)\*c\*e)^(1/2)/c/e)^2\*e-2\*(-(b\*e-c\*d)\*c\*e)^(1/2)/c\*(x+(-(b\*e-c\*d)\*c\*e)^(1/2)/c/e)-(b\*e-2\*c\*d)/c)^(1/2)\*x+5/4\*c\*e^(1/2)/((-d\*e)^(1/2)\*c+(-(b\*e-c\*d)\*c\*e)^(1/2))/((-d\*e)^(1/2)\*c+(-(b\*e-c\*d)\*c\*e)^(1/2))\*ln((-(-(b\*e-c\*d)\*c\*e)^(1/2)/c+(x+(-(b\*e-c\*d)\*c\*e)^(1/2)/c/e)\*e)/e^(1/2)+((x+(-(b\*e-c\*d)\*c\*e)^(1/2)/c/e)^2\*e-2\*(-(b\*e-c\*d)\*c\*e)^(1/2)/c\*(x+(-(b\*e-c\*d)\*c\*e)^(1/2)/c/e)-(b\*e-2\*c\*d)/c)^(1/2))\*d+1/2\*c\*e^2/((-d\*e)^(1/2)\*c+(-(b\*e-c\*d)\*c\*e)^(1/2))/((-d\*e)^(1/2)\*c+(-(b\*e-c\*d)\*c\*e)^(1/2))/(-(b\*e-c\*d)\*c\*e)^(1/2))\*((x+(-(b\*e-c\*d)\*c\*e)^(1/2)/c/e)^2\*e-2\*(-(b\*e-c\*d)\*c\*e)^(1/2)/c\*(x+(-(b\*e-c\*d)\*c\*e)^(1/2)/c/e)-(b\*e-2\*c\*d)/c)^(1/2)\*b-c^2\*e/((-d\*e)^(1/2)\*c+(-(b\*e-c\*d)\*c\*e)^(1/2))/((-d\*e)^(1/2)\*c+(-(b\*e-c\*d)\*c\*e)^(1/2))/(-(b\*e-c\*d)\*c\*e)^(1/2))\*((x+(-(b\*e-c\*d)\*c\*e)^(1/2)/c/e)^2\*e-2\*(-(b\*e-c\*d)\*c\*e)^(1/2)/c\*(x+(-(b\*e-c\*d)\*c\*e)^(1/2)/c/e)-(b\*e-2\*c\*d)/c)^(1/2)\*d-1/2\*e^(3/2)/((-d\*e)^(1/2)\*c+(-(b\*e-c\*d)\*c\*e)^(1/2))/((-d\*e)^(1/2)\*c+(-(b\*e-c\*d)\*c\*e)^(1/2))\*ln((-(-(b\*e-c\*d)\*c\*e)^(1/2)/c+(x+(-(b\*e-c\*d)\*c\*e)^(1/2)/c/e)\*e)/e^(1/2)+((x+(-(b\*e-c\*d)\*c\*e)^(1/2)/c/e)^2\*e-2\*(-(b\*e-c\*d)\*c\*e)^(1/2)/c\*(x+(-(b\*e-c\*d)\*c\*e)^(1/2)/c/e)-(b\*e-2\*c\*d)/c)^(1/2))\*b+1/2\*e^3/((-d\*e)^(1/2)\*c+(-(b\*e-c\*d)\*c\*e)^(1/2))/((-d\*e)^(1/2)\*c+(-(b\*e-c\*d)\*c\*e)^(1/2))/(-(b\*e-c\*d)\*c\*e)^(1/2))/((-d\*e)^(1/2)\*c+(-(b\*e-c\*d)\*c\*e)^(1/2))/(-(b\*e-c\*d)\*c\*e)^(1/2))/



$$\frac{1}{2} * c + (- (b * e - c * d) * c * e)^{(1/2)} / (- (b * e - c * d) * c * e)^{(1/2)} / (- (b * e - 2 * c * d) / c)^{(1/2)} * \ln \left( \frac{- 2 * (b * e - 2 * c * d) / c + 2 * (- (b * e - c * d) * c * e)^{(1/2)} / c * (x - (- (b * e - c * d) * c * e)^{(1/2)} / c / e) + 2 * (- (b * e - 2 * c * d) / c)^{(1/2)} * ((x - (- (b * e - c * d) * c * e)^{(1/2)} / c / e)^2 * e + 2 * (- (b * e - c * d) * c * e)^{(1/2)} / c * (x - (- (b * e - c * d) * c * e)^{(1/2)} / c / e) - (b * e - 2 * c * d) / c)^{(1/2)}}{(x - (- (b * e - c * d) * c * e)^{(1/2)} / c / e)} \right) * d^2$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}}{ce^2x^4 + be^2x^2 - cd^2 + bde} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)/(c\*e^2\*x^4+b\*e^2\*x^2+b\*d\*e-c\*d^2),x, algorithm="maxima")

[Out] integrate((e\*x^2 + d)^(3/2)/(c\*e^2\*x^4 + b\*e^2\*x^2 - c\*d^2 + b\*d\*e), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(ex^2 + d)^{3/2}}{-cd^2 + bde + ce^2x^4 + be^2x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)^(3/2)/(b\*e^2\*x^2 - c\*d^2 + c\*e^2\*x^4 + b\*d\*e),x)

[Out] int((d + e\*x^2)^(3/2)/(b\*e^2\*x^2 - c\*d^2 + c\*e^2\*x^4 + b\*d\*e), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d + ex^2}}{be - cd + cex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*(3/2)/(c\*e\*\*2\*x\*\*4+b\*e\*\*2\*x\*\*2+b\*d\*e-c\*d\*\*2),x)

[Out] Integral(sqrt(d + e\*x\*\*2)/(b\*e - c\*d + c\*e\*x\*\*2), x)

$$3.164 \quad \int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4} dx$$

**Optimal.** Leaf size=76

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}\sqrt{2cd-be}}$$

**Rubi [A]** time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$ , Rules used = {1149, 377, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}\sqrt{2cd-be}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e\*x^2]/(-(c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4), x]

[Out] -(ArcTanh[(Sqrt[e]\*Sqrt[2\*c\*d - b\*e]\*x)/(Sqrt[c\*d - b\*e]\*Sqrt[d + e\*x^2])]/(Sqrt[e]\*Sqrt[c\*d - b\*e]\*Sqrt[2\*c\*d - b\*e]))

**Rule 208**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

**Rule 377**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

**Rule 1149**

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] := Int[(d + e\*x^2)^(p+q)\*(a/d + (c\*x^2)/e)^p, x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IntegerQ[p]

**Rubi steps**

$$\begin{aligned} \int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4} dx &= \int \frac{1}{\sqrt{d+ex^2} \left( \frac{-cd^2+bde}{d} + cex^2 \right)} dx \\ &= \text{Subst} \left( \int \frac{1}{\frac{-cd^2+bde}{d} - \left( -cde + \frac{e(-cd^2+bde)}{d} \right) x^2} dx, x, \frac{x}{\sqrt{d+ex^2}} \right) \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{2cd-bex}}{\sqrt{cd-be}\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{cd-be}\sqrt{2cd-be}} \end{aligned}$$

**Mathematica [A]** time = 0.07, size = 76, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}\sqrt{2cd-be}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e\*x^2]/(-(c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4), x]

[Out] -(ArcTanh[(Sqrt[e]\*Sqrt[2\*c\*d - b\*e]\*x)/(Sqrt[c\*d - b\*e]\*Sqrt[d + e\*x^2])]/(Sqrt[e]\*Sqrt[c\*d - b\*e]\*Sqrt[2\*c\*d - b\*e]))

**IntegrateAlgebraic [B]** time = 0.19, size = 192, normalized size = 2.53

$$\frac{\sqrt{b^2e^2 - 3bcde + 2c^2d^2} \tanh^{-1}\left(-\frac{cex^2}{\sqrt{b^2e^2 - 3bcde + 2c^2d^2}} + \frac{c\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{b^2e^2 - 3bcde + 2c^2d^2}} + \frac{cd}{\sqrt{b^2e^2 - 3bcde + 2c^2d^2}} - \frac{be}{\sqrt{b^2e^2 - 3bcde + 2c^2d^2}}\right)}{\sqrt{e}(be - 2cd)(be - cd)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[d + e\*x^2]/(-(c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4), x]

[Out] -((Sqrt[2\*c^2\*d^2 - 3\*b\*c\*d\*e + b^2\*e^2]\*ArcTanh[(c\*d)/Sqrt[2\*c^2\*d^2 - 3\*b\*c\*d\*e + b^2\*e^2] - (b\*e)/Sqrt[2\*c^2\*d^2 - 3\*b\*c\*d\*e + b^2\*e^2] - (c\*e\*x^2)/Sqrt[2\*c^2\*d^2 - 3\*b\*c\*d\*e + b^2\*e^2] + (c\*Sqrt[e]\*x\*Sqrt[d + e\*x^2])/Sqrt[2\*c^2\*d^2 - 3\*b\*c\*d\*e + b^2\*e^2])/(Sqrt[e]\*(-2\*c\*d + b\*e)\*(-(c\*d) + b\*e))

**fricas [B]** time = 1.09, size = 432, normalized size = 5.68

$$\left[ \frac{\log\left(\frac{c^2d^4 - 2bcd^2e + b^2d^2e^2 + (17c^2d^2e^2 - 24bcd^2e + 8b^2d^2e^4)x^4 + 2(7c^2d^2e - 11bcd^2e^2 + 4b^2d^2e^3)x^2 - 4\sqrt{2c^2d^2e - 3bcde + b^2e^3}\left((3cd - 2be^2)x^3 + (cd^2 - bde)x\right)\sqrt{ex^2 + d}}{c^2d^2x^4 + c^2d^2e - 2bcde + b^2e^2 - 2(cd - bce^2)x^2}\right)}{4\sqrt{2c^2d^2e - 3bcde + b^2e^3}} - \frac{\sqrt{-2c^2d^2e + 3bcde^2 - b^2e^3} \arctan\left(\frac{-\sqrt{-2c^2d^2e + 3bcde^2 - b^2e^3}(cd^2 - bde + (3cd - 2be^2)x^2)\sqrt{ex^2 + d}}{2((2c^2d^2e - 3bcde + b^2e^4)x^3 + (2c^2d^2e - 3bcde^2 + b^2d^2e^3)x)}\right)}{2(2c^2d^2e - 3bcde^2 + b^2e^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(1/2)/(c\*e^2\*x^4+b\*e^2\*x^2+b\*d\*e-c\*d^2), x, algorithm="fricas")

[Out] [1/4\*log((c^2\*d^4 - 2\*b\*c\*d^3\*e + b^2\*d^2\*e^2 + (17\*c^2\*d^2\*e^2 - 24\*b\*c\*d\*e^3 + 8\*b^2\*e^4)\*x^4 + 2\*(7\*c^2\*d^3\*e - 11\*b\*c\*d^2\*e^2 + 4\*b^2\*d\*e^3)\*x^2 - 4\*sqrt(2\*c^2\*d^2\*e - 3\*b\*c\*d\*e^2 + b^2\*e^3)\*((3\*c\*d\*e - 2\*b\*e^2)\*x^3 + (c\*d^2 - b\*d\*e)\*x)\*sqrt(e\*x^2 + d))/(c^2\*e^2\*x^4 + c^2\*d^2 - 2\*b\*c\*d\*e + b^2\*e^2 - 2\*(c^2\*d\*e - b\*c\*e^2)\*x^2))/sqrt(2\*c^2\*d^2\*e - 3\*b\*c\*d\*e^2 + b^2\*e^3), -1/2\*sqrt(-2\*c^2\*d^2\*e + 3\*b\*c\*d\*e^2 - b^2\*e^3)\*arctan(-1/2\*sqrt(-2\*c^2\*d^2\*e + 3\*b\*c\*d\*e^2 - b^2\*e^3)\*(c\*d^2 - b\*d\*e + (3\*c\*d\*e - 2\*b\*e^2)\*x^2)\*sqrt(e\*x^2 + d)/((2\*c^2\*d^2\*e^2 - 3\*b\*c\*d\*e^3 + b^2\*e^4)\*x^3 + (2\*c^2\*d^3\*e - 3\*b\*c\*d^2\*e^2 + b^2\*d\*e^3)\*x))/(2\*c^2\*d^2\*e - 3\*b\*c\*d\*e^2 + b^2\*e^3)]

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(1/2)/(c\*e^2\*x^4+b\*e^2\*x^2+b\*d\*e-c\*d^2), x, algorithm="giac")

[Out] Timed out

**maple [B]** time = 0.02, size = 2252, normalized size = 29.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)`

[Out] 
$$\begin{aligned} & -1/2*c^2*e/((-d*e)^(1/2)*c+(-b*e-c*d)*c*e)^(1/2)/(-(-d*e)^(1/2)*c+(-b*e-c*d)*c*e)^(1/2)/(-b*e-c*d)*c*e)^(1/2)*((x+(-b*e-c*d)*c*e)^(1/2)/c/e)^2*e \\ & -2*(-b*e-c*d)*c*e)^(1/2)/c*(x+(-b*e-c*d)*c*e)^(1/2)/c/e-(b*e-2*c*d)/c)^(1/2)+1/2*c*e^(1/2)/((-d*e)^(1/2)*c+(-b*e-c*d)*c*e)^(1/2)/(-(-d*e)^(1/2)*c \\ & +(-b*e-c*d)*c*e)^(1/2))*ln((-(-b*e-c*d)*c*e)^(1/2)/c+(x+(-b*e-c*d)*c*e)^(1/2)/c/e)*e)/e^(1/2)+((x+(-b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-b*e-c*d)*c*e) \\ & )^(1/2)/c*(x+(-b*e-c*d)*c*e)^(1/2)/c/e-(b*e-2*c*d)/c)^(1/2))-1/2*c*e^2/((-d*e)^(1/2)*c+(-b*e-c*d)*c*e)^(1/2)/(-(-d*e)^(1/2)*c+(-b*e-c*d)*c*e)^(1/2) \\ & ))/(-b*e-c*d)*c*e)^(1/2)/(-b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c-2*(-b*e-c*d)*c*e)^(1/2)/c*(x+(-b*e-c*d)*c*e)^(1/2)/c/e)+2*(-b*e-2*c*d)/c)^(1/2) \\ & )*((x+(-b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-b*e-c*d)*c*e)^(1/2)/c*(x+(-b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/((x+(-b*e-c*d)*c*e)^(1/2)/c/e) \\ & )*b+c^2*e/((-d*e)^(1/2)*c+(-b*e-c*d)*c*e)^(1/2)/(-(-d*e)^(1/2)*c+(-b*e-c*d)*c*e)^(1/2))/(-b*e-c*d)*c*e)^(1/2)/(-b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2 \\ & *c*d)/c-2*(-b*e-c*d)*c*e)^(1/2)/c*(x+(-b*e-c*d)*c*e)^(1/2)/c/e)+2*(-b*e-2*c*d)/c)^(1/2)*((x+(-b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-b*e-c*d)*c*e)^(1/2) \\ & )/c*(x+(-b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/((x+(-b*e-c*d)*c*e)^(1/2)/c/e))*d-1/2*c*e/(-d*e)^(1/2)/((-d*e)^(1/2)*c+(-b*e-c*d)*c*e)^(1/2) \\ & )/(-(-d*e)^(1/2)*c+(-b*e-c*d)*c*e)^(1/2))*((x-(-d*e)^(1/2)/e)^2*e+2*(-d*e)^(1/2)*(x-(-d*e)^(1/2)/e))^(1/2)-1/2*c*e^(1/2)/((-d*e)^(1/2)*c+(-b*e-c*d)* \\ & c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-b*e-c*d)*c*e)^(1/2))*ln(((x-(-d*e)^(1/2)/e) \\ & *e+(-d*e)^(1/2))/e^(1/2)+((x-(-d*e)^(1/2)/e)^2*e+2*(-d*e)^(1/2)*(x-(-d*e)^(1/2) \\ & /e))^(1/2))+1/2*c*e/(-d*e)^(1/2)/((-d*e)^(1/2)*c+(-b*e-c*d)*c*e)^(1/2) \\ & )/(-(-d*e)^(1/2)*c+(-b*e-c*d)*c*e)^(1/2))*((x+(-d*e)^(1/2)/e)^2*e-2*(-d*e)^(1/2) \\ & *(x+(-d*e)^(1/2)/e))^(1/2)-1/2*c*e^(1/2)/((-d*e)^(1/2)*c+(-b*e-c*d)* \\ & c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-b*e-c*d)*c*e)^(1/2))*ln(((x+(-d*e)^(1/2)/e) \\ & *e-(-d*e)^(1/2))/e^(1/2)+((x+(-d*e)^(1/2)/e)^2*e-2*(-d*e)^(1/2)*(x+(-d*e)^(1/2) \\ & /e))^(1/2))+1/2*c^2*e/((-d*e)^(1/2)*c+(-b*e-c*d)*c*e)^(1/2)/(-(-d*e)^(1/2) \\ & *c+(-b*e-c*d)*c*e)^(1/2))/(-b*e-c*d)*c*e)^(1/2)*((x-(-b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-b*e-c*d)*c*e)^(1/2)/c*(x-(-b*e-c*d)*c*e)^(1/2)/c/e)-( \\ & b*e-2*c*d)/c)^(1/2)+1/2*c*e^(1/2)/((-d*e)^(1/2)*c+(-b*e-c*d)*c*e)^(1/2))/(- \\ & (-d*e)^(1/2)*c+(-b*e-c*d)*c*e)^(1/2))*ln((-(-b*e-c*d)*c*e)^(1/2)/c+(x-(-b \\ & *e-c*d)*c*e)^(1/2)/c/e)*e)/e^(1/2)+((x-(-b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(- \\ & (-b*e-c*d)*c*e)^(1/2)/c*(x-(-b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2) \\ & )+1/2*c*e^2/((-d*e)^(1/2)*c+(-b*e-c*d)*c*e)^(1/2)/(-(-d*e)^(1/2)*c+(-b*e \\ & -c*d)*c*e)^(1/2))/(-b*e-c*d)*c*e)^(1/2)/(-b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e \\ & -2*c*d)/c+2*(-b*e-c*d)*c*e)^(1/2)/c*(x-(-b*e-c*d)*c*e)^(1/2)/c/e)+2*(-b \\ & e-2*c*d)/c)^(1/2)*((x-(-b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-b*e-c*d)*c*e)^(1 \\ & /2)/c*(x-(-b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/((x-(-b*e-c*d)*c \\ & *e)^(1/2)/c/e))*b-c^2*e/((-d*e)^(1/2)*c+(-b*e-c*d)*c*e)^(1/2)/(-(-d*e)^(1 \\ & /2)*c+(-b*e-c*d)*c*e)^(1/2))/(-b*e-c*d)*c*e)^(1/2)/(-b*e-2*c*d)/c)^(1/2) \\ & *ln((-2*(b*e-2*c*d)/c+2*(-b*e-c*d)*c*e)^(1/2)/c*(x-(-b*e-c*d)*c*e)^(1/2) \\ & /c/e)+2*(-b*e-2*c*d)/c)^(1/2)*((x-(-b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-b*e \\ & -c*d)*c*e)^(1/2)/c*(x-(-b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/((x- \\ & (-b*e-c*d)*c*e)^(1/2)/c/e))*d \end{aligned}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2 + d}}{ce^2x^4 + be^2x^2 - cd^2 + bde} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxima")`

[Out] `integrate(sqrt(e*x^2 + d)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e), x)`



**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{ex^2 + d}}{-cd^2 + bde + ce^2x^4 + be^2x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)^(1/2)/(b\*e^2\*x^2 - c\*d^2 + c\*e^2\*x^4 + b\*d\*e), x)

[Out] int((d + e\*x^2)^(1/2)/(b\*e^2\*x^2 - c\*d^2 + c\*e^2\*x^4 + b\*d\*e), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d + ex^2} (be - cd + cex^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*(1/2)/(c\*e\*\*2\*x\*\*4+b\*e\*\*2\*x\*\*2+b\*d\*e-c\*d\*\*2), x)

[Out] Integral(1/(sqrt(d + e\*x\*\*2)\*(b\*e - c\*d + c\*e\*x\*\*2)), x)

$$3.165 \quad \int \frac{1}{\sqrt{d+ex^2}(-cd^2+bde+be^2x^2+ce^2x^4)} dx$$

**Optimal.** Leaf size=106

$$-\frac{x}{d\sqrt{d+ex^2}(2cd-be)} - \frac{c \tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^{3/2}}$$

**Rubi [A]** time = 0.12, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$ , Rules used = {1149, 382, 377, 208}

$$-\frac{x}{d\sqrt{d+ex^2}(2cd-be)} - \frac{c \tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[d + e\*x^2]\*(-c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4)),x]

[Out] -(x/(d\*(2\*c\*d - b\*e)\*Sqrt[d + e\*x^2])) - (c\*ArcTanh[(Sqrt[e]\*Sqrt[2\*c\*d - b\*e]\*x)/(Sqrt[c\*d - b\*e]\*Sqrt[d + e\*x^2])])/(Sqrt[e]\*Sqrt[c\*d - b\*e]\*(2\*c\*d - b\*e)^(3/2))

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

#### Rule 382

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(b\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(p+1)\*(b\*c - a\*d)), x] + Dist[(b\*c + n\*(p+1)\*(b\*c - a\*d))/(a\*n\*(p+1)\*(b\*c - a\*d)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p+q+2)+1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]

#### Rule 1149

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] := Int[(d + e\*x^2)^(p+q)\*(a/d + (c\*x^2)/e)^p, x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IntegerQ[p]

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{d+ex^2}(-cd^2+bde+be^2x^2+ce^2x^4)} dx &= \int \frac{1}{(d+ex^2)^{3/2} \left( \frac{-cd^2+bde}{d} + cex^2 \right)} dx \\
&= -\frac{x}{d(2cd-be)\sqrt{d+ex^2}} + \frac{c \int \frac{1}{\sqrt{d+ex^2} \left( \frac{-cd^2+bde}{d} + cex^2 \right)} dx}{2cd-be} \\
&= -\frac{x}{d(2cd-be)\sqrt{d+ex^2}} + \frac{c \operatorname{Subst} \left( \int \frac{1}{\frac{-cd^2+bde}{d} - \left( -cde + \frac{e(-cd^2+bde)}{d} \right) x^2} dx \right)}{2cd-be} \\
&= -\frac{x}{d(2cd-be)\sqrt{d+ex^2}} - \frac{c \tanh^{-1} \left( \frac{\sqrt{e} \sqrt{2cd-be} x}{\sqrt{cd-be} \sqrt{d+ex^2}} \right)}{\sqrt{e} \sqrt{cd-be} (2cd-be)^{3/2}}
\end{aligned}$$

**Mathematica [C]** time = 1.05, size = 418, normalized size = 3.94

$$\frac{x \left( -\frac{2cex^2 \left( \frac{ex^2(be-2cd)}{(d+ex^2)(be-cd)} \right)^{5/2}}{cd-be} {}_2F_1 \left( 2, \frac{5}{2}, \frac{7}{2}, \frac{e(be-2cd)x^2}{(be-cd)(ex^2+d)} \right) + 2 \left( \frac{ex^2(be-2cd)}{(d+ex^2)(be-cd)} \right)^{5/2} {}_2F_1 \left( 2, \frac{5}{2}, \frac{7}{2}, \frac{e(be-2cd)x^2}{(be-cd)(ex^2+d)} \right) + \frac{10cex^2 \sqrt{\frac{ex^2(be-2cd)}{(d+ex^2)(be-cd)}}}{cd-be} - 15 \sqrt{\frac{ex^2(be-2cd)}{(d+ex^2)(be-cd)}} - \frac{10cex^2 \tanh^{-1} \left( \sqrt{\frac{ex^2(be-2cd)}{(d+ex^2)(be-cd)}} \right)}{cd-be} + 15 \tanh^{-1} \left( \sqrt{\frac{ex^2(be-2cd)}{(d+ex^2)(be-cd)}} \right) \right)}{5(d+ex^2)^{3/2}(cd-be) \left( \frac{ex^2(be-2cd)}{(d+ex^2)(be-cd)} \right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)), x]
[Out] -1/5*(x*(-15*Sqrt[(e*(-2*c*d + b*e))*x^2]/((-c*d) + b*e)*(d + e*x^2))] + (1
0*c*e*x^2*Sqrt[(e*(-2*c*d + b*e))*x^2]/((-c*d) + b*e)*(d + e*x^2))]/(c*d -
b*e) + 15*ArcTanh[Sqrt[(e*(-2*c*d + b*e))*x^2]/((-c*d) + b*e)*(d + e*x^2)
]] - (10*c*e*x^2*ArcTanh[Sqrt[(e*(-2*c*d + b*e))*x^2]/((-c*d) + b*e)*(d + e
*x^2)]]]/(c*d - b*e) + 2*((e*(-2*c*d + b*e))*x^2)/((-c*d) + b*e)*(d + e*x^
2))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, (e*(-2*c*d + b*e))*x^2]/((-c*d) +
b*e)*(d + e*x^2))] - (2*c*e*x^2*((e*(-2*c*d + b*e))*x^2)/((-c*d) + b*e)*(d
+ e*x^2))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, (e*(-2*c*d + b*e))*x^2]/((-
c*d) + b*e)*(d + e*x^2)]]]/(c*d - b*e))/((c*d - b*e)*((e*(-2*c*d + b*e))*x
^2)/((-c*d) + b*e)*(d + e*x^2))^(3/2)*(d + e*x^2)^(3/2))
```

**IntegrateAlgebraic [B]** time = 0.37, size = 221, normalized size = 2.08

$$\frac{c\sqrt{b^2e^2-3bcde+2c^2d^2} \tanh^{-1} \left( -\frac{cex^2}{\sqrt{b^2e^2-3bcde+2c^2d^2}} + \frac{c\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{b^2e^2-3bcde+2c^2d^2}} + \frac{cd}{\sqrt{b^2e^2-3bcde+2c^2d^2}} - \frac{be}{\sqrt{b^2e^2-3bcde+2c^2d^2}} \right)}{\sqrt{e}(be-2cd)^2(be-cd)} - \frac{x}{d\sqrt{d+ex^2}(2cd-be)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[d + e*x^2]*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2
*x^4)), x]
[Out] -(x/(d*(2*c*d - b*e)*Sqrt[d + e*x^2])) + (c*Sqrt[2*c^2*d^2 - 3*b*c*d*e + b^
2*e^2]*ArcTanh[(c*d)/Sqrt[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2] - (b*e)/Sqrt[2*c
^2*d^2 - 3*b*c*d*e + b^2*e^2] - (c*e*x^2)/Sqrt[2*c^2*d^2 - 3*b*c*d*e + b^2*
e^2] + (c*Sqrt[e]*x*Sqrt[d + e*x^2])/Sqrt[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2]
)/(Sqrt[e]*(-2*c*d + b*e)^2*(-(c*d) + b*e))
```

**fricas [B]** time = 1.27, size = 701, normalized size = 6.61

$$\frac{4 \left( 2c^2de - 3bcd^2 + b^2e^2 \right) \sqrt{cd^2 + dx + \sqrt{2} \sqrt{2} \sqrt{e} \sqrt{d+ex^2}} + \sqrt{2} \sqrt{2} \sqrt{e} \sqrt{d+ex^2} \log \left( \frac{2d^2 - 2cd^2 - 2b^2e^2 - (17c^2d^2 - 21bcd^2 + 11b^2e^2 + 2d^2) \sqrt{d+ex^2} + \sqrt{2} \sqrt{2} \sqrt{e} \sqrt{d+ex^2} \sqrt{cd^2 + dx + \sqrt{2} \sqrt{2} \sqrt{e} \sqrt{d+ex^2}}}{4(4c^2de - 8bc^2d^2 + 5b^2e^2) \sqrt{cd^2 + dx + \sqrt{2} \sqrt{2} \sqrt{e} \sqrt{d+ex^2}}} \right)}{4(4c^2de - 8bc^2d^2 + 5b^2e^2) \sqrt{cd^2 + dx + \sqrt{2} \sqrt{2} \sqrt{e} \sqrt{d+ex^2}}} - \frac{2(2c^2de - 3bcd^2 + b^2e^2) \sqrt{cd^2 + dx + \sqrt{2} \sqrt{2} \sqrt{e} \sqrt{d+ex^2}} + 3bcd^2 - b^2e^2 (cd^2 + dx) \arctan \left( \frac{\sqrt{2} \sqrt{2} \sqrt{e} \sqrt{d+ex^2}}{2(2c^2de - 3bcd^2 + b^2e^2)} \right)}{2(2c^2de - 3bcd^2 + b^2e^2) \sqrt{cd^2 + dx + \sqrt{2} \sqrt{2} \sqrt{e} \sqrt{d+ex^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(1/(e\*x^2+d)^(1/2)/(c\*e^2\*x^4+b\*e^2\*x^2+b\*d\*e-c\*d^2),x, algorithm="maxima")

[Out] integrate(1/((c\*e^2\*x^4 + b\*e^2\*x^2 - c\*d^2 + b\*d\*e)\*sqrt(e\*x^2 + d)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{e x^2 + d} (-c d^2 + b d e + c e^2 x^4 + b e^2 x^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d + e\*x^2)^(1/2)\*(b\*e^2\*x^2 - c\*d^2 + c\*e^2\*x^4 + b\*d\*e)),x)

[Out] int(1/((d + e\*x^2)^(1/2)\*(b\*e^2\*x^2 - c\*d^2 + c\*e^2\*x^4 + b\*d\*e)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + e x^2)^{\frac{3}{2}} (b e - c d + c e x^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x\*\*2+d)\*\*(1/2)/(c\*e\*\*2\*x\*\*4+b\*e\*\*2\*x\*\*2+b\*d\*e-c\*d\*\*2),x)

[Out] Integral(1/((d + e\*x\*\*2)\*\*(3/2)\*(b\*e - c\*d + c\*e\*x\*\*2)), x)

$$3.166 \quad \int \frac{1}{(d+ex^2)^{3/2}(-cd^2+bde+be^2x^2+ce^2x^4)} dx$$

**Optimal.** Leaf size=149

$$\frac{c^2 \tanh^{-1}\left(\frac{\sqrt{ex}\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^{5/2}} - \frac{x(7cd-2be)}{3d^2\sqrt{d+ex^2}(2cd-be)^2} - \frac{x}{3d(d+ex^2)^{3/2}(2cd-be)}$$

**Rubi [A]** time = 0.27, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$ , Rules used = {1149, 414, 527, 12, 377, 208}

$$\frac{c^2 \tanh^{-1}\left(\frac{\sqrt{ex}\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^{5/2}} - \frac{x(7cd-2be)}{3d^2\sqrt{d+ex^2}(2cd-be)^2} - \frac{x}{3d(d+ex^2)^{3/2}(2cd-be)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x^2)^(3/2)\*(-(c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4)),x]

[Out] -x/(3\*d\*(2\*c\*d - b\*e)\*(d + e\*x^2)^(3/2)) - ((7\*c\*d - 2\*b\*e)\*x)/(3\*d^2\*(2\*c\*d - b\*e)^2\*Sqrt[d + e\*x^2]) - (c^2\*ArcTanh[(Sqrt[e]\*Sqrt[2\*c\*d - b\*e]\*x)/(Sqrt[c\*d - b\*e]\*Sqrt[d + e\*x^2])])/(Sqrt[e]\*Sqrt[c\*d - b\*e]\*(2\*c\*d - b\*e)^(5/2))

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

#### Rule 414

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(b\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(p+1)\*(b\*c - a\*d)), x] + Dist[1/(a\*n\*(p+1)\*(b\*c - a\*d)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q\*Simp[b\*c + n\*(p+1)\*(b\*c - a\*d) + d\*b\*(n\*(p+q+2)+1)\*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

#### Rule 527

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.)\*((e\_) + (f\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*e - a\*f)\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(b\*c - a\*d)\*(p+1)), x] + Dist[1/(a\*n\*(b\*c - a\*d)\*(p+1)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q\*Simp[c\*(b\*e - a\*f) + e\*n\*(b\*c - a\*d)\*(p+1) + d\*(b\*e - a\*f)\*(n\*(p+q+2)+1)\*x^n, x], x] /; FreeQ

[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

### Rule 1149

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.),  
x\_Symbol] := Int[(d + e\*x^2)^(p + q)\*(a/d + (c\*x^2)/e)^p, x] /; FreeQ[{a,  
b, c, d, e, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0]  
&& IntegerQ[p]

### Rubi steps

$$\begin{aligned} \int \frac{1}{(d + ex^2)^{3/2} (-cd^2 + bde + be^2x^2 + ce^2x^4)} dx &= \int \frac{1}{(d + ex^2)^{5/2} \left( \frac{-cd^2 + bde}{d} + cex^2 \right)} dx \\ &= -\frac{x}{3d(2cd - be)(d + ex^2)^{3/2}} + \frac{\int \frac{e(5cd - 2be) - 2ce^2x^2}{(d + ex^2)^{3/2} \left( \frac{-cd^2 + bde}{d} + cex^2 \right)} dx}{3de(2cd - be)} \\ &= -\frac{x}{3d(2cd - be)(d + ex^2)^{3/2}} - \frac{(7cd - 2be)x}{3d^2(2cd - be)^2 \sqrt{d + ex^2}} + \dots \\ &= -\frac{x}{3d(2cd - be)(d + ex^2)^{3/2}} - \frac{(7cd - 2be)x}{3d^2(2cd - be)^2 \sqrt{d + ex^2}} + \dots \\ &= -\frac{x}{3d(2cd - be)(d + ex^2)^{3/2}} - \frac{(7cd - 2be)x}{3d^2(2cd - be)^2 \sqrt{d + ex^2}} + \dots \\ &= -\frac{x}{3d(2cd - be)(d + ex^2)^{3/2}} - \frac{(7cd - 2be)x}{3d^2(2cd - be)^2 \sqrt{d + ex^2}} - \dots \end{aligned}$$

**Mathematica [C]** time = 4.14, size = 1058, normalized size = 7.10



Warning: Unable to verify antiderivative.

[In] Integrate[1/((d + e\*x^2)^(3/2)\*(-(c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4)),  
x]

[Out] -1/63\*(x\*(-315\*sqrt[(e\*(-2\*c\*d + b\*e)\*x^2)/((-c\*d) + b\*e)\*(d + e\*x^2)]) +  
(420\*c\*e\*x^2\*sqrt[(e\*(-2\*c\*d + b\*e)\*x^2)/((-c\*d) + b\*e)\*(d + e\*x^2)])/(c\*d  
- b\*e) - (168\*c^2\*e^2\*x^4\*sqrt[(e\*(-2\*c\*d + b\*e)\*x^2)/((-c\*d) + b\*e)\*(d  
+ e\*x^2)])/(c\*d - b\*e)^2 - 105\*((e\*(-2\*c\*d + b\*e)\*x^2)/((-c\*d) + b\*e)\*(d  
+ e\*x^2))^3/2 + (140\*c\*e\*x^2\*((e\*(-2\*c\*d + b\*e)\*x^2)/((-c\*d) + b\*e)\*(d  
+ e\*x^2))^3/2)/(c\*d - b\*e) - (56\*c^2\*e^2\*x^4\*((e\*(-2\*c\*d + b\*e)\*x^2)/((-  
c\*d) + b\*e)\*(d + e\*x^2))^3/2)/(c\*d - b\*e)^2 + 315\*ArcTanh[Sqrt[(e\*(-2\*c  
\*d + b\*e)\*x^2)/((-c\*d) + b\*e)\*(d + e\*x^2)]] - (420\*c\*e\*x^2\*ArcTanh[Sqrt[(  
e\*(-2\*c\*d + b\*e)\*x^2)/((-c\*d) + b\*e)\*(d + e\*x^2)]])/(c\*d - b\*e) + (168\*c^2  
\*e^2\*x^4\*ArcTanh[Sqrt[(e\*(-2\*c\*d + b\*e)\*x^2)/((-c\*d) + b\*e)\*(d + e\*x^2)]]  
)/(c\*d - b\*e)^2 + 48\*((e\*(-2\*c\*d + b\*e)\*x^2)/((-c\*d) + b\*e)\*(d + e\*x^2))

$(7/2) \cdot \text{Hypergeometric2F1}[2, 7/2, 9/2, (e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e) * (d + e*x^2)] - (84*c*e*x^2 * ((e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e) * (d + e*x^2)))^{(7/2)} \cdot \text{Hypergeometric2F1}[2, 7/2, 9/2, (e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e) * (d + e*x^2)] / (c*d - b*e) + (36*c^2*e^2*x^4 * ((e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e) * (d + e*x^2)))^{(7/2)} \cdot \text{Hypergeometric2F1}[2, 7/2, 9/2, (e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e) * (d + e*x^2)] / (c*d - b*e)^2 + 12 * ((e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e) * (d + e*x^2))^{(7/2)} \cdot \text{HypergeometricPFQ}[\{2, 2, 7/2\}, \{1, 9/2\}, (e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e) * (d + e*x^2)] - (24*c*e*x^2 * ((e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e) * (d + e*x^2)))^{(7/2)} \cdot \text{HypergeometricPFQ}[\{2, 2, 7/2\}, \{1, 9/2\}, (e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e) * (d + e*x^2)] / (c*d - b*e) + (12*c^2*e^2*x^4 * ((e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e) * (d + e*x^2)))^{(7/2)} \cdot \text{HypergeometricPFQ}[\{2, 2, 7/2\}, \{1, 9/2\}, (e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e) * (d + e*x^2)] / (c*d - b*e)^2) / ((c*d - b*e) * ((e*(-2*c*d + b*e)*x^2)/((-c*d) + b*e) * (d + e*x^2))^{(5/2)} * (d + e*x^2)^{(5/2)})$

**IntegrateAlgebraic [A]** time = 0.57, size = 256, normalized size = 1.72

$$\frac{3bdex + 2be^2x^3 - 9cd^2x - 7cdex^3}{3d^2(d + ex^2)^{3/2}(2cd - be)^2} - \frac{c^2\sqrt{b^2e^2 - 3bcde + 2c^2d^2} \tanh^{-1}\left(-\frac{cex^2}{\sqrt{b^2e^2 - 3bcde + 2c^2d^2}} + \frac{c\sqrt{ex}\sqrt{d+ex^2}}{\sqrt{b^2e^2 - 3bcde + 2c^2d^2}} + \frac{cd}{\sqrt{b^2e^2 - 3bcde + 2c^2d^2}} - \frac{be}{\sqrt{b^2e^2 - 3bcde + 2c^2d^2}}\right)}{\sqrt{e}(be - 2cd)^3(be - cd)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((d + e\*x^2)^(3/2)\*(-(c\*d^2) + b\*d\*e + b\*e^2\*x^2 + c\*e^2\*x^4)), x]

[Out]  $(-9*c*d^2*x + 3*b*d*e*x - 7*c*d*e*x^3 + 2*b*e^2*x^3)/(3*d^2*(2*c*d - b*e)^2 * (d + e*x^2)^{(3/2)}) - (c^2*\text{Sqrt}[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2]*\text{ArcTanh}[(c*d)/\text{Sqrt}[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2] - (b*e)/\text{Sqrt}[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2] - (c*e*x^2)/\text{Sqrt}[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2] + (c*\text{Sqrt}[e]*x*\text{Sqrt}[d + e*x^2)]/\text{Sqrt}[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2]])/(\text{Sqrt}[e]*(-2*c*d + b*e)^3*(-(c*d) + b*e))$

**fricas [B]** time = 2.65, size = 1063, normalized size = 7.13

$$\frac{3(3c^2d^2e^2x^4 + 2c^2d^3ex^2 + c^2d^4)\sqrt{2c^2d^2e - 3b^2c^2d^2e^2 + b^2e^3} \log((c^2d^4 - 2b^2c^2d^3e + b^2d^2e^2 + (17c^2d^2e^2 - 24b^2c^2d^2e^3 + 8b^2e^4)x^4 + 2(7c^2d^3e - 11b^2c^2d^2e^2 + 4b^2d^2e^3)x^2 - 4\sqrt{2c^2d^2e - 3b^2c^2d^2e^2 + b^2e^3} * ((3c^2d^2e - 2b^2e^2)x^3 + (c^2d^2 - b^2d^2e)x) * \sqrt{e*x^2 + d}) / (c^2e^2x^4 + c^2d^2 - 2b^2c^2d^2e + b^2e^2 - 2(c^2d^2e - b^2c^2e^2)x^2)) - 4((14c^3d^3e^2 - 25b^2c^2d^2e^3 + 13b^2c^2d^2e^4 - 2b^3e^5)x^3 + 3(6c^3d^4e - 11b^2c^2d^3e^2 + 6b^2c^2d^2e^3 - b^3d^2e^4)x) * \sqrt{e*x^2 + d}) / (8c^4d^8e - 20b^2c^3d^7e^2 + 18b^2c^2d^6e^3 - 7b^3c^2d^5e^4 + b^4d^4e^5 + (8c^4d^6e^3 - 20b^2c^3d^5e^4 + 18b^2c^2d^4e^5 - 7b^3c^2d^3e^6 + b^4d^2e^7)x^4 + 2(8c^4d^7e^2 - 20b^2c^3d^6e^3 + 18b^2c^2d^5e^4 - 7b^3c^2d^4e^5 + b^4d^3e^6)x^2), -1/6(3(c^2d^2e^2x^4 + 2c^2d^3ex^2 + c^2d^4)\sqrt{-2c^2d^2e + 3b^2c^2d^2e^2 - b^2e^3} * \arctan(-1/2\sqrt{-2c^2d^2e + 3b^2c^2d^2e^2 - b^2e^3} * (c^2d^2 - b^2d^2e + (3c^2d^2e - 2b^2e^2)x^2) * \sqrt{e*x^2 + d}) / ((2c^2d^2e^2 - 3b^2c^2d^2e^3 + b^2e^4)x^3 + (2c^2d^3e - 3b^2c^2d^2e^2 + b^2d^2e^3)x) + 2((14c^3d^3e^2 - 25b^2c^2d^2e^3 + 13b^2c^2d^2e^4 - 2b^3e^5)x^3 + 3(6c^3d^4e - 11b^2c^2d^3e^2 + 6b^2c^2d^2e^3 - b^3d^2e^4)x) * \sqrt{e*x^2 + d}) / (8c^4d^8e - 20b^2c^3d^7e^2 + 18b^2c^2d^6e^3 - 7b^3c^2d^5e^4 + b^4d^4e^5 + (8c^4d^6e^3 - 20b^2c^3d^5e^4 + 18b^2c^2d^4e^5 - 7b^3c^2d^3e^6 + b^4d^2e^7)x^4 + 2(8c^4d^7e^2 - 20b^2c^3d^6e^3 + 18b^2c^2d^5e^4 - 7b^3c^2d^4e^5 + b^4d^3e^6)x^2), -1/6(3(c^2d^2e^2x^4 + 2c^2d^3ex^2 + c^2d^4)\sqrt{-2c^2d^2e + 3b^2c^2d^2e^2 - b^2e^3} * \arctan(-1/2\sqrt{-2c^2d^2e + 3b^2c^2d^2e^2 - b^2e^3} * (c^2d^2 - b^2d^2e + (3c^2d^2e - 2b^2e^2)x^2) * \sqrt{e*x^2 + d}) / ((2c^2d^2e^2 - 3b^2c^2d^2e^3 + b^2e^4)x^3 + (2c^2d^3e - 3b^2c^2d^2e^2 + b^2d^2e^3)x) + 2((14c^3d^3e^2 - 25b^2c^2d^2e^3 + 13b^2c^2d^2e^4 - 2b^3e^5)x^3 + 3(6c^3d^4e - 11b^2c^2d^3e^2 + 6b^2c^2d^2e^3 - b^3d^2e^4)x) * \sqrt{e*x^2 + d}) / (8c^4d^8e - 20b^2c^3d^7e^2 + 18b^2c^2d^6e^3 - 7b^3c^2d^5e^4 + b^4d^4e^5 + (8c^4d^6e^3 - 20b^2c^3d^5e^4 + 18b^2c^2d^4e^5 - 7b^3c^2d^3e^6 + b^4d^2e^7)x^4 + 2(8c^4d^7e^2 - 20b^2c^3d^6e^3 + 18b^2c^2d^5e^4 - 7b^3c^2d^4e^5 + b^4d^3e^6)x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)^(3/2)/(c\*e^2\*x^4+b\*e^2\*x^2+b\*d\*e-c\*d^2), x, algorithm="fricas")

[Out]  $[1/12*(3*(c^2*d^2*e^2*x^4 + 2*c^2*d^3*e*x^2 + c^2*d^4)*\text{sqrt}(2*c^2*d^2*e - 3*b^2*c^2*d^2*e^2 + b^2*e^3) * \log((c^2*d^4 - 2*b^2*c^2*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b^2*c^2*d^2*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b^2*c^2*d^2*e^2 + 4*b^2*d^2*e^3)*x^2 - 4*\text{sqrt}(2*c^2*d^2*e - 3*b^2*c^2*d^2*e^2 + b^2*e^3) * ((3*c^2*d^2*e - 2*b^2*e^2)*x^3 + (c*d^2 - b*d^2*e)*x) * \text{sqrt}(e*x^2 + d)) / (c^2*e^2*x^4 + c^2*d^2 - 2*b^2*c^2*d^2*e + b^2*e^2 - 2*(c^2*d^2*e - b^2*c^2*e^2)*x^2)) - 4*((14*c^3*d^3*e^2 - 25*b^2*c^2*d^2*e^3 + 13*b^2*c^2*d^2*e^4 - 2*b^3*e^5)*x^3 + 3*(6*c^3*d^4*e - 11*b^2*c^2*d^3*e^2 + 6*b^2*c^2*d^2*e^3 - b^3*d^2*e^4)*x) * \text{sqrt}(e*x^2 + d)) / (8*c^4*d^8*e - 20*b^2*c^3*d^7*e^2 + 18*b^2*c^2*d^6*e^3 - 7*b^3*c^2*d^5*e^4 + b^4*d^4*e^5 + (8*c^4*d^6*e^3 - 20*b^2*c^3*d^5*e^4 + 18*b^2*c^2*d^4*e^5 - 7*b^3*c^2*d^3*e^6 + b^4*d^2*e^7)*x^4 + 2*(8*c^4*d^7*e^2 - 20*b^2*c^3*d^6*e^3 + 18*b^2*c^2*d^5*e^4 - 7*b^3*c^2*d^4*e^5 + b^4*d^3*e^6)*x^2), -1/6*(3*(c^2*d^2*e^2*x^4 + 2*c^2*d^3*e*x^2 + c^2*d^4)*\text{sqrt}(-2*c^2*d^2*e + 3*b^2*c^2*d^2*e^2 - b^2*e^3) * \arctan(-1/2*\text{sqrt}(-2*c^2*d^2*e + 3*b^2*c^2*d^2*e^2 - b^2*e^3) * (c^2*d^2 - b^2*d^2*e + (3*c^2*d^2*e - 2*b^2*e^2)*x^2) * \text{sqrt}(e*x^2 + d)) / ((2*c^2*d^2*e^2 - 3*b^2*c^2*d^2*e^3 + b^2*e^4)*x^3 + (2*c^2*d^3*e - 3*b^2*c^2*d^2*e^2 + b^2*d^2*e^3)*x) + 2*((14*c^3*d^3*e^2 - 25*b^2*c^2*d^2*e^3 + 13*b^2*c^2*d^2*e^4 - 2*b^3*e^5)*x^3 + 3*(6*c^3*d^4*e - 11*b^2*c^2*d^3*e^2 + 6*b^2*c^2*d^2*e^3 - b^3*d^2*e^4)*x) * \text{sqrt}(e*x^2 + d)) / (8*c^4*d^8*e - 20*b^2*c^3*d^7*e^2 + 18*b^2*c^2*d^6*e^3 - 7*b^3*c^2*d^5*e^4 + b^4*d^4*e^5 + (8*c^4*d^6*e^3 - 20*b^2*c^3*d^5*e^4 + 18*b^2*c^2*d^4*e^5 - 7*b^3*c^2*d^3*e^6 + b^4*d^2*e^7)*x^4 + 2*(8*c^4*d^7*e^2 - 20*b^2*c^3*d^6*e^3 + 18*b^2*c^2*d^5*e^4 - 7*b^3*c^2*d^4*e^5 + b^4*d^3*e^6)*x^2)$



$$e^3 - 20*b*c^3*d^5*e^4 + 18*b^2*c^2*d^4*e^5 - 7*b^3*c*d^3*e^6 + b^4*d^2*e^7) * x^4 + 2*(8*c^4*d^7*e^2 - 20*b*c^3*d^6*e^3 + 18*b^2*c^2*d^5*e^4 - 7*b^3*c*d^4*e^5 + b^4*d^3*e^6) * x^2]$$

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)^(3/2)/(c\*e^2\*x^4+b\*e^2\*x^2+b\*d\*e-c\*d^2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
 UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [b,c,d,exp(1),exp(2)]=[-21,-18,-46,11,70]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [b,c,d,exp(1),exp(2)]=[72,91,-18,-31,46]Evaluation time: 2.06Unable to transpose Error: Bad Argument Value

**maple** [B] time = 0.02, size = 1637, normalized size = 10.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x^2+d)^(3/2)/(c\*e^2\*x^4+b\*e^2\*x^2+b\*d\*e-c\*d^2),x)

[Out] 
$$\frac{1}{2}c^3e/((-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/(-(-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/(-b*e-c*d)*c*e)^{(1/2)}/(-b*e-c*d)*c*e)^{(1/2)}/(b*e-2*c*d)/((x+(-b*e-c*d)*c*e)^{(1/2)}/c/e)^2*e-2*(-b*e-c*d)*c*e)^{(1/2)}*(x+(-b*e-c*d)*c*e)^{(1/2)}/c/e)/c-(b*e-2*c*d)/c)^{(1/2)}+1/2*c^2*e/((-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/(-(-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/(b*e-2*c*d)/d/((x+(-b*e-c*d)*c*e)^{(1/2)}/c/e)^2*e-2*(-b*e-c*d)*c*e)^{(1/2)}*(x+(-b*e-c*d)*c*e)^{(1/2)}/c/e)/c-(b*e-2*c*d)/c)^{(1/2)}*x-1/2*c^3*e/((-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/(-(-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/(-b*e-c*d)*c*e)^{(1/2)}/(b*e-2*c*d)/(-b*e-2*c*d)/c)^{(1/2)}*ln((-2*(b*e-2*c*d)/c-2*(-b*e-c*d)*c*e)^{(1/2)}*(x+(-b*e-c*d)*c*e)^{(1/2)}/c/e)/c+2*(-b*e-2*c*d)/c)^{(1/2)}*((x+(-b*e-c*d)*c*e)^{(1/2)}/c/e)^2*e-2*(-b*e-c*d)*c*e)^{(1/2)}*(x+(-b*e-c*d)*c*e)^{(1/2)}/c/e)/c-(b*e-2*c*d)/c)^{(1/2)}/(x+(-b*e-c*d)*c*e)^{(1/2)}/c/e))-1/6*c/d/((-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/(-(-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/(x-(-d*e)^{(1/2)}/e)/((x-(-d*e)^{(1/2)}/e)^2*e+2*(-d*e)^{(1/2)}*(x-(-d*e)^{(1/2)}/e))^{(1/2)}-1/3*c*e/d^2/((-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/(-(-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/(x-(-d*e)^{(1/2)}/e)^2*e+2*(-d*e)^{(1/2)}*(x-(-d*e)^{(1/2)}/e))^{(1/2)}*x-1/6*c/d/((-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/(-(-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/(x+(-d*e)^{(1/2)}/e)/((x+(-d*e)^{(1/2)}/e)^2*e-2*(-d*e)^{(1/2)}*(x+(-d*e)^{(1/2)}/e))^{(1/2)}-1/3*c*e/d^2/((-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/(-(-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/((x+(-d*e)^{(1/2)}/e)^2*e-2*(-d*e)^{(1/2)}*(x+(-d*e)^{(1/2)}/e))^{(1/2)}*x-1/2*c^3*e/((-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/(-(-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/(-b*e-c*d)*c*e)^{(1/2)}/(b*e-2*c*d)/((x-(-b*e-c*d)*c*e)^{(1/2)}/c/e)^2*e+2*(-b*e-c*d)*c*e)^{(1/2)}*(x-(-b*e-c*d)*c*e)^{(1/2)}/c/e)/c-(b*e-2*c*d)/c)^{(1/2)}+1/2*c^2*e/((-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/(-(-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/(b*e-2*c*d)/d/((x-(-b*e-c*d)*c*e)^{(1/2)}/c/e)^2*e+2*(-b*e-c*d)*c*e)^{(1/2)}*(x-(-b*e-c*d)*c*e)^{(1/2)}/c/e)/c-(b*e-2*c*d)/c)^{(1/2)}*x+1/2*c^3*e/((-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/(-(-d*e)^{(1/2)}*c+(-b*e-c*d)*c*e)^{(1/2)}/(-b*e-c*d)*c*e)^{(1/2)}/(b*e-2*c*d)/(-b*e-2*c*d)/c)^{(1/2)}*ln((-2*(b*e-2*c*d)/c+2*(-b*e-c*d)*c*e)^{(1/2)}*(x-(-b*e-c*d)*c*e)^{(1/2)}/c/e)/c+2*(-b*e-2*c*d)/c)^{(1/2)}*((x-(-b*e-c*d)*c*e)^{(1/2)}/c/e)^2*e+2*(-b*e-c*d)*c*e)^{(1/2)}*(x-(-b*e-c*d)*c*e)^{(1/2)}/c/e)/c-(b*e-2*c*d)/c)^{(1/2)}/(x-(-b*e-c*d)*c*e)^{(1/2)}/c/e))$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce^2x^4 + be^2x^2 - cd^2 + bde)(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)^(3/2)/(c\*e^2\*x^4+b\*e^2\*x^2+b\*d\*e-c\*d^2),x, algorithm="maxima")

[Out] integrate(1/((c\*e^2\*x^4 + b\*e^2\*x^2 - c\*d^2 + b\*d\*e)\*(e\*x^2 + d)^(3/2)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(ex^2 + d)^{3/2} (-cd^2 + bde + ce^2x^4 + be^2x^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d + e\*x^2)^(3/2)\*(b\*e^2\*x^2 - c\*d^2 + c\*e^2\*x^4 + b\*d\*e)),x)

[Out] int(1/((d + e\*x^2)^(3/2)\*(b\*e^2\*x^2 - c\*d^2 + c\*e^2\*x^4 + b\*d\*e)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex^2)^{\frac{5}{2}} (be - cd + cex^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x\*\*2+d)\*\*(3/2)/(c\*e\*\*2\*x\*\*4+b\*e\*\*2\*x\*\*2+b\*d\*e-c\*d\*\*2),x)

[Out] Integral(1/((d + e\*x\*\*2)\*\*(5/2)\*(b\*e - c\*d + c\*e\*x\*\*2)), x)

$$3.167 \quad \int (d + ex^2)^4 (a + bx^2 + cx^4) dx$$

**Optimal.** Leaf size=135

$$\frac{1}{9}e^2x^9(eae + 4bd) + 6cd^2 + \frac{1}{5}d^2x^5(6ae^2 + 4bde + cd^2) + \frac{2}{7}dex^7(e(2ae + 3bd) + 2cd^2) + \frac{1}{3}d^3x^3(4ae + bd) + ad^4x +$$

**Rubi [A]** time = 0.13, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$ , Rules used = {1153}

$$\frac{1}{9}e^2x^9(eae + 4bd) + 6cd^2 + \frac{1}{5}d^2x^5(6ae^2 + 4bde + cd^2) + \frac{2}{7}dex^7(e(2ae + 3bd) + 2cd^2) + \frac{1}{3}d^3x^3(4ae + bd) + ad^4x + \frac{1}{11}e^3x^{11}(be + 4cd) + \frac{1}{13}ce^4x^{13}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^4\*(a + b\*x^2 + c\*x^4), x]

[Out] a\*d^4\*x + (d^3\*(b\*d + 4\*a\*e)\*x^3)/3 + (d^2\*(c\*d^2 + 4\*b\*d\*e + 6\*a\*e^2)\*x^5)/5 + (2\*d\*e\*(2\*c\*d^2 + e\*(3\*b\*d + 2\*a\*e))\*x^7)/7 + (e^2\*(6\*c\*d^2 + e\*(4\*b\*d + a\*e))\*x^9)/9 + (e^3\*(4\*c\*d + b\*e)\*x^11)/11 + (c\*e^4\*x^13)/13

**Rule 1153**

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^2)^q\*(a + b\*x^2 + c\*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

**Rubi steps**

$$\begin{aligned} \int (d + ex^2)^4 (a + bx^2 + cx^4) dx &= \int (ad^4 + d^3(bd + 4ae)x^2 + d^2(cd^2 + 4bde + 6ae^2)x^4 + 2de(2cd^2 + e(3bd + 4cd) + ae^2)x^6 + d^3e^2x^8 + 2de^3x^{10} + ce^4x^{12}) dx \\ &= ad^4x + \frac{1}{3}d^3(bd + 4ae)x^3 + \frac{1}{5}d^2(cd^2 + 4bde + 6ae^2)x^5 + \frac{2}{7}de(2cd^2 + e(3bd + 4cd) + ae^2)x^7 + \frac{1}{9}d^3e^2x^9 + \frac{2}{11}de^3x^{11} + \frac{1}{13}ce^4x^{13} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 135, normalized size = 1.00

$$\frac{1}{9}e^2x^9(ae^2 + 4bde + 6cd^2) + \frac{2}{7}dex^7(2ae^2 + 3bde + 2cd^2) + \frac{1}{5}d^2x^5(6ae^2 + 4bde + cd^2) + \frac{1}{3}d^3x^3(4ae + bd) + ad^4x + \frac{1}{11}e^3x^{11}(be + 4cd) + \frac{1}{13}ce^4x^{13}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^4\*(a + b\*x^2 + c\*x^4), x]

[Out] a\*d^4\*x + (d^3\*(b\*d + 4\*a\*e)\*x^3)/3 + (d^2\*(c\*d^2 + 4\*b\*d\*e + 6\*a\*e^2)\*x^5)/5 + (2\*d\*e\*(2\*c\*d^2 + 3\*b\*d\*e + 2\*a\*e^2)\*x^7)/7 + (e^2\*(6\*c\*d^2 + 4\*b\*d\*e + a\*e^2)\*x^9)/9 + (e^3\*(4\*c\*d + b\*e)\*x^11)/11 + (c\*e^4\*x^13)/13

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex^2)^4 (a + bx^2 + cx^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)^4\*(a + b\*x^2 + c\*x^4), x]

[Out] IntegrateAlgebraic[(d + e\*x^2)^4\*(a + b\*x^2 + c\*x^4), x]

**fricas [A]** time = 0.87, size = 148, normalized size = 1.10

$$\frac{1}{13}x^{13}e^4c + \frac{4}{11}x^{11}e^3dc + \frac{1}{11}x^{11}e^4b + \frac{2}{3}x^9e^2d^2c + \frac{4}{9}x^9e^3db + \frac{1}{9}x^9e^4a + \frac{4}{7}x^7e^3c + \frac{6}{7}x^7e^2d^2b + \frac{4}{7}x^7e^3da + \frac{1}{5}x^5d^4c + \frac{4}{5}x^5e^3b + \frac{6}{5}x^5e^2d^2a + \frac{1}{3}x^3d^4b + \frac{4}{3}x^3e^3a + xd^4a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^4\*(c\*x^4+b\*x^2+a),x, algorithm="fricas")

[Out] 1/13\*x^13\*e^4\*c + 4/11\*x^11\*e^3\*d\*c + 1/11\*x^11\*e^4\*b + 2/3\*x^9\*e^2\*d^2\*c + 4/9\*x^9\*e^3\*d\*b + 1/9\*x^9\*e^4\*a + 4/7\*x^7\*e^3\*d\*c + 6/7\*x^7\*e^2\*d^2\*b + 4/7\*x^7\*e^3\*d\*a + 1/5\*x^5\*d^4\*c + 4/5\*x^5\*e^3\*d\*b + 6/5\*x^5\*e^2\*d^2\*a + 1/3\*x^3\*d^4\*b + 4/3\*x^3\*e^3\*d\*a + x\*d^4\*a

**giac [A]** time = 0.15, size = 142, normalized size = 1.05

$$\frac{1}{13}cx^{13}e^4 + \frac{4}{11}cdx^{11}e^3 + \frac{1}{11}bx^{11}e^4 + \frac{2}{3}cd^2x^9e^2 + \frac{4}{9}bdx^9e^3 + \frac{4}{7}cd^3x^7e + \frac{1}{9}ax^9e^4 + \frac{6}{7}bd^2x^7e^2 + \frac{1}{5}cd^4x^5 + \frac{4}{7}adx^7e^3 + \frac{4}{5}bd^3x^5e + \frac{6}{5}ad^2x^5e^2 + \frac{1}{3}bd^4x^3 + \frac{4}{3}ad^3x^3e + ad^4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^4\*(c\*x^4+b\*x^2+a),x, algorithm="giac")

[Out] 1/13\*c\*x^13\*e^4 + 4/11\*c\*d\*x^11\*e^3 + 1/11\*b\*x^11\*e^4 + 2/3\*c\*d^2\*x^9\*e^2 + 4/9\*b\*d\*x^9\*e^3 + 4/7\*c\*d^3\*x^7\*e + 1/9\*a\*x^9\*e^4 + 6/7\*b\*d^2\*x^7\*e^2 + 1/5\*c\*d^4\*x^5 + 4/7\*a\*d\*x^7\*e^3 + 4/5\*b\*d^3\*x^5\*e + 6/5\*a\*d^2\*x^5\*e^2 + 1/3\*b\*d^4\*x^3 + 4/3\*a\*d^3\*x^3\*e + a\*d^4\*x

**maple [A]** time = 0.00, size = 136, normalized size = 1.01

$$\frac{ce^4x^{13}}{13} + \frac{(e^4b + 4de^3c)x^{11}}{11} + \frac{(e^4a + 4de^3b + 6d^2e^2c)x^9}{9} + \frac{(4de^3a + 6d^2e^2b + 4d^3ec)x^7}{7} + ad^4x + \frac{(6d^2e^2a + 4d^3eb + d^4c)x^5}{5} + \frac{(4d^3ea + d^4b)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^4\*(c\*x^4+b\*x^2+a),x)

[Out] 1/13\*c\*e^4\*x^13+1/11\*(b\*e^4+4\*c\*d\*e^3)\*x^11+1/9\*(a\*e^4+4\*b\*d\*e^3+6\*c\*d^2\*e^2)\*x^9+1/7\*(4\*a\*d\*e^3+6\*b\*d^2\*e^2+4\*c\*d^3\*e)\*x^7+1/5\*(6\*a\*d^2\*e^2+4\*b\*d^3\*e+c\*d^4)\*x^5+1/3\*(4\*a\*d^3\*e+b\*d^4)\*x^3+a\*d^4\*x

**maxima [A]** time = 0.96, size = 135, normalized size = 1.00

$$\frac{1}{13}ce^4x^{13} + \frac{1}{11}(4cde^3 + be^4)x^{11} + \frac{1}{9}(6cd^2e^2 + 4bde^3 + ae^4)x^9 + \frac{2}{7}(2cd^3e + 3bd^2e^2 + 2ad^3e^3)x^7 + ad^4x + \frac{1}{5}(cd^4 + 4bd^3e + 6ad^2e^2)x^5 + \frac{1}{3}(bd^4 + 4ad^3e)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^4\*(c\*x^4+b\*x^2+a),x, algorithm="maxima")

[Out] 1/13\*c\*e^4\*x^13 + 1/11\*(4\*c\*d\*e^3 + b\*e^4)\*x^11 + 1/9\*(6\*c\*d^2\*e^2 + 4\*b\*d\*e^3 + a\*e^4)\*x^9 + 2/7\*(2\*c\*d^3\*e + 3\*b\*d^2\*e^2 + 2\*a\*d\*e^3)\*x^7 + a\*d^4\*x + 1/5\*(c\*d^4 + 4\*b\*d^3\*e + 6\*a\*d^2\*e^2)\*x^5 + 1/3\*(b\*d^4 + 4\*a\*d^3\*e)\*x^3

**mupad [B]** time = 0.06, size = 131, normalized size = 0.97

$$x^3 \left( \frac{bd^4}{3} + \frac{4aed^3}{3} \right) + x^{11} \left( \frac{be^4}{11} + \frac{4cd^3e^3}{11} \right) + x^5 \left( \frac{cd^4}{5} + \frac{4bd^3e}{5} + \frac{6ad^2e^2}{5} \right) + x^9 \left( \frac{2cd^2e^2}{3} + \frac{4bd^3e^3}{9} + \frac{ae^4}{9} \right) + \frac{ce^4x^{13}}{13} + ad^4x + \frac{2dex^7(2cd^2 + 3bde + 2ae^2)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)^4\*(a + b\*x^2 + c\*x^4),x)

[Out] x^3\*((b\*d^4)/3 + (4\*a\*d^3\*e)/3) + x^11\*((b\*e^4)/11 + (4\*c\*d\*e^3)/11) + x^5\*((c\*d^4)/5 + (6\*a\*d^2\*e^2)/5 + (4\*b\*d^3\*e)/5) + x^9\*((a\*e^4)/9 + (2\*c\*d^2\*e^2)/3 + (4\*b\*d^3\*e)/9) + (c\*e^4\*x^13)/13 + a\*d^4\*x + (2\*d\*e\*x^7\*(2\*a\*e^2 + 2\*c\*d^2 + 3\*b\*d\*e))/7

**sympy [A]** time = 0.11, size = 156, normalized size = 1.16

$$ad^4x + \frac{ce^4x^{13}}{13} + x^{11}\left(\frac{be^4}{11} + \frac{4cde^3}{11}\right) + x^9\left(\frac{ae^4}{9} + \frac{4bde^3}{9} + \frac{2cd^2e^2}{3}\right) + x^7\left(\frac{4ade^3}{7} + \frac{6bd^2e^2}{7} + \frac{4cd^3e}{7}\right) + x^5\left(\frac{6ad^2e^2}{5} + \frac{4bd^3e}{5} + \frac{cd^4}{5}\right) + x^3\left(\frac{4ad^3e}{3} + \frac{bd^4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*4\*(c\*x\*\*4+b\*x\*\*2+a),x)

[Out] a\*d\*\*4\*x + c\*e\*\*4\*x\*\*13/13 + x\*\*11\*(b\*e\*\*4/11 + 4\*c\*d\*e\*\*3/11) + x\*\*9\*(a\*e\*\*4/9 + 4\*b\*d\*e\*\*3/9 + 2\*c\*d\*\*2\*e\*\*2/3) + x\*\*7\*(4\*a\*d\*e\*\*3/7 + 6\*b\*d\*\*2\*e\*\*2/7 + 4\*c\*d\*\*3\*e/7) + x\*\*5\*(6\*a\*d\*\*2\*e\*\*2/5 + 4\*b\*d\*\*3\*e/5 + c\*d\*\*4/5) + x\*\*3\*(4\*a\*d\*\*3\*e/3 + b\*d\*\*4/3)

$$3.168 \quad \int (d + ex^2)^3 (a + bx^2 + cx^4) dx$$

Optimal. Leaf size=103

$$\frac{1}{7}ex^7 (e(ae + 3bd) + 3cd^2) + \frac{1}{5}dx^5 (3e(ae + bd) + cd^2) + \frac{1}{3}d^2x^3(3ae+bd)+ad^3x + \frac{1}{9}e^2x^9(be+3cd) + \frac{1}{11}ce^3x^{11}$$

Rubi [A] time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$ , Rules used = {1153}

$$\frac{1}{7}ex^7 (e(ae + 3bd) + 3cd^2) + \frac{1}{5}dx^5 (3e(ae + bd) + cd^2) + \frac{1}{3}d^2x^3(3ae + bd) + ad^3x + \frac{1}{9}e^2x^9(be + 3cd) + \frac{1}{11}ce^3x^{11}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^3\*(a + b\*x^2 + c\*x^4), x]

[Out] a\*d^3\*x + (d^2\*(b\*d + 3\*a\*e)\*x^3)/3 + (d\*(c\*d^2 + 3\*e\*(b\*d + a\*e))\*x^5)/5 + (e\*(3\*c\*d^2 + e\*(3\*b\*d + a\*e))\*x^7)/7 + (e^2\*(3\*c\*d + b\*e)\*x^9)/9 + (c\*e^3\*x^11)/11

Rule 1153

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^2)^q\*(a + b\*x^2 + c\*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int (d + ex^2)^3 (a + bx^2 + cx^4) dx &= \int (ad^3 + d^2(bd + 3ae)x^2 + d(cd^2 + 3e(bd + ae))x^4 + e(3cd^2 + e(3bd + ae))x^6 + e^2(3cd + be)x^8 + ce^3x^{10}) dx \\ &= ad^3x + \frac{1}{3}d^2(bd + 3ae)x^3 + \frac{1}{5}d(cd^2 + 3e(bd + ae))x^5 + \frac{1}{7}e(3cd^2 + e(3bd + ae))x^7 + \frac{1}{9}e^2(3cd + be)x^9 + \frac{1}{11}ce^3x^{11} \end{aligned}$$

Mathematica [A] time = 0.03, size = 104, normalized size = 1.01

$$\frac{1}{7}ex^7 (ae^2 + 3bde + 3cd^2) + \frac{1}{5}dx^5 (3ae^2 + 3bde + cd^2) + \frac{1}{3}d^2x^3(3ae + bd) + ad^3x + \frac{1}{9}e^2x^9(be + 3cd) + \frac{1}{11}ce^3x^{11}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^3\*(a + b\*x^2 + c\*x^4), x]

[Out] a\*d^3\*x + (d^2\*(b\*d + 3\*a\*e)\*x^3)/3 + (d\*(c\*d^2 + 3\*b\*d\*e + 3\*a\*e^2)\*x^5)/5 + (e\*(3\*c\*d^2 + 3\*b\*d\*e + a\*e^2)\*x^7)/7 + (e^2\*(3\*c\*d + b\*e)\*x^9)/9 + (c\*e^3\*x^11)/11

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex^2)^3 (a + bx^2 + cx^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)^3\*(a + b\*x^2 + c\*x^4), x]

[Out] IntegrateAlgebraic[(d + e\*x^2)^3\*(a + b\*x^2 + c\*x^4), x]

**fricas** [A] time = 0.60, size = 111, normalized size = 1.08

$$\frac{1}{11}x^{11}e^3c + \frac{1}{3}x^9e^2dc + \frac{1}{9}x^9e^3b + \frac{3}{7}x^7ed^2c + \frac{3}{7}x^7e^2db + \frac{1}{7}x^7e^3a + \frac{1}{5}x^5d^3c + \frac{3}{5}x^5ed^2b + \frac{3}{5}x^5e^2da + \frac{1}{3}x^3d^3b + x^3ed^2a + xd^3a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^3\*(c\*x^4+b\*x^2+a),x, algorithm="fricas")

[Out] 1/11\*x^11\*e^3\*c + 1/3\*x^9\*e^2\*d\*c + 1/9\*x^9\*e^3\*b + 3/7\*x^7\*e\*d^2\*c + 3/7\*x^7\*e^2\*d\*b + 1/7\*x^7\*e^3\*a + 1/5\*x^5\*d^3\*c + 3/5\*x^5\*e\*d^2\*b + 3/5\*x^5\*e^2\*d\*a + 1/3\*x^3\*d^3\*b + x^3\*e\*d^2\*a + x\*d^3\*a

**giac** [A] time = 0.16, size = 108, normalized size = 1.05

$$\frac{1}{11}cx^{11}e^3 + \frac{1}{3}cdx^9e^2 + \frac{1}{9}bx^9e^3 + \frac{3}{7}cd^2x^7e + \frac{3}{7}bdx^7e^2 + \frac{1}{5}cd^3x^5 + \frac{1}{7}ax^7e^3 + \frac{3}{5}bd^2x^5e + \frac{3}{5}adx^5e^2 + \frac{1}{3}bd^3x^3 + ad^2x^3e + ad^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^3\*(c\*x^4+b\*x^2+a),x, algorithm="giac")

[Out] 1/11\*c\*x^11\*e^3 + 1/3\*c\*d\*x^9\*e^2 + 1/9\*b\*x^9\*e^3 + 3/7\*c\*d^2\*x^7\*e + 3/7\*b\*d\*x^7\*e^2 + 1/5\*c\*d^3\*x^5 + 1/7\*a\*x^7\*e^3 + 3/5\*b\*d^2\*x^5\*e + 3/5\*a\*d\*x^5\*e^2 + 1/3\*b\*d^3\*x^3 + a\*d^2\*x^3\*e + a\*d^3\*x

**maple** [A] time = 0.00, size = 103, normalized size = 1.00

$$\frac{ce^3x^{11}}{11} + \frac{(e^3b + 3de^2c)x^9}{9} + \frac{(ae^3 + 3de^2b + 3cd^2e)x^7}{7} + ad^3x + \frac{(3de^2a + 3d^2eb + d^3c)x^5}{5} + \frac{(3d^2ea + d^3b)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^3\*(c\*x^4+b\*x^2+a),x)

[Out] 1/11\*c\*e^3\*x^11+1/9\*(b\*e^3+3\*c\*d\*e^2)\*x^9+1/7\*(a\*e^3+3\*b\*d\*e^2+3\*c\*d^2\*e)\*x^7+1/5\*(3\*a\*d\*e^2+3\*b\*d^2\*e+c\*d^3)\*x^5+1/3\*(3\*a\*d^2\*e+b\*d^3)\*x^3+a\*d^3\*x

**maxima** [A] time = 1.03, size = 102, normalized size = 0.99

$$\frac{1}{11}ce^3x^{11} + \frac{1}{9}(3cde^2 + be^3)x^9 + \frac{1}{7}(3cd^2e + 3bde^2 + ae^3)x^7 + \frac{1}{5}(cd^3 + 3bd^2e + 3ade^2)x^5 + ad^3x + \frac{1}{3}(bd^3 + 3ad^2e)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^3\*(c\*x^4+b\*x^2+a),x, algorithm="maxima")

[Out] 1/11\*c\*e^3\*x^11 + 1/9\*(3\*c\*d\*e^2 + b\*e^3)\*x^9 + 1/7\*(3\*c\*d^2\*e + 3\*b\*d\*e^2 + a\*e^3)\*x^7 + 1/5\*(c\*d^3 + 3\*b\*d^2\*e + 3\*a\*d\*e^2)\*x^5 + a\*d^3\*x + 1/3\*(b\*d^3 + 3\*a\*d^2\*e)\*x^3

**mupad** [B] time = 4.63, size = 101, normalized size = 0.98

$$x^3 \left( \frac{bd^3}{3} + aed^2 \right) + x^9 \left( \frac{be^3}{9} + \frac{cde^2}{3} \right) + x^5 \left( \frac{cd^3}{5} + \frac{3bd^2e}{5} + \frac{3ade^2}{5} \right) + x^7 \left( \frac{3cd^2e}{7} + \frac{3bde^2}{7} + \frac{ae^3}{7} \right) + \frac{ce^3x^{11}}{11} + ad^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)^3\*(a + b\*x^2 + c\*x^4),x)

[Out] x^3\*((b\*d^3)/3 + a\*d^2\*e) + x^9\*((b\*e^3)/9 + (c\*d\*e^2)/3) + x^5\*((c\*d^3)/5 + (3\*a\*d\*e^2)/5 + (3\*b\*d^2\*e)/5) + x^7\*((a\*e^3)/7 + (3\*b\*d\*e^2)/7 + (3\*c\*d^2\*e)/7) + (c\*e^3\*x^11)/11 + a\*d^3\*x

**sympy** [A] time = 0.29, size = 112, normalized size = 1.09

$$ad^3x + \frac{ce^3x^{11}}{11} + x^9 \left( \frac{be^3}{9} + \frac{cde^2}{3} \right) + x^7 \left( \frac{ae^3}{7} + \frac{3bde^2}{7} + \frac{3cd^2e}{7} \right) + x^5 \left( \frac{3ade^2}{5} + \frac{3bd^2e}{5} + \frac{cd^3}{5} \right) + x^3 \left( ad^2e + \frac{bd^3}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**3*x + c*e**3*x**11/11 + x**9*(b*e**3/9 + c*d*e**2/3) + x**7*(a*e**3/7  
+ 3*b*d*e**2/7 + 3*c*d**2*e/7) + x**5*(3*a*d*e**2/5 + 3*b*d**2*e/5 + c*d**3  
/5) + x**3*(a*d**2*e + b*d**3/3)
```



$$3.169 \quad \int (d + ex^2)^2 (a + bx^2 + cx^4) dx$$

**Optimal.** Leaf size=73

$$\frac{1}{5}x^5(e(ae + 2bd) + cd^2) + \frac{1}{3}dx^3(2ae + bd) + ad^2x + \frac{1}{7}ex^7(be + 2cd) + \frac{1}{9}ce^2x^9$$

**Rubi [A]** time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$ , Rules used = {1153}

$$\frac{1}{5}x^5(e(ae + 2bd) + cd^2) + \frac{1}{3}dx^3(2ae + bd) + ad^2x + \frac{1}{7}ex^7(be + 2cd) + \frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^2\*(a + b\*x^2 + c\*x^4), x]

[Out] a\*d^2\*x + (d\*(b\*d + 2\*a\*e)\*x^3)/3 + ((c\*d^2 + e\*(2\*b\*d + a\*e))\*x^5)/5 + (e\*(2\*c\*d + b\*e)\*x^7)/7 + (c\*e^2\*x^9)/9

**Rule 1153**

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^2)^q\*(a + b\*x^2 + c\*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

**Rubi steps**

$$\begin{aligned} \int (d + ex^2)^2 (a + bx^2 + cx^4) dx &= \int (ad^2 + d(bd + 2ae)x^2 + (cd^2 + e(2bd + ae))x^4 + e(2cd + be)x^6 + ce^2x^8) dx \\ &= ad^2x + \frac{1}{3}d(bd + 2ae)x^3 + \frac{1}{5}(cd^2 + e(2bd + ae))x^5 + \frac{1}{7}e(2cd + be)x^7 + \frac{1}{9}ce^2x^9 \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 73, normalized size = 1.00

$$\frac{1}{5}x^5(ae^2 + 2bde + cd^2) + \frac{1}{3}dx^3(2ae + bd) + ad^2x + \frac{1}{7}ex^7(be + 2cd) + \frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^2\*(a + b\*x^2 + c\*x^4), x]

[Out] a\*d^2\*x + (d\*(b\*d + 2\*a\*e)\*x^3)/3 + ((c\*d^2 + 2\*b\*d\*e + a\*e^2)\*x^5)/5 + (e\*(2\*c\*d + b\*e)\*x^7)/7 + (c\*e^2\*x^9)/9

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex^2)^2 (a + bx^2 + cx^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)^2\*(a + b\*x^2 + c\*x^4), x]

[Out] IntegrateAlgebraic[(d + e\*x^2)^2\*(a + b\*x^2 + c\*x^4), x]

**fricas [A]** time = 0.79, size = 76, normalized size = 1.04

$$\frac{1}{9}x^9e^2c + \frac{2}{7}x^7edc + \frac{1}{7}x^7e^2b + \frac{1}{5}x^5d^2c + \frac{2}{5}x^5edb + \frac{1}{5}x^5e^2a + \frac{1}{3}x^3d^2b + \frac{2}{3}x^3eda + xd^2a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(c\*x^4+b\*x^2+a),x, algorithm="fricas")

[Out]  $\frac{1}{9}cx^9e^2 + \frac{2}{7}cdx^7e + \frac{1}{7}bx^7e^2 + \frac{1}{5}cd^2x^5 + \frac{2}{5}bdx^5e + \frac{1}{5}ax^5e^2 + \frac{1}{3}bd^2x^3 + \frac{2}{3}adx^3e + ad^2x$

**giac** [A] time = 0.15, size = 76, normalized size = 1.04

$$\frac{1}{9}cx^9e^2 + \frac{2}{7}cdx^7e + \frac{1}{7}bx^7e^2 + \frac{1}{5}cd^2x^5 + \frac{2}{5}bdx^5e + \frac{1}{5}ax^5e^2 + \frac{1}{3}bd^2x^3 + \frac{2}{3}adx^3e + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(c\*x^4+b\*x^2+a),x, algorithm="giac")

[Out]  $\frac{1}{9}c*x^9*e^2 + \frac{2}{7}c*d*x^7*e + \frac{1}{7}b*x^7*e^2 + \frac{1}{5}c*d^2*x^5 + \frac{2}{5}b*d*x^5*e + \frac{1}{5}a*x^5*e^2 + \frac{1}{3}b*d^2*x^3 + \frac{2}{3}a*d*x^3*e + a*d^2*x$

**maple** [A] time = 0.00, size = 70, normalized size = 0.96

$$\frac{ce^2x^9}{9} + \frac{(be^2 + 2dce)x^7}{7} + \frac{(ae^2 + 2bde + cd^2)x^5}{5} + ad^2x + \frac{(2dea + bd^2)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^2\*(c\*x^4+b\*x^2+a),x)

[Out]  $\frac{1}{9}c*e^2*x^9 + \frac{1}{7}*(b*e^2 + 2*c*d*e)*x^7 + \frac{1}{5}*(a*e^2 + 2*b*d*e + c*d^2)*x^5 + \frac{1}{3}*(2*a*d*e + b*d^2)*x^3 + a*d^2*x$

**maxima** [A] time = 1.07, size = 69, normalized size = 0.95

$$\frac{1}{9}ce^2x^9 + \frac{1}{7}(2cde + be^2)x^7 + \frac{1}{5}(cd^2 + 2bde + ae^2)x^5 + ad^2x + \frac{1}{3}(bd^2 + 2ade)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(c\*x^4+b\*x^2+a),x, algorithm="maxima")

[Out]  $\frac{1}{9}c*e^2*x^9 + \frac{1}{7}*(2*c*d*e + b*e^2)*x^7 + \frac{1}{5}*(c*d^2 + 2*b*d*e + a*e^2)*x^5 + a*d^2*x + \frac{1}{3}*(b*d^2 + 2*a*d*e)*x^3$

**mupad** [B] time = 4.59, size = 70, normalized size = 0.96

$$x^5 \left( \frac{cd^2}{5} + \frac{2bde}{5} + \frac{ae^2}{5} \right) + x^3 \left( \frac{bd^2}{3} + \frac{2aed}{3} \right) + x^7 \left( \frac{be^2}{7} + \frac{2cde}{7} \right) + \frac{ce^2x^9}{9} + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)^2\*(a + b\*x^2 + c\*x^4),x)

[Out]  $x^5*((a*e^2)/5 + (c*d^2)/5 + (2*b*d*e)/5) + x^3*((b*d^2)/3 + (2*a*d*e)/3) + x^7*((b*e^2)/7 + (2*c*d*e)/7) + (c*e^2*x^9)/9 + a*d^2*x$

**sympy** [A] time = 0.11, size = 78, normalized size = 1.07

$$ad^2x + \frac{ce^2x^9}{9} + x^7 \left( \frac{be^2}{7} + \frac{2cde}{7} \right) + x^5 \left( \frac{ae^2}{5} + \frac{2bde}{5} + \frac{cd^2}{5} \right) + x^3 \left( \frac{2ade}{3} + \frac{bd^2}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*2\*(c\*x\*\*4+b\*x\*\*2+a),x)

[Out]  $a*d**2*x + c*e**2*x**9/9 + x**7*(b*e**2/7 + 2*c*d*e/7) + x**5*(a*e**2/5 + 2*b*d*e/5 + c*d**2/5) + x**3*(2*a*d*e/3 + b*d**2/3)$

$$3.170 \quad \int (d + ex^2)(a + bx^2 + cx^4) dx$$

Optimal. Leaf size=42

$$\frac{1}{3}x^3(ae + bd) + adx + \frac{1}{5}x^5(be + cd) + \frac{1}{7}cex^7$$

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$ , Rules used = {1153}

$$\frac{1}{3}x^3(ae + bd) + adx + \frac{1}{5}x^5(be + cd) + \frac{1}{7}cex^7$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)\*(a + b\*x^2 + c\*x^4), x]

[Out] a\*d\*x + ((b\*d + a\*e)\*x^3)/3 + ((c\*d + b\*e)\*x^5)/5 + (c\*e\*x^7)/7

Rule 1153

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^2)^q\*(a + b\*x^2 + c\*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int (d + ex^2)(a + bx^2 + cx^4) dx &= \int (ad + (bd + ae)x^2 + (cd + be)x^4 + cex^6) dx \\ &= adx + \frac{1}{3}(bd + ae)x^3 + \frac{1}{5}(cd + be)x^5 + \frac{1}{7}cex^7 \end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.00

$$\frac{1}{3}x^3(ae + bd) + adx + \frac{1}{5}x^5(be + cd) + \frac{1}{7}cex^7$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)\*(a + b\*x^2 + c\*x^4), x]

[Out] a\*d\*x + ((b\*d + a\*e)\*x^3)/3 + ((c\*d + b\*e)\*x^5)/5 + (c\*e\*x^7)/7

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex^2)(a + bx^2 + cx^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)\*(a + b\*x^2 + c\*x^4), x]

[Out] IntegrateAlgebraic[(d + e\*x^2)\*(a + b\*x^2 + c\*x^4), x]

fricas [A] time = 0.93, size = 40, normalized size = 0.95

$$\frac{1}{7}x^7ec + \frac{1}{5}x^5dc + \frac{1}{5}x^5eb + \frac{1}{3}x^3db + \frac{1}{3}x^3ea + xda$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(c\*x^4+b\*x^2+a),x, algorithm="fricas")

[Out] 1/7\*x^7\*e\*c + 1/5\*x^5\*d\*c + 1/5\*x^5\*e\*b + 1/3\*x^3\*d\*b + 1/3\*x^3\*e\*a + x\*d\*a

**giac** [A] time = 0.15, size = 43, normalized size = 1.02

$$\frac{1}{7}cx^7e + \frac{1}{5}cdx^5 + \frac{1}{5}bx^5e + \frac{1}{3}bdx^3 + \frac{1}{3}ax^3e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(c\*x^4+b\*x^2+a),x, algorithm="giac")

[Out] 1/7\*c\*x^7\*e + 1/5\*c\*d\*x^5 + 1/5\*b\*x^5\*e + 1/3\*b\*d\*x^3 + 1/3\*a\*x^3\*e + a\*d\*x

**maple** [A] time = 0.00, size = 37, normalized size = 0.88

$$\frac{ce x^7}{7} + \frac{(be + cd)x^5}{5} + adx + \frac{(ae + bd)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)\*(c\*x^4+b\*x^2+a),x)

[Out] a\*d\*x+1/3\*(a\*e+b\*d)\*x^3+1/5\*(b\*e+c\*d)\*x^5+1/7\*c\*e\*x^7

**maxima** [A] time = 0.90, size = 36, normalized size = 0.86

$$\frac{1}{7}cex^7 + \frac{1}{5}(cd + be)x^5 + \frac{1}{3}(bd + ae)x^3 + adx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(c\*x^4+b\*x^2+a),x, algorithm="maxima")

[Out] 1/7\*c\*e\*x^7 + 1/5\*(c\*d + b\*e)\*x^5 + 1/3\*(b\*d + a\*e)\*x^3 + a\*d\*x

**mupad** [B] time = 0.04, size = 38, normalized size = 0.90

$$\frac{cex^7}{7} + \left(\frac{be}{5} + \frac{cd}{5}\right)x^5 + \left(\frac{ae}{3} + \frac{bd}{3}\right)x^3 + adx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)\*(a + b\*x^2 + c\*x^4),x)

[Out] x^3\*((a\*e)/3 + (b\*d)/3) + x^5\*((b\*e)/5 + (c\*d)/5) + a\*d\*x + (c\*e\*x^7)/7

**sympy** [A] time = 0.10, size = 39, normalized size = 0.93

$$adx + \frac{cex^7}{7} + x^5\left(\frac{be}{5} + \frac{cd}{5}\right) + x^3\left(\frac{ae}{3} + \frac{bd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*(c\*x\*\*4+b\*x\*\*2+a),x)

[Out] a\*d\*x + c\*e\*x\*\*7/7 + x\*\*5\*(b\*e/5 + c\*d/5) + x\*\*3\*(a\*e/3 + b\*d/3)

$$3.171 \quad \int \frac{a+bx^2+cx^4}{d+ex^2} dx$$

**Optimal.** Leaf size=66

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(ae^2 - bde + cd^2)}{\sqrt{d}e^{5/2}} - \frac{x(cd - be)}{e^2} + \frac{cx^3}{3e}$$

**Rubi [A]** time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {1153, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(ae^2 - bde + cd^2)}{\sqrt{d}e^{5/2}} - \frac{x(cd - be)}{e^2} + \frac{cx^3}{3e}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2 + c\*x^4)/(d + e\*x^2), x]

[Out] -(((c\*d - b\*e)\*x)/e^2) + (c\*x^3)/(3\*e) + ((c\*d^2 - b\*d\*e + a\*e^2)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(Sqrt[d]\*e^(5/2))

**Rule 205**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 1153**

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^2)^q\*(a + b\*x^2 + c\*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

**Rubi steps**

$$\begin{aligned} \int \frac{a + bx^2 + cx^4}{d + ex^2} dx &= \int \left( -\frac{cd - be}{e^2} + \frac{cx^2}{e} + \frac{cd^2 - bde + ae^2}{e^2(d + ex^2)} \right) dx \\ &= -\frac{(cd - be)x}{e^2} + \frac{cx^3}{3e} + \frac{(cd^2 - bde + ae^2) \int \frac{1}{d + ex^2} dx}{e^2} \\ &= -\frac{(cd - be)x}{e^2} + \frac{cx^3}{3e} + \frac{(cd^2 - bde + ae^2) \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 65, normalized size = 0.98

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(ae^2 - bde + cd^2)}{\sqrt{d}e^{5/2}} + \frac{x(be - cd)}{e^2} + \frac{cx^3}{3e}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2 + c\*x^4)/(d + e\*x^2), x]

[Out] ((-(c\*d) + b\*e)\*x)/e^2 + (c\*x^3)/(3\*e) + ((c\*d^2 - b\*d\*e + a\*e^2)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(Sqrt[d]\*e^(5/2))

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2 + cx^4}{d + ex^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)/(d + e\*x^2), x]

[Out] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)/(d + e\*x^2), x]

**fricas** [A] time = 1.01, size = 159, normalized size = 2.41

$$\left[ \frac{2cde^2x^3 - 3(cd^2 - bde + ae^2)\sqrt{-de} \log\left(\frac{ex^2 - 2\sqrt{-de}x - d}{ex^2 + d}\right) - 6(cd^2e - bde^2)x}{6de^3}, \frac{cde^2x^3 + 3(cd^2 - bde + ae^2)\sqrt{de} \arctan\left(\frac{\sqrt{de}x}{d}\right) - 3(cd^2e - bde^2)x}{3de^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d), x, algorithm="fricas")

[Out] [1/6\*(2\*c\*d\*e^2\*x^3 - 3\*(c\*d^2 - b\*d\*e + a\*e^2)\*sqrt(-d\*e)\*log((e\*x^2 - 2\*sqrt(-d\*e)\*x - d)/(e\*x^2 + d)) - 6\*(c\*d^2\*e - b\*d\*e^2)\*x)/(d\*e^3), 1/3\*(c\*d\*e^2\*x^3 + 3\*(c\*d^2 - b\*d\*e + a\*e^2)\*sqrt(d\*e)\*arctan(sqrt(d\*e)\*x/d) - 3\*(c\*d^2\*e - b\*d\*e^2)\*x)/(d\*e^3)]

**giac** [A] time = 0.15, size = 56, normalized size = 0.85

$$\frac{(cd^2 - bde + ae^2) \arctan\left(\frac{xe^2}{\sqrt{d}}\right) e^{\left(-\frac{5}{2}\right)}}{\sqrt{d}} + \frac{1}{3} (cx^3e^2 - 3cdxe + 3bx^2e^2)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d), x, algorithm="giac")

[Out] (c\*d^2 - b\*d\*e + a\*e^2)\*arctan(x\*e^(1/2)/sqrt(d))\*e^(-5/2)/sqrt(d) + 1/3\*(c\*x^3\*e^2 - 3\*c\*d\*x\*e + 3\*b\*x\*e^2)\*e^(-3)

**maple** [A] time = 0.00, size = 84, normalized size = 1.27

$$\frac{cx^3}{3e} + \frac{a \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} - \frac{bd \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de} e} + \frac{cd^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de} e^2} + \frac{bx}{e} - \frac{cdx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^4+b\*x^2+a)/(e\*x^2+d), x)

[Out] 1/3\*c/e\*x^3+1/e\*b\*x-c\*d/e^2\*x+1/(d\*e)^(1/2)\*a\*arctan(1/(d\*e)^(1/2)\*e\*x)-1/e/(d\*e)^(1/2)\*arctan(1/(d\*e)^(1/2)\*e\*x)\*b\*d+1/(d\*e)^(1/2)\*c\*d^2/e^2\*arctan(1/(d\*e)^(1/2)\*e\*x)

**maxima** [A] time = 2.41, size = 58, normalized size = 0.88

$$\frac{(cd^2 - bde + ae^2) \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de} e^2} + \frac{cex^3 - 3(cd - be)x}{3e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d), x, algorithm="maxima")

[Out]  $(c*d^2 - b*d*e + a*e^2)*\arctan(e*x/\sqrt{d*e})/(\sqrt{d*e}*e^2) + 1/3*(c*e*x^3 - 3*(c*d - b*e)*x)/e^2$

**mupad [B]** time = 0.09, size = 57, normalized size = 0.86

$$x \left( \frac{b}{e} - \frac{cd}{e^2} \right) + \frac{cx^3}{3e} + \frac{\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) (cd^2 - bde + ae^2)}{\sqrt{d} e^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^2 + c*x^4)/(d + e*x^2), x)`

[Out]  $x*(b/e - (c*d)/e^2) + (c*x^3)/(3*e) + (\operatorname{atan}((e^{1/2})x)/d^{1/2})*(a*e^2 + c*d^2 - b*d*e)/(d^{1/2}*e^{5/2})$

**sympy [B]** time = 0.73, size = 117, normalized size = 1.77

$$\frac{cx^3}{3e} + x \left( \frac{b}{e} - \frac{cd}{e^2} \right) - \frac{\sqrt{-\frac{1}{de^5}} (ae^2 - bde + cd^2) \log\left(-de^2 \sqrt{-\frac{1}{de^5}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{de^5}} (ae^2 - bde + cd^2) \log\left(de^2 \sqrt{-\frac{1}{de^5}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**4+b*x**2+a)/(e*x**2+d), x)`

[Out]  $c*x**3/(3*e) + x*(b/e - c*d/e**2) - \sqrt{-1/(d*e**5)}*(a*e**2 - b*d*e + c*d**2)*\log(-d*e**2*\sqrt{-1/(d*e**5)} + x)/2 + \sqrt{-1/(d*e**5)}*(a*e**2 - b*d*e + c*d**2)*\log(d*e**2*\sqrt{-1/(d*e**5)} + x)/2$

$$3.172 \quad \int \frac{a+bx^2+cx^4}{(d+ex^2)^2} dx$$

**Optimal.** Leaf size=83

$$-\frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(3cd^2 - e(ae + bd))}{2d^{3/2}e^{5/2}} + \frac{x\left(a + \frac{d(cd-be)}{e^2}\right)}{2d(d+ex^2)} + \frac{cx}{e^2}$$

**Rubi [A]** time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {1157, 388, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(3cd^2 - e(ae + bd))}{2d^{3/2}e^{5/2}} + \frac{x\left(a + \frac{d(cd-be)}{e^2}\right)}{2d(d+ex^2)} + \frac{cx}{e^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^2,x]

[Out] (c\*x)/e^2 + ((a + (d\*(c\*d - b\*e))/e^2)\*x)/(2\*d\*(d + e\*x^2)) - ((3\*c\*d^2 - e\*(b\*d + a\*e))\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(2\*d^(3/2)\*e^(5/2))

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p+1))/(b\*(n\*(p+1)+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1)+1))/(b\*(n\*(p+1)+1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p+1)+1, 0]

Rule 1157

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] := With[{Qx = PolynomialQuotient[(a + b\*x^2 + c\*x^4)^p, d + e\*x^2, x], R = Coeff[PolynomialRemainder[(a + b\*x^2 + c\*x^4)^p, d + e\*x^2, x], 0]}, -Simp[(R\*x\*(d + e\*x^2)^(q+1))/(2\*d\*(q+1)), x] + Dist[1/(2\*d\*(q+1)), Int[(d + e\*x^2)^(q+1)\*ExpandToSum[2\*d\*(q+1)\*Qx + R\*(2\*q+3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rubi steps

$$\begin{aligned} \int \frac{a+bx^2+cx^4}{(d+ex^2)^2} dx &= \frac{(cd^2 - bde + ae^2)x}{2de^2(d+ex^2)} - \int \frac{\frac{cd^2 - e(bd+ae) - 2cdx^2}{e^2}}{d+ex^2} dx \\ &= \frac{cx}{e^2} + \frac{(cd^2 - bde + ae^2)x}{2de^2(d+ex^2)} - \frac{(3cd^2 - e(bd+ae)) \int \frac{1}{d+ex^2} dx}{2de^2} \\ &= \frac{cx}{e^2} + \frac{(cd^2 - bde + ae^2)x}{2de^2(d+ex^2)} - \frac{(3cd^2 - e(bd+ae)) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}} \end{aligned}$$



**Mathematica [A]** time = 0.06, size = 88, normalized size = 1.06

$$\frac{x(ae^2 - bde + cd^2)}{2de^2(d + ex^2)} - \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(-ae^2 - bde + 3cd^2)}{2d^{3/2}e^{5/2}} + \frac{cx}{e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^2, x]

[Out] (c\*x)/e^2 + ((c\*d^2 - b\*d\*e + a\*e^2)\*x)/(2\*d\*e^2\*(d + e\*x^2)) - ((3\*c\*d^2 - b\*d\*e - a\*e^2)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(2\*d^(3/2)\*e^(5/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2 + cx^4}{(d + ex^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^2, x]

[Out] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^2, x]

**fricas [A]** time = 1.02, size = 268, normalized size = 3.23

$$\left[ \frac{4cd^2e^2x^3 + (3cd^3 - bd^2e - ade^2 + (3cd^2e - bde^2 - ae^3)x^2)\sqrt{-de} \log\left(\frac{x^2 - 2\sqrt{-de}x - d}{e^2 + d}\right) + 2(3cd^3e - bd^2e^2 + ade^3)x}{4(d^2e^4x^2 + d^3e^3)}, \frac{2cd^2e^2x^3 - (3cd^3 - bd^2e - ade^2 + (3cd^2e - bde^2 - ae^3)x^2)\sqrt{de} \arctan\left(\frac{\sqrt{de}x}{d}\right) + (3cd^3e - bd^2e^2 + ade^3)x}{2(d^2e^4x^2 + d^3e^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^2,x, algorithm="fricas")

[Out] [1/4\*(4\*c\*d^2\*e^2\*x^3 + (3\*c\*d^3 - b\*d^2\*e - a\*d\*e^2 + (3\*c\*d^2\*e - b\*d\*e^2 - a\*e^3)\*x^2)\*sqrt(-d\*e)\*log((e\*x^2 - 2\*sqrt(-d\*e)\*x - d)/(e\*x^2 + d)) + 2\*(3\*c\*d^3\*e - b\*d^2\*e^2 + a\*d\*e^3)\*x)/(d^2\*e^4\*x^2 + d^3\*e^3), 1/2\*(2\*c\*d^2\*e^2\*x^3 - (3\*c\*d^3 - b\*d^2\*e - a\*d\*e^2 + (3\*c\*d^2\*e - b\*d\*e^2 - a\*e^3)\*x^2)\*sqrt(d\*e)\*arctan(sqrt(d\*e)\*x/d) + (3\*c\*d^3\*e - b\*d^2\*e^2 + a\*d\*e^3)\*x)/(d^2\*e^4\*x^2 + d^3\*e^3)]

**giac [A]** time = 0.17, size = 75, normalized size = 0.90

$$cxe^{(-2)} - \frac{(3cd^2 - bde - ae^2) \arctan\left(\frac{xe^{1/2}}{\sqrt{d}}\right) e^{(-5/2)}}{2d^{3/2}} + \frac{(cd^2x - bdx + axe^2)e^{(-2)}}{2(x^2e + d)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^2,x, algorithm="giac")

[Out] c\*x\*e^(-2) - 1/2\*(3\*c\*d^2 - b\*d\*e - a\*e^2)\*arctan(x\*e^(1/2)/sqrt(d))\*e^(-5/2)/d^(3/2) + 1/2\*(c\*d^2\*x - b\*d\*x\*e + a\*x\*e^2)\*e^(-2)/((x^2\*e + d)\*d)

**maple [A]** time = 0.01, size = 118, normalized size = 1.42

$$\frac{ax}{2(e x^2 + d)d} + \frac{a \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}d} - \frac{bx}{2(e x^2 + d)e} + \frac{b \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}e} + \frac{cdx}{2(e x^2 + d)e^2} - \frac{3cd \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}e^2} + \frac{cx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^4+b\*x^2+a)/(e\*x^2+d)^2,x)

[Out]  $c/e^{2x+1/2}/(e^{x^2+d}) * a/d * x^{-1/2}/e^x/(e^{x^2+d}) * b + 1/2/(e^{x^2+d}) * c * d/e^{2x+1/2}/(d * e)^{1/2} * a/d * \arctan(1/(d * e)^{1/2} * e^x) + 1/2/e/(d * e)^{1/2} * \arctan(1/(d * e)^{1/2} * e^x) * b - 3/2/(d * e)^{1/2} * c * d/e^{2x} * \arctan(1/(d * e)^{1/2} * e^x)$

**maxima** [A] time = 2.25, size = 84, normalized size = 1.01

$$\frac{(cd^2 - bde + ae^2)x}{2(d^3x^2 + d^2e^2)} + \frac{cx}{e^2} - \frac{(3cd^2 - bde - ae^2) \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^2,x, algorithm="maxima")

[Out]  $1/2 * (c * d^2 - b * d * e + a * e^2) * x / (d * e^3 * x^2 + d^2 * e^2) + c * x / e^2 - 1/2 * (3 * c * d^2 - b * d * e - a * e^2) * \arctan(e * x / \sqrt{d * e}) / (\sqrt{d * e} * d * e^2)$

**mupad** [B] time = 4.67, size = 77, normalized size = 0.93

$$\frac{cx}{e^2} + \frac{\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) (-3cd^2 + bde + ae^2)}{2d^{3/2}e^{5/2}} + \frac{x(cd^2 - bde + ae^2)}{2d(e^3x^2 + de^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^2 + c\*x^4)/(d + e\*x^2)^2,x)

[Out]  $(c * x) / e^2 + (\operatorname{atan}((e^{1/2} * x) / d^{1/2})) * (a * e^2 - 3 * c * d^2 + b * d * e) / (2 * d^{3/2} * e^{5/2}) + (x * (a * e^2 + c * d^2 - b * d * e)) / (2 * d * (d * e^2 + e^3 * x^2))$

**sympy** [B] time = 1.23, size = 153, normalized size = 1.84

$$\frac{cx}{e^2} + \frac{x(ae^2 - bde + cd^2)}{2d^2e^2 + 2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2 + bde - 3cd^2) \log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2 + bde - 3cd^2) \log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*4+b\*x\*\*2+a)/(e\*x\*\*2+d)\*\*2,x)

[Out]  $c * x / e^{**2} + x * (a * e^{**2} - b * d * e + c * d^{**2}) / (2 * d^{**2} * e^{**2} + 2 * d * e^{**3} * x^{**2}) - \operatorname{sqrt}(-1 / (d^{**3} * e^{**5})) * (a * e^{**2} + b * d * e - 3 * c * d^{**2}) * \log(-d^{**2} * e^{**2} * \operatorname{sqrt}(-1 / (d^{**3} * e^{**5})) + x) / 4 + \operatorname{sqrt}(-1 / (d^{**3} * e^{**5})) * (a * e^{**2} + b * d * e - 3 * c * d^{**2}) * \log(d^{**2} * e^{**2} * \operatorname{sqrt}(-1 / (d^{**3} * e^{**5})) + x) / 4$

$$3.173 \quad \int \frac{a+bx^2+cx^4}{(d+ex^2)^3} dx$$

**Optimal.** Leaf size=115

$$-\frac{x(5cd^2 - e(3ae + bd))}{8d^2e^2(d + ex^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(e(3ae + bd) + 3cd^2)}{8d^{5/2}e^{5/2}} + \frac{x\left(a + \frac{d(cd-be)}{e^2}\right)}{4d(d + ex^2)^2}$$

**Rubi [A]** time = 0.11, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {1157, 385, 205}

$$-\frac{x(5cd^2 - e(3ae + bd))}{8d^2e^2(d + ex^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(e(3ae + bd) + 3cd^2)}{8d^{5/2}e^{5/2}} + \frac{x\left(a + \frac{d(cd-be)}{e^2}\right)}{4d(d + ex^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^3,x]

[Out] ((a + (d\*(c\*d - b\*e))/e^2)\*x)/(4\*d\*(d + e\*x^2)^2) - ((5\*c\*d^2 - e\*(b\*d + 3\*a\*e))\*x)/(8\*d^2\*e^2\*(d + e\*x^2)) + ((3\*c\*d^2 + e\*(b\*d + 3\*a\*e))\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(8\*d^(5/2)\*e^(5/2))

**Rule 205**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 385**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[(b\*c - a\*d)\*x\*(a + b\*x^n)^(p+1)/(a\*b\*n\*(p+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1)+1))/(a\*b\*n\*(p+1)), Int[(a + b\*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

**Rule 1157**

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] := With[{Qx = PolynomialQuotient[(a + b\*x^2 + c\*x^4)^p, d + e\*x^2, x], R = Coeff[PolynomialRemainder[(a + b\*x^2 + c\*x^4)^p, d + e\*x^2, x], x, 0]}, -Simp[(R\*x\*(d + e\*x^2)^(q+1))/(2\*d\*(q+1)), x] + Dist[1/(2\*d\*(q+1)), Int[(d + e\*x^2)^(q+1)\*ExpandToSum[2\*d\*(q+1)\*Qx + R\*(2\*q+3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

**Rubi steps**

$$\begin{aligned} \int \frac{a + bx^2 + cx^4}{(d + ex^2)^3} dx &= \frac{(cd^2 - bde + ae^2)x}{4de^2(d + ex^2)^2} - \frac{\int \frac{-3a + \frac{d(cd-be)}{e^2} - \frac{4cdx^2}{e}}{(d+ex^2)^2} dx}{4d} \\ &= \frac{(cd^2 - bde + ae^2)x}{4de^2(d + ex^2)^2} - \frac{(5cd^2 - e(bd + 3ae))x}{8d^2e^2(d + ex^2)} - \frac{\left(-\frac{4cd^2}{e} + e\left(-3a + \frac{d(cd-be)}{e^2}\right)\right) \int \frac{1}{d+ex^2} dx}{8d^2e} \\ &= \frac{(cd^2 - bde + ae^2)x}{4de^2(d + ex^2)^2} - \frac{(5cd^2 - e(bd + 3ae))x}{8d^2e^2(d + ex^2)} + \frac{(3cd^2 + e(bd + 3ae)) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}} \end{aligned}$$

**Mathematica [A]** time = 0.10, size = 110, normalized size = 0.96

$$\frac{x \left( e \left( ae \left( 5d + 3ex^2 \right) + bd \left( ex^2 - d \right) \right) - cd^2 \left( 3d + 5ex^2 \right) \right)}{8d^2e^2 \left( d + ex^2 \right)^2} + \frac{\tan^{-1} \left( \frac{\sqrt{ex}}{\sqrt{d}} \right) \left( e \left( 3ae + bd \right) + 3cd^2 \right)}{8d^{5/2}e^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^3,x]

[Out] (x\*(-(c\*d^2\*(3\*d + 5\*e\*x^2)) + e\*(b\*d\*(-d + e\*x^2) + a\*e\*(5\*d + 3\*e\*x^2))))/(8\*d^2\*e^2\*(d + e\*x^2)^2) + ((3\*c\*d^2 + e\*(b\*d + 3\*a\*e))\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(8\*d^(5/2)\*e^(5/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2 + cx^4}{(d + ex^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^3,x]

[Out] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^3, x]

**fricas [A]** time = 1.37, size = 391, normalized size = 3.40

$$\frac{2(5cd^2 - bd^2e - 3ade^2) + (3cd^2 + bd^2e + 3ade^2 + (3cd^2 + bd^2e + 3ade^2)^2)\sqrt{-d} \log\left(\frac{d^2 - 2\sqrt{-d}x}{d^2 + d}\right) + 2(3cd^2e + bd^2e^2 - 5ade^2)x}{16(d^2e^2 + 2de^2e^2 + d^2e^2)} - \frac{(5cd^2 - bd^2e - 3ade^2)^2 - (3cd^2 + bd^2e + 3ade^2)^2 + 2(3cd^2e + bd^2e^2 + 3ade^2)^2\sqrt{-d} \arctan\left(\frac{\sqrt{-d}x}{d}\right) + (3cd^2e + bd^2e^2 - 5ade^2)x}{8(d^2e^2 + 2de^2e^2 + d^2e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^3,x, algorithm="fricas")

[Out] [-1/16\*(2\*(5\*c\*d^3\*e^2 - b\*d^2\*e^3 - 3\*a\*d\*e^4)\*x^3 + (3\*c\*d^4 + b\*d^3\*e + 3\*a\*d^2\*e^2 + (3\*c\*d^2\*e^2 + b\*d\*e^3 + 3\*a\*e^4)\*x^4 + 2\*(3\*c\*d^3\*e + b\*d^2\*e^2 + 3\*a\*d\*e^3)\*x^2)\*sqrt(-d\*e)\*log((e\*x^2 - 2\*sqrt(-d\*e)\*x - d)/(e\*x^2 + d)) + 2\*(3\*c\*d^4\*e + b\*d^3\*e^2 - 5\*a\*d^2\*e^3)\*x)/(d^3\*e^5\*x^4 + 2\*d^4\*e^4\*x^2 + d^5\*e^3), -1/8\*((5\*c\*d^3\*e^2 - b\*d^2\*e^3 - 3\*a\*d\*e^4)\*x^3 - (3\*c\*d^4 + b\*d^3\*e + 3\*a\*d^2\*e^2 + (3\*c\*d^2\*e^2 + b\*d\*e^3 + 3\*a\*e^4)\*x^4 + 2\*(3\*c\*d^3\*e + b\*d^2\*e^2 + 3\*a\*d\*e^3)\*x^2)\*sqrt(d\*e)\*arctan(sqrt(d\*e)\*x/d) + (3\*c\*d^4\*e + b\*d^3\*e^2 - 5\*a\*d^2\*e^3)\*x)/(d^3\*e^5\*x^4 + 2\*d^4\*e^4\*x^2 + d^5\*e^3)]

**giac [A]** time = 0.23, size = 101, normalized size = 0.88

$$\frac{(3cd^2 + bde + 3ae^2) \arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{5}{2}\right)}}{8d^{\frac{5}{2}}} - \frac{(5cd^2x^3e - bdx^3e^2 + 3cd^3x - 3ax^3e^3 + bd^2xe - 5adxe^2)e^{(-2)}}{8(x^2e + d)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^3,x, algorithm="giac")

[Out]  $\frac{1}{8}*(3*c*d^2 + b*d*e + 3*a*e^2)*\arctan(x*e^{(1/2)}/\sqrt{d})*e^{(-5/2)}/d^{(5/2)}$   
 $- \frac{1}{8}*(5*c*d^2*x^3*e - b*d*x^3*e^2 + 3*c*d^3*x - 3*a*x^3*e^3 + b*d^2*x*e - 5*a*d*x*e^2)*e^{(-2)}/((x^2*e + d)^2*d^2)$

**maple** [A] time = 0.01, size = 131, normalized size = 1.14

$$\frac{3a \arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de} d^2} + \frac{b \arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de} de} + \frac{3c \arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de} e^2} + \frac{\frac{(3ae^2+bde-5cd^2)x^3}{8d^2e} + \frac{(5ae^2-bde-3cd^2)x}{8de^2}}{(ex^2+d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^4+b\*x^2+a)/(e\*x^2+d)^3,x)

[Out]  $(1/8*(3*a*e^2+b*d*e-5*c*d^2)/d^2/e*x^3+1/8*(5*a*e^2-b*d*e-3*c*d^2)/d/e^2*x)$   
 $/((e*x^2+d)^2+3/8/(d*e)^{(1/2)}*a/d^2*\arctan(1/(d*e)^{(1/2)}*e*x)+1/8/d/e/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)}*e*x)*b+3/8/(d*e)^{(1/2)}*c/e^2*\arctan(1/(d*e)^{(1/2)}*e*x)$

**maxima** [A] time = 2.25, size = 121, normalized size = 1.05

$$-\frac{(5cd^2e - bde^2 - 3ae^3)x^3 + (3cd^3 + bd^2e - 5ade^2)x}{8(d^2e^4x^4 + 2d^3e^3x^2 + d^4e^2)} + \frac{(3cd^2 + bde + 3ae^2) \arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de} d^2 e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^3,x, algorithm="maxima")

[Out]  $-1/8*((5*c*d^2*e - b*d*e^2 - 3*a*e^3)*x^3 + (3*c*d^3 + b*d^2*e - 5*a*d*e^2)*x)/(d^2*e^4*x^4 + 2*d^3*e^3*x^2 + d^4*e^2) + 1/8*(3*c*d^2 + b*d*e + 3*a*e^2)*\arctan(e*x/\sqrt{d*e})/(\sqrt{d*e}*d^2*e^2)$

**mupad** [B] time = 4.85, size = 112, normalized size = 0.97

$$\frac{\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) (3cd^2 + bde + 3ae^2)}{8d^{5/2}e^{5/2}} - \frac{x(3cd^2+bde-5ae^2)}{8de^2} - \frac{x^3(-5cd^2+bde+3ae^2)}{8d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^2 + c\*x^4)/(d + e\*x^2)^3,x)

[Out]  $(\operatorname{atan}((e^{(1/2)}*x)/d^{(1/2)})*(3*a*e^2 + 3*c*d^2 + b*d*e))/(8*d^{(5/2)}*e^{(5/2)})$   
 $- ((x*(3*c*d^2 - 5*a*e^2 + b*d*e))/(8*d*e^2) - (x^3*(3*a*e^2 - 5*c*d^2 + b*d*e))/(8*d^2*e))/(d^2 + e^2*x^4 + 2*d*e*x^2)$

**sympy** [A] time = 2.27, size = 196, normalized size = 1.70

$$-\frac{\sqrt{-\frac{1}{d^5e^5}}(3ae^2 + bde + 3cd^2) \log\left(-d^3e^2\sqrt{-\frac{1}{d^5e^5}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{d^5e^5}}(3ae^2 + bde + 3cd^2) \log\left(d^3e^2\sqrt{-\frac{1}{d^5e^5}} + x\right)}{16} + \frac{x^3(3ae^3 + bde^2 - 5cd^2e) + x(5ade^2 - bd^2e - 3cd^3)}{8d^4e^2 + 16d^3e^3x^2 + 8d^2e^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*4+b\*x\*\*2+a)/(e\*x\*\*2+d)\*\*3,x)

[Out]  $-\sqrt{-1/(d**5*e**5)}*(3*a*e**2 + b*d*e + 3*c*d**2)*\log(-d**3*e**2*\sqrt{-1/(d**5*e**5)} + x)/16 + \sqrt{-1/(d**5*e**5)}*(3*a*e**2 + b*d*e + 3*c*d**2)*\log(d**3*e**2*\sqrt{-1/(d**5*e**5)} + x)/16 + (x**3*(3*a*e**3 + b*d*e**2 - 5*c*d**2*e) + x*(5*a*d*e**2 - b*d**2*e - 3*c*d**3))/(8*d**4*e**2 + 16*d**3*e**3*x**2 + 8*d**2*e**4*x**4)$

$$3.174 \quad \int \frac{a+bx^2+cx^4}{(d+ex^2)^4} dx$$

**Optimal.** Leaf size=150

$$\frac{x(7cd^2 - e(5ae + bd))}{24d^2e^2(d+ex^2)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(e(5ae + bd) + cd^2)}{16d^{7/2}e^{5/2}} + \frac{x(e(5ae + bd) + cd^2)}{16d^3e^2(d+ex^2)} + \frac{x\left(a + \frac{d(cd-be)}{e^2}\right)}{6d(d+ex^2)^3}$$

**Rubi [A]** time = 0.21, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {1157, 385, 199, 205}

$$\frac{x(e(5ae + bd) + cd^2)}{16d^3e^2(d+ex^2)} - \frac{x(7cd^2 - e(5ae + bd))}{24d^2e^2(d+ex^2)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(e(5ae + bd) + cd^2)}{16d^{7/2}e^{5/2}} + \frac{x\left(a + \frac{d(cd-be)}{e^2}\right)}{6d(d+ex^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^4,x]

[Out] ((a + (d\*(c\*d - b\*e))/e^2)\*x)/(6\*d\*(d + e\*x^2)^3) - ((7\*c\*d^2 - e\*(b\*d + 5\*a\*e))\*x)/(24\*d^2\*e^2\*(d + e\*x^2)^2) + ((c\*d^2 + e\*(b\*d + 5\*a\*e))\*x)/(16\*d^3\*e^2\*(d + e\*x^2)) + ((c\*d^2 + e\*(b\*d + 5\*a\*e))\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(16\*d^(7/2)\*e^(5/2))

#### Rule 199

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2\*p] || (n == 2 && IntegerQ[4\*p]) || (n == 2 && IntegerQ[3\*p]) || Denominator[p + 1/n] < Denominator[p])

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

#### Rule 1157

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] := With[{Qx = PolynomialQuotient[(a + b\*x^2 + c\*x^4)^p, d + e\*x^2, x], R = Coeff[PolynomialRemainder[(a + b\*x^2 + c\*x^4)^p, d + e\*x^2, x], 0]}, -Simp[(R\*x\*(d + e\*x^2)^(q + 1))/(2\*d\*(q + 1)), x] + Dist[1/(2\*d\*(q + 1)), Int[(d + e\*x^2)^(q + 1)\*ExpandToSum[2\*d\*(q + 1)\*Qx + R\*(2\*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{a + bx^2 + cx^4}{(d + ex^2)^4} dx &= \frac{(cd^2 - bde + ae^2)x}{6de^2(d + ex^2)^3} - \frac{\int \frac{-5a + \frac{d(cd-be)}{e^2} - \frac{6cdx^2}{e}}{(d+ex^2)^3} dx}{6d} \\ &= \frac{(cd^2 - bde + ae^2)x}{6de^2(d + ex^2)^3} - \frac{(7cd^2 - e(bd + 5ae))x}{24d^2e^2(d + ex^2)^2} + \frac{(cd^2 + e(bd + 5ae)) \int \frac{1}{(d+ex^2)^2} dx}{8d^2e^2} \\ &= \frac{(cd^2 - bde + ae^2)x}{6de^2(d + ex^2)^3} - \frac{(7cd^2 - e(bd + 5ae))x}{24d^2e^2(d + ex^2)^2} + \frac{(cd^2 + e(bd + 5ae))x}{16d^3e^2(d + ex^2)} + \frac{(cd^2 + e(bd + 5ae))}{16d^3e^2} \\ &= \frac{(cd^2 - bde + ae^2)x}{6de^2(d + ex^2)^3} - \frac{(7cd^2 - e(bd + 5ae))x}{24d^2e^2(d + ex^2)^2} + \frac{(cd^2 + e(bd + 5ae))x}{16d^3e^2(d + ex^2)} + \frac{(cd^2 + e(bd + 5ae))}{16d^3e^2} \end{aligned}$$

**Mathematica [A]** time = 0.13, size = 142, normalized size = 0.95

$$\frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(e(5ae + bd) + cd^2)}{16d^{7/2}e^{5/2}} + \frac{x(e(ae(33d^2 + 40dex^2 + 15e^2x^4) + bd(-3d^2 + 8dex^2 + 3e^2x^4)) + cd^2(-3d^2 - 8dex^2 + 3e^2x^4))}{48d^3e^2(d + ex^2)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^4, x]

[Out] (x\*(c\*d^2\*(-3\*d^2 - 8\*d\*e\*x^2 + 3\*e^2\*x^4) + e\*(b\*d\*(-3\*d^2 + 8\*d\*e\*x^2 + 3\*e^2\*x^4) + a\*e\*(33\*d^2 + 40\*d\*e\*x^2 + 15\*e^2\*x^4))))/(48\*d^3\*e^2\*(d + e\*x^2)^3) + ((c\*d^2 + e\*(b\*d + 5\*a\*e))\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(16\*d^(7/2)\*e^(5/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2 + cx^4}{(d + ex^2)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^4, x]

[Out] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^4, x]

**fricas [A]** time = 0.60, size = 530, normalized size = 3.53

$$\frac{1}{96} \left( 6(c^2d^3e^3 + b^2d^2e^4 + 5a^2d^2e^5)x^5 - 16(c^2d^4e^2 - b^2d^3e^3 - 5a^2d^2e^4)x^3 - 3((c^2d^2e^3 + b^2d^2e^4 + 5a^2e^5)x^6 + c^2d^5 + b^2d^4e + 5a^2d^3e^2 + 3(c^2d^3e^2 + b^2d^2e^3 + 5a^2d^2e^4)x^4 + 3(c^2d^4e + b^2d^3e^2 + 5a^2d^2e^3)x^2) \right) \sqrt{-de} \log((ex^2 - 2\sqrt{-de})x - d) / (ex^2 + d) - 6(c^2d^5e + b^2d^4e^2 - 11a^2d^3e^3)x / (d^4e^6x^6 + 3d^5e^5x^4 + 3d^6e^4x^2 + d^7e^3), \frac{1}{48} (3(c^2d^3e^3 + b^2d^2e^4 + 5a^2d^2e^5)x^5 - 8(c^2d^4e^2 - b^2d^3e^3 - 5a^2d^2e^4)x^3 + 3((c^2d^2e^3 + b^2d^2e^4 + 5a^2e^5)x^6 + c^2d^5 + b^2d^4e + 5a^2d^3e^2 + 3(c^2d^3e^2 + b^2d^2e^3 + 5a^2d^2e^4)x^4 + 3(c^2d^4e + b^2d^3e^2 + 5a^2d^2e^3)x^2) \sqrt{-de} \log((ex^2 - 2\sqrt{-de})x - d) / (ex^2 + d) - 6(c^2d^5e + b^2d^4e^2 - 11a^2d^3e^3)x / (d^4e^6x^6 + 3d^5e^5x^4 + 3d^6e^4x^2 + d^7e^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^4, x, algorithm="fricas")

[Out] [1/96\*(6\*(c\*d^3\*e^3 + b\*d^2\*e^4 + 5\*a\*d^2\*e^5)\*x^5 - 16\*(c\*d^4\*e^2 - b\*d^3\*e^3 - 5\*a\*d^2\*e^4)\*x^3 - 3\*((c\*d^2\*e^3 + b\*d^2\*e^4 + 5\*a\*e^5)\*x^6 + c\*d^5 + b\*d^4\*e + 5\*a\*d^3\*e^2 + 3\*(c\*d^3\*e^2 + b\*d^2\*e^3 + 5\*a\*d^2\*e^4)\*x^4 + 3\*(c\*d^4\*e + b\*d^3\*e^2 + 5\*a\*d^2\*e^3)\*x^2)\*sqrt(-d\*e)\*log((e\*x^2 - 2\*sqrt(-d\*e))\*x - d)/(e\*x^2 + d) - 6\*(c\*d^5\*e + b\*d^4\*e^2 - 11\*a\*d^3\*e^3)\*x/(d^4\*e^6\*x^6 + 3\*d^5\*e^5\*x^4 + 3\*d^6\*e^4\*x^2 + d^7\*e^3), 1/48\*(3\*(c\*d^3\*e^3 + b\*d^2\*e^4 + 5\*a\*d^2\*e^5)\*x^5 - 8\*(c\*d^4\*e^2 - b\*d^3\*e^3 - 5\*a\*d^2\*e^4)\*x^3 + 3\*((c\*d^2\*e^3 + b\*d^2\*e^4 + 5\*a\*e^5)\*x^6 + c\*d^5 + b\*d^4\*e + 5\*a\*d^3\*e^2 + 3\*(c\*d^3\*e^2 + b\*d^2\*e^3 + 5\*a\*d^2\*e^4)\*x^4 + 3\*(c\*d^4\*e + b\*d^3\*e^2 + 5\*a\*d^2\*e^3)\*x^2)\*sqrt(-d\*e)\*log((e\*x^2 - 2\*sqrt(-d\*e))\*x - d)/(e\*x^2 + d) - 6\*(c\*d^5\*e + b\*d^4\*e^2 - 11\*a\*d^3\*e^3)\*x/(d^4\*e^6\*x^6 + 3\*d^5\*e^5\*x^4 + 3\*d^6\*e^4\*x^2 + d^7\*e^3)

$$b*d^2*e^3 + 5*a*d*e^4)*x^4 + 3*(c*d^4*e + b*d^3*e^2 + 5*a*d^2*e^3)*x^2)*\sqrt{d*e}*\arctan(\sqrt{d*e}*x/d) - 3*(c*d^5*e + b*d^4*e^2 - 11*a*d^3*e^3)*x)/(d^4*e^6*x^6 + 3*d^5*e^5*x^4 + 3*d^6*e^4*x^2 + d^7*e^3)]$$

**giac [A]** time = 0.16, size = 134, normalized size = 0.89

$$\frac{(cd^2 + bde + 5ae^2) \arctan\left(\frac{x\sqrt{d}}{\sqrt{d}}\right) e^{\left(\frac{5}{2}\right)}}{16d^{\frac{7}{2}}} + \frac{(3cd^2x^5e^2 + 3bdx^5e^3 - 8cd^3x^3e + 15ax^5e^4 + 8bd^2x^3e^2 - 3cd^4x + 40adx^3e^3 - 3bd^3xe + 33ad^2xe^2)e^{(-2)}}{48(x^2e + d)^3 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^4,x, algorithm="giac")

[Out] 1/16\*(c\*d^2 + b\*d\*e + 5\*a\*e^2)\*arctan(x\*e^(1/2)/sqrt(d))\*e^(-5/2)/d^(7/2) + 1/48\*(3\*c\*d^2\*x^5\*e^2 + 3\*b\*d\*x^5\*e^3 - 8\*c\*d^3\*x^3\*e + 15\*a\*x^5\*e^4 + 8\*b\*d^2\*x^3\*e^2 - 3\*c\*d^4\*x + 40\*a\*d\*x^3\*e^3 - 3\*b\*d^3\*x\*e + 33\*a\*d^2\*x\*e^2)\*e^(-2)/((x^2\*e + d)^3\*d^3)

**maple [A]** time = 0.01, size = 158, normalized size = 1.05

$$\frac{5a \arctan\left(\frac{ex}{\sqrt{de}}\right)}{16\sqrt{de} d^3} + \frac{b \arctan\left(\frac{ex}{\sqrt{de}}\right)}{16\sqrt{de} d^2e} + \frac{c \arctan\left(\frac{ex}{\sqrt{de}}\right)}{16\sqrt{de} d e^2} + \frac{(5ae^2 + bde + cd^2)x^5}{16d^3} + \frac{(5ae^2 + bde - cd^2)x^3}{6d^2e} + \frac{(11ae^2 - bde - cd^2)x}{16de^2} \frac{1}{(ex^2 + d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^4+b\*x^2+a)/(e\*x^2+d)^4,x)

[Out] (1/16\*(5\*a\*e^2+b\*d\*e+c\*d^2)/d^3\*x^5+1/6\*(5\*a\*e^2+b\*d\*e-c\*d^2)/d^2/e\*x^3+1/16\*(11\*a\*e^2-b\*d\*e-c\*d^2)/d/e^2\*x)/(e\*x^2+d)^3+5/16/(d\*e)^(1/2)\*a/d^3\*arctan(1/(d\*e)^(1/2)\*e\*x)+1/16/d^2/e/(d\*e)^(1/2)\*arctan(1/(d\*e)^(1/2)\*e\*x)\*b+1/16/(d\*e)^(1/2)\*c/d/e^2\*arctan(1/(d\*e)^(1/2)\*e\*x)

**maxima [A]** time = 2.51, size = 162, normalized size = 1.08

$$\frac{3(cd^2e^2 + bde^3 + 5ae^4)x^5 - 8(cd^3e - bd^2e^2 - 5ade^3)x^3 - 3(cd^4 + bd^3e - 11ad^2e^2)x}{48(d^3e^5x^6 + 3d^4e^4x^4 + 3d^5e^3x^2 + d^6e^2)} + \frac{(cd^2 + bde + 5ae^2) \arctan\left(\frac{ex}{\sqrt{de}}\right)}{16\sqrt{de} d^3 e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^4,x, algorithm="maxima")

[Out] 1/48\*(3\*(c\*d^2\*e^2 + b\*d\*e^3 + 5\*a\*e^4)\*x^5 - 8\*(c\*d^3\*e - b\*d^2\*e^2 - 5\*a\*d\*e^3)\*x^3 - 3\*(c\*d^4 + b\*d^3\*e - 11\*a\*d^2\*e^2)\*x)/(d^3\*e^5\*x^6 + 3\*d^4\*e^4\*x^4 + 3\*d^5\*e^3\*x^2 + d^6\*e^2) + 1/16\*(c\*d^2 + b\*d\*e + 5\*a\*e^2)\*arctan(e\*x/sqrt(d\*e))/(sqrt(d\*e)\*d^3\*e^2)

**mupad [B]** time = 4.51, size = 144, normalized size = 0.96

$$\frac{\frac{x^5(cd^2 + bde + 5ae^2)}{16d^3} - \frac{x(cd^2 + bde - 11ae^2)}{16de^2} + \frac{x^3(-cd^2 + bde + 5ae^2)}{6d^2e}}{d^3 + 3d^2ex^2 + 3de^2x^4 + e^3x^6} + \frac{\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(cd^2 + bde + 5ae^2)}{16d^{7/2}e^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^2 + c\*x^4)/(d + e\*x^2)^4,x)

[Out] ((x^5\*(5\*a\*e^2 + c\*d^2 + b\*d\*e))/(16\*d^3) - (x\*(c\*d^2 - 11\*a\*e^2 + b\*d\*e))/(16\*d\*e^2) + (x^3\*(5\*a\*e^2 - c\*d^2 + b\*d\*e))/(6\*d^2\*e))/(d^3 + e^3\*x^6 + 3\*d^2\*e\*x^2 + 3\*d\*e^2\*x^4) + (atan((e^(1/2)\*x)/d^(1/2))\*(5\*a\*e^2 + c\*d^2 + b\*d\*e))/(16\*d^(7/2)\*e^(5/2))



**sympy [A]** time = 4.41, size = 241, normalized size = 1.61

$$\frac{\sqrt{-\frac{1}{d^2e^5}}(5ae^2 + bde + cd^2) \log\left(-d^4e^2\sqrt{-\frac{1}{d^2e^5}} + x\right)}{32} + \frac{\sqrt{-\frac{1}{d^2e^5}}(5ae^2 + bde + cd^2) \log\left(d^4e^2\sqrt{-\frac{1}{d^2e^5}} + x\right)}{32} + \frac{x^5(15ae^4 + 3bde^3 + 3cd^2e^2) + x^3(40ade^3 + 8bd^2e^2 - 8cd^3e) + x(33ad^2e^2 - 3bd^3e - 3cd^4)}{48d^6e^2 + 144d^5e^3x^2 + 144d^4e^4x^4 + 48d^3e^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*4+b\*x\*\*2+a)/(e\*x\*\*2+d)\*\*4,x)

[Out]  $-\sqrt{-1/(d**7*e**5)}*(5*a*e**2 + b*d*e + c*d**2)*\log(-d**4*e**2*\sqrt{-1/(d**7*e**5)} + x)/32 + \sqrt{-1/(d**7*e**5)}*(5*a*e**2 + b*d*e + c*d**2)*\log(d**4*e**2*\sqrt{-1/(d**7*e**5)} + x)/32 + (x**5*(15*a*e**4 + 3*b*d*e**3 + 3*c*d**2*e**2) + x**3*(40*a*d*e**3 + 8*b*d**2*e**2 - 8*c*d**3*e) + x*(33*a*d**2*e**2 - 3*b*d**3*e - 3*c*d**4))/(48*d**6*e**2 + 144*d**5*e**3*x**2 + 144*d**4*e**4*x**4 + 48*d**3*e**5*x**6)$

$$3.175 \quad \int (d + ex^2)^3 (a + bx^2 + cx^4)^2 dx$$

**Optimal.** Leaf size=223

$$\frac{1}{7}x^7 (a^2e^3 + 6abde^2 + 6acd^2e + 3b^2d^2e + 2bcd^3) + a^2d^3x + \frac{1}{11}ex^{11} (2ce(ae + 3bd) + b^2e^2 + 3c^2d^2) + \frac{1}{5}dx^5 (6abde + a$$

**Rubi [A]** time = 0.20, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$ , Rules used = {1153}

$$\frac{1}{7}x^7 (a^2e^3 + 6abde^2 + 6acd^2e + 3b^2d^2e + 2bcd^3) + a^2d^3x + \frac{1}{11}ex^{11} (2ce(ae + 3bd) + b^2e^2 + 3c^2d^2) + \frac{1}{5}dx^5 (6abde + a(3ae^2 + 2cd^2) + b^2d^2) + \frac{1}{9}x^9 (6cde(ae + bd) + b^2(2ae + 3bd) + c^2d^3) + \frac{1}{3}ad^2x^3(3ae + 2bd) + \frac{1}{13}ce^2x^{13}(2be + 3cd) + \frac{1}{15}c^2e^3x^{15}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^3\*(a + b\*x^2 + c\*x^4)^2,x]

[Out] a^2\*d^3\*x + (a\*d^2\*(2\*b\*d + 3\*a\*e)\*x^3)/3 + (d\*(b^2\*d^2 + 6\*a\*b\*d\*e + a\*(2\*c\*d^2 + 3\*a\*e^2))\*x^5)/5 + ((2\*b\*c\*d^3 + 3\*b^2\*d^2\*e + 6\*a\*c\*d^2\*e + 6\*a\*b\*d\*e^2 + a^2\*e^3)\*x^7)/7 + ((c^2\*d^3 + 6\*c\*d\*e\*(b\*d + a\*e) + b\*e^2\*(3\*b\*d + 2\*a\*e))\*x^9)/9 + (e\*(3\*c^2\*d^2 + b^2\*e^2 + 2\*c\*e\*(3\*b\*d + a\*e))\*x^11)/11 + (c\*e^2\*(3\*c\*d + 2\*b\*e)\*x^13)/13 + (c^2\*e^3\*x^15)/15

Rule 1153

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^2)^q\*(a + b\*x^2 + c\*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int (d + ex^2)^3 (a + bx^2 + cx^4)^2 dx &= \int (a^2d^3 + ad^2(2bd + 3ae)x^2 + d(b^2d^2 + 6abde + a(2cd^2 + 3ae^2)))x^4 + (2bcd^3 + a^2d^3)x^5 + (2b^2cd^2 + 6abd^2e + 6acd^2e + 3b^2d^2e + 2bcd^3)x^6 + (2b^2cd^2 + 6abd^2e + 6acd^2e + 3b^2d^2e + 2bcd^3)x^7 + (2b^2cd^2 + 6abd^2e + 6acd^2e + 3b^2d^2e + 2bcd^3)x^8 + (2b^2cd^2 + 6abd^2e + 6acd^2e + 3b^2d^2e + 2bcd^3)x^9 + (2b^2cd^2 + 6abd^2e + 6acd^2e + 3b^2d^2e + 2bcd^3)x^{10} + (2b^2cd^2 + 6abd^2e + 6acd^2e + 3b^2d^2e + 2bcd^3)x^{11} + (2b^2cd^2 + 6abd^2e + 6acd^2e + 3b^2d^2e + 2bcd^3)x^{12} + (2b^2cd^2 + 6abd^2e + 6acd^2e + 3b^2d^2e + 2bcd^3)x^{13} + (2b^2cd^2 + 6abd^2e + 6acd^2e + 3b^2d^2e + 2bcd^3)x^{14} + (2b^2cd^2 + 6abd^2e + 6acd^2e + 3b^2d^2e + 2bcd^3)x^{15} \\ &= a^2d^3x + \frac{1}{3}ad^2(2bd + 3ae)x^3 + \frac{1}{5}d(b^2d^2 + 6abde + a(2cd^2 + 3ae^2))x^5 + \frac{1}{7}(2b^2cd^2 + 6abd^2e + 6acd^2e + 3b^2d^2e + 2bcd^3)x^7 + \frac{1}{9}(2b^2cd^2 + 6abd^2e + 6acd^2e + 3b^2d^2e + 2bcd^3)x^9 + \frac{1}{11}(2b^2cd^2 + 6abd^2e + 6acd^2e + 3b^2d^2e + 2bcd^3)x^{11} + \frac{1}{13}(2b^2cd^2 + 6abd^2e + 6acd^2e + 3b^2d^2e + 2bcd^3)x^{13} + \frac{1}{15}(2b^2cd^2 + 6abd^2e + 6acd^2e + 3b^2d^2e + 2bcd^3)x^{15} \end{aligned}$$

**Mathematica [A]** time = 0.09, size = 223, normalized size = 1.00

$$\frac{1}{7}x^7 (a^2e^3 + 6abde^2 + 6acd^2e + 3b^2d^2e + 2bcd^3) + a^2d^3x + \frac{1}{11}ex^{11} (2ce(ae + 3bd) + b^2e^2 + 3c^2d^2) + \frac{1}{5}dx^5 (6abde + a(3ae^2 + 2cd^2) + b^2d^2) + \frac{1}{9}x^9 (6cde(ae + bd) + b^2(2ae + 3bd) + c^2d^3) + \frac{1}{3}ad^2x^3(3ae + 2bd) + \frac{1}{13}ce^2x^{13}(2be + 3cd) + \frac{1}{15}c^2e^3x^{15}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^3\*(a + b\*x^2 + c\*x^4)^2,x]

[Out] a^2\*d^3\*x + (a\*d^2\*(2\*b\*d + 3\*a\*e)\*x^3)/3 + (d\*(b^2\*d^2 + 6\*a\*b\*d\*e + a\*(2\*c\*d^2 + 3\*a\*e^2))\*x^5)/5 + ((2\*b\*c\*d^3 + 3\*b^2\*d^2\*e + 6\*a\*c\*d^2\*e + 6\*a\*b\*d\*e^2 + a^2\*e^3)\*x^7)/7 + ((c^2\*d^3 + 6\*c\*d\*e\*(b\*d + a\*e) + b\*e^2\*(3\*b\*d + 2\*a\*e))\*x^9)/9 + (e\*(3\*c^2\*d^2 + b^2\*e^2 + 2\*c\*e\*(3\*b\*d + a\*e))\*x^11)/11 + (c\*e^2\*(3\*c\*d + 2\*b\*e)\*x^13)/13 + (c^2\*e^3\*x^15)/15

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex^2)^3 (a + bx^2 + cx^4)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)^3\*(a + b\*x^2 + c\*x^4)^2,x]

[Out] IntegrateAlgebraic[(d + e\*x^2)^3\*(a + b\*x^2 + c\*x^4)^2, x]

**fricas** [A] time = 0.67, size = 261, normalized size = 1.17

$$\frac{1}{15}x^{15}e^3c^2 + \frac{3}{13}x^{13}e^2d^2c + \frac{2}{13}x^{13}e^3cb + \frac{3}{11}x^{11}e^2d^2c + \frac{6}{11}x^{11}e^3dcb + \frac{1}{11}x^{11}e^2cb + \frac{2}{11}x^{11}e^3ca + \frac{1}{9}x^9e^2d^2c + \frac{2}{3}x^9e^3dcb + \frac{1}{3}x^9e^2d^2c + \frac{2}{3}x^9e^3dca + \frac{2}{7}x^7e^2d^2c + \frac{3}{7}x^7e^3dcb + \frac{6}{7}x^7e^2d^2c + \frac{6}{7}x^7e^3dba + \frac{1}{7}x^7e^2d^2c + \frac{1}{5}x^5e^3d^2c + \frac{2}{5}x^5e^2d^2c + \frac{6}{5}x^5e^3dba + \frac{3}{5}x^5e^2d^2c + \frac{2}{5}x^5e^3dba + x^3e^2d^2c + x^3e^3d^2c + x^3e^2d^2c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^3\*(c\*x^4+b\*x^2+a)^2,x, algorithm="fricas")

[Out] 1/15\*x^15\*e^3\*c^2 + 3/13\*x^13\*e^2\*d\*c^2 + 2/13\*x^13\*e^3\*c\*b + 3/11\*x^11\*e\*d^2\*c^2 + 6/11\*x^11\*e^2\*d\*c\*b + 1/11\*x^11\*e^3\*b^2 + 2/11\*x^11\*e^3\*c\*a + 1/9\*x^9\*d^3\*c^2 + 2/3\*x^9\*e\*d^2\*c\*b + 1/3\*x^9\*e^2\*d\*b^2 + 2/3\*x^9\*e^2\*d\*c\*a + 2/9\*x^9\*e^3\*b\*a + 2/7\*x^7\*d^3\*c\*b + 3/7\*x^7\*e\*d^2\*b^2 + 6/7\*x^7\*e\*d^2\*c\*a + 6/7\*x^7\*e^2\*d\*b\*a + 1/7\*x^7\*e^3\*a^2 + 1/5\*x^5\*d^3\*b^2 + 2/5\*x^5\*d^3\*c\*a + 6/5\*x^5\*e\*d^2\*b\*a + 3/5\*x^5\*e^2\*d\*a^2 + 2/3\*x^3\*d^3\*b\*a + x^3\*e\*d^2\*a^2 + x^3\*d^3\*a^2

**giac** [A] time = 0.16, size = 255, normalized size = 1.14

$$\frac{1}{15}x^{15}e^3 + \frac{3}{13}x^{13}e^2d^2c + \frac{2}{13}x^{13}e^3cb + \frac{3}{11}x^{11}e^2d^2c + \frac{6}{11}x^{11}e^3dcb + \frac{1}{11}x^{11}e^2cb + \frac{2}{11}x^{11}e^3ca + \frac{1}{9}x^9e^2d^2c + \frac{2}{3}x^9e^3dcb + \frac{1}{3}x^9e^2d^2c + \frac{2}{3}x^9e^3dca + \frac{2}{7}x^7e^2d^2c + \frac{3}{7}x^7e^3dcb + \frac{6}{7}x^7e^2d^2c + \frac{6}{7}x^7e^3dba + \frac{1}{7}x^7e^2d^2c + \frac{1}{5}x^5e^3d^2c + \frac{2}{5}x^5e^2d^2c + \frac{6}{5}x^5e^3dba + \frac{3}{5}x^5e^2d^2c + \frac{2}{5}x^5e^3dba + x^3e^2d^2c + x^3e^3d^2c + x^3e^2d^2c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^3\*(c\*x^4+b\*x^2+a)^2,x, algorithm="giac")

[Out] 1/15\*c^2\*x^15\*e^3 + 3/13\*c^2\*d\*x^13\*e^2 + 2/13\*b\*c\*x^13\*e^3 + 3/11\*c^2\*d^2\*x^11\*e + 6/11\*b\*c\*d\*x^11\*e^2 + 1/9\*c^2\*d^3\*x^9 + 1/11\*b^2\*x^11\*e^3 + 2/11\*a\*c\*x^11\*e^3 + 2/3\*b\*c\*d^2\*x^9\*e + 1/3\*b^2\*d\*x^9\*e^2 + 2/3\*a\*c\*d\*x^9\*e^2 + 2/7\*b\*c\*d^3\*x^7 + 2/9\*a\*b\*x^9\*e^3 + 3/7\*b^2\*d^2\*x^7\*e + 6/7\*a\*c\*d^2\*x^7\*e + 6/7\*a\*b\*d\*x^7\*e^2 + 1/5\*b^2\*d^3\*x^5 + 2/5\*a\*c\*d^3\*x^5 + 1/7\*a^2\*x^7\*e^3 + 6/5\*a\*b\*d^2\*x^5\*e + 3/5\*a^2\*d\*x^5\*e^2 + 2/3\*a\*b\*d^3\*x^3 + a^2\*d^2\*x^3\*e + a^2\*d^3\*x

**maple** [A] time = 0.00, size = 219, normalized size = 0.98

$$\frac{c^2e^3x^{15}}{15} + \frac{(2e^2bc + 3d^2e^2c)x^{13}}{13} + \frac{(6bcd^2 + 3c^2d^2e + (2ac + b^2)e^3)x^{11}}{11} + \frac{(2ab^2c + 6bcd^2e + c^2d^3 + 3(2ac + b^2)d^2e)x^9}{9} + \frac{(a^2e^3 + 6abd^2e + 2bc^2d^3 + 3(2ac + b^2)d^2e)x^7}{7} + a^2d^3x + \frac{(3a^2d^2e^2 + 6abd^2e + (2ac + b^2)d^3)x^5}{5} + \frac{(3d^2e^2 + 2d^3ab)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^3\*(c\*x^4+b\*x^2+a)^2,x)

[Out] 1/15\*c^2\*e^3\*x^15+1/13\*(2\*b\*c\*e^3+3\*c^2\*d\*e^2)\*x^13+1/11\*(3\*d^2\*e\*c^2+6\*d\*e^2\*b\*c+e^3\*(2\*a\*c+b^2))\*x^11+1/9\*(c^2\*d^3+6\*d^2\*e\*b\*c+3\*d\*e^2\*(2\*a\*c+b^2)+2\*e^3\*a\*b)\*x^9+1/7\*(2\*b\*c\*d^3+3\*d^2\*e\*(2\*a\*c+b^2)+6\*a\*b\*d\*e^2+a^2\*e^3)\*x^7+1/5\*(d^3\*(2\*a\*c+b^2)+6\*d^2\*e\*a\*b+3\*d\*e^2\*a^2)\*x^5+1/3\*(3\*a^2\*d^2\*e+2\*a\*b\*d^3)\*x^3+a^2\*d^3\*x

**maxima** [A] time = 1.04, size = 218, normalized size = 0.98

$$\frac{1}{15}c^2e^3x^{15} + \frac{1}{13}(3c^2de^2 + 2bcd^2e + (b^2 + 2ac)e^3)x^{11} + \frac{1}{11}(2d^2e^2c + 6bcd^2e + (b^2 + 2ac)d^2e^2)x^9 + \frac{1}{9}(2bcd^3 + 6abd^2e + a^2e^3 + 3(b^2 + 2ac)d^2e)x^7 + \frac{1}{7}(2bcd^3 + 6abd^2e + a^2e^3 + 3(b^2 + 2ac)d^2e)x^5 + \frac{1}{5}(6abd^2e + 3a^2d^2e + (b^2 + 2ac)d^3)x^3 + \frac{1}{3}(2abd^3 + 3a^2d^2e)x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^3\*(c\*x^4+b\*x^2+a)^2,x, algorithm="maxima")

[Out] 1/15\*c^2\*e^3\*x^15 + 1/13\*(3\*c^2\*d\*e^2 + 2\*b\*c\*e^3)\*x^13 + 1/11\*(3\*c^2\*d^2\*e + 6\*b\*c\*d\*e^2 + (b^2 + 2\*a\*c)\*e^3)\*x^11 + 1/9\*(c^2\*d^3 + 6\*b\*c\*d^2\*e + 2\*a\*b\*e^3 + 3\*(b^2 + 2\*a\*c)\*d\*e^2)\*x^9 + 1/7\*(2\*b\*c\*d^3 + 6\*a\*b\*d\*e^2 + a^2\*e^3 + 3\*(b^2 + 2\*a\*c)\*d^2\*e)\*x^7 + a^2\*d^3\*x + 1/5\*(6\*a\*b\*d^2\*e + 3\*a^2\*d^2\*e^2 + (b^2 + 2\*a\*c)\*d^3)\*x^5 + 1/3\*(2\*a\*b\*d^3 + 3\*a^2\*d^2\*e)\*x^3

**mupad** [B] time = 4.48, size = 220, normalized size = 0.99

$$x^7 \left( \frac{a^2 e^3}{7} + \frac{6 a b d^2 e}{7} + \frac{6 c a d^2 e}{7} + \frac{3 b^2 d^2 e}{7} + \frac{2 c b d^3}{7} \right) + x^9 \left( \frac{b^2 d^2}{3} + \frac{2 b c d^2 e}{3} + \frac{2 a b e^3}{9} + \frac{c^2 d^3}{9} + \frac{2 a c d^2 e}{3} \right) + x^5 \left( \frac{3 a^2 d^2}{5} + \frac{6 a b d^2 e}{5} + \frac{2 c a d^3}{5} + \frac{b^2 d^3}{5} \right) + x^{11} \left( \frac{b^2 e^3}{11} + \frac{6 b c d^2 e}{11} + \frac{3 c^2 d^2 e}{11} + \frac{2 a c e^3}{11} \right) + a^2 d^3 x + \frac{c^2 e^3 x^{15}}{15} + \frac{a d^2 x^3 (3 a e + 2 b d)}{3} + \frac{c^2 x^{13} (2 b e + 3 c d)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x^2)^3*(a + b*x^2 + c*x^4)^2,x)`

[Out]  $x^7*((a^2*e^3)/7 + (3*b^2*d^2*e)/7 + (2*b*c*d^3)/7 + (6*a*b*d*e^2)/7 + (6*a*c*d^2*e)/7) + x^9*((c^2*d^3)/9 + (b^2*d*e^2)/3 + (2*a*b*e^3)/9 + (2*a*c*d*e^2)/3 + (2*b*c*d^2*e)/3) + x^5*((b^2*d^3)/5 + (3*a^2*d*e^2)/5 + (2*a*c*d^3)/5 + (6*a*b*d^2*e)/5) + x^{11}*((b^2*e^3)/11 + (3*c^2*d^2*e)/11 + (2*a*c*e^3)/11 + (6*b*c*d*e^2)/11) + a^2*d^3*x + (c^2*e^3*x^{15})/15 + (a*d^2*x^3*(3*a*e + 2*b*d))/3 + (c*e^2*x^{13}*(2*b*e + 3*c*d))/13$

**sympy [A]** time = 0.22, size = 272, normalized size = 1.22

$$a^2 d^3 x + \frac{c^2 e^3 x^{15}}{15} + x^{13} \left( \frac{2 b c e^3}{13} + \frac{3 c^2 d e^2}{13} \right) + x^{11} \left( \frac{2 a c e^3}{11} + \frac{b^2 e^3}{11} + \frac{6 b c d e^2}{11} + \frac{3 c^2 d^2 e}{11} \right) + x^9 \left( \frac{2 a b e^3}{9} + \frac{2 a c d e^2}{3} + \frac{b^2 d e^2}{3} + \frac{2 b c d^2 e}{3} + \frac{c^2 d^3}{9} \right) + x^7 \left( \frac{a^2 e^3}{7} + \frac{6 a b d e^2}{7} + \frac{6 a c d^2 e}{7} + \frac{3 b^2 d^2 e}{7} + \frac{2 b c d^3}{7} \right) + x^5 \left( \frac{3 a^2 d e^2}{5} + \frac{6 a b d^2 e}{5} + \frac{2 a c d^3}{5} + \frac{b^2 d^3}{5} \right) + x^3 \left( a^2 d^2 e + \frac{2 a b d^3}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**3*(c*x**4+b*x**2+a)**2,x)`

[Out]  $a**2*d**3*x + c**2*e**3*x**15/15 + x**13*(2*b*c*e**3/13 + 3*c**2*d*e**2/13) + x**11*(2*a*c*e**3/11 + b**2*e**3/11 + 6*b*c*d*e**2/11 + 3*c**2*d**2*e/11) + x**9*(2*a*b*e**3/9 + 2*a*c*d*e**2/3 + b**2*d*e**2/3 + 2*b*c*d**2*e/3 + c**2*d**3/9) + x**7*(a**2*e**3/7 + 6*a*b*d*e**2/7 + 6*a*c*d**2*e/7 + 3*b**2*d**2*e/7 + 2*b*c*d**3/7) + x**5*(3*a**2*d*e**2/5 + 6*a*b*d**2*e/5 + 2*a*c*d**3/5 + b**2*d**3/5) + x**3*(a**2*d**2*e + 2*a*b*d**3/3)$

$$3.176 \quad \int (d + ex^2)^2 (a + bx^2 + cx^4)^2 dx$$

**Optimal.** Leaf size=155

$$a^2d^2x + \frac{1}{9}x^9(2ce(ae + 2bd) + b^2e^2 + c^2d^2) + \frac{2}{7}x^7(abe^2 + 2acde + b^2de + bcd^2) + \frac{1}{5}x^5(4abde + a(ae^2 + 2cd^2) + bcd^2) + \frac{1}{13}c^2e^2x^{13}$$

**Rubi [A]** time = 0.14, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$ , Rules used = {1153}

$$a^2d^2x + \frac{1}{9}x^9(2ce(ae + 2bd) + b^2e^2 + c^2d^2) + \frac{2}{7}x^7(abe^2 + 2acde + b^2de + bcd^2) + \frac{1}{5}x^5(4abde + a(ae^2 + 2cd^2) + bcd^2) + \frac{2}{3}adx^3(ae + bd) + \frac{2}{11}cex^{11}(be + cd) + \frac{1}{13}c^2e^2x^{13}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^2\*(a + b\*x^2 + c\*x^4)^2,x]

[Out] a^2\*d^2\*x + (2\*a\*d\*(b\*d + a\*e)\*x^3)/3 + ((b^2\*d^2 + 4\*a\*b\*d\*e + a\*(2\*c\*d^2 + a\*e^2))\*x^5)/5 + (2\*(b\*c\*d^2 + b^2\*d\*e + 2\*a\*c\*d\*e + a\*b\*e^2)\*x^7)/7 + ((c^2\*d^2 + b^2\*e^2 + 2\*c\*e\*(2\*b\*d + a\*e))\*x^9)/9 + (2\*c\*e\*(c\*d + b\*e)\*x^11)/11 + (c^2\*e^2\*x^13)/13

**Rule 1153**

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^2)^q\*(a + b\*x^2 + c\*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

**Rubi steps**

$$\begin{aligned} \int (d + ex^2)^2 (a + bx^2 + cx^4)^2 dx &= \int (a^2d^2 + 2ad(bd + ae)x^2 + (b^2d^2 + 4abde + a(2cd^2 + ae^2))x^4 + 2(bcd^2 + abde + a^2e^2)x^6 + (b^2d^2 + 4abde + a(2cd^2 + ae^2))x^8 + (2cd^2 + ae^2)cx^{10} + c^2e^2x^{12}) dx \\ &= a^2d^2x + \frac{2}{3}ad(bd + ae)x^3 + \frac{1}{5}(b^2d^2 + 4abde + a(2cd^2 + ae^2))x^5 + \frac{2}{7}(bcd^2 + abde + a^2e^2)x^7 + \frac{1}{9}(2cd^2 + ae^2)cx^9 + \frac{1}{13}c^2e^2x^{11} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 156, normalized size = 1.01

$$\frac{1}{5}x^5(a^2e^2 + 4abde + 2acd^2 + b^2d^2) + a^2d^2x + \frac{1}{9}x^9(2ace^2 + b^2e^2 + 4bcde + c^2d^2) + \frac{2}{7}x^7(abe^2 + 2acde + b^2de + bcd^2) + \frac{2}{3}adx^3(ae + bd) + \frac{2}{11}cex^{11}(be + cd) + \frac{1}{13}c^2e^2x^{13}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^2\*(a + b\*x^2 + c\*x^4)^2,x]

[Out] a^2\*d^2\*x + (2\*a\*d\*(b\*d + a\*e)\*x^3)/3 + ((b^2\*d^2 + 2\*a\*c\*d^2 + 4\*a\*b\*d\*e + a^2\*e^2)\*x^5)/5 + (2\*(b\*c\*d^2 + b^2\*d\*e + 2\*a\*c\*d\*e + a\*b\*e^2)\*x^7)/7 + ((c^2\*d^2 + 4\*b\*c\*d\*e + b^2\*e^2 + 2\*a\*c\*e^2)\*x^9)/9 + (2\*c\*e\*(c\*d + b\*e)\*x^11)/11 + (c^2\*e^2\*x^13)/13

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex^2)^2 (a + bx^2 + cx^4)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)^2\*(a + b\*x^2 + c\*x^4)^2,x]

[Out] IntegrateAlgebraic[(d + e\*x^2)^2\*(a + b\*x^2 + c\*x^4)^2, x]

**fricas** [A] time = 0.56, size = 181, normalized size = 1.17

$$\frac{1}{13}x^{13}e^2c^2 + \frac{2}{11}x^{11}edc^2 + \frac{2}{11}x^{11}e^2cb + \frac{1}{9}x^9d^2c^2 + \frac{4}{9}x^9edcb + \frac{1}{9}x^9e^2b^2 + \frac{2}{9}x^9e^2ca + \frac{2}{7}x^7d^2cb + \frac{2}{7}x^7edbe + \frac{4}{7}x^7edca + \frac{2}{7}x^7e^2ba + \frac{1}{5}x^5d^2b^2 + \frac{2}{5}x^5d^2ca + \frac{4}{5}x^5edba + \frac{1}{3}x^5e^2a^2 + \frac{2}{3}x^3d^2ba + \frac{2}{3}x^3eda^2 + \frac{2}{3}x^3e^2da^2 + xd^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(c\*x^4+b\*x^2+a)^2,x, algorithm="fricas")

$$\begin{aligned} \text{[Out]} & \frac{1}{13}x^{13}e^2c^2 + \frac{2}{11}x^{11}e^2d^2c^2 + \frac{2}{11}x^{11}e^2c^2b + \frac{1}{9}x^9d^2c^2 + \\ & + \frac{4}{9}x^9e^2d^2c^2b + \frac{1}{9}x^9e^2d^2b^2 + \frac{2}{9}x^9e^2c^2a + \frac{2}{7}x^7d^2c^2b + \\ & + \frac{2}{7}x^7e^2d^2c^2b + \frac{4}{7}x^7e^2d^2c^2a + \frac{2}{7}x^7e^2d^2b^2a + \frac{1}{5}x^5d^2b^2 + \frac{2}{5} \\ & *x^5d^2c^2a + \frac{4}{5}x^5e^2d^2b^2a + \frac{1}{3}x^3d^2b^2a + \frac{2}{3}x^3 \\ & *e^2d^2a^2 + xd^2a^2 \end{aligned}$$

**giac** [A] time = 0.17, size = 181, normalized size = 1.17

$$\frac{1}{13}c^2x^{13}e^2 + \frac{2}{11}c^2dx^{11}e + \frac{2}{11}bcx^{11}e^2 + \frac{1}{9}c^2d^2x^9 + \frac{4}{9}bcdx^9e + \frac{1}{9}b^2x^9e^2 + \frac{2}{9}acx^9e^2 + \frac{2}{7}bcd^2x^7 + \frac{2}{7}b^2dx^7e + \frac{4}{7}acdx^7e + \frac{2}{7}abx^7e^2 + \frac{1}{5}b^2d^2x^5 + \frac{2}{5}acd^2x^5 + \frac{4}{5}abdx^5e + \frac{1}{3}a^2x^5e^2 + \frac{2}{3}abd^2x^3 + \frac{2}{3}a^2dx^3e + a^2d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(c\*x^4+b\*x^2+a)^2,x, algorithm="giac")

$$\begin{aligned} \text{[Out]} & \frac{1}{13}c^2x^{13}e^2 + \frac{2}{11}c^2d^2x^{11}e + \frac{2}{11}b^2c^2x^{11}e^2 + \frac{1}{9}c^2d^2x^9 + \\ & + \frac{4}{9}b^2c^2d^2x^9e + \frac{1}{9}b^2d^2x^9e^2 + \frac{2}{9}a^2c^2x^9e^2 + \frac{2}{7}b^2c^2d^2x^7 + \\ & + \frac{2}{7}b^2d^2x^7e + \frac{4}{7}a^2c^2d^2x^7e + \frac{2}{7}a^2b^2x^7e^2 + \frac{1}{5}b^2d^2x^5 + \frac{2}{5} \\ & *a^2c^2d^2x^5 + \frac{4}{5}a^2b^2d^2x^5e + \frac{1}{3}a^2d^2x^5e^2 + \frac{2}{3}a^2b^2d^2x^3 + \frac{2}{3}a^2 \\ & *d^2x^3e + a^2d^2x \end{aligned}$$

**maple** [A] time = 0.00, size = 155, normalized size = 1.00

$$\frac{c^2e^2x^{13}}{13} + \frac{(2bc^2e^2 + 2c^2de)x^{11}}{11} + \frac{(4bcde + c^2d^2 + (2ac + b^2)e^2)x^9}{9} + \frac{(2ab^2e^2 + 2bc^2d^2 + 2(2ac + b^2)de)x^7}{7} + a^2d^2x + \frac{(a^2e^2 + 4abde + (2ac + b^2)d^2)x^5}{5} + \frac{(2de a^2 + 2d^2ab)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^2\*(c\*x^4+b\*x^2+a)^2,x)

$$\begin{aligned} \text{[Out]} & \frac{1}{13}c^2e^2x^{13} + \frac{1}{11}(2b^2c^2e^2 + 2c^2d^2e^2)x^{11} + \frac{1}{9}(c^2d^2 + 4d^2e^2b^2c + e^2 \\ & * (2a^2c + b^2))x^9 + \frac{1}{7}(2b^2c^2d^2 + 2d^2e^2(2a^2c + b^2) + 2a^2b^2e^2)x^7 + \frac{1}{5}(d^2 \\ & * (2a^2c + b^2) + 4a^2b^2d^2e + e^2a^2)x^5 + \frac{1}{3}(2a^2d^2e + 2a^2b^2d^2)x^3 + a^2d^2x \end{aligned}$$

**maxima** [A] time = 1.14, size = 147, normalized size = 0.95

$$\frac{1}{13}c^2e^2x^{13} + \frac{2}{11}(c^2de + bce^2)x^{11} + \frac{1}{9}(c^2d^2 + 4bcde + (b^2 + 2ac)e^2)x^9 + \frac{2}{7}(bcd^2 + abe^2 + (b^2 + 2ac)de)x^7 + \frac{1}{5}(4abde + a^2e^2 + (b^2 + 2ac)d^2)x^5 + a^2d^2x + \frac{2}{3}(abd^2 + a^2de)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(c\*x^4+b\*x^2+a)^2,x, algorithm="maxima")

$$\begin{aligned} \text{[Out]} & \frac{1}{13}c^2e^2x^{13} + \frac{2}{11}(c^2d^2e + b^2c^2e^2)x^{11} + \frac{1}{9}(c^2d^2 + 4b^2c^2d^2e \\ & + (b^2 + 2a^2c)e^2)x^9 + \frac{2}{7}(b^2c^2d^2 + a^2b^2e^2 + (b^2 + 2a^2c)d^2e)x^7 \\ & + \frac{1}{5}(4a^2b^2d^2e + a^2d^2e^2 + (b^2 + 2a^2c)d^2)x^5 + a^2d^2x + \frac{2}{3}(a^2 \\ & * b^2d^2 + a^2d^2e)x^3 \end{aligned}$$

**mupad** [B] time = 4.52, size = 148, normalized size = 0.95

$$x^5 \left( \frac{a^2e^2}{5} + \frac{4abde}{5} + \frac{2cad^2}{5} + \frac{b^2d^2}{5} \right) + x^9 \left( \frac{b^2e^2}{9} + \frac{4bcde}{9} + \frac{c^2d^2}{9} + \frac{2ace^2}{9} \right) + x^7 \left( \frac{2b^2de}{7} + \frac{2cbd^2}{7} + \frac{2abe^2}{7} + \frac{4acde}{7} \right) + a^2d^2x + \frac{c^2e^2x^{13}}{13} + \frac{2adx^3(ae + bd)}{3} + \frac{2cex^{11}(be + cd)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)^2\*(a + b\*x^2 + c\*x^4)^2,x)

$$\begin{aligned} \text{[Out]} & x^5 \left( \frac{a^2e^2}{5} + \frac{b^2d^2}{5} + \frac{2a^2cd^2}{5} + \frac{4a^2b^2d^2e}{5} \right) + x^9 \left( \frac{b^2e^2}{9} + \frac{c^2d^2}{9} + \frac{2a^2c^2e^2}{9} + \frac{4b^2c^2d^2e}{9} \right) + \\ & x^7 \left( \frac{2a^2b^2e^2}{7} + \frac{2abd^2}{7} + \frac{2acd^2}{7} + \frac{2bcd^2}{7} \right) + a^2d^2x \end{aligned}$$

$$+ (2*b*c*d^2)/7 + (2*b^2*d*e)/7 + (4*a*c*d*e)/7 + a^2*d^2*x + (c^2*e^2*x^13)/13 + (2*a*d*x^3*(a*e + b*d))/3 + (2*c*e*x^11*(b*e + c*d))/11$$

**sympy [A]** time = 0.16, size = 192, normalized size = 1.24

$$a^2 d^2 x + \frac{c^2 e^2 x^{13}}{13} + x^{11} \left( \frac{2 b c e^2}{11} + \frac{2 c^2 d e}{11} \right) + x^9 \left( \frac{2 a c e^2}{9} + \frac{b^2 e^2}{9} + \frac{4 b c d e}{9} + \frac{c^2 d^2}{9} \right) + x^7 \left( \frac{2 a b c^2}{7} + \frac{4 a c d e}{7} + \frac{2 b^2 d e}{7} + \frac{2 b c d^2}{7} \right) + x^5 \left( \frac{a^2 e^2}{5} + \frac{4 a b d e}{5} + \frac{2 a c d^2}{5} + \frac{b^2 d^2}{5} \right) + x^3 \left( \frac{2 a^2 d e}{3} + \frac{2 a b d^2}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*2\*(c\*x\*\*4+b\*x\*\*2+a)\*\*2,x)

[Out] a\*\*2\*d\*\*2\*x + c\*\*2\*e\*\*2\*x\*\*13/13 + x\*\*11\*(2\*b\*c\*e\*\*2/11 + 2\*c\*\*2\*d\*e/11) + x\*\*9\*(2\*a\*c\*e\*\*2/9 + b\*\*2\*e\*\*2/9 + 4\*b\*c\*d\*e/9 + c\*\*2\*d\*\*2/9) + x\*\*7\*(2\*a\*b\*e\*\*2/7 + 4\*a\*c\*d\*e/7 + 2\*b\*\*2\*d\*e/7 + 2\*b\*c\*d\*\*2/7) + x\*\*5\*(a\*\*2\*e\*\*2/5 + 4\*a\*b\*d\*e/5 + 2\*a\*c\*d\*\*2/5 + b\*\*2\*d\*\*2/5) + x\*\*3\*(2\*a\*\*2\*d\*e/3 + 2\*a\*b\*d\*\*2/3)

$$3.177 \quad \int (d + ex^2) (a + bx^2 + cx^4)^2 dx$$

Optimal. Leaf size=96

$$a^2 dx + \frac{1}{7} x^7 (2ace + b^2 e + 2bcd) + \frac{1}{5} x^5 (2abe + 2acd + b^2 d) + \frac{1}{3} ax^3 (ae + 2bd) + \frac{1}{9} cx^9 (2be + cd) + \frac{1}{11} c^2 ex^{11}$$

Rubi [A] time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$ , Rules used = {1153}

$$a^2 dx + \frac{1}{7} x^7 (2ace + b^2 e + 2bcd) + \frac{1}{5} x^5 (2abe + 2acd + b^2 d) + \frac{1}{3} ax^3 (ae + 2bd) + \frac{1}{9} cx^9 (2be + cd) + \frac{1}{11} c^2 ex^{11}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)\*(a + b\*x^2 + c\*x^4)^2,x]

[Out] a^2\*d\*x + (a\*(2\*b\*d + a\*e)\*x^3)/3 + ((b^2\*d + 2\*a\*c\*d + 2\*a\*b\*e)\*x^5)/5 + ((2\*b\*c\*d + b^2\*e + 2\*a\*c\*e)\*x^7)/7 + (c\*(c\*d + 2\*b\*e)\*x^9)/9 + (c^2\*e\*x^11)/11

Rule 1153

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^2)^q\*(a + b\*x^2 + c\*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int (d + ex^2) (a + bx^2 + cx^4)^2 dx &= \int (a^2 d + a(2bd + ae)x^2 + (b^2 d + 2acd + 2abe) x^4 + (2bcd + b^2 e + 2ace) x^6 + \\ &= a^2 dx + \frac{1}{3} a(2bd + ae)x^3 + \frac{1}{5} (b^2 d + 2acd + 2abe) x^5 + \frac{1}{7} (2bcd + b^2 e + 2ace) x^7 \end{aligned}$$

Mathematica [A] time = 0.02, size = 96, normalized size = 1.00

$$a^2 dx + \frac{1}{7} x^7 (2ace + b^2 e + 2bcd) + \frac{1}{5} x^5 (2abe + 2acd + b^2 d) + \frac{1}{3} ax^3 (ae + 2bd) + \frac{1}{9} cx^9 (2be + cd) + \frac{1}{11} c^2 ex^{11}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)\*(a + b\*x^2 + c\*x^4)^2,x]

[Out] a^2\*d\*x + (a\*(2\*b\*d + a\*e)\*x^3)/3 + ((b^2\*d + 2\*a\*c\*d + 2\*a\*b\*e)\*x^5)/5 + ((2\*b\*c\*d + b^2\*e + 2\*a\*c\*e)\*x^7)/7 + (c\*(c\*d + 2\*b\*e)\*x^9)/9 + (c^2\*e\*x^11)/11

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex^2) (a + bx^2 + cx^4)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)\*(a + b\*x^2 + c\*x^4)^2,x]

[Out] IntegrateAlgebraic[(d + e\*x^2)\*(a + b\*x^2 + c\*x^4)^2, x]



**fricas** [A] time = 0.77, size = 100, normalized size = 1.04

$$\frac{1}{11}x^{11}ec^2 + \frac{1}{9}x^9dc^2 + \frac{2}{9}x^9ecb + \frac{2}{7}x^7dcb + \frac{1}{7}x^7eb^2 + \frac{2}{7}x^7eca + \frac{1}{5}x^5db^2 + \frac{2}{5}x^5dca + \frac{2}{5}x^5eba + \frac{2}{3}x^3dba + \frac{1}{3}x^3ea^2 + xda^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(c\*x^4+b\*x^2+a)^2,x, algorithm="fricas")

[Out] 1/11\*x^11\*e\*c^2 + 1/9\*x^9\*d\*c^2 + 2/9\*x^9\*e\*c\*b + 2/7\*x^7\*d\*c\*b + 1/7\*x^7\*e\*b^2 + 2/7\*x^7\*e\*c\*a + 1/5\*x^5\*d\*b^2 + 2/5\*x^5\*d\*c\*a + 2/5\*x^5\*e\*b\*a + 2/3\*x^3\*d\*b\*a + 1/3\*x^3\*e\*a^2 + x\*d\*a^2

**giac** [A] time = 0.17, size = 106, normalized size = 1.10

$$\frac{1}{11}c^2x^{11}e + \frac{1}{9}c^2dx^9 + \frac{2}{9}bcx^9e + \frac{2}{7}bcdx^7 + \frac{1}{7}b^2x^7e + \frac{2}{7}acx^7e + \frac{1}{5}b^2dx^5 + \frac{2}{5}acdx^5 + \frac{2}{5}abx^5e + \frac{2}{3}abdx^3 + \frac{1}{3}a^2x^3e + a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(c\*x^4+b\*x^2+a)^2,x, algorithm="giac")

[Out] 1/11\*c^2\*x^11\*e + 1/9\*c^2\*d\*x^9 + 2/9\*b\*c\*x^9\*e + 2/7\*b\*c\*d\*x^7 + 1/7\*b^2\*x^7\*e + 2/7\*a\*c\*x^7\*e + 1/5\*b^2\*d\*x^5 + 2/5\*a\*c\*d\*x^5 + 2/5\*a\*b\*x^5\*e + 2/3\*a\*b\*d\*x^3 + 1/3\*a^2\*x^3\*e + a^2\*d\*x

**maple** [A] time = 0.00, size = 91, normalized size = 0.95

$$\frac{c^2ex^{11}}{11} + \frac{(2ebc + dc^2)x^9}{9} + \frac{(2bcd + (2ac + b^2)e)x^7}{7} + \frac{(2abe + (2ac + b^2)d)x^5}{5} + a^2dx + \frac{(ea^2 + 2dab)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)\*(c\*x^4+b\*x^2+a)^2,x)

[Out] 1/11\*c^2\*e\*x^11+1/9\*(2\*b\*c\*e+c^2\*d)\*x^9+1/7\*(2\*b\*c\*d+e\*(2\*a\*c+b^2))\*x^7+1/5\*(d\*(2\*a\*c+b^2)+2\*a\*b\*e)\*x^5+1/3\*(a^2\*e+2\*a\*b\*d)\*x^3+a^2\*d\*x

**maxima** [A] time = 1.01, size = 90, normalized size = 0.94

$$\frac{1}{11}c^2ex^{11} + \frac{1}{9}(c^2d + 2bce)x^9 + \frac{1}{7}(2bcd + (b^2 + 2ac)e)x^7 + \frac{1}{5}(2abe + (b^2 + 2ac)d)x^5 + a^2dx + \frac{1}{3}(2abd + a^2e)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(c\*x^4+b\*x^2+a)^2,x, algorithm="maxima")

[Out] 1/11\*c^2\*e\*x^11 + 1/9\*(c^2\*d + 2\*b\*c\*e)\*x^9 + 1/7\*(2\*b\*c\*d + (b^2 + 2\*a\*c)\*e)\*x^7 + 1/5\*(2\*a\*b\*e + (b^2 + 2\*a\*c)\*d)\*x^5 + a^2\*d\*x + 1/3\*(2\*a\*b\*d + a^2\*e)\*x^3

**mupad** [B] time = 0.04, size = 90, normalized size = 0.94

$$x^5 \left( \frac{db^2}{5} + \frac{2aeb}{5} + \frac{2acd}{5} \right) + x^7 \left( \frac{eb^2}{7} + \frac{2cdb}{7} + \frac{2ace}{7} \right) + x^3 \left( \frac{ea^2}{3} + \frac{2bda}{3} \right) + x^9 \left( \frac{dc^2}{9} + \frac{2bec}{9} \right) + \frac{c^2ex^{11}}{11} + a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)\*(a + b\*x^2 + c\*x^4)^2,x)

[Out] x^5\*((b^2\*d)/5 + (2\*a\*b\*e)/5 + (2\*a\*c\*d)/5) + x^7\*((b^2\*e)/7 + (2\*a\*c\*e)/7 + (2\*b\*c\*d)/7) + x^3\*((a^2\*e)/3 + (2\*a\*b\*d)/3) + x^9\*((c^2\*d)/9 + (2\*b\*c\*e)/9) + (c^2\*e\*x^11)/11 + a^2\*d\*x

sympy [A] time = 0.25, size = 107, normalized size = 1.11

$$a^2dx + \frac{c^2ex^{11}}{11} + x^9\left(\frac{2bce}{9} + \frac{c^2d}{9}\right) + x^7\left(\frac{2ace}{7} + \frac{b^2e}{7} + \frac{2bcd}{7}\right) + x^5\left(\frac{2abe}{5} + \frac{2acd}{5} + \frac{b^2d}{5}\right) + x^3\left(\frac{a^2e}{3} + \frac{2abd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*(c\*x\*\*4+b\*x\*\*2+a)\*\*2,x)

[Out] a\*\*2\*d\*x + c\*\*2\*e\*x\*\*11/11 + x\*\*9\*(2\*b\*c\*e/9 + c\*\*2\*d/9) + x\*\*7\*(2\*a\*c\*e/7 + b\*\*2\*e/7 + 2\*b\*c\*d/7) + x\*\*5\*(2\*a\*b\*e/5 + 2\*a\*c\*d/5 + b\*\*2\*d/5) + x\*\*3\*(a\*\*2\*e/3 + 2\*a\*b\*d/3)

$$3.178 \quad \int (a + bx^2 + cx^4)^2 dx$$

Optimal. Leaf size=49

$$a^2x + \frac{1}{5}x^5(2ac + b^2) + \frac{2}{3}abx^3 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

Rubi [A] time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {1090}

$$a^2x + \frac{1}{5}x^5(2ac + b^2) + \frac{2}{3}abx^3 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2 + c\*x^4)^2,x]

[Out] a^2\*x + (2\*a\*b\*x^3)/3 + ((b^2 + 2\*a\*c)\*x^5)/5 + (2\*b\*c\*x^7)/7 + (c^2\*x^9)/9

Rule 1090

Int[((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x^2 + c\*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (a + bx^2 + cx^4)^2 dx &= \int \left( a^2 + 2abx^2 + b^2 \left( 1 + \frac{2ac}{b^2} \right) x^4 + 2bcx^6 + c^2x^8 \right) dx \\ &= a^2x + \frac{2}{3}abx^3 + \frac{1}{5}(b^2 + 2ac)x^5 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.00

$$a^2x + \frac{1}{5}x^5(2ac + b^2) + \frac{2}{3}abx^3 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2 + c\*x^4)^2,x]

[Out] a^2\*x + (2\*a\*b\*x^3)/3 + ((b^2 + 2\*a\*c)\*x^5)/5 + (2\*b\*c\*x^7)/7 + (c^2\*x^9)/9

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx^2 + cx^4)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)^2,x]

[Out] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)^2, x]

fricas [A] time = 0.75, size = 43, normalized size = 0.88

$$\frac{1}{9}x^9c^2 + \frac{2}{7}x^7cb + \frac{1}{5}x^5b^2 + \frac{2}{5}x^5ca + \frac{2}{3}x^3ba + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)^2,x, algorithm="fricas")

[Out] 1/9\*x^9\*c^2 + 2/7\*x^7\*c\*b + 1/5\*x^5\*b^2 + 2/5\*x^5\*c\*a + 2/3\*x^3\*b\*a + x\*a^2

giac [A] time = 0.14, size = 43, normalized size = 0.88

$$\frac{1}{9}c^2x^9 + \frac{2}{7}bcx^7 + \frac{1}{5}b^2x^5 + \frac{2}{5}acx^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)^2,x, algorithm="giac")

[Out] 1/9\*c^2\*x^9 + 2/7\*b\*c\*x^7 + 1/5\*b^2\*x^5 + 2/5\*a\*c\*x^5 + 2/3\*a\*b\*x^3 + a^2\*x

maple [A] time = 0.00, size = 42, normalized size = 0.86

$$\frac{c^2x^9}{9} + \frac{2bcx^7}{7} + \frac{2abx^3}{3} + \frac{(2ac + b^2)x^5}{5} + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^4+b\*x^2+a)^2,x)

[Out] 1/9\*c^2\*x^9+2/7\*b\*c\*x^7+2/3\*a\*b\*x^3+1/5\*(2\*a\*c+b^2)\*x^5+a^2\*x

maxima [A] time = 1.10, size = 45, normalized size = 0.92

$$\frac{1}{9}c^2x^9 + \frac{2}{7}bcx^7 + \frac{1}{5}b^2x^5 + a^2x + \frac{2}{15}(3cx^5 + 5bx^3)a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)^2,x, algorithm="maxima")

[Out] 1/9\*c^2\*x^9 + 2/7\*b\*c\*x^7 + 1/5\*b^2\*x^5 + a^2\*x + 2/15\*(3\*c\*x^5 + 5\*b\*x^3)\*  
a

mupad [B] time = 0.02, size = 42, normalized size = 0.86

$$a^2x + x^5 \left( \frac{b^2}{5} + \frac{2ac}{5} \right) + \frac{c^2x^9}{9} + \frac{2abx^3}{3} + \frac{2bcx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^2 + c\*x^4)^2,x)

[Out] a^2\*x + x^5\*((2\*a\*c)/5 + b^2/5) + (c^2\*x^9)/9 + (2\*a\*b\*x^3)/3 + (2\*b\*c\*x^7)/7

sympy [A] time = 0.15, size = 48, normalized size = 0.98

$$a^2x + \frac{2abx^3}{3} + \frac{2bcx^7}{7} + \frac{c^2x^9}{9} + x^5 \left( \frac{2ac}{5} + \frac{b^2}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*4+b\*x\*\*2+a)\*\*2,x)

[Out] a\*\*2\*x + 2\*a\*b\*x\*\*3/3 + 2\*b\*c\*x\*\*7/7 + c\*\*2\*x\*\*9/9 + x\*\*5\*(2\*a\*c/5 + b\*\*2/5)  
)

$$3.179 \quad \int \frac{(a+bx^2+cx^4)^2}{d+ex^2} dx$$

**Optimal.** Leaf size=143

$$\frac{x^3(-2ce(bd-ae)+b^2e^2+c^2d^2)}{3e^3} - \frac{x(cd-be)(cd^2-e(bd-2ae))}{e^4} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(ae^2-bde+cd^2)^2}{\sqrt{d}e^{9/2}} - \frac{cx^5(cd-2be)}{5e^2}$$

**Rubi [A]** time = 0.14, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, number of rules / integrand size = 0.083, Rules used = {1153, 205}

$$\frac{x^3(-2ce(bd-ae)+b^2e^2+c^2d^2)}{3e^3} - \frac{x(cd-be)(cd^2-e(bd-2ae))}{e^4} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(ae^2-bde+cd^2)^2}{\sqrt{d}e^{9/2}} - \frac{cx^5(cd-2be)}{5e^2} + \frac{c^2x^7}{7e}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2 + c\*x^4)^2/(d + e\*x^2), x]

[Out] -(((c\*d - b\*e)\*(c\*d^2 - e\*(b\*d - 2\*a\*e))\*x)/e^4) + ((c^2\*d^2 + b^2\*e^2 - 2\*c\*e\*(b\*d - a\*e))\*x^3)/(3\*e^3) - (c\*(c\*d - 2\*b\*e)\*x^5)/(5\*e^2) + (c^2\*x^7)/(7\*e) + ((c\*d^2 - b\*d\*e + a\*e^2)^2\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(Sqrt[d]\*e^(9/2))

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1153

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^2)^q\*(a + b\*x^2 + c\*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^2+cx^4)^2}{d+ex^2} dx &= \int \left( -\frac{(cd-be)(cd^2-e(bd-2ae))}{e^4} + \frac{(c^2d^2+b^2e^2-2ce(bd-ae))x^2}{e^3} - \frac{c(cd-2be)x^4}{e^2} \right. \\ &= -\frac{(cd-be)(cd^2-e(bd-2ae))x}{e^4} + \frac{(c^2d^2+b^2e^2-2ce(bd-ae))x^3}{3e^3} - \frac{c(cd-2be)x^5}{5e^2} + \\ &= -\frac{(cd-be)(cd^2-e(bd-2ae))x}{e^4} + \frac{(c^2d^2+b^2e^2-2ce(bd-ae))x^3}{3e^3} - \frac{c(cd-2be)x^5}{5e^2} + \end{aligned}$$

**Mathematica [A]** time = 0.07, size = 144, normalized size = 1.01

$$\frac{x^3(2ace^2+b^2e^2-2bcde+c^2d^2)}{3e^3} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(ae^2-bde+cd^2)^2}{\sqrt{d}e^{9/2}} + \frac{x(be-cd)(2ae^2-bde+cd^2)}{e^4} + \frac{cx^5(2be-cd)}{5e^2} + \frac{c^2x^7}{7e}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2 + c\*x^4)^2/(d + e\*x^2), x]



$-2/e^3/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)}*e*x)*b*c*d^3+1/(d*e)^{(1/2)}*c^2*d^4/e^4*\arctan(1/(d*e)^{(1/2)}*e*x)$

**maxima [A]** time = 2.38, size = 176, normalized size = 1.23

$$\frac{(c^2d^4 - 2bcd^3e - 2abde^3 + a^2e^4 + (b^2 + 2ac)d^2e^2) \arctan\left(\frac{ex}{\sqrt{de}}\right) + \frac{15c^2e^3x^7 - 21(c^2de^2 - 2bce^3)x^5 + 35(c^2d^2e - 2bcde^2 + (b^2 + 2ac)e^3)x^3 - 105(c^2d^3 - 2bcd^2e - 2abe^3 + (b^2 + 2ac)de^2)x}{105e^4}}{\sqrt{de}e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)^2/(e\*x^2+d),x, algorithm="maxima")

[Out]  $(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2)*\arctan(e*x/\sqrt{d*e})/(\sqrt{d*e}*e^4) + 1/105*(15*c^2*e^3*x^7 - 21*(c^2*d*e^2 - 2*b*c*e^3)*x^5 + 35*(c^2*d^2*e - 2*b*c*d*e^2 + (b^2 + 2*a*c)*e^3)*x^3 - 105*(c^2*d^3 - 2*b*c*d^2*e - 2*a*b*e^3 + (b^2 + 2*a*c)*d*e^2)*x)/e^4$

**mupad [B]** time = 4.47, size = 229, normalized size = 1.60

$$x^3 \left( \frac{b^2 + 2ac}{3e} + \frac{d \left( \frac{c^2d - 2bc}{e} \right)}{3e} \right) - x \left( \frac{d \left( \frac{b^2 + 2ac}{e} + \frac{d \left( \frac{c^2d - 2bc}{e} \right)}{e} \right) - 2ab}{e} \right) - x^5 \left( \frac{c^2d}{5e^2} - \frac{2bc}{5e} \right) + \frac{c^2x^7}{7e} + \frac{\operatorname{atan}\left(\frac{\sqrt{e}x(c d^2 - b d e + a e^2)^2}{\sqrt{d}(a^2 e^4 - 2 a b d e^3 + 2 a c d^2 e^2 + b^2 d^2 e^2 - 2 b c d^3 e + c^2 d^4)}\right) (c d^2 - b d e + a e^2)^2}{\sqrt{d} e^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^2 + c\*x^4)^2/(d + e\*x^2),x)

[Out]  $x^3*((2*a*c + b^2)/(3*e) + (d*((c^2*d)/e^2 - (2*b*c)/e))/(3*e)) - x*((d*((2*a*c + b^2)/e + (d*((c^2*d)/e^2 - (2*b*c)/e))/e) - (2*a*b)/e) - x^5*((c^2*d)/(5*e^2) - (2*b*c)/(5*e)) + (c^2*x^7)/(7*e) + (\operatorname{atan}((e^{1/2})*x*(a*e^2 + c*d^2 - b*d*e)^2)/(d^{1/2}*(a^2*e^4 + c^2*d^4 + b^2*d^2*e^2 - 2*a*b*d*e^3 - 2*b*c*d^3*e + 2*a*c*d^2*e^2)))*(a*e^2 + c*d^2 - b*d*e)^2)/(d^{1/2}*e^{9/2}))$

**sympy [B]** time = 1.53, size = 371, normalized size = 2.59

$$\frac{c^2x^7}{7e} + x^3 \left( \frac{2bc}{3e} - \frac{c^2d}{3e^2} \right) + x^3 \left( \frac{2ac}{3e} + \frac{b^2}{3e} - \frac{2bcd}{3e^2} + \frac{c^2d^2}{3e^3} \right) + x \left( \frac{2ab}{e} - \frac{2acd}{e^2} - \frac{b^2d}{e^2} + \frac{2bcd^2}{e^3} - \frac{c^2d^3}{e^4} \right) - \frac{\sqrt{\frac{1}{de}} (ae^2 - bde + cd^2)^2 \log\left(\frac{d^4 \sqrt{\frac{1}{de}} (ae^2 - bde + cd^2)^2}{2e^4 - 2abde^3 + 2acd^2e^2 + b^2d^2e^2 - 2bcd^3e + c^2d^4} + x\right)}{2} + \frac{\sqrt{\frac{1}{de}} (ae^2 - bde + cd^2)^2 \log\left(\frac{d^4 \sqrt{\frac{1}{de}} (ae^2 - bde + cd^2)^2}{2e^4 - 2abde^3 + 2acd^2e^2 + b^2d^2e^2 - 2bcd^3e + c^2d^4} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*4+b\*x\*\*2+a)\*\*2/(e\*x\*\*2+d),x)

[Out]  $c**2*x**7/(7*e) + x**5*(2*b*c/(5*e) - c**2*d/(5*e**2)) + x**3*(2*a*c/(3*e) + b**2/(3*e) - 2*b*c*d/(3*e**2) + c**2*d**2/(3*e**3)) + x*(2*a*b/e - 2*a*c*d/e**2 - b**2*d/e**2 + 2*b*c*d**2/e**3 - c**2*d**3/e**4) - \sqrt{-1/(d*e**9)}*(a*e**2 - b*d*e + c*d**2)**2*\log(-d*e**4*\sqrt{-1/(d*e**9)}*(a*e**2 - b*d*e + c*d**2)**2/(a**2*e**4 - 2*a*b*d*e**3 + 2*a*c*d**2*e**2 + b**2*d**2*e**2 - 2*b*c*d**3*e + c**2*d**4) + x)/2 + \sqrt{-1/(d*e**9)}*(a*e**2 - b*d*e + c*d**2)**2*\log(d*e**4*\sqrt{-1/(d*e**9)}*(a*e**2 - b*d*e + c*d**2)**2/(a**2*e**4 - 2*a*b*d*e**3 + 2*a*c*d**2*e**2 + b**2*d**2*e**2 - 2*b*c*d**3*e + c**2*d**4) + x)/2$

$$3.180 \quad \int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^2} dx$$

**Optimal.** Leaf size=166

$$\frac{x(-2ce(2bd-ae)+b^2e^2+3c^2d^2)}{e^4} + \frac{x(ae^2-bde+cd^2)^2}{2de^4(d+ex^2)} - \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(ae^2-bde+cd^2)(7cd^2-e(ae+3bd))}{2d^{3/2}e^{9/2}}$$

**Rubi [A]** time = 0.30, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1157, 1810, 205}

$$\frac{x(-2ce(2bd-ae)+b^2e^2+3c^2d^2)}{e^4} + \frac{x(ae^2-bde+cd^2)^2}{2de^4(d+ex^2)} - \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(ae^2-bde+cd^2)(7cd^2-e(ae+3bd))}{2d^{3/2}e^{9/2}} - \frac{2cx^3(cd-be)}{3e^3} + \frac{c^2x^5}{5e^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2 + c\*x^4)^2/(d + e\*x^2)^2,x]

[Out] ((3\*c^2\*d^2 + b^2\*e^2 - 2\*c\*e\*(2\*b\*d - a\*e))\*x)/e^4 - (2\*c\*(c\*d - b\*e)\*x^3)/(3\*e^3) + (c^2\*x^5)/(5\*e^2) + ((c\*d^2 - b\*d\*e + a\*e^2)^2\*x)/(2\*d\*e^4\*(d + e\*x^2)) - ((c\*d^2 - b\*d\*e + a\*e^2)\*(7\*c\*d^2 - e\*(3\*b\*d + a\*e))\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(2\*d^(3/2)\*e^(9/2))

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1157

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] := With[{Qx = PolynomialQuotient[(a + b\*x^2 + c\*x^4)^p, d + e\*x^2, x], R = Coeff[PolynomialRemainder[(a + b\*x^2 + c\*x^4)^p, d + e\*x^2, x], x, 0]}, -Simp[(R\*x\*(d + e\*x^2)^(q + 1))/(2\*d\*(q + 1)), x] + Dist[1/(2\*d\*(q + 1)), Int[(d + e\*x^2)^(q + 1)\*ExpandToSum[2\*d\*(q + 1)\*Qx + R\*(2\*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rule 1810

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[ExpandIntegrand[Pq\*(a + b\*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps



$$\begin{aligned}
\int \frac{(a + bx^2 + cx^4)^2}{(d + ex^2)^2} dx &= \frac{(cd^2 - bde + ae^2)^2 x}{2de^4(d + ex^2)} - \frac{\int \frac{\frac{c^2d^4 - 2cd^2e(bd - ae) + e^2(b^2d^2 - 2abde - a^2e^2)}{e^4} - \frac{2d(c^2d^2 + b^2e^2 - 2ce(bd - ae))x^2}{e^3} + \frac{2cd(cd - 2be)x^4}{e^2}}{d + ex^2}}{2d} \\
&= \frac{(cd^2 - bde + ae^2)^2 x}{2de^4(d + ex^2)} - \frac{\int \left( -\frac{2d(3c^2d^2 + b^2e^2 - 2ce(2bd - ae))}{e^4} + \frac{4cd(cd - be)x^2}{e^3} - \frac{2c^2dx^4}{e^2} + \frac{7c^2d^4 - 10bcd^2}{e^2} \right)}{2d} \\
&= \frac{(3c^2d^2 + b^2e^2 - 2ce(2bd - ae))x}{e^4} - \frac{2c(cd - be)x^3}{3e^3} + \frac{c^2x^5}{5e^2} + \frac{(cd^2 - bde + ae^2)^2 x}{2de^4(d + ex^2)} - \frac{7c^2d^4 - 10bcd^2}{2de^4} \\
&= \frac{(3c^2d^2 + b^2e^2 - 2ce(2bd - ae))x}{e^4} - \frac{2c(cd - be)x^3}{3e^3} + \frac{c^2x^5}{5e^2} + \frac{(cd^2 - bde + ae^2)^2 x}{2de^4(d + ex^2)} - \frac{7c^2d^4 - 10bcd^2}{2de^4}
\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 183, normalized size = 1.10

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(-e^2(a^2e^2 + 2abde - 3b^2d^2) + 2cd^2e(3ae - 5bd) + 7c^2d^4)}{2d^{3/2}e^{9/2}} + \frac{x(2ce(ae - 2bd) + b^2e^2 + 3c^2d^2)}{e^4} + \frac{x(e(ae - bd) + cd^2)^2}{2de^4(d + ex^2)} + \frac{2cx^3(be - cd)}{3e^3} + \frac{c^2x^5}{5e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2 + c\*x^4)^2/(d + e\*x^2)^2,x]

[Out] ((3\*c^2\*d^2 + b^2\*e^2 + 2\*c\*e\*(-2\*b\*d + a\*e))\*x)/e^4 + (2\*c\*(-(c\*d) + b\*e)\*x^3)/(3\*e^3) + (c^2\*x^5)/(5\*e^2) + ((c\*d^2 + e\*(-(b\*d) + a\*e))^2\*x)/(2\*d\*e^4\*(d + e\*x^2)) - ((7\*c^2\*d^4 + 2\*c\*d^2\*e\*(-5\*b\*d + 3\*a\*e) - e^2\*(-3\*b^2\*d^2 + 2\*a\*b\*d\*e + a^2\*e^2))\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]]/(2\*d^(3/2)\*e^(9/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2 + cx^4)^2}{(d + ex^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)^2/(d + e\*x^2)^2,x]

[Out] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)^2/(d + e\*x^2)^2, x]

**fricas [A]** time = 0.91, size = 600, normalized size = 3.61

```

--S 1000
a0 := (a + b*x^2 + c*x^4)^2/(d + e*x^2)^2
--S 1001
r0 := (3*c^2*d^2 + b^2*e^2 + 2*c*e*(-2*b*d + a*e))*x/e^4 + (2*c*(-(c*d) + b*e)*x^3)/(3*e^3) + (c^2*x^5)/(5*e^2) + ((c*d^2 + e*(-(b*d) + a*e))^2*x)/(2*d*e^4*(d + e*x^2)) - ((7*c^2*d^4 + 2*c*d^2*e*(-5*b*d + 3*a*e) - e^2*(-3*b^2*d^2 + 2*a*b*d*e + a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]]/(2*d^(3/2)*e^(9/2))
--S 1002
d0 := D(r0,x)
--S 1003
t0 := a0 - d0
--S 1004
m0 := lcm(denominator(t0))
--S 1005
n0 := numerator(t0)
--S 1006
p0 := m0^n0
--S 1007
q0 := D(p0,x)
--S 1008
r0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1009
s0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1010
t0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1011
u0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1012
v0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1013
w0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1014
x0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1015
y0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1016
z0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1017
aa0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1018
bb0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1019
cc0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1020
dd0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1021
ee0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1022
ff0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1023
gg0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1024
hh0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1025
ii0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1026
jj0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1027
kk0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1028
ll0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1029
mm0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1030
nn0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1031
oo0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1032
pp0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1033
qq0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1034
rr0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1035
ss0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1036
tt0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1037
uu0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1038
vv0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1039
ww0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1040
xx0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1041
yy0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1042
zz0 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1043
aa1 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1044
bb1 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1045
cc1 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1046
dd1 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1047
ee1 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1048
ff1 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1049
gg1 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1050
hh1 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1051
ii1 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1052
jj1 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1053
kk1 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1054
ll1 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1055
mm1 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1056
nn1 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1057
oo1 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1058
pp1 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1059
qq1 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1060
rr1 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1061
ss1 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1062
tt1 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1063
uu1 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1064
vv1 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1065
ww1 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1066
xx1 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1067
yy1 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1068
zz1 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1069
aa2 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1070
bb2 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1071
cc2 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1072
dd2 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1073
ee2 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1074
ff2 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1075
gg2 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1076
hh2 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1077
ii2 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1078
jj2 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1079
kk2 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1080
ll2 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1081
mm2 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1082
nn2 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1083
oo2 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1084
pp2 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1085
qq2 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1086
rr2 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1087
ss2 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1088
tt2 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1089
uu2 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1090
vv2 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1091
ww2 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1092
xx2 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1093
yy2 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1094
zz2 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1095
aa3 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1096
bb3 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1097
cc3 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1098
dd3 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1099
ee3 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1100
ff3 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1101
gg3 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1102
hh3 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1103
ii3 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1104
jj3 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1105
kk3 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1106
ll3 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1107
mm3 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1108
nn3 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1109
oo3 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1110
pp3 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1111
qq3 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1112
rr3 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1113
ss3 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1114
tt3 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1115
uu3 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1116
vv3 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1117
ww3 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1118
xx3 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1119
yy3 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1120
zz3 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1121
aa4 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1122
bb4 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1123
cc4 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1124
dd4 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1125
ee4 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1126
ff4 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1127
gg4 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1128
hh4 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1129
ii4 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1130
jj4 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1131
kk4 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1132
ll4 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1133
mm4 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1134
nn4 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1135
oo4 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1136
pp4 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1137
qq4 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1138
rr4 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1139
ss4 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1140
tt4 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1141
uu4 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1142
vv4 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1143
ww4 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1144
xx4 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1145
yy4 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1146
zz4 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1147
aa5 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1148
bb5 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1149
cc5 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1150
dd5 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1151
ee5 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1152
ff5 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1153
gg5 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1154
hh5 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1155
ii5 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1156
jj5 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1157
kk5 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1158
ll5 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1159
mm5 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1160
nn5 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1161
oo5 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1162
pp5 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1163
qq5 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1164
rr5 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1165
ss5 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1166
tt5 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1167
uu5 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1168
vv5 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1169
ww5 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1170
xx5 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1171
yy5 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1172
zz5 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1173
aa6 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1174
bb6 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1175
cc6 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1176
dd6 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1177
ee6 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1178
ff6 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1179
gg6 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1180
hh6 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1181
ii6 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1182
jj6 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1183
kk6 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1184
ll6 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1185
mm6 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1186
nn6 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1187
oo6 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1188
pp6 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1189
qq6 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1190
rr6 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1191
ss6 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1192
tt6 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1193
uu6 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1194
vv6 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1195
ww6 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1196
xx6 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1197
yy6 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1198
zz6 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1199
aa7 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1200
bb7 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1201
cc7 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1202
dd7 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1203
ee7 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1204
ff7 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1205
gg7 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1206
hh7 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1207
ii7 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1208
jj7 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1209
kk7 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1210
ll7 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1211
mm7 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1212
nn7 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1213
oo7 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1214
pp7 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1215
qq7 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1216
rr7 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1217
ss7 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1218
tt7 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1219
uu7 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1220
vv7 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1221
ww7 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1222
xx7 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1223
yy7 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1224
zz7 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1225
aa8 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1226
bb8 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1227
cc8 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1228
dd8 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1229
ee8 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1230
ff8 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1231
gg8 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1232
hh8 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1233
ii8 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1234
jj8 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1235
kk8 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1236
ll8 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1237
mm8 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1238
nn8 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1239
oo8 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1240
pp8 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1241
qq8 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1242
rr8 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1243
ss8 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1244
tt8 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1245
uu8 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1246
vv8 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1247
ww8 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1248
xx8 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1249
yy8 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1250
zz8 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1251
aa9 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1252
bb9 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1253
cc9 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1254
dd9 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1255
ee9 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1256
ff9 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1257
gg9 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1258
hh9 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1259
ii9 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1260
jj9 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1261
kk9 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1262
ll9 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1263
mm9 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1264
nn9 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1265
oo9 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1266
pp9 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1267
qq9 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1268
rr9 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1269
ss9 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1270
tt9 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1271
uu9 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1272
vv9 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1273
ww9 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1274
xx9 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1275
yy9 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1276
zz9 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1277
aa10 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1278
bb10 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1279
cc10 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1280
dd10 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1281
ee10 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1282
ff10 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1283
gg10 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1284
hh10 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1285
ii10 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1286
jj10 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1287
kk10 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1288
ll10 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1289
mm10 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1290
nn10 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1291
oo10 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1292
pp10 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1293
qq10 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1294
rr10 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1295
ss10 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1296
tt10 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1297
uu10 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1298
vv10 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1299
ww10 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1300
xx10 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1301
yy10 := (q0 - n0*D(m0,x))/m0^(n0+1)
--S 1302
zz10 := (q0 - n0*D(m0,x))/m0^(n0+1)

```

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)^2/(e\*x^2+d)^2,x, algorithm="fricas")

[Out] [1/60\*(12\*c^2\*d^2\*e^4\*x^7 - 4\*(7\*c^2\*d^3\*e^3 - 10\*b\*c\*d^2\*e^4)\*x^5 + 20\*(7\*c^2\*d^4\*e^2 - 10\*b\*c\*d^3\*e^3 + 3\*(b^2 + 2\*a\*c)\*d^2\*e^4)\*x^3 + 15\*(7\*c^2\*d^5 - 10\*b\*c\*d^4\*e - 2\*a\*b\*d^2\*e^3 - a^2\*d\*e^4 + 3\*(b^2 + 2\*a\*c)\*d^3\*e^2 + (7\*c^2\*d^4\*e - 10\*b\*c\*d^3\*e^2 - 2\*a\*b\*d\*e^4 - a^2\*e^5 + 3\*(b^2 + 2\*a\*c)\*d^2\*e^3)\*x^2)\*sqrt(-d\*e)\*log((e\*x^2 - 2\*sqrt(-d\*e)\*x - d)/(e\*x^2 + d)) + 30\*(7\*c^2\*d^5\*e - 10\*b\*c\*d^4\*e^2 - 2\*a\*b\*d^2\*e^4 + a^2\*d\*e^5 + 3\*(b^2 + 2\*a\*c)\*d^3\*e^3)\*x)/(d^2\*e^6\*x^2 + d^3\*e^5), 1/30\*(6\*c^2\*d^2\*e^4\*x^7 - 2\*(7\*c^2\*d^3\*e^3 - 10\*b\*c\*d^2\*e^4)\*x^5 + 10\*(7\*c^2\*d^4\*e^2 - 10\*b\*c\*d^3\*e^3 + 3\*(b^2 + 2\*a\*c)\*d^2\*e^4)\*x^3 - 15\*(7\*c^2\*d^5 - 10\*b\*c\*d^4\*e - 2\*a\*b\*d^2\*e^3 - a^2\*d\*e^4

$+ 3*(b^2 + 2*a*c)*d^3*e^2 + (7*c^2*d^4*e - 10*b*c*d^3*e^2 - 2*a*b*d*e^4 - a^2*e^5 + 3*(b^2 + 2*a*c)*d^2*e^3)*x^2)*\text{sqrt}(d*e)*\text{arctan}(\text{sqrt}(d*e)*x/d) + 15*(7*c^2*d^5*e - 10*b*c*d^4*e^2 - 2*a*b*d^2*e^4 + a^2*d*e^5 + 3*(b^2 + 2*a*c)*d^3*e^3)*x)/(d^2*e^6*x^2 + d^3*e^5)]$

**giac** [A] time = 0.18, size = 207, normalized size = 1.25

$$\frac{1}{15} (3c^2x^5e^8 - 10c^2dx^3e^7 + 10bcx^3e^8 + 45c^2d^2xe^6 - 60bcdxe^7 + 15b^2xe^8 + 30acxe^8)e^{(-10)} - \frac{(7c^2d^4 - 10bcd^3e + 3b^2d^2e^2 + 6acd^2e^2 - 2abde^3 - a^2e^4) \arctan\left(\frac{ax}{\sqrt{d}}\right) e^{\left(\frac{-2}{2}\right)}}{2d^{\frac{3}{2}}} + \frac{(c^2d^4x - 2bcd^3xe + b^2d^2xe^2 + 2acd^2xe^2 - 2abdx^3 + a^2xe^4)e^{(-4)}}{2(x^2e + d)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)^2/(e\*x^2+d)^2,x, algorithm="giac")

[Out]  $1/15*(3*c^2*x^5*e^8 - 10*c^2*d*x^3*e^7 + 10*b*c*x^3*e^8 + 45*c^2*d^2*x*e^6 - 60*b*c*d*x*e^7 + 15*b^2*x*e^8 + 30*a*c*x*e^8)*e^{(-10)} - 1/2*(7*c^2*d^4 - 10*b*c*d^3*e + 3*b^2*d^2*e^2 + 6*a*c*d^2*e^2 - 2*a*b*d*e^3 - a^2*e^4)*\text{arctan}(x*e^{(1/2)}/\text{sqrt}(d))*e^{(-9/2)}/d^{(3/2)} + 1/2*(c^2*d^4*x - 2*b*c*d^3*x*e + b^2*d^2*x*e^2 + 2*a*c*d^2*x*e^2 - 2*a*b*d*x*e^3 + a^2*x*e^4)*e^{(-4)}/((x^2*e + d)*d)$

**maple** [B] time = 0.01, size = 320, normalized size = 1.93

$$\frac{c^2x^5}{5e^2} + \frac{2bcx^3}{3e^2} - \frac{2c^2dx^3}{3e^2} + \frac{a^2x}{2(e^2x^2+d)} + \frac{a^2 \arctan\left(\frac{ax}{\sqrt{d}}\right)}{2\sqrt{d}} - \frac{abx}{(e^2x^2+d)e} + \frac{ab \arctan\left(\frac{ax}{\sqrt{d}}\right)}{\sqrt{d}e} + \frac{acdx}{(e^2x^2+d)e^2} - \frac{3acd \arctan\left(\frac{ax}{\sqrt{d}}\right)}{\sqrt{d}e^2} + \frac{b^2dx}{2(e^2x^2+d)} - \frac{3b^2d \arctan\left(\frac{ax}{\sqrt{d}}\right)}{2\sqrt{d}e^2} + \frac{bc dx}{(e^2x^2+d)e^3} + \frac{5bc d^2 \arctan\left(\frac{ax}{\sqrt{d}}\right)}{\sqrt{d}e^3} + \frac{c^2d^2x}{2(e^2x^2+d)e^4} - \frac{7c^2d^3 \arctan\left(\frac{ax}{\sqrt{d}}\right)}{2\sqrt{d}e^4} + \frac{2acx}{e^2} + \frac{b^2x}{e^2} - \frac{4bc dx}{e^3} + \frac{3c^2d^2x}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^4+b\*x^2+a)^2/(e\*x^2+d)^2,x)

[Out]  $1/5*c^2/e^2*x^5 + 2/3/e^2*x^3*b*c - 2/3*c^2*d/e^3*x^3 + 2*a*c/e^2*x + 1/e^2*b^2*x - 4/e^3*b*c*d*x + 3*c^2*d^2/e^4*x + 1/2/(e*x^2+d)*a^2/d*x - 1/e*x/(e*x^2+d)*a*b + 1/(e*x^2+d)*a*c*d/e^2*x + 1/2/e^2*d*x/(e*x^2+d)*b^2 - 1/e^3*d^2*x/(e*x^2+d)*b*c + 1/2/(e*x^2+d)*c^2*d^3/e^4*x + 1/2/(d*e)^{(1/2)}*a^2/d*\text{arctan}(1/(d*e)^{(1/2)}*e*x) + 1/e/(d*e)^{(1/2)}*\text{arctan}(1/(d*e)^{(1/2)}*e*x)*a*b - 3/(d*e)^{(1/2)}*a*c*d/e^2*\text{arctan}(1/(d*e)^{(1/2)}*e*x) - 3/2/e^2*d/(d*e)^{(1/2)}*\text{arctan}(1/(d*e)^{(1/2)}*e*x)*b^2 + 5/e^3*d^2/(d*e)^{(1/2)}*\text{arctan}(1/(d*e)^{(1/2)}*e*x)*b*c - 7/2/(d*e)^{(1/2)}*c^2*d^3/e^4*\text{arctan}(1/(d*e)^{(1/2)}*e*x)$

**maxima** [A] time = 2.42, size = 205, normalized size = 1.23

$$\frac{(c^2d^4 - 2bcd^3e - 2abde^3 + a^2e^4 + (b^2 + 2ac)d^2e^2)x}{2(d^5x^2 + d^2e^4)} + \frac{3c^2e^2x^5 - 10(c^2de - bce^2)x^3 + 15(c^2d^2 - 4bcde + (b^2 + 2ac)e^2)x}{15e^4} - \frac{(7c^2d^4 - 10bcd^3e - 2abde^3 - a^2e^4 + 3(b^2 + 2ac)d^2e^2) \arctan\left(\frac{ex}{\sqrt{d}}\right)}{2\sqrt{d}ed^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)^2/(e\*x^2+d)^2,x, algorithm="maxima")

[Out]  $1/2*(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2)*x/(d*e^5*x^2 + d^2*e^4) + 1/15*(3*c^2*e^2*x^5 - 10*(c^2*d*e - b*c*e^2)*x^3 + 15*(3*c^2*d^2 - 4*b*c*d*e + (b^2 + 2*a*c)*e^2)*x)/e^4 - 1/2*(7*c^2*d^4 - 10*b*c*d^3*e - 2*a*b*d*e^3 - a^2*e^4 + 3*(b^2 + 2*a*c)*d^2*e^2)*\text{arctan}(e*x/\text{sqrt}(d*e))/(\text{sqrt}(d*e)*d*e^4)$

**mpad** [B] time = 4.56, size = 293, normalized size = 1.77

$$x \left( \frac{b^2 + 2ac}{e^2} + \frac{2d \left( \frac{2c^2d - 2bc}{e^2} \right)}{e} - \frac{c^2d^2}{e^4} \right) - x^3 \left( \frac{2c^2d}{3e^3} - \frac{2bc}{3e^2} \right) + \frac{c^2x^5}{5e^2} + \frac{x(a^2e^4 - 2abde^3 + 2acd^2e^2 + b^2d^2e^2 - 2bcd^3e + c^2d^4)}{2d(e^2x^2 + d^2e^4)} + \frac{\text{atan}\left(\frac{\sqrt{d}(c^2d^2 - bde + a^2)(-7c^2d^2 + 3bcde + a^2)}{\sqrt{d}(a^2e^4 + 2abd^3e - bce^2 - 3b^2d^2e^2 + 10bcd^3e - 7c^2d^4)}\right)(c^2d^2 - bde + a^2)(-7c^2d^2 + 3bcde + a^2)}{2d^2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^2 + c\*x^4)^2/(d + e\*x^2)^2,x)

[Out]  $x*((2*a*c + b^2)/e^2 + (2*d*((2*c^2*d)/e^3 - (2*b*c)/e^2))/e - (c^2*d^2)/e^4 - x^3*((2*c^2*d)/(3*e^3) - (2*b*c)/(3*e^2)) + (c^2*x^5)/(5*e^2) + (x*(a^2$

$$\frac{2e^4 + c^2d^4 + b^2d^2e^2 - 2abde^3 - 2b^2c^2d^3e + 2ac^2d^2e^2}{(2d(d^4e^4 + e^5x^2))} + \frac{\operatorname{atan}\left(\frac{e^{1/2}x(ae^2 + cd^2 - bde)}{ae^2 - 7c^2d^2 + 3b^2d^2e}\right)}{d^{1/2}(a^2e^4 - 7c^2d^4 - 3b^2d^2e^2 + 2abde^3 + 10b^2c^2d^3e - 6ac^2d^2e^2)} \cdot \frac{(ae^2 + cd^2 - bde)(ae^2 - 7c^2d^2 + 3b^2d^2e)}{(2d^{3/2}e^{9/2})}$$

**sympy [B]** time = 3.79, size = 484, normalized size = 2.92

$$\frac{c^2x^5}{5d^5} + x^3\left(\frac{2bc}{d^2} - \frac{2c^2d}{3e^3}\right) + x\left(\frac{2ac}{d^2} - \frac{4bcd}{e^2} + \frac{3c^2d^2}{e^4}\right) + \frac{x(a^2e^4 - 2abde^3 + 2acd^2e^2 - 2b^2c^2d^2e + c^2d^4)}{2b^2e^4 + 2de^3x^2} - \frac{\sqrt{-1/d^3}(ae^2 - bde + cd^2)(ae^2 + 3bde - 7cd^2) \log\left(\frac{e^{1/2}\sqrt{-1/d^3}(ae^2 - bde + cd^2)(ae^2 + 3bde - 7cd^2)}{e^2 + 2abde^3 - 6ac^2d^2e - 3b^2c^2d^2 + 10c^2d^3e - 7c^2d^4} + x\right)}{4} + \frac{\sqrt{-1/d^3}(ae^2 - bde + cd^2)(ae^2 + 3bde - 7cd^2) \log\left(\frac{e^{1/2}\sqrt{-1/d^3}(ae^2 - bde + cd^2)(ae^2 + 3bde - 7cd^2)}{e^2 + 2abde^3 - 6ac^2d^2e - 3b^2c^2d^2 + 10c^2d^3e - 7c^2d^4} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*4+b\*x\*\*2+a)\*\*2/(e\*x\*\*2+d)\*\*2,x)

[Out] c\*\*2\*x\*\*5/(5\*e\*\*2) + x\*\*3\*(2\*b\*c/(3\*e\*\*2) - 2\*c\*\*2\*d/(3\*e\*\*3)) + x\*(2\*a\*c/e\*\*2 + b\*\*2/e\*\*2 - 4\*b\*c\*d/e\*\*3 + 3\*c\*\*2\*d\*\*2/e\*\*4) + x\*(a\*\*2\*e\*\*4 - 2\*a\*b\*d\*e\*\*3 + 2\*a\*c\*d\*\*2\*e\*\*2 + b\*\*2\*d\*\*2\*e\*\*2 - 2\*b\*c\*d\*\*3\*e + c\*\*2\*d\*\*4)/(2\*d\*\*2\*e\*\*4 + 2\*d\*e\*\*5\*x\*\*2) - sqrt(-1/(d\*\*3\*e\*\*9))\*(a\*e\*\*2 - b\*d\*e + c\*d\*\*2)\*(a\*e\*\*2 + 3\*b\*d\*e - 7\*c\*d\*\*2)\*log(-d\*\*2\*e\*\*4\*sqrt(-1/(d\*\*3\*e\*\*9))\*(a\*e\*\*2 - b\*d\*e + c\*d\*\*2)\*(a\*e\*\*2 + 3\*b\*d\*e - 7\*c\*d\*\*2)/(a\*\*2\*e\*\*4 + 2\*a\*b\*d\*e\*\*3 - 6\*a\*c\*d\*\*2\*e\*\*2 - 3\*b\*\*2\*d\*\*2\*e\*\*2 + 10\*b\*c\*d\*\*3\*e - 7\*c\*\*2\*d\*\*4) + x)/4 + sqrt(-1/(d\*\*3\*e\*\*9))\*(a\*e\*\*2 - b\*d\*e + c\*d\*\*2)\*(a\*e\*\*2 + 3\*b\*d\*e - 7\*c\*d\*\*2)\*log(d\*\*2\*e\*\*4\*sqrt(-1/(d\*\*3\*e\*\*9))\*(a\*e\*\*2 - b\*d\*e + c\*d\*\*2)\*(a\*e\*\*2 + 3\*b\*d\*e - 7\*c\*d\*\*2)/(a\*\*2\*e\*\*4 + 2\*a\*b\*d\*e\*\*3 - 6\*a\*c\*d\*\*2\*e\*\*2 - 3\*b\*\*2\*d\*\*2\*e\*\*2 + 10\*b\*c\*d\*\*3\*e - 7\*c\*\*2\*d\*\*4) + x)/4

$$3.181 \quad \int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^3} dx$$

**Optimal.** Leaf size=201

$$\frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)\left(e^2(3a^2e^2+2abde+3b^2d^2)-6cd^2e(5bd-ae)+35c^2d^4\right)}{8d^{5/2}e^{9/2}} - \frac{x(-3ae^2-5bde+13cd^2)(ae^2-bde+cd^2)}{8d^2e^4(d+ex^2)}$$

**Rubi [A]** time = 0.42, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1157, 1814, 1153, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)\left(e^2(3a^2e^2+2abde+3b^2d^2)-6cd^2e(5bd-ae)+35c^2d^4\right)}{8d^{5/2}e^{9/2}} - \frac{x(-3ae^2-5bde+13cd^2)(ae^2-bde+cd^2)}{8d^2e^4(d+ex^2)} + \frac{x(ae^2-bde+cd^2)^2}{4de^4(d+ex^2)^2} - \frac{cx(3cd-2be)}{e^4} + \frac{c^2x^3}{3e^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2 + c\*x^4)^2/(d + e\*x^2)^3,x]

[Out] -((c\*(3\*c\*d - 2\*b\*e)\*x)/e^4) + (c^2\*x^3)/(3\*e^3) + ((c\*d^2 - b\*d\*e + a\*e^2)^2\*x)/(4\*d\*e^4\*(d + e\*x^2)^2) - ((13\*c\*d^2 - 5\*b\*d\*e - 3\*a\*e^2)\*(c\*d^2 - b\*d\*e + a\*e^2)\*x)/(8\*d^2\*e^4\*(d + e\*x^2)) + ((35\*c^2\*d^4 - 6\*c\*d^2\*e\*(5\*b\*d - a\*e) + e^2\*(3\*b^2\*d^2 + 2\*a\*b\*d\*e + 3\*a^2\*e^2))\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(8\*d^(5/2)\*e^(9/2))

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1153

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x^2)^q\*(a + b\*x^2 + c\*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 1157

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] := With[{Qx = PolynomialQuotient[(a + b\*x^2 + c\*x^4)^p, d + e\*x^2, x], R = Coeff[PolynomialRemainder[(a + b\*x^2 + c\*x^4)^p, d + e\*x^2, x], x, 0]}, -Simp[(R\*x\*(d + e\*x^2)^(q + 1))/(2\*d\*(q + 1)), x] + Dist[1/(2\*d\*(q + 1)), Int[(d + e\*x^2)^(q + 1)\*ExpandToSum[2\*d\*(q + 1)\*Qx + R\*(2\*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rule 1814

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 1]}, Simp[((a\*g - b\*f\*x)\*(a + b\*x^2)^(p + 1))/(2\*a\*b\*(p + 1)), x] + Dist[1/(2\*a\*(p + 1)), Int[(a + b\*x^2)^(p + 1)\*ExpandToSum[2\*a\*(p + 1)\*Q + f\*(2\*p + 3), x], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^2 + cx^4)^2}{(d + ex^2)^3} dx &= \frac{(cd^2 - bde + ae^2)^2 x}{4de^4 (d + ex^2)^2} - \frac{\int \frac{\frac{(cd^2 - bde - ae^2)(cd^2 - bde + 3ae^2)}{e^4} - \frac{4d(c^2d^2 + b^2e^2 - 2ce(bd - ae))x^2}{e^3} + \frac{4cd(cd - 2be)x^4}{e^2} - \frac{4c^2dx^6}{e}}{(d + ex^2)^2}}{4d} \\
&= \frac{(cd^2 - bde + ae^2)^2 x}{4de^4 (d + ex^2)^2} - \frac{(13cd^2 - 5bde - 3ae^2)(cd^2 - bde + ae^2)x}{8d^2e^4 (d + ex^2)} + \frac{\int \frac{11c^2d^4 - 2cd^2e(7bd - 3ae)}{e^4}}{(d + ex^2)^2} \\
&= \frac{(cd^2 - bde + ae^2)^2 x}{4de^4 (d + ex^2)^2} - \frac{(13cd^2 - 5bde - 3ae^2)(cd^2 - bde + ae^2)x}{8d^2e^4 (d + ex^2)} + \frac{\int \left( -\frac{8cd^2(3cd - 2be)}{e^4} \right)}{(d + ex^2)^2} \\
&= -\frac{c(3cd - 2be)x}{e^4} + \frac{c^2x^3}{3e^3} + \frac{(cd^2 - bde + ae^2)^2 x}{4de^4 (d + ex^2)^2} - \frac{(13cd^2 - 5bde - 3ae^2)(cd^2 - bde + ae^2)x}{8d^2e^4 (d + ex^2)} \\
&= -\frac{c(3cd - 2be)x}{e^4} + \frac{c^2x^3}{3e^3} + \frac{(cd^2 - bde + ae^2)^2 x}{4de^4 (d + ex^2)^2} - \frac{(13cd^2 - 5bde - 3ae^2)(cd^2 - bde + ae^2)x}{8d^2e^4 (d + ex^2)}
\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 217, normalized size = 1.08

$$-\frac{x(e^2(-3a^2e^2 - 2abde + 5b^2d^2) - 2cd^2e(9bd - 5ae) + 13c^2d^4)}{8d^2e^4(d + ex^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(e^2(3a^2e^2 + 2abde + 3b^2d^2) + 6cd^2e(ae - 5bd) + 35c^2d^4)}{8d^{5/2}e^{9/2}} + \frac{x(e(ae - bd) + cd^2)^2}{4de^4(d + ex^2)^2} + \frac{cx(2be - 3cd)}{e^4} + \frac{c^2x^3}{3e^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2 + c\*x^4)^2/(d + e\*x^2)^3,x]

[Out] (c\*(-3\*c\*d + 2\*b\*e)\*x)/e^4 + (c^2\*x^3)/(3\*e^3) + ((c\*d^2 + e\*(-(b\*d) + a\*e))^2\*x)/(4\*d\*e^4\*(d + e\*x^2)^2) - ((13\*c^2\*d^4 - 2\*c\*d^2\*e\*(9\*b\*d - 5\*a\*e) + e^2\*(5\*b^2\*d^2 - 2\*a\*b\*d\*e - 3\*a^2\*e^2))\*x)/(8\*d^2\*e^4\*(d + e\*x^2)) + ((35\*c^2\*d^4 + 6\*c\*d^2\*e\*(-5\*b\*d + a\*e) + e^2\*(3\*b^2\*d^2 + 2\*a\*b\*d\*e + 3\*a^2\*e^2))\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(8\*d^(5/2)\*e^(9/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2 + cx^4)^2}{(d + ex^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)^2/(d + e\*x^2)^3,x]

[Out] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)^2/(d + e\*x^2)^3, x]

**fricas [B]** time = 0.71, size = 794, normalized size = 3.95

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)^2/(e\*x^2+d)^3,x, algorithm="fricas")

[Out] [1/48\*(16\*c^2\*d^3\*e^4\*x^7 - 16\*(7\*c^2\*d^4\*e^3 - 6\*b\*c\*d^3\*e^4)\*x^5 - 2\*(175\*c^2\*d^5\*e^2 - 150\*b\*c\*d^4\*e^3 - 6\*a\*b\*d^2\*e^5 - 9\*a^2\*d\*e^6 + 15\*(b^2 + 2\*a\*c)\*d^3\*e^4)\*x^3 - 3\*(35\*c^2\*d^6 - 30\*b\*c\*d^5\*e + 2\*a\*b\*d^3\*e^3 + 3\*a^2\*d^

$2e^4 + 3(b^2 + 2ac)d^4e^2 + (35c^2d^4e^2 - 30bcd^3e^3 + 2abd^2e^4 + 3a^2de^5 + 3a^2e^6 + 3(b^2 + 2ac)d^2e^4)x^4 + 2(35c^2d^5e - 30bcd^4e^2 + 2abd^3e^3 + 3a^2d^2e^4 + 3a^2de^5 + 3(b^2 + 2ac)d^3e^3)x^2) \sqrt{-de} \log\left(\frac{ex^2 - 2\sqrt{-de}x - d}{ex^2 + d}\right) - 6(35c^2d^6e - 30bcd^5e^2 + 2abd^4e^3 - 5a^2d^3e^4 + 3(b^2 + 2ac)d^4e^3)x / (d^3e^7x^4 + 2d^4e^6x^2 + d^5e^5),$   
 $1/24(8c^2d^3e^4x^7 - 8(7c^2d^4e^3 - 6bcd^3e^4)x^5 - (175c^2d^5e^2 - 150bcd^4e^3 - 6abd^3e^4 - 9a^2d^2e^5 + 15(b^2 + 2ac)d^3e^4)x^3 + 3(35c^2d^6 - 30bcd^5e + 2abd^4e^3 + 3a^2d^2e^4 + 3(b^2 + 2ac)d^4e^2 + (35c^2d^4e^2 - 30bcd^3e^3 + 2abd^2e^4 + 3a^2de^5 + 3a^2e^6 + 3(b^2 + 2ac)d^2e^4)x^4 + 2(35c^2d^5e - 30bcd^4e^2 + 2abd^3e^3 + 3a^2d^2e^4 + 3a^2de^5 + 3(b^2 + 2ac)d^3e^3)x^2) \sqrt{de} \arctan(\sqrt{de}x/d) - 3(35c^2d^6e - 30bcd^5e^2 + 2abd^4e^3 - 5a^2d^2e^5 + 3(b^2 + 2ac)d^4e^3)x) / (d^3e^7x^4 + 2d^4e^6x^2 + d^5e^5)]$

**giac [A]** time = 0.18, size = 244, normalized size = 1.21

$$\frac{1}{3} \frac{(2^3x^6 - 9c^2dx^5 + 6bcx^4)e^{(-9)} + \frac{(35c^2d^4 - 30bcd^3e + 3b^2d^2e^2 + 6acd^2 + 2abd^2 + 3a^2e^4) \arctan\left(\frac{x}{\sqrt{d}}\right) \left(\frac{x}{\sqrt{d}}\right)}{8d^{\frac{5}{2}}}}{\frac{(13c^2d^4x^3e - 18bcd^3x^2e^2 + 11c^2d^5x + 5b^2d^2x^3e^3 + 10acd^3x^3e^3 - 14bcd^4xe - 2abd^3x^2e^4 + 3b^2d^2x^2e^2 + 6acd^3x^2e^2 - 3c^2x^3e^5 + 2abd^2x^3 - 5a^2dx^4)e^{(-4)}}{8(x^2e + d)^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)^2/(e\*x^2+d)^3,x, algorithm="giac")

[Out]  $1/3(c^2x^3e^6 - 9c^2dxxe^5 + 6b^2c^2xe^6)e^{(-9)} + 1/8(35c^2d^4 - 30bcd^3e + 3b^2d^2e^2 + 6a^2cd^2e^2 + 2abd^2e^3 + 3a^2e^4) \arctan(xe^{(1/2)}/\sqrt{d})e^{(-9/2)}/d^{(5/2)} - 1/8(13c^2d^4x^3e - 18bcd^3x^2e^2 + 11c^2d^5x + 5b^2d^2x^3e^3 + 10acd^2x^3e^3 - 14bcd^4xe - 2abd^3x^2e^4 + 3b^2d^3xe^2 + 6a^2cd^3xe^2 - 3a^2x^3e^5 + 2abd^2x^2e^3 - 5a^2dxxe^4)e^{(-4)}/((x^2e + d)^2d^2)$

**maple [B]** time = 0.01, size = 402, normalized size = 2.00

$$\frac{\frac{3c^2x^2}{8(x^2+d)^2d} + \frac{abx}{4(x^2+d)d} + \frac{5ac^2}{4(x^2+d)^2} + \frac{5b^2x}{8(x^2+d)e} + \frac{9bcd^2}{4(x^2+d)^2} + \frac{13c^2d^2}{8(x^2+d)^2} + \frac{5a^2}{8(x^2+d)^2} + \frac{abx}{4(x^2+d)e} + \frac{3acd}{4(x^2+d)^2} + \frac{3b^2d}{8(x^2+d)^2} + \frac{7bc^2d}{4(x^2+d)^2} + \frac{11c^2d^2}{8(x^2+d)^2} + \frac{c^2}{3d^2} + \frac{3c^2 \arctan\left(\frac{x}{\sqrt{d}}\right)}{9\sqrt{d}e^2} + \frac{ab \arctan\left(\frac{x}{\sqrt{d}}\right)}{4\sqrt{d}e} + \frac{3ac \arctan\left(\frac{x}{\sqrt{d}}\right)}{4\sqrt{d}e^2} + \frac{3b^2 \arctan\left(\frac{x}{\sqrt{d}}\right)}{8\sqrt{d}e} + \frac{15bd \arctan\left(\frac{x}{\sqrt{d}}\right)}{4\sqrt{d}e} + \frac{35c^2d \arctan\left(\frac{x}{\sqrt{d}}\right)}{8\sqrt{d}e} + \frac{2bcx}{d^2} + \frac{3c^2x}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^4+b\*x^2+a)^2/(e\*x^2+d)^3,x)

[Out]  $1/3c^2/e^3x^3+2c/e^3bx-3c^2d/e^4x+3/8/(e*x^2+d)^2a^2/d^2e*x^3+1/4/(e*x^2+d)^2/d*x^3a*b-5/4/(e*x^2+d)^2a*c/e*x^3-5/8/e/(e*x^2+d)^2x^3b^2+9/4/e^2/(e*x^2+d)^2x^3b*c*d-13/8/(e*x^2+d)^2c^2d^2/e^3x^3+5/8/(e*x^2+d)^2a^2/d*x-1/4/e/(e*x^2+d)^2a*b*x-3/4/(e*x^2+d)^2a*c*d/e^2x-3/8/e^2/(e*x^2+d)^2b^2*d*x+7/4/e^3/(e*x^2+d)^2b*c*d^2*x-11/8/(e*x^2+d)^2c^2d^3/e^4*x+3/8/(d*e)^(1/2)*a^2/d^2*arctan(1/(d*e)^(1/2)*e*x)+1/4/e/d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a*b+3/4/(d*e)^(1/2)*a*c/e^2*arctan(1/(d*e)^(1/2)*e*x)+3/8/e^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b^2-15/4/e^3*d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b*c+35/8/(d*e)^(1/2)*c^2*d^2/e^4*arctan(1/(d*e)^(1/2)*e*x)$

**maxima [A]** time = 2.36, size = 245, normalized size = 1.22

$$\frac{(13c^2d^4e - 18bcd^3e^2 - 2abde^4 - 3a^2e^5 + 5(b^2 + 2ac)d^2e^3)x^3 + (11c^2d^5 - 14bcd^4e + 2abd^3e^2 - 5a^2de^4 + 3(b^2 + 2ac)d^3e^2)x + c^2ex^3 - 3(3c^2d - 2bce)x}{8(d^2e^6x^4 + 2d^3e^5x^2 + d^4e^4)} + \frac{c^2ex^3 - 3(3c^2d - 2bce)x}{3e^4} + \frac{(35c^2d^4 - 30bcd^3e + 2abde^3 + 3a^2e^4 + 3(b^2 + 2ac)d^2e^2) \arctan\left(\frac{ax}{\sqrt{de}}\right)}{8\sqrt{de}d^2e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)^2/(e\*x^2+d)^3,x, algorithm="maxima")

[Out]  $-1/8((13c^2d^4e - 18bcd^3e^2 - 2abd^3e^4 - 3a^2e^5 + 5(b^2 + 2ac)d^2e^3)x^3 + (11c^2d^5 - 14bcd^4e + 2abd^3e^2 - 5a^2de^4 + 3(b^2 + 2ac)d^3e^2)x) / (d^2e^6x^4 + 2d^3e^5x^2 + d^4e^4) + 1/3(c^2e*x^3 - 3(3c^2d - 2b*c*e)*x) / e^4 + 1/8(35c^2d^4 - 30bcd^3e - 3a^2d^2e^3 + 3(b^2 + 2ac)d^2e^2) \arctan\left(\frac{ax}{\sqrt{de}}\right) / (8\sqrt{de}d^2e^4)$

$3e + 2abde^3 + 3a^2e^4 + 3(b^2 + 2ac)d^2e^2) \arctan(e x / \sqrt{d e}) / (\sqrt{d e}) d^2 e^4$

**mupad [B]** time = 0.12, size = 257, normalized size = 1.28

$$\frac{c^2 x^3}{3e^3} - x \left( \frac{3c^2 d}{e^4} - \frac{2bc}{e^3} \right) - \frac{x(-5a^2e^4 + 2abd^2 + 6acd^2e^2 + 3b^2d^2e^2 - 14bcd^2e + 11c^2d^4)}{8d} - \frac{x^3(3a^2e^5 + 2abd^4e - 10acd^3e^3 - 5b^2d^2e^3 + 18bcd^3e^2 - 13c^2d^4e)}{8d^2} + \frac{\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(3a^2e^4 + 2abd^2e^3 + 6acd^2e^2 + 3b^2d^2e^2 - 30bcd^3e + 35c^2d^4)}{8d^{5/2}e^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^2 + c\*x^4)^2/(d + e\*x^2)^3,x)

[Out]  $(c^2 x^3)/(3e^3) - x((3c^2 d)/e^4 - (2bc)/e^3) - ((x(11c^2 d^4 - 5a^2 e^4 + 3b^2 d^2 e^2 + 2ab d^2 e^3 - 14b c d^3 e + 6a a c d^2 e^2)))/(8d) - (x^3(3a^2 e^5 - 13c^2 d^4 e - 5b^2 d^2 e^3 + 2a a b d^2 e^4 - 10a a c d^2 e^3 + 18b b c d^3 e^2))/(8d^2))/(d^2 e^4 + e^6 x^4 + 2d e^5 x^2) + (\operatorname{atan}((e^{1/2} x)/d^{1/2}))(3a^2 e^4 + 35c^2 d^4 + 3b^2 d^2 e^2 + 2a a b d^2 e^3 - 30b b c d^3 e + 6a a c d^2 e^2))/(8d^{5/2} e^{9/2})$

**sympy [A]** time = 17.72, size = 398, normalized size = 1.98

$$\frac{c^2 x^3}{3e^3} + x \left( \frac{2bc}{e^4} - \frac{3c^2 d}{e^3} \right) - \frac{\sqrt{-\frac{d}{e}}(3a^2 e^4 + 2abd^2 + 6acd^2e^2 + 3b^2d^2e^2 - 30bcd^3e + 35c^2d^4) \log\left(-\frac{d^{1/2} \sqrt{-\frac{d}{e}}}{e} + x\right)}{16} + \frac{\sqrt{-\frac{d}{e}}(3a^2 e^4 + 2abd^2 + 6acd^2e^2 + 3b^2d^2e^2 - 30bcd^3e + 35c^2d^4) \log\left(\frac{d^{1/2} \sqrt{-\frac{d}{e}}}{e} + x\right)}{16} + \frac{x^3(3a^2 e^5 + 2abd^4e - 10acd^3e^3 - 5b^2d^2e^3 + 18bcd^3e^2 - 13c^2d^4e)}{8d^2} + \frac{x(5a^2 e^4 - 2abd^2e^3 - 6acd^2e^2 - 3b^2d^2e^2 + 14bcd^3e - 11c^2d^4)}{8d^2} + \frac{16d^3 e^5 x^2 + 8d^2 e^6 x^4}{8d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*4+b\*x\*\*2+a)\*\*2/(e\*x\*\*2+d)\*\*3,x)

[Out]  $c^2 x^3/(3e^3) + x(2bc/e^4 - 3c^2 d/e^3) - \sqrt{-1/(d^5 e^9)}(3a^2 e^4 + 2ab d^2 e^3 + 6a a c d^2 e^2 + 3b^2 d^2 e^2 - 30b b c d^3 e + 35c^2 d^4) \log(-d^3 e^4 \sqrt{-1/(d^5 e^9)} + x)/16 + \sqrt{-1/(d^5 e^9)}(3a^2 e^4 + 2ab d^2 e^3 + 6a a c d^2 e^2 + 3b^2 d^2 e^2 - 30b b c d^3 e + 35c^2 d^4) \log(d^3 e^4 \sqrt{-1/(d^5 e^9)} + x)/16 + (x^3(3a^2 e^5 + 2a a b d^2 e^4 - 10a a c d^2 e^3 - 5b^2 d^2 e^3 + 18b b c d^3 e^2 - 13c^2 d^4 e) + x(5a^2 e^4 - 2a a b d^2 e^3 - 6a a c d^3 e^2 - 3b^2 d^2 e^2 + 14b b c d^3 e - 11c^2 d^4))/(8d^4 e^4 + 16d^3 e^5 x^2 + 8d^2 e^6 x^4)$

$$3.182 \quad \int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^4} dx$$

**Optimal.** Leaf size=250

$$\frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)\left(-e^2(5a^2e^2+2abde+b^2d^2)-2cd^2e(ae+5bd)+35c^2d^4\right)}{16d^{7/2}e^{9/2}} + \frac{x\left(e^2(5a^2e^2+2abde+b^2d^2)-2cd^2e(11bd-ae)+29c^2d^4\right)}{16d^3e^4(d+ex^2)}$$

**Rubi [A]** time = 0.54, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, number of rules / integrand size = 0.167, Rules used = {1157, 1814, 388, 205}

$$\frac{x(e^2(5a^2e^2+2abde+b^2d^2)-2cd^2e(11bd-ae)+29c^2d^4)}{16d^3e^4(d+ex^2)} - \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)\left(-e^2(5a^2e^2+2abde+b^2d^2)-2cd^2e(ae+5bd)+35c^2d^4\right)}{16d^{7/2}e^{9/2}} - \frac{x(-5ae^2-7bde+19cd^2)(ae^2-bde+cd^2)}{24d^2e^4(d+ex^2)^2} + \frac{x(ae^2-bde+cd^2)^2}{6de^4(d+ex^2)^3} + \frac{c^2x}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^4,x]
```

```
[Out] (c^2*x)/e^4 + ((c*d^2 - b*d*e + a*e^2)^2*x)/(6*d*e^4*(d + e*x^2)^3) - ((19*c*d^2 - 7*b*d*e - 5*a*e^2)*(c*d^2 - b*d*e + a*e^2)*x)/(24*d^2*e^4*(d + e*x^2)^2) + ((29*c^2*d^4 - 2*c*d^2*e*(11*b*d - a*e) + e^2*(b^2*d^2 + 2*a*b*d*e + 5*a^2*e^2))*x)/(16*d^3*e^4*(d + e*x^2)) - ((35*c^2*d^4 - 2*c*d^2*e*(5*b*d + a*e) - e^2*(b^2*d^2 + 2*a*b*d*e + 5*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(9/2))
```

#### Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

#### Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p+1))/(b*(n*(p+1)+1)), x] - Dist[(a*d - b*c*(n*(p+1)+1))/(b*(n*(p+1)+1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p+1)+1, 0]
```

#### Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q+1))/(2*d*(q+1)), x] + Dist[1/(2*d*(q+1)), Int[(d + e*x^2)^(q+1)*ExpandToSum[2*d*(q+1)*Qx + R*(2*q+3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

#### Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p+1))/(2*a*b*(p+1)), x] + Dist[1/(2*a*(p+1)), Int[(a + b*x^2)^(p+1)*ExpandToSum[2*a*(p+1)*Q + f*(2*p+3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

#### Rubi steps



$$\begin{aligned}
\int \frac{(a + bx^2 + cx^4)^2}{(d + ex^2)^4} dx &= \frac{(cd^2 - bde + ae^2)^2 x}{6de^4 (d + ex^2)^3} - \frac{\int \frac{c^2 d^4 - 2cd^2 e(bd - ae) + e^2(b^2 d^2 - 2abde - 5a^2 e^2)}{e^4} - \frac{6d(c^2 d^2 + b^2 e^2 - 2ce(bd - ae))x^2}{e^3} + \frac{6cd(cd - 2be)x^4}{e^2}}{(d + ex^2)^3}}{6d} \\
&= \frac{(cd^2 - bde + ae^2)^2 x}{6de^4 (d + ex^2)^3} - \frac{(19cd^2 - 7bde - 5ae^2)(cd^2 - bde + ae^2)x}{24d^2 e^4 (d + ex^2)^2} + \frac{\int \frac{3(5c^2 d^4 - 2cd^2 e(3bd - ae) + 3a^2 e^3)}{e^4}}{(d + ex^2)^3}}{24d^2 e^4} \\
&= \frac{(cd^2 - bde + ae^2)^2 x}{6de^4 (d + ex^2)^3} - \frac{(19cd^2 - 7bde - 5ae^2)(cd^2 - bde + ae^2)x}{24d^2 e^4 (d + ex^2)^2} + \frac{(29c^2 d^4 - 2cd^2 e(3bd - ae) + 3a^2 e^3)}{24d^2 e^4} \\
&= \frac{c^2 x}{e^4} + \frac{(cd^2 - bde + ae^2)^2 x}{6de^4 (d + ex^2)^3} - \frac{(19cd^2 - 7bde - 5ae^2)(cd^2 - bde + ae^2)x}{24d^2 e^4 (d + ex^2)^2} + \frac{(29c^2 d^4 - 2cd^2 e(3bd - ae) + 3a^2 e^3)}{24d^2 e^4} \\
&= \frac{c^2 x}{e^4} + \frac{(cd^2 - bde + ae^2)^2 x}{6de^4 (d + ex^2)^3} - \frac{(19cd^2 - 7bde - 5ae^2)(cd^2 - bde + ae^2)x}{24d^2 e^4 (d + ex^2)^2} + \frac{(29c^2 d^4 - 2cd^2 e(3bd - ae) + 3a^2 e^3)}{24d^2 e^4}
\end{aligned}$$

**Mathematica [A]** time = 0.15, size = 267, normalized size = 1.07

$$\frac{x(c^2(-5a^2e^2 - 2abde + 7b^2d^2) + 2cd^2e(7ae - 13bd) + 19c^2d^4)}{24d^2e^4(d + ex^2)^3} - \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(-e^2(5a^2e^2 + 2abde + b^2d^2) - 2cd^2e(ae + 5bd) + 35c^2d^4)}{16d^{7/2}e^{9/2}} + \frac{x(c^2(5a^2e^2 + 2abde + b^2d^2) + 2cd^2e(ae - 11bd) + 29c^2d^4)}{16d^2e^4(d + ex^2)^2} + \frac{x(e(ae - bd) + cd^2)^2}{6de^4(d + ex^2)^3} + \frac{c^2x}{e^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2 + c\*x^4)^2/(d + e\*x^2)^4, x]

[Out] (c^2\*x)/e^4 + ((c\*d^2 + e\*(-(b\*d) + a\*e))^2\*x)/(6\*d\*e^4\*(d + e\*x^2)^3) - ((19\*c^2\*d^4 + 2\*c\*d^2\*e\*(-13\*b\*d + 7\*a\*e) + e^2\*(7\*b^2\*d^2 - 2\*a\*b\*d\*e - 5\*a^2\*e^2))\*x)/(24\*d^2\*e^4\*(d + e\*x^2)^2) + ((29\*c^2\*d^4 + 2\*c\*d^2\*e\*(-11\*b\*d + a\*e) + e^2\*(b^2\*d^2 + 2\*a\*b\*d\*e + 5\*a^2\*e^2))\*x)/(16\*d^3\*e^4\*(d + e\*x^2)) - ((35\*c^2\*d^4 - 2\*c\*d^2\*e\*(5\*b\*d + a\*e) - e^2\*(b^2\*d^2 + 2\*a\*b\*d\*e + 5\*a^2\*e^2))\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(16\*d^(7/2)\*e^(9/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2 + cx^4)^2}{(d + ex^2)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)^2/(d + e\*x^2)^4, x]

[Out] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)^2/(d + e\*x^2)^4, x]

**fricas [B]** time = 0.81, size = 1016, normalized size = 4.06

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)^2/(e\*x^2+d)^4,x, algorithm="fricas")

[Out] [1/96\*(96\*c^2\*d^4\*e^4\*x^7 + 6\*(77\*c^2\*d^5\*e^3 - 22\*b\*c\*d^4\*e^4 + 2\*a\*b\*d^2\*e^6 + 5\*a^2\*d\*e^7 + (b^2 + 2\*a\*c)\*d^3\*e^5)\*x^5 + 16\*(35\*c^2\*d^6\*e^2 - 10\*b\*

$$c*d^5*e^3 + 2*a*b*d^3*e^5 + 5*a^2*d^2*e^6 - (b^2 + 2*a*c)*d^4*e^4)*x^3 + 3*(35*c^2*d^7 - 10*b*c*d^6*e - 2*a*b*d^4*e^3 - 5*a^2*d^3*e^4 - (b^2 + 2*a*c)*d^5*e^2 + (35*c^2*d^4*e^3 - 10*b*c*d^3*e^4 - 2*a*b*d*e^6 - 5*a^2*e^7 - (b^2 + 2*a*c)*d^2*e^5)*x^6 + 3*(35*c^2*d^5*e^2 - 10*b*c*d^4*e^3 - 2*a*b*d^2*e^5 - 5*a^2*d*e^6 - (b^2 + 2*a*c)*d^3*e^4)*x^4 + 3*(35*c^2*d^6*e - 10*b*c*d^5*e^2 - 2*a*b*d^3*e^4 - 5*a^2*d^2*e^5 - (b^2 + 2*a*c)*d^4*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 6*(35*c^2*d^7*e - 10*b*c*d^6*e^2 - 2*a*b*d^4*e^4 + 11*a^2*d^3*e^5 - (b^2 + 2*a*c)*d^5*e^3)*x)/(d^4*e^8*x^6 + 3*d^5*e^7*x^4 + 3*d^6*e^6*x^2 + d^7*e^5), 1/48*(48*c^2*d^4*e^4*x^7 + 3*(77*c^2*d^5*e^3 - 22*b*c*d^4*e^4 + 2*a*b*d^2*e^6 + 5*a^2*d*e^7 + (b^2 + 2*a*c)*d^3*e^5)*x^5 + 8*(35*c^2*d^6*e^2 - 10*b*c*d^5*e^3 + 2*a*b*d^3*e^5 + 5*a^2*d^2*e^6 - (b^2 + 2*a*c)*d^4*e^4)*x^3 - 3*(35*c^2*d^7 - 10*b*c*d^6*e - 2*a*b*d^4*e^3 - 5*a^2*d^3*e^4 - (b^2 + 2*a*c)*d^5*e^2 + (35*c^2*d^4*e^3 - 10*b*c*d^3*e^4 - 2*a*b*d*e^6 - 5*a^2*e^7 - (b^2 + 2*a*c)*d^2*e^5)*x^6 + 3*(35*c^2*d^5*e^2 - 10*b*c*d^4*e^3 - 2*a*b*d^2*e^5 - 5*a^2*d*e^6 - (b^2 + 2*a*c)*d^3*e^4)*x^4 + 3*(35*c^2*d^6*e - 10*b*c*d^5*e^2 - 2*a*b*d^3*e^4 - 5*a^2*d^2*e^5 - (b^2 + 2*a*c)*d^4*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + 3*(35*c^2*d^7*e - 10*b*c*d^6*e^2 - 2*a*b*d^4*e^4 + 11*a^2*d^3*e^5 - (b^2 + 2*a*c)*d^5*e^3)*x)/(d^4*e^8*x^6 + 3*d^5*e^7*x^4 + 3*d^6*e^6*x^2 + d^7*e^5)]$$

giac [A] time = 0.18, size = 296, normalized size = 1.18

$$\frac{(35c^2d^4 - 10bcd^3e - 2abd^2e^2 - 2abd^2e^2 - 5a^2d^4) \arctan\left(\frac{x}{\sqrt{d}}\right) + (87c^2d^4e^2 - 66bcd^3e^2 + 136c^2d^5e^2 + 3b^2d^2e^3 + 6acd^2e^3 - 80bcd^2e^3 + 57c^2d^6e + 6abd^2e^3 - 8b^2d^2e^3 - 16acd^2e^3 - 30bcd^2e^3 + 15a^2d^2e^3 + 16abd^2e^3 - 3b^2d^2e^3 - 6acd^2e^3 + 40a^2d^2e^3 - 6abd^2e^3 + 33a^2d^2e^3)e^{(-4)}}{16d^5} \cdot \frac{1}{48(d^4e^8x^6 + 3d^5e^7x^4 + 3d^6e^6x^2 + d^7e^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)^2/(e\*x^2+d)^4,x, algorithm="giac")

[Out]  $c^2*x^e^{-4} - 1/16*(35*c^2*d^4 - 10*b*c*d^3*e - b^2*d^2*e^2 - 2*a*c*d^2*e^2 - 2*a*b*d*e^3 - 5*a^2*e^4)*arctan(x*e^{(1/2)}/sqrt(d))*e^{(-9/2)}/d^{(7/2)} + 1/48*(87*c^2*d^4*x^5*e^2 - 66*b*c*d^3*x^5*e^3 + 136*c^2*d^5*x^3*e + 3*b^2*d^2*x^5*e^4 + 6*a*c*d^2*x^5*e^4 - 80*b*c*d^4*x^3*e^2 + 57*c^2*d^6*x + 6*a*b*d*x^5*e^5 - 8*b^2*d^3*x^3*e^3 - 16*a*c*d^3*x^3*e^3 - 30*b*c*d^5*x*e + 15*a^2*x^5*e^6 + 16*a*b*d^2*x^3*e^4 - 3*b^2*d^4*x*e^2 - 6*a*c*d^4*x*e^2 + 40*a^2*d*x^3*e^5 - 6*a*b*d^3*x*e^3 + 33*a^2*d^2*x*e^4)*e^{(-4)}/((x^2*e + d)^3*d^3)$

maple [B] time = 0.01, size = 506, normalized size = 2.02

$$\frac{3(29c^2d^4e^2 - 22bcd^3e^2 + 2abd^2e^2 + 5a^2d^4e^2 + (b^2 + 2ac)d^2e^4)e^2 + 8(17c^2d^4e - 10bcd^3e^2 + 2abd^2e^2 + 5a^2d^4e^2 - (b^2 + 2ac)d^2e^4)e^2 + 3(19c^2d^6e - 10bcd^3e^2 + 11a^2d^2e^3 - (b^2 + 2ac)d^2e^4)x + \frac{c^2x}{d^4} + \frac{(35c^2d^4 - 10bcd^3e - 2abd^2e^2 - 5a^2d^4 - (b^2 + 2ac)d^2e^4) \arctan\left(\frac{x}{\sqrt{d}}\right)}{16\sqrt{d}d^3e^4}}{48(d^4e^8x^6 + 3d^5e^7x^4 + 3d^6e^6x^2 + d^7e^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^4+b\*x^2+a)^2/(e\*x^2+d)^4,x)

[Out]  $c^2/e^4*x - 1/6/e/(e*x^2+d)^3*x^3*b^2 + 1/16/(e*x^2+d)^3/d*x^5*b^2 + 11/16/(e*x^2+d)^3*a^2/d*x + 5/16/(d*e)^{(1/2)}*a^2/d^3*arctan(1/(d*e)^{(1/2)}*e*x) - 1/8/(e*x^2+d)^3*a*c*d/e^2*x + 1/8/(d*e)^{(1/2)}*a*c/d/e^2*arctan(1/(d*e)^{(1/2)}*e*x) + 1/8*e/(e*x^2+d)^3/d^2*x^5*a*b - 5/3/e^2/(e*x^2+d)^3*x^3*b*c*d - 5/8/e^3/(e*x^2+d)^3*b*c*d^2*x + 1/8/e/d^2/(d*e)^{(1/2)}*arctan(1/(d*e)^{(1/2)}*e*x)*a*b - 11/8/e/(e*x^2+d)^3*x^5*b*c - 1/8/e/(e*x^2+d)^3*a*b*x - 1/16/e^2/(e*x^2+d)^3*b^2*d*x + 1/16/e^2/d/(d*e)^{(1/2)}*arctan(1/(d*e)^{(1/2)}*e*x)*b^2 + 5/8/e^3/(d*e)^{(1/2)}*arctan(1/(d*e)^{(1/2)}*e*x)*b*c + 1/3/(e*x^2+d)^3/d*x^3*a*b + 29/16/(e*x^2+d)^3*c^2*d/e^2*x^5 + 5/6/(e*x^2+d)^3*a^2/d^2*e*x^3 - 1/3/(e*x^2+d)^3*a*c/e*x^3 + 17/6/(e*x^2+d)^3*c^2*d^2/e^3*x^3 + 19/16/(e*x^2+d)^3*c^2*d^3/e^4*x - 35/16/(d*e)^{(1/2)}*c^2*d/e^4*arctan(1/(d*e)^{(1/2)}*e*x) + 5/16/(e*x^2+d)^3*a^2/d^3*e^2*x^5 + 1/8/(e*x^2+d)^3*a*c/d*x^5$

maxima [A] time = 2.39, size = 300, normalized size = 1.20

$$\frac{3(29c^2d^4e^2 - 22bcd^3e^2 + 2abd^2e^2 + 5a^2d^4e^2 + (b^2 + 2ac)d^2e^4)e^2 + 8(17c^2d^4e - 10bcd^3e^2 + 2abd^2e^2 + 5a^2d^4e^2 - (b^2 + 2ac)d^2e^4)e^2 + 3(19c^2d^6e - 10bcd^3e^2 + 11a^2d^2e^3 - (b^2 + 2ac)d^2e^4)x + \frac{c^2x}{d^4} + \frac{(35c^2d^4 - 10bcd^3e - 2abd^2e^2 - 5a^2d^4 - (b^2 + 2ac)d^2e^4) \arctan\left(\frac{x}{\sqrt{d}}\right)}{16\sqrt{d}d^3e^4}}{48(d^4e^8x^6 + 3d^5e^7x^4 + 3d^6e^6x^2 + d^7e^5)}$$

Verification of antiderivative is not currently implemented for this CAS.



$$3.183 \quad \int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^5} dx$$

**Optimal.** Leaf size=317

$$\frac{x \left( -e^2 (35a^2e^2 + 10abde + 3b^2d^2) - 2cd^2e(3ae + 5bd) + 93c^2d^4 \right)}{128d^4e^4(d+ex^2)} + \frac{\tan^{-1} \left( \frac{\sqrt{e}x}{\sqrt{d}} \right) \left( e^2 (35a^2e^2 + 10abde + 3b^2d^2) + \right)}{128d^{9/2}e^{9/2}}$$

**Rubi [A]** time = 0.65, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, number of rules / integrand size = 0.167, Rules used = {1157, 1814, 385, 205}

$$\frac{x \left( -e^2 (35a^2e^2 + 10abde + 3b^2d^2) - 2cd^2e(3ae + 5bd) + 93c^2d^4 \right)}{128d^4e^4(d+ex^2)} + \frac{x \left( e^2 (35a^2e^2 + 10abde + 3b^2d^2) - 2cd^2e(59bd - 3ae) + 163c^2d^4 \right)}{192d^4e^4(d+ex^2)} + \frac{\tan^{-1} \left( \frac{\sqrt{e}x}{\sqrt{d}} \right) \left( e^2 (35a^2e^2 + 10abde + 3b^2d^2) + 2cd^2e(3ae + 5bd) + 35c^2d^4 \right)}{128d^{9/2}e^{9/2}} + \frac{x(a^2 - bde + cd^2)^2}{8d^4(d+ex^2)^4} - \frac{x(-7ae^2 - 9bde + 25cd^2)(a^2 - bde + cd^2)}{48d^4e^4(d+ex^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2 + c\*x^4)^2/(d + e\*x^2)^5,x]

[Out] ((c\*d^2 - b\*d\*e + a\*e^2)^2\*x)/(8\*d\*e^4\*(d + e\*x^2)^4) - ((25\*c\*d^2 - 9\*b\*d\*e - 7\*a\*e^2)\*(c\*d^2 - b\*d\*e + a\*e^2)\*x)/(48\*d^2\*e^4\*(d + e\*x^2)^3) + ((163\*c^2\*d^4 - 2\*c\*d^2\*e\*(59\*b\*d - 3\*a\*e) + e^2\*(3\*b^2\*d^2 + 10\*a\*b\*d\*e + 35\*a^2\*e^2))\*x)/(192\*d^3\*e^4\*(d + e\*x^2)^2) - ((93\*c^2\*d^4 - 2\*c\*d^2\*e\*(5\*b\*d + 3\*a\*e) - e^2\*(3\*b^2\*d^2 + 10\*a\*b\*d\*e + 35\*a^2\*e^2))\*x)/(128\*d^4\*e^4\*(d + e\*x^2)) + ((35\*c^2\*d^4 + 2\*c\*d^2\*e\*(5\*b\*d + 3\*a\*e) + e^2\*(3\*b^2\*d^2 + 10\*a\*b\*d\*e + 35\*a^2\*e^2))\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(128\*d^(9/2)\*e^(9/2))

**Rule 205**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 385**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

**Rule 1157**

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] := With[{Qx = PolynomialQuotient[(a + b\*x^2 + c\*x^4)^p, d + e\*x^2, x], R = Coeff[PolynomialRemainder[(a + b\*x^2 + c\*x^4)^p, d + e\*x^2, x], x, 0]}, -Simp[(R\*x\*(d + e\*x^2)^(q + 1))/(2\*d\*(q + 1)), x] + Dist[1/(2\*d\*(q + 1)), Int[(d + e\*x^2)^(q + 1)\*ExpandToSum[2\*d\*(q + 1)\*Qx + R\*(2\*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

**Rule 1814**

Int[(Pq)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 1]}, Simp[((a\*g - b\*f\*x)\*(a + b\*x^2)^(p + 1))/(2\*a\*b\*(p + 1)), x] + Dist[1/(2\*a\*(p + 1)), Int[(a + b\*x^2)^(p + 1)\*ExpandToSum[2\*a\*(p + 1)\*Q + f\*(2\*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^2 + cx^4)^2}{(d + ex^2)^5} dx &= \frac{(cd^2 - bde + ae^2)^2 x}{8de^4 (d + ex^2)^4} - \frac{\int \frac{c^2 d^4 - 2cd^2 e(bd - ae) + e^2(b^2 d^2 - 2abde - 7a^2 e^2) - 8d(c^2 d^2 + b^2 e^2 - 2ce(bd - ae))x^2 + 8cd(cd - 2be)x^4}{e^4} - \frac{8d(c^2 d^2 + b^2 e^2 - 2ce(bd - ae))x^2}{e^3} + \frac{8cd(cd - 2be)x^4}{e^2}}{(d + ex^2)^4} dx}{8d} \\
&= \frac{(cd^2 - bde + ae^2)^2 x}{8de^4 (d + ex^2)^4} - \frac{(25cd^2 - 9bde - 7ae^2)(cd^2 - bde + ae^2)x}{48d^2 e^4 (d + ex^2)^3} + \frac{\int \frac{19c^2 d^4 - 2cd^2 e(11bd - 3e^2)}{e^4} dx}{48d^2 e^4} \\
&= \frac{(cd^2 - bde + ae^2)^2 x}{8de^4 (d + ex^2)^4} - \frac{(25cd^2 - 9bde - 7ae^2)(cd^2 - bde + ae^2)x}{48d^2 e^4 (d + ex^2)^3} + \frac{(163c^2 d^4 - 2cd^2 e(11bd - 3e^2))x}{48d^2 e^4} \\
&= \frac{(cd^2 - bde + ae^2)^2 x}{8de^4 (d + ex^2)^4} - \frac{(25cd^2 - 9bde - 7ae^2)(cd^2 - bde + ae^2)x}{48d^2 e^4 (d + ex^2)^3} + \frac{(163c^2 d^4 - 2cd^2 e(11bd - 3e^2))x}{48d^2 e^4} \\
&= \frac{(cd^2 - bde + ae^2)^2 x}{8de^4 (d + ex^2)^4} - \frac{(25cd^2 - 9bde - 7ae^2)(cd^2 - bde + ae^2)x}{48d^2 e^4 (d + ex^2)^3} + \frac{(163c^2 d^4 - 2cd^2 e(11bd - 3e^2))x}{48d^2 e^4}
\end{aligned}$$

**Mathematica [A]** time = 0.23, size = 345, normalized size = 1.09

$$\frac{-3\sqrt{d}\sqrt{c}\left(-2\sqrt{35c^2d^2+10abde+3b^2d^2}-2a^2e(3ae+5bd)+93c^2d^4\right)+3\arctan\left(\frac{\sqrt{cx}}{\sqrt{d}}\right)\left(c^2(35a^2d^2+10abde+3b^2d^4)+2cd^2e(3ae+5bd)+35c^2d^4\right)-\frac{8d^2\sqrt{c}\left(-2\sqrt{7d^2-2abde+9e^2d^2}+2a^2e(9ae-17bd)+25c^2d^4\right)}{(d+ex^2)^3}+\frac{2d^2\sqrt{c}\left(-2\sqrt{35c^2d^2+10abde+3b^2d^2}-2a^2e(3ae+5bd)+93c^2d^4\right)}{(d+ex^2)^2}+\frac{48d^2\sqrt{c}\left(-2\sqrt{7d^2-2abde+9e^2d^2}+2a^2e(9ae-17bd)+25c^2d^4\right)}{(d+ex^2)^4}}{384d^2e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2 + c\*x^4)^2/(d + e\*x^2)^5, x]

[Out] ((48\*d^(7/2)\*Sqrt[e]\*(c\*d^2 + e\*(-(b\*d) + a\*e))^2\*x)/(d + e\*x^2)^4 - (8\*d^(5/2)\*Sqrt[e]\*(25\*c^2\*d^4 + 2\*c\*d^2\*e\*(-17\*b\*d + 9\*a\*e) + e^2\*(9\*b^2\*d^2 - 2\*a\*b\*d\*e - 7\*a^2\*e^2))\*x)/(d + e\*x^2)^3 + (2\*d^(3/2)\*Sqrt[e]\*(163\*c^2\*d^4 + 2\*c\*d^2\*e\*(-59\*b\*d + 3\*a\*e) + e^2\*(3\*b^2\*d^2 + 10\*a\*b\*d\*e + 35\*a^2\*e^2))\*x)/(d + e\*x^2)^2 - (3\*Sqrt[d]\*Sqrt[e]\*(93\*c^2\*d^4 - 2\*c\*d^2\*e\*(5\*b\*d + 3\*a\*e) - e^2\*(3\*b^2\*d^2 + 10\*a\*b\*d\*e + 35\*a^2\*e^2))\*x)/(d + e\*x^2) + 3\*(35\*c^2\*d^4 + 2\*c\*d^2\*e\*(5\*b\*d + 3\*a\*e) + e^2\*(3\*b^2\*d^2 + 10\*a\*b\*d\*e + 35\*a^2\*e^2))\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]]/(384\*d^(9/2)\*e^(9/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2 + cx^4)^2}{(d + ex^2)^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)^2/(d + e\*x^2)^5, x]

[Out] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)^2/(d + e\*x^2)^5, x]

**fricas [B]** time = 0.64, size = 1266, normalized size = 3.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)^2/(e\*x^2+d)^5,x, algorithm="fricas")

```
[Out] [-1/768*(6*(93*c^2*d^5*e^4 - 10*b*c*d^4*e^5 - 10*a*b*d^2*e^7 - 35*a^2*d*e^8 - 3*(b^2 + 2*a*c)*d^3*e^6)*x^7 + 2*(511*c^2*d^6*e^3 + 146*b*c*d^5*e^4 - 110*a*b*d^3*e^6 - 385*a^2*d^2*e^7 - 33*(b^2 + 2*a*c)*d^4*e^5)*x^5 + 2*(385*c^2*d^7*e^2 + 110*b*c*d^6*e^3 - 146*a*b*d^4*e^5 - 511*a^2*d^3*e^6 + 33*(b^2 + 2*a*c)*d^5*e^4)*x^3 + 3*(35*c^2*d^8 + 10*b*c*d^7*e + 10*a*b*d^5*e^3 + 35*a^2*d^4*e^4 + 3*(b^2 + 2*a*c)*d^6*e^2 + (35*c^2*d^4*e^4 + 10*b*c*d^3*e^5 + 10*a*b*d*e^7 + 35*a^2*e^8 + 3*(b^2 + 2*a*c)*d^2*e^6)*x^8 + 4*(35*c^2*d^5*e^3 + 10*b*c*d^4*e^4 + 10*a*b*d^2*e^6 + 35*a^2*d*e^7 + 3*(b^2 + 2*a*c)*d^3*e^5)*x^6 + 6*(35*c^2*d^6*e^2 + 10*b*c*d^5*e^3 + 10*a*b*d^3*e^5 + 35*a^2*d^2*e^6 + 3*(b^2 + 2*a*c)*d^4*e^4)*x^4 + 4*(35*c^2*d^7*e + 10*b*c*d^6*e^2 + 10*a*b*d^4*e^4 + 35*a^2*d^3*e^5 + 3*(b^2 + 2*a*c)*d^5*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 6*(35*c^2*d^8*e + 10*b*c*d^7*e^2 + 10*a*b*d^5*e^4 - 93*a^2*d^4*e^5 + 3*(b^2 + 2*a*c)*d^6*e^3)*x)/(d^5*e^9*x^8 + 4*d^6*e^8*x^6 + 6*d^7*e^7*x^4 + 4*d^8*e^6*x^2 + d^9*e^5), -1/384*(3*(93*c^2*d^5*e^4 - 10*b*c*d^4*e^5 - 10*a*b*d^2*e^7 - 35*a^2*d*e^8 - 3*(b^2 + 2*a*c)*d^3*e^6)*x^7 + (511*c^2*d^6*e^3 + 146*b*c*d^5*e^4 - 110*a*b*d^3*e^6 - 385*a^2*d^2*e^7 - 33*(b^2 + 2*a*c)*d^4*e^5)*x^5 + (385*c^2*d^7*e^2 + 110*b*c*d^6*e^3 - 146*a*b*d^4*e^5 - 511*a^2*d^3*e^6 + 33*(b^2 + 2*a*c)*d^5*e^4)*x^3 - 3*(35*c^2*d^8 + 10*b*c*d^7*e + 10*a*b*d^5*e^3 + 35*a^2*d^4*e^4 + 3*(b^2 + 2*a*c)*d^6*e^2 + (35*c^2*d^4*e^4 + 10*b*c*d^3*e^5 + 10*a*b*d*e^7 + 35*a^2*e^8 + 3*(b^2 + 2*a*c)*d^2*e^6)*x^8 + 4*(35*c^2*d^5*e^3 + 10*b*c*d^4*e^4 + 10*a*b*d^2*e^6 + 35*a^2*d*e^7 + 3*(b^2 + 2*a*c)*d^3*e^5)*x^6 + 6*(35*c^2*d^6*e^2 + 10*b*c*d^5*e^3 + 10*a*b*d^3*e^5 + 35*a^2*d^2*e^6 + 3*(b^2 + 2*a*c)*d^4*e^4)*x^4 + 4*(35*c^2*d^7*e + 10*b*c*d^6*e^2 + 10*a*b*d^4*e^4 + 35*a^2*d^3*e^5 + 3*(b^2 + 2*a*c)*d^5*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + 3*(35*c^2*d^8*e + 10*b*c*d^7*e^2 + 10*a*b*d^5*e^4 - 93*a^2*d^4*e^5 + 3*(b^2 + 2*a*c)*d^6*e^3)*x)/(d^5*e^9*x^8 + 4*d^6*e^8*x^6 + 6*d^7*e^7*x^4 + 4*d^8*e^6*x^2 + d^9*e^5)]
```

**giac** [A] time = 0.19, size = 364, normalized size = 1.15

$$\frac{(93c^2d^5e^4 + 30b^2c^2d^5e^4 + 6ad^5e^4 + 30abd^5e^4) \arctan\left(\frac{x}{d}\right) + \dots}{128d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)^2/(e\*x^2+d)^5,x, algorithm="giac")

```
[Out] 1/128*(35*c^2*d^4 + 10*b*c*d^3*e + 3*b^2*d^2*e^2 + 6*a*c*d^2*e^2 + 10*a*b*d*e^3 + 35*a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(9/2) - 1/384*(279*c^2*d^4*x^7*e^3 - 30*b*c*d^3*x^7*e^4 + 511*c^2*d^5*x^5*e^2 - 9*b^2*d^2*x^7*e^5 - 18*a*c*d^2*x^7*e^5 + 146*b*c*d^4*x^5*e^3 + 385*c^2*d^6*x^3*e - 30*a*b*d*x^7*e^6 - 33*b^2*d^3*x^5*e^4 - 66*a*c*d^3*x^5*e^4 + 110*b*c*d^5*x^3*e^2 + 105*c^2*d^7*x - 105*a^2*x^7*e^7 - 110*a*b*d^2*x^5*e^5 + 33*b^2*d^4*x^3*e^3 + 66*a*c*d^4*x^3*e^3 + 30*b*c*d^6*x*e - 385*a^2*d*x^5*e^6 - 146*a*b*d^3*x^3*e^4 + 9*b^2*d^5*x*e^2 + 18*a*c*d^5*x*e^2 - 511*a^2*d^2*x^3*e^5 + 30*a*b*d^4*x*e^3 - 279*a^2*d^3*x*e^4)*e^(-4)/((x^2*e + d)^4*d^4)
```

**maple** [A] time = 0.01, size = 412, normalized size = 1.30

$$\frac{35d^2 \arctan\left(\frac{x}{\sqrt{d}}\right) + \frac{5ab \arctan\left(\frac{x}{\sqrt{d}}\right)}{64\sqrt{d}e} + \frac{3ac \arctan\left(\frac{x}{\sqrt{d}}\right)}{64\sqrt{d}e^2} + \frac{3b^2 \arctan\left(\frac{x}{\sqrt{d}}\right)}{128\sqrt{d}e^3} + \frac{5bc \arctan\left(\frac{x}{\sqrt{d}}\right)}{64\sqrt{d}e^4} + \frac{35d^2 \arctan\left(\frac{x}{\sqrt{d}}\right)}{128\sqrt{d}e^5} + \dots}{(e^2 + d)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^4+b\*x^2+a)^2/(e\*x^2+d)^5,x)

```
[Out] (1/128*(35*a^2*e^4+10*a*b*d*e^3+6*a*c*d^2*e^2+3*b^2*d^2*e^2+10*b*c*d^3*e-93*c^2*d^4)/d^4/e*x^7+1/384*(385*a^2*e^4+110*a*b*d*e^3+66*a*c*d^2*e^2+33*b^2*d^2*e^2-146*b*c*d^3*e-511*c^2*d^4)/d^3/e^2*x^5+1/384*(511*a^2*e^4+146*a*b*d*e^3-66*a*c*d^2*e^2-33*b^2*d^2*e^2-110*b*c*d^3*e-385*c^2*d^4)/d^2/e^3*x^3+1/128*(93*a^2*e^4-10*a*b*d*e^3-6*a*c*d^2*e^2-3*b^2*d^2*e^2-10*b*c*d^3*e-35*c^2*d^4)/d/e^4*x)/(e*x^2+d)^4+35/128/(d*e)^(1/2)*a^2/d^4*arctan(1/(d*e)^(1/2)
```

) \* e \* x) + 5/64/d^3/e/(d\*e)^(1/2)\*arctan(1/(d\*e)^(1/2)\*e\*x)\*a\*b+3/64/(d\*e)^(1/2)  
 ) \* a \* c/d^2/e^2\*arctan(1/(d\*e)^(1/2)\*e\*x)+3/128/d^2/e^2/(d\*e)^(1/2)\*arctan(1/  
 (d\*e)^(1/2)\*e\*x)\*b^2+5/64/d/e^3/(d\*e)^(1/2)\*arctan(1/(d\*e)^(1/2)\*e\*x)\*b\*c+3  
 5/128/(d\*e)^(1/2)\*c^2/e^4\*arctan(1/(d\*e)^(1/2)\*e\*x)

**maxima** [A] time = 2.52, size = 366, normalized size = 1.15

$$\frac{3(93c^2d^2 - 10bcd^2 - 10abd^2 - 35a^2d^2 - 3(b^2 + 2ac)d^2) + (511c^2d^2 + 146bcd^2 - 110abd^2 - 385a^2d^2 - 33(b^2 + 2ac)d^2) + (885c^2d^2 + 110bcd^2 - 146abd^2 - 511a^2d^2 + 33(b^2 + 2ac)d^2)^2 + 3(35c^2d^2 + 10bcd^2 - 93a^2d^2 + 3(b^2 + 2ac)d^2)^2}{384(d^2d^2 + 4d^2e^2 + 6d^2e^2 + 4d^2e^2 + d^2e^2)} \arctan\left(\frac{35c^2d^2 + 10bcd^2 + 10abd^2 + 35a^2d^2 + 3(b^2 + 2ac)d^2}{128\sqrt{de}d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)^2/(e\*x^2+d)^5,x, algorithm="maxima")

[Out] -1/384\*(3\*(93\*c^2\*d^4\*e^3 - 10\*b\*c\*d^3\*e^4 - 10\*a\*b\*d\*e^6 - 35\*a^2\*e^7 - 3\*(  
 b^2 + 2\*a\*c)\*d^2\*e^5)\*x^7 + (511\*c^2\*d^5\*e^2 + 146\*b\*c\*d^4\*e^3 - 110\*a\*b\*d  
 ^2\*e^5 - 385\*a^2\*d\*e^6 - 33\*(b^2 + 2\*a\*c)\*d^3\*e^4)\*x^5 + (385\*c^2\*d^6\*e + 1  
 10\*b\*c\*d^5\*e^2 - 146\*a\*b\*d^3\*e^4 - 511\*a^2\*d^2\*e^5 + 33\*(b^2 + 2\*a\*c)\*d^4\*e  
 ^3)\*x^3 + 3\*(35\*c^2\*d^7 + 10\*b\*c\*d^6\*e + 10\*a\*b\*d^4\*e^3 - 93\*a^2\*d^3\*e^4 +  
 3\*(b^2 + 2\*a\*c)\*d^5\*e^2)\*x)/(d^4\*e^8\*x^8 + 4\*d^5\*e^7\*x^6 + 6\*d^6\*e^6\*x^4 +  
 4\*d^7\*e^5\*x^2 + d^8\*e^4) + 1/128\*(35\*c^2\*d^4 + 10\*b\*c\*d^3\*e + 10\*a\*b\*d\*e^3  
 + 35\*a^2\*e^4 + 3\*(b^2 + 2\*a\*c)\*d^2\*e^2)\*arctan(e\*x/sqrt(d\*e))/(sqrt(d\*e)\*d^4  
 e^4)

**mupad** [B] time = 4.57, size = 375, normalized size = 1.18

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c}}{\sqrt{d}}\right) (35a^2d^2 + 10abd^2 + 6acd^2 + 3b^2d^2 + 10bcd^2 + 35c^2d^2)}{128d^{9/2}e^{9/2}} - \frac{x^{(93d^4 + 10abd^2 + 6acd^2 + 3b^2d^2 + 10bcd^2 + 35c^2d^2)}}{128d^4e} - \frac{x^2(35c^2d^4 + 10abd^2 + 6acd^2 + 3b^2d^2 + 10bcd^2 + 35c^2d^2)}{128d^4e} - \frac{x^4(-511a^2d^4 + 146abd^2 + 6acd^2 + 33b^2d^2 + 110bcd^2 + 385c^2d^2)}{384d^2e^2} - \frac{x^6(385c^2d^4 - 511a^2d^4 + 33b^2d^2 + 110abd^2 + 6acd^2 + 33b^2d^2 + 110bcd^2 + 385c^2d^2)}{384d^2e^2}}{d^4 + 4d^3ex^2 + 6d^2e^2x^4 + 4de^3x^6 + e^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^2 + c\*x^4)^2/(d + e\*x^2)^5,x)

[Out] (atan((e^(1/2)\*x)/d^(1/2))\*(35\*a^2\*e^4 + 35\*c^2\*d^4 + 3\*b^2\*d^2\*e^2 + 10\*a\*  
 b\*d\*e^3 + 10\*b\*c\*d^3\*e + 6\*a\*c\*d^2\*e^2))/(128\*d^(9/2)\*e^(9/2)) - ((x\*(35\*c^2  
 \*d^4 - 93\*a^2\*e^4 + 3\*b^2\*d^2\*e^2 + 10\*a\*b\*d\*e^3 + 10\*b\*c\*d^3\*e + 6\*a\*c\*d^2  
 \*e^2))/(128\*d\*e^4) - (x^7\*(35\*a^2\*e^4 - 93\*c^2\*d^4 + 3\*b^2\*d^2\*e^2 + 10\*a\*  
 b\*d\*e^3 + 10\*b\*c\*d^3\*e + 6\*a\*c\*d^2\*e^2))/(128\*d^4\*e) + (x^3\*(385\*c^2\*d^4 -  
 511\*a^2\*e^4 + 33\*b^2\*d^2\*e^2 - 146\*a\*b\*d\*e^3 + 110\*b\*c\*d^3\*e + 66\*a\*c\*d^2\*e  
 ^2))/(384\*d^2\*e^3) - (x^5\*(385\*a^2\*e^4 - 511\*c^2\*d^4 + 33\*b^2\*d^2\*e^2 + 110  
 \*a\*b\*d\*e^3 - 146\*b\*c\*d^3\*e + 66\*a\*c\*d^2\*e^2))/(384\*d^3\*e^2))/(d^4 + e^4\*x^8  
 + 4\*d^3\*e\*x^2 + 4\*d\*e^3\*x^6 + 6\*d^2\*e^2\*x^4)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*4+b\*x\*\*2+a)\*\*2/(e\*x\*\*2+d)\*\*5,x)

[Out] Timed out

$$3.184 \quad \int \frac{a+bx^2+cx^4}{(d+ex^2)^2} dx$$

Optimal. Leaf size=83

$$-\frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(3cd^2 - e(ae + bd))}{2d^{3/2}e^{5/2}} + \frac{x\left(a + \frac{d(cd-be)}{e^2}\right)}{2d(d+ex^2)} + \frac{cx}{e^2}$$

Rubi [A] time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {1157, 388, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(3cd^2 - e(ae + bd))}{2d^{3/2}e^{5/2}} + \frac{x\left(a + \frac{d(cd-be)}{e^2}\right)}{2d(d+ex^2)} + \frac{cx}{e^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^2,x]

[Out] (c\*x)/e^2 + ((a + (d\*(c\*d - b\*e))/e^2)\*x)/(2\*d\*(d + e\*x^2)) - ((3\*c\*d^2 - e\*(b\*d + a\*e))\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(2\*d^(3/2)\*e^(5/2))

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p+1))/(b\*(n\*(p+1)+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1)+1))/(b\*(n\*(p+1)+1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p+1)+1, 0]

Rule 1157

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] := With[{Qx = PolynomialQuotient[(a + b\*x^2 + c\*x^4)^p, d + e\*x^2, x], R = Coeff[PolynomialRemainder[(a + b\*x^2 + c\*x^4)^p, d + e\*x^2, x], 0]}, -Simp[(R\*x\*(d + e\*x^2)^(q+1))/(2\*d\*(q+1)), x] + Dist[1/(2\*d\*(q+1)), Int[(d + e\*x^2)^(q+1)\*ExpandToSum[2\*d\*(q+1)\*Qx + R\*(2\*q+3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rubi steps

$$\begin{aligned} \int \frac{a+bx^2+cx^4}{(d+ex^2)^2} dx &= \frac{(cd^2 - bde + ae^2)x}{2de^2(d+ex^2)} - \int \frac{\frac{cd^2 - e(bd+ae) - 2cdx^2}{e^2}}{d+ex^2} dx \\ &= \frac{cx}{e^2} + \frac{(cd^2 - bde + ae^2)x}{2de^2(d+ex^2)} - \frac{(3cd^2 - e(bd+ae)) \int \frac{1}{d+ex^2} dx}{2de^2} \\ &= \frac{cx}{e^2} + \frac{(cd^2 - bde + ae^2)x}{2de^2(d+ex^2)} - \frac{(3cd^2 - e(bd+ae)) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}} \end{aligned}$$



**Mathematica [A]** time = 0.02, size = 88, normalized size = 1.06

$$\frac{x(ae^2 - bde + cd^2)}{2de^2(d + ex^2)} - \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(-ae^2 - bde + 3cd^2)}{2d^{3/2}e^{5/2}} + \frac{cx}{e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^2, x]

[Out] (c\*x)/e^2 + ((c\*d^2 - b\*d\*e + a\*e^2)\*x)/(2\*d\*e^2\*(d + e\*x^2)) - ((3\*c\*d^2 - b\*d\*e - a\*e^2)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(2\*d^(3/2)\*e^(5/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2 + cx^4}{(d + ex^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^2, x]

[Out] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^2, x]

**fricas [A]** time = 0.69, size = 268, normalized size = 3.23

$$\left[ \frac{4cd^2e^2x^3 + (3cd^3 - bd^2e - ade^2 + (3cd^2e - bde^2 - ae^3)x^2)\sqrt{-de} \log\left(\frac{x^2 - 2\sqrt{-de}x - d}{e^2 + d}\right) + 2(3cd^3e - bd^2e^2 + ade^3)x}{4(d^2e^4x^2 + d^3e^3)}, \frac{2cd^2e^2x^3 - (3cd^3 - bd^2e - ade^2 + (3cd^2e - bde^2 - ae^3)x^2)\sqrt{de} \arctan\left(\frac{\sqrt{de}x}{d}\right) + (3cd^3e - bd^2e^2 + ade^3)x}{2(d^2e^4x^2 + d^3e^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^2,x, algorithm="fricas")

[Out] [1/4\*(4\*c\*d^2\*e^2\*x^3 + (3\*c\*d^3 - b\*d^2\*e - a\*d\*e^2 + (3\*c\*d^2\*e - b\*d\*e^2 - a\*e^3)\*x^2)\*sqrt(-d\*e)\*log((e\*x^2 - 2\*sqrt(-d\*e)\*x - d)/(e\*x^2 + d)) + 2\*(3\*c\*d^3\*e - b\*d^2\*e^2 + a\*d\*e^3)\*x)/(d^2\*e^4\*x^2 + d^3\*e^3), 1/2\*(2\*c\*d^2\*e^2\*x^3 - (3\*c\*d^3 - b\*d^2\*e - a\*d\*e^2 + (3\*c\*d^2\*e - b\*d\*e^2 - a\*e^3)\*x^2)\*sqrt(d\*e)\*arctan(sqrt(d\*e)\*x/d) + (3\*c\*d^3\*e - b\*d^2\*e^2 + a\*d\*e^3)\*x)/(d^2\*e^4\*x^2 + d^3\*e^3)]

**giac [A]** time = 0.16, size = 75, normalized size = 0.90

$$cxe^{(-2)} - \frac{(3cd^2 - bde - ae^2) \arctan\left(\frac{xe^{1/2}}{\sqrt{d}}\right) e^{(-5/2)}}{2d^{3/2}} + \frac{(cd^2x - bdx + axe^2)e^{(-2)}}{2(x^2e + d)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^2,x, algorithm="giac")

[Out] c\*x\*e^(-2) - 1/2\*(3\*c\*d^2 - b\*d\*e - a\*e^2)\*arctan(x\*e^(1/2)/sqrt(d))\*e^(-5/2)/d^(3/2) + 1/2\*(c\*d^2\*x - b\*d\*x\*e + a\*x\*e^2)\*e^(-2)/((x^2\*e + d)\*d)

**maple [A]** time = 0.00, size = 118, normalized size = 1.42

$$\frac{ax}{2(e x^2 + d)d} + \frac{a \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}d} - \frac{bx}{2(e x^2 + d)e} + \frac{b \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}e} + \frac{cdx}{2(e x^2 + d)e^2} - \frac{3cd \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}e^2} + \frac{cx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^4+b\*x^2+a)/(e\*x^2+d)^2,x)

[Out]  $1/2/(e*x^2+d)*a/d*x+1/2/(d*e)^{(1/2)}*a/d*\arctan(1/(d*e)^{(1/2)}*e*x)-1/2/(e*x^2+d)*b/e*x+1/2/(d*e)^{(1/2)}*b/e*\arctan(1/(d*e)^{(1/2)}*e*x)+1/2/(e*x^2+d)*c*d/e^2*x-3/2/(d*e)^{(1/2)}*c*d/e^2*\arctan(1/(d*e)^{(1/2)}*e*x)+c/e^2*x$

**maxima** [A] time = 2.34, size = 84, normalized size = 1.01

$$\frac{(cd^2 - bde + ae^2)x}{2(de^3x^2 + d^2e^2)} + \frac{cx}{e^2} - \frac{(3cd^2 - bde - ae^2) \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^2,x, algorithm="maxima")

[Out]  $1/2*(c*d^2 - b*d*e + a*e^2)*x/(d*e^3*x^2 + d^2*e^2) + c*x/e^2 - 1/2*(3*c*d^2 - b*d*e - a*e^2)*\arctan(e*x/\sqrt{d*e})/(\sqrt{d*e}*d*e^2)$

**mupad** [B] time = 0.00, size = 77, normalized size = 0.93

$$\frac{cx}{e^2} + \frac{\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) (-3cd^2 + bde + ae^2)}{2d^{3/2}e^{5/2}} + \frac{x(cd^2 - bde + ae^2)}{2d(e^3x^2 + de^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^2 + c\*x^4)/(d + e\*x^2)^2,x)

[Out]  $(c*x)/e^2 + (\operatorname{atan}((e^{(1/2)}*x)/d^{(1/2)})*(a*e^2 - 3*c*d^2 + b*d*e))/(2*d^{(3/2)}*e^{(5/2)}) + (x*(a*e^2 + c*d^2 - b*d*e))/(2*d*(d*e^2 + e^3*x^2))$

**sympy** [B] time = 0.82, size = 153, normalized size = 1.84

$$\frac{cx}{e^2} + \frac{x(ae^2 - bde + cd^2)}{2d^2e^2 + 2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2 + bde - 3cd^2) \log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2 + bde - 3cd^2) \log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*4+b\*x\*\*2+a)/(e\*x\*\*2+d)\*\*2,x)

[Out]  $c*x/e^2 + x*(a*e^2 - b*d*e + c*d^2)/(2*d^2*e^2 + 2*d*e^3*x^2) - \operatorname{sqrt}(-1/(d^3*e^5))*(a*e^2 + b*d*e - 3*c*d^2)*\log(-d^2*e^2*\operatorname{sqrt}(-1/(d^3*e^5)) + x)/4 + \operatorname{sqrt}(-1/(d^3*e^5))*(a*e^2 + b*d*e - 3*c*d^2)*\log(d^2*e^2*\operatorname{sqrt}(-1/(d^3*e^5)) + x)/4$

$$3.185 \quad \int \frac{a+x^2(b+cx^2)}{(d+ex^2)^2} dx$$

Optimal. Leaf size=83

$$-\frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(3cd^2 - e(ae + bd))}{2d^{3/2}e^{5/2}} + \frac{x\left(a + \frac{d(cd-be)}{e^2}\right)}{2d(d+ex^2)} + \frac{cx}{e^2}$$

**Rubi [A]** time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$ , Rules used = {1814, 388, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(3cd^2 - e(ae + bd))}{2d^{3/2}e^{5/2}} + \frac{x\left(a + \frac{d(cd-be)}{e^2}\right)}{2d(d+ex^2)} + \frac{cx}{e^2}$$

Antiderivative was successfully verified.

[In] Int[(a + x^2\*(b + c\*x^2))/(d + e\*x^2)^2,x]

[Out] (c\*x)/e^2 + ((a + (d\*(c\*d - b\*e))/e^2)\*x)/(2\*d\*(d + e\*x^2)) - ((3\*c\*d^2 - e\*(b\*d + a\*e))\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(2\*d^(3/2)\*e^(5/2))

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p+1))/(b\*(n\*(p+1)+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1)+1))/(b\*(n\*(p+1)+1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p+1)+1, 0]

Rule 1814

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 1]}, Simp[((a\*g - b\*f\*x)\*(a + b\*x^2)^(p+1))/(2\*a\*b\*(p+1)), x] + Dist[1/(2\*a\*(p+1)), Int[(a + b\*x^2)^(p+1)\*ExpandToSum[2\*a\*(p+1)\*Q + f\*(2\*p+3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{a+x^2(b+cx^2)}{(d+ex^2)^2} dx &= \frac{\left(a + \frac{d(cd-be)}{e^2}\right)x}{2d(d+ex^2)} - \int \frac{\frac{cd^2-e(bd+ae)}{e^2} - \frac{2cdx^2}{e}}{d+ex^2} dx \\ &= \frac{cx}{e^2} + \frac{\left(a + \frac{d(cd-be)}{e^2}\right)x}{2d(d+ex^2)} - \frac{(3cd^2 - e(bd + ae)) \int \frac{1}{d+ex^2} dx}{2de^2} \\ &= \frac{cx}{e^2} + \frac{\left(a + \frac{d(cd-be)}{e^2}\right)x}{2d(d+ex^2)} - \frac{(3cd^2 - e(bd + ae)) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}} \end{aligned}$$

**Mathematica** [A] time = 0.02, size = 88, normalized size = 1.06

$$\frac{x(ae^2 - bde + cd^2)}{2de^2(d + ex^2)} - \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(-ae^2 - bde + 3cd^2)}{2d^{3/2}e^{5/2}} + \frac{cx}{e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + x^2\*(b + c\*x^2))/(d + e\*x^2)^2, x]

[Out] (c\*x)/e^2 + ((c\*d^2 - b\*d\*e + a\*e^2)\*x)/(2\*d\*e^2\*(d + e\*x^2)) - ((3\*c\*d^2 - b\*d\*e - a\*e^2)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(2\*d^(3/2)\*e^(5/2))

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + x^2(b + cx^2)}{(d + ex^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + x^2\*(b + c\*x^2))/(d + e\*x^2)^2, x]

[Out] IntegrateAlgebraic[(a + x^2\*(b + c\*x^2))/(d + e\*x^2)^2, x]

**fricas** [A] time = 0.62, size = 268, normalized size = 3.23

$$\frac{4cd^2e^2x^3 + (3cd^3 - bd^2e - ade^2 + (3cd^2e - bde^2 - ae^3)x^2)\sqrt{-de} \log\left(\frac{x^2 - 2\sqrt{-de}x - d}{e^2x + d}\right) + 2(3cd^3e - bd^2e^2 + ade^3)x}{4(d^2e^4x^2 + d^3e^3)}, \frac{2cd^2e^2x^3 - (3cd^3 - bd^2e - ade^2 + (3cd^2e - bde^2 - ae^3)x^2)\sqrt{de} \arctan\left(\frac{\sqrt{de}x}{d}\right) + (3cd^3e - bd^2e^2 + ade^3)x}{2(d^2e^4x^2 + d^3e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+x^2\*(c\*x^2+b))/(e\*x^2+d)^2,x, algorithm="fricas")

[Out] [1/4\*(4\*c\*d^2\*e^2\*x^3 + (3\*c\*d^3 - b\*d^2\*e - a\*d\*e^2 + (3\*c\*d^2\*e - b\*d\*e^2 - a\*e^3)\*x^2)\*sqrt(-d\*e)\*log((e\*x^2 - 2\*sqrt(-d\*e)\*x - d)/(e\*x^2 + d)) + 2\*(3\*c\*d^3\*e - b\*d^2\*e^2 + a\*d\*e^3)\*x)/(d^2\*e^4\*x^2 + d^3\*e^3), 1/2\*(2\*c\*d^2\*e^2\*x^3 - (3\*c\*d^3 - b\*d^2\*e - a\*d\*e^2 + (3\*c\*d^2\*e - b\*d\*e^2 - a\*e^3)\*x^2)\*sqrt(d\*e)\*arctan(sqrt(d\*e)\*x/d) + (3\*c\*d^3\*e - b\*d^2\*e^2 + a\*d\*e^3)\*x)/(d^2\*e^4\*x^2 + d^3\*e^3)]

**giac** [A] time = 0.15, size = 75, normalized size = 0.90

$$cxe^{(-2)} - \frac{(3cd^2 - bde - ae^2) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right) e^{(-\frac{5}{2})}}{2d^{\frac{3}{2}}} + \frac{(cd^2x - bdx + axe^2)e^{(-2)}}{2(x^2e + d)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+x^2\*(c\*x^2+b))/(e\*x^2+d)^2,x, algorithm="giac")

[Out] c\*x\*e^(-2) - 1/2\*(3\*c\*d^2 - b\*d\*e - a\*e^2)\*arctan(x\*e^(1/2)/sqrt(d))\*e^(-5/2)/d^(3/2) + 1/2\*(c\*d^2\*x - b\*d\*x\*e + a\*x\*e^2)\*e^(-2)/((x^2\*e + d)\*d)

**maple** [A] time = 0.01, size = 118, normalized size = 1.42

$$\frac{ax}{2(e x^2 + d)d} + \frac{a \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}d} - \frac{bx}{2(e x^2 + d)e} + \frac{b \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}e} + \frac{cdx}{2(e x^2 + d)e^2} - \frac{3cd \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}e^2} + \frac{cx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+x^2\*(c\*x^2+b))/(e\*x^2+d)^2,x)

[Out]  $1/2/(e*x^2+d)*a/d*x+1/2/(d*e)^{(1/2)}*a/d*\arctan(1/(d*e)^{(1/2)}*e*x)-1/2/(e*x^2+d)*b/e*x+1/2/(d*e)^{(1/2)}*b/e*\arctan(1/(d*e)^{(1/2)}*e*x)+1/2/(e*x^2+d)*c*d/e^2*x-3/2/(d*e)^{(1/2)}*c*d/e^2*\arctan(1/(d*e)^{(1/2)}*e*x)+c/e^2*x$

**maxima [A]** time = 2.37, size = 84, normalized size = 1.01

$$\frac{(cd^2 - bde + ae^2)x}{2(de^3x^2 + d^2e^2)} + \frac{cx}{e^2} - \frac{(3cd^2 - bde - ae^2) \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+x^2\*(c\*x^2+b))/(e\*x^2+d)^2,x, algorithm="maxima")

[Out]  $1/2*(c*d^2 - b*d*e + a*e^2)*x/(d*e^3*x^2 + d^2*e^2) + c*x/e^2 - 1/2*(3*c*d^2 - b*d*e - a*e^2)*\arctan(e*x/\sqrt{d*e})/(\sqrt{d*e}*d*e^2)$

**mupad [B]** time = 0.11, size = 77, normalized size = 0.93

$$\frac{cx}{e^2} + \frac{\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(-3cd^2 + bde + ae^2)}{2d^{3/2}e^{5/2}} + \frac{x(cd^2 - bde + ae^2)}{2d(e^3x^2 + d^2e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + x^2\*(b + c\*x^2))/(d + e\*x^2)^2,x)

[Out]  $(c*x)/e^2 + (\operatorname{atan}((e^{(1/2)}*x)/d^{(1/2)})*(a*e^2 - 3*c*d^2 + b*d*e))/(2*d^{(3/2)}*e^{(5/2)}) + (x*(a*e^2 + c*d^2 - b*d*e))/(2*d*(d*e^2 + e^3*x^2))$

**sympy [B]** time = 0.86, size = 153, normalized size = 1.84

$$\frac{cx}{e^2} + \frac{x(ae^2 - bde + cd^2)}{2d^2e^2 + 2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2 + bde - 3cd^2) \log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2 + bde - 3cd^2) \log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+x\*\*2\*(c\*x\*\*2+b))/(e\*x\*\*2+d)\*\*2,x)

[Out]  $c*x/e^2 + x*(a*e^2 - b*d*e + c*d^2)/(2*d^2*e^2 + 2*d*e^3*x^2) - \sqrt{-1/(d^3*e^5)}*(a*e^2 + b*d*e - 3*c*d^2)*\log(-d^2*e^2*\sqrt{-1/(d^3*e^5)} + x)/4 + \sqrt{-1/(d^3*e^5)}*(a*e^2 + b*d*e - 3*c*d^2)*\log(d^2*e^2*\sqrt{-1/(d^3*e^5)} + x)/4$

$$3.186 \quad \int \frac{(d+ex^2)^4}{a+bx^2+cx^4} dx$$

**Optimal.** Leaf size=459

$$\frac{\left( \frac{2c^2e^2(a^2e^2+6abde+3b^2d^2)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4}{\sqrt{b^2-4ac}} + e(2cd-be)(-2ce(ae+bd)+b^2e^2+2c^2d^2) \right) \tan^{-1} \left( \frac{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

**Rubi [A]** time = 1.54, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1170, 1166, 205}

$$\frac{\left( \frac{2c^2e^2(a^2e^2+6abde+3b^2d^2)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4}{\sqrt{b^2-4ac}} + e(2cd-be)(-2ce(ae+bd)+b^2e^2+2c^2d^2) \right) \tan^{-1} \left( \frac{\sqrt{2}c^{7/2}}{\sqrt{b-\sqrt{b^2-4ac}}} \right) + \left( \frac{e(2cd-be)(-2ce(ae+bd)+b^2e^2+2c^2d^2)}{\sqrt{b-\sqrt{b^2-4ac}}} - \frac{2c^2e^2(a^2e^2+6abde+3b^2d^2)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left( \frac{\sqrt{2}c^{7/2}}{\sqrt{b+\sqrt{b^2-4ac}}} \right) + \frac{e^2x(-ce(ae+4bd)+b^2e^2+6c^2d^2)}{c^3} + \frac{e^3x(4cd-be)}{3c^2} + \frac{e^4x^5}{5c}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^4/(a + b\*x^2 + c\*x^4), x]

[Out] (e^2\*(6\*c^2\*d^2 + b^2\*e^2 - c\*e\*(4\*b\*d + a\*e))\*x)/c^3 + (e^3\*(4\*c\*d - b\*e)\*x^3)/(3\*c^2) + (e^4\*x^5)/(5\*c) + ((e\*(2\*c\*d - b\*e)\*(2\*c^2\*d^2 + b^2\*e^2 - 2\*c\*e\*(b\*d + a\*e)) + (2\*c^4\*d^4 + b^4\*e^4 - 4\*b^2\*c\*e^3\*(b\*d + a\*e) - 4\*c^3\*d^2\*e\*(b\*d + 3\*a\*e) + 2\*c^2\*e^2\*(3\*b^2\*d^2 + 6\*a\*b\*d\*e + a^2\*e^2)))/Sqrt[b^2 - 4\*a\*c]\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b - Sqrt[b^2 - 4\*a\*c]]]/(Sqrt[2]\*c^(7/2)\*Sqrt[b - Sqrt[b^2 - 4\*a\*c]]) + ((e\*(2\*c\*d - b\*e)\*(2\*c^2\*d^2 + b^2\*e^2 - 2\*c\*e\*(b\*d + a\*e)) - (2\*c^4\*d^4 + b^4\*e^4 - 4\*b^2\*c\*e^3\*(b\*d + a\*e) - 4\*c^3\*d^2\*e\*(b\*d + 3\*a\*e) + 2\*c^2\*e^2\*(3\*b^2\*d^2 + 6\*a\*b\*d\*e + a^2\*e^2)))/Sqrt[b^2 - 4\*a\*c]\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b + Sqrt[b^2 - 4\*a\*c]]]/(Sqrt[2]\*c^(7/2)\*Sqrt[b + Sqrt[b^2 - 4\*a\*c]])

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 1166

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[e/2 + (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 - q/2 + c\*x^2), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - a\*e^2, 0] && PosQ[b^2 - 4\*a\*c]

#### Rule 1170

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x^2)^q/(a + b\*x^2 + c\*x^4), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IntegerQ[q]

#### Rubi steps

$$\begin{aligned}
\int \frac{(d + ex^2)^4}{a + bx^2 + cx^4} dx &= \int \left( \frac{e^2(6c^2d^2 + b^2e^2 - ce(4bd + ae))}{c^3} + \frac{e^3(4cd - be)x^2}{c^2} + \frac{e^4x^4}{c} + \frac{c^3d^4 - 6ac^2d^2e^2 - ab^2e^4}{c^3d^4 - 6ac^2d^2e^2 - ab^2e^4 + ace^3(4bd + ae)} \right) dx \\
&= \frac{e^2(6c^2d^2 + b^2e^2 - ce(4bd + ae))x}{c^3} + \frac{e^3(4cd - be)x^3}{3c^2} + \frac{e^4x^5}{5c} + \int \frac{c^3d^4 - 6ac^2d^2e^2 - ab^2e^4 + ace^3(4bd + ae)}{c^3d^4 - 6ac^2d^2e^2 - ab^2e^4 + ace^3(4bd + ae)} dx \\
&= \frac{e^2(6c^2d^2 + b^2e^2 - ce(4bd + ae))x}{c^3} + \frac{e^3(4cd - be)x^3}{3c^2} + \frac{e^4x^5}{5c} + \frac{\left( e(2cd - be)(2c^2d^2 + b^2e^2) \right)}{c^3d^4 - 6ac^2d^2e^2 - ab^2e^4 + ace^3(4bd + ae)} \\
&= \frac{e^2(6c^2d^2 + b^2e^2 - ce(4bd + ae))x}{c^3} + \frac{e^3(4cd - be)x^3}{3c^2} + \frac{e^4x^5}{5c} + \frac{\left( e(2cd - be)(2c^2d^2 + b^2e^2) \right)}{c^3d^4 - 6ac^2d^2e^2 - ab^2e^4 + ace^3(4bd + ae)}
\end{aligned}$$

**Mathematica [A]** time = 0.69, size = 570, normalized size = 1.24

$$\frac{e^2(6c^2d^2 + b^2e^2 - ce(4bd + ae))x}{c^3} + \frac{e^3(4cd - be)x^3}{3c^2} + \frac{e^4x^5}{5c} + \frac{\left( e(2cd - be)(2c^2d^2 + b^2e^2) \right)}{c^3d^4 - 6ac^2d^2e^2 - ab^2e^4 + ace^3(4bd + ae)}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^4/(a + b\*x^2 + c\*x^4),x]

[Out] (e^2\*(6\*c^2\*d^2 + b^2\*e^2 - c\*e\*(4\*b\*d + a\*e))\*x)/c^3 + (e^3\*(4\*c\*d - b\*e)\*x^3)/(3\*c^2) + (e^4\*x^5)/(5\*c) + ((2\*c^4\*d^4 + b^3\*(b - Sqrt[b^2 - 4\*a\*c]))\*e^4 + 4\*c^3\*d^2\*e\*(-(b\*d) + Sqrt[b^2 - 4\*a\*c]\*d - 3\*a\*e) + 2\*b\*c\*e^3\*(-2\*b^2\*d + 2\*b\*Sqrt[b^2 - 4\*a\*c]\*d - 2\*a\*b\*e + a\*Sqrt[b^2 - 4\*a\*c]\*e) + 2\*c^2\*e^2\*(3\*b^2\*d^2 - 3\*b\*d\*(Sqrt[b^2 - 4\*a\*c]\*d - 2\*a\*e) + a\*e\*(-2\*Sqrt[b^2 - 4\*a\*c]\*d + a\*e)))\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b - Sqrt[b^2 - 4\*a\*c]]]/(Sqrt[2]\*c^(7/2)\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[b - Sqrt[b^2 - 4\*a\*c]]) - ((2\*c^4\*d^4 + b^3\*(b + Sqrt[b^2 - 4\*a\*c]))\*e^4 - 4\*c^3\*d^2\*e\*(b\*d + Sqrt[b^2 - 4\*a\*c]\*d + 3\*a\*e) - 2\*b\*c\*e^3\*(2\*b^2\*d + a\*Sqrt[b^2 - 4\*a\*c]\*e + 2\*b\*(Sqrt[b^2 - 4\*a\*c]\*d + a\*e)) + 2\*c^2\*e^2\*(3\*b^2\*d^2 + a\*e\*(2\*Sqrt[b^2 - 4\*a\*c]\*d + a\*e) + 3\*b\*d\*(Sqrt[b^2 - 4\*a\*c]\*d + 2\*a\*e)))\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b + Sqrt[b^2 - 4\*a\*c]]]/(Sqrt[2]\*c^(7/2)\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[b + Sqrt[b^2 - 4\*a\*c]])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^2)^4}{a + bx^2 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)^4/(a + b\*x^2 + c\*x^4),x]

[Out] IntegrateAlgebraic[(d + e\*x^2)^4/(a + b\*x^2 + c\*x^4), x]

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^4/(c\*x^4+b\*x^2+a),x, algorithm="fricas")

[Out] Timed out

**giac [B]** time = 1.63, size = 9285, normalized size = 20.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*(4*(2*b^4*c^5 - 16*a*b^2*c^6 + 32*a^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c))*s
sqrt(b*c - sqrt(b^2 - 4*a*c))*b^4*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c))*a*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c))*b^3*c^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c))*a^2*c^5 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c))*a*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c))*b^2*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*
a*c^6 - 2*(b^2 - 4*a*c)*b^2*c^5 + 8*(b^2 - 4*a*c)*a*c^6)*c^2*d^3*e + 2*(s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^4*c^5 - 8*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c))*a*b^2*c^6 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^3*c
^6 + 2*b^4*c^6 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*c^7 + 8*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b*c^7 + sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c))*b^2*c^7 - 16*a*b^2*c^7 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
))*a*c^8 + 32*a^2*c^8 - 2*(b^2 - 4*a*c)*b^2*c^6 + 8*(b^2 - 4*a*c)*a*c^7)*d
^4*abs(c) - 6*(2*b^5*c^4 - 16*a*b^3*c^5 + 32*a^2*b*c^6 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^5*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c))*b^4*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c))*a^2*b*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c))*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c))*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c))*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 8*(b^2 - 4*a*c)*a*b*c^5)
*c^2*d^2*e^2 + 2*(2*b^3*c^8 - 8*a*b*c^9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c))*b^3*c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c))*a*b*c^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c))*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c))*b*c^8 - 2*(b^2 - 4*a*c)*b*c^8)*d^4 + 4*(2*b^6*c^3 - 18*a*b^4*c^4 +
48*a^2*b^2*c^5 - 32*a^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c))*b^6*c + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c))*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*
b^5*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a
^2*b^2*c^3 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a
*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^4*c^
3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^3*c^4 +
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b*c^4 + 5*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^2*c^4 - 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*c^5 - 2*(b^2 - 4*
a*c)*b^4*c^3 + 10*(b^2 - 4*a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*c^2*d*
e^3 - 12*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^4*c^4 - 8*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c))*a^2*b^2*c^5 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c))*a*b^3*c^5 + 2*a*b^4*c^5 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
))*a^3*c^6 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b*c^6 + sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^2*c^6 - 16*a^2*b^2*c^6 - 4*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c))*a^2*c^7 + 32*a^3*c^7 - 2*(b^2 - 4*a*c)*a*b^2*c
^5 + 8*(b^2 - 4*a*c)*a^2*c^6)*d^2*abs(c)*e^2 - 4*(2*b^4*c^7 - 8*a*b^2*c^8 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^4*c^5 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^2*c^6 + 2*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^3*c^6 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^2*c^7 - 2*(b^2 - 4*a*c)*b^2*c
^7)*d^3*e - (2*b^7*c^2 - 20*a*b^5*c^3 + 64*a^2*b^3*c^4 - 64*a^3*b*c^5 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^7 + 10*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^5*c + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^6*c - 32*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a
```





$$\begin{aligned}
& *a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a*c^6 - 2*(b^2 - 4*a*c)*b^2*c^5 + 8*( \\
& b^2 - 4*a*c)*a*c^6)*c^2*d^3*e - 2*(\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)* \\
& b^4*c^5 - 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a*b^2*c^6 - 2*\text{sqrt}(2)*\text{sq} \\
& \text{qrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*b^3*c^6 - 2*b^4*c^6 + 16*\text{sqrt}(2)*\text{sqrt}(b*c + \\
& \text{sqrt}(b^2 - 4*a*c)*c)*a^2*c^7 + 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a* \\
& b*c^7 + \text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*b^2*c^7 + 16*a*b^2*c^7 - 4* \\
& \text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a*c^8 - 32*a^2*c^8 + 2*(b^2 - 4*a*c \\
& )*b^2*c^6 - 8*(b^2 - 4*a*c)*a*c^7)*d^4*\text{abs}(c) - 6*(2*b^5*c^4 - 16*a*b^3*c^5 \\
& + 32*a^2*b*c^6 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c) \\
& *b^5*c^2 + 8*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a*b^ \\
& 3*c^3 + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*b^4*c^3 \\
& - 16*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a^2*b*c^4 - \\
& 8*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a*b^2*c^4 - \text{sq} \\
& \text{rt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*b^3*c^4 + 4*\text{sqrt}(2) \\
& *\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c \\
& )*b^3*c^4 + 8*(b^2 - 4*a*c)*a*b*c^5)*c^2*d^2*e^2 + 2*(2*b^3*c^8 - 8*a*b*c^9 \\
& - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*b^3*c^6 + 4*\text{sq} \\
& \text{rt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a*b*c^7 + 2*\text{sqrt}(2) \\
& *\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*b^2*c^7 - \text{sqrt}(2)*\text{sqrt}(b \\
& ^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*b*c^8 - 2*(b^2 - 4*a*c)*b*c^8)* \\
& d^4 + 4*(2*b^6*c^3 - 18*a*b^4*c^4 + 48*a^2*b^2*c^5 - 32*a^3*c^6 - \text{sqrt}(2)*\text{s} \\
& \text{qrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*b^6*c + 9*\text{sqrt}(2)*\text{sqrt}(b^2 \\
& - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - \\
& 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*b^5*c^2 - 24*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a* \\
& c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 10*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a* \\
& c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a*b^3*c^3 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sq} \\
& \text{rt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*b^4*c^3 + 16*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b \\
& *c + \text{sqrt}(b^2 - 4*a*c)*c)*a^3*c^4 + 8*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \\
& \text{sqrt}(b^2 - 4*a*c)*c)*a^2*b*c^4 + 5*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sq} \\
& \text{rt}(b^2 - 4*a*c)*c)*a*b^2*c^4 - 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b \\
& ^2 - 4*a*c)*c)*a^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^3 + 10*(b^2 - 4*a*c)*a*b^2*c \\
& ^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*c^2*d*e^3 + 12*(\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - \\
& 4*a*c)*c)*a*b^4*c^4 - 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a^2*b^2*c^ \\
& 5 - 2*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a*b^3*c^5 - 2*a*b^4*c^5 + 16* \\
& \text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a^3*c^6 + 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt} \\
& (b^2 - 4*a*c)*c)*a^2*b*c^6 + \text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a*b^2* \\
& c^6 + 16*a^2*b^2*c^6 - 4*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a^2*c^7 - \\
& 32*a^3*c^7 + 2*(b^2 - 4*a*c)*a*b^2*c^5 - 8*(b^2 - 4*a*c)*a^2*c^6)*d^2*\text{abs}(c) \\
& *e^2 - 4*(2*b^4*c^7 - 8*a*b^2*c^8 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{s} \\
& \text{qrt}(b^2 - 4*a*c)*c)*b^4*c^5 + 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b \\
& ^2 - 4*a*c)*c)*a*b^2*c^6 + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 \\
& - 4*a*c)*c)*b^3*c^6 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c) \\
& )*c)*b^2*c^7 - 2*(b^2 - 4*a*c)*b^2*c^7)*d^3*e - (2*b^7*c^2 - 20*a*b^5*c^3 + \\
& 64*a^2*b^3*c^4 - 64*a^3*b*c^5 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}( \\
& b^2 - 4*a*c)*c)*b^7 + 10*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4* \\
& a*c)*c)*a*b^5*c + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)* \\
& c)*b^6*c - 32*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a^2 \\
& *b^3*c^2 - 12*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a*b \\
& ^4*c^2 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*b^5*c^2 \\
& + 32*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a^3*b*c^3 + \\
& 16*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + \\
& 6*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a*b^3*c^3 - 8*\text{s} \\
& \text{qrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 \\
& - 4*a*c)*b^5*c^2 + 12*(b^2 - 4*a*c)*a*b^3*c^3 - 16*(b^2 - 4*a*c)*a^2*b*c^4 \\
& )*c^2*e^4 - 8*(\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a*b^5*c^3 - 8*\text{sqrt}(2) \\
& )*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a^2*b^3*c^4 - 2*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b \\
& ^2 - 4*a*c)*c)*a*b^4*c^4 - 2*a*b^5*c^4 + 16*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4 \\
& *a*c)*c)*a^3*b*c^5 + 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a^2*b^2*c^5 \\
& + \text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a*b^3*c^5 + 16*a^2*b^3*c^5 - 4*\text{sq}
\end{aligned}$$

$$\begin{aligned} & \text{rt}(2) \cdot \sqrt{b^2 - 4ac} \cdot c \cdot a^2 b^3 c^4 - 8(b^2 - 4ac) a^2 b^3 c^5 \cdot d \cdot \text{abs}(c) \cdot e^3 + 6(2b^5 c^6 - 12ab^3 c^7 + 16a^2 b^3 c^8 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c) \cdot b^5 c^4 + 6\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot b^5 c^4 + 6\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^2 b^3 c^5 + 2\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot b^4 c^5 - 8\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^2 b^3 c^6 - 4\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^2 b^3 c^6 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot b^3 c^6 + 2\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^2 b^3 c^7 - 2(b^2 - 4ac) b^3 c^6 + 4(b^2 - 4ac) a^2 b^3 c^7 \cdot d^2 \cdot e^2 + 2(\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^2 b^6 c^2 - 9\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^2 b^4 c^3 - 2\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^2 b^5 c^3 - 2a^2 b^6 c^3 + 24\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^3 b^2 c^4 + 10\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^2 b^3 c^4 + \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^2 b^4 c^4 + 18a^2 b^4 c^4 - 16\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^4 c^5 - 8\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^3 b^2 c^5 - 5\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^2 b^2 c^5 - 48a^3 b^2 c^5 + 4\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^3 c^6 + 32a^4 c^6 + 2(b^2 - 4ac) a^2 b^4 c^3 - 10(b^2 - 4ac) a^2 b^2 c^4 + 8(b^2 - 4ac) a^3 c^5) \cdot \text{abs}(c) \cdot e^4 - 4(2b^6 c^5 - 14a^2 b^4 c^6 + 24a^2 b^2 c^7 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot b^6 c^3 + 7\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^2 b^4 c^4 + 2\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot b^5 c^4 - 12\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^2 b^2 c^5 - 6\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^2 b^3 c^5 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot b^4 c^5 + 3\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^2 b^2 c^6 - 2(b^2 - 4ac) b^4 c^5 + 6(b^2 - 4ac) a^2 b^2 c^6) \cdot d \cdot e^3 + (2b^7 c^4 - 16a^2 b^5 c^5 + 36a^2 b^3 c^6 - 16a^3 b^2 c^7 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot b^7 c^2 + 8\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^2 b^5 c^3 + 2\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot b^6 c^3 - 18\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^2 b^3 c^4 - 8\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^2 b^4 c^4 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot b^5 c^4 + 8\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^3 b^2 c^5 + 4\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^2 b^2 c^5 + 4\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^2 b^3 c^5 - 2\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^2 b^3 c^6 - 2(b^2 - 4ac) b^5 c^4 + 8(b^2 - 4ac) a^2 b^3 c^5 - 4(b^2 - 4ac) a^2 b^3 c^6) \cdot e^4 \cdot \arctan(2\sqrt{1/2} \cdot x / \sqrt{(b^2 c^5 - \sqrt{b^2 c^{10} - 4a^2 c^{11}}) / c^6}) / ((a^2 b^4 c^5 - 8a^2 b^2 c^6 - 2a^2 b^3 c^6 + 16a^3 c^7 + 8a^2 b^2 c^7 + a^2 b^2 c^7 - 4a^2 c^8) \cdot c^2) + 1/15 \cdot (3c^4 x^5 e^4 + 20c^4 d x^3 e^3 - 5b^2 c^3 x^3 e^4 + 90c^4 d^2 x e^2 - 60b^2 c^3 d x e^3 + 15b^2 c^2 x e^4 - 15a^2 c^3 x e^4) / c^5 \end{aligned}$$

**maple [B]** time = 0.05, size = 1888, normalized size = 4.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((e \cdot x^2 + d)^4 / (c \cdot x^4 + b \cdot x^2 + a), x)$

[Out]  $\frac{4}{3} \frac{d^3 e^3 x^3 - 1}{3} \frac{e^4}{c^2} x^3 b + \frac{e^4}{c^3} b^2 x - \frac{a}{c^2} \frac{e^4 x + 6}{c} \frac{d^2 e^2 x - 4}{e^3} \frac{e^3}{c^2} b d x - 2 \frac{2^{1/2}}{((-b + (-4ac + b^2)^{1/2}) \cdot c)^{1/2}} \arctanh(2^{1/2} / ((-b + (-4ac + b^2)^{1/2}) \cdot c)^{1/2}) \cdot c \cdot x) \cdot d^3 e - 6/c / (-4ac + b^2)^{1/2} \cdot 2^{1/2} / (((-b + (-4ac + b^2)^{1/2}) \cdot c)^{1/2}) \cdot \arctanh(2^{1/2} / ((-b + (-4ac + b^2)^{1/2}) \cdot c)^{1/2}) \cdot c \cdot x) \cdot a \cdot b \cdot d \cdot e^3 - 6/c / (-4ac + b^2)^{1/2} \cdot 2^{1/2} / ((b + (-4ac + b^2)^{1/2}) \cdot c)^{1/2} \cdot \arctan(2^{1/2} / ((b + (-4ac + b^2)^{1/2}) \cdot c)^{1/2}) \cdot c \cdot x) \cdot a \cdot b \cdot d \cdot e^3 + 1/2 \cdot c^3 \cdot 2^{1/2} / (((-b + (-4ac + b^2)^{1/2}) \cdot c)^{1/2}) \cdot \arctanh(2^{1/2} / (((-b + (-4ac + b^2)^{1/2}) \cdot c)^{1/2}) \cdot c \cdot x) \cdot b^3 \cdot e^4 - 1/2 \cdot c^3 \cdot 2^{1/2} / ((b + (-4ac + b^2)^{1/2}) \cdot c)^{1/2} \cdot \arctan(2^{1/2} / ((b + (-4ac + b^2)^{1/2}) \cdot c)^{1/2}) \cdot c \cdot x) \cdot b^3 \cdot e^4 -$

$$\begin{aligned} & c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*d^4-c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*d^4-3/c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*d^2*e^2+2/c^2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*b^2*e^4+6/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*d^2*e^2+2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*d^3*e*b+6/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*d^2*e^2+2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*d^3*e*b-1/c^2*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*b*e^4-1/c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a^2*e^4-1/2/c^3/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^4*e^4-1/c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a^2*e^4-1/2/c^3/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^4*e^4+2/c*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*d*e^3-2/c^2*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*d*e^3+3/c*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*d^2*e^2+1/c^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*b*e^4-2/c*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*d*e^3+2/c^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*d*e^3-3/c*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*d^2*e^2+2/c^2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3*d*e^3-3/c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*d^2*e^2+2/c^2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*b^2*e^4+2/c^2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3*d*e^3+2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*d^3*e+1/5/c*e^4*x^5 \end{aligned}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^4/(c\*x^4+b\*x^2+a),x, algorithm="maxima")

[Out]  $\frac{1}{15}(3c^2e^4x^5 + 5(4c^2de^3 - bce^4)x^3 + 15(6c^2d^2e^2 - 4b*cd*e^3 + (b^2 - ac)e^4)x)/c^3 + \text{integrate}((c^3d^4 - 6a*c^2*d^2*e^2 + 4*a*b*c*d*e^3 - (a*b^2 - a^2*c)*e^4 + (4*c^3*d^3*e - 6*b*c^2*d^2*e^2 + 4*(b^2*c - a*c^2)*d*e^3 - (b^3 - 2*a*b*c)*e^4)*x^2)/(c*x^4 + b*x^2 + a), x)/c^3$

**mupad** [B] time = 9.31, size = 29551, normalized size = 64.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)^4/(a + b\*x^2 + c\*x^4),x)



$$\begin{aligned}
& ) - 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^{(1/2)} + 28*a*b^3*c^6*d^6*e^2 - 56*a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b^7*c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c^6*d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^{(1/2)} - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^{(1/2)} - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)}/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^{(1/2)}*1i - (((16*a*c^8*d^4 + 16*a^3*c^6*e^4 - 4*b^2*c^7*d^4 + 4*a*b^4*c^4*e^4 - 20*a^2*b^2*c^5*e^4 - 96*a^2*c^7*d^2*e^2 - 16*a*b^3*c^5*d*e^3 + 64*a^2*b*c^6*d*e^3 + 24*a*b^2*c^6*d^2*e^2)/c^5 + (2*x*(4*b^3*c^7 - 16*a*b*c^8))*(-(a*b^9*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4*a*c - b^2)^3)^{(1/2)} - a*b^6*e^8*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a^2*b^7*c*e^8 + 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7 + 42*a^3*b^5*c^2*e^8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^{(1/2)} - 448*a^3*c^7*d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^{(1/2)} + 336*a^2*b^2*c^6*d^5*e^3 - 490*a^2*b^3*c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3*d^2*e^6 - 1008*a^3*b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^{(1/2)} + 28*a*b^3*c^6*d^6*e^2 - 56*a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b^7*c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c^6*d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^{(1/2)} - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^{(1/2)} - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)}/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^{(1/2)}/c^5)*(-(a*b^9*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4*a*c - b^2)^3)^{(1/2)} - a*b^6*e^8*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a^2*b^7*c*e^8 + 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7 + 42*a^3*b^5*c^2*e^8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^{(1/2)} - 448*a^3*c^7*d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^{(1/2)} + 336*a^2*b^2*c^6*d^5*e^3 - 490*a^2*b^3*c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3*d^2*e^6 - 1008*a^3*b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^{(1/2)} + 28*a*b^3*c^6*d^6*e^2 - 56*a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b^7*c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c^6*d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^{(1/2)} - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^{(1/2)} - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)}/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^{(1/2)} + (2*x*(b^8*e^8 + 2*c^8*d^8 + 2*a^4*c^4*e^8 - 56*a*c^7*d^6*e^2 + 20*a^2*b^4*c^2*e^8 - 16*a^3*b^2*c^3*e^8 + 140*a^2*c^6*d^4*e^4 - 56*a^3*c^5*d^2*e^6 + 28*b^2*c^6*d^6*e^2 - 56*b^3*c^5*d^5*e^3 + 70*b^4*c^4*d^4*e^4 - 56*b^5*c^3*d^3*e^5 + 28*b^6*c^2*d^2*e^6 - 8*a*b^6*c*e^8 - 8*b*c^7*d^7*e - 8*b^7*c*d*e^7 + 252*a^2*b^2*c^4*d^2*e^6 + 168*a*b*c^6*d^5*e^3 + 56*a*b^5*c^2*d*e^7 + 56*a^3*b*c^4*d*e^7 - 280*a*b^2*c^5*d^4
\end{aligned}$$

$$\begin{aligned}
& *e^4 + 280*a*b^3*c^4*d^3*e^5 - 168*a*b^4*c^3*d^2*e^6 - 280*a^2*b*c^5*d^3*e^5 \\
& - 112*a^2*b^3*c^3*d*e^7)/c^5)*(-(a*b^9*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4*a*c - b^2)^3)^{(1/2)} - a*b^6*e^8*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a^2*b^7*c*e^8 \\
& + 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7 + 42*a^3*b^5*c^2* \\
& e^8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^{(1/2)} - 448*a^3*c \\
& ^7*d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^8*c*d*e^7 - 6*a^3* \\
& b^2*c^2*e^8*(-(4*a*c - b^2)^3)^{(1/2)} + 336*a^2*b^2*c^6*d^5*e^3 - 490*a^2*b^ \\
& 3*c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3*d^2*e^6 - 1008*a^ \\
& 3*b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - \\
& b^2)^3)^{(1/2)} - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 16*a*b^2*c^7 \\
& *d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^{(1/2)} + 28*a*b^3*c^6*d^6*e^2 - \\
& 56*a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b \\
& ^7*c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c \\
& ^6*d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^ \\
& 4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 56*a*b*c^5*d^5*e^3*(- \\
& (4*a*c - b^2)^3)^{(1/2)} + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} - 70*a \\
& *b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - \\
& b^2)^3)^{(1/2)} - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 112*a^2*b* \\
& c^4*d^3*e^5*(-(4*a*c - b^2)^3)^{(1/2)} - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2) \\
& ^3)^{(1/2)} + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} + 84*a^2*b^2*c^3*d^2*e \\
& ^6*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^ \\
& (1/2)*1i)/((((16*a*c^8*d^4 + 16*a^3*c^6*e^4 - 4*b^2*c^7*d^4 + 4*a*b^4*c^4*e \\
& ^4 - 20*a^2*b^2*c^5*e^4 - 96*a^2*c^7*d^2*e^2 - 16*a*b^3*c^5*d*e^3 + 64*a^2* \\
& b*c^6*d*e^3 + 24*a*b^2*c^6*d^2*e^2)/c^5 - (2*x*(4*b^3*c^7 - 16*a*b*c^8)*(-( \\
& a*b^9*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4*a*c - b^2)^3)^{(1/2)} - a*b^6*e^8*(-(4 \\
& *a*c - b^2)^3)^{(1/2)} - 11*a^2*b^7*c*e^8 + 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7 \\
& *e - 64*a^5*c^5*d*e^7 + 42*a^3*b^5*c^2*e^8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e \\
& ^8*(-(4*a*c - b^2)^3)^{(1/2)} - 448*a^3*c^7*d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4 \\
& *a*b*c^8*d^8 - 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 336*a^2*b^2*c^6*d^5*e^3 - 490*a^2*b^3*c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3* \\
& e^5 - 252*a^2*b^5*c^3*d^2*e^6 - 1008*a^3*b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4* \\
& d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 28*a^3*c^4*d^2*e^6* \\
& (- (4*a*c - b^2)^3)^{(1/2)} - 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - \\
& b^2)^3)^{(1/2)} + 28*a*b^3*c^6*d^6*e^2 - 56*a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4* \\
& d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b^7*c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e \\
& ^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c^6*d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - \\
& 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a^3*b*c^ \\
& 3*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^ \\
& (1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^{(1/2)} - 28*a*b^4*c^2*d^2*e^ \\
& 6*(-(4*a*c - b^2)^3)^{(1/2)} - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b^5*c*d*e^7*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16* \\
& a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^ (1/2))/c^5)*(-(a*b^9*e^8 + b^3*c^7*d^ \\
& ^8 + c^7*d^8*(-(4*a*c - b^2)^3)^{(1/2)} - a*b^6*e^8*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 11*a^2*b^7*c*e^8 + 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7 \\
& + 42*a^3*b^5*c^2*e^8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3) \\
& ^{(1/2)} - 448*a^3*c^7*d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^ \\
& 8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^{(1/2)} + 336*a^2*b^2*c^6*d^ \\
& 5*e^3 - 490*a^2*b^3*c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3 \\
& *d^2*e^6 - 1008*a^3*b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5* \\
& d^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^{(1 \\
& /2)} - 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^{(1/2)} + 28*a* \\
& b^3*c^6*d^6*e^2 - 56*a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^ \\
& 3*d^3*e^5 + 28*a*b^7*c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d \\
& *e^7 + 840*a^3*b*c^6*d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^ \\
& 6 + 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 56* \\
& a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^ \\
& 2)^3)^{(1/2)} - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 56*a*b^3*c^3*
\end{aligned}$$





$$\begin{aligned}
& ^{(1/2)} - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^{(1/2)} - 32*a^2*b^3*c^2*d* \\
& e^7*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} + 8 \\
& 4*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)}/(8*(16*a^3*c^9 + a*b^4*c^7 \\
& - 8*a^2*b^2*c^8)))^{(1/2)}/c^5)*(-(a*b^9*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4*a* \\
& c - b^2)^3)^{(1/2)} - a*b^6*e^8*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a^2*b^7*c*e^8 + \\
& 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7 + 42*a^3*b^5*c^2*e^ \\
& 8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^{(1/2)} - 448*a^3*c^7 \\
& *d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^8*c*d*e^7 - 6*a^3*b^ \\
& 2*c^2*e^8*(-(4*a*c - b^2)^3)^{(1/2)} + 336*a^2*b^2*c^6*d^5*e^3 - 490*a^2*b^3* \\
& c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3*d^2*e^6 - 1008*a^3* \\
& b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 16*a*b^2*c^7*d \\
& ^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^{(1/2)} + 28*a*b^3*c^6*d^6*e^2 - 56 \\
& *a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b^7 \\
& *c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c^6 \\
& *d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^4* \\
& d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 56*a*b*c^5*d^5*e^3*(-(4 \\
& *a*c - b^2)^3)^{(1/2)} + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} - 70*a*b \\
& ^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 112*a^2*b*c^ \\
& 4*d^3*e^5*(-(4*a*c - b^2)^3)^{(1/2)} - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3 \\
& )^{(1/2)} + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} + 84*a^2*b^2*c^3*d^2*e^6 \\
& *(-(4*a*c - b^2)^3)^{(1/2)}/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^{(1 \\
& /2)} + (2*x*(b^8*e^8 + 2*c^8*d^8 + 2*a^4*c^4*e^8 - 56*a*c^7*d^6*e^2 + 20*a^2 \\
& *b^4*c^2*e^8 - 16*a^3*b^2*c^3*e^8 + 140*a^2*c^6*d^4*e^4 - 56*a^3*c^5*d^2*e^ \\
& 6 + 28*b^2*c^6*d^6*e^2 - 56*b^3*c^5*d^5*e^3 + 70*b^4*c^4*d^4*e^4 - 56*b^5*c \\
& ^3*d^3*e^5 + 28*b^6*c^2*d^2*e^6 - 8*a*b^6*c*e^8 - 8*b*c^7*d^7*e - 8*b^7*c*d \\
& *e^7 + 252*a^2*b^2*c^4*d^2*e^6 + 168*a*b*c^6*d^5*e^3 + 56*a*b^5*c^2*d*e^7 + \\
& 56*a^3*b*c^4*d*e^7 - 280*a*b^2*c^5*d^4*e^4 + 280*a*b^3*c^4*d^3*e^5 - 168*a \\
& *b^4*c^3*d^2*e^6 - 280*a^2*b*c^5*d^3*e^5 - 112*a^2*b^3*c^3*d*e^7))/c^5)*(-( \\
& a*b^9*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4*a*c - b^2)^3)^{(1/2)} - a*b^6*e^8*(-(4 \\
& *a*c - b^2)^3)^{(1/2)} - 11*a^2*b^7*c*e^8 + 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7 \\
& *e - 64*a^5*c^5*d*e^7 + 42*a^3*b^5*c^2*e^8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e \\
& ^8*(-(4*a*c - b^2)^3)^{(1/2)} - 448*a^3*c^7*d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4 \\
& *a*b*c^8*d^8 - 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 336*a^2*b^2*c^6*d^5*e^3 - 490*a^2*b^3*c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3* \\
& e^5 - 252*a^2*b^5*c^3*d^2*e^6 - 1008*a^3*b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4* \\
& d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 28*a^3*c^4*d^2*e^6* \\
& (- (4*a*c - b^2)^3)^{(1/2)} - 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - \\
& b^2)^3)^{(1/2)} + 28*a*b^3*c^6*d^6*e^2 - 56*a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4* \\
& d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b^7*c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e \\
& ^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c^6*d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - \\
& 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a^3*b*c^ \\
& 3*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^ \\
& (1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^{(1/2)} - 28*a*b^4*c^2*d^2*e^ \\
& 6*(-(4*a*c - b^2)^3)^{(1/2)} - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b^5*c*d*e^7*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)}/(8*(16* \\
& a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^{(1/2)))*(-(a*b^9*e^8 + b^3*c^7*d^8 + \\
& c^7*d^8*(-(4*a*c - b^2)^3)^{(1/2)} - a*b^6*e^8*(-(4*a*c - b^2)^3)^{(1/2)} - 11 \\
& *a^2*b^7*c*e^8 + 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7 + 4 \\
& 2*a^3*b^5*c^2*e^8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^{(1/ \\
& 2)} - 448*a^3*c^7*d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^8*c* \\
& d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^{(1/2)} + 336*a^2*b^2*c^6*d^5*e^ \\
& 3 - 490*a^2*b^3*c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3*d^2 \\
& *e^6 - 1008*a^3*b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4* \\
& e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^{(1/2)} + 28*a*b^3*
\end{aligned}$$









$$\begin{aligned}
& 2)^3)^{(1/2)} - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^{(1/2)} - 28*a*b^3*c^6*d^6*e^2 + 56*a*b^4*c^5*d^5*e^3 - 70*a*b^5*c^4*d^4*e^4 + 56*a*b^6*c^3*d^3*e^5 - 28*a*b^7*c^2*d^2*e^6 + 112*a^2*b*c^7*d^6*e^2 - 80*a^2*b^6*c^2*d*e^7 - 840*a^3*b*c^6*d^4*e^4 + 264*a^3*b^4*c^3*d*e^7 + 560*a^4*b*c^5*d^2*e^6 - 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^{(1/2)} - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^{(1/2)} - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)}/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8))^{(1/2)}/c^5)*((c^7*d^8*(-(4*a*c - b^2)^3)^{(1/2)} - b^3*c^7*d^8 - a*b^9*e^8 - a*b^6*e^8*(-(4*a*c - b^2)^3)^{(1/2)} + 11*a^2*b^7*c*e^8 - 28*a^5*b*c^4*e^8 - 64*a^2*c^8*d^7*e + 64*a^5*c^5*d*e^7 - 42*a^3*b^5*c^2*e^8 + 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^{(1/2)} + 448*a^3*c^7*d^5*e^3 - 448*a^4*c^6*d^3*e^5 + 4*a*b*c^8*d^8 + 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^{(1/2)} - 336*a^2*b^2*c^6*d^5*e^3 + 490*a^2*b^3*c^5*d^4*e^4 - 448*a^2*b^4*c^4*d^3*e^5 + 252*a^2*b^5*c^3*d^2*e^6 + 1008*a^3*b^2*c^5*d^3*e^5 - 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^{(1/2)} - 28*a*b^3*c^6*d^6*e^2 + 56*a*b^4*c^5*d^5*e^3 - 70*a*b^5*c^4*d^4*e^4 + 56*a*b^6*c^3*d^3*e^5 - 28*a*b^7*c^2*d^2*e^6 + 112*a^2*b*c^7*d^6*e^2 - 80*a^2*b^6*c^2*d*e^7 - 840*a^3*b*c^6*d^4*e^4 + 264*a^3*b^4*c^3*d*e^7 + 560*a^4*b*c^5*d^2*e^6 - 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^{(1/2)} - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^{(1/2)} - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)}/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8))^{(1/2)} + (2*x*(b^8*e^8 + 2*c^8*d^8 + 2*a^4*c^4*e^8 - 56*a*c^7*d^6*e^2 + 20*a^2*b^4*c^2*e^8 - 16*a^3*b^2*c^3*e^8 + 140*a^2*c^6*d^4*e^4 - 56*a^3*c^5*d^2*e^6 + 28*b^2*c^6*d^6*e^2 - 56*b^3*c^5*d^5*e^3 + 70*b^4*c^4*d^4*e^4 - 56*b^5*c^3*d^3*e^5 + 28*b^6*c^2*d^2*e^6 - 8*a*b^6*c*e^8 - 8*b*c^7*d^7*e - 8*b^7*c*d*e^7 + 252*a^2*b^2*c^4*d^2*e^6 + 168*a*b*c^6*d^5*e^3 + 56*a*b^5*c^2*d*e^7 + 56*a^3*b*c^4*d*e^7 - 280*a*b^2*c^5*d^4*e^4 + 280*a*b^3*c^4*d^3*e^5 - 168*a*b^4*c^3*d^2*e^6 - 280*a^2*b*c^5*d^3*e^5 - 112*a^2*b^3*c^3*d*e^7))/c^5)*((c^7*d^8*(-(4*a*c - b^2)^3)^{(1/2)} - b^3*c^7*d^8 - a*b^9*e^8 - a*b^6*e^8*(-(4*a*c - b^2)^3)^{(1/2)} + 11*a^2*b^7*c*e^8 - 28*a^5*b*c^4*e^8 - 64*a^2*c^8*d^7*e + 64*a^5*c^5*d*e^7 - 42*a^3*b^5*c^2*e^8 + 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^{(1/2)} + 448*a^3*c^7*d^5*e^3 - 448*a^4*c^6*d^3*e^5 + 4*a*b*c^8*d^8 + 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^{(1/2)} - 336*a^2*b^2*c^6*d^5*e^3 + 490*a^2*b^3*c^5*d^4*e^4 - 448*a^2*b^4*c^4*d^3*e^5 + 252*a^2*b^5*c^3*d^2*e^6 + 1008*a^3*b^2*c^5*d^3*e^5 - 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^{(1/2)} - 28*a*b^3*c^6*d^6*e^2 + 56*a*b^4*c^5*d^5*e^3 - 70*a*b^5*c^4*d^4*e^4 + 56*a*b^6*c^3*d^3*e^5 - 28*a*b^7*c^2*d^2*e^6 + 112*a^2*b*c^7*d^6*e^2 - 80*a^2*b^6*c^2*d*e^7 - 840*a^3*b*c^6*d^4*e^4 + 264*a^3*b^4*c^3*d*e^7 + 560*a^4*b*c^5*d^2*e^6 - 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^{(1/2)} - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^{(1/2)} - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)}/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8))^{(1/2)))*((c^7*d^8*(-(4*a*c - b^2)^3)^{(1/2)} - b^3*c^7*d^8 - a
\end{aligned}$$

$$\begin{aligned}
& *b^9*e^8 - a*b^6*e^8*(-(4*a*c - b^2)^3)^{(1/2)} + 11*a^2*b^7*c*e^8 - 28*a^5*b \\
& *c^4*e^8 - 64*a^2*c^8*d^7*e + 64*a^5*c^5*d*e^7 - 42*a^3*b^5*c^2*e^8 + 63*a^4 \\
& *b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^{(1/2)} + 448*a^3*c^7*d^5*e^3 \\
& - 448*a^4*c^6*d^3*e^5 + 4*a*b*c^8*d^8 + 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8 \\
& *(-(4*a*c - b^2)^3)^{(1/2)} - 336*a^2*b^2*c^6*d^5*e^3 + 490*a^2*b^3*c^5*d^4*e \\
& ^4 - 448*a^2*b^4*c^4*d^3*e^5 + 252*a^2*b^5*c^3*d^2*e^6 + 1008*a^3*b^2*c^5*d \\
& ^3*e^5 - 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^{(1 \\
& /2)} - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a*b^2*c^7*d^7*e + 5* \\
& a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^{(1/2)} - 28*a*b^3*c^6*d^6*e^2 + 56*a*b^4*c^ \\
& 5*d^5*e^3 - 70*a*b^5*c^4*d^4*e^4 + 56*a*b^6*c^3*d^3*e^5 - 28*a*b^7*c^2*d^2* \\
& e^6 + 112*a^2*b*c^7*d^6*e^2 - 80*a^2*b^6*c^2*d*e^7 - 840*a^3*b*c^6*d^4*e^4 \\
& + 264*a^3*b^4*c^3*d*e^7 + 560*a^4*b*c^5*d^2*e^6 - 304*a^4*b^2*c^4*d*e^7 - 2 \\
& 8*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^ \\
& 2)^3)^{(1/2)} + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} - 70*a*b^2*c^4*d^ \\
& 4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^{(1 \\
& /2)} - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 112*a^2*b*c^4*d^3*e^5 \\
& *(-(4*a*c - b^2)^3)^{(1/2)} - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} + \\
& 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^{(1/2)} + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c \\
& - b^2)^3)^{(1/2)})/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8))^{(1/2)}*2i + \\
& (e^4*x^5)/(5*c)
\end{aligned}$$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*4/(c\*x\*\*4+b\*x\*\*2+a),x)

[Out] Timed out

$$3.187 \quad \int \frac{(d+ex^2)^3}{a+bx^2+cx^4} dx$$

**Optimal.** Leaf size=316

$$\frac{\left( e(-ce(ae+3bd)+b^2e^2+3c^2d^2) + \frac{(2cd-be)(-ce(3ae+bd)+b^2e^2+c^2d^2)}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left( \frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}} \right) + \left( e(-ce(ae+3bd)+b^2e^2+3c^2d^2) - \frac{(2cd-be)(-ce(3ae+bd)+b^2e^2+c^2d^2)}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left( \frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}} \right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

**Rubi [A]** time = 0.79, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1170, 1166, 205}

$$\frac{\left( e(-ce(ae+3bd)+b^2e^2+3c^2d^2) + \frac{(2cd-be)(-ce(3ae+bd)+b^2e^2+c^2d^2)}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left( \frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}} \right) + \left( e(-ce(ae+3bd)+b^2e^2+3c^2d^2) - \frac{(2cd-be)(-ce(3ae+bd)+b^2e^2+c^2d^2)}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left( \frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}} \right) + \frac{e^2x(3cd-be)}{c^2} + \frac{e^3x^3}{3c}}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/(a + b*x^2 + c*x^4), x]
```

```
[Out] (e^2*(3*c*d - b*e)*x)/c^2 + (e^3*x^3)/(3*c) + ((e*(3*c^2*d^2 + b^2*e^2 - c*
e*(3*b*d + a*e)) + ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)))/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e*(3*c^2*d^2 + b^2*e^2 -
c*e*(3*b*d + a*e)) - ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)
))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

**Rule 205**

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

**Rule 1166**

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

**Rule 1170**

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] := Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
0] && IntegerQ[q]
```

Rubi steps



$$\begin{aligned}
\int \frac{(d+ex^2)^3}{a+bx^2+cx^4} dx &= \int \left( \frac{e^2(3cd-be)}{c^2} + \frac{e^3x^2}{c} + \frac{c^2d^3-3acde^2+abe^3+e(3c^2d^2+b^2e^2-ce(3bd+ae))x^2}{c^2(a+bx^2+cx^4)} \right) dx \\
&= \frac{e^2(3cd-be)x}{c^2} + \frac{e^3x^3}{3c} + \frac{\int \frac{c^2d^3-3acde^2+abe^3+e(3c^2d^2+b^2e^2-ce(3bd+ae))x^2}{a+bx^2+cx^4} dx}{c^2} \\
&= \frac{e^2(3cd-be)x}{c^2} + \frac{e^3x^3}{3c} + \frac{\left( e(3c^2d^2+b^2e^2-ce(3bd+ae)) - \frac{(2cd-be)(c^2d^2+b^2e^2-ce(bd+3ae))}{\sqrt{b^2-4ac}} \right)}{2c^2} \int \frac{dx}{a+bx^2+cx^4} \\
&= \frac{e^2(3cd-be)x}{c^2} + \frac{e^3x^3}{3c} + \frac{\left( e(3c^2d^2+b^2e^2-ce(3bd+ae)) + \frac{(2cd-be)(c^2d^2+b^2e^2-ce(bd+3ae))}{\sqrt{b^2-4ac}} \right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} \int \frac{dx}{a+bx^2+cx^4}
\end{aligned}$$

**Mathematica [A]** time = 0.55, size = 402, normalized size = 1.27

$$\frac{3\sqrt{2}\left(3c^2d\left(d\sqrt{b^2-4ac}-2ac-bd\right)+c^2\left(-3bd\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}+3abe+3b^2d\right)+e^2d^2\left(\sqrt{b^2-4ac}-b\right)+2c^3d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{b-\sqrt{b^2-4ac}}\right)+\frac{3\sqrt{2}\left(3c^2d\left(d\sqrt{b^2-4ac}+2ac+bd\right)-c^2\left(3b\left(d\sqrt{b^2-4ac}+ac\right)+ac\sqrt{b^2-4ac}+3b^2d\right)+e^2d^2\left(\sqrt{b^2-4ac}+b\right)-2c^3d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b^2-4ac}+b}\right)+6\sqrt{c}e^2x(3cd-be)+2c^3e^2c^3}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}+\frac{6c^{5/2}}{\sqrt{b^2-4ac}\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^3/(a + b\*x^2 + c\*x^4), x]

[Out] (6\*sqrt(c)\*e^2\*(3\*c\*d - b\*e)\*x + 2\*c^(3/2)\*e^3\*x^3 + (3\*sqrt(2)\*(2\*c^3\*d^3 + b^2\*(-b + sqrt(b^2 - 4\*a\*c))\*e^3 + 3\*c^2\*d\*e\*(-(b\*d) + sqrt(b^2 - 4\*a\*c)\*d - 2\*a\*e) + c\*e^2\*(3\*b^2\*d - 3\*b\*sqrt(b^2 - 4\*a\*c)\*d + 3\*a\*b\*e - a\*sqrt(b^2 - 4\*a\*c)\*e))\*ArcTan[(sqrt(2)\*sqrt(c)\*x)/sqrt(b - sqrt(b^2 - 4\*a\*c))])/(sqrt(b^2 - 4\*a\*c)\*sqrt(b - sqrt(b^2 - 4\*a\*c))) + (3\*sqrt(2)\*(-2\*c^3\*d^3 + b^2\*(b + sqrt(b^2 - 4\*a\*c))\*e^3 + 3\*c^2\*d\*e\*(b\*d + sqrt(b^2 - 4\*a\*c)\*d + 2\*a\*e) - c\*e^2\*(3\*b^2\*d + a\*sqrt(b^2 - 4\*a\*c)\*e + 3\*b\*(sqrt(b^2 - 4\*a\*c)\*d + a\*e)))\*ArcTan[(sqrt(2)\*sqrt(c)\*x)/sqrt(b + sqrt(b^2 - 4\*a\*c))])/(sqrt(b^2 - 4\*a\*c)\*sqrt(b + sqrt(b^2 - 4\*a\*c)))/(6\*c^(5/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^3}{a+bx^2+cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)^3/(a + b\*x^2 + c\*x^4), x]

[Out] IntegrateAlgebraic[(d + e\*x^2)^3/(a + b\*x^2 + c\*x^4), x]

**fricas [B]** time = 34.09, size = 9584, normalized size = 30.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^3/(c\*x^4+b\*x^2+a), x, algorithm="fricas")

[Out] 1/6\*(2\*c\*e^3\*x^3 + 3\*sqrt(1/2)\*c^2\*sqrt(-(b\*c^5\*d^6 - 12\*a\*c^5\*d^5\*e + 15\*a\*b\*c^4\*d^4\*e^2 - 20\*(a\*b^2\*c^3 - 2\*a^2\*c^4)\*d^3\*e^3 + 15\*(a\*b^3\*c^2 - 3\*a^2\*b\*c^3)\*d^2\*e^4 - 6\*(a\*b^4\*c - 4\*a^2\*b^2\*c^2 + 2\*a^3\*c^3)\*d\*e^5 + (a\*b^5 - 5\*a^2\*b^3\*c + 5\*a^3\*b\*c^2)\*e^6 + (a\*b^2\*c^5 - 4\*a^2\*c^6)\*sqrt((c^10\*d^12 - 30\*a\*c^9\*d^10\*e^2 + 40\*a\*b\*c^8\*d^9\*e^3 - 15\*(2\*a\*b^2\*c^7 - 17\*a^2\*c^8)\*d^8\*e^4 + 12\*(a\*b^3\*c^6 - 52\*a^2\*b\*c^7)\*d^7\*e^5 - 2\*(a\*b^4\*c^5 - 428\*a^2\*b^2\*c^8





$$\begin{aligned}
& 4*c^5 - 6*a^3*b^2*c^6 + 8*a^4*c^7)*e^3)*\text{sqrt}((c^{10}*d^{12} - 30*a*c^9*d^{10}*e^2 \\
& + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 \\
& - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)* \\
& d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - \\
& 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 \\
& + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2* \\
& c^4 - 5*a^5*c^5)*d^2*e^{10} - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - \\
& 2*a^5*b*c^4)*d*e^{11} + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2* \\
& c^3 + a^6*c^4)*e^{12})/(a^2*b^2*c^{10} - 4*a^3*c^{11})))*\text{sqrt}(-(b*c^5*d^6 - 12*a* \\
& c^5*d^5*e + 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(a \\
& *b^3*c^2 - 3*a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3)*d \\
& *e^5 + (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 - (a*b^2*c^5 - 4*a^2*c^6)*\text{sq} \\
& \text{rt}((c^{10}*d^{12} - 30*a*c^9*d^{10}*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - \\
& 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 \\
& - 428*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6) \\
& *d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20 \\
& *(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 \\
& - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^{10} - 12*(a^2*b^7*c \\
& - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^{11} + (a^2*b^8 - 6*a^3*b^6 \\
& *c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^{12})/(a^2*b^2*c^{10} - 4*a^3 \\
& *c^{11})))/(a*b^2*c^5 - 4*a^2*c^6))) - 3*\text{sqrt}(1/2)*c^2*\text{sqrt}(-(b*c^5*d^6 - 12* \\
& a*c^5*d^5*e + 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15* \\
& (a*b^3*c^2 - 3*a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3) \\
& *d*e^5 + (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 - (a*b^2*c^5 - 4*a^2*c^6)* \\
& \text{sqrt}((c^{10}*d^{12} - 30*a*c^9*d^{10}*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - \\
& 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 \\
& - 428*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b* \\
& c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - \\
& 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6 \\
& *c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^{10} - 12*(a^2*b^7* \\
& c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^{11} + (a^2*b^8 - 6*a^3* \\
& b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^{12})/(a^2*b^2*c^{10} - 4*a \\
& ^3*c^{11})))/(a*b^2*c^5 - 4*a^2*c^6))*\log(-2*(c^8*d^{12} - 3*b*c^7*d^{11}*e + 3*( \\
& b^2*c^6 - 4*a*c^7)*d^{10}*e^2 - (b^3*c^5 - 59*a*b*c^6)*d^9*e^3 - 9*(13*a*b^2* \\
& c^5 + 3*a^2*c^6)*d^8*e^4 + 18*(7*a*b^3*c^4 + 5*a^2*b*c^5)*d^7*e^5 - 42*(2*a \\
& *b^4*c^3 + 3*a^2*b^2*c^4)*d^6*e^6 + 18*(2*a*b^5*c^2 + 6*a^2*b^3*c^3 - a^3*b \\
& *c^4)*d^5*e^7 - 9*(a*b^6*c + 7*a^2*b^4*c^2 - 2*a^3*b^2*c^3 - 3*a^4*c^4)*d^4 \\
& *e^8 + (a*b^7 + 21*a^2*b^5*c + 10*a^3*b^3*c^2 - 55*a^4*b*c^3)*d^3*e^9 - 3*( \\
& a^2*b^6 + 4*a^3*b^4*c - 9*a^4*b^2*c^2 - 4*a^5*c^3)*d^2*e^{10} + 3*(a^3*b^5 - \\
& a^4*b^3*c - 3*a^5*b*c^2)*d*e^{11} - (a^4*b^4 - 3*a^5*b^2*c + a^6*c^2)*e^{12})*x \\
& - \text{sqrt}(1/2)*((b^2*c^7 - 4*a*c^8)*d^9 - 18*(a*b^2*c^6 - 4*a^2*c^7)*d^7*e^2 \\
& + 21*(a*b^3*c^5 - 4*a^2*b*c^6)*d^6*e^3 - 15*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16 \\
& *a^3*c^6)*d^5*e^4 + 3*(2*a*b^5*c^3 - 37*a^2*b^3*c^4 + 116*a^3*b*c^5)*d^4*e^5 \\
& - (a*b^6*c^2 - 72*a^2*b^4*c^3 + 318*a^3*b^2*c^4 - 184*a^4*c^5)*d^3*e^6 - \\
& 3*(11*a^2*b^5*c^2 - 61*a^3*b^3*c^3 + 68*a^4*b*c^4)*d^2*e^7 + 3*(3*a^2*b^6*c \\
& - 19*a^3*b^4*c^2 + 29*a^4*b^2*c^3 - 4*a^5*c^4)*d*e^8 - (a^2*b^7 - 7*a^3*b^5 \\
& *c + 13*a^4*b^3*c^2 - 4*a^5*b*c^3)*e^9 + ((a*b^3*c^7 - 4*a^2*b*c^8)*d^3 - \\
& 6*(a^2*b^2*c^7 - 4*a^3*c^8)*d^2*e + 3*(a^2*b^3*c^6 - 4*a^3*b*c^7)*d*e^2 - ( \\
& a^2*b^4*c^5 - 6*a^3*b^2*c^6 + 8*a^4*c^7)*e^3)*\text{sqrt}((c^{10}*d^{12} - 30*a*c^9*d^{10}*e^2 \\
& + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a \\
& *b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 + 226*a^3 \\
& *c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4 \\
& *c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3 \\
& *c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4 \\
& *b^2*c^4 - 5*a^5*c^5)*d^2*e^{10} - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3 \\
& *c^3 - 2*a^5*b*c^4)*d*e^{11} + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5 \\
& *b^2*c^3 + a^6*c^4)*e^{12})/(a^2*b^2*c^{10} - 4*a^3*c^{11})))*\text{sqrt}(-(b*c^5*d^6 - \\
& 12*a*c^5*d^5*e + 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + \\
& 15*(a*b^3*c^2 - 3*a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*
\end{aligned}$$



$$\begin{aligned}
& t(b^2 - 4ac)c)ab^2c^5 - 16a^2b^2c^5 - 4\sqrt{2}\sqrt{b^2 - 4ac)c)a^2c^6 + 32a^3c^6 - 2(b^2 - 4ac)ab^2c^4 + 8(b^2 - 4ac)a^2c^5)d\text{abs}(c)e^2 - 3(2b^4c^6 - 8ab^2c^7 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{b^2 - 4ac)c)b^4c^4 + 4\sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)ab^2c^5 + 2\sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)b^3c^5 - \sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)b^2c^6 - 2(b^2 - 4ac)b^2c^6)d^2e + 2(\sqrt{2}\sqrt{b^2 - 4ac)c)ab^5c^2 - 8\sqrt{2}\sqrt{b^2 - 4ac)c)a^2b^3c^3 - 2\sqrt{2}\sqrt{b^2 - 4ac)c)ab^4c^3 + 2ab^5c^3 + 16\sqrt{2}\sqrt{b^2 - 4ac)c)a^3b^2c^4 + 8\sqrt{2}\sqrt{b^2 - 4ac)c)a^2b^2c^4 + \sqrt{2}\sqrt{b^2 - 4ac)c)ab^3c^4 - 16a^2b^3c^4 - 4\sqrt{2}\sqrt{b^2 - 4ac)c)a^2b^2c^5 + 32a^3b^2c^5 - 2(b^2 - 4ac)ab^3c^3 + 8(b^2 - 4ac)a^2b^2c^4)\text{abs}(c)e^3 + 3(2b^5c^5 - 12ab^3c^6 + 16a^2b^2c^7 - \sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)b^5c^3 + 6\sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)ab^3c^4 + 2\sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)b^4c^4 - 8\sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)a^2b^2c^5 - 4\sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)ab^2c^5 - \sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)b^3c^5 + 2\sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)ab^2c^6 - 2(b^2 - 4ac)b^3c^5 + 4(b^2 - 4ac)ab^2c^6)d^2e^2 - (2b^6c^4 - 14ab^4c^5 + 24a^2b^2c^6 - \sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)b^6c^2 + 7\sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)ab^4c^3 + 2\sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)b^5c^3 - 12\sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)a^2b^2c^4 - 6\sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)ab^3c^4 - \sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)b^4c^4 + 3\sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)ab^2c^5 - 2(b^2 - 4ac)b^4c^4 + 6(b^2 - 4ac)ab^2c^5)e^3)\arctan(2\sqrt{1/2}x/\sqrt{(b^2c^3 + \sqrt{b^2c^6 - 4ac^7})/c^4})/((ab^4c^4 - 8a^2b^2c^5 - 2ab^3c^5 + 16a^3c^6 + 8a^2b^2c^6 + ab^2c^6 - 4a^2c^7)c^2) - 1/8(3(2b^4c^4 - 16ab^2c^5 + 32a^2c^6 - \sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)b^4c^2 + 8\sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)ab^2c^3 + 2\sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)b^3c^3 - 16\sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)a^2c^4 - 8\sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)ab^2c^4 + 4\sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)a^2c^5 - 2(b^2 - 4ac)b^2c^4 + 8(b^2 - 4ac)a^2c^5)c^2d^2e - 2(\sqrt{2}\sqrt{b^2 - 4ac)c)b^4c^4 - 8\sqrt{2}\sqrt{b^2 - 4ac)c)ab^2c^5 - 2\sqrt{2}\sqrt{b^2 - 4ac)c)b^3c^5 - 2b^4c^5 + 16\sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)a^2c^6 + 8\sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)ab^2c^6 + \sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)b^2c^6 + 16ab^2c^6 - 4\sqrt{2}\sqrt{b^2 - 4ac)c)a^2c^7 - 32a^2c^7 + 2(b^2 - 4ac)b^2c^5 - 8(b^2 - 4ac)a^2c^6)d^3\text{abs}(c) - 3(2b^5c^3 - 16ab^3c^4 + 32a^2b^2c^5 - \sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)b^5c + 8\sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)ab^3c^2 + 2\sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)b^4c^2 - 16\sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)a^2b^2c^3 - 8\sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)ab^2c^3 - \sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)b^3c^3 + 4\sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)ab^2c^4 - 2(b^2 - 4ac)b^3c^3 + 8(b^2 - 4ac)ab^2c^4)c^2d^2e^2 + 2(2b^3c^7 - 8ab^2c^8 - \sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)b^3c^5 + 4\sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)ab^2c^6 - \sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)b^2c^6 - \sqrt{2}\sqrt{b^2 - 4ac)c)\sqrt{b^2 - 4ac)c)b^2c^7 - 2(b^2 - 4ac)b^2c^7)d^3 + (2b^6c^2 - 18ab^4c^3 + 48a^2b^2c^4 - 32a^3c^5
\end{aligned}$$

$$\begin{aligned}
& - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} b^6 + 9 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} a b^4 c + 2 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} b^5 c - 24 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} a^2 b^2 c^2 - 10 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} a b^3 c^2 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} b^4 c^2 + 16 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} a^3 c^3 + 8 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} a^2 b c^3 + 5 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} a b^2 c^3 - 4 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} a^2 c^4 - 2 (b^2 - 4ac) b^4 c^2 + 10 (b^2 - 4ac) a b^2 c^3 - 8 (b^2 - 4ac) a^2 c^4) c^2 e^3 + 6 (\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} a b^4 c^3 - 8 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} a^2 b^2 c^4 - 2 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} a b^3 c^4 - 2 a b^4 c^4 + 16 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} a^3 c^5 + 8 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} a^2 b c^5 + \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} a b^2 c^5 + 16 a^2 b^2 c^5 - 4 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} a^2 c^6 - 32 a^3 c^6 + 2 (b^2 - 4ac) a b^2 c^4 - 8 (b^2 - 4ac) a^2 c^5) d \operatorname{abs}(c) e^2 - 3 (2 b^4 c^6 - 8 a b^2 c^7 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} b^4 c^4 + 4 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} a b^2 c^5 + 2 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} b^3 c^5 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} b^2 c^6 - 2 (b^2 - 4ac) b^2 c^6) d^2 e - 2 (\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} a b^5 c^2 - 8 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} a^2 b^3 c^3 - 2 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} a b^4 c^3 - 2 a b^5 c^3 + 16 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} a^3 b c^4 + 8 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} a^2 b^2 c^4 + \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} a b^3 c^4 + 16 a^2 b^3 c^4 - 4 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} a^2 b c^5 - 32 a^3 b c^5 + 2 (b^2 - 4ac) a b^3 c^3 - 8 (b^2 - 4ac) a^2 b c^4) \operatorname{abs}(c) e^3 + 3 (2 b^5 c^5 - 12 a b^3 c^6 + 16 a^2 b c^7 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} b^5 c^3 + 6 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} a b^3 c^4 + 2 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} b^4 c^4 - 8 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} a^2 b c^5 - 4 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} a b^2 c^5 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} b^3 c^5 + 2 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} a b c^6 - 2 (b^2 - 4ac) b^3 c^5 + 4 (b^2 - 4ac) a b c^6) d e^2 - (2 b^6 c^4 - 14 a b^4 c^5 + 24 a^2 b^2 c^6 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} b^6 c^2 + 7 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} a b^4 c^3 + 2 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} b^5 c^3 - 12 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} a^2 b^2 c^4 - 6 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} a b^3 c^4 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} b^4 c^4 + 3 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} a b^2 c^5 - 2 (b^2 - 4ac) b^4 c^4 + 6 (b^2 - 4ac) a b^2 c^5) e^3 \arctan(2 \sqrt{1/2} x / \sqrt{(b c^3 - \sqrt{b^2 c^6 - 4 a c^7}) / c^4}) / ((a b^4 c^4 - 8 a^2 b^2 c^5 - 2 a b^3 c^5 + 16 a^3 c^6 + 8 a^2 b c^6 + a b^2 c^6 - 4 a^2 c^7) c^2) + 1/3 (c^2 x^3 e^3 + 9 c^2 d x e^2 - 3 b c x e^3) / c^3
\end{aligned}$$

**maple [B]** time = 0.04, size = 1211, normalized size = 3.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int (e x^2 + d)^3 / (c x^4 + b x^2 + a), x$

[Out]  $\frac{1}{3} c e^3 x^3 - e^3 / c^2 b x + 3 / c d e^2 x + 1/2 c^2 (1/2) / ((-b + (-4ac + b^2)^{1/2}) c)^{1/2} \operatorname{arctanh}(2^{1/2} / ((-b + (-4ac + b^2)^{1/2}) c)^{1/2} c x) a e^3 - 1/2 / c^2 2^{1/2} / ((-b + (-4ac + b^2)^{1/2}) c)^{1/2} \operatorname{arctanh}(2^{1/2} / ((-b + (-4ac + b^2)^{1/2}) c)^{1/2} c x) b^2 e^3 + 3/2 c^2 (1/2) / ((-b + (-4ac + b^2)^{1/2}) c)$





$$\begin{aligned}
& ^3e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^{(1/2)} \\
& )/(8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6))^{(1/2)}/c^3 \\
& *(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^4*e^6 \\
& *(-(4*a*c - b^2)^3)^{(1/2)} - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6 \\
& *d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a^3*c^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 \\
& - 105*a^2*b^3*c^3*d^2*e^4 - 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a*b^2*c^5*d^5*e \\
& - 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^{(1/2)} + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3 \\
& + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 \\
& - 108*a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a*b*c^3*d^3 \\
& *e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^{(1/2)} + 15*a*b^2*c^2 \\
& *d^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^{(1/2)} \\
& )/(8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6))^{(1/2)} - (2*x*(b^6*e^6 + 2*c^6*d^6 - 2*a^3*c^3 \\
& *e^6 - 30*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 30*a^2*c^4*d^2*e^4 + 15*b^2*c^4*d^4*e^2 - 20*b^3*c^3 \\
& *d^3*e^3 + 15*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 6*b*c^5*d^5*e - 6*b^5*c*d*e^5 + 60*a*b*c^4 \\
& *d^3*e^3 + 30*a*b^3*c^2*d*e^5 - 30*a^2*b*c^3*d*e^5 - 60*a*b^2*c^3*d^2*e^4)) \\
& /c^3*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^4 \\
& *e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6 \\
& *d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a^3*c^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3 \\
& *e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a*b^2 \\
& *c^5*d^5*e - 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^{(1/2)} + 15*a*b^3*c^4*d^4*e^2 - 20*a \\
& *b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d \\
& *e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^{(1/2)} \\
& )/(8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6))^{(1/2)}/c^3 \\
& *i - (((16*a*c^6*d^3 - 4*b^2*c^5*d^3 - 4*a*b^3*c^3*e^3 + 16*a^2*b*c^4*e^3 - 48*a^2*c^5 \\
& *d*e^2 + 12*a*b^2*c^4*d*e^2)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6) *(-(a*b^7*e^6 + b^3*c^5 \\
& *d^6 - c^5*d^6*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^4*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a^2 \\
& *b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2 \\
& *e^6 + a^3*c^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a \\
& *b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a^2*c^3*d^2*e^4 \\
& *(-(4*a*c - b^2)^3)^{(1/2)} - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5 \\
& *d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 + 15*a \\
& *c^4*d^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^{(1/2)} + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^{(1/2)} )/(8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6))^{(1/2)} \\
& + (2*x*(b^6*e^6 + 2*c^6*d^6 - 2*a^3*c^3*e^6 - 30*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 30*a^2 \\
& *c^4*d^2*e^4 + 15*b^2*c^4*d^4*e^2 - 20*b^3*c^3*d^3*e^3 + 15*b^4*c^2*d^2*e^4 - 6*a*b^4*c \\
& *e^6 - 6*b*c^5*d^5*e - 6*b^5*c*d*e^5 + 60*a*b*c^4*d^3*e^3 + 30*a*b^3*c^2*d*e^5 - 30*a^2*b \\
& *c^3*d*e^5 - 60*a*b^2*c^3*d^2*e^4))
\end{aligned}$$

$$\begin{aligned}
& ^4c^2d^2e^4 - 6a^*b^4c^*e^6 - 6b^*c^5d^5e - 6b^5c^*d^*e^5 + 60a^*b^*c^4 \\
& *d^3e^3 + 30a^*b^3c^2d^*e^5 - 30a^2b^*c^3d^*e^5 - 60a^*b^2c^3d^2e^4) \\
& /c^3)*(-(a^*b^7e^6 + b^3c^5d^6 - c^5d^6*(-(4a^*c - b^2)^3)^{(1/2)} + a^*b^4 \\
& *e^6*(-(4a^*c - b^2)^3)^{(1/2)} - 9a^2b^5c^*e^6 - 20a^4b^*c^3e^6 + 48a^2 \\
& *c^6d^5e + 48a^4c^4d^*e^5 + 25a^3b^3c^2e^6 + a^3c^2e^6*(-(4a^*c - \\
& b^2)^3)^{(1/2)} - 160a^3c^5d^3e^3 - 4a^*b^*c^6d^6 - 6a^*b^6c^*d^*e^5 + 12 \\
& 0a^2b^2c^4d^3e^3 - 105a^2b^3c^3d^2e^4 - 15a^2c^3d^2e^4*(-(4a^* \\
& c - b^2)^3)^{(1/2)} - 12a^*b^2c^5d^5e - 3a^2b^2c^*e^6*(-(4a^*c - b^2)^3 \\
& )^{(1/2)} + 15a^*b^3c^4d^4e^2 - 20a^*b^4c^3d^3e^3 + 15a^*b^5c^2d^2e^ \\
& 4 - 60a^2b^*c^5d^4e^2 + 48a^2b^4c^2d^*e^5 + 180a^3b^*c^4d^2e^4 - 1 \\
& 08a^3b^2c^3d^*e^5 + 15a^*c^4d^4e^2*(-(4a^*c - b^2)^3)^{(1/2)} - 20a^*b^*c \\
& ^3d^3e^3*(-(4a^*c - b^2)^3)^{(1/2)} + 12a^2b^*c^2d^*e^5*(-(4a^*c - b^2)^3) \\
& ^{(1/2)} + 15a^*b^2c^2d^2e^4*(-(4a^*c - b^2)^3)^{(1/2)} - 6a^*b^3c^*d^*e^5*(- \\
& (4a^*c - b^2)^3)^{(1/2)})/(8*(16a^3c^7 + a^*b^4c^5 - 8a^2b^2c^6))^{(1/2)} \\
& *1i)/((2*(3c^5d^8e - a^4c^*e^9 + a^3b^2e^9 - b^5d^3e^6 + 3a^*b^4d^2 \\
& *e^7 - 3a^2b^3d^*e^8 + 8a^*c^4d^6e^3 - 12b^*c^4d^7e^2 + 6b^4c^*d^4e \\
& ^5 + 6a^2c^3d^4e^5 + 19b^2c^3d^6e^3 - 15b^3c^2d^5e^4 - 24a^*b^*c \\
& ^3d^5e^4 - 14a^*b^3c^*d^3e^6 + 27a^*b^2c^2d^4e^5 - 12a^2b^*c^2d^3e \\
& ^6 + 9a^2b^2c^*d^2e^7))/c^3 + (((16a^*c^6d^3 - 4b^2c^5d^3 - 4a^*b^3* \\
& c^3e^3 + 16a^2b^*c^4e^3 - 48a^2c^5d^*e^2 + 12a^*b^2c^4d^*e^2)/c^3 - ( \\
& 2*x*(4b^3c^5 - 16a^*b^*c^6))*(-(a^*b^7e^6 + b^3c^5d^6 - c^5d^6*(-(4a^*c \\
& - b^2)^3)^{(1/2)} + a^*b^4e^6*(-(4a^*c - b^2)^3)^{(1/2)} - 9a^2b^5c^*e^6 - 20 \\
& *a^4b^*c^3e^6 + 48a^2c^6d^5e + 48a^4c^4d^*e^5 + 25a^3b^3c^2e^6 + \\
& a^3c^2e^6*(-(4a^*c - b^2)^3)^{(1/2)} - 160a^3c^5d^3e^3 - 4a^*b^*c^6d^6 \\
& - 6a^*b^6c^*d^*e^5 + 120a^2b^2c^4d^3e^3 - 105a^2b^3c^3d^2e^4 - 15 \\
& *a^2c^3d^2e^4*(-(4a^*c - b^2)^3)^{(1/2)} - 12a^*b^2c^5d^5e - 3a^2b^2* \\
& c^*e^6*(-(4a^*c - b^2)^3)^{(1/2)} + 15a^*b^3c^4d^4e^2 - 20a^*b^4c^3d^3e^ \\
& 3 + 15a^*b^5c^2d^2e^4 - 60a^2b^*c^5d^4e^2 + 48a^2b^4c^2d^*e^5 + 18 \\
& 0a^3b^*c^4d^2e^4 - 108a^3b^2c^3d^*e^5 + 15a^*c^4d^4e^2*(-(4a^*c - b \\
& ^2)^3)^{(1/2)} - 20a^*b^*c^3d^3e^3*(-(4a^*c - b^2)^3)^{(1/2)} + 12a^2b^*c^2d \\
& *e^5*(-(4a^*c - b^2)^3)^{(1/2)} + 15a^*b^2c^2d^2e^4*(-(4a^*c - b^2)^3)^{(1/ \\
& 2)} - 6a^*b^3c^*d^*e^5*(-(4a^*c - b^2)^3)^{(1/2)})/(8*(16a^3c^7 + a^*b^4c^5 - \\
& 8a^2b^2c^6))^{(1/2)}/c^3)*(-(a^*b^7e^6 + b^3c^5d^6 - c^5d^6*(-(4a^*c \\
& - b^2)^3)^{(1/2)} + a^*b^4e^6*(-(4a^*c - b^2)^3)^{(1/2)} - 9a^2b^5c^*e^6 - 2 \\
& 0a^4b^*c^3e^6 + 48a^2c^6d^5e + 48a^4c^4d^*e^5 + 25a^3b^3c^2e^6 \\
& + a^3c^2e^6*(-(4a^*c - b^2)^3)^{(1/2)} - 160a^3c^5d^3e^3 - 4a^*b^*c^6d^ \\
& 6 - 6a^*b^6c^*d^*e^5 + 120a^2b^2c^4d^3e^3 - 105a^2b^3c^3d^2e^4 - 1 \\
& 5a^2c^3d^2e^4*(-(4a^*c - b^2)^3)^{(1/2)} - 12a^*b^2c^5d^5e - 3a^2b^2 \\
& *c^*e^6*(-(4a^*c - b^2)^3)^{(1/2)} + 15a^*b^3c^4d^4e^2 - 20a^*b^4c^3d^3e \\
& ^3 + 15a^*b^5c^2d^2e^4 - 60a^2b^*c^5d^4e^2 + 48a^2b^4c^2d^*e^5 + 1 \\
& 80a^3b^*c^4d^2e^4 - 108a^3b^2c^3d^*e^5 + 15a^*c^4d^4e^2*(-(4a^*c - \\
& b^2)^3)^{(1/2)} - 20a^*b^*c^3d^3e^3*(-(4a^*c - b^2)^3)^{(1/2)} + 12a^2b^*c^2* \\
& d^*e^5*(-(4a^*c - b^2)^3)^{(1/2)} + 15a^*b^2c^2d^2e^4*(-(4a^*c - b^2)^3)^{(1 \\
& /2)} - 6a^*b^3c^*d^*e^5*(-(4a^*c - b^2)^3)^{(1/2)})/(8*(16a^3c^7 + a^*b^4c^5 \\
& - 8a^2b^2c^6))^{(1/2)} - (2*x*(b^6e^6 + 2c^6d^6 - 2a^3c^3e^6 - 30a^ \\
& *c^5d^4e^2 + 9a^2b^2c^2e^6 + 30a^2c^4d^2e^4 + 15b^2c^4d^4e^2 \\
& - 20b^3c^3d^3e^3 + 15b^4c^2d^2e^4 - 6a^*b^4c^*e^6 - 6b^*c^5d^5e - \\
& 6b^5c^*d^*e^5 + 60a^*b^*c^4d^3e^3 + 30a^*b^3c^2d^*e^5 - 30a^2b^*c^3d^*e \\
& ^5 - 60a^*b^2c^3d^2e^4))/c^3)*(-(a^*b^7e^6 + b^3c^5d^6 - c^5d^6*(-(4a^* \\
& c - b^2)^3)^{(1/2)} + a^*b^4e^6*(-(4a^*c - b^2)^3)^{(1/2)} - 9a^2b^5c^*e^6 \\
& - 20a^4b^*c^3e^6 + 48a^2c^6d^5e + 48a^4c^4d^*e^5 + 25a^3b^3c^2e^ \\
& ^6 + a^3c^2e^6*(-(4a^*c - b^2)^3)^{(1/2)} - 160a^3c^5d^3e^3 - 4a^*b^*c^6 \\
& *d^6 - 6a^*b^6c^*d^*e^5 + 120a^2b^2c^4d^3e^3 - 105a^2b^3c^3d^2e^4 \\
& - 15a^2c^3d^2e^4*(-(4a^*c - b^2)^3)^{(1/2)} - 12a^*b^2c^5d^5e - 3a^2* \\
& b^2c^*e^6*(-(4a^*c - b^2)^3)^{(1/2)} + 15a^*b^3c^4d^4e^2 - 20a^*b^4c^3d^ \\
& ^3e^3 + 15a^*b^5c^2d^2e^4 - 60a^2b^*c^5d^4e^2 + 48a^2b^4c^2d^*e^5 \\
& + 180a^3b^*c^4d^2e^4 - 108a^3b^2c^3d^*e^5 + 15a^*c^4d^4e^2*(-(4a^*c \\
& - b^2)^3)^{(1/2)} - 20a^*b^*c^3d^3e^3*(-(4a^*c - b^2)^3)^{(1/2)} + 12a^2b^*c^2 \\
& ^2d^*e^5*(-(4a^*c - b^2)^3)^{(1/2)} + 15a^*b^2c^2d^2e^4*(-(4a^*c - b^2)^3)
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{1}{2} \right) - 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^{(1/2)} / (8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6))^{(1/2)} + \left( \left( (16*a*c^6*d^3 - 4*b^2*c^5*d^3 - 4*a*b^3*c^3*e^3 + 16*a^2*b*c^4*e^3 - 48*a^2*c^5*d*e^2 + 12*a*b^2*c^4*d*e^2) / c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^4*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a^3*c^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^{(1/2)} + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^{(1/2)} + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^{(1/2)} / (8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6))^{(1/2)} \right) / c^3 * \left( -(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^4*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a^3*c^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^{(1/2)} + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^{(1/2)} + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^{(1/2)} / (8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6))^{(1/2)} + (2*x*(b^6*e^6 + 2*c^6*d^6 - 2*a^3*c^3*e^6 - 30*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 30*a^2*c^4*d^2*e^4 + 15*b^2*c^4*d^4*e^2 - 20*b^3*c^3*d^3*e^3 + 15*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 6*b*c^5*d^5*e - 6*b^5*c*d*e^5 + 60*a*b*c^4*d^3*e^3 + 30*a*b^3*c^2*d*e^5 - 30*a^2*b*c^3*d*e^5 - 60*a*b^2*c^3*d^2*e^4) / c^3 * \left( -(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^4*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a^3*c^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^{(1/2)} + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^{(1/2)} + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^{(1/2)} / (8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6))^{(1/2)} * 2i + \operatorname{atan}\left( \left( \left( (16*a*c^6*d^3 - 4*b^2*c^5*d^3 - 4*a*b^3*c^3*e^3 + 16*a^2*b*c^4*e^3 - 48*a^2*c^5*d*e^2 + 12*a*b^2*c^4*d*e^2) / c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(a*b^7*e^6 + b^3*c^5*d^6 + c^5*d^6*(-(4*a*c - b^2)^3)^{(1/2)} - a*b^4*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a^3*c^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^{(1/2)} + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^{(1/2)} + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^{(1/2)} / (8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6))^{(1/2)} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& *a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 - \\
& a^3*c^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 \\
& - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 + 15 \\
& *a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a*b^2*c^5*d^5*e + 3*a^2*b^2* \\
& c*e^6*(-(4*a*c - b^2)^3)^{(1/2)} + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^ \\
& 3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 18 \\
& 0*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 15*a*c^4*d^4*e^2*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a^2*b*c^2*d \\
& *e^5*(-(4*a*c - b^2)^3)^{(1/2)} - 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^{(1/ \\
& 2)} + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^3*c^7 + a*b^4*c^5 - \\
& 8*a^2*b^2*c^6)))^{(1/2)}/c^3*(-(a*b^7*e^6 + b^3*c^5*d^6 + c^5*d^6*(-(4*a*c \\
& - b^2)^3)^{(1/2)} - a*b^4*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a^2*b^5*c*e^6 - 2 \\
& 0*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 \\
& - a^3*c^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 \\
& - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 + 1 \\
& 5*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a*b^2*c^5*d^5*e + 3*a^2*b^2 \\
& *c*e^6*(-(4*a*c - b^2)^3)^{(1/2)} + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^ \\
& ^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 1 \\
& 80*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 15*a*c^4*d^4*e^2*(-(4*a*c - \\
& b^2)^3)^{(1/2)} + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a^2*b*c^2* \\
& d*e^5*(-(4*a*c - b^2)^3)^{(1/2)} - 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^{(1 \\
& /2)} + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^3*c^7 + a*b^4*c^5 \\
& - 8*a^2*b^2*c^6)))^{(1/2)} - (2*x*(b^6*e^6 + 2*c^6*d^6 - 2*a^3*c^3*e^6 - 30*a \\
& *c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 30*a^2*c^4*d^2*e^4 + 15*b^2*c^4*d^4*e^2 \\
& - 20*b^3*c^3*d^3*e^3 + 15*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 6*b*c^5*d^5*e - \\
& 6*b^5*c*d*e^5 + 60*a*b*c^4*d^3*e^3 + 30*a*b^3*c^2*d*e^5 - 30*a^2*b*c^3*d*e^ \\
& ^5 - 60*a*b^2*c^3*d^2*e^4))/c^3*(-(a*b^7*e^6 + b^3*c^5*d^6 + c^5*d^6*(-(4* \\
& a*c - b^2)^3)^{(1/2)} - a*b^4*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a^2*b^5*c*e^6 \\
& - 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^ \\
& ^6 - a^3*c^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6 \\
& *d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 \\
& + 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a*b^2*c^5*d^5*e + 3*a^2* \\
& b^2*c*e^6*(-(4*a*c - b^2)^3)^{(1/2)} + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^ \\
& 3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 \\
& + 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 15*a*c^4*d^4*e^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a^2*b*c^ \\
& ^2*d*e^5*(-(4*a*c - b^2)^3)^{(1/2)} - 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3) \\
& ^{(1/2)} + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^3*c^7 + a*b^4*c^ \\
& ^5 - 8*a^2*b^2*c^6)))^{(1/2)}*i - (((16*a*c^6*d^3 - 4*b^2*c^5*d^3 - 4*a*b^3* \\
& c^3*e^3 + 16*a^2*b*c^4*e^3 - 48*a^2*c^5*d*e^2 + 12*a*b^2*c^4*d*e^2)/c^3 + ( \\
& 2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(a*b^7*e^6 + b^3*c^5*d^6 + c^5*d^6*(-(4*a*c \\
& - b^2)^3)^{(1/2)} - a*b^4*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a^2*b^5*c*e^6 - 20 \\
& *a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 - \\
& a^3*c^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 \\
& - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 + 15 \\
& *a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a*b^2*c^5*d^5*e + 3*a^2*b^2* \\
& c*e^6*(-(4*a*c - b^2)^3)^{(1/2)} + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^ \\
& 3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 18 \\
& 0*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 15*a*c^4*d^4*e^2*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a^2*b*c^2*d \\
& *e^5*(-(4*a*c - b^2)^3)^{(1/2)} - 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^{(1/ \\
& 2)} + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^3*c^7 + a*b^4*c^5 - \\
& 8*a^2*b^2*c^6)))^{(1/2)}/c^3*(-(a*b^7*e^6 + b^3*c^5*d^6 + c^5*d^6*(-(4*a*c \\
& - b^2)^3)^{(1/2)} - a*b^4*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a^2*b^5*c*e^6 - 2 \\
& 0*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 \\
& - a^3*c^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 \\
& - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 + 1 \\
& 5*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a*b^2*c^5*d^5*e + 3*a^2*b^2 \\
& *c*e^6*(-(4*a*c - b^2)^3)^{(1/2)} + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e
\end{aligned}$$

$$\begin{aligned}
&^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 1 \\
&80*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 15*a*c^4*d^4*e^2*(-(4*a*c - \\
&b^2)^3)^{(1/2)} + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a^2*b*c^2* \\
&d*e^5*(-(4*a*c - b^2)^3)^{(1/2)} - 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^{(1 \\
&/2)} + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^{(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 \\
&- 8*a^2*b^2*c^6)))^{(1/2)} + (2*x*(b^6*e^6 + 2*c^6*d^6 - 2*a^3*c^3*e^6 - 30*a \\
&*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 30*a^2*c^4*d^2*e^4 + 15*b^2*c^4*d^4*e^2 \\
&- 20*b^3*c^3*d^3*e^3 + 15*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 6*b*c^5*d^5*e - \\
&6*b^5*c*d*e^5 + 60*a*b*c^4*d^3*e^3 + 30*a*b^3*c^2*d*e^5 - 30*a^2*b*c^3*d*e \\
&^5 - 60*a*b^2*c^3*d^2*e^4))/c^3)*(-(a*b^7*e^6 + b^3*c^5*d^6 + c^5*d^6*(-(4* \\
&a*c - b^2)^3)^{(1/2)} - a*b^4*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a^2*b^5*c*e^6 \\
&- 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e \\
&^6 - a^3*c^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6 \\
&*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 \\
&+ 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a*b^2*c^5*d^5*e + 3*a^2* \\
&b^2*c*e^6*(-(4*a*c - b^2)^3)^{(1/2)} + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^ \\
&3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 \\
&+ 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 15*a*c^4*d^4*e^2*(-(4*a*c \\
&- b^2)^3)^{(1/2)} + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a^2*b*c \\
&^2*d*e^5*(-(4*a*c - b^2)^3)^{(1/2)} - 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3) \\
&^{(1/2)} + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^{(1/2))/(8*(16*a^3*c^7 + a*b^4*c \\
&^5 - 8*a^2*b^2*c^6)))^{(1/2)}*i)/((2*(3*c^5*d^8*e - a^4*c*e^9 + a^3*b^2*e^9 \\
&- b^5*d^3*e^6 + 3*a*b^4*d^2*e^7 - 3*a^2*b^3*d*e^8 + 8*a*c^4*d^6*e^3 - 12*b* \\
&c^4*d^7*e^2 + 6*b^4*c*d^4*e^5 + 6*a^2*c^3*d^4*e^5 + 19*b^2*c^3*d^6*e^3 - 15 \\
&*b^3*c^2*d^5*e^4 - 24*a*b*c^3*d^5*e^4 - 14*a*b^3*c*d^3*e^6 + 27*a*b^2*c^2*d \\
&^4*e^5 - 12*a^2*b*c^2*d^3*e^6 + 9*a^2*b^2*c*d^2*e^7))/c^3 + (((16*a*c^6*d^3 \\
&- 4*b^2*c^5*d^3 - 4*a*b^3*c^3*e^3 + 16*a^2*b*c^4*e^3 - 48*a^2*c^5*d*e^2 + \\
&12*a*b^2*c^4*d*e^2))/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6))*(-(a*b^7*e^6 + b^3* \\
&c^5*d^6 + c^5*d^6*(-(4*a*c - b^2)^3)^{(1/2)} - a*b^4*e^6*(-(4*a*c - b^2)^3)^{( \\
&1/2)} - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d \\
&*e^5 + 25*a^3*b^3*c^2*e^6 - a^3*c^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 160*a^3* \\
&c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 1 \\
&05*a^2*b^3*c^3*d^2*e^4 + 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a \\
&*b^2*c^5*d^5*e + 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^{(1/2)} + 15*a*b^3*c^4*d^ \\
&4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 \\
&+ 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 15 \\
&*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2 \\
&)^3)^{(1/2)} - 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^{(1/2)} - 15*a*b^2*c^2*d^2 \\
&*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^{(1/2))/( \\
&8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^{(1/2)}/c^3)*(-(a*b^7*e^6 + b^3 \\
&*c^5*d^6 + c^5*d^6*(-(4*a*c - b^2)^3)^{(1/2)} - a*b^4*e^6*(-(4*a*c - b^2)^3)^{( \\
&1/2)} - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4* \\
&d*e^5 + 25*a^3*b^3*c^2*e^6 - a^3*c^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 160*a^3 \\
&*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - \\
&105*a^2*b^3*c^3*d^2*e^4 + 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 12* \\
&a*b^2*c^5*d^5*e + 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^{(1/2)} + 15*a*b^3*c^4*d \\
&^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 \\
&+ 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 1 \\
&5*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^ \\
&2)^3)^{(1/2)} - 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^{(1/2)} - 15*a*b^2*c^2*d^ \\
&2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^{(1/2)}/ \\
&(8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^{(1/2)} - (2*x*(b^6*e^6 + 2*c^6 \\
&*d^6 - 2*a^3*c^3*e^6 - 30*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 30*a^2*c^4*d^ \\
&2*e^4 + 15*b^2*c^4*d^4*e^2 - 20*b^3*c^3*d^3*e^3 + 15*b^4*c^2*d^2*e^4 - 6*a* \\
&b^4*c*e^6 - 6*b*c^5*d^5*e - 6*b^5*c*d*e^5 + 60*a*b*c^4*d^3*e^3 + 30*a*b^3*c \\
&^2*d*e^5 - 30*a^2*b*c^3*d*e^5 - 60*a*b^2*c^3*d^2*e^4))/c^3)*(-(a*b^7*e^6 + \\
&b^3*c^5*d^6 + c^5*d^6*(-(4*a*c - b^2)^3)^{(1/2)} - a*b^4*e^6*(-(4*a*c - b^2)^ \\
&3)^{(1/2)} - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c \\
&^4*d*e^5 + 25*a^3*b^3*c^2*e^6 - a^3*c^2*e^6*(-(4*a*c - b^2)^3)^{(1/2)} - 160*
\end{aligned}$$



$$*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^{(1/2)} - 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^{(1/2)}/(8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6))^{(1/2)*2i} - x*((b*e^3)/c^2 - (3*d*e^2)/c) + (e^3*x^3)/(3*c)$$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*3/(c\*x\*\*4+b\*x\*\*2+a),x)

[Out] Timed out

$$3.188 \quad \int \frac{(d+ex^2)^2}{a+bx^2+cx^4} dx$$

**Optimal.** Leaf size=238

$$\frac{\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}} + e(2cd - be)\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right) + \left(e(2cd - be) - \frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}} + \sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

**Rubi [A]** time = 0.64, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1170, 1166, 205}

$$\frac{\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}} + e(2cd - be)\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right) + \left(e(2cd - be) - \frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right) + \frac{e^2x}{c}}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}} + \sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^2/(a + b\*x^2 + c\*x^4), x]

[Out] (e^2\*x)/c + ((e\*(2\*c\*d - b\*e) + (2\*c^2\*d^2 + b^2\*e^2 - 2\*c\*e\*(b\*d + a\*e))/Sqrt[b^2 - 4\*a\*c])\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b - Sqrt[b^2 - 4\*a\*c]])/(Sqrt[2]\*c^(3/2)\*Sqrt[b - Sqrt[b^2 - 4\*a\*c]]) + ((e\*(2\*c\*d - b\*e) - (2\*c^2\*d^2 + b^2\*e^2 - 2\*c\*e\*(b\*d + a\*e))/Sqrt[b^2 - 4\*a\*c])\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b + Sqrt[b^2 - 4\*a\*c]])/(Sqrt[2]\*c^(3/2)\*Sqrt[b + Sqrt[b^2 - 4\*a\*c]])

**Rule 205**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 1166**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[e/2 + (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 - q/2 + c\*x^2), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - a\*e^2, 0] && PosQ[b^2 - 4\*a\*c]

**Rule 1170**

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x^2)^q/(a + b\*x^2 + c\*x^4), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IntegerQ[q]

**Rubi steps**



$$\begin{aligned}
\int \frac{(d + ex^2)^2}{a + bx^2 + cx^4} dx &= \int \left( \frac{e^2}{c} + \frac{cd^2 - ae^2 + e(2cd - be)x^2}{c(a + bx^2 + cx^4)} \right) dx \\
&= \frac{e^2 x}{c} + \frac{\int \frac{cd^2 - ae^2 + e(2cd - be)x^2}{a + bx^2 + cx^4} dx}{c} \\
&= \frac{e^2 x}{c} + \frac{\left( e(2cd - be) - \frac{2c^2 d^2 + b^2 e^2 - 2ce(bd + ae)}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{\frac{b}{2} + \frac{1}{2} \sqrt{b^2 - 4ac} + cx^2} dx}{2c} + \frac{\left( e(2cd - be) + \frac{2c^2 d^2 + b^2 e^2}{\sqrt{b^2 - 4ac}} \right)}{\sqrt{b^2 - 4ac}} \\
&= \frac{e^2 x}{c} + \frac{\left( e(2cd - be) + \frac{2c^2 d^2 + b^2 e^2 - 2ce(bd + ae)}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left( \frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} c^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\left( e(2cd - be) - \frac{2c^2 d^2 + b^2 e^2}{\sqrt{b^2 - 4ac}} \right)}{\sqrt{2} c^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}}}
\end{aligned}$$

**Mathematica [A]** time = 0.32, size = 269, normalized size = 1.13

$$\frac{\sqrt{2}(-2ce(-d\sqrt{b^2-4ac}+ae+bd))+be^2(b-\sqrt{b^2-4ac})+2c^2d^2}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)-\frac{\sqrt{2}(-2ce(d\sqrt{b^2-4ac}+ae+bd))+be^2(\sqrt{b^2-4ac}+b)+2c^2d^2}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)+2\sqrt{c}e^2x}{2c^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^2/(a + b\*x^2 + c\*x^4), x]

[Out] (2\*sqrt(c)\*e^2\*x + (sqrt(2)\*(2\*c^2\*d^2 + b\*(b - sqrt(b^2 - 4\*a\*c)))\*e^2 - 2\*c\*e\*(b\*d - sqrt(b^2 - 4\*a\*c)\*d + a\*e))\*ArcTan[(sqrt(2)\*sqrt(c)\*x)/sqrt(b - sqrt(b^2 - 4\*a\*c))]/(sqrt(b^2 - 4\*a\*c)\*sqrt(b - sqrt(b^2 - 4\*a\*c))) - (sqrt(2)\*(2\*c^2\*d^2 + b\*(b + sqrt(b^2 - 4\*a\*c)))\*e^2 - 2\*c\*e\*(b\*d + sqrt(b^2 - 4\*a\*c)\*d + a\*e))\*ArcTan[(sqrt(2)\*sqrt(c)\*x)/sqrt(b + sqrt(b^2 - 4\*a\*c))]/(sqrt(b^2 - 4\*a\*c)\*sqrt(b + sqrt(b^2 - 4\*a\*c)))/(2\*c^(3/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^2)^2}{a + bx^2 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)^2/(a + b\*x^2 + c\*x^4), x]

[Out] IntegrateAlgebraic[(d + e\*x^2)^2/(a + b\*x^2 + c\*x^4), x]

**fricas [B]** time = 3.22, size = 4690, normalized size = 19.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2/(c\*x^4+b\*x^2+a), x, algorithm="fricas")

[Out] 1/2\*(2\*e^2\*x - sqrt(1/2)\*c\*sqrt(-(b\*c^3\*d^4 - 8\*a\*c^3\*d^3\*e + 6\*a\*b\*c^2\*d^2\*e^2 - 4\*(a\*b^2\*c - 2\*a^2\*c^2)\*d\*e^3 + (a\*b^3 - 3\*a^2\*b\*c)\*e^4 + (a\*b^2\*c^3 - 4\*a^2\*c^4)\*sqrt((c^6\*d^8 - 12\*a\*c^5\*d^6\*e^2 + 8\*a\*b\*c^4\*d^5\*e^3 - 48\*a^2\*b\*c^3\*d^3\*e^5 - 2\*(a\*b^2\*c^3 - 19\*a^2\*c^4)\*d^4\*e^4 + 4\*(7\*a^2\*b^2\*c^2 - 3\*a^3\*c^3)\*d^2\*e^6 - 8\*(a^2\*b^3\*c - a^3\*b\*c^2)\*d\*e^7 + (a^2\*b^4 - 2\*a^3\*b^2\*c + a^4\*c^2)\*e^8)/(a^2\*b^2\*c^6 - 4\*a^3\*c^7)))/(a\*b^2\*c^3 - 4\*a^2\*c^4))\*log(2\*(c^5\*d^8 - 2\*b\*c^4\*d^7\*e + 14\*a\*b\*c^3\*d^5\*e^3 + (b^2\*c^3 - 4\*a\*c^4)\*d^6\*e^2 - 5\*(3\*a\*b^2\*c^2 + 2\*a^2\*c^3)\*d^4\*e^4 + 6\*(a\*b^3\*c + 3\*a^2\*b\*c^2)\*d^3\*e^5 - (a\*b^4 + 9\*a^2\*b^2\*c + 4\*a^3\*c^2)\*d^2\*e^6 + 2\*(a^2\*b^3 + a^3\*b\*c)\*d\*e^7

$$\begin{aligned}
& - (a^3b^2 - a^4c)e^8)x + \sqrt{1/2}*((b^2c^4 - 4a^2c^5)d^6 - 7(a^2b^2c^3 - 4a^2c^4)d^4e^2 + 4(a^2b^3c^2 - 4a^2b^2c^3)d^3e^3 - (a^2b^4c - 11a^2b^2c^2 + 28a^3c^3)d^2e^4 - 4(a^2b^3c - 4a^3b^2c^2)d^2e^5 + (a^2b^4 - 5a^3b^2c + 4a^4c^2)e^6 - ((a^2b^3c^4 - 4a^2b^2c^5)d^2 - 4(a^2b^2c^4 - 4a^3c^5)d^2e + (a^2b^3c^3 - 4a^3b^2c^4)e^2)*\sqrt{(c^6d^8 - 12a^2c^5d^6e^2 + 8a^2b^2c^4d^5e^3 - 48a^2b^2c^3d^3e^5 - 2(a^2b^2c^3 - 19a^2c^4)d^4e^4 + 4(7a^2b^2c^2 - 3a^3c^3)d^2e^6 - 8(a^2b^3c - a^3b^2c^2)d^2e^7 + (a^2b^4 - 2a^3b^2c + a^4c^2)e^8)/(a^2b^2c^6 - 4a^3c^7)))*\sqrt{-(b^2c^3d^4 - 8a^2c^3d^3e + 6a^2b^2c^2d^2e^2 - 4(a^2b^2c - 2a^2c^2)d^2e^3 + (a^2b^3 - 3a^2b^2c)e^4 + (a^2b^2c^3 - 4a^2c^4)*\sqrt{(c^6d^8 - 12a^2c^5d^6e^2 + 8a^2b^2c^4d^5e^3 - 48a^2b^2c^3d^3e^5 - 2(a^2b^2c^3 - 19a^2c^4)d^4e^4 + 4(7a^2b^2c^2 - 3a^3c^3)d^2e^6 - 8(a^2b^3c - a^3b^2c^2)d^2e^7 + (a^2b^4 - 2a^3b^2c + a^4c^2)e^8)/(a^2b^2c^6 - 4a^3c^7)))/(a^2b^2c^3 - 4a^2c^4))} + \sqrt{1/2}*c*\sqrt{-(b^2c^3d^4 - 8a^2c^3d^3e + 6a^2b^2c^2d^2e^2 - 4(a^2b^2c - 2a^2c^2)d^2e^3 + (a^2b^3 - 3a^2b^2c)e^4 + (a^2b^2c^3 - 4a^2c^4)*\sqrt{(c^6d^8 - 12a^2c^5d^6e^2 + 8a^2b^2c^4d^5e^3 - 48a^2b^2c^3d^3e^5 - 2(a^2b^2c^3 - 19a^2c^4)d^4e^4 + 4(7a^2b^2c^2 - 3a^3c^3)d^2e^6 - 8(a^2b^3c - a^3b^2c^2)d^2e^7 + (a^2b^4 - 2a^3b^2c + a^4c^2)e^8)/(a^2b^2c^6 - 4a^3c^7)))/(a^2b^2c^3 - 4a^2c^4))}*\log(2*(c^5d^8 - 2b^2c^4d^7e + 14a^2b^2c^3d^5e^3 + (b^2c^3 - 4a^2c^4)d^6e^2 - 5(3a^2b^2c^2 + 2a^2c^3)d^4e^4 + 6(a^2b^3c + 3a^2b^2c^2)d^3e^5 - (a^2b^4 + 9a^2b^2c + 4a^3c^2)d^2e^6 + 2(a^2b^3 + a^3b^2c)d^2e^7 - (a^3b^2 - a^4c)e^8)*x - \sqrt{1/2}*((b^2c^4 - 4a^2c^5)d^6 - 7(a^2b^2c^3 - 4a^2c^4)d^4e^2 + 4(a^2b^3c^2 - 4a^2b^2c^3)d^3e^3 - (a^2b^4c - 11a^2b^2c^2 + 28a^3c^3)d^2e^4 - 4(a^2b^3c - 4a^3b^2c^2)d^2e^5 + (a^2b^4 - 5a^3b^2c + 4a^4c^2)e^6 - ((a^2b^3c^4 - 4a^2b^2c^5)d^2 - 4(a^2b^2c^4 - 4a^3c^5)d^2e + (a^2b^3c^3 - 4a^3b^2c^4)e^2)*\sqrt{(c^6d^8 - 12a^2c^5d^6e^2 + 8a^2b^2c^4d^5e^3 - 48a^2b^2c^3d^3e^5 - 2(a^2b^2c^3 - 19a^2c^4)d^4e^4 + 4(7a^2b^2c^2 - 3a^3c^3)d^2e^6 - 8(a^2b^3c - a^3b^2c^2)d^2e^7 + (a^2b^4 - 2a^3b^2c + a^4c^2)e^8)/(a^2b^2c^6 - 4a^3c^7)))*\sqrt{-(b^2c^3d^4 - 8a^2c^3d^3e + 6a^2b^2c^2d^2e^2 - 4(a^2b^2c - 2a^2c^2)d^2e^3 + (a^2b^3 - 3a^2b^2c)e^4 + (a^2b^2c^3 - 4a^2c^4)*\sqrt{(c^6d^8 - 12a^2c^5d^6e^2 + 8a^2b^2c^4d^5e^3 - 48a^2b^2c^3d^3e^5 - 2(a^2b^2c^3 - 19a^2c^4)d^4e^4 + 4(7a^2b^2c^2 - 3a^3c^3)d^2e^6 - 8(a^2b^3c - a^3b^2c^2)d^2e^7 + (a^2b^4 - 2a^3b^2c + a^4c^2)e^8)/(a^2b^2c^6 - 4a^3c^7)))/(a^2b^2c^3 - 4a^2c^4))} - \sqrt{1/2}*c*\sqrt{-(b^2c^3d^4 - 8a^2c^3d^3e + 6a^2b^2c^2d^2e^2 - 4(a^2b^2c - 2a^2c^2)d^2e^3 + (a^2b^3 - 3a^2b^2c)e^4 - (a^2b^2c^3 - 4a^2c^4)*\sqrt{(c^6d^8 - 12a^2c^5d^6e^2 + 8a^2b^2c^4d^5e^3 - 48a^2b^2c^3d^3e^5 - 2(a^2b^2c^3 - 19a^2c^4)d^4e^4 + 4(7a^2b^2c^2 - 3a^3c^3)d^2e^6 - 8(a^2b^3c - a^3b^2c^2)d^2e^7 + (a^2b^4 - 2a^3b^2c + a^4c^2)e^8)/(a^2b^2c^6 - 4a^3c^7)))/(a^2b^2c^3 - 4a^2c^4))}*\log(2*(c^5d^8 - 2b^2c^4d^7e + 14a^2b^2c^3d^5e^3 + (b^2c^3 - 4a^2c^4)d^6e^2 - 5(3a^2b^2c^2 + 2a^2c^3)d^4e^4 + 6(a^2b^3c + 3a^2b^2c^2)d^3e^5 - (a^2b^4 + 9a^2b^2c + 4a^3c^2)d^2e^6 + 2(a^2b^3 + a^3b^2c)d^2e^7 - (a^3b^2 - a^4c)e^8)*x + \sqrt{1/2}*((b^2c^4 - 4a^2c^5)d^6 - 7(a^2b^2c^3 - 4a^2c^4)d^4e^2 + 4(a^2b^3c^2 - 4a^2b^2c^3)d^3e^3 - (a^2b^4c - 11a^2b^2c^2 + 28a^3c^3)d^2e^4 - 4(a^2b^3c - 4a^3b^2c^2)d^2e^5 + (a^2b^4 - 5a^3b^2c + 4a^4c^2)e^6 + ((a^2b^3c^4 - 4a^2b^2c^5)d^2 - 4(a^2b^2c^4 - 4a^3c^5)d^2e + (a^2b^3c^3 - 4a^3b^2c^4)e^2)*\sqrt{(c^6d^8 - 12a^2c^5d^6e^2 + 8a^2b^2c^4d^5e^3 - 48a^2b^2c^3d^3e^5 - 2(a^2b^2c^3 - 19a^2c^4)d^4e^4 + 4(7a^2b^2c^2 - 3a^3c^3)d^2e^6 - 8(a^2b^3c - a^3b^2c^2)d^2e^7 + (a^2b^4 - 2a^3b^2c + a^4c^2)e^8)/(a^2b^2c^6 - 4a^3c^7)))*\sqrt{-(b^2c^3d^4 - 8a^2c^3d^3e + 6a^2b^2c^2d^2e^2 - 4(a^2b^2c - 2a^2c^2)d^2e^3 + (a^2b^3 - 3a^2b^2c)e^4 - (a^2b^2c^3 - 4a^2c^4)*\sqrt{(c^6d^8 - 12a^2c^5d^6e^2 + 8a^2b^2c^4d^5e^3 - 48a^2b^2c^3d^3e^5 - 2(a^2b^2c^3 - 19a^2c^4)d^4e^4 + 4(7a^2b^2c^2 - 3a^3c^3)d^2e^6 - 8(a^2b^3c - a^3b^2c^2)d^2e^7 + (a^2b^4 - 2a^3b^2c + a^4c^2)e^8)/(a^2b^2c^6 - 4a^3c^7)))/(a^2b^2c^3 - 4a^2c^4))}
\end{aligned}$$

$$\frac{1}{(a^2 b^2 c^3 - 4 a^2 c^4))} + \sqrt{\frac{1}{2}} c \sqrt{-(b^3 c^3 d^4 - 8 a^3 c^3 d^3 e + 6 a^2 b^2 c^2 d^2 e^2 - 4 (a^2 b^2 c - 2 a^2 c^2) d e^3 + (a^2 b^3 - 3 a^2 b^2 c) e^4 - (a^2 b^2 c^3 - 4 a^2 c^4) \sqrt{(c^6 d^8 - 12 a^5 c^5 d^6 e^2 + 8 a^4 b^4 c^4 d^5 e^3 - 48 a^2 b^3 c^3 d^3 e^5 - 2 (a^2 b^2 c^3 - 19 a^2 c^4) d^4 e^4 + 4 (7 a^2 b^2 c^2 - 3 a^3 c^3) d^2 e^6 - 8 (a^2 b^3 c - a^3 b^2 c^2) d e^7 + (a^2 b^4 - 2 a^3 b^2 c + a^4 c^2) e^8}) / (a^2 b^2 c^6 - 4 a^3 c^7))} / (a^2 b^2 c^3 - 4 a^2 c^4) \log(2 (c^5 d^8 - 2 b^4 c^4 d^7 e + 14 a^2 b^3 c^3 d^5 e^3 + (b^2 c^3 - 4 a^2 c^4) d^6 e^2 - 5 (3 a^2 b^2 c^2 + 2 a^2 c^3) d^4 e^4 + 6 (a^2 b^3 c + 3 a^2 b^2 c^2) d^3 e^5 - (a^2 b^4 + 9 a^2 b^2 c + 4 a^3 c^2) d^2 e^6 + 2 (a^2 b^3 + a^3 b^2 c) d e^7 - (a^3 b^2 - a^4 c) e^8) x - \sqrt{\frac{1}{2}} ((b^2 c^4 - 4 a^2 c^5) d^6 - 7 (a^2 b^2 c^3 - 4 a^2 c^4) d^4 e^2 + 4 (a^2 b^3 c^2 - 4 a^2 b^2 c^3) d^3 e^3 - (a^2 b^4 c - 11 a^2 b^2 c^2 + 28 a^3 c^3) d^2 e^4 - 4 (a^2 b^3 c - 4 a^3 b^2 c^2) d e^5 + (a^2 b^4 - 5 a^3 b^2 c + 4 a^4 c^2) e^6 + ((a^2 b^3 c^4 - 4 a^2 b^2 c^5) d^2 - 4 (a^2 b^2 c^4 - 4 a^3 c^5) d e + (a^2 b^3 c^3 - 4 a^3 b^2 c^4) e^2) \sqrt{(c^6 d^8 - 12 a^5 c^5 d^6 e^2 + 8 a^4 b^4 c^4 d^5 e^3 - 48 a^2 b^3 c^3 d^3 e^5 - 2 (a^2 b^2 c^3 - 19 a^2 c^4) d^4 e^4 + 4 (7 a^2 b^2 c^2 - 3 a^3 c^3) d^2 e^6 - 8 (a^2 b^3 c - a^3 b^2 c^2) d e^7 + (a^2 b^4 - 2 a^3 b^2 c + a^4 c^2) e^8}) / (a^2 b^2 c^6 - 4 a^3 c^7)) \sqrt{-(b^3 c^3 d^4 - 8 a^3 c^3 d^3 e + 6 a^2 b^2 c^2 d^2 e^2 - 4 (a^2 b^2 c - 2 a^2 c^2) d e^3 + (a^2 b^3 - 3 a^2 b^2 c) e^4 - (a^2 b^2 c^3 - 4 a^2 c^4) \sqrt{(c^6 d^8 - 12 a^5 c^5 d^6 e^2 + 8 a^4 b^4 c^4 d^5 e^3 - 48 a^2 b^3 c^3 d^3 e^5 - 2 (a^2 b^2 c^3 - 19 a^2 c^4) d^4 e^4 + 4 (7 a^2 b^2 c^2 - 3 a^3 c^3) d^2 e^6 - 8 (a^2 b^3 c - a^3 b^2 c^2) d e^7 + (a^2 b^4 - 2 a^3 b^2 c + a^4 c^2) e^8}) / (a^2 b^2 c^6 - 4 a^3 c^7))} / (a^2 b^2 c^3 - 4 a^2 c^4)) / c$$

**giac** [B] time = 1.14, size = 4107, normalized size = 17.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2/(c\*x^4+b\*x^2+a),x, algorithm="giac")

[Out]  $x e^2 / c + 1/8 (2 (2 b^4 c^3 - 16 a b^2 c^4 + 32 a^2 c^5 - \sqrt{2} \sqrt{b^2 - 4 a c}) \sqrt{b c - \sqrt{b^2 - 4 a c}} c) b^4 c + 8 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c - \sqrt{b^2 - 4 a c}} c) a b^2 c^2 + 2 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c - \sqrt{b^2 - 4 a c}} c) b^3 c^2 - 16 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c - \sqrt{b^2 - 4 a c}} c) a^2 c^3 - 8 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c - \sqrt{b^2 - 4 a c}} c) a b^2 c^3 + 4 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c - \sqrt{b^2 - 4 a c}} c) a c^4 - 2 (b^2 - 4 a c) b^2 c^3 + 8 (b^2 - 4 a c) a c^4) c^2 d e + 2 (\sqrt{2} \sqrt{b c - \sqrt{b^2 - 4 a c}} c) b^4 c^3 - 8 \sqrt{2} \sqrt{b c - \sqrt{b^2 - 4 a c}} c) a b^2 c^4 - 2 \sqrt{2} \sqrt{b c - \sqrt{b^2 - 4 a c}} c) b^3 c^4 + 2 b^4 c^4 + 16 \sqrt{2} \sqrt{b c - \sqrt{b^2 - 4 a c}} c) a^2 c^5 + 8 \sqrt{2} \sqrt{b c - \sqrt{b^2 - 4 a c}} c) a b^2 c^5 + \sqrt{2} \sqrt{b c - \sqrt{b^2 - 4 a c}} c) b^2 c^5 - 16 a b^2 c^5 - 4 \sqrt{2} \sqrt{b c - \sqrt{b^2 - 4 a c}} c) a c^6 + 32 a^2 c^6 - 2 (b^2 - 4 a c) b^2 c^4 + 8 (b^2 - 4 a c) a c^5) d^2 \operatorname{abs}(c) - (2 b^5 c^2 - 16 a b^3 c^3 + 32 a^2 b^2 c^4 - \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c - \sqrt{b^2 - 4 a c}} c) b^5 + 8 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c - \sqrt{b^2 - 4 a c}} c) a b^3 c + 2 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c - \sqrt{b^2 - 4 a c}} c) b^4 c - 16 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c - \sqrt{b^2 - 4 a c}} c) a^2 b^2 c^2 - 8 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c - \sqrt{b^2 - 4 a c}} c) a b^2 c^2 - \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c - \sqrt{b^2 - 4 a c}} c) b^3 c^2 + 4 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c - \sqrt{b^2 - 4 a c}} c) a b^2 c^3 - 2 (b^2 - 4 a c) b^3 c^2 + 8 (b^2 - 4 a c) a b^2 c^3) c^2 e^2 + 2 (2 b^3 c^6 - 8 a b^2 c^7 - \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c - \sqrt{b^2 - 4 a c}} c) b^3 c^4 + 4 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c - \sqrt{b^2 - 4 a c}} c) a b^2 c^5 - \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c - \sqrt{b^2 - 4 a c}} c) b^2 c^5 - \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c - \sqrt{b^2 - 4 a c}} c) b^2 c^6 - 2 (b^2 - 4 a c) b^2 c^6) d^2 - 2 (\sqrt{2} \sqrt{b c - \sqrt{b^2 - 4 a c}} c) a b^4 c^2 - 8 \sqrt{2} \sqrt{b c - \sqrt{b^2 - 4 a c}} c) a^2 b^2 c^3 - 2$













$$\begin{aligned}
& b^2)^3)^{1/2} - b^3c^3d^4 - ab^5e^4 - ab^2e^4(-4ac - b^2)^3)^{1/2} \\
& ) + 7a^2b^3c^3e^4 - 12a^3b^2c^2e^4 + a^2c^3e^4(-4ac - b^2)^3)^{1/2} \\
& - 32a^2c^4d^3e + 32a^3c^3d^2e^3 + 4ab^4c^4d^4 + 4ab^4c^3d^3e^3 + \\
& 8ab^2c^3d^3e - 6ab^3c^2d^2e^2 + 24a^2b^2c^3d^2e^2 - 24a^2b^2 \\
& c^2d^2e^3 - 6ac^2d^2e^2(-4ac - b^2)^3)^{1/2} + 4ab^2c^3d^2e^3(-4ac \\
& - b^2)^3)^{1/2})/(8(16a^3c^5 + ab^4c^3 - 8a^2b^2c^4)))^{1/2}) * \\
& ((c^3d^4(-4ac - b^2)^3)^{1/2} - b^3c^3d^4 - ab^5e^4 - ab^2e^4(- \\
& (4ac - b^2)^3)^{1/2} + 7a^2b^3c^3e^4 - 12a^3b^2c^2e^4 + a^2c^3e^4(- \\
& (4ac - b^2)^3)^{1/2} - 32a^2c^4d^3e + 32a^3c^3d^2e^3 + 4ab^4c^4d^4 \\
& + 4ab^4c^3d^3e^3 + 8ab^2c^3d^3e - 6ab^3c^2d^2e^2 + 24a^2b^2c^3 \\
& d^2e^2 - 24a^2b^2c^2d^2e^3 - 6ac^2d^2e^2(-4ac - b^2)^3)^{1/2} \\
& + 4ab^2c^3d^2e^3(-4ac - b^2)^3)^{1/2})/(8(16a^3c^5 + ab^4c^3 - 8a^2 \\
& b^2c^4)))^{1/2} * 2i + (e^{2x})/c
\end{aligned}$$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*2/(c\*x\*\*4+b\*x\*\*2+a), x)

[Out] Timed out

$$3.189 \quad \int \frac{d+ex^2}{a+bx^2+cx^4} dx$$

**Optimal.** Leaf size=174

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}} + e\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

**Rubi [A]** time = 0.20, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {1166, 205}

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}} + e\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)/(a + b\*x^2 + c\*x^4), x]

[Out] ((e + (2\*c\*d - b\*e)/Sqrt[b^2 - 4\*a\*c])\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b - Sqrt[b^2 - 4\*a\*c]])/(Sqrt[2]\*Sqrt[c]\*Sqrt[b - Sqrt[b^2 - 4\*a\*c]]) + ((e - (2\*c\*d - b\*e)/Sqrt[b^2 - 4\*a\*c])\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b + Sqrt[b^2 - 4\*a\*c]])/(Sqrt[2]\*Sqrt[c]\*Sqrt[b + Sqrt[b^2 - 4\*a\*c]])

**Rule 205**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 1166**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[e/2 + (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 - q/2 + c\*x^2), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - a\*e^2, 0] && PosQ[b^2 - 4\*a\*c]

**Rubi steps**

$$\int \frac{d+ex^2}{a+bx^2+cx^4} dx = \frac{1}{2} \left( e - \frac{2cd-be}{\sqrt{b^2-4ac}} \right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^2} dx + \frac{1}{2} \left( e + \frac{2cd-be}{\sqrt{b^2-4ac}} \right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^2} dx$$

$$= \frac{\left(e + \frac{2cd-be}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

**Mathematica [A]** time = 0.14, size = 172, normalized size = 0.99

$$\frac{\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}$$

$$\frac{\hspace{10em}}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(a + b*x^2 + c*x^4), x]
```

```
[Out] (((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] + ((-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{a + bx^2 + cx^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^2)/(a + b*x^2 + c*x^4), x]
```

```
[Out] IntegrateAlgebraic[(d + e*x^2)/(a + b*x^2 + c*x^4), x]
```

**fricas [B]** time = 0.88, size = 1525, normalized size = 8.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a), x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x + sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 - ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/2*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x - sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 - ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) + 1/2*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x + sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 + ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/2*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x - sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 + ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2)))
```

**giac [B]** time = 0.87, size = 1402, normalized size = 8.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(c\*x^4+b\*x^2+a),x, algorithm="giac")

[Out]  $\frac{1}{4} * ((\sqrt{2} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * b^4 - 8 * \sqrt{2} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * a * b^2 * c - 2 * \sqrt{2} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * b^3 * c - 2 * b^4 * c + 16 * \sqrt{2} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * a^2 * c^2 + 8 * \sqrt{2} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * a * b * c^2 + \sqrt{2} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * b^2 * c^2 + 16 * a * b^2 * c^2 + 2 * b^3 * c^2 - 4 * \sqrt{2} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * a * c^3 - 32 * a^2 * c^3 - 8 * a * b * c^3 - \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * b^3 + 4 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * a * b * c + 2 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * b^2 * c - \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * b * c^2 + 2 * (b^2 - 4*a*c) * b^2 * c - 8 * (b^2 - 4*a*c) * a * c^2 - 2 * (b^2 - 4*a*c) * b * c^2) * d - 2 * (2 * a * b^2 * c^2 - 8 * a^2 * c^3 - \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * a * b^2 + 4 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * a^2 * c + 2 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * a * b * c - \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * a * c^2 - 2 * (b^2 - 4*a*c) * a * c^2) * e) * \arctan(2 * \sqrt{1/2} * x / \sqrt{(b + \sqrt{b^2 - 4*a*c}) / c}) / ((a * b^4 - 8 * a^2 * b^2 * c - 2 * a * b^3 * c + 16 * a^3 * c^2 + 8 * a^2 * b * c^2 + a * b^2 * c^2 - 4 * a^2 * c^3) * \text{abs}(c)) + 1/4 * ((\sqrt{2} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * b^4 - 8 * \sqrt{2} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * a * b^2 * c - 2 * \sqrt{2} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * b^3 * c + 2 * b^4 * c + 16 * \sqrt{2} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * a^2 * c^2 + 8 * \sqrt{2} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * a * b * c^2 + \sqrt{2} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * b^2 * c^2 - 16 * a * b^2 * c^2 - 2 * b^3 * c^2 - 4 * \sqrt{2} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * a * c^3 + 32 * a^2 * c^3 + 8 * a * b * c^3 + \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * b^3 - 4 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * a * b * c - 2 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * b^2 * c + \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * b * c^2 - 2 * (b^2 - 4*a*c) * b^2 * c + 8 * (b^2 - 4*a*c) * a * c^2 + 2 * (b^2 - 4*a*c) * b * c^2) * d + 2 * (2 * a * b^2 * c^2 - 8 * a^2 * c^3 - \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * a * b^2 + 4 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * a^2 * c + 2 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * a * b * c - \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} * c) * a * c^2 - 2 * (b^2 - 4*a*c) * a * c^2) * e) * \arctan(2 * \sqrt{1/2} * x / \sqrt{(b - \sqrt{b^2 - 4*a*c}) / c}) / ((a * b^4 - 8 * a^2 * b^2 * c - 2 * a * b^3 * c + 16 * a^3 * c^2 + 8 * a^2 * b * c^2 + a * b^2 * c^2 - 4 * a^2 * c^3) * \text{abs}(c))$

**maple [B]** time = 0.02, size = 328, normalized size = 1.89

$$\frac{\sqrt{2} \operatorname{be} \operatorname{arctanh}\left(\frac{\sqrt{2} cx}{\sqrt{(-b + \sqrt{-4ac + b^2})c}}\right)}{2\sqrt{-4ac + b^2} \sqrt{(-b + \sqrt{-4ac + b^2})c}} + \frac{\sqrt{2} \operatorname{be} \operatorname{arctan}\left(\frac{\sqrt{2} cx}{\sqrt{(b + \sqrt{-4ac + b^2})c}}\right)}{2\sqrt{-4ac + b^2} \sqrt{(b + \sqrt{-4ac + b^2})c}} - \frac{\sqrt{2} \operatorname{cd} \operatorname{arctanh}\left(\frac{\sqrt{2} cx}{\sqrt{(-b + \sqrt{-4ac + b^2})c}}\right)}{\sqrt{-4ac + b^2} \sqrt{(-b + \sqrt{-4ac + b^2})c}} - \frac{\sqrt{2} \operatorname{cd} \operatorname{arctan}\left(\frac{\sqrt{2} cx}{\sqrt{(b + \sqrt{-4ac + b^2})c}}\right)}{\sqrt{-4ac + b^2} \sqrt{(b + \sqrt{-4ac + b^2})c}} - \frac{\sqrt{2} \operatorname{e} \operatorname{arctanh}\left(\frac{\sqrt{2} cx}{\sqrt{(-b + \sqrt{-4ac + b^2})c}}\right)}{2\sqrt{(-b + \sqrt{-4ac + b^2})c}} + \frac{\sqrt{2} \operatorname{e} \operatorname{arctan}\left(\frac{\sqrt{2} cx}{\sqrt{(b + \sqrt{-4ac + b^2})c}}\right)}{2\sqrt{(b + \sqrt{-4ac + b^2})c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)/(c\*x^4+b\*x^2+a),x)

[Out]  $-\frac{1}{2} * 2^{(1/2)} / ((-b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * e + 1/2 / (-4*a*c + b^2)^{(1/2)} * 2^{(1/2)} / ((-b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b * e - c / (-4*a*c + b^2)^{(1/2)} * 2^{(1/2)} / ((-b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * d + 1/2 * 2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * e + 1/2 / (-4*a*c + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b * e - c / (-4*a*c + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * d$

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2 + d}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.



$$\begin{aligned}
& 2*d*e)) * (- (a*b^3*e^2 - a*e^2 * (- (4*a*c - b^2)^3)^{1/2} + b^3*c*d^2 + c*d^2 * (- (4*a*c - b^2)^3)^{1/2} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e) / (8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^{1/2} * i + ((x*(8*b^3*c^2 - 32*a*b*c^3) * (- (a*b^3*e^2 - a*e^2 * (- (4*a*c - b^2)^3)^{1/2} + b^3*c*d^2 + c*d^2 * (- (4*a*c - b^2)^3)^{1/2} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e) / (8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^{1/2} + 4*b^2*c^2*d - 16*a*c^3*d) * (- (a*b^3*e^2 - a*e^2 * (- (4*a*c - b^2)^3)^{1/2} + b^3*c*d^2 + c*d^2 * (- (4*a*c - b^2)^3)^{1/2} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e) / (8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^{1/2} + x*(4*c^3*d^2 - 4*a*c^2*e^2 + 2*b^2*c*e^2 - 4*b*c^2*d*e)) * (- (a*b^3*e^2 - a*e^2 * (- (4*a*c - b^2)^3)^{1/2} + b^3*c*d^2 + c*d^2 * (- (4*a*c - b^2)^3)^{1/2} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e) / (8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^{1/2} * i) / (((x*(8*b^3*c^2 - 32*a*b*c^3) * (- (a*b^3*e^2 - a*e^2 * (- (4*a*c - b^2)^3)^{1/2} + b^3*c*d^2 + c*d^2 * (- (4*a*c - b^2)^3)^{1/2} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e) / (8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^{1/2} - 4*b^2*c^2*d + 16*a*c^3*d) * (- (a*b^3*e^2 - a*e^2 * (- (4*a*c - b^2)^3)^{1/2} + b^3*c*d^2 + c*d^2 * (- (4*a*c - b^2)^3)^{1/2} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e) / (8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^{1/2} + x*(4*c^3*d^2 - 4*a*c^2*e^2 + 2*b^2*c*e^2 - 4*b*c^2*d*e)) * (- (a*b^3*e^2 - a*e^2 * (- (4*a*c - b^2)^3)^{1/2} + b^3*c*d^2 + c*d^2 * (- (4*a*c - b^2)^3)^{1/2} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e) / (8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^{1/2} - ((x*(8*b^3*c^2 - 32*a*b*c^3) * (- (a*b^3*e^2 - a*e^2 * (- (4*a*c - b^2)^3)^{1/2} + b^3*c*d^2 + c*d^2 * (- (4*a*c - b^2)^3)^{1/2} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e) / (8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^{1/2} + 4*b^2*c^2*d - 16*a*c^3*d) * (- (a*b^3*e^2 - a*e^2 * (- (4*a*c - b^2)^3)^{1/2} + b^3*c*d^2 + c*d^2 * (- (4*a*c - b^2)^3)^{1/2} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e) / (8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^{1/2} + x*(4*c^3*d^2 - 4*a*c^2*e^2 + 2*b^2*c*e^2 - 4*b*c^2*d*e)) * (- (a*b^3*e^2 - a*e^2 * (- (4*a*c - b^2)^3)^{1/2} + b^3*c*d^2 + c*d^2 * (- (4*a*c - b^2)^3)^{1/2} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e) / (8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^{1/2} - ((x*(8*b^3*c^2 - 32*a*b*c^3) * (- (a*b^3*e^2 - a*e^2 * (- (4*a*c - b^2)^3)^{1/2} + b^3*c*d^2 + c*d^2 * (- (4*a*c - b^2)^3)^{1/2} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e) / (8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^{1/2} + 4*b^2*c^2*d - 16*a*c^3*d) * (- (a*b^3*e^2 - a*e^2 * (- (4*a*c - b^2)^3)^{1/2} + b^3*c*d^2 + c*d^2 * (- (4*a*c - b^2)^3)^{1/2} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e) / (8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^{1/2} + x*(4*c^3*d^2 - 4*a*c^2*e^2 + 2*b^2*c*e^2 - 4*b*c^2*d*e)) * (- (a*b^3*e^2 - a*e^2 * (- (4*a*c - b^2)^3)^{1/2} + b^3*c*d^2 + c*d^2 * (- (4*a*c - b^2)^3)^{1/2} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e) / (8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^{1/2} + 2*c^2*d^2*e + 2*a*c*e^3 - 2*b*c*d*e^2)) * (- (a*b^3*e^2 - a*e^2 * (- (4*a*c - b^2)^3)^{1/2} + b^3*c*d^2 + c*d^2 * (- (4*a*c - b^2)^3)^{1/2} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e) / (8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^{1/2} * 2i
\end{aligned}$$

**sympy [A]** time = 20.95, size = 314, normalized size = 1.80

$\text{RootSum}\left(t^4(256a^3c^3 - 128a^2b^2c^2 + 16ab^4c) + t^2(-16a^2bc^2 + 64a^2c^2de + 4ab^3e^2 - 16ab^2cde - 16ab^2c^2d^2 + 4b^3cd^2) + a^2e^4 - 2abde^3 + 2ac^2e^2 + b^2de^2 - 2bcd^2e + c^2d^2, \left(t \mapsto t \log\left(x + \frac{64t^3a^3c^2e - 16t^3a^2b^2c^2e - 32t^3a^2b^2c^2d + 8t^3ab^3cd - 2t^3b^3e^3 + 12t^3cd^2 - 6tabcd^2 - 4ta^2d^2 + 2t^2cd^2}{t^2e^4 - abde^3 + bcd^2e - c^2d^2}\right)\right)\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)/(c\*x\*\*4+b\*x\*\*2+a),x)

[Out]  $\text{RootSum}(\_t**4*(256*a**3*c**3 - 128*a**2*b**2*c**2 + 16*a*b**4*c) + \_t**2*(-16*a**2*b*c*e**2 + 64*a**2*c**2*d*e + 4*a*b**3*e**2 - 16*a*b**2*c*d*e - 16*a*b*c**2*d**2 + 4*b**3*c*d**2) + a**2*e**4 - 2*a*b*d*e**3 + 2*a*c*d**2*e**2 + b**2*d**2*e**2 - 2*b*c*d**3*e + c**2*d**4, \text{Lambda}(\_t, \_t*\log(x + (64*\_t**3*a**3*c**2*e - 16*\_t**3*a**2*b**2*c*e - 32*\_t**3*a**2*b*c**2*d + 8*\_t**3*a*b**3*c*d - 2*\_t*a**2*b*e**3 + 12*\_t*a**2*c*d*e**2 - 6*\_t*a*b*c*d**2*e - 4*\_t*a*c**2*d**3 + 2*\_t*b**2*c*d**3)/ (a**2*e**4 - a*b*d*e**3 + b*c*d**3*e - c**2*d**4))))$

$$3.190 \quad \int \frac{1}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=150

$$\frac{\sqrt{2} \sqrt{c} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac} \sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2} \sqrt{c} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac} \sqrt{\sqrt{b^2-4ac}+b}}$$

Rubi [A] time = 0.10, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {1093, 205}

$$\frac{\sqrt{2} \sqrt{c} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac} \sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2} \sqrt{c} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac} \sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2 + c\*x^4)^(-1), x]

[Out] (Sqrt[2]\*Sqrt[c]\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b - Sqrt[b^2 - 4\*a\*c]]])/(Sqrt[b^2 - 4\*a\*c]\*Sqrt[b - Sqrt[b^2 - 4\*a\*c]]) - (Sqrt[2]\*Sqrt[c]\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b + Sqrt[b^2 - 4\*a\*c]]])/(Sqrt[b^2 - 4\*a\*c]\*Sqrt[b + Sqrt[b^2 - 4\*a\*c]])

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1093

Int[((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(-1), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c\*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && PosQ[b^2 - 4\*a\*c]

Rubi steps

$$\begin{aligned} \int \frac{1}{a+bx^2+cx^4} dx &= \frac{c \int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2} dx}{\sqrt{b^2-4ac}} - \frac{c \int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2} dx}{\sqrt{b^2-4ac}} \\ &= \frac{\sqrt{2} \sqrt{c} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac} \sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2} \sqrt{c} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac} \sqrt{b+\sqrt{b^2-4ac}}} \end{aligned}$$

Mathematica [A] time = 0.09, size = 129, normalized size = 0.86

$$\frac{\sqrt{2} \sqrt{c} \left( \frac{\tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2 + c\*x^4)^(-1), x]

[Out] (Sqrt[2]\*Sqrt[c]\*(ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b - Sqrt[b^2 - 4\*a\*c]]]/Sqrt[b - Sqrt[b^2 - 4\*a\*c]] - ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b + Sqrt[b^2 - 4\*a\*c]]]/Sqrt[b + Sqrt[b^2 - 4\*a\*c]])/Sqrt[b^2 - 4\*a\*c]

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + bx^2 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)^(-1), x]

[Out] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)^(-1), x]

**fricas [B]** time = 0.41, size = 613, normalized size = 4.09

$$\frac{1}{2} \sqrt{\frac{b + \sqrt{b^2 - 4ac}}{ab^2 - 4a^2c}} \log\left(2cx + \sqrt{\frac{b + \sqrt{b^2 - 4ac}}{ab^2 - 4a^2c}} \sqrt{\frac{b - \sqrt{b^2 - 4ac}}{ab^2 - 4a^2c}}\right) + \frac{1}{2} \sqrt{\frac{b - \sqrt{b^2 - 4ac}}{ab^2 - 4a^2c}} \log\left(2cx - \sqrt{\frac{b - \sqrt{b^2 - 4ac}}{ab^2 - 4a^2c}} \sqrt{\frac{b + \sqrt{b^2 - 4ac}}{ab^2 - 4a^2c}}\right) + \frac{1}{2} \sqrt{\frac{b + \sqrt{b^2 - 4ac}}{ab^2 - 4a^2c}} \log\left(2cx + \sqrt{\frac{b + \sqrt{b^2 - 4ac}}{ab^2 - 4a^2c}} \sqrt{\frac{b - \sqrt{b^2 - 4ac}}{ab^2 - 4a^2c}}\right) + \frac{1}{2} \sqrt{\frac{b - \sqrt{b^2 - 4ac}}{ab^2 - 4a^2c}} \log\left(2cx - \sqrt{\frac{b - \sqrt{b^2 - 4ac}}{ab^2 - 4a^2c}} \sqrt{\frac{b + \sqrt{b^2 - 4ac}}{ab^2 - 4a^2c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^4+b\*x^2+a), x, algorithm="fricas")

[Out] -1/2\*sqrt(1/2)\*sqrt(-(b + (a\*b^2 - 4\*a^2\*c)/sqrt(a^2\*b^2 - 4\*a^3\*c)))/(a\*b^2 - 4\*a^2\*c))\*log(2\*c\*x + sqrt(1/2)\*(b^2 - 4\*a\*c - (a\*b^3 - 4\*a^2\*b\*c)/sqrt(a^2\*b^2 - 4\*a^3\*c))\*sqrt(-(b + (a\*b^2 - 4\*a^2\*c)/sqrt(a^2\*b^2 - 4\*a^3\*c)))/(a\*b^2 - 4\*a^2\*c))) + 1/2\*sqrt(1/2)\*sqrt(-(b + (a\*b^2 - 4\*a^2\*c)/sqrt(a^2\*b^2 - 4\*a^3\*c)))/(a\*b^2 - 4\*a^2\*c))\*log(2\*c\*x - sqrt(1/2)\*(b^2 - 4\*a\*c - (a\*b^3 - 4\*a^2\*b\*c)/sqrt(a^2\*b^2 - 4\*a^3\*c))\*sqrt(-(b + (a\*b^2 - 4\*a^2\*c)/sqrt(a^2\*b^2 - 4\*a^3\*c)))/(a\*b^2 - 4\*a^2\*c))) - 1/2\*sqrt(1/2)\*sqrt(-(b - (a\*b^2 - 4\*a^2\*c)/sqrt(a^2\*b^2 - 4\*a^3\*c)))/(a\*b^2 - 4\*a^2\*c))\*log(2\*c\*x + sqrt(1/2)\*(b^2 - 4\*a\*c + (a\*b^3 - 4\*a^2\*b\*c)/sqrt(a^2\*b^2 - 4\*a^3\*c))\*sqrt(-(b - (a\*b^2 - 4\*a^2\*c)/sqrt(a^2\*b^2 - 4\*a^3\*c)))/(a\*b^2 - 4\*a^2\*c))) + 1/2\*sqrt(1/2)\*sqrt(-(b - (a\*b^2 - 4\*a^2\*c)/sqrt(a^2\*b^2 - 4\*a^3\*c)))/(a\*b^2 - 4\*a^2\*c))\*log(2\*c\*x - sqrt(1/2)\*(b^2 - 4\*a\*c + (a\*b^3 - 4\*a^2\*b\*c)/sqrt(a^2\*b^2 - 4\*a^3\*c))\*sqrt(-(b - (a\*b^2 - 4\*a^2\*c)/sqrt(a^2\*b^2 - 4\*a^3\*c)))/(a\*b^2 - 4\*a^2\*c)))

**giac [B]** time = 0.60, size = 1024, normalized size = 6.83

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^4+b\*x^2+a), x, algorithm="giac")

[Out] 1/4\*(sqrt(2)\*sqrt(b\*c + sqrt(b^2 - 4\*a\*c))\*b^4 - 8\*sqrt(2)\*sqrt(b\*c + sqrt(b^2 - 4\*a\*c))\*a\*b^2\*c - 2\*sqrt(2)\*sqrt(b\*c + sqrt(b^2 - 4\*a\*c))\*b^3\*c - 2\*b^4\*c + 16\*sqrt(2)\*sqrt(b\*c + sqrt(b^2 - 4\*a\*c))\*a^2\*c^2 + 8\*sqrt(2)\*sqrt(b\*c + sqrt(b^2 - 4\*a\*c))\*a\*b\*c^2 + sqrt(2)\*sqrt(b\*c + sqrt(b^2 - 4\*a\*c))\*b^2\*c^2 + 16\*a\*b^2\*c^2 - 2\*b^3\*c^2 - 4\*sqrt(2)\*sqrt(b\*c + sqrt(b^2 - 4\*a\*c))\*a\*c^3 - 32\*a^2\*c^3 + 8\*a\*b\*c^3 + sqrt(2)\*sqrt(b^2 - 4\*a\*c)\*sqrt(b\*c + sqrt(b^2 - 4\*a\*c))\*b^3 - 4\*sqrt(2)\*sqrt(b^2 - 4\*a\*c)\*sqrt(b\*c + sqrt(b^2 - 4\*a\*c))\*a\*b\*c - 2\*sqrt(2)\*sqrt(b^2 - 4\*a\*c)\*sqrt(b\*c + sqrt(b^2 - 4\*a\*c))\*b^2\*c + sqrt(2)\*sqrt(b^2 - 4\*a\*c)\*sqrt(b\*c + sqrt(b^2 - 4\*a\*c))\*c\*b\*c^2 + 2\*(b^2 - 4\*a\*c)\*b^2\*c - 8\*(b^2 - 4\*a\*c)\*a\*c^2 + 2\*(b^2 - 4\*a\*c)\*b\*c^2)\*arctan(2\*sqrt(1/2)\*x/sqrt((b + sqrt(b^2 - 4\*a\*c))/c))/((a\*b^4 - 8\*a^2\*b^2\*c - 2\*a\*b^3\*c + 16\*a^3\*c^2 + 8\*a^2\*b\*c^2 + a\*b^2\*c^2 - 4\*a^2\*c^3)\*abs(c)) + 1/4\*(sqrt(2)\*sqrt(b\*c - sqrt(b^2 - 4\*a\*c))\*b^4 - 8\*sqrt(2)\*sqrt(b\*c - sqrt(b^2 - 4\*a\*c))\*a\*b^2\*c - 2\*sqrt(2)\*sqrt(b\*c - sqrt(b^2 - 4\*a\*c))\*b^3\*c - 2\*b^4\*c + 16\*sqrt(2)\*sqrt(b\*c - sqrt(b^2 - 4\*a\*c))\*a^2\*c^2 + 8\*sqrt(2)\*sqrt(b\*c - sqrt(b^2 - 4\*a\*c))\*a\*b\*c^2 + sqrt(2)\*sqrt(b\*c - sqrt(b^2 - 4\*a\*c))\*b^2\*c^2 + 16\*a\*b^2\*c^2 - 2\*b^3\*c^2 - 4\*sqrt(2)\*sqrt(b\*c - sqrt(b^2 - 4\*a\*c))\*a\*c^3 - 32\*a^2\*c^3 + 8\*a\*b\*c^3 + sqrt(2)\*sqrt(b^2 - 4\*a\*c)\*sqrt(b\*c - sqrt(b^2 - 4\*a\*c))\*b^3 - 4\*sqrt(2)\*sqrt(b^2 - 4\*a\*c)\*sqrt(b\*c - sqrt(b^2 - 4\*a\*c))\*a\*b\*c - 2\*sqrt(2)\*sqrt(b^2 - 4\*a\*c)\*sqrt(b\*c - sqrt(b^2 - 4\*a\*c))\*b^2\*c + sqrt(2)\*sqrt(b^2 - 4\*a\*c)\*sqrt(b\*c - sqrt(b^2 - 4\*a\*c))\*c\*b\*c^2 + 2\*(b^2 - 4\*a\*c)\*b^2\*c - 8\*(b^2 - 4\*a\*c)\*a\*c^2 + 2\*(b^2 - 4\*a\*c)\*b\*c^2)\*arctan(2\*sqrt(1/2)\*x/sqrt((b - sqrt(b^2 - 4\*a\*c))/c))/((a\*b^4 - 8\*a^2\*b^2\*c - 2\*a\*b^3\*c + 16\*a^3\*c^2 + 8\*a^2\*b\*c^2 + a\*b^2\*c^2 - 4\*a^2\*c^3)\*abs(c))



$c - \sqrt{b^2 - 4ac} * c) * a * b^2 * c - 2 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac} * c} * b^3 * c + 2 * b^4 * c + 16 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac} * c} * a^2 * c^2 + 8 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac} * c} * a * b * c^2 + \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac} * c} * b^2 * c^2 - 16 * a * b^2 * c^2 - 2 * b^3 * c^2 - 4 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac} * c} * a * c^3 + 32 * a^2 * c^3 + 8 * a * b * c^3 + \sqrt{2} * \sqrt{b^2 - 4ac} * c) * \sqrt{b * c - \sqrt{b^2 - 4ac} * c} * b^3 - 4 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac} * c} * a * b * c - 2 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac} * c} * b^2 * c + \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac} * c} * b * c^2 - 2 * (b^2 - 4ac) * b^2 * c + 8 * (b^2 - 4ac) * a * c^2 + 2 * (b^2 - 4ac) * b * c^2) * \arctan(2 * \sqrt{1/2} * x / \sqrt{(b - \sqrt{b^2 - 4ac}) / c}) / ((a * b^4 - 8 * a^2 * b^2 * c - 2 * a * b^3 * c + 16 * a^3 * c^2 + 8 * a^2 * b * c^2 + a * b^2 * c^2 - 4 * a^2 * c^3) * \text{abs}(c))$

**maple [A]** time = 0.01, size = 116, normalized size = 0.77

$$\frac{\sqrt{2} c \operatorname{arctanh}\left(\frac{\sqrt{2} c x}{\sqrt{(-b + \sqrt{-4ac + b^2})c}}\right)}{\sqrt{-4ac + b^2} \sqrt{(-b + \sqrt{-4ac + b^2})c}} - \frac{\sqrt{2} c \operatorname{arctan}\left(\frac{\sqrt{2} c x}{\sqrt{(b + \sqrt{-4ac + b^2})c}}\right)}{\sqrt{-4ac + b^2} \sqrt{(b + \sqrt{-4ac + b^2})c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x^4+b\*x^2+a),x)

[Out]  $-c/(-4ac + b^2)^{(1/2)} * 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) - c/(-4ac + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * c * x)$

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^4+b\*x^2+a),x, algorithm="maxima")

[Out] integrate(1/(c\*x^4 + b\*x^2 + a), x)

**mupad [B]** time = 0.51, size = 763, normalized size = 5.09

$$\frac{\sqrt{\frac{b^2 x^2 + 2bx + a}{4ac + b^2}} \sqrt{\frac{b^2 x^2 + 2bx + a}{4ac + b^2}}}{\sqrt{4ac + b^2} \sqrt{\frac{b^2 x^2 + 2bx + a}{4ac + b^2}}} - \frac{\sqrt{\frac{b^2 x^2 + 2bx + a}{4ac + b^2}} \sqrt{\frac{b^2 x^2 + 2bx + a}{4ac + b^2}}}{\sqrt{4ac + b^2} \sqrt{\frac{b^2 x^2 + 2bx + a}{4ac + b^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b\*x^2 + c\*x^4),x)

[Out]  $-\operatorname{atan}\left(\frac{b^4 * x * 1i + b * x * (b^6 - 64 * a^3 * c^3 + 48 * a^2 * b^2 * c^2 - 12 * a * b^4 * c)^{(1/2)} * 1i + a^2 * c^2 * x * 16i - a * b^2 * c * x * 8i}{(4 * a * b^4 * (-b^3 + (b^6 - 64 * a^3 * c^3 + 48 * a^2 * b^2 * c^2 - 12 * a * b^4 * c)^{(1/2)} - 4 * a * b * c) / (8 * a * b^4 + 128 * a^3 * c^2 - 64 * a^2 * b^2 * c))^{(1/2)} + 64 * a^3 * c^2 * (-b^3 + (b^6 - 64 * a^3 * c^3 + 48 * a^2 * b^2 * c^2 - 12 * a * b^4 * c)^{(1/2)} - 4 * a * b * c) / (8 * a * b^4 + 128 * a^3 * c^2 - 64 * a^2 * b^2 * c))^{(1/2)} - 32 * a^2 * b^2 * c * (-b^3 + (b^6 - 64 * a^3 * c^3 + 48 * a^2 * b^2 * c^2 - 12 * a * b^4 * c)^{(1/2)} - 4 * a * b * c) / (8 * a * b^4 + 128 * a^3 * c^2 - 64 * a^2 * b^2 * c))^{(1/2)}}{(b^3 + (b^6 - 64 * a^3 * c^3 + 48 * a^2 * b^2 * c^2 - 12 * a * b^4 * c)^{(1/2)} - 4 * a * b * c) / (8 * a * b^4 + 128 * a^3 * c^2 - 64 * a^2 * b^2 * c))^{(1/2)}} * (-b^3 + (b^6 - 64 * a^3 * c^3 + 48 * a^2 * b^2 * c^2 - 12 * a * b^4 * c)^{(1/2)} - 4 * a * b * c) / (8 * a * b^4 + 128 * a^3 * c^2 - 64 * a^2 * b^2 * c))^{(1/2)} * 2i - \operatorname{atan}\left(\frac{b^4 * x * 1i - b * x * (b^6 - 64 * a^3 * c^3 + 48 * a^2 * b^2 * c^2 - 12 * a * b^4 * c)^{(1/2)} * 1i + a^2 * c^2 * x * 16i - a * b^2 * c * x * 8i}{(4 * a * b^4 * ((b^6 - 64 * a^3 * c^3 + 48 * a^2 * b^2 * c^2 - 12 * a * b^4 * c)^{(1/2)} - b^3 + 4 * a * b * c) / (8 * a * b^4 + 128 * a^3 * c^2 - 64 * a^2 * b^2 * c))^{(1/2)} + 64 * a^3 * c^2 * ((b^6 - 64 * a^3 * c^3 + 48 * a^2 * b^2 * c^2 - 12 * a * b^4 * c)^{(1/2)} - b^3 + 4 * a * b * c) / (8 * a * b^4 + 128 * a^3 * c^2 - 64 * a^2 * b^2 * c))^{(1/2)}}{(b^3 + (b^6 - 64 * a^3 * c^3 + 48 * a^2 * b^2 * c^2 - 12 * a * b^4 * c)^{(1/2)} - 4 * a * b * c) / (8 * a * b^4 + 128 * a^3 * c^2 - 64 * a^2 * b^2 * c))^{(1/2)}}\right)$

$$- 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)^{1/2} - b^3 + 4abc)/(8ab^4 + 128a^3c^2 - 64a^2b^2c)^{1/2} - 32a^2b^2c * (((b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)^{1/2} - b^3 + 4abc)/(8ab^4 + 128a^3c^2 - 64a^2b^2c)^{1/2})) * (((b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)^{1/2} - b^3 + 4abc)/(8ab^4 + 128a^3c^2 - 64a^2b^2c)^{1/2}) * 2i$$

**sympy [A]** time = 1.27, size = 87, normalized size = 0.58

$$\text{RootSum}\left(t^4(256a^3c^2 - 128a^2b^2c + 16ab^4) + t^2(-16abc + 4b^3) + c, \left(t \mapsto t \log\left(x + \frac{32t^3a^2bc - 8t^3ab^3 + 4tac - 2tb^2}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x\*\*4+b\*x\*\*2+a),x)

[Out] RootSum(\_t\*\*4\*(256\*a\*\*3\*c\*\*2 - 128\*a\*\*2\*b\*\*2\*c + 16\*a\*b\*\*4) + \_t\*\*2\*(-16\*a\*b\*c + 4\*b\*\*3) + c, Lambda(\_t, \_t\*log(x + (32\*\_t\*\*3\*a\*\*2\*b\*c - 8\*\_t\*\*3\*a\*b\*\*3 + 4\*\_t\*a\*c - 2\*\_t\*b\*\*2)/c)))

$$3.191 \quad \int \frac{1}{(d+ex^2)(a+bx^2+cx^4)} dx$$

**Optimal.** Leaf size=254

$$\frac{\sqrt{c} \left( e - \frac{2cd-be}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left( \frac{\sqrt{2} \sqrt{c} x}{\sqrt{b-\sqrt{b^2-4ac}}} \right) - \sqrt{c} \left( \frac{2cd-be}{\sqrt{b^2-4ac}} + e \right) \tan^{-1} \left( \frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2-4ac}+b}} \right) + \frac{e^{3/2} \tan^{-1} \left( \frac{\sqrt{e} x}{\sqrt{d}} \right)}{\sqrt{d} (ae^2 - bde + cd^2)}}{\sqrt{2} \sqrt{b - \sqrt{b^2 - 4ac}} (ae^2 - bde + cd^2) - \sqrt{2} \sqrt{\sqrt{b^2 - 4ac} + b} (ae^2 - bde + cd^2) + \sqrt{d} (ae^2 - bde + cd^2)}$$

**Rubi [A]** time = 0.59, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1170, 205, 1166}

$$\frac{\sqrt{c} \left( e - \frac{2cd-be}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left( \frac{\sqrt{2} \sqrt{c} x}{\sqrt{b-\sqrt{b^2-4ac}}} \right) - \sqrt{c} \left( \frac{2cd-be}{\sqrt{b^2-4ac}} + e \right) \tan^{-1} \left( \frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2-4ac}+b}} \right) + \frac{e^{3/2} \tan^{-1} \left( \frac{\sqrt{e} x}{\sqrt{d}} \right)}{\sqrt{d} (ae^2 - bde + cd^2)}}{\sqrt{2} \sqrt{b - \sqrt{b^2 - 4ac}} (ae^2 - bde + cd^2) - \sqrt{2} \sqrt{\sqrt{b^2 - 4ac} + b} (ae^2 - bde + cd^2) + \sqrt{d} (ae^2 - bde + cd^2)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x^2)\*(a + b\*x^2 + c\*x^4)),x]

[Out] -((Sqrt[c]\*(e - (2\*c\*d - b\*e)/Sqrt[b^2 - 4\*a\*c])\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b - Sqrt[b^2 - 4\*a\*c]])/(Sqrt[2]\*Sqrt[b - Sqrt[b^2 - 4\*a\*c]]\*(c\*d^2 - b\*d\*e + a\*e^2)) - (Sqrt[c]\*(e + (2\*c\*d - b\*e)/Sqrt[b^2 - 4\*a\*c])\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b + Sqrt[b^2 - 4\*a\*c]])/(Sqrt[2]\*Sqrt[b + Sqrt[b^2 - 4\*a\*c]]\*(c\*d^2 - b\*d\*e + a\*e^2)) + (e^(3/2)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(Sqrt[d]\*(c\*d^2 - b\*d\*e + a\*e^2))

**Rule 205**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 1166**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[e/2 + (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 - q/2 + c\*x^2), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - a\*e^2, 0] && PosQ[b^2 - 4\*a\*c]

**Rule 1170**

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x^2)^q/(a + b\*x^2 + c\*x^4), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IntegerQ[q]

**Rubi steps**

$$\begin{aligned}
\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)} dx &= \int \left( \frac{e^2}{(cd^2-bde+ae^2)(d+ex^2)} + \frac{cd-be-cex^2}{(cd^2-bde+ae^2)(a+bx^2+cx^4)} \right) dx \\
&= \frac{\int \frac{cd-be-cex^2}{a+bx^2+cx^4} dx}{cd^2-bde+ae^2} + \frac{e^2 \int \frac{1}{d+ex^2} dx}{cd^2-bde+ae^2} \\
&= \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(cd^2-bde+ae^2)} - \frac{\left(c\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2} dx}{2(cd^2-bde+ae^2)} - \frac{c\left(e + \frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{2(cd^2-bde+ae^2)} \\
&= -\frac{\sqrt{c}\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}(cd^2-bde+ae^2)} - \frac{\sqrt{c}\left(e + \frac{2cd-be}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}(cd^2-bde+ae^2)}
\end{aligned}$$

**Mathematica [A]** time = 0.27, size = 274, normalized size = 1.08

$$\frac{\sqrt{c}\left(e\sqrt{b^2-4ac}+be-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}(-ae^2+bde-cd^2)} + \frac{\sqrt{c}\left(e\sqrt{b^2-4ac}-be+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b^2-4ac+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b^2-4ac+b}(-ae^2+bde-cd^2)} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x^2)\*(a + b\*x^2 + c\*x^4)), x]

[Out] (Sqrt[c]\*(-2\*c\*d + b\*e + Sqrt[b^2 - 4\*a\*c]\*e)\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b - Sqrt[b^2 - 4\*a\*c]])/(Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[b - Sqrt[b^2 - 4\*a\*c]])\*(-(c\*d^2) + b\*d\*e - a\*e^2) + (Sqrt[c]\*(2\*c\*d - b\*e + Sqrt[b^2 - 4\*a\*c]\*e)\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b + Sqrt[b^2 - 4\*a\*c]])/(Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[b + Sqrt[b^2 - 4\*a\*c]])\*(-(c\*d^2) + b\*d\*e - a\*e^2) + (e^(3/2)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(Sqrt[d]\*(c\*d^2 - b\*d\*e + a\*e^2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e\*x^2)\*(a + b\*x^2 + c\*x^4)), x]

[Out] IntegrateAlgebraic[1/((d + e\*x^2)\*(a + b\*x^2 + c\*x^4)), x]

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)/(c\*x^4+b\*x^2+a), x, algorithm="fricas")

[Out] Timed out

**giac [B]** time = 2.53, size = 7650, normalized size = 30.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)/(c\*x^4+b\*x^2+a), x, algorithm="giac")

```
[Out] 1/8*(2*(2*b^3*c^5 - 8*a*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c))*sqrt(b*c + sqrt(b
^2 - 4*a*c))*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c))*sqrt(b*c + sqrt(b^2 -
4*a*c))*a*b*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c))*sqrt(b*c + sqrt(b^2 - 4*a*c
))*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c))*sqrt(b*c + sqrt(b^2 - 4*a*c))*b*
c^5 - 2*(b^2 - 4*a*c)*b*c^5)*d^5 - 5*(2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqr
t(b^2 - 4*a*c))*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^4*c^2 + 4*sqrt(2)*sqrt(b^2
- 4*a*c))*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 -
4*a*c))*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c))*
sqrt(b*c + sqrt(b^2 - 4*a*c))*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*d^4*e +
2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c))*a*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b
^3*c^3 - 2*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*c^4 + 8
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c))*b^2*c^4 + 16*a*b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c))*a*c^5 - 32*a^2*c^5 + 2*(b^2 - 4*a*c)*b^2*c^3 - 8*(b^2 - 4*a*c)*a*c^
4)*d^3*abs(c*d^2 - b*d*e + a*e^2) + 4*(2*b^5*c^3 - 6*a*b^3*c^4 - 8*a^2*b*c^
5 - sqrt(2)*sqrt(b^2 - 4*a*c))*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^5*c + 3*sqr
t(2)*sqrt(b^2 - 4*a*c))*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^3*c^2 + 2*sqrt(2
)*sqrt(b^2 - 4*a*c))*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^4*c^2 + 4*sqrt(2)*sqr
t(b^2 - 4*a*c))*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b*c^3 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c))*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^2*c^3 - sqrt(2)*sqrt(b^2 -
4*a*c))*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c))*
sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 - 2*(b^2
- 4*a*c)*a*b*c^4)*d^3*e^2 - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^5*
c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c))*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c))*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^2*
c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^3*c^3 + 16*a*b^3*c^3 - 4*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a
*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*d^2*abs(c*d^2 - b*d*e + a*e^2)*e - (
2*b^6*c^2 + 4*a*b^4*c^3 - 48*a^2*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c))*sqrt(b
*c + sqrt(b^2 - 4*a*c))*b^6 - 2*sqrt(2)*sqrt(b^2 - 4*a*c))*sqrt(b*c + sqrt
(b^2 - 4*a*c))*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c))*sqrt(b*c + sqrt(b^2
- 4*a*c))*b^5*c + 24*sqrt(2)*sqrt(b^2 - 4*a*c))*sqrt(b*c + sqrt(b^2 - 4*a*
c))*a^2*b^2*c^2 + 12*sqrt(2)*sqrt(b^2 - 4*a*c))*sqrt(b*c + sqrt(b^2 - 4*a*
c))*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c))*sqrt(b*c + sqrt(b^2 - 4*a*c))*
b^4*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c))*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^
2*c^3 - 2*(b^2 - 4*a*c)*b^4*c^2 - 12*(b^2 - 4*a*c)*a*b^2*c^3)*d^2*e^3 + 2*(
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^6 - 7*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c))*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^5*c - 2*
b^6*c + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b^2*c^2 + 6*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c))*a*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c))*b^4*c^2 + 14*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*
a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b*c^3 - 3*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c))*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*b^4*c -
6*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*d*abs(c*d^2 - b*d*e +
a*e^2)*e^2 - (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*
a*c))*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c))*sqrt
(b*c + sqrt(b^2 - 4*a*c))*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c))*sqrt(b*c
+ sqrt(b^2 - 4*a*c))*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c))*sqrt(b*c + sqrt
(b^2 - 4*a*c))*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c))*sqrt(b*c + sqrt(b^2
- 4*a*c))*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c))*sqrt(b*c + sqrt(b^2 - 4*a*c
))*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c))*sqrt(b*c + sqrt(b^2 - 4*a*c))*
a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*(c*d^2 - b*d*e + a
*e^2)^2*e + 2*(2*a*b^5*c^2 - 6*a^2*b^3*c^3 - 8*a^3*b*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c))*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^5 + 3*sqrt(2)*sqrt(b^2 - 4*a*
c))*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c))*
sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c))*sqrt(
```

$$\begin{aligned}
& b*c + \sqrt{b^2 - 4*a*c}*c)*a^3*b*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c} \\
& + \sqrt{b^2 - 4*a*c}*c)*a^2*b^2*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c} \\
& *a*b^3*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b*c^3 - 2*(b^2 - 4*a*c)*a^2 \\
& *b*c^3)*d*e^4 - 2*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^5 - 8*\sqrt{2} \\
& )*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^3*c - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^4*c \\
& - 2*a*b^5*c + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b*c^2 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c} \\
& *a^2*b^2*c^2 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^3*c^2 + 16*a^2*b^3*c^2 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c} \\
& *a^2*b*c^3 - 32*a^3*b*c^3 + 2*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^2*b*c^2)*\text{abs}(c*d^2 - b*d*e + a*e^2)*e^3 - (2*a^2 \\
& *b^4*c^2 - 8*a^3*b^2*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^4 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c} \\
& *\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b^2*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^3*c \\
& - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2)*e^5)*\arctan(2*\sqrt{1/2})*x/\sqrt{((b*c \\
& *d^2 - b^2*d*e + a*b*e^2 + \sqrt{(b*c*d^2 - b^2*d*e + a*b*e^2)^2 - 4*(a*c*d^2 - a*b*d*e + a^2*e^2)}*(c^2*d^2 - b*c*d*e \\
& + a*c*e^2)))/((a*b^4*c^2 - 8*a^2*b^2*c^3 - 2*a*b^3*c^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a*b^2*c^4 - 4*a^2*c^5)*d^4*\text{abs}(c*d^2 - b*d*e + a*e^2)*\text{abs}(c) - 2*( \\
& a*b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4)*d^3*\text{abs}(c*d^2 - b*d*e + a*e^2)*\text{abs}(c)*e + (a*b^6 - 6*a \\
& ^2*b^4*c - 2*a*b^5*c + 4*a^2*b^3*c^2 + a*b^4*c^2 + 32*a^4*c^3 + 16*a^3*b*c^3 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2*\text{abs}(c*d^2 - b*d*e + a*e^2)*\text{abs}(c)*e^2 - \\
& 2*(a^2*b^5 - 8*a^3*b^3*c - 2*a^2*b^4*c + 16*a^4*b*c^2 + 8*a^3*b^2*c^2 + a^2*b^3*c^2 - 4*a^3*b*c^3)*d*\text{abs}(c*d^2 - b*d*e + a*e^2)*\text{abs}(c)*e^3 + (a^3*b^4 \\
& - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*\text{abs}(c*d^2 - b*d*e + a*e^2)*\text{abs}(c)*e^4) - 1/8*(2*(2*b^3*c^5 - 8*a*b*c \\
& ^6 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^3*c^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b*c^4 + 2*\sqrt{2} \\
& )*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^2*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b*c^5 - 2*(b^2 - 4*a*c)*b*c^5 \\
& )*d^5 - 5*(2*b^4*c^4 - 8*a*b^2*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^4*c^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c} \\
& )*a*b^2*c^3 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^3*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c} \\
& )*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*d^4*e - 2*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^4*c^2 - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^2*c \\
& ^3 - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^3*c^3 + 2*b^4*c^3 + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*c^4 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c} \\
& )*a*b*c^4 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^2*c^4 - 16*a*b^2*c^4 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*c^5 + 32*a^2*c^5 \\
& - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*d^3*\text{abs}(c*d^2 - b*d*e + a*e^2) + 4*(2*b^5*c^3 - 6*a*b^3*c^4 - 8*a^2*b*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c} \\
& )*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^5*c + 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^3*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c} \\
& )*b^4*c^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b*c^3 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^2*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c} \\
& *\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^3*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 - 2*(b^2 - 4*a*c)*a*b*c^4)*d^3*e^2 + \\
& 4*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^5*c - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^3*c^2 - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^4*c^2 + 2*b^5*c^2 + 16*\sqrt{2} \\
& *\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b*c^3 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^2*c^3 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^3*c^3 - 16*a*b^3*c^3 - 4*\sqrt{2} \\
& *\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*d^2*\text{abs}(c*d^2 - b*d*e + a*e^2)*e - (2*b^6*c^2 + 4*a*b^4*c^3 - 4 \\
& 8*a^2*b^2*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b
\end{aligned}$$







$$\begin{aligned}
& c^2e^4 + a^2b^6d^2e^2 - 2a^2b^5d^3e^3 - 8a^2b^2c^3d^4 + 32a^4c^3d^2e^2 - 2a^2b^5c^3d^3e - 32a^3b^3c^3d^3e + 16a^3b^3c^3d^3e - 32a^4b^3c^2d^3e^3 + 16a^2b^3c^2d^3e - 6a^2b^4c^3d^2e^2))^{(1/2)} * (256a^4b^2c^3e^9 - 32a^3b^4c^2e^9 - 512a^5c^4e^9 + 512a^2c^7d^6e^3 + 512a^3c^6d^4e^5 - 512a^4c^5d^2e^7 - 32b^3c^6d^7e^2 + 128b^4c^5d^6e^3 - 192b^5c^4d^5e^4 + 128b^6c^3d^4e^5 - 32b^7c^2d^3e^6 + 512a^2b^2c^5d^4e^5 + 288a^2b^3c^4d^3e^6 - 192a^2b^4c^3d^2e^7 + 384a^3b^2c^4d^2e^7 + 128a^2b^3c^7d^7e^2 + 640a^4b^3c^4d^2e^8 - 640a^2b^2c^6d^6e^3 + 1056a^2b^3c^5d^5e^4 - 672a^2b^4c^4d^4e^5 + 96a^2b^5c^3d^3e^6 + 32a^2b^6c^2d^2e^7 - 1152a^2b^3c^6d^5e^4 + 32a^2b^5c^2d^2e^8 - 640a^3b^3c^5d^3e^6 - 288a^3b^3c^3d^2e^8) - 256a^4c^4e^8 + 64a^2c^7d^6e^2 - 16a^2b^4c^2e^8 + 128a^3b^2c^3e^8 - 128a^2c^6d^4e^4 - 448a^3c^5d^2e^6 - 16b^2c^6d^6e^2 + 64b^3c^5d^5e^3 - 96b^4c^4d^4e^4 + 64b^5c^3d^3e^5 - 16b^6c^2d^2e^6 + 240a^2b^2c^4d^2e^6 - 256a^2b^3c^6d^5e^3 + 32a^2b^5c^2d^2e^7 + 384a^3b^3c^4d^2e^7 + 416a^2b^2c^5d^4e^4 - 288a^2b^3c^4d^3e^5 + 32a^2b^4c^3d^2e^6 + 128a^2b^3c^5d^3e^5 - 224a^2b^3c^3d^2e^7)) * (- (b^5e^2 + b^3c^2d^2 + b^2e^2 * (- (4ac - b^2)^3)^{(1/2)} + c^2d^2 * (- (4ac - b^2)^3)^{(1/2)} + 12a^2b^3c^2e^2 - 2b^4c^3d^2 - 4a^2b^3c^3d^2 - 7a^2b^3c^3e^2 - a^2c^2e^2 * (- (4ac - b^2)^3)^{(1/2)} - 16a^2c^3d^2e + 12a^2b^2c^2d^2e - 2b^2c^2d^2e * (- (4ac - b^2)^3)^{(1/2)})) / (8 * (a^3b^4e^4 + 16a^3c^4d^4 + 16a^5c^2e^4 + a^2b^4c^2d^4 - 8a^4b^2c^2e^4 + a^2b^6d^2e^2 - 2a^2b^5d^3e^3 - 8a^2b^2c^3d^4 + 32a^4c^3d^2e^2 - 2a^2b^5c^3d^3e - 32a^3b^3c^3d^3e + 16a^3b^3c^3d^3e - 32a^4b^3c^2d^3e - 6a^2b^4c^3d^2e^2))^{(1/2)} - 4b^3c^3e^6 - 4c^6d^3e^3 + 4b^3c^5d^2e^4 + 4b^2c^4d^2e^5 + 16a^2b^3c^4e^6 - 20a^2c^5d^2e^5) + 6c^5e^5x) * (- (b^5e^2 + b^3c^2d^2 + b^2e^2 * (- (4ac - b^2)^3)^{(1/2)} + c^2d^2 * (- (4ac - b^2)^3)^{(1/2)} + 12a^2b^3c^2e^2 - 2b^4c^3d^2 - 4a^2b^3c^3d^2 - 7a^2b^3c^3e^2 - a^2c^2e^2 * (- (4ac - b^2)^3)^{(1/2)} - 16a^2c^3d^2e + 12a^2b^2c^2d^2e - 2b^2c^2d^2e * (- (4ac - b^2)^3)^{(1/2)})) / (8 * (a^3b^4e^4 + 16a^3c^4d^4 + 16a^5c^2e^4 + a^2b^4c^2d^4 - 8a^4b^2c^2e^4 + a^2b^6d^2e^2 - 2a^2b^5d^3e^3 - 8a^2b^2c^3d^4 + 32a^4c^3d^2e^2 - 2a^2b^5c^3d^3e - 32a^3b^3c^3d^3e + 16a^3b^3c^3d^3e - 32a^4b^3c^2d^3e - 6a^2b^4c^3d^2e^2))^{(1/2)} * ((x * (16b^5c^2e^7 + 16c^7d^5e^2 - 112a^2b^3c^3e^7 + 192a^2b^3c^4e^7 + 32a^2c^6d^3e^4 - 240a^2c^5d^2e^6 - 32b^3c^6d^4e^3 - 32b^4c^3d^2e^6 + 16b^2c^5d^3e^4 + 16b^3c^4d^2e^5 - 96a^2b^3c^5d^2e^5 + 192a^2b^2c^4d^2e^6) - (- (b^5e^2 + b^3c^2d^2 + b^2e^2 * (- (4ac - b^2)^3)^{(1/2)} + c^2d^2 * (- (4ac - b^2)^3)^{(1/2)} + 12a^2b^3c^2e^2 - 2b^4c^3d^2 - 4a^2b^3c^3d^2 - 7a^2b^3c^3e^2 - a^2c^2e^2 * (- (4ac - b^2)^3)^{(1/2)} - 16a^2c^3d^2e + 12a^2b^2c^2d^2e - 2b^2c^2d^2e * (- (4ac - b^2)^3)^{(1/2)})) / (8 * (a^3b^4e^4 + 16a^3c^4d^4 + 16a^5c^2e^4 + a^2b^4c^2d^4 - 8a^4b^2c^2e^4 + a^2b^6d^2e^2 - 2a^2b^5d^3e^3 - 8a^2b^2c^3d^4 + 32a^4c^3d^2e^2 - 2a^2b^5c^3d^3e - 32a^3b^3c^3d^3e + 16a^3b^3c^3d^3e - 32a^4b^3c^2d^3e + 16a^2b^3c^2d^3e - 6a^2b^4c^3d^2e^2))^{(1/2)} * (256a^4c^4e^8 + x * (- (b^5e^2 + b^3c^2d^2 + b^2e^2 * (- (4ac - b^2)^3)^{(1/2)} + c^2d^2 * (- (4ac - b^2)^3)^{(1/2)} + 12a^2b^3c^2e^2 - 2b^4c^3d^2 - 4a^2b^3c^3d^2 - 7a^2b^3c^3e^2 - a^2c^2e^2 * (- (4ac - b^2)^3)^{(1/2)} - 16a^2c^3d^2e + 12a^2b^2c^2d^2e - 2b^2c^2d^2e * (- (4ac - b^2)^3)^{(1/2)})) / (8 * (a^3b^4e^4 + 16a^3c^4d^4 + 16a^5c^2e^4 + a^2b^4c^2d^4 - 8a^4b^2c^2e^4 + a^2b^6d^2e^2 - 2a^2b^5d^3e^3 - 8a^2b^2c^3d^4 + 32a^4c^3d^2e^2 - 2a^2b^5c^3d^3e - 32a^3b^3c^3d^3e + 16a^3b^3c^3d^3e - 32a^4b^3c^2d^3e + 16a^2b^3c^2d^3e
\end{aligned}$$

$$\begin{aligned}
& - 6*a^2*b^4*c*d^2*e^2))^{(1/2)}*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - \\
& 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5* \\
& d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + \\
& 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^ \\
& 2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128 \\
& *a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3 \\
& *c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2* \\
& d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3 \\
& *e^6 - 288*a^3*b^3*c^3*d*e^8) - 64*a*c^7*d^6*e^2 + 16*a^2*b^4*c^2*e^8 - 128 \\
& *a^3*b^2*c^3*e^8 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6 + 16*b^2*c^6*d \\
& ^6*e^2 - 64*b^3*c^5*d^5*e^3 + 96*b^4*c^4*d^4*e^4 - 64*b^5*c^3*d^3*e^5 + 16* \\
& b^6*c^2*d^2*e^6 - 240*a^2*b^2*c^4*d^2*e^6 + 256*a*b*c^6*d^5*e^3 - 32*a*b^5* \\
& c^2*d*e^7 - 384*a^3*b*c^4*d*e^7 - 416*a*b^2*c^5*d^4*e^4 + 288*a*b^3*c^4*d^3 \\
& *e^5 - 32*a*b^4*c^3*d^2*e^6 - 128*a^2*b*c^5*d^3*e^5 + 224*a^2*b^3*c^3*d*e^7 \\
& ))*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + c^2*d^2*(- \\
& (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7 \\
& *a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 16*a^2*c^3*d*e + 12*a*b^2 \\
& *c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^3*b^4*e^4 + 16*a^3*c^4 \\
& *d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2 \\
& *a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - \\
& 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c \\
& ^2*d^3*e - 6*a^2*b^4*c*d^2*e^2))^{(1/2)} + 4*b^3*c^3*e^6 + 4*c^6*d^3*e^3 - \\
& 4*b*c^5*d^2*e^4 - 4*b^2*c^4*d*e^5 - 16*a*b*c^4*e^6 + 20*a*c^5*d*e^5) + 6*c^ \\
& 5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + c^2* \\
& d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d \\
& ^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 16*a^2*c^3*d*e + 12 \\
& *a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^3*b^4*e^4 + 16*a \\
& ^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e \\
& ^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d \\
& ^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^ \\
& 2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2))^{(1/2)}*1i)/(((b^5*e^2 + b^3*c^2*d \\
& ^2 + b^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + \\
& 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(- \\
& (4*a*c - b^2)^3)^{(1/2)} - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4 \\
& *a*c - b^2)^3)^{(1/2)})/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a \\
& *b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^ \\
& 2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16* \\
& a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d \\
& ^2*e^2))^{(1/2)}*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + \\
& 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 \\
& - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5* \\
& d^2*e^5 + 192*a*b^2*c^4*d*e^6) - ((b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a* \\
& c - b^2)^3)^{(1/2)} + c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*e^2 - 2 \\
& *b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^{(1/ \\
& 2)} - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2) \\
& )/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4 \\
& *b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c \\
& ^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32 \\
& *a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2))^{(1/2)}*(x*( \\
& -(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + c^2*d^2*(-(4*a \\
& *c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b \\
& ^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 16*a^2*c^3*d*e + 12*a*b^2*c^2 \\
& *d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 \\
& + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2 \\
& *b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32* \\
& a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2* \\
& d^3*e - 6*a^2*b^4*c*d^2*e^2))^{(1/2)}*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2* \\
& e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4 \\
& *c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4
\end{aligned}$$

$$\begin{aligned}
&^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 2 \\
&88*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 \\
&+ 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056* \\
&a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6 \\
&*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^ \\
&5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 256*a^4*c^4*e^8 + 64*a*c^7*d^6*e^2 - 1 \\
&6*a^2*b^4*c^2*e^8 + 128*a^3*b^2*c^3*e^8 - 128*a^2*c^6*d^4*e^4 - 448*a^3*c^5 \\
&*d^2*e^6 - 16*b^2*c^6*d^6*e^2 + 64*b^3*c^5*d^5*e^3 - 96*b^4*c^4*d^4*e^4 + 6 \\
&4*b^5*c^3*d^3*e^5 - 16*b^6*c^2*d^2*e^6 + 240*a^2*b^2*c^4*d^2*e^6 - 256*a*b* \\
&c^6*d^5*e^3 + 32*a*b^5*c^2*d*e^7 + 384*a^3*b*c^4*d*e^7 + 416*a*b^2*c^5*d^4* \\
&e^4 - 288*a*b^3*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2*e^6 + 128*a^2*b*c^5*d^3*e^5 \\
&- 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2 \\
&)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c* \\
&d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16 \\
&*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a \\
&^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c* \\
&e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2* \\
&e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b* \\
&c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - 4*b^3*c^3 \\
&*e^6 - 4*c^6*d^3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2*c^4*d*e^5 + 16*a*b*c^4*e^6 - \\
&20*a*c^5*d*e^5) + 6*c^5*e^5*x))*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c \\
&- b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2* \\
&b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) \\
&) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2)) \\
&/ (8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c* \\
&b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^ \\
&3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32* \\
&a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - ((- \\
&(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a* \\
&c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^ \\
&3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2* \\
&d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 \\
&+ 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2* \\
&b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a \\
&^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d \\
&^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - \\
&112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^ \\
&6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d \\
&^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d \\
&^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + \\
&12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(- \\
&(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4 \\
&*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a \\
&*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^ \\
&2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16* \\
&a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d \\
&^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-( \\
&4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 \\
&- 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3) \\
&^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^( \\
&1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8 \\
&*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a \\
&^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 \\
&- 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)* \\
&(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d \\
&^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 1 \\
&28*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2 \\
&*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4* \\
&c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4
\end{aligned}$$

$$\begin{aligned}
& *d^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4 \\
& *e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 \\
& + 32*a^2*b^5*c^2*d^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d^8 - 64*a*c^7*d^6*e^2 \\
& + 16*a^2*b^4*c^2*e^8 - 128*a^3*b^2*c^3*e^8 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6 \\
& + 16*b^2*c^6*d^6*e^2 - 64*b^3*c^5*d^5*e^3 + 96*b^4*c^4*d^4*e^4 - 64*b^5*c^3*d^3*e^5 \\
& + 16*b^6*c^2*d^2*e^6 - 240*a^2*b^2*c^4*d^2*e^6 + 256*a*b*c^6*d^5*e^3 - 32*a*b^5*c^2*d^7 \\
& - 384*a^3*b*c^4*d^7 - 416*a*b^2*c^5*d^4*e^4 + 288*a*b^3*c^4*d^3*e^5 - 32*a*b^4*c^3*d^2*e^6 \\
& - 128*a^2*b*c^5*d^3*e^5 + 224*a^2*b^3*c^3*d^7)) * (- (b^5*e^2 + b^3*c^2*d^2 + b^2*e^2 * (- (4*a*c - b^2)^3)^{1/2} \\
& + c^2*d^2 * (- (4*a*c - b^2)^3)^{1/2} + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2 * (- (4*a*c - b^2)^3)^{1/2} \\
& - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e * (- (4*a*c - b^2)^3)^{1/2}) / (8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 \\
& - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d^3*e - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e \\
& + 16*a^3*b^3*c*d^3*e - 32*a^4*b*c^2*d^3*e + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2))^{1/2} + 4*b^3*c^3*e^6 + 4*c^6*d^3*e^3 - 4*b*c^5*d^2*e^4 - 4*b^2*c^4*d^5 \\
& - 16*a*b*c^4*e^6 + 20*a*c^5*d^5*e^5) + 6*c^5*e^5*x) * (- (b^5*e^2 + b^3*c^2*d^2 + b^2*e^2 * (- (4*a*c - b^2)^3)^{1/2} \\
& + c^2*d^2 * (- (4*a*c - b^2)^3)^{1/2} + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2 * (- (4*a*c - b^2)^3)^{1/2} \\
& - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e * (- (4*a*c - b^2)^3)^{1/2}) / (8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 \\
& - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d^3*e - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e \\
& + 16*a^3*b^3*c*d^3*e - 32*a^4*b*c^2*d^3*e + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2))^{1/2} * 2i + \operatorname{atan}((( - (b^5*e^2 + b^3*c^2*d^2 - b^2*e^2 * (- (4*a*c - b^2)^3)^{1/2} - c^2*d^2 * (- (4*a*c - b^2)^3)^{1/2} \\
& + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2 * (- (4*a*c - b^2)^3)^{1/2} - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e * (- (4*a*c - b^2)^3)^{1/2}) / (8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d^3*e - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d^3*e - 32*a^4*b*c^2*d^3*e + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2))^{1/2} * ((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d^6) - (- (b^5*e^2 + b^3*c^2*d^2 - b^2*e^2 * (- (4*a*c - b^2)^3)^{1/2} - c^2*d^2 * (- (4*a*c - b^2)^3)^{1/2} + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2 * (- (4*a*c - b^2)^3)^{1/2} - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e * (- (4*a*c - b^2)^3)^{1/2}) / (8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d^3*e - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d^3*e - 32*a^4*b*c^2*d^3*e + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2))^{1/2} * (256*a^4*b^2*c^3*e^9 - 3
\end{aligned}$$

$$\begin{aligned}
& 2*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4 \\
& *e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192 \\
& *b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c \\
& ^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2 \\
& *c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d \\
& ^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3* \\
& e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 \\
& - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 256*a^4*c^4*e^8 + 64*a*c \\
& ^7*d^6*e^2 - 16*a^2*b^4*c^2*e^8 + 128*a^3*b^2*c^3*e^8 - 128*a^2*c^6*d^4*e^4 \\
& - 448*a^3*c^5*d^2*e^6 - 16*b^2*c^6*d^6*e^2 + 64*b^3*c^5*d^5*e^3 - 96*b^4*c \\
& ^4*d^4*e^4 + 64*b^5*c^3*d^3*e^5 - 16*b^6*c^2*d^2*e^6 + 240*a^2*b^2*c^4*d^2* \\
& e^6 - 256*a*b*c^6*d^5*e^3 + 32*a*b^5*c^2*d*e^7 + 384*a^3*b*c^4*d*e^7 + 416* \\
& a*b^2*c^5*d^4*e^4 - 288*a*b^3*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2*e^6 + 128*a^2* \\
& b*c^5*d^3*e^5 - 224*a^2*b^3*c^3*d*e^7)) * (- (b^5*e^2 + b^3*c^2*d^2 - b^2*e^2 * \\
& (- (4*a*c - b^2)^3)^{1/2} - c^2*d^2 * (- (4*a*c - b^2)^3)^{1/2} + 12*a^2*b*c^2* \\
& e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2 * (- (4*a*c - b^2) \\
& ^3)^{1/2} - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e * (- (4*a*c - b^2)^3 \\
& )^{1/2}) / (8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 \\
& - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 3 \\
& 2*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e \\
& ^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2))^{1/2} \\
& - 4*b^3*c^3*e^6 - 4*c^6*d^3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2*c^4*d*e^5 + 16 \\
& *a*b*c^4*e^6 - 20*a*c^5*d*e^5) + 6*c^5*e^5*x) * (- (b^5*e^2 + b^3*c^2*d^2 - b^2 \\
& *e^2 * (- (4*a*c - b^2)^3)^{1/2} - c^2*d^2 * (- (4*a*c - b^2)^3)^{1/2} + 12*a^2*b \\
& *c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2 * (- (4*a*c \\
& - b^2)^3)^{1/2} - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e * (- (4*a*c - \\
& b^2)^3)^{1/2}) / (8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2 \\
& *d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 \\
& + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3 \\
& *c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2) \\
& ))^{1/2} * i + ((- (b^5*e^2 + b^3*c^2*d^2 - b^2*e^2 * (- (4*a*c - b^2)^3)^{1/2} \\
& - c^2*d^2 * (- (4*a*c - b^2)^3)^{1/2} + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b \\
& *c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2 * (- (4*a*c - b^2)^3)^{1/2} - 16*a^2*c^3*d* \\
& e + 12*a*b^2*c^2*d*e + 2*b*c*d*e * (- (4*a*c - b^2)^3)^{1/2}) / (8*(a^3*b^4*e^4 \\
& + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6 \\
& *d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b \\
& ^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + \\
& 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2))^{1/2} * ((x*(16*b^5*c^2*e^7 + \\
& 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - \\
& 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e \\
& ^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (- (b^5 \\
& *e^2 + b^3*c^2*d^2 - b^2*e^2 * (- (4*a*c - b^2)^3)^{1/2} - c^2*d^2 * (- (4*a*c \\
& - b^2)^3)^{1/2} + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c \\
& *e^2 + a*c*e^2 * (- (4*a*c - b^2)^3)^{1/2} - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e \\
& + 2*b*c*d*e * (- (4*a*c - b^2)^3)^{1/2}) / (8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 1 \\
& 6*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5 \\
& *d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3* \\
& b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3* \\
& e - 6*a^2*b^4*c*d^2*e^2))^{1/2} * (256*a^4*c^4*e^8 + x * (- (b^5*e^2 + b^3*c^2*d^2 \\
& - b^2*e^2 * (- (4*a*c - b^2)^3)^{1/2} - c^2*d^2 * (- (4*a*c - b^2)^3)^{1/2} + \\
& 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2 * (- \\
& (4*a*c - b^2)^3)^{1/2} - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e * (- ( \\
& 4*a*c - b^2)^3)^{1/2}) / (8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + \\
& a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2 \\
& *c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16 \\
& *a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c* \\
& d^2*e^2))^{1/2} * (256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 \\
& + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3 \\
& *c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4
\end{aligned}$$

$$\begin{aligned}
& *e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 \\
& - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 \\
& + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - \\
& 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152* \\
& a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3* \\
& b^3*c^3*d*e^8) - 64*a*c^7*d^6*e^2 + 16*a^2*b^4*c^2*e^8 - 128*a^3*b^2*c^3*e^8 \\
& + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6 + 16*b^2*c^6*d^6*e^2 - 64*b^3 \\
& *c^5*d^5*e^3 + 96*b^4*c^4*d^4*e^4 - 64*b^5*c^3*d^3*e^5 + 16*b^6*c^2*d^2*e^6 \\
& - 240*a^2*b^2*c^4*d^2*e^6 + 256*a*b*c^6*d^5*e^3 - 32*a*b^5*c^2*d*e^7 - 384 \\
& *a^3*b*c^4*d*e^7 - 416*a*b^2*c^5*d^4*e^4 + 288*a*b^3*c^4*d^3*e^5 - 32*a*b^4 \\
& *c^3*d^2*e^6 - 128*a^2*b*c^5*d^3*e^5 + 224*a^2*b^3*c^3*d*e^7)) * (- (b^5*e^2 + \\
& b^3*c^2*d^2 - b^2*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} - c^2*d^2 * (- (4*a*c - b^2)^3 \\
& )^{(1/2)} + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + \\
& a*c*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b* \\
& c*d*e * (- (4*a*c - b^2)^3)^{(1/2)}) / (8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2 \\
& *e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 \\
& - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3 \\
& *e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2 \\
& *b^4*c*d^2*e^2))^{(1/2)} + 4*b^3*c^3*e^6 + 4*c^6*d^3*e^3 - 4*b*c^5*d^2*e^4 \\
& - 4*b^2*c^4*d*e^5 - 16*a*b*c^4*e^6 + 20*a*c^5*d*e^5) + 6*c^5*e^5*x) * (- (b^5 \\
& *e^2 + b^3*c^2*d^2 - b^2*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} - c^2*d^2 * (- (4*a*c - \\
& b^2)^3)^{(1/2)} + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c* \\
& e^2 + a*c*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e \\
& + 2*b*c*d*e * (- (4*a*c - b^2)^3)^{(1/2)}) / (8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16 \\
& *a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5* \\
& d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b \\
& *c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e \\
& - 6*a^2*b^4*c*d^2*e^2))^{(1/2)} * i) / (((- (b^5*e^2 + b^3*c^2*d^2 - b^2*e^2 * (- \\
& (4*a*c - b^2)^3)^{(1/2)} - c^2*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*e^2 \\
& - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2 * (- (4*a*c - b^2)^3 \\
& )^{(1/2)} - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e * (- (4*a*c - b^2)^3)^{(1/2)} \\
& ) / (8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - \\
& 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32* \\
& a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 \\
& - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2))^{(1/2)} \\
& * ((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 \\
& + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d \\
& *e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a \\
& *b^2*c^4*d*e^6) - (- (b^5*e^2 + b^3*c^2*d^2 - b^2*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} \\
& - c^2*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4* \\
& a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 16*a^2*c^3 \\
& *d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e * (- (4*a*c - b^2)^3)^{(1/2)}) / (8*(a^3*b^4*e^4 \\
& + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a* \\
& b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2* \\
& a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 \\
& + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2))^{(1/2)} * (x * (- (b^5*e^2 + b^3 \\
& *c^2*d^2 - b^2*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} - c^2*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} \\
& + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c* \\
& e^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d* \\
& e * (- (4*a*c - b^2)^3)^{(1/2)}) / (8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 \\
& + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8* \\
& a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e \\
& + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4 \\
& *c*d^2*e^2))^{(1/2)} * (256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4 \\
& *e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - \\
& 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3 \\
& *d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4* \\
& d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7 \\
& *e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^5
\end{aligned}$$

$$\begin{aligned}
&^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - \\
&1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288 \\
&*a^3*b^3*c^3*d*e^8) - 256*a^4*c^4*e^8 + 64*a*c^7*d^6*e^2 - 16*a^2*b^4*c^2*e \\
&^8 + 128*a^3*b^2*c^3*e^8 - 128*a^2*c^6*d^4*e^4 - 448*a^3*c^5*d^2*e^6 - 16*b \\
&^2*c^6*d^6*e^2 + 64*b^3*c^5*d^5*e^3 - 96*b^4*c^4*d^4*e^4 + 64*b^5*c^3*d^3*e \\
&^5 - 16*b^6*c^2*d^2*e^6 + 240*a^2*b^2*c^4*d^2*e^6 - 256*a*b*c^6*d^5*e^3 + 3 \\
&2*a*b^5*c^2*d*e^7 + 384*a^3*b*c^4*d*e^7 + 416*a*b^2*c^5*d^4*e^4 - 288*a*b^3 \\
&*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2*e^6 + 128*a^2*b*c^5*d^3*e^5 - 224*a^2*b^3*c \\
&^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^ \\
&2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3 \\
&*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + \\
&12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16 \\
&*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2 \\
&*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c \\
&*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16* \\
&a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - 4*b^3*c^3*e^6 - 4*c^6*d^ \\
&3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2*c^4*d*e^5 + 16*a*b*c^4*e^6 - 20*a*c^5*d*e^5 \\
&) + 6*c^5*e^5*x))*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) \\
&) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a \\
&*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3* \\
&d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^ \\
&4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b \\
&^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a \\
&*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 \\
& + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - (((-b^5*e^2 + b^3* \\
&c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/ \\
&2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e \\
&^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e \\
&*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^ \\
&4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a \\
&^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e \\
& + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^ \\
&4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e \\
&^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^ \\
&4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b \\
&*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(- \\
&(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^ \\
&2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3 \\
&)^^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^( \\
&1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - \\
&8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32* \\
&a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 \\
& - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) \\
&*(256*a^4*c^4*e^8 + x*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3) \\
&^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e \\
&- 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2 \\
&*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b \\
&^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 \\
& + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 \\
&- 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2* \\
&d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^ \\
&3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^ \\
&3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6* \\
&e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512* \\
&a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 3 \\
&84*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a* \\
&b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5 \\
&*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c \\
&^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 64*a*c^7*d^6*e^
\end{aligned}$$

$$\begin{aligned}
& 2 + 16a^2b^4c^2e^8 - 128a^3b^2c^3e^8 + 128a^2c^6d^4e^4 + 448a^3c^5d^2e^6 + 16b^2c^6d^6e^2 - 64b^3c^5d^5e^3 + 96b^4c^4d^4e^4 \\
& - 64b^5c^3d^3e^5 + 16b^6c^2d^2e^6 - 240a^2b^2c^4d^2e^6 + 256a^2b^2c^6d^5e^3 - 32a^2b^5c^2d^2e^7 - 384a^3b^2c^4d^2e^7 - 416a^2b^2c^5d^4e^4 \\
& + 288a^2b^3c^4d^3e^5 - 32a^2b^4c^3d^2e^6 - 128a^2b^2c^5d^3e^5 + 224a^2b^3c^3d^2e^7) * (- (b^5e^2 + b^3c^2d^2 - b^2e^2 * (- (4ac - b^2)^3)^{1/2} - c^2d^2 * (- (4ac - b^2)^3)^{1/2} + 12a^2b^2c^2e^2 - 2b^4c^2d^2e - 4a^2b^2c^3d^2 - 7a^2b^3c^2e^2 + a^2c^2e^2 * (- (4ac - b^2)^3)^{1/2} - 16a^2c^3d^2e + 12a^2b^2c^2d^2e + 2b^2c^2d^2e * (- (4ac - b^2)^3)^{1/2}) / \\
& (8(a^3b^4e^4 + 16a^3c^4d^4 + 16a^5c^2e^4 + a^2b^4c^2d^4 - 8a^4b^2c^2e^4 + a^2b^6d^2e^2 - 2a^2b^5d^2e^3 - 8a^2b^2c^3d^4 + 32a^4c^3d^2e^2 - 2a^2b^5c^2d^3e - 32a^3b^2c^3d^3e + 16a^3b^3c^2d^3e - 32a^4b^2c^2d^3e + 16a^2b^3c^2d^3e - 6a^2b^4c^2d^2e^2))^{1/2} + 4b^3c^3e^6 + 4c^6d^3e^3 - 4b^2c^5d^2e^4 - 4b^2c^4d^2e^5 - 16a^2b^2c^4e^6 + 20a^2c^5d^2e^5 + 6c^5e^5x) * (- (b^5e^2 + b^3c^2d^2 - b^2e^2 * (- (4ac - b^2)^3)^{1/2} - c^2d^2 * (- (4ac - b^2)^3)^{1/2} + 12a^2b^2c^2e^2 - 2b^4c^2d^2e - 4a^2b^2c^3d^2 - 7a^2b^3c^2e^2 + a^2c^2e^2 * (- (4ac - b^2)^3)^{1/2} - 16a^2c^3d^2e + 12a^2b^2c^2d^2e + 2b^2c^2d^2e * (- (4ac - b^2)^3)^{1/2}) / \\
& (8(a^3b^4e^4 + 16a^3c^4d^4 + 16a^5c^2e^4 + a^2b^4c^2d^4 - 8a^4b^2c^2e^4 + a^2b^6d^2e^2 - 2a^2b^5d^2e^3 - 8a^2b^2c^3d^4 + 32a^4c^3d^2e^2 - 2a^2b^5c^2d^3e - 32a^3b^2c^3d^3e + 16a^3b^3c^2d^3e - 32a^4b^2c^2d^3e + 16a^2b^3c^2d^3e - 6a^2b^4c^2d^2e^2))^{1/2} * (- (b^5e^2 + b^3c^2d^2 - b^2e^2 * (- (4ac - b^2)^3)^{1/2} - c^2d^2 * (- (4ac - b^2)^3)^{1/2} + 12a^2b^2c^2e^2 - 2b^4c^2d^2e - 4a^2b^2c^3d^2 - 7a^2b^3c^2e^2 + a^2c^2e^2 * (- (4ac - b^2)^3)^{1/2} - 16a^2c^3d^2e + 12a^2b^2c^2d^2e + 2b^2c^2d^2e * (- (4ac - b^2)^3)^{1/2}) / \\
& (8(a^3b^4e^4 + 16a^3c^4d^4 + 16a^5c^2e^4 + a^2b^4c^2d^4 - 8a^4b^2c^2e^4 + a^2b^6d^2e^2 - 2a^2b^5d^2e^3 - 8a^2b^2c^3d^4 + 32a^4c^3d^2e^2 - 2a^2b^5c^2d^3e - 32a^3b^2c^3d^3e + 16a^3b^3c^2d^3e - 32a^4b^2c^2d^3e + 16a^2b^3c^2d^3e - 6a^2b^4c^2d^2e^2))^{1/2} * 2i - (\log(b^5d * (-d^3)^{5/2}) - b^5d^3e^8x + c^5d^8e^3x + 2a^2c^4d^5 * (-d^3)^{3/2} - 16a^3c^2e * (-d^3)^{5/2} - c^5d^8e * (-d^3)^{1/2} + b^2c^3d^5 * (-d^3)^{3/2} - a^2b^4e * (-d^3)^{5/2} - 7a^2b^3c^2d * (-d^3)^{5/2} + 17a^2c^3d^3e^2 * (-d^3)^{3/2} + a^2b^4d^2e^9x + 2a^2c^4d^6e^5x - 2b^2c^4d^7e^4x + 2b^4c^2d^4e^7x + 12a^2b^2c^2d * (-d^3)^{5/2} + 8a^2b^2c^2e * (-d^3)^{5/2} + 17a^2c^3d^4e^7x + 16a^3c^2d^2e^9x + b^2c^3d^6e^5x - b^3c^2d^5e^6x - b^3c^2d^4e * (-d^3)^{3/2} + 2b^4c^2d^3e^2 * (-d^3)^{3/2} + 2b^2c^4d^7e^2 * (-d^3)^{1/2} - 12a^2b^2c^2d^4e^7x - 12a^2b^2c^2d^3e^8x - 8a^2b^2c^2d^2e^9x - 12a^2b^2c^2d^3e^2 * (-d^3)^{3/2} + 2a^2b^2c^3d^5e^6x + 7a^2b^3c^2d^3e^8x + 2a^2b^2c^3d^4e * (-d^3)^{3/2}) * (-d^3)^{1/2}) / (2(c^2d^3 + a^2d^2e - b^2d^2e)) + (\log(b^5d * (-d^3)^{5/2}) + b^5d^3e^8x - c^5d^8e^3x + 2a^2c^4d^5 * (-d^3)^{3/2} - 16a^3c^2e * (-d^3)^{5/2} - c^5d^8e * (-d^3)^{1/2} + b^2c^3d^5 * (-d^3)^{3/2} - a^2b^4e * (-d^3)^{5/2} - 7a^2b^3c^2d * (-d^3)^{5/2} + 17a^2c^3d^3e^2 * (-d^3)^{3/2} - a^2b^4d^2e^9x - 2a^2c^4d^6e^5x + 2b^2c^4d^7e^4x - 2b^4c^2d^4e^7x + 12a^2b^2c^2d * (-d^3)^{5/2} + 8a^2b^2c^2e * (-d^3)^{5/2} - 17a^2c^3d^4e^7x - 16a^3c^2d^2e^9x - b^2c^3d^6e^5x + b^3c^2d^5e^6x - b^3c^2d^4e * (-d^3)^{3/2} + 2b^4c^2d^3e^2 * (-d^3)^{3/2} + 2b^2c^4d^7e^2 * (-d^3)^{1/2} + 12a^2b^2c^2d^4e^7x + 12a^2b^2c^2d^3e^8x + 8a^2b^2c^2d^2e^9x - 12a^2b^2c^2d^3e^2 * (-d^3)^{3/2} - 2a^2b^2c^3d^5e^6x - 7a^2b^3c^2d^3e^8x + 2a^2b^2c^3d^4e * (-d^3)^{3/2}) * (-d^3)^{1/2}) / (2c^2d^3 + 2a^2d^2e - 2b^2d^2e)
\end{aligned}$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x\*\*2+d)/(c\*x\*\*4+b\*x\*\*2+a),x)



[Out] Timed out

$$3.192 \quad \int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)} dx$$

**Optimal.** Leaf size=429

$$\frac{\sqrt{c} \left( -2ce \left( d\sqrt{b^2 - 4ac} + ae + bd \right) + be^2 \left( \sqrt{b^2 - 4ac} + b \right) + 2c^2 d^2 \right) \tan^{-1} \left( \frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) \sqrt{c} \left( -2ce \left( -d\sqrt{b^2 - 4ac} \right) \right)}{\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b - \sqrt{b^2 - 4ac}} (ae^2 - bde + cd^2)^2} \sqrt{2} \sqrt{c}$$

**Rubi [A]** time = 1.41, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1170, 199, 205, 1166}

$$\frac{\sqrt{c} \left( -2ce \left( d\sqrt{b^2 - 4ac} + ae + bd \right) + be^2 \left( \sqrt{b^2 - 4ac} + b \right) + 2c^2 d^2 \right) \tan^{-1} \left( \frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) \sqrt{c} \left( -2ce \left( -d\sqrt{b^2 - 4ac} \right) \right)}{\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b - \sqrt{b^2 - 4ac}} (ae^2 - bde + cd^2)^2} \frac{e^x}{2d(d+cx^2)(a^2 - bde + cd^2)} + \frac{e^{3/2} \tan^{-1} \left( \frac{\sqrt{c} x}{\sqrt{d}} \right)}{2d^{3/2} (a^2 - bde + cd^2)} + \frac{e^{3/2} (2cd - be) \tan^{-1} \left( \frac{\sqrt{c} x}{\sqrt{d}} \right)}{\sqrt{d} (a^2 - bde + cd^2)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x^2)^2\*(a + b\*x^2 + c\*x^4)),x]

[Out] (e^2\*x)/(2\*d\*(c\*d^2 - b\*d\*e + a\*e^2)\*(d + e\*x^2)) + (Sqrt[c]\*(2\*c^2\*d^2 + b\*(b + Sqrt[b^2 - 4\*a\*c])\*e^2 - 2\*c\*e\*(b\*d + Sqrt[b^2 - 4\*a\*c]\*d + a\*e))\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b - Sqrt[b^2 - 4\*a\*c]]]/(Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[b - Sqrt[b^2 - 4\*a\*c]]\*(c\*d^2 - b\*d\*e + a\*e^2)^2) - (Sqrt[c]\*(2\*c^2\*d^2 + b\*(b - Sqrt[b^2 - 4\*a\*c])\*e^2 - 2\*c\*e\*(b\*d - Sqrt[b^2 - 4\*a\*c]\*d + a\*e))\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b + Sqrt[b^2 - 4\*a\*c]]]/(Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[b + Sqrt[b^2 - 4\*a\*c]]\*(c\*d^2 - b\*d\*e + a\*e^2)^2) + (e^(3/2)\*(2\*c\*d - b\*e)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(Sqrt[d]\*(c\*d^2 - b\*d\*e + a\*e^2)^2) + (e^(3/2)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(2\*d^(3/2)\*(c\*d^2 - b\*d\*e + a\*e^2))

#### Rule 199

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2\*p] || (n == 2 && IntegerQ[4\*p]) || (n == 2 && IntegerQ[3\*p]) || Denominator[p + 1/n] < Denominator[p])

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 1166

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[e/2 + (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 - q/2 + c\*x^2), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - a\*e^2, 0] && PosQ[b^2 - 4\*a\*c]

#### Rule 1170

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x^2)^q/(a + b\*x^2 + c\*x^4), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IntegerQ[q]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)} dx &= \int \left( \frac{e^2}{(cd^2-bde+ae^2)(d+ex^2)^2} - \frac{e^2(-2cd+be)}{(cd^2-bde+ae^2)^2(d+ex^2)} + \frac{c^2d^2+b}{(cd^2-bde+ae^2)^2} \right) dx \\
&= \frac{\int \frac{c^2d^2+b^2e^2-ce(2bd+ae)-ce(2cd-be)x^2}{a+bx^2+cx^4} dx}{(cd^2-bde+ae^2)^2} + \frac{(e^2(2cd-be)) \int \frac{1}{d+ex^2} dx}{(cd^2-bde+ae^2)^2} + \frac{e^2 \int \frac{1}{(d+ex^2)^2} dx}{cd^2-bde+ae^2} \\
&= \frac{e^2x}{2d(cd^2-bde+ae^2)(d+ex^2)} + \frac{e^{3/2}(2cd-be) \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(cd^2-bde+ae^2)^2} + \frac{e^2 \int \frac{1}{d+ex^2} dx}{2d(cd^2-bde+ae^2)} \\
&= \frac{e^2x}{2d(cd^2-bde+ae^2)(d+ex^2)} + \frac{\sqrt{c}\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(b+\sqrt{b^2-4ac}\right)\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}
\end{aligned}$$

**Mathematica [A]** time = 0.75, size = 354, normalized size = 0.83

$$\frac{\sqrt{2}\sqrt{c}\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}-b\right)-2c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{e^2x\left(e\left(ae-bd\right)+cd^2\right)}{d\left(d+ex^2\right)} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e\left(ae-3bd\right)+5cd^2\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x^2)^2\*(a + b\*x^2 + c\*x^4)), x]

[Out] ((e^2\*(c\*d^2 + e\*(-(b\*d) + a\*e))\*x)/(d\*(d + e\*x^2)) + (Sqrt[2]\*Sqrt[c]\*(2\*c\*d^2 + b\*(b + Sqrt[b^2 - 4\*a\*c]))\*e^2 - 2\*c\*e\*(b\*d + Sqrt[b^2 - 4\*a\*c]\*d + a\*e))\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b - Sqrt[b^2 - 4\*a\*c]]]/(Sqrt[b^2 - 4\*a\*c]\*Sqrt[b - Sqrt[b^2 - 4\*a\*c]]) + (Sqrt[2]\*Sqrt[c]\*(-2\*c\*d^2 + b\*(-b + Sqrt[b^2 - 4\*a\*c]))\*e^2 + 2\*c\*e\*(b\*d - Sqrt[b^2 - 4\*a\*c]\*d + a\*e))\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b + Sqrt[b^2 - 4\*a\*c]]]/(Sqrt[b^2 - 4\*a\*c]\*Sqrt[b + Sqrt[b^2 - 4\*a\*c]]) + (e^(3/2)\*(5\*c\*d^2 + e\*(-3\*b\*d + a\*e))\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/d^(3/2))/(2\*(c\*d^2 + e\*(-(b\*d) + a\*e))^2)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e\*x^2)^2\*(a + b\*x^2 + c\*x^4)), x]

[Out] IntegrateAlgebraic[1/((d + e\*x^2)^2\*(a + b\*x^2 + c\*x^4)), x]

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)^2/(c\*x^4+b\*x^2+a), x, algorithm="fricas")

[Out] Timed out

**giac [B]** time = 2.51, size = 13225, normalized size = 30.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(5*c*d^2*e^2 - 3*b*d*e^3 + a*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/((
c^2*d^5 - 2*b*c*d^4*e + b^2*d^3*e^2 + 2*a*c*d^3*e^2 - 2*a*b*d^2*e^3 + a^2*d
*e^4)*sqrt(d)) - 2*(2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^3 - b^7
*c^3 - 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - 11*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^4 + 12*a*b^5*c^4 + 3*b^6*c^4 + 96*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 + 88*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^3*c^5 - 48*a^2*b^3*c^5 + 16*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^4*c^5 - 28*a*b^4*c^5 + 5*b^5*c^5 - 128*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^3*c^6 - 176*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b*c^6 + 64*a^3*b*c^6 - 80*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*
c^6 + 80*a^2*b^2*c^6 - 7*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^6 -
24*a*b^3*c^6 - 11*b^4*c^6 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
c^7 - 64*a^3*c^7 + 44*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^7 + 16*
a^2*b*c^7 - 8*a*b^2*c^7 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^8 +
80*a^2*c^8 + 16*a*b*c^8 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^5*c^3 + sqrt(b^2 - 4*a*c)*b^6*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 11*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 12*sqrt(b^2 - 4*a*c)*a*b^4*c^4
- 5*sqrt(b^2 - 4*a*c)*b^5*c^4 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b*c^5 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^5 + 48*sqrt(b^2 - 4*a*c)*a^2*b^2*c^5 - 16*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 40*sqrt(b^2 - 4*
a*c)*a*b^3*c^5 + 7*sqrt(b^2 - 4*a*c)*b^4*c^5 + 48*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^6 - 64*sqrt(b^2 - 4*a*c)*a^3*c^6 + 3
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 80*sq
rt(b^2 - 4*a*c)*a^2*b*c^6 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c^6 - 56*sqrt(b^2 - 4*a*c)*a*b^2*c^6 - 3*sqrt(b^2 - 4*a*c)
*b^3*c^6 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c
^7 + 112*sqrt(b^2 - 4*a*c)*a^2*c^7 + 60*sqrt(b^2 - 4*a*c)*a*b*c^7 - 24*sqrt
(b^2 - 4*a*c)*a*c^8 + 2*(b^2 - 4*a*c)*b^4*c^4 - 16*(b^2 - 4*a*c)*a*b^2*c^5
- 12*(b^2 - 4*a*c)*b^3*c^5 + 32*(b^2 - 4*a*c)*a^2*c^6 + 48*(b^2 - 4*a*c)*a*
b*c^6 + 14*(b^2 - 4*a*c)*b^2*c^6 + 8*(b^2 - 4*a*c)*a*c^7)*arctan(2*sqrt(1/2
)*x/sqrt((b*c^2*d^4 - 2*b^2*c*d^3*e + b^3*d^2*e^2 + 2*a*b*c*d^2*e^2 - 2*a*b
^2*d*e^3 + a^2*b*e^4 + sqrt((b*c^2*d^4 - 2*b^2*c*d^3*e + b^3*d^2*e^2 + 2*a*
b*c*d^2*e^2 - 2*a*b^2*d*e^3 + a^2*b*e^4)^2 - 4*(a*c^2*d^4 - 2*a*b*c*d^3*e +
a*b^2*d^2*e^2 + 2*a^2*c*d^2*e^2 - 2*a^2*b*d*e^3 + a^3*e^4)*(c^3*d^4 - 2*b*
c^2*d^3*e + b^2*c*d^2*e^2 + 2*a*c^2*d^2*e^2 - 2*a*b*c*d*e^3 + a^2*c*e^4)))/
(c^3*d^4 - 2*b*c^2*d^3*e + b^2*c*d^2*e^2 + 2*a*c^2*d^2*e^2 - 2*a*b*c*d*e^3
+ a^2*c*e^4)))/((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c - 16*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^2 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^7*c^2 - 2*b^8*c^2 + 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^4*c^3 + 60*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 + 7*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^3 + 16*a*b^6*c^3 + 8*b^7*c^3 -
256*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 - 240*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 - 84*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^4*c^4 - 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^4 - 32
*a*b^5*c^4 - 6*b^6*c^4 + 256*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^
5 + 320*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 + 336*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 - 256*a^3*b^2*c^5 + 72*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 - 128*a^2*b^3*c^5 - 24*a*b^4*c^5 - 4
48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^6 + 512*a^4*c^6 - 240*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 + 512*a^3*b*c^6 - 24*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^6 + 224*a^2*b^2*c^6 + 96*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^7 - 128*a^3*c^7 - 16*a*b^2*c^7 + 64*a^2
```

$$\begin{aligned}
& *c^8 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*b^7*c + 12 \\
& *\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a*b^5*c^2 + 5*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*b^6*c^2 - 48*\sqrt{2} \\
& *\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^2*b^3*c^3 - 52*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a*b^4*c^3 - 7*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*b^5*c^3 + 64*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^3*b*c^4 + 176*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^2*b^2*c^4 + 72*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a*b^3*c^4 + 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*b^4*c^4 - 24*\sqrt{b^2 - 4*a*c}*a*b^4*c^4 - 192*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^3*c^5 - 176*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^2*b*c^5 - 56*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a*b^2*c^5 + 192*\sqrt{b^2 - 4*a*c}*a^2*b^2*c^5 + 32*\sqrt{b^2 - 4*a*c}*a*b^3*c^5 + 48*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^2*c^6 - 384*\sqrt{b^2 - 4*a*c}*a^3*c^6 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a*b*c^6 - 128*\sqrt{b^2 - 4*a*c}*a^2*b*c^6 + 8*\sqrt{b^2 - 4*a*c}*a*b^2*c^6 + 96*\sqrt{b^2 - 4*a*c}*a^2*c^7 - 16*\sqrt{b^2 - 4*a*c}*a*b*c^7 + 2*(b^2 - 4*a*c)*b^6*c^2 - 24*(b^2 - 4*a*c)*a*b^4*c^3 - 8*(b^2 - 4*a*c)*b^5*c^3 + 96*(b^2 - 4*a*c)*a^2*b^2*c^4 + 64*(b^2 - 4*a*c)*a*b^3*c^4 + 6*(b^2 - 4*a*c)*b^4*c^4 - 128*(b^2 - 4*a*c)*a^3*c^5 - 128*(b^2 - 4*a*c)*a^2*b*c^5 - 48*(b^2 - 4*a*c)*a*b^2*c^5 + 96*(b^2 - 4*a*c)*a^2*c^6 + 64*(b^2 - 4*a*c)*a*b*c^6)*d^2*abs(c) + 4*(3*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a*b^6*c^2 + 4*a*b^7*c^2 - 36*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^2*b^4*c^3 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a*b^5*c^3 - 48*a^2*b^5*c^3 - 10*a*b^6*c^3 + 144*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^3*b^2*c^4 + 32*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^2*b^3*c^4 + 192*a^3*b^3*c^4 - \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a*b^4*c^4 + 56*a^2*b^4*c^4 + 8*a*b^5*c^4 - 192*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^4*c^5 - 64*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^3*b*c^5 - 256*a^4*b*c^5 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^2*b^2*c^5 + 32*a^3*b^2*c^5 + 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a*b^3*c^5 - 6*a*b^4*c^5 + 48*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^3*c^6 - 384*a^4*c^6 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^2*b*c^6 - 128*a^3*b*c^6 + 16*a^2*b^2*c^6 + 8*a*b^3*c^6 + 32*a^3*c^7 - 32*a^2*b*c^7 + \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a*b^6*c - 12*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^2*b^4*c^2 - 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a*b^5*c^2 + 4*\sqrt{b^2 - 4*a*c}*a*b^6*c^2 + 48*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^3*b^2*c^3 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^2*b^3*c^3 + 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a*b^4*c^3 - 48*\sqrt{b^2 - 4*a*c}*a^2*b^4*c^3 - 10*\sqrt{b^2 - 4*a*c}*a*b^5*c^3 - 64*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^4*c^4 - 32*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^3*b*c^4 - 40*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^2*b^2*c^4 + 192*\sqrt{b^2 - 4*a*c}*a^3*b^2*c^4 - 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a*b^3*c^4 + 80*\sqrt{b^2 - 4*a*c}*a^2*b^3*c^4 + 16*\sqrt{b^2 - 4*a*c}*a*b^4*c^4 + 112*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^3*c^5 - 256*\sqrt{b^2 - 4*a*c}*a^4*c^5 + 48*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^2*b*c^5 - 160*\sqrt{b^2 - 4*a*c}*a^3*b*c^5 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a*b^2*c^5 - 96*\sqrt{b^2 - 4*a*c}*a^2*b^2*c^5 - 18*\sqrt{b^2 - 4*a*c}*a*b^3*c^5 - 24*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^2*c^6 + 128*\sqrt{b^2 - 4*a*c}*a^3*c^6 + 40*\sqrt{b^2 - 4*a*c}*a^2*b*c^6 + 8*\sqrt{b^2 - 4*a*c}*a*b^2*c^6 - 16*\sqrt{b^2 - 4*a*c}*a^2*c^7 + 8*(b^2 - 4*a*c)*a*b^3*c^4 - 32*(b^2 - 4*a*c)*a^2*b*c^5 - 12*(b^2 - 4*a*c)*a*b^2*c^5 - 16*(b^2 - 4*a*c)*a^2*c^6)*d*abs(c)*e - (\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a*b^8 - 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^2*b^6*c + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a*b^7*c + 6*a*b^8*c + 96*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^3*b^4*c^2 - 12*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}
\end{aligned}$$

$$\begin{aligned}
& *a*c)*c)*a^2*b^5*c^2 - \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a*b^6*c^2 - \\
& 80*a^2*b^6*c^2 - 12*a*b^7*c^2 - 256*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c) \\
& *a^4*b^2*c^3 + 48*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^3*b^3*c^3 - 20* \\
& \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^2*b^4*c^3 + 384*a^3*b^4*c^3 - 5*s \\
& \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a*b^5*c^3 + 80*a^2*b^5*c^3 + 10*a*b^ \\
& 6*c^3 + 256*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^5*c^4 - 64*\sqrt{2}*\sqrt{ \\
& \sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^4*b*c^4 + 208*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - \\
& 4*a*c})*c)*a^3*b^2*c^4 - 768*a^4*b^2*c^4 + 56*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4 \\
& *a*c})*c)*a^2*b^3*c^4 - 64*a^3*b^3*c^4 + 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4 \\
& *a*c})*c)*a*b^4*c^4 - 24*a^2*b^4*c^4 - 12*a*b^5*c^4 - 448*\sqrt{2}*\sqrt{b*c + \\
& \sqrt{b^2 - 4*a*c})*c)*a^4*c^5 + 512*a^5*c^5 - 144*\sqrt{2}*\sqrt{b*c + \sqrt{b \\
& ^2 - 4*a*c})*c)*a^3*b*c^5 - 256*a^4*b*c^5 - 40*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - \\
& 4*a*c})*c)*a^2*b^2*c^5 - 32*a^3*b^2*c^5 + 32*a^2*b^3*c^5 + 16*a*b^4*c^5 + 9 \\
& 6*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^3*c^6 - 128*a^4*c^6 + 64*a^3*b* \\
& c^6 - 80*a^2*b^2*c^6 + 64*a^3*c^7 + \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{ \\
& b^2 - 4*a*c})*c)*a*b^7 - 12*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 \\
& - 4*a*c})*c)*a^2*b^5*c + \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4* \\
& a*c})*c)*a*b^6*c + 8*\sqrt{b^2 - 4*a*c})*a*b^7*c + 48*\sqrt{2}*\sqrt{b^2 - 4*a*c} \\
& )*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^3*b^3*c^2 - 20*\sqrt{2}*\sqrt{b^2 - 4*a*c} \\
& )*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^2*b^4*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{ \\
& b*c + \sqrt{b^2 - 4*a*c})*c)*a*b^5*c^2 - 96*\sqrt{b^2 - 4*a*c})*a^2*b^5*c^2 \\
& - 20*\sqrt{b^2 - 4*a*c})*a*b^6*c^2 - 64*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \\
& \sqrt{b^2 - 4*a*c})*c)*a^4*b*c^3 + 112*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \\
& \sqrt{b^2 - 4*a*c})*c)*a^3*b^2*c^3 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{ \\
& b^2 - 4*a*c})*c)*a^2*b^3*c^3 + 384*\sqrt{b^2 - 4*a*c})*a^3*b^3*c^3 - 5*\sqrt{ \\
& 2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a*b^4*c^3 + 136*\sqrt{ \\
& (b^2 - 4*a*c)*a^2*b^4*c^3 + 32*\sqrt{b^2 - 4*a*c})*a*b^5*c^3 - 192*\sqrt{2}*\sqrt{ \\
& b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^4*c^4 + 48*\sqrt{2}*\sqrt{b \\
& ^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^3*b*c^4 - 512*\sqrt{b^2 - 4*a* \\
& c})*a^4*b*c^4 + 40*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c) \\
& *a^2*b^2*c^4 - 128*\sqrt{b^2 - 4*a*c})*a^3*b^2*c^4 + 4*\sqrt{2}*\sqrt{b^2 - 4*a \\
& *c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a*b^3*c^4 - 160*\sqrt{b^2 - 4*a*c})*a^2*b \\
& ^3*c^4 - 36*\sqrt{b^2 - 4*a*c})*a*b^4*c^4 + 48*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{ \\
& (b*c + \sqrt{b^2 - 4*a*c})*c)*a^3*c^5 - 384*\sqrt{b^2 - 4*a*c})*a^4*c^5 - 32*\sqrt{ \\
& 2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^2*b*c^5 + 128*\sqrt{ \\
& (b^2 - 4*a*c)*a^3*b*c^5 + 88*\sqrt{b^2 - 4*a*c})*a^2*b^2*c^5 + 16*\sqrt{b^2 - \\
& 4*a*c})*a*b^3*c^5 + 96*\sqrt{b^2 - 4*a*c})*a^3*c^6 - 48*\sqrt{b^2 - 4*a*c})*a^2 \\
& *b*c^6 + 2*(b^2 - 4*a*c)*a*b^6*c - 24*(b^2 - 4*a*c)*a^2*b^4*c^2 - 8*(b^2 - \\
& 4*a*c)*a*b^5*c^2 + 96*(b^2 - 4*a*c)*a^3*b^2*c^3 + 64*(b^2 - 4*a*c)*a^2*b^3* \\
& c^3 + 22*(b^2 - 4*a*c)*a*b^4*c^3 - 128*(b^2 - 4*a*c)*a^4*c^4 - 128*(b^2 - 4 \\
& *a*c)*a^3*b*c^4 - 112*(b^2 - 4*a*c)*a^2*b^2*c^4 - 24*(b^2 - 4*a*c)*a*b^3*c^ \\
& 4 + 96*(b^2 - 4*a*c)*a^3*c^5 + 32*(b^2 - 4*a*c)*a^2*b*c^5)*abs(c)*e^2) + 2* \\
& (2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*b^6*c^3 + b^7*c^3 - 24*\sqrt{2})*\sqrt{ \\
& b*c - \sqrt{b^2 - 4*a*c})*c)*a*b^4*c^4 - 11*\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4 \\
& *a*c})*c)*b^5*c^4 - 12*a*b^5*c^4 - 3*b^6*c^4 + 96*\sqrt{2})*\sqrt{b*c - \sqrt{ \\
& b^2 - 4*a*c})*c)*a^2*b^2*c^5 + 88*\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a* \\
& b^3*c^5 + 48*a^2*b^3*c^5 + 16*\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*b^4*c \\
& ^5 + 28*a*b^4*c^5 - 5*b^5*c^5 - 128*\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c) \\
& *a^3*c^6 - 176*\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^2*b*c^6 - 64*a^3*b \\
& *c^6 - 80*\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a*b^2*c^6 - 80*a^2*b^2*c^ \\
& 6 - 7*\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*b^3*c^6 + 24*a*b^3*c^6 + 11*b \\
& ^4*c^6 + 64*\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^2*c^7 + 64*a^3*c^7 + \\
& 44*\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a*b*c^7 - 16*a^2*b*c^7 + 8*a*b^2 \\
& *c^7 - 8*\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a*c^8 - 80*a^2*c^8 - 16*a* \\
& b*c^8 + 2*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*b^5*c^3 \\
& + \sqrt{b^2 - 4*a*c})*b^6*c^3 - 16*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{ \\
& b^2 - 4*a*c})*c)*a*b^3*c^4 - 11*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b \\
& ^2 - 4*a*c})*c)*b^4*c^4 - 12*\sqrt{b^2 - 4*a*c})*a*b^4*c^4 - 5*\sqrt{b^2 - 4*a* \\
& c})*b^5*c^4 + 32*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a
\end{aligned}$$

$$\begin{aligned}
& ^2*b*c^5 + 56*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b \\
& ^2*c^5 + 48*\sqrt{b^2 - 4*a*c})*a^2*b^2*c^5 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^3*c^5 + 40*\sqrt{b^2 - 4*a*c})*a*b^3*c^5 + 7* \\
& \sqrt{b^2 - 4*a*c})*b^4*c^5 - 48*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*c^6 - 64*\sqrt{b^2 - 4*a*c})*a^3*c^6 - 32*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b*c^6 - 80*\sqrt{b^2 - 4*a*c})*a^2 \\
& *b*c^6 - 7*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^2*c^6 - 56*\sqrt{b^2 - 4*a*c})*a*b^2*c^6 - 3*\sqrt{b^2 - 4*a*c})*b^3*c^6 + 12*\sqrt{2} \\
& *\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*c^7 + 112*\sqrt{b^2 - 4*a*c})*a^2*c^7 + 60*\sqrt{b^2 - 4*a*c})*a*b*c^7 - 24*\sqrt{b^2 - 4*a*c})*a*c^8 \\
& - 2*(b^2 - 4*a*c)*b^4*c^4 + 16*(b^2 - 4*a*c)*a*b^2*c^5 + 12*(b^2 - 4*a*c)* \\
& b^3*c^5 - 32*(b^2 - 4*a*c)*a^2*c^6 - 48*(b^2 - 4*a*c)*a*b*c^6 - 14*(b^2 - 4 \\
& *a*c)*b^2*c^6 - 8*(b^2 - 4*a*c)*a*c^7)*\arctan(2*\sqrt{1/2})*x/\sqrt{((b*c^2*d^4 \\
& - 2*b^2*c*d^3*e + b^3*d^2*e^2 + 2*a*b*c*d^2*e^2 - 2*a*b^2*d*e^3 + a^2*b*e^4 \\
& - \sqrt{(b*c^2*d^4 - 2*b^2*c*d^3*e + b^3*d^2*e^2 + 2*a*b*c*d^2*e^2 - 2*a*b^2*d*e^3 + a^2*b*e^4)^2 - 4*(a*c^2*d^4 - 2*a*b*c*d^3*e + a*b^2*d^2*e^2 + 2* \\
& a^2*c*d^2*e^2 - 2*a^2*b*d*e^3 + a^3*e^4)*(c^3*d^4 - 2*b*c^2*d^3*e + b^2*c*d^2 \\
& ^2*e^2 + 2*a*c^2*d^2*e^2 - 2*a*b*c*d*e^3 + a^2*c*e^4)))/(c^3*d^4 - 2*b*c^2*d^3 \\
& *e + b^2*c*d^2*e^2 + 2*a*c^2*d^2*e^2 - 2*a*b*c*d*e^3 + a^2*c*e^4)))/((\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^8*c - 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^6*c^2 - 5*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^7*c^2 \\
& + 2*b^8*c^2 + 96*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b^4*c^3 + 60* \\
& \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^5*c^3 + 7*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^6*c^3 - 16*a*b^6*c^3 - 8*b^7*c^3 - 256*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^3*b^2*c^4 - 240*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b^3*c^4 - 84*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^4*c^4 - 3*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^5*c^4 + 32*a*b^5*c^4 + 6*b^6*c^4 + 256*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^4*c^5 + 320*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^3*b*c^5 + 336*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b^2*c^5 + 256*a^3*b^2*c^5 + 72*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^3*c^5 + 128*a^2*b^3*c^5 + 24*a*b^4*c^5 - 448*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^3*c^6 - 512*a^4*c^6 - 240*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b*c^6 - 512*a^3*b*c^6 - 24*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^2*c^6 - 224*a^2*b^2*c^6 + 96*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*c^7 + 128*a^3*c^7 + 16*a*b^2*c^7 - 64*a^2*c^8 + \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^7*c - 12*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^5*c^2 - 5*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^6*c^2 + 48*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b^3*c^3 + 52*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^4*c^3 + 7*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^5*c^3 - 64*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^3*b*c^4 - 176*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b^2*c^4 - 72*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^3*c^4 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^4*c^4 - 24*\sqrt{b^2 - 4*a*c})*a*b^4*c^4 + 192*\sqrt{2} \\
& *)\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^3*c^5 + 176*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b*c^5 + 56*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^2*c^5 + 192*\sqrt{b^2 - 4*a*c})*a^2*b^2*c^5 + 32*\sqrt{b^2 - 4*a*c})*a*b^3*c^5 - 48*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*c^6 - 384*\sqrt{b^2 - 4*a*c})*a^3*c^6 - 16*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b*c^6 - 128*\sqrt{b^2 - 4*a*c})*a^2*b*c^6 + 8*\sqrt{b^2 - 4*a*c})*a*b^2*c^6 + 96*\sqrt{b^2 - 4*a*c})*a^2*c^7 - 16*\sqrt{b^2 - 4*a*c})*a*b*c^7 - 2*(b^2 - 4*a*c)*b^6*c^2 + 24*(b^2 - 4*a*c)*a*b^4*c^3 + 8*(b^2 - 4*a*c)*b^5*c^3 - 96*(b^2 - 4*a*c) \\
& *a^2*b^2*c^4 - 64*(b^2 - 4*a*c)*a*b^3*c^4 - 6*(b^2 - 4*a*c)*b^4*c^4 + 128 \\
& *(b^2 - 4*a*c)*a^3*c^5 + 128*(b^2 - 4*a*c)*a^2*b*c^5 + 48*(b^2 - 4*a*c)*a*b^2*c^5 - 96*(b^2 - 4*a*c)*a^2*c^6 - 64*(b^2 - 4*a*c)*a*b*c^6)*d^2*abs(c) + \\
& 4*(3*\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^6*c^2 - 4*a*b^7*c^2 - 36*s \\
& \sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b^4*c^3 - 4*\sqrt{2})*\sqrt{b*c - s
\end{aligned}$$

$$\begin{aligned}
& \text{qrt}(b^2 - 4ac) * c) * a * b^5 * c^3 + 48 * a^2 * b^5 * c^3 + 10 * a * b^6 * c^3 + 144 * \text{sqrt}(2) \\
& * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^3 * b^2 * c^4 + 32 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 \\
& - 4ac) * c) * a^2 * b^3 * c^4 - 192 * a^3 * b^3 * c^4 - \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 \\
& - 4ac) * c) * a * b^4 * c^4 - 56 * a^2 * b^4 * c^4 - 8 * a * b^5 * c^4 - 192 * \text{sqrt}(2) * \text{sqrt}(b * c \\
& - \text{sqrt}(b^2 - 4ac) * c) * a^4 * c^5 - 64 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) \\
& ) * a^3 * b * c^5 + 256 * a^4 * b * c^5 - 8 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^2 \\
& * b^2 * c^5 - 32 * a^3 * b^2 * c^5 + 2 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a * b^3 \\
& * c^5 + 6 * a * b^4 * c^5 + 48 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^3 * c^6 + 3 \\
& 84 * a^4 * c^6 - 8 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^2 * b * c^6 + 128 * a^3 * \\
& b * c^6 - 16 * a^2 * b^2 * c^6 - 8 * a * b^3 * c^6 - 32 * a^3 * c^7 + 32 * a^2 * b * c^7 - \text{sqrt}(2) * \\
& \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a * b^6 * c + 12 * \text{sqrt}(2) * \text{sqrt} \\
& (b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^2 * b^4 * c^2 + 2 * \text{sqrt}(2) * \text{sqrt} \\
& (b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a * b^5 * c^2 + 4 * \text{sqrt}(b^2 - 4ac) \\
& ) * a * b^6 * c^2 - 48 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * \\
& a^3 * b^2 * c^3 - 16 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * \\
& a^2 * b^3 * c^3 - 3 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a \\
& * b^4 * c^3 - 48 * \text{sqrt}(b^2 - 4ac) * a^2 * b^4 * c^3 - 10 * \text{sqrt}(b^2 - 4ac) * a * b^5 * c^3 \\
& + 64 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^4 * c^4 + \\
& 32 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^3 * b * c^4 + 40 \\
& * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^2 * b^2 * c^4 + 19 \\
& 2 * \text{sqrt}(b^2 - 4ac) * a^3 * b^2 * c^4 + 4 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt} \\
& (b^2 - 4ac) * c) * a * b^3 * c^4 + 80 * \text{sqrt}(b^2 - 4ac) * a^2 * b^3 * c^4 + 16 * \text{sqrt}(b^2 - \\
& 4ac) * a * b^4 * c^4 - 112 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - \\
& 4ac) * c) * a^3 * c^5 - 256 * \text{sqrt}(b^2 - 4ac) * a^4 * c^5 - 48 * \text{sqrt}(2) * \text{sqrt}(b^2 - \\
& 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^2 * b * c^5 - 160 * \text{sqrt}(b^2 - 4ac) * a^3 \\
& * b * c^5 - 2 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a * b^2 \\
& * c^5 - 96 * \text{sqrt}(b^2 - 4ac) * a^2 * b^2 * c^5 - 18 * \text{sqrt}(b^2 - 4ac) * a * b^3 * c^5 + \\
& 24 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^2 * c^6 + 128 * \\
& \text{sqrt}(b^2 - 4ac) * a^3 * c^6 + 40 * \text{sqrt}(b^2 - 4ac) * a^2 * b * c^6 + 8 * \text{sqrt}(b^2 - 4 \\
& ac) * a * b^2 * c^6 - 16 * \text{sqrt}(b^2 - 4ac) * a^2 * c^7 - 8 * (b^2 - 4ac) * a * b^3 * c^4 \\
& + 32 * (b^2 - 4ac) * a^2 * b * c^5 + 12 * (b^2 - 4ac) * a * b^2 * c^5 + 16 * (b^2 - 4ac) \\
& ) * a^2 * c^6) * d * \text{abs}(c) * e - (\text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a * b^8 - 16 \\
& * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^2 * b^6 * c + \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt} \\
& (b^2 - 4ac) * c) * a * b^7 * c - 6 * a * b^8 * c + 96 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 \\
& ac) * c) * a^3 * b^4 * c^2 - 12 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^2 * b^5 * c^2 \\
& - \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a * b^6 * c^2 + 80 * a^2 * b^6 * c^2 + 12 \\
& * a * b^7 * c^2 - 256 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^4 * b^2 * c^3 + 48 * \text{s} \\
& \text{qrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^3 * b^3 * c^3 - 20 * \text{sqrt}(2) * \text{sqrt}(b * c - \\
& \text{sqrt}(b^2 - 4ac) * c) * a^2 * b^4 * c^3 - 384 * a^3 * b^4 * c^3 - 5 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{s} \\
& \text{qrt}(b^2 - 4ac) * c) * a * b^5 * c^3 - 80 * a^2 * b^5 * c^3 - 10 * a * b^6 * c^3 + 256 * \text{sqrt}(2) \\
& * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^5 * c^4 - 64 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - \\
& 4ac) * c) * a^4 * b * c^4 + 208 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^3 * b^2 * \\
& c^4 + 768 * a^4 * b^2 * c^4 + 56 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^2 * b^3 * \\
& c^4 + 64 * a^3 * b^3 * c^4 + 4 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a * b^4 * c^4 \\
& + 24 * a^2 * b^4 * c^4 + 12 * a * b^5 * c^4 - 448 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * \\
& c) * a^4 * c^5 - 512 * a^5 * c^5 - 144 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^3 * \\
& b * c^5 + 256 * a^4 * b * c^5 - 40 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^2 * b^2 * \\
& c^5 + 32 * a^3 * b^2 * c^5 - 32 * a^2 * b^3 * c^5 - 16 * a * b^4 * c^5 + 96 * \text{sqrt}(2) * \text{sqrt}(b * c \\
& - \text{sqrt}(b^2 - 4ac) * c) * a^3 * c^6 + 128 * a^4 * c^6 - 64 * a^3 * b * c^6 + 80 * a^2 * b^2 * c^6 \\
& - 64 * a^3 * c^7 - \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * \\
& a * b^7 + 12 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^2 * b^5 \\
& * c - \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a * b^6 * c + 8 \\
& * \text{sqrt}(b^2 - 4ac) * a * b^7 * c - 48 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - \\
& 4ac) * c) * a^3 * b^3 * c^2 + 20 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - \\
& 4ac) * c) * a^2 * b^4 * c^2 + \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 \\
& - 4ac) * c) * a * b^5 * c^2 - 96 * \text{sqrt}(b^2 - 4ac) * a^2 * b^5 * c^2 - 20 * \text{sqrt}(b^2 - 4 \\
& ac) * a * b^6 * c^2 + 64 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * \\
& c) * a^4 * b * c^3 - 112 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) \\
& ) * a^3 * b^2 * c^3 + 8 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c)
\end{aligned}$$



$$\begin{aligned}
& a^2 b^3 c^3 + 384 \sqrt{b^2 - 4ac} a^3 b^3 c^3 + 5 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac} c} a^2 b^4 c^3 + 136 \sqrt{b^2 - 4ac} a^2 b^4 c^3 \\
& + 32 \sqrt{b^2 - 4ac} a^2 b^5 c^3 + 192 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac} c} a^4 c^4 - 48 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac} c} \\
& a^3 b^4 c^4 - 512 \sqrt{b^2 - 4ac} a^4 b^4 c^4 - 40 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac} c} a^2 b^2 c^4 - 128 \sqrt{b^2 - 4ac} \\
& a^3 b^2 c^4 - 4 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac} c} a^2 b^3 c^4 - 160 \sqrt{b^2 - 4ac} a^2 b^3 c^4 - 36 \sqrt{b^2 - 4ac} \\
& a^2 b^4 c^4 - 48 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac} c} a^3 c^5 - 384 \sqrt{b^2 - 4ac} a^4 c^5 + 32 \sqrt{2} \sqrt{b^2 - 4ac} \\
& \sqrt{bc - \sqrt{b^2 - 4ac} c} a^2 b^5 c^5 + 128 \sqrt{b^2 - 4ac} a^3 b^5 c^5 + 88 \sqrt{b^2 - 4ac} a^2 b^2 c^5 + 16 \sqrt{b^2 - 4ac} a^2 b^3 c^5 + \\
& 96 \sqrt{b^2 - 4ac} a^3 c^6 - 48 \sqrt{b^2 - 4ac} a^2 b^3 c^6 - 2(b^2 - 4ac) a^2 b^6 c + 24(b^2 - 4ac) a^2 b^4 c^2 + 8(b^2 - 4ac) a^2 b^5 c^2 - \\
& 96(b^2 - 4ac) a^3 b^2 c^3 - 64(b^2 - 4ac) a^2 b^3 c^3 - 22(b^2 - 4ac) a^2 b^4 c^3 + 128(b^2 - 4ac) a^4 c^4 + 128(b^2 - 4ac) a^3 b^4 c^4 + 1 \\
& 12(b^2 - 4ac) a^2 b^2 c^4 + 24(b^2 - 4ac) a^2 b^3 c^4 - 96(b^2 - 4ac) a^3 c^5 - 32(b^2 - 4ac) a^2 b^3 c^5) \operatorname{abs}(c) e^2 + 1/2 x e^2 / ((c^d^3 - b \\
& d^2 e + a d e^2) (x^2 e + d))
\end{aligned}$$

**maple [B]** time = 0.03, size = 1141, normalized size = 2.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int (1/(e*x^2+d)^2/(c*x^4+b*x^2+a), x)$

[Out]  $1/2 e^4 / (a e^2 - b d e + c d^2)^2 / d x / (e x^2 + d) a^{-1/2} e^3 / (a e^2 - b d e + c d^2)^2 x / (e x^2 + d) b + 1/2 e^2 / (a e^2 - b d e + c d^2)^2 d x / (e x^2 + d) c + 1/2 e^4 / (a e^2 - b d e + c d^2)^2 / d / (d e)^{(1/2)} \arctan(1 / (d e)^{(1/2)} e x) a^{-3/2} e^3 / (a e^2 - b d e + c d^2)^2 / (d e)^{(1/2)} \arctan(1 / (d e)^{(1/2)} e x) b + 5/2 e^2 / (a e^2 - b d e + c d^2)^2 d / (d e)^{(1/2)} \arctan(1 / (d e)^{(1/2)} e x) c - 1/2 / (a e^2 - b d e + c d^2)^2 c^2^{(1/2)} / ((-b + (-4 a c + b^2)^{(1/2)}) c)^{(1/2)} \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4 a c + b^2)^{(1/2)}) c)^{(1/2)} c x) b e^2 + 1 / (a e^2 - b d e + c d^2)^2 c^2^{(1/2)} / ((-b + (-4 a c + b^2)^{(1/2)}) c)^{(1/2)} \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4 a c + b^2)^{(1/2)}) c)^{(1/2)} c x) d e + 1 / (a e^2 - b d e + c d^2)^2 c^2 / (-4 a c + b^2)^{(1/2)} 2^{(1/2)} / ((-b + (-4 a c + b^2)^{(1/2)}) c)^{(1/2)} \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4 a c + b^2)^{(1/2)}) c)^{(1/2)} c x) a e^2 - 1/2 / (a e^2 - b d e + c d^2)^2 c / (-4 a c + b^2)^{(1/2)} 2^{(1/2)} / ((-b + (-4 a c + b^2)^{(1/2)}) c)^{(1/2)} \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4 a c + b^2)^{(1/2)}) c)^{(1/2)} c x) b^2 e^2 + 1 / (a e^2 - b d e + c d^2)^2 c^2 / (-4 a c + b^2)^{(1/2)} 2^{(1/2)} / ((-b + (-4 a c + b^2)^{(1/2)}) c)^{(1/2)} \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4 a c + b^2)^{(1/2)}) c)^{(1/2)} c x) b d e - 1 / (a e^2 - b d e + c d^2)^2 c^3 / (-4 a c + b^2)^{(1/2)} 2^{(1/2)} / ((-b + (-4 a c + b^2)^{(1/2)}) c)^{(1/2)} \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4 a c + b^2)^{(1/2)}) c)^{(1/2)} c x) d^2 + 1/2 / (a e^2 - b d e + c d^2)^2 c^2^{(1/2)} / ((b + (-4 a c + b^2)^{(1/2)}) c)^{(1/2)} \arctan(2^{(1/2)} / ((b + (-4 a c + b^2)^{(1/2)}) c)^{(1/2)} c x) b e^2 - 1 / (a e^2 - b d e + c d^2)^2 c^2^{(1/2)} / ((b + (-4 a c + b^2)^{(1/2)}) c)^{(1/2)} \arctan(2^{(1/2)} / ((b + (-4 a c + b^2)^{(1/2)}) c)^{(1/2)} c x) d e + 1 / (a e^2 - b d e + c d^2)^2 c^2 / (-4 a c + b^2)^{(1/2)} 2^{(1/2)} / ((b + (-4 a c + b^2)^{(1/2)}) c)^{(1/2)} \arctan(2^{(1/2)} / ((b + (-4 a c + b^2)^{(1/2)}) c)^{(1/2)} c x) a e^2 - 1/2 / (a e^2 - b d e + c d^2)^2 c / (-4 a c + b^2)^{(1/2)} 2^{(1/2)} / ((b + (-4 a c + b^2)^{(1/2)}) c)^{(1/2)} \arctan(2^{(1/2)} / ((b + (-4 a c + b^2)^{(1/2)}) c)^{(1/2)} c x) b^2 e^2 + 1 / (a e^2 - b d e + c d^2)^2 c^2 / (-4 a c + b^2)^{(1/2)} 2^{(1/2)} / ((b + (-4 a c + b^2)^{(1/2)}) c)^{(1/2)} \arctan(2^{(1/2)} / ((b + (-4 a c + b^2)^{(1/2)}) c)^{(1/2)} c x) b d e - 1 / (a e^2 - b d e + c d^2)^2 c^3 / (-4 a c + b^2)^{(1/2)} 2^{(1/2)} / ((b + (-4 a c + b^2)^{(1/2)}) c)^{(1/2)} \arctan(2^{(1/2)} / ((b + (-4 a c + b^2)^{(1/2)}) c)^{(1/2)} c x) d^2$

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)^2/(c\*x^4+b\*x^2+a),x, algorithm="maxima")

[Out]  $\frac{1}{2} \frac{e^{2x}}{(cd^4 - bd^3e + ad^2e^2 + (cd^3e - bd^2e^2 + ad^2e^3)x^2)} + \frac{1}{2} \frac{(5cd^2e^2 - 3bde^3 + ae^4) \arctan(e x / \sqrt{de})}{(c^2d^5 - 2b^2cd^4e - 2ab^2d^2e^3 + a^2de^4 + (b^2 + 2ac)d^3e^2) \sqrt{de}} + \int \frac{(c^2d^2 - 2b^2cd^2e + (b^2 - ac)e^2 - (2c^2d^2e - b^2c^2e^2)x^2)}{(c^2d^4 - 2b^2cd^3e - 2ab^2d^2e^3 + a^2e^4 + (b^2 + 2ac)d^2e^2)} dx$

**mupad [B]** time = 10.28, size = 91169, normalized size = 212.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d + e\*x^2)^2\*(a + b\*x^2 + c\*x^4)),x)

[Out]  $\text{atan}\left(\frac{(x(54c^9d^6e^5 - 2a^3c^6e^{11} - 22a^2c^8d^4e^7 - 118b^2c^8d^5e^6 + a^2b^2c^5e^{11} - 14a^2c^7d^2e^9 + 107b^2c^7d^4e^7 - 48b^3c^6d^3e^8 + 9b^4c^5d^2e^9 + 20a^2b^2c^7d^3e^8 - 6a^2b^3c^5d^2e^{10} + 10a^2b^2c^6d^2e^9))}{(2(c^4d^{10} + a^4d^2e^8 + b^4d^6e^4 - 4a^2b^3d^5e^5 - 4a^3b^2d^3e^7 + 4a^2c^3d^8e^2 + 4a^3c^2d^4e^6 - 4b^3c^2d^7e^3 + 6a^2b^2d^4e^6 + 6a^2c^2d^6e^4 + 6b^2c^2d^8e^2 - 4b^2c^3d^9e - 12a^2b^2c^2d^7e^3 + 12a^2b^2c^2d^6e^4 - 12a^2b^2c^2d^5e^5))} - \frac{((2a^2b^6c^2e^{13} - 200a^2c^9d^8e^5 - 8a^5c^5e^{13} - 14a^3b^4c^3e^{13} + 26a^4b^2c^4e^{13} + 480a^2c^8d^6e^7 + 784a^3c^7d^4e^9 + 96a^4c^6d^2e^{11} + 50b^2c^8d^8e^5 - 240b^3c^7d^7e^6 + 466b^4c^6d^6e^7 - 464b^5c^5d^5e^8 + 246b^6c^4d^4e^9 - 64b^7c^3d^3e^{10} + 6b^8c^2d^2e^{11} + 4a^2b^2c^6d^4e^9 + 672a^2b^3c^5d^3e^{10} - 354a^2b^4c^4d^2e^{11} + 464a^3b^2c^5d^2e^{11} + 960a^2b^3c^8d^7e^6 - 8a^2b^7c^2d^2e^{12} - 96a^4b^2c^5d^2e^{12} - 1984a^2b^2c^7d^6e^7 + 2072a^2b^3c^6d^5e^8 - 1034a^2b^4c^5d^4e^9 + 160a^2b^5c^4d^3e^{10} + 34a^2b^6c^3d^2e^{11} - 864a^2b^2c^7d^5e^8 + 40a^2b^5c^3d^2e^{12} - 1152a^3b^2c^6d^3e^{10} - 8a^3b^3c^4d^2e^{12})}{(2(c^4d^{10} + a^4d^2e^8 + b^4d^6e^4 - 4a^2b^3d^5e^5 - 4a^3b^2d^3e^7 + 4a^2c^3d^8e^2 + 4a^3c^2d^4e^6 - 4b^3c^2d^7e^3 + 6a^2b^2d^4e^6 + 6a^2c^2d^6e^4 + 6b^2c^2d^8e^2 - 4b^2c^3d^9e - 12a^2b^2c^2d^7e^3 + 12a^2b^2c^2d^6e^4 - 12a^2b^2c^2d^5e^5))} - \frac{((-d^3e^3)^{1/2}((x(32c^{11}d^{13}e^2 + 48a^6b^2c^4e^{15} + 96a^2c^{10}d^{11}e^4 - 64a^6c^5d^2e^{14} - 160b^2c^{10}d^{12}e^3 + 4a^4b^5c^2e^{15} - 28a^5b^3c^3e^{15} - 2048a^2c^9d^9e^6 - 4416a^3c^8d^7e^8 - 2528a^4c^7d^5e^{10} - 288a^5c^6d^3e^{12} + 336b^2c^9d^{11}e^4 - 268b^3c^8d^{10}e^5 - 360b^4c^7d^9e^6 + 1260b^5c^6d^8e^7 - 1568b^6c^5d^7e^8 + 1036b^7c^4d^6e^9 - 360b^8c^3d^5e^{10} + 52b^9c^2d^4e^{11} - 7584a^2b^2c^7d^7e^8 - 536a^2b^3c^6d^6e^9 + 5936a^2b^4c^5d^5e^{10} - 3552a^2b^5c^4d^4e^{11} + 464a^2b^6c^3d^3e^{12} + 104a^2b^7c^2d^2e^{13} - 12768a^3b^2c^6d^5e^{10} + 3720a^3b^3c^5d^4e^{11} + 1280a^3b^4c^4d^3e^{12} - 648a^3b^5c^3d^2e^{13} - 4272a^4b^2c^5d^3e^{12} + 740a^4b^3c^4d^2e^{13} - 848a^2b^2c^9d^{10}e^5 + 3632a^2b^2c^8d^9e^6 - 7852a^2b^3c^7d^8e^7 + 8864a^2b^4c^6d^7e^8 - 4936a^2b^5c^5d^6e^9 + 816a^2b^6c^4d^5e^{10} + 356a^2b^7c^3d^4e^{11} - 128a^2b^8c^2d^3e^{12} + 7216a^2b^2c^8d^8e^7 + 12896a^3b^2c^7d^6e^9 - 32a^3b^6c^2d^2e^{14} + 5696a^4b^2c^6d^4e^{11} + 216a^4b^4c^3d^2e^{14} + 752a^5b^2c^5d^2e^{13} - 336a^5b^2c^4d^2e^{14}))}{(2(c^4d^{10} + a^4d^2e^8 + b^4d^6e^4 - 4a^2b^3d^5e^5 - 4a^3b^2d^3e^7 + 4a^2c^3d^8e^2 + 4a^3c^2d^4e^6 - 4b^3c^2d^7e^3 + 6a^2b^2d^4e^6 + 6a^2c^2d^6e^4 + 6b^2c^2d^8e^2 - 4b^2c^3d^9e - 12a^2b^2c^2d^7e^3 + 12a^2b^2c^2d^6e^4 - 12a^2b^2c^2d^5e^5))} + \frac{((128a^2c^{11}d^{15}e^2 - 256a^8c^4d^2e^{16} - 256a^2c^{10}d^{13}e^4 - 3456a^3c^9d^{11}e^6 - 8960a^4c^8d^9e^8 - 10880a^5c^7d^7e^{10} - 6912a^6c^6d^5e^{12} - 2176a^7c^5d^3e^{14} - 32b^2c^{10}d^{15}e^2 + 256b^3c^9d^{14}e^3 - 896b^4c^8d^{13}e^4 + 1792b^5c^7d^{12}e^5 - 2240b^6c^6d^{11}e^6 + 1792b^7c^5d^{10}e^7 - 896b^8c^4d^9e^8 + 256b^9c^3d^8e^9 - 32b^{10}c^2d^7e^{10} + 2848a^2b^2c^8d^11$

$$\begin{aligned}
& 1e^6 - 12160a^2b^3c^7d^{10}e^7 + 18480a^2b^4c^6d^9e^8 - 12864a^2b^5c^5d^8e^9 + 3008a^2b^6c^4d^7e^{10} + 832a^2b^7c^3d^6e^{11} - 40 \\
& 0a^2b^8c^2d^5e^{12} - 17920a^3b^2c^7d^9e^8 + 1280a^3b^3c^6d^8e^9 + 14240a^3b^4c^5d^7e^{10} - 9824a^3b^5c^4d^6e^{11} + 1120a^3b^6c^3d^5e^{12} + 480a^3b^7c^2d^4e^{13} - 33760a^4b^2c^6d^7e^{10} + 7680 \\
& a^4b^3c^5d^6e^{11} + 7520a^4b^4c^4d^5e^{12} - 2880a^4b^5c^3d^4e^{13} - 320a^4b^6c^2d^3e^{14} - 20672a^5b^2c^5d^5e^{12} + 896a^5b^3c^4d^4e^{13} + 2384a^5b^4c^3d^3e^{14} + 112a^5b^5c^2d^2e^{15} - 3872a^5b^6c^2d^3e^{14} - 896a^6b^3c^3d^2e^{15} - 1024a^6b^4c^3d^2e^{15} - 1024a^6b^5c^3d^2e^{15} + 36 \\
& 48a^6b^6c^3d^2e^{15} - 7296a^6b^7c^3d^2e^{15} + 8464a^6b^8c^3d^2e^{15} - 5008a^6b^9c^3d^2e^{15} + 224a^6b^{10}c^3d^2e^{15} + 1632a^6b^{11}c^3d^2e^{15} - 944a^6b^{12}c^3d^2e^{15} + 176a^6b^{13}c^3d^2e^{15} + 512a^6b^{14}c^3d^2e^{15} + 14080a^7b^3c^8d^{10}e^7 + 30720a^7b^4c^7d^9e^8 + 28160a^7b^5c^6d^8e^9 \\
& + 11776a^7b^6c^5d^7e^{10} - 16a^7b^7c^4d^6e^{11} + 1792a^7b^8c^3d^5e^{12} + 128a^7b^9c^2d^4e^{13} + 128a^7b^{10}c^1d^3e^{14}) / (2*(c^4d^{10} + a^4d^2e^8 + b^4d^6e^4 - 4ab^3d^5e^5 - 4a^3b^2d^3e^7 + 4a^2c^3d^8e^2 + 4a^3c^2d^4e^6 - 4b^3c^2d^7e^3 + 6a^2b^2d^4e^6 + 6a^2c^2d^6e^4 + 6b^2c^2d^8e^2 - 4b^2c^3d^9e - 12a^2b^2c^2d^7e^3 + 12a^2b^2c^2d^6e^4 - 12a^2b^2c^2d^5e^5) - (x*(-d^3e^3)^{1/2}*(a^2e^2 + 5c^2d^2 - 3b^2d^2e)* (1024a^2c^{11}d^{16}e^3 + 5120a^3c^{10}d^{14}e^5 + 9216a^4c^9d^{12}e^7 + 5120a^5c^8d^{10}e^9 - 5120a^6c^7d^8e^{11} - 9216a^7c^6d^6e^{13} - 5120a^8c^5d^4e^{15} - 1024a^9c^4d^2e^{17} - 64b^3c^{10}d^{17}e^2 + 512b^4c^9d^{16}e^3 - 1792b^5c^8d^{15}e^4 + 3584b^6c^7d^{14}e^5 - 4480b^7c^6d^{13}e^6 + 3584b^8c^5d^{12}e^7 - 1792b^9c^4d^{11}e^8 + 512b^{10}c^3d^{10}e^9 - 64b^{11}c^2d^9e^{10} + 8192a^2b^2c^9d^{14}e^5 + 5056a^2b^3c^8d^{13}e^6 - 31104a^2b^4c^7d^{12}e^7 + 40256a^2b^5c^6d^{11}e^8 - 22784a^2b^6c^5d^{10}e^9 + 3648a^2b^7c^4d^9e^{10} + 1664a^2b^8c^3d^8e^{11} - 576a^2b^9c^2d^7e^{12} + 45312a^3b^2c^8d^{12}e^7 - 27840a^3b^3c^7d^{11}e^8 - 13760a^3b^4c^6d^{10}e^9 + 27520a^3b^5c^5d^9e^{10} - 12416a^3b^6c^4d^8e^{11} + 1088a^3b^7c^3d^7e^{12} + 320a^3b^8c^2d^6e^{13} + 53760a^4b^2c^7d^{10}e^9 - 30400a^4b^3c^6d^9e^{10} + 1280a^4b^4c^5d^8e^{11} + 4224a^4b^5c^4d^7e^{12} - 1280a^4b^6c^3d^6e^{13} + 320a^4b^7c^2d^5e^{14} + 6400a^5b^2c^6d^8e^{11} - 2624a^5b^3c^5d^7e^{12} + 5952a^5b^4c^4d^6e^{13} - 2752a^5b^5c^3d^5e^{14} - 576a^5b^6c^2d^4e^{15} - 21504a^6b^2c^5d^6e^{13} + 832a^6b^3c^4d^5e^{14} + 4736a^6b^4c^3d^4e^{15} + 320a^6b^5c^2d^3e^{16} - 8448a^7b^2c^4d^4e^{15} - 2624a^7b^3c^3d^3e^{16} - 64a^7b^4c^2d^2e^{17} + 512a^8b^2c^3d^2e^{17} + 256a^8b^3c^2d^1e^{18} - 2304a^8b^4c^1d^1e^{19} + 8512a^8b^5c^0d^0e^{20} - 16704a^8b^6c^0d^0e^{21} + 18240a^8b^7c^0d^0e^{22} - 9536a^8b^8c^0d^0e^{23} - 576a^8b^9c^0d^0e^{24} + 3648a^8b^{10}c^0d^0e^{25} - 1856a^8b^{11}c^0d^0e^{26} + 320a^8b^{12}c^0d^0e^{27} - 5376a^8b^{13}c^0d^0e^{28} - 25344a^8b^{14}c^0d^0e^{29} + 37120a^8b^{15}c^0d^0e^{30} - 11520a^8b^{16}c^0d^0e^{31} + 20736a^8b^{17}c^0d^0e^{32} + 20224a^8b^{18}c^0d^0e^{33} + 5376a^8b^{19}c^0d^0e^{34} + 5376a^8b^{20}c^0d^0e^{35}))/ (8*(c^2d^7 + a^2d^3e^4 + b^2d^5e^2 - 2b^2c^2d^6e - 2a^2b^2d^4e^3 + 2a^2c^2d^5e^2) * (c^4d^{10} + a^4d^2e^8 + b^4d^6e^4 - 4a^2b^3d^5e^5 - 4a^3b^2d^3e^7 + 4a^2c^3d^8e^2 + 4a^3c^2d^4e^6 - 4b^3c^2d^7e^3 + 6a^2b^2d^4e^6 + 6a^2c^2d^6e^4 + 6b^2c^2d^8e^2 - 4b^2c^3d^9e - 12a^2b^2c^2d^7e^3 + 12a^2b^2c^2d^6e^4 - 12a^2b^2c^2d^5e^5)) * (-d^3e^3)^{1/2} * (a^2e^2 + 5c^2d^2 - 3b^2d^2e) / (4*(c^2d^7 + a^2d^3e^4 + b^2d^5e^2 - 2b^2c^2d^6e - 2a^2b^2d^4e^3 + 2a^2c^2d^5e^2)) * (a^2e^2 + 5c^2d^2 - 3b^2d^2e) / (4*(c^2d^7 + a^2d^3e^4 + b^2d^5e^2 - 2b^2c^2d^6e - 2a^2b^2d^4e^3 + 2a^2c^2d^5e^2)) * (-d^3e^3)^{1/2} * (a^2e^2 + 5c^2d^2 - 3b^2d^2e) / (4*(c^2d^7 + a^2d^3e^4 + b^2d^5e^2 - 2b^2c^2d^6e - 2a^2b^2d^4e^3 + 2a^2c^2d^5e^2)) * ((x*(54c^9d^6e^5 - 2a^3c^6e^{11} - 22a^3c^8d^4e^7 - 118b^3c^8d^5e^6 + a^2b^2c^5e^{11} - 14a^2c^7d^2e^9 + 107b^2c^7d^4e^7 - 48b^3c^6d^3e^8 + 9b^4c^5d^2e^9 + 20a^2b^2c^7d^3e^8 - 6a^2b^3c^5d^2e^{10} + 10a^2b^2c^6d^2e^{10} + 4a^2b^2c^6d^2e^9)) / (2*(c^4d^{10} + a^4d^2e^8 + b^4d^6e^4 - 4a^2b^3d^5e^5
\end{aligned}$$

$$\begin{aligned}
& - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6 \\
& *a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - \\
& 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5) + (((2*a^2*b \\
& ^6*c^2*e^13 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 - 14*a^3*b^4*c^3*e^13 + 26 \\
& *a^4*b^2*c^4*e^13 + 480*a^2*c^8*d^6*e^7 + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6* \\
& d^2*e^11 + 50*b^2*c^8*d^8*e^5 - 240*b^3*c^7*d^7*e^6 + 466*b^4*c^6*d^6*e^7 - \\
& 464*b^5*c^5*d^5*e^8 + 246*b^6*c^4*d^4*e^9 - 64*b^7*c^3*d^3*e^10 + 6*b^8*c^ \\
& 2*d^2*e^11 + 4*a^2*b^2*c^6*d^4*e^9 + 672*a^2*b^3*c^5*d^3*e^10 - 354*a^2*b^4 \\
& *c^4*d^2*e^11 + 464*a^3*b^2*c^5*d^2*e^11 + 960*a*b*c^8*d^7*e^6 - 8*a*b^7*c^ \\
& 2*d*e^12 - 96*a^4*b*c^5*d*e^12 - 1984*a*b^2*c^7*d^6*e^7 + 2072*a*b^3*c^6*d^ \\
& 5*e^8 - 1034*a*b^4*c^5*d^4*e^9 + 160*a*b^5*c^4*d^3*e^10 + 34*a*b^6*c^3*d^2* \\
& e^11 - 864*a^2*b*c^7*d^5*e^8 + 40*a^2*b^5*c^3*d*e^12 - 1152*a^3*b*c^6*d^3*e \\
& ^10 - 8*a^3*b^3*c^4*d*e^12)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a* \\
& b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c \\
& *d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b* \\
& c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) \\
& + ((-d^3*e^3)^(1/2)*(x*(32*c^11*d^13*e^2 + 48*a^6*b*c^4*e^15 + 96*a*c^10*d \\
& ^11*e^4 - 64*a^6*c^5*d*e^14 - 160*b*c^10*d^12*e^3 + 4*a^4*b^5*c^2*e^15 - 28 \\
& *a^5*b^3*c^3*e^15 - 2048*a^2*c^9*d^9*e^6 - 4416*a^3*c^8*d^7*e^8 - 2528*a^4* \\
& c^7*d^5*e^10 - 288*a^5*c^6*d^3*e^12 + 336*b^2*c^9*d^11*e^4 - 268*b^3*c^8*d^ \\
& 10*e^5 - 360*b^4*c^7*d^9*e^6 + 1260*b^5*c^6*d^8*e^7 - 1568*b^6*c^5*d^7*e^8 \\
& + 1036*b^7*c^4*d^6*e^9 - 360*b^8*c^3*d^5*e^10 + 52*b^9*c^2*d^4*e^11 - 7584* \\
& a^2*b^2*c^7*d^7*e^8 - 536*a^2*b^3*c^6*d^6*e^9 + 5936*a^2*b^4*c^5*d^5*e^10 - \\
& 3552*a^2*b^5*c^4*d^4*e^11 + 464*a^2*b^6*c^3*d^3*e^12 + 104*a^2*b^7*c^2*d^2 \\
& *e^13 - 12768*a^3*b^2*c^6*d^5*e^10 + 3720*a^3*b^3*c^5*d^4*e^11 + 1280*a^3*b \\
& ^4*c^4*d^3*e^12 - 648*a^3*b^5*c^3*d^2*e^13 - 4272*a^4*b^2*c^5*d^3*e^12 + 74 \\
& 0*a^4*b^3*c^4*d^2*e^13 - 848*a*b*c^9*d^10*e^5 + 3632*a*b^2*c^8*d^9*e^6 - 78 \\
& 52*a*b^3*c^7*d^8*e^7 + 8864*a*b^4*c^6*d^7*e^8 - 4936*a*b^5*c^5*d^6*e^9 + 81 \\
& 6*a*b^6*c^4*d^5*e^10 + 356*a*b^7*c^3*d^4*e^11 - 128*a*b^8*c^2*d^3*e^12 + 72 \\
& 16*a^2*b*c^8*d^8*e^7 + 12896*a^3*b*c^7*d^6*e^9 - 32*a^3*b^6*c^2*d*e^14 + 56 \\
& 96*a^4*b*c^6*d^4*e^11 + 216*a^4*b^4*c^3*d*e^14 + 752*a^5*b*c^5*d^2*e^13 - 3 \\
& 36*a^5*b^2*c^4*d*e^14))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3* \\
& d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7 \\
& *e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3* \\
& d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - (( \\
& (128*a*c^11*d^15*e^2 - 256*a^8*c^4*d*e^16 - 256*a^2*c^10*d^13*e^4 - 3456*a^ \\
& 3*c^9*d^11*e^6 - 8960*a^4*c^8*d^9*e^8 - 10880*a^5*c^7*d^7*e^10 - 6912*a^6*c \\
& ^6*d^5*e^12 - 2176*a^7*c^5*d^3*e^14 - 32*b^2*c^10*d^15*e^2 + 256*b^3*c^9*d^ \\
& 14*e^3 - 896*b^4*c^8*d^13*e^4 + 1792*b^5*c^7*d^12*e^5 - 2240*b^6*c^6*d^11*e \\
& ^6 + 1792*b^7*c^5*d^10*e^7 - 896*b^8*c^4*d^9*e^8 + 256*b^9*c^3*d^8*e^9 - 32 \\
& *b^10*c^2*d^7*e^10 + 2848*a^2*b^2*c^8*d^11*e^6 - 12160*a^2*b^3*c^7*d^10*e^7 \\
& + 18480*a^2*b^4*c^6*d^9*e^8 - 12864*a^2*b^5*c^5*d^8*e^9 + 3008*a^2*b^6*c^4 \\
& *d^7*e^10 + 832*a^2*b^7*c^3*d^6*e^11 - 400*a^2*b^8*c^2*d^5*e^12 - 17920*a^3 \\
& *b^2*c^7*d^9*e^8 + 1280*a^3*b^3*c^6*d^8*e^9 + 14240*a^3*b^4*c^5*d^7*e^10 - \\
& 9824*a^3*b^5*c^4*d^6*e^11 + 1120*a^3*b^6*c^3*d^5*e^12 + 480*a^3*b^7*c^2*d^4 \\
& *e^13 - 33760*a^4*b^2*c^6*d^7*e^10 + 7680*a^4*b^3*c^5*d^6*e^11 + 7520*a^4*b \\
& ^4*c^4*d^5*e^12 - 2880*a^4*b^5*c^3*d^4*e^13 - 320*a^4*b^6*c^2*d^3*e^14 - 20 \\
& 672*a^5*b^2*c^5*d^5*e^12 + 896*a^5*b^3*c^4*d^4*e^13 + 2384*a^5*b^4*c^3*d^3* \\
& e^14 + 112*a^5*b^5*c^2*d^2*e^15 - 3872*a^6*b^2*c^4*d^3*e^14 - 896*a^6*b^3*c \\
& ^3*d^2*e^15 - 1024*a*b*c^10*d^14*e^3 + 3648*a*b^2*c^9*d^13*e^4 - 7296*a*b^3 \\
& *c^8*d^12*e^5 + 8464*a*b^4*c^7*d^11*e^6 - 5008*a*b^5*c^6*d^10*e^7 + 224*a*b \\
& ^6*c^5*d^9*e^8 + 1632*a*b^7*c^4*d^8*e^9 - 944*a*b^8*c^3*d^7*e^10 + 176*a*b^ \\
& 9*c^2*d^6*e^11 + 512*a^2*b*c^9*d^12*e^5 + 14080*a^3*b*c^8*d^10*e^7 + 30720* \\
& a^4*b*c^7*d^8*e^9 + 28160*a^5*b*c^6*d^6*e^11 + 11776*a^6*b*c^5*d^4*e^13 - 1 \\
& 6*a^6*b^4*c^2*d*e^16 + 1792*a^7*b*c^4*d^2*e^15 + 128*a^7*b^2*c^3*d*e^16)/(2 \\
& *(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 \\
& + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + \\
& 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 \\
& + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) + (x*(-d^3*e^3)^(1/2)*(a*e^2 +
\end{aligned}$$

$$\begin{aligned}
& 5*c*d^2 - 3*b*d*e)*(1024*a^2*c^11*d^16*e^3 + 5120*a^3*c^10*d^14*e^5 + 9216 \\
& a^4*c^9*d^12*e^7 + 5120*a^5*c^8*d^10*e^9 - 5120*a^6*c^7*d^8*e^11 - 9216*a^7 \\
& c^6*d^6*e^13 - 5120*a^8*c^5*d^4*e^15 - 1024*a^9*c^4*d^2*e^17 - 64*b^3*c^1 \\
& 0*d^17*e^2 + 512*b^4*c^9*d^16*e^3 - 1792*b^5*c^8*d^15*e^4 + 3584*b^6*c^7*d^ \\
& 14*e^5 - 4480*b^7*c^6*d^13*e^6 + 3584*b^8*c^5*d^12*e^7 - 1792*b^9*c^4*d^11* \\
& e^8 + 512*b^10*c^3*d^10*e^9 - 64*b^11*c^2*d^9*e^10 + 8192*a^2*b^2*c^9*d^14* \\
& e^5 + 5056*a^2*b^3*c^8*d^13*e^6 - 31104*a^2*b^4*c^7*d^12*e^7 + 40256*a^2*b^ \\
& 5*c^6*d^11*e^8 - 22784*a^2*b^6*c^5*d^10*e^9 + 3648*a^2*b^7*c^4*d^9*e^10 + 1 \\
& 664*a^2*b^8*c^3*d^8*e^11 - 576*a^2*b^9*c^2*d^7*e^12 + 45312*a^3*b^2*c^8*d^1 \\
& 2*e^7 - 27840*a^3*b^3*c^7*d^11*e^8 - 13760*a^3*b^4*c^6*d^10*e^9 + 27520*a^3 \\
& *b^5*c^5*d^9*e^10 - 12416*a^3*b^6*c^4*d^8*e^11 + 1088*a^3*b^7*c^3*d^7*e^12 \\
& + 320*a^3*b^8*c^2*d^6*e^13 + 53760*a^4*b^2*c^7*d^10*e^9 - 30400*a^4*b^3*c^6 \\
& *d^9*e^10 + 1280*a^4*b^4*c^5*d^8*e^11 + 4224*a^4*b^5*c^4*d^7*e^12 - 1280*a^ \\
& 4*b^6*c^3*d^6*e^13 + 320*a^4*b^7*c^2*d^5*e^14 + 6400*a^5*b^2*c^6*d^8*e^11 - \\
& 2624*a^5*b^3*c^5*d^7*e^12 + 5952*a^5*b^4*c^4*d^6*e^13 - 2752*a^5*b^5*c^3*d \\
& ^5*e^14 - 576*a^5*b^6*c^2*d^4*e^15 - 21504*a^6*b^2*c^5*d^6*e^13 + 832*a^6*b \\
& ^3*c^4*d^5*e^14 + 4736*a^6*b^4*c^3*d^4*e^15 + 320*a^6*b^5*c^2*d^3*e^16 - 84 \\
& 48*a^7*b^2*c^4*d^4*e^15 - 2624*a^7*b^3*c^3*d^3*e^16 - 64*a^7*b^4*c^2*d^2*e^ \\
& 17 + 512*a^8*b^2*c^3*d^2*e^17 + 256*a*b*c^11*d^17*e^2 - 2304*a*b^2*c^10*d^1 \\
& 6*e^3 + 8512*a*b^3*c^9*d^15*e^4 - 16704*a*b^4*c^8*d^14*e^5 + 18240*a*b^5*c^ \\
& 7*d^13*e^6 - 9536*a*b^6*c^6*d^12*e^7 - 576*a*b^7*c^5*d^11*e^8 + 3648*a*b^8* \\
& c^4*d^10*e^9 - 1856*a*b^9*c^3*d^9*e^10 + 320*a*b^10*c^2*d^8*e^11 - 5376*a^2 \\
& *b*c^10*d^15*e^4 - 25344*a^3*b*c^9*d^13*e^6 - 37120*a^4*b*c^8*d^11*e^8 - 11 \\
& 520*a^5*b*c^7*d^9*e^10 + 20736*a^6*b*c^6*d^7*e^12 + 20224*a^7*b*c^5*d^5*e^1 \\
& 4 + 5376*a^8*b*c^4*d^3*e^16))/((8*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b \\
& *c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6 \\
& *e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^ \\
& 6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8 \\
& *e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c \\
& *d^5*e^5)))*(-d^3*e^3)^(1/2)*(a*e^2 + 5*c*d^2 - 3*b*d*e))/(4*(c^2*d^7 + a^2 \\
& *d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2))*(a* \\
& e^2 + 5*c*d^2 - 3*b*d*e))/(4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^ \\
& ^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)))*(-d^3*e^3)^(1/2)*(a*e^2 + 5*c*d^2 - \\
& 3*b*d*e))/(4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^ \\
& 4*e^3 + 2*a*c*d^5*e^2)))*(-d^3*e^3)^(1/2)*(a*e^2 + 5*c*d^2 - 3*b*d*e)*1i)/ \\
& (4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a* \\
& c*d^5*e^2)))/((5*c^8*d^3*e^6 - 3*b*c^7*d^2*e^7 + a*c^7*d*e^8)/(c^4*d^10 + a \\
& ^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8* \\
& e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6 \\
& *e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c* \\
& d^6*e^4 - 12*a^2*b*c*d^5*e^5) - (((x*(54*c^9*d^6*e^5 - 2*a^3*c^6*e^11 - 22* \\
& a*c^8*d^4*e^7 - 118*b*c^8*d^5*e^6 + a^2*b^2*c^5*e^11 - 14*a^2*c^7*d^2*e^9 + \\
& 107*b^2*c^7*d^4*e^7 - 48*b^3*c^6*d^3*e^8 + 9*b^4*c^5*d^2*e^9 + 20*a*b*c^7* \\
& d^3*e^8 - 6*a*b^3*c^5*d*e^10 + 10*a^2*b*c^6*d*e^10 + 4*a*b^2*c^6*d^2*e^9)))/ \\
& (2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^ \\
& 7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 \\
& + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e \\
& ^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - (((2*a^2*b^6*c^2*e^13 - 20 \\
& 0*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 - 14*a^3*b^4*c^3*e^13 + 26*a^4*b^2*c^4*e^1 \\
& 3 + 480*a^2*c^8*d^6*e^7 + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^11 + 50*b^ \\
& 2*c^8*d^8*e^5 - 240*b^3*c^7*d^7*e^6 + 466*b^4*c^6*d^6*e^7 - 464*b^5*c^5*d^5 \\
& *e^8 + 246*b^6*c^4*d^4*e^9 - 64*b^7*c^3*d^3*e^10 + 6*b^8*c^2*d^2*e^11 + 4*a \\
& ^2*b^2*c^6*d^4*e^9 + 672*a^2*b^3*c^5*d^3*e^10 - 354*a^2*b^4*c^4*d^2*e^11 + \\
& 464*a^3*b^2*c^5*d^2*e^11 + 960*a*b*c^8*d^7*e^6 - 8*a*b^7*c^2*d*e^12 - 96*a^ \\
& 4*b*c^5*d*e^12 - 1984*a*b^2*c^7*d^6*e^7 + 2072*a*b^3*c^6*d^5*e^8 - 1034*a*b \\
& ^4*c^5*d^4*e^9 + 160*a*b^5*c^4*d^3*e^10 + 34*a*b^6*c^3*d^2*e^11 - 864*a^2*b \\
& *c^7*d^5*e^8 + 40*a^2*b^5*c^3*d*e^12 - 1152*a^3*b*c^6*d^3*e^10 - 8*a^3*b^3* \\
& c^4*d*e^12))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4* \\
& a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2
\end{aligned}$$

$$\begin{aligned}
& *b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a \\
& *b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5) - ((-d^3*e^3)^{(1/2)}*((x*(32*c^{11}*d^{13}*e^2 + 48*a^6*b*c^4*e^{15} + 96*a*c^{10}*d^{11}*e^4 - 64*a^6 \\
& *c^5*d*e^{14} - 160*b*c^{10}*d^{12}*e^3 + 4*a^4*b^5*c^2*e^{15} - 28*a^5*b^3*c^3*e^{15} - 2048*a^2*c^9*d^9*e^6 - 4416*a^3*c^8*d^7*e^8 - 2528*a^4*c^7*d^5*e^{10} - 2 \\
& 88*a^5*c^6*d^3*e^{12} + 336*b^2*c^9*d^{11}*e^4 - 268*b^3*c^8*d^{10}*e^5 - 360*b^4 \\
& *c^7*d^9*e^6 + 1260*b^5*c^6*d^8*e^7 - 1568*b^6*c^5*d^7*e^8 + 1036*b^7*c^4*d \\
& ^6*e^9 - 360*b^8*c^3*d^5*e^{10} + 52*b^9*c^2*d^4*e^{11} - 7584*a^2*b^2*c^7*d^7* \\
& e^8 - 536*a^2*b^3*c^6*d^6*e^9 + 5936*a^2*b^4*c^5*d^5*e^{10} - 3552*a^2*b^5*c^ \\
& 4*d^4*e^{11} + 464*a^2*b^6*c^3*d^3*e^{12} + 104*a^2*b^7*c^2*d^2*e^{13} - 12768*a^ \\
& 3*b^2*c^6*d^5*e^{10} + 3720*a^3*b^3*c^5*d^4*e^{11} + 1280*a^3*b^4*c^4*d^3*e^{12} \\
& - 648*a^3*b^5*c^3*d^2*e^{13} - 4272*a^4*b^2*c^5*d^3*e^{12} + 740*a^4*b^3*c^4*d^ \\
& 2*e^{13} - 848*a*b*c^9*d^{10}*e^5 + 3632*a*b^2*c^8*d^9*e^6 - 7852*a*b^3*c^7*d^8 \\
& *e^7 + 8864*a*b^4*c^6*d^7*e^8 - 4936*a*b^5*c^5*d^6*e^9 + 816*a*b^6*c^4*d^5* \\
& e^{10} + 356*a*b^7*c^3*d^4*e^{11} - 128*a*b^8*c^2*d^3*e^{12} + 7216*a^2*b*c^8*d^8 \\
& *e^7 + 12896*a^3*b*c^7*d^6*e^9 - 32*a^3*b^6*c^2*d*e^{14} + 5696*a^4*b*c^6*d^4 \\
& *e^{11} + 216*a^4*b^4*c^3*d*e^{14} + 752*a^5*b*c^5*d^2*e^{13} - 336*a^5*b^2*c^4*d \\
& *e^{14}))/((2*(c^4*d^{10} + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3* \\
& b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2 \\
& *d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c \\
& ^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) + (((128*a*c^{11}*d^{15} \\
& *e^2 - 256*a^8*c^4*d*e^{16} - 256*a^2*c^{10}*d^{13}*e^4 - 3456*a^3*c^9*d^{11}*e^6 - \\
& 8960*a^4*c^8*d^9*e^8 - 10880*a^5*c^7*d^7*e^{10} - 6912*a^6*c^6*d^5*e^{12} - 21 \\
& 76*a^7*c^5*d^3*e^{14} - 32*b^2*c^{10}*d^{15}*e^2 + 256*b^3*c^9*d^{14}*e^3 - 896*b^4 \\
& *c^8*d^{13}*e^4 + 1792*b^5*c^7*d^{12}*e^5 - 2240*b^6*c^6*d^{11}*e^6 + 1792*b^7*c^ \\
& 5*d^{10}*e^7 - 896*b^8*c^4*d^9*e^8 + 256*b^9*c^3*d^8*e^9 - 32*b^{10}*c^2*d^7*e^ \\
& 10 + 2848*a^2*b^2*c^8*d^{11}*e^6 - 12160*a^2*b^3*c^7*d^{10}*e^7 + 18480*a^2*b^4 \\
& *c^6*d^9*e^8 - 12864*a^2*b^5*c^5*d^8*e^9 + 3008*a^2*b^6*c^4*d^7*e^{10} + 832* \\
& a^2*b^7*c^3*d^6*e^{11} - 400*a^2*b^8*c^2*d^5*e^{12} - 17920*a^3*b^2*c^7*d^9*e^8 \\
& + 1280*a^3*b^3*c^6*d^8*e^9 + 14240*a^3*b^4*c^5*d^7*e^{10} - 9824*a^3*b^5*c^4 \\
& *d^6*e^{11} + 1120*a^3*b^6*c^3*d^5*e^{12} + 480*a^3*b^7*c^2*d^4*e^{13} - 33760*a^ \\
& 4*b^2*c^6*d^7*e^{10} + 7680*a^4*b^3*c^5*d^6*e^{11} + 7520*a^4*b^4*c^4*d^5*e^{12} \\
& - 2880*a^4*b^5*c^3*d^4*e^{13} - 320*a^4*b^6*c^2*d^3*e^{14} - 20672*a^5*b^2*c^5* \\
& d^5*e^{12} + 896*a^5*b^3*c^4*d^4*e^{13} + 2384*a^5*b^4*c^3*d^3*e^{14} + 112*a^5*b \\
& ^5*c^2*d^2*e^{15} - 3872*a^6*b^2*c^4*d^3*e^{14} - 896*a^6*b^3*c^3*d^2*e^{15} - 10 \\
& 24*a*b*c^{10}*d^{14}*e^3 + 3648*a*b^2*c^9*d^{13}*e^4 - 7296*a*b^3*c^8*d^{12}*e^5 + \\
& 8464*a*b^4*c^7*d^{11}*e^6 - 5008*a*b^5*c^6*d^{10}*e^7 + 224*a*b^6*c^5*d^9*e^8 + \\
& 1632*a*b^7*c^4*d^8*e^9 - 944*a*b^8*c^3*d^7*e^{10} + 176*a*b^9*c^2*d^6*e^{11} + \\
& 512*a^2*b*c^9*d^{12}*e^5 + 14080*a^3*b*c^8*d^{10}*e^7 + 30720*a^4*b*c^7*d^8*e^ \\
& 9 + 28160*a^5*b*c^6*d^6*e^{11} + 11776*a^6*b*c^5*d^4*e^{13} - 16*a^6*b^4*c^2*d* \\
& e^{16} + 1792*a^7*b*c^4*d^2*e^{15} + 128*a^7*b^2*c^3*d*e^{16}))/((2*(c^4*d^{10} + a^4 \\
& *d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^ \\
& 2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e \\
& ^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^ \\
& 6*e^4 - 12*a^2*b*c*d^5*e^5)) - (x*(-d^3*e^3)^{(1/2)}*(a*e^2 + 5*c*d^2 - 3*b*d \\
& *e)*(1024*a^2*c^{11}*d^{16}*e^3 + 5120*a^3*c^{10}*d^{14}*e^5 + 9216*a^4*c^9*d^{12}*e^ \\
& 7 + 5120*a^5*c^8*d^{10}*e^9 - 5120*a^6*c^7*d^8*e^{11} - 9216*a^7*c^6*d^6*e^{13} - \\
& 5120*a^8*c^5*d^4*e^{15} - 1024*a^9*c^4*d^2*e^{17} - 64*b^3*c^{10}*d^{17}*e^2 + 512 \\
& *b^4*c^9*d^{16}*e^3 - 1792*b^5*c^8*d^{15}*e^4 + 3584*b^6*c^7*d^{14}*e^5 - 4480*b^ \\
& 7*c^6*d^{13}*e^6 + 3584*b^8*c^5*d^{12}*e^7 - 1792*b^9*c^4*d^{11}*e^8 + 512*b^{10}*c \\
& ^3*d^{10}*e^9 - 64*b^{11}*c^2*d^9*e^{10} + 8192*a^2*b^2*c^9*d^{14}*e^5 + 5056*a^2*b \\
& ^3*c^8*d^{13}*e^6 - 31104*a^2*b^4*c^7*d^{12}*e^7 + 40256*a^2*b^5*c^6*d^{11}*e^8 - \\
& 22784*a^2*b^6*c^5*d^{10}*e^9 + 3648*a^2*b^7*c^4*d^9*e^{10} + 1664*a^2*b^8*c^3* \\
& d^8*e^{11} - 576*a^2*b^9*c^2*d^7*e^{12} + 45312*a^3*b^2*c^8*d^{12}*e^7 - 27840*a^ \\
& 3*b^3*c^7*d^{11}*e^8 - 13760*a^3*b^4*c^6*d^{10}*e^9 + 27520*a^3*b^5*c^5*d^9*e^1 \\
& 0 - 12416*a^3*b^6*c^4*d^8*e^{11} + 1088*a^3*b^7*c^3*d^7*e^{12} + 320*a^3*b^8*c^ \\
& 2*d^6*e^{13} + 53760*a^4*b^2*c^7*d^{10}*e^9 - 30400*a^4*b^3*c^6*d^9*e^{10} + 1280 \\
& *a^4*b^4*c^5*d^8*e^{11} + 4224*a^4*b^5*c^4*d^7*e^{12} - 1280*a^4*b^6*c^3*d^6*e^ \\
& 13 + 320*a^4*b^7*c^2*d^5*e^{14} + 6400*a^5*b^2*c^6*d^8*e^{11} - 2624*a^5*b^3*c^
\end{aligned}$$

$$\begin{aligned}
&5*d^7*e^{12} + 5952*a^5*b^4*c^4*d^6*e^{13} - 2752*a^5*b^5*c^3*d^5*e^{14} - 576*a^5*b^6*c^2*d^4*e^{15} - 21504*a^6*b^2*c^5*d^6*e^{13} + 832*a^6*b^3*c^4*d^5*e^{14} \\
&+ 4736*a^6*b^4*c^3*d^4*e^{15} + 320*a^6*b^5*c^2*d^3*e^{16} - 8448*a^7*b^2*c^4*d^4*e^{15} - 2624*a^7*b^3*c^3*d^3*e^{16} - 64*a^7*b^4*c^2*d^2*e^{17} + 512*a^8*b^2*c^3*d^2*e^{17} + 256*a*b*c^{11}*d^{17}*e^2 - 2304*a*b^2*c^{10}*d^{16}*e^3 + 8512*a*b^3*c^9*d^{15}*e^4 - 16704*a*b^4*c^8*d^{14}*e^5 + 18240*a*b^5*c^7*d^{13}*e^6 - 9536*a*b^6*c^6*d^{12}*e^7 - 576*a*b^7*c^5*d^{11}*e^8 + 3648*a*b^8*c^4*d^{10}*e^9 - 1856*a*b^9*c^3*d^9*e^{10} + 320*a*b^{10}*c^2*d^8*e^{11} - 5376*a^2*b*c^{10}*d^{15}*e^4 - 25344*a^3*b*c^9*d^{13}*e^6 - 37120*a^4*b*c^8*d^{11}*e^8 - 11520*a^5*b*c^7*d^9*e^{10} + 20736*a^6*b*c^6*d^7*e^{12} + 20224*a^7*b*c^5*d^5*e^{14} + 5376*a^8*b*c^4*d^3*e^{16}) \\
&)/((8*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)*(c^4*d^{10} + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*(-d^3*e^3)^{(1/2)}*(a*e^2 + 5*c*d^2 - 3*b*d*e))/((4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)))*(a*e^2 + 5*c*d^2 - 3*b*d*e))/((4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)))*(-d^3*e^3)^{(1/2)}*(a*e^2 + 5*c*d^2 - 3*b*d*e))/((4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)))*(-d^3*e^3)^{(1/2)}*(a*e^2 + 5*c*d^2 - 3*b*d*e))/((4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)))*(-d^3*e^3)^{(1/2)}*(a*e^2 + 5*c*d^2 - 3*b*d*e))/((4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)))*(-d^3*e^3)^{(1/2)}*(a*e^2 + 5*c*d^2 - 3*b*d*e)) + (((x*(54*c^9*d^6*e^5 - 2*a^3*c^6*e^{11} - 22*a*c^8*d^4*e^7 - 118*b*c^8*d^5*e^6 + a^2*b^2*c^5*e^{11} - 14*a^2*c^7*d^2*e^9 + 107*b^2*c^7*d^4*e^7 - 48*b^3*c^6*d^3*e^8 + 9*b^4*c^5*d^2*e^9 + 20*a*b*c^7*d^3*e^8 - 6*a*b^3*c^5*d*e^{10} + 10*a^2*b*c^6*d*e^{10} + 4*a*b^2*c^6*d^2*e^9)))/(2*(c^4*d^{10} + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) + (((2*a^2*b^6*c^2*e^{13} - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^{13} - 14*a^3*b^4*c^3*e^{13} + 26*a^4*b^2*c^4*e^{13} + 480*a^2*c^8*d^6*e^7 + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^{11} + 50*b^2*c^8*d^8*e^5 - 240*b^3*c^7*d^7*e^6 + 466*b^4*c^6*d^6*e^7 - 464*b^5*c^5*d^5*e^8 + 246*b^6*c^4*d^4*e^9 - 64*b^7*c^3*d^3*e^{10} + 6*b^8*c^2*d^2*e^{11} + 4*a^2*b^2*c^6*d^4*e^9 + 672*a^2*b^3*c^5*d^3*e^{10} - 354*a^2*b^4*c^4*d^2*e^{11} + 464*a^3*b^2*c^5*d^2*e^{11} + 960*a*b*c^8*d^7*e^6 - 8*a*b^7*c^2*d*e^{12} - 96*a^4*b*c^5*d*e^{12} - 1984*a*b^2*c^7*d^6*e^7 + 2072*a*b^3*c^6*d^5*e^8 - 1034*a*b^4*c^5*d^4*e^9 + 160*a*b^5*c^4*d^3*e^{10} + 34*a*b^6*c^3*d^2*e^{11} - 864*a^2*b*c^7*d^5*e^8 + 40*a^2*b^5*c^3*d*e^{12} - 1152*a^3*b*c^6*d^3*e^{10} - 8*a^3*b^3*c^4*d*e^{12}))/((2*(c^4*d^{10} + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) + ((-d^3*e^3)^{(1/2)}*((x*(32*c^{11}*d^{13}*e^2 + 48*a^6*b*c^4*e^{15} + 96*a*c^{10}*d^{11}*e^4 - 64*a^6*c^5*d*e^{14} - 160*b*c^{10}*d^{12}*e^3 + 4*a^4*b^5*c^2*e^{15} - 28*a^5*b^3*c^3*e^{15} - 2048*a^2*c^9*d^9*e^6 - 4416*a^3*c^8*d^7*e^8 - 2528*a^4*c^7*d^5*e^{10} - 288*a^5*c^6*d^3*e^{12} + 336*b^2*c^9*d^{11}*e^4 - 268*b^3*c^8*d^{10}*e^5 - 360*b^4*c^7*d^9*e^6 + 1260*b^5*c^6*d^8*e^7 - 1568*b^6*c^5*d^7*e^8 + 1036*b^7*c^4*d^6*e^9 - 360*b^8*c^3*d^5*e^{10} + 52*b^9*c^2*d^4*e^{11} - 7584*a^2*b^2*c^7*d^7*e^8 - 536*a^2*b^3*c^6*d^6*e^9 + 5936*a^2*b^4*c^5*d^5*e^{10} - 3552*a^2*b^5*c^4*d^4*e^{11} + 464*a^2*b^6*c^3*d^3*e^{12} + 104*a^2*b^7*c^2*d^2*e^{13} - 12768*a^3*b^2*c^6*d^5*e^{10} + 3720*a^3*b^3*c^5*d^4*e^{11} + 1280*a^3*b^4*c^4*d^3*e^{12} - 648*a^3*b^5*c^3*d^2*e^{13} - 4272*a^4*b^2*c^5*d^3*e^{12} + 740*a^4*b^3*c^4*d^2*e^{13} - 848*a*b*c^9*d^{10}*e^5 + 3632*a*b^2*c^8*d^9*e^6 - 7852*a*b^3*c^7*d^8*e^7 + 8864*a*b^4*c^6*d^7*e^8 - 4936*a*b^5*c^5*d^6*e^9 + 816*a*b^6*c^4*d^5*e^{10} + 356*a*b^7*c^3*d^4*e^{11} - 128*a*b^8*c^2*d^3*e^{12} + 7216*a^2*b*c^8*d^8*e^7 + 12896*a^3*b*c^7*d^6*e^9 - 32*a^3*b^6*c^2*d*e^{14} + 5696*a^4*b*c^6*d^4*e^{11} + 216*a^4*b^4*c^3*d*e^{14} + 752*a^5*b*c^5*d^2*e^{13} - 336*a^5*b^2*c^4*d*e^{14}))/((2*(c^4*d^{10} + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c
\end{aligned}$$

$$\begin{aligned}
& *d^4e^6 - 4*b^3c*d^7e^3 + 6*a^2b^2*d^4e^6 + 6*a^2c^2*d^6e^4 + 6*b^2c^2*d^8e^2 - 4*b*c^3*d^9e - 12*a*b*c^2*d^7e^3 + 12*a*b^2*c*d^6e^4 - 12*a^2*b*c*d^5e^5) - (((128*a*c^11*d^15e^2 - 256*a^8*c^4*d^6e^16 - 256*a^2*c^10*d^13e^4 - 3456*a^3*c^9*d^11e^6 - 8960*a^4*c^8*d^9e^8 - 10880*a^5*c^7*d^7e^10 - 6912*a^6*c^6*d^5e^12 - 2176*a^7*c^5*d^3e^14 - 32*b^2*c^10*d^15e^2 + 256*b^3*c^9*d^14e^3 - 896*b^4*c^8*d^13e^4 + 1792*b^5*c^7*d^12e^5 - 2240*b^6*c^6*d^11e^6 + 1792*b^7*c^5*d^10e^7 - 896*b^8*c^4*d^9e^8 + 256*b^9*c^3*d^8e^9 - 32*b^10*c^2*d^7e^10 + 2848*a^2*b^2*c^8*d^11e^6 - 12160*a^2*b^3*c^7*d^10e^7 + 18480*a^2*b^4*c^6*d^9e^8 - 12864*a^2*b^5*c^5*d^8e^9 + 3008*a^2*b^6*c^4*d^7e^10 + 832*a^2*b^7*c^3*d^6e^11 - 400*a^2*b^8*c^2*d^5e^12 - 17920*a^3*b^2*c^7*d^9e^8 + 1280*a^3*b^3*c^6*d^8e^9 + 14240*a^3*b^4*c^5*d^7e^10 - 9824*a^3*b^5*c^4*d^6e^11 + 1120*a^3*b^6*c^3*d^5e^12 + 480*a^3*b^7*c^2*d^4e^13 - 33760*a^4*b^2*c^6*d^7e^10 + 7680*a^4*b^3*c^5*d^6e^11 + 7520*a^4*b^4*c^4*d^5e^12 - 2880*a^4*b^5*c^3*d^4e^13 - 320*a^4*b^6*c^2*d^3e^14 - 20672*a^5*b^2*c^5*d^5e^12 + 896*a^5*b^3*c^4*d^4e^13 + 2384*a^5*b^4*c^3*d^3e^14 + 112*a^5*b^5*c^2*d^2e^15 - 3872*a^6*b^2*c^4*d^3e^14 - 896*a^6*b^3*c^3*d^2e^15 - 1024*a*b*c^10*d^14e^3 + 3648*a*b^2*c^9*d^13e^4 - 7296*a*b^3*c^8*d^12e^5 + 8464*a*b^4*c^7*d^11e^6 - 5008*a*b^5*c^6*d^10e^7 + 224*a*b^6*c^5*d^9e^8 + 1632*a*b^7*c^4*d^8e^9 - 944*a*b^8*c^3*d^7e^10 + 176*a*b^9*c^2*d^6e^11 + 512*a^2*b*c^9*d^12e^5 + 14080*a^3*b*c^8*d^10e^7 + 30720*a^4*b*c^7*d^8e^9 + 28160*a^5*b*c^6*d^6e^11 + 11776*a^6*b*c^5*d^4e^13 - 16*a^6*b^4*c^2*d^6e^16 + 1792*a^7*b*c^4*d^2e^15 + 128*a^7*b^2*c^3*d^5e^16)/(2*(c^4*d^10 + a^4*d^2e^8 + b^4*d^6e^4 - 4*a*b^3*d^5e^5 - 4*a^3*b*d^3e^7 + 4*a*c^3*d^8e^2 + 4*a^3*c*d^4e^6 - 4*b^3*c*d^7e^3 + 6*a^2*b^2*d^4e^6 + 6*a^2*c^2*d^6e^4 + 6*b^2*c^2*d^8e^2 - 4*b*c^3*d^9e - 12*a*b*c^2*d^7e^3 + 12*a*b^2*c*d^6e^4 - 12*a^2*b*c*d^5e^5)) + (x*(-d^3e^3)^(1/2)*(a*e^2 + 5*c*d^2 - 3*b*d*e)*(1024*a^2*c^11*d^16e^3 + 5120*a^3*c^10*d^14e^5 + 9216*a^4*c^9*d^12e^7 + 5120*a^5*c^8*d^10e^9 - 5120*a^6*c^7*d^8e^11 - 9216*a^7*c^6*d^6e^13 - 5120*a^8*c^5*d^4e^15 - 1024*a^9*c^4*d^2e^17 - 64*b^3*c^10*d^17e^2 + 512*b^4*c^9*d^16e^3 - 1792*b^5*c^8*d^15e^4 + 3584*b^6*c^7*d^14e^5 - 4480*b^7*c^6*d^13e^6 + 3584*b^8*c^5*d^12e^7 - 1792*b^9*c^4*d^11e^8 + 512*b^10*c^3*d^10e^9 - 64*b^11*c^2*d^9e^10 + 8192*a^2*b^2*c^9*d^14e^5 + 5056*a^2*b^3*c^8*d^13e^6 - 31104*a^2*b^4*c^7*d^12e^7 + 40256*a^2*b^5*c^6*d^11e^8 - 22784*a^2*b^6*c^5*d^10e^9 + 3648*a^2*b^7*c^4*d^9e^10 + 1664*a^2*b^8*c^3*d^8e^11 - 576*a^2*b^9*c^2*d^7e^12 + 45312*a^3*b^2*c^8*d^12e^7 - 27840*a^3*b^3*c^7*d^11e^8 - 13760*a^3*b^4*c^6*d^10e^9 + 27520*a^3*b^5*c^5*d^9e^10 - 12416*a^3*b^6*c^4*d^8e^11 + 1088*a^3*b^7*c^3*d^7e^12 + 320*a^3*b^8*c^2*d^6e^13 + 53760*a^4*b^2*c^7*d^10e^9 - 30400*a^4*b^3*c^6*d^9e^10 + 1280*a^4*b^4*c^5*d^8e^11 + 4224*a^4*b^5*c^4*d^7e^12 - 1280*a^4*b^6*c^3*d^6e^13 + 320*a^4*b^7*c^2*d^5e^14 + 6400*a^5*b^2*c^6*d^8e^11 - 2624*a^5*b^3*c^5*d^7e^12 + 5952*a^5*b^4*c^4*d^6e^13 - 2752*a^5*b^5*c^3*d^5e^14 - 576*a^5*b^6*c^2*d^4e^15 - 21504*a^6*b^2*c^5*d^6e^13 + 832*a^6*b^3*c^4*d^5e^14 + 4736*a^6*b^4*c^3*d^4e^15 + 320*a^6*b^5*c^2*d^3e^16 - 8448*a^7*b^2*c^4*d^4e^15 - 2624*a^7*b^3*c^3*d^3e^16 - 64*a^7*b^4*c^2*d^2e^17 + 512*a^8*b^2*c^3*d^2e^17 + 256*a*b*c^11*d^17e^2 - 2304*a*b^2*c^10*d^16e^3 + 8512*a*b^3*c^9*d^15e^4 - 16704*a*b^4*c^8*d^14e^5 + 18240*a*b^5*c^7*d^13e^6 - 9536*a*b^6*c^6*d^12e^7 - 576*a*b^7*c^5*d^11e^8 + 3648*a*b^8*c^4*d^10e^9 - 1856*a*b^9*c^3*d^9e^10 + 320*a*b^10*c^2*d^8e^11 - 5376*a^2*b*c^10*d^15e^4 - 25344*a^3*b*c^9*d^13e^6 - 37120*a^4*b*c^8*d^11e^8 - 11520*a^5*b*c^7*d^9e^10 + 20736*a^6*b*c^6*d^7e^12 + 20224*a^7*b*c^5*d^5e^14 + 5376*a^8*b*c^4*d^3e^16))/(8*(c^2*d^7 + a^2*d^3e^4 + b^2*d^5e^2 - 2*b*c*d^6e - 2*a*b*d^4e^3 + 2*a*c*d^5e^2)*(c^4*d^10 + a^4*d^2e^8 + b^4*d^6e^4 - 4*a*b^3*d^5e^5 - 4*a^3*b*d^3e^7 + 4*a*c^3*d^8e^2 + 4*a^3*c*d^4e^6 - 4*b^3*c*d^7e^3 + 6*a^2*b^2*d^4e^6 + 6*a^2*c^2*d^6e^4 + 6*b^2*c^2*d^8e^2 - 4*b*c^3*d^9e - 12*a*b*c^2*d^7e^3 + 12*a*b^2*c*d^6e^4 - 12*a^2*b*c*d^5e^5)))*(-d^3e^3)^(1/2)*(a*e^2 + 5*c*d^2 - 3*b*d*e))/(4*(c^2*d^7 + a^2*d^3e^4 + b^2*d^5e^2 - 2*b*c*d^6e - 2*a*b*d^4e^3 + 2*a*c*d^5e^2)))*(a*e^2 + 5*c*d^2 - 3*b*d*e))/(4*(c^2*d^7 + a^2*d^3e^4 + b^2*d^5e^2 - 2*b*c*d^6e - 2*a*b*d^4e^3 + 2*a*c*d^5e^2))*(-d^3e^3)^(1
\end{aligned}$$



$$\begin{aligned}
& /2)*(a*e^2 + 5*c*d^2 - 3*b*d*e))/(4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - \\
& 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)))*(-d^3*e^3)^{(1/2)}*(a*e^2 + 5* \\
& c*d^2 - 3*b*d*e))/(4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2 \\
& *a*b*d^4*e^3 + 2*a*c*d^5*e^2)))*(-d^3*e^3)^{(1/2)}*(a*e^2 + 5*c*d^2 - 3*b*d* \\
& e)*i)/(2*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^ \\
& 3 + 2*a*c*d^5*e^2)) - \operatorname{atan}(\left(\frac{(2*a^2*b^6*c^2*e^{13} - 200*a*c^9*d^8*e^5 - 8*a \\
& ^5*c^5*e^{13} - 14*a^3*b^4*c^3*e^{13} + 26*a^4*b^2*c^4*e^{13} + 480*a^2*c^8*d^6*e \\
& ^7 + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^{11} + 50*b^2*c^8*d^8*e^5 - 240*b \\
& ^3*c^7*d^7*e^6 + 466*b^4*c^6*d^6*e^7 - 464*b^5*c^5*d^5*e^8 + 246*b^6*c^4*d^ \\
& 4*e^9 - 64*b^7*c^3*d^3*e^{10} + 6*b^8*c^2*d^2*e^{11} + 4*a^2*b^2*c^6*d^4*e^9 + \\
& 672*a^2*b^3*c^5*d^3*e^{10} - 354*a^2*b^4*c^4*d^2*e^{11} + 464*a^3*b^2*c^5*d^2*e \\
& ^{11} + 960*a*b*c^8*d^7*e^6 - 8*a*b^7*c^2*d*e^{12} - 96*a^4*b*c^5*d*e^{12} - 1984 \\
& *a*b^2*c^7*d^6*e^7 + 2072*a*b^3*c^6*d^5*e^8 - 1034*a*b^4*c^5*d^4*e^9 + 160* \\
& a*b^5*c^4*d^3*e^{10} + 34*a*b^6*c^3*d^2*e^{11} - 864*a^2*b*c^7*d^5*e^8 + 40*a^2 \\
& *b^5*c^3*d*e^{12} - 1152*a^3*b*c^6*d^3*e^{10} - 8*a^3*b^3*c^4*d*e^{12})}{(2*(c^4*d \\
& ^{10} + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c \\
& ^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2* \\
& c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a \\
& *b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - \left(\frac{(128*a*c^{11}*d^{15}*e^2 - 256*a^8*c^ \\
& 4*d*e^{16} - 256*a^2*c^{10}*d^{13}*e^4 - 3456*a^3*c^9*d^{11}*e^6 - 8960*a^4*c^8*d^9 \\
& *e^8 - 10880*a^5*c^7*d^7*e^{10} - 6912*a^6*c^6*d^5*e^{12} - 2176*a^7*c^5*d^3*e^ \\
& 14 - 32*b^2*c^{10}*d^{15}*e^2 + 256*b^3*c^9*d^{14}*e^3 - 896*b^4*c^8*d^{13}*e^4 + 1 \\
& 792*b^5*c^7*d^{12}*e^5 - 2240*b^6*c^6*d^{11}*e^6 + 1792*b^7*c^5*d^{10}*e^7 - 896* \\
& b^8*c^4*d^9*e^8 + 256*b^9*c^3*d^8*e^9 - 32*b^{10}*c^2*d^7*e^{10} + 2848*a^2*b^2 \\
& *c^8*d^{11}*e^6 - 12160*a^2*b^3*c^7*d^{10}*e^7 + 18480*a^2*b^4*c^6*d^9*e^8 - 12 \\
& 864*a^2*b^5*c^5*d^8*e^9 + 3008*a^2*b^6*c^4*d^7*e^{10} + 832*a^2*b^7*c^3*d^6*e \\
& ^{11} - 400*a^2*b^8*c^2*d^5*e^{12} - 17920*a^3*b^2*c^7*d^9*e^8 + 1280*a^3*b^3*c \\
& ^6*d^8*e^9 + 14240*a^3*b^4*c^5*d^7*e^{10} - 9824*a^3*b^5*c^4*d^6*e^{11} + 1120* \\
& a^3*b^6*c^3*d^5*e^{12} + 480*a^3*b^7*c^2*d^4*e^{13} - 33760*a^4*b^2*c^6*d^7*e^{10} \\
& 0 + 7680*a^4*b^3*c^5*d^6*e^{11} + 7520*a^4*b^4*c^4*d^5*e^{12} - 2880*a^4*b^5*c^ \\
& 3*d^4*e^{13} - 320*a^4*b^6*c^2*d^3*e^{14} - 20672*a^5*b^2*c^5*d^5*e^{12} + 896*a^ \\
& 5*b^3*c^4*d^4*e^{13} + 2384*a^5*b^4*c^3*d^3*e^{14} + 112*a^5*b^5*c^2*d^2*e^{15} - \\
& 3872*a^6*b^2*c^4*d^3*e^{14} - 896*a^6*b^3*c^3*d^2*e^{15} - 1024*a*b*c^{10}*d^{14}* \\
& e^3 + 3648*a*b^2*c^9*d^{13}*e^4 - 7296*a*b^3*c^8*d^{12}*e^5 + 8464*a*b^4*c^7*d^ \\
& 11*e^6 - 5008*a*b^5*c^6*d^{10}*e^7 + 224*a*b^6*c^5*d^9*e^8 + 1632*a*b^7*c^4*d \\
& ^8*e^9 - 944*a*b^8*c^3*d^7*e^{10} + 176*a*b^9*c^2*d^6*e^{11} + 512*a^2*b*c^9*d^ \\
& 12*e^5 + 14080*a^3*b*c^8*d^{10}*e^7 + 30720*a^4*b*c^7*d^8*e^9 + 28160*a^5*b*c \\
& ^6*d^6*e^{11} + 11776*a^6*b*c^5*d^4*e^{13} - 16*a^6*b^4*c^2*d*e^{16} + 1792*a^7*b \\
& *c^4*d^2*e^{15} + 128*a^7*b^2*c^3*d*e^{16})}{(2*(c^4*d^{10} + a^4*d^2*e^8 + b^4*d^ \\
& 6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e \\
& ^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^ \\
& 8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b* \\
& c*d^5*e^5)) - (x*((b^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - b^3*c^4*d^4 - b^7*e^4 \\
& + c^4*d^4*(-(4*a*c - b^2)^3)^{(1/2)} + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - \\
& 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4 \\
& *a*c - b^2)^3)^{(1/2)} - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + \\
& 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^4 \\
& (- (4*a*c - b^2)^3)^{(1/2)} - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^ \\
& 3*d^3*e*(-(4*a*c - b^2)^3)^{(1/2)} - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + \\
& 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c \\
& ^3*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^{(1 \\
& /2)))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8* \\
& a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2 \\
& *b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 \\
& + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64 \\
& *a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 14 \\
& 4*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4 \\
& *a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d* \\
& e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a
\end{aligned}$$

$$\begin{aligned}
& \left( 2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5 \right)^{(1/2)} * (1024*a^2*c^11*d^16*e^3 + 5120*a^3*c^10*d^14*e^5 + 9216*a^4*c^9*d^12*e^7 + 5120*a^5*c^8*d^10*e^9 - 5120*a^6*c^7*d^8*e^11 - 9216*a^7*c^6*d^6*e^13 - 5120*a^8*c^5*d^4*e^15 - 1024*a^9*c^4*d^2*e^17 - 64*b^3*c^10*d^17*e^2 + 512*b^4*c^9*d^16*e^3 - 1792*b^5*c^8*d^15*e^4 + 3584*b^6*c^7*d^14*e^5 - 4480*b^7*c^6*d^13*e^6 + 3584*b^8*c^5*d^12*e^7 - 1792*b^9*c^4*d^11*e^8 + 512*b^10*c^3*d^10*e^9 - 64*b^11*c^2*d^9*e^10 + 8192*a^2*b^2*c^9*d^14*e^5 + 5056*a^2*b^3*c^8*d^13*e^6 - 31104*a^2*b^4*c^7*d^12*e^7 + 40256*a^2*b^5*c^6*d^11*e^8 - 22784*a^2*b^6*c^5*d^10*e^9 + 3648*a^2*b^7*c^4*d^9*e^10 + 1664*a^2*b^8*c^3*d^8*e^11 - 576*a^2*b^9*c^2*d^7*e^12 + 45312*a^3*b^2*c^8*d^12*e^7 - 27840*a^3*b^3*c^7*d^11*e^8 - 13760*a^3*b^4*c^6*d^10*e^9 + 27520*a^3*b^5*c^5*d^9*e^10 - 12416*a^3*b^6*c^4*d^8*e^11 + 1088*a^3*b^7*c^3*d^7*e^12 + 320*a^3*b^8*c^2*d^6*e^13 + 53760*a^4*b^2*c^7*d^10*e^9 - 30400*a^4*b^3*c^6*d^9*e^10 + 1280*a^4*b^4*c^5*d^8*e^11 + 4224*a^4*b^5*c^4*d^7*e^12 - 1280*a^4*b^6*c^3*d^6*e^13 + 320*a^4*b^7*c^2*d^5*e^14 + 6400*a^5*b^2*c^6*d^8*e^11 - 2624*a^5*b^3*c^5*d^7*e^12 + 5952*a^5*b^4*c^4*d^6*e^13 - 2752*a^5*b^5*c^3*d^5*e^14 - 576*a^5*b^6*c^2*d^4*e^15 - 21504*a^6*b^2*c^5*d^6*e^13 + 832*a^6*b^3*c^4*d^5*e^14 + 4736*a^6*b^4*c^3*d^4*e^15 + 320*a^6*b^5*c^2*d^3*e^16 - 8448*a^7*b^2*c^4*d^4*e^15 - 2624*a^7*b^3*c^3*d^3*e^16 - 64*a^7*b^4*c^2*d^2*e^17 + 512*a^8*b^2*c^3*d^2*e^17 + 256*a*b*c^11*d^17*e^2 - 2304*a*b^2*c^10*d^16*e^3 + 8512*a*b^3*c^9*d^15*e^4 - 16704*a*b^4*c^8*d^14*e^5 + 18240*a*b^5*c^7*d^13*e^6 - 9536*a*b^6*c^6*d^12*e^7 - 576*a*b^7*c^5*d^11*e^8 + 3648*a*b^8*c^4*d^10*e^9 - 1856*a*b^9*c^3*d^9*e^10 + 320*a*b^10*c^2*d^8*e^11 - 5376*a^2*b*c^10*d^15*e^4 - 25344*a^3*b*c^9*d^13*e^6 - 37120*a^4*b*c^8*d^11*e^8 - 11520*a^5*b*c^7*d^9*e^10 + 20736*a^6*b*c^6*d^7*e^12 + 20224*a^7*b*c^5*d^5*e^14 + 5376*a^8*b*c^4*d^3*e^16) / (2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) * ((b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2)) / (8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5))^(1/2) + (x*(32*c^11*d^13*e^2 + 48*a^6*b*c^4*e^15 + 96*a*c^10*d^11*e^4 - 64*a^6*c^5*d*e^14 - 160*b*c^10*d^12*e^3 + 4*a^4*b^5*c^2*e^15 - 28*a^5*b^3*c^3*e^15 - 2048*a^2*c^9*d^9*e^6 - 4416*a^3*c^8*d^7*e^8 - 2528*a^4*c^7*d^5*e^10 - 288*a^5*c^6*d^3*e^12 + 336*b^2*c^9*d^11*e^4 - 268*b^3*c^8*d^10*e^5 - 360*b^4*c^7*d^9*e^6 + 1260*b^5*c^6*d^8*e^7 - 1568*b^6*c^5*d^7*e^8 + 1036*b^7*c^4*d^6*e^9 - 360*b^8*c^3*d^5*e^10 + 52*b^9*c^2*d^4*e^11 - 7584*a^2*b^2*c^7*d^7*e^8 - 536*a^2*b^3*c^6*d^6*e^9 + 5936*a^2*b^4*c^5*d^5*e^10 - 3552*a^2*b^5*c^4*d^4*e^11 + 464*a^2*b^6*c^3*d^3*e^12 + 104*a^2*b^7*c^2*d^2*e^13 - 12768*a^3*b^2*c^6*d^5*e^10 + 3720*a^3*b^3*c^5*d^4*e^11 + 1280*a^3*b^4*c^4*d^3*e^12 - 648*a^3*b^5*c^3*d^2*e^13 - 4272*a^4*b^2*c^5*d^3*e^12 + 740*a^4*b^3*c^4*d^2*e^13 - 848*a*b*c^9*d^10*e^5 + 3632*a*b^2*c^8*d^9*e^6 - 7852*a*b^3*c^7*d^8*e^7 + 8864*a*b^4*c^6*
\end{aligned}$$

$$\begin{aligned}
& d^7 e^8 - 4936 a b^5 c^5 d^6 e^9 + 816 a^2 b^6 c^4 d^5 e^{10} + 356 a^3 b^7 c^3 d^4 e^{11} - 128 a^4 b^8 c^2 d^3 e^{12} + 7216 a^5 b^9 c^1 d^2 e^{13} \\
& + 12896 a^6 b^{10} c^0 d^1 e^{14} - 32 a^7 b^{11} c^{-1} d^0 e^{15} + 5696 a^8 b^{12} c^{-2} d^{-1} e^{16} + 216 a^9 b^{13} c^{-3} d^{-2} e^{17} \\
& + 752 a^{10} b^{14} c^{-4} d^{-3} e^{18} - 336 a^{11} b^{15} c^{-5} d^{-4} e^{19} + 64 a^{12} b^{16} c^{-6} d^{-5} e^{20} - 64 a^{13} b^{17} c^{-7} d^{-6} e^{21} \\
& + 256 a^{14} b^{18} c^{-8} d^{-7} e^{22} - 512 a^{15} b^{19} c^{-9} d^{-8} e^{23} + 512 a^{16} b^{20} c^{-10} d^{-9} e^{24} - 256 a^{17} b^{21} c^{-11} d^{-10} e^{25} \\
& + 64 a^{18} b^{22} c^{-12} d^{-11} e^{26} - 64 a^{19} b^{23} c^{-13} d^{-12} e^{27} + 64 a^{20} b^{24} c^{-14} d^{-13} e^{28} - 64 a^{21} b^{25} c^{-15} d^{-14} e^{29} \\
& + 64 a^{22} b^{26} c^{-16} d^{-15} e^{30} - 64 a^{23} b^{27} c^{-17} d^{-16} e^{31} + 64 a^{24} b^{28} c^{-18} d^{-17} e^{32} - 64 a^{25} b^{29} c^{-19} d^{-18} e^{33} \\
& + 64 a^{26} b^{30} c^{-20} d^{-19} e^{34} - 64 a^{27} b^{31} c^{-21} d^{-20} e^{35} + 64 a^{28} b^{32} c^{-22} d^{-21} e^{36} - 64 a^{29} b^{33} c^{-23} d^{-22} e^{37} \\
& + 64 a^{30} b^{34} c^{-24} d^{-23} e^{38} - 64 a^{31} b^{35} c^{-25} d^{-24} e^{39} + 64 a^{32} b^{36} c^{-26} d^{-25} e^{40} - 64 a^{33} b^{37} c^{-27} d^{-26} e^{41} \\
& + 64 a^{34} b^{38} c^{-28} d^{-27} e^{42} - 64 a^{35} b^{39} c^{-29} d^{-28} e^{43} + 64 a^{36} b^{40} c^{-30} d^{-29} e^{44} - 64 a^{37} b^{41} c^{-31} d^{-30} e^{45} \\
& + 64 a^{38} b^{42} c^{-32} d^{-31} e^{46} - 64 a^{39} b^{43} c^{-33} d^{-32} e^{47} + 64 a^{40} b^{44} c^{-34} d^{-33} e^{48} - 64 a^{41} b^{45} c^{-35} d^{-34} e^{49} \\
& + 64 a^{42} b^{46} c^{-36} d^{-35} e^{50} - 64 a^{43} b^{47} c^{-37} d^{-36} e^{51} + 64 a^{44} b^{48} c^{-38} d^{-37} e^{52} - 64 a^{45} b^{49} c^{-39} d^{-38} e^{53} \\
& + 64 a^{46} b^{50} c^{-40} d^{-39} e^{54} - 64 a^{47} b^{51} c^{-41} d^{-40} e^{55} + 64 a^{48} b^{52} c^{-42} d^{-41} e^{56} - 64 a^{49} b^{53} c^{-43} d^{-42} e^{57} \\
& + 64 a^{50} b^{54} c^{-44} d^{-43} e^{58} - 64 a^{51} b^{55} c^{-45} d^{-44} e^{59} + 64 a^{52} b^{56} c^{-46} d^{-45} e^{60} - 64 a^{53} b^{57} c^{-47} d^{-46} e^{61} \\
& + 64 a^{54} b^{58} c^{-48} d^{-47} e^{62} - 64 a^{55} b^{59} c^{-49} d^{-48} e^{63} + 64 a^{56} b^{60} c^{-50} d^{-49} e^{64} - 64 a^{57} b^{61} c^{-51} d^{-50} e^{65} \\
& + 64 a^{58} b^{62} c^{-52} d^{-51} e^{66} - 64 a^{59} b^{63} c^{-53} d^{-52} e^{67} + 64 a^{60} b^{64} c^{-54} d^{-53} e^{68} - 64 a^{61} b^{65} c^{-55} d^{-54} e^{69} \\
& + 64 a^{62} b^{66} c^{-56} d^{-55} e^{70} - 64 a^{63} b^{67} c^{-57} d^{-56} e^{71} + 64 a^{64} b^{68} c^{-58} d^{-57} e^{72} - 64 a^{65} b^{69} c^{-59} d^{-58} e^{73} \\
& + 64 a^{66} b^{70} c^{-60} d^{-59} e^{74} - 64 a^{67} b^{71} c^{-61} d^{-60} e^{75} + 64 a^{68} b^{72} c^{-62} d^{-61} e^{76} - 64 a^{69} b^{73} c^{-63} d^{-62} e^{77} \\
& + 64 a^{70} b^{74} c^{-64} d^{-63} e^{78} - 64 a^{71} b^{75} c^{-65} d^{-64} e^{79} + 64 a^{72} b^{76} c^{-66} d^{-65} e^{80} - 64 a^{73} b^{77} c^{-67} d^{-66} e^{81} \\
& + 64 a^{74} b^{78} c^{-68} d^{-67} e^{82} - 64 a^{75} b^{79} c^{-69} d^{-68} e^{83} + 64 a^{76} b^{80} c^{-70} d^{-69} e^{84} - 64 a^{77} b^{81} c^{-71} d^{-70} e^{85} \\
& + 64 a^{78} b^{82} c^{-72} d^{-71} e^{86} - 64 a^{79} b^{83} c^{-73} d^{-72} e^{87} + 64 a^{80} b^{84} c^{-74} d^{-73} e^{88} - 64 a^{81} b^{85} c^{-75} d^{-74} e^{89} \\
& + 64 a^{82} b^{86} c^{-76} d^{-75} e^{90} - 64 a^{83} b^{87} c^{-77} d^{-76} e^{91} + 64 a^{84} b^{88} c^{-78} d^{-77} e^{92} - 64 a^{85} b^{89} c^{-79} d^{-78} e^{93} \\
& + 64 a^{86} b^{90} c^{-80} d^{-79} e^{94} - 64 a^{87} b^{91} c^{-81} d^{-80} e^{95} + 64 a^{88} b^{92} c^{-82} d^{-81} e^{96} - 64 a^{89} b^{93} c^{-83} d^{-82} e^{97} \\
& + 64 a^{90} b^{94} c^{-84} d^{-83} e^{98} - 64 a^{91} b^{95} c^{-85} d^{-84} e^{99} + 64 a^{92} b^{96} c^{-86} d^{-85} e^{100} - 64 a^{93} b^{97} c^{-87} d^{-86} e^{101} \\
& + 64 a^{94} b^{98} c^{-88} d^{-87} e^{102} - 64 a^{95} b^{99} c^{-89} d^{-88} e^{103} + 64 a^{96} b^{100} c^{-90} d^{-89} e^{104} - 64 a^{97} b^{101} c^{-91} d^{-90} e^{105} \\
& + 64 a^{98} b^{102} c^{-92} d^{-91} e^{106} - 64 a^{99} b^{103} c^{-93} d^{-92} e^{107} + 64 a^{100} b^{104} c^{-94} d^{-93} e^{108} - 64 a^{101} b^{105} c^{-95} d^{-94} e^{109} \\
& + 64 a^{102} b^{106} c^{-96} d^{-95} e^{110} - 64 a^{103} b^{107} c^{-97} d^{-96} e^{111} + 64 a^{104} b^{108} c^{-98} d^{-97} e^{112} - 64 a^{105} b^{109} c^{-99} d^{-98} e^{113} \\
& + 64 a^{106} b^{110} c^{-100} d^{-99} e^{114} - 64 a^{107} b^{111} c^{-101} d^{-100} e^{115} + 64 a^{108} b^{112} c^{-102} d^{-101} e^{116} - 64 a^{109} b^{113} c^{-103} d^{-102} e^{117} \\
& + 64 a^{110} b^{114} c^{-104} d^{-103} e^{118} - 64 a^{111} b^{115} c^{-105} d^{-104} e^{119} + 64 a^{112} b^{116} c^{-106} d^{-105} e^{120} - 64 a^{113} b^{117} c^{-107} d^{-106} e^{121} \\
& + 64 a^{114} b^{118} c^{-108} d^{-107} e^{122} - 64 a^{115} b^{119} c^{-109} d^{-108} e^{123} + 64 a^{116} b^{120} c^{-110} d^{-109} e^{124} - 64 a^{117} b^{121} c^{-111} d^{-110} e^{125} \\
& + 64 a^{118} b^{122} c^{-112} d^{-111} e^{126} - 64 a^{119} b^{123} c^{-113} d^{-112} e^{127} + 64 a^{120} b^{124} c^{-114} d^{-113} e^{128} - 64 a^{121} b^{125} c^{-115} d^{-114} e^{129} \\
& + 64 a^{122} b^{126} c^{-116} d^{-115} e^{130} - 64 a^{123} b^{127} c^{-117} d^{-116} e^{131} + 64 a^{124} b^{128} c^{-118} d^{-117} e^{132} - 64 a^{125} b^{129} c^{-119} d^{-118} e^{133} \\
& + 64 a^{126} b^{130} c^{-120} d^{-119} e^{134} - 64 a^{127} b^{131} c^{-121} d^{-120} e^{135} + 64 a^{128} b^{132} c^{-122} d^{-121} e^{136} - 64 a^{129} b^{133} c^{-123} d^{-122} e^{137} \\
& + 64 a^{130} b^{134} c^{-124} d^{-123} e^{138} - 64 a^{131} b^{135} c^{-125} d^{-124} e^{139} + 64 a^{132} b^{136} c^{-126} d^{-125} e^{140} - 64 a^{133} b^{137} c^{-127} d^{-126} e^{141} \\
& + 64 a^{134} b^{138} c^{-128} d^{-127} e^{142} - 64 a^{135} b^{139} c^{-129} d^{-128} e^{143} + 64 a^{136} b^{140} c^{-130} d^{-129} e^{144} - 64 a^{137} b^{141} c^{-131} d^{-130} e^{145} \\
& + 64 a^{138} b^{142} c^{-132} d^{-131} e^{146} - 64 a^{139} b^{143} c^{-133} d^{-132} e^{147} + 64 a^{140} b^{144} c^{-134} d^{-133} e^{148} - 64 a^{141} b^{145} c^{-135} d^{-134} e^{149} \\
& + 64 a^{142} b^{146} c^{-136} d^{-135} e^{150} - 64 a^{143} b^{147} c^{-137} d^{-136} e^{151} + 64 a^{144} b^{148} c^{-138} d^{-137} e^{152} - 64 a^{145} b^{149} c^{-139} d^{-138} e^{153} \\
& + 64 a^{146} b^{150} c^{-140} d^{-139} e^{154} - 64 a^{147} b^{151} c^{-141} d^{-140} e^{155} + 64 a^{148} b^{152} c^{-142} d^{-141} e^{156} - 64 a^{149} b^{153} c^{-143} d^{-142} e^{157} \\
& + 64 a^{150} b^{154} c^{-144} d^{-143} e^{158} - 64 a^{151} b^{155} c^{-145} d^{-144} e^{159} + 64 a^{152} b^{156} c^{-146} d^{-145} e^{160} - 64 a^{153} b^{157} c^{-147} d^{-146} e^{161} \\
& + 64 a^{154} b^{158} c^{-148} d^{-147} e^{162} - 64 a^{155} b^{159} c^{-149} d^{-148} e^{163} + 64 a^{156} b^{160} c^{-150} d^{-149} e^{164} - 64 a^{157} b^{161} c^{-151} d^{-150} e^{165} \\
& + 64 a^{158} b^{162} c^{-152} d^{-151} e^{166} - 64 a^{159} b^{163} c^{-153} d^{-152} e^{167} + 64 a^{160} b^{164} c^{-154} d^{-153} e^{168} - 64 a^{161} b^{165} c^{-155} d^{-154} e^{169} \\
& + 64 a^{162} b^{166} c^{-156} d^{-155} e^{170} - 64 a^{163} b^{167} c^{-157} d^{-156} e^{171} + 64 a^{164} b^{168} c^{-158} d^{-157} e^{172} - 64 a^{165} b^{169} c^{-159} d^{-158} e^{173} \\
& + 64 a^{166} b^{170} c^{-160} d^{-159} e^{174} - 64 a^{167} b^{171} c^{-161} d^{-160} e^{175} + 64 a^{168} b^{172} c^{-162} d^{-161} e^{176} - 64 a^{169} b^{173} c^{-163} d^{-162} e^{177} \\
& + 64 a^{170} b^{174} c^{-164} d^{-163} e^{178} - 64 a^{171} b^{175} c^{-165} d^{-164} e^{179} + 64 a^{172} b^{176} c^{-166} d^{-165} e^{180} - 64 a^{173} b^{177} c^{-167} d^{-166} e^{181} \\
& + 64 a^{174} b^{178} c^{-168} d^{-167} e^{182} - 64 a^{175} b^{179} c^{-169} d^{-168} e^{183} + 64 a^{176} b^{180} c^{-170} d^{-169} e^{184} - 64 a^{177} b^{181} c^{-171} d^{-170} e^{185} \\
& + 64 a^{178} b^{182} c^{-172} d^{-171} e^{186} - 64 a^{179} b^{183} c^{-173} d^{-172} e^{187} + 64 a^{180} b^{184} c^{-174} d^{-173} e^{188} - 64 a^{181} b^{185} c^{-175} d^{-174} e^{189} \\
& + 64 a^{182} b^{186} c^{-176} d^{-175} e^{190} - 64 a^{183} b^{187} c^{-177} d^{-176} e^{191} + 64 a^{184} b^{188} c^{-178} d^{-177} e^{192} - 64 a^{185} b^{189} c^{-179} d^{-178} e^{193} \\
& + 64 a^{186} b^{190} c^{-180} d^{-179} e^{194} - 64 a^{187} b^{191} c^{-181} d^{-180} e^{195} + 64 a^{188} b^{192} c^{-182} d^{-181} e^{196} - 64 a^{189} b^{193} c^{-183} d^{-182} e^{197} \\
& + 64 a^{190} b^{194} c^{-184} d^{-183} e^{198} - 64 a^{191} b^{195} c^{-185} d^{-184} e^{199} + 64 a^{192} b^{196} c^{-186} d^{-185} e^{200} - 64 a^{193} b^{197} c^{-187} d^{-186} e^{201} \\
& + 64 a^{194} b^{198} c^{-188} d^{-187} e^{202} - 64 a^{195} b^{199} c^{-189} d^{-188} e^{203} + 64 a^{196} b^{200} c^{-190} d^{-189} e^{204} - 64 a^{197} b^{201} c^{-191} d^{-190} e^{205} \\
& + 64 a^{198} b^{202} c^{-192} d^{-191} e^{206} - 64 a^{199} b^{203} c^{-193} d^{-192} e^{207} + 64 a^{200} b^{204} c^{-194} d^{-193} e^{208} - 64 a^{201} b^{205} c^{-195} d^{-194} e^{209} \\
& + 64 a^{202} b^{206} c^{-196} d^{-195} e^{210} - 64 a^{203} b^{207} c^{-197} d^{-196} e^{211} + 64 a^{204} b^{208} c^{-198} d^{-197} e^{212} - 64 a^{205} b^{209} c^{-199} d^{-198} e^{213} \\
& + 64 a^{206} b^{210} c^{-200} d^{-199} e^{214} - 64 a^{207} b^{211} c^{-201} d^{-200} e^{215} + 64 a^{208} b^{212} c^{-202} d^{-201} e^{216} - 64 a^{209} b^{213} c^{-203} d^{-202} e^{217} \\
& + 64 a^{210} b^{214} c^{-204} d^{-203} e^{218} - 64 a^{211} b^{215} c^{-205} d^{-204} e^{219} + 64 a^{212} b^{216} c^{-206} d^{-205} e^{220} - 64 a^{213} b^{217} c^{-207} d^{-206} e^{221} \\
& + 64 a^{214} b^{218} c^{-208} d^{-207} e^{222} - 64 a^{215} b^{219} c^{-209} d^{-208} e^{223} + 64 a^{216} b^{220} c^{-210} d^{-209} e^{224} - 64 a^{217} b^{221} c^{-211} d^{-210} e^{225} \\
& + 64 a^{218} b^{222} c^{-212} d^{-211} e^{226} - 64 a^{219} b^{223} c^{-213} d^{-212} e^{227} + 64 a^{220} b^{224} c^{-214} d^{-213} e^{228} - 64 a^{221} b^{225} c^{-215} d^{-214} e^{229} \\
& + 64 a^{222} b^{226} c^{-216} d^{-215} e^{230} - 64 a^{223} b^{227} c^{-217} d^{-216} e^{231} + 64 a^{224} b^{228} c^{-218} d^{-217} e^{232} - 64 a^{225} b^{229} c^{-219} d^{-218} e^{233} \\
& + 64 a^{226} b^{230} c^{-220} d^{-219} e^{234} - 64 a^{227} b^{231} c^{-221} d^{-220} e^{235} + 64 a^{228} b^{232} c^{-222} d^{-221} e^{236} - 64 a^{229} b^{233} c^{-223} d^{-222} e^{237} \\
& + 64 a^{230} b^{234} c^{-224} d^{-223} e^{238} - 64 a^{231} b^{235} c^{-225} d^{-224} e^{239} + 64 a^{232} b^{236} c^{-226} d^{-225} e^{240} - 64 a^{233} b^{237} c^{-227} d^{-226} e^{241} \\
& + 64 a^{234} b^{238} c^{-228} d^{-227} e^{242} - 64 a^{235} b^{239} c^{-229} d^{-228} e^{243} + 64 a^{236} b^{240} c^{-230} d^{-229} e^{244} - 64 a^{237} b^{241} c^{-231} d^{-230} e^{245} \\
& + 64 a^{238} b^{242} c^{-232} d^{-231} e^{246} - 64 a^{239} b^{243} c^{-233} d^{-232} e^{247} + 64 a^{240} b^{244} c^{-234} d^{-233} e^{248} - 64 a^{241} b^{245} c^{-235} d^{-234} e^{249} \\
& + 64 a^{242} b^{246} c^{-236} d^{-235} e^{250} - 64 a^{243} b^{247} c^{-237} d^{-236} e^{251} + 64 a^{244} b^{248} c^{-238} d^{-237} e^{252} - 64 a^{245} b^{249} c^{-239} d^{-238} e^{253} \\
& + 64 a^{246} b^{250} c^{-240} d^{-239} e^{254} - 64 a^{247} b^{251} c^{-241} d^{-240} e^{255} + 64 a^{248} b^{252} c^{-242} d^{-241} e^{256} - 64 a^{249} b^{253} c^{-243} d^{-242} e^{257} \\
& + 64 a^{250} b^{254} c^{-244} d^{-243} e^{258} - 64 a^{251} b^{255} c^{-245} d^{-244} e^{259} + 64 a^{252} b^{256} c^{-246} d^{-245} e^{260} - 64 a^{253} b^{257} c^{-247} d^{-246} e^{261} \\
& + 64 a^{254} b^{258} c^{-248} d^{-247} e^{262} - 64 a^{255} b^{259} c^{-249} d^{-248} e^{263} + 64 a^{256} b^{260} c^{-250} d^{-249} e^{264} - 64 a^{257} b^{261} c^{-251} d^{-250} e^{265} \\
& + 64 a^{258} b^{262} c^{-252} d^{-251} e^{266} - 64 a^{259} b^{263} c^{-253} d^{-252} e^{267} + 64 a^{260} b^{264} c^{-254} d^{-253} e^{268} - 64 a^{261} b^{265} c^{-255} d^{-254} e^{269} \\
& + 64 a^{262} b^{266} c^{-256} d^{-255} e^{270} - 64 a^{263} b^{267} c^{-257} d^{-256} e^{271} + 64 a^{264} b^{268} c^{-258} d^{-257} e^{272} - 64 a^{265} b^{269} c^{-259} d^{-258} e^{273} \\
& + 64 a^{266} b^{270} c^{-260} d^{-259} e^{274} - 64 a^{267} b^{271} c^{-261} d^{-260} e^{275} + 64 a^{268} b^{272} c^{-262} d^{-261} e^{276} - 64 a^{269} b^{273} c^{-263} d^{-262} e^{277} \\
& + 64 a^{270} b^{274} c^{-264} d^{-263} e^{278} - 64 a^{271} b^{275} c^{-265} d^{-264} e^{279} + 64 a^{272} b^{276} c^{-266} d^{-265} e^{280} - 64 a^{273} b^{277} c^{-267} d^{-266} e^{281} \\
& + 64 a^{274} b^{278} c^{-268} d^{-267} e^{282} - 64 a^{275} b^{279} c^{-269} d^{-268} e^{283} + 64 a^{276} b^{280} c^{-270} d^{-269} e^{284} - 64 a^{277} b^{281} c^{-271} d^{-270} e^{285} \\
& + 64 a^{278} b^{282} c^{-272} d^{-271} e^{286} - 64 a^{279} b^{283} c^{-273} d^{-272} e^{287} + 64 a^{280} b^{284} c^{-274} d^{-273} e^{288} - 64 a^{281} b^{285} c^{-275} d^{-274} e^{289} \\
& + 64 a^{282} b^{286} c^{-276} d^{-275} e^{290} - 64 a^{283} b^{287} c^{-277} d^{-276} e^{291} + 64 a^{284} b^{288} c^{-278} d^{-277} e^{292} - 64 a^{285} b^{289} c^{-279} d^{-278} e^{293} \\
& + 64 a^{286} b^{290} c^{-280} d^{-279} e^{294} - 64 a^{287} b^{291} c^{-281} d^{-280} e^{295} + 64 a^{288} b^{292} c^{-282} d^{-281} e^{296} - 64 a^{289} b^{293} c^{-283} d^{-282} e^{297} \\
& + 64 a^{290} b^{294} c^{-284} d^{-283} e^{298} - 64 a^{291} b^{295} c^{-285} d^{-284} e^{299} + 64 a^{292} b^{296} c^{-286} d^{-285} e^{300} - 64 a^{293} b^{297} c^{-287} d^{-286} e^{301} \\
& + 64 a^{294} b^{298} c^{-288} d^{-287} e^{302} - 64 a^{295} b^{299} c^{-289} d^{-288} e^{303} + 64 a^{296} b^{300} c^{-290} d^{-289} e^{304} - 64 a^{297} b^{301} c^{-291} d^{-290} e^{305} \\
& + 64 a^{298} b^{302} c^{-292} d^{-291} e^{306} - 64 a^{299} b^{303} c^{-293} d^{-292} e^{307} + 64 a^{300} b^{304} c^{-294} d^{-293} e^{308} - 64 a^{301} b^{305} c^{-295} d^{-294} e^{309} \\
& + 64 a^{302} b^{306} c^{-296} d^{-295} e^{310} - 64 a^{303} b^{307} c^{-297} d^{-296} e^{311} + 64 a^{304} b^{308} c^{-298} d^{-297} e^{312} - 64 a^{305} b^{309} c^{-299} d^{-298} e^{313} \\
& + 64 a^{306} b^{310} c^{-300} d^{-299} e^{314} - 64 a^{307} b^{311} c^{-301} d^{-300} e^{315} + 64 a^{308} b^{312} c^{-302} d^{-301} e^{316} - 64 a^{309} b^{313} c^{-303} d^{-302} e^{317} \\
& + 64 a^{310} b^{314} c^{-304} d^{-303} e^{318} - 64 a^{311} b^{315} c^{-305} d^{-304} e^{319} + 64 a^{312} b^{316} c^{-306} d^{-305} e^{320} - 64 a^{313} b^{317} c^{-307} d^{-306} e^{321} \\
& + 64 a^{314} b^{318} c^{-308} d^{-307} e^{322} - 64 a^{315} b^{319} c^{-309} d^{-308} e^{323} + 64 a^{316} b^{320} c^{-310} d^{-309} e^{324} - 64 a^{317} b^{321} c^{-311} d^{-310} e^{325} \\
& + 64 a^{318} b^{322} c^{-312} d^{-311} e^{326} - 64 a^{319} b^{323} c^{-313} d^{-312} e^{327} + 64 a^{320} b^{324} c^{-314} d^{-313} e^{328} - 64 a^{321} b^{325} c^{-315} d^{-314} e^{329} \\
& + 64 a^{322} b^{326} c^{-316} d^{-315} e^{330} - 64 a^{323} b^{327} c^{-317} d^{-316} e^{331} + 64 a^{324} b^{328} c^{-318} d^{-317} e^{332} - 64 a^{325} b^{329} c^{-319} d^{-318} e^{333} \\
& + 64 a^{326} b^{330} c^{-320} d^{-319} e^{334} - 64 a^{327} b^{331} c^{-321} d^{-320} e^{335} + 64 a^{328} b^{332} c^{-322} d^{-321} e^{336} - 64 a^{329} b^{333} c^{-323} d^{-322} e^{337} \\
& + 64 a^{330} b^{334} c^{-324} d^{-323} e^{338} - 64 a^{331} b^{335} c^{-325} d^{-324} e^{339} + 64 a^{332} b^{336} c^{-326} d^{-325} e^{340} - 64 a^{333} b^{337} c^{-327} d^{-326} e^{341} \\
& + 64 a^{334} b^{338} c^{-328} d^{-327} e^{342} - 64 a^{335} b^{339} c^{-329} d^{-328} e^{343} + 64 a^{336} b^{340} c^{-330} d^{-329} e^{344} - 64 a^{337} b^{341} c^{-331} d^{-330} e^{345} \\
& + 64 a^{338} b^{342} c^{-332} d^{-331} e^{346} - 64 a^{339} b^{343} c^{-333} d^{-332} e^{347} + 64 a^{340} b^{344} c^{-334} d^{-333} e^{348} - 64 a^{341} b^{345} c^{-335} d^{-334} e^{349} \\
& + 64 a^{342} b^{346} c^{-336} d^{-335} e^{350} - 64 a^{343} b^{347} c^{-337} d^{-336} e^{351} + 64 a^{344} b^{348} c^{-338} d^{-337} e^{352} - 64 a^{345} b^{349} c^{-339} d^{-338} e^{353} \\
& + 64 a^{346} b^{350} c^{-340} d^{-339} e^{354} - 64 a^{347} b^{351} c^{-341} d^{-340} e^{355} + 64 a^{348} b^{352} c^{-342} d^{-341} e^{356} - 64 a^{349} b^{353} c^{-343} d^{-342} e^{357} \\
& + 64 a^{350} b^{354} c^{-344} d^{-343} e^{358} - 64 a^{351} b^{355} c^{-345} d^{-344} e^{359} + 64 a^{352} b^{356} c^{-346} d^{-345} e^{360} - 64 a^{353} b^{357} c^{-347} d^{-346} e^{361} \\
& + 64 a^{354} b^{358} c^{-348} d^{-347} e^{362} - 64 a^{355} b^{359} c^{-349} d^{-348} e^{363} + 64 a^{356} b^{360} c^{-350} d^{-349} e^{364} - 64 a^{357} b^{361} c^{-351} d^{-350} e^{365} \\
& + 64 a^{358} b^{362} c^{-352} d^{-351} e^{366} - 64 a^{359} b^{363} c^{-353} d^{-352} e^{367} + 64 a^{360} b^{364} c^{-354} d^{-353} e^{368} - 64 a^{361} b^{365} c^{-355} d^{-354} e^{369} \\
& + 64 a^{362} b^{366} c^{-356} d^{-355} e^{370} - 64 a^{363} b^{367} c^{-357} d^{-356} e^{371} + 64 a^{364} b^{368} c^{-358} d^{-357} e^{372} - 64 a^{365} b^{369} c^{-359} d^{-358} e^{373} \\
& + 64 a^{366} b^{370} c^{-360} d^{-359} e^{374} - 64 a^{367} b^{371} c^{-361} d^{-360} e^{375} + 64 a^{36$$

$$\begin{aligned}
& 3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)}/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5))^{(1/2)}*i - (((2*a^2*b^6*c^2*e^13 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 - 14*a^3*b^4*c^3*e^13 + 26*a^4*b^2*c^4*e^13 + 480*a^2*c^8*d^6*e^7 + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^11 + 50*b^2*c^8*d^8*e^5 - 240*b^3*c^7*d^7*e^6 + 466*b^4*c^6*d^6*e^7 - 464*b^5*c^5*d^5*e^8 + 246*b^6*c^4*d^4*e^9 - 64*b^7*c^3*d^3*e^10 + 6*b^8*c^2*d^2*e^11 + 4*a^2*b^2*c^6*d^4*e^9 + 672*a^2*b^3*c^5*d^3*e^10 - 354*a^2*b^4*c^4*d^2*e^11 + 464*a^3*b^2*c^5*d^2*e^11 + 960*a*b*c^8*d^7*e^6 - 8*a*b^7*c^2*d*e^12 - 96*a^4*b*c^5*d*e^12 - 1984*a*b^2*c^7*d^6*e^7 + 2072*a*b^3*c^6*d^5*e^8 - 1034*a*b^4*c^5*d^4*e^9 + 160*a*b^5*c^4*d^3*e^10 + 34*a*b^6*c^3*d^2*e^11 - 864*a^2*b*c^7*d^5*e^8 + 40*a^2*b^5*c^3*d*e^12 - 1152*a^3*b*c^6*d^3*e^10 - 8*a^3*b^3*c^4*d*e^12)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - (((128*a*c^11*d^15*e^2 - 256*a^8*c^4*d*e^16 - 256*a^2*c^10*d^13*e^4 - 3456*a^3*c^9*d^11*e^6 - 8960*a^4*c^8*d^9*e^8 - 10880*a^5*c^7*d^7*e^10 - 6912*a^6*c^6*d^5*e^12 - 2176*a^7*c^5*d^3*e^14 - 32*b^2*c^10*d^15*e^2 + 256*b^3*c^9*d^14*e^3 - 896*b^4*c^8*d^13*e^4 + 1792*b^5*c^7*d^12*e^5 - 2240*b^6*c^6*d^11*e^6 + 1792*b^7*c^5*d^10*e^7 - 896*b^8*c^4*d^9*e^8 + 256*b^9*c^3*d^8*e^9 - 32*b^10*c^2*d^7*e^10 + 2848*a^2*b^2*c^8*d^11*e^6 - 12160*a^2*b^3*c^7*d^10*e^7 + 18480*a^2*b^4*c^6*d^9*e^8 - 12864*a^2*b^5*c^5*d^8*e^9 + 3008*a^2*b^6*c^4*d^7*e^10 + 832*a^2*b^7*c^3*d^6*e^11 - 400*a^2*b^8*c^2*d^5*e^12 - 17920*a^3*b^2*c^7*d^9*e^8 + 1280*a^3*b^3*c^6*d^8*e^9 + 14240*a^3*b^4*c^5*d^7*e^10 - 9824*a^3*b^5*c^4*d^6*e^11 + 1120*a^3*b^6*c^3*d^5*e^12 + 480*a^3*b^7*c^2*d^4*e^13 - 33760*a^4*b^2*c^6*d^7*e^10 + 7680*a^4*b^3*c^5*d^6*e^11 + 7520*a^4*b^4*c^4*d^5*e^12 - 2880*a^4*b^5*c^3*d^4*e^13 - 320*a^4*b^6*c^2*d^3*e^14 - 20672*a^5*b^2*c^5*d^5*e^12 + 896*a^5*b^3*c^4*d^4*e^13 + 2384*a^5*b^4*c^3*d^3*e^14 + 112*a^5*b^5*c^2*d^2*e^15 - 3872*a^6*b^2*c^4*d^3*e^14 - 896*a^6*b^3*c^3*d^2*e^15 - 1024*a*b*c^10*d^14*e^3 + 3648*a*b^2*c^9*d^13*e^4 - 7296*a*b^3*c^8*d^12*e^5 + 8464*a*b^4*c^7*d^11*e^6 - 5008*a*b^5*c^6*d^10*e^7 + 224*a*b^6*c^5*d^9*e^8 + 1632*a*b^7*c^4*d^8*e^9 - 944*a*b^8*c^3*d^7*e^10 + 176*a*b^9*c^2*d^6*e^11 + 512*a^2*b*c^9*d^12*e^5 + 14080*a^3*b*c^8*d^10*e^7 + 30720*a^4*b*c^7*d^8*e^9 + 28160*a^5*b*c^6*d^6*e^11 + 11776*a^6*b*c^5*d^4*e^13 - 16*a^6*b^4*c^2*d*e^16 + 1792*a^7*b*c^4*d^2*e^15 + 128*a^7*b^2*c^3*d*e^16)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) + (x*((b^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^{(1/2)} + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^{(1/2)} - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*
\end{aligned}$$

$$\begin{aligned}
& a^4c^5d^6e^2 + 96a^5c^4d^4e^4 + 64a^6c^3d^2e^6 - 44a^2b^4c^3d^6e^2 + 20a^2b^5c^2d^5e^3 + 64a^3b^2c^4d^6e^2 + 32a^3b^3c^3d^5e^3 \\
& - 74a^3b^4c^2d^4e^4 + 144a^4b^2c^3d^4e^4 + 32a^4b^3c^2d^3e^5 + 64a^5b^2c^2d^2e^6 - 4a^*b^5c^3d^7e - 4a^*b^7c^*d^5e^3 - \\
& 64a^3b^*c^5d^7e + 32a^5b^3c^*d^e^7 - 64a^6b^*c^2d^e^7 + 6a^*b^6c^2d^6e^2 + 32a^2b^3c^4d^7e + 4a^2b^6c^*d^4e^4 + 20a^3b^5c^*d^3e^5 \\
& - 192a^4b^*c^4d^5e^3 - 44a^4b^4c^*d^2e^6 - 192a^5b^*c^3d^3e^5)) \\
& ^{(1/2)} * (1024a^2c^{11}d^{16}e^3 + 5120a^3c^{10}d^{14}e^5 + 9216a^4c^9d^{12}e^7 + 5120a^5c^8d^{10}e^9 - 5120a^6c^7d^8e^{11} - 9216a^7c^6d^6e^{13} \\
& - 5120a^8c^5d^4e^{15} - 1024a^9c^4d^2e^{17} - 64b^3c^{10}d^{17}e^2 + 512b^4c^9d^{16}e^3 - 1792b^5c^8d^{15}e^4 + 3584b^6c^7d^{14}e^5 - 4480 \\
& b^7c^6d^{13}e^6 + 3584b^8c^5d^{12}e^7 - 1792b^9c^4d^{11}e^8 + 512b^{10}c^3d^{10}e^9 - 64b^{11}c^2d^9e^{10} + 8192a^2b^2c^9d^{14}e^5 + 5056a^2 \\
& b^3c^8d^{13}e^6 - 31104a^2b^4c^7d^{12}e^7 + 40256a^2b^5c^6d^{11}e^8 - 22784a^2b^6c^5d^{10}e^9 + 3648a^2b^7c^4d^9e^{10} + 1664a^2b^8c^3 \\
& d^8e^{11} - 576a^2b^9c^2d^7e^{12} + 45312a^3b^2c^8d^{12}e^7 - 27840a^3b^3c^7d^{11}e^8 - 13760a^3b^4c^6d^{10}e^9 + 27520a^3b^5c^5d^9e^{10} \\
& - 12416a^3b^6c^4d^8e^{11} + 1088a^3b^7c^3d^7e^{12} + 320a^3b^8c^2d^6e^{13} + 53760a^4b^2c^7d^{10}e^9 - 30400a^4b^3c^6d^9e^{10} + 1 \\
& 280a^4b^4c^5d^8e^{11} + 4224a^4b^5c^4d^7e^{12} - 1280a^4b^6c^3d^6e^{13} + 320a^4b^7c^2d^5e^{14} + 6400a^5b^2c^6d^8e^{11} - 2624a^5b^3 \\
& c^5d^7e^{12} + 5952a^5b^4c^4d^6e^{13} - 2752a^5b^5c^3d^5e^{14} - 576a^5b^6c^2d^4e^{15} - 21504a^6b^2c^5d^6e^{13} + 832a^6b^3c^4d^5e^{14} \\
& + 4736a^6b^4c^3d^4e^{15} + 320a^6b^5c^2d^3e^{16} - 8448a^7b^2c^4d^4e^{15} - 2624a^7b^3c^3d^3e^{16} - 64a^7b^4c^2d^2e^{17} + 512a^8b^2 \\
& c^3d^2e^{17} + 256a^*b^*c^{11}d^{17}e^2 - 2304a^*b^2c^{10}d^{16}e^3 + 8512a^*b^3c^9d^{15}e^4 - 16704a^*b^4c^8d^{14}e^5 + 18240a^*b^5c^7d^{13}e^6 - \\
& 9536a^*b^6c^6d^{12}e^7 - 576a^*b^7c^5d^{11}e^8 + 3648a^*b^8c^4d^{10}e^9 - 1856a^*b^9c^3d^9e^{10} + 320a^*b^{10}c^2d^8e^{11} - 5376a^2b^*c^{10}d^{15}e^4 \\
& - 25344a^3b^*c^9d^{13}e^6 - 37120a^4b^*c^8d^{11}e^8 - 11520a^5b^*c^7d^9e^{10} + 20736a^6b^*c^6d^7e^{12} + 20224a^7b^*c^5d^5e^{14} + 5376a^8b^* \\
& c^4d^3e^{16})) / (2*(c^4d^{10} + a^4d^2e^8 + b^4d^6e^4 - 4a^*b^3d^5e^5 - 4a^3b^*d^3e^7 + 4a^*c^3d^8e^2 + 4a^3c^*d^4e^6 - 4b^3c^*d^7e^3 + \\
& 6a^2b^2d^4e^6 + 6a^2c^2d^6e^4 + 6b^2c^2d^8e^2 - 4b^*c^3d^9e - 12a^*b^*c^2d^7e^3 + 12a^*b^2c^*d^6e^4 - 12a^2b^*c^*d^5e^5))) * ((b^4e^4 * \\
& (- (4a^*c - b^2)^3)^{(1/2)} - b^3c^4d^4 - b^7e^4 + c^4d^4 * (- (4a^*c - b^2)^3)^{(1/2)} + 20a^3b^*c^3e^4 + 32a^2c^5d^3e - 32a^3c^4d^*e^3 + 4b^4c^3 \\
& d^3e - 25a^2b^3c^2e^4 + a^2c^2e^4 * (- (4a^*c - b^2)^3)^{(1/2)} - 6b^5c^2d^2e^2 + 4a^*b^*c^5d^4 + 9a^*b^5c^*e^4 + 4b^6c^*d^*e^3 + 6b^2c^2d^2 \\
& e^2 * (- (4a^*c - b^2)^3)^{(1/2)} - 3a^*b^2c^*e^4 * (- (4a^*c - b^2)^3)^{(1/2)} - 24a^*b^2c^4d^3e - 32a^*b^4c^2d^*e^3 - 4b^*c^3d^3e * (- (4a^*c - b^2)^3)^{(1/2)} \\
& - 4b^3c^*d^*e^3 * (- (4a^*c - b^2)^3)^{(1/2)} + 42a^*b^3c^3d^2e^2 - 72a^2b^*c^4d^2e^2 + 72a^2b^2c^3d^*e^3 - 6a^*c^3d^2e^2 * (- (4a^*c - b^2)^3)^{(1/2)} \\
& + 8a^*b^*c^2d^*e^3 * (- (4a^*c - b^2)^3)^{(1/2})) / (8*(16a^3c^6d^8 + a^5b^4e^8 + 16a^7c^2e^8 + a^*b^4c^4d^8 - 8a^6b^2c^*e^8 + a^*b^8d^4e^4 \\
& - 4a^4b^5d^*e^7 - 8a^2b^2c^5d^8 - 4a^2b^7d^3e^5 + 6a^3b^6d^2e^6 + 64a^4c^5d^6e^2 + 96a^5c^4d^4e^4 + 64a^6c^3d^2e^6 - 44a^2b^4c^3d^6e^2 + 20a^2b^5c^2d^5e^3 + 64a^3b^2c^4d^6e^2 + 32a^3 \\
& b^3c^3d^5e^3 - 74a^3b^4c^2d^4e^4 + 144a^4b^2c^3d^4e^4 + 32a^4b^3c^2d^3e^5 + 64a^5b^2c^2d^2e^6 - 4a^*b^5c^3d^7e - 4a^*b^7c^*d^5e^3 - 64a^3b^*c^5d^7e + 32a^5b^3c^*d^e^7 - 64a^6b^*c^2d^e^7 + \\
& 6a^*b^6c^2d^6e^2 + 32a^2b^3c^4d^7e + 4a^2b^6c^*d^4e^4 + 20a^3b^5c^*d^3e^5 - 192a^4b^*c^4d^5e^3 - 44a^4b^4c^*d^2e^6 - 192a^5b^*c^3d^3e^5)) \\
& ^{(1/2)} - (x*(32c^{11}d^{13}e^2 + 48a^6b^*c^4e^{15} + 96a^*c^{10}d^{11}e^4 - 64a^6c^5d^*e^{14} - 160b^*c^{10}d^{12}e^3 + 4a^4b^5c^2e^{15} - 28a^5b^3c^3e^{15} - 2048a^2c^9d^9e^6 - 4416a^3c^8d^7e^8 - 2528a^4c^7 \\
& d^5e^{10} - 288a^5c^6d^3e^{12} + 336b^2c^9d^{11}e^4 - 268b^3c^8d^{10}e^5 - 360b^4c^7d^9e^6 + 1260b^5c^6d^8e^7 - 1568b^6c^5d^7e^8 + 1036b^7c^4d^6e^9 - 360b^8c^3d^5e^{10} + 52b^9c^2d^4e^{11} - 7584a
\end{aligned}$$

$$\begin{aligned}
& \cdot 2*b^2*c^7*d^7*e^8 - 536*a^2*b^3*c^6*d^6*e^9 + 5936*a^2*b^4*c^5*d^5*e^{10} - \\
& 3552*a^2*b^5*c^4*d^4*e^{11} + 464*a^2*b^6*c^3*d^3*e^{12} + 104*a^2*b^7*c^2*d^2* \\
& e^{13} - 12768*a^3*b^2*c^6*d^5*e^{10} + 3720*a^3*b^3*c^5*d^4*e^{11} + 1280*a^3*b^4* \\
& c^4*d^3*e^{12} - 648*a^3*b^5*c^3*d^2*e^{13} - 4272*a^4*b^2*c^5*d^3*e^{12} + 740 \\
& *a^4*b^3*c^4*d^2*e^{13} - 848*a*b*c^9*d^{10}*e^5 + 3632*a*b^2*c^8*d^9*e^6 - 785 \\
& 2*a*b^3*c^7*d^8*e^7 + 8864*a*b^4*c^6*d^7*e^8 - 4936*a*b^5*c^5*d^6*e^9 + 816 \\
& *a*b^6*c^4*d^5*e^{10} + 356*a*b^7*c^3*d^4*e^{11} - 128*a*b^8*c^2*d^3*e^{12} + 721 \\
& 6*a^2*b*c^8*d^8*e^7 + 12896*a^3*b*c^7*d^6*e^9 - 32*a^3*b^6*c^2*d*e^{14} + 569 \\
& 6*a^4*b*c^6*d^4*e^{11} + 216*a^4*b^4*c^3*d*e^{14} + 752*a^5*b*c^5*d^2*e^{13} - 33 \\
& 6*a^5*b^2*c^4*d*e^{14}))/((2*(c^4*d^{10} + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d \\
& ^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7* \\
& e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d \\
& ^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*((b^ \\
& 4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - \\
& b^2)^3)^{(1/2)} + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4 \\
& *b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2 \\
& *c^2*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^{(1 \\
& /2)} - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^ \\
& 2)^3)^{(1/2)} - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 42*a*b^3*c^3*d^2*e^2 \\
& - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - \\
& b^2)^3)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^3*c^6*d \\
& ^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8 \\
& *d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3* \\
& b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 \\
& - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 \\
& + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 \\
& + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4* \\
& a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d* \\
& e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20 \\
& *a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5 \\
& *b*c^3*d^3*e^5))^{(1/2)})*((b^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - b^3*c^4*d^4 - \\
& b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^{(1/2)} + 20*a^3*b*c^3*e^4 + 32*a^2*c^5 \\
& *d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2* \\
& e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5* \\
& c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^ \\
& 2*c*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 \\
& - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^{(1/2)} - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3) \\
& ^{(1/2)} + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 \\
& - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4*a*c - b^ \\
& 2)^3)^{(1/2)))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4* \\
& d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 \\
& - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4* \\
& d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5* \\
& e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4* \\
& e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2 \\
& *e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5* \\
& b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7 \\
& *e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 4 \\
& 4*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5))^{(1/2)} + (x*(54*c^9*d^6*e^5 - \\
& 2*a^3*c^6*e^{11} - 22*a*c^8*d^4*e^7 - 118*b*c^8*d^5*e^6 + a^2*b^2*c^5*e^{11} - \\
& 14*a^2*c^7*d^2*e^9 + 107*b^2*c^7*d^4*e^7 - 48*b^3*c^6*d^3*e^8 + 9*b^4*c^5* \\
& d^2*e^9 + 20*a*b*c^7*d^3*e^8 - 6*a*b^3*c^5*d*e^{10} + 10*a^2*b*c^6*d*e^{10} + 4 \\
& *a*b^2*c^6*d^2*e^9))/(2*(c^4*d^{10} + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5 \\
& *e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^ \\
& 3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9 \\
& *e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*((b^4* \\
& e^4*(-(4*a*c - b^2)^3)^{(1/2)} - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b
\end{aligned}$$

$$\begin{aligned}
& ^4c^3d^3e - 25a^2b^3c^2e^4 + a^2c^2e^4(-4ac - b^2)^3)^{(1/2)} - \\
& 6b^5c^2d^2e^2 + 4a^2b^3c^5d^4 + 9a^2b^5c^2e^4 + 4b^6c^2d^2e^3 + 6b^2c^2d^2e^2(-4ac - b^2)^3)^{(1/2)} - 3a^2b^2c^2e^4(-4ac - b^2)^3)^{(1/2)} \\
& ) - 24a^2b^2c^4d^3e - 32a^2b^4c^2d^2e^3 - 4b^3c^3d^3e(-4ac - b^2)^3)^{(1/2)} - 4b^3c^3d^2e^3(-4ac - b^2)^3)^{(1/2)} + 42a^2b^3c^3d^2e^2 - \\
& 72a^2b^2c^4d^2e^2 + 72a^2b^2c^3d^2e^3 - 6a^2c^3d^2e^2(-4ac - b^2)^3)^{(1/2)} + 8a^2b^2c^2d^2e^3(-4ac - b^2)^3)^{(1/2)})/(8(16a^3c^6d^8 \\
& + a^5b^4e^8 + 16a^7c^2e^8 + a^2b^4c^4d^8 - 8a^6b^2c^2e^8 + a^2b^8d^4e^4 - 4a^4b^5d^2e^7 - 8a^2b^2c^5d^8 - 4a^2b^7d^3e^5 + 6a^3b^6 \\
& d^2e^6 + 64a^4c^5d^6e^2 + 96a^5c^4d^4e^4 + 64a^6c^3d^2e^6 - 44a^2b^4c^3d^6e^2 + 20a^2b^5c^2d^5e^3 + 64a^3b^2c^4d^6e^2 + \\
& 32a^3b^3c^3d^5e^3 - 74a^3b^4c^2d^4e^4 + 144a^4b^2c^3d^4e^4 + 32a^4b^3c^2d^3e^5 + 64a^5b^2c^2d^2e^6 - 4a^2b^5c^3d^7e - 4a^2 \\
& b^7c^2d^5e^3 - 64a^3b^3c^5d^7e + 32a^5b^3c^2d^2e^7 - 64a^6b^2c^2d^2e^7 + 6a^2b^6c^2d^6e^2 + 32a^2b^3c^4d^7e + 4a^2b^6c^2d^4e^4 + 20a^3 \\
& b^5c^2d^3e^5 - 192a^4b^3c^4d^5e^3 - 44a^4b^4c^2d^2e^6 - 192a^5b^3c^3d^3e^5))^{(1/2)}*i)/((5c^8d^3e^6 - 3b^7c^7d^2e^7 + a^7c^7d^8e^8)/ \\
& (c^4d^10 + a^4d^2e^8 + b^4d^6e^4 - 4a^2b^3d^5e^5 - 4a^3b^2d^3e^7 + 4a^2c^3d^8e^2 + 4a^3c^2d^4e^6 - 4b^3c^2d^7e^3 + 6a^2b^2d^4e^6 + \\
& 6a^2c^2d^6e^4 + 6b^2c^2d^8e^2 - 4b^3c^3d^9e - 12a^2b^2c^2d^7e^3 + 12a^2b^2c^2d^6e^4 - 12a^2b^2c^2d^5e^5) + (((2a^2b^6c^2e^13 - 200a^2 \\
& c^9d^8e^5 - 8a^5c^5e^13 - 14a^3b^4c^3e^13 + 26a^4b^2c^4e^13 + 480a^2c^8d^6e^7 + 784a^3c^7d^4e^9 + 96a^4c^6d^2e^11 + 50b^2c^8 \\
& d^8e^5 - 240b^3c^7d^7e^6 + 466b^4c^6d^6e^7 - 464b^5c^5d^5e^8 + 246b^6c^4d^4e^9 - 64b^7c^3d^3e^10 + 6b^8c^2d^2e^11 + 4a^2b^2 \\
& b^2c^6d^4e^9 + 672a^2b^3c^5d^3e^10 - 354a^2b^4c^4d^2e^11 + 464a^3b^2c^5d^2e^11 + 960a^2b^3c^8d^7e^6 - 8a^2b^7c^2d^2e^12 - 96a^4b^2 \\
& c^5d^2e^12 - 1984a^2b^2c^7d^6e^7 + 2072a^2b^3c^6d^5e^8 - 1034a^2b^4c^5d^4e^9 + 160a^2b^5c^4d^3e^10 + 34a^2b^6c^3d^2e^11 - 864a^2b^7c^2 \\
& d^5e^8 + 40a^2b^5c^3d^2e^12 - 1152a^3b^2c^6d^3e^10 - 8a^3b^3c^4d^2e^12)/(2(c^4d^10 + a^4d^2e^8 + b^4d^6e^4 - 4a^2b^3d^5e^5 - 4a^3b^2 \\
& b^2d^3e^7 + 4a^2c^3d^8e^2 + 4a^3c^2d^4e^6 - 4b^3c^2d^7e^3 + 6a^2b^2d^4e^6 + 6a^2c^2d^6e^4 + 6b^2c^2d^8e^2 - 4b^3c^3d^9e - 12a^2b^2c^2 \\
& d^7e^3 + 12a^2b^2c^2d^6e^4 - 12a^2b^2c^2d^5e^5)) - (((128a^2c^11d^15 \\
& e^2 - 256a^8c^4d^2e^16 - 256a^2c^10d^13e^4 - 3456a^3c^9d^11e^6 - 8960a^4c^8d^9e^8 - 10880a^5c^7d^7e^10 - 6912a^6c^6d^5e^12 - 21 \\
& 76a^7c^5d^3e^14 - 32b^2c^10d^15e^2 + 256b^3c^9d^14e^3 - 896b^4c^8d^13e^4 + 1792b^5c^7d^12e^5 - 2240b^6c^6d^11e^6 + 1792b^7c^5 \\
& d^10e^7 - 896b^8c^4d^9e^8 + 256b^9c^3d^8e^9 - 32b^10c^2d^7e^10 + 2848a^2b^2c^8d^11e^6 - 12160a^2b^3c^7d^10e^7 + 18480a^2b^4c^6 \\
& d^9e^8 - 12864a^2b^5c^5d^8e^9 + 3008a^2b^6c^4d^7e^10 + 832a^2b^7c^3d^6e^11 - 400a^2b^8c^2d^5e^12 - 17920a^3b^2c^7d^9e^8 \\
& + 1280a^3b^3c^6d^8e^9 + 14240a^3b^4c^5d^7e^10 - 9824a^3b^5c^4d^6e^11 + 1120a^3b^6c^3d^5e^12 + 480a^3b^7c^2d^4e^13 - 33760a^4 \\
& b^2c^6d^7e^10 + 7680a^4b^3c^5d^6e^11 + 7520a^4b^4c^4d^5e^12 - 2880a^4b^5c^3d^4e^13 - 320a^4b^6c^2d^3e^14 - 20672a^5b^2c^5d^5 \\
& e^12 + 896a^5b^3c^4d^4e^13 + 2384a^5b^4c^3d^3e^14 + 112a^5b^5c^2d^2e^15 - 3872a^6b^2c^4d^3e^14 - 896a^6b^3c^3d^2e^15 - 10 \\
& 24a^2b^3c^10d^14e^3 + 3648a^2b^2c^9d^13e^4 - 7296a^2b^3c^8d^12e^5 + 8464a^2b^4c^7d^11e^6 - 5008a^2b^5c^6d^10e^7 + 224a^2b^6c^5d^9e^8 + \\
& 1632a^2b^7c^4d^8e^9 - 944a^2b^8c^3d^7e^10 + 176a^2b^9c^2d^6e^11 + 512a^2b^10c^1d^5e^12 + 14080a^3b^2c^8d^10e^7 + 30720a^4b^2c^7d^8e^9 \\
& + 28160a^5b^2c^6d^6e^11 + 11776a^6b^2c^5d^4e^13 - 16a^6b^4c^2d^2e^16 + 1792a^7b^2c^4d^2e^15 + 128a^7b^2c^3d^2e^16)/(2(c^4d^10 + a^4 \\
& d^2e^8 + b^4d^6e^4 - 4a^2b^3d^5e^5 - 4a^3b^2d^3e^7 + 4a^2c^3d^8e^2 + 4a^3c^2d^4e^6 - 4b^3c^2d^7e^3 + 6a^2b^2d^4e^6 + 6a^2c^2d^6e^4 \\
& + 6b^2c^2d^8e^2 - 4b^3c^3d^9e - 12a^2b^2c^2d^7e^3 + 12a^2b^2c^2d^6e^4 - 12a^2b^2c^2d^5e^5)) - (x((b^4e^4(-4ac - b^2)^3)^{(1/2)} - b^3c^4 \\
& d^4 - b^7e^4 + c^4d^4(-4ac - b^2)^3)^{(1/2)} + 20a^3b^3c^3e^4 + 3
\end{aligned}$$

$$\begin{aligned}
& 2*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + \\
& a^2*c^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + \\
& 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - \\
& 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^{(1/2)} - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + \\
& 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + \\
& 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a \\
& *b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - \\
& 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - \\
& 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - \\
& 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - \\
& 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - \\
& 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + \\
& 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^{(1/2)} * \\
& (1024*a^2*c^11*d^16*e^3 + 5120*a^3*c^10*d^14*e^5 + 9216*a^4*c^9*d^12*e^7 + 5120*a^5*c^8*d^10*e^9 - \\
& 5120*a^6*c^7*d^8*e^11 - 9216*a^7*c^6*d^6*e^13 - 5120*a^8*c^5*d^4*e^15 - 1024*a^9*c^4*d^2*e^17 - \\
& 64*b^3*c^10*d^17*e^2 + 512*b^4*c^9*d^16*e^3 - 1792*b^5*c^8*d^15*e^4 + 3584*b^6*c^7*d^14*e^5 - \\
& 4480*b^7*c^6*d^13*e^6 + 3584*b^8*c^5*d^12*e^7 - 1792*b^9*c^4*d^11*e^8 + 512*b^10*c^3*d^10*e^9 - \\
& 64*b^11*c^2*d^9*e^10 + 8192*a^2*b^2*c^9*d^14*e^5 + 5056*a^2*b^3*c^8*d^13*e^6 - 31104*a^2*b^4*c^7*d^12*e^7 + \\
& 40256*a^2*b^5*c^6*d^11*e^8 - 22784*a^2*b^6*c^5*d^10*e^9 + 3648*a^2*b^7*c^4*d^9*e^10 + 1664*a^2*b^8*c^3*d^8*e^11 - \\
& 576*a^2*b^9*c^2*d^7*e^12 + 45312*a^3*b^2*c^8*d^12*e^7 - 27840*a^3*b^3*c^7*d^11*e^8 - 13760*a^3*b^4*c^6*d^10*e^9 + \\
& 27520*a^3*b^5*c^5*d^9*e^10 - 12416*a^3*b^6*c^4*d^8*e^11 + 1088*a^3*b^7*c^3*d^7*e^12 + 320*a^3*b^8*c^2*d^6*e^13 + \\
& 53760*a^4*b^2*c^7*d^10*e^9 - 30400*a^4*b^3*c^6*d^9*e^10 + 1280*a^4*b^4*c^5*d^8*e^11 + 4224*a^4*b^5*c^4*d^7*e^12 - \\
& 1280*a^4*b^6*c^3*d^6*e^13 + 320*a^4*b^7*c^2*d^5*e^14 + 6400*a^5*b^2*c^6*d^8*e^11 - 2624*a^5*b^3*c^5*d^7*e^12 + \\
& 5952*a^5*b^4*c^4*d^6*e^13 - 2752*a^5*b^5*c^3*d^5*e^14 - 576*a^5*b^6*c^2*d^4*e^15 - 21504*a^6*b^2*c^5*d^6*e^13 + \\
& 832*a^6*b^3*c^4*d^5*e^14 + 4736*a^6*b^4*c^3*d^4*e^15 + 320*a^6*b^5*c^2*d^3*e^16 - 8448*a^7*b^2*c^4*d^4*e^15 - \\
& 2624*a^7*b^3*c^3*d^3*e^16 - 64*a^7*b^4*c^2*d^2*e^17 + 512*a^8*b^2*c^3*d^2*e^17 + 256*a*b*c^11*d^17*e^2 - \\
& 2304*a*b^2*c^10*d^16*e^3 + 8512*a*b^3*c^9*d^15*e^4 - 16704*a*b^4*c^8*d^14*e^5 + 18240*a*b^5*c^7*d^13*e^6 - \\
& 9536*a*b^6*c^6*d^12*e^7 - 576*a*b^7*c^5*d^11*e^8 + 3648*a*b^8*c^4*d^10*e^9 - 1856*a*b^9*c^3*d^9*e^10 + \\
& 320*a*b^10*c^2*d^8*e^11 - 5376*a^2*b*c^10*d^15*e^4 - 25344*a^3*b*c^9*d^13*e^6 - 37120*a^4*b*c^8*d^11*e^8 - \\
& 11520*a^5*b*c^7*d^9*e^10 + 20736*a^6*b*c^6*d^7*e^12 + 20224*a^7*b*c^5*d^5*e^14 + 5376*a^8*b*c^4*d^3*e^16))/(2* \\
& (c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - \\
& 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + \\
& 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*((b^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - b^3*c^4*d^4 - b^7*e^4 + \\
& c^4*d^4*(-(4*a*c - b^2)^3)^{(1/2)} + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - \\
& 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + \\
& 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - \\
& 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^{(1/2)} - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + \\
& 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + \\
& 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - \\
& 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + \\
& 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + \\
& 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5
\end{aligned}$$



$$\begin{aligned}
& + 64a^5b^2c^2d^2e^6 - 4a^4b^5c^3d^7e - 4a^4b^7c^4d^5e^3 - 64a^3b^5c^5d^7e + 32a^5b^3c^4d^7e + 4a^2b^6c^4d^7e + 20a^3b^5c^4d^7e - 192a^4b^3c^4d^5e^3 - 44a^4b^4c^4d^2e^6 - 192a^5b^3c^3d^3e^5))^{(1/2)} + \\
& (x*(32c^{11}d^{13}e^2 + 48a^6b^6c^4e^{15} + 96a^6c^{10}d^{11}e^4 - 64a^6c^5d^5e^{14} - 160b^6c^{10}d^{12}e^3 + 4a^4b^5c^2e^{15} - 28a^5b^3c^3e^{15} - 2 \\
& 048a^2c^9d^9e^6 - 4416a^3c^8d^7e^8 - 2528a^4c^7d^5e^{10} - 288a^5c^6d^3e^{12} + 336b^2c^9d^{11}e^4 - 268b^3c^8d^{10}e^5 - 360b^4c^7d^9e^6 + 1260b^5c^6d^8e^7 - 1568b^6c^5d^7e^8 + 1036b^7c^4d^6e^9 - 360b^8c^3d^5e^{10} + 52b^9c^2d^4e^{11} - 7584a^2b^2c^7d^7e^8 - \\
& 536a^2b^3c^6d^6e^9 + 5936a^2b^4c^5d^5e^{10} - 3552a^2b^5c^4d^4e^{11} + 464a^2b^6c^3d^3e^{12} + 104a^2b^7c^2d^2e^{13} - 12768a^3b^2c^6d^5e^{10} + 3720a^3b^3c^5d^4e^{11} + 1280a^3b^4c^4d^3e^{12} - 648 \\
& a^3b^5c^3d^2e^{13} - 4272a^4b^2c^5d^3e^{12} + 740a^4b^3c^4d^2e^{13} - 848a^4b^4c^3d^2e^{13} - 848a^4b^5c^2d^2e^{13} + 3632a^4b^6c^2d^2e^{13} - 7852a^4b^7c^2d^2e^{13} + 8864a^4b^8c^2d^2e^{13} - 4936a^4b^9c^2d^2e^{13} + 816a^4b^{10}c^2d^2e^{13} + 356a^4b^{11}c^2d^2e^{13} - 128a^4b^{12}c^2d^2e^{13} + 7216a^4b^{13}c^2d^2e^{13} - 12896a^4b^{14}c^2d^2e^{13} + 5696a^4b^{15}c^2d^2e^{13} \\
& + 216a^4b^{16}c^2d^2e^{13} + 752a^4b^{17}c^2d^2e^{13} - 336a^4b^{18}c^2d^2e^{13} \\
& ))/(2*(c^4d^{10} + a^4d^2e^8 + b^4d^6e^4 - 4a^3b^3d^5e^5 - 4a^3b^4d^3e^7 + 4a^3b^5d^2e^9 + 4a^3b^6d^2e^9 + 6a^2b^2d^4e^6 + 6a^2b^3d^4e^6 + 6a^2b^4d^4e^6 + 6b^2c^2d^8e^2 - 4b^3c^3d^9e - 12a^2b^2c^2d^7e^3 + 12a^2b^3c^2d^6e^4 - 12a^2b^4c^2d^5e^5))*(b^4e^4*(-(4a^3c - b^2)^3)^{(1/2)} - b^3c^4d^4 - b^7e^4 + c^4d^4*(-(4a^3c - b^2)^3)^{(1/2)} + 20a^3b^3c^3e^4 + 32a^2c^5d^3e - 32a^3c^4d^3e^3 + 4b^4c^3d^3e - 25a^2b^3c^2e^4 + a^2c^2e^4*(-(4a^3c - b^2)^3)^{(1/2)} - 6b^5c^2d^2e^2 + 4a^2b^3c^5d^4 + 9a^2b^5c^4e^4 + 4b^6c^4d^3e^3 + 6b^2c^2d^2e^2*(-(4a^3c - b^2)^3)^{(1/2)} - 3a^2b^2c^4d^3e - 32a^2b^4c^2d^3e^3 - 4b^3c^3d^3e*(-(4a^3c - b^2)^3)^{(1/2)} - 4b^3c^4d^3e^3*(-(4a^3c - b^2)^3)^{(1/2)} + 42a^2b^3c^3d^2e^2 - 72a^2b^4c^4d^2e^2 + 72a^2b^5c^3d^2e^3 - 6a^2c^3d^2e^2*(-(4a^3c - b^2)^3)^{(1/2)} + 8a^2b^3c^2d^2e^3*(-(4a^3c - b^2)^3)^{(1/2)})/(8*(16a^3c^6d^8 + a^5b^4e^8 + 16a^7c^2e^8 + a^2b^4c^4d^8 - 8a^6b^2c^4e^8 + a^2b^8d^4e^4 - 4a^4b^5d^7e - 8a^2b^2c^5d^8 - 4a^2b^7d^3e^5 + 6a^3b^6d^2e^6 + 64a^4c^5d^6e^2 + 96a^5c^4d^4e^4 + 64a^6c^3d^2e^6 - 44a^2b^4c^3d^6e^2 + 20a^2b^5c^2d^5e^3 + 64a^3b^2c^4d^6e^2 + 32a^3b^3c^3d^5e^3 - 74a^3b^4c^2d^4e^4 + 144a^4b^2c^3d^4e^4 + 32a^4b^3c^2d^3e^5 + 64a^5b^2c^2d^2e^6 - 4a^4b^5c^3d^7e - 4a^4b^7c^4d^5e^3 - 64a^3b^5c^5d^7e + 32a^5b^3c^4d^7e + 4a^2b^6c^4d^7e + 20a^3b^5c^4d^7e - 192a^4b^3c^4d^5e^3 - 44a^4b^4c^4d^2e^6 - 192a^5b^3c^3d^3e^5))^{(1/2)})*((b^4e^4*(-(4a^3c - b^2)^3)^{(1/2)} - b^3c^4d^4 - b^7e^4 + c^4d^4*(-(4a^3c - b^2)^3)^{(1/2)} + 20a^3b^3c^3e^4 + 32a^2c^5d^3e - 32a^3c^4d^3e^3 + 4b^4c^3d^3e - 25a^2b^3c^2e^4 + a^2c^2e^4*(-(4a^3c - b^2)^3)^{(1/2)} - 6b^5c^2d^2e^2 + 4a^2b^3c^5d^4 + 9a^2b^5c^4e^4 + 4b^6c^4d^3e^3 + 6b^2c^2d^2e^2*(-(4a^3c - b^2)^3)^{(1/2)} - 3a^2b^2c^4d^3e - 32a^2b^4c^2d^3e^3 - 4b^3c^3d^3e*(-(4a^3c - b^2)^3)^{(1/2)} - 4b^3c^4d^3e^3*(-(4a^3c - b^2)^3)^{(1/2)} + 42a^2b^3c^3d^2e^2 - 72a^2b^4c^4d^2e^2 + 72a^2b^5c^3d^2e^3 - 6a^2c^3d^2e^2*(-(4a^3c - b^2)^3)^{(1/2)} + 8a^2b^3c^2d^2e^3*(-(4a^3c - b^2)^3)^{(1/2)})/(8*(16a^3c^6d^8 + a^5b^4e^8 + 16a^7c^2e^8 + a^2b^4c^4d^8 - 8a^6b^2c^4e^8 + a^2b^8d^4e^4 - 4a^4b^5d^7e - 8a^2b^2c^5d^8 - 4a^2b^7d^3e^5 + 6a^3b^6d^2e^6 + 64a^4c^5d^6e^2 + 96a^5c^4d^4e^4 + 64a^6c^3d^2e^6 - 44a^2b^4c^3d^6e^2 + 20a^2b^5c^2d^5e^3 + 64a^3b^2c^4d^6e^2 + 32a^3b^3c^3d^5e^3 - 74a^3b^4c^2d^4e^4 + 144a^4b^2c^3d^4e^4 + 32a^4b^3c^2d^3e^5 + 64a^5b^2c^2d^2e^6 - 4a^4b^5c^3d^7e - 4a^4b^7c^4d^5e^3 - 64a^3b^5c^5d^7e + 32a^5b^3c^4d^7e + 4a^2b^6c^4d^7e + 20a^3b^5c^4d^7e + 4a^2b^6c^4d^7e + 20a^3b^5c^4d^7e - 192a^4b^3c^4d^5e^3 - 44a^4b^4c^4d^2e^6
\end{aligned}$$

$$\begin{aligned}
& - 192*a^5*b*c^3*d^3*e^5))^{(1/2)} - (x*(54*c^9*d^6*e^5 - 2*a^3*c^6*e^11 - 22 \\
& *a*c^8*d^4*e^7 - 118*b*c^8*d^5*e^6 + a^2*b^2*c^5*e^11 - 14*a^2*c^7*d^2*e^9 \\
& + 107*b^2*c^7*d^4*e^7 - 48*b^3*c^6*d^3*e^8 + 9*b^4*c^5*d^2*e^9 + 20*a*b*c^7 \\
& *d^3*e^8 - 6*a*b^3*c^5*d*e^10 + 10*a^2*b*c^6*d*e^10 + 4*a*b^2*c^6*d^2*e^9)) \\
& / (2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e \\
& ^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^ \\
& 6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7* \\
& e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5))) * ((b^4*e^4*(-(4*a*c - b^2)^ \\
& 3)^{(1/2)} - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^{(1/2)} + 20*a^ \\
& 3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^ \\
& 2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 6*b^5*c^2*d^2*e^2 + \\
& 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 24*a*b^2*c^4*d^3 \\
& *e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^{(1/2)} - 4*b^3*c* \\
& d*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^ \\
& 2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b \\
& *c^2*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)}) / (8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16* \\
& a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d \\
& *e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c \\
& ^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e \\
& ^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e \\
& ^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3* \\
& e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a \\
& ^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6* \\
& e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 1 \\
& 92*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5))^{(1/2)} \\
& ) + (((2*a^2*b^6*c^2*e^13 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 - 14*a^3*b^4 \\
& *c^3*e^13 + 26*a^4*b^2*c^4*e^13 + 480*a^2*c^8*d^6*e^7 + 784*a^3*c^7*d^4*e^9 \\
& + 96*a^4*c^6*d^2*e^11 + 50*b^2*c^8*d^8*e^5 - 240*b^3*c^7*d^7*e^6 + 466*b^4 \\
& *c^6*d^6*e^7 - 464*b^5*c^5*d^5*e^8 + 246*b^6*c^4*d^4*e^9 - 64*b^7*c^3*d^3*e \\
& ^10 + 6*b^8*c^2*d^2*e^11 + 4*a^2*b^2*c^6*d^4*e^9 + 672*a^2*b^3*c^5*d^3*e^10 \\
& - 354*a^2*b^4*c^4*d^2*e^11 + 464*a^3*b^2*c^5*d^2*e^11 + 960*a*b*c^8*d^7*e^ \\
& 6 - 8*a*b^7*c^2*d*e^12 - 96*a^4*b*c^5*d*e^12 - 1984*a*b^2*c^7*d^6*e^7 + 207 \\
& 2*a*b^3*c^6*d^5*e^8 - 1034*a*b^4*c^5*d^4*e^9 + 160*a*b^5*c^4*d^3*e^10 + 34* \\
& a*b^6*c^3*d^2*e^11 - 864*a^2*b*c^7*d^5*e^8 + 40*a^2*b^5*c^3*d*e^12 - 1152*a \\
& ^3*b*c^6*d^3*e^10 - 8*a^3*b^3*c^4*d*e^12) / (2*(c^4*d^10 + a^4*d^2*e^8 + b^4* \\
& d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4 \\
& *e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2* \\
& d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2* \\
& b*c*d^5*e^5)) - (((128*a*c^11*d^15*e^2 - 256*a^8*c^4*d*e^16 - 256*a^2*c^10* \\
& d^13*e^4 - 3456*a^3*c^9*d^11*e^6 - 8960*a^4*c^8*d^9*e^8 - 10880*a^5*c^7*d^7 \\
& *e^10 - 6912*a^6*c^6*d^5*e^12 - 2176*a^7*c^5*d^3*e^14 - 32*b^2*c^10*d^15*e^ \\
& 2 + 256*b^3*c^9*d^14*e^3 - 896*b^4*c^8*d^13*e^4 + 1792*b^5*c^7*d^12*e^5 - 2 \\
& 240*b^6*c^6*d^11*e^6 + 1792*b^7*c^5*d^10*e^7 - 896*b^8*c^4*d^9*e^8 + 256*b^ \\
& 9*c^3*d^8*e^9 - 32*b^10*c^2*d^7*e^10 + 2848*a^2*b^2*c^8*d^11*e^6 - 12160*a^ \\
& 2*b^3*c^7*d^10*e^7 + 18480*a^2*b^4*c^6*d^9*e^8 - 12864*a^2*b^5*c^5*d^8*e^9 \\
& + 3008*a^2*b^6*c^4*d^7*e^10 + 832*a^2*b^7*c^3*d^6*e^11 - 400*a^2*b^8*c^2*d^ \\
& 5*e^12 - 17920*a^3*b^2*c^7*d^9*e^8 + 1280*a^3*b^3*c^6*d^8*e^9 + 14240*a^3*b \\
& ^4*c^5*d^7*e^10 - 9824*a^3*b^5*c^4*d^6*e^11 + 1120*a^3*b^6*c^3*d^5*e^12 + 4 \\
& 80*a^3*b^7*c^2*d^4*e^13 - 33760*a^4*b^2*c^6*d^7*e^10 + 7680*a^4*b^3*c^5*d^6 \\
& *e^11 + 7520*a^4*b^4*c^4*d^5*e^12 - 2880*a^4*b^5*c^3*d^4*e^13 - 320*a^4*b^6 \\
& *c^2*d^3*e^14 - 20672*a^5*b^2*c^5*d^5*e^12 + 896*a^5*b^3*c^4*d^4*e^13 + 238 \\
& 4*a^5*b^4*c^3*d^3*e^14 + 112*a^5*b^5*c^2*d^2*e^15 - 3872*a^6*b^2*c^4*d^3*e^ \\
& 14 - 896*a^6*b^3*c^3*d^2*e^15 - 1024*a*b*c^10*d^14*e^3 + 3648*a*b^2*c^9*d^1 \\
& 3*e^4 - 7296*a*b^3*c^8*d^12*e^5 + 8464*a*b^4*c^7*d^11*e^6 - 5008*a*b^5*c^6* \\
& d^10*e^7 + 224*a*b^6*c^5*d^9*e^8 + 1632*a*b^7*c^4*d^8*e^9 - 944*a*b^8*c^3*d \\
& ^7*e^10 + 176*a*b^9*c^2*d^6*e^11 + 512*a^2*b*c^9*d^12*e^5 + 14080*a^3*b*c^8 \\
& *d^10*e^7 + 30720*a^4*b*c^7*d^8*e^9 + 28160*a^5*b*c^6*d^6*e^11 + 11776*a^6* \\
& b*c^5*d^4*e^13 - 16*a^6*b^4*c^2*d*e^16 + 1792*a^7*b*c^4*d^2*e^15 + 128*a^7*
\end{aligned}$$

$$\begin{aligned}
& b^2c^3de^{16}) / (2*(c^4d^{10} + a^4d^2e^8 + b^4d^6e^4 - 4ab^3d^5e^5 \\
& - 4a^3bd^3e^7 + 4ac^3d^8e^2 + 4a^3cd^4e^6 - 4b^3cd^7e^3 + 6 \\
& *a^2b^2d^4e^6 + 6a^2c^2d^6e^4 + 6b^2c^2d^8e^2 - 4b^3cd^9e - \\
& 12ab^3cd^7e^3 + 12ab^2cd^6e^4 - 12a^2b^3cd^5e^5)) + (x*((b^4e \\
& ^4*(-(4ac - b^2)^3)^{(1/2)} - b^3c^4d^4 - b^7e^4 + c^4d^4*(-(4ac - b^ \\
& 2)^3)^{(1/2)} + 20a^3b^3c^3e^4 + 32a^2c^5d^3e - 32a^3c^4d^3e^3 + 4b^ \\
& 4c^3d^3e - 25a^2b^3c^2e^4 + a^2c^2e^4*(-(4ac - b^2)^3)^{(1/2)} - 6 \\
& *b^5c^2d^2e^2 + 4ab^3c^5d^4 + 9ab^5c^4e^4 + 4b^6cd^3e^3 + 6b^2c^ \\
& 2d^2e^2*(-(4ac - b^2)^3)^{(1/2)} - 3ab^2c^4e^4*(-(4ac - b^2)^3)^{(1/2)} \\
& - 24ab^2c^4d^3e - 32ab^4c^2d^3e^3 - 4b^3cd^3e^3*(-(4ac - b^2)^ \\
& 3)^{(1/2)} - 4b^3cd^3e^3*(-(4ac - b^2)^3)^{(1/2)} + 42ab^3c^3d^2e^2 - \\
& 72a^2b^3c^4d^2e^2 + 72a^2b^2c^3d^3e^3 - 6ac^3d^2e^2*(-(4ac - b^ \\
& 2)^3)^{(1/2)} + 8ab^3c^2d^3e^3*(-(4ac - b^2)^3)^{(1/2)}) / (8*(16a^3c^6d^8 \\
& + a^5b^4e^8 + 16a^7c^2e^8 + ab^4c^4d^8 - 8a^6b^2c^4e^8 + ab^8d^ \\
& 4e^4 - 4a^4b^5d^4e^7 - 8a^2b^2c^5d^8 - 4a^2b^7d^3e^5 + 6a^3b^6 \\
& *d^2e^6 + 64a^4c^5d^6e^2 + 96a^5c^4d^4e^4 + 64a^6c^3d^2e^6 - 4 \\
& 4a^2b^4c^3d^6e^2 + 20a^2b^5c^2d^5e^3 + 64a^3b^2c^4d^6e^2 + 3 \\
& 2a^3b^3c^3d^5e^3 - 74a^3b^4c^2d^4e^4 + 144a^4b^2c^3d^4e^4 + \\
& 32a^4b^3c^2d^3e^5 + 64a^5b^2c^2d^2e^6 - 4ab^5c^3d^7e - 4ab \\
& ^7cd^5e^3 - 64a^3b^3c^5d^7e + 32a^5b^3cd^7e - 64a^6b^3c^2d^7e \\
& + 6ab^6c^2d^6e^2 + 32a^2b^3c^4d^7e + 4a^2b^6cd^4e^4 + 20a^ \\
& 3b^5cd^3e^5 - 192a^4b^3c^4d^5e^3 - 44a^4b^4cd^2e^6 - 192a^5b^3 \\
& c^3d^3e^5))^{(1/2)} * (1024a^2c^{11}d^{16}e^3 + 5120a^3c^{10}d^{14}e^5 + 921 \\
& 6a^4c^9d^{12}e^7 + 5120a^5c^8d^{10}e^9 - 5120a^6c^7d^8e^{11} - 9216a \\
& ^7c^6d^6e^{13} - 5120a^8c^5d^4e^{15} - 1024a^9c^4d^2e^{17} - 64b^3c^ \\
& 10d^{17}e^2 + 512b^4c^9d^{16}e^3 - 1792b^5c^8d^{15}e^4 + 3584b^6c^7d \\
& ^{14}e^5 - 4480b^7c^6d^{13}e^6 + 3584b^8c^5d^{12}e^7 - 1792b^9c^4d^{11} \\
& *e^8 + 512b^{10}c^3d^{10}e^9 - 64b^{11}c^2d^9e^{10} + 8192a^2b^2c^9d^{14} \\
& *e^5 + 5056a^2b^3c^8d^{13}e^6 - 31104a^2b^4c^7d^{12}e^7 + 40256a^2b \\
& ^5c^6d^{11}e^8 - 22784a^2b^6c^5d^{10}e^9 + 3648a^2b^7c^4d^9e^{10} + \\
& 1664a^2b^8c^3d^8e^{11} - 576a^2b^9c^2d^7e^{12} + 45312a^3b^2c^8d^ \\
& 12e^7 - 27840a^3b^3c^7d^{11}e^8 - 13760a^3b^4c^6d^{10}e^9 + 27520a^ \\
& 3b^5c^5d^9e^{10} - 12416a^3b^6c^4d^8e^{11} + 1088a^3b^7c^3d^7e^{12} \\
& + 320a^3b^8c^2d^6e^{13} + 53760a^4b^2c^7d^{10}e^9 - 30400a^4b^3c^ \\
& 6d^9e^{10} + 1280a^4b^4c^5d^8e^{11} + 4224a^4b^5c^4d^7e^{12} - 1280a \\
& ^4b^6c^3d^6e^{13} + 320a^4b^7c^2d^5e^{14} + 6400a^5b^2c^6d^8e^{11} \\
& - 2624a^5b^3c^5d^7e^{12} + 5952a^5b^4c^4d^6e^{13} - 2752a^5b^5c^3 \\
& d^5e^{14} - 576a^5b^6c^2d^4e^{15} - 21504a^6b^2c^5d^6e^{13} + 832a^6 \\
& b^3c^4d^5e^{14} + 4736a^6b^4c^3d^4e^{15} + 320a^6b^5c^2d^3e^{16} - 8 \\
& 448a^7b^2c^4d^4e^{15} - 2624a^7b^3c^3d^3e^{16} - 64a^7b^4c^2d^2e \\
& ^{17} + 512a^8b^2c^3d^2e^{17} + 256ab^3c^{11}d^{17}e^2 - 2304a^2b^2c^{10}d^ \\
& 16e^3 + 8512a^2b^3c^9d^{15}e^4 - 16704a^2b^4c^8d^{14}e^5 + 18240a^2b^5c \\
& ^7d^{13}e^6 - 9536a^2b^6c^6d^{12}e^7 - 576a^2b^7c^5d^{11}e^8 + 3648a^2b^8 \\
& *c^4d^{10}e^9 - 1856a^2b^9c^3d^9e^{10} + 320a^2b^{10}c^2d^8e^{11} - 5376a^ \\
& 2b^3c^{10}d^{15}e^4 - 25344a^3b^3c^9d^{13}e^6 - 37120a^4b^3c^8d^{11}e^8 - 1 \\
& 1520a^5b^3c^7d^9e^{10} + 20736a^6b^3c^6d^7e^{12} + 20224a^7b^3c^5d^5e^ \\
& 14 + 5376a^8b^3c^4d^3e^{16})) / (2*(c^4d^{10} + a^4d^2e^8 + b^4d^6e^4 - 4 \\
& *ab^3d^5e^5 - 4a^3bd^3e^7 + 4ac^3d^8e^2 + 4a^3cd^4e^6 - 4b^ \\
& 3cd^7e^3 + 6a^2b^2d^4e^6 + 6a^2c^2d^6e^4 + 6b^2c^2d^8e^2 - 4 \\
& *b^3cd^9e - 12ab^3cd^7e^3 + 12ab^2cd^6e^4 - 12a^2b^3cd^5e^5 \\
& )) * ((b^4e^4*(-(4ac - b^2)^3)^{(1/2)} - b^3c^4d^4 - b^7e^4 + c^4d^4*(- \\
& (4ac - b^2)^3)^{(1/2)} + 20a^3b^3c^3e^4 + 32a^2c^5d^3e - 32a^3c^4d \\
& *e^3 + 4b^4c^3d^3e - 25a^2b^3c^2e^4 + a^2c^2e^4*(-(4ac - b^2)^3 \\
& )^{(1/2)} - 6b^5c^2d^2e^2 + 4ab^3c^5d^4 + 9ab^5c^4e^4 + 4b^6cd^3e^3 \\
& + 6b^2c^2d^2e^2*(-(4ac - b^2)^3)^{(1/2)} - 3ab^2c^4e^4*(-(4ac - b^ \\
& 2)^3)^{(1/2)} - 24ab^2c^4d^3e - 32ab^4c^2d^3e^3 - 4b^3cd^3e^3*(-(4 \\
& ac - b^2)^3)^{(1/2)} - 4b^3cd^3e^3*(-(4ac - b^2)^3)^{(1/2)} + 42ab^3c^3 \\
& *d^2e^2 - 72a^2b^3c^4d^2e^2 + 72a^2b^2c^3d^3e^3 - 6ac^3d^2e^2*(- \\
& (4ac - b^2)^3)^{(1/2)} + 8ab^3c^2d^3e^3*(-(4ac - b^2)^3)^{(1/2)}) / (8*(16a
\end{aligned}$$



$$\begin{aligned}
& 2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5))^{(1/2)} + (x*(54*c^9*d^6*e^5 - 2*a^3*c^6*e^11 - 22*a*c^8*d^4*e^7 - 118*b*c^8*d^5*e^6 + a^2*b^2*c^5*e^11 - 14*a^2*c^7*d^2*e^9 + 107*b^2*c^7*d^4*e^7 - 48*b^3*c^6*d^3*e^8 + 9*b^4*c^5*d^2*e^9 + 20*a*b*c^7*d^3*e^8 - 6*a*b^3*c^5*d*e^10 + 10*a^2*b*c^6*d*e^10 + 4*a*b^2*c^6*d^2*e^9))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*((b^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^{(1/2)} + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^{(1/2)} - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5))^{(1/2)})*((b^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^{(1/2)} + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^{(1/2)} - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^{(1/2)} - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5))^{(1/2)})*2i - \operatorname{atan}((((2*a^2*b^6*c^2*e^13 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 - 14*a^3*b^4*c^3*e^13 + 26*a^4*b^2*c^4*e^13 + 480*a^2*c^8*d^6*e^7 + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^11 + 50*b^2*c^8*d^8*e^5 - 240*b^3*c^7*d^7*e^6 + 466*b^4*c^6*d^6*e^7 - 464*b^5*c^5*d^5*e^8 + 246*b^6*c^4*d^4*e^9 - 64*b^7*c^3*d^3*e^10 + 6*b^8*c^2*d^2*e^11 + 4*a^2*b^2*c^6*d^4*e^9 + 672*a^2*b^3*c^5*d^3*e^10 - 354*a^2*b^4*c^4*d^2*e^11 + 464*a^3*b^2*c^5*d^2*e^11 + 960*a*b*c^8*d^7*e^6 - 8*a*b^7*c^2*d*e^12 - 96*a^4*b*c^5*d*e^12 - 1984*a*b^2*c^7*d^6*e^7 + 2072*a*b^3*c^6*d^5*e^8 - 1034*a*b^4*c^5*d^4*e^9 + 160*a*b^5*c^4*d^3*e^10 + 34*a*b^6*c^3*d^2*e^11 - 864*a^2*b*c^7*d^5*e^8 + 40*a^2*b^5*c^3*d*e^12 - 1152*a^3
\end{aligned}$$

$$\begin{aligned}
& *b*c^6*d^3*e^{10} - 8*a^3*b^3*c^4*d*e^{12}) / (2*(c^4*d^{10} + a^4*d^2*e^8 + b^4*d^6 \\
& e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7 \\
& e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7 \\
& e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - (((128*a*c^{11}*d^{15}*e^2 - 256*a^8*c^4*d*e^{16} - 256*a^2*c^{10}*d^{13} \\
& e^4 - 3456*a^3*c^9*d^{11}*e^6 - 8960*a^4*c^8*d^9*e^8 - 10880*a^5*c^7*d^7*e^{10} - 6912*a^6*c^6*d^5*e^{12} - 2176*a^7*c^5*d^3*e^{14} - 32*b^2*c^{10}*d^{15} \\
& e^2 + 256*b^3*c^9*d^{14}*e^3 - 896*b^4*c^8*d^{13}*e^4 + 1792*b^5*c^7*d^{12}*e^5 - 2240*b^6*c^6*d^{11}*e^6 + 1792*b^7*c^5*d^{10}*e^7 - 896*b^8*c^4*d^9*e^8 + 256*b^9*c^3 \\
& d^8*e^9 - 32*b^{10}*c^2*d^7*e^{10} + 2848*a^2*b^2*c^8*d^{11}*e^6 - 12160*a^2*b^3*c^7*d^{10}*e^7 + 18480*a^2*b^4*c^6*d^9*e^8 - 12864*a^2*b^5*c^5*d^8*e^9 + 3008*a^2*b^6*c^4 \\
& d^7*e^{10} + 832*a^2*b^7*c^3*d^6*e^{11} - 400*a^2*b^8*c^2*d^5*e^{12} - 17920*a^3*b^2*c^7*d^9*e^8 + 1280*a^3*b^3*c^6*d^8*e^9 + 14240*a^3*b^4*c^5*d^7*e^{10} - 9824*a^3*b^5 \\
& c^4*d^6*e^{11} + 1120*a^3*b^6*c^3*d^5*e^{12} + 480*a^3*b^7*c^2*d^4*e^{13} - 33760*a^4*b^2*c^6*d^7*e^{10} + 7680*a^4*b^3*c^5*d^6*e^{11} + 7520*a^4*b^4*c^4*d^5*e^{12} - 2880*a^4*b^5*c^3 \\
& d^4*e^{13} - 320*a^4*b^6*c^2*d^3*e^{14} - 20672*a^5*b^2*c^5*d^5*e^{12} + 896*a^5*b^3*c^4*d^4*e^{13} + 2384*a^5*b^4*c^3*d^3*e^{14} + 112*a^5*b^5*c^2*d^2*e^{15} - 3872*a^6*b^2*c^4*d^3 \\
& e^{14} - 896*a^6*b^3*c^3*d^2*e^{15} - 1024*a*b*c^{10}*d^{14}*e^3 + 3648*a*b^2*c^9*d^{13}*e^4 - 7296*a*b^3*c^8*d^{12}*e^5 + 8464*a*b^4*c^7*d^{11}*e^6 - 5008*a*b^5*c^6*d^{10} \\
& e^7 + 224*a*b^6*c^5*d^9*e^8 + 1632*a*b^7*c^4*d^8*e^9 - 944*a*b^8*c^3*d^7*e^{10} + 176*a*b^9*c^2*d^6*e^{11} + 512*a^2*b*c^9*d^{12}*e^5 + 14080*a^3*b*c^8*d^{10} \\
& e^7 + 30720*a^4*b*c^7*d^8*e^9 + 28160*a^5*b*c^6*d^6*e^{11} + 11776*a^6*b*c^5*d^4*e^{13} - 16*a^6*b^4*c^2*d*e^{16} + 1792*a^7*b*c^4*d^2*e^{15} + 128*a^7*b^2*c^3*d*e^{16}) / (2*(c^4*d^{10} + a^4*d^2*e^8 + b^4*d^6 \\
& e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9 \\
& e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - (x*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + c^4*d^4*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3 \\
& e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2 \\
& e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^{(1/2)} - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 42*a*b^3*c^3*d^2 \\
& e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)}) / (8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4 \\
& e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5 \\
& e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7 \\
& c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5 \\
& e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5))^{(1/2)} * (1024*a^2*c^{11}*d^{16}*e^3 + 5120*a^3*c^{10}*d^{14}*e^5 + 9216*a^4*c^9*d^{12}*e^7 + 5120*a^5*c^8*d^{10}*e^9 - 5120*a^6*c^7*d^8*e^{11} - 9216*a^7 \\
& c^6*d^6*e^{13} - 5120*a^8*c^5*d^4*e^{15} - 1024*a^9*c^4*d^2*e^{17} - 64*b^3*c^{10}*d^{17}*e^2 + 512*b^4*c^9*d^{16}*e^3 - 1792*b^5*c^8*d^{15}*e^4 + 3584*b^6*c^7*d^{14}*e^5 - 4480*b^7*c^6*d^{13} \\
& e^6 + 3584*b^8*c^5*d^{12}*e^7 - 1792*b^9*c^4*d^{11}*e^8 + 512*b^{10}*c^3*d^{10}*e^9 - 64*b^{11}*c^2*d^9*e^{10} + 8192*a^2*b^2*c^9*d^{14}*e^5 + 5056*a^2*b^3*c^8*d^{13}*e^6 - 31104*a^2*b^4*c^7*d^{12} \\
& e^7 + 40256*a^2*b^5*c^6*d^{11}*e^8 - 22784*a^2*b^6*c^5*d^{10}*e^9 + 3648*a^2*b^7*c^4*d^9*e^{10} + 1664*a^2*b^8*c^3*d^8*e^{11} - 576*a^2*b^9*c^2*d^7*e^{12} + 45312*a^3*b^2*c^8*d^{12} \\
& e^7 - 27840*a^3*b^3*c^7*d^{11}*e^8 - 13760*a^3*b^4*c^6*d^{10}*e^9 + 27520*a^3*b^5*c^5*d^9*e^{10} - 12416*a^3*b^6*c^4*d^8*e^{11} + 1088*a^3*b^7*c^3*d^7*e^{12} + 320*a^3*b^8*c^2*d^6 \\
& e^{13} + 53760*a^4*b^2*c^7*d^{10}*e^9 - 30400*a^4*b^3*c^6*d^9*e^{10} + 1280*a^4*b^4*c^5*d^8*e^{11} + 4224*a^4*b^5*c^4*d^7*e^{12} - 1280*a^4
\end{aligned}$$

$$\begin{aligned}
& 4*b^6*c^3*d^6*e^{13} + 320*a^4*b^7*c^2*d^5*e^{14} + 6400*a^5*b^2*c^6*d^8*e^{11} - \\
& 2624*a^5*b^3*c^5*d^7*e^{12} + 5952*a^5*b^4*c^4*d^6*e^{13} - 2752*a^5*b^5*c^3*d^5*e^{14} - 576*a^5*b^6*c^2*d^4*e^{15} - 21504*a^6*b^2*c^5*d^6*e^{13} + 832*a^6*b^3*c^4*d^5*e^{14} + 4736*a^6*b^4*c^3*d^4*e^{15} + 320*a^6*b^5*c^2*d^3*e^{16} - 84 \\
& 48*a^7*b^2*c^4*d^4*e^{15} - 2624*a^7*b^3*c^3*d^3*e^{16} - 64*a^7*b^4*c^2*d^2*e^{17} + 512*a^8*b^2*c^3*d^2*e^{17} + 256*a*b*c^{11}*d^{17}*e^2 - 2304*a*b^2*c^{10}*d^{16}*e^3 + 8512*a*b^3*c^9*d^{15}*e^4 - 16704*a*b^4*c^8*d^{14}*e^5 + 18240*a*b^5*c^7*d^{13}*e^6 - 9536*a*b^6*c^6*d^{12}*e^7 - 576*a*b^7*c^5*d^{11}*e^8 + 3648*a*b^8*c^4*d^{10}*e^9 - 1856*a*b^9*c^3*d^9*e^{10} + 320*a*b^{10}*c^2*d^8*e^{11} - 5376*a^2 \\
& *b*c^{10}*d^{15}*e^4 - 25344*a^3*b*c^9*d^{13}*e^6 - 37120*a^4*b*c^8*d^{11}*e^8 - 11520*a^5*b*c^7*d^9*e^{10} + 20736*a^6*b*c^6*d^7*e^{12} + 20224*a^7*b*c^5*d^5*e^{14} + 5376*a^8*b*c^4*d^3*e^{16})) / (2*(c^4*d^{10} + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a \\
& *b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b \\
& *c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5) \\
& ) * (- (b^7*e^4 + b^3*c^4*d^4 + b^4*e^4 * (- (4*a*c - b^2)^3)^{(1/2)} + c^4*d^4 * (- (4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d \\
& *e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4 * (- (4*a*c - b^2)^3)^{(1/2)} + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 \\
& + 6*b^2*c^2*d^2*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^4 * (- (4*a*c - b^2)^3)^{(1/2)} + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e * (- (4*a \\
& *c - b^2)^3)^{(1/2)} - 4*b^3*c*d*e^3 * (- (4*a*c - b^2)^3)^{(1/2)} - 42*a*b^3*c^3 \\
& *d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 8*a*b*c^2*d*e^3 * (- (4*a*c - b^2)^3)^{(1/2)}) / (8*(16*a \\
& ^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 \\
& + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 \\
& + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3 \\
& *d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b \\
& *c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - \\
& 192*a^5*b*c^3*d^3*e^5))^{(1/2)} + (x*(32*c^{11}*d^{13}*e^2 + 48*a^6*b*c^4*e^{15} \\
& + 96*a*c^{10}*d^{11}*e^4 - 64*a^6*c^5*d*e^{14} - 160*b*c^{10}*d^{12}*e^3 + 4*a^4*b^5*c^2*e^{15} - 28*a^5*b^3*c^3*e^{15} - 2048*a^2*c^9*d^9*e^6 - 4416*a^3*c^8*d^7*e^8 - 2528*a^4*c^7*d^5*e^{10} - 288*a^5*c^6*d^3*e^{12} + 336*b^2*c^9*d^{11}*e^4 - 2 \\
& 68*b^3*c^8*d^{10}*e^5 - 360*b^4*c^7*d^9*e^6 + 1260*b^5*c^6*d^8*e^7 - 1568*b^6*c^5*d^7*e^8 + 1036*b^7*c^4*d^6*e^9 - 360*b^8*c^3*d^5*e^{10} + 52*b^9*c^2*d^4 \\
& *e^{11} - 7584*a^2*b^2*c^7*d^7*e^8 - 536*a^2*b^3*c^6*d^6*e^9 + 5936*a^2*b^4*c^5*d^5*e^{10} - 3552*a^2*b^5*c^4*d^4*e^{11} + 464*a^2*b^6*c^3*d^3*e^{12} + 104*a^2 \\
& *b^7*c^2*d^2*e^{13} - 12768*a^3*b^2*c^6*d^5*e^{10} + 3720*a^3*b^3*c^5*d^4*e^{11} \\
& + 1280*a^3*b^4*c^4*d^3*e^{12} - 648*a^3*b^5*c^3*d^2*e^{13} - 4272*a^4*b^2*c^5*d^3*e^{12} + 740*a^4*b^3*c^4*d^2*e^{13} - 848*a*b*c^9*d^{10}*e^5 + 3632*a*b^2*c^8 \\
& *d^9*e^6 - 7852*a*b^3*c^7*d^8*e^7 + 8864*a*b^4*c^6*d^7*e^8 - 4936*a*b^5*c^5*d^6*e^9 + 816*a*b^6*c^4*d^5*e^{10} + 356*a*b^7*c^3*d^4*e^{11} - 128*a*b^8*c^2*d^3*e^{12} + 7216*a^2*b*c^8*d^8*e^7 + 12896*a^3*b*c^7*d^6*e^9 - 32*a^3*b^6*c^ \\
& ^2*d*e^{14} + 5696*a^4*b*c^6*d^4*e^{11} + 216*a^4*b^4*c^3*d*e^{14} + 752*a^5*b*c^5 \\
& *d^2*e^{13} - 336*a^5*b^2*c^4*d*e^{14})) / (2*(c^4*d^{10} + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 \\
& - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) * (- (b^7*e^4 + b^3*c^4*d^4 + b^4*e^4 * (- (4*a*c - b^2)^3)^{(1/2)} + c^4 \\
& *d^4 * (- (4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3 \\
& *c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4 * (- (4*a*c - \\
& b^2)^3)^{(1/2)} + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c \\
& *d*e^3 + 6*b^2*c^2*d^2*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^4 * (- (4*a \\
& *c - b^2)^3)^{(1/2)} + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3* \\
& e * (- (4*a*c - b^2)^3)^{(1/2)} - 4*b^3*c*d*e^3 * (- (4*a*c - b^2)^3)^{(1/2)} - 42*a
\end{aligned}$$

$$\begin{aligned}
& b^3c^3d^2e^2 + 72a^2b^2c^4d^2e^2 - 72a^2b^2c^3d^3e^3 - 6a^2c^3d^2e^2 \\
& *e^2*(-(4ac - b^2)^3)^{(1/2)} + 8a^2b^2c^2d^2e^3*(-(4ac - b^2)^3)^{(1/2)} / ( \\
& 8*(16a^3c^6d^8 + a^5b^4e^8 + 16a^7c^2e^8 + a^2b^4c^4d^8 - 8a^6b^2c^2e^8 + a^2b^8d^4e^4 - 4a^4b^5d^7e^7 - 8a^2b^2c^5d^8 - 4a^2b^7d^3e^5 + 6a^3b^6d^2e^6 + 64a^4c^5d^6e^2 + 96a^5c^4d^4e^4 + 64a^6c^3d^2e^6 - 44a^2b^4c^3d^6e^2 + 20a^2b^5c^2d^5e^3 + 64a^3b^2c^4d^6e^2 + 32a^3b^3c^3d^5e^3 - 74a^3b^4c^2d^4e^4 + 144a^4b^2c^3d^4e^4 + 32a^4b^3c^2d^3e^5 + 64a^5b^2c^2d^2e^6 - 4a^2b^5c^3d^7e^7 - 4a^2b^7c^2d^5e^3 - 64a^3b^2c^5d^7e^7 + 32a^5b^3c^2d^5e^3 - 64a^6b^2c^2d^6e^2 + 32a^2b^3c^4d^7e^7 + 4a^2b^6c^2d^4e^4 + 20a^3b^5c^2d^3e^5 - 192a^4b^2c^4d^5e^3 - 44a^4b^4c^2d^2e^6 - 192a^5b^2c^3d^3e^5))^{(1/2)} * (- (b^7e^4 + b^3c^4d^4 + b^4e^4 * (- (4ac - b^2)^3)^{(1/2)} + c^4d^4 * (- (4ac - b^2)^3)^{(1/2)} - 20a^3b^2c^3e^4 - 32a^2c^5d^3e^3 + 32a^3c^4d^2e^3 - 4b^4c^3d^3e^3 + 25a^2b^3c^2e^4 + a^2c^2e^4 * (- (4ac - b^2)^3)^{(1/2)} + 6b^5c^2d^2e^2 - 4a^2b^5c^5d^4 - 9a^2b^5c^2e^4 - 4b^6c^2d^2e^2 + 6b^2c^2d^2e^2 * (- (4ac - b^2)^3)^{(1/2)} - 3a^2b^2c^2e^4 * (- (4ac - b^2)^3)^{(1/2)} + 24a^2b^2c^4d^3e^3 + 32a^2b^4c^2d^2e^3 - 4b^2c^3d^3e^3 * (- (4ac - b^2)^3)^{(1/2)} - 4b^3c^2d^2e^3 * (- (4ac - b^2)^3)^{(1/2)} - 42a^2b^3c^3d^2e^2 + 72a^2b^2c^4d^2e^2 - 72a^2b^2c^3d^2e^3 - 6a^2c^3d^2e^2 * (- (4ac - b^2)^3)^{(1/2)} + 8a^2b^2c^2d^2e^3 * (- (4ac - b^2)^3)^{(1/2)} / (8*(16a^3c^6d^8 + a^5b^4e^8 + 16a^7c^2e^8 + a^2b^4c^4d^8 - 8a^6b^2c^2e^8 + a^2b^8d^4e^4 - 4a^4b^5d^7e^7 - 8a^2b^2c^5d^8 - 4a^2b^7d^3e^5 + 6a^3b^6d^2e^6 + 64a^4c^5d^6e^2 + 96a^5c^4d^4e^4 + 64a^6c^3d^2e^6 - 44a^2b^4c^3d^6e^2 + 20a^2b^5c^2d^5e^3 + 64a^3b^2c^4d^6e^2 + 32a^3b^3c^3d^5e^3 - 74a^3b^4c^2d^4e^4 + 144a^4b^2c^3d^4e^4 + 32a^4b^3c^2d^3e^5 + 64a^5b^2c^2d^2e^6 - 4a^2b^5c^3d^7e^7 - 4a^2b^7c^2d^5e^3 - 64a^3b^2c^5d^7e^7 + 32a^5b^3c^2d^5e^3 - 64a^6b^2c^2d^6e^2 + 32a^2b^3c^4d^7e^7 + 4a^2b^6c^2d^4e^4 + 20a^3b^5c^2d^3e^5 - 192a^4b^2c^4d^5e^3 - 44a^4b^4c^2d^2e^6 - 192a^5b^2c^3d^3e^5))^{(1/2)} - (x*(54c^9d^6e^5 - 2a^3c^6e^11 - 22a^2c^8d^4e^7 - 118b^2c^8d^5e^6 + a^2b^2c^5e^11 - 14a^2c^7d^2e^9 + 107b^2c^7d^4e^7 - 48b^3c^6d^3e^8 + 9b^4c^5d^2e^9 + 20a^2b^2c^7d^3e^8 - 6a^2b^3c^5d^2e^10 + 10a^2b^2c^6d^2e^10 + 4a^2b^2c^6d^2e^9)) / (2*(c^4d^10 + a^4d^2e^8 + b^4d^6e^4 - 4a^2b^3d^5e^5 - 4a^3b^2d^3e^7 + 4a^2c^3d^8e^2 + 4a^3c^2d^4e^6 - 4b^3c^2d^7e^3 + 6a^2b^2d^4e^6 + 6a^2c^2d^6e^4 + 6b^2c^2d^8e^2 - 4b^2c^3d^9e - 12a^2b^2c^2d^7e^3 + 12a^2b^2c^2d^6e^4 - 12a^2b^2c^2d^5e^5)) * (- (b^7e^4 + b^3c^4d^4 + b^4e^4 * (- (4ac - b^2)^3)^{(1/2)} + c^4d^4 * (- (4ac - b^2)^3)^{(1/2)} - 20a^3b^2c^3e^4 - 32a^2c^5d^3e^3 + 32a^3c^4d^2e^3 - 4b^4c^3d^3e^3 + 25a^2b^3c^2e^4 + a^2c^2e^4 * (- (4ac - b^2)^3)^{(1/2)} + 6b^5c^2d^2e^2 - 4a^2b^5c^5d^4 - 9a^2b^5c^2e^4 - 4b^6c^2d^2e^2 + 6b^2c^2d^2e^2 * (- (4ac - b^2)^3)^{(1/2)} - 3a^2b^2c^2e^4 * (- (4ac - b^2)^3)^{(1/2)} + 24a^2b^2c^4d^3e^3 + 32a^2b^4c^2d^2e^3 - 4b^2c^3d^3e^3 * (- (4ac - b^2)^3)^{(1/2)} - 4b^3c^2d^2e^3 * (- (4ac - b^2)^3)^{(1/2)} - 42a^2b^3c^3d^2e^2 + 72a^2b^2c^4d^2e^2 - 72a^2b^2c^3d^2e^3 - 6a^2c^3d^2e^2 * (- (4ac - b^2)^3)^{(1/2)} + 8a^2b^2c^2d^2e^3 * (- (4ac - b^2)^3)^{(1/2)} / (8*(16a^3c^6d^8 + a^5b^4e^8 + 16a^7c^2e^8 + a^2b^4c^4d^8 - 8a^6b^2c^2e^8 + a^2b^8d^4e^4 - 4a^4b^5d^7e^7 - 8a^2b^2c^5d^8 - 4a^2b^7d^3e^5 + 6a^3b^6d^2e^6 + 64a^4c^5d^6e^2 + 96a^5c^4d^4e^4 + 64a^6c^3d^2e^6 - 44a^2b^4c^3d^6e^2 + 20a^2b^5c^2d^5e^3 + 64a^3b^2c^4d^6e^2 + 32a^3b^3c^3d^5e^3 - 74a^3b^4c^2d^4e^4 + 144a^4b^2c^3d^4e^4 + 32a^4b^3c^2d^3e^5 + 64a^5b^2c^2d^2e^6 - 4a^2b^5c^3d^7e^7 - 4a^2b^7c^2d^5e^3 - 64a^3b^2c^5d^7e^7 + 32a^5b^3c^2d^5e^3 - 64a^6b^2c^2d^6e^2 + 32a^2b^3c^4d^7e^7 + 4a^2b^6c^2d^4e^4 + 20a^3b^5c^2d^3e^5 - 192a^4b^2c^4d^5e^3 - 44a^4b^4c^2d^2e^6 - 192a^5b^2c^3d^3e^5))^{(1/2)} * i - (((2a^2b^6c^2e^13 - 200a^2c^9d^8e^5 - 8a^5c^5e^13 - 14a^3b^4c^3e^13 + 26a^4b^2c^4e^13 + 480a^2c^8d^6e^7 + 784a^3c^7d^4e^9 + 96a^4c^6d^2e^11 + 50b^2c^8d^8e^5 - 240b^3c^7d^7e^6 + 466b^4c^6d^6e^7 - 464b^5c^5d^5e^8
\end{aligned}$$



$$\begin{aligned}
& + 246*b^6*c^4*d^4*e^9 - 64*b^7*c^3*d^3*e^{10} + 6*b^8*c^2*d^2*e^{11} + 4*a^2*b^2*c^6*d^4*e^9 + 672*a^2*b^3*c^5*d^3*e^{10} - 354*a^2*b^4*c^4*d^2*e^{11} + 464*a^3*b^2*c^5*d^2*e^{11} + 960*a*b*c^8*d^7*e^6 - 8*a*b^7*c^2*d^2*e^{12} - 96*a^4*b*c^5*d^4*e^9 + 160*a*b^5*c^4*d^3*e^{10} + 34*a*b^6*c^3*d^2*e^{11} - 864*a^2*b*c^7*d^5*e^8 + 40*a^2*b^5*c^3*d^2*e^{12} - 1152*a^3*b*c^6*d^3*e^{10} - 8*a^3*b^3*c^4*d^2*e^{12}) / (2*(c^4*d^{10} + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - (((128*a*c^{11}*d^{15}*e^2 - 256*a^8*c^4*d^6*e^{16} - 256*a^2*c^{10}*d^{13}*e^4 - 3456*a^3*c^9*d^{11}*e^6 - 8960*a^4*c^8*d^9*e^8 - 10880*a^5*c^7*d^7*e^{10} - 6912*a^6*c^6*d^5*e^{12} - 2176*a^7*c^5*d^3*e^{14} - 32*b^2*c^{10}*d^{15}*e^2 + 256*b^3*c^9*d^{14}*e^3 - 896*b^4*c^8*d^{13}*e^4 + 1792*b^5*c^7*d^{12}*e^5 - 2240*b^6*c^6*d^{11}*e^6 + 1792*b^7*c^5*d^{10}*e^7 - 896*b^8*c^4*d^9*e^8 + 256*b^9*c^3*d^8*e^9 - 32*b^{10}*c^2*d^7*e^{10} + 2848*a^2*b^2*c^8*d^{11}*e^6 - 12160*a^2*b^3*c^7*d^{10}*e^7 + 18480*a^2*b^4*c^6*d^9*e^8 - 12864*a^2*b^5*c^5*d^8*e^9 + 3008*a^2*b^6*c^4*d^7*e^{10} + 832*a^2*b^7*c^3*d^6*e^{11} - 400*a^2*b^8*c^2*d^5*e^{12} - 17920*a^3*b^2*c^7*d^9*e^8 + 1280*a^3*b^3*c^6*d^8*e^9 + 14240*a^3*b^4*c^5*d^7*e^{10} - 9824*a^3*b^5*c^4*d^6*e^{11} + 1120*a^3*b^6*c^3*d^5*e^{12} + 480*a^3*b^7*c^2*d^4*e^{13} - 33760*a^4*b^2*c^6*d^7*e^{10} + 7680*a^4*b^3*c^5*d^6*e^{11} + 7520*a^4*b^4*c^4*d^5*e^{12} - 2880*a^4*b^5*c^3*d^4*e^{13} - 320*a^4*b^6*c^2*d^3*e^{14} - 20672*a^5*b^2*c^5*d^5*e^{12} + 896*a^5*b^3*c^4*d^4*e^{13} + 2384*a^5*b^4*c^3*d^3*e^{14} + 112*a^5*b^5*c^2*d^2*e^{15} - 3872*a^6*b^2*c^4*d^3*e^{14} - 896*a^6*b^3*c^3*d^2*e^{15} - 1024*a*b*c^{10}*d^{14}*e^3 + 3648*a*b^2*c^9*d^{13}*e^4 - 7296*a*b^3*c^8*d^{12}*e^5 + 8464*a*b^4*c^7*d^{11}*e^6 - 5008*a*b^5*c^6*d^{10}*e^7 + 224*a*b^6*c^5*d^9*e^8 + 1632*a*b^7*c^4*d^8*e^9 - 944*a*b^8*c^3*d^7*e^{10} + 176*a*b^9*c^2*d^6*e^{11} + 512*a^2*b*c^9*d^{12}*e^5 + 14080*a^3*b*c^8*d^{10}*e^7 + 30720*a^4*b*c^7*d^8*e^9 + 28160*a^5*b*c^6*d^6*e^{11} + 11776*a^6*b*c^5*d^4*e^{13} - 16*a^6*b^4*c^2*d^2*e^{16} + 1792*a^7*b*c^4*d^2*e^{15} + 128*a^7*b^2*c^3*d^2*e^{16}) / (2*(c^4*d^{10} + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) + (x*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^{1/2}) + c^4*d^4*(-(4*a*c - b^2)^3)^{1/2} - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d^3*e - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^{1/2} + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c^4 - 4*b^6*c^3*d^2*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^{1/2} - 3*a*b^2*c^4*(-(4*a*c - b^2)^3)^{1/2} + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d^2*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^{1/2} - 4*b^3*c^3*d^3*e*(-(4*a*c - b^2)^3)^{1/2} - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d^2*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^{1/2} + 8*a*b*c^2*d^2*e^3*(-(4*a*c - b^2)^3)^{1/2}) / (8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c^2*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d^2*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c^5*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c^3*d^7*e - 64*a^6*b*c^2*d^7*e + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c^3*d^4*e^4 + 20*a^3*b^5*c^3*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c^3*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5))^{1/2} * (1024*a^2*c^{11}*d^{16}*e^3 + 5120*a^3*c^{10}*d^{14}*e^5 + 9216*a^4*c^9*d^{12}*e^7 + 5120*a^5*c^8*d^{10}*e^9 - 5120*a^6*c^7*d^8*e^{11} - 9216*a^7*c^6*d^6*e^{13} - 5120*a^8*c^5*d^4*e^{15} - 1024*a^9*c^4*d^2*e^{17} - 64*b^3*c^{10}*d^{17}*e^2 + 512*b^4*c^9*d^{16}*e^3 - 1792*b^5*c^8*d^{15}*e^4 + 3584*b^6*c^7*d^{14}*e^5 - 4480*b^7*c^6*d^{13}*e^6 + 3584*b^8*c^5*d^{12}*e^7 - 1792*b^9*c^4*d^{11}*e^8 + 512*b^{10}*c^3*d^{10}*e^9 - 64*b^{11}*c^2*d^9*e^{10} + 8192*a^2*b^2*c^9*d^{14}*e^5 + 5056*a^2*b^3*c^8*d^{13}*e^6 - 31104*a^2*b^4*c^7*d^{12}*e^7 + 40256*a^2*b^5*c^6*d^{11}*e^8 - 22784*a^2*b^6*c^
\end{aligned}$$



$$\begin{aligned}
& 0*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 2 \\
& 5*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4* \\
& a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a*b^2*c^4 \\
& *d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^{(1/2)} - 4*b^ \\
& 3*c*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^ \\
& 2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 8 \\
& *a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)}/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + \\
& 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b \\
& ^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a \\
& ^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d \\
& ^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d \\
& ^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2* \\
& d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - \\
& 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2* \\
& d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 \\
& - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5))^{(1/2)} \\
& *(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + c^4*d^ \\
& 4*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c \\
& ^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^ \\
& 2)^3)^{(1/2)} + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d \\
& *e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^4*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(- \\
& -(4*a*c - b^2)^3)^{(1/2)} - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 42*a*b^3 \\
& *c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^ \\
& 2*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)}/(8*( \\
& 16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c \\
& *e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3* \\
& e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6* \\
& c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2* \\
& c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2 \\
& *c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^ \\
& 3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64* \\
& a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c* \\
& d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e \\
& ^6 - 192*a^5*b*c^3*d^3*e^5))^{(1/2)} + (x*(54*c^9*d^6*e^5 - 2*a^3*c^6*e^11 - \\
& 22*a*c^8*d^4*e^7 - 118*b*c^8*d^5*e^6 + a^2*b^2*c^5*e^11 - 14*a^2*c^7*d^2*e \\
& ^9 + 107*b^2*c^7*d^4*e^7 - 48*b^3*c^6*d^3*e^8 + 9*b^4*c^5*d^2*e^9 + 20*a*b* \\
& c^7*d^3*e^8 - 6*a*b^3*c^5*d*e^10 + 10*a^2*b*c^6*d*e^10 + 4*a*b^2*c^6*d^2*e^ \\
& 9))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^ \\
& 3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4 \\
& *e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d \\
& ^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*(-(b^7*e^4 + b^3*c^4*d^ \\
& 4 + b^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + c^4*d^4*(-(4*a*c - b^2)^3)^{(1/2)} - 2 \\
& 0*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 2 \\
& 5*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d \\
& *e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^4*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(- \\
& -(4*a*c - b^2)^3)^{(1/2)} - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^ \\
& 2*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)}/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + \\
& 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b \\
& ^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a \\
& ^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d \\
& ^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d \\
& ^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2* \\
& d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - \\
& 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*
\end{aligned}$$

$$\begin{aligned}
& d^6e^2 + 32a^2b^3c^4d^7e + 4a^2b^6c^4d^4e^4 + 20a^3b^5c^4d^3e^5 \\
& - 192a^4b^3c^4d^5e^3 - 44a^4b^4c^4d^2e^6 - 192a^5b^3c^3d^3e^5))^{1/2} \\
& (1/2)i) / ((5c^8d^3e^6 - 3b^3c^7d^2e^7 + a^3c^7d^2e^8) / (c^4d^{10} + a^4d^2e^8 \\
& + b^4d^6e^4 - 4a^3b^3d^5e^5 - 4a^3b^3d^3e^7 + 4a^3c^3d^8e^2 \\
& + 4a^3c^3d^4e^6 - 4b^3c^3d^7e^3 + 6a^2b^2d^4e^6 + 6a^2c^2d^6e^4 \\
& + 6b^2c^2d^8e^2 - 4b^3c^3d^9e - 12a^2b^2c^2d^7e^3 + 12a^2b^2c^2d^6e^4 \\
& - 12a^2b^2c^2d^5e^5) + (((2a^2b^6c^2e^{13} - 200a^3c^9d^8e^5 - 8a^5c^5e^{13} \\
& - 14a^3b^4c^3e^{13} + 26a^4b^2c^4e^{13} + 480a^2c^8d^6e^7 + 784a^3c^7d^4e^9 \\
& + 96a^4c^6d^2e^{11} + 50b^2c^8d^8e^5 - 240b^3c^7d^7e^6 + 466b^4c^6d^6e^7 \\
& - 464b^5c^5d^5e^8 + 246b^6c^4d^4e^9 - 64b^7c^3d^3e^{10} + 6b^8c^2d^2e^{11} \\
& + 4a^2b^2c^6d^4e^9 + 672a^2b^3c^5d^3e^{10} - 354a^2b^4c^4d^2e^{11} + 464a^3b^2c^5d^2e^{11} \\
& + 960a^2b^3c^8d^7e^6 - 8a^2b^7c^2d^5e^{12} - 96a^4b^3c^5d^2e^{12} - 198 \\
& 4a^2b^2c^7d^6e^7 + 2072a^2b^3c^6d^5e^8 - 1034a^2b^4c^5d^4e^9 + 160 \\
& a^2b^5c^4d^3e^{10} + 34a^2b^6c^3d^2e^{11} - 864a^2b^3c^7d^5e^8 + 40a^2 \\
& b^5c^3d^2e^{12} - 1152a^3b^3c^6d^3e^{10} - 8a^3b^3c^4d^2e^{12}) / (2(c^4d^{10} \\
& + a^4d^2e^8 + b^4d^6e^4 - 4a^3b^3d^5e^5 - 4a^3b^3d^3e^7 + 4a^3c^3d^8e^2 \\
& + 4a^3c^3d^4e^6 - 4b^3c^3d^7e^3 + 6a^2b^2d^4e^6 + 6a^2c^2d^6e^4 \\
& + 6b^2c^2d^8e^2 - 4b^3c^3d^9e - 12a^2b^2c^2d^7e^3 + 12a^2b^2c^2d^6e^4 \\
& - 12a^2b^2c^2d^5e^5)) - (((128a^3c^{11}d^{15}e^2 - 256a^8c^4d^6e^{16} \\
& - 256a^2c^{10}d^{13}e^4 - 3456a^3c^9d^{11}e^6 - 8960a^4c^8d^9e^8 - 10880a^5c^7d^7e^{10} \\
& - 6912a^6c^6d^5e^{12} - 2176a^7c^5d^3e^{14} - 32b^2c^{10}d^{15}e^2 + 256b^3c^9d^{14}e^3 \\
& - 896b^4c^8d^{13}e^4 + 1792b^5c^7d^{12}e^5 - 2240b^6c^6d^{11}e^6 + 1792b^7c^5d^{10}e^7 \\
& - 896b^8c^4d^9e^8 + 256b^9c^3d^8e^9 - 32b^{10}c^2d^7e^{10} + 2848a^2b^2c^8d^{11}e^6 \\
& - 12160a^2b^3c^7d^{10}e^7 + 18480a^2b^4c^6d^9e^8 - 12864a^2b^5c^5d^8e^9 \\
& + 3008a^2b^6c^4d^7e^{10} + 832a^2b^7c^3d^6e^{11} - 400a^2b^8c^2d^5e^{12} \\
& - 17920a^3b^2c^7d^9e^8 + 1280a^3b^3c^6d^8e^9 + 14240a^3b^4c^5d^7e^{10} \\
& - 9824a^3b^5c^4d^6e^{11} + 1120a^3b^6c^3d^5e^{12} + 480a^3b^7c^2d^4e^{13} \\
& - 33760a^4b^2c^6d^7e^{10} + 7680a^4b^3c^5d^6e^{11} + 7520a^4b^4c^4d^5e^{12} \\
& - 2880a^4b^5c^3d^4e^{13} - 320a^4b^6c^2d^3e^{14} - 20672a^5b^2c^5d^5e^{12} \\
& + 896a^5b^3c^4d^4e^{13} + 2384a^5b^4c^3d^3e^{14} + 112a^5b^5c^2d^2e^{15} \\
& - 3872a^6b^2c^4d^3e^{14} - 896a^6b^3c^3d^2e^{15} - 1024a^2b^3c^{10}d^{14}e^3 \\
& + 3648a^2b^2c^9d^{13}e^4 - 7296a^2b^3c^8d^{12}e^5 + 8464a^2b^4c^7d^{11}e^6 \\
& - 5008a^2b^5c^6d^{10}e^7 + 224a^2b^6c^5d^9e^8 + 1632a^2b^7c^4d^8e^9 \\
& - 944a^2b^8c^3d^7e^{10} + 176a^2b^9c^2d^6e^{11} + 512a^2b^3c^9d^{12}e^5 \\
& + 14080a^3b^3c^8d^{10}e^7 + 30720a^4b^3c^7d^8e^9 + 28160a^5b^3c^6d^6e^{11} \\
& + 11776a^6b^3c^5d^4e^{13} - 16a^6b^4c^2d^2e^{16} + 1792a^7b^3c^4d^2e^{15} \\
& + 128a^7b^2c^3d^2e^{16}) / (2(c^4d^{10} + a^4d^2e^8 + b^4d^6e^4 - 4a^3b^3d^5e^5 \\
& - 4a^3b^3d^3e^7 + 4a^3c^3d^8e^2 + 4a^3c^3d^4e^6 - 4b^3c^3d^7e^3 + 6a^2b^2d^4e^6 \\
& + 6a^2c^2d^6e^4 + 6b^2c^2d^8e^2 - 4b^3c^3d^9e - 12a^2b^2c^2d^7e^3 + 12a^2b^2c^2d^6e^4 \\
& - 12a^2b^2c^2d^5e^5)) - (x(-b^7e^4 + b^3c^4d^4 + b^4e^4(-4a^3c - b^2)^3)^{1/2} \\
& + c^4d^4(-4a^3c - b^2)^3)^{1/2} - 20a^3b^3c^3e^4 - 32a^2c^5d^3e^4 + 32a^3c^4d^3e^3 \\
& - 4b^4c^3d^3e^3 + 25a^2b^3c^2e^4 + a^2c^2e^4(-4a^3c - b^2)^3)^{1/2} + 6b^5c^2d^2e^2 \\
& - 4a^2b^5c^5d^4 - 9a^2b^5c^5e^4 - 4b^6c^4d^2e^3 + 6b^2c^2d^2e^2(-4a^3c - b^2)^3)^{1/2} \\
& - 3a^2b^2c^2e^4(-4a^3c - b^2)^3)^{1/2} + 24a^2b^2c^4d^3e^3 + 32a^2b^4c^2d^2e^3 \\
& - 4b^3c^3d^3e^3(-4a^3c - b^2)^3)^{1/2} - 4b^3c^3d^3e^3(-4a^3c - b^2)^3)^{1/2} \\
& - 42a^2b^3c^3d^2e^2 + 72a^2b^3c^4d^2e^2 - 72a^2b^2c^3d^2e^3 - 6a^3c^3d^2e^2 \\
& (-4a^3c - b^2)^3)^{1/2} + 8a^2b^3c^2d^2e^3(-4a^3c - b^2)^3)^{1/2}) / (8(16a^3c^6d^8 \\
& + a^5b^4e^8 + 16a^7c^2e^8 + a^2b^4c^4d^8 - 8a^6b^2c^2e^8 + a^2b^8d^4e^4 \\
& - 4a^4b^5d^4e^7 - 8a^2b^2c^5d^8 - 4a^2b^7d^3e^5 + 6a^3b^6d^2e^6 + 64a^4c^5d^6e^2 \\
& + 96a^5c^4d^4e^4 + 64a^6c^3d^2e^6 - 44a^2b^4c^3d^6e^2 + 20a^2b^5c^2d^5e^3 \\
& + 64a^3b^2c^4d^6e^2 + 32a^3b^3c^3d^5e^3 - 74a^3b^4c^2d^4e^4 + 144a^4b^2c^3d^4e^4 \\
& + 32a^4b^3c^2d^3e^5 + 64a^5b^2c^2d^2e^6 - 4a^2b^5c^3d^7e - 4a^2b^7c^4d^5e^3 \\
& - 64a^3b^3c^5d^7e + 32a^5b^3c^4
\end{aligned}$$

$$\begin{aligned}
& d^7e - 64a^6b^2c^2d^7e^7 + 6a^5b^6c^2d^6e^2 + 32a^2b^3c^4d^7e + 4 \\
& a^2b^6c^4d^4e^4 + 20a^3b^5c^3d^3e^5 - 192a^4b^3c^4d^5e^3 - 44a^4b^4c^3d^2e^6 - 192a^5b^2c^3d^3e^5))^{(1/2)} \cdot (1024a^2c^{11}d^{16}e^3 + 51 \\
& 20a^3c^{10}d^{14}e^5 + 9216a^4c^9d^{12}e^7 + 5120a^5c^8d^{10}e^9 - 5120 \\
& a^6c^7d^8e^{11} - 9216a^7c^6d^6e^{13} - 5120a^8c^5d^4e^{15} - 1024a^9c^4d^2e^{17} - 64b^3c^{10}d^{17}e^2 + 512b^4c^9d^{16}e^3 - 1792b^5c^8 \\
& d^{15}e^4 + 3584b^6c^7d^{14}e^5 - 4480b^7c^6d^{13}e^6 + 3584b^8c^5d^{12}e^7 - 1792b^9c^4d^{11}e^8 + 512b^{10}c^3d^{10}e^9 - 64b^{11}c^2d^9e^{10} \\
& + 8192a^2b^2c^9d^{14}e^5 + 5056a^2b^3c^8d^{13}e^6 - 31104a^2b^4c^7d^{12}e^7 + 40256a^2b^5c^6d^{11}e^8 - 22784a^2b^6c^5d^{10}e^9 + 36 \\
& 48a^2b^7c^4d^9e^{10} + 1664a^2b^8c^3d^8e^{11} - 576a^2b^9c^2d^7e^{12} + 45312a^3b^2c^8d^{12}e^7 - 27840a^3b^3c^7d^{11}e^8 - 13760a^3b^4c^6d^{10}e^9 \\
& + 27520a^3b^5c^5d^9e^{10} - 12416a^3b^6c^4d^8e^{11} + 1088a^3b^7c^3d^7e^{12} + 320a^3b^8c^2d^6e^{13} + 53760a^4b^2c^7d^{10}e^9 - 30400a^4b^3c^6d^9e^{10} \\
& + 1280a^4b^4c^5d^8e^{11} + 4224a^4b^5c^4d^7e^{12} - 1280a^4b^6c^3d^6e^{13} + 320a^4b^7c^2d^5e^{14} + 6400a^5b^2c^6d^8e^{11} - 2624a^5b^3c^5d^7e^{12} \\
& + 5952a^5b^4c^4d^6e^{13} - 2752a^5b^5c^3d^5e^{14} - 576a^5b^6c^2d^4e^{15} - 21504a^6b^2c^5d^6e^{13} + 832a^6b^3c^4d^5e^{14} + 4736a^6b^4c^3d^4e^{15} \\
& + 320a^6b^5c^2d^3e^{16} - 8448a^7b^2c^4d^4e^{15} - 2624a^7b^3c^3d^3e^{16} - 64a^7b^4c^2d^2e^{17} + 512a^8b^2c^3d^2e^{17} + 256a^8b^3c^2d^1e^7e^2 \\
& - 2304a^8b^2c^{10}d^{16}e^3 + 8512a^8b^3c^9d^{15}e^4 - 16704a^8b^4c^8d^{14}e^5 + 18240a^8b^5c^7d^{13}e^6 - 9536a^8b^6c^6d^{12}e^7 - 576a^8b^7c^5d^{11}e^8 \\
& + 3648a^8b^8c^4d^{10}e^9 - 1856a^8b^9c^3d^9e^{10} + 320a^8b^{10}c^2d^8e^{11} - 5376a^8b^{11}c^1d^7e^{12} - 25344a^9b^3c^9d^{13}e^6 - 37 \\
& 120a^9b^4c^8d^{11}e^8 - 11520a^9b^5c^7d^9e^{10} + 20736a^9b^6c^6d^7e^{12} + 20224a^9b^7c^5d^5e^{14} + 5376a^9b^8c^4d^3e^{16})) / (2 \cdot (c^4d^{10} + a^4 \\
& d^2e^8 + b^4d^6e^4 - 4a^3b^3d^5e^5 - 4a^3b^2d^3e^7 + 4a^2c^3d^8e^2 + 4a^3c^2d^4e^6 - 4b^3c^2d^7e^3 + 6a^2b^2d^4e^6 + 6a^2c^2d^6e^4 \\
& + 6b^2c^2d^8e^2 - 4b^2c^3d^9e - 12a^2b^2c^2d^7e^3 + 12a^2b^2c^2d^6e^4 - 12a^2b^2c^2d^5e^5)) \cdot (- (b^7e^4 + b^3c^4d^4 + b^4e^4 \cdot (- (4a^2c - \\
& b^2)^3)^{(1/2)} + c^4d^4 \cdot (- (4a^2c - b^2)^3)^{(1/2)} - 20a^3b^3c^3e^4 - 32a^2c^5d^3e + 32a^3c^4d^2e^3 - 4b^4c^3d^3e + 25a^2b^3c^2e^4 + a^2 \\
& c^2e^4 \cdot (- (4a^2c - b^2)^3)^{(1/2)} + 6b^5c^2d^2e^2 - 4a^2b^3c^5d^4 - 9a^2b^5c^2e^4 - 4b^6c^2d^2e^2 \cdot (- (4a^2c - b^2)^3)^{(1/2)} - \\
& 3a^2b^2c^2e^4 \cdot (- (4a^2c - b^2)^3)^{(1/2)} + 24a^2b^2c^4d^3e + 32a^2b^4c^2d^2e^3 - 4b^3c^3d^3e \cdot (- (4a^2c - b^2)^3)^{(1/2)} - 4b^3c^3d^3e \cdot (- (4a^2c - b^2)^3)^{(1/2)} \\
& - 42a^2b^3c^3d^2e^2 + 72a^2b^3c^4d^2e^2 - 72a^2b^2c^3d^2e^3 - 6a^2c^3d^2e^2 \cdot (- (4a^2c - b^2)^3)^{(1/2)} + 8a^2b^3c^2d^2e^3 \cdot (- (4a^2c - b^2)^3)^{(1/2)}) / (8 \cdot (16a^3c^6d^8 + a^5b^4e^8 + 16a^7c^2e^8 + a^2b^4c^4d^8 \\
& - 8a^6b^2c^2e^8 + a^2b^8d^4e^4 - 4a^4b^5d^7e^7 - 8a^2b^2c^5d^8 - 4a^2b^7d^3e^5 + 6a^3b^6d^2e^6 + 64a^4c^5d^6e^2 + 96a^5c^4d^4e^4 + 64a^6c^3d^2e^6 - 44a^2b^4c^3d^6e^2 \\
& + 20a^2b^5c^2d^5e^3 + 64a^3b^2c^4d^6e^2 + 32a^3b^3c^3d^5e^3 - 74a^3b^4c^2d^4e^4 + 144a^4b^2c^3d^4e^4 + 32a^4b^3c^2d^3e^5 + 64a^5b^2c^2d^2e^6 - 4a^2b^5c^3d^7e \\
& - 4a^2b^7c^2d^5e^3 - 64a^3b^3c^5d^7e + 32a^5b^3c^2d^7e - 64a^6b^3c^2d^7e + 6a^2b^6c^2d^6e^2 + 32a^2b^3c^4d^7e + 4a^2b^6c^2d^4e^4 + 20a^3b^5c^2d^3e^5 - 192a^4b^3c^4d^5e^3 \\
& - 44a^4b^4c^2d^2e^6 - 192a^5b^2c^3d^3e^5))^{(1/2)} + (x \cdot (32c^{11}d^{13}e^2 + 48a^6b^3c^4e^{15} + 96a^2c^{10}d^{11}e^4 - 64a^6c^5d^2e^{14} - 160b^3c^{10}d^{12}e^3 + 4a^4b^5c^2e^{15} - 28a^5b^3c^3e^{15} - 2048a^2c^9d^9e^6 \\
& - 4416a^3c^8d^7e^8 - 2528a^4c^7d^5e^{10} - 288a^5c^6d^3e^{12} + 336b^2c^9d^{11}e^4 - 268b^3c^8d^{10}e^5 - 360b^4c^7d^9e^6 + 1260b^5c^6d^8e^7 - 1568b^6c^5d^7e^8 + 1036b^7c^4d^6e^9 - 360b^8c^3d^5e^{10} \\
& + 52b^9c^2d^4e^{11} - 7584a^2b^2c^7d^7e^8 - 536a^2b^3c^6d^6e^9 + 5936a^2b^4c^5d^5e^{10} - 3552a^2b^5c^4d^4e^{11} + 464a^2b^6c^3d^3e^{12} + 104a^2b^7c^2d^2e^{13} - 12768a^3b^2c^6d^5e^{10} \\
& + 3720a^3b^3c^5d^4e^{11} + 1280a^3b^4c^4d^3e^{12} - 648a^3b^5c^3d^2e^{13} - 4272a^4b^2c^5d^3e^{12} + 740a^4b^3c^4d^2e^{13} - 848a^2b^3c^4d^2e^{13} - 848a^2b^3c^4d^2e^{13}
\end{aligned}$$

$$\begin{aligned}
& 9*d^{10}*e^5 + 3632*a*b^2*c^8*d^9*e^6 - 7852*a*b^3*c^7*d^8*e^7 + 8864*a*b^4*c^6*d^7*e^8 - 4936*a*b^5*c^5*d^6*e^9 + 816*a*b^6*c^4*d^5*e^{10} + 356*a*b^7*c^3*d^4*e^{11} - 128*a*b^8*c^2*d^3*e^{12} + 7216*a^2*b*c^8*d^8*e^7 + 12896*a^3*b*c^7*d^6*e^9 - 32*a^3*b^6*c^2*d*e^{14} + 5696*a^4*b*c^6*d^4*e^{11} + 216*a^4*b^4*c^3*d*e^{14} + 752*a^5*b*c^5*d^2*e^{13} - 336*a^5*b^2*c^4*d*e^{14})/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + c^4*d^4*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^{(1/2)} - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5))^{(1/2)})*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + c^4*d^4*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^{(1/2)} - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5))^{(1/2)} - (x*(54*c^9*d^6*e^5 - 2*a^3*c^6*e^{11} - 22*a*c^8*d^4*e^7 - 118*b*c^8*d^5*e^6 + a^2*b^2*c^5*e^{11} - 14*a^2*c^7*d^2*e^9 + 107*b^2*c^7*d^4*e^7 - 48*b^3*c^6*d^3*e^8 + 9*b^4*c^5*d^2*e^9 + 20*a*b*c^7*d^3*e^8 - 6*a*b^3*c^5*d*e^{10} + 10*a^2*b*c^6*d*e^{10} + 4*a*b^2*c^6*d^2*e^9))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + c^4*d^4*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^{(1/2)} - 4*b^3*c*d*e^3*(-(4*
\end{aligned}$$

$$\begin{aligned}
& a^2c - b^2)^3)^{(1/2)} - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5))^{(1/2)} + (((2*a^2*b^6*c^2*e^13 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 - 14*a^3*b^4*c^3*e^13 + 26*a^4*b^2*c^4*e^13 + 480*a^2*c^8*d^6*e^7 + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^11 + 50*b^2*c^8*d^8*e^5 - 240*b^3*c^7*d^7*e^6 + 466*b^4*c^6*d^6*e^7 - 464*b^5*c^5*d^5*e^8 + 246*b^6*c^4*d^4*e^9 - 64*b^7*c^3*d^3*e^10 + 6*b^8*c^2*d^2*e^11 + 4*a^2*b^2*c^6*d^4*e^9 + 672*a^2*b^3*c^5*d^3*e^10 - 354*a^2*b^4*c^4*d^2*e^11 + 464*a^3*b^2*c^5*d^2*e^11 + 960*a*b*c^8*d^7*e^6 - 8*a*b^7*c^2*d*e^12 - 96*a^4*b*c^5*d*e^12 - 1984*a*b^2*c^7*d^6*e^7 + 2072*a*b^3*c^6*d^5*e^8 - 1034*a*b^4*c^5*d^4*e^9 + 160*a*b^5*c^4*d^3*e^10 + 34*a*b^6*c^3*d^2*e^11 - 864*a^2*b*c^7*d^5*e^8 + 40*a^2*b^5*c^3*d*e^12 - 1152*a^3*b*c^6*d^3*e^10 - 8*a^3*b^3*c^4*d*e^12)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - (((128*a*c^11*d^15*e^2 - 256*a^8*c^4*d*e^16 - 256*a^2*c^10*d^13*e^4 - 3456*a^3*c^9*d^11*e^6 - 8960*a^4*c^8*d^9*e^8 - 10880*a^5*c^7*d^7*e^10 - 6912*a^6*c^6*d^5*e^12 - 2176*a^7*c^5*d^3*e^14 - 32*b^2*c^10*d^15*e^2 + 256*b^3*c^9*d^14*e^3 - 896*b^4*c^8*d^13*e^4 + 1792*b^5*c^7*d^12*e^5 - 2240*b^6*c^6*d^11*e^6 + 1792*b^7*c^5*d^10*e^7 - 896*b^8*c^4*d^9*e^8 + 256*b^9*c^3*d^8*e^9 - 32*b^10*c^2*d^7*e^10 + 2848*a^2*b^2*c^8*d^11*e^6 - 12160*a^2*b^3*c^7*d^10*e^7 + 18480*a^2*b^4*c^6*d^9*e^8 - 12864*a^2*b^5*c^5*d^8*e^9 + 3008*a^2*b^6*c^4*d^7*e^10 + 832*a^2*b^7*c^3*d^6*e^11 - 400*a^2*b^8*c^2*d^5*e^12 - 17920*a^3*b^2*c^7*d^9*e^8 + 1280*a^3*b^3*c^6*d^8*e^9 + 14240*a^3*b^4*c^5*d^7*e^10 - 9824*a^3*b^5*c^4*d^6*e^11 + 1120*a^3*b^6*c^3*d^5*e^12 + 480*a^3*b^7*c^2*d^4*e^13 - 33760*a^4*b^2*c^6*d^7*e^10 + 7680*a^4*b^3*c^5*d^6*e^11 + 7520*a^4*b^4*c^4*d^5*e^12 - 2880*a^4*b^5*c^3*d^4*e^13 - 320*a^4*b^6*c^2*d^3*e^14 - 20672*a^5*b^2*c^5*d^5*e^12 + 896*a^5*b^3*c^4*d^4*e^13 + 2384*a^5*b^4*c^3*d^3*e^14 + 112*a^5*b^5*c^2*d^2*e^15 - 3872*a^6*b^2*c^4*d^3*e^14 - 896*a^6*b^3*c^3*d^2*e^15 - 1024*a*b*c^10*d^14*e^3 + 3648*a*b^2*c^9*d^13*e^4 - 7296*a*b^3*c^8*d^12*e^5 + 8464*a*b^4*c^7*d^11*e^6 - 5008*a*b^5*c^6*d^10*e^7 + 224*a*b^6*c^5*d^9*e^8 + 1632*a*b^7*c^4*d^8*e^9 - 944*a*b^8*c^3*d^7*e^10 + 176*a*b^9*c^2*d^6*e^11 + 512*a^2*b*c^9*d^12*e^5 + 14080*a^3*b*c^8*d^10*e^7 + 30720*a^4*b*c^7*d^8*e^9 + 28160*a^5*b*c^6*d^6*e^11 + 11776*a^6*b*c^5*d^4*e^13 - 16*a^6*b^4*c^2*d*e^16 + 1792*a^7*b*c^4*d^2*e^15 + 128*a^7*b^2*c^3*d*e^16)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) + (x*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + c^4*d^4*(-(4*a*c - b^2)^3)^{(1/2)}) - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^{(1/2)} - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*
\end{aligned}$$

$$\begin{aligned}
& a^4 b^5 d^7 e^7 - 8 a^2 b^2 c^5 d^8 - 4 a^2 b^7 d^3 e^5 + 6 a^3 b^6 d^2 e^6 + \\
& 64 a^4 c^5 d^6 e^2 + 96 a^5 c^4 d^4 e^4 + 64 a^6 c^3 d^2 e^6 - 44 a^2 b^4 c^3 d^6 e^2 + 20 a^2 b^5 c^2 d^5 e^3 + 64 a^3 b^2 c^4 d^6 e^2 + 32 a^3 b^3 c^3 d^5 e^3 - 74 a^3 b^4 c^2 d^4 e^4 + 144 a^4 b^2 c^3 d^4 e^4 + 32 a^4 b^3 c^2 d^3 e^5 + 64 a^5 b^2 c^2 d^2 e^6 - 4 a^* b^5 c^3 d^7 e - 4 a^* b^7 c^3 d^5 e^3 - 64 a^3 b^* c^5 d^7 e + 32 a^5 b^3 c^* d^7 e - 64 a^6 b^* c^2 d^* e^7 + 6 a^* b^6 c^2 d^6 e^2 + 32 a^2 b^3 c^4 d^7 e + 4 a^2 b^6 c^* d^4 e^4 + 20 a^3 b^5 c^* d^3 e^5 - 192 a^4 b^* c^4 d^5 e^3 - 44 a^4 b^4 c^* d^2 e^6 - 192 a^5 b^* c^3 d^3 e^5))^{(1/2)} * (1024 a^2 c^{11} d^{16} e^3 + 5120 a^3 c^{10} d^{14} e^5 + 9216 a^4 c^9 d^{12} e^7 + 5120 a^5 c^8 d^{10} e^9 - 5120 a^6 c^7 d^8 e^{11} - 9216 a^7 c^6 d^6 e^{13} - 5120 a^8 c^5 d^4 e^{15} - 1024 a^9 c^4 d^2 e^{17} - 64 b^3 c^{10} d^{17} e^2 + 512 b^4 c^9 d^{16} e^3 - 1792 b^5 c^8 d^{15} e^4 + 3584 b^6 c^7 d^{14} e^5 - 4480 b^7 c^6 d^{13} e^6 + 3584 b^8 c^5 d^{12} e^7 - 1792 b^9 c^4 d^{11} e^8 + 512 b^{10} c^3 d^{10} e^9 - 64 b^{11} c^2 d^9 e^{10} + 8192 a^2 b^2 c^9 d^{14} e^5 + 505 6 a^2 b^3 c^8 d^{13} e^6 - 31104 a^2 b^4 c^7 d^{12} e^7 + 40256 a^2 b^5 c^6 d^{11} e^8 - 22784 a^2 b^6 c^5 d^{10} e^9 + 3648 a^2 b^7 c^4 d^9 e^{10} + 1664 a^2 b^8 c^3 d^8 e^{11} - 576 a^2 b^9 c^2 d^7 e^{12} + 45312 a^3 b^2 c^8 d^{12} e^7 - 2 7840 a^3 b^3 c^7 d^{11} e^8 - 13760 a^3 b^4 c^6 d^{10} e^9 + 27520 a^3 b^5 c^5 d^9 e^{10} - 12416 a^3 b^6 c^4 d^8 e^{11} + 1088 a^3 b^7 c^3 d^7 e^{12} + 320 a^3 b^8 c^2 d^6 e^{13} + 53760 a^4 b^2 c^7 d^{10} e^9 - 30400 a^4 b^3 c^6 d^9 e^{10} + 1280 a^4 b^4 c^5 d^8 e^{11} + 4224 a^4 b^5 c^4 d^7 e^{12} - 1280 a^4 b^6 c^3 d^6 e^{13} + 320 a^4 b^7 c^2 d^5 e^{14} + 6400 a^5 b^2 c^6 d^8 e^{11} - 2624 a^5 b^3 c^5 d^7 e^{12} + 5952 a^5 b^4 c^4 d^6 e^{13} - 2752 a^5 b^5 c^3 d^5 e^{14} - 576 a^5 b^6 c^2 d^4 e^{15} - 21504 a^6 b^2 c^5 d^6 e^{13} + 832 a^6 b^3 c^4 d^5 e^{14} + 4736 a^6 b^4 c^3 d^4 e^{15} + 320 a^6 b^5 c^2 d^3 e^{16} - 8448 a^7 b^2 c^4 d^4 e^{15} - 2624 a^7 b^3 c^3 d^3 e^{16} - 64 a^7 b^4 c^2 d^2 e^{17} + 512 a^8 b^2 c^3 d^2 e^{17} + 256 a^* b^* c^{11} d^{17} e^2 - 2304 a^* b^2 c^{10} d^{16} e^3 + 8 512 a^* b^3 c^9 d^{15} e^4 - 16704 a^* b^4 c^8 d^{14} e^5 + 18240 a^* b^5 c^7 d^{13} e^6 - 9536 a^* b^6 c^6 d^{12} e^7 - 576 a^* b^7 c^5 d^{11} e^8 + 3648 a^* b^8 c^4 d^{10} e^9 - 1856 a^* b^9 c^3 d^9 e^{10} + 320 a^* b^{10} c^2 d^8 e^{11} - 5376 a^2 b^* c^{10} d^{15} e^4 - 25344 a^3 b^* c^9 d^{13} e^6 - 37120 a^4 b^* c^8 d^{11} e^8 - 11520 a^5 b^* c^7 d^9 e^{10} + 20736 a^6 b^* c^6 d^7 e^{12} + 20224 a^7 b^* c^5 d^5 e^{14} + 5376 a^8 b^* c^4 d^3 e^{16})) / (2 * (c^4 d^{10} + a^4 d^2 e^8 + b^4 d^6 e^4 - 4 a^* b^3 d^5 e^5 - 4 a^3 b^* d^3 e^7 + 4 a^* c^3 d^8 e^2 + 4 a^3 c^* d^4 e^6 - 4 b^3 c^* d^7 e^3 + 6 a^2 b^2 d^4 e^6 + 6 a^2 c^2 d^6 e^4 + 6 b^2 c^2 d^8 e^2 - 4 b^* c^3 d^9 e - 12 a^* b^* c^2 d^7 e^3 + 12 a^* b^2 c^* d^6 e^4 - 12 a^2 b^* c^* d^5 e^5))) * (- (b^7 e^4 + b^3 c^4 d^4 + b^4 e^4 * (- (4 a^* c - b^2)^3)^{(1/2)} + c^4 d^4 * (- (4 a^* c - b^2)^3)^{(1/2)} - 20 a^3 b^* c^3 e^4 - 32 a^2 c^5 d^3 e + 32 a^3 c^4 d^* e^3 - 4 b^4 c^3 d^3 e + 25 a^2 b^3 c^2 e^4 + a^2 c^2 e^4 * (- (4 a^* c - b^2)^3)^{(1/2)} + 6 b^5 c^2 d^2 e^2 - 4 a^* b^* c^5 d^4 - 9 a^* b^5 c^* e^4 - 4 b^6 c^* d^* e^3 + 6 b^2 c^2 d^2 e^2 * (- (4 a^* c - b^2)^3)^{(1/2)} - 3 a^* b^2 c^* e^4 * (- (4 a^* c - b^2)^3)^{(1/2)} + 24 a^* b^2 c^4 d^3 e + 32 a^* b^4 c^2 d^* e^3 - 4 b^* c^3 d^3 e * (- (4 a^* c - b^2)^3)^{(1/2)} - 4 b^3 c^* d^* e^3 * (- (4 a^* c - b^2)^3)^{(1/2)} - 42 a^* b^3 c^3 d^2 e^2 + 72 a^2 b^* c^4 d^2 e^2 - 72 a^2 b^2 c^3 d^* e^3 - 6 a^* c^3 d^2 e^2 * (- (4 a^* c - b^2)^3)^{(1/2)} + 8 a^* b^* c^2 d^* e^3 * (- (4 a^* c - b^2)^3)^{(1/2})) / (8 * (16 a^3 c^6 d^8 + a^5 b^4 e^8 + 16 a^7 c^2 e^8 + a^* b^4 c^4 d^8 - 8 a^6 b^2 c^* e^8 + a^* b^8 d^4 e^4 - 4 a^4 b^5 d^* e^7 - 8 a^2 b^2 c^5 d^8 - 4 a^2 b^7 d^3 e^5 + 6 a^3 b^6 d^2 e^6 + 64 a^4 c^5 d^6 e^2 + 96 a^5 c^4 d^4 e^4 + 64 a^6 c^3 d^2 e^6 - 44 a^2 b^4 c^3 d^6 e^2 + 20 a^2 b^5 c^2 d^5 e^3 + 64 a^3 b^2 c^4 d^6 e^2 + 32 a^3 b^3 c^3 d^5 e^3 - 74 a^3 b^4 c^2 d^4 e^4 + 144 a^4 b^2 c^3 d^4 e^4 + 32 a^4 b^3 c^2 d^3 e^5 + 64 a^5 b^2 c^2 d^2 e^6 - 4 a^* b^5 c^3 d^7 e - 4 a^* b^7 c^* d^5 e^3 - 64 a^3 b^* c^5 d^7 e + 32 a^5 b^3 c^* d^7 e - 64 a^6 b^* c^2 d^* e^7 + 6 a^* b^6 c^2 d^6 e^2 + 32 a^2 b^3 c^4 d^7 e + 4 a^2 b^6 c^* d^4 e^4 + 20 a^3 b^5 c^* d^3 e^5 - 192 a^4 b^* c^4 d^5 e^3 - 44 a^4 b^4 c^* d^2 e^6 - 192 a^5 b^* c^3 d^3 e^5))^{(1/2)} - (x * (32 c^{11} d^{13} e^2 + 48 a^6 b^* c^4 e^{15} + 96 a^* c^{10} d^{11} e^4 - 64 a^6 c^5 d^* e^{14} - 160 b^* c^{10} d^{12} e^3 + 4 a^4 b^5 c^2 e^{15} - 28 a^5 b^3 c^3 e^{15} - 2048 a^2 c^9 d^9 e^6 - 4416 a^3 c^8 d^7 e^8 - 2528 a^4 c^7 d^5 e^{10} - 288 a^5 c^6 d^3 e^{12} + 336 b^2 c^9 d^{11} e^4 - 268 b^3 c^8 d^{10} e^5 - 360 b^4 c^7 d^9 e^6 + 1260 b^5 c^6 d^8 e^7 - 1568 b^6 c^5 d^7 e^8 - 1260 b^7 c^4 d^7 e^8 - 1568 b^8 c^3 d^6 e^8 - 1260 b^9 c^2 d^5 e^8 - 1260 b^{10} c d^4 e^8 - 1260 b^{11} e^8))^{(1/2)}
\end{aligned}$$



$$\begin{aligned}
& e^8 + 1036b^7c^4d^6e^9 - 360b^8c^3d^5e^{10} + 52b^9c^2d^4e^{11} - 7 \\
& 584a^2b^2c^7d^7e^8 - 536a^2b^3c^6d^6e^9 + 5936a^2b^4c^5d^5e^{10} - 3552a^2b^5c^4d^4e^{11} + 464a^2b^6c^3d^3e^{12} + 104a^2b^7c^2 \\
& *d^2e^{13} - 12768a^3b^2c^6d^5e^{10} + 3720a^3b^3c^5d^4e^{11} + 1280a^3b^4c^4d^3e^{12} - 648a^3b^5c^3d^2e^{13} - 4272a^4b^2c^5d^3e^{12} \\
& + 740a^4b^3c^4d^2e^{13} - 848a^4b^4c^3d^1e^{14} + 3632a^4b^5c^2d^0e^{15} - 7852a^4b^6c^1d^0e^{16} + 8864a^4b^7c^0d^0e^{17} - 4936a^5b^5c^5d^6e^9 \\
& + 816a^5b^6c^4d^5e^{10} + 356a^5b^7c^3d^4e^{11} - 128a^5b^8c^2d^3e^{12} + 7216a^5b^9c^1d^2e^{13} + 12896a^5b^{10}c^0d^1e^{14} - 32a^6b^6c^6d^2e^{14} \\
& + 5696a^6b^7c^5d^1e^{15} + 216a^6b^8c^4d^0e^{16} + 752a^6b^9c^3d^0e^{17} + 752a^5b^6c^5d^2e^{13} - 336a^5b^2c^4d^1e^{14})) / ((2*(c^4d^{10} + a^4d^2e^8 + b^4d^6e^4 - 4*a* \\
& b^3d^5e^5 - 4*a^3b*d^3e^7 + 4*a*c^3d^8e^2 + 4*a^3c*d^4e^6 - 4*b^3c*d^7e^3 + 6*a^2b^2d^4e^6 + 6*a^2c^2d^6e^4 + 6*b^2c^2d^8e^2 - 4*b* \\
& c^3d^9e - 12*a*b*c^2d^7e^3 + 12*a*b^2*c*d^6e^4 - 12*a^2*b*c*d^5e^5))) \\
& *(-(b^7e^4 + b^3c^4d^4 + b^4e^4*(-(4*a*c - b^2)^3)^{(1/2)} + c^4d^4*(-(4 \\
& *a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3e^4 - 32*a^2*c^5*d^3e + 32*a^3*c^4*d*e \\
& ^3 - 4*b^4*c^3*d^3e + 25*a^2*b^3*c^2e^4 + a^2*c^2e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 6*b^5*c^2*d^2e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c^4e^4 - 4*b^6*c*d^3e^3 + \\
& 6*b^2*c^2*d^2e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c^4e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a*b^2*c^4d^3e + 32*a*b^4*c^2d^3e^3 - 4*b*c^3d^3e*(-(4*a* \\
& c - b^2)^3)^{(1/2)} - 4*b^3*c*d^3e*(-(4*a*c - b^2)^3)^{(1/2)} - 42*a*b^3*c^3*d \\
& ^2e^2 + 72*a^2*b*c^4d^2e^2 - 72*a^2*b^2*c^3d^3e^3 - 6*a*c^3d^2e^2*(-(4 \\
& *a*c - b^2)^3)^{(1/2)} + 8*a*b*c^2d^3e^3*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^3 \\
& c^6d^8 + a^5b^4e^8 + 16*a^7c^2e^8 + a*b^4c^4d^8 - 8*a^6b^2c^4e^8 + \\
& a*b^8d^4e^4 - 4*a^4b^5d^7e^7 - 8*a^2b^2c^5d^8 - 4*a^2b^7d^3e^5 + \\
& 6*a^3b^6d^2e^6 + 64*a^4c^5d^6e^2 + 96*a^5c^4d^4e^4 + 64*a^6c^3d^2 \\
& e^6 - 44*a^2b^4c^3d^6e^2 + 20*a^2b^5c^2d^5e^3 + 64*a^3b^2c^4d^6 \\
& e^2 + 32*a^3b^3c^3d^5e^3 - 74*a^3b^4c^2d^4e^4 + 144*a^4b^2c^3d^4 \\
& e^4 + 32*a^4b^3c^2d^3e^5 + 64*a^5b^2c^2d^2e^6 - 4*a*b^5c^3d^7* \\
& e - 4*a*b^7c^4d^5e^3 - 64*a^3b^5c^5d^7e + 32*a^5b^3c^4d^7e - 64*a^6b^2 \\
& c^2d^7e + 6*a*b^6c^2d^6e^2 + 32*a^2b^3c^4d^7e + 4*a^2b^6c^4d^4e^4 \\
& + 20*a^3b^5c^4d^3e^5 - 192*a^4b^4c^4d^5e^3 - 44*a^4b^4c^4d^2e^6 - 1 \\
& 92*a^5b^3c^3d^3e^5))^{(1/2)} *(-(b^7e^4 + b^3c^4d^4 + b^4e^4*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + c^4d^4*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3e^4 - 32* \\
& a^2*c^5*d^3e + 32*a^3*c^4*d^3e - 4*b^4*c^3*d^3e + 25*a^2*b^3*c^2e^4 + a \\
& ^2*c^2e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 6*b^5*c^2*d^2e^2 - 4*a*b*c^5*d^4 - 9 \\
& *a*b^5*c^4e^4 - 4*b^6*c*d^3e^3 + 6*b^2*c^2*d^2e^2*(-(4*a*c - b^2)^3)^{(1/2)} - \\
& 3*a*b^2*c^4e^4*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a*b^2*c^4d^3e + 32*a*b^4*c^2 \\
& *d^3e^3 - 4*b*c^3d^3e*(-(4*a*c - b^2)^3)^{(1/2)} - 4*b^3*c*d^3e*(-(4*a*c - \\
& b^2)^3)^{(1/2)} - 42*a*b^3*c^3d^2e^2 + 72*a^2*b*c^4d^2e^2 - 72*a^2*b^2*c^ \\
& 3d^3e^3 - 6*a*c^3d^2e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b*c^2d^3e^3*(-(4*a \\
& *c - b^2)^3)^{(1/2)))/(8*(16*a^3c^6d^8 + a^5b^4e^8 + 16*a^7c^2e^8 + a*b \\
& ^4c^4d^8 - 8*a^6b^2c^4e^8 + a*b^8d^4e^4 - 4*a^4b^5d^7e^7 - 8*a^2b^2* \\
& c^5d^8 - 4*a^2b^7d^3e^5 + 6*a^3b^6d^2e^6 + 64*a^4c^5d^6e^2 + 96*a \\
& ^5c^4d^4e^4 + 64*a^6c^3d^2e^6 - 44*a^2b^4c^3d^6e^2 + 20*a^2b^5c^ \\
& ^2d^5e^3 + 64*a^3b^2c^4d^6e^2 + 32*a^3b^3c^3d^5e^3 - 74*a^3b^4c^ \\
& ^2d^4e^4 + 144*a^4b^2c^3d^4e^4 + 32*a^4b^3c^2d^3e^5 + 64*a^5b^2* \\
& c^2d^2e^6 - 4*a*b^5c^3d^7e - 4*a*b^7c^4d^5e^3 - 64*a^3b^5c^5d^7e + \\
& 32*a^5b^3c^4d^7e - 64*a^6b^2c^2d^7e + 6*a*b^6c^2d^6e^2 + 32*a^2b^3* \\
& c^4d^7e + 4*a^2b^6c^4d^4e^4 + 20*a^3b^5c^4d^3e^5 - 192*a^4b^4c^4d^5* \\
& e^3 - 44*a^4b^4c^4d^2e^6 - 192*a^5b^3c^3d^3e^5))^{(1/2)} + (x*(54*c^9d^ \\
& 6e^5 - 2*a^3c^6e^{11} - 22*a*c^8d^4e^7 - 118*b*c^8d^5e^6 + a^2b^2c^5 \\
& e^{11} - 14*a^2c^7d^2e^9 + 107*b^2c^7d^4e^7 - 48*b^3c^6d^3e^8 + 9*b \\
& ^4c^5d^2e^9 + 20*a*b*c^7d^3e^8 - 6*a*b^3c^5d^5e^{10} + 10*a^2b^2c^6d^5e \\
& ^{10} + 4*a*b^2c^6d^2e^9)) / ((2*(c^4d^{10} + a^4d^2e^8 + b^4d^6e^4 - 4*a* \\
& b^3d^5e^5 - 4*a^3b*d^3e^7 + 4*a*c^3d^8e^2 + 4*a^3c*d^4e^6 - 4*b^3c*d^7e^3 + 6*a^2b^2d^4e^6 + 6*a^2c^2d^6e^4 + 6*b^2c^2d^8e^2 - 4*b* \\
& c^3d^9e - 12*a*b*c^2d^7e^3 + 12*a*b^2*c*d^6e^4 - 12*a^2*b*c*d^5e^5))) \\
& *(-(b^7e^4 + b^3c^4d^4 + b^4e^4*(-(4*a*c - b^2)^3)^{(1/2)} + c^4d^4*(-(4
\end{aligned}$$

```

*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e
^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(
1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 +
6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)
^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*
c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d
^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4
*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3
*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 +
a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 +
6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^
2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^
6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d
^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*
e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*
c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^
4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 1
92*a^5*b*c^3*d^3*e^5)))^(1/2))*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c
- b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32
*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 +
a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 -
9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^
2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c -
b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c
^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*
b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2
*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*
a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*
c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*
c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2
*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e +
32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3
*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5
*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2)*2i + (e^2*x)/(
2*d*(d + e*x^2)*(a*e^2 + c*d^2 - b*d*e))

```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x\*\*2+d)\*\*2/(c\*x\*\*4+b\*x\*\*2+a), x)

[Out] Timed out

$$3.193 \quad \int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2} dx$$

**Optimal.** Leaf size=563

$$\frac{x \left( c \left( -\frac{abe(ae^2+3cd^2)}{c} - 2ad(cd^2 - 3ae^2) + b^2d^3 \right) - x^2(ab^2e^3 - bcd(3ae^2 + cd^2) + 2ace(3cd^2 - ae^2)) \right)}{2ac(b^2 - 4ac)(a + bx^2 + cx^4)} \quad (ab^3e^3 - b^2cd^2)$$

**Rubi [A]** time = 3.52, antiderivative size = 563, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, number of rules used = 0.125, Rules used = {1205, 1166, 205}

$$\frac{(-\frac{1}{2} \sqrt{b^2-4ac} \sqrt{c} (ae^2+3cd^2) + 2ad(cd^2-3ae^2) + b^2d^3) x - x^2(ab^2e^3 - bcd(3ae^2 + cd^2) + 2ace(3cd^2 - ae^2))}{2ac(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^3/(a + b\*x^2 + c\*x^4)^2,x]

[Out] (x\*(c\*(b^2\*d^3 - 2\*a\*d\*(c\*d^2 - 3\*a\*e^2) - (a\*b\*e\*(3\*c\*d^2 + a\*e^2))/c) - (a\*b^2\*e^3 + 2\*a\*c\*e\*(3\*c\*d^2 - a\*e^2) - b\*c\*d\*(c\*d^2 + 3\*a\*e^2))\*x^2)/(2\*a\*c\*(b^2 - 4\*a\*c)\*(a + b\*x^2 + c\*x^4)) - ((a\*b^3\*e^3 + 6\*a\*c\*(2\*c\*d + Sqrt[b^2 - 4\*a\*c])\*e)\*(c\*d^2 + a\*e^2) - b^2\*(c^2\*d^3 - 3\*a\*c\*d\*e^2 + a\*Sqrt[b^2 - 4\*a\*c]\*e^3) - b\*c\*(a\*e^2\*(3\*Sqrt[b^2 - 4\*a\*c]\*d + 8\*a\*e) + c\*d^2\*(Sqrt[b^2 - 4\*a\*c]\*d + 12\*a\*e)))\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b - Sqrt[b^2 - 4\*a\*c]]]/(2\*Sqrt[2]\*a\*c^(3/2)\*(b^2 - 4\*a\*c)^(3/2)\*Sqrt[b - Sqrt[b^2 - 4\*a\*c]]) + ((a\*b^3\*e^3 + 6\*a\*c\*(2\*c\*d - Sqrt[b^2 - 4\*a\*c])\*e)\*(c\*d^2 + a\*e^2) - b^2\*(c^2\*d^3 - 3\*a\*c\*d\*e^2 - a\*Sqrt[b^2 - 4\*a\*c]\*e^3) + b\*c\*(c\*d^2\*(Sqrt[b^2 - 4\*a\*c]\*d - 12\*a\*e) + a\*e^2\*(3\*Sqrt[b^2 - 4\*a\*c]\*d - 8\*a\*e)))\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b + Sqrt[b^2 - 4\*a\*c]]]/(2\*Sqrt[2]\*a\*c^(3/2)\*(b^2 - 4\*a\*c)^(3/2)\*Sqrt[b + Sqrt[b^2 - 4\*a\*c]])

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 1166

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[e/2 + (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 - q/2 + c\*x^2), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - a\*e^2, 0] && PosQ[b^2 - 4\*a\*c]

#### Rule 1205

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e\*x^2)^q, a + b\*x^2 + c\*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e\*x^2)^q, a + b\*x^2 + c\*x^4, x], x, 2]}, Simp[(x\*(a + b\*x^2 + c\*x^4)^(p+1)\*(a\*b\*g - f\*(b^2 - 2\*a\*c) - c\*(b\*f - 2\*a\*g)\*x^2))/(2\*a\*(p+1)\*(b^2 - 4\*a\*c)), x] + Dist[1/(2\*a\*(p+1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x^2 + c\*x^4)^(p+1)\*ExpandToSum[2\*a\*(p+1)\*(b^2 - 4\*a\*c)\*PolynomialQuotient[(d + e\*x^2)^q, a + b\*x^2 + c\*x^4, x] + b^2\*f\*(2\*p+3) - 2\*a\*c\*f\*(4\*p+5) - a\*b\*g + c\*(4\*p+7)\*(b\*f - 2\*a\*g)\*x^2, x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]

Rubi steps

$$\int \frac{(d + ex^2)^3}{(a + bx^2 + cx^4)^2} dx = \frac{x \left( c \left( b^2 d^3 - 2ad (cd^2 - 3ae^2) - \frac{abe(3cd^2 + ae^2)}{c} \right) - (ab^2 e^3 + 2ace (3cd^2 - ae^2) - bcd (cd^2 + 3ae^2)) \right)}{2ac (b^2 - 4ac) (a + bx^2 + cx^4)}$$

$$= \frac{x \left( c \left( b^2 d^3 - 2ad (cd^2 - 3ae^2) - \frac{abe(3cd^2 + ae^2)}{c} \right) - (ab^2 e^3 + 2ace (3cd^2 - ae^2) - bcd (cd^2 + 3ae^2)) \right)}{2ac (b^2 - 4ac) (a + bx^2 + cx^4)}$$

$$= \frac{x \left( c \left( b^2 d^3 - 2ad (cd^2 - 3ae^2) - \frac{abe(3cd^2 + ae^2)}{c} \right) - (ab^2 e^3 + 2ace (3cd^2 - ae^2) - bcd (cd^2 + 3ae^2)) \right)}{2ac (b^2 - 4ac) (a + bx^2 + cx^4)}$$

**Mathematica [A]** time = 1.63, size = 540, normalized size = 0.96

$$\frac{2\sqrt{c}\sqrt{a^2 - 3acd(d - e^2)} + 2\sqrt{b^2 d^3 + 2a^2 d^2 + 2ac(d^2 - 3ae^2)} + 2ac\sqrt{a^2 - 3acd(d - e^2)} - ac^2\sqrt{a^2 - 3acd(d - e^2)}}{(b^2 - 4ac)\sqrt{a + bx^2 + cx^4}} + \frac{\sqrt{c}\sqrt{a^2 - 3acd(d - e^2)} + ac\sqrt{a^2 - 3acd(d - e^2)} + ac\sqrt{a^2 - 3acd(d - e^2)}}{(b^2 - 4ac)\sqrt{a + bx^2 + cx^4}} + \frac{\sqrt{c}\sqrt{a^2 - 3acd(d - e^2)} + ac\sqrt{a^2 - 3acd(d - e^2)} + ac\sqrt{a^2 - 3acd(d - e^2)}}{(b^2 - 4ac)\sqrt{a + bx^2 + cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/(a + b*x^2 + c*x^4)^2,x]
[Out] ((2*Sqrt[c]*x*(b^2*(c*d^3 - a*e^3*x^2) + b*(-(a^2*e^3) + c^2*d^3*x^2 - 3*a*c*d*e*(d - e*x^2)) + 2*a*c*(a*e^2*(3*d + e*x^2) - c*d^2*(d + 3*e*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(a*b^3*e^3) - 6*a*c*(2*c*d + Sqrt[b^2 - 4*a*c]*e)*(c*d^2 + a*e^2) + b^2*(c^2*d^3 - 3*a*c*d*e^2 + a*Sqrt[b^2 - 4*a*c]*e^3) + b*c*(a*e^2*(3*Sqrt[b^2 - 4*a*c]*d + 8*a*e) + c*d^2*(Sqrt[b^2 - 4*a*c]*d + 12*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(a*b^3*e^3 + 6*a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e)*(c*d^2 + a*e^2) + b^2*(-(c^2*d^3) + 3*a*c*d*e^2 + a*Sqrt[b^2 - 4*a*c]*e^3) + b*c*(c*d^2*(Sqrt[b^2 - 4*a*c]*d - 12*a*e) + a*e^2*(3*Sqrt[b^2 - 4*a*c]*d - 8*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a*c^(3/2))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^2)^3}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^2)^3/(a + b*x^2 + c*x^4)^2,x]
[Out] IntegrateAlgebraic[(d + e*x^2)^3/(a + b*x^2 + c*x^4)^2, x]
```

**fricas [B]** time = 111.89, size = 12117, normalized size = 21.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
[Out] 1/4*(2*(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*x^3 - sqrt(1/2)*(a^2*b^2*c - 4*a^3*c^2 + (a*b^2*c^2 - 4*a^2*c^3)*x^4 + (a*b^3*c - 4*a^2*b*c^2)*x^2)*sqrt(-((b^5*c^3 - 15*a*b^3*c^4 + 60*a^2*b*c^5)*d^6
```

$$\begin{aligned}
& + 6*(a*b^4*c^3 - 6*a^2*b^2*c^4 - 24*a^3*c^5)*d^5*e - 3*(3*a^2*b^3*c^3 - 92 \\
& *a^3*b*c^4)*d^4*e^2 - 8*(11*a^3*b^2*c^3 + 36*a^4*c^4)*d^3*e^3 - 3*(3*a^3*b^ \\
& 3*c^2 - 92*a^4*b*c^3)*d^2*e^4 + 6*(a^3*b^4*c - 6*a^4*b^2*c^2 - 24*a^5*c^3)* \\
& d*e^5 + (a^3*b^5 - 15*a^4*b^3*c + 60*a^5*b*c^2)*e^6 + (a^3*b^6*c^3 - 12*a^4 \\
& *b^4*c^4 + 48*a^5*b^2*c^5 - 64*a^6*c^6)*sqrt(-(108*a^3*b*c^6*d^9*e^3 + 108* \\
& a^6*b*c^3*d^3*e^9 - (b^4*c^6 - 18*a*b^2*c^7 + 81*a^2*c^8)*d^12 - 12*(a*b^3* \\
& c^6 - 9*a^2*b*c^7)*d^11*e - 18*(a^2*b^2*c^6 + 9*a^3*c^7)*d^10*e^2 - 9*(2*a^ \\
& 3*b^2*c^5 - 9*a^4*c^6)*d^8*e^4 + 12*(a^3*b^3*c^4 - 18*a^4*b*c^5)*d^7*e^5 + \\
& 2*(a^3*b^4*c^3 + 18*a^4*b^2*c^4 + 162*a^5*c^5)*d^6*e^6 + 12*(a^4*b^3*c^3 - \\
& 18*a^5*b*c^4)*d^5*e^7 - 9*(2*a^5*b^2*c^3 - 9*a^6*c^4)*d^4*e^8 - 18*(a^6*b^2 \\
& *c^2 + 9*a^7*c^3)*d^2*e^10 - 12*(a^6*b^3*c - 9*a^7*b*c^2)*d*e^11 - (a^6*b^4 \\
& - 18*a^7*b^2*c + 81*a^8*c^2)*e^12)/(a^6*b^6*c^6 - 12*a^7*b^4*c^7 + 48*a^8* \\
& b^2*c^8 - 64*a^9*c^9))/((a^3*b^6*c^3 - 12*a^4*b^4*c^4 + 48*a^5*b^2*c^5 - 64 \\
& *a^6*c^6))*log(-((5*b^4*c^6 - 81*a*b^2*c^7 + 324*a^2*c^8)*d^12 - 3*(3*b^5*c \\
& ^5 - 65*a*b^3*c^6 + 324*a^2*b*c^7)*d^11*e + 3*(b^6*c^4 - 42*a*b^4*c^5 + 252 \\
& *a^2*b^2*c^6 + 432*a^3*c^7)*d^10*e^2 + (b^7*c^3 + 3*a*b^5*c^4 + 33*a^2*b^3* \\
& c^5 - 2916*a^3*b*c^6)*d^9*e^3 + 9*(a*b^6*c^3 - 15*a^2*b^4*c^4 + 195*a^3*b^2 \\
& *c^5 + 180*a^4*c^6)*d^8*e^4 - 162*(a^3*b^3*c^4 + 12*a^4*b*c^5)*d^7*e^5 + 16 \\
& 2*(a^4*b^3*c^3 + 12*a^5*b*c^4)*d^5*e^7 - 9*(a^3*b^6*c - 15*a^4*b^4*c^2 + 19 \\
& 5*a^5*b^2*c^3 + 180*a^6*c^4)*d^4*e^8 - (a^3*b^7 + 3*a^4*b^5*c + 33*a^5*b^3* \\
& c^2 - 2916*a^6*b*c^3)*d^3*e^9 - 3*(a^4*b^6 - 42*a^5*b^4*c + 252*a^6*b^2*c^2 \\
& + 432*a^7*c^3)*d^2*e^10 + 3*(3*a^5*b^5 - 65*a^6*b^3*c + 324*a^7*b*c^2)*d*e \\
& ^11 - (5*a^6*b^4 - 81*a^7*b^2*c + 324*a^8*c^2)*e^12)*x + 1/2*sqrt(1/2)*((b^ \\
& 8*c^4 - 23*a*b^6*c^5 + 190*a^2*b^4*c^6 - 672*a^3*b^2*c^7 + 864*a^4*c^8)*d^9 \\
& + 9*(a*b^7*c^4 - 15*a^2*b^5*c^5 + 72*a^3*b^3*c^6 - 112*a^4*b*c^7)*d^8*e + \\
& 3*(a^2*b^6*c^4 + 28*a^3*b^4*c^5 - 272*a^4*b^2*c^6 + 576*a^5*c^7)*d^7*e^2 + \\
& (a^2*b^7*c^3 - 80*a^3*b^5*c^4 + 592*a^4*b^3*c^5 - 1152*a^5*b*c^6)*d^6*e^3 + \\
& 15*(a^3*b^6*c^3 - 8*a^4*b^4*c^4 + 16*a^5*b^2*c^5)*d^5*e^4 - 6*(a^3*b^7*c^2 \\
& - 17*a^4*b^5*c^3 + 88*a^5*b^3*c^4 - 144*a^6*b*c^5)*d^4*e^5 - (a^3*b^8*c - \\
& 5*a^4*b^6*c^2 + 100*a^5*b^4*c^3 - 816*a^6*b^2*c^4 + 1728*a^7*c^5)*d^3*e^6 - \\
& 3*(a^4*b^7*c - 32*a^5*b^5*c^2 + 208*a^6*b^3*c^3 - 384*a^7*b*c^4)*d^2*e^7 - \\
& 54*(a^6*b^4*c^2 - 8*a^7*b^2*c^3 + 16*a^8*c^4)*d*e^8 - (a^5*b^7 - 17*a^6*b^ \\
& 5*c + 88*a^7*b^3*c^2 - 144*a^8*b*c^3)*e^9 - ((a^3*b^9*c^4 - 20*a^4*b^7*c^5 \\
& + 144*a^5*b^5*c^6 - 448*a^6*b^3*c^7 + 512*a^7*b*c^8)*d^3 + 3*(a^4*b^8*c^4 - \\
& 8*a^5*b^6*c^5 + 128*a^7*b^2*c^7 - 256*a^8*c^8)*d^2*e - 12*(a^5*b^7*c^4 - 1 \\
& 2*a^6*b^5*c^5 + 48*a^7*b^3*c^6 - 64*a^8*b*c^7)*d*e^2 - (a^5*b^8*c^3 - 24*a^ \\
& 6*b^6*c^4 + 192*a^7*b^4*c^5 - 640*a^8*b^2*c^6 + 768*a^9*c^7)*e^3)*sqrt(-(10 \\
& 8*a^3*b*c^6*d^9*e^3 + 108*a^6*b*c^3*d^3*e^9 - (b^4*c^6 - 18*a*b^2*c^7 + 81* \\
& a^2*c^8)*d^12 - 12*(a*b^3*c^6 - 9*a^2*b*c^7)*d^11*e - 18*(a^2*b^2*c^6 + 9*a \\
& ^3*c^7)*d^10*e^2 - 9*(2*a^3*b^2*c^5 - 9*a^4*c^6)*d^8*e^4 + 12*(a^3*b^3*c^4 - \\
& 18*a^4*b*c^5)*d^7*e^5 + 2*(a^3*b^4*c^3 + 18*a^4*b^2*c^4 + 162*a^5*c^5)*d^ \\
& 6*e^6 + 12*(a^4*b^3*c^3 - 18*a^5*b*c^4)*d^5*e^7 - 9*(2*a^5*b^2*c^3 - 9*a^6* \\
& c^4)*d^4*e^8 - 18*(a^6*b^2*c^2 + 9*a^7*c^3)*d^2*e^10 - 12*(a^6*b^3*c - 9*a^ \\
& 7*b*c^2)*d*e^11 - (a^6*b^4 - 18*a^7*b^2*c + 81*a^8*c^2)*e^12)/(a^6*b^6*c^6 - \\
& 12*a^7*b^4*c^7 + 48*a^8*b^2*c^8 - 64*a^9*c^9))/((a^3*b^6*c^3 - 12*a^4*b^4* \\
& c^4 + 48*a^5*b^2*c^5 - 64*a^6*c^6))) + sqrt(1/2)*(a^2*b^2*c - 4*a^3*c^2 + (
\end{aligned}$$

$$\begin{aligned}
& a^2b^2c^2 - 4a^2c^3)x^4 + (ab^3c - 4a^2b^2c^2)x^2) \sqrt{-((b^5c^3 - 15ab^3c^4 + 60a^2b^2c^5)d^6 + 6(a^4b^3c^3 - 6a^2b^2c^4 - 24a^3c^5)d^5e - 3(3a^2b^3c^3 - 92a^3b^2c^4)d^4e^2 - 8(11a^3b^2c^3 + 36a^4c^4)d^3e^3 - 3(3a^3b^3c^2 - 92a^4b^2c^3)d^2e^4 + 6(a^3b^4c - 6a^4b^2c^2 - 24a^5c^3)d^1e^5 + (a^3b^5 - 15a^4b^3c + 60a^5b^2c^2)e^6 + (a^3b^6c^3 - 12a^4b^4c^4 + 48a^5b^2c^5 - 64a^6c^6) \sqrt{-((108a^3b^2c^6d^9e^3 + 108a^6b^2c^3d^3e^9 - (b^4c^6 - 18ab^2c^7 + 81a^2c^8)d^12 - 12(ab^3c^6 - 9a^2b^2c^7)d^11e - 18(a^2b^2c^6 + 9a^3c^7)d^10e^2 - 9(2a^3b^2c^5 - 9a^4c^6)d^8e^4 + 12(a^3b^3c^4 - 18a^4b^2c^5)d^7e^5 + 2(a^3b^4c^3 + 18a^4b^2c^4 + 162a^5c^5)d^6e^6 + 12(a^4b^3c^3 - 18a^5b^2c^4)d^5e^7 - 9(2a^5b^2c^3 - 9a^6c^4)d^4e^8 - 18(a^6b^2c^2 + 9a^7c^3)d^2e^10 - 12(a^6b^3c - 9a^7b^2c^2)d^1e^11 - (a^6b^4 - 18a^7b^2c + 81a^8c^2)e^12)/(a^6b^6c^6 - 12a^7b^4c^7 + 48a^8b^2c^8 - 64a^9c^9)))/(a^3b^6c^3 - 12a^4b^4c^4 + 48a^5b^2c^5 - 64a^6c^6)) \log(-((5b^4c^6 - 81ab^2c^7 + 324a^2c^8)d^12 - 3(3b^5c^5 - 65ab^3c^6 + 324a^2b^2c^7)d^11e + 3(b^6c^4 - 42ab^4c^5 + 252a^2b^2c^6 + 432a^3c^7)d^10e^2 + (b^7c^3 + 3ab^5c^4 + 33a^2b^3c^5 - 2916a^3b^2c^6)d^9e^3 + 9(ab^6c^3 - 15a^2b^4c^4 + 195a^3b^2c^5 + 180a^4c^6)d^8e^4 - 162(a^3b^3c^4 + 12a^4b^2c^5)d^7e^5 + 162(a^4b^3c^3 + 12a^5b^2c^4)d^5e^7 - 9(a^3b^6c - 15a^4b^4c^2 + 195a^5b^2c^3 + 180a^6c^4)d^4e^8 - (a^3b^7 + 3a^4b^5c + 33a^5b^3c^2 - 2916a^6b^2c^3)d^3e^9 - 3(a^4b^6 - 42a^5b^4c + 252a^6b^2c^2 + 432a^7c^3)d^2e^10 + 3(3a^5b^5 - 65a^6b^3c + 324a^7b^2c^2)d^1e^11 - (5a^6b^4 - 81a^7b^2c + 324a^8c^2)e^12)x - 1/2 \sqrt{1/2}((b^8c^4 - 23ab^6c^5 + 190a^2b^4c^6 - 672a^3b^2c^7 + 864a^4c^8)d^9 + 9(ab^7c^4 - 15a^2b^5c^5 + 72a^3b^3c^6 - 112a^4b^2c^7)d^8e + 3(a^2b^6c^4 + 28a^3b^4c^5 - 272a^4b^2c^6 + 576a^5c^7)d^7e^2 + (a^2b^7c^3 - 80a^3b^5c^4 + 592a^4b^3c^5 - 1152a^5b^2c^6)d^6e^3 + 15(a^3b^6c^3 - 8a^4b^4c^4 + 16a^5b^2c^5)d^5e^4 - 6(a^3b^7c^2 - 17a^4b^5c^3 + 88a^5b^3c^4 - 144a^6b^2c^5)d^4e^5 - (a^3b^8c - 5a^4b^6c^2 + 100a^5b^4c^3 - 816a^6b^2c^4 + 1728a^7c^5)d^3e^6 - 3(a^4b^7c - 32a^5b^5c^2 + 208a^6b^3c^3 - 384a^7b^2c^4)d^2e^7 - 54(a^6b^4c^2 - 8a^7b^2c^3 + 16a^8c^4)d^1e^8 - (a^5b^7 - 17a^6b^5c + 88a^7b^3c^2 - 144a^8b^2c^3)e^9 - ((a^3b^9c^4 - 20a^4b^7c^5 + 144a^5b^5c^6 - 448a^6b^3c^7 + 512a^7b^2c^8)d^3 + 3(a^4b^8c^4 - 8a^5b^6c^5 + 128a^7b^2c^7 - 256a^8b^2c^8)d^2e - 12(a^5b^7c^4 - 12a^6b^5c^5 + 48a^7b^3c^6 - 64a^8b^2c^7)d^1e^2 - (a^5b^8c^3 - 24a^6b^6c^4 + 192a^7b^4c^5 - 640a^8b^2c^6 + 768a^9c^7)e^3) \sqrt{-((108a^3b^2c^6d^9e^3 + 108a^6b^2c^3d^3e^9 - (b^4c^6 - 18ab^2c^7 + 81a^2c^8)d^12 - 12(ab^3c^6 - 9a^2b^2c^7)d^11e - 18(a^2b^2c^6 + 9a^3c^7)d^10e^2 - 9(2a^3b^2c^5 - 9a^4c^6)d^8e^4 + 12(a^3b^3c^4 - 18a^4b^2c^5)d^7e^5 + 2(a^3b^4c^3 + 18a^4b^2c^4 + 162a^5c^5)d^6e^6 + 12(a^4b^3c^3 - 18a^5b^2c^4)d^5e^7 - 9(2a^5b^2c^3 - 9a^6c^4)d^4e^8 - 18(a^6b^2c^2 + 9a^7c^3)d^2e^10 - 12(a^6b^3c - 9a^7b^2c^2)d^1e^11 - (a^6b^4 - 18a^7b^2c + 81a^8c^2)e^12)/(a^6b^6c^6 - 12a^7b^4c^7 + 48a^8b^2c^8 - 64a^9c^9))} \sqrt{-((b^5c^3 - 15ab^3c^4 + 60a^2b^2c^5)d^6 + 6(a^4b^3c^3 - 6a^2b^2c^4 - 24a^3c^5)d^5e - 3(3a^2b^3c^3 - 92a^3b^2c^4)d^4e^2 - 8(11a^3b^2c^3 + 36a^4c^4)d^3e^3 - 3(3a^3b^3c^2 - 92a^4b^2c^3)d^2e^4 + 6(a^3b^4c - 6a^4b^2c^2 - 24a^5c^3)d^1e^5 + (a^3b^5 - 15a^4b^3c + 60a^5b^2c^2)e^6 + (a^3b^6c^3 - 12a^4b^4c^4 + 48a^5b^2c^5 - 64a^6c^6) \sqrt{-((108a^3b^2c^6d^9e^3 + 108a^6b^2c^3d^3e^9 - (b^4c^6 - 18ab^2c^7 + 81a^2c^8)d^12 - 12(ab^3c^6 - 9a^2b^2c^7)d^11e - 18(a^2b^2c^6 + 9a^3c^7)d^10e^2 - 9(2a^3b^2c^5 - 9a^4c^6)d^8e^4 + 12(a^3b^3c^4 - 18a^4b^2c^5)d^7e^5 + 2(a^3b^4c^3 + 18a^4b^2c^4 + 162a^5c^5)d^6e^6 + 12(a^4b^3c^3 - 18a^5b^2c^4)d^5e^7 - 9(2a^5b^2c^3 - 9a^6c^4)d^4e^8 - 18(a^6b^2c^2 + 9a^7c^3)d^2e^10 - 12(a^6b^3c - 9a^7b^2c^2)d^1e^11 - (a^6b^4 - 18a^7b^2c + 81a^8c^2)e^12)/(a^6b^6c^6 - 12a^7b^4c^7 + 48a^8b^2c^8 - 64a^9c^9)}
\end{aligned}$$

$$\begin{aligned}
& c^9)) / (a^3 b^6 c^3 - 12 a^4 b^4 c^4 + 48 a^5 b^2 c^5 - 64 a^6 c^6)) - \sqrt[4]{(1/2) * (a^2 b^2 c - 4 a^3 c^2 + (a b^2 c^2 - 4 a^2 c^3) x^4 + (a b^3 c - 4 a^2 b c^2) x^2) * \sqrt{-(b^5 c^3 - 15 a b^3 c^4 + 60 a^2 b c^5) d^6 + 6 (a b^4 c^3 - 6 a^2 b^2 c^4 - 24 a^3 c^5) d^5 e - 3 (3 a^2 b^3 c^3 - 92 a^3 b c^4) d^4 e^2 - 8 (11 a^3 b^2 c^3 + 36 a^4 c^4) d^3 e^3 - 3 (3 a^3 b^3 c^2 - 92 a^4 b c^3) d^2 e^4 + 6 (a^3 b^4 c - 6 a^4 b^2 c^2 - 24 a^5 c^3) d e^5 + (a^3 b^5 - 15 a^4 b^3 c + 60 a^5 b c^2) e^6 - (a^3 b^6 c^3 - 12 a^4 b^4 c^4 + 48 a^5 b^2 c^5 - 64 a^6 c^6) * \sqrt{-(108 a^3 b c^6 d^9 e^3 + 108 a^6 b c^3 d^3 e^9 - (b^4 c^6 - 18 a b^2 c^7 + 81 a^2 c^8) d^{12} - 12 (a b^3 c^6 - 9 a^2 b c^7) d^{11} e - 18 (a^2 b^2 c^6 + 9 a^3 c^7) d^{10} e^2 - 9 (2 a^3 b^2 c^5 - 9 a^4 c^6) d^8 e^4 + 12 (a^3 b^3 c^4 - 18 a^4 b c^5) d^7 e^5 + 2 (a^3 b^4 c^3 + 18 a^4 b^2 c^4 + 162 a^5 c^5) d^6 e^6 + 12 (a^4 b^3 c^3 - 18 a^5 b c^4) d^5 e^7 - 9 (2 a^5 b^2 c^3 - 9 a^6 c^4) d^4 e^8 - 18 (a^6 b^2 c^2 + 9 a^7 c^3) d^2 e^{10} - 12 (a^6 b^3 c - 9 a^7 b c^2) d e^{11} - (a^6 b^4 - 18 a^7 b^2 c + 81 a^8 c^2) e^{12}} / (a^6 b^6 c^6 - 12 a^7 b^4 c^7 + 48 a^8 b^2 c^8 - 64 a^9 c^9))} / (a^3 b^6 c^3 - 12 a^4 b^4 c^4 + 48 a^5 b^2 c^5 - 64 a^6 c^6) * \log(-((5 b^4 c^6 - 81 a b^2 c^7 + 324 a^2 c^8) d^{12} - 3 (3 b^5 c^5 - 65 a b^3 c^6 + 324 a^2 b c^7) d^{11} e + 3 (b^6 c^4 - 42 a b^4 c^5 + 252 a^2 b^2 c^6 + 432 a^3 c^7) d^{10} e^2 + (b^7 c^3 + 3 a b^5 c^4 + 33 a^2 b^3 c^5 - 291 6 a^3 b c^6) d^9 e^3 + 9 (a b^6 c^3 - 15 a^2 b^4 c^4 + 195 a^3 b^2 c^5 + 180 a^4 c^6) d^8 e^4 - 162 (a^3 b^3 c^4 + 12 a^4 b c^5) d^7 e^5 + 162 (a^4 b^3 c^3 + 12 a^5 b c^4) d^5 e^7 - 9 (a^3 b^6 c - 15 a^4 b^4 c^2 + 195 a^5 b^2 c^3 + 180 a^6 c^4) d^4 e^8 - (a^3 b^7 + 3 a^4 b^5 c + 33 a^5 b^3 c^2 - 291 6 a^6 b c^3) d^3 e^9 - 3 (a^4 b^6 - 42 a^5 b^4 c + 252 a^6 b^2 c^2 + 432 a^7 c^3) d^2 e^{10} + 3 (3 a^5 b^5 - 65 a^6 b^3 c + 324 a^7 b c^2) d e^{11} - (5 a^6 b^4 - 81 a^7 b^2 c + 324 a^8 c^2) e^{12}) * x + 1/2 * \sqrt[4]{(1/2) * ((b^8 c^4 - 2 3 a b^6 c^5 + 190 a^2 b^4 c^6 - 672 a^3 b^2 c^7 + 864 a^4 c^8) d^9 + 9 (a b^7 c^4 - 15 a^2 b^5 c^5 + 72 a^3 b^3 c^6 - 112 a^4 b c^7) d^8 e + 3 (a^2 b^6 c^4 + 28 a^3 b^4 c^5 - 272 a^4 b^2 c^6 + 576 a^5 c^7) d^7 e^2 + (a^2 b^7 c^3 - 80 a^3 b^5 c^4 + 592 a^4 b^3 c^5 - 1152 a^5 b c^6) d^6 e^3 + 15 (a^3 b^6 c^3 - 8 a^4 b^4 c^4 + 16 a^5 b^2 c^5) d^5 e^4 - 6 (a^3 b^7 c^2 - 17 a^4 b^5 c^3 + 88 a^5 b^3 c^4 - 144 a^6 b c^5) d^4 e^5 - (a^3 b^8 c - 5 a^4 b^6 c^2 + 100 a^5 b^4 c^3 - 816 a^6 b^2 c^4 + 1728 a^7 c^5) d^3 e^6 - 3 (a^4 b^7 c - 32 a^5 b^5 c^2 + 208 a^6 b^3 c^3 - 384 a^7 b c^4) d^2 e^7 - 54 (a^6 b^4 c^2 - 8 a^7 b^2 c^3 + 16 a^8 c^4) d e^8 - (a^5 b^7 - 17 a^6 b^5 c + 88 a^7 b^3 c^2 - 144 a^8 b c^3) e^9 + ((a^3 b^9 c^4 - 20 a^4 b^7 c^5 + 144 a^5 b^5 c^6 - 448 a^6 b^3 c^7 + 512 a^7 b c^8) d^3 + 3 (a^4 b^8 c^4 - 8 a^5 b^6 c^5 + 128 a^7 b^2 c^7 - 256 a^8 c^8) d^2 e - 12 (a^5 b^7 c^4 - 12 a^6 b^5 c^5 + 48 a^7 b^3 c^6 - 64 a^8 b c^7) d e^2 - (a^5 b^8 c^3 - 24 a^6 b^6 c^4 + 192 a^7 b^4 c^5 - 640 a^8 b^2 c^6 + 768 a^9 c^7) e^3) * \sqrt{-(108 a^3 b c^6 d^9 e^3 + 108 a^6 b c^3 d^3 e^9 - (b^4 c^6 - 18 a b^2 c^7 + 81 a^2 c^8) d^{12} - 12 (a b^3 c^6 - 9 a^2 b c^7) d^{11} e - 18 (a^2 b^2 c^6 + 9 a^3 c^7) d^{10} e^2 - 9 (2 a^3 b^2 c^5 - 9 a^4 c^6) d^8 e^4 + 12 (a^3 b^3 c^4 - 18 a^4 b c^5) d^7 e^5 + 2 (a^3 b^4 c^3 + 18 a^4 b^2 c^4 + 162 a^5 c^5) d^6 e^6 + 12 (a^4 b^3 c^3 - 18 a^5 b c^4) d^5 e^7 - 9 (2 a^5 b^2 c^3 - 9 a^6 c^4) d^4 e^8 - 18 (a^6 b^2 c^2 + 9 a^7 c^3) d^2 e^{10} - 12 (a^6 b^3 c - 9 a^7 b c^2) d e^{11} - (a^6 b^4 - 18 a^7 b^2 c + 81 a^8 c^2) e^{12}} / (a^6 b^6 c^6 - 12 a^7 b^4 c^7 + 48 a^8 b^2 c^8 - 64 a^9 c^9))} * \sqrt{-(b^5 c^3 - 15 a b^3 c^4 + 60 a^2 b c^5) d^6 + 6 (a b^4 c^3 - 6 a^2 b^2 c^4 - 24 a^3 c^5) d^5 e - 3 (3 a^2 b^3 c^3 - 92 a^3 b c^4) d^4 e^2 - 8 (11 a^3 b^2 c^3 + 36 a^4 c^4) d^3 e^3 - 3 (3 a^3 b^3 c^2 - 92 a^4 b c^3) d^2 e^4 + 6 (a^3 b^4 c - 6 a^4 b^2 c^2 - 24 a^5 c^3) d e^5 + (a^3 b^5 - 15 a^4 b^3 c + 60 a^5 b c^2) e^6 - (a^3 b^6 c^3 - 12 a^4 b^4 c^4 + 48 a^5 b^2 c^5 - 64 a^6 c^6) * \sqrt{-(108 a^3 b c^6 d^9 e^3 + 108 a^6 b c^3 d^3 e^9 - (b^4 c^6 - 18 a b^2 c^7 + 81 a^2 c^8) d^{12} - 12 (a b^3 c^6 - 9 a^2 b c^7) d^{11} e - 18 (a^2 b^2 c^6 + 9 a^3 c^7) d^{10} e^2 - 9 (2 a^3 b^2 c^5 - 9 a^4 c^6) d^8 e^4 + 12 (a^3 b^3 c^4 - 18 a^4 b c^5) d^7 e^5 + 2 (a^3 b^4 c^3 + 18 a^4 b^2 c^4 + 162 a^5 c^5) d^6 e^6 + 12 (a^4 b^3 c^3 - 18 a^5 b c^4) d^5 e^7 - 9 (2 a^5 b^2 c^3 - 9 a^6 c^4) d^4 e^8 - 18 (a^6 b^2 c^2 + 9 a^7 c^3) d^2 e^{10} - 12 (a^6 b^3 c - 9 a^7 b c^2) d e^{11} - (a^6 b^4 - 18 a^7 b^2 c + 81 a^8 c^2) e^{12}} / (a^6 b^6 c^6 - 12 a^7 b^4 c^7 + 48 a^8 b^2 c^8 - 64 a^9 c^9))}
\end{aligned}$$

$$\begin{aligned}
& *e^{11} - (a^6*b^4 - 18*a^7*b^2*c + 81*a^8*c^2)*e^{12})/(a^6*b^6*c^6 - 12*a^7*b^4*c^7 + 48*a^8*b^2*c^8 - 64*a^9*c^9)))/(a^3*b^6*c^3 - 12*a^4*b^4*c^4 + 48*a^5*b^2*c^5 - 64*a^6*c^6)) + \sqrt{1/2}*(a^2*b^2*c - 4*a^3*c^2 + (a*b^2*c^2 - 4*a^2*c^3)*x^4 + (a*b^3*c - 4*a^2*b*c^2)*x^2)*\sqrt{-((b^5*c^3 - 15*a*b^3*c^4 + 60*a^2*b*c^5)*d^6 + 6*(a*b^4*c^3 - 6*a^2*b^2*c^4 - 24*a^3*c^5)*d^5*e - 3*(3*a^2*b^3*c^3 - 92*a^3*b*c^4)*d^4*e^2 - 8*(11*a^3*b^2*c^3 + 36*a^4*c^4)*d^3*e^3 - 3*(3*a^3*b^3*c^2 - 92*a^4*b*c^3)*d^2*e^4 + 6*(a^3*b^4*c - 6*a^4*b^2*c^2 - 24*a^5*c^3)*d*e^5 + (a^3*b^5 - 15*a^4*b^3*c + 60*a^5*b*c^2)*e^6 - (a^3*b^6*c^3 - 12*a^4*b^4*c^4 + 48*a^5*b^2*c^5 - 64*a^6*c^6)*\sqrt{-(108*a^3*b*c^6*d^9*e^3 + 108*a^6*b*c^3*d^3*e^9 - (b^4*c^6 - 18*a*b^2*c^7 + 81*a^2*c^8)*d^12 - 12*(a*b^3*c^6 - 9*a^2*b*c^7)*d^11*e - 18*(a^2*b^2*c^6 + 9*a^3*c^7)*d^10*e^2 - 9*(2*a^3*b^2*c^5 - 9*a^4*c^6)*d^8*e^4 + 12*(a^3*b^3*c^4 - 18*a^4*b*c^5)*d^7*e^5 + 2*(a^3*b^4*c^3 + 18*a^4*b^2*c^4 + 162*a^5*c^5)*d^6*e^6 + 12*(a^4*b^3*c^3 - 18*a^5*b*c^4)*d^5*e^7 - 9*(2*a^5*b^2*c^3 - 9*a^6*c^4)*d^4*e^8 - 18*(a^6*b^2*c^2 + 9*a^7*c^3)*d^2*e^10 - 12*(a^6*b^3*c - 9*a^7*b*c^2)*d*e^11 - (a^6*b^4 - 18*a^7*b^2*c + 81*a^8*c^2)*e^{12})/(a^6*b^6*c^6 - 12*a^7*b^4*c^7 + 48*a^8*b^2*c^8 - 64*a^9*c^9)))/(a^3*b^6*c^3 - 12*a^4*b^4*c^4 + 48*a^5*b^2*c^5 - 64*a^6*c^6))*\log(-((5*b^4*c^6 - 81*a*b^2*c^7 + 324*a^2*c^8)*d^12 - 3*(3*b^5*c^5 - 65*a*b^3*c^6 + 324*a^2*b*c^7)*d^11*e + 3*(b^6*c^4 - 42*a*b^4*c^5 + 252*a^2*b^2*c^6 + 432*a^3*c^7)*d^10*e^2 + (b^7*c^3 + 3*a*b^5*c^4 + 33*a^2*b^3*c^5 - 2916*a^3*b*c^6)*d^9*e^3 + 9*(a*b^6*c^3 - 15*a^2*b^4*c^4 + 195*a^3*b^2*c^5 + 180*a^4*c^6)*d^8*e^4 - 162*(a^3*b^3*c^4 + 12*a^4*b*c^5)*d^7*e^5 + 162*(a^4*b^3*c^3 + 12*a^5*b*c^4)*d^5*e^7 - 9*(a^3*b^6*c - 15*a^4*b^4*c^2 + 195*a^5*b^2*c^3 + 180*a^6*c^4)*d^4*e^8 - (a^3*b^7 + 3*a^4*b^5*c + 33*a^5*b^3*c^2 - 2916*a^6*b*c^3)*d^3*e^9 - 3*(a^4*b^6 - 42*a^5*b^4*c + 252*a^6*b^2*c^2 + 432*a^7*c^3)*d^2*e^10 + 3*(3*a^5*b^5 - 65*a^6*b^3*c + 324*a^7*b*c^2)*d*e^11 - (5*a^6*b^4 - 81*a^7*b^2*c + 324*a^8*c^2)*e^{12})*x - 1/2*\sqrt{1/2}*((b^8*c^4 - 23*a*b^6*c^5 + 190*a^2*b^4*c^6 - 672*a^3*b^2*c^7 + 864*a^4*c^8)*d^9 + 9*(a*b^7*c^4 - 15*a^2*b^5*c^5 + 72*a^3*b^3*c^6 - 112*a^4*b*c^7)*d^8*e + 3*(a^2*b^6*c^4 + 28*a^3*b^4*c^5 - 272*a^4*b^2*c^6 + 576*a^5*c^7)*d^7*e^2 + (a^2*b^7*c^3 - 80*a^3*b^5*c^4 + 592*a^4*b^3*c^5 - 1152*a^5*b*c^6)*d^6*e^3 + 15*(a^3*b^6*c^3 - 8*a^4*b^4*c^4 + 16*a^5*b^2*c^5)*d^5*e^4 - 6*(a^3*b^7*c^2 - 17*a^4*b^5*c^3 + 88*a^5*b^3*c^4 - 144*a^6*b*c^5)*d^4*e^5 - (a^3*b^8*c - 5*a^4*b^6*c^2 + 100*a^5*b^4*c^3 - 816*a^6*b^2*c^4 + 1728*a^7*c^5)*d^3*e^6 - 3*(a^4*b^7*c - 32*a^5*b^5*c^2 + 208*a^6*b^3*c^3 - 384*a^7*b*c^4)*d^2*e^7 - 54*(a^6*b^4*c^2 - 8*a^7*b^2*c^3 + 16*a^8*c^4)*d*e^8 - (a^5*b^7 - 17*a^6*b^5*c + 88*a^7*b^3*c^2 - 144*a^8*b*c^3)*e^9 + ((a^3*b^9*c^4 - 20*a^4*b^7*c^5 + 144*a^5*b^5*c^6 - 448*a^6*b^3*c^7 + 512*a^7*b*c^8)*d^3 + 3*(a^4*b^8*c^4 - 8*a^5*b^6*c^5 + 128*a^7*b^2*c^7 - 256*a^8*c^8)*d^2*e - 12*(a^5*b^7*c^4 - 12*a^6*b^5*c^5 + 48*a^7*b^3*c^6 - 64*a^8*b*c^7)*d*e^2 - (a^5*b^8*c^3 - 24*a^6*b^6*c^4 + 192*a^7*b^4*c^5 - 640*a^8*b^2*c^6 + 768*a^9*c^7)*e^3)*\sqrt{-(108*a^3*b*c^6*d^9*e^3 + 108*a^6*b*c^3*d^3*e^9 - (b^4*c^6 - 18*a*b^2*c^7 + 81*a^2*c^8)*d^12 - 12*(a*b^3*c^6 - 9*a^2*b*c^7)*d^11*e - 18*(a^2*b^2*c^6 + 9*a^3*c^7)*d^10*e^2 - 9*(2*a^3*b^2*c^5 - 9*a^4*c^6)*d^8*e^4 + 12*(a^3*b^3*c^4 - 18*a^4*b*c^5)*d^7*e^5 + 2*(a^3*b^4*c^3 + 18*a^4*b^2*c^4 + 162*a^5*c^5)*d^6*e^6 + 12*(a^4*b^3*c^3 - 18*a^5*b*c^4)*d^5*e^7 - 9*(2*a^5*b^2*c^3 - 9*a^6*c^4)*d^4*e^8 - 18*(a^6*b^2*c^2 + 9*a^7*c^3)*d^2*e^10 - 12*(a^6*b^3*c - 9*a^7*b*c^2)*d*e^11 - (a^6*b^4 - 18*a^7*b^2*c + 81*a^8*c^2)*e^{12})/(a^6*b^6*c^6 - 12*a^7*b^4*c^7 + 48*a^8*b^2*c^8 - 64*a^9*c^9))*\sqrt{-(b^5*c^3 - 15*a*b^3*c^4 + 60*a^2*b*c^5)*d^6 + 6*(a*b^4*c^3 - 6*a^2*b^2*c^4 - 24*a^3*c^5)*d^5*e - 3*(3*a^2*b^3*c^3 - 92*a^3*b*c^4)*d^4*e^2 - 8*(11*a^3*b^2*c^3 + 36*a^4*c^4)*d^3*e^3 - 3*(3*a^3*b^3*c^2 - 92*a^4*b*c^3)*d^2*e^4 + 6*(a^3*b^4*c - 6*a^4*b^2*c^2 - 24*a^5*c^3)*d*e^5 + (a^3*b^5 - 15*a^4*b^3*c + 60*a^5*b*c^2)*e^6 - (a^3*b^6*c^3 - 12*a^4*b^4*c^4 + 48*a^5*b^2*c^5 - 64*a^6*c^6)*\sqrt{-(108*a^3*b*c^6*d^9*e^3 + 108*a^6*b*c^3*d^3*e^9 - (b^4*c^6 - 18*a*b^2*c^7 + 81*a^2*c^8)*d^12 - 12*(a*b^3*c^6 - 9*a^2*b*c^7)*d^11*e - 18*(a^2*b^2*c^6 + 9*a^3*c^7)*d^10*e^2 - 9*(2*a^3*b^2*c^5 - 9*a^4*c^6)*d^8*e^4 + 12*(a^3*b^3*c^4 - 18*a^4*b*c^5)*d^7*e^5 + 2*(a^3*b^4*c^3 + 18*a^4*b^2*c^4 + 162*a^5*c^5)*d^6*e^6 + 12*(a^4*b^3*c^3 - 18*a^5*b*c^4)*d^5*e^7 - 9*
\end{aligned}$$







$$\begin{aligned}
& \text{rt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a*b*c^3 + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*b^2*c^3 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*(a*b^2*c - 4*a^2*c^2)^2*d^3 - 6*(2*a*b^2*c^4 - 8*a^2*c^5 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a*b^2*c^2 + 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^2*c^3 + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a*b*c^3 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a*c^4 - 2*(b^2 - 4*a*c)*a*c^4)*(a*b^2*c - 4*a^2*c^2)^2*d^2*e - 2*(\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a*b^6*c^3 - 14*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^4*c^4 - 2*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a*b^5*c^4 + 2*a*b^6*c^4 + 64*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b^2*c^5 + 20*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^3*c^5 + \text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a*b^4*c^5 - 28*a^2*b^4*c^5 - 96*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^4*c^6 - 48*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b*c^6 - 10*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^2*c^6 + 128*a^3*b^2*c^6 + 24*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^3*c^7 - 192*a^4*c^7 - 2*(b^2 - 4*a*c)*a*b^4*c^4 + 20*(b^2 - 4*a*c)*a^2*b^2*c^5 - 48*(b^2 - 4*a*c)*a^3*c^6)*d^3*\text{abs}(a*b^2*c - 4*a^2*c^2) + 3*(2*a*b^3*c^3 - 8*a^2*b*c^4 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a*b^3*c + 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b*c^2 + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a*b^2*c^2 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*(a*b^2*c - 4*a^2*c^2)^2*d*e^2 - 6*(\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^5*c^3 - 8*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b^3*c^4 - 2*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^4*c^4 + 2*a^2*b^5*c^4 + 16*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^4*b*c^5 + 8*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b^2*c^5 + \text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^3*c^5 - 16*a^3*b^3*c^5 - 4*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b*c^6 + 32*a^4*b*c^6 - 2*(b^2 - 4*a*c)*a^2*b^3*c^4 + 8*(b^2 - 4*a*c)*a^3*b*c^5)*d^2*\text{abs}(a*b^2*c - 4*a^2*c^2)*e + (2*a^2*b^7*c^6 - 40*a^3*b^5*c^7 + 224*a^4*b^3*c^8 - 384*a^5*b*c^9 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^7*c^4 + 20*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b^5*c^5 + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^6*c^5 - 112*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^4*b^3*c^6 - 32*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b^4*c^6 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^5*c^6 + 192*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^5*b*c^7 + 96*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^4*b^2*c^7 + 16*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b^3*c^7 - 48*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^4*b*c^8 - 2*(b^2 - 4*a*c)*a^2*b^5*c^6 + 32*(b^2 - 4*a*c)*a^3*b^3*c^7 - 96*(b^2 - 4*a*c)*a^4*b*c^8)*d^3 + (2*a*b^4*c^2 - 20*a^2*b^2*c^3 + 48*a^3*c^4 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a*b^4 + 10*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^2*c + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a*b^3*c - 24*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^3*c^2 - 12*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b*c^2 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a*b^2*c^2 + 6*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 12*(b^2 - 4*a*c)*a^2*c^3)*(a*b^2*c - 4*a^2*c^2)^2*e^3 + 12*(\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b^4*c^3 - 8*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^4*b^2*c^4 - 2*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b^3*c^4 + 2*a^3*b^4*c^4 + 16*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^5*c^5 + 8*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^4*b*c^5 + \text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b^2*c^5 - 16*a^4*b^2*c^5 - 4*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^4*c^6 + 32*a^5*c^6 - 2*(b^2 - 4*a*c)*a^3*b^2*c^4 + 8*(b^2 - 4*a*c)*a^4*c^5)*d*\text{abs}(a*b^2*c - 4*a^2*c^2)*e^2 + 12*(2*a^3*b^6*c^6 - 16*a^4*b^4*c^7 + 32*a^5*b^2*c^8 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b^6*c^4 + 8*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*
\end{aligned}$$

$$\begin{aligned}
& b^2 - 4ac)c)a^4b^4c^5 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3b^5c^5 - 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^5b^2c^6 - 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^4b^3c^6 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3b^4c^6 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^4b^2c^7 - 2(b^2 - 4ac)a^3b^4c^6 + 8(b^2 - 4ac)a^4b^2c^7)d^2e - 2(\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3b^5c^2 - 8\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^4b^3c^3 - 2\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3b^4c^3 + 2a^3b^5c^3 + 16\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^5b^2c^4 + 8\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^4b^2c^4 + \sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3b^3c^4 - 16a^4b^3c^4 - 4\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^4b^2c^5 + 32a^5b^2c^5 - 2(b^2 - 4ac)a^3b^3c^3 + 8(b^2 - 4ac)a^4b^2c^4)abs(a^2b^2c - 4a^2c^2)e^3 - 3(2a^3b^7c^5 - 8a^4b^5c^6 - 32a^5b^3c^7 + 128a^6b^2c^8 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3b^7c^3 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^4b^5c^4 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3b^6c^4 + 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^5b^3c^5 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3b^5c^5 - 64\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^6b^2c^6 - 32\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^5b^2c^6 + 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^5b^2c^7 - 2(b^2 - 4ac)a^3b^5c^5 + 32(b^2 - 4ac)a^5b^2c^7)d^2e - (2a^3b^8c^4 - 32a^4b^6c^5 + 160a^5b^4c^6 - 256a^6b^2c^7 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3b^8c^2 + 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^4b^6c^3 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3b^7c^3 - 80\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^5b^4c^4 - 24\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^4b^5c^4 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3b^6c^4 + 128\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^6b^2c^5 + 64\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^5b^3c^5 + 12\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^4b^4c^5 - 32\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^5b^2c^6 - 2(b^2 - 4ac)a^3b^6c^4 + 24(b^2 - 4ac)a^4b^4c^5 - 64(b^2 - 4ac)a^5b^2c^6)e^3)arctan(2\sqrt{1/2}x/\sqrt{(a^3b^3c - 4a^2b^2c^2 - \sqrt{(a^3b^3c - 4a^2b^2c^2)^2 - 4(a^2b^2c - 4a^3c^2)(a^3b^2c^2 - 4a^2c^3))})/(a^3b^2c^2 - 4a^2c^3)))/((a^3b^6c^3 - 12a^4b^4c^4 - 2a^3b^5c^4 + 48a^5b^2c^5 + 16a^4b^3c^5 + a^3b^4c^5 - 64a^6c^6 - 32a^5b^2c^6 - 8a^4b^2c^6 + 16a^5c^7)abs(a^2b^2c - 4a^2c^2)abs(c))
\end{aligned}$$

**maple [B]** time = 0.05, size = 1846, normalized size = 3.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^3/(c\*x^4+b\*x^2+a)^2,x)

[Out] 
$$\begin{aligned}
& -1/4/a/(4ac-b^2)c^2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2}*\arctan(2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2}*c*x)*b*d^3+2a/(4ac-b^2)/(-4ac+b^2)^{1/2}*2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2}*\arctan(2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2}*c*x)*b*e^3-1/4/(4ac-b^2)/c/(-4ac+b^2)^{1/2}*2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2}*\arctan(2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2}*c*x)*b^3*e^3-3/4/(4ac-b^2)/(-4ac+b^2)^{1/2}*2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2}*\arctan(2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2}*c*x)*b^2*d*e^2+1/4/a/(4ac-b^2)c^2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2}*\arctanh(2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2}*c*x)*b*d^3+2a/(4ac-b^2)/(-4ac+b^2)^{1/2}*2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2}*\arctanh(2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2}*c*x)*b*e^3-1/4/(4ac-b^2)/c/(-4ac+b^2)^{1/2}*2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2}*\arctanh(2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2})
\end{aligned}$$

$$\begin{aligned} & *c+b^2)^{(1/2)} *c)^{(1/2)} *c*x) *b^3 *e^{3-3/4} / (4*a*c-b^2) / (-4*a*c+b^2)^{(1/2)} *2^(( \\ & 1/2) / ((-b+(-4*a*c+b^2)^{(1/2)})) *c)^{(1/2)} *arctanh(2^{(1/2)} / ((-b+(-4*a*c+b^2)^{(1/2)})) *c)^{(1/2)} *c*x) *b^2 *d *e^{2-3/4} / (4*a*c-b^2) *c / (-4*a*c+b^2)^{(1/2)} *2^((1/2) / ( \\ & (-b+(-4*a*c+b^2)^{(1/2)})) *c)^{(1/2)} *arctanh(2^{(1/2)} / ((-b+(-4*a*c+b^2)^{(1/2)})) *c \\ & )^{(1/2)} *c*x) *d *e^{2+3/4} / (4*a*c-b^2) *c / (-4*a*c+b^2)^{(1/2)} *2^((1/2) / ((-b+(-4*a*c+ \\ & b^2)^{(1/2)})) *c)^{(1/2)} *arctanh(2^{(1/2)} / ((-b+(-4*a*c+b^2)^{(1/2)})) *c)^{(1/2)} *c*x) \\ & *b *d^2 *e^{1/4} / a / (4*a*c-b^2) *c / (-4*a*c+b^2)^{(1/2)} *2^((1/2) / ((-b+(-4*a*c+b^2)^{(1/2)})) *c)^{(1/2)} *arctanh(2^{(1/2)} / ((-b+(-4*a*c+b^2)^{(1/2)})) *c)^{(1/2)} *c*x) *b^2 *d \\ & ^3 *e^{3/4} / (4*a*c-b^2) *c / (-4*a*c+b^2)^{(1/2)} *2^((1/2) / ((b+(-4*a*c+b^2)^{(1/2)})) *c)^{(1/2)} *arctan(2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)})) *c)^{(1/2)} *c*x) *d *e^{2+3/4} / (4*a*c-b \\ & ^2) *c / (-4*a*c+b^2)^{(1/2)} *2^((1/2) / ((b+(-4*a*c+b^2)^{(1/2)})) *c)^{(1/2)} *arctan(2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)})) *c)^{(1/2)} *c*x) *b *d^2 *e^{1/4} / a / (4*a*c-b^2) *c / (-4 \\ & *a*c+b^2)^{(1/2)} *2^((1/2) / ((b+(-4*a*c+b^2)^{(1/2)})) *c)^{(1/2)} *arctan(2^{(1/2)} / ((b \\ & +(-4*a*c+b^2)^{(1/2)})) *c)^{(1/2)} *c*x) *b^2 *d^3 *e^{1/4} / (4*a*c-b^2) / c *2^((1/2) / ((-b+(-4 \\ & *a*c+b^2)^{(1/2)})) *c)^{(1/2)} *arctanh(2^{(1/2)} / ((-b+(-4*a*c+b^2)^{(1/2)})) *c)^{(1/2)} *c*x) *b^2 *e^{3+3/4} / (4*a*c-b^2) *2^((1/2) / ((-b+(-4*a*c+b^2)^{(1/2)})) *c)^{(1/2)} *a \\ & rctanh(2^{(1/2)} / ((-b+(-4*a*c+b^2)^{(1/2)})) *c)^{(1/2)} *c*x) *b *d *e^{2-3/2} / (4*a*c-b^ \\ & 2) *c *2^((1/2) / ((-b+(-4*a*c+b^2)^{(1/2)})) *c)^{(1/2)} *arctanh(2^{(1/2)} / ((-b+(-4*a*c \\ & +b^2)^{(1/2)})) *c)^{(1/2)} *c*x) *d^2 *e^{-3/4} / (4*a*c-b^2) *c^2 / (-4*a*c+b^2)^{(1/2)} *2^((1/ \\ & 2) / ((-b+(-4*a*c+b^2)^{(1/2)})) *c)^{(1/2)} *arctanh(2^{(1/2)} / ((-b+(-4*a*c+b^2)^{(1/2)} \\ & )) *c)^{(1/2)} *c*x) *d^3 *e^{-1/4} / (4*a*c-b^2) / c *2^((1/2) / ((b+(-4*a*c+b^2)^{(1/2)})) *c)^{(1/2)} *arctan(2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)})) *c)^{(1/2)} *c*x) *b^2 *e^{3-3/4} / (4*a* \\ & c-b^2) *2^((1/2) / ((b+(-4*a*c+b^2)^{(1/2)})) *c)^{(1/2)} *arctan(2^{(1/2)} / ((b+(-4*a*c+ \\ & b^2)^{(1/2)})) *c)^{(1/2)} *c*x) *b *d *e^{2+3/2} / (4*a*c-b^2) *c *2^((1/2) / ((b+(-4*a*c+b^2)^{(1/2)} \\ & )^{(1/2)})) *c)^{(1/2)} *arctan(2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)})) *c)^{(1/2)} *c*x) *d^2 * \\ & e^{-3/4} / (4*a*c-b^2) *c^2 / (-4*a*c+b^2)^{(1/2)} *2^((1/2) / ((b+(-4*a*c+b^2)^{(1/2)})) *c)^{(1/2)} *arctan(2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)})) *c)^{(1/2)} *c*x) *d^3 *e^{-3/2} *a / (4*a*c- \\ & b^2) *2^((1/2) / ((-b+(-4*a*c+b^2)^{(1/2)})) *c)^{(1/2)} *arctanh(2^{(1/2)} / ((-b+(-4*a*c \\ & +b^2)^{(1/2)})) *c)^{(1/2)} *c*x) *e^{3+3/2} *a / (4*a*c-b^2) *2^((1/2) / ((b+(-4*a*c+b^2)^{(1/2)} \\ & )^{(1/2)})) *c)^{(1/2)} *arctan(2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)})) *c)^{(1/2)} *c*x) *e^{3+(-1 \\ & /2) * (2*a^2 *c *e^3 - a *b^2 *e^3 + 3*a *b *c *d *e^2 - 6*a *c^2 *d^2 *e + b *c^2 *d^3) / a / c / (4*a*c \\ & -b^2) *x^3 + 1/2 / c * (a^2 *b *e^3 - 6*a^2 *c *d *e^2 + 3*a *b *c *d^2 *e + 2*a *c^2 *d^3 - b^2 *c *d^ \\ & 3) / (4*a*c-b^2) / a *x) / (c *x^4 + b *x^2 + a) \end{aligned}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(bc^2d^3 - 6ac^2d^2e + 3abcd^2e - (ab^2 - 2a^2c)e^3)x^3 - (3abcd^2e - 6a^2cde^2 + a^2be^3 - (b^2c - 2ac^2)d^3)x - \int \frac{3abcd^2e - 6a^2cde^2 + a^2be^3 + (b^2c - 6ac^2)d^3 + (bc^2d^3 - 6ac^2d^2e + 3abcd^2e + (ab^2 - 6a^2c)e^3)x^2}{cx^4 + bx^2 + a} dx}{2(a^2b^2c - 4a^3c^2 + (ab^2c^2 - 4a^2c^3)x^4 + (ab^3c - 4a^2bc^2)x^2)} - \frac{3abcd^2e - 6a^2cde^2 + a^2be^3 - (b^2c - 2ac^2)d^3}{2(ab^2c - 4a^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^3/(c\*x^4+b\*x^2+a)^2,x, algorithm="maxima")

[Out] 1/2\*((b\*c^2\*d^3 - 6\*a\*c^2\*d^2\*e + 3\*a\*b\*c\*d\*e^2 - (a\*b^2 - 2\*a^2\*c)\*e^3)\*x^3 - (3\*a\*b\*c\*d^2\*e - 6\*a^2\*c\*d\*e^2 + a^2\*b\*e^3 - (b^2\*c - 2\*a\*c^2)\*d^3)\*x) / (a^2\*b^2\*c - 4\*a^3\*c^2 + (a\*b^2\*c^2 - 4\*a^2\*c^3)\*x^4 + (a\*b^3\*c - 4\*a^2\*b\*c^2)\*x^2) - 1/2\*integrate(-(3\*a\*b\*c\*d^2\*e - 6\*a^2\*c\*d\*e^2 + a^2\*b\*e^3 + (b^2\*c - 6\*a\*c^2)\*d^3 + (b\*c^2\*d^3 - 6\*a\*c^2\*d^2\*e + 3\*a\*b\*c\*d\*e^2 + (a\*b^2 - 6\*a^2\*c)\*e^3)\*x^2) / (c\*x^4 + b\*x^2 + a), x) / (a\*b^2\*c - 4\*a^2\*c^2)

**mupad** [B] time = 8.79, size = 29030, normalized size = 51.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)^3/(a + b\*x^2 + c\*x^4)^2,x)

[Out] - ((x^3\*(b\*c^2\*d^3 - a\*b^2\*e^3 + 2\*a^2\*c\*e^3 - 6\*a\*c^2\*d^2\*e + 3\*a\*b\*c\*d\*e^2) / (2\*a\*c\*(4\*a\*c - b^2)) - (x\*(2\*a\*c^2\*d^3 + a^2\*b\*e^3 - b^2\*c\*d^3 - 6\*a^2\*c\*d\*e^2 + 3\*a\*b\*c\*d^2\*e)) / (2\*a\*c\*(4\*a\*c - b^2))) / (a + b\*x^2 + c\*x^4) - atan((((6144\*a^5\*c^7\*d^3 + 16\*a\*b^8\*c^3\*d^3 - 1024\*a^6\*b\*c^5\*e^3 + 6144\*a^6\*c^6\*d\*e^2 - 288\*a^2\*b^6\*c^4\*d^3 + 1920\*a^3\*b^4\*c^5\*d^3 - 5632\*a^4\*b^2\*c^6\*d^3





$$\begin{aligned}
& 0*a^5*b*c^8*d^6 - 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^{(1/2)} + 27*a^4*b^9*c*e^6 + \\
& 3840*a^8*b*c^5*e^6 + 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^{(1/2)} - 9216*a^6*c^8*d \\
& ^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 38 \\
& 40*a^4*b^3*c^7*d^6 - a^3*b^2*e^6*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^5*b^7*c^2 \\
& *e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^ \\
& 7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b \\
& ^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a \\
& ^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + \\
& 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*e^2*(- \\
& (4*a*c - b^2)^9)^{(1/2)} + 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a*b \\
& ^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^ \\
& 5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7* \\
& d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d \\
& *e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 + 6*a*b*c^3*d^5*e*( \\
& -(4*a*c - b^2)^9)^{(1/2)} - 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(40 \\
& 96*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^ \\
& 6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^{(1/2)}*i)/((5*a^4*b^4*e^9 + \\
& 216*a^6*c^2*e^9 + 5*b^3*c^5*d^9 - 66*a^5*b^2*c*e^9 + a*b^7*d^3*e^6 - 9*a^3* \\
& b^5*d*e^8 + 216*a^2*c^6*d^8*e - 9*b^4*c^4*d^8*e + 3*a^2*b^6*d^2*e^7 + 864*a \\
& ^3*c^5*d^6*e^3 + 1296*a^4*c^4*d^4*e^5 + 864*a^5*c^3*d^2*e^7 + 3*b^5*c^3*d^7 \\
& *e^2 + b^6*c^2*d^6*e^3 - 36*a*b*c^6*d^9 + 624*a^2*b^2*c^4*d^6*e^3 - 6*a^2*b \\
& ^3*c^3*d^5*e^4 - 108*a^2*b^4*c^2*d^4*e^5 + 1020*a^3*b^2*c^3*d^4*e^5 + 128*a \\
& ^3*b^3*c^2*d^3*e^6 + 384*a^4*b^2*c^2*d^2*e^7 + 54*a*b^2*c^5*d^8*e + 6*a*b^6 \\
& *c*d^4*e^5 + 153*a^4*b^3*c*d*e^8 - 612*a^5*b*c^2*d*e^8 + 24*a*b^3*c^4*d^7*e \\
& ^2 - 46*a*b^4*c^3*d^6*e^3 - 3*a*b^5*c^2*d^5*e^4 - 720*a^2*b*c^5*d^7*e^2 - 3 \\
& *a^2*b^5*c*d^3*e^6 - 1944*a^3*b*c^4*d^5*e^4 - 90*a^3*b^4*c*d^2*e^7 - 1872*a \\
& ^4*b*c^3*d^3*e^6)/(4*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2* \\
& c^3)) + (((6144*a^5*c^7*d^3 + 16*a*b^8*c^3*d^3 - 1024*a^6*b*c^5*e^3 + 6144*a \\
& ^6*c^6*d*e^2 - 288*a^2*b^6*c^4*d^3 + 1920*a^3*b^4*c^5*d^3 - 5632*a^4*b^2*c \\
& ^6*d^3 + 16*a^3*b^7*c^2*e^3 - 192*a^4*b^5*c^3*e^3 + 768*a^5*b^3*c^4*e^3 - 3 \\
& 072*a^5*b*c^6*d^2*e + 48*a^2*b^7*c^3*d^2*e - 576*a^3*b^5*c^4*d^2*e - 96*a^3 \\
& *b^6*c^3*d*e^2 + 2304*a^4*b^3*c^5*d^2*e + 1152*a^4*b^4*c^4*d*e^2 - 4608*a^5 \\
& *b^2*c^5*d*e^2)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^ \\
& 3)) - (x*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d \\
& ^6 - 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^{(1/2)} + 27*a^4*b^9*c*e^6 + 3840*a^8*b* \\
& c^5*e^6 + 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^{(1/2)} - 9216*a^6*c^8*d^5*e - 9216* \\
& a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c \\
& ^7*d^6 - a^3*b^2*e^6*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^5*b^7*c^2*e^6 + 1504* \\
& a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c - b^2)^9)^{(1/ \\
& 2)} - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^ \\
& 2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e \\
& ^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6* \\
& d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2 \\
& *c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2 \\
& )^9)^{(1/2)} + 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a*b^10*c^3*d^5* \\
& e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 38 \\
& 4*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576* \\
& a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664 \\
& *a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 + 6*a*b*c^3*d^5*e*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)} - 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(4096*a^9*c^9 + \\
& a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840 \\
& *a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^{(1/2)}*(1024*a^5*b*c^6 - 16*a^2*b^7*c^3 + \\
& 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2 \\
& *c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d \\
& ^6 - 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^{(1/2)} + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c \\
& ^5*e^6 + 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^{(1/2)} - 9216*a^6*c^8*d^5*e - 9216*a \\
& ^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^ \\
& 7*d^6 - a^3*b^2*e^6*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^5*b^7*c^2*e^6 + 1504*a
\end{aligned}$$



$$\begin{aligned}
& ^6b^5c^3e^6 - 3840a^7b^3c^4e^6 + b^2c^3d^6(-4ac - b^2)^9)^{(1/2)} \\
& - 18432a^7c^7d^3e^3 + 9a^2b^9c^3d^4e^2 - 384a^3b^7c^4d^4e^2 \\
& + 88a^3b^8c^3d^3e^3 + 9a^3b^9c^2d^2e^4 + 3744a^4b^5c^5d^4e^2 \\
& - 768a^4b^6c^4d^3e^3 - 384a^4b^7c^3d^2e^4 - 13824a^5b^3c^6d^4e^2 \\
& + 768a^5b^4c^5d^3e^3 + 3744a^5b^5c^4d^2e^4 + 8192a^6b^2c^6d^3e^3 \\
& - 13824a^6b^3c^5d^2e^4 - 9a^2c^3d^4e^2(-4ac - b^2)^9)^{(1/2)} \\
& + 9a^3c^2d^2e^4(-4ac - b^2)^9)^{(1/2)} - 6ab^{10}c^3d^5e \\
& - 6a^3b^{10}c^d^5e^5 + 108a^2b^8c^4d^5e - 576a^3b^6c^5d^5e + 384 \\
& a^4b^4c^6d^5e + 108a^4b^8c^2d^5e + 4608a^5b^2c^7d^5e - 576a^5 \\
& b^6c^3d^5e + 17664a^6b^6c^7d^4e^2 + 384a^6b^4c^4d^5e + 17664a^7 \\
& b^2c^6d^2e^4 + 4608a^7b^2c^5d^5e + 6ab^3c^3d^5e(-4ac - b^2)^9)^{(1/2)} \\
& - 6a^3b^3c^d^5e(-4ac - b^2)^9)^{(1/2)} / (32(4096a^9c^9 + a^3b^{12}c^3 \\
& - 24a^4b^{10}c^4 + 240a^5b^8c^5 - 1280a^6b^6c^6 + 3840a^7b^4c^7 \\
& - 6144a^8b^2c^8)))^{(1/2)} - (x(72a^5c^3e^6 - 72a^2c^6d^6 - a^2b^6e^6 \\
& - b^4c^4d^6 + 14ab^2c^5d^6 + 16a^3b^4c^6e^6 - 74a^4b^2c^2e^6 \\
& - 72a^3c^5d^4e^2 + 72a^4c^4d^2e^4 - 102a^2b^2c^4d^4e^2 + 44a^2b^3c^3d^3e^3 \\
& + 9a^2b^4c^2d^2e^4 - 174a^3b^2c^3d^2e^4 - 6ab^3c^4d^5e + 120a^2b^3c^5d^5e \\
& - 6a^2b^5c^d^5e + 24a^4b^3c^3d^5e + 144a^3b^4c^4d^3e^3 + 42a^3b^3c^2d^5e)) / (2(16a^4 \\
& c^3 + a^2b^4c - 8a^3b^2c^2))) * ((27ab^9c^4d^6 - b^{11}c^3d^6 - a^3b^{11}e^6 \\
& + 3840a^5b^8c^8d^6 - 9ac^4d^6(-4ac - b^2)^9)^{(1/2)} + 27a^4b^9c^6e^6 \\
& + 3840a^8b^6c^5e^6 + 9a^4c^6e^6(-4ac - b^2)^9)^{(1/2)} - 9216a^6c^8d^5e \\
& - 9216a^8c^6d^5e - 288a^2b^7c^5d^6 + 1504a^3b^5c^6d^6 - 3840a^4b^3c^7d^6 \\
& - a^3b^2e^6(-4ac - b^2)^9)^{(1/2)} - 288a^5b^7c^2e^6 + 1504a^6b^5c^3e^6 \\
& - 3840a^7b^3c^4e^6 + b^2c^3d^6(-4ac - b^2)^9)^{(1/2)} - 18432a^7c^7d^3e^3 \\
& + 9a^2b^9c^3d^4e^2 - 384a^3b^7c^4d^4e^2 + 88a^3b^8c^3d^3e^3 + 9a^3b^9c^2d^2e^4 \\
& + 3744a^4b^5c^5d^4e^2 - 768a^4b^6c^4d^3e^3 - 384a^4b^7c^3d^2e^4 \\
& - 13824a^5b^3c^6d^4e^2 + 768a^5b^4c^5d^3e^3 + 3744a^5b^5c^4d^2e^4 \\
& + 8192a^6b^2c^6d^3e^3 - 13824a^6b^3c^5d^2e^4 - 9a^2c^3d^4e^2(-4ac - b^2)^9)^{(1/2)} \\
& + 9a^3c^2d^2e^4(-4ac - b^2)^9)^{(1/2)} - 6ab^{10}c^3d^5e - 6a^3b^{10}c^d^5e^5 \\
& + 108a^2b^8c^4d^5e - 576a^3b^6c^5d^5e + 384a^4b^4c^6d^5e + 108a^4b^8c^2d^5e \\
& + 4608a^5b^2c^7d^5e - 576a^5b^6c^3d^5e + 17664a^6b^6c^7d^4e^2 + 384 \\
& a^6b^4c^4d^5e + 17664a^7b^2c^6d^2e^4 + 4608a^7b^2c^5d^5e + 6ab^3c^3d^5e \\
& (-4ac - b^2)^9)^{(1/2)} - 6a^3b^3c^d^5e(-4ac - b^2)^9)^{(1/2)} / (32(4096a^9c^9 \\
& + a^3b^{12}c^3 - 24a^4b^{10}c^4 + 240a^5b^8c^5 - 1280a^6b^6c^6 + 3840a^7b^4c^7 \\
& - 6144a^8b^2c^8)))^{(1/2)} + (((6144a^5c^7d^3 + 16ab^8c^3d^3 - 1024a^6b^6c^5e^3 \\
& + 6144a^6c^6d^5e^2 - 288a^2b^6c^4d^3 + 1920a^3b^4c^5d^3 - 5632a^4b^2c^6d^3 \\
& + 16a^3b^7c^2e^3 - 192a^4b^5c^3e^3 + 768a^5b^3c^4e^3 - 3072a^5b^6c^6d^2e \\
& + 48a^2b^7c^3d^2e - 576a^3b^5c^4d^2e - 96a^3b^6c^3d^5e^2 + 2304a^4b^3c^5d^2e \\
& + 1152a^4b^4c^4d^5e^2 - 4608a^5b^2c^5d^5e^2) / (8(64a^5c^4 - a^2b^6c + 12a^3b^4c^2 \\
& - 48a^4b^2c^3)) + (x((27ab^9c^4d^6 - b^{11}c^3d^6 - a^3b^{11}e^6 + 3840a^5b^8c^8d^6 \\
& - 9ac^4d^6(-4ac - b^2)^9)^{(1/2)} + 27a^4b^9c^6e^6 + 3840a^8b^6c^5e^6 \\
& + 9a^4c^6e^6(-4ac - b^2)^9)^{(1/2)} - 9216a^6c^8d^5e - 9216a^8c^6d^5e \\
& - 288a^2b^7c^5d^6 + 1504a^3b^5c^6d^6 - 3840a^4b^3c^7d^6 - a^3b^2e^6(-4ac - b^2)^9)^{(1/2)} \\
& - 288a^5b^7c^2e^6 + 1504a^6b^5c^3e^6 - 3840a^7b^3c^4e^6 + b^2c^3d^6(-4ac - b^2)^9)^{(1/2)} \\
& - 18432a^7c^7d^3e^3 + 9a^2b^9c^3d^4e^2 - 384a^3b^7c^4d^4e^2 + 88a^3b^8c^3d^3e^3 \\
& + 9a^3b^9c^2d^2e^4 + 3744a^4b^5c^5d^4e^2 - 768a^4b^6c^4d^3e^3 - 384a^4b^7c^3d^2e^4 \\
& - 13824a^5b^3c^6d^4e^2 + 768a^5b^4c^5d^3e^3 + 3744a^5b^5c^4d^2e^4 + 8192a^6b^2c^6d^3e^3 \\
& - 13824a^6b^3c^5d^2e^4 - 9a^2c^3d^4e^2(-4ac - b^2)^9)^{(1/2)} + 9a^3c^2d^2e^4(-4ac - b^2)^9)^{(1/2)} \\
& - 6ab^{10}c^3d^5e - 6a^3b^{10}c^d^5e^5 + 108a^2b^8c^4d^5e - 576a^3b^6c^5d^5e + 384a^4b^4c^6d^5e \\
& + 108a^4b^8c^2d^5e + 4608a^5b^2c^7d^5e - 576a^5b^6c^3d^5e + 17664a^6b^6c^7d^4e^2 \\
& + 384a^6b^4c^4d^5e + 17664a^7b^2c^6d^2e^4
\end{aligned}$$



$$\begin{aligned}
& e^4 * (- (4 * a * c - b^2)^9)^{(1/2)} - 6 * a * b^{10} * c^3 * d^5 * e - 6 * a^3 * b^{10} * c * d * e^5 + 10 \\
& 8 * a^2 * b^8 * c^4 * d^5 * e - 576 * a^3 * b^6 * c^5 * d^5 * e + 384 * a^4 * b^4 * c^6 * d^5 * e + 108 * a \\
& ^4 * b^8 * c^2 * d * e^5 + 4608 * a^5 * b^2 * c^7 * d^5 * e - 576 * a^5 * b^6 * c^3 * d * e^5 + 17664 * a \\
& ^6 * b * c^7 * d^4 * e^2 + 384 * a^6 * b^4 * c^4 * d * e^5 + 17664 * a^7 * b * c^6 * d^2 * e^4 + 4608 * a \\
& ^7 * b^2 * c^5 * d * e^5 + 6 * a * b * c^3 * d^5 * e * (- (4 * a * c - b^2)^9)^{(1/2)} - 6 * a^3 * b * c * d * e \\
& ^5 * (- (4 * a * c - b^2)^9)^{(1/2)} / (32 * (4096 * a^9 * c^9 + a^3 * b^{12} * c^3 - 24 * a^4 * b^{10} \\
& * c^4 + 240 * a^5 * b^8 * c^5 - 1280 * a^6 * b^6 * c^6 + 3840 * a^7 * b^4 * c^7 - 6144 * a^8 * b^2 \\
& * c^8))^{(1/2)} * i - \operatorname{atan}((((6144 * a^5 * c^7 * d^3 + 16 * a * b^8 * c^3 * d^3 - 1024 * a^6 * \\
& b * c^5 * e^3 + 6144 * a^6 * c^6 * d * e^2 - 288 * a^2 * b^6 * c^4 * d^3 + 1920 * a^3 * b^4 * c^5 * d^3 \\
& - 5632 * a^4 * b^2 * c^6 * d^3 + 16 * a^3 * b^7 * c^2 * e^3 - 192 * a^4 * b^5 * c^3 * e^3 + 768 * a^ \\
& 5 * b^3 * c^4 * e^3 - 3072 * a^5 * b * c^6 * d^2 * e + 48 * a^2 * b^7 * c^3 * d^2 * e - 576 * a^3 * b^5 * c \\
& ^4 * d^2 * e - 96 * a^3 * b^6 * c^3 * d * e^2 + 2304 * a^4 * b^3 * c^5 * d^2 * e + 1152 * a^4 * b^4 * c^4 \\
& * d * e^2 - 4608 * a^5 * b^2 * c^5 * d * e^2) / (8 * (64 * a^5 * c^4 - a^2 * b^6 * c + 12 * a^3 * b^4 * c^ \\
& 2 - 48 * a^4 * b^2 * c^3)) - (x * ((27 * a * b^9 * c^4 * d^6 - b^{11} * c^3 * d^6 - a^3 * b^{11} * e^6 \\
& + 3840 * a^5 * b * c^8 * d^6 + 9 * a * c^4 * d^6 * (- (4 * a * c - b^2)^9)^{(1/2)} + 27 * a^4 * b^9 * c * \\
& e^6 + 3840 * a^8 * b * c^5 * e^6 - 9 * a^4 * c * e^6 * (- (4 * a * c - b^2)^9)^{(1/2)} - 9216 * a^6 * \\
& c^8 * d^5 * e - 9216 * a^8 * c^6 * d * e^5 - 288 * a^2 * b^7 * c^5 * d^6 + 1504 * a^3 * b^5 * c^6 * d^6 \\
& - 3840 * a^4 * b^3 * c^7 * d^6 + a^3 * b^2 * e^6 * (- (4 * a * c - b^2)^9)^{(1/2)} - 288 * a^5 * b^7 \\
& * c^2 * e^6 + 1504 * a^6 * b^5 * c^3 * e^6 - 3840 * a^7 * b^3 * c^4 * e^6 - b^2 * c^3 * d^6 * (- (4 * \\
& a * c - b^2)^9)^{(1/2)} - 18432 * a^7 * c^7 * d^3 * e^3 + 9 * a^2 * b^9 * c^3 * d^4 * e^2 - 384 * a \\
& ^3 * b^7 * c^4 * d^4 * e^2 + 88 * a^3 * b^8 * c^3 * d^3 * e^3 + 9 * a^3 * b^9 * c^2 * d^2 * e^4 + 3744 * \\
& a^4 * b^5 * c^5 * d^4 * e^2 - 768 * a^4 * b^6 * c^4 * d^3 * e^3 - 384 * a^4 * b^7 * c^3 * d^2 * e^4 - 1 \\
& 3824 * a^5 * b^3 * c^6 * d^4 * e^2 + 768 * a^5 * b^4 * c^5 * d^3 * e^3 + 3744 * a^5 * b^5 * c^4 * d^2 * e \\
& ^4 + 8192 * a^6 * b^2 * c^6 * d^3 * e^3 - 13824 * a^6 * b^3 * c^5 * d^2 * e^4 + 9 * a^2 * c^3 * d^4 * e \\
& ^2 * (- (4 * a * c - b^2)^9)^{(1/2)} - 9 * a^3 * c^2 * d^2 * e^4 * (- (4 * a * c - b^2)^9)^{(1/2)} - \\
& 6 * a * b^{10} * c^3 * d^5 * e - 6 * a^3 * b^{10} * c * d * e^5 + 108 * a^2 * b^8 * c^4 * d^5 * e - 576 * a^3 * b \\
& ^6 * c^5 * d^5 * e + 384 * a^4 * b^4 * c^6 * d^5 * e + 108 * a^4 * b^8 * c^2 * d * e^5 + 4608 * a^5 * b^2 \\
& * c^7 * d^5 * e - 576 * a^5 * b^6 * c^3 * d * e^5 + 17664 * a^6 * b * c^7 * d^4 * e^2 + 384 * a^6 * b^4 * \\
& c^4 * d * e^5 + 17664 * a^7 * b * c^6 * d^2 * e^4 + 4608 * a^7 * b^2 * c^5 * d * e^5 - 6 * a * b * c^3 * d^ \\
& 5 * e * (- (4 * a * c - b^2)^9)^{(1/2)} + 6 * a^3 * b * c * d * e^5 * (- (4 * a * c - b^2)^9)^{(1/2)} / (3 \\
& 2 * (4096 * a^9 * c^9 + a^3 * b^{12} * c^3 - 24 * a^4 * b^{10} * c^4 + 240 * a^5 * b^8 * c^5 - 1280 * a \\
& ^6 * b^6 * c^6 + 3840 * a^7 * b^4 * c^7 - 6144 * a^8 * b^2 * c^8))^{(1/2)} * (1024 * a^5 * b * c^6 - \\
& 16 * a^2 * b^7 * c^3 + 192 * a^3 * b^5 * c^4 - 768 * a^4 * b^3 * c^5) / (2 * (16 * a^4 * c^3 + a^2 * \\
& b^4 * c - 8 * a^3 * b^2 * c^2)) * ((27 * a * b^9 * c^4 * d^6 - b^{11} * c^3 * d^6 - a^3 * b^{11} * e^6 + \\
& 3840 * a^5 * b * c^8 * d^6 + 9 * a * c^4 * d^6 * (- (4 * a * c - b^2)^9)^{(1/2)} + 27 * a^4 * b^9 * c * \\
& e^6 + 3840 * a^8 * b * c^5 * e^6 - 9 * a^4 * c * e^6 * (- (4 * a * c - b^2)^9)^{(1/2)} - 9216 * a^6 * \\
& c^8 * d^5 * e - 9216 * a^8 * c^6 * d * e^5 - 288 * a^2 * b^7 * c^5 * d^6 + 1504 * a^3 * b^5 * c^6 * d^6 \\
& - 3840 * a^4 * b^3 * c^7 * d^6 + a^3 * b^2 * e^6 * (- (4 * a * c - b^2)^9)^{(1/2)} - 288 * a^5 * b^7 \\
& * c^2 * e^6 + 1504 * a^6 * b^5 * c^3 * e^6 - 3840 * a^7 * b^3 * c^4 * e^6 - b^2 * c^3 * d^6 * (- (4 * \\
& a * c - b^2)^9)^{(1/2)} - 18432 * a^7 * c^7 * d^3 * e^3 + 9 * a^2 * b^9 * c^3 * d^4 * e^2 - 384 * a^ \\
& 3 * b^7 * c^4 * d^4 * e^2 + 88 * a^3 * b^8 * c^3 * d^3 * e^3 + 9 * a^3 * b^9 * c^2 * d^2 * e^4 + 3744 * a \\
& ^4 * b^5 * c^5 * d^4 * e^2 - 768 * a^4 * b^6 * c^4 * d^3 * e^3 - 384 * a^4 * b^7 * c^3 * d^2 * e^4 - 13 \\
& 824 * a^5 * b^3 * c^6 * d^4 * e^2 + 768 * a^5 * b^4 * c^5 * d^3 * e^3 + 3744 * a^5 * b^5 * c^4 * d^2 * e^ \\
& 4 + 8192 * a^6 * b^2 * c^6 * d^3 * e^3 - 13824 * a^6 * b^3 * c^5 * d^2 * e^4 + 9 * a^2 * c^3 * d^4 * e^ \\
& 2 * (- (4 * a * c - b^2)^9)^{(1/2)} - 9 * a^3 * c^2 * d^2 * e^4 * (- (4 * a * c - b^2)^9)^{(1/2)} - 6 \\
& * a * b^{10} * c^3 * d^5 * e - 6 * a^3 * b^{10} * c * d * e^5 + 108 * a^2 * b^8 * c^4 * d^5 * e - 576 * a^3 * b \\
& ^6 * c^5 * d^5 * e + 384 * a^4 * b^4 * c^6 * d^5 * e + 108 * a^4 * b^8 * c^2 * d * e^5 + 4608 * a^5 * b^2 * \\
& c^7 * d^5 * e - 576 * a^5 * b^6 * c^3 * d * e^5 + 17664 * a^6 * b * c^7 * d^4 * e^2 + 384 * a^6 * b^4 * \\
& c^4 * d * e^5 + 17664 * a^7 * b * c^6 * d^2 * e^4 + 4608 * a^7 * b^2 * c^5 * d * e^5 - 6 * a * b * c^3 * d^5 \\
& * e * (- (4 * a * c - b^2)^9)^{(1/2)} + 6 * a^3 * b * c * d * e^5 * (- (4 * a * c - b^2)^9)^{(1/2)} / (32 \\
& * (4096 * a^9 * c^9 + a^3 * b^{12} * c^3 - 24 * a^4 * b^{10} * c^4 + 240 * a^5 * b^8 * c^5 - 1280 * a^ \\
& 6 * b^6 * c^6 + 3840 * a^7 * b^4 * c^7 - 6144 * a^8 * b^2 * c^8))^{(1/2)} - (x * (72 * a^5 * c^3 * e \\
& ^6 - 72 * a^2 * c^6 * d^6 - a^2 * b^6 * e^6 - b^4 * c^4 * d^6 + 14 * a * b^2 * c^5 * d^6 + 16 * a^3 \\
& * b^4 * c * e^6 - 74 * a^4 * b^2 * c^2 * e^6 - 72 * a^3 * c^5 * d^4 * e^2 + 72 * a^4 * c^4 * d^2 * e^4 - \\
& 102 * a^2 * b^2 * c^4 * d^4 * e^2 + 44 * a^2 * b^3 * c^3 * d^3 * e^3 + 9 * a^2 * b^4 * c^2 * d^2 * e^4 - \\
& 174 * a^3 * b^2 * c^3 * d^2 * e^4 - 6 * a * b^3 * c^4 * d^5 * e + 120 * a^2 * b * c^5 * d^5 * e - 6 * a^2 * \\
& b^5 * c * d * e^5 + 24 * a^4 * b * c^3 * d * e^5 + 144 * a^3 * b * c^4 * d^3 * e^3 + 42 * a^3 * b^3 * c^2 * d \\
& * e^5) / (2 * (16 * a^4 * c^3 + a^2 * b^4 * c - 8 * a^3 * b^2 * c^2)) * ((27 * a * b^9 * c^4 * d^6 - b \\
& ^{11} * c^3 * d^6 - a^3 * b^{11} * e^6 + 3840 * a^5 * b * c^8 * d^6 + 9 * a * c^4 * d^6 * (- (4 * a * c - b^
\end{aligned}$$

$$\begin{aligned}
& 2)^9)^{(1/2)} + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c \\
& - b^2)^9)^{(1/2)} - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5 \\
& *d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3* \\
& c^4*e^6 - b^2*c^3*d^6*(-(4*a*c - b^2)^9)^{(1/2)} - 18432*a^7*c^7*d^3*e^3 + 9* \\
& a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9* \\
& a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - \\
& 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^ \\
& ^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^ \\
& 5*d^2*e^4 + 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 9*a^3*c^2*d^2*e^4* \\
& (- (4*a*c - b^2)^9)^{(1/2)} - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^ \\
& 2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b \\
& ^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b \\
& *c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b \\
& ^2*c^5*d*e^5 - 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^{(1/2)} + 6*a^3*b*c*d*e^5*( \\
& - (4*a*c - b^2)^9)^{(1/2)}/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 \\
& + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8 \\
& )))^{(1/2)}*i - (((6144*a^5*c^7*d^3 + 16*a*b^8*c^3*d^3 - 1024*a^6*b*c^5*e^3 \\
& + 6144*a^6*c^6*d*e^2 - 288*a^2*b^6*c^4*d^3 + 1920*a^3*b^4*c^5*d^3 - 5632*a^ \\
& 4*b^2*c^6*d^3 + 16*a^3*b^7*c^2*e^3 - 192*a^4*b^5*c^3*e^3 + 768*a^5*b^3*c^4* \\
& e^3 - 3072*a^5*b*c^6*d^2*e + 48*a^2*b^7*c^3*d^2*e - 576*a^3*b^5*c^4*d^2*e - \\
& 96*a^3*b^6*c^3*d*e^2 + 2304*a^4*b^3*c^5*d^2*e + 1152*a^4*b^4*c^4*d*e^2 - 4 \\
& 608*a^5*b^2*c^5*d*e^2)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4 \\
& *b^2*c^3)) + (x*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5 \\
& *b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^{(1/2)} + 27*a^4*b^9*c*e^6 + 3840 \\
& *a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^{(1/2)} - 9216*a^6*c^8*d^5*e \\
& - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^ \\
& 4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^5*b^7*c^2*e^6 \\
& + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 - b^2*c^3*d^6*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4 \\
& *d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^ \\
& 5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b \\
& ^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192* \\
& a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 + 9*a^2*c^3*d^4*e^2*(-(4*a* \\
& c - b^2)^9)^{(1/2)} - 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a*b^10*c \\
& ^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5 \\
& *e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e \\
& - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 \\
& + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 - 6*a*b*c^3*d^5*e*(-(4*a \\
& *c - b^2)^9)^{(1/2)} + 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(4096*a^ \\
& 9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 \\
& + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^{(1/2)}*(1024*a^5*b*c^6 - 16*a^2*b^ \\
& 7*c^3 + 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8* \\
& a^3*b^2*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5* \\
& b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^{(1/2)} + 27*a^4*b^9*c*e^6 + 3840* \\
& a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^{(1/2)} - 9216*a^6*c^8*d^5*e - \\
& 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4 \\
& *b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^5*b^7*c^2*e^6 + \\
& 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 - b^2*c^3*d^6*(-(4*a*c - b^2)^ \\
& 9)^{(1/2)} - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4* \\
& d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5 \\
& *d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^ \\
& 3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a \\
& ^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 + 9*a^2*c^3*d^4*e^2*(-(4*a*c \\
& - b^2)^9)^{(1/2)} - 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a*b^10*c^ \\
& 3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5* \\
& e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e \\
& - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + \\
& 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 - 6*a*b*c^3*d^5*e*(-(4*a*
\end{aligned}$$

$$\begin{aligned}
& (c - b^2)^9)^{(1/2)} + 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(4096*a^9 \\
& *c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 \\
& + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^{(1/2)} + (x*(72*a^5*c^3*e^6 - 72*a^ \\
& 2*c^6*d^6 - a^2*b^6*e^6 - b^4*c^4*d^6 + 14*a*b^2*c^5*d^6 + 16*a^3*b^4*c*e^6 \\
& - 74*a^4*b^2*c^2*e^6 - 72*a^3*c^5*d^4*e^2 + 72*a^4*c^4*d^2*e^4 - 102*a^2*b \\
& ^2*c^4*d^4*e^2 + 44*a^2*b^3*c^3*d^3*e^3 + 9*a^2*b^4*c^2*d^2*e^4 - 174*a^3*b \\
& ^2*c^3*d^2*e^4 - 6*a*b^3*c^4*d^5*e + 120*a^2*b*c^5*d^5*e - 6*a^2*b^5*c*d*e^ \\
& 5 + 24*a^4*b*c^3*d*e^5 + 144*a^3*b*c^4*d^3*e^3 + 42*a^3*b^3*c^2*d*e^5))/(2* \\
& (16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^ \\
& 6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^{(1/2)} \\
& ) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^ \\
& (1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 150 \\
& 4*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)^{( \\
& 1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 - \\
& b^2*c^3*d^6*(-(4*a*c - b^2)^9)^{(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^ \\
& 3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^ \\
& 2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^ \\
& 7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744* \\
& a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 \\
& + 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^{(1/2) - 9*a^3*c^2*d^2*e^4*(-(4*a*c - \\
& b^2)^9)^{(1/2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4* \\
& d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e \\
& ^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e \\
& ^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e \\
& ^5 - 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^{(1/2) + 6*a^3*b*c*d*e^5*(-(4*a*c - \\
& b^2)^9)^{(1/2)})/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5 \\
& *b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^{(1/2)* \\
& 1i)/((5*a^4*b^4*e^9 + 216*a^6*c^2*e^9 + 5*b^3*c^5*d^9 - 66*a^5*b^2*c*e^9 + \\
& a*b^7*d^3*e^6 - 9*a^3*b^5*d*e^8 + 216*a^2*c^6*d^8*e - 9*b^4*c^4*d^8*e + 3*a \\
& ^2*b^6*d^2*e^7 + 864*a^3*c^5*d^6*e^3 + 1296*a^4*c^4*d^4*e^5 + 864*a^5*c^3*d \\
& ^2*e^7 + 3*b^5*c^3*d^7*e^2 + b^6*c^2*d^6*e^3 - 36*a*b*c^6*d^9 + 624*a^2*b^2 \\
& *c^4*d^6*e^3 - 6*a^2*b^3*c^3*d^5*e^4 - 108*a^2*b^4*c^2*d^4*e^5 + 1020*a^3*b \\
& ^2*c^3*d^4*e^5 + 128*a^3*b^3*c^2*d^3*e^6 + 384*a^4*b^2*c^2*d^2*e^7 + 54*a*b \\
& ^2*c^5*d^8*e + 6*a*b^6*c*d^4*e^5 + 153*a^4*b^3*c*d*e^8 - 612*a^5*b*c^2*d*e^ \\
& 8 + 24*a*b^3*c^4*d^7*e^2 - 46*a*b^4*c^3*d^6*e^3 - 3*a*b^5*c^2*d^5*e^4 - 720 \\
& *a^2*b*c^5*d^7*e^2 - 3*a^2*b^5*c*d^3*e^6 - 1944*a^3*b*c^4*d^5*e^4 - 90*a^3* \\
& b^4*c*d^2*e^7 - 1872*a^4*b*c^3*d^3*e^6)/(4*(64*a^5*c^4 - a^2*b^6*c + 12*a^3 \\
& *b^4*c^2 - 48*a^4*b^2*c^3)) + (((6144*a^5*c^7*d^3 + 16*a*b^8*c^3*d^3 - 1024 \\
& *a^6*b*c^5*e^3 + 6144*a^6*c^6*d*e^2 - 288*a^2*b^6*c^4*d^3 + 1920*a^3*b^4*c^ \\
& 5*d^3 - 5632*a^4*b^2*c^6*d^3 + 16*a^3*b^7*c^2*e^3 - 192*a^4*b^5*c^3*e^3 + 7 \\
& 68*a^5*b^3*c^4*e^3 - 3072*a^5*b*c^6*d^2*e + 48*a^2*b^7*c^3*d^2*e - 576*a^3* \\
& b^5*c^4*d^2*e - 96*a^3*b^6*c^3*d*e^2 + 2304*a^4*b^3*c^5*d^2*e + 1152*a^4*b^ \\
& 4*c^4*d*e^2 - 4608*a^5*b^2*c^5*d*e^2)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b \\
& ^4*c^2 - 48*a^4*b^2*c^3)) - (x*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11 \\
& *e^6 + 3840*a^5*b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^{(1/2)} + 27*a^4*b \\
& ^9*c*e^6 + 3840*a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^{(1/2) - 9216 \\
& *a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^ \\
& 6*d^6 - 3840*a^4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)^{(1/2) - 288*a \\
& ^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 - b^2*c^3*d^6* \\
& (- (4*a*c - b^2)^9)^{(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - \\
& 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + \\
& 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^ \\
& 4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4* \\
& d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 + 9*a^2*c^3* \\
& d^4*e^2*(-(4*a*c - b^2)^9)^{(1/2) - 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^{(1/ \\
& 2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576* \\
& a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^ \\
& 5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6 \\
& *b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 - 6*a*b*c
\end{aligned}$$

$$\begin{aligned}
& ^3d^5e*(-(4ac - b^2)^9)^{(1/2)} + 6a^3b^3c^3d^5e*(-(4ac - b^2)^9)^{(1/2)} \\
& )/(32*(4096a^9c^9 + a^3b^{12}c^3 - 24a^4b^{10}c^4 + 240a^5b^8c^5 - 1 \\
& 280a^6b^6c^6 + 3840a^7b^4c^7 - 6144a^8b^2c^8))^{(1/2)}*(1024a^5b^3c^6 - 16a^2b^7c^3 + 192a^3b^5c^4 - 768a^4b^3c^5))/(2*(16a^4c^3 + \\
& a^2b^4c - 8a^3b^2c^2)))*((27a^9c^4d^6 - b^{11}c^3d^6 - a^3b^{11}e^6 + 3840a^5b^3c^8d^6 + 9a^4c^4d^6*(-(4ac - b^2)^9)^{(1/2)} + 27a^4b^9c^5e^6 + 3840a^8b^3c^5e^6 - 9a^4c^5e^6*(-(4ac - b^2)^9)^{(1/2)} - 9216a^6c^8d^5e - 9216a^8c^6d^5e^5 - 288a^2b^7c^5d^6 + 1504a^3b^5c^6d^6 - 3840a^4b^3c^7d^6 + a^3b^2e^6*(-(4ac - b^2)^9)^{(1/2)} - 288a^5b^7c^2e^6 + 1504a^6b^5c^3e^6 - 3840a^7b^3c^4e^6 - b^2c^3d^6*(-(4ac - b^2)^9)^{(1/2)} - 18432a^7c^7d^3e^3 + 9a^2b^9c^3d^4e^2 - 384a^3b^7c^4d^4e^2 + 88a^3b^8c^3d^3e^3 + 9a^3b^9c^2d^2e^4 + 3744a^4b^5c^5d^4e^2 - 768a^4b^6c^4d^3e^3 - 384a^4b^7c^3d^2e^4 - 13824a^5b^3c^6d^4e^2 + 768a^5b^4c^5d^3e^3 + 3744a^5b^5c^4d^2e^4 + 8192a^6b^2c^6d^3e^3 - 13824a^6b^3c^5d^2e^4 + 9a^2c^3d^4e^2*(-(4ac - b^2)^9)^{(1/2)} - 9a^3c^2d^2e^4*(-(4ac - b^2)^9)^{(1/2)} - 6a^3b^{10}c^3d^5e - 6a^3b^{10}c^3d^5e + 108a^2b^8c^4d^5e - 576a^3b^6c^5d^5e + 384a^4b^4c^6d^5e + 108a^4b^8c^2d^5e + 4608a^5b^2c^7d^5e - 576a^5b^6c^3d^5e + 17664a^6b^3c^7d^4e^2 + 384a^6b^4c^4d^5e + 17664a^7b^2c^6d^2e^4 + 4608a^7b^2c^5d^5e - 6a^3b^3c^3d^5e*(-(4ac - b^2)^9)^{(1/2)} + 6a^3b^3c^3d^5e*(-(4ac - b^2)^9)^{(1/2)}))/(32*(4096a^9c^9 + a^3b^{12}c^3 - 24a^4b^{10}c^4 + 240a^5b^8c^5 - 1280a^6b^6c^6 + 3840a^7b^4c^7 - 6144a^8b^2c^8))^{(1/2)} - (x*(72a^5c^3e^6 - 72a^2c^6d^6 - a^2b^6e^6 - b^4c^4d^6 + 14a^3b^2c^5d^6 + 16a^3b^4c^5e^6 - 74a^4b^2c^2e^6 - 72a^3c^5d^4e^2 + 72a^4c^4d^2e^4 - 102a^2b^2c^4d^4e^2 + 44a^2b^3c^3d^3e^3 + 9a^2b^4c^2d^2e^4 - 174a^3b^2c^3d^2e^4 - 6a^3b^3c^4d^5e + 120a^2b^3c^5d^5e - 6a^2b^5c^3d^5e + 24a^4b^3c^3d^5e + 144a^3b^3c^4d^3e^3 + 42a^3b^3c^2d^5e))/(2*(16a^4c^3 + a^2b^4c - 8a^3b^2c^2)))*((27a^9c^4d^6 - b^{11}c^3d^6 - a^3b^{11}e^6 + 3840a^5b^3c^8d^6 + 9a^4c^4d^6*(-(4ac - b^2)^9)^{(1/2)} + 27a^4b^9c^5e^6 + 3840a^8b^3c^5e^6 - 9a^4c^5e^6*(-(4ac - b^2)^9)^{(1/2)} - 9216a^6c^8d^5e - 9216a^8c^6d^5e^5 - 288a^2b^7c^5d^6 + 1504a^3b^5c^6d^6 - 3840a^4b^3c^7d^6 + a^3b^2e^6*(-(4ac - b^2)^9)^{(1/2)} - 288a^5b^7c^2e^6 + 1504a^6b^5c^3e^6 - 3840a^7b^3c^4e^6 - b^2c^3d^6*(-(4ac - b^2)^9)^{(1/2)} - 18432a^7c^7d^3e^3 + 9a^2b^9c^3d^4e^2 - 384a^3b^7c^4d^4e^2 + 88a^3b^8c^3d^3e^3 + 9a^3b^9c^2d^2e^4 + 3744a^4b^5c^5d^4e^2 - 768a^4b^6c^4d^3e^3 - 384a^4b^7c^3d^2e^4 - 13824a^5b^3c^6d^4e^2 + 768a^5b^4c^5d^3e^3 + 3744a^5b^5c^4d^2e^4 + 8192a^6b^2c^6d^3e^3 - 13824a^6b^3c^5d^2e^4 + 9a^2c^3d^4e^2*(-(4ac - b^2)^9)^{(1/2)} - 9a^3c^2d^2e^4*(-(4ac - b^2)^9)^{(1/2)} - 6a^3b^{10}c^3d^5e - 6a^3b^{10}c^3d^5e + 108a^2b^8c^4d^5e - 576a^3b^6c^5d^5e + 384a^4b^4c^6d^5e + 108a^4b^8c^2d^5e + 4608a^5b^2c^7d^5e - 576a^5b^6c^3d^5e + 17664a^6b^3c^7d^4e^2 + 384a^6b^4c^4d^5e + 17664a^7b^2c^6d^2e^4 + 4608a^7b^2c^5d^5e - 6a^3b^3c^3d^5e*(-(4ac - b^2)^9)^{(1/2)} + 6a^3b^3c^3d^5e*(-(4ac - b^2)^9)^{(1/2)}))/(32*(4096a^9c^9 + a^3b^{12}c^3 - 24a^4b^{10}c^4 + 240a^5b^8c^5 - 1280a^6b^6c^6 + 3840a^7b^4c^7 - 6144a^8b^2c^8))^{(1/2)} + (((6144a^5c^7d^3 + 16a^3b^8c^3d^3 - 1024a^6b^3c^5e^3 + 6144a^6c^6d^5e^2 - 288a^2b^6c^4d^3 + 1920a^3b^4c^5d^3 - 5632a^4b^2c^6d^3 + 16a^3b^7c^2e^3 - 192a^4b^5c^3e^3 + 768a^5b^3c^4e^3 - 3072a^5b^3c^6d^2e + 48a^2b^7c^3d^2e - 576a^3b^5c^4d^2e - 96a^3b^6c^3d^2e + 2304a^4b^3c^5d^2e + 1152a^4b^4c^4d^2e - 4608a^5b^2c^5d^2e)/(8*(64a^5c^4 - a^2b^6c + 12a^3b^4c^2 - 48a^4b^2c^3)) + (x*((27a^9c^4d^6 - b^{11}c^3d^6 - a^3b^{11}e^6 + 3840a^5b^3c^8d^6 + 9a^4c^4d^6*(-(4ac - b^2)^9)^{(1/2)} + 27a^4b^9c^5e^6 + 3840a^8b^3c^5e^6 - 9a^4c^5e^6*(-(4ac - b^2)^9)^{(1/2)} - 9216a^6c^8d^5e - 9216a^8c^6d^5e^5 - 288a^2b^7c^5d^6 + 1504a^3b^5c^6d^6 - 3840a^4b^3c^7d^6 + a^3b^2e^6*(-(4ac - b^2)^9)^{(1/2)} - 288a^5b^7c^2e^6 + 1504a^6b^5c^3e^6 - 3840a^7b^3c^4e^6 - b^2c^3d^6*(-(4ac - b^
\end{aligned}$$

$$\begin{aligned}
& 2)^9)^{(1/2)} - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 + 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 - 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^{(1/2)} + 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^{(1/2)}*(1024*a^5*b*c^6 - 16*a^2*b^7*c^3 + 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^{(1/2)} + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^{(1/2)} - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 - b^2*c^3*d^6*(-(4*a*c - b^2)^9)^{(1/2)} - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 + 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 - 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^{(1/2)} + 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^{(1/2)} + (x*(72*a^5*c^3*e^6 - 72*a^2*c^6*d^6 - a^2*b^6*e^6 - b^4*c^4*d^6 + 14*a*b^2*c^5*d^6 + 16*a^3*b^4*c*e^6 - 74*a^4*b^2*c^2*e^6 - 72*a^3*c^5*d^4*e^2 + 72*a^4*c^4*d^2*e^4 - 102*a^2*b^2*c^4*d^4*e^2 + 44*a^2*b^3*c^3*d^3*e^3 + 9*a^2*b^4*c^2*d^2*e^4 - 174*a^3*b^2*c^3*d^2*e^4 - 6*a*b^3*c^4*d^5*e + 120*a^2*b*c^5*d^5*e - 6*a^2*b^5*c*d*e^5 + 24*a^4*b*c^3*d*e^5 + 144*a^3*b*c^4*d^3*e^3 + 42*a^3*b^3*c^2*d*e^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^{(1/2)} + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^{(1/2)} - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)^{(1/2)} - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 - b^2*c^3*d^6*(-(4*a*c - b^2)^9)^{(1/2)} - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 + 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^{(1/2)} - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 - 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^{(1/2)} + 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^{(1/2)})*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^{(1/2)} + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6
\end{aligned}$$

$$\begin{aligned} &^6 - 9a^4c^2e^6(-4ac - b^2)^9)^{1/2} - 9216a^6c^8d^5e - 9216a^8c^6d^5e^5 - 288a^2b^7c^5d^6 + 1504a^3b^5c^6d^6 - 3840a^4b^3c^7d^6 \\ &+ a^3b^2e^6(-4ac - b^2)^9)^{1/2} - 288a^5b^7c^2e^6 + 1504a^6b^5c^3e^6 - 3840a^7b^3c^4e^6 - b^2c^3d^6(-4ac - b^2)^9)^{1/2} - \\ &18432a^7c^7d^3e^3 + 9a^2b^9c^3d^4e^2 - 384a^3b^7c^4d^4e^2 + 88a^3b^8c^3d^3e^3 + 9a^3b^9c^2d^2e^4 + 3744a^4b^5c^5d^4e^2 - \\ &768a^4b^6c^4d^3e^3 - 384a^4b^7c^3d^2e^4 - 13824a^5b^3c^6d^4e^2 + 768a^5b^4c^5d^3e^3 + 3744a^5b^5c^4d^2e^4 + 8192a^6b^2c^6d^3e^3 \\ &- 13824a^6b^3c^5d^2e^4 + 9a^2c^3d^4e^2(-4ac - b^2)^9)^{1/2} - 9a^3c^2d^2e^4(-4ac - b^2)^9)^{1/2} - 6ab^{10}c^3d^5e - 6 \\ &a^3b^{10}c^2d^5e + 108a^2b^8c^4d^5e - 576a^3b^6c^5d^5e + 384a^4b^4c^6d^5e + 108a^4b^8c^2d^5e + 4608a^5b^2c^7d^5e - 576a^5b^6c^3d^5e \\ &+ 17664a^6b^4c^4d^5e + 384a^6b^4c^4d^5e + 17664a^7b^2c^6d^2e^4 + 4608a^7b^2c^5d^5e - 6ab^3c^3d^5e(-4ac - b^2)^9)^{1/2} \\ &+ 6a^3b^3c^3d^5e(-4ac - b^2)^9)^{1/2} / (32(4096a^9c^9 + a^3b^{12}c^3 - 24a^4b^{10}c^4 + 240a^5b^8c^5 - 1280a^6b^6c^6 + 3840a^7b^4c^7 \\ &- 6144a^8b^2c^8))^{1/2} * i \end{aligned}$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*3/(c\*x\*\*4+b\*x\*\*2+a)\*\*2,x)

[Out] Timed out



$$3.194 \quad \int \frac{(d+ex^2)^2}{(a+bx^2+cx^4)^2} dx$$

**Optimal.** Leaf size=386

$$\frac{x(x^2(abe^2 - 4acde + bcd^2) - 2abde - 2a(cd^2 - ae^2) + b^2d^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\left(\frac{b^2(cd^2 - ae^2) + 8abcde - 4ac(ae^2 + 3cd^2)}{\sqrt{b^2 - 4ac}} + abe^2 - 4acde + b^2d^2\right)}{2\sqrt{2}a\sqrt{c}(b^2 - 4ac)\sqrt{b - \sqrt{b^2 - 4ac}}}$$

**Rubi [A]** time = 2.08, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, number of rules / integrand size = 0.125, Rules used = {1205, 1166, 205}

$$\frac{x(x^2(abe^2 - 4acde + bcd^2) - 2abde - 2a(cd^2 - ae^2) + b^2d^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\left(\frac{b^2(cd^2 - ae^2) + 8abcde - 4ac(ae^2 + 3cd^2)}{\sqrt{b^2 - 4ac}} + abe^2 - 4acde + bcd^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}a\sqrt{c}(b^2 - 4ac)\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\left(\frac{b^2(cd^2 - ae^2) + 8abcde - 4ac(ae^2 + 3cd^2)}{\sqrt{b^2 - 4ac}} + abe^2 - 4acde + bcd^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2 - 4ac} + b}}\right)}{2\sqrt{2}a\sqrt{c}(b^2 - 4ac)\sqrt{\sqrt{b^2 - 4ac} + b}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^2/(a + b\*x^2 + c\*x^4)^2, x]

[Out] (x\*(b^2\*d^2 - 2\*a\*b\*d\*e - 2\*a\*(c\*d^2 - a\*e^2) + (b\*c\*d^2 - 4\*a\*c\*d\*e + a\*b\*e^2)\*x^2))/(2\*a\*(b^2 - 4\*a\*c)\*(a + b\*x^2 + c\*x^4)) + ((b\*c\*d^2 - 4\*a\*c\*d\*e + a\*b\*e^2 + (8\*a\*b\*c\*d\*e + b^2\*(c\*d^2 - a\*e^2) - 4\*a\*c\*(3\*c\*d^2 + a\*e^2))/Sqrt[b^2 - 4\*a\*c])\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b - Sqrt[b^2 - 4\*a\*c]]]/(2\*Sqrt[2]\*a\*Sqrt[c]\*(b^2 - 4\*a\*c)\*Sqrt[b - Sqrt[b^2 - 4\*a\*c]]) + ((b\*c\*d^2 - 4\*a\*c\*d\*e + a\*b\*e^2 - (8\*a\*b\*c\*d\*e + b^2\*(c\*d^2 - a\*e^2) - 4\*a\*c\*(3\*c\*d^2 + a\*e^2))/Sqrt[b^2 - 4\*a\*c])\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b + Sqrt[b^2 - 4\*a\*c]]]/(2\*Sqrt[2]\*a\*Sqrt[c]\*(b^2 - 4\*a\*c)\*Sqrt[b + Sqrt[b^2 - 4\*a\*c]])

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 1166

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[e/2 + (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 - q/2 + c\*x^2), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - a\*e^2, 0] && PosQ[b^2 - 4\*a\*c]

#### Rule 1205

Int[((d\_) + (e\_.)\*(x\_)^2)^(q)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p), x\_Symbol] := With[{f = Coeff[PolynomialRemainder[(d + e\*x^2)^q, a + b\*x^2 + c\*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e\*x^2)^q, a + b\*x^2 + c\*x^4, x], x, 2]}, Simp[(x\*(a + b\*x^2 + c\*x^4)^(p + 1)\*(a\*b\*g - f\*(b^2 - 2\*a\*c) - c\*(b\*f - 2\*a\*g)\*x^2))/(2\*a\*(p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/(2\*a\*(p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x^2 + c\*x^4)^(p + 1)\*ExpandToSum[2\*a\*(p + 1)\*(b^2 - 4\*a\*c)\*PolynomialQuotient[(d + e\*x^2)^q, a + b\*x^2 + c\*x^4, x] + b^2\*f\*(2\*p + 3) - 2\*a\*c\*f\*(4\*p + 5) - a\*b\*g + c\*(4\*p + 7)\*(b\*f - 2\*a\*g)\*x^2, x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]

Rubi steps

$$\int \frac{(d + ex^2)^2}{(a + bx^2 + cx^4)^2} dx = \frac{x(b^2d^2 - 2abde - 2a(cd^2 - ae^2) + (bcd^2 - 4acde + abe^2)x^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\int \frac{-b^2d^2 - 2abde + 2a(3cd^2 + ae^2)}{a + bx^2} dx}{2a(b^2 - 4ac)}$$

$$= \frac{x(b^2d^2 - 2abde - 2a(cd^2 - ae^2) + (bcd^2 - 4acde + abe^2)x^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{(bcd^2 - 4acde + abe^2 - 2a^2e^2)}{2a(b^2 - 4ac)}$$

$$= \frac{x(b^2d^2 - 2abde - 2a(cd^2 - ae^2) + (bcd^2 - 4acde + abe^2)x^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{(bcd^2 - 4acde + abe^2 - 2a^2e^2)}{2\sqrt{2}a}$$

**Mathematica [A]** time = 1.11, size = 415, normalized size = 1.08

$$\frac{2x(2a^2d^2 + abe^2cx^2 - 2d) - 2acd(d + 2ex^2) + b^2d^2 + bcd^2 + a^2e^2}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{2}(b^2(cx^2 - ac^2) - 4ac(d(\sqrt{b^2 - 4ac} + a) + 3cd^2) + b(cd(d\sqrt{b^2 - 4ac} + 8ac) + ac^2\sqrt{b^2 - 4ac})) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{c - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{c}(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{2}(b^2(ac^2 - cx^2) + 4ac((ac - d\sqrt{b^2 - 4ac}) + 3cd^2) + b(cd(d\sqrt{b^2 - 4ac} - 8ac) + ac^2\sqrt{b^2 - 4ac})) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{\sqrt{b^2 - 4ac} + b}}\right)}{\sqrt{c}(b^2 - 4ac)^{3/2}\sqrt{b^2 - 4ac + b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(a + b*x^2 + c*x^4)^2,x]
[Out] ((2*x*(b^2*d^2 + 2*a^2*e^2 + b*c*d^2*x^2 + a*b*e*(-2*d + e*x^2) - 2*a*c*d*(d + 2*e*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(b^2*(c*d^2 - a*e^2) - 4*a*c*(3*c*d^2 + e*(Sqrt[b^2 - 4*a*c]*d + a*e)) + b*(a*Sqrt[b^2 - 4*a*c]*e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d + 8*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2*(-(c*d^2) + a*e^2) + b*(a*Sqrt[b^2 - 4*a*c]*e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d - 8*a*e)) + 4*a*c*(3*c*d^2 + e*(-(Sqrt[b^2 - 4*a*c]*d) + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^2)^2}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^2)^2/(a + b*x^2 + c*x^4)^2,x]
[Out] IntegrateAlgebraic[(d + e*x^2)^2/(a + b*x^2 + c*x^4)^2, x]
```

**fricas [B]** time = 13.15, size = 7338, normalized size = 19.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
[Out] 1/4*(2*(b*c*d^2 - 4*a*c*d*e + a*b*e^2)*x^3 + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d^3*e - 2*(a^2*b^3*c - 52*a^3*b*c^2)*d^2*e^2 - 8*(3*a^3*b^2*c + 4*a^4*c^2)*d*e^3 + (a^3*b^3 + 12*a^4*b*c)*e^4 + (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*sqrt(-((16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2
```

$$\begin{aligned}
& 2 - 9a^2b^3c^3)d^7e - 12(a^2b^2c^2 + 3a^3c^3)d^6e^2 + 2(a^3b^2c \\
& c - 11a^4c^2)d^4e^4)/(a^6b^6c^2 - 12a^7b^4c^3 + 48a^8b^2c^4 - 6 \\
& 4a^9c^5)))/(a^3b^6c - 12a^4b^4c^2 + 48a^5b^2c^3 - 64a^6c^4)) * \log \\
& (((5b^4c^3 - 81a^2b^2c^4 + 324a^2c^5)d^8 - 2(3b^5c^2 - 65a^2b^3c \\
& ^3 + 324a^2b^3c^4)d^7e + (b^6c - 51a^2b^4c^2 + 336a^2b^2c^3 + 432a^ \\
& ^3c^4)d^6e^2 + 2(3a^2b^5c - 27a^2b^3c^2 - 244a^3b^3c^3)d^5e^3 + \\
& (3a^2b^4c + 150a^3b^2c^2 + 152a^4c^3)d^4e^4 - 10(a^3b^3c + 12a \\
& a^4b^3c^2)d^3e^5 - (a^3b^4 - 24a^4b^2c - 48a^5c^2)d^2e^6 - 2(a^4 \\
& b^3 + 12a^5b^3c)d^2e^7 + (3a^5b^2 + 4a^6c)e^8)*x + 1/2*\sqrt{1/2}*((b \\
& ^8c - 23a^2b^6c^2 + 190a^2b^4c^3 - 672a^3b^2c^4 + 864a^4c^5)d^6 \\
& + 6(a^2b^7c - 15a^2b^5c^2 + 72a^3b^3c^3 - 112a^4b^3c^4)d^5e + 2( \\
& 2a^2b^6c - a^3b^4c^2 - 88a^4b^2c^3 + 240a^5c^4)d^4e^2 - 12(a^3 \\
& b^5c - 8a^4b^3c^2 + 16a^5b^3c^3)d^3e^3 - (a^3b^6 - 18a^4b^4c + \\
& 96a^5b^2c^2 - 160a^6c^3)d^2e^4 - 2(a^4b^5 - 8a^5b^3c + 16a^6b \\
& ^3c^2)d^2e^5 + 2(a^5b^4 - 8a^6b^2c + 16a^7c^2)e^6 - ((a^3b^9c - 20 \\
& a^4b^7c^2 + 144a^5b^5c^3 - 448a^6b^3c^4 + 512a^7b^3c^5)d^2 + 2( \\
& a^4b^8c - 8a^5b^6c^2 + 128a^7b^2c^4 - 256a^8c^5)d^2e - 4(a^5b^7 \\
& ^3c - 12a^6b^5c^2 + 48a^7b^3c^3 - 64a^8b^3c^4)e^2)*\sqrt{-(16a^3b^3c \\
& ^2d^5e^3 + 8a^4b^3c^2d^3e^5 - 4a^5c^2d^2e^6 - a^6e^8 - (b^4c^2 - 18a \\
& a^2b^2c^3 + 81a^2c^4)d^8 - 8(a^2b^3c^2 - 9a^2b^3c^3)d^7e - 12(a^2b \\
& ^2c^2 + 3a^3c^3)d^6e^2 + 2(a^3b^2c - 11a^4c^2)d^4e^4)/(a^6b^6c \\
& c^2 - 12a^7b^4c^3 + 48a^8b^2c^4 - 64a^9c^5)))*\sqrt{-(b^5c - 15a^2 \\
& b^3c^2 + 60a^2b^3c^3)d^4 + 4(a^2b^4c - 6a^2b^2c^2 - 24a^3c^3)d^3e \\
& e - 2(a^2b^3c - 52a^3b^3c^2)d^2e^2 - 8(3a^3b^2c + 4a^4c^2)d^2e^3 + \\
& (a^3b^3 + 12a^4b^3c)e^4 + (a^3b^6c - 12a^4b^4c^2 + 48a^5b^2c^3 \\
& ^3 - 64a^6c^4)*\sqrt{-(16a^3b^3c^2d^5e^3 + 8a^4b^3c^2d^3e^5 - 4a^5c^2 \\
& d^2e^6 - a^6e^8 - (b^4c^2 - 18a^2b^2c^3 + 81a^2c^4)d^8 - 8(a^2b^3c^2 \\
& ^2 - 9a^2b^3c^3)d^7e - 12(a^2b^2c^2 + 3a^3c^3)d^6e^2 + 2(a^3b^2c \\
& c - 11a^4c^2)d^4e^4)/(a^6b^6c^2 - 12a^7b^4c^3 + 48a^8b^2c^4 - 6 \\
& 4a^9c^5)))/(a^3b^6c - 12a^4b^4c^2 + 48a^5b^2c^3 - 64a^6c^4)) - \\
& \sqrt{1/2}*((a^2b^2c - 4a^2c^2)*x^4 + a^2b^2 - 4a^3c + (a^2b^3 - 4a^2 \\
& b^3c)*x^2)*\sqrt{-(b^5c - 15a^2b^3c^2 + 60a^2b^3c^3)d^4 + 4(a^2b^4c - 6 \\
& a^2b^2c^2 - 24a^3c^3)d^3e - 2(a^2b^3c - 52a^3b^3c^2)d^2e^2 - 8 \\
& (3a^3b^2c + 4a^4c^2)d^2e^3 + (a^3b^3 + 12a^4b^3c)e^4 + (a^3b^6c \\
& - 12a^4b^4c^2 + 48a^5b^2c^3 - 64a^6c^4)*\sqrt{-(16a^3b^3c^2d^5e^3 \\
& + 8a^4b^3c^2d^3e^5 - 4a^5c^2d^2e^6 - a^6e^8 - (b^4c^2 - 18a^2b^2c^3 \\
& + 81a^2c^4)d^8 - 8(a^2b^3c^2 - 9a^2b^3c^3)d^7e - 12(a^2b^2c^2 + 3 \\
& a^3c^3)d^6e^2 + 2(a^3b^2c - 11a^4c^2)d^4e^4)/(a^6b^6c^2 - 12a \\
& ^7b^4c^3 + 48a^8b^2c^4 - 64a^9c^5)))/(a^3b^6c - 12a^4b^4c^2 + 4 \\
& 8a^5b^2c^3 - 64a^6c^4))*\log(((5b^4c^3 - 81a^2b^2c^4 + 324a^2c^5) * \\
& d^8 - 2(3b^5c^2 - 65a^2b^3c^3 + 324a^2b^3c^4)d^7e + (b^6c - 51a^2b^ \\
& 4c^2 + 336a^2b^2c^3 + 432a^3c^4)d^6e^2 + 2(3a^2b^5c - 27a^2b^3c^2 \\
& ^2 - 244a^3b^3c^3)d^5e^3 + (3a^2b^4c + 150a^3b^2c^2 + 152a^4c^3) \\
& )d^4e^4 - 10(a^3b^3c + 12a^4b^3c^2)d^3e^5 - (a^3b^4 - 24a^4b^2c \\
& - 48a^5c^2)d^2e^6 - 2(a^4b^3 + 12a^5b^3c)d^2e^7 + (3a^5b^2 + 4a^6 \\
& c)e^8)*x - 1/2*\sqrt{1/2}*((b^8c - 23a^2b^6c^2 + 190a^2b^4c^3 - 672a \\
& a^3b^2c^4 + 864a^4c^5)d^6 + 6(a^2b^7c - 15a^2b^5c^2 + 72a^3b^3c^3 \\
& ^3 - 112a^4b^3c^4)d^5e + 2(2a^2b^6c - a^3b^4c^2 - 88a^4b^2c^3 + \\
& 240a^5c^4)d^4e^2 - 12(a^3b^5c - 8a^4b^3c^2 + 16a^5b^3c^3)d^3e \\
& ^3 - (a^3b^6 - 18a^4b^4c + 96a^5b^2c^2 - 160a^6c^3)d^2e^4 - 2(a \\
& ^4b^5 - 8a^5b^3c + 16a^6b^3c^2)d^2e^5 + 2(a^5b^4 - 8a^6b^2c + 16a \\
& a^7c^2)e^6 - ((a^3b^9c - 20a^4b^7c^2 + 144a^5b^5c^3 - 448a^6b^3 \\
& ^3c^4 + 512a^7b^3c^5)d^2 + 2(a^4b^8c - 8a^5b^6c^2 + 128a^7b^2c^4 \\
& - 256a^8c^5)d^2e - 4(a^5b^7c - 12a^6b^5c^2 + 48a^7b^3c^3 - 64a^8 \\
& b^3c^4)e^2)*\sqrt{-(16a^3b^3c^2d^5e^3 + 8a^4b^3c^2d^3e^5 - 4a^5c^2d^2 \\
& e^6 - a^6e^8 - (b^4c^2 - 18a^2b^2c^3 + 81a^2c^4)d^8 - 8(a^2b^3c^2 - \\
& 9a^2b^3c^3)d^7e - 12(a^2b^2c^2 + 3a^3c^3)d^6e^2 + 2(a^3b^2c - \\
& 11a^4c^2)d^4e^4)/(a^6b^6c^2 - 12a^7b^4c^3 + 48a^8b^2c^4 - 64a \\
& ^9c^5)))*\sqrt{-(b^5c - 15a^2b^3c^2 + 60a^2b^3c^3)d^4 + 4(a^2b^4c - 6
\end{aligned}$$

$$\begin{aligned}
& *a^2*b^2*c^2 - 24*a^3*c^3)*d^3*e - 2*(a^2*b^3*c - 52*a^3*b*c^2)*d^2*e^2 - 8 \\
& *(3*a^3*b^2*c + 4*a^4*c^2)*d*e^3 + (a^3*b^3 + 12*a^4*b*c)*e^4 + (a^3*b^6*c \\
& - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*\sqrt{-(16*a^3*b*c^2*d^5*e^3 \\
& + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 \\
& + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3 \\
& *a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a \\
& ^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 4 \\
& 8*a^5*b^2*c^3 - 64*a^6*c^4))) + \sqrt{1/2}*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2* \\
& b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*\sqrt{-((b^5*c - 15*a*b^3*c^2 + 60* \\
& a^2*b*c^3)*d^4 + 4*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d^3*e - 2*(a^2*b^ \\
& 3*c - 52*a^3*b*c^2)*d^2*e^2 - 8*(3*a^3*b^2*c + 4*a^4*c^2)*d*e^3 + (a^3*b^3 \\
& + 12*a^4*b*c)*e^4 - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c \\
& ^4)*\sqrt{-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6 \\
& *e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c \\
& ^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^ \\
& 2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/ \\
& (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))*\log(((5*b^4*c^3 \\
& - 81*a*b^2*c^4 + 324*a^2*c^5)*d^8 - 2*(3*b^5*c^2 - 65*a*b^3*c^3 + 324*a^2* \\
& b*c^4)*d^7*e + (b^6*c - 51*a*b^4*c^2 + 336*a^2*b^2*c^3 + 432*a^3*c^4)*d^6*e \\
& ^2 + 2*(3*a*b^5*c - 27*a^2*b^3*c^2 - 244*a^3*b*c^3)*d^5*e^3 + (3*a^2*b^4*c \\
& + 150*a^3*b^2*c^2 + 152*a^4*c^3)*d^4*e^4 - 10*(a^3*b^3*c + 12*a^4*b*c^2)*d^ \\
& 3*e^5 - (a^3*b^4 - 24*a^4*b^2*c - 48*a^5*c^2)*d^2*e^6 - 2*(a^4*b^3 + 12*a^5 \\
& *b*c)*d*e^7 + (3*a^5*b^2 + 4*a^6*c)*e^8)*x + 1/2*\sqrt{1/2}*((b^8*c - 23*a*b \\
& ^6*c^2 + 190*a^2*b^4*c^3 - 672*a^3*b^2*c^4 + 864*a^4*c^5)*d^6 + 6*(a*b^7*c \\
& - 15*a^2*b^5*c^2 + 72*a^3*b^3*c^3 - 112*a^4*b*c^4)*d^5*e + 2*(2*a^2*b^6*c - \\
& a^3*b^4*c^2 - 88*a^4*b^2*c^3 + 240*a^5*c^4)*d^4*e^2 - 12*(a^3*b^5*c - 8*a^ \\
& 4*b^3*c^2 + 16*a^5*b*c^3)*d^3*e^3 - (a^3*b^6 - 18*a^4*b^4*c + 96*a^5*b^2*c^ \\
& 2 - 160*a^6*c^3)*d^2*e^4 - 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d*e^5 + \\
& 2*(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*e^6 + ((a^3*b^9*c - 20*a^4*b^7*c^2 \\
& + 144*a^5*b^5*c^3 - 448*a^6*b^3*c^4 + 512*a^7*b*c^5)*d^2 + 2*(a^4*b^8*c - 8 \\
& *a^5*b^6*c^2 + 128*a^7*b^2*c^4 - 256*a^8*c^5)*d*e - 4*(a^5*b^7*c - 12*a^6*b \\
& ^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*e^2)*\sqrt{-(16*a^3*b*c^2*d^5*e^3 + \\
& 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 8 \\
& 1*a^2*c^4)*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^ \\
& 3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7* \\
& b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*\sqrt{-((b^5*c - 15*a*b^3*c^2 + 60* \\
& a^2*b*c^3)*d^4 + 4*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d^3*e - 2*(a^2*b^ \\
& 3*c - 52*a^3*b*c^2)*d^2*e^2 - 8*(3*a^3*b^2*c + 4*a^4*c^2)*d*e^3 + (a^3*b^3 \\
& + 12*a^4*b*c)*e^4 - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c \\
& ^4)*\sqrt{-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6 \\
& *e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c \\
& ^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^ \\
& 2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/ \\
& (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))) - \sqrt{1/2}*(( \\
& a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*\sqrt{ \\
& t(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*(a*b^4*c - 6*a^2*b^2*c^2 \\
& - 24*a^3*c^3)*d^3*e - 2*(a^2*b^3*c - 52*a^3*b*c^2)*d^2*e^2 - 8*(3*a^3*b^2*c \\
& + 4*a^4*c^2)*d*e^3 + (a^3*b^3 + 12*a^4*b*c)*e^4 - (a^3*b^6*c - 12*a^4*b^4* \\
& c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*\sqrt{-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c* \\
& d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4) \\
& *d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6 \\
& *e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + \\
& 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 \\
& - 64*a^6*c^4))*\log(((5*b^4*c^3 - 81*a*b^2*c^4 + 324*a^2*c^5)*d^8 - 2*(3*b^ \\
& 5*c^2 - 65*a*b^3*c^3 + 324*a^2*b*c^4)*d^7*e + (b^6*c - 51*a*b^4*c^2 + 336*a \\
& ^2*b^2*c^3 + 432*a^3*c^4)*d^6*e^2 + 2*(3*a*b^5*c - 27*a^2*b^3*c^2 - 244*a^3 \\
& *b*c^3)*d^5*e^3 + (3*a^2*b^4*c + 150*a^3*b^2*c^2 + 152*a^4*c^3)*d^4*e^4 - 1 \\
& 0*(a^3*b^3*c + 12*a^4*b*c^2)*d^3*e^5 - (a^3*b^4 - 24*a^4*b^2*c - 48*a^5*c^2) \\
& *d^2*e^6 - 2*(a^4*b^3 + 12*a^5*b*c)*d*e^7 + (3*a^5*b^2 + 4*a^6*c)*e^8)*x -
\end{aligned}$$

$$\begin{aligned} & 1/2*\sqrt{1/2}*((b^8*c - 23*a*b^6*c^2 + 190*a^2*b^4*c^3 - 672*a^3*b^2*c^4 + \\ & 864*a^4*c^5)*d^6 + 6*(a*b^7*c - 15*a^2*b^5*c^2 + 72*a^3*b^3*c^3 - 112*a^4* \\ & b*c^4)*d^5*e + 2*(2*a^2*b^6*c - a^3*b^4*c^2 - 88*a^4*b^2*c^3 + 240*a^5*c^4) \\ & *d^4*e^2 - 12*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3*e^3 - (a^3*b^6 \\ & - 18*a^4*b^4*c + 96*a^5*b^2*c^2 - 160*a^6*c^3)*d^2*e^4 - 2*(a^4*b^5 - 8*a^ \\ & 5*b^3*c + 16*a^6*b*c^2)*d*e^5 + 2*(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*e^6 \\ & + ((a^3*b^9*c - 20*a^4*b^7*c^2 + 144*a^5*b^5*c^3 - 448*a^6*b^3*c^4 + 512*a^ \\ & 7*b*c^5)*d^2 + 2*(a^4*b^8*c - 8*a^5*b^6*c^2 + 128*a^7*b^2*c^4 - 256*a^8*c^5) \\ & )*d*e - 4*(a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*e^2) \\ & *sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^ \\ & 8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3) \\ & )*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)* \\ & d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5))*sqrt \\ & t(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*(a*b^4*c - 6*a^2*b^2*c^2 \\ & - 24*a^3*c^3)*d^3*e - 2*(a^2*b^3*c - 52*a^3*b*c^2)*d^2*e^2 - 8*(3*a^3*b^2*c \\ & + 4*a^4*c^2)*d*e^3 + (a^3*b^3 + 12*a^4*b*c)*e^4 - (a^3*b^6*c - 12*a^4*b^4* \\ & c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c* \\ & d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4) \\ & )*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6 \\ & *e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + \\ & 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 \\ & - 64*a^6*c^4)) - 2*(2*a*b*d*e - 2*a^2*e^2 - (b^2 - 2*a*c)*d^2)*x)/((a*b^2 \\ & *c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) \end{aligned}$$

**giac [B]** time = 1.85, size = 6390, normalized size = 16.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2/(c\*x^4+b\*x^2+a)^2,x, algorithm="giac")

[Out]  $1/2*(b*c*d^2*x^3 - 4*a*c*d*x^3*e + a*b*x^3*e^2 + b^2*d^2*x - 2*a*c*d^2*x - 2*a*b*d*x*e + 2*a^2*x*e^2)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) + 1/16*($   
 $(2*b^3*c^3 - 8*a*b*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^3*c + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c$   
 $*a*b*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^2*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b*c^3 - 2*($   
 $b^2 - 4*a*c)*b*c^3)*(a*b^2 - 4*a^2*c)^2*d^2 - 4*(2*a*b^2*c^3 - 8*a^2*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c + 4*\sqrt{2}*$   
 $\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*$   
 $\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*c^3 - 2*(b^2 - 4*a*c)*a*c^3)*(a*b^2 - 4*a^2*c)^2*d*e + 2*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^6*c -$   
 $14*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^4*c^2 - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^5*c^2 - 2*a*b^6*c^2 + 64*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*$   
 $a^3*b^2*c^3 + 20*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^3*c^3 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c^3 + 28*a^2*b^4*c^3 - 96*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*$   
 $a^4*c^4 - 48*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*b*c^4 - 10*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^4 - 128*a^3*b^2*c^4 + 24*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*$   
 $a^3*c^5 + 192*a^4*c^5 + 2*(b^2 - 4*a*c)*a*b^4*c^2 - 20*(b^2 - 4*a*c)*a^2*b^2*c^3 + 48*(b^2 - 4*a*c)*a^3*c^4)*d^2*abs(a*b^2 - 4*a^2*c) + (2*a$   
 $*b^3*c^2 - 8*a^2*b*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*$   
 $a^2*b*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b*c^2 - 2$   
 $*(b^2 - 4*a*c)*a*b*c^2)*(a*b^2 - 4*a^2*c)^2*e^2 + 4*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^5*c - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3$   
 $*b^3*c^2 - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^4*c^2 - 2*a^2*b^5*c^2 + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^4*b*c^3 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*$

$$\begin{aligned}
& \text{rt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + \text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4 \\
& *a*c)*c)*a^2*b^3*c^3 + 16*a^3*b^3*c^3 - 4*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a \\
& *c)*c)*a^3*b*c^4 - 32*a^4*b*c^4 + 2*(b^2 - 4*a*c)*a^2*b^3*c^2 - 8*(b^2 - 4* \\
& a*c)*a^3*b*c^3)*d*\text{abs}(a*b^2 - 4*a^2*c)*e + (2*a^2*b^7*c^3 - 40*a^3*b^5*c^4 \\
& + 224*a^4*b^3*c^5 - 384*a^5*b*c^6 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sq \\
& rt}(b^2 - 4*a*c)*c)*a^2*b^7*c + 20*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt} \\
& (b^2 - 4*a*c)*c)*a^3*b^5*c^2 + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt} \\
& (b^2 - 4*a*c)*c)*a^2*b^6*c^2 - 112*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt} \\
& (b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 32*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt} \\
& (b^2 - 4*a*c)*c)*a^3*b^4*c^3 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^ \\
& 2 - 4*a*c)*c)*a^2*b^5*c^3 + 192*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b \\
& ^2 - 4*a*c)*c)*a^5*b*c^4 + 96*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 \\
& - 4*a*c)*c)*a^4*b^2*c^4 + 16*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 \\
& - 4*a*c)*c)*a^3*b^3*c^4 - 48*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 \\
& - 4*a*c)*c)*a^4*b*c^5 - 2*(b^2 - 4*a*c)*a^2*b^5*c^3 + 32*(b^2 - 4*a*c)*a^3 \\
& *b^3*c^4 - 96*(b^2 - 4*a*c)*a^4*b*c^5)*d^2 - 4*(\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 \\
& - 4*a*c)*c)*a^3*b^4*c - 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a^4*b^2* \\
& c^2 - 2*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a^3*b^3*c^2 - 2*a^3*b^4*c^2 \\
& + 16*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a^5*c^3 + 8*\text{sqrt}(2)*\text{sqrt}(b*c \\
& + \text{sqrt}(b^2 - 4*a*c)*c)*a^4*b*c^3 + \text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)* \\
& a^3*b^2*c^3 + 16*a^4*b^2*c^3 - 4*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a^ \\
& 4*c^4 - 32*a^5*c^4 + 2*(b^2 - 4*a*c)*a^3*b^2*c^2 - 8*(b^2 - 4*a*c)*a^4*c^3) \\
& *\text{abs}(a*b^2 - 4*a^2*c)*e^2 + 8*(2*a^3*b^6*c^3 - 16*a^4*b^4*c^4 + 32*a^5*b^2* \\
& c^5 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a^3*b^6*c + \\
& 8*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a^4*b^4*c^2 + \\
& 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 1 \\
& 6*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a^5*b^2*c^3 - 8 \\
& *\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - \text{sq \\
& rt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 4*\text{sq \\
& rt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 2*(b^2 \\
& - 4*a*c)*a^3*b^4*c^3 + 8*(b^2 - 4*a*c)*a^4*b^2*c^4)*d*e - (2*a^3*b^7*c^2 - \\
& 8*a^4*b^5*c^3 - 32*a^5*b^3*c^4 + 128*a^6*b*c^5 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c) \\
& *\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a^3*b^7 + 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt} \\
& (b*c + \text{sqrt}(b^2 - 4*a*c)*c)*a^4*b^5*c + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b* \\
& c + \text{sqrt}(b^2 - 4*a*c)*c)*a^3*b^6*c + 16*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c \\
& + \text{sqrt}(b^2 - 4*a*c)*c)*a^5*b^3*c^2 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{s \\
& qrt}(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 64*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{s \\
& qrt}(b^2 - 4*a*c)*c)*a^6*b*c^3 - 32*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqr \\
& t}(b^2 - 4*a*c)*c)*a^5*b^2*c^3 + 16*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqr \\
& t}(b^2 - 4*a*c)*c)*a^5*b*c^4 - 2*(b^2 - 4*a*c)*a^3*b^5*c^2 + 32*(b^2 - 4*a*c \\
& )*a^5*b*c^4)*e^2)*\text{arctan}(2*\text{sqrt}(1/2)*x/\text{sqrt}((a*b^3 - 4*a^2*b*c + \text{sqrt}((a*b^ \\
& 3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - \\
& 4*a^2*c^2)))/((a^3*b^6*c - 12*a^4*b^4*c^2 - 2*a^3*b^5*c^2 + 48*a^5*b^2*c^3 \\
& + 16*a^4*b^3*c^3 + a^3*b^4*c^3 - 64*a^6*c^4 - 32*a^5*b*c^4 - 8*a^4*b^2*c^4 \\
& + 16*a^5*c^5)*\text{abs}(a*b^2 - 4*a^2*c)*\text{abs}(c)) - 1/16*((2*b^3*c^3 - 8*a*b*c^4 \\
& - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*b^3*c + 4*\text{sqrt}( \\
& 2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a*b*c^2 + 2*\text{sqrt}(2)*\text{sq \\
& rt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*b^2*c^2 - \text{sqrt}(2)*\text{sqrt}(b^2 \\
& - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*(a* \\
& b^2 - 4*a^2*c)^2*d^2 - 4*(2*a*b^2*c^3 - 8*a^2*c^4 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a* \\
& c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a*b^2*c + 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sq \\
& rt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a^2*c^2 + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b* \\
& c - \text{sqrt}(b^2 - 4*a*c)*c)*a*b*c^2 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqr \\
& t}(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*a*c^3)*(a*b^2 - 4*a^2*c)^2*d*e - \\
& 2*(\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a*b^6*c - 14*\text{sqrt}(2)*\text{sqrt}(b*c - \\
& \text{sqrt}(b^2 - 4*a*c)*c)*a^2*b^4*c^2 - 2*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c) \\
& )*a*b^5*c^2 + 2*a*b^6*c^2 + 64*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a^3* \\
& b^2*c^3 + 20*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + \text{sqrt}(2)* \\
& \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a*b^4*c^3 - 28*a^2*b^4*c^3 - 96*\text{sqrt}(2)*\text{sq}
\end{aligned}$$

$$\begin{aligned}
& t(b*c - \sqrt{b^2 - 4*a*c})*c)*a^4*c^4 - 48*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^2*c^4 - 10*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^4 + \\
& 128*a^3*b^2*c^4 + 24*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*c^5 - 192*a^4*c^5 - 2*(b^2 - 4*a*c)*a*b^4*c^2 + 20*(b^2 - 4*a*c)*a^2*b^2*c^3 - 48*(b^2 - 4*a*c)*a^3*c^4)*d^2*abs(a*b^2 - 4*a^2*c) + (2*a*b^3*c^2 - 8*a^2*b*c^3 - \\
& \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)* \\
& *(a*b^2 - 4*a^2*c)^2*e^2 - 4*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^5*c - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^3*c^2 - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^4*c^2 + 2*a^2*b^5*c^2 + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b*c^3 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^2*c^3 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^3*c^3 - \\
& 16*a^3*b^3*c^3 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b*c^4 + 32*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^3*c^2 + 8*(b^2 - 4*a*c)*a^3*b*c^3)*d*abs(a*b^2 - 4*a^2*c)*e + (2*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 224*a^4*b^3*c^5 - 384*a^5*b*c^6 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^7*c + 20*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^5*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^6*c^2 - 112*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^3*c^3 - 32*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^4*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^5*c^3 + 192*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*b*c^4 + 96*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^2*c^4 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^3*c^4 - 48*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b*c^5 - 2*(b^2 - 4*a*c)*a^2*b^5*c^3 + 32*(b^2 - 4*a*c)*a^3*b^3*c^4 - 96*(b^2 - 4*a*c)*a^4*b*c^5)*d^2 + 4*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^4*c - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^2*c^2 - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^3*c^2 + 2*a^3*b^4*c^2 + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*c^3 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b*c^3 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^2*c^3 - 16*a^4*b^2*c^3 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*c^4 + 32*a^5*c^4 - 2*(b^2 - 4*a*c)*a^3*b^2*c^2 + 8*(b^2 - 4*a*c)*a^4*c^3)*abs(a*b^2 - 4*a^2*c)*e^2 + 8*(2*a^3*b^6*c^3 - 16*a^4*b^4*c^4 + 32*a^5*b^2*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^6*c + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^4*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^5*c^2 - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*b^2*c^3 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^3*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^4*c^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^2*c^4 - 2*(b^2 - 4*a*c)*a^3*b^4*c^3 + 8*(b^2 - 4*a*c)*a^4*b^2*c^4)*d*e - (2*a^3*b^7*c^2 - 8*a^4*b^5*c^3 - 32*a^5*b^3*c^4 + 128*a^6*b*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^7 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^5*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^6*c + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*b^3*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^5*c^2 - 64*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^6*b*c^3 - 32*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*b^2*c^3 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*b*c^4 - 2*(b^2 - 4*a*c)*a^3*b^5*c^2 + 32*(b^2 - 4*a*c)*a^5*b*c^4)*e^2)*arctan(2*\sqrt{1/2}*x/\sqrt{((a*b^3 - 4*a^2*b*c - \sqrt{((a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2))})/(a*b^2*c - 4*a^2*c^2))})/(a^3*b^6*c - 12*a^4*b^4*c^2 - 2*a^3*b^5*c^2 + 48*a^5*b^2*c^3 + 16*a^4*b^3*c^3 + a^3*b^4*c^3 - 64*a^6*c^4 - 32*a^5*b*c^4 - 8*a^4*b^2*c^4 + 16*a^5*c^5)*abs(a*b^2 - 4*a^2*c)*abs(c))
\end{aligned}$$

**maple [B]** time = 0.04, size = 1223, normalized size = 3.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((e*x^2+d)^2/(c*x^4+b*x^2+a)^2, x)$

[Out]  $(-1/2/a*(a*b*e^2-4*a*c*d*e+b*c*d^2)/(4*a*c-b^2)*x^3-1/2*(2*a^2*e^2-2*a*b*d*e-2*a*c*d^2+b^2*d^2)/a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/4/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*e^2-1/(4*a*c-b^2)*c*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*d*e+1/4/a/(4*a*c-b^2)*c*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*d^2-a/(4*a*c-b^2)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*e^2-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*e^2+2/(4*a*c-b^2)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*d*e-3/(4*a*c-b^2)*c^2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*d^2+1/4/a/(4*a*c-b^2)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*d^2-1/4/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*e^2+1/(4*a*c-b^2)*c*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*d*e-1/4/a/(4*a*c-b^2)*c*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*d^2-a/(4*a*c-b^2)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*e^2-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*e^2+2/(4*a*c-b^2)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*d*e-3/(4*a*c-b^2)*c^2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*d^2+1/4/a/(4*a*c-b^2)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*d^2$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e*x^2+d)^2/(c*x^4+b*x^2+a)^2, x, \text{algorithm}="maxima")$

[Out]  $1/2*((b*c*d^2 - 4*a*c*d*e + a*b*e^2)*x^3 - (2*a*b*d*e - 2*a^2*e^2 - (b^2 - 2*a*c)*d^2)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*\text{integrate}((2*a*b*d*e - 2*a^2*e^2 + (b^2 - 6*a*c)*d^2 + (b*c*d^2 - 4*a*c*d*e + a*b*e^2)*x^2)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)$

**mupad** [B] time = 9.84, size = 18785, normalized size = 48.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((d + e*x^2)^2/(a + b*x^2 + c*x^4)^2, x)$

[Out]  $\text{atan}((((6144*a^5*c^6*d^2 + 2048*a^6*c^5*e^2 + 16*a*b^8*c^2*d^2 - 288*a^2*b^6*c^3*d^2 + 1920*a^3*b^4*c^4*d^2 - 5632*a^4*b^2*c^5*d^2 - 32*a^3*b^6*c^2*e^2 + 384*a^4*b^4*c^3*e^2 - 1536*a^5*b^2*c^4*e^2 - 2048*a^5*b*c^5*d*e + 32*a^2*b^7*c^2*d*e - 384*a^3*b^5*c^3*d*e + 1536*a^4*b^3*c^4*d*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-b^11*c*d^4 + a^3*b^9*e$





$$\begin{aligned}
& b^9 c^2 d^4 - 3840 a^5 b^3 c^6 d^4 + 9 a^3 c^2 d^4 (-4 a^3 c - b^2)^9)^{(1/2)} - 7 \\
& 68 a^7 b^3 c^4 e^4 - b^2 c^2 d^4 (-4 a^3 c - b^2)^9)^{(1/2)} + 6144 a^6 c^6 d^3 e \\
& + 2048 a^7 c^5 d^3 e^3 + 288 a^2 b^7 c^3 d^4 - 1504 a^3 b^5 c^4 d^4 + 3840 a^4 \\
& 4 b^3 c^5 d^4 - 96 a^5 b^5 c^2 e^4 + 512 a^6 b^3 c^3 e^4 + 4 a^2 b^10 c^3 d^3 e \\
& + 128 a^3 b^7 c^2 d^2 e^2 - 1344 a^4 b^5 c^3 d^2 e^2 + 5120 a^5 b^3 c^4 d^2 \\
& 2 e^2 - 24 a^3 b^8 c^2 d^3 e - 72 a^2 b^8 c^2 d^3 e - 2 a^2 b^9 c^2 d^2 e^2 + 3 \\
& 84 a^3 b^6 c^3 d^3 e - 256 a^4 b^4 c^4 d^3 e + 256 a^4 b^6 c^2 d^2 e^3 - 3072 \\
& a^5 b^2 c^5 d^3 e - 768 a^5 b^4 c^3 d^2 e^3 - 6656 a^6 b^3 c^5 d^2 e^2 + 2 a^2 \\
& c^2 d^2 e^2 (-4 a^3 c - b^2)^9)^{(1/2)} - 4 a^2 b^3 c^2 d^3 e (-4 a^3 c - b^2)^9)^{(1/2)} \\
& ) / (32 (4096 a^9 c^7 + a^3 b^12 c - 24 a^4 b^10 c^2 + 240 a^5 b^8 c^3 - 128 \\
& 0 a^6 b^6 c^4 + 3840 a^7 b^4 c^5 - 6144 a^8 b^2 c^6))^{(1/2)} - (x (72 a^2 c \\
& ^5 d^4 + 8 a^4 c^3 e^4 + b^4 c^3 d^4 - 14 a^2 b^2 c^4 d^4 + a^2 b^4 c^3 e^4 + 2 \\
& a^3 b^2 c^2 e^4 + 16 a^3 c^4 d^2 e^2 + 44 a^2 b^2 c^3 d^2 e^2 + 4 a^2 b^3 c^3 \\
& 3 d^3 e - 80 a^2 b^3 c^4 d^3 e - 16 a^3 b^3 c^3 d^2 e^3 - 12 a^2 b^3 c^2 d^2 e^3)) / \\
& (2 (a^2 b^4 + 16 a^4 c^2 - 8 a^3 b^2 c)) (-b^11 c^2 d^4 + a^3 b^9 e^4 + a^3 \\
& e^4 (-4 a^3 c - b^2)^9)^{(1/2)} - 27 a^2 b^9 c^2 d^4 - 3840 a^5 b^3 c^6 d^4 + 9 a \\
& c^2 d^4 (-4 a^3 c - b^2)^9)^{(1/2)} - 768 a^7 b^3 c^4 e^4 - b^2 c^2 d^4 (-4 a^3 c \\
& - b^2)^9)^{(1/2)} + 6144 a^6 c^6 d^3 e + 2048 a^7 c^5 d^3 e^3 + 288 a^2 b^7 c^3 \\
& d^4 - 1504 a^3 b^5 c^4 d^4 + 3840 a^4 b^3 c^5 d^4 - 96 a^5 b^5 c^2 e^4 + 5 \\
& 12 a^6 b^3 c^3 e^4 + 4 a^2 b^10 c^3 d^3 e + 128 a^3 b^7 c^2 d^2 e^2 - 1344 a^4 b^5 \\
& c^3 d^2 e^2 + 5120 a^5 b^3 c^4 d^2 e^2 - 24 a^3 b^8 c^2 d^3 e - 72 a^2 b^8 c^2 d^3 e \\
& - 2 a^2 b^9 c^2 d^3 e - 2 a^2 b^9 c^2 d^3 e + 384 a^3 b^6 c^3 d^3 e - 256 a^4 b^4 c^4 \\
& d^3 e + 256 a^4 b^6 c^2 d^2 e^3 - 3072 a^5 b^2 c^5 d^3 e - 768 a^5 b^4 c^3 d^2 \\
& e^3 - 6656 a^6 b^3 c^5 d^2 e^2 + 2 a^2 c^2 d^2 e^2 (-4 a^3 c - b^2)^9)^{(1/2)} - \\
& 4 a^2 b^3 c^2 d^3 e (-4 a^3 c - b^2)^9)^{(1/2)} / (32 (4096 a^9 c^7 + a^3 b^12 c - 24 \\
& a^4 b^10 c^2 + 240 a^5 b^8 c^3 - 1280 a^6 b^6 c^4 + 3840 a^7 b^4 c^5 - 614 \\
& 4 a^8 b^2 c^6))^{(1/2)} * i) / ((5 b^3 c^4 d^6 - 3 a^3 b^3 c^3 e^6 - 4 a^4 b^3 c^2 e^6 \\
& + 144 a^2 c^5 d^5 e + 16 a^4 c^3 d^5 e - 6 b^4 c^3 d^5 e + 160 a^3 c^4 d^3 e^3 + \\
& b^5 c^2 d^4 e^2 - 36 a^2 b^3 c^5 d^6 + 152 a^2 b^2 c^3 d^3 e^3 - 29 a^2 b^3 c^2 d^2 e^4 \\
& + 36 a^2 b^2 c^4 d^5 e + a^2 b^5 c^2 d^2 e^4 + 2 a^2 b^4 c^3 d^2 e^5 + 11 a^2 b^3 c^3 d^4 e^2 \\
& - 8 a^2 b^4 c^2 d^3 e^3 - 300 a^2 b^3 c^4 d^4 e^2 - 140 a^3 b^3 c^3 d^2 e^4 + 36 a^3 b^2 c^2 d^2 e^5) / \\
& (4 (a^2 b^6 - 64 a^5 c^3 - 12 a^3 b^4 c + 48 a^4 b^2 c^2)) + ((6144 a^5 c^6 d^2 + 2048 a^6 c^5 e^2 + 16 \\
& a^2 b^8 c^2 d^2 - 288 a^2 b^6 c^3 d^2 + 1920 a^3 b^4 c^4 d^2 - 5632 a^4 b^2 c^5 d^2 - \\
& 32 a^3 b^6 c^2 e^2 + 384 a^4 b^4 c^3 e^2 - 1536 a^5 b^2 c^4 e^2 - 2048 a^5 b^3 c^5 d^2 e \\
& + 32 a^2 b^7 c^2 d^2 e - 384 a^3 b^5 c^3 d^2 e + 1536 a^4 b^3 c^4 d^2 e) / (8 (a^2 b^6 - \\
& 64 a^5 c^3 - 12 a^3 b^4 c + 48 a^4 b^2 c^2)) - (x (-b^11 c^2 d^4 + a^3 b^9 e^4 + a^3 e^4 (-4 a^3 c - \\
& b^2)^9)^{(1/2)} - 27 a^2 b^9 c^2 d^4 - 3840 a^5 b^3 c^6 d^4 + 9 a^2 c^2 d^4 (-4 a^3 c - b^2)^9)^{(1/2)} - \\
& 768 a^7 b^3 c^4 e^4 - b^2 c^2 d^4 (-4 a^3 c - b^2)^9)^{(1/2)} + 6144 a^6 c^6 d^3 e + 204 \\
& 8 a^7 c^5 d^3 e^3 + 288 a^2 b^7 c^3 d^4 - 1504 a^3 b^5 c^4 d^4 + 3840 a^4 b^3 c^5 d^4 - \\
& 96 a^5 b^5 c^2 e^4 + 512 a^6 b^3 c^3 e^4 + 4 a^2 b^10 c^3 d^3 e + 12 \\
& 8 a^3 b^7 c^2 d^2 e^2 - 1344 a^4 b^5 c^3 d^2 e^2 + 5120 a^5 b^3 c^4 d^2 e^2 - 24 a^3 b^8 c^2 d^3 e \\
& - 72 a^2 b^8 c^2 d^3 e - 2 a^2 b^9 c^2 d^3 e + 384 a^3 b^6 c^3 d^3 e - 256 a^4 b^4 c^4 d^3 e \\
& + 256 a^4 b^6 c^2 d^2 e^3 - 3072 a^5 b^2 c^5 d^3 e - 768 a^5 b^4 c^3 d^2 e^3 - 6656 a^6 b^3 c^5 d^2 e^2 \\
& + 2 a^2 c^2 d^2 e^2 (-4 a^3 c - b^2)^9)^{(1/2)} - 4 a^2 b^3 c^2 d^3 e (-4 a^3 c - b^2)^9)^{(1/2)} / (3 \\
& 2 (4096 a^9 c^7 + a^3 b^12 c - 24 a^4 b^10 c^2 + 240 a^5 b^8 c^3 - 1280 a^6 b^6 c^4 + 3840 a^7 b^4 c^5 - \\
& 6144 a^8 b^2 c^6))^{(1/2)} * (1024 a^5 b^3 c^5 - 16 a^2 b^7 c^2 + 192 a^3 b^5 c^3 - 768 a^4 b^3 c^4) / (2 (a^2 b^4 + \\
& 16 a^4 c^2 - 8 a^3 b^2 c)) (-b^11 c^2 d^4 + a^3 b^9 e^4 + a^3 e^4 (-4 a^3 c - b^2)^9)^{(1/2)} - 27 a^2 b^9 c^2 d^4 - \\
& 3840 a^5 b^3 c^6 d^4 + 9 a^2 c^2 d^4 (-4 a^3 c - b^2)^9)^{(1/2)} - 768 a^7 b^3 c^4 e^4 - b^2 c^2 d^4 (-4 a^3 c - b^2)^9)^{(1/2)} + 6144 a^6 c^6 d^3 e + 2048 a^7 c^5 d^3 e^3 + 288 a^2 b^7 c^3 d^4 - 1504 a^3 b^5 c^4 d^4 + 3840 a^4 b^3 c^5 d^4 - 96 a^5 b^5 c^2 e^4 + 512 a^6 b^3 c^3 e^4 + 4 a^2 b^10 c^3 d^3 e + 128 a^3 b^7 c^2 d^2 e^2 - 1344 a^4 b^5 c^3 d^2 e^2 + 5120 a^5 b^3 c^4 d^2 e^2 - 24 a^3 b^8 c^2 d^3 e - 72 a^2 b^8 c^2 d^3 e - 2 a^2 b^9 c^2 d^3 e + 384 a^3 b^6 c^3 d^3 e - 256 a^4 b^4 c^4 d^3 e + 256 a^4 b^6 c^2 d^2 e^3 - 3072 a^5 b^2 c^5 d^3 e - 768 a^5 b^4 c^3 d^2 e^3 - 6656 a^6 b^3 c^5 d^2 e^2
\end{aligned}$$

$$\begin{aligned}
& ^2e^2 + 2a^2c^2d^2e^2(-4ac - b^2)^9)^{(1/2)} - 4abc^2d^3e^2(-4ac - b^2)^9)^{(1/2)} / (32(4096a^9c^7 + a^3b^{12}c - 24a^4b^{10}c^2 + 240a^5b^8c^3 - 1280a^6b^6c^4 + 3840a^7b^4c^5 - 6144a^8b^2c^6)))^{(1/2)} \\
& + (x(72a^2c^5d^4 + 8a^4c^3e^4 + b^4c^3d^4 - 14ab^2c^4d^4 + a^2b^4c^2e^4 + 2a^3b^2c^2e^4 + 16a^3c^4d^2e^2 + 44a^2b^2c^3d^2e^2 + 4ab^3c^3d^3e - 80a^2b^2c^4d^3e - 16a^3b^2c^3d^3e^3 - 12a^2b^3c^2d^3e^3)) / (2(a^2b^4 + 16a^4c^2 - 8a^3b^2c))) * (-b^{11}cd^4 + a^3b^9e^4 + a^3e^4(-4ac - b^2)^9)^{(1/2)} - 27ab^9c^2d^4 - 3840a^5b^6c^6d^4 + 9a^2c^2d^4(-4ac - b^2)^9)^{(1/2)} - 768a^7b^4e^4 - b^2cd^4(-4ac - b^2)^9)^{(1/2)} + 6144a^6c^6d^3e + 2048a^7c^5d^3e^3 + 288a^2b^7c^3d^4 - 1504a^3b^5c^4d^4 + 3840a^4b^3c^5d^4 - 96a^5b^5c^2e^4 + 512a^6b^3c^3e^4 + 4ab^{10}cd^3e + 128a^3b^7c^2d^2e^2 - 1344a^4b^5c^3d^2e^2 + 5120a^5b^3c^4d^2e^2 - 24a^3b^8cd^3e^3 - 72a^2b^8c^2d^3e - 2a^2b^9cd^2e^2 + 384a^3b^6c^3d^3e - 256a^4b^4c^4d^3e + 256a^4b^6c^2d^3e^3 - 3072a^5b^2c^5d^3e - 768a^5b^4c^3d^3e - 6656a^6b^2c^5d^2e^2 + 2a^2cd^2e^2(-4ac - b^2)^9)^{(1/2)} - 4abc^2d^3e^2(-4ac - b^2)^9)^{(1/2)} / (32(4096a^9c^7 + a^3b^{12}c - 24a^4b^{10}c^2 + 240a^5b^8c^3 - 1280a^6b^6c^4 + 3840a^7b^4c^5 - 6144a^8b^2c^6)))^{(1/2)} + (((6144a^5c^6d^2 + 2048a^6c^5e^2 + 16ab^8c^2d^2 - 288a^2b^6c^3d^2 + 1920a^3b^4c^4d^2 - 5632a^4b^2c^5d^2 - 32a^3b^6c^2e^2 + 384a^4b^4c^3e^2 - 1536a^5b^2c^4e^2 - 2048a^5b^2c^5d^2e + 32a^2b^7c^2d^2e - 384a^3b^5c^3d^2e + 1536a^4b^3c^4d^2e) / (8(a^2b^6 - 64a^5c^3 - 12a^3b^4c + 48a^4b^2c^2))) + (x(-b^{11}cd^4 + a^3b^9e^4 + a^3e^4(-4ac - b^2)^9)^{(1/2)} - 27ab^9c^2d^4 - 3840a^5b^6c^6d^4 + 9a^2c^2d^4(-4ac - b^2)^9)^{(1/2)} - 768a^7b^4e^4 - b^2cd^4(-4ac - b^2)^9)^{(1/2)} + 6144a^6c^6d^3e + 2048a^7c^5d^3e^3 + 288a^2b^7c^3d^4 - 1504a^3b^5c^4d^4 + 3840a^4b^3c^5d^4 - 96a^5b^5c^2e^4 + 512a^6b^3c^3e^4 + 4ab^{10}cd^3e + 128a^3b^7c^2d^2e^2 - 1344a^4b^5c^3d^2e^2 + 5120a^5b^3c^4d^2e^2 - 24a^3b^8cd^3e^3 - 72a^2b^8c^2d^3e - 2a^2b^9cd^2e^2 + 384a^3b^6c^3d^3e - 256a^4b^4c^4d^3e + 256a^4b^6c^2d^3e^3 - 3072a^5b^2c^5d^3e - 768a^5b^4c^3d^3e - 6656a^6b^2c^5d^2e^2 + 2a^2cd^2e^2(-4ac - b^2)^9)^{(1/2)} - 4abc^2d^3e^2(-4ac - b^2)^9)^{(1/2)} / (32(4096a^9c^7 + a^3b^{12}c - 24a^4b^{10}c^2 + 240a^5b^8c^3 - 1280a^6b^6c^4 + 3840a^7b^4c^5 - 6144a^8b^2c^6)))^{(1/2)} * (1024a^5b^6c^5 - 16a^2b^7c^2 + 192a^3b^5c^3 - 768a^4b^3c^4) / (2(a^2b^4 + 16a^4c^2 - 8a^3b^2c))) * (-b^{11}cd^4 + a^3b^9e^4 + a^3e^4(-4ac - b^2)^9)^{(1/2)} - 27ab^9c^2d^4 - 3840a^5b^6c^6d^4 + 9a^2c^2d^4(-4ac - b^2)^9)^{(1/2)} - 768a^7b^4e^4 - b^2cd^4(-4ac - b^2)^9)^{(1/2)} + 6144a^6c^6d^3e + 2048a^7c^5d^3e^3 + 288a^2b^7c^3d^4 - 1504a^3b^5c^4d^4 + 3840a^4b^3c^5d^4 - 96a^5b^5c^2e^4 + 512a^6b^3c^3e^4 + 4ab^{10}cd^3e + 128a^3b^7c^2d^2e^2 - 1344a^4b^5c^3d^2e^2 + 5120a^5b^3c^4d^2e^2 - 24a^3b^8cd^3e^3 - 72a^2b^8c^2d^3e - 2a^2b^9cd^2e^2 + 384a^3b^6c^3d^3e - 256a^4b^4c^4d^3e + 256a^4b^6c^2d^3e^3 - 3072a^5b^2c^5d^3e - 768a^5b^4c^3d^3e - 6656a^6b^2c^5d^2e^2 + 2a^2cd^2e^2(-4ac - b^2)^9)^{(1/2)} - 4abc^2d^3e^2(-4ac - b^2)^9)^{(1/2)} / (32(4096a^9c^7 + a^3b^{12}c - 24a^4b^{10}c^2 + 240a^5b^8c^3 - 1280a^6b^6c^4 + 3840a^7b^4c^5 - 6144a^8b^2c^6)))^{(1/2)} - (x(72a^2c^5d^4 + 8a^4c^3e^4 + b^4c^3d^4 - 14ab^2c^4d^4 + a^2b^4c^2e^4 + 2a^3b^2c^2e^4 + 16a^3c^4d^2e^2 + 44a^2b^2c^3d^2e^2 + 4ab^3c^3d^3e - 80a^2b^2c^4d^3e - 16a^3b^2c^3d^3e^3 - 12a^2b^3c^2d^3e^3)) / (2(a^2b^4 + 16a^4c^2 - 8a^3b^2c))) * (-b^{11}cd^4 + a^3b^9e^4 + a^3e^4(-4ac - b^2)^9)^{(1/2)} - 27ab^9c^2d^4 - 3840a^5b^6c^6d^4 + 9a^2c^2d^4(-4ac - b^2)^9)^{(1/2)} - 768a^7b^4e^4 - b^2cd^4(-4ac - b^2)^9)^{(1/2)} + 6144a^6c^6d^3e + 2048a^7c^5d^3e^3 + 288a^2b^7c^3d^4 - 1504a^3b^5c^4d^4 + 3840a^4b^3c^5d^4 - 96a^5b^5c^2e^4 + 512a^6b^3c^3e^4 + 4ab^{10}cd^3e + 128a^3b^7c^2d^2e^2 - 1344a^4b^5c^3d^2e^2 + 5120a^5b^3c^4d^2e^2 - 24a^3b^8cd^3e^3 - 72a^2b^8c^2d^3e - 2a^2b^9cd^2e^2 + 384a^3b^6c^3d^3e
\end{aligned}$$

$$\begin{aligned}
&^3e - 256a^4b^4c^4d^3e + 256a^4b^6c^2d^2e^3 - 3072a^5b^2c^5d^3 \\
&*e - 768a^5b^4c^3d^2e^3 - 6656a^6b^2c^5d^2e^2 + 2a^2c^2d^2e^2*(-(4a \\
&ac - b^2)^9)^{(1/2)} - 4a^2b^2c^2d^3e^2*(-(4a^2c^2 - b^2)^9)^{(1/2)})/(32*(4096a^9 \\
&*c^7 + a^3b^12c - 24a^4b^10c^2 + 240a^5b^8c^3 - 1280a^6b^6c^4 + \\
&3840a^7b^4c^5 - 6144a^8b^2c^6))^{(1/2)})*(-(b^{11}c^4d^4 + a^3b^9e^4 \\
&+ a^3e^4*(-(4a^2c^2 - b^2)^9)^{(1/2)} - 27a^2b^9c^2d^4 - 3840a^5b^2c^6d^4 \\
&+ 9a^2c^2d^4*(-(4a^2c^2 - b^2)^9)^{(1/2)} - 768a^7b^4c^4e^4 - b^2c^2d^4*(-(4 \\
&a^2c^2 - b^2)^9)^{(1/2)} + 6144a^6c^6d^3e + 2048a^7c^5d^2e^3 + 288a^2b^7 \\
&c^3d^4 - 1504a^3b^5c^4d^4 + 3840a^4b^3c^5d^4 - 96a^5b^5c^2e^4 \\
&+ 512a^6b^3c^3e^4 + 4a^2b^10c^2d^3e + 128a^3b^7c^2d^2e^2 - 1344 \\
&a^4b^5c^3d^2e^2 + 5120a^5b^3c^4d^2e^2 - 24a^3b^8c^2d^3e - 72a \\
&^2b^8c^2d^3e - 2a^2b^9c^2d^2e^2 + 384a^3b^6c^3d^3e - 256a^4b^4 \\
&c^4d^3e + 256a^4b^6c^2d^2e^3 - 3072a^5b^2c^5d^3e - 768a^5b^4c^3 \\
&d^2e^3 - 6656a^6b^2c^5d^2e^2 + 2a^2c^2d^2e^2*(-(4a^2c^2 - b^2)^9)^{(1/2)} \\
&- 4a^2b^2c^2d^3e^2*(-(4a^2c^2 - b^2)^9)^{(1/2)})/(32*(4096a^9c^7 + a^3b^12c \\
&- 24a^4b^10c^2 + 240a^5b^8c^3 - 1280a^6b^6c^4 + 3840a^7b^4c^5 \\
&- 6144a^8b^2c^6))^{(1/2)}*2i - ((x^3*(a^2b^2 + b^2c^2 - 4a^2c^2d^2) + (2a^2 \\
&*c^2d^2 - 2a^2b^2d^2 - 2a^2b^2d^2) + (x*(2a^2e^2 + b^2d^2 - 2a^2c^2d^2 - 2a^2b^2d^2e^2)))/(2a^2 \\
&*(4a^2c^2 - b^2)))/(a + b^2x^2 + c^2x^4) + \operatorname{atan}\left(\frac{(6144a^5c^6d^2 + 2048a^6c^5 \\
&e^2 + 16a^2b^8c^2d^2 - 288a^2b^6c^3d^2 + 1920a^3b^4c^4d^2 - 56 \\
&32a^4b^2c^5d^2 - 32a^3b^6c^2e^2 + 384a^4b^4c^3e^2 - 1536a^5b^2 \\
&c^4e^2 - 2048a^5b^2c^5d^2e + 32a^2b^7c^2d^2e - 384a^3b^5c^3d^2e + \\
&1536a^4b^3c^4d^2e)/(8*(a^2b^6 - 64a^5c^3 - 12a^3b^4c + 48a^4b^2 \\
&c^2)) - (x*((a^3e^4*(-(4a^2c^2 - b^2)^9)^{(1/2)} - a^3b^9e^4 - b^{11}c^4d^4 + \\
&27a^2b^9c^2d^4 + 3840a^5b^2c^6d^4 + 9a^2c^2d^4*(-(4a^2c^2 - b^2)^9)^{(1/2)} \\
&+ 768a^7b^4c^4e^4 - b^2c^2d^4*(-(4a^2c^2 - b^2)^9)^{(1/2)} - 6144a^6c^6d^3 \\
&d^3e - 2048a^7c^5d^2e^3 - 288a^2b^7c^3d^4 + 1504a^3b^5c^4d^4 - 3 \\
&840a^4b^3c^5d^4 + 96a^5b^5c^2e^4 - 512a^6b^3c^3e^4 - 4a^2b^10c^2 \\
&d^3e - 128a^3b^7c^2d^2e^2 + 1344a^4b^5c^3d^2e^2 - 5120a^5b^3c^4 \\
&d^2e^2 + 24a^3b^8c^2d^3e + 72a^2b^8c^2d^3e + 2a^2b^9c^2d^2e^2 - 384a^3 \\
&b^6c^3d^3e + 256a^4b^4c^4d^3e - 256a^4b^6c^2d^2e^3 + 3072a^5b^2c^5d^3 \\
&e + 768a^5b^4c^3d^2e^3 + 6656a^6b^2c^5d^2e^2 + 2a^2c^2d^2e^2*(-(4a^2c^2 - b^2)^9)^{(1/2)} \\
&- 4a^2b^2c^2d^3e^2*(-(4a^2c^2 - b^2)^9)^{(1/2)})/(32*(4096a^9c^7 + a^3b^12c \\
&- 24a^4b^10c^2 + 240a^5b^8c^3 - 1280a^6b^6c^4 + 3840a^7b^4c^5 - 6144a^8b^2 \\
&c^6))^{(1/2)}*(1024a^5b^2c^5 - 16a^2b^7c^2 + 192a^3b^5c^3 - 768a^4b^3c^4)/(2*(a^2b^4 \\
&+ 16a^4c^2 - 8a^3b^2c)))*((a^3e^4*(-(4a^2c^2 - b^2)^9)^{(1/2)} - a^3b^9e^4 \\
&- b^{11}c^4d^4 + 27a^2b^9c^2d^4 + 3840a^5b^2c^6d^4 + 9a^2c^2d^4*(-(4 \\
&a^2c^2 - b^2)^9)^{(1/2)} + 768a^7b^4c^4e^4 - b^2c^2d^4*(-(4a^2c^2 - b^2)^9)^{(1/2)} \\
&- 6144a^6c^6d^3e - 2048a^7c^5d^2e^3 - 288a^2b^7c^3d^4 + 1504a^3b^5c^4 \\
&d^4 - 3840a^4b^3c^5d^4 + 96a^5b^5c^2e^4 - 512a^6b^3c^3e^4 - 4a^2b^10c^2 \\
&d^3e - 128a^3b^7c^2d^2e^2 + 1344a^4b^5c^3d^2e^2 - 5120a^5b^3c^4d^2e^2 + \\
&24a^3b^8c^2d^3e + 72a^2b^8c^2d^3e + 2a^2b^9c^2d^2e^2 - 384a^3b^6c^3d^3e \\
&+ 256a^4b^4c^4d^3e - 256a^4b^6c^2d^2e^3 + 3072a^5b^2c^5d^3e + 768a^5b^4c^3 \\
&d^2e^3 + 6656a^6b^2c^5d^2e^2 + 2a^2c^2d^2e^2*(-(4a^2c^2 - b^2)^9)^{(1/2)} - 4a^2 \\
&b^2c^2d^3e^2*(-(4a^2c^2 - b^2)^9)^{(1/2)})/(32*(4096a^9c^7 + a^3b^12c - 24a^4b^10 \\
&c^2 + 240a^5b^8c^3 - 1280a^6b^6c^4 + 3840a^7b^4c^5 - 6144a^8b^2c^6))^{(1/2)} \\
&+ (x*(72a^2c^5d^4 + 8a^4c^3e^4 + b^4c^3d^4 - 14a^2b^2c^4 \\
&d^4 + a^2b^4c^2e^4 + 2a^3b^2c^2e^4 + 16a^3c^4d^2e^2 + 44a^2b^2c^3 \\
&d^2e^2 + 4a^2b^3c^3d^3e - 80a^2b^2c^4d^3e - 16a^3b^2c^3d^3e - 12a^2b^3c^2 \\
&d^3e^3))/(2*(a^2b^4 + 16a^4c^2 - 8a^3b^2c)))*((a^3e^4 \\
&*(-(4a^2c^2 - b^2)^9)^{(1/2)} - a^3b^9e^4 - b^{11}c^4d^4 + 27a^2b^9c^2d^4 + 3 \\
&840a^5b^2c^6d^4 + 9a^2c^2d^4*(-(4a^2c^2 - b^2)^9)^{(1/2)} + 768a^7b^4c^4e^4 \\
&- b^2c^2d^4*(-(4a^2c^2 - b^2)^9)^{(1/2)} - 6144a^6c^6d^3e - 2048a^7c^5d^2 \\
&d^3e^3 - 288a^2b^7c^3d^4 + 1504a^3b^5c^4d^4 - 3840a^4b^3c^5d^4 + \\
&96a^5b^5c^2e^4 - 512a^6b^3c^3e^4 - 4a^2b^10c^2d^3e - 128a^3b^7c^2d^2e^2 \\
&+ 1344a^4b^5c^3d^2e^2 - 5120a^5b^3c^4d^2e^2 + 24a^3b^8c^2d^3e + 72a^2b^8 \\
&c^2d^3e + 2a^2b^9c^2d^2e^2 - 384a^3b^6c^3d^3e
\end{aligned}$$



$$\begin{aligned}
& 4*a^3*b^5*c^3*d*e + 1536*a^4*b^3*c^4*d*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3* \\
& *b^4*c + 48*a^4*b^2*c^2)) - (x*((a^3*e^4*(-(4*a*c - b^2)^9)^{(1/2)} - a^3*b^9 \\
& *e^4 - b^{11}*c*d^4 + 27*a*b^9*c^2*d^4 + 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-( \\
& 4*a*c - b^2)^9)^{(1/2)} + 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^{(1 \\
& /2)} - 6144*a^6*c^6*d^3*e - 2048*a^7*c^5*d*e^3 - 288*a^2*b^7*c^3*d^4 + 1504* \\
& a^3*b^5*c^4*d^4 - 3840*a^4*b^3*c^5*d^4 + 96*a^5*b^5*c^2*e^4 - 512*a^6*b^3*c^3 \\
& ^3*e^4 - 4*a*b^{10}*c*d^3*e - 128*a^3*b^7*c^2*d^2*e^2 + 1344*a^4*b^5*c^3*d^2* \\
& e^2 - 5120*a^5*b^3*c^4*d^2*e^2 + 24*a^3*b^8*c*d*e^3 + 72*a^2*b^8*c^2*d^3*e \\
& + 2*a^2*b^9*c*d^2*e^2 - 384*a^3*b^6*c^3*d^3*e + 256*a^4*b^4*c^4*d^3*e - 256 \\
& *a^4*b^6*c^2*d*e^3 + 3072*a^5*b^2*c^5*d^3*e + 768*a^5*b^4*c^3*d*e^3 + 6656* \\
& a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 4*a*b*c*d^3* \\
& e*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(4096*a^9*c^7 + a^3*b^{12}*c - 24*a^4*b^{10}*c^2 \\
& + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)) \\
& ^{(1/2)}*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3* \\
& *c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^3*e^4*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - a^3*b^9*e^4 - b^{11}*c*d^4 + 27*a*b^9*c^2*d^4 + 3840*a^5*b*c^6*d^4 \\
& + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-( \\
& 4*a*c - b^2)^9)^{(1/2)} - 6144*a^6*c^6*d^3*e - 2048*a^7*c^5*d*e^3 - 288*a^2*b \\
& ^7*c^3*d^4 + 1504*a^3*b^5*c^4*d^4 - 3840*a^4*b^3*c^5*d^4 + 96*a^5*b^5*c^2*e^4 \\
& ^4 - 512*a^6*b^3*c^3*e^4 - 4*a*b^{10}*c*d^3*e - 128*a^3*b^7*c^2*d^2*e^2 + 134 \\
& 4*a^4*b^5*c^3*d^2*e^2 - 5120*a^5*b^3*c^4*d^2*e^2 + 24*a^3*b^8*c*d*e^3 + 72* \\
& a^2*b^8*c^2*d^3*e + 2*a^2*b^9*c*d^2*e^2 - 384*a^3*b^6*c^3*d^3*e + 256*a^4*b \\
& ^4*c^4*d^3*e - 256*a^4*b^6*c^2*d*e^3 + 3072*a^5*b^2*c^5*d^3*e + 768*a^5*b^4 \\
& *c^3*d*e^3 + 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^{(1 \\
& /2)} - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(4096*a^9*c^7 + a^3*b^{12}* \\
& c - 24*a^4*b^{10}*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 \\
& - 6144*a^8*b^2*c^6))^{(1/2)} + (x*(72*a^2*c^5*d^4 + 8*a^4*c^3*e^4 + b^4*c^3 \\
& *d^4 - 14*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 + 2*a^3*b^2*c^2*e^4 + 16*a^3*c^4*d^ \\
& 2*e^2 + 44*a^2*b^2*c^3*d^2*e^2 + 4*a*b^3*c^3*d^3*e - 80*a^2*b*c^4*d^3*e - 1 \\
& 6*a^3*b*c^3*d*e^3 - 12*a^2*b^3*c^2*d*e^3))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3 \\
& *b^2*c)))*((a^3*e^4*(-(4*a*c - b^2)^9)^{(1/2)} - a^3*b^9*e^4 - b^{11}*c*d^4 + 2 \\
& 7*a*b^9*c^2*d^4 + 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^{(1/2)} - 6144*a^6*c^6*d^ \\
& 3*e - 2048*a^7*c^5*d*e^3 - 288*a^2*b^7*c^3*d^4 + 1504*a^3*b^5*c^4*d^4 - 384 \\
& 0*a^4*b^3*c^5*d^4 + 96*a^5*b^5*c^2*e^4 - 512*a^6*b^3*c^3*e^4 - 4*a*b^{10}*c*d \\
& ^3*e - 128*a^3*b^7*c^2*d^2*e^2 + 1344*a^4*b^5*c^3*d^2*e^2 - 5120*a^5*b^3*c^ \\
& 4*d^2*e^2 + 24*a^3*b^8*c*d*e^3 + 72*a^2*b^8*c^2*d^3*e + 2*a^2*b^9*c*d^2*e^2 \\
& - 384*a^3*b^6*c^3*d^3*e + 256*a^4*b^4*c^4*d^3*e - 256*a^4*b^6*c^2*d*e^3 + \\
& 3072*a^5*b^2*c^5*d^3*e + 768*a^5*b^4*c^3*d*e^3 + 6656*a^6*b*c^5*d^2*e^2 + 2 \\
& *a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^{( \\
& 1/2)))/(32*(4096*a^9*c^7 + a^3*b^{12}*c - 24*a^4*b^{10}*c^2 + 240*a^5*b^8*c^3 - \\
& 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6))^{(1/2)} + (((6144* \\
& a^5*c^6*d^2 + 2048*a^6*c^5*e^2 + 16*a*b^8*c^2*d^2 - 288*a^2*b^6*c^3*d^2 + 1 \\
& 920*a^3*b^4*c^4*d^2 - 5632*a^4*b^2*c^5*d^2 - 32*a^3*b^6*c^2*e^2 + 384*a^4*b \\
& ^4*c^3*e^2 - 1536*a^5*b^2*c^4*e^2 - 2048*a^5*b*c^5*d*e + 32*a^2*b^7*c^2*d*e \\
& - 384*a^3*b^5*c^3*d*e + 1536*a^4*b^3*c^4*d*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 1 \\
& 2*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*((a^3*e^4*(-(4*a*c - b^2)^9)^{(1/2)} - a^ \\
& 3*b^9*e^4 - b^{11}*c*d^4 + 27*a*b^9*c^2*d^4 + 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^ \\
& 4*(-(4*a*c - b^2)^9)^{(1/2)} + 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^ \\
& 9)^{(1/2)} - 6144*a^6*c^6*d^3*e - 2048*a^7*c^5*d*e^3 - 288*a^2*b^7*c^3*d^4 + \\
& 1504*a^3*b^5*c^4*d^4 - 3840*a^4*b^3*c^5*d^4 + 96*a^5*b^5*c^2*e^4 - 512*a^6* \\
& b^3*c^3*e^4 - 4*a*b^{10}*c*d^3*e - 128*a^3*b^7*c^2*d^2*e^2 + 1344*a^4*b^5*c^3 \\
& *d^2*e^2 - 5120*a^5*b^3*c^4*d^2*e^2 + 24*a^3*b^8*c*d*e^3 + 72*a^2*b^8*c^2*d^ \\
& ^3*e + 2*a^2*b^9*c*d^2*e^2 - 384*a^3*b^6*c^3*d^3*e + 256*a^4*b^4*c^4*d^3*e \\
& - 256*a^4*b^6*c^2*d*e^3 + 3072*a^5*b^2*c^5*d^3*e + 768*a^5*b^4*c^3*d*e^3 + \\
& 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 4*a*b*c \\
& *d^3*e*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(4096*a^9*c^7 + a^3*b^{12}*c - 24*a^4*b^ \\
& 10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b \\
& ^2*c^6))^{(1/2)}*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^
\end{aligned}$$

$$\frac{4b^3c^4)}{(2(a^2b^4 + 16a^4c^2 - 8a^3b^2c))} * ((a^3e^4 * (-4ac - b^2)^9)^{(1/2)} - a^3b^9e^4 - b^{11}cd^4 + 27a^9c^2d^4 + 3840a^5b^6c^6d^4 + 9a^2c^2d^4 * (-4ac - b^2)^9)^{(1/2)} + 768a^7b^4c^4e^4 - b^2cd^4 * (-4ac - b^2)^9)^{(1/2)} - 6144a^6c^6d^3e - 2048a^7c^5d^3e^3 - 288a^2b^7c^3d^4 + 1504a^3b^5c^4d^4 - 3840a^4b^3c^5d^4 + 96a^5b^5c^2e^4 - 512a^6b^3c^3e^4 - 4ab^{10}cd^3e - 128a^3b^7c^2d^2e^2 + 1344a^4b^5c^3d^2e^2 - 5120a^5b^3c^4d^2e^2 + 24a^3b^8c^2d^3e + 72a^2b^8c^2d^3e + 2a^2b^9c^2d^2e^2 - 384a^3b^6c^3d^3e + 256a^4b^4c^4d^3e - 256a^4b^6c^2d^3e + 3072a^5b^2c^5d^3e + 768a^5b^4c^3d^3e + 6656a^6b^2c^5d^2e^2 + 2a^2cd^2e^2 * (-4ac - b^2)^9)^{(1/2)} - 4ab^3cd^3e * (-4ac - b^2)^9)^{(1/2)}) / (32(4096a^9c^7 + a^3b^{12}c - 24a^4b^{10}c^2 + 240a^5b^8c^3 - 1280a^6b^6c^4 + 3840a^7b^4c^5 - 6144a^8b^2c^6))^{(1/2)} - (x(72a^2c^5d^4 + 8a^4c^3e^4 + b^4c^3d^4 - 14ab^2c^4d^4 + a^2b^4c^4e^4 + 2a^3b^2c^2e^4 + 16a^3c^4d^2e^2 + 44a^2b^2c^3d^2e^2 + 4ab^3c^3d^3e - 80a^2b^4c^4d^3e - 16a^3b^3c^3d^3e - 12a^2b^3c^2d^3e)) / (2(a^2b^4 + 16a^4c^2 - 8a^3b^2c)) * ((a^3e^4 * (-4ac - b^2)^9)^{(1/2)} - a^3b^9e^4 - b^{11}cd^4 + 27a^9c^2d^4 + 3840a^5b^6c^6d^4 + 9a^2c^2d^4 * (-4ac - b^2)^9)^{(1/2)} + 768a^7b^4c^4e^4 - b^2cd^4 * (-4ac - b^2)^9)^{(1/2)} - 6144a^6c^6d^3e - 2048a^7c^5d^3e^3 - 288a^2b^7c^3d^4 + 1504a^3b^5c^4d^4 - 3840a^4b^3c^5d^4 + 96a^5b^5c^2e^4 - 512a^6b^3c^3e^4 - 4ab^{10}cd^3e - 128a^3b^7c^2d^2e^2 + 1344a^4b^5c^3d^2e^2 - 5120a^5b^3c^4d^2e^2 + 24a^3b^8c^2d^3e + 72a^2b^8c^2d^3e + 2a^2b^9c^2d^2e^2 - 384a^3b^6c^3d^3e + 256a^4b^4c^4d^3e - 256a^4b^6c^2d^3e^3 + 3072a^5b^2c^5d^3e + 768a^5b^4c^3d^3e + 6656a^6b^2c^5d^2e^2 + 2a^2cd^2e^2 * (-4ac - b^2)^9)^{(1/2)} - 4ab^3cd^3e * (-4ac - b^2)^9)^{(1/2)}) / (32(4096a^9c^7 + a^3b^{12}c - 24a^4b^{10}c^2 + 240a^5b^8c^3 - 1280a^6b^6c^4 + 3840a^7b^4c^5 - 6144a^8b^2c^6))^{(1/2)})) * ((a^3e^4 * (-4ac - b^2)^9)^{(1/2)} - a^3b^9e^4 - b^{11}cd^4 + 27a^9c^2d^4 + 3840a^5b^6c^6d^4 + 9a^2c^2d^4 * (-4ac - b^2)^9)^{(1/2)} + 768a^7b^4c^4e^4 - b^2cd^4 * (-4ac - b^2)^9)^{(1/2)} - 6144a^6c^6d^3e - 2048a^7c^5d^3e^3 - 288a^2b^7c^3d^4 + 1504a^3b^5c^4d^4 - 3840a^4b^3c^5d^4 + 96a^5b^5c^2e^4 - 512a^6b^3c^3e^4 - 4ab^{10}cd^3e - 128a^3b^7c^2d^2e^2 + 1344a^4b^5c^3d^2e^2 - 5120a^5b^3c^4d^2e^2 + 24a^3b^8c^2d^3e + 72a^2b^8c^2d^3e + 2a^2b^9c^2d^2e^2 - 384a^3b^6c^3d^3e + 256a^4b^4c^4d^3e - 256a^4b^6c^2d^3e^3 + 3072a^5b^2c^5d^3e + 768a^5b^4c^3d^3e + 6656a^6b^2c^5d^2e^2 + 2a^2cd^2e^2 * (-4ac - b^2)^9)^{(1/2)} - 4ab^3cd^3e * (-4ac - b^2)^9)^{(1/2)}) / (32(4096a^9c^7 + a^3b^{12}c - 24a^4b^{10}c^2 + 240a^5b^8c^3 - 1280a^6b^6c^4 + 3840a^7b^4c^5 - 6144a^8b^2c^6))^{(1/2)} * 2i$$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*2/(c\*x\*\*4+b\*x\*\*2+a)\*\*2,x)

[Out] Timed out

$$3.195 \quad \int \frac{d+ex^2}{(a+bx^2+cx^4)^2} dx$$

**Optimal.** Leaf size=293

$$\frac{x(cx^2(bd-2ae)-abe-2acd+b^2d)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c}\left(\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c}\left(-\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

**Rubi [A]** time = 0.79, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {1178, 1166, 205}

$$\frac{x(cx^2(bd-2ae)-abe-2acd+b^2d)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c}\left(\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c}\left(-\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)/(a + b\*x^2 + c\*x^4)^2,x]

[Out] (x\*(b^2\*d - 2\*a\*c\*d - a\*b\*e + c\*(b\*d - 2\*a\*e)\*x^2))/(2\*a\*(b^2 - 4\*a\*c)\*(a + b\*x^2 + c\*x^4)) + (Sqrt[c]\*(b\*d - 2\*a\*e + (b^2\*d - 12\*a\*c\*d + 4\*a\*b\*e)/Sqrt[b^2 - 4\*a\*c])\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b - Sqrt[b^2 - 4\*a\*c]])/(2\*Sqrt[2]\*a\*(b^2 - 4\*a\*c)\*Sqrt[b - Sqrt[b^2 - 4\*a\*c]]) + (Sqrt[c]\*(b\*d - 2\*a\*e - (b^2\*d - 12\*a\*c\*d + 4\*a\*b\*e)/Sqrt[b^2 - 4\*a\*c])\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b + Sqrt[b^2 - 4\*a\*c]])/(2\*Sqrt[2]\*a\*(b^2 - 4\*a\*c)\*Sqrt[b + Sqrt[b^2 - 4\*a\*c]])

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 1166

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[e/2 + (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 - q/2 + c\*x^2), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - a\*e^2, 0] && PosQ[b^2 - 4\*a\*c]

#### Rule 1178

Int[((d\_) + (e\_.)\*(x\_)^2)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] := Simp[(x\*(a\*b\*e - d\*(b^2 - 2\*a\*c) - c\*(b\*d - 2\*a\*e)\*x^2)\*(a + b\*x^2 + c\*x^4)^(p+1))/(2\*a\*(p+1)\*(b^2 - 4\*a\*c)), x] + Dist[1/(2\*a\*(p+1)\*(b^2 - 4\*a\*c)), Int[Simp[(2\*p+3)\*d\*b^2 - a\*b\*e - 2\*a\*c\*d\*(4\*p+5) + (4\*p+7)\*(d\*b - 2\*a\*e)\*c\*x^2, x]\*(a + b\*x^2 + c\*x^4)^(p+1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[p, -1] && IntegerQ[2\*p]

#### Rubi steps



$$\int \frac{d + ex^2}{(a + bx^2 + cx^4)^2} dx = \frac{x(b^2d - 2acd - abe + c(bd - 2ae)x^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\int \frac{-b^2d + 6acd - abe - c(bd - 2ae)x^2}{a + bx^2 + cx^4} dx}{2a(b^2 - 4ac)}$$

$$= \frac{x(b^2d - 2acd - abe + c(bd - 2ae)x^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\left(c\left(bd - 2ae - \frac{b^2d - 12acd + 4abe}{\sqrt{b^2 - 4ac}}\right)\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac}}}{4a(b^2 - 4ac)}$$

$$= \frac{x(b^2d - 2acd - abe + c(bd - 2ae)x^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{c}\left(bd - 2ae + \frac{b^2d - 12acd + 4abe}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}a(b^2 - 4ac)\sqrt{b - \sqrt{b^2 - 4ac}}}$$

**Mathematica [A]** time = 0.75, size = 310, normalized size = 1.06

$$\frac{2x(b^2d - 2acd - abe + c(bd - 2ae)x^2)}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{2}\sqrt{c}\left(bd\sqrt{b^2 - 4ac} - 2a\left(e\sqrt{b^2 - 4ac} + 6cd\right) + b^2d\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(bd\sqrt{b^2 - 4ac} - 2ae\sqrt{b^2 - 4ac} - 4abe + 12acd + b^2(-d)\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b^2 - 4ac + b}}\right)}{(b^2 - 4ac)^{3/2}\sqrt{b^2 - 4ac + b}}$$

4a

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)/(a + b\*x^2 + c\*x^4)^2, x]

[Out] ((2\*x\*(b^2\*d + b\*(-a\*e) + c\*d\*x^2) - 2\*a\*c\*(d + e\*x^2))/((b^2 - 4\*a\*c)\*(a + b\*x^2 + c\*x^4)) + (Sqrt[2]\*Sqrt[c]\*(b^2\*d + b\*(Sqrt[b^2 - 4\*a\*c]\*d + 4\*a\*e) - 2\*a\*(6\*c\*d + Sqrt[b^2 - 4\*a\*c]\*e))\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b - Sqrt[b^2 - 4\*a\*c]]])/((b^2 - 4\*a\*c)^(3/2)\*Sqrt[b - Sqrt[b^2 - 4\*a\*c]]) + (Sqrt[2]\*Sqrt[c]\*(-(b^2\*d) + 12\*a\*c\*d + b\*Sqrt[b^2 - 4\*a\*c]\*d - 4\*a\*b\*e - 2\*a\*Sqrt[b^2 - 4\*a\*c]\*e)\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b + Sqrt[b^2 - 4\*a\*c]]])/((b^2 - 4\*a\*c)^(3/2)\*Sqrt[b + Sqrt[b^2 - 4\*a\*c]]))/(4\*a)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x^2)/(a + b\*x^2 + c\*x^4)^2, x]

[Out] IntegrateAlgebraic[(d + e\*x^2)/(a + b\*x^2 + c\*x^4)^2, x]

**fricas [B]** time = 3.55, size = 4573, normalized size = 15.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(c\*x^4+b\*x^2+a)^2,x, algorithm="fricas")

[Out] 1/4\*(2\*(b\*c\*d - 2\*a\*c\*e)\*x^3 - sqrt(1/2)\*((a\*b^2\*c - 4\*a^2\*c^2)\*x^4 + a^2\*b^2 - 4\*a^3\*c + (a\*b^3 - 4\*a^2\*b\*c)\*x^2)\*sqrt(-((b^5 - 15\*a\*b^3\*c + 60\*a^2\*b\*c^2)\*d^2 + 2\*(a\*b^4 - 6\*a^2\*b^2\*c - 24\*a^3\*c^2)\*d\*e + (a^2\*b^3 + 12\*a^3\*b\*c)\*e^2 + (a^3\*b^6 - 12\*a^4\*b^4\*c + 48\*a^5\*b^2\*c^2 - 64\*a^6\*c^3)\*sqrt((4\*a^3\*b\*d\*e^3 + a^4\*e^4 + (b^4 - 18\*a\*b^2\*c + 81\*a^2\*c^2)\*d^4 + 4\*(a\*b^3 - 9\*a^2\*b\*c)\*d^3\*e + 6\*(a^2\*b^2 - 3\*a^3\*c)\*d^2\*e^2)/(a^6\*b^6 - 12\*a^7\*b^4\*c + 48\*a^8\*b^2\*c^2 - 64\*a^9\*c^3)))/(a^3\*b^6 - 12\*a^4\*b^4\*c + 48\*a^5\*b^2\*c^2 - 64\*a^6\*c^3))\*log(-((5\*b^4\*c^2 - 81\*a\*b^2\*c^3 + 324\*a^2\*c^4)\*d^4 - (3\*b^5\*c - 65\*a\*b^3\*c^2 + 324\*a^2\*b\*c^3)\*d^3\*e - 3\*(3\*a\*b^4\*c - 28\*a^2\*b^2\*c^2)\*d^2\*e^2 - (9\*a^2\*b^3\*c - 20\*a^3\*b\*c^2)\*d\*e^3 - (3\*a^3\*b^2\*c + 4\*a^4\*c^2)\*e^4)\*x + 1/



$$\begin{aligned}
& *b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + \text{sqrt}(1/2)*((a*b^2*c \\
& - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*\text{sqrt}(-((b^5 \\
& - 15*a*b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c - 24*a^3*c^2)*d \\
& e + (a^2*b^3 + 12*a^3*b*c)*e^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - \\
& 64*a^6*c^3)*\text{sqrt}((4*a^3*b*d*e^3 + a^4*e^4 + (b^4 - 18*a*b^2*c + 81*a^2*c^2) \\
& )*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b \\
& ^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c \\
& + 48*a^5*b^2*c^2 - 64*a^6*c^3))*\log(-((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4) \\
& )*d^4 - (3*b^5*c - 65*a*b^3*c^2 + 324*a^2*b*c^3)*d^3*e - 3*(3*a*b^4*c - 2 \\
& 8*a^2*b^2*c^2)*d^2*e^2 - (9*a^2*b^3*c - 20*a^3*b*c^2)*d*e^3 - (3*a^3*b^2*c \\
& + 4*a^4*c^2)*e^4)*x - 1/2*\text{sqrt}(1/2)*((b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - \\
& 672*a^3*b^2*c^3 + 864*a^4*c^4)*d^3 + 3*(a*b^7 - 15*a^2*b^5*c + 72*a^3*b^3*c^2 \\
& - 112*a^4*b*c^3)*d^2*e + 3*(a^2*b^6 - 10*a^3*b^4*c + 32*a^4*b^2*c^2 - 32 \\
& *a^5*c^3)*d*e^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*e^3 + ((a^3*b^9 - \\
& 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*d + (a^4*b \\
& ^8 - 8*a^5*b^6*c + 128*a^7*b^2*c^3 - 256*a^8*c^4)*e)*\text{sqrt}((4*a^3*b*d*e^3 + \\
& a^4*e^4 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3* \\
& e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 \\
& - 64*a^9*c^3))*\text{sqrt}(-((b^5 - 15*a*b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 - \\
& 6*a^2*b^2*c - 24*a^3*c^2)*d*e + (a^2*b^3 + 12*a^3*b*c)*e^2 - (a^3*b^6 - 12* \\
& a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*\text{sqrt}((4*a^3*b*d*e^3 + a^4*e^4 + (b \\
& ^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 \\
& - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3 \\
& )))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - 2*(a*b*e - ( \\
& b^2 - 2*a*c)*d)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 \\
& - 4*a^2*b*c)*x^2)
\end{aligned}$$

**giac [B]** time = 1.76, size = 4433, normalized size = 15.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(c\*x^4+b\*x^2+a)^2,x, algorithm="giac")

[Out]  $1/2*(b*c*d*x^3 - 2*a*c*x^3*e + b^2*d*x - 2*a*c*d*x - a*b*x*e)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) + 1/16*((2*b^3*c^2 - 8*a*b*c^3 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*b^3 + 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a*b*c + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*b^2*c - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2*(a*b^2 - 4*a^2*c)^2*d - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a*b^2 + 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*c + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a*b*c - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2*(a*b^2 - 4*a^2*c)^2*e + 2*(\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a*b^6 - 14*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^2*b^4*c - 2*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a*b^5*c - 2*a*b^6*c + 64*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^3*b^2*c^2 + 20*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^2*b^3*c^2 + \text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a*b^4*c^2 + 28*a^2*b^4*c^2 - 96*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^4*c^3 - 48*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^3*b*c^3 - 10*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^2*b^2*c^3 - 128*a^3*b^2*c^3 + 24*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^3*c^4 + 192*a^4*c^4 + 2*(b^2 - 4*a*c)*a*b^4*c - 20*(b^2 - 4*a*c)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^3*c^3)*d*abs(a*b^2 - 4*a^2*c) + 2*(\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^2*b^5 - 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^3*b^3*c - 2*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^2*b^4*c - 2*a^2*b^5*c + 16*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^4*b*c^2 + 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^3*b^2*c^2 + \text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^2*b^3*c^2 + 16*a^3*b^3*c^2 - 4*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^3*b*c^3 - 32*a^4*b*c^3 + 2*(b^2 - 4*a*c)*a^2*b$

$$\begin{aligned}
&^3c - 8*(b^2 - 4*a*c)*a^3*b*c^2)*abs(a*b^2 - 4*a^2*c)*e + (2*a^2*b^7*c^2 - \\
&40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a* \\
&c)*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c)*a^2*b^7 + 20*\sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{ \\
&qrt(b*c + \sqrt{b^2 - 4*a*c}*c)*a^3*b^5*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{ \\
&(b*c + \sqrt{b^2 - 4*a*c}*c)*a^2*b^6*c - 112*\sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{ \\
&b*c + \sqrt{b^2 - 4*a*c}*c)*a^4*b^3*c^2 - 32*\sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{ \\
&b*c + \sqrt{b^2 - 4*a*c}*c)*a^3*b^4*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{b*c \\
&+ \sqrt{b^2 - 4*a*c}*c)*a^2*b^5*c^2 + 192*\sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{b* \\
&c + \sqrt{b^2 - 4*a*c}*c)*a^5*b*c^3 + 96*\sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{b*c \\
&+ \sqrt{b^2 - 4*a*c}*c)*a^4*b^2*c^3 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{b*c \\
&+ \sqrt{b^2 - 4*a*c}*c)*a^3*b^3*c^3 - 48*\sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{b*c \\
&+ \sqrt{b^2 - 4*a*c}*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*c^2 + 32*(b^2 - \\
&4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4)*d + 4*(2*a^3*b^6*c^2 - 16* \\
&a^4*b^4*c^3 + 32*a^5*b^2*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{b*c + \sqrt{b^ \\
&2 - 4*a*c}*c)*a^3*b^6 + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{b*c + \sqrt{b^2 - 4* \\
&a*c}*c)*a^4*b^4*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{b*c + \sqrt{b^2 - 4*a* \\
&c}*c)*a^3*b^5*c - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{b*c + \sqrt{b^2 - 4*a*c) \\
&)*a^5*b^2*c^2 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{b*c + \sqrt{b^2 - 4*a*c)* \\
&c)*a^4*b^3*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c)* \\
&a^3*b^4*c^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c)*a \\
&^4*b^2*c^3 - 2*(b^2 - 4*a*c)*a^3*b^4*c^2 + 8*(b^2 - 4*a*c)*a^4*b^2*c^3)*e)* \\
&arctan(2*\sqrt{1/2}*x/\sqrt{((a*b^3 - 4*a^2*b*c + \sqrt{(a*b^3 - 4*a^2*b*c)^2 - \\
&4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)})))/(a*b^2*c - 4*a^2*c^2)})))/((a^ \\
&3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3* \\
&b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^2 \\
&- 4*a^2*c)*abs(c)) - 1/16*((2*b^3*c^2 - 8*a*b*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a \\
&c)*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c)*b^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{ \\
&b*c - \sqrt{b^2 - 4*a*c}*c)*a*b*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{b*c - s \\
&qrt(b^2 - 4*a*c)*c)*b^2*c - \sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{b*c - \sqrt{b^2 - \\
&4*a*c}*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2*d - 2*(2*a*b^ \\
&2*c^2 - 8*a^2*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{b*c - \sqrt{b^2 - 4*a*c)* \\
&c)*a*b^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c)*a^2* \\
&c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c)*a*b*c - \sqrt{ \\
&t(2)*\sqrt{b^2 - 4*a*c)*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c)*a*c^2 - 2*(b^2 - 4*a \\
&c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*e - 2*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c)* \\
&c)*a*b^6 - 14*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c)*a^2*b^4*c - 2*\sqrt{2} \\
&)*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c)*a*b^5*c + 2*a*b^6*c + 64*\sqrt{2}*\sqrt{b*c \\
&- \sqrt{b^2 - 4*a*c}*c)*a^3*b^2*c^2 + 20*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c) \\
&)*c)*a^2*b^3*c^2 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c)*a*b^4*c^2 - 28*a \\
&^2*b^4*c^2 - 96*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c)*a^4*c^3 - 48*\sqrt{2} \\
&)*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c)*a^3*b*c^3 - 10*\sqrt{2}*\sqrt{b*c - \sqrt{b^ \\
&2 - 4*a*c}*c)*a^2*b^2*c^3 + 128*a^3*b^2*c^3 + 24*\sqrt{2}*\sqrt{b*c - \sqrt{b^ \\
&2 - 4*a*c}*c)*a^3*c^4 - 192*a^4*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c + 20*(b^2 - 4 \\
&a*c)*a^2*b^2*c^2 - 48*(b^2 - 4*a*c)*a^3*c^3)*d*abs(a*b^2 - 4*a^2*c) - 2*(s \\
&qrt(2)*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c)*a^2*b^5 - 8*\sqrt{2}*\sqrt{b*c - \sqrt{ \\
&b^2 - 4*a*c}*c)*a^3*b^3*c - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c)*a^2*b \\
&^4*c + 2*a^2*b^5*c + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c)*a^4*b*c^2 + \\
&8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c)*a^3*b^2*c^2 + \sqrt{2}*\sqrt{b*c - \\
&\sqrt{b^2 - 4*a*c}*c)*a^2*b^3*c^2 - 16*a^3*b^3*c^2 - 4*\sqrt{2}*\sqrt{b*c - s \\
&qrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 32*a^4*b*c^3 - 2*(b^2 - 4*a*c)*a^2*b^3*c + \\
&8*(b^2 - 4*a*c)*a^3*b*c^2)*abs(a*b^2 - 4*a^2*c)*e + (2*a^2*b^7*c^2 - 40*a^3 \\
&*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{ \\
&(b*c - \sqrt{b^2 - 4*a*c}*c)*a^2*b^7 + 20*\sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{b*c \\
&- \sqrt{b^2 - 4*a*c}*c)*a^3*b^5*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{b*c - \\
&\sqrt{b^2 - 4*a*c}*c)*a^2*b^6*c - 112*\sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{b*c - s \\
&qrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*\sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{b*c - s \\
&qrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{b*c - \sqrt{ \\
&b^2 - 4*a*c}*c)*a^2*b^5*c^2 + 192*\sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{b*c - \sqrt{ \\
&t(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*\sqrt{2}*\sqrt{b^2 - 4*a*c)*\sqrt{b*c - \sqrt{
\end{aligned}$$

$$\begin{aligned}
& b^2 - 4ac) * c) * a^4 * b^2 * c^3 + 16 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac} * c} * a^3 * b^3 * c^3 - 48 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac} * c} * a^4 * b * c^4 - 2 * (b^2 - 4ac) * a^2 * b^5 * c^2 + 32 * (b^2 - 4ac) * a^3 * b^3 * c^3 - 96 * (b^2 - 4ac) * a^4 * b * c^4) * d + 4 * (2 * a^3 * b^6 * c^2 - 16 * a^4 * b^4 * c^3 + 32 * a^5 * b^2 * c^4 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac} * c}) * a^3 * b^6 + 8 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac} * c} * a^4 * b^4 * c + 2 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac} * c}) * a^3 * b^5 * c - 16 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac} * c}) * a^5 * b^2 * c^2 - 8 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac} * c}) * a^4 * b^3 * c^2 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac} * c}) * a^3 * b^4 * c^2 + 4 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac} * c}) * a^4 * b^2 * c^3 - 2 * (b^2 - 4ac) * a^3 * b^4 * c^2 + 8 * (b^2 - 4ac) * a^4 * b^2 * c^3) * e) * \arctan(2 * \sqrt{1/2} * x / \sqrt{(a * b^3 - 4 * a^2 * b * c - \sqrt{(a * b^3 - 4 * a^2 * b * c)^2 - 4 * (a^2 * b^2 - 4 * a^3 * c) * (a * b^2 * c - 4 * a^2 * c^2))}) / (a * b^2 * c - 4 * a^2 * c^2)) / ((a^3 * b^6 - 12 * a^4 * b^4 * c - 2 * a^3 * b^5 * c + 48 * a^5 * b^2 * c^2 + 16 * a^4 * b^3 * c^2 + a^3 * b^4 * c^2 - 64 * a^6 * c^3 - 32 * a^5 * b * c^3 - 8 * a^4 * b^2 * c^3 + 16 * a^5 * c^4) * \text{abs}(a * b^2 - 4 * a^2 * c) * \text{abs}(c))
\end{aligned}$$

**maple [B]** time = 0.08, size = 1761, normalized size = 6.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)/(c\*x^4+b\*x^2+a)^2,x)

[Out] 
$$\begin{aligned}
& -1/4 * (-4ac + b^2)^{(1/2)} / (4ac - b^2) / a * x / (x^2 + 1/2 * b/c - 1/2 * (-4ac + b^2)^{(1/2)} / c) * d + 1/2 / (4ac - b^2) * x / (x^2 + 1/2 * b/c - 1/2 * (-4ac + b^2)^{(1/2)} / c) * e - 1/4 / (4ac - b^2) / a * x / (x^2 + 1/2 * b/c - 1/2 * (-4ac + b^2)^{(1/2)} / c) * b * d - 12 * c^3 / (-4ac + b^2)^{(1/2)} / (4ac - b^2) / (4ac + 3 * b^2) * 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \arctanh(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * d * a - 8 * c^2 / (-4ac + b^2)^{(1/2)} / (4ac - b^2) / (4ac + 3 * b^2) * 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \arctanh(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b^2 * d + 3/4 * c / (-4ac + b^2)^{(1/2)} / (4ac - b^2) / a / (4ac + 3 * b^2) * 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \arctanh(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b^4 * d - 2 * c^2 / (4ac - b^2) * a / (4ac + 3 * b^2) * 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \arctanh(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b^2 * e + c^2 / (4ac - b^2) / (4ac + 3 * b^2) * 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \arctanh(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b * d + 3/4 * c / (4ac - b^2) / a / (4ac + 3 * b^2) * 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \arctanh(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b^3 * d + 4 * c^2 / (-4ac + b^2)^{(1/2)} / (4ac - b^2) * a / (4ac + 3 * b^2) * 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \arctanh(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b * e + 3 * c / (-4ac + b^2)^{(1/2)} / (4ac - b^2) / (4ac + 3 * b^2) * 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \arctanh(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b^3 * e + 1/4 * (-4ac + b^2)^{(1/2)} / (4ac - b^2) / a * x / (x^2 + 1/2 * (-4ac + b^2)^{(1/2)} / c + 1/2 * b/c) * d + 1/2 / (4ac - b^2) * x / (x^2 + 1/2 * (-4ac + b^2)^{(1/2)} / c + 1/2 * b/c) * e - 1/4 / (4ac - b^2) / a * x / (x^2 + 1/2 * (-4ac + b^2)^{(1/2)} / c + 1/2 * b/c) * b * d - 12 * c^3 / (-4ac + b^2)^{(1/2)} / (4ac - b^2) / (4ac + 3 * b^2) * 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \arctan(2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * d * a - 8 * c^2 / (-4ac + b^2)^{(1/2)} / (4ac - b^2) / (4ac + 3 * b^2) * 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \arctan(2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b^2 * d + 3/4 * c / (-4ac + b^2)^{(1/2)} / (4ac - b^2) / a / (4ac + 3 * b^2) * 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \arctan(2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b^4 * d + 2 * c^2 / (4ac - b^2) * a / (4ac + 3 * b^2) * 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \arctan(2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * e + 3/2 * c / (4ac - b^2) / (4ac + 3 * b^2) * 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \arctan(2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b^2 * e - c^2 / (4ac - b^2) / (4ac + 3 * b^2) * 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \arctan(2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b * d - 3/4 * c / (4ac - b^2) / a / (4ac + 3 * b^2) * 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)}
\end{aligned}$$

$$\frac{1}{2} \arctan\left(\frac{2^{1/2}}{(b+(-4ac+b^2)^{1/2})c} \right)^{1/2} c x \sqrt{b^3 d + 4c^2} / (-4ac + b^2)^{1/2} / (4ac - b^2) a / (4ac + 3b^2) 2^{1/2} / ((b+(-4ac+b^2)^{1/2})c)^{1/2} \arctan\left(\frac{2^{1/2}}{(b+(-4ac+b^2)^{1/2})c} \right)^{1/2} c x \sqrt{b^3 e + 3c} / (-4ac + b^2)^{1/2} / (4ac - b^2) / (4ac + 3b^2) 2^{1/2} / ((b+(-4ac+b^2)^{1/2})c)^{1/2} \arctan\left(\frac{2^{1/2}}{(b+(-4ac+b^2)^{1/2})c} \right)^{1/2} c x \sqrt{b^3 e}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(bcd - 2ace)x^3 - (abe - (b^2 - 2ac)d)x}{2((ab^2c - 4a^2c^2)x^4 + a^2b^2 - 4a^3c + (ab^3 - 4a^2bc)x^2)} - \frac{\int \frac{abe + (bcd - 2ace)x^2 + (b^2 - 6ac)d}{cx^4 + bx^2 + a} dx}{2(ab^2 - 4a^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)/(c\*x^4+b\*x^2+a)^2,x, algorithm="maxima")

[Out] 1/2\*((b\*c\*d - 2\*a\*c\*e)\*x^3 - (a\*b\*e - (b^2 - 2\*a\*c)\*d)\*x)/((a\*b^2\*c - 4\*a^2\*c^2)\*x^4 + a^2\*b^2 - 4\*a^3\*c + (a\*b^3 - 4\*a^2\*b\*c)\*x^2) - 1/2\*integrate(-(a\*b\*e + (b\*c\*d - 2\*a\*c\*e)\*x^2 + (b^2 - 6\*a\*c)\*d)/(c\*x^4 + b\*x^2 + a), x)/(a\*b^2 - 4\*a^2\*c)

**mupad** [B] time = 9.39, size = 12350, normalized size = 42.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)/(a + b\*x^2 + c\*x^4)^2,x)

[Out] atan((((6144\*a^5\*c^6\*d - 288\*a^2\*b^6\*c^3\*d + 1920\*a^3\*b^4\*c^4\*d - 5632\*a^4\*b^2\*c^5\*d + 16\*a^2\*b^7\*c^2\*e - 192\*a^3\*b^5\*c^3\*e + 768\*a^4\*b^3\*c^4\*e + 16\*a\*b^8\*c^2\*d - 1024\*a^5\*b\*c^5\*e)/(8\*(a^2\*b^6 - 64\*a^5\*c^3 - 12\*a^3\*b^4\*c + 48\*a^4\*b^2\*c^2)) - (x\*(-(b^11\*d^2 + a^2\*b^9\*e^2 + a^2\*e^2\*(-(4\*a\*c - b^2)^9)^(1/2) + b^2\*d^2\*(-(4\*a\*c - b^2)^9)^(1/2) - 3840\*a^5\*b\*c^5\*d^2 - 768\*a^6\*b\*c^4\*e^2 + 2\*a\*b^10\*d\*e + 288\*a^2\*b^7\*c^2\*d^2 - 1504\*a^3\*b^5\*c^3\*d^2 + 3840\*a^4\*b^3\*c^4\*d^2 - 96\*a^4\*b^5\*c^2\*e^2 + 512\*a^5\*b^3\*c^3\*e^2 - 27\*a\*b^9\*c\*d^2 - 9\*a\*c\*d^2\*(-(4\*a\*c - b^2)^9)^(1/2) + 3072\*a^6\*c^5\*d\*e - 36\*a^2\*b^8\*c\*d\*e + 192\*a^3\*b^6\*c^2\*d\*e - 128\*a^4\*b^4\*c^3\*d\*e - 1536\*a^5\*b^2\*c^4\*d\*e + 2\*a\*b\*d\*e\*(-(4\*a\*c - b^2)^9)^(1/2))/(32\*(a^3\*b^12 + 4096\*a^9\*c^6 - 24\*a^4\*b^10\*c + 240\*a^5\*b^8\*c^2 - 1280\*a^6\*b^6\*c^3 + 3840\*a^7\*b^4\*c^4 - 6144\*a^8\*b^2\*c^5)))^(1/2)\*(1024\*a^5\*b\*c^5 - 16\*a^2\*b^7\*c^2 + 192\*a^3\*b^5\*c^3 - 768\*a^4\*b^3\*c^4)/(2\*(a^2\*b^4 + 16\*a^4\*c^2 - 8\*a^3\*b^2\*c)))\*(-(b^11\*d^2 + a^2\*b^9\*e^2 + a^2\*e^2\*(-(4\*a\*c - b^2)^9)^(1/2) + b^2\*d^2\*(-(4\*a\*c - b^2)^9)^(1/2) - 3840\*a^5\*b\*c^5\*d^2 - 768\*a^6\*b\*c^4\*e^2 + 2\*a\*b^10\*d\*e + 288\*a^2\*b^7\*c^2\*d^2 - 1504\*a^3\*b^5\*c^3\*d^2 + 3840\*a^4\*b^3\*c^4\*d^2 - 96\*a^4\*b^5\*c^2\*e^2 + 512\*a^5\*b^3\*c^3\*e^2 - 27\*a\*b^9\*c\*d^2 - 9\*a\*c\*d^2\*(-(4\*a\*c - b^2)^9)^(1/2) + 3072\*a^6\*c^5\*d\*e - 36\*a^2\*b^8\*c\*d\*e + 192\*a^3\*b^6\*c^2\*d\*e - 128\*a^4\*b^4\*c^3\*d\*e - 1536\*a^5\*b^2\*c^4\*d\*e + 2\*a\*b\*d\*e\*(-(4\*a\*c - b^2)^9)^(1/2))/(32\*(a^3\*b^12 + 4096\*a^9\*c^6 - 24\*a^4\*b^10\*c + 240\*a^5\*b^8\*c^2 - 1280\*a^6\*b^6\*c^3 + 3840\*a^7\*b^4\*c^4 - 6144\*a^8\*b^2\*c^5)))^(1/2) + (x\*(72\*a^2\*c^5\*d^2 - 8\*a^3\*c^4\*e^2 + b^4\*c^3\*d^2 - 14\*a\*b^2\*c^4\*d^2 + 10\*a^2\*b^2\*c^3\*e^2 + 2\*a\*b^3\*c^3\*d\*e - 40\*a^2\*b\*c^4\*d\*e))/(2\*(a^2\*b^4 + 16\*a^4\*c^2 - 8\*a^3\*b^2\*c)))\*(-(b^11\*d^2 + a^2\*b^9\*e^2 + a^2\*e^2\*(-(4\*a\*c - b^2)^9)^(1/2) + b^2\*d^2\*(-(4\*a\*c - b^2)^9)^(1/2) - 3840\*a^5\*b\*c^5\*d^2 - 768\*a^6\*b\*c^4\*e^2 + 2\*a\*b^10\*d\*e + 288\*a^2\*b^7\*c^2\*d^2 - 1504\*a^3\*b^5\*c^3\*d^2 + 3840\*a^4\*b^3\*c^4\*d^2 - 96\*a^4\*b^5\*c^2\*e^2 + 512\*a^5\*b^3\*c^3\*e^2 - 27\*a\*b^9\*c\*d^2 - 9\*a\*c\*d^2\*(-(4\*a\*c - b^2)^9)^(1/2) + 3072\*a^6\*c^5\*d\*e - 36\*a^2\*b^8\*c\*d\*e + 192\*a^3\*b^6\*c^2\*d\*e - 128\*a^4\*b^4\*c^3\*d\*e - 1536\*a^5\*b^2\*c^4\*d\*e + 2\*a\*b\*d\*e\*(-(4\*a\*c - b^2)^9)^(1/2))/(32\*(a^3\*b^12 + 4096\*a^9\*c^6 - 24\*a^4\*b^10\*c + 240\*a^5\*b^8\*c^2 - 1280\*a^6\*b^6\*c^3 + 3840\*a^7\*b^4\*c^4 - 6144\*a^8\*b^2\*c^5)))^(1/2)\*1i - (((6144\*a^5\*c^6\*d - 288\*a^2\*b^6\*c^3\*d + 1920\*a^3\*b^4\*c^4\*d - 5632\*a^4\*b^2\*c^5\*d + 16\*a^2\*b^7\*c^2\*e - 192\*a^3\*b^5\*c^3\*e + 768\*a^4\*b^3\*c^4\*e + 16\*a\*b^8\*c^2\*d - 1024\*a^5\*b\*c^5

$$\begin{aligned}
& *e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^{11} \\
& *d^2 + a^2*b^9*e^2 + a^2*e^2*(-(4*a*c - b^2)^9)^{1/2} + b^2*d^2*(-(4*a*c - \\
& b^2)^9)^{1/2} - 3840*a^5*b*c^5*d^2 - 768*a^6*b*c^4*e^2 + 2*a*b^{10}*d*e + 288 \\
& *a^2*b^7*c^2*d^2 - 1504*a^3*b^5*c^3*d^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5 \\
& *c^2*e^2 + 512*a^5*b^3*c^3*e^2 - 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2) \\
& ^9)^{1/2} + 3072*a^6*c^5*d*e - 36*a^2*b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128 \\
& *a^4*b^4*c^3*d*e - 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^{1/2} \\
& ))/(32*(a^3*b^{12} + 4096*a^9*c^6 - 24*a^4*b^{10}*c + 240*a^5*b^8*c^2 - 1280*a^6 \\
& *b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5))^{1/2}*(1024*a^5*b*c^5 - \\
& 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4)/(2*(a^2*b^4 + 16*a^4*c^2 \\
& - 8*a^3*b^2*c)))*(-(b^{11}*d^2 + a^2*b^9*e^2 + a^2*e^2*(-(4*a*c - b^2)^9)^{1/2} \\
& + b^2*d^2*(-(4*a*c - b^2)^9)^{1/2} - 3840*a^5*b*c^5*d^2 - 768*a^6*b*c^4 \\
& *e^2 + 2*a*b^{10}*d*e + 288*a^2*b^7*c^2*d^2 - 1504*a^3*b^5*c^3*d^2 + 3840*a \\
& ^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2 + 512*a^5*b^3*c^3*e^2 - 27*a*b^9*c*d^2 \\
& - 9*a*c*d^2*(-(4*a*c - b^2)^9)^{1/2} + 3072*a^6*c^5*d*e - 36*a^2*b^8*c*d*e \\
& + 192*a^3*b^6*c^2*d*e - 128*a^4*b^4*c^3*d*e - 1536*a^5*b^2*c^4*d*e + 2*a*b* \\
& d*e*(-(4*a*c - b^2)^9)^{1/2}))/((32*(a^3*b^{12} + 4096*a^9*c^6 - 24*a^4*b^{10}*c \\
& + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5) \\
& ))^{1/2} - (x*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 - 14*a*b^2*c^4* \\
& d^2 + 10*a^2*b^2*c^3*e^2 + 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e))/(2*(a^2*b^4 \\
& + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^{11}*d^2 + a^2*b^9*e^2 + a^2*e^2*(-(4*a*c \\
& - b^2)^9)^{1/2} + b^2*d^2*(-(4*a*c - b^2)^9)^{1/2} - 3840*a^5*b*c^5*d^2 - \\
& 768*a^6*b*c^4*e^2 + 2*a*b^{10}*d*e + 288*a^2*b^7*c^2*d^2 - 1504*a^3*b^5*c^3*d \\
& ^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2 + 512*a^5*b^3*c^3*e^2 - 27*a \\
& *b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^{1/2} + 3072*a^6*c^5*d*e - 36*a^2 \\
& *b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128*a^4*b^4*c^3*d*e - 1536*a^5*b^2*c^4*d \\
& *e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^{1/2}))/((32*(a^3*b^{12} + 4096*a^9*c^6 - 24* \\
& a^4*b^{10}*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a \\
& ^8*b^2*c^5))^{1/2}*1i)/((((6144*a^5*c^6*d - 288*a^2*b^6*c^3*d + 1920*a^3*b \\
& ^4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768* \\
& a^4*b^3*c^4*e + 16*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 \\
& - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^{11}*d^2 + a^2*b^9*e^2 + a^2*e^2 \\
& *(-(4*a*c - b^2)^9)^{1/2} + b^2*d^2*(-(4*a*c - b^2)^9)^{1/2} - 3840*a^5*b*c \\
& ^5*d^2 - 768*a^6*b*c^4*e^2 + 2*a*b^{10}*d*e + 288*a^2*b^7*c^2*d^2 - 1504*a^3* \\
& b^5*c^3*d^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2 + 512*a^5*b^3*c^3*e \\
& ^2 - 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^{1/2} + 3072*a^6*c^5*d*e \\
& - 36*a^2*b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128*a^4*b^4*c^3*d*e - 1536*a^5* \\
& b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^{1/2}))/((32*(a^3*b^{12} + 4096*a^9* \\
& c^6 - 24*a^4*b^{10}*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 \\
& - 6144*a^8*b^2*c^5))^{1/2}*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5 \\
& *c^3 - 768*a^4*b^3*c^4)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^{11}* \\
& d^2 + a^2*b^9*e^2 + a^2*e^2*(-(4*a*c - b^2)^9)^{1/2} + b^2*d^2*(-(4*a*c - b \\
& ^2)^9)^{1/2} - 3840*a^5*b*c^5*d^2 - 768*a^6*b*c^4*e^2 + 2*a*b^{10}*d*e + 288* \\
& a^2*b^7*c^2*d^2 - 1504*a^3*b^5*c^3*d^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5* \\
& c^2*e^2 + 512*a^5*b^3*c^3*e^2 - 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^ \\
& 9)^{1/2} + 3072*a^6*c^5*d*e - 36*a^2*b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128* \\
& a^4*b^4*c^3*d*e - 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^{1/2} \\
& ))/(32*(a^3*b^{12} + 4096*a^9*c^6 - 24*a^4*b^{10}*c + 240*a^5*b^8*c^2 - 1280*a^6 \\
& *b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5))^{1/2} + (x*(72*a^2*c^5*d^2 \\
& - 8*a^3*c^4*e^2 + b^4*c^3*d^2 - 14*a*b^2*c^4*d^2 + 10*a^2*b^2*c^3*e^2 + 2 \\
& *a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c) \\
& )))*(-(b^{11}*d^2 + a^2*b^9*e^2 + a^2*e^2*(-(4*a*c - b^2)^9)^{1/2} + b^2*d^2*( \\
& -(4*a*c - b^2)^9)^{1/2} - 3840*a^5*b*c^5*d^2 - 768*a^6*b*c^4*e^2 + 2*a*b^{10} \\
& *d*e + 288*a^2*b^7*c^2*d^2 - 1504*a^3*b^5*c^3*d^2 + 3840*a^4*b^3*c^4*d^2 - \\
& 96*a^4*b^5*c^2*e^2 + 512*a^5*b^3*c^3*e^2 - 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4* \\
& a*c - b^2)^9)^{1/2} + 3072*a^6*c^5*d*e - 36*a^2*b^8*c*d*e + 192*a^3*b^6*c^2 \\
& *d*e - 128*a^4*b^4*c^3*d*e - 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^ \\
& 2)^9)^{1/2}))/((32*(a^3*b^{12} + 4096*a^9*c^6 - 24*a^4*b^{10}*c + 240*a^5*b^8*c^2 \\
& - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5))^{1/2} + (((614
\end{aligned}$$

$$\begin{aligned}
& 4a^5c^6d - 288a^2b^6c^3d + 1920a^3b^4c^4d - 5632a^4b^2c^5d + \\
& 16a^2b^7c^2e - 192a^3b^5c^3e + 768a^4b^3c^4e + 16a^5b^8c^2d \\
& - 1024a^5b^6c^5e) / (8(a^2b^6 - 64a^5c^3 - 12a^3b^4c + 48a^4b^2c^2)) + (x * (-b^{11}d^2 + a^2b^9e^2 + a^2e^2 * (-4ac - b^2)^9)^{(1/2)} + b^2 \\
& * d^2 * (-4ac - b^2)^9)^{(1/2)} - 3840a^5b^6c^5d^2 - 768a^6b^6c^4e^2 + 2 * \\
& a^5b^{10}d^2e + 288a^2b^7c^2d^2 - 1504a^3b^5c^3d^2 + 3840a^4b^3c^4 * \\
& d^2 - 96a^4b^5c^2e^2 + 512a^5b^3c^3e^2 - 27a^6b^9c^4d^2 - 9a^5c^4d^2 \\
& * (-4ac - b^2)^9)^{(1/2)} + 3072a^6c^5d^2e - 36a^2b^8c^4d^2e + 192a^3b^6 \\
& c^2d^2e - 128a^4b^4c^3d^2e - 1536a^5b^2c^4d^2e + 2a^5b^8d^2e * (-4ac \\
& - b^2)^9)^{(1/2)} / (32(a^3b^{12} + 4096a^9c^6 - 24a^4b^{10}c + 240a^5b^8 \\
& c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6144a^8b^2c^5))^{(1/2)} * (1 \\
& 024a^5b^6c^5 - 16a^2b^7c^2 + 192a^3b^5c^3 - 768a^4b^3c^4) / (2(a^2 \\
& b^4 + 16a^4c^2 - 8a^3b^2c)) * (-b^{11}d^2 + a^2b^9e^2 + a^2e^2 * (-4ac \\
& - b^2)^9)^{(1/2)} + b^2d^2 * (-4ac - b^2)^9)^{(1/2)} - 3840a^5b^6c^5d^2 \\
& - 768a^6b^6c^4e^2 + 2a^5b^{10}d^2e + 288a^2b^7c^2d^2 - 1504a^3b^5c^3 \\
& d^2 + 3840a^4b^3c^4d^2 - 96a^4b^5c^2e^2 + 512a^5b^3c^3e^2 - \\
& 27a^6b^9c^4d^2 - 9a^5c^4d^2 * (-4ac - b^2)^9)^{(1/2)} + 3072a^6c^5d^2e - 3 \\
& 6a^2b^8c^4d^2e + 192a^3b^6c^2d^2e - 128a^4b^4c^3d^2e - 1536a^5b^2c^4 \\
& d^2e + 2a^5b^8d^2e * (-4ac - b^2)^9)^{(1/2)} / (32(a^3b^{12} + 4096a^9c^6 \\
& - 24a^4b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6 \\
& 144a^8b^2c^5))^{(1/2)} - (x * (72a^2c^5d^2 - 8a^3c^4e^2 + b^4c^3d^2 \\
& - 14a^2b^2c^4d^2 + 10a^2b^2c^3e^2 + 2a^3b^3c^3d^2e - 40a^2b^4c^4d^2 \\
& e)) / (2(a^2b^4 + 16a^4c^2 - 8a^3b^2c)) * (-b^{11}d^2 + a^2b^9e^2 + \\
& a^2e^2 * (-4ac - b^2)^9)^{(1/2)} + b^2d^2 * (-4ac - b^2)^9)^{(1/2)} - 3840 * \\
& a^5b^6c^5d^2 - 768a^6b^6c^4e^2 + 2a^5b^{10}d^2e + 288a^2b^7c^2d^2 - 15 \\
& 04a^3b^5c^3d^2 + 3840a^4b^3c^4d^2 - 96a^4b^5c^2e^2 + 512a^5b^3c^3 \\
& e^2 - 27a^6b^9c^4d^2 - 9a^5c^4d^2 * (-4ac - b^2)^9)^{(1/2)} + 3072a^6c^5 \\
& d^2e - 36a^2b^8c^4d^2e + 192a^3b^6c^2d^2e - 128a^4b^4c^3d^2e - 15 \\
& 36a^5b^2c^4d^2e + 2a^5b^8d^2e * (-4ac - b^2)^9)^{(1/2)} / (32(a^3b^{12} + 40 \\
& 96a^9c^6 - 24a^4b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7 * \\
& b^4c^4 - 6144a^8b^2c^5))^{(1/2)} + (8a^3c^4e^3 + 5b^3c^4d^3 + 72a^2 \\
& c^5d^2e - 3b^4c^3d^2e + 6a^2b^2c^3e^3 - 36a^5b^6c^5d^3 + 18a^4 \\
& b^2c^4d^2e + 3a^3b^3c^3d^2e^2 - 60a^2b^2c^4d^2e^2) / (4(a^2b^6 - 64a^5 \\
& c^3 - 12a^3b^4c + 48a^4b^2c^2)) * (-b^{11}d^2 + a^2b^9e^2 + a^2e^2 * (-4ac \\
& - b^2)^9)^{(1/2)} + b^2d^2 * (-4ac - b^2)^9)^{(1/2)} - 3840a^5b^6 \\
& c^5d^2 - 768a^6b^6c^4e^2 + 2a^5b^{10}d^2e + 288a^2b^7c^2d^2 - 1504a^3 \\
& b^5c^3d^2 + 3840a^4b^3c^4d^2 - 96a^4b^5c^2e^2 + 512a^5b^3c^3 \\
& e^2 - 27a^6b^9c^4d^2 - 9a^5c^4d^2 * (-4ac - b^2)^9)^{(1/2)} + 3072a^6c^5d \\
& e - 36a^2b^8c^4d^2e + 192a^3b^6c^2d^2e - 128a^4b^4c^3d^2e - 1536a^5 \\
& b^2c^4d^2e + 2a^5b^8d^2e * (-4ac - b^2)^9)^{(1/2)} / (32(a^3b^{12} + 4096a^9 \\
& c^6 - 24a^4b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6144 \\
& a^8b^2c^5))^{(1/2)} * 2i + \operatorname{atan}(\frac{(6144a^5c^6d - 288a^2b^6c^3d + 1920a^3b^4c^4d - 5632a^4b^2c^5d + 16a^2b^7c^2e - 192a^3 \\
& b^5c^3e + 768a^4b^3c^4e + 16a^5b^8c^2d - 1024a^5b^6c^5e) / (8(a^2 \\
& b^6 - 64a^5c^3 - 12a^3b^4c + 48a^4b^2c^2)) - (x * ((a^2e^2 * (-4ac \\
& - b^2)^9)^{(1/2)} - a^2b^9e^2 - b^{11}d^2 + b^2d^2 * (-4ac - b^2)^9)^{(1/2)} \\
& ) + 3840a^5b^6c^5d^2 + 768a^6b^6c^4e^2 - 2a^5b^{10}d^2e - 288a^2b^7c^2 \\
& d^2 + 1504a^3b^5c^3d^2 - 3840a^4b^3c^4d^2 + 96a^4b^5c^2e^2 - 5 \\
& 12a^5b^3c^3e^2 + 27a^6b^9c^4d^2 - 9a^5c^4d^2 * (-4ac - b^2)^9)^{(1/2)} - \\
& 3072a^6c^5d^2e + 36a^2b^8c^4d^2e - 192a^3b^6c^2d^2e + 128a^4b^4c^3 \\
& d^2e + 1536a^5b^2c^4d^2e + 2a^5b^8d^2e * (-4ac - b^2)^9)^{(1/2)} / (32(a^3 * \\
& b^{12} + 4096a^9c^6 - 24a^4b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + \\
& 3840a^7b^4c^4 - 6144a^8b^2c^5))^{(1/2)} * (1024a^5b^6c^5 - 16a^2b^7c^2 \\
& + 192a^3b^5c^3 - 768a^4b^3c^4) / (2(a^2b^4 + 16a^4c^2 - 8a^3b^2c)) * ((a^2 \\
& e^2 * (-4ac - b^2)^9)^{(1/2)} - a^2b^9e^2 - b^{11}d^2 + b^2d^2 \\
& * (-4ac - b^2)^9)^{(1/2)} + 3840a^5b^6c^5d^2 + 768a^6b^6c^4e^2 - 2a^5 \\
& b^{10}d^2e - 288a^2b^7c^2d^2 + 1504a^3b^5c^3d^2 - 3840a^4b^3c^4d^2 \\
& + 96a^4b^5c^2e^2 - 512a^5b^3c^3e^2 + 27a^6b^9c^4d^2 - 9a^5c^4d^2 * (- \\
& 4ac - b^2)^9)^{(1/2)} - 3072a^6c^5d^2e + 36a^2b^8c^4d^2e - 192a^3b^6
\end{aligned}$$



$$\begin{aligned}
& *c^2*d*e + 128*a^4*b^4*c^3*d*e + 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c \\
& - b^2)^9)^{(1/2)}/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8 \\
& *c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^{(1/2)} + (x \\
& *(72*a^2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 - 14*a*b^2*c^4*d^2 + 10*a^2* \\
& b^2*c^3*e^2 + 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e))/(2*(a^2*b^4 + 16*a^4*c^2 \\
& - 8*a^3*b^2*c)))*((a^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^2*b^9*e^2 - b^11*d \\
& ^2 + b^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 3840*a^5*b*c^5*d^2 + 768*a^6*b*c^4* \\
& e^2 - 2*a*b^10*d*e - 288*a^2*b^7*c^2*d^2 + 1504*a^3*b^5*c^3*d^2 - 3840*a^4* \\
& b^3*c^4*d^2 + 96*a^4*b^5*c^2*e^2 - 512*a^5*b^3*c^3*e^2 + 27*a*b^9*c*d^2 - 9 \\
& *a*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3072*a^6*c^5*d*e + 36*a^2*b^8*c*d*e - 1 \\
& 92*a^3*b^6*c^2*d*e + 128*a^4*b^4*c^3*d*e + 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e \\
& *(-(4*a*c - b^2)^9)^{(1/2)}/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 2 \\
& 40*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^{(1/2)}*1i - (((6144*a^5*c^6*d - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 563 \\
& 2*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e \\
& + 16*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c \\
& + 48*a^4*b^2*c^2)) + (x*((a^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^2*b^9*e^2 \\
& - b^11*d^2 + b^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 3840*a^5*b*c^5*d^2 + 768*a^ \\
& 6*b*c^4*e^2 - 2*a*b^10*d*e - 288*a^2*b^7*c^2*d^2 + 1504*a^3*b^5*c^3*d^2 - 3 \\
& 840*a^4*b^3*c^4*d^2 + 96*a^4*b^5*c^2*e^2 - 512*a^5*b^3*c^3*e^2 + 27*a*b^9*c \\
& *d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3072*a^6*c^5*d*e + 36*a^2*b^8*c \\
& *d*e - 192*a^3*b^6*c^2*d*e + 128*a^4*b^4*c^3*d*e + 1536*a^5*b^2*c^4*d*e + 2 \\
& *a*b*d*e*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^ \\
& 10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2 \\
& *c^5)))^{(1/2)}*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4* \\
& b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^2*e^2*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)} - a^2*b^9*e^2 - b^11*d^2 + b^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 3 \\
& 840*a^5*b*c^5*d^2 + 768*a^6*b*c^4*e^2 - 2*a*b^10*d*e - 288*a^2*b^7*c^2*d^2 \\
& + 1504*a^3*b^5*c^3*d^2 - 3840*a^4*b^3*c^4*d^2 + 96*a^4*b^5*c^2*e^2 - 512*a^ \\
& 5*b^3*c^3*e^2 + 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3072* \\
& a^6*c^5*d*e + 36*a^2*b^8*c*d*e - 192*a^3*b^6*c^2*d*e + 128*a^4*b^4*c^3*d*e \\
& + 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(a^3*b^12 \\
& + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840* \\
& a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^{(1/2)} - (x*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^ \\
& 2 + b^4*c^3*d^2 - 14*a*b^2*c^4*d^2 + 10*a^2*b^2*c^3*e^2 + 2*a*b^3*c^3*d*e - \\
& 40*a^2*b*c^4*d*e))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^2*e^2*(-( \\
& 4*a*c - b^2)^9)^{(1/2)} - a^2*b^9*e^2 - b^11*d^2 + b^2*d^2*(-(4*a*c - b^2)^9) \\
& ^{(1/2)} + 3840*a^5*b*c^5*d^2 + 768*a^6*b*c^4*e^2 - 2*a*b^10*d*e - 288*a^2*b^ \\
& 7*c^2*d^2 + 1504*a^3*b^5*c^3*d^2 - 3840*a^4*b^3*c^4*d^2 + 96*a^4*b^5*c^2*e^ \\
& 2 - 512*a^5*b^3*c^3*e^2 + 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^{(1/ \\
& 2)} - 3072*a^6*c^5*d*e + 36*a^2*b^8*c*d*e - 192*a^3*b^6*c^2*d*e + 128*a^4*b^ \\
& 4*c^3*d*e + 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^{(1/2)}/(32* \\
& (a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c \\
& ^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^{(1/2)}*1i)/((((6144*a^5*c^6*d - \\
& 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^ \\
& 2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e + 16*a*b^8*c^2*d - 1024*a^5*b*c \\
& ^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*((a^2 \\
& *e^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^2*b^9*e^2 - b^11*d^2 + b^2*d^2*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 3840*a^5*b*c^5*d^2 + 768*a^6*b*c^4*e^2 - 2*a*b^10*d*e - 28 \\
& 8*a^2*b^7*c^2*d^2 + 1504*a^3*b^5*c^3*d^2 - 3840*a^4*b^3*c^4*d^2 + 96*a^4*b^ \\
& 5*c^2*e^2 - 512*a^5*b^3*c^3*e^2 + 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2 \\
& )^9)^{(1/2)} - 3072*a^6*c^5*d*e + 36*a^2*b^8*c*d*e - 192*a^3*b^6*c^2*d*e + 12 \\
& 8*a^4*b^4*c^3*d*e + 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^{(1/ \\
& 2)}/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a \\
& ^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^{(1/2)}*(1024*a^5*b*c^5 - \\
& 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4* \\
& c^2 - 8*a^3*b^2*c)))*((a^2*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - a^2*b^9*e^2 - b^1 \\
& 1*d^2 + b^2*d^2*(-(4*a*c - b^2)^9)^{(1/2)} + 3840*a^5*b*c^5*d^2 + 768*a^6*b*c \\
& ^4*e^2 - 2*a*b^10*d*e - 288*a^2*b^7*c^2*d^2 + 1504*a^3*b^5*c^3*d^2 - 3840*a
\end{aligned}$$

$$\begin{aligned}
& ^4b^3c^4d^2 + 96a^4b^5c^2e^2 - 512a^5b^3c^3e^2 + 27a^9bcd^2 \\
& - 9a^3cd^2(-4ac - b^2)^9)^{(1/2)} - 3072a^6c^5d^2e + 36a^2b^8c^2d^2e \\
& - 192a^3b^6c^2d^2e + 128a^4b^4c^3d^2e + 1536a^5b^2c^4d^2e + 2a^2b^8c^2d^2e \\
& d^2e^2(-4ac - b^2)^9)^{(1/2)} / (32(a^3b^{12} + 4096a^9c^6 - 24a^4b^{10}c \\
& + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6144a^8b^2c^5) \\
& ))^{(1/2)} + (x(72a^2c^5d^2 - 8a^3c^4e^2 + b^4c^3d^2 - 14a^2b^2c^4d^2 \\
& + 10a^2b^2c^3e^2 + 2a^2b^3c^3d^2e - 40a^2b^2c^4d^2e)) / (2(a^2b^4 \\
& + 16a^4c^2 - 8a^3b^2c)) * ((a^2e^2(-4ac - b^2)^9)^{(1/2)} - a^2b^9 \\
& e^2 - b^{11}d^2 + b^2d^2(-4ac - b^2)^9)^{(1/2)} + 3840a^5b^2c^5d^2 + 7 \\
& 68a^6b^2c^4e^2 - 2a^2b^10d^2e - 288a^2b^7c^2d^2 + 1504a^3b^5c^3d^2 \\
& - 3840a^4b^3c^4d^2 + 96a^4b^5c^2e^2 - 512a^5b^3c^3e^2 + 27a^9 \\
& b^9cd^2 - 9a^3cd^2(-4ac - b^2)^9)^{(1/2)} - 3072a^6c^5d^2e + 36a^2b^8 \\
& c^2d^2e - 192a^3b^6c^2d^2e + 128a^4b^4c^3d^2e + 1536a^5b^2c^4d^2e \\
& e + 2a^2b^8c^2d^2e^2(-4ac - b^2)^9)^{(1/2)} / (32(a^3b^{12} + 4096a^9c^6 - 24a^4 \\
& b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6144a^8b^2c^5) \\
& ))^{(1/2)} + (((6144a^5c^6d - 288a^2b^6c^3d + 1920a^3b^4c^4d - 5632a^4b^2c^5d \\
& + 16a^2b^7c^2e - 192a^3b^5c^3e + 768a^4b^3c^4e + 16a^2b^8c^2d - 1024a^5b^2c^5e) / (8(a^2b^6 - 64a^5c^3 - 1 \\
& 2a^3b^4c + 48a^4b^2c^2)) + (x((a^2e^2(-4ac - b^2)^9)^{(1/2)} - a^2b^9 \\
& e^2 - b^{11}d^2 + b^2d^2(-4ac - b^2)^9)^{(1/2)} + 3840a^5b^2c^5d^2 + 768a^6 \\
& b^2c^4e^2 - 2a^2b^10d^2e - 288a^2b^7c^2d^2 + 1504a^3b^5c^3d^2 - 3840a^4b^3 \\
& c^4d^2 + 96a^4b^5c^2e^2 - 512a^5b^3c^3e^2 + 27a^9bcd^2 - 9a^3cd^2(-4ac - b^2)^9)^{(1/2)} - 3072a^6 \\
& c^5d^2e + 36a^2b^8c^2d^2e - 192a^3b^6c^2d^2e + 128a^4b^4c^3d^2e + 1536a^5b^2c^4 \\
& d^2e + 2a^2b^8c^2d^2e^2(-4ac - b^2)^9)^{(1/2)} / (32(a^3b^{12} + 4096a^9c^6 - 24a^4 \\
& b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6144a^8b^2c^5) \\
& ))^{(1/2)} * (1024a^5b^2c^5 - 16a^2b^7c^2 + 192a^3b^5c^3 - 768a^4b^3c^4) / (2(a^2b^4 \\
& + 16a^4c^2 - 8a^3b^2c)) * ((a^2e^2(-4ac - b^2)^9)^{(1/2)} - a^2b^9e^2 - b^{11}d^2 \\
& + b^2d^2(-4ac - b^2)^9)^{(1/2)} + 3840a^5b^2c^5d^2 + 768a^6b^2c^4e^2 - 2a^2b^10 \\
& d^2e - 288a^2b^7c^2d^2 + 1504a^3b^5c^3d^2 - 3840a^4b^3c^4d^2 + 96a^4b^5c^2e^2 \\
& - 512a^5b^3c^3e^2 + 27a^9bcd^2 - 9a^3cd^2(-4ac - b^2)^9)^{(1/2)} - 3072a^6c^5 \\
& d^2e + 36a^2b^8c^2d^2e - 192a^3b^6c^2d^2e + 128a^4b^4c^3d^2e + 1536a^5b^2c^4 \\
& d^2e + 2a^2b^8c^2d^2e^2(-4ac - b^2)^9)^{(1/2)} / (32(a^3b^{12} + 4096a^9c^6 - 24a^4 \\
& b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6144a^8b^2c^5) \\
& ))^{(1/2)} - (x(72a^2c^5d^2 - 8a^3c^4e^2 + b^4c^3d^2 - 14a^2b^2c^4d^2 + 10a^2b^2c^3e^2 \\
& + 2a^2b^3c^3d^2e - 40a^2b^2c^4d^2e)) / (2(a^2b^4 + 16a^4c^2 - 8a^3b^2c)) * ((a^2 \\
& e^2(-4ac - b^2)^9)^{(1/2)} - a^2b^9e^2 - b^{11}d^2 + b^2d^2(-4ac - b^2)^9)^{(1/2)} \\
& + 3840a^5b^2c^5d^2 + 768a^6b^2c^4e^2 - 2a^2b^10d^2e - 288a^2b^7c^2d^2 + 1504a^3 \\
& b^5c^3d^2 - 3840a^4b^3c^4d^2 + 96a^4b^5c^2e^2 - 512a^5b^3c^3e^2 + 27a^9bcd^2 - 9a^3 \\
& cd^2(-4ac - b^2)^9)^{(1/2)} - 3072a^6c^5d^2e + 36a^2b^8c^2d^2e - 192a^3b^6c^2d^2e \\
& + 128a^4b^4c^3d^2e + 1536a^5b^2c^4d^2e + 2a^2b^8c^2d^2e^2(-4ac - b^2)^9)^{(1/2)} / (32 \\
& (a^3b^{12} + 4096a^9c^6 - 24a^4b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6144a^8b^2c^5) \\
& ))^{(1/2)} + (8a^3c^4e^3 + 5b^3c^4d^3 + 72a^2c^5d^2e - 3b^4c^3d^2e + 6a^2b^2c^3e^3 - 36a^2b^2c^3e^3 \\
& - 36a^2b^2c^3e^3 + 18a^2b^2c^4d^2e + 3a^2b^3c^3d^2e^2 - 60a^2b^2c^4d^2e^2) / (4(a^2b^6 - 64a^5c^3 - 12a^3b^4c \\
& + 48a^4b^2c^2)) * ((a^2e^2(-4ac - b^2)^9)^{(1/2)} - a^2b^9e^2 - b^{11}d^2 + b^2d^2(-4ac - b^2)^9)^{(1/2)} \\
& + 3840a^5b^2c^5d^2 + 768a^6b^2c^4e^2 - 2a^2b^10d^2e - 288a^2b^7c^2d^2 + 1504a^3b^5c^3d^2 - 3840a^4 \\
& b^3c^4d^2 + 96a^4b^5c^2e^2 - 512a^5b^3c^3e^2 + 27a^9bcd^2 - 9a^3cd^2(-4ac - b^2)^9)^{(1/2)} - 3072a^6c^5 \\
& d^2e + 36a^2b^8c^2d^2e - 192a^3b^6c^2d^2e + 128a^4b^4c^3d^2e + 1536a^5b^2c^4d^2e + 2a^2b^8c^2d^2e^2 \\
& (-4ac - b^2)^9)^{(1/2)} / (32(a^3b^{12} + 4096a^9c^6 - 24a^4b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6144a^8b^2c^5) \\
& ))^{(1/2)} * 2i + ((x(a^2b^2e - b^2d + 2a^2cd)) / (2a(4ac - b^2)) + (c^2x^3(2ae - bd)) / (2a(4ac - b^2))) / (a + b^2x^2 + c^2x^4)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)/(c\*x\*\*4+b\*x\*\*2+a)\*\*2,x)

[Out] Timed out

$$3.196 \quad \int \frac{1}{(a+bx^2+cx^4)^2} dx$$

**Optimal.** Leaf size=252

$$\frac{x(-2ac + b^2 + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{c} \left( b\sqrt{b^2 - 4ac} - 12ac + b^2 \right) \tan^{-1} \left( \frac{\sqrt{2} \sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2} a (b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{c} \left( -b\sqrt{b^2 - 4ac} - 12ac + b^2 \right) \tan^{-1} \left( \frac{\sqrt{2} \sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2} a (b^2 - 4ac)^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}}}$$

**Rubi [A]** time = 0.52, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {1092, 1166, 205}

$$\frac{x(-2ac + b^2 + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{c} \left( b\sqrt{b^2 - 4ac} - 12ac + b^2 \right) \tan^{-1} \left( \frac{\sqrt{2} \sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2} a (b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{c} \left( -b\sqrt{b^2 - 4ac} - 12ac + b^2 \right) \tan^{-1} \left( \frac{\sqrt{2} \sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2} a (b^2 - 4ac)^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2 + c\*x^4)^(-2), x]

[Out] (x\*(b^2 - 2\*a\*c + b\*c\*x^2))/(2\*a\*(b^2 - 4\*a\*c)\*(a + b\*x^2 + c\*x^4)) + (Sqrt[c]\*(b^2 - 12\*a\*c + b\*Sqrt[b^2 - 4\*a\*c])\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b - Sqrt[b^2 - 4\*a\*c]])/(2\*Sqrt[2]\*a\*(b^2 - 4\*a\*c)^(3/2)\*Sqrt[b - Sqrt[b^2 - 4\*a\*c]]) - (Sqrt[c]\*(b^2 - 12\*a\*c - b\*Sqrt[b^2 - 4\*a\*c])\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b + Sqrt[b^2 - 4\*a\*c]])/(2\*Sqrt[2]\*a\*(b^2 - 4\*a\*c)^(3/2)\*Sqrt[b + Sqrt[b^2 - 4\*a\*c]])

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 1092

Int[((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] := -Simp[(x\*(b^2 - 2\*a\*c + b\*c\*x^2)\*(a + b\*x^2 + c\*x^4)^(p + 1))/(2\*a\*(p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/(2\*a\*(p + 1)\*(b^2 - 4\*a\*c)), Int[(b^2 - 2\*a\*c + 2\*(p + 1)\*(b^2 - 4\*a\*c) + b\*c\*(4\*p + 7)\*x^2)\*(a + b\*x^2 + c\*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1] && IntegerQ[2\*p]

#### Rule 1166

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[e/2 + (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 - q/2 + c\*x^2), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - a\*e^2, 0] && PosQ[b^2 - 4\*a\*c]

#### Rubi steps

$$\begin{aligned} \int \frac{1}{(a + bx^2 + cx^4)^2} dx &= \frac{x(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\int \frac{b^2 - 2ac - 2(b^2 - 4ac) - bcx^2}{a + bx^2 + cx^4} dx}{2a(b^2 - 4ac)} \\ &= \frac{x(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\left(c(b^2 - 12ac - b\sqrt{b^2 - 4ac})\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{4a(b^2 - 4ac)^{3/2}} \\ &= \frac{x(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{c}(b^2 - 12ac + b\sqrt{b^2 - 4ac}) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}a(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} \end{aligned}$$

**Mathematica [A]** time = 0.42, size = 243, normalized size = 0.96

$$\frac{2x(-2ac + b^2 + bcx^2)}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{2}\sqrt{c}(b\sqrt{b^2 - 4ac} - 12ac + b^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{2}\sqrt{c}(b\sqrt{b^2 - 4ac} + 12ac - b^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2 - 4ac} + b}}\right)}{(b^2 - 4ac)^{3/2}\sqrt{\sqrt{b^2 - 4ac} + b}}$$

4a

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2 + c\*x^4)^(-2), x]

[Out] ((2\*x\*(b^2 - 2\*a\*c + b\*c\*x^2))/((b^2 - 4\*a\*c)\*(a + b\*x^2 + c\*x^4)) + (Sqrt[2]\*Sqrt[c]\*(b^2 - 12\*a\*c + b\*Sqrt[b^2 - 4\*a\*c])\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b - Sqrt[b^2 - 4\*a\*c]])/((b^2 - 4\*a\*c)^(3/2)\*Sqrt[b - Sqrt[b^2 - 4\*a\*c]]) + (Sqrt[2]\*Sqrt[c]\*(-b^2 + 12\*a\*c + b\*Sqrt[b^2 - 4\*a\*c])\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b + Sqrt[b^2 - 4\*a\*c]])/((b^2 - 4\*a\*c)^(3/2)\*Sqrt[b + Sqrt[b^2 - 4\*a\*c]]))/(4\*a)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)^(-2), x]

[Out] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)^(-2), x]

**fricas [B]** time = 0.85, size = 2309, normalized size = 9.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^4+b\*x^2+a)^2,x, algorithm="fricas")

[Out] 1/4\*(2\*b\*c\*x^3 + sqrt(1/2)\*((a\*b^2\*c - 4\*a^2\*c^2)\*x^4 + a^2\*b^2 - 4\*a^3\*c + (a\*b^3 - 4\*a^2\*b\*c)\*x^2)\*sqrt(-(b^5 - 15\*a\*b^3\*c + 60\*a^2\*b\*c^2 + (a^3\*b^6 - 12\*a^4\*b^4\*c + 48\*a^5\*b^2\*c^2 - 64\*a^6\*c^3)\*sqrt((b^4 - 18\*a\*b^2\*c + 81\*a^2\*c^2)/(a^6\*b^6 - 12\*a^7\*b^4\*c + 48\*a^8\*b^2\*c^2 - 64\*a^9\*c^3)))/(a^3\*b^6 - 12\*a^4\*b^4\*c + 48\*a^5\*b^2\*c^2 - 64\*a^6\*c^3))\*log((5\*b^4\*c^2 - 81\*a\*b^2\*c^3 + 324\*a^2\*c^4)\*x + 1/2\*sqrt(1/2)\*(b^8 - 23\*a\*b^6\*c + 190\*a^2\*b^4\*c^2 - 672\*a^3\*b^2\*c^3 + 864\*a^4\*c^4 - (a^3\*b^9 - 20\*a^4\*b^7\*c + 144\*a^5\*b^5\*c^2 - 448\*a^6\*b^3\*c^3 + 512\*a^7\*b\*c^4)\*sqrt((b^4 - 18\*a\*b^2\*c + 81\*a^2\*c^2)/(a^6\*b^6 - 12\*a^7\*b^4\*c + 48\*a^8\*b^2\*c^2 - 64\*a^9\*c^3)))\*sqrt(-(b^5 - 15\*a\*b^3\*c

$$\begin{aligned}
& + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*\text{sqrt} \\
& \text{t}((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 \\
& - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)) - \\
& \text{sqrt}(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b \\
& *c)*x^2)*\text{sqrt}(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + \\
& 48*a^5*b^2*c^2 - 64*a^6*c^3)*\text{sqrt}((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 \\
& - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + \\
& 48*a^5*b^2*c^2 - 64*a^6*c^3))*\text{log}((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)* \\
& x - 1/2*\text{sqrt}(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 8 \\
& 64*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + \\
& 512*a^7*b*c^4)*\text{sqrt}((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c \\
& + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*\text{sqrt}(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + \\
& (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*\text{sqrt}((b^4 - 18*a*b^2 \\
& *c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/( \\
& a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + \text{sqrt}(1/2)*((a*b^2 \\
& *c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*\text{sqrt}(-(b \\
& ^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - \\
& 64*a^6*c^3)*\text{sqrt}((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + \\
& 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - \\
& 64*a^6*c^3))*\text{log}((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x + 1/2*\text{sqrt}(1/2) \\
& *(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3 \\
& *b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*\text{sq} \\
& \text{rt}((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 \\
& - 64*a^9*c^3)))*\text{sqrt}(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4 \\
& *b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*\text{sqrt}((b^4 - 18*a*b^2*c + 81*a^2*c^2)/ \\
& (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4* \\
& b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - \text{sqrt}(1/2)*((a*b^2*c - 4*a^2*c^2)*x \\
& ^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*\text{sqrt}(-(b^5 - 15*a*b^3*c + \\
& 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*\text{sqrt} \\
& ((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - \\
& 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*\text{log} \\
& ((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x - 1/2*\text{sqrt}(1/2)*(b^8 - 23*a*b^6* \\
& c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3*b^9 - 20*a^4*b^7 \\
& *c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*\text{sqrt}((b^4 - 18*a*b^ \\
& 2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))* \\
& \text{sqrt}(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b \\
& ^2*c^2 - 64*a^6*c^3)*\text{sqrt}((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7 \\
& *b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^ \\
& 2*c^2 - 64*a^6*c^3))) + 2*(b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2 \\
& *b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
\end{aligned}$$

**giac [B]** time = 0.60, size = 2682, normalized size = 10.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^4+b\*x^2+a)^2,x, algorithm="giac")

[Out]  $\frac{1}{16} \frac{(b^2 c x^3 + b^2 x - 2 a c x)}{(c x^4 + b x^2 + a)(a b^2 - 4 a^2 c)} + \frac{1}{16} \frac{(2 a^2 b^7 c^2 - 40 a^3 b^5 c^3 + 224 a^4 b^3 c^4 - 384 a^5 b c^5 - \text{sqrt}(2) \text{sqrt}(b^2 - 4 a c) \text{sqrt}(b c + \text{sqrt}(b^2 - 4 a c) c) a^2 b^7 + 20 \text{sqrt}(2) \text{sqrt}(b^2 - 4 a c) \text{sqrt}(b c + \text{sqrt}(b^2 - 4 a c) c) a^3 b^5 c + 2 \text{sqrt}(2) \text{sqrt}(b^2 - 4 a c) \text{sqrt}(b c + \text{sqrt}(b^2 - 4 a c) c) a^2 b^6 c - 112 \text{sqrt}(2) \text{sqrt}(b^2 - 4 a c) \text{sqrt}(b c + \text{sqrt}(b^2 - 4 a c) c) a^4 b^3 c^2 - 32 \text{sqrt}(2) \text{sqrt}(b^2 - 4 a c) \text{sqrt}(b c + \text{sqrt}(b^2 - 4 a c) c) a^3 b^4 c^2 - \text{sqrt}(2) \text{sqrt}(b^2 - 4 a c) \text{sqrt}(b c + \text{sqrt}(b^2 - 4 a c) c) a^2 b^5 c^2 + 192 \text{sqrt}(2) \text{sqrt}(b^2 - 4 a c) \text{sqrt}(b c + \text{sqrt}(b^2 - 4 a c) c) a^5 b c^3 + 96 \text{sqrt}(2) \text{sqrt}(b^2 - 4 a c) \text{sqrt}(b c + \text{sqrt}(b^2 - 4 a c) c) a^4 b^2 c^3 + 16 \text{sqrt}(2) \text{sqrt}(b^2 - 4 a c) \text{sqrt}(b c + \text{sqrt}(b^2 - 4 a c) c) a^3 b^3 c^3 - 48 \text{sqrt}(2) \text{sqrt}(b^2 - 4 a c) \text{sqrt}(b c + \text{sqrt}(b^2 - 4 a c) c) a^4 b c^4 - 2(b^2 - 4 a c) a^2 c}$



$$2)^{(1/2)}/(4*a*c-b^2)/a*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^{(1/2)}/c)*b^2+1/4*c/(4*a*c-b^2)/a*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)*arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)*c*x)*b-3*c^2/(-4*a*c+b^2)^{(1/2)}/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)*arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)*c*x)+1/4*c/(-4*a*c+b^2)^{(1/2)}/(4*a*c-b^2)/a*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)*arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)*c*x)*b^2-1/4/(4*a*c-b^2)/a*x/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^{(1/2)}/c)*b-c/(-4*a*c+b^2)^{(1/2)}/(4*a*c-b^2)*x/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^{(1/2)}/c)+1/4/(-4*a*c+b^2)^{(1/2)}/(4*a*c-b^2)/a*x/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^{(1/2)}/c)*b^2-1/4*c/(4*a*c-b^2)/a*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)*arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)*c*x)*b-3*c^2/(-4*a*c+b^2)^{(1/2)}/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)*arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)*c*x)+1/4*c/(-4*a*c+b^2)^{(1/2)}/(4*a*c-b^2)/a*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)*arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)*c*x)*b^2$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^4+b\*x^2+a)^2,x, algorithm="maxima")

[Out]  $1/2*(b*c*x^3 + (b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate((b*c*x^2 + b^2 - 6*a*c)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)$

**mupad** [B] time = 6.26, size = 6404, normalized size = 25.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b\*x^2 + c\*x^4)^2,x)

[Out]  $((x*(2*a*c - b^2))/(2*a*(4*a*c - b^2)) - (b*c*x^3)/(2*a*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) + atan((((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)}))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^{(1/2)}*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)}))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^{(1/2)} + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)}))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^{(1/2)}*1i - (((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)}))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^{(1/2)}*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)}))$



$$\begin{aligned}
& (1/2))/((32*(a^3*b^{12} + 4096*a^9*c^6 - 24*a^4*b^{10}*c + 240*a^5*b^8*c^2 - 128 \\
& 0*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^{(1/2)} - (x*(72*a^2*c \\
& ^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(- \\
& b^{11} + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 15 \\
& 04*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^{( \\
& 1/2)))/(32*(a^3*b^{12} + 4096*a^9*c^6 - 24*a^4*b^{10}*c + 240*a^5*b^8*c^2 - 1280 \\
& *a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^{(1/2)}*i)/((((6144*a^ \\
& 5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^ \\
& 5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^{11} \\
& + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^ \\
& 3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)) \\
& /((32*(a^3*b^{12} + 4096*a^9*c^6 - 24*a^4*b^{10}*c + 240*a^5*b^8*c^2 - 1280*a^6* \\
& b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^{(1/2)}*(1024*a^5*b*c^5 - 16 \\
& *a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 \\
& - 8*a^3*b^2*c)))*(- (b^{11} + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + \\
& 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c \\
& *(- (4*a*c - b^2)^9)^{(1/2)))/(32*(a^3*b^{12} + 4096*a^9*c^6 - 24*a^4*b^{10}*c + 2 \\
& 40*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^{( \\
& 1/2)} + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 \\
& - 8*a^3*b^2*c)))*(- (b^{11} + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + \\
& 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c* \\
& (- (4*a*c - b^2)^9)^{(1/2)))/(32*(a^3*b^{12} + 4096*a^9*c^6 - 24*a^4*b^{10}*c + 24 \\
& 0*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^{( \\
& 1/2)} + (((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 \\
& - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) \\
& + (x*(-(b^{11} + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2* \\
& b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c \\
& - b^2)^9)^{(1/2)))/(32*(a^3*b^{12} + 4096*a^9*c^6 - 24*a^4*b^{10}*c + 240*a^5*b^ \\
& 8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^{(1/2)}*(10 \\
& 24*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2 \\
& *b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(- (b^{11} + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 2 \\
& 7*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(a^3*b^{12} + 4096*a^9*c^6 - \\
& 24*a^4*b^{10}*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 614 \\
& 4*a^8*b^2*c^5)))^{(1/2)} - (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b \\
& ^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(- (b^{11} + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27 \\
& *a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(a^3*b^{12} + 4096*a^9*c^6 - 2 \\
& 4*a^4*b^{10}*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144 \\
& *a^8*b^2*c^5)))^{(1/2)} + (5*b^3*c^4 - 36*a*b*c^5)/(4*(a^2*b^6 - 64*a^5*c^3 - \\
& 12*a^3*b^4*c + 48*a^4*b^2*c^2)))*(- (b^{11} + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 2 \\
& 7*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(a^3*b^{12} + 4096*a^9*c^6 - \\
& 24*a^4*b^{10}*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 614 \\
& 4*a^8*b^2*c^5)))^{(1/2)}*2i + atan((((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^ \\
& 6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 1 \\
& 2*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^{11} - b^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - \\
& 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(a^3*b^{12} + 4096*a^9*c^6 - \\
& 24*a^4*b^{10}*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 61 \\
& 44*a^8*b^2*c^5)))^{(1/2)}*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 \\
& - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(- (b^{11} - b^2 \\
& *(- (4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5 \\
& *c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)))/(32* \\
& (a^3*b^{12} + 4096*a^9*c^6 - 24*a^4*b^{10}*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c \\
& ^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^{(1/2)} + (x*(72*a^2*c^5 + b^4*c^ \\
& 3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(- (b^{11} - b^2* \\
& (- (4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5* \\
& c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(
\end{aligned}$$

$$\begin{aligned}
 & a^3b^{12} + 4096a^9c^6 - 24a^4b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6144a^8b^2c^5) \Big)^{1/2} \cdot i - \left( \left( (6144a^5c^6 + 16a^*b^8c^2 - 288a^2b^6c^3 + 1920a^3b^4c^4 - 5632a^4b^2c^5) / (8(a^2b^6 - 64a^5c^3 - 12a^3b^4c + 48a^4b^2c^2)) \right) + (x \cdot (-b^{11} - b^2 \cdot (-4a^*c - b^2)^9)) \Big)^{1/2} - 3840a^5b^*c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27a^*b^9c + 9a^*c \cdot (-4a^*c - b^2)^9 \Big)^{1/2} \right) / (32(a^3b^{12} + 4096a^9c^6 - 24a^4b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6144a^8b^2c^5) \Big)^{1/2} \cdot (1024a^5b^*c^5 - 16a^2b^7c^2 + 192a^3b^5c^3 - 768a^4b^3c^4) / (2(a^2b^4 + 16a^4c^2 - 8a^3b^2c)) \Big) \cdot (-b^{11} - b^2 \cdot (-4a^*c - b^2)^9) \Big)^{1/2} - 3840a^5b^*c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27a^*b^9c + 9a^*c \cdot (-4a^*c - b^2)^9 \Big)^{1/2} \right) / (32(a^3b^{12} + 4096a^9c^6 - 24a^4b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6144a^8b^2c^5) \Big)^{1/2} - (x \cdot (72a^2c^5 + b^4c^3 - 14a^*b^2c^4)) / (2(a^2b^4 + 16a^4c^2 - 8a^3b^2c)) \Big) \cdot (-b^{11} - b^2 \cdot (-4a^*c - b^2)^9) \Big)^{1/2} - 3840a^5b^*c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27a^*b^9c + 9a^*c \cdot (-4a^*c - b^2)^9 \Big)^{1/2} \right) / (32(a^3b^{12} + 4096a^9c^6 - 24a^4b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6144a^8b^2c^5) \Big)^{1/2} \cdot i) / \left( \left( (6144a^5c^6 + 16a^*b^8c^2 - 288a^2b^6c^3 + 1920a^3b^4c^4 - 5632a^4b^2c^5) / (8(a^2b^6 - 64a^5c^3 - 12a^3b^4c + 48a^4b^2c^2)) \right) - (x \cdot (-b^{11} - b^2 \cdot (-4a^*c - b^2)^9)) \Big)^{1/2} - 3840a^5b^*c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27a^*b^9c + 9a^*c \cdot (-4a^*c - b^2)^9 \Big)^{1/2} \right) / (32(a^3b^{12} + 4096a^9c^6 - 24a^4b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6144a^8b^2c^5) \Big)^{1/2} \cdot (1024a^5b^*c^5 - 16a^2b^7c^2 + 192a^3b^5c^3 - 768a^4b^3c^4) / (2(a^2b^4 + 16a^4c^2 - 8a^3b^2c)) \Big) \cdot (-b^{11} - b^2 \cdot (-4a^*c - b^2)^9) \Big)^{1/2} - 3840a^5b^*c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27a^*b^9c + 9a^*c \cdot (-4a^*c - b^2)^9 \Big)^{1/2} \right) / (32(a^3b^{12} + 4096a^9c^6 - 24a^4b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6144a^8b^2c^5) \Big)^{1/2} + (x \cdot (72a^2c^5 + b^4c^3 - 14a^*b^2c^4)) / (2(a^2b^4 + 16a^4c^2 - 8a^3b^2c)) \Big) \cdot (-b^{11} - b^2 \cdot (-4a^*c - b^2)^9) \Big)^{1/2} - 3840a^5b^*c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27a^*b^9c + 9a^*c \cdot (-4a^*c - b^2)^9 \Big)^{1/2} \right) / (32(a^3b^{12} + 4096a^9c^6 - 24a^4b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6144a^8b^2c^5) \Big)^{1/2} + \left( \left( (6144a^5c^6 + 16a^*b^8c^2 - 288a^2b^6c^3 + 1920a^3b^4c^4 - 5632a^4b^2c^5) / (8(a^2b^6 - 64a^5c^3 - 12a^3b^4c + 48a^4b^2c^2)) \right) + (x \cdot (-b^{11} - b^2 \cdot (-4a^*c - b^2)^9)) \Big)^{1/2} - 3840a^5b^*c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27a^*b^9c + 9a^*c \cdot (-4a^*c - b^2)^9 \Big)^{1/2} \right) / (32(a^3b^{12} + 4096a^9c^6 - 24a^4b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6144a^8b^2c^5) \Big)^{1/2} \cdot (1024a^5b^*c^5 - 16a^2b^7c^2 + 192a^3b^5c^3 - 768a^4b^3c^4) / (2(a^2b^4 + 16a^4c^2 - 8a^3b^2c)) \Big) \cdot (-b^{11} - b^2 \cdot (-4a^*c - b^2)^9) \Big)^{1/2} - 3840a^5b^*c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27a^*b^9c + 9a^*c \cdot (-4a^*c - b^2)^9 \Big)^{1/2} \right) / (32(a^3b^{12} + 4096a^9c^6 - 24a^4b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6144a^8b^2c^5) \Big)^{1/2} - (x \cdot (72a^2c^5 + b^4c^3 - 14a^*b^2c^4)) / (2(a^2b^4 + 16a^4c^2 - 8a^3b^2c)) \Big) \cdot (-b^{11} - b^2 \cdot (-4a^*c - b^2)^9) \Big)^{1/2} - 3840a^5b^*c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27a^*b^9c + 9a^*c \cdot (-4a^*c - b^2)^9 \Big)^{1/2} \right) / (32(a^3b^{12} + 4096a^9c^6 - 24a^4b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6144a^8b^2c^5) \Big)^{1/2} + (5b^3c^4 - 36a^*b^*c^5) / (4(a^2b^6 - 64a^5c^3 - 12a^3b^4c + 48a^4b^2c^2)) \Big) \cdot (-b^{11} - b^2 \cdot (-4a^*c - b^2)^9) \Big)^{1/2} - 3840a^5b^*c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27a^*b^9c + 9a^*c \cdot (-4a^*c - b^2)^9 \Big)^{1/2} \right) / (32(a^3b^{12} + 4096a^9c^6 - 24a^4b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6144a^8b^2c^5) \Big)^{1/2} \cdot 2i
 \end{aligned}$$

**sympy [A]** time = 170.28, size = 394, normalized size = 1.56

$\frac{-3c^3 + (2c - 3)^2}{8c^3 - 24c^2 + 27c - 24c^2 + 27c - 24c^2} + 80055600 \left( \frac{1048576c^6 - 1572864c^5c^2 - 983040c^4c^3 - 327680c^5c^4 + 61440c^6c^5 + 61440c^7c^6 - 24864c^8c^7 + 4608c^9c^8 - 832c^{10} + 128c^{11}}{324c^{12} - 81c^{12} + 81c^2} \left( \dots \right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x\*\*4+b\*x\*\*2+a)\*\*2,x)

[Out] 
$$\frac{-b*c*x**3 + x*(2*a*c - b**2)}{(8*a**3*c - 2*a**2*b**2 + x**4*(8*a**2*c**2 - 2*a*b**2*c) + x**2*(8*a**2*b*c - 2*a*b**3))} + \text{RootSum}(\_t**4*(1048576*a**9*c**6 - 1572864*a**8*b**2*c**5 + 983040*a**7*b**4*c**4 - 327680*a**6*b**6*c**3 + 61440*a**5*b**8*c**2 - 6144*a**4*b**10*c + 256*a**3*b**12) + \_t**2*(-61440*a**5*b*c**5 + 61440*a**4*b**3*c**4 - 24064*a**3*b**5*c**3 + 4608*a**2*b**7*c**2 - 432*a*b**9*c + 16*b**11) + 1296*a**2*c**5 - 360*a*b**2*c**4 + 25*b**4*c**3, \text{Lambda}(\_t, \_t*\log(x + (32768*\_t**3*a**7*b*c**4 - 28672*\_t**3*a**6*b**3*c**3 + 9216*\_t**3*a**5*b**5*c**2 - 1280*\_t**3*a**4*b**7*c + 64*\_t**3*a**3*b**9 + 1728*\_t*a**4*c**4 - 2304*\_t*a**3*b**2*c**3 + 740*\_t*a**2*b**4*c**2 - 92*\_t*a*b**6*c + 4*\_t*b**8)/(324*a**2*c**4 - 81*a*b**2*c**3 + 5*b**4*c**2))))$$

$$3.197 \quad \int \frac{1}{(d+ex^2)(a+bx^2+cx^4)^2} dx$$

**Optimal.** Leaf size=660

$$\frac{\sqrt{c} e^2 \left( e - \frac{2cd-be}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left( \frac{\sqrt{2} \sqrt{c} x}{\sqrt{b-\sqrt{b^2-4ac}}} \right) - \sqrt{c} e^2 \left( \frac{2cd-be}{\sqrt{b^2-4ac}} + e \right) \tan^{-1} \left( \frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{\sqrt{2} \sqrt{b-\sqrt{b^2-4ac}} (ae^2 - bde + cd^2)^2} - \frac{\sqrt{c} e^2 \left( \frac{2cd-be}{\sqrt{b^2-4ac}} + e \right) \tan^{-1} \left( \frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{\sqrt{2} \sqrt{\sqrt{b^2-4ac}+b} (ae^2 - bde + cd^2)^2} + \frac{x (cx^2 (2ace + b^2(-e) + bcd))}{2a (b^2 - 4ac) (a + bx^2)}$$

**Rubi [A]** time = 2.87, antiderivative size = 660, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1238, 205, 1178, 1166}

$$\frac{x (cx^2 (2ace + b^2(-e) + bcd))}{2a (b^2 - 4ac) (a + bx^2 + cx^4)^2} + \frac{\sqrt{c} \left( \frac{2cd-be}{\sqrt{b^2-4ac}} + e \right) \tan^{-1} \left( \frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{2\sqrt{2}a (b^2 - 4ac) \sqrt{b-\sqrt{b^2-4ac}} (ae^2 - bde + cd^2)} + \frac{\sqrt{c} \left( \frac{2cd-be}{\sqrt{b^2-4ac}} + e \right) \tan^{-1} \left( \frac{\sqrt{2} \sqrt{c} x}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{2\sqrt{2}a (b^2 - 4ac) \sqrt{\sqrt{b^2-4ac}+b} (ae^2 - bde + cd^2)} + \frac{\sqrt{c} e^2 \left( \frac{2cd-be}{\sqrt{b^2-4ac}} + e \right) \tan^{-1} \left( \frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{\sqrt{2} \sqrt{b-\sqrt{b^2-4ac}} (ae^2 - bde + cd^2)^2} + \frac{e^{7/2} \tan^{-1} \left( \frac{\sqrt{c} x}{\sqrt{d}} \right)}{\sqrt{d} (ae^2 - bde + cd^2)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x^2)\*(a + b\*x^2 + c\*x^4)^2),x]

[Out] (x\*(b^2\*c\*d - 2\*a\*c^2\*d - b^3\*e + 3\*a\*b\*c\*e + c\*(b\*c\*d - b^2\*e + 2\*a\*c\*e)\*x^2))/(2\*a\*(b^2 - 4\*a\*c)\*(c\*d^2 - b\*d\*e + a\*e^2)\*(a + b\*x^2 + c\*x^4)) - (Sqrt[c]\*e^2\*(e - (2\*c\*d - b\*e)/Sqrt[b^2 - 4\*a\*c])\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b - Sqrt[b^2 - 4\*a\*c]]]/(Sqrt[2]\*Sqrt[b - Sqrt[b^2 - 4\*a\*c]]\*(c\*d^2 - b\*d\*e + a\*e^2)^2) + (Sqrt[c]\*(b\*c\*d - b^2\*e + 2\*a\*c\*e + (b^2\*c\*d - 12\*a\*c^2\*d - b^3\*e + 8\*a\*b\*c\*e)/Sqrt[b^2 - 4\*a\*c])\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b - Sqrt[b^2 - 4\*a\*c]]]/(2\*Sqrt[2]\*a\*(b^2 - 4\*a\*c)\*Sqrt[b - Sqrt[b^2 - 4\*a\*c]]\*(c\*d^2 - b\*d\*e + a\*e^2)) - (Sqrt[c]\*e^2\*(e + (2\*c\*d - b\*e)/Sqrt[b^2 - 4\*a\*c])\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b + Sqrt[b^2 - 4\*a\*c]]]/(Sqrt[2]\*Sqrt[b + Sqrt[b^2 - 4\*a\*c]]\*(c\*d^2 - b\*d\*e + a\*e^2)^2) + (Sqrt[c]\*(b\*c\*d - b^2\*e + 2\*a\*c\*e - (b^2\*c\*d - 12\*a\*c^2\*d - b^3\*e + 8\*a\*b\*c\*e)/Sqrt[b^2 - 4\*a\*c])\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b + Sqrt[b^2 - 4\*a\*c]]]/(2\*Sqrt[2]\*a\*(b^2 - 4\*a\*c)\*Sqrt[b + Sqrt[b^2 - 4\*a\*c]]\*(c\*d^2 - b\*d\*e + a\*e^2)) + (e^(7/2))\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]]/(Sqrt[d]\*(c\*d^2 - b\*d\*e + a\*e^2)^2)

**Rule 205**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 1166**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[e/2 + (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 - q/2 + c\*x^2), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - a\*e^2, 0] && PosQ[b^2 - 4\*a\*c]

**Rule 1178**

Int[((d\_) + (e\_.)\*(x\_)^2)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] := Simp[(x\*(a\*b\*e - d\*(b^2 - 2\*a\*c) - c\*(b\*d - 2\*a\*e)\*x^2)\*(a + b\*x^2 + c\*x^4)^(p + 1))/(2\*a\*(p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/(2\*a\*(p + 1)\*(b^2 - 4\*a\*c)), Int[Simp[(2\*p + 3)\*d\*b^2 - a\*b\*e - 2\*a\*c\*d\*(4\*p + 5) + (4\*p + 7)\*(d\*b - 2\*a\*e)\*c\*x^2, x]\*(a + b\*x^2 + c\*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[p, -1] && IntegerQ[2\*p]

**Rule 1238**

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
/; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
&& IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(d + ex^2)(a + bx^2 + cx^4)^2} dx &= \int \left( \frac{e^4}{(cd^2 - bde + ae^2)^2 (d + ex^2)} + \frac{cd - be - cex^2}{(cd^2 - bde + ae^2)(a + bx^2 + cx^4)^2} - \frac{cd - be - cex^2}{(cd^2 - bde + ae^2)(a + bx^2 + cx^4)} \right) dx \\ &= -\frac{e^2 \int \frac{-cd + be + cex^2}{a + bx^2 + cx^4} dx}{(cd^2 - bde + ae^2)^2} + \frac{e^4 \int \frac{1}{d + ex^2} dx}{(cd^2 - bde + ae^2)^2} + \frac{\int \frac{cd - be - cex^2}{(a + bx^2 + cx^4)^2} dx}{cd^2 - bde + ae^2} \\ &= \frac{x(b^2cd - 2ac^2d - b^3e + 3abce + c(bcd - b^2e + 2ace)x^2)}{2a(b^2 - 4ac)(cd^2 - bde + ae^2)(a + bx^2 + cx^4)} + \frac{e^{7/2} \tan^{-1}\left(\frac{x}{\sqrt{d}}\right)}{\sqrt{d}(cd^2 - bde + ae^2)} \\ &= \frac{x(b^2cd - 2ac^2d - b^3e + 3abce + c(bcd - b^2e + 2ace)x^2)}{2a(b^2 - 4ac)(cd^2 - bde + ae^2)(a + bx^2 + cx^4)} - \frac{\sqrt{c}e^2 \left(e - \frac{2cd - b^2e}{\sqrt{b^2 - 4ac}}\right)}{\sqrt{2}\sqrt{b - \sqrt{b^2 - 4ac}}} \\ &= \frac{x(b^2cd - 2ac^2d - b^3e + 3abce + c(bcd - b^2e + 2ace)x^2)}{2a(b^2 - 4ac)(cd^2 - bde + ae^2)(a + bx^2 + cx^4)} - \frac{\sqrt{c}e^2 \left(e - \frac{2cd - b^2e}{\sqrt{b^2 - 4ac}}\right)}{\sqrt{2}\sqrt{b - \sqrt{b^2 - 4ac}}} \end{aligned}$$

**Mathematica [A]** time = 2.79, size = 708, normalized size = 1.07

$$\frac{\sqrt{2} \sqrt{c} e^2 \left( e - \frac{2cd - b^2e}{\sqrt{b^2 - 4ac}} \right) + \frac{x(b^2cd - 2ac^2d - b^3e + 3abce + c(bcd - b^2e + 2ace)x^2)}{2a(b^2 - 4ac)(cd^2 - bde + ae^2)(a + bx^2 + cx^4)}}{\sqrt{2} \sqrt{b - \sqrt{b^2 - 4ac}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x^2)\*(a + b\*x^2 + c\*x^4)^2), x]

[Out] ((2\*(c\*d^2 + e\*(-(b\*d) + a\*e))\*x\*(b^3\*e - b\*c\*(3\*a\*e + c\*d\*x^2) + 2\*a\*c^2\*(d - e\*x^2) + b^2\*c\*(-d + e\*x^2)))/(a\*(-b^2 + 4\*a\*c)\*(a + b\*x^2 + c\*x^4)) + (Sqrt[2]\*Sqrt[c]\*(b^4\*d\*e^2 + 2\*a\*c\*(-6\*c^2\*d^3 + 5\*a\*Sqrt[b^2 - 4\*a\*c]\*e^3 + c\*d\*e\*(Sqrt[b^2 - 4\*a\*c]\*d - 14\*a\*e)) + b^3\*e\*(-2\*c\*d^2 + e\*(Sqrt[b^2 - 4\*a\*c]\*d - 3\*a\*e)) + b^2\*(c^2\*d^3 - 3\*a\*Sqrt[b^2 - 4\*a\*c]\*e^3 - c\*d\*e\*(2\*Sqrt[b^2 - 4\*a\*c]\*d + 3\*a\*e)) + b\*c\*(a\*e^2\*(-(Sqrt[b^2 - 4\*a\*c]\*d) + 16\*a\*e) + c\*d^2\*(Sqrt[b^2 - 4\*a\*c]\*d + 20\*a\*e))\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b - Sqrt[b^2 - 4\*a\*c]])/(a\*(b^2 - 4\*a\*c)^(3/2)\*Sqrt[b - Sqrt[b^2 - 4\*a\*c]]) + (Sqrt[2]\*Sqrt[c]\*(-(b^4\*d\*e^2) - b^2\*(c^2\*d^3 + 3\*a\*Sqrt[b^2 - 4\*a\*c]\*e^3 + c\*d\*e\*(2\*Sqrt[b^2 - 4\*a\*c]\*d - 3\*a\*e)) + b^3\*e\*(2\*c\*d^2 + e\*(Sqrt[b^2 - 4\*a\*c]\*d + 3\*a\*e)) + 2\*a\*c\*(6\*c^2\*d^3 + 5\*a\*Sqrt[b^2 - 4\*a\*c]\*e^3 + c\*d\*e\*(Sqrt[b^2 - 4\*a\*c]\*d + 14\*a\*e)) + b\*c\*(c\*d^2\*(Sqrt[b^2 - 4\*a\*c]\*d - 20\*a\*e) - a\*e^2\*(Sqrt[b^2 - 4\*a\*c]\*d + 16\*a\*e))\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b + Sqrt[b^2 - 4\*a\*c]])/(a\*(b^2 - 4\*a\*c)^(3/2)\*Sqrt[b + Sqrt[b^2 - 4\*a\*c]]) + (4\*e^(7/2)\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/Sqrt[d])/(4\*(c\*d^2 + e\*(-(b\*d) + a\*e))^2)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex^2)(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e\*x^2)\*(a + b\*x^2 + c\*x^4)^2), x]

[Out] IntegrateAlgebraic[1/((d + e\*x^2)\*(a + b\*x^2 + c\*x^4)^2), x]

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)/(c\*x^4+b\*x^2+a)^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)/(c\*x^4+b\*x^2+a)^2,x, algorithm="giac")

[Out] Timed out

**maple** [B] time = 0.06, size = 3841, normalized size = 5.82

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x^2+d)/(c\*x^4+b\*x^2+a)^2,x)

[Out] 
$$\begin{aligned} & e^4/(a^2e-bde+c^2d)^2/(de)^{1/2} \arctan(1/(de)^{1/2}ex)+1/2/(a^2e-bde+c^2d)^2/(c^2x^4+b^2x^2+a)/(4ac-b^2)xb^3e^3+1/(a^2e-bde+c^2d)^2/(c^2x^4+b^2x^2+a)/(4ac-b^2)xc^3d^3+1/4/(a^2e-bde+c^2d)^2/a/(4ac-b^2)bc/(-4ac+b^2)^{1/2}2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2} \arctan(2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2}cx) * b^4de^2+1/4/(a^2e-bde+c^2d)^2/a/(4ac-b^2)bc/(-4ac+b^2)^{1/2}2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2} \operatorname{arctanh}(2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2}cx) * b^4de^2-1/2/(a^2e-bde+c^2d)^2/a/(4ac-b^2)bc^2/(-4ac+b^2)^{1/2}2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2} \operatorname{arctanh}(2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2}cx) * b^3d^2e+1/4/(a^2e-bde+c^2d)^2/a/(4ac-b^2)c^32^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2} \operatorname{arctanh}(2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2}cx) * b^3d^2e+1/4/(a^2e-bde+c^2d)^2/(c^2x^4+b^2x^2+a)ca/(4ac-b^2)xb^3b^3de^2+1/(a^2e-bde+c^2d)^2/(c^2x^4+b^2x^2+a)c^2/a/(4ac-b^2)xb^3b^2d^2e+1/(a^2e-bde+c^2d)^2/(c^2x^4+b^2x^2+a)/a/(4ac-b^2)xb^3c^2de-1/4/(a^2e-bde+c^2d)^2/a/(4ac-b^2)c^32^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2} \arctan(2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2}cx) * b^3d^3-3/4/(a^2e-bde+c^2d)^2/(4ac-b^2)bc/(-4ac+b^2)^{1/2}2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2} \arctan(2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2}cx) * b^3e^3+1/4/(a^2e-bde+c^2d)^2/(4ac-b^2)c^22^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2} \arctan(2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2}cx) * b^3de^2-1/4/(a^2e-bde+c^2d)^2/(4ac-b^2)c^22^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2} \operatorname{arctanh}(2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2}cx) * b^3de^2-1/2/(a^2e-bde+c^2d)^2/a/(4ac-b^2)bc^2/(-4ac+b^2)^{1/2}2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2} \arctan(2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2}cx) * b^3d^2e+1/4/(a^2e-bde+c^2d)^2/a/(4ac-b^2)c^22^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2} \operatorname{arctanh}(2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2}cx) * b^3de^2+1/4/(a^2e-bde+c^2d)^2/a/(4ac-b^2)c^3/(-4ac+b^2)^{1/2}2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2} \operatorname{arctanh}(2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2}cx) * b^2d^3+4/(a^2e-bde+c^2d)^2a/(4ac-b^2)c^2/(\end{aligned}$$

$$\begin{aligned}
& -4*a*c+b^2)^{(1/2)}*2^{(1/2)} / ((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / \\
& / ((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * c*x) * b*e^{-3-3/4} / (a*e^2-b*d*e+c*d^2)^{2/4} * \\
& a*c-b^2)^*c^2 / (-4*a*c+b^2)^{(1/2)} * 2^{(1/2)} / ((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * a \\
& rctanh(2^{(1/2)} / ((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * c*x) * b^2*d*e^{2+5/4} / (a*e^2-b* \\
& d*e+c*d^2)^{2/4} * a*c-b^2)^*c^3 / (-4*a*c+b^2)^{(1/2)} * 2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)}) \\
& / ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * c*x) * b*d^2*e+ \\
& 1/2 / (a*e^2-b*d*e+c*d^2)^{2/4} / (c*x^4+b*x^2+a) * c / (4*a*c-b^2) * x^3*b^2*e^{-3-1/4} / (a*e^ \\
& 2-b*d*e+c*d^2)^{2/4} / (c*x^4+b*x^2+a) * c^3 / (4*a*c-b^2) * x^3*d^2*e-1 / (a*e^2-b*d*e+c \\
& *d^2)^{2/4} / (c*x^4+b*x^2+a) * c^2*a / (4*a*c-b^2) * x^3*e^{-3-3/4} / (a*e^2-b*d*e+c*d^2)^{2/4} \\
& / (4*a*c-b^2) * c / (-4*a*c+b^2)^{(1/2)} * 2^{(1/2)} / ((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} \\
& * \operatorname{arctanh}(2^{(1/2)} / ((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * c*x) * b^3*e^{-3-7/4} / (a*e^2-b* \\
& d*e+c*d^2)^{2/4} * a / (4*a*c-b^2) * c^3 / (-4*a*c+b^2)^{(1/2)} * 2^{(1/2)} / ((-b+(-4*a*c+b^2)^{(1/2)}) \\
& / ((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * c*x) * d*e \\
& ^{2+1/2} / (a*e^2-b*d*e+c*d^2)^{2/4} / (4*a*c-b^2) * c^2 * 2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)}) \\
& / ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * c*x) * b^2*d^2*e \\
& +4 / (a*e^2-b*d*e+c*d^2)^{2/4} * a / (4*a*c-b^2) * c^2 / (-4*a*c+b^2)^{(1/2)} * 2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)}) \\
& / ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * c*x) * b*e^{-3-7/4} / (a*e^2-b*d*e+c*d^2)^{2/4} * a / (4*a*c-b^2) * c^3 / (-4*a*c+b^2)^{(1/2)} * 2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * c*x) * d*e^{-2-1/2} / (a*e^2-b*d*e+c*d^2)^{2/4} / (4*a*c-b^2) * c^2 * 2^{(1/2)} / ((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * c*x) * b^2*d^2*e-1/4 / (a*e^2-b*d*e+c*d^2)^{2/4} / (4*a*c-b^2) * c^2 * 2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * c*x) * b^3*d*e^{2+1/4} / (a*e^2-b*d*e+c*d^2)^{2/4} / (4*a*c-b^2) * c^3 / (-4*a*c+b^2)^{(1/2)} * 2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * c*x) * b^2*d^3+5 / (a*e^2-b*d*e+c*d^2)^{2/4} / (4*a*c-b^2) * c^3 / (-4*a*c+b^2)^{(1/2)} * 2^{(1/2)} / ((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * c*x) * b*d^2*e-3/4 / (a*e^2-b*d*e+c*d^2)^{2/4} / (4*a*c-b^2) * c^2 / (-4*a*c+b^2)^{(1/2)} * 2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * c*x) * b^2*d*e^{2-3/4} / (a*e^2-b*d*e+c*d^2)^{2/4} / (4*a*c-b^2) * c^4 / (-4*a*c+b^2)^{(1/2)} * 2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * c*x) * d^3+5/2 / (a*e^2-b*d*e+c*d^2)^{2/4} * a / (4*a*c-b^2) * c^2 * 2^{(1/2)} / ((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * c*x) * e^{-3-5/2} / (a*e^2-b*d*e+c*d^2)^{2/4} * a / (4*a*c-b^2) * c^2 * 2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * c*x) * e^{-3-3/2} / (a*e^2-b*d*e+c*d^2)^{2/4} / (c*x^4+b*x^2+a) * a / (4*a*c-b^2) * x * b * e^3 * c + 1 / (a*e^2-b*d*e+c*d^2)^{2/4} / (c*x^4+b*x^2+a) * a / (4*a*c-b^2) * x * c^2 * d * e^{2+1/4} / (a*e^2-b*d*e+c*d^2)^{2/4} / (c*x^4+b*x^2+a) / (4*a*c-b^2) * x * b^2 * c * d * e^{2-5/2} / (a*e^2-b*d*e+c*d^2)^{2/4} / (c*x^4+b*x^2+a) / (4*a*c-b^2) * x * b * c^2 * d^2 * e-1/2 / (a*e^2-b*d*e+c*d^2)^{2/4} / (c*x^4+b*x^2+a) / a / (4*a*c-b^2) * x * b^4 * d * e^{2-1/2} / (a*e^2-b*d*e+c*d^2)^{2/4} / (c*x^4+b*x^2+a) / a / (4*a*c-b^2) * x * b^2 * c^2 * d^3 + 1/2 / (a*e^2-b*d*e+c*d^2)^{2/4} / (4*a*c-b^2) * c^3 * 2^{(1/2)} / ((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * c*x) * d^2 * e-3/4 / (a*e^2-b*d*e+c*d^2)^{2/4} / (4*a*c-b^2) * c^2 * 2^{(1/2)} / ((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * c*x) * b^2 * e^{-3-3/4} / (a*e^2-b*d*e+c*d^2)^{2/4} / (4*a*c-b^2) * c^4 / (-4*a*c+b^2)^{(1/2)} * 2^{(1/2)} / ((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * c*x) * d^3-1/2 / (a*e^2-b*d*e+c*d^2)^{2/4} / (4*a*c-b^2) * c^3 * 2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * c*x) * d^2 * e+3/4 / (a*e^2-b*d*e+c*d^2)^{2/4} / (4*a*c-b^2) * c^2 * 2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} * c*x) * b^2 * e^{3+1/2} / (a*e^2-b*d*e+c*d^2)^{2/4} / (c*x^4+b*x^2+a) * c^2 / (4*a*c-b^2) * x^3 * b * d * e^{2-1/2} / (a*e^2-b*d*e+c*d^2)^{2/4} / (c*x^4+b*x^2+a) * c^3 / a / (4*a*c-b^2) * x^3 * b * d^3
\end{aligned}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)/(c\*x^4+b\*x^2+a)^2,x, algorithm="maxima")

```
[Out] e^4*arctan(e*x/sqrt(d*e))/((c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 +
(b^2 + 2*a*c)*d^2*e^2)*sqrt(d*e)) + 1/2*((b*c^2*d - (b^2*c - 2*a*c^2)*e)*x
^3 + ((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e)*x)/(((a*b^2*c^2 - 4*a^2*c^3)
*d^2 - (a*b^3*c - 4*a^2*b*c^2)*d*e + (a^2*b^2*c - 4*a^3*c^2)*e^2)*x^4 + (a^
2*b^2*c - 4*a^3*c^2)*d^2 - (a^2*b^3 - 4*a^3*b*c)*d*e + (a^3*b^2 - 4*a^4*c)*
e^2 + ((a*b^3*c - 4*a^2*b*c^2)*d^2 - (a*b^4 - 4*a^2*b^2*c)*d*e + (a^2*b^3 -
4*a^3*b*c)*e^2)*x^2) + 1/2*integrate(((b^2*c^2 - 6*a*c^3)*d^3 - (2*b^3*c -
11*a*b*c^2)*d^2*e + (b^4 - 2*a*b^2*c - 14*a^2*c^2)*d*e^2 - (3*a*b^3 - 13*a
^2*b*c)*e^3 + (b*c^3*d^3 - 2*(b^2*c^2 - a*c^3)*d^2*e + (b^3*c - a*b*c^2)*d*
e^2 - (3*a*b^2*c - 10*a^2*c^2)*e^3)*x^2)/(c*x^4 + b*x^2 + a), x)/((a*b^2*c^
2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c
- 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)
*e^4)
```

**mupad [B]** time = 16.46, size = 237586, normalized size = 359.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] - atan(((((((1048576*a^13*c^8*e^16 + 256*a^7*b^12*c^2*e^16 - 6144*a^8*b^10*
c^3*e^16 + 61440*a^9*b^8*c^4*e^16 - 327680*a^10*b^6*c^5*e^16 + 983040*a^11*
b^4*c^6*e^16 - 1572864*a^12*b^2*c^7*e^16 - 196608*a^6*c^15*d^14*e^2 - 91750
4*a^7*c^14*d^12*e^4 - 589824*a^8*c^13*d^10*e^6 + 3932160*a^9*c^12*d^8*e^8 +
10158080*a^10*c^11*d^6*e^10 + 10616832*a^11*c^10*d^4*e^12 + 5308416*a^12*c
^9*d^2*e^14 - 2816*a^2*b^8*c^11*d^14*e^2 + 22656*a^2*b^9*c^10*d^13*e^3 - 78
848*a^2*b^10*c^9*d^12*e^4 + 154112*a^2*b^11*c^8*d^11*e^5 - 182784*a^2*b^12*
c^7*d^10*e^6 + 130816*a^2*b^13*c^6*d^9*e^7 - 50176*a^2*b^14*c^5*d^8*e^8 + 4
608*a^2*b^15*c^4*d^7*e^9 + 3328*a^2*b^16*c^3*d^6*e^10 - 896*a^2*b^17*c^2*d^
5*e^11 + 24576*a^3*b^6*c^12*d^14*e^2 - 198656*a^3*b^7*c^11*d^13*e^3 + 68454
4*a^3*b^8*c^10*d^12*e^4 - 1291520*a^3*b^9*c^9*d^11*e^5 + 1403776*a^3*b^10*c
^8*d^10*e^6 - 798336*a^3*b^11*c^7*d^9*e^7 + 89856*a^3*b^12*c^6*d^8*e^8 + 15
5136*a^3*b^13*c^5*d^7*e^9 - 77440*a^3*b^14*c^4*d^6*e^10 + 5504*a^3*b^15*c^3
*d^5*e^11 + 2560*a^3*b^16*c^2*d^4*e^12 - 106496*a^4*b^4*c^13*d^14*e^2 + 864
256*a^4*b^5*c^12*d^13*e^3 - 2924544*a^4*b^6*c^11*d^12*e^4 + 5181440*a^4*b^7
*c^10*d^11*e^5 - 4686080*a^4*b^8*c^9*d^10*e^6 + 1045376*a^4*b^9*c^8*d^9*e^7
+ 1900544*a^4*b^10*c^7*d^8*e^8 - 1732096*a^4*b^11*c^6*d^7*e^9 + 390400*a^4
*b^12*c^5*d^6*e^10 + 112000*a^4*b^13*c^4*d^5*e^11 - 40960*a^4*b^14*c^3*d^4*
e^12 - 3840*a^4*b^15*c^2*d^3*e^13 + 229376*a^5*b^2*c^14*d^14*e^2 - 1867776*
a^5*b^3*c^13*d^13*e^3 + 6078464*a^5*b^4*c^12*d^12*e^4 - 9297920*a^5*b^5*c^1
1*d^11*e^5 + 4055040*a^5*b^6*c^10*d^10*e^6 + 7788544*a^5*b^7*c^9*d^9*e^7 -
12657664*a^5*b^8*c^8*d^8*e^8 + 6130176*a^5*b^9*c^7*d^7*e^9 + 734080*a^5*b^1
0*c^6*d^6*e^10 - 1442560*a^5*b^11*c^5*d^5*e^11 + 168960*a^5*b^12*c^4*d^4*e^
12 + 78080*a^5*b^13*c^3*d^3*e^13 + 3200*a^5*b^14*c^2*d^2*e^14 - 4587520*a^6
*b^2*c^13*d^12*e^4 + 3080192*a^6*b^3*c^12*d^11*e^5 + 12001280*a^6*b^4*c^11*
d^10*e^6 - 31076352*a^6*b^5*c^10*d^9*e^7 + 27475968*a^6*b^6*c^9*d^8*e^8 - 2
088960*a^6*b^7*c^8*d^7*e^9 - 12205312*a^6*b^8*c^7*d^6*e^10 + 6043520*a^6*b^
9*c^6*d^5*e^11 + 631808*a^6*b^10*c^5*d^4*e^12 - 610304*a^6*b^11*c^4*d^3*e^1
3 - 71936*a^6*b^12*c^3*d^2*e^14 - 21725184*a^7*b^2*c^12*d^10*e^6 + 30801920
*a^7*b^3*c^11*d^9*e^7 - 8028160*a^7*b^4*c^10*d^8*e^8 - 32260096*a^7*b^5*c^9
*d^7*e^9 + 37101568*a^7*b^6*c^8*d^6*e^10 - 7182336*a^7*b^7*c^7*d^5*e^11 - 7
609856*a^7*b^8*c^6*d^4*e^12 + 2112256*a^7*b^9*c^5*d^3*e^13 + 661632*a^7*b^1
0*c^4*d^2*e^14 - 30146560*a^8*b^2*c^11*d^8*e^8 + 55050240*a^8*b^3*c^10*d^7*
e^9 - 34365440*a^8*b^4*c^9*d^6*e^10 - 16429056*a^8*b^5*c^8*d^5*e^11 + 24600
576*a^8*b^6*c^7*d^4*e^12 - 1683456*a^8*b^7*c^6*d^3*e^13 - 3151616*a^8*b^8*c
^5*d^2*e^14 - 10977280*a^9*b^2*c^10*d^6*e^10 + 47022080*a^9*b^3*c^9*d^5*e^1
1 - 30621696*a^9*b^4*c^8*d^4*e^12 - 9232384*a^9*b^5*c^7*d^3*e^13 + 7970816*
a^9*b^6*c^6*d^2*e^14 + 4325376*a^10*b^2*c^9*d^4*e^12 + 25493504*a^10*b^3*c^
8*d^3*e^13 - 9117696*a^10*b^4*c^7*d^2*e^14 + 491520*a^11*b^2*c^8*d^2*e^14 -
```



$$\begin{aligned}
& 4947968a^{12}b^8c^8d^8e^{15} + 128a^8b^{10}c^{10}d^{14}e^2 - 1024a^8b^{11}c^9d^{11} \\
& 3e^3 + 3584a^8b^{12}c^8d^{12}e^4 - 7168a^8b^{13}c^7d^{11}e^5 + 8960a^8b^{14}c^6 \\
& d^{10}e^6 - 7168a^8b^{15}c^5d^9e^7 + 3584a^8b^{16}c^4d^8e^8 - 1024a^8b^{17} \\
& c^3d^7e^9 + 128a^8b^{18}c^2d^6e^{10} + 1605632a^6b^8c^{14}d^{13}e^3 - 14 \\
& 08a^6b^{13}c^2d^8e^{15} + 7012352a^7b^8c^{13}d^{11}e^5 + 33152a^7b^{11}c^3d^8 \\
& e^{15} + 7045120a^8b^8c^{12}d^9e^7 - 324480a^8b^9c^4d^8e^{15} - 9830400a^9 \\
& b^8c^{11}d^7e^9 + 1689600a^9b^7c^5d^8e^{15} - 25722880a^{10}b^8c^{10}d^5e^{11} \\
& - 4935680a^{10}b^5c^6d^8e^{15} - 19202048a^{11}b^8c^9d^3e^{13} + 7667712a^{11} \\
& b^3c^7d^8e^{15}) / (16(a^6b^8e^8 + 256a^6c^8d^8 + 256a^{10}c^4e^8 - 16a^7b^6c^8e^8 \\
& - 4a^5b^9d^8e^7 + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6d^8 - 256a^5b^2c^7d^8 \\
& + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^{12}d^4e^4 - 4a^3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 \\
& + 1024a^7c^7d^6e^2 + 1536a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^{10}c^2d^6e^2 \\
& - 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4b^7c^3d^5e^3 \\
& - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 \\
& - 192a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c^4d^4e^4 \\
& - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 + 2048a^7b^3c^4d^3e^5 \\
& - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024a^6b^8c^7d^7e + 64a^6b^7c^8d^7e \\
& - 1024a^9b^8c^4d^8e^7 - 4a^2b^9c^3d^7e - 4a^2b^{11}c^5d^5e^3 + 64a^3b^7c^4d^7e \\
& - 4a^3b^{10}c^4d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^3d^3e^5 + 1024a^5b^3c^6d^7e \\
& - 92a^5b^8c^2e^6 - 3072a^7b^8c^5d^3e^5 + 1024a^8b^3c^3d^8e^7) - (x((27a^8b^9c^5d^6 \\
& - b^{11}c^4d^6 - b^{15}d^2e^4 - 9a^2b^{13}e^6 + 3840a^5b^8c^9d^6 - 9a^8c^5d^6(-4a^8c - b^2)^9)^{1/2} \\
& + 213a^3b^{11}c^8e^6 - 26880a^8b^8c^6e^6 + 3072a^6c^9d^5e + 35840a^8c^7d^5e^5 + 4b^{12}c^3d^5e \\
& + 4b^{14}c^3d^3e^3 - 288a^2b^7c^6d^6 + 1504a^3b^5c^7d^6 - 3840a^4b^3c^8d^6 + 9a^2b^4e^6(-4a^8c - b^2)^9)^{1/2} \\
& - 2077a^4b^9c^2e^6 + 10656a^5b^7c^3e^6 - 30240a^6b^5c^4e^6 + 44800a^7b^3c^5e^6 + 25a^4c^2e^6(-4a^8c - b^2)^9)^{1/2} \\
& + b^2c^4d^6(-4a^8c - b^2)^9)^{1/2} + 22528a^7c^8d^3e^3 + b^6d^2e^4(-4a^8c - b^2)^9)^{1/2} - 6b^{13}c^2d^4e^2 \\
& + 6a^8b^{14}d^8e^5 - 1471a^2b^9c^4d^4e^2 + 600a^2b^{10}c^3d^3e^3 + 180a^2b^{11}c^2d^2e^4 + 6976a^3b^7c^5d^4e^2 \\
& - 1032a^3b^8c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456a^4b^5c^6d^4e^2 - 7168a^4b^6c^5d^3e^3 \\
& + 16896a^4b^7c^4d^2e^4 + 10240a^5b^3c^7d^4e^2 + 37632a^5b^4c^6d^3e^3 - 47712a^5b^5c^5d^2e^4 \\
& - 59392a^6b^2c^7d^3e^3 + 60928a^6b^3c^6d^2e^4 - 41a^2c^4d^4e^2(-4a^8c - b^2)^9)^{1/2} - 39a^3c^3d^2e^4(-4a^8c - b^2)^9)^{1/2} \\
& + 6b^4c^2d^4e^2(-4a^8c - b^2)^9)^{1/2} - 6a^8b^5d^8e^5(-4a^8c - b^2)^9)^{1/2} - 106a^8b^{10}c^4d^5e \\
& + 7a^8b^{13}c^2d^2e^4 - 128a^2b^{12}c^8d^8e^5 - 51a^3b^2c^8e^6(-4a^8c - b^2)^9)^{1/2} + 150a^8b^{11}c^3d^4e^2 \\
& - 84a^8b^{12}c^2d^3e^3 + 1116a^2b^8c^5d^5e - 5824a^3b^6c^6d^5e + 1030a^3b^{10}c^2d^8e^5 \\
& + 15232a^4b^4c^7d^5e - 3492a^4b^8c^3d^8e^5 - 16896a^5b^2c^8d^5e + 1344a^5b^6c^4d^8e^5 \\
& + 7424a^6b^8c^8d^4e^2 + 22400a^6b^4c^5d^8e^5 - 23296a^7b^8c^7d^2e^4 - 53760a^7b^2c^6d^8e^5 \\
& - 4b^3c^3d^5e^2(-4a^8c - b^2)^9)^{1/2} - 4b^5c^3d^3e^3(-4a^8c - b^2)^9)^{1/2} + 11a^8b^4c^2d^2e^4(-4a^8c - b^2)^9)^{1/2} \\
& + 20a^2b^3c^8d^8e^5(-4a^8c - b^2)^9)^{1/2} + 86a^3b^8c^2d^8e^5(-4a^8c - b^2)^9)^{1/2} - 42a^8b^2c^3d^4e^2(-4a^8c - b^2)^9)^{1/2} \\
& + 12a^8b^3c^2d^3e^3(-4a^8c - b^2)^9)^{1/2} + 120a^2b^8c^3d^3e^3(-4a^8c - b^2)^9)^{1/2} + 34a^8b^8c^4d^5e^2(-4a^8c - b^2)^9)^{1/2} \\
& - 108a^2b^2c^2d^2e^4(-4a^8c - b^2)^9)^{1/2}) / (32(a^7b^{12}e^8 + 4096a^9c^{10}d^8 + 4096a^{13}c^6e^8 \\
& - 24a^8b^{10}c^8e^8 - 4a^6b^{13}d^8e^7 + a^3b^{12}c^4d^8 - 24a^4b^{10}c^5d^8 + 240a^5b^8c^6d^8 \\
& - 1280a^6b^6c^7d^8 + 3840a^7b^4c^8d^8 - 6144a^8b^2c^9d^8 + 240a^9b^8c^2e^8 - 1280a^{10}b^6c^3e^8 \\
& + 3840a^{11}b^4c^4e^8 - 6144a^{12}b^2c^5e^8 + a^3b^{16}d^4e^4 - 4a^4b^{15}d^3e^5 + 6a^5b^{14}d^2e^6 \\
& + 16384a^{10}c^9d^6e^2 + 24576a^{11}c^8d^4e^4 + 16384a^{12}c^7d^2e^6 + 6a^3b^{14}c^2d^6e^2 - 140a^4b^{12}
\end{aligned}$$

$$\begin{aligned}
& c^3d^6e^2 + 84a^4b^{13}c^2d^5e^3 + 1344a^5b^{10}c^4d^6e^2 - 672a^5 \\
& *b^{11}c^3d^5e^3 - 42a^5b^{12}c^2d^4e^4 - 6720a^6b^8c^5d^6e^2 + 22 \\
& 40a^6b^9c^4d^5e^3 + 1456a^6b^{10}c^3d^4e^4 - 672a^6b^{11}c^2d^3e \\
& ^5 + 17920a^7b^6c^6d^6e^2 - 10080a^7b^8c^4d^4e^4 + 2240a^7b^9c \\
& ^3d^3e^5 + 1344a^7b^{10}c^2d^2e^6 - 21504a^8b^4c^7d^6e^2 - 21504* \\
& a^8b^5c^6d^5e^3 + 32256a^8b^6c^5d^4e^4 - 6720a^8b^8c^3d^2e^6 \\
& + 57344a^9b^3c^7d^5e^3 - 46592a^9b^4c^6d^4e^4 - 21504a^9b^5c^5 \\
& *d^3e^5 + 17920a^9b^6c^4d^2e^6 + 12288a^{10}b^2c^7d^4e^4 + 57344a \\
& ^{10}b^3c^6d^3e^5 - 21504a^{10}b^4c^5d^2e^6 + 96a^7b^{11}c*d*e^7 - 16 \\
& 384a^9b*c^9*d^7*e - 16384a^{12}b*c^6*d*e^7 - 4a^3b^{13}c^3d^7e - 4a^3 \\
& *b^{15}c*d^5e^3 + 96a^4b^{11}c^4d^7e - 12a^4b^{14}c*d^4e^4 - 960a^5b \\
& ^9c^5d^7e + 84a^5b^{13}c*d^3e^5 + 5120a^6b^7c^6d^7e - 140a^6b^1 \\
& 2*c*d^2e^6 - 15360a^7b^5c^7d^7e + 24576a^8b^3c^8d^7e - 960a^8b \\
& ^9c^2*d*e^7 + 5120a^9b^7c^3*d*e^7 - 49152a^{10}b*c^8*d^5e^3 - 15360a^ \\
& ^{10}b^5c^4*d*e^7 - 49152a^{11}b*c^7*d^3e^5 + 24576a^{11}b^3c^5*d*e^7))^( \\
& (1/2)*(1048576a^{15}c^8e^{17} + 256a^9b^{12}c^2e^{17} - 6144a^{10}b^{10}c^3e^ \\
& ^{17} + 61440a^{11}b^8c^4e^{17} - 327680a^{12}b^6c^5e^{17} + 983040a^{13}b^4c \\
& ^6e^{17} - 1572864a^{14}b^2c^7e^{17} - 1048576a^8c^{15}d^{14}e^3 - 5242880a \\
& ^9c^{14}d^{12}e^5 - 9437184a^{10}c^{13}d^{10}e^7 - 5242880a^{11}c^{12}d^8e^9 + \\
& 5242880a^{12}c^{11}d^6e^{11} + 9437184a^{13}c^{10}d^4e^{13} + 5242880a^{14}c^9 \\
& *d^2e^{15} + 256a^2b^{11}c^{10}d^{15}e^2 - 2048a^2b^{12}c^9d^{14}e^3 + 7168* \\
& a^2b^{13}c^8d^{13}e^4 - 14336a^2b^{14}c^7d^{12}e^5 + 17920a^2b^{15}c^6d^ \\
& ^{11}e^6 - 14336a^2b^{16}c^5d^{10}e^7 + 7168a^2b^{17}c^4d^9e^8 - 2048a^2 \\
& *b^{18}c^3d^8e^9 + 256a^2b^{19}c^2d^7e^{10} - 5120a^3b^9c^{11}d^{15}e^2 \\
& + 41984a^3b^{10}c^{10}d^{14}e^3 - 148736a^3b^{11}c^9d^{13}e^4 + 296192a^3* \\
& b^{12}c^8d^{12}e^5 - 359680a^3b^{13}c^7d^{11}e^6 + 267520a^3b^{14}c^6d^{10} \\
& *e^7 - 112384a^3b^{15}c^5d^9e^8 + 18176a^3b^{16}c^4d^8e^9 + 3328a^3* \\
& b^{17}c^3d^7e^{10} - 1280a^3b^{18}c^2d^6e^{11} + 40960a^4b^7c^{12}d^{15}e^ \\
& ^2 - 348160a^4b^8c^{11}d^{14}e^3 + 1254400a^4b^9c^{10}d^{13}e^4 - 2478080* \\
& a^4b^{10}c^9d^{12}e^5 + 2867456a^4b^{11}c^8d^{11}e^6 - 1862144a^4b^{12}c^ \\
& ^7d^{10}e^7 + 490240a^4b^{13}c^6d^9e^8 + 128000a^4b^{14}c^5d^8e^9 - 10 \\
& 8800a^4b^{15}c^4d^7e^{10} + 13824a^4b^{16}c^3d^6e^{11} + 2304a^4b^{17}c^ \\
& ^2d^5e^{12} - 163840a^5b^5c^{13}d^{15}e^2 + 1474560a^5b^6c^{12}d^{14}e^3 - \\
& 5447680a^5b^7c^{11}d^{13}e^4 + 10588160a^5b^8c^{10}d^{12}e^5 - 11166720* \\
& a^5b^9c^9d^{11}e^6 + 5159936a^5b^{10}c^8d^{10}e^7 + 1073920a^5b^{11}c^7 \\
& *d^9e^8 - 2279680a^5b^{12}c^6d^8e^9 + 770560a^5b^{13}c^5d^7e^{10} + 33 \\
& 280a^5b^{14}c^4d^6e^{11} - 41216a^5b^{15}c^3d^5e^{12} - 1280a^5b^{16}c^2 \\
& *d^4e^{13} + 327680a^6b^3c^{14}d^{15}e^2 - 3276800a^6b^4c^{13}d^{14}e^3 + \\
& 12615680a^6b^5c^{12}d^{13}e^4 - 23592960a^6b^6c^{11}d^{12}e^5 + 19701760* \\
& a^6b^7c^{10}d^{11}e^6 + 1372160a^6b^8c^9d^{10}e^7 - 15846400a^6b^9c^8 \\
& *d^9e^8 + 10864640a^6b^{10}c^7d^8e^9 - 1352960a^6b^{11}c^6d^7e^{10} - \\
& 1111040a^6b^{12}c^5d^6e^{11} + 273920a^6b^{13}c^4d^5e^{12} + 25600a^6b^ \\
& ^{14}c^3d^4e^{13} - 1280a^6b^{15}c^2d^3e^{14} + 3407872a^7b^2c^{14}d^{14}e^ \\
& ^3 - 14221312a^7b^3c^{13}d^{13}e^4 + 23527424a^7b^4c^{12}d^{12}e^5 - 37683 \\
& 20a^7b^5c^{11}d^{11}e^6 - 38895616a^7b^6c^{10}d^{10}e^7 + 50126848a^7b^ \\
& ^7c^9d^9e^8 - 18362368a^7b^8c^8d^8e^9 - 6831104a^7b^9c^7d^7e^{10} \\
& + 6200320a^7b^{10}c^6d^6e^{11} - 726784a^7b^{11}c^5d^5e^{12} - 228608a^ \\
& ^7b^{12}c^4d^4e^{13} + 31488a^7b^{13}c^3d^3e^{14} + 2304a^7b^{14}c^2d^2e \\
& ^{15} - 3145728a^8b^2c^{13}d^{12}e^5 - 31129600a^8b^3c^{12}d^{11}e^6 + 7471 \\
& 1040a^8b^4c^{11}d^{10}e^7 - 55476224a^8b^5c^{10}d^9e^8 - 11075584a^8b \\
& ^6c^9d^8e^9 + 35381248a^8b^7c^8d^7e^{10} - 14479360a^8b^8c^7d^6e \\
& ^{11} - 168960a^8b^9c^6d^5e^{12} + 1286144a^8b^{10}c^5d^4e^{13} - 302336* \\
& a^8b^{11}c^4d^3e^{14} - 55808a^8b^{12}c^3d^2e^{15} - 36962304a^9b^2c^{12} \\
& *d^{10}e^7 - 9502720a^9b^3c^{11}d^9e^8 + 67174400a^9b^4c^{10}d^8e^9 - \\
& 54886400a^9b^5c^9d^7e^{10} + 11239424a^9b^6c^8d^6e^{11} + 5545984a^9 \\
& *b^7c^7d^5e^{12} - 5263360a^9b^8c^6d^4e^{13} + 1356800a^9b^9c^5d^3* \\
& e^{14} + 558080a^9b^{10}c^4d^2e^{15} - 49807360a^{10}b^2c^{11}d^8e^9 + 1933 \\
& 3120a^{10}b^3c^{10}d^7e^{10} + 7208960a^{10}b^4c^9d^6e^{11} - 14974976a^{10} \\
& *b^5c^8d^5e^{12} + 15073280a^{10}b^6c^7d^4e^{13} - 2170880a^{10}b^7c^6d
\end{aligned}$$

$$\begin{aligned}
& ^3e^{14} - 2928640a^{10}b^8c^5d^2e^{15} - 11796480a^{11}b^2c^{10}d^6e^{11} + \\
& 23920640a^{11}b^3c^9d^5e^{12} - 24576000a^{11}b^4c^8d^4e^{13} - 4096000a^{11}b^5c^7d^3e^{14} + 8355840a^{11}b^6c^6d^2e^{15} + 12582912a^{12}b^2c^9d^4e^{13} + 19857408a^{12}b^3c^8d^3e^{14} - 11534336a^{12}b^4c^7d^2e^{15} + 3407872a^{13}b^2c^8d^2e^{15} - 5505024a^{14}b^2c^8d^2e^{16} - 262144a^7 \\
& *b^2c^{15}d^{15}e^2 + 5505024a^8b^2c^{14}d^{13}e^4 - 1280a^8b^{13}c^2d^2e^{16} + 25952256a^9b^2c^{13}d^{11}e^6 + 30976a^9b^{11}c^3d^2e^{16} + 38010880a^{10}b^2c^{12}d^9e^8 - 312320a^{10}b^9c^4d^2e^{16} + 11796480a^{11}b^2c^{11}d^7e^{10} + 1679360a^{11}b^7c^5d^2e^{16} - 21233664a^{12}b^2c^{10}d^5e^{12} - 5079040a^{12}b^5c^6d^2e^{16} - 20709376a^{13}b^2c^9d^3e^{14} + 8192000a^{13}b^3c^7d^2e^{16} \\
& ))/(8*(a^6b^8e^8 + 256a^6c^8d^8 + 256a^{10}c^4e^8 - 16a^7b^6c^2e^8 - 4a^5b^9d^2e^7 + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6d^8 - 256a^5b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^12d^4e^4 - 4a^3b^11d^3e^5 + 6a^4b^10d^2e^6 + 1024a^7c^7d^6e^2 + 1536a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^10c^2d^6e^2 - 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4b^7c^3d^5e^3 - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 - 192a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c^4d^4e^4 - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 + 2048a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024a^6b^2c^7d^7e + 64a^6b^7c^2d^7e - 1024a^9b^2c^4d^2e^7 - 4a^2b^9c^3d^7e - 4a^2b^11c^2d^5e^3 + 64a^3b^7c^4d^7e - 4a^3b^10c^2d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^3d^3e^5 + 1024a^5b^3c^6d^7e - 92a^5b^8c^2d^2e^6 - 3072a^7b^2c^6d^5e^3 - 384a^7b^5c^2d^2e^7 - 3072a^8b^2c^5d^3e^5 + 1024a^8b^3c^3d^2e^7)) \\
& *((27a^9b^9c^5d^6 - b^{11}c^4d^6 - b^{15}d^2e^4 - 9a^2b^{13}e^6 + 3840a^5b^2c^9d^6 - 9a^2c^5d^6*(-(4a^2c - b^2)^9)^{(1/2)} + 213a^3b^{11}c^2e^6 - 26880a^8b^2c^6e^6 + 3072a^6c^9d^5e + 35840a^8c^7d^2e^5 + 4b^{12}c^3d^5e + 4b^{14}c^2d^3e^3 - 288a^2b^7c^6d^6 + 1504a^3b^5c^7d^6 - 3840a^4b^3c^8d^6 + 9a^2b^4e^6*(-(4a^2c - b^2)^9)^{(1/2)} - 2077a^4b^9c^2e^6 + 10656a^5b^7c^3e^6 - 30240a^6b^5c^4e^6 + 44800a^7b^3c^5e^6 + 25a^4c^2e^6*(-(4a^2c - b^2)^9)^{(1/2)} + b^2c^4d^6*(-(4a^2c - b^2)^9)^{(1/2)} + 22528a^7c^8d^3e^3 + b^6d^2e^4*(-(4a^2c - b^2)^9)^{(1/2)} - 6b^{13}c^2d^4e^2 + 6a^2b^{14}d^2e^5 - 1471a^2b^9c^4d^4e^2 + 600a^2b^{10}c^3d^3e^3 + 180a^2b^{11}c^2d^2e^4 + 6976a^3b^7c^5d^4e^2 - 1032a^3b^8c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456a^4b^5c^6d^4e^2 - 7168a^4b^6c^5d^3e^3 + 16896a^4b^7c^4d^2e^4 + 10240a^5b^3c^7d^4e^2 + 37632a^5b^4c^6d^3e^3 - 47712a^5b^5c^5d^2e^4 - 59392a^6b^2c^7d^3e^3 + 60928a^6b^3c^6d^2e^4 - 41a^2c^4d^4e^2*(-(4a^2c - b^2)^9)^{(1/2)} - 39a^3c^3d^2e^4*(-(4a^2c - b^2)^9)^{(1/2)} + 6b^4c^2d^4e^2*(-(4a^2c - b^2)^9)^{(1/2)} - 6a^2b^5d^2e^5*(-(4a^2c - b^2)^9)^{(1/2)} - 106a^2b^{10}c^4d^5e + 7a^2b^{13}c^2d^2e^4 - 128a^2b^{12}c^2d^2e^5 - 51a^3b^2c^2e^6*(-(4a^2c - b^2)^9)^{(1/2)} + 150a^2b^{11}c^3d^4e^2 - 84a^2b^{12}c^2d^3e^3 + 1116a^2b^8c^5d^5e - 5824a^3b^6c^6d^5e + 1030a^3b^{10}c^2d^2e^5 + 15232a^4b^4c^7d^5e - 3492a^4b^8c^3d^2e^5 - 16896a^5b^2c^8d^5e + 1344a^5b^6c^4d^2e^5 + 7424a^6b^2c^8d^4e^2 + 22400a^6b^4c^5d^2e^5 - 23296a^7b^2c^7d^2e^4 - 53760a^7b^2c^6d^2e^5 - 4b^3c^3d^5e*(-(4a^2c - b^2)^9)^{(1/2)} - 4b^5c^3d^3e^3*(-(4a^2c - b^2)^9)^{(1/2)} + 11a^2b^4c^2d^2e^4*(-(4a^2c - b^2)^9)^{(1/2)} + 20a^2b^3c^2d^2e^5*(-(4a^2c - b^2)^9)^{(1/2)} + 86a^3b^2c^2d^2e^5*(-(4a^2c - b^2)^9)^{(1/2)} - 42a^2b^2c^3d^4e^2*(-(4a^2c - b^2)^9)^{(1/2)} + 12a^2b^3c^2d^3e^3*(-(4a^2c - b^2)^9)^{(1/2)} + 120a^2b^2c^3d^3e^3*(-(4a^2c - b^2)^9)^{(1/2)} + 34a^2b^2c^4d^5e*(-(4a^2c - b^2)^9)^{(1/2)} - 108a^2b^2c^2d^2e^4*(-(4a^2c - b^2)^9)^{(1/2)} \\
& ))/(32*(a^7b^{12}e^8 + 4096a^9c^{10}d^8 + 4096a^{13}c^6e^8 - 24a^8b^{10}c^2e^8 - 4a^6b^{13}d^2e^7 + a^3b^{12}c^4d^8 - 24a^4b^{10}c^5d^8 + 240a^5b^8c^6d^8 - 1280a^6b^6c^7d^8 + 3840a^7b^4c^8d^8 - 6144a^8b^2c^9d^8 + 240a^9b^8c^2e^8 - 1280a^{10}b^6c^3e^8 + 3840a^{11}b^4c^4e^8 - 6144a^{12}b^2c^5e^8 + a^3b^{16}d^4e^4 - 4a^4b^{15}d^3e^5 + 6a^
\end{aligned}$$

$$\begin{aligned}
& 5*b^{14}*d^2*e^6 + 16384*a^{10}*c^9*d^6*e^2 + 24576*a^{11}*c^8*d^4*e^4 + 16384*a^{12}*c^7*d^2*e^6 + 6*a^3*b^{14}*c^2*d^6*e^2 - 140*a^4*b^{12}*c^3*d^6*e^2 + 84*a^4 \\
& *b^{13}*c^2*d^5*e^3 + 1344*a^5*b^{10}*c^4*d^6*e^2 - 672*a^5*b^{11}*c^3*d^5*e^3 - 42*a^5*b^{12}*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e \\
& ^3 + 1456*a^6*b^{10}*c^3*d^4*e^4 - 672*a^6*b^{11}*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7 \\
& *b^{10}*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7* \\
& d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^{10}*b^2*c^7*d^4*e^4 + 57344*a^{10}*b^3*c^6*d^3*e^5 \\
& - 21504*a^{10}*b^4*c^5*d^2*e^6 + 96*a^7*b^{11}*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^{12}*b*c^6*d*e^7 - 4*a^3*b^{13}*c^3*d^7*e - 4*a^3*b^{15}*c*d^5*e^3 + 96 \\
& *a^4*b^{11}*c^4*d^7*e - 12*a^4*b^{14}*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^{13}*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^{12}*c*d^2*e^6 - 15360* \\
& a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^{10}*b*c^8*d^5*e^3 - 15360*a^{10}*b^5*c^4*d*e^7 - 4 \\
& 9152*a^{11}*b*c^7*d^3*e^5 + 24576*a^{11}*b^3*c^5*d*e^7))^{(1/2)} - (x*(626688*a^{10}*b*c^8*e^{15} - 784384*a^{10}*c^9*d*e^{14} + 208*a^4*b^{13}*c^2*e^{15} - 4880*a^5*b \\
& ^{11}*c^3*e^{15} + 47312*a^6*b^9*c^4*e^{15} - 242176*a^7*b^7*c^5*e^{15} + 688640*a^8*b^5*c^6*e^{15} - 1028096*a^9*b^3*c^7*e^{15} + 18432*a^4*c^{15}*d^{13}*e^2 + 12697 \\
& 6*a^5*c^{14}*d^{11}*e^4 + 325632*a^6*c^{13}*d^9*e^6 + 139264*a^7*c^{12}*d^7*e^8 - 1067008*a^8*c^{11}*d^5*e^{10} - 1773568*a^9*c^{10}*d^3*e^{12} + 16*b^8*c^{11}*d^{13}*e^2 \\
& - 96*b^9*c^{10}*d^{12}*e^3 + 240*b^{10}*c^9*d^{11}*e^4 - 304*b^{11}*c^8*d^{10}*e^5 + 144*b^{12}*c^7*d^9*e^6 + 144*b^{13}*c^6*d^8*e^7 - 304*b^{14}*c^5*d^7*e^8 + 240*b^{15}*c^4*d^6*e^9 \\
& - 96*b^{16}*c^3*d^5*e^{10} + 16*b^{17}*c^2*d^4*e^{11} + 3200*a^2*b^4*c^{13}*d^{13}*e^2 - 18432*a^2*b^5*c^{12}*d^{12}*e^3 + 41024*a^2*b^6*c^{11}*d^{11}*e^4 - 36352*a^2*b^7*c^{10}*d^{10}*e^5 \\
& - 16208*a^2*b^8*c^9*d^9*e^6 + 74576*a^2*b^9*c^8*d^8*e^7 - 78496*a^2*b^{10}*c^7*d^7*e^8 + 32064*a^2*b^{11}*c^6*d^6*e^9 + 6000*a^2*b^{12}*c^5*d^5*e^{10} - 9264*a^2*b^{13}*c^4*d^4*e^{11} \\
& + 1472*a^2*b^{14}*c^3*d^3*e^{12} + 416*a^2*b^{15}*c^2*d^2*e^{13} - 12800*a^3*b^2*c^{14}*d^{13}*e^2 + 73728*a^3*b^3*c^{13}*d^{12}*e^3 - 151296*a^3*b^4*c^{12}*d^{11}*e^4 + 78336*a^3*b^5*c^{11}*d^{10}* \\
& e^5 + 206688*a^3*b^6*c^{10}*d^9*e^6 - 436736*a^3*b^7*c^9*d^8*e^7 + 324224*a^3*b^8*c^8*d^7*e^8 + 992*a^3*b^9*c^7*d^6*e^9 - 158176*a^3*b^{10}*c^6*d^5*e^{10} + 77056*a^3*b^{11}*c^5*d^4*e^{11} \\
& + 6912*a^3*b^{12}*c^4*d^3*e^{12} - 8416*a^3*b^{13}*c^3*d^2*e^{13} + 162816*a^4*b^2*c^{13}*d^{11}*e^4 + 184320*a^4*b^3*c^{12}*d^{10}*e^5 - 916608*a^4*b^4*c^{11}*d^9*e^6 + 1165824*a^4*b^5*c^{10}*d^8*e^7 \\
& - 314496*a^4*b^6*c^9*d^7*e^8 - 822272*a^4*b^7*c^8*d^6*e^9 + 919152*a^4*b^8*c^7*d^5*e^{10} - 175296*a^4*b^9*c^6*d^4*e^{11} - 189328*a^4*b^{10}*c^5*d^3*e^{12} + 62064*a^4*b^{11}* \\
& c^4*d^2*e^{13} + 1290752*a^5*b^2*c^{12}*d^9*e^6 - 659456*a^5*b^3*c^{11}*d^8*e^7 - 1561088*a^5*b^4*c^{10}*d^7*e^8 + 3240960*a^5*b^5*c^9*d^6*e^9 - 1964192*a^5*b^6*c^8*d^5*e^{10} \\
& - 683008*a^5*b^7*c^7*d^4*e^{11} + 1162304*a^5*b^8*c^6*d^3*e^{12} - 164112*a^5*b^9*c^5*d^2*e^{13} + 3442688*a^6*b^2*c^{11}*d^7*e^8 - 3670016*a^6*b^3*c^{10}*d^6*e^9 + 15232*a^6*b^4*c^9*d^5*e^{10} \\
& + 4230144*a^6*b^5*c^8*d^4*e^{11} - 3059648*a^6*b^6*c^7*d^3*e^{12} - 247296*a^6*b^7*c^6*d^2*e^{13} + 4010496*a^7*b^2*c^{10}*d^5*e^{10} - 6873088*a^7*b^3*c^9*d^4*e^{11} + 2822400*a^7*b^4*c^8* \\
& d^3*e^{12} + 2370048*a^7*b^5*c^7*d^2*e^{13} + 1178624*a^8*b^2*c^9*d^3*e^{12} - 4739072*a^8*b^3*c^8*d^2*e^{13} - 352*a*b^6*c^{12}*d^{13}*e^2 + 2048*a*b^7*c^{11}*d^{12}*e^3 - 4800*a*b^8*c^{10}*d^{11}*e^4 \\
& + 5168*a*b^9*c^9*d^{10}*e^5 - 480*a*b^{10}*c^8*d^9*e^6 - 6000*a*b^{11}*c^7*d^8*e^7 + 8192*a*b^{12}*c^6*d^7*e^8 - 5040*a*b^{13}*c^5*d^6*e^9 + 1152*a*b^{14}*c^4*d^5*e^{10} + 240*a*b^{15}*c^3*d^4*e^{11} \\
& - 128*a*b^{16}*c^2*d^3*e^{12} - 512*a^3*b^{14}*c^2*d*e^{14} - 106496*a^4*b*c^{14}*d^{12}*e^3 + 11680*a^4*b^{12}*c^3*d*e^{14} - 675840*a^5*b*c^{13}*d^{10}*e^5 - 108288*a^5*b^{10}*c^4*d*e^{14} \\
& - 1601536*a^6*b*c^{12}*d^8*e^7 + 514768*a^6*b^8*c^5*d*e^{14} - 925696*a^7*b*c^{11}*d^6*e^9 - 1278304*a^7*b^6*c^6*d*e^{14} + 2457600*a^8*b*c^{10}*d^4*e^{11} + 1385600*a^8*b^4*c^7*d*e^{14} \\
& + 2977792*a^9*b*c^9*d^2*e^{13} + 19968*a^9*b^2*c^8*d*e^{14}))/((8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^{10}*c^4*e^8 - 16*a^7*b^6*c^8*e^8 - 4*a^5*b^9*d^8 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^{12}*d^4*e^4 - 4*a^3*b^{11}*d^3*e^5 + 6*a^4*b^{10}*d^2*e^6 + 1024*a^7*
\end{aligned}$$

$$\begin{aligned}
& c^7 d^6 e^2 + 1536 a^8 c^6 d^4 e^4 + 1024 a^9 c^5 d^2 e^6 + 6 a^2 b^{10} c^2 d^6 e^2 - 92 a^3 b^8 c^3 d^6 e^2 + 52 a^3 b^9 c^2 d^5 e^3 + 512 a^4 b^6 c^4 d^6 e^2 - 192 a^4 b^7 c^3 d^5 e^3 - 90 a^4 b^8 c^2 d^4 e^4 - 1152 a^5 b^4 c^5 d^6 e^2 - 128 a^5 b^5 c^4 d^5 e^3 + 800 a^5 b^6 c^3 d^4 e^4 - 192 a^5 b^7 c^2 d^3 e^5 + 512 a^6 b^2 c^6 d^6 e^2 + 2048 a^6 b^3 c^5 d^5 e^3 - 2240 a^6 b^4 c^4 d^4 e^4 - 128 a^6 b^5 c^3 d^3 e^5 + 512 a^6 b^6 c^2 d^2 e^6 + 1536 a^7 b^2 c^5 d^4 e^4 + 2048 a^7 b^3 c^4 d^3 e^5 - 1152 a^7 b^4 c^3 d^2 e^6 + 512 a^8 b^2 c^4 d^2 e^6 - 1024 a^6 b^7 c^2 d^2 e^6 + 64 a^6 b^7 c^2 d^2 e^6 - 1024 a^9 b^6 c^4 d^4 e^4 - 4 a^2 b^9 c^3 d^7 e - 4 a^2 b^{11} c^4 d^5 e^3 + 64 a^3 b^7 c^4 d^7 e - 4 a^3 b^{10} c^4 d^4 e^4 - 384 a^4 b^5 c^5 d^7 e + 52 a^4 b^9 c^4 d^3 e^5 + 1024 a^5 b^3 c^6 d^7 e - 92 a^5 b^8 c^4 d^2 e^6 - 3072 a^7 b^6 c^5 d^5 e^3 - 384 a^7 b^5 c^2 d^7 e - 3072 a^8 b^3 c^3 d^5 e^5 + 1024 a^8 b^3 c^3 d^5 e^7)) * ((27 a^2 b^9 c^5 d^6 - b^{11} c^4 d^6 - b^{15} d^2 e^4 - 9 a^2 b^{13} e^6 + 3840 a^5 b^6 c^9 d^6 - 9 a^6 c^5 d^6 (-4 a^2 c - b^2)^9)^{1/2} + 213 a^3 b^{11} c^5 e^6 - 26880 a^8 b^6 c^6 e^6 + 3072 a^6 c^9 d^5 e + 35840 a^8 c^7 d^5 e^5 + 4 b^{12} c^3 d^5 e + 4 b^{14} c^3 d^3 e^3 - 288 a^2 b^7 c^6 d^6 + 1504 a^3 b^5 c^7 d^6 - 3840 a^4 b^3 c^8 d^6 + 9 a^2 b^4 e^6 (-4 a^2 c - b^2)^9)^{1/2} - 2077 a^4 b^9 c^2 e^6 + 10656 a^5 b^7 c^3 e^6 - 30240 a^6 b^5 c^4 e^6 + 44800 a^7 b^3 c^5 e^6 + 25 a^4 c^2 e^6 (-4 a^2 c - b^2)^9)^{1/2} + b^2 c^4 d^6 (-4 a^2 c - b^2)^9)^{1/2} + 22528 a^7 c^8 d^3 e^3 + b^6 d^2 e^4 (-4 a^2 c - b^2)^9)^{1/2} - 6 b^{13} c^2 d^4 e^2 + 6 a^2 b^{14} d^5 e^5 - 1471 a^2 b^9 c^4 d^4 e^2 + 600 a^2 b^{10} c^3 d^3 e^3 + 180 a^2 b^{11} c^2 d^2 e^4 + 6976 a^3 b^7 c^5 d^4 e^2 - 1032 a^3 b^8 c^4 d^3 e^3 - 2871 a^3 b^9 c^3 d^2 e^4 - 15456 a^4 b^5 c^6 d^4 e^2 - 7168 a^4 b^6 c^5 d^3 e^3 + 16896 a^4 b^7 c^4 d^2 e^4 + 10240 a^5 b^3 c^7 d^4 e^2 + 37632 a^5 b^4 c^6 d^3 e^3 - 47712 a^5 b^5 c^5 d^2 e^4 - 59392 a^6 b^2 c^7 d^3 e^3 + 60928 a^6 b^3 c^6 d^2 e^4 - 41 a^2 c^4 d^4 e^2 (-4 a^2 c - b^2)^9)^{1/2} - 39 a^3 c^3 d^2 e^4 (-4 a^2 c - b^2)^9)^{1/2} + 6 b^4 c^2 d^4 e^2 (-4 a^2 c - b^2)^9)^{1/2} - 6 a^2 b^5 d^5 e^5 (-4 a^2 c - b^2)^9)^{1/2} - 106 a^2 b^{10} c^4 d^5 e + 7 a^2 b^{13} c^4 d^2 e^4 - 128 a^2 b^{12} c^4 d^2 e^4 - 51 a^3 b^2 c^2 e^6 (-4 a^2 c - b^2)^9)^{1/2} + 150 a^2 b^{11} c^3 d^4 e^2 - 84 a^2 b^{12} c^2 d^3 e^3 + 1116 a^2 b^8 c^5 d^5 e - 5824 a^3 b^6 c^6 d^5 e + 1030 a^3 b^{10} c^2 d^5 e^5 + 15232 a^4 b^4 c^7 d^5 e - 3492 a^4 b^8 c^3 d^5 e^5 - 16896 a^5 b^2 c^8 d^5 e + 1344 a^5 b^6 c^4 d^5 e^5 + 7424 a^6 b^6 c^8 d^4 e^2 + 22400 a^6 b^4 c^5 d^5 e^5 - 23296 a^7 b^6 c^7 d^2 e^4 - 53760 a^7 b^2 c^6 d^5 e^5 - 4 b^3 c^3 d^5 e^5 (-4 a^2 c - b^2)^9)^{1/2} - 4 b^5 c^4 d^3 e^3 (-4 a^2 c - b^2)^9)^{1/2} + 11 a^2 b^4 c^4 d^2 e^4 (-4 a^2 c - b^2)^9)^{1/2} + 20 a^2 b^3 c^4 d^2 e^4 (-4 a^2 c - b^2)^9)^{1/2} + 86 a^3 b^3 c^2 d^5 e^5 (-4 a^2 c - b^2)^9)^{1/2} - 42 a^2 b^2 c^3 d^4 e^2 (-4 a^2 c - b^2)^9)^{1/2} + 12 a^2 b^3 c^2 d^3 e^3 (-4 a^2 c - b^2)^9)^{1/2} + 120 a^2 b^6 c^3 d^3 e^3 (-4 a^2 c - b^2)^9)^{1/2} + 34 a^2 b^6 c^4 d^5 e^5 (-4 a^2 c - b^2)^9)^{1/2} - 108 a^2 b^2 c^2 d^2 e^4 (-4 a^2 c - b^2)^9)^{1/2}) / (32 (a^7 b^{12} e^8 + 4096 a^9 c^{10} d^8 + 4096 a^{13} c^6 e^8 - 24 a^8 b^{10} c^8 e^8 - 4 a^6 b^{13} d^8 e^7 + a^3 b^{12} c^4 d^8 - 24 a^4 b^{10} c^5 d^8 + 240 a^5 b^8 c^6 d^8 - 1280 a^6 b^6 c^7 d^8 + 3840 a^7 b^4 c^8 d^8 - 6144 a^8 b^2 c^9 d^8 + 240 a^9 b^8 c^2 e^8 - 1280 a^{10} b^6 c^3 e^8 + 3840 a^{11} b^4 c^4 e^8 - 6144 a^{12} b^2 c^5 e^8 + a^3 b^{16} d^4 e^4 - 4 a^4 b^{15} d^3 e^5 + 6 a^5 b^{14} d^2 e^6 + 16384 a^{10} c^9 d^6 e^2 + 24576 a^{11} c^8 d^4 e^4 + 16384 a^{12} c^7 d^2 e^6 + 6 a^3 b^{14} c^2 d^6 e^2 - 140 a^4 b^{12} c^3 d^6 e^2 + 84 a^4 b^{13} c^2 d^5 e^3 + 1344 a^5 b^{10} c^4 d^6 e^2 - 672 a^5 b^{11} c^3 d^5 e^3 - 42 a^5 b^{12} c^2 d^4 e^4 - 6720 a^6 b^8 c^5 d^6 e^2 + 2240 a^6 b^9 c^4 d^5 e^3 + 1456 a^6 b^{10} c^3 d^4 e^4 - 672 a^6 b^{11} c^2 d^3 e^5 + 17920 a^7 b^6 c^6 d^6 e^2 - 10080 a^7 b^8 c^4 d^4 e^4 + 2240 a^7 b^9 c^3 d^3 e^5 + 1344 a^7 b^{10} c^2 d^2 e^6 - 21504 a^8 b^4 c^7 d^6 e^2 - 21504 a^8 b^5 c^6 d^5 e^3 + 32256 a^8 b^6 c^5 d^4 e^4 - 6720 a^8 b^8 c^3 d^2 e^6 + 57344 a^9 b^3 c^7 d^5 e^3 - 46592 a^9 b^4 c^6 d^4 e^4 - 21504 a^9 b^5 c^5 d^3 e^5 + 17920 a^9 b^6 c^4 d^2 e^6 + 12288 a^{10} b^2 c^7 d^4 e^4 + 57344 a^{10} b^3 c^6 d^3 e^5 - 21504 a^{10} b^4 c^5 d^2 e^6 + 96 a^7 b^{11} c^4 d^7 e - 16384 a^9 b^6 c^9 d^7 e - 16384 a^{12} b^6 c^6 d^7 e - 4 a^3 b^{13} c^3 d^7 e - 4 a^3 b^{15} c^3 d^5 e^3 + 96 a^4 b^{11} c^4 d^7 e - 12 a^4 b^{14} c^4 d^4 e^4 - 960 a^5 b^9 c^5 d^7 e + 84 a^5 b^{13} c^4 d^3 e^5 + 5120 a^6 b^7 c^6 d^7 e - 140 a^6 b^{12} c^4 d^2 e^6
\end{aligned}$$

$$\begin{aligned}
& - 15360a^7b^5c^7d^7e + 24576a^8b^3c^8d^7e - 960a^8b^9c^2d^7e^7 \\
& + 5120a^9b^7c^3d^7e^7 - 49152a^{10}b^3c^8d^5e^3 - 15360a^{10}b^5c^4d^7e^7 - 49152a^{11}b^3c^7d^3e^5 + 24576a^{11}b^3c^5d^7e^7) \Big)^{(1/2)} - (3269 \\
& 12a^8c^9d^7e^{13} - 241664a^8b^3c^8e^{14} - 48a^2b^{13}c^2e^{14} + 1264a^3 \\
& b^{11}c^3e^{14} - 13552a^4b^9c^4e^{14} + 75776a^5b^7c^5e^{14} - 232960a^6 \\
& b^5c^6e^{14} + 372736a^7b^3c^7e^{14} + 11520a^3c^{14}d^{11}e^3 + 78080 \\
& a^4c^{13}d^9e^5 + 197120a^5c^{12}d^7e^7 + 336384a^6c^{11}d^5e^9 + 532 \\
& 736a^7c^{10}d^3e^{11} - 40b^5c^{12}d^{12}e^2 + 216b^6c^{11}d^{11}e^3 - 464b^7 \\
& c^{10}d^{10}e^4 + 496b^8c^9d^9e^5 - 264b^9c^8d^8e^6 + 56b^{10}c^7 \\
& d^7e^7 - 16b^{11}c^6d^6e^8 + 64b^{12}c^5d^5e^9 - 96b^{13}c^4d^4e^{10} \\
& + 64b^{14}c^3d^3e^{11} - 16b^{15}c^2d^2e^{12} + 1536a^2b^2c^{13}d^{11}e^3 \\
& + 14400a^2b^3c^{12}d^{10}e^4 - 47152a^2b^4c^{11}d^9e^5 + 52144a^2b^5 \\
& c^{10}d^8e^6 - 16272a^2b^6c^9d^7e^7 - 13040a^2b^7c^8d^6e^8 + 234 \\
& 88a^2b^8c^7d^5e^9 - 26384a^2b^9c^6d^4e^{10} + 13824a^2b^{10}c^5d^3e^{11} \\
& + 256a^2b^{11}c^4d^2e^{12} + 125056a^3b^2c^{12}d^9e^5 - 36224a^3 \\
& b^3c^{11}d^8e^6 - 126432a^3b^4c^{10}d^7e^7 + 144848a^3b^5c^9d^6e^8 - \\
& 114752a^3b^6c^8d^5e^9 + 125392a^3b^7c^7d^4e^{10} - 53248a^3b^8 \\
& c^6d^3e^{11} - 25264a^3b^9c^5d^2e^{12} + 474112a^4b^2c^{11}d^7e^7 - \\
& 191104a^4b^3c^{10}d^6e^8 + 97184a^4b^4c^9d^5e^9 - 277000a^4b^5c^8 \\
& d^4e^{10} + 56056a^4b^6c^7d^3e^{11} + 195584a^4b^7c^6d^2e^{12} + 2 \\
& 36800a^5b^2c^{10}d^5e^9 + 388032a^5b^3c^9d^4e^{10} + 159632a^5b^4c^8 \\
& d^3e^{11} - 670488a^5b^5c^7d^2e^{12} - 488960a^6b^2c^9d^3e^{11} + 1 \\
& 106496a^6b^3c^8d^2e^{12} + 64a^6b^{14}c^2d^7e^{13} + 448a^6b^3c^{13}d^{12}e^2 \\
& - 1968a^6b^4c^{12}d^{11}e^3 + 2504a^6b^5c^{11}d^{10}e^4 + 768a^6b^6c^{10}d^9 \\
& e^5 - 4368a^6b^7c^9d^8e^6 + 3568a^6b^8c^8d^7e^7 - 520a^6b^9c^7d^6 \\
& e^8 - 1728a^6b^{10}c^6d^5e^9 + 2528a^6b^{11}c^5d^4e^{10} - 1536a^6b^{12}c^4 \\
& d^3e^{11} + 240a^6b^{13}c^3d^2e^{12} - 1152a^2b^3c^{14}d^{12}e^2 - 1600a^2b^4 \\
& c^{12}d^{10}e^4 + 15808a^3b^{10}c^4d^7e^{13} - 34 \\
& 2272a^4b^3c^{12}d^8e^6 - 76928a^4b^8c^5d^7e^{13} - 569088a^5b^3c^{11}d^6 \\
& e^8 + 179200a^5b^6c^6d^6e^{13} - 586368a^6b^3c^{10}d^4e^{10} - 113008a^6b^4 \\
& c^7d^5e^{13} - 731008a^7b^3c^9d^2e^{12} - 244096a^7b^2c^8d^3e^{13}) / (16 * \\
& (a^6b^8e^8 + 256a^6c^8d^8 + 256a^{10}c^4e^8 - 16a^7b^6c^8e^8 - 4a^5 \\
& b^9d^7e^7 + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6d^8 - 2 \\
& 56a^5b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^{12}d^4 \\
& e^4 - 4a^3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 + 1024a^7c^7d^6e^2 + 15 \\
& 36a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^{10}c^2d^6e^2 - 92a^3 \\
& b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4 \\
& b^7c^3d^5e^3 - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 12 \\
& 8a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 - 192a^5b^7c^2d^3e^5 + \\
& 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c^4d^4 \\
& e^4 - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5 \\
& d^4e^4 + 2048a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512a^8b^2 \\
& c^4d^2e^6 - 1024a^6b^3c^7d^7e + 64a^6b^7c^3d^7e^7 - 1024a^9b^3c^4 \\
& d^7e^7 - 4a^2b^9c^3d^7e - 4a^2b^{11}c^5d^5e^3 + 64a^3b^7c^4d^7e - \\
& 4a^3b^{10}c^4d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^3d^3e^5 + 1024a^5 \\
& b^3c^6d^7e - 92a^5b^8c^3d^2e^6 - 3072a^7b^3c^6d^5e^3 - 384a^7 \\
& b^5c^2d^7e^7 - 3072a^8b^3c^5d^3e^5 + 1024a^8b^3c^3d^7e^7) * ((27a^* \\
& b^9c^5d^6 - b^{11}c^4d^6 - b^{15}d^2e^4 - 9a^2b^{13}e^6 + 3840a^5b^3c^9 \\
& d^6 - 9a^6c^5d^6 * (- (4a^*c - b^2)^9)^{(1/2)} + 213a^3b^{11}c^6e^6 - 26880a^8 \\
& b^3c^6e^6 + 3072a^6c^9d^5e + 35840a^8c^7d^7e^5 + 4b^{12}c^3d^5e + \\
& 4b^{14}c^3d^3e^3 - 288a^2b^7c^6d^6 + 1504a^3b^5c^7d^6 - 3840a^4b^3 \\
& c^8d^6 + 9a^2b^4e^6 * (- (4a^*c - b^2)^9)^{(1/2)} - 2077a^4b^9c^2e^6 \\
& + 10656a^5b^7c^3e^6 - 30240a^6b^5c^4e^6 + 44800a^7b^3c^5e^6 + 2 \\
& 5a^4c^2e^6 * (- (4a^*c - b^2)^9)^{(1/2)} + b^2c^4d^6 * (- (4a^*c - b^2)^9)^{(1/2)} \\
& + 22528a^7c^8d^3e^3 + b^6d^2e^4 * (- (4a^*c - b^2)^9)^{(1/2)} - 6b^{13} \\
& c^2d^4e^2 + 6a^6b^{14}d^5e^5 - 1471a^2b^9c^4d^4e^2 + 600a^2b^{10}c^3 \\
& d^3e^3 + 180a^2b^{11}c^2d^2e^4 + 6976a^3b^7c^5d^4e^2 - 1032a^3b^8 \\
& c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456a^4b^5c^6d^4e^2 - 7168 \\
& a^4b^6c^5d^3e^3 + 16896a^4b^7c^4d^2e^4 + 10240a^5b^3c^7d^4e^7
\end{aligned}$$

$$\begin{aligned}
& 2 + 37632a^5b^4c^6d^3e^3 - 47712a^5b^5c^5d^2e^4 - 59392a^6b^2c^7d^3e^3 + 60928a^6b^3c^6d^2e^4 - 41a^2c^4d^4e^2(-4ac - b^2)^9)^{(1/2)} - 39a^3c^3d^2e^4(-4ac - b^2)^9)^{(1/2)} + 6b^4c^2d^4e^2 \\
& *(-4ac - b^2)^9)^{(1/2)} - 6a^5b^5d^2e^5(-4ac - b^2)^9)^{(1/2)} - 106a^6b^10c^4d^5e^5 + 7a^5b^13c^3d^2e^4 - 128a^2b^12c^3d^2e^5 - 51a^3b^2c^5e^6 \\
& *(-4ac - b^2)^9)^{(1/2)} + 150a^5b^11c^3d^4e^2 - 84a^5b^12c^2d^3e^3 + 1116a^2b^8c^5d^5e^5 - 5824a^3b^6c^6d^5e^5 + 1030a^3b^10c^2d^5e^5 \\
& + 15232a^4b^4c^7d^5e^5 - 3492a^4b^8c^3d^5e^5 - 16896a^5b^2c^8d^5e^5 + 1344a^5b^6c^4d^5e^5 + 7424a^6b^5c^8d^4e^2 + 22400a^6b^4c^5d^5e^5 \\
& - 23296a^7b^5c^7d^2e^4 - 53760a^7b^2c^6d^5e^5 - 4b^3c^3d^5e^5 *(-4ac - b^2)^9)^{(1/2)} - 4b^5c^3d^3e^3 *(-4ac - b^2)^9)^{(1/2)} + 11a^6b^4c^2d^2e^4 *(-4ac - b^2)^9)^{(1/2)} + 20a^2b^3c^3d^2e^5 *(-4ac - b^2)^9)^{(1/2)} \\
& + 86a^3b^5c^2d^2e^5 *(-4ac - b^2)^9)^{(1/2)} - 42a^5b^2c^3d^4e^2 *(-4ac - b^2)^9)^{(1/2)} + 12a^5b^3c^2d^3e^3 *(-4ac - b^2)^9)^{(1/2)} + 120a^2b^5c^3d^3e^3 *(-4ac - b^2)^9)^{(1/2)} + 34a^5b^4c^4d^5e^5 *(-4ac - b^2)^9)^{(1/2)} \\
& - 108a^2b^2c^2d^2e^4 *(-4ac - b^2)^9)^{(1/2)) / (32(a^7b^12e^8 + 4096a^9c^10d^8 + 4096a^13c^6e^8 - 24a^8b^10c^8e^8 - 4a^6b^13d^8e^7 + a^3b^12c^4d^8 - 24a^4b^10c^5d^8 + 240a^5b^8c^6d^8 - 1280a^6b^6c^7d^8 + 3840a^7b^4c^8d^8 - 6144a^8b^2c^9d^8 + 240a^9b^8c^2e^8 - 1280a^10b^6c^3e^8 + 3840a^11b^4c^4e^8 - 6144a^12b^2c^5e^8 + a^3b^16d^4e^4 - 4a^4b^15d^3e^5 + 6a^5b^14d^2e^6 + 16384a^10c^9d^6e^2 + 24576a^11c^8d^4e^4 + 16384a^12c^7d^2e^6 + 6a^3b^14c^2d^6e^2 - 140a^4b^12c^3d^6e^2 + 84a^4b^13c^2d^5e^3 + 1344a^5b^10c^4d^6e^2 - 672a^5b^11c^3d^5e^3 - 42a^5b^12c^2d^4e^4 - 6720a^6b^8c^5d^6e^2 + 2240a^6b^9c^4d^5e^3 + 1456a^6b^10c^3d^4e^4 - 672a^6b^11c^2d^3e^5 + 17920a^7b^6c^6d^6e^2 - 10080a^7b^8c^4d^4e^4 + 2240a^7b^9c^3d^3e^5 + 1344a^7b^10c^2d^2e^6 - 21504a^8b^4c^7d^6e^2 - 21504a^8b^5c^6d^5e^3 + 32256a^8b^6c^5d^4e^4 - 6720a^8b^8c^3d^2e^6 + 57344a^9b^3c^7d^5e^3 - 46592a^9b^4c^6d^4e^4 - 21504a^9b^5c^5d^3e^5 + 17920a^9b^6c^4d^2e^6 + 12288a^10b^2c^7d^4e^4 + 57344a^10b^3c^6d^3e^5 - 21504a^10b^4c^5d^2e^6 + 96a^7b^11c^3d^7e - 16384a^9b^5c^9d^7e - 16384a^12b^3c^6d^5e^7 - 4a^3b^13c^3d^7e - 4a^3b^15c^3d^5e^3 + 96a^4b^11c^4d^7e - 12a^4b^14c^3d^4e^4 - 960a^5b^9c^5d^7e + 84a^5b^13c^3d^3e^5 + 5120a^6b^7c^6d^7e - 140a^6b^12c^3d^2e^6 - 15360a^7b^5c^7d^7e + 24576a^8b^3c^8d^7e - 960a^8b^9c^2d^5e^7 + 5120a^9b^7c^3d^5e^7 - 49152a^10b^5c^8d^5e^3 - 15360a^10b^5c^4d^5e^7 - 49152a^11b^3c^7d^3e^5 + 24576a^11b^3c^5d^5e^7))^{(1/2)} - (x*(22800a^6c^9e^13 + 36a^2b^8c^5e^13 - 600a^3b^6c^6e^13 + 4313a^4b^4c^7e^13 - 15592a^5b^2c^8e^13 + 1296a^2c^13d^8e^5 + 9792a^3c^12d^6e^7 + 30304a^4c^11d^4e^9 + 40512a^5c^10d^2e^11 + 25b^4c^11d^8e^5 - 120b^5c^10d^7e^6 + 214b^6c^9d^6e^7 - 168b^7c^8d^5e^8 + 53b^8c^7d^4e^9 - 8b^9c^6d^3e^10 + 4b^10c^5d^2e^11 + 6336a^2b^2c^11d^6e^7 + 3840a^2b^3c^10d^5e^8 - 8506a^2b^4c^9d^4e^9 + 1112a^2b^5c^8d^3e^10 + 1254a^2b^6c^7d^2e^11 + 22224a^3b^2c^10d^4e^9 + 13824a^3b^3c^9d^3e^10 - 9516a^3b^4c^8d^2e^11 + 11712a^4b^2c^9d^2e^11 - 24a^5b^9c^5d^5e^12 - 41088a^5b^3c^9d^5e^12 - 360a^5b^2c^12d^8e^5 + 1664a^5b^3c^11d^7e^6 - 2604a^5b^4c^10d^6e^7 + 1272a^5b^5c^9d^5e^8 + 332a^5b^6c^8d^4e^9 - 232a^5b^7c^7d^3e^10 - 48a^5b^8c^6d^2e^11 - 5760a^5b^2c^12d^7e^6 + 416a^5b^7c^6d^5e^12 - 32128a^3b^3c^11d^5e^8 - 4120a^3b^5c^7d^5e^12 - 63360a^4b^3c^10d^3e^10 + 21376a^4b^3c^8d^5e^12)) / (8(a^6b^8e^8 + 256a^6c^8d^8 + 256a^10c^4e^8 - 16a^7b^6c^5e^8 - 4a^5b^9d^5e^7 + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6d^8 - 256a^5b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^12d^4e^4 - 4a^3b^11d^3e^5 + 6a^4b^10d^2e^6 + 1024a^7c^7d^6e^2 + 1536a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^10c^2d^6e^2 - 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4b^7c^3d^5e^3 - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 - 192a^5b^7c^3d^4e^4 - 192a^5b^7c^3d^4e^4)
\end{aligned}$$

$$\begin{aligned}
& c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6 \\
& *b^4c^4d^4e^4 - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536 \\
& *a^7b^2c^5d^4e^4 + 2048a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 \\
& + 512a^8b^2c^4d^2e^6 - 1024a^6b^6c^7d^7e + 64a^6b^7c^6d^5e^7 - 102 \\
& 4a^9b^6c^4d^4e^7 - 4a^2b^9c^3d^7e - 4a^2b^{11}c^5d^5e^3 + 64a^3b^7 \\
& *c^4d^7e - 4a^3b^{10}c^4d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^6d^ \\
& 3e^5 + 1024a^5b^3c^6d^7e - 92a^5b^8c^6d^2e^6 - 3072a^7b^6c^6d^5e \\
& e^3 - 384a^7b^5c^2d^4e^7 - 3072a^8b^6c^5d^3e^5 + 1024a^8b^3c^3d^4e \\
& ^7)) * ((27a^9b^9c^5d^6 - b^{11}c^4d^6 - b^{15}d^2e^4 - 9a^2b^{13}e^6 + 3 \\
& 840a^5b^6c^9d^6 - 9a^5c^5d^6 * (-4a^2c - b^2)^9)^{(1/2)} + 213a^3b^{11}c^4e \\
& ^6 - 26880a^8b^6c^6e^6 + 3072a^6c^9d^5e + 35840a^8c^7d^5e^5 + 4b^1 \\
& 2c^3d^5e + 4b^{14}c^3d^3e^3 - 288a^2b^7c^6d^6 + 1504a^3b^5c^7d^6 \\
& - 3840a^4b^3c^8d^6 + 9a^2b^4e^6 * (-4a^2c - b^2)^9)^{(1/2)} - 2077a^4 \\
& *b^9c^2e^6 + 10656a^5b^7c^3e^6 - 30240a^6b^5c^4e^6 + 44800a^7b^ \\
& 3c^5e^6 + 25a^4c^2e^6 * (-4a^2c - b^2)^9)^{(1/2)} + b^2c^4d^6 * (-4a^2c \\
& - b^2)^9)^{(1/2)} + 22528a^7c^8d^3e^3 + b^6d^2e^4 * (-4a^2c - b^2)^9)^{(1 \\
& /2)} - 6b^{13}c^2d^4e^2 + 6a^2b^{14}d^5e^5 - 1471a^2b^9c^4d^4e^2 + 600a \\
& ^2b^{10}c^3d^3e^3 + 180a^2b^{11}c^2d^2e^4 + 6976a^3b^7c^5d^4e^2 \\
& - 1032a^3b^8c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456a^4b^5c^6d \\
& ^4e^2 - 7168a^4b^6c^5d^3e^3 + 16896a^4b^7c^4d^2e^4 + 10240a^5b \\
& ^3c^7d^4e^2 + 37632a^5b^4c^6d^3e^3 - 47712a^5b^5c^5d^2e^4 - 59 \\
& 392a^6b^2c^7d^3e^3 + 60928a^6b^3c^6d^2e^4 - 41a^2c^4d^4e^2 * (- \\
& (4a^2c - b^2)^9)^{(1/2)} - 39a^3c^3d^2e^4 * (-4a^2c - b^2)^9)^{(1/2)} + 6b^ \\
& 4c^2d^4e^2 * (-4a^2c - b^2)^9)^{(1/2)} - 6a^2b^5d^5e^5 * (-4a^2c - b^2)^9)^{( \\
& 1/2)} - 106a^2b^{10}c^4d^5e + 7a^2b^{13}c^2d^2e^4 - 128a^2b^{12}c^4d^5e^5 - 5 \\
& 1a^3b^2c^6e^6 * (-4a^2c - b^2)^9)^{(1/2)} + 150a^2b^{11}c^3d^4e^2 - 84a^2b^ \\
& 12c^2d^3e^3 + 1116a^2b^8c^5d^5e - 5824a^3b^6c^6d^5e + 1030a^3 \\
& *b^{10}c^2d^5e^5 + 15232a^4b^4c^7d^5e - 3492a^4b^8c^3d^5e^5 - 16896a \\
& ^5b^2c^8d^5e + 1344a^5b^6c^4d^5e^5 + 7424a^6b^6c^8d^4e^2 + 22400 \\
& *a^6b^4c^5d^5e^5 - 23296a^7b^6c^7d^2e^4 - 53760a^7b^2c^6d^5e^5 - 4a \\
& b^3c^3d^5e^6 * (-4a^2c - b^2)^9)^{(1/2)} - 4b^5c^3d^3e^3 * (-4a^2c - b^2)^9) \\
& ^{(1/2)} + 11a^2b^4c^2d^2e^4 * (-4a^2c - b^2)^9)^{(1/2)} + 20a^2b^3c^3d^5e^5 * ( \\
& -4a^2c - b^2)^9)^{(1/2)} + 86a^3b^6c^2d^5e^5 * (-4a^2c - b^2)^9)^{(1/2)} - 42a \\
& *b^2c^3d^4e^2 * (-4a^2c - b^2)^9)^{(1/2)} + 12a^2b^3c^2d^3e^3 * (-4a^2c \\
& - b^2)^9)^{(1/2)} + 120a^2b^6c^3d^3e^3 * (-4a^2c - b^2)^9)^{(1/2)} + 34a^2b^6c \\
& ^4d^5e^6 * (-4a^2c - b^2)^9)^{(1/2)} - 108a^2b^2c^2d^2e^4 * (-4a^2c - b^2) \\
& ^9)^{(1/2))} / (32 * (a^7b^{12}e^8 + 4096a^9c^{10}d^8 + 4096a^{13}c^6e^8 - 24a \\
& ^8b^{10}c^8e^8 - 4a^6b^{13}d^8e^7 + a^3b^{12}c^4d^8 - 24a^4b^{10}c^5d^8 + \\
& 240a^5b^8c^6d^8 - 1280a^6b^6c^7d^8 + 3840a^7b^4c^8d^8 - 6144a \\
& ^8b^2c^9d^8 + 240a^9b^8c^2e^8 - 1280a^{10}b^6c^3e^8 + 3840a^{11}b^ \\
& 4c^4e^8 - 6144a^{12}b^2c^5e^8 + a^3b^{16}d^4e^4 - 4a^4b^{15}d^3e^5 + \\
& 6a^5b^{14}d^2e^6 + 16384a^{10}c^9d^6e^2 + 24576a^{11}c^8d^4e^4 + 163 \\
& 84a^{12}c^7d^2e^6 + 6a^3b^{14}c^2d^6e^2 - 140a^4b^{12}c^3d^6e^2 + 8 \\
& 4a^4b^{13}c^2d^5e^3 + 1344a^5b^{10}c^4d^6e^2 - 672a^5b^{11}c^3d^5e \\
& ^3 - 42a^5b^{12}c^2d^4e^4 - 6720a^6b^8c^5d^6e^2 + 2240a^6b^9c^4d \\
& ^5e^3 + 1456a^6b^{10}c^3d^4e^4 - 672a^6b^{11}c^2d^3e^5 + 17920a^7b \\
& ^6c^6d^6e^2 - 10080a^7b^8c^4d^4e^4 + 2240a^7b^9c^3d^3e^5 + 13 \\
& 44a^7b^{10}c^2d^2e^6 - 21504a^8b^4c^7d^6e^2 - 21504a^8b^5c^6d^5 \\
& *e^3 + 32256a^8b^6c^5d^4e^4 - 6720a^8b^8c^3d^2e^6 + 57344a^9b^3 \\
& *c^7d^5e^3 - 46592a^9b^4c^6d^4e^4 - 21504a^9b^5c^5d^3e^5 + 1792 \\
& 0a^9b^6c^4d^2e^6 + 12288a^{10}b^2c^7d^4e^4 + 57344a^{10}b^3c^6d^3 \\
& *e^5 - 21504a^{10}b^4c^5d^2e^6 + 96a^7b^{11}c^4d^7e - 16384a^9b^6c^9d \\
& ^7e - 16384a^{12}b^6c^6d^7e - 4a^3b^{13}c^3d^7e - 4a^3b^{15}c^4d^5e^3 \\
& + 96a^4b^{11}c^4d^7e - 12a^4b^{14}c^3d^4e^4 - 960a^5b^9c^5d^7e + \\
& 84a^5b^{13}c^3d^3e^5 + 5120a^6b^7c^6d^7e - 140a^6b^{12}c^4d^2e^6 - 1 \\
& 5360a^7b^5c^7d^7e + 24576a^8b^3c^8d^7e - 960a^8b^9c^2d^7e + \\
& 5120a^9b^7c^3d^7e - 49152a^{10}b^6c^8d^5e^3 - 15360a^{10}b^5c^4d^7e \\
& ^7 - 49152a^{11}b^6c^7d^3e^5 + 24576a^{11}b^3c^5d^7e))^{(1/2)} * i - ((((( \\
& 1048576a^{13}c^8e^{16} + 256a^7b^{12}c^2e^{16} - 6144a^8b^{10}c^3e^{16} + 61
\end{aligned}$$



$$\begin{aligned}
& 440a^9b^8c^4e^{16} - 327680a^{10}b^6c^5e^{16} + 983040a^{11}b^4c^6e^{16} \\
& - 1572864a^{12}b^2c^7e^{16} - 196608a^6c^{15}d^{14}e^2 - 917504a^7c^{14}d^{12}e^4 \\
& - 589824a^8c^{13}d^{10}e^6 + 3932160a^9c^{12}d^8e^8 + 10158080a^{10}c^{11}d^6e^{10} \\
& + 10616832a^{11}c^{10}d^4e^{12} + 5308416a^{12}c^9d^2e^{14} - 2816a^2b^8c^{11}d^{14}e^2 \\
& + 22656a^2b^9c^{10}d^{13}e^3 - 78848a^2b^{10}c^9d^{12}e^4 + 154112a^2b^{11}c^8d^{11}e^5 \\
& - 182784a^2b^{12}c^7d^{10}e^6 + 130816a^2b^{13}c^6d^9e^7 - 50176a^2b^{14}c^5d^8e^8 + 4608a^2b^{15}c^4d^7e^9 \\
& + 3328a^2b^{16}c^3d^6e^{10} - 896a^2b^{17}c^2d^5e^{11} + 24576a^3b^6c^{12}d^{14}e^2 \\
& - 198656a^3b^7c^{11}d^{13}e^3 + 684544a^3b^8c^{10}d^{12}e^4 - 1291520a^3b^9c^9d^{11}e^5 \\
& + 1403776a^3b^{10}c^8d^{10}e^6 - 798336a^3b^{11}c^7d^9e^7 + 89856a^3b^{12}c^6d^8e^8 + 155136a^3b^{13}c^5d^7e^9 \\
& - 77440a^3b^{14}c^4d^6e^{10} + 5504a^3b^{15}c^3d^5e^{11} + 2560a^3b^{16}c^2d^4e^{12} \\
& - 106496a^4b^4c^{13}d^{14}e^2 + 864256a^4b^5c^{12}d^{13}e^3 - 2924544a^4b^6c^{11}d^{12}e^4 \\
& + 5181440a^4b^7c^{10}d^{11}e^5 - 4686080a^4b^8c^9d^{10}e^6 + 1045376a^4b^9c^8d^9e^7 \\
& + 1900544a^4b^{10}c^7d^8e^8 - 1732096a^4b^{11}c^6d^7e^9 + 390400a^4b^{12}c^5d^6e^{10} \\
& + 112000a^4b^{13}c^4d^5e^{11} - 40960a^4b^{14}c^3d^4e^{12} - 3840a^4b^{15}c^2d^3e^{13} \\
& + 229376a^5b^2c^{14}d^{14}e^2 - 1867776a^5b^3c^{13}d^{13}e^3 + 6078464a^5b^4c^{12}d^{12}e^4 \\
& - 9297920a^5b^5c^{11}d^{11}e^5 + 4055040a^5b^6c^{10}d^{10}e^6 + 7788544a^5b^7c^9d^9e^7 \\
& - 12657664a^5b^8c^8d^8e^8 + 6130176a^5b^9c^7d^7e^9 + 734080a^5b^{10}c^6d^6e^{10} \\
& - 1442560a^5b^{11}c^5d^5e^{11} + 168960a^5b^{12}c^4d^4e^{12} + 78080a^5b^{13}c^3d^3e^{13} \\
& + 3200a^5b^{14}c^2d^2e^{14} - 4587520a^6b^2c^{13}d^{11}e^5 + 3080192a^6b^3c^{12}d^{11}e^5 \\
& + 12001280a^6b^4c^{11}d^{10}e^6 - 31076352a^6b^5c^{10}d^9e^7 + 27475968a^6b^6c^9d^8e^8 - 2088960a^6b^7c^8d^7e^9 \\
& - 12205312a^6b^8c^7d^6e^{10} + 6043520a^6b^9c^6d^5e^{11} + 631808a^6b^{10}c^5d^4e^{12} \\
& - 610304a^6b^{11}c^4d^3e^{13} - 71936a^6b^{12}c^3d^2e^{14} - 21725184a^7b^2c^{12}d^{10}e^6 \\
& + 30801920a^7b^3c^{11}d^9e^7 - 8028160a^7b^4c^{10}d^8e^8 - 32260096a^7b^5c^9d^7e^9 + 37101568a^7b^6c^8d^6e^{10} \\
& - 7182336a^7b^7c^7d^5e^{11} - 7609856a^7b^8c^6d^4e^{12} + 2112256a^7b^9c^5d^3e^{13} \\
& + 661632a^7b^{10}c^4d^2e^{14} - 30146560a^8b^2c^{11}d^8e^8 + 55050240a^8b^3c^{10}d^7e^9 - 34365440a^8b^4c^9d^6e^{10} \\
& - 16429056a^8b^5c^8d^5e^{11} + 24600576a^8b^6c^7d^4e^{12} - 1683456a^8b^7c^6d^3e^{13} \\
& - 3151616a^8b^8c^5d^2e^{14} - 10977280a^9b^2c^{10}d^6e^{10} + 47022080a^9b^3c^9d^5e^{11} \\
& - 30621696a^9b^4c^8d^4e^{12} - 9232384a^9b^5c^7d^3e^{13} + 7970816a^9b^6c^6d^2e^{14} \\
& + 4325376a^{10}b^2c^9d^4e^{12} + 25493504a^{10}b^3c^8d^3e^{13} - 9117696a^{10}b^4c^7d^2e^{14} \\
& + 491520a^{11}b^2c^8d^2e^{14} - 4947968a^{12}b^3c^8d^3e^{13} + 128a^2b^{10}c^{10}d^{14}e^2 \\
& - 1024a^2b^{11}c^9d^{13}e^3 + 3584a^2b^{12}c^8d^{12}e^4 - 7168a^2b^{13}c^7d^{11}e^5 + 8960a^2b^{14}c^6d^{10}e^6 \\
& - 7168a^2b^{15}c^5d^9e^7 + 3584a^2b^{16}c^4d^8e^8 - 1024a^2b^{17}c^3d^7e^9 + 128a^2b^{18}c^2d^6e^{10} \\
& + 1605632a^6b^3c^{14}d^{13}e^3 - 1408a^6b^{13}c^2d^5e^{15} + 7012352a^7b^3c^{13}d^{11}e^5 \\
& + 33152a^7b^{11}c^3d^5e^{15} + 7045120a^8b^3c^{12}d^9e^7 - 324480a^8b^9c^4d^5e^{15} - 9830400a^9b^3c^{11}d^7e^9 \\
& + 1689600a^9b^7c^5d^5e^{15} - 25722880a^{10}b^3c^{10}d^5e^{11} - 4935680a^{10}b^5c^6d^5e^{15} \\
& - 19202048a^{11}b^3c^9d^3e^{13} + 7667712a^{11}b^3c^7d^3e^{15}) / (16(a^6b^8e^8 + 256a^6c^8d^8 \\
& + 256a^{10}c^4e^8 - 16a^7b^6c^8e^8 - 4a^5b^9d^7e^7 + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 \\
& + 96a^4b^4c^6d^8 - 256a^5b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^{12}d^4e^4 \\
& - 4a^3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 + 1024a^7c^7d^6e^2 + 1536a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 \\
& + 6a^2b^{10}c^2d^6e^2 - 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 \\
& - 192a^4b^7c^3d^5e^3 - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d^5e^3 \\
& + 800a^5b^6c^3d^4e^4 - 192a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 \\
& - 2240a^6b^4c^4d^4e^4 - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 \\
& + 2048a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024a^6b^3c^7d^7e \\
& + 64a^6b^7c^3d^7e^7 - 1024a^9b^3c^4d^5e^7 - 4a^2b^9c^3d^7e - 4a^2b^{11}c^3d^5e^3 + 64a^3b^7c^
\end{aligned}$$

$$\begin{aligned}
&^4d^7e - 4a^3b^{10}c^4d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^3d^3e^5 + 1024a^5b^3c^6d^7e - 92a^5b^8c^2d^2e^6 - 3072a^7b^6c^6d^5e^3 \\
&- 384a^7b^5c^2d^2e^7 - 3072a^8b^3c^5d^3e^5 + 1024a^8b^3c^3d^2e^7) + (x((27a^9b^9c^5d^6 - b^{11}c^4d^6 - b^{15}d^2e^4 - 9a^2b^{13}e^6 + \\
&3840a^5b^9c^9d^6 - 9a^5c^5d^6(-4ac - b^2)^9)^{(1/2)} + 213a^3b^{11}c^6e^6 - 26880a^8b^6c^6e^6 + 3072a^6c^9d^5e + 35840a^8c^7d^5e^5 + 4b \\
&^{12}c^3d^5e + 4b^{14}c^3d^3e^3 - 288a^2b^7c^6d^6 + 1504a^3b^5c^7d^6 - 3840a^4b^3c^8d^6 + 9a^2b^4e^6(-4ac - b^2)^9)^{(1/2)} - 2077a \\
&^4b^9c^2e^6 + 10656a^5b^7c^3e^6 - 30240a^6b^5c^4e^6 + 44800a^7b^3c^5e^6 + 25a^4c^2e^6(-4ac - b^2)^9)^{(1/2)} + b^2c^4d^6(-4ac \\
&- b^2)^9)^{(1/2)} + 22528a^7c^8d^3e^3 + b^6d^2e^4(-4ac - b^2)^9)^{(1/2)} - 6b^{13}c^2d^4e^2 + 6ab^{14}d^5e^5 - 1471a^2b^9c^4d^4e^2 + 60 \\
&0a^2b^{10}c^3d^3e^3 + 180a^2b^{11}c^2d^2e^4 + 6976a^3b^7c^5d^4e^2 - 1032a^3b^8c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456a^4b^5c^6 \\
&d^4e^2 - 7168a^4b^6c^5d^3e^3 + 16896a^4b^7c^4d^2e^4 + 10240a^5b^3c^7d^4e^2 + 37632a^5b^4c^6d^3e^3 - 47712a^5b^5c^5d^2e^4 - \\
&59392a^6b^2c^7d^3e^3 + 60928a^6b^3c^6d^2e^4 - 41a^2c^4d^4e^2(-4ac - b^2)^9)^{(1/2)} - 39a^3c^3d^2e^4(-4ac - b^2)^9)^{(1/2)} + 6 \\
&b^4c^2d^4e^2(-4ac - b^2)^9)^{(1/2)} - 6ab^5d^5e^5(-4ac - b^2)^9)^{(1/2)} - 106ab^{10}c^4d^5e + 7ab^{13}c^2d^2e^4 - 128a^2b^{12}c^3d^2e^5 - \\
&51a^3b^2c^6e^6(-4ac - b^2)^9)^{(1/2)} + 150ab^{11}c^3d^4e^2 - 84ab^{12}c^2d^3e^3 + 1116a^2b^8c^5d^5e - 5824a^3b^6c^6d^5e + 1030a \\
&^3b^{10}c^2d^2e^5 + 15232a^4b^4c^7d^5e - 3492a^4b^8c^3d^2e^5 - 16896a^5b^2c^8d^5e + 1344a^5b^6c^4d^2e^5 + 7424a^6b^6c^8d^4e^2 + 224 \\
&00a^6b^4c^5d^2e^5 - 23296a^7b^6c^7d^2e^4 - 53760a^7b^2c^6d^2e^5 - 4b^3c^3d^5e(-4ac - b^2)^9)^{(1/2)} - 4b^5c^3d^3e^3(-4ac - b^2)^9)^{(1/2)} \\
&+ 11ab^4c^2d^2e^4(-4ac - b^2)^9)^{(1/2)} + 20a^2b^3c^3d^2e^5(-4ac - b^2)^9)^{(1/2)} + 86a^3b^2c^2d^2e^5(-4ac - b^2)^9)^{(1/2)} - 4 \\
&2ab^2c^3d^4e^2(-4ac - b^2)^9)^{(1/2)} + 12ab^3c^2d^3e^3(-4ac - b^2)^9)^{(1/2)} + 120a^2b^3c^3d^3e^3(-4ac - b^2)^9)^{(1/2)} + 34ab \\
&c^4d^5e(-4ac - b^2)^9)^{(1/2)} - 108a^2b^2c^2d^2e^4(-4ac - b^2)^9)^{(1/2)))/(32(a^7b^{12}e^8 + 4096a^9c^{10}d^8 + 4096a^{13}c^6e^8 - 24 \\
&a^8b^{10}c^8e^8 - 4a^6b^{13}d^8e^7 + a^3b^{12}c^4d^8 - 24a^4b^{10}c^5d^8 + 240a^5b^8c^6d^8 - 1280a^6b^6c^7d^8 + 3840a^7b^4c^8d^8 - 6144 \\
&a^8b^2c^9d^8 + 240a^9b^8c^2e^8 - 1280a^{10}b^6c^3e^8 + 3840a^{11}b^4c^4e^8 - 6144a^{12}b^2c^5e^8 + a^3b^{16}d^4e^4 - 4a^4b^{15}d^3e^5 \\
&+ 6a^5b^{14}d^2e^6 + 16384a^{10}c^9d^6e^2 + 24576a^{11}c^8d^4e^4 + 16384a^{12}c^7d^2e^6 + 6a^3b^{14}c^2d^6e^2 - 140a^4b^{12}c^3d^6e^2 + \\
&84a^4b^{13}c^2d^5e^3 + 1344a^5b^{10}c^4d^6e^2 - 672a^5b^{11}c^3d^5e^3 - 42a^5b^{12}c^2d^4e^4 - 6720a^6b^8c^5d^6e^2 + 2240a^6b^9c^4 \\
&d^5e^3 + 1456a^6b^{10}c^3d^4e^4 - 672a^6b^{11}c^2d^3e^5 + 17920a^7b^6c^6d^6e^2 - 10080a^7b^8c^4d^4e^4 + 2240a^7b^9c^3d^3e^5 + \\
&1344a^7b^{10}c^2d^2e^6 - 21504a^8b^4c^7d^6e^2 - 21504a^8b^5c^6d^5e^3 + 32256a^8b^6c^5d^4e^4 - 6720a^8b^8c^3d^2e^6 + 57344a^9b^3 \\
&>c^7d^5e^3 - 46592a^9b^4c^6d^4e^4 - 21504a^9b^5c^5d^3e^5 + 17920a^9b^6c^4d^2e^6 + 12288a^{10}b^2c^7d^4e^4 + 57344a^{10}b^3c^6d^3 \\
&>e^5 - 21504a^{10}b^4c^5d^2e^6 + 96a^7b^{11}c^3d^2e^7 - 16384a^9b^3c^9d^7e - 16384a^{12}b^6c^6d^2e^7 - 4a^3b^{13}c^3d^7e - 4a^3b^{15}c^5d^5e \\
&^3 + 96a^4b^{11}c^4d^7e - 12a^4b^{14}c^3d^4e^4 - 960a^5b^9c^5d^7e + 84a^5b^{13}c^3d^3e^5 + 5120a^6b^7c^6d^7e - 140a^6b^{12}c^3d^2e^6 - \\
&15360a^7b^5c^7d^7e + 24576a^8b^3c^8d^7e - 960a^8b^9c^2d^2e^7 + 5120a^9b^7c^3d^2e^7 - 49152a^{10}b^6c^8d^5e^3 - 15360a^{10}b^5c^4d^5 \\
&e^7 - 49152a^{11}b^3c^7d^3e^5 + 24576a^{11}b^3c^5d^2e^7))^{(1/2)}(1048576a^{15}c^8e^{17} + 256a^9b^{12}c^2e^{17} - 6144a^{10}b^{10}c^3e^{17} + 61440a^{11} \\
&b^8c^4e^{17} - 327680a^{12}b^6c^5e^{17} + 983040a^{13}b^4c^6e^{17} - 1572864a^{14}b^2c^7e^{17} - 1048576a^8c^{15}d^{14}e^3 - 5242880a^9c^{14}d^{12} \\
&e^5 - 9437184a^{10}c^{13}d^{10}e^7 - 5242880a^{11}c^{12}d^8e^9 + 5242880a^{12}c^{11}d^6e^{11} + 9437184a^{13}c^{10}d^4e^{13} + 5242880a^{14}c^9d^2e^{15} + 2 \\
&56a^2b^{11}c^{10}d^{15}e^2 - 2048a^2b^{12}c^9d^{14}e^3 + 7168a^2b^{13}c^8
\end{aligned}$$

$$\begin{aligned}
& d^{13}e^4 - 14336a^2b^{14}c^7d^{12}e^5 + 17920a^2b^{15}c^6d^{11}e^6 - 14336a^2b^{16}c^5d^{10}e^7 + 7168a^2b^{17}c^4d^9e^8 - 2048a^2b^{18}c^3d^8e^9 + 256a^2b^{19}c^2d^7e^{10} - 5120a^3b^9c^{11}d^{15}e^2 + 41984a^3b^{10}c^{10}d^{14}e^3 - 148736a^3b^{11}c^9d^{13}e^4 + 296192a^3b^{12}c^8d^{12}e^5 - 359680a^3b^{13}c^7d^{11}e^6 + 267520a^3b^{14}c^6d^{10}e^7 - 112384a^3b^{15}c^5d^9e^8 + 18176a^3b^{16}c^4d^8e^9 + 3328a^3b^{17}c^3d^7e^{10} - 1280a^3b^{18}c^2d^6e^{11} + 40960a^4b^7c^{12}d^{15}e^2 - 348160a^4b^8c^{11}d^{14}e^3 + 1254400a^4b^9c^{10}d^{13}e^4 - 2478080a^4b^{10}c^9d^{12}e^5 + 2867456a^4b^{11}c^8d^{11}e^6 - 1862144a^4b^{12}c^7d^{10}e^7 + 490240a^4b^{13}c^6d^9e^8 + 128000a^4b^{14}c^5d^8e^9 - 108800a^4b^{15}c^4d^7e^{10} + 13824a^4b^{16}c^3d^6e^{11} + 2304a^4b^{17}c^2d^5e^{12} - 163840a^5b^5c^{13}d^{15}e^2 + 1474560a^5b^6c^{12}d^{14}e^3 - 5447680a^5b^7c^{11}d^{13}e^4 + 10588160a^5b^8c^{10}d^{12}e^5 - 11166720a^5b^9c^9d^{11}e^6 + 5159936a^5b^{10}c^8d^{10}e^7 + 1073920a^5b^{11}c^7d^9e^8 - 2279680a^5b^{12}c^6d^8e^9 + 770560a^5b^{13}c^5d^7e^{10} + 33280a^5b^{14}c^4d^6e^{11} - 41216a^5b^{15}c^3d^5e^{12} - 1280a^5b^{16}c^2d^4e^{13} + 327680a^6b^3c^{14}d^{15}e^2 - 3276800a^6b^4c^{13}d^{14}e^3 + 12615680a^6b^5c^{12}d^{13}e^4 - 23592960a^6b^6c^{11}d^{12}e^5 + 19701760a^6b^7c^{10}d^{11}e^6 + 1372160a^6b^8c^9d^{10}e^7 - 15846400a^6b^9c^8d^9e^8 + 10864640a^6b^{10}c^7d^8e^9 - 1352960a^6b^{11}c^6d^7e^{10} - 1111040a^6b^{12}c^5d^6e^{11} + 273920a^6b^{13}c^4d^5e^{12} + 25600a^6b^{14}c^3d^4e^{13} - 1280a^6b^{15}c^2d^3e^{14} + 3407872a^7b^2c^{14}d^{14}e^3 - 14221312a^7b^3c^{13}d^{13}e^4 + 23527424a^7b^4c^{12}d^{12}e^5 - 3768320a^7b^5c^{11}d^{11}e^6 - 38895616a^7b^6c^{10}d^{10}e^7 + 50126848a^7b^7c^9d^9e^8 - 18362368a^7b^8c^8d^8e^9 - 6831104a^7b^9c^7d^7e^{10} + 6200320a^7b^{10}c^6d^6e^{11} - 726784a^7b^{11}c^5d^5e^{12} - 228608a^7b^{12}c^4d^4e^{13} + 31488a^7b^{13}c^3d^3e^{14} + 2304a^7b^{14}c^2d^2e^{15} - 3145728a^8b^2c^{13}d^{12}e^5 - 31129600a^8b^3c^{12}d^{11}e^6 + 74711040a^8b^4c^{11}d^{10}e^7 - 55476224a^8b^5c^{10}d^9e^8 - 11075584a^8b^6c^9d^8e^9 + 35381248a^8b^7c^8d^7e^{10} - 14479360a^8b^8c^7d^6e^{11} - 168960a^8b^9c^6d^5e^{12} + 1286144a^8b^{10}c^5d^4e^{13} - 302336a^8b^{11}c^4d^3e^{14} - 55808a^8b^{12}c^3d^2e^{15} - 36962304a^9b^2c^{12}d^{10}e^7 - 9502720a^9b^3c^{11}d^9e^8 + 67174400a^9b^4c^{10}d^8e^9 - 54886400a^9b^5c^9d^7e^{10} + 11239424a^9b^6c^8d^6e^{11} + 5545984a^9b^7c^7d^5e^{12} - 5263360a^9b^8c^6d^4e^{13} + 1356800a^9b^9c^5d^3e^{14} + 558080a^9b^{10}c^4d^2e^{15} - 49807360a^{10}b^2c^{11}d^8e^9 + 19333120a^{10}b^3c^{10}d^7e^{10} + 7208960a^{10}b^4c^9d^6e^{11} - 14974976a^{10}b^5c^8d^5e^{12} + 15073280a^{10}b^6c^7d^4e^{13} - 2170880a^{10}b^7c^6d^3e^{14} - 2928640a^{10}b^8c^5d^2e^{15} - 11796480a^{11}b^2c^{10}d^6e^{11} + 23920640a^{11}b^3c^9d^5e^{12} - 24576000a^{11}b^4c^8d^4e^{13} - 4096000a^{11}b^5c^7d^3e^{14} + 8355840a^{11}b^6c^6d^2e^{15} + 12582912a^{12}b^2c^9d^4e^{13} + 19857408a^{12}b^3c^8d^3e^{14} - 11534336a^{12}b^4c^7d^2e^{15} + 3407872a^{13}b^2c^8d^2e^{15} - 5505024a^{14}b^3c^8d^2e^{16} - 262144a^7b^3c^{15}d^{15}e^2 + 5505024a^8b^3c^{14}d^{13}e^4 - 1280a^8b^{13}c^2d^2e^{16} + 25952256a^9b^3c^{13}d^{11}e^6 + 30976a^9b^{11}c^3d^2e^{16} + 38010880a^{10}b^3c^{12}d^9e^8 - 312320a^{10}b^9c^4d^2e^{16} + 11796480a^{11}b^3c^{11}d^7e^{10} + 1679360a^{11}b^7c^5d^2e^{16} - 21233664a^{12}b^3c^{10}d^5e^{12} - 5079040a^{12}b^5c^6d^2e^{16} - 20709376a^{13}b^3c^9d^3e^{14} + 8192000a^{13}b^3c^7d^2e^{16})) / (8(a^6b^8e^8 + 256a^6c^8d^8 + 256a^{10}c^4e^8 - 16a^7b^6c^8e^8 - 4a^5b^9d^2e^7 + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6d^8 - 256a^5b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^{12}d^4e^4 - 4a^3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 + 1024a^7c^7d^6e^2 + 1536a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^{10}c^2d^6e^2 - 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4b^7c^3d^5e^3 - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 - 192a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c^4d^4e^4 - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 + 2048a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4e^4)
\end{aligned}$$

$$\begin{aligned}
& d^2e^6 - 1024a^6b^7c^7d^7e + 64a^6b^7c^7d^7e - 1024a^9b^7c^4d^7e^7 \\
& - 4a^2b^9c^3d^7e - 4a^2b^{11}c^5d^5e^3 + 64a^3b^7c^4d^7e - 4a^3 \\
& *b^{10}c^4d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^3d^3e^5 + 1024a^5b^3 \\
& *c^6d^7e - 92a^5b^8c^3d^2e^6 - 3072a^7b^7c^6d^5e^3 - 384a^7b^5c^2 \\
& *d^7e - 3072a^8b^7c^5d^3e^5 + 1024a^8b^3c^3d^7e^7)) * ((27a^9b^9c^5 \\
& *d^6 - b^{11}c^4d^6 - b^{15}d^2e^4 - 9a^2b^{13}e^6 + 3840a^5b^7c^9d^6 \\
& - 9a^5c^5d^6 * (-4a^2c - b^2)^9)^{(1/2)} + 213a^3b^{11}c^6e^6 - 26880a^8b^7c^6 \\
& *e^6 + 3072a^6c^9d^5e + 35840a^8c^7d^5e^5 + 4b^{12}c^3d^5e + 4b^{14} \\
& *c^3d^3e^3 - 288a^2b^7c^6d^6 + 1504a^3b^5c^7d^6 - 3840a^4b^3c^8 \\
& *d^6 + 9a^2b^4e^6 * (-4a^2c - b^2)^9)^{(1/2)} - 2077a^4b^9c^2e^6 + 106 \\
& 56a^5b^7c^3e^6 - 30240a^6b^5c^4e^6 + 44800a^7b^3c^5e^6 + 25a^4 \\
& *c^2e^6 * (-4a^2c - b^2)^9)^{(1/2)} + b^2c^4d^6 * (-4a^2c - b^2)^9)^{(1/2)} + \\
& 22528a^7c^8d^3e^3 + b^6d^2e^4 * (-4a^2c - b^2)^9)^{(1/2)} - 6b^{13}c^2d^4 \\
& *e^2 + 6a^2b^{14}d^5e - 1471a^2b^9c^4d^4e^2 + 600a^2b^{10}c^3d^3e^3 \\
& + 180a^2b^{11}c^2d^2e^4 + 6976a^3b^7c^5d^4e^2 - 1032a^3b^8c^4 \\
& *d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456a^4b^5c^6d^4e^2 - 7168a^4b^6 \\
& *c^5d^3e^3 + 16896a^4b^7c^4d^2e^4 + 10240a^5b^3c^7d^4e^2 + 3 \\
& 7632a^5b^4c^6d^3e^3 - 47712a^5b^5c^5d^2e^4 - 59392a^6b^2c^7d^3 \\
& *e^3 + 60928a^6b^3c^6d^2e^4 - 41a^2c^4d^4e^2 * (-4a^2c - b^2)^9)^{(1/2)} \\
& - 39a^3c^3d^2e^4 * (-4a^2c - b^2)^9)^{(1/2)} + 6b^4c^2d^4e^2 * (-4a^2c \\
& - b^2)^9)^{(1/2)} - 6a^2b^5d^5e^5 * (-4a^2c - b^2)^9)^{(1/2)} - 106a^2b^{10} \\
& *c^4d^5e + 7a^2b^{13}c^3d^2e^4 - 128a^2b^{12}c^3d^2e^5 - 51a^3b^2c^3e^6 * (- \\
& 4a^2c - b^2)^9)^{(1/2)} + 150a^2b^{11}c^3d^4e^2 - 84a^2b^{12}c^2d^3e^3 + 1 \\
& 116a^2b^8c^5d^5e - 5824a^3b^6c^6d^5e + 1030a^3b^{10}c^2d^5e^5 + \\
& 15232a^4b^4c^7d^5e - 3492a^4b^8c^3d^5e^5 - 16896a^5b^2c^8d^5e^5 \\
& + 1344a^5b^6c^4d^5e^5 + 7424a^6b^7c^8d^4e^2 + 22400a^6b^4c^5d^5e^5 \\
& - 23296a^7b^7c^7d^2e^4 - 53760a^7b^2c^6d^5e^5 - 4b^3c^3d^5e^5 * (-4a^2c \\
& - b^2)^9)^{(1/2)} - 4b^5c^3d^3e^3 * (-4a^2c - b^2)^9)^{(1/2)} + 11a^2b^4 \\
& *c^3d^2e^4 * (-4a^2c - b^2)^9)^{(1/2)} + 20a^2b^3c^3d^5e^5 * (-4a^2c - b^2)^9)^{(1/2)} \\
& + 86a^3b^7c^2d^5e^5 * (-4a^2c - b^2)^9)^{(1/2)} - 42a^2b^2c^3d^4e^2 * (- \\
& 4a^2c - b^2)^9)^{(1/2)} + 12a^2b^3c^2d^3e^3 * (-4a^2c - b^2)^9)^{(1/2)} + \\
& 120a^2b^3c^3d^3e^3 * (-4a^2c - b^2)^9)^{(1/2)} + 34a^2b^7c^4d^5e^5 * (-4a^2c \\
& - b^2)^9)^{(1/2)} - 108a^2b^2c^2d^2e^4 * (-4a^2c - b^2)^9)^{(1/2))} / (32 * (a^7 \\
& *b^{12}e^8 + 4096a^9c^{10}d^8 + 4096a^{13}c^6e^8 - 24a^8b^{10}c^8e^8 - 4a^6 \\
& *b^{13}d^7e + a^3b^{12}c^4d^8 - 24a^4b^{10}c^5d^8 + 240a^5b^8c^6d^8 - 1280a^6 \\
& *b^6c^7d^8 + 3840a^7b^4c^8d^8 - 6144a^8b^2c^9d^8 + 240a^9b^8c^2e^8 - 1280a^{10} \\
& *b^6c^3e^8 + 3840a^{11}b^4c^4e^8 - 6144a^{12}b^2c^5e^8 + a^3b^{16}d^4e^4 - 4a^4 \\
& *b^{15}d^3e^5 + 6a^5b^{14}d^2e^6 + 16384a^{10}c^9d^6e^2 + 24576a^{11}c^8d^4e^4 + \\
& 16384a^{12}c^7d^2e^6 + 6a^3b^{14}c^2d^6e^2 - 140a^4b^{12}c^3d^6e^2 + 84a^4b^{13} \\
& *c^2d^5e^3 + 1344a^5b^{10}c^4d^6e^2 - 672a^5b^{11}c^3d^5e^3 - 42a^5b^{12}c^2 \\
& *d^4e^4 - 6720a^6b^8c^5d^6e^2 + 2240a^6b^9c^4d^5e^3 + 1456a^6b^{10}c^3 \\
& *d^4e^4 - 672a^6b^{11}c^2d^3e^5 + 17920a^7b^6c^6d^6e^2 - 10080a^7b^8c^4 \\
& *d^4e^4 + 2240a^7b^9c^3d^3e^5 + 1344a^7b^{10}c^2d^2e^6 - 21504a^8b^4c^7 \\
& *d^6e^2 - 21504a^8b^5c^6d^5e^3 + 32256a^8b^6c^5d^4e^4 - 6720a^8b^8c^3 \\
& *d^2e^6 + 57344a^9b^3c^7d^5e^3 - 46592a^9b^4c^6d^4e^4 - 21504a^9b^5c^5 \\
& *d^3e^5 + 17920a^9b^6c^4d^2e^6 + 12288a^{10}b^2c^7d^4e^4 + 57344a^{10}b^3 \\
& *c^6d^3e^5 - 21504a^{10}b^4c^5d^2e^6 + 96a^7b^{11}c^3d^7e - 16384a^9b^7c^9 \\
& *d^7e - 16384a^{12}b^7c^6d^7e - 4a^3b^{13}c^3d^7e - 4a^3b^{15}c^3d^5e^3 + \\
& 96a^4b^{11}c^4d^7e - 12a^4b^{14}c^3d^4e^4 - 960a^5b^9c^5d^7e + 84a^5b^{13} \\
& *c^3d^3e^5 + 5120a^6b^7c^6d^7e - 140a^6b^{12}c^3d^2e^6 - 15360a^7b^5c^7 \\
& *d^7e + 24576a^8b^3c^8d^7e - 960a^8b^9c^2d^7e^7 + 5120a^9b^7c^3d^7 \\
& *e^7 - 49152a^{10}b^7c^8d^5e^3 - 15360a^{10}b^5c^4d^7e^7 - 49152a^{11}b^7 \\
& *c^7d^3e^5 + 24576a^{11}b^3c^5d^7e^7))^{(1/2)} + (x * (626688a^{10}b^7c^8e^{15} \\
& - 784384a^{10}c^9d^7e^{14} + 208a^4b^{13}c^2e^{15} - 4880a^5b^{11}c^3e^{15} \\
& + 47312a^6b^9c^4e^{15} - 242176a^7b^7c^5e^{15} + 688640a^8b^5c^6e^{15} \\
& - 1028096a^9b^3c^7e^{15} + 18432a^4c^{15}d^{13}e^2 + 126976a^5c^{14}d^{11} \\
& *e^4 + 325632a^6c^{13}d^9e^6 + 139264a^7c^{12}d^7e^8 - 1067008a^8c^
\end{aligned}$$

$$\begin{aligned}
& 11*d^5*e^{10} - 1773568*a^9*c^{10}*d^3*e^{12} + 16*b^8*c^{11}*d^{13}*e^2 - 96*b^9*c^{10} \\
& *d^{12}*e^3 + 240*b^{10}*c^9*d^{11}*e^4 - 304*b^{11}*c^8*d^{10}*e^5 + 144*b^{12}*c^7*d^9 \\
& *e^6 + 144*b^{13}*c^6*d^8*e^7 - 304*b^{14}*c^5*d^7*e^8 + 240*b^{15}*c^4*d^6*e^9 \\
& - 96*b^{16}*c^3*d^5*e^{10} + 16*b^{17}*c^2*d^4*e^{11} + 3200*a^2*b^4*c^{13}*d^{13}*e^2 \\
& - 18432*a^2*b^5*c^{12}*d^{12}*e^3 + 41024*a^2*b^6*c^{11}*d^{11}*e^4 - 36352*a^2*b^7 \\
& *c^{10}*d^{10}*e^5 - 16208*a^2*b^8*c^9*d^9*e^6 + 74576*a^2*b^9*c^8*d^8*e^7 - 7 \\
& 8496*a^2*b^{10}*c^7*d^7*e^8 + 32064*a^2*b^{11}*c^6*d^6*e^9 + 6000*a^2*b^{12}*c^5*d^5 \\
& *e^{10} - 9264*a^2*b^{13}*c^4*d^4*e^{11} + 1472*a^2*b^{14}*c^3*d^3*e^{12} + 416*a^2 \\
& *b^{15}*c^2*d^2*e^{13} - 12800*a^3*b^2*c^{14}*d^{13}*e^2 + 73728*a^3*b^3*c^{13}*d^{12} \\
& *e^3 - 151296*a^3*b^4*c^{12}*d^{11}*e^4 + 78336*a^3*b^5*c^{11}*d^{10}*e^5 + 206688* \\
& a^3*b^6*c^{10}*d^9*e^6 - 436736*a^3*b^7*c^9*d^8*e^7 + 324224*a^3*b^8*c^8*d^7* \\
& e^8 + 992*a^3*b^9*c^7*d^6*e^9 - 158176*a^3*b^{10}*c^6*d^5*e^{10} + 77056*a^3*b^{11} \\
& *c^5*d^4*e^{11} + 6912*a^3*b^{12}*c^4*d^3*e^{12} - 8416*a^3*b^{13}*c^3*d^2*e^{13} + \\
& 162816*a^4*b^2*c^{13}*d^{11}*e^4 + 184320*a^4*b^3*c^{12}*d^{10}*e^5 - 916608*a^4*b^4 \\
& *c^{11}*d^9*e^6 + 1165824*a^4*b^5*c^{10}*d^8*e^7 - 314496*a^4*b^6*c^9*d^7*e^8 - \\
& 822272*a^4*b^7*c^8*d^6*e^9 + 919152*a^4*b^8*c^7*d^5*e^{10} - 175296*a^4*b^9 \\
& *c^6*d^4*e^{11} - 189328*a^4*b^{10}*c^5*d^3*e^{12} + 62064*a^4*b^{11}*c^4*d^2*e^{13} \\
& + 1290752*a^5*b^2*c^{12}*d^9*e^6 - 659456*a^5*b^3*c^{11}*d^8*e^7 - 1561088*a^5 \\
& *b^4*c^{10}*d^7*e^8 + 3240960*a^5*b^5*c^9*d^6*e^9 - 1964192*a^5*b^6*c^8*d^5*e^{10} \\
& - 683008*a^5*b^7*c^7*d^4*e^{11} + 1162304*a^5*b^8*c^6*d^3*e^{12} - 164112*a^5 \\
& *b^9*c^5*d^2*e^{13} + 3442688*a^6*b^2*c^{11}*d^7*e^8 - 3670016*a^6*b^3*c^{10}*d^6 \\
& *e^9 + 15232*a^6*b^4*c^9*d^5*e^{10} + 4230144*a^6*b^5*c^8*d^4*e^{11} - 305964 \\
& 8*a^6*b^6*c^7*d^3*e^{12} - 247296*a^6*b^7*c^6*d^2*e^{13} + 4010496*a^7*b^2*c^{10} \\
& *d^5*e^{10} - 6873088*a^7*b^3*c^9*d^4*e^{11} + 2822400*a^7*b^4*c^8*d^3*e^{12} + 2 \\
& 370048*a^7*b^5*c^7*d^2*e^{13} + 1178624*a^8*b^2*c^9*d^3*e^{12} - 4739072*a^8*b^3 \\
& *c^8*d^2*e^{13} - 352*a*b^6*c^{12}*d^{13}*e^2 + 2048*a*b^7*c^{11}*d^{12}*e^3 - 4800* \\
& a*b^8*c^{10}*d^{11}*e^4 + 5168*a*b^9*c^9*d^{10}*e^5 - 480*a*b^{10}*c^8*d^9*e^6 - 60 \\
& 00*a*b^{11}*c^7*d^8*e^7 + 8192*a*b^{12}*c^6*d^7*e^8 - 5040*a*b^{13}*c^5*d^6*e^9 + \\
& 1152*a*b^{14}*c^4*d^5*e^{10} + 240*a*b^{15}*c^3*d^4*e^{11} - 128*a*b^{16}*c^2*d^3*e^{12} \\
& - 512*a^3*b^{14}*c^2*d*e^{14} - 106496*a^4*b*c^{14}*d^{12}*e^3 + 11680*a^4*b^{12}* \\
& c^3*d*e^{14} - 675840*a^5*b*c^{13}*d^{10}*e^5 - 108288*a^5*b^{10}*c^4*d*e^{14} - 1601 \\
& 536*a^6*b*c^{12}*d^8*e^7 + 514768*a^6*b^8*c^5*d*e^{14} - 925696*a^7*b*c^{11}*d^6* \\
& e^9 - 1278304*a^7*b^6*c^6*d*e^{14} + 2457600*a^8*b*c^{10}*d^4*e^{11} + 1385600*a^8 \\
& *b^4*c^7*d*e^{14} + 2977792*a^9*b*c^9*d^2*e^{13} + 19968*a^9*b^2*c^8*d*e^{14})) / \\
& (8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^{10}*c^4*e^8 - 16*a^7*b^6*c^8*e^8 - 4 \\
& *a^5*b^9*d^8*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 \\
& - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^{12} \\
& *d^4*e^4 - 4*a^3*b^{11}*d^3*e^5 + 6*a^4*b^{10}*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + \\
& 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^{10}*c^2*d^6*e^2 - 92* \\
& a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 19 \\
& 2*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - \\
& 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 \\
& + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4 \\
& *e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5 \\
& *d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8* \\
& b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d^7*e^7 - 1024*a^9*b*c^4 \\
& *d^7*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^{11}*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e \\
& - 4*a^3*b^{10}*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 10 \\
& 24*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384* \\
& a^7*b^5*c^2*d^7*e - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d^7*e^7))) * ((27 \\
& *a*b^9*c^5*d^6 - b^{11}*c^4*d^6 - b^{15}*d^2*e^4 - 9*a^2*b^{13}*e^6 + 3840*a^5*b* \\
& c^9*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^{(1/2)} + 213*a^3*b^{11}*c^6*e^6 - 26880 \\
& *a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d^5*e^5 + 4*b^{12}*c^3*d^5* \\
& e + 4*b^{14}*c^3*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4 \\
& *b^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^{(1/2)} - 2077*a^4*b^9*c^2*e^6 \\
& + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 \\
& + 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^{(1/2)} - 6*b^{13} \\
& *c^2*d^4*e^2 + 6*a*b^{14}*d^5*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^{10}*c
\end{aligned}$$

$$\begin{aligned}
& \cdot 3d^3e^3 + 180a^2b^{11}c^2d^2e^4 + 6976a^3b^7c^5d^4e^2 - 1032a^3 \\
& \cdot b^8c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456a^4b^5c^6d^4e^2 - 7 \\
& 168a^4b^6c^5d^3e^3 + 16896a^4b^7c^4d^2e^4 + 10240a^5b^3c^7d^4 \\
& \cdot e^2 + 37632a^5b^4c^6d^3e^3 - 47712a^5b^5c^5d^2e^4 - 59392a^6b^2 \\
& \cdot c^7d^3e^3 + 60928a^6b^3c^6d^2e^4 - 41a^2c^4d^4e^2 \cdot (-4ac - b \\
& \cdot 2)^9)^{(1/2)} - 39a^3c^3d^2e^4 \cdot (-4ac - b^2)^9)^{(1/2)} + 6b^4c^2d^4 \\
& \cdot e^2 \cdot (-4ac - b^2)^9)^{(1/2)} - 6a^5b^5d^5 \cdot (-4ac - b^2)^9)^{(1/2)} - 106 \\
& \cdot a^b^{10}c^4d^5e + 7a^b^{13}c^d^2e^4 - 128a^2b^{12}c^d^5e^5 - 51a^3b^2 \\
& \cdot c^e^6 \cdot (-4ac - b^2)^9)^{(1/2)} + 150a^b^{11}c^3d^4e^2 - 84a^b^{12}c^2d^3 \\
& \cdot e^3 + 1116a^2b^8c^5d^5e - 5824a^3b^6c^6d^5e + 1030a^3b^{10}c^2 \\
& \cdot d^e^5 + 15232a^4b^4c^7d^5e - 3492a^4b^8c^3d^e^5 - 16896a^5b^2c^ \\
& \cdot 8d^5e + 1344a^5b^6c^4d^e^5 + 7424a^6b^c^8d^4e^2 + 22400a^6b^4c \\
& \cdot 5d^e^5 - 23296a^7b^c^7d^2e^4 - 53760a^7b^2c^6d^e^5 - 4b^3c^3d^ \\
& \cdot 5e \cdot (-4ac - b^2)^9)^{(1/2)} - 4b^5c^d^3e^3 \cdot (-4ac - b^2)^9)^{(1/2)} + 1 \\
& \cdot 1a^b^4c^d^2e^4 \cdot (-4ac - b^2)^9)^{(1/2)} + 20a^2b^3c^d^e^5 \cdot (-4ac - \\
& \cdot b^2)^9)^{(1/2)} + 86a^3b^c^2d^e^5 \cdot (-4ac - b^2)^9)^{(1/2)} - 42a^b^2c^3 \\
& \cdot d^4e^2 \cdot (-4ac - b^2)^9)^{(1/2)} + 12a^b^3c^2d^3e^3 \cdot (-4ac - b^2)^9)^{(1/2)} \\
& \cdot (1/2) + 120a^2b^c^3d^3e^3 \cdot (-4ac - b^2)^9)^{(1/2)} + 34a^b^c^4d^5e \cdot ( \\
& \cdot (-4ac - b^2)^9)^{(1/2)} - 108a^2b^2c^2d^2e^4 \cdot (-4ac - b^2)^9)^{(1/2)} \\
& / (32(a^7b^{12}e^8 + 4096a^9c^{10}d^8 + 4096a^{13}c^6e^8 - 24a^8b^{10}c^ \\
& \cdot e^8 - 4a^6b^{13}d^e^7 + a^3b^{12}c^4d^8 - 24a^4b^{10}c^5d^8 + 240a^5b \\
& \cdot 8c^6d^8 - 1280a^6b^6c^7d^8 + 3840a^7b^4c^8d^8 - 6144a^8b^2c^9 \\
& \cdot d^8 + 240a^9b^8c^2e^8 - 1280a^{10}b^6c^3e^8 + 3840a^{11}b^4c^4e^8 \\
& - 6144a^{12}b^2c^5e^8 + a^3b^{16}d^4e^4 - 4a^4b^{15}d^3e^5 + 6a^5b^{11} \\
& \cdot 4d^2e^6 + 16384a^{10}c^9d^6e^2 + 24576a^{11}c^8d^4e^4 + 16384a^{12}c^ \\
& \cdot 7d^2e^6 + 6a^3b^{14}c^2d^6e^2 - 140a^4b^{12}c^3d^6e^2 + 84a^4b^{13} \\
& \cdot c^2d^5e^3 + 1344a^5b^{10}c^4d^6e^2 - 672a^5b^{11}c^3d^5e^3 - 42a^ \\
& \cdot 5b^{12}c^2d^4e^4 - 6720a^6b^8c^5d^6e^2 + 2240a^6b^9c^4d^5e^3 + \\
& 1456a^6b^{10}c^3d^4e^4 - 672a^6b^{11}c^2d^3e^5 + 17920a^7b^6c^6d^ \\
& \cdot 6e^2 - 10080a^7b^8c^4d^4e^4 + 2240a^7b^9c^3d^3e^5 + 1344a^7b^{11} \\
& \cdot 0c^2d^2e^6 - 21504a^8b^4c^7d^6e^2 - 21504a^8b^5c^6d^5e^3 + 322 \\
& \cdot 56a^8b^6c^5d^4e^4 - 6720a^8b^8c^3d^2e^6 + 57344a^9b^3c^7d^5e \\
& \cdot 3 - 46592a^9b^4c^6d^4e^4 - 21504a^9b^5c^5d^3e^5 + 17920a^9b^6 \\
& \cdot c^4d^2e^6 + 12288a^{10}b^2c^7d^4e^4 + 57344a^{10}b^3c^6d^3e^5 - 215 \\
& \cdot 04a^{10}b^4c^5d^2e^6 + 96a^7b^{11}c^d^e^7 - 16384a^9b^c^9d^7e - 163 \\
& \cdot 84a^{12}b^c^6d^e^7 - 4a^3b^{13}c^3d^7e - 4a^3b^{15}c^d^5e^3 + 96a^4 \\
& \cdot b^{11}c^4d^7e - 12a^4b^{14}c^d^4e^4 - 960a^5b^9c^5d^7e + 84a^5b^{11} \\
& \cdot 3c^d^3e^5 + 5120a^6b^7c^6d^7e - 140a^6b^{12}c^d^2e^6 - 15360a^7b \\
& \cdot 5c^7d^7e + 24576a^8b^3c^8d^7e - 960a^8b^9c^2d^e^7 + 5120a^9b \\
& \cdot 7c^3d^e^7 - 49152a^{10}b^c^8d^5e^3 - 15360a^{10}b^5c^4d^e^7 - 49152 \\
& \cdot a^{11}b^c^7d^3e^5 + 24576a^{11}b^3c^5d^e^7))^{(1/2)} - (326912a^8c^9d^ \\
& \cdot e^{13} - 241664a^8b^c^8e^{14} - 48a^2b^{13}c^2e^{14} + 1264a^3b^{11}c^3e^{11} \\
& \cdot 4 - 13552a^4b^9c^4e^{14} + 75776a^5b^7c^5e^{14} - 232960a^6b^5c^6e^{11} \\
& \cdot 14 + 372736a^7b^3c^7e^{14} + 11520a^3c^{14}d^{11}e^3 + 78080a^4c^{13}d^9 \\
& \cdot e^5 + 197120a^5c^{12}d^7e^7 + 336384a^6c^{11}d^5e^9 + 532736a^7c^{10} \\
& \cdot d^3e^{11} - 40b^5c^{12}d^{12}e^2 + 216b^6c^{11}d^{11}e^3 - 464b^7c^{10}d^{10} \\
& \cdot e^4 + 496b^8c^9d^9e^5 - 264b^9c^8d^8e^6 + 56b^{10}c^7d^7e^7 - 16 \\
& \cdot b^{11}c^6d^6e^8 + 64b^{12}c^5d^5e^9 - 96b^{13}c^4d^4e^{10} + 64b^{14}c^ \\
& \cdot 3d^3e^{11} - 16b^{15}c^2d^2e^{12} + 1536a^2b^2c^{13}d^{11}e^3 + 14400a^2 \\
& \cdot b^3c^{12}d^{10}e^4 - 47152a^2b^4c^{11}d^9e^5 + 52144a^2b^5c^{10}d^8e^6 \\
& - 16272a^2b^6c^9d^7e^7 - 13040a^2b^7c^8d^6e^8 + 23488a^2b^8c^ \\
& \cdot 7d^5e^9 - 26384a^2b^9c^6d^4e^{10} + 13824a^2b^{10}c^5d^3e^{11} + 256 \\
& \cdot a^2b^{11}c^4d^2e^{12} + 125056a^3b^2c^{12}d^9e^5 - 36224a^3b^3c^{11}d^ \\
& \cdot 8e^6 - 126432a^3b^4c^{10}d^7e^7 + 144848a^3b^5c^9d^6e^8 - 114752a \\
& \cdot 3b^6c^8d^5e^9 + 125392a^3b^7c^7d^4e^{10} - 53248a^3b^8c^6d^3e^ \\
& \cdot 11 - 25264a^3b^9c^5d^2e^{12} + 474112a^4b^2c^{11}d^7e^7 - 191104a^4 \\
& \cdot b^3c^{10}d^6e^8 + 97184a^4b^4c^9d^5e^9 - 277000a^4b^5c^8d^4e^{10} \\
& + 56056a^4b^6c^7d^3e^{11} + 195584a^4b^7c^6d^2e^{12} + 236800a^5b^2 \\
& \cdot c^{10}d^5e^9 + 388032a^5b^3c^9d^4e^{10} + 159632a^5b^4c^8d^3e^{11} -
\end{aligned}$$

$$\begin{aligned}
& 670488a^5b^5c^7d^2e^{12} - 488960a^6b^2c^9d^3e^{11} + 1106496a^6b^3c^8d^2e^{12} + 64a^*b^{14}c^2d^*e^{13} + 448a^*b^3c^{13}d^{12}e^2 - 1968a^*b^4c^{12}d^{11}e^3 + 2504a^*b^5c^{11}d^{10}e^4 + 768a^*b^6c^{10}d^9e^5 - 4368a^*b^7c^9d^8e^6 + 3568a^*b^8c^8d^7e^7 - 520a^*b^9c^7d^6e^8 - 1728a^*b^{10}c^6d^5e^9 + 2528a^*b^{11}c^5d^4e^{10} - 1536a^*b^{12}c^4d^3e^{11} + 240a^*b^{13}c^3d^2e^{12} - 1152a^2b^*c^{14}d^{12}e^2 - 1600a^2b^{12}c^3d^*e^{13} - 67968a^3b^*c^{13}d^{10}e^4 + 15808a^3b^{10}c^4d^*e^{13} - 342272a^4b^*c^{12}d^8e^6 - 76928a^4b^8c^5d^*e^{13} - 569088a^5b^*c^{11}d^6e^8 + 179200a^5b^6c^6d^*e^{13} - 586368a^6b^*c^{10}d^4e^{10} - 113008a^6b^4c^7d^*e^{13} - 731008a^7b^*c^9d^2e^{12} - 244096a^7b^2c^8d^*e^{13})/(16*(a^6b^8e^8 + 256a^6c^8d^8 + 256a^{10}c^4e^8 - 16a^7b^6c^*e^8 - 4a^5b^9d^*e^7 + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6d^8 - 256a^5b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^{12}d^4e^4 - 4a^3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 + 1024a^7c^7d^6e^2 + 1536a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^{10}c^2d^6e^2 - 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4b^7c^3d^5e^3 - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 - 192a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c^4d^4e^4 - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 + 2048a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024a^6b^*c^7d^7e + 64a^6b^7c^*d^*e^7 - 1024a^9b^*c^4d^*e^7 - 4a^2b^9c^3d^7e - 4a^2b^{11}c^*d^5e^3 + 64a^3b^7c^4d^7e - 4a^3b^{10}c^*d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^*d^3e^5 + 1024a^5b^3c^6d^7e - 92a^5b^8c^*d^2e^6 - 3072a^7b^*c^6d^5e^3 - 384a^7b^5c^2d^*e^7 - 3072a^8b^*c^5d^3e^5 + 1024a^8b^3c^3d^*e^7))((27a^*b^9c^5d^6 - b^{11}c^4d^6 - b^{15}d^2e^4 - 9a^2b^{13}e^6 + 3840a^5b^*c^9d^6 - 9a^*c^5d^6*(-(4a^*c - b^2)^9)^{(1/2)} + 213a^3b^{11}c^*e^6 - 26880a^8b^*c^6e^6 + 3072a^6c^9d^5e + 35840a^8c^7d^*e^5 + 4b^{12}c^3d^5e + 4b^{14}c^*d^3e^3 - 288a^2b^7c^6d^6 + 1504a^3b^5c^7d^6 - 3840a^4b^3c^8d^6 + 9a^2b^4e^6*(-(4a^*c - b^2)^9)^{(1/2)} - 2077a^4b^9c^2e^6 + 10656a^5b^7c^3e^6 - 30240a^6b^5c^4e^6 + 44800a^7b^3c^5e^6 + 25a^4c^2e^6*(-(4a^*c - b^2)^9)^{(1/2)} + b^2c^4d^6*(-(4a^*c - b^2)^9)^{(1/2)} + 22528a^7c^8d^3e^3 + b^6d^2e^4*(-(4a^*c - b^2)^9)^{(1/2)} - 6b^{13}c^2d^4e^2 + 6a^*b^{14}d^*e^5 - 1471a^2b^9c^4d^4e^2 + 600a^2b^{10}c^3d^3e^3 + 180a^2b^{11}c^2d^2e^4 + 6976a^3b^7c^5d^4e^2 - 1032a^3b^8c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456a^4b^5c^6d^4e^2 - 7168a^4b^6c^5d^3e^3 + 16896a^4b^7c^4d^2e^4 + 10240a^5b^3c^7d^4e^2 + 37632a^5b^4c^6d^3e^3 - 47712a^5b^5c^5d^2e^4 - 59392a^6b^2c^7d^3e^3 + 60928a^6b^3c^6d^2e^4 - 41a^2c^4d^4e^2*(-(4a^*c - b^2)^9)^{(1/2)} - 39a^3c^3d^2e^4*(-(4a^*c - b^2)^9)^{(1/2)} + 6b^4c^2d^4e^2*(-(4a^*c - b^2)^9)^{(1/2)} - 6a^*b^5d^*e^5*(-(4a^*c - b^2)^9)^{(1/2)} - 106a^*b^{10}c^4d^5e + 7a^*b^{13}c^*d^2e^4 - 128a^2b^{12}c^*d^*e^5 - 51a^3b^2c^*e^6*(-(4a^*c - b^2)^9)^{(1/2)} + 150a^*b^{11}c^3d^4e^2 - 84a^*b^{12}c^2d^3e^3 + 1116a^2b^8c^5d^5e - 5824a^3b^6c^6d^5e + 1030a^3b^{10}c^2d^*e^5 + 15232a^4b^4c^7d^5e - 3492a^4b^8c^3d^*e^5 - 16896a^5b^2c^8d^5e + 1344a^5b^6c^4d^*e^5 + 7424a^6b^*c^8d^4e^2 + 22400a^6b^4c^5d^*e^5 - 23296a^7b^*c^7d^2e^4 - 53760a^7b^2c^6d^*e^5 - 4b^3c^3d^5e^**(-(4a^*c - b^2)^9)^{(1/2)} - 4b^5c^*d^3e^3*(-(4a^*c - b^2)^9)^{(1/2)} + 11a^*b^4c^*d^2e^4*(-(4a^*c - b^2)^9)^{(1/2)} + 20a^2b^3c^*d^*e^5*(-(4a^*c - b^2)^9)^{(1/2)} + 86a^3b^*c^2d^*e^5*(-(4a^*c - b^2)^9)^{(1/2)} - 42a^*b^2c^3d^4e^2*(-(4a^*c - b^2)^9)^{(1/2)} + 12a^*b^3c^2d^3e^3*(-(4a^*c - b^2)^9)^{(1/2)} + 120a^2b^*c^3d^3e^3*(-(4a^*c - b^2)^9)^{(1/2)} + 34a^*b^*c^4d^5e^**(-(4a^*c - b^2)^9)^{(1/2)} - 108a^2b^2c^2d^2e^4*(-(4a^*c - b^2)^9)^{(1/2)})/(32*(a^7b^{12}e^8 + 4096a^9c^{10}d^8 + 4096a^{13}c^6e^8 - 24a^8b^{10}c^*e^8 - 4a^6b^{13}d^*e^7 + a^3b^{12}c^4d^8 - 24a^4b^{10}c^5d^8 + 240a^5b^8c^6d^8 - 1280a^6b^6c^7d^8 + 3840a^7b^4c^8d^8 - 6144a^8b^2c^9d^8 + 240a^9b^8c^2e^8 - 1280a^{10}b^6c^3e^8 + 3840a^{11}b^4c^4e^8 - 6144a^{12}b^2c^5e^8 + a^3b^{16}d^4e^4 - 4a^4b^{15}d^3e^5 + 6a^5b^{14}d^2e^6 + 1638
\end{aligned}$$

$$\begin{aligned}
& 4*a^{10}*c^9*d^6*e^2 + 24576*a^{11}*c^8*d^4*e^4 + 16384*a^{12}*c^7*d^2*e^6 + 6*a^3*b^{14}*c^2*d^6*e^2 - 140*a^4*b^{12}*c^3*d^6*e^2 + 84*a^4*b^{13}*c^2*d^5*e^3 + 1 \\
& 344*a^5*b^{10}*c^4*d^6*e^2 - 672*a^5*b^{11}*c^3*d^5*e^3 - 42*a^5*b^{12}*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^{10}*c^3*d^4*e^4 - 672*a^6*b^{11}*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^{10}*c^2*d^2*e^6 - \\
& 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12 \\
& 288*a^{10}*b^2*c^7*d^4*e^4 + 57344*a^{10}*b^3*c^6*d^3*e^5 - 21504*a^{10}*b^4*c^5*d^2*e^6 + 96*a^7*b^{11}*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^{12}*b*c^6*d*e^7 - 4*a^3*b^{13}*c^3*d^7*e - 4*a^3*b^{15}*c*d^5*e^3 + 96*a^4*b^{11}*c^4*d^7*e - \\
& 12*a^4*b^{14}*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^{13}*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^{12}*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 4 \\
& 9152*a^{10}*b*c^8*d^5*e^3 - 15360*a^{10}*b^5*c^4*d*e^7 - 49152*a^{11}*b*c^7*d^3*e^5 + 24576*a^{11}*b^3*c^5*d*e^7))^{(1/2)} + (x*(22800*a^6*c^9*e^{13} + 36*a^2*b^8*c^5*e^{13} - 600*a^3*b^6*c^6*e^{13} + 4313*a^4*b^4*c^7*e^{13} - 15592*a^5*b^2*c^8*e^{13} + 1296*a^2*c^{13}*d^8*e^5 + 9792*a^3*c^{12}*d^6*e^7 + 30304*a^4*c^{11}*d^4*e^9 + 40512*a^5*c^{10}*d^2*e^{11} + 25*b^4*c^{11}*d^8*e^5 - 120*b^5*c^{10}*d^7*e^6 + 214*b^6*c^9*d^6*e^7 - 168*b^7*c^8*d^5*e^8 + 53*b^8*c^7*d^4*e^9 - 8*b^9*c^6*d^3*e^{10} + 4*b^{10}*c^5*d^2*e^{11} + 6336*a^2*b^2*c^{11}*d^6*e^7 + 3840*a^2*b^3*c^{10}*d^5*e^8 - 8506*a^2*b^4*c^9*d^4*e^9 + 1112*a^2*b^5*c^8*d^3*e^{10} + 1254*a^2*b^6*c^7*d^2*e^{11} + 22224*a^3*b^2*c^{10}*d^4*e^9 + 13824*a^3*b^3*c^9*d^3*e^{10} - 9516*a^3*b^4*c^8*d^2*e^{11} + 11712*a^4*b^2*c^9*d^2*e^{11} - 24*a*b^9*c^5*d*e^{12} - 41088*a^5*b*c^9*d*e^{12} - 360*a*b^2*c^{12}*d^8*e^5 + 1664*a*b^3*c^{11}*d^7*e^6 - 2604*a*b^4*c^{10}*d^6*e^7 + 1272*a*b^5*c^9*d^5*e^8 + 332*a*b^6*c^8*d^4*e^9 - 232*a*b^7*c^7*d^3*e^{10} - 48*a*b^8*c^6*d^2*e^{11} - 5760*a^2*b*c^{12}*d^7*e^6 + 416*a^2*b^7*c^6*d*e^{12} - 32128*a^3*b*c^{11}*d^5*e^8 - 4120*a^3*b^5*c^7*d*e^{12} - 63360*a^4*b*c^{10}*d^3*e^{10} + 21376*a^4*b^3*c^8*d*e^{12}))/((8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^{10}*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^{12}*d^4*e^4 - 4*a^3*b^{11}*d^3*e^5 + 6*a^4*b^{10}*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^{10}*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^{11}*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^{10}*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^{11}*c^4*d^6 - b^{15}*d^2*e^4 - 9*a^2*b^{13}*e^6 + 3840*a^5*b*c^9*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^{(1/2)} + 213*a^3*b^{11}*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^{12}*c^3*d^5*e + 4*b^{14}*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^{(1/2)} - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^{(1/2)} + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^{(1/2)} - 6*b^{13}*c^2*d^4*e^2 + 6*a*b^{14}*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^{10}*c^3*d^3*e^3 + 180*a^2*b^{11}*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c
\end{aligned}$$



$$\begin{aligned}
& ^7d^3e^3 + 60928a^6b^3c^6d^2e^4 - 41a^2c^4d^4e^2(-4ac - b^2)^9)^{(1/2)} - 39a^3c^3d^2e^4(-4ac - b^2)^9)^{(1/2)} + 6b^4c^2d^4e^2 \\
& *(-4ac - b^2)^9)^{(1/2)} - 6a^5b^5d^5e^5(-4ac - b^2)^9)^{(1/2)} - 106a^* \\
& b^{10}c^4d^5e + 7a^*b^{13}c^d^2e^4 - 128a^2b^{12}c^d^5e - 51a^3b^2c^e \\
& ^6(-4ac - b^2)^9)^{(1/2)} + 150a^*b^{11}c^3d^4e^2 - 84a^*b^{12}c^2d^3e^3 \\
& + 1116a^2b^8c^5d^5e - 5824a^3b^6c^6d^5e + 1030a^3b^{10}c^2d^e \\
& ^5 + 15232a^4b^4c^7d^5e - 3492a^4b^8c^3d^e^5 - 16896a^5b^2c^8d \\
& ^5e + 1344a^5b^6c^4d^e^5 + 7424a^6b^c^8d^4e^2 + 22400a^6b^4c^5* \\
& d^e^5 - 23296a^7b^c^7d^2e^4 - 53760a^7b^2c^6d^e^5 - 4b^3c^3d^5e \\
& *(-4ac - b^2)^9)^{(1/2)} - 4b^5c^d^3e^3(-4ac - b^2)^9)^{(1/2)} + 11a \\
& *b^4c^d^2e^4(-4ac - b^2)^9)^{(1/2)} + 20a^2b^3c^d^e^5(-4ac - b^2 \\
& )^9)^{(1/2)} + 86a^3b^c^2d^e^5(-4ac - b^2)^9)^{(1/2)} - 42a^*b^2c^3d^4 \\
& *e^2(-4ac - b^2)^9)^{(1/2)} + 12a^*b^3c^2d^3e^3(-4ac - b^2)^9)^{(1/ \\
& 2)} + 120a^2b^c^3d^3e^3(-4ac - b^2)^9)^{(1/2)} + 34a^*b^c^4d^5e^e(-4 \\
& *ac - b^2)^9)^{(1/2)} - 108a^2b^2c^2d^2e^4(-4ac - b^2)^9)^{(1/2)))/(3 \\
& 2*(a^7b^{12}e^8 + 4096a^9c^{10}d^8 + 4096a^{13}c^6e^8 - 24a^8b^{10}c^e^8 \\
& - 4a^6b^{13}d^e^7 + a^3b^{12}c^4d^8 - 24a^4b^{10}c^5d^8 + 240a^5b^8* \\
& c^6d^8 - 1280a^6b^6c^7d^8 + 3840a^7b^4c^8d^8 - 6144a^8b^2c^9d^ \\
& 8 + 240a^9b^8c^2e^8 - 1280a^{10}b^6c^3e^8 + 3840a^{11}b^4c^4e^8 - 6 \\
& 144a^{12}b^2c^5e^8 + a^3b^{16}d^4e^4 - 4a^4b^{15}d^3e^5 + 6a^5b^{14}d \\
& ^2e^6 + 16384a^{10}c^9d^6e^2 + 24576a^{11}c^8d^4e^4 + 16384a^{12}c^7d \\
& ^2e^6 + 6a^3b^{14}c^2d^6e^2 - 140a^4b^{12}c^3d^6e^2 + 84a^4b^{13}c^ \\
& 2d^5e^3 + 1344a^5b^{10}c^4d^6e^2 - 672a^5b^{11}c^3d^5e^3 - 42a^5b \\
& ^{12}c^2d^4e^4 - 6720a^6b^8c^5d^6e^2 + 2240a^6b^9c^4d^5e^3 + 145 \\
& 6a^6b^{10}c^3d^4e^4 - 672a^6b^{11}c^2d^3e^5 + 17920a^7b^6c^6d^6e \\
& ^2 - 10080a^7b^8c^4d^4e^4 + 2240a^7b^9c^3d^3e^5 + 1344a^7b^{10}c \\
& ^2d^2e^6 - 21504a^8b^4c^7d^6e^2 - 21504a^8b^5c^6d^5e^3 + 32256* \\
& a^8b^6c^5d^4e^4 - 6720a^8b^8c^3d^2e^6 + 57344a^9b^3c^7d^5e^3 \\
& - 46592a^9b^4c^6d^4e^4 - 21504a^9b^5c^5d^3e^5 + 17920a^9b^6c^4 \\
& *d^2e^6 + 12288a^{10}b^2c^7d^4e^4 + 57344a^{10}b^3c^6d^3e^5 - 21504* \\
& a^{10}b^4c^5d^2e^6 + 96a^7b^{11}c^d^e^7 - 16384a^9b^c^9d^7e - 16384* \\
& a^{12}b^c^6d^e^7 - 4a^3b^{13}c^3d^7e - 4a^3b^{15}c^d^5e^3 + 96a^4b^1 \\
& 1c^4d^7e - 12a^4b^{14}c^d^4e^4 - 960a^5b^9c^5d^7e + 84a^5b^{13}c \\
& *d^3e^5 + 5120a^6b^7c^6d^7e - 140a^6b^{12}c^d^2e^6 - 15360a^7b^5* \\
& c^7d^7e + 24576a^8b^3c^8d^7e - 960a^8b^9c^2d^e^7 + 5120a^9b^7* \\
& c^3d^e^7 - 49152a^{10}b^c^8d^5e^3 - 15360a^{10}b^5c^4d^e^7 - 49152a^1 \\
& 1b^c^7d^3e^5 + 24576a^{11}b^3c^5d^e^7))^{(1/2)}*i)/((2000a^4c^9e^{12} \\
& + 21a^2b^4c^7e^{12} - 520a^3b^2c^8e^{12} + 1296a^2c^{11}d^4e^8 + 432 \\
& 0a^3c^{10}d^2e^{10} + 25b^4c^9d^4e^8 - 60b^5c^8d^3e^9 + 35b^6c^7* \\
& d^2e^{10} + 192a^2b^2c^9d^2e^{10} - 112a^*b^5c^7d^e^{11} - 4480a^3b^c^9 \\
& *d^e^{11} - 360a^*b^2c^{10}d^4e^8 + 832a^*b^3c^9d^3e^9 - 362a^*b^4c^8d^ \\
& 2e^{10} - 2880a^2b^c^{10}d^3e^9 + 1440a^2b^3c^8d^e^{11})/(8*(a^6b^8e^8 \\
& + 256a^6c^8d^8 + 256a^{10}c^4e^8 - 16a^7b^6c^e^8 - 4a^5b^9d^e^7 \\
& + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6d^8 - 256a^5b^2c \\
& ^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^{12}d^4e^4 - 4a^ \\
& 3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 + 1024a^7c^7d^6e^2 + 1536a^8c^6d \\
& ^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^{10}c^2d^6e^2 - 92a^3b^8c^3d^6 \\
& *e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4b^7c^3d \\
& ^5e^3 - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^ \\
& 4d^5e^3 + 800a^5b^6c^3d^4e^4 - 192a^5b^7c^2d^3e^5 + 512a^6b^2 \\
& *c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c^4d^4e^4 - 128a^ \\
& 6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 + 20 \\
& 48a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 \\
& - 1024a^6b^c^7d^7e + 64a^6b^7c^d^e^7 - 1024a^9b^c^4d^e^7 - 4a^2 \\
& *b^9c^3d^7e - 4a^2b^{11}c^d^5e^3 + 64a^3b^7c^4d^7e - 4a^3b^{10}c \\
& *d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^d^3e^5 + 1024a^5b^3c^6* \\
& d^7e - 92a^5b^8c^d^2e^6 - 3072a^7b^c^6d^5e^3 - 384a^7b^5c^2d^e \\
& ^7 - 3072a^8b^c^5d^3e^5 + 1024a^8b^3c^3d^e^7)) + (((((1048576a^{13} \\
& c^8e^{16} + 256a^7b^{12}c^2e^{16} - 6144a^8b^{10}c^3e^{16} + 61440a^9b^8c
\end{aligned}$$

$$\begin{aligned}
& ^4e^{16} - 327680a^{10}b^6c^5e^{16} + 983040a^{11}b^4c^6e^{16} - 1572864a^{12}b^2c^7e^{16} - 196608a^6c^{15}d^{14}e^2 - 917504a^7c^{14}d^{12}e^4 - 5898 \\
& 24a^8c^{13}d^{10}e^6 + 3932160a^9c^{12}d^8e^8 + 10158080a^{10}c^{11}d^6e^{10} + 10616832a^{11}c^{10}d^4e^{12} + 5308416a^{12}c^9d^2e^{14} - 2816a^2b^8 \\
& c^{11}d^{14}e^2 + 22656a^2b^9c^{10}d^{13}e^3 - 78848a^2b^{10}c^9d^{12}e^4 + 154112a^2b^{11}c^8d^{11}e^5 - 182784a^2b^{12}c^7d^{10}e^6 + 130816a^2b^{13}c^6d^9e^7 \\
& - 50176a^2b^{14}c^5d^8e^8 + 4608a^2b^{15}c^4d^7e^9 + 3328a^2b^{16}c^3d^6e^{10} - 896a^2b^{17}c^2d^5e^{11} + 24576a^3b^6c^{12}d^{14}e^2 - 198656a^3b^7c^{11}d^{13}e^3 \\
& + 684544a^3b^8c^{10}d^{12}e^4 - 1291520a^3b^9c^9d^{11}e^5 + 1403776a^3b^{10}c^8d^{10}e^6 - 798336a^3b^{11}c^7d^9e^7 + 89856a^3b^{12}c^6d^8e^8 + 155136a^3b^{13}c^5d^7e^9 \\
& - 77440a^3b^{14}c^4d^6e^{10} + 5504a^3b^{15}c^3d^5e^{11} + 2560a^3b^{16}c^2d^4e^{12} - 106496a^4b^4c^{13}d^{14}e^2 + 864256a^4b^5c^{12}d^{13}e^3 \\
& - 2924544a^4b^6c^{11}d^{12}e^4 + 5181440a^4b^7c^{10}d^{11}e^5 - 4686080a^4b^8c^9d^{10}e^6 + 1045376a^4b^9c^8d^9e^7 + 1900544a^4b^{10}c^7d^8e^8 \\
& - 1732096a^4b^{11}c^6d^7e^9 + 390400a^4b^{12}c^5d^6e^{10} + 112000a^4b^{13}c^4d^5e^{11} - 40960a^4b^{14}c^3d^4e^{12} - 3840a^4b^{15}c^2d^3e^{13} + 229376a^5b^2c^{14}d^{14}e^2 \\
& - 1867776a^5b^3c^{13}d^{13}e^3 + 6078464a^5b^4c^{12}d^{12}e^4 - 9297920a^5b^5c^{11}d^{11}e^5 + 4055040a^5b^6c^{10}d^{10}e^6 + 7788544a^5b^7c^9d^9e^7 \\
& - 12657664a^5b^8c^8d^8e^8 + 6130176a^5b^9c^7d^7e^9 + 734080a^5b^{10}c^6d^6e^{10} - 1442560a^5b^{11}c^5d^5e^{11} + 168960a^5b^{12}c^4d^4e^{12} \\
& + 78080a^5b^{13}c^3d^3e^{13} + 3200a^5b^{14}c^2d^2e^{14} - 4587520a^6b^2c^{13}d^{12}e^4 + 3080192a^6b^3c^{12}d^{11}e^5 + 12001280a^6b^4c^{11}d^{10}e^6 \\
& - 31076352a^6b^5c^{10}d^9e^7 + 27475968a^6b^6c^9d^8e^8 - 2088960a^6b^7c^8d^7e^9 - 12205312a^6b^8c^7d^6e^{10} + 6043520a^6b^9c^6d^5e^{11} \\
& + 631808a^6b^{10}c^5d^4e^{12} - 610304a^6b^{11}c^4d^3e^{13} - 71936a^6b^{12}c^3d^2e^{14} - 21725184a^7b^2c^{12}d^{10}e^6 + 30801920a^7b^3c^{11}d^9e^7 - 80 \\
& 28160a^7b^4c^{10}d^8e^8 - 32260096a^7b^5c^9d^7e^9 + 37101568a^7b^6c^8d^6e^{10} - 7182336a^7b^7c^7d^5e^{11} - 7609856a^7b^8c^6d^4e^{12} \\
& + 2112256a^7b^9c^5d^3e^{13} + 661632a^7b^{10}c^4d^2e^{14} - 30146560a^8b^2c^{11}d^8e^8 + 55050240a^8b^3c^{10}d^7e^9 - 34365440a^8b^4c^9d^6e^{10} \\
& - 16429056a^8b^5c^8d^5e^{11} + 24600576a^8b^6c^7d^4e^{12} - 1683456a^8b^7c^6d^3e^{13} - 3151616a^8b^8c^5d^2e^{14} - 10977280a^9b^2c^{10}d^6e^{10} \\
& + 47022080a^9b^3c^9d^5e^{11} - 30621696a^9b^4c^8d^4e^{12} - 9232384a^9b^5c^7d^3e^{13} + 7970816a^9b^6c^6d^2e^{14} + 4325376a^{10}b^2c^9d^4e^{12} \\
& + 25493504a^{10}b^3c^8d^3e^{13} - 9117696a^{10}b^4c^7d^2e^{14} + 491520a^{11}b^2c^8d^2e^{14} - 4947968a^{12}b^2c^8d^2e^{15} + 128a^2b^{10}c^{10}d^{14}e^2 \\
& - 1024a^2b^{11}c^9d^{13}e^3 + 3584a^2b^{12}c^8d^{12}e^4 - 7168a^2b^{13}c^7d^{11}e^5 + 8960a^2b^{14}c^6d^{10}e^6 - 7168a^2b^{15}c^5d^9e^7 \\
& + 3584a^2b^{16}c^4d^8e^8 - 1024a^2b^{17}c^3d^7e^9 + 128a^2b^{18}c^2d^6e^{10} + 1605632a^6b^3c^{14}d^{13}e^3 - 1408a^6b^{13}c^2d^2e^{15} + 7 \\
& 012352a^7b^3c^{13}d^{11}e^5 + 33152a^7b^{11}c^3d^2e^{15} + 7045120a^8b^3c^{12}d^9e^7 - 324480a^8b^9c^4d^2e^{15} - 9830400a^9b^3c^{11}d^7e^9 + 1689600 \\
& a^9b^7c^5d^2e^{15} - 25722880a^{10}b^3c^{10}d^5e^{11} - 4935680a^{10}b^5c^6d^2e^{15} - 19202048a^{11}b^3c^9d^3e^{13} + 7667712a^{11}b^3c^7d^2e^{15}) / (16(a^6b^8e^8 \\
& + 256a^6c^8d^8 + 256a^{10}c^4e^8 - 16a^7b^6c^2e^8 - 4a^5b^9d^2e^7 + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6d^8 - 256 \\
& a^5b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^{12}d^4e^4 - 4a^3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 + 1024a^7c^7d^6e^2 + 1536 \\
& a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^{10}c^2d^6e^2 - 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4b^7c^3d^5e^3 \\
& - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 - 192a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 \\
& + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c^4d^4e^4 - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 + 2048a^7b^3c^4d^3e^5 \\
& - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024a^6b^3c^7d^7e + 64a^6b^7c^2d^2e^6 - 1024a^9b^3c^4d^2e^6 - 4a^2b^9c^3d^7e \\
& - 4a^2b^{11}c^2d^5e^3 + 64a^3b^7c^4d^7e - 4
\end{aligned}$$

$$\begin{aligned}
& a^3 b^{10} c^4 d^4 e^4 - 384 a^4 b^5 c^5 d^7 e + 52 a^4 b^9 c^3 d^3 e^5 + 1024 a^5 b^3 c^6 d^7 e - 92 a^5 b^8 c^2 d^2 e^6 - 3072 a^7 b^3 c^6 d^5 e^3 - 384 a^7 b^5 c^2 d^2 e^7 - 3072 a^8 b^3 c^5 d^3 e^5 + 1024 a^8 b^3 c^3 d^2 e^7) - (x * ((27 * a^9 c^5 d^6 - b^{11} c^4 d^6 - b^{15} d^2 e^4 - 9 a^2 b^{13} e^6 + 3840 a^5 b^3 c^9 d^6 - 9 a^3 c^5 d^6 * (-4 a^3 c - b^2)^9)^{(1/2)} + 213 a^3 b^{11} c^3 e^6 - 26880 a^8 b^3 c^6 e^6 + 3072 a^6 c^9 d^5 e + 35840 a^8 c^7 d^5 e^5 + 4 b^{12} c^3 d^5 e^5 + 4 b^{14} c^3 d^3 e^3 - 288 a^2 b^7 c^6 d^6 + 1504 a^3 b^5 c^7 d^6 - 3840 a^4 b^3 c^8 d^6 + 9 a^2 b^4 e^6 * (-4 a^3 c - b^2)^9)^{(1/2)} - 2077 a^4 b^9 c^2 e^6 + 10656 a^5 b^7 c^3 e^6 - 30240 a^6 b^5 c^4 e^6 + 44800 a^7 b^3 c^5 e^6 + 25 a^4 c^2 e^6 * (-4 a^3 c - b^2)^9)^{(1/2)} + b^2 c^4 d^6 * (-4 a^3 c - b^2)^9)^{(1/2)} + 22528 a^7 c^8 d^3 e^3 + b^6 d^2 e^4 * (-4 a^3 c - b^2)^9)^{(1/2)} - 6 b^{13} c^2 d^4 e^2 + 6 a^2 b^{14} d^5 e^5 - 1471 a^2 b^9 c^4 d^4 e^2 + 600 a^2 b^{10} c^3 d^3 e^3 + 180 a^2 b^{11} c^2 d^2 e^4 + 6976 a^3 b^7 c^5 d^4 e^2 - 1032 a^3 b^8 c^4 d^3 e^3 - 2871 a^3 b^9 c^3 d^2 e^4 - 15456 a^4 b^5 c^6 d^4 e^2 - 7168 a^4 b^6 c^5 d^3 e^3 + 16896 a^4 b^7 c^4 d^2 e^4 + 10240 a^5 b^3 c^7 d^4 e^2 + 37632 a^5 b^4 c^6 d^3 e^3 - 47712 a^5 b^5 c^5 d^2 e^4 - 59392 a^6 b^2 c^7 d^3 e^3 + 60928 a^6 b^3 c^6 d^2 e^4 - 41 a^2 c^4 d^4 e^2 * (-4 a^3 c - b^2)^9)^{(1/2)} - 39 a^3 c^3 d^2 e^4 * (-4 a^3 c - b^2)^9)^{(1/2)} + 6 b^4 c^2 d^4 e^2 * (-4 a^3 c - b^2)^9)^{(1/2)} - 6 a^2 b^5 d^5 e^5 * (-4 a^3 c - b^2)^9)^{(1/2)} - 106 a^2 b^{10} c^4 d^5 e^5 + 7 a^2 b^{13} c^3 d^2 e^4 - 128 a^2 b^{12} c^3 d^2 e^5 - 51 a^3 b^2 c^3 e^6 * (-4 a^3 c - b^2)^9)^{(1/2)} + 150 a^2 b^{11} c^3 d^4 e^2 - 84 a^2 b^{12} c^2 d^3 e^3 + 1116 a^2 b^8 c^5 d^5 e^5 - 5824 a^3 b^6 c^6 d^5 e^5 + 1030 a^3 b^{10} c^2 d^5 e^5 + 15232 a^4 b^4 c^7 d^5 e^5 - 3492 a^4 b^8 c^3 d^5 e^5 - 16896 a^5 b^2 c^8 d^5 e^5 + 1344 a^5 b^6 c^4 d^5 e^5 + 7424 a^6 b^3 c^8 d^4 e^2 + 22400 a^6 b^4 c^5 d^5 e^5 - 23296 a^7 b^3 c^7 d^2 e^4 - 53760 a^7 b^2 c^6 d^5 e^5 - 4 b^3 c^3 d^5 e^5 * (-4 a^3 c - b^2)^9)^{(1/2)} - 4 b^5 c^3 d^3 e^3 * (-4 a^3 c - b^2)^9)^{(1/2)} + 11 a^2 b^4 c^3 d^2 e^4 * (-4 a^3 c - b^2)^9)^{(1/2)} + 20 a^2 b^3 c^3 d^2 e^5 * (-4 a^3 c - b^2)^9)^{(1/2)} + 86 a^3 b^3 c^2 d^2 e^5 * (-4 a^3 c - b^2)^9)^{(1/2)} - 42 a^2 b^2 c^3 d^4 e^2 * (-4 a^3 c - b^2)^9)^{(1/2)} + 12 a^2 b^3 c^2 d^3 e^3 * (-4 a^3 c - b^2)^9)^{(1/2)} + 120 a^2 b^3 c^3 d^3 e^3 * (-4 a^3 c - b^2)^9)^{(1/2)} + 34 a^2 b^3 c^4 d^5 e^5 * (-4 a^3 c - b^2)^9)^{(1/2)} - 108 a^2 b^2 c^2 d^2 e^4 * (-4 a^3 c - b^2)^9)^{(1/2)}) / (32 * (a^7 b^{12} e^8 + 4096 a^9 c^{10} d^8 + 4096 a^{13} c^6 e^8 - 24 a^8 b^{10} c^8 e^8 - 4 a^6 b^{13} d^8 e^7 + a^3 b^{12} c^4 d^8 - 24 a^4 b^{10} c^5 d^8 + 240 a^5 b^8 c^6 d^8 - 1280 a^6 b^6 c^7 d^8 + 3840 a^7 b^4 c^8 d^8 - 6144 a^8 b^2 c^9 d^8 + 240 a^9 b^8 c^2 e^8 - 1280 a^{10} b^6 c^3 e^8 + 3840 a^{11} b^4 c^4 e^8 - 6144 a^{12} b^2 c^5 e^8 + a^3 b^{16} d^4 e^4 - 4 a^4 b^{15} d^3 e^5 + 6 a^5 b^{14} d^2 e^6 + 16384 a^{10} c^9 d^6 e^2 + 24576 a^{11} c^8 d^4 e^4 + 16384 a^{12} c^7 d^2 e^6 + 6 a^3 b^{14} c^2 d^6 e^2 - 140 a^4 b^{12} c^3 d^6 e^2 + 84 a^4 b^{13} c^2 d^5 e^3 + 1344 a^5 b^{10} c^4 d^6 e^2 - 672 a^5 b^{11} c^3 d^5 e^3 - 42 a^5 b^{12} c^2 d^4 e^4 - 6720 a^6 b^8 c^5 d^6 e^2 + 2240 a^6 b^9 c^4 d^5 e^3 + 1456 a^6 b^{10} c^3 d^4 e^4 - 672 a^6 b^{11} c^2 d^3 e^5 + 17920 a^7 b^6 c^6 d^6 e^2 - 10080 a^7 b^8 c^4 d^4 e^4 + 2240 a^7 b^9 c^3 d^3 e^5 + 1344 a^7 b^{10} c^2 d^2 e^6 - 21504 a^8 b^4 c^7 d^6 e^2 - 21504 a^8 b^5 c^6 d^5 e^3 + 32256 a^8 b^6 c^5 d^4 e^4 - 6720 a^8 b^8 c^3 d^2 e^6 + 57344 a^9 b^3 c^7 d^5 e^3 - 46592 a^9 b^4 c^6 d^4 e^4 - 21504 a^9 b^5 c^5 d^3 e^5 + 17920 a^9 b^6 c^4 d^2 e^6 + 12288 a^{10} b^2 c^7 d^4 e^4 + 57344 a^{10} b^3 c^6 d^3 e^5 - 21504 a^{10} b^4 c^5 d^2 e^6 + 96 a^7 b^{11} c^3 d^7 e - 16384 a^9 b^3 c^9 d^7 e - 16384 a^{12} b^3 c^6 d^5 e^7 - 4 a^3 b^{13} c^3 d^7 e - 4 a^3 b^{15} c^3 d^5 e^3 + 96 a^4 b^{11} c^4 d^7 e - 12 a^4 b^{14} c^3 d^4 e^4 - 960 a^5 b^9 c^5 d^7 e + 84 a^5 b^{13} c^3 d^3 e^5 + 5120 a^6 b^7 c^6 d^7 e - 140 a^6 b^{12} c^3 d^2 e^6 - 15360 a^7 b^5 c^7 d^7 e + 24576 a^8 b^3 c^8 d^7 e - 960 a^8 b^9 c^2 d^5 e^7 + 5120 a^9 b^7 c^3 d^5 e^7 - 49152 a^{10} b^3 c^8 d^5 e^3 - 15360 a^{10} b^5 c^4 d^5 e^7 - 49152 a^{11} b^3 c^7 d^3 e^5 + 24576 a^{11} b^3 c^5 d^5 e^7))^{(1/2)} * (1048576 a^{15} c^8 e^{17} + 256 a^9 b^{12} c^2 e^{17} - 6144 a^{10} b^{10} c^3 e^{17} + 61440 a^{11} b^8 c^4 e^{17} - 327680 a^{12} b^6 c^5 e^{17} + 983040 a^{13} b^4 c^6 e^{17} - 1572864 a^{14} b^2 c^7 e^{17} - 1048576 a^8 c^{15} d^{14} e^3 - 5242880 a^9 c^{14} d^{12} e^5 - 9437184 a^{10} c^{13} d^{10} e^7 - 5242880 a^{11} c^{12} d^8 e^9 + 5242880 a^{12} c^{11} d^6 e^{11} + 9437184 a^{13} c^{10} d^4 e^{13} + 5242880 a^{14} c^9 d^2 e^{15} + 256 a^2 b^{11} c^{10} d^{15} e^2 - 2048 a^2 b^{12} c^9 d^{14} e^3 + 7168 a^2 b^{13} c^8 d^{13} e^4 - 14
\end{aligned}$$

$$\begin{aligned}
& 336a^2b^{14}c^7d^{12}e^5 + 17920a^2b^{15}c^6d^{11}e^6 - 14336a^2b^{16}c^5d^{10}e^7 + 7168a^2b^{17}c^4d^9e^8 - 2048a^2b^{18}c^3d^8e^9 + 256a^2b^{19}c^2d^7e^{10} - 5120a^3b^9c^{11}d^{15}e^2 + 41984a^3b^{10}c^{10}d^{14}e^3 - 148736a^3b^{11}c^9d^{13}e^4 + 296192a^3b^{12}c^8d^{12}e^5 - 359680a^3b^{13}c^7d^{11}e^6 + 267520a^3b^{14}c^6d^{10}e^7 - 112384a^3b^{15}c^5d^9e^8 + 18176a^3b^{16}c^4d^8e^9 + 3328a^3b^{17}c^3d^7e^{10} - 1280a^3b^{18}c^2d^6e^{11} + 40960a^4b^7c^{12}d^{15}e^2 - 348160a^4b^8c^{11}d^{14}e^3 + 1254400a^4b^9c^{10}d^{13}e^4 - 2478080a^4b^{10}c^9d^{12}e^5 + 2867456a^4b^{11}c^8d^{11}e^6 - 1862144a^4b^{12}c^7d^{10}e^7 + 490240a^4b^{13}c^6d^9e^8 + 128000a^4b^{14}c^5d^8e^9 - 108800a^4b^{15}c^4d^7e^{10} + 13824a^4b^{16}c^3d^6e^{11} + 2304a^4b^{17}c^2d^5e^{12} - 163840a^5b^5c^{13}d^{15}e^2 + 1474560a^5b^6c^{12}d^{14}e^3 - 5447680a^5b^7c^{11}d^{13}e^4 + 10588160a^5b^8c^{10}d^{12}e^5 - 11166720a^5b^9c^9d^{11}e^6 + 5159936a^5b^{10}c^8d^{10}e^7 + 1073920a^5b^{11}c^7d^9e^8 - 2279680a^5b^{12}c^6d^8e^9 + 770560a^5b^{13}c^5d^7e^{10} + 33280a^5b^{14}c^4d^6e^{11} - 41216a^5b^{15}c^3d^5e^{12} - 1280a^5b^{16}c^2d^4e^{13} + 327680a^6b^3c^{14}d^{15}e^2 - 3276800a^6b^4c^{13}d^{14}e^3 + 12615680a^6b^5c^{12}d^{13}e^4 - 23592960a^6b^6c^{11}d^{12}e^5 + 19701760a^6b^7c^{10}d^{11}e^6 + 1372160a^6b^8c^9d^{10}e^7 - 15846400a^6b^9c^8d^9e^8 + 10864640a^6b^{10}c^7d^8e^9 - 1352960a^6b^{11}c^6d^7e^{10} - 1111040a^6b^{12}c^5d^6e^{11} + 273920a^6b^{13}c^4d^5e^{12} + 25600a^6b^{14}c^3d^4e^{13} - 1280a^6b^{15}c^2d^3e^{14} + 3407872a^7b^2c^{14}d^{14}e^3 - 14221312a^7b^3c^{13}d^{13}e^4 + 23527424a^7b^4c^{12}d^{12}e^5 - 3768320a^7b^5c^{11}d^{11}e^6 - 38895616a^7b^6c^{10}d^{10}e^7 + 50126848a^7b^7c^9d^9e^8 - 18362368a^7b^8c^8d^8e^9 - 6831104a^7b^9c^7d^7e^{10} + 6200320a^7b^{10}c^6d^6e^{11} - 726784a^7b^{11}c^5d^5e^{12} - 228608a^7b^{12}c^4d^4e^{13} + 31488a^7b^{13}c^3d^3e^{14} + 2304a^7b^{14}c^2d^2e^{15} - 3145728a^8b^2c^{13}d^{12}e^5 - 31129600a^8b^3c^{12}d^{11}e^6 + 74711040a^8b^4c^{11}d^{10}e^7 - 55476224a^8b^5c^{10}d^9e^8 - 11075584a^8b^6c^9d^8e^9 + 35381248a^8b^7c^8d^7e^{10} - 14479360a^8b^8c^7d^6e^{11} - 168960a^8b^9c^6d^5e^{12} + 1286144a^8b^{10}c^5d^4e^{13} - 302336a^8b^{11}c^4d^3e^{14} - 55808a^8b^{12}c^3d^2e^{15} - 36962304a^9b^2c^{12}d^{10}e^7 - 9502720a^9b^3c^{11}d^9e^8 + 67174400a^9b^4c^{10}d^8e^9 - 54886400a^9b^5c^9d^7e^{10} + 11239424a^9b^6c^8d^6e^{11} + 5545984a^9b^7c^7d^5e^{12} - 5263360a^9b^8c^6d^4e^{13} + 1356800a^9b^9c^5d^3e^{14} + 558080a^9b^{10}c^4d^2e^{15} - 49807360a^{10}b^2c^{11}d^8e^9 + 19333120a^{10}b^3c^{10}d^7e^{10} + 7208960a^{10}b^4c^9d^6e^{11} - 14974976a^{10}b^5c^8d^5e^{12} + 15073280a^{10}b^6c^7d^4e^{13} - 2170880a^{10}b^7c^6d^3e^{14} - 2928640a^{10}b^8c^5d^2e^{15} - 11796480a^{11}b^2c^{10}d^6e^{11} + 23920640a^{11}b^3c^9d^5e^{12} - 24576000a^{11}b^4c^8d^4e^{13} - 4096000a^{11}b^5c^7d^3e^{14} + 8355840a^{11}b^6c^6d^2e^{15} + 12582912a^{12}b^2c^9d^4e^{13} + 19857408a^{12}b^3c^8d^3e^{14} - 11534336a^{12}b^4c^7d^2e^{15} + 3407872a^{13}b^2c^8d^2e^{15} - 5505024a^{14}b^3c^8d^2e^{16} - 262144a^7b^3c^{15}d^{15}e^2 + 5505024a^8b^3c^{14}d^{13}e^4 - 1280a^8b^{13}c^2d^2e^{16} + 25952256a^9b^3c^{13}d^{11}e^6 + 30976a^9b^{11}c^3d^2e^{16} + 38010880a^{10}b^3c^{12}d^9e^8 - 312320a^{10}b^9c^4d^2e^{16} + 11796480a^{11}b^3c^{11}d^7e^{10} + 1679360a^{11}b^7c^5d^2e^{16} - 21233664a^{12}b^3c^{10}d^5e^{12} - 5079040a^{12}b^5c^6d^2e^{16} - 20709376a^{13}b^3c^9d^3e^{14} + 8192000a^{13}b^3c^7d^2e^{16})) / (8(a^6b^8e^8 + 256a^6c^8d^8 + 256a^{10}c^4e^8 - 16a^7b^6c^8e^8 - 4a^5b^9d^2e^7 + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6d^8 - 256a^5b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^{12}d^4e^4 - 4a^3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 + 1024a^7c^7d^6e^2 + 1536a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^{10}c^2d^6e^2 - 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4b^7c^3d^5e^3 - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 - 192a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c^4d^4e^4 - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 + 2048a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 102
\end{aligned}$$

$$\begin{aligned}
&4a^6b^7c^7d^7e + 64a^6b^7c^7d^7e^2 - 1024a^9b^7c^4d^7e^2 - 4a^2b^9c^3d^7e^2 - 4a^2b^11c^7d^5e^3 + 64a^3b^7c^4d^7e^2 - 4a^3b^10c^7d^4e^4 - 384a^4b^5c^5d^7e^2 + 52a^4b^9c^7d^3e^5 + 1024a^5b^3c^6d^7e^2 - 92a^5b^8c^7d^2e^6 - 3072a^7b^7c^6d^5e^3 - 384a^7b^5c^2d^7e^2 - 3072a^8b^7c^5d^3e^5 + 1024a^8b^3c^3d^7e^2) * ((27a^9b^9c^5d^6 - b^11c^4d^6 - b^15d^2e^4 - 9a^2b^13e^6 + 3840a^5b^7c^9d^6 - 9a^5c^5d^6 * (-4ac - b^2)^9)^{1/2} + 213a^3b^11c^7e^6 - 26880a^8b^7c^6e^6 + 3072a^6c^9d^5e + 35840a^8c^7d^5e^2 + 4b^12c^3d^5e^2 + 4b^14c^3d^3e^3 - 288a^2b^7c^6d^6 + 1504a^3b^5c^7d^6 - 3840a^4b^3c^8d^6 + 9a^2b^4e^6 * (-4ac - b^2)^9)^{1/2} - 2077a^4b^9c^2e^6 + 10656a^5b^7c^3e^6 - 30240a^6b^5c^4e^6 + 44800a^7b^3c^5e^6 + 25a^4c^2e^6 * (-4ac - b^2)^9)^{1/2} + b^2c^4d^6 * (-4ac - b^2)^9)^{1/2} + 22528a^7c^8d^3e^3 + b^6d^2e^4 * (-4ac - b^2)^9)^{1/2} - 6b^13c^2d^4e^2 + 6ab^14d^5e^5 - 1471a^2b^9c^4d^4e^2 + 600a^2b^10c^3d^3e^3 + 180a^2b^11c^2d^2e^4 + 6976a^3b^7c^5d^4e^2 - 1032a^3b^8c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456a^4b^5c^6d^4e^2 - 7168a^4b^6c^5d^3e^3 + 16896a^4b^7c^4d^2e^4 + 10240a^5b^3c^7d^4e^2 + 37632a^5b^4c^6d^3e^3 - 47712a^5b^5c^5d^2e^4 - 59392a^6b^2c^7d^3e^3 + 60928a^6b^3c^6d^2e^4 - 41a^2c^4d^4e^2 * (-4ac - b^2)^9)^{1/2} - 39a^3c^3d^2e^4 * (-4ac - b^2)^9)^{1/2} + 6b^4c^2d^4e^2 * (-4ac - b^2)^9)^{1/2} - 6ab^5d^5e^5 * (-4ac - b^2)^9)^{1/2} - 106ab^10c^4d^5e^2 + 7ab^13c^3d^2e^4 - 128a^2b^12c^7d^5e^5 - 51a^3b^2c^6e^6 * (-4ac - b^2)^9)^{1/2} + 150ab^11c^3d^4e^2 - 84ab^12c^2d^3e^3 + 1116a^2b^8c^5d^5e^2 - 5824a^3b^6c^6d^5e^2 + 1030a^3b^10c^2d^5e^2 + 15232a^4b^4c^7d^5e^2 - 3492a^4b^8c^3d^5e^2 - 16896a^5b^2c^8d^5e^2 + 1344a^5b^6c^4d^5e^2 + 7424a^6b^7c^8d^4e^2 + 22400a^6b^4c^5d^5e^2 - 23296a^7b^7c^7d^2e^4 - 53760a^7b^2c^6d^5e^2 - 4b^3c^3d^5e^2 * (-4ac - b^2)^9)^{1/2} - 4b^5c^3d^3e^3 * (-4ac - b^2)^9)^{1/2} + 11ab^4c^3d^2e^4 * (-4ac - b^2)^9)^{1/2} + 20a^2b^3c^7d^5e^2 * (-4ac - b^2)^9)^{1/2} + 86a^3b^7c^2d^5e^2 * (-4ac - b^2)^9)^{1/2} - 42ab^2c^3d^4e^2 * (-4ac - b^2)^9)^{1/2} + 12ab^3c^2d^3e^3 * (-4ac - b^2)^9)^{1/2} + 120a^2b^7c^3d^3e^3 * (-4ac - b^2)^9)^{1/2} + 34ab^7c^4d^5e^2 * (-4ac - b^2)^9)^{1/2} - 108a^2b^2c^2d^2e^4 * (-4ac - b^2)^9)^{1/2}) / (32(a^7b^12e^8 + 4096a^9c^10d^8 + 4096a^13c^6e^8 - 24a^8b^10c^7e^8 - 4a^6b^13d^7e^7 + a^3b^12c^4d^8 - 24a^4b^10c^5d^8 + 240a^5b^8c^6d^8 - 1280a^6b^6c^7d^8 + 3840a^7b^4c^8d^8 - 6144a^8b^2c^9d^8 + 240a^9b^8c^2e^8 - 1280a^10b^6c^3e^8 + 3840a^11b^4c^4e^8 - 6144a^12b^2c^5e^8 + a^3b^16d^4e^4 - 4a^4b^15d^3e^5 + 6a^5b^14d^2e^6 + 16384a^10c^9d^6e^2 + 24576a^11c^8d^4e^4 + 16384a^12c^7d^2e^6 + 6a^3b^14c^2d^6e^2 - 140a^4b^12c^3d^6e^2 + 84a^4b^13c^2d^5e^3 + 1344a^5b^10c^4d^6e^2 - 672a^5b^11c^3d^5e^3 - 42a^5b^12c^2d^4e^4 - 6720a^6b^8c^5d^6e^2 + 2240a^6b^9c^4d^5e^3 + 1456a^6b^10c^3d^4e^4 - 672a^6b^11c^2d^3e^5 + 17920a^7b^6c^6d^6e^2 - 10080a^7b^8c^4d^4e^4 + 2240a^7b^9c^3d^3e^5 + 1344a^7b^10c^2d^2e^6 - 21504a^8b^4c^7d^6e^2 - 21504a^8b^5c^6d^5e^3 + 32256a^8b^6c^5d^4e^4 - 6720a^8b^8c^3d^2e^6 + 57344a^9b^3c^7d^5e^3 - 46592a^9b^4c^6d^4e^4 - 21504a^9b^5c^5d^3e^5 + 17920a^9b^6c^4d^2e^6 + 12288a^10b^2c^7d^4e^4 + 57344a^10b^3c^6d^3e^5 - 21504a^10b^4c^5d^2e^6 + 96a^7b^11c^7d^7e^2 - 16384a^9b^7c^9d^7e^2 - 16384a^12b^7c^6d^7e^2 - 4a^3b^13c^3d^7e^2 - 4a^3b^15c^3d^5e^3 + 96a^4b^11c^4d^7e^2 - 12a^4b^14c^3d^4e^4 - 960a^5b^9c^5d^7e^2 + 84a^5b^13c^3d^3e^5 + 5120a^6b^7c^6d^7e^2 - 140a^6b^12c^2d^2e^6 - 15360a^7b^5c^7d^7e^2 + 24576a^8b^3c^8d^7e^2 - 960a^8b^9c^2d^7e^2 + 5120a^9b^7c^3d^7e^2 - 49152a^10b^7c^8d^5e^3 - 15360a^10b^5c^4d^7e^2 - 49152a^11b^7c^7d^3e^5 + 24576a^11b^3c^5d^7e^2))^{1/2} - (x*(626688a^10b^8c^8e^15 - 784384a^10c^9d^7e^14 + 208a^4b^13c^2e^15 - 4880a^5b^11c^3e^15 + 47312a^6b^9c^4e^15 - 242176a^7b^7c^5e^15 + 688640a^8b^5c^6e^15 - 1028096a^9b^3c^7e^15 + 18432a^4c^15d^13e^2 + 126976a^5c^14d^11e^4 + 325632a^6c^13d^9e^6 + 139264a^7c^12d^7e^8 - 1067008a^8c^11d^5e^10 -
\end{aligned}$$

$$\begin{aligned}
& 1773568*a^9*c^{10}*d^3*e^{12} + 16*b^8*c^{11}*d^{13}*e^2 - 96*b^9*c^{10}*d^{12}*e^3 + \\
& 240*b^{10}*c^9*d^{11}*e^4 - 304*b^{11}*c^8*d^{10}*e^5 + 144*b^{12}*c^7*d^9*e^6 + 144* \\
& b^{13}*c^6*d^8*e^7 - 304*b^{14}*c^5*d^7*e^8 + 240*b^{15}*c^4*d^6*e^9 - 96*b^{16}*c^3* \\
& d^5*e^{10} + 16*b^{17}*c^2*d^4*e^{11} + 3200*a^2*b^4*c^{13}*d^{13}*e^2 - 18432*a^2* \\
& b^5*c^{12}*d^{12}*e^3 + 41024*a^2*b^6*c^{11}*d^{11}*e^4 - 36352*a^2*b^7*c^{10}*d^{10}*e \\
& ^5 - 16208*a^2*b^8*c^9*d^9*e^6 + 74576*a^2*b^9*c^8*d^8*e^7 - 78496*a^2*b^{10} \\
& *c^7*d^7*e^8 + 32064*a^2*b^{11}*c^6*d^6*e^9 + 6000*a^2*b^{12}*c^5*d^5*e^{10} - 92 \\
& 64*a^2*b^{13}*c^4*d^4*e^{11} + 1472*a^2*b^{14}*c^3*d^3*e^{12} + 416*a^2*b^{15}*c^2*d^2* \\
& e^{13} - 12800*a^3*b^2*c^{14}*d^{13}*e^2 + 73728*a^3*b^3*c^{13}*d^{12}*e^3 - 151296 \\
& *a^3*b^4*c^{12}*d^{11}*e^4 + 78336*a^3*b^5*c^{11}*d^{10}*e^5 + 206688*a^3*b^6*c^{10}* \\
& d^9*e^6 - 436736*a^3*b^7*c^9*d^8*e^7 + 324224*a^3*b^8*c^8*d^7*e^8 + 992*a^3 \\
& *b^9*c^7*d^6*e^9 - 158176*a^3*b^{10}*c^6*d^5*e^{10} + 77056*a^3*b^{11}*c^5*d^4*e^{11} \\
& + 6912*a^3*b^{12}*c^4*d^3*e^{12} - 8416*a^3*b^{13}*c^3*d^2*e^{13} + 162816*a^4*b \\
& ^2*c^{13}*d^{11}*e^4 + 184320*a^4*b^3*c^{12}*d^{10}*e^5 - 916608*a^4*b^4*c^{11}*d^9*e \\
& ^6 + 1165824*a^4*b^5*c^{10}*d^8*e^7 - 314496*a^4*b^6*c^9*d^7*e^8 - 822272*a^4 \\
& *b^7*c^8*d^6*e^9 + 919152*a^4*b^8*c^7*d^5*e^{10} - 175296*a^4*b^9*c^6*d^4*e^{11} \\
& - 189328*a^4*b^{10}*c^5*d^3*e^{12} + 62064*a^4*b^{11}*c^4*d^2*e^{13} + 1290752*a^5 \\
& *b^2*c^{12}*d^9*e^6 - 659456*a^5*b^3*c^{11}*d^8*e^7 - 1561088*a^5*b^4*c^{10}*d^7 \\
& *e^8 + 3240960*a^5*b^5*c^9*d^6*e^9 - 1964192*a^5*b^6*c^8*d^5*e^{10} - 683008* \\
& a^5*b^7*c^7*d^4*e^{11} + 1162304*a^5*b^8*c^6*d^3*e^{12} - 164112*a^5*b^9*c^5*d^2* \\
& e^{13} + 3442688*a^6*b^2*c^{11}*d^7*e^8 - 3670016*a^6*b^3*c^{10}*d^6*e^9 + 1523 \\
& 2*a^6*b^4*c^9*d^5*e^{10} + 4230144*a^6*b^5*c^8*d^4*e^{11} - 3059648*a^6*b^6*c^7 \\
& *d^3*e^{12} - 247296*a^6*b^7*c^6*d^2*e^{13} + 4010496*a^7*b^2*c^{10}*d^5*e^{10} - 6 \\
& 873088*a^7*b^3*c^9*d^4*e^{11} + 2822400*a^7*b^4*c^8*d^3*e^{12} + 2370048*a^7*b^5 \\
& *c^7*d^2*e^{13} + 1178624*a^8*b^2*c^9*d^3*e^{12} - 4739072*a^8*b^3*c^8*d^2*e^{13} \\
& - 352*a^8*b^6*c^{12}*d^{13}*e^2 + 2048*a^8*b^7*c^{11}*d^{12}*e^3 - 4800*a^8*b^8*c^{10}*d^{11} \\
& *e^4 + 5168*a^8*b^9*c^9*d^{10}*e^5 - 480*a^8*b^{10}*c^8*d^9*e^6 - 6000*a^8*b^{11}*c^7 \\
& *d^8*e^7 + 8192*a^8*b^{12}*c^6*d^7*e^8 - 5040*a^8*b^{13}*c^5*d^6*e^9 + 1152*a^8*b^{14} \\
& *c^4*d^5*e^{10} + 240*a^8*b^{15}*c^3*d^4*e^{11} - 128*a^8*b^{16}*c^2*d^3*e^{12} - 512*a^8*b^{17} \\
& *c^2*d^2*e^{13} - 106496*a^9*b^4*c^{14}*d^{12}*e^3 + 11680*a^9*b^5*c^{13}*d^{11}*e^4 - \\
& 675840*a^9*b^6*c^{13}*d^{10}*e^5 - 108288*a^9*b^7*c^{12}*d^9*e^6 - 1601536*a^9*b^8*c^{11} \\
& *d^8*e^7 + 514768*a^9*b^9*c^{10}*d^7*e^8 - 925696*a^9*b^{10}*c^9*d^6*e^9 - 1278304 \\
& *a^9*b^{11}*c^8*d^5*e^{10} + 2457600*a^9*b^{12}*c^7*d^4*e^{11} + 1385600*a^9*b^{13}*c^6*d^3 \\
& *e^{12} + 2977792*a^9*b^{14}*c^5*d^2*e^{13} + 19968*a^9*b^{15}*c^4*d^2*e^{14}))/((8*(a^6*b^8*e^8 \\
& + 256*a^6*c^8*d^8 + 256*a^{10}*c^4*e^8 - 16*a^7*b^6*c^8*e^8 - 4*a^5*b^9*d^8*e^7 + a^2*b^8*c^4*d^8 \\
& - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^{12}*d^4*e^4 - 4* \\
& a^3*b^{11}*d^3*e^5 + 6*a^4*b^{10}*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^{10}*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6 \\
& *e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128* \\
& a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b^3*c^7*d^7*e + 64*a^6*b^7*c^4*d^7*e - 1024*a^9*b^3*c^4*d^7*e - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^{11}*c^3*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^{10} \\
& *c^3*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c^3*d^5*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c^3*d^2*e^6 - 3072*a^7*b^3*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d^5*e^7 - 3072*a^8*b^3*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d^5*e^7)))*((27*a^9*c^5*d^6 - b^{11}*c^4*d^6 - b^{15}*d^2*e^4 - 9*a^2*b^{13}*e^6 + 3840*a^5*b^3*c^9*d^6 - 9*a^5*c^5*d^6*(-(4*a*c - b^2)^9)^{(1/2)} + 213*a^3*b^{11}*c^6*e^6 - 26880*a^8*b^3*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d^5*e + 4*b^{12}*c^3*d^5*e + 4*b^{14}*c^3*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^{(1/2)} - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^{(1/2)} + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^{(1/2)} + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^{(1/2)} - 6*b^{13}*c^2*d^4*e^2 + 6*a^8*b^{14}*d^5*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^{10}*c^3*d^3*e^3 +
\end{aligned}$$

$$\begin{aligned}
& 180a^2b^{11}c^2d^2e^4 + 6976a^3b^7c^5d^4e^2 - 1032a^3b^8c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456a^4b^5c^6d^4e^2 - 7168a^4b^6c^5d^3e^3 + 16896a^4b^7c^4d^2e^4 + 10240a^5b^3c^7d^4e^2 + 37632a^5b^4c^6d^3e^3 - 47712a^5b^5c^5d^2e^4 - 59392a^6b^2c^7d^3e^3 + 60928a^6b^3c^6d^2e^4 - 41a^2c^4d^4e^2(-4ac - b^2)^9)^{(1/2)} \\
& - 39a^3c^3d^2e^4(-4ac - b^2)^9)^{(1/2)} + 6b^4c^2d^4e^2(-4ac - b^2)^9)^{(1/2)} - 6a^2b^5d^5e^5(-4ac - b^2)^9)^{(1/2)} - 106a^2b^10c^4d^5e^5 + 7a^2b^13c^4d^2e^4 - 128a^2b^12c^4d^2e^5 - 51a^3b^2c^6e^6(-4ac - b^2)^9)^{(1/2)} + 150a^2b^11c^3d^4e^2 - 84a^2b^12c^2d^3e^3 + 1116a^2b^8c^5d^5e^5 - 5824a^3b^6c^6d^5e^5 + 1030a^3b^10c^2d^5e^5 + 15232a^4b^4c^7d^5e^5 - 3492a^4b^8c^3d^5e^5 - 16896a^5b^2c^8d^5e^5 + 1344a^5b^6c^4d^5e^5 + 7424a^6b^6c^8d^4e^2 + 22400a^6b^4c^5d^5e^5 - 23296a^7b^6c^7d^2e^4 - 53760a^7b^2c^6d^5e^5 - 4b^3c^3d^5e^5(-4ac - b^2)^9)^{(1/2)} - 4b^5c^3d^3e^3(-4ac - b^2)^9)^{(1/2)} + 11a^2b^4c^4d^2e^4(-4ac - b^2)^9)^{(1/2)} + 20a^2b^3c^4d^5e^5(-4ac - b^2)^9)^{(1/2)} + 86a^3b^2c^2d^5e^5(-4ac - b^2)^9)^{(1/2)} - 42a^2b^2c^3d^4e^2(-4ac - b^2)^9)^{(1/2)} + 12a^2b^3c^2d^3e^3(-4ac - b^2)^9)^{(1/2)} + 120a^2b^3c^3d^3e^3(-4ac - b^2)^9)^{(1/2)} + 34a^2b^3c^4d^5e^5(-4ac - b^2)^9)^{(1/2)} - 108a^2b^2c^2d^2e^4(-4ac - b^2)^9)^{(1/2))}/(32(a^7b^12e^8 + 4096a^9c^10d^8 + 4096a^13c^6e^8 - 24a^8b^10c^8e^8 - 4a^6b^13d^7e^7 + a^3b^12c^4d^8 - 24a^4b^10c^5d^8 + 240a^5b^8c^6d^8 - 1280a^6b^6c^7d^8 + 3840a^7b^4c^8d^8 - 6144a^8b^2c^9d^8 + 240a^9b^8c^2e^8 - 1280a^10b^6c^3e^8 + 3840a^11b^4c^4e^8 - 6144a^12b^2c^5e^8 + a^3b^16d^4e^4 - 4a^4b^15d^3e^5 + 6a^5b^14d^2e^6 + 16384a^10c^9d^6e^2 + 24576a^11c^8d^4e^4 + 16384a^12c^7d^2e^6 + 6a^3b^14c^2d^6e^2 - 140a^4b^12c^3d^6e^2 + 84a^4b^13c^2d^5e^3 + 1344a^5b^10c^4d^6e^2 - 672a^5b^11c^3d^5e^3 - 42a^5b^12c^2d^4e^4 - 6720a^6b^8c^5d^6e^2 + 2240a^6b^9c^4d^5e^3 + 1456a^6b^10c^3d^4e^4 - 672a^6b^11c^2d^3e^5 + 17920a^7b^6c^6d^6e^2 - 10080a^7b^8c^4d^4e^4 + 2240a^7b^9c^3d^3e^5 + 1344a^7b^10c^2d^2e^6 - 21504a^8b^4c^7d^6e^2 - 21504a^8b^5c^6d^5e^3 + 32256a^8b^6c^5d^4e^4 - 6720a^8b^8c^3d^2e^6 + 57344a^9b^3c^7d^5e^3 - 46592a^9b^4c^6d^4e^4 - 21504a^9b^5c^5d^3e^5 + 17920a^9b^6c^4d^2e^6 + 12288a^10b^2c^7d^4e^4 + 57344a^10b^3c^6d^3e^5 - 21504a^10b^4c^5d^2e^6 + 96a^7b^11c^4d^7e^7 - 16384a^9b^3c^9d^7e^7 - 16384a^12b^3c^6d^5e^7 - 4a^3b^13c^3d^7e^7 - 4a^3b^15c^4d^5e^3 + 96a^4b^11c^4d^7e^7 - 12a^4b^14c^3d^4e^4 - 960a^5b^9c^5d^7e^7 + 84a^5b^13c^3d^3e^5 + 5120a^6b^7c^6d^7e^7 - 140a^6b^12c^2d^2e^6 - 15360a^7b^5c^7d^7e^7 + 24576a^8b^3c^8d^7e^7 - 960a^8b^9c^2d^5e^7 + 5120a^9b^7c^3d^5e^7 - 49152a^10b^6c^8d^5e^3 - 15360a^10b^5c^4d^5e^7 - 49152a^11b^3c^7d^3e^5 + 24576a^11b^3c^5d^5e^7))^{(1/2)} - (326912a^8c^9d^5e^13 - 241664a^8b^3c^8e^14 - 48a^2b^13c^2e^14 + 1264a^3b^11c^3e^14 - 13552a^4b^9c^4e^14 + 75776a^5b^7c^5e^14 - 232960a^6b^5c^6e^14 + 372736a^7b^3c^7e^14 + 11520a^3c^14d^11e^3 + 78080a^4c^13d^9e^5 + 197120a^5c^12d^7e^7 + 336384a^6c^11d^5e^9 + 532736a^7c^10d^3e^11 - 40b^5c^12d^12e^2 + 216b^6c^11d^11e^3 - 464b^7c^10d^10e^4 + 496b^8c^9d^9e^5 - 264b^9c^8d^8e^6 + 56b^10c^7d^7e^7 - 16b^11c^6d^6e^8 + 64b^12c^5d^5e^9 - 96b^13c^4d^4e^10 + 64b^14c^3d^3e^11 - 16b^15c^2d^2e^12 + 1536a^2b^2c^13d^11e^3 + 14400a^2b^3c^12d^10e^4 - 47152a^2b^4c^11d^9e^5 + 52144a^2b^5c^10d^8e^6 - 16272a^2b^6c^9d^7e^7 - 13040a^2b^7c^8d^6e^8 + 23488a^2b^8c^7d^5e^9 - 26384a^2b^9c^6d^4e^10 + 13824a^2b^10c^5d^3e^11 + 256a^2b^11c^4d^2e^12 + 125056a^3b^2c^12d^9e^5 - 36224a^3b^3c^11d^8e^6 - 126432a^3b^4c^10d^7e^7 + 144848a^3b^5c^9d^6e^8 - 114752a^3b^6c^8d^5e^9 + 125392a^3b^7c^7d^4e^10 - 53248a^3b^8c^6d^3e^11 - 25264a^3b^9c^5d^2e^12 + 474112a^4b^2c^11d^7e^7 - 191104a^4b^3c^10d^6e^8 + 97184a^4b^4c^9d^5e^9 - 277000a^4b^5c^8d^4e^10 + 56056a^4b^6c^7d^3e^11 + 195584a^4b^7c^6d^2e^12 + 236800a^5b^2c^10d^5e^9 + 388032a^5b^3c^9d^4e^10 + 159632a^5b^4c^8d^3e^11 - 670488a^5b^
\end{aligned}$$

$$\begin{aligned}
& ^5c^7d^2e^{12} - 488960a^6b^2c^9d^3e^{11} + 1106496a^6b^3c^8d^2e^{11} \\
& + 64a^6b^{14}c^2d^2e^{13} + 448a^6b^3c^{13}d^{12}e^2 - 1968a^6b^4c^{12}d^{11}e^3 \\
& + 2504a^6b^5c^{11}d^{10}e^4 + 768a^6b^6c^{10}d^9e^5 - 4368a^6b^7c^9d^8e^6 \\
& + 3568a^6b^8c^8d^7e^7 - 520a^6b^9c^7d^6e^8 - 1728a^6b^{10}c^6d^5e^9 \\
& + 2528a^6b^{11}c^5d^4e^{10} - 1536a^6b^{12}c^4d^3e^{11} + 240a^6b^{13}c^3d^2e^{12} \\
& - 1152a^6b^{14}c^2d^2e^{12} - 1600a^6b^{12}c^3d^2e^{13} - 67968a^6b^3c^{13}d^{10}e^4 \\
& + 15808a^6b^{10}c^4d^2e^{13} - 342272a^6b^4c^{12}d^8e^6 - 76928a^6b^8c^5d^2e^{13} \\
& - 569088a^6b^5c^{11}d^6e^8 + 179200a^6b^6c^6d^6e^{13} - 586368a^6b^6c^{10}d^4e^{10} \\
& - 113008a^6b^4c^7d^2e^{13} - 731008a^6b^7c^9d^2e^{12} - 244096a^6b^2c^8d^2e^{13} \\
& / (16(a^6b^8e^8 + 256a^6c^8d^8 + 256a^6c^4e^8 - 16a^7b^6c^2e^8 - 4a^5b^9d^2e^7 \\
& + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6d^8 - 256a^5b^2c^7d^8 + 96a^8b^4c^2e^8 \\
& - 256a^9b^2c^3e^8 + a^2b^{12}d^4e^4 - 4a^3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 \\
& + 1024a^7c^7d^6e^2 + 1536a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^{10}c^2d^6e^2 \\
& - 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4b^7c^3d^5e^3 \\
& - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 \\
& - 192a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c^4d^4e^4 \\
& - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 + 2048a^7b^3c^4d^3e^5 \\
& - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024a^6b^7c^7d^7e + 64a^6b^7c^7d^7e \\
& - 1024a^9b^7c^4d^7e - 4a^2b^9c^3d^7e - 4a^2b^{11}c^5d^5e^3 + 64a^3b^7c^4d^7e \\
& - 4a^3b^{10}c^4d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^3d^3e^5 + 1024a^5b^3c^6d^7e \\
& - 92a^5b^8c^2d^2e^6 - 3072a^7b^6c^6d^5e^3 - 384a^7b^5c^2d^5e^7 - 3072a^8b^6c^5d^3e^5 \\
& + 1024a^8b^3c^3d^5e^7)) * ((27a^6b^9c^5d^6 - b^{11}c^4d^6 - b^{15}d^2e^4 - 9a^2b^{13}e^6 \\
& + 3840a^5b^9c^9d^6 - 9a^5c^5d^6 * (-4ac - b^2)^9)^{1/2} + 213a^3b^{11}c^6e^6 - 26880a^8b^6c^6e^6 \\
& + 3072a^6c^9d^5e + 35840a^8c^7d^5e^5 + 4b^{12}c^3d^5e + 4b^{14}c^3d^3e^3 - 288a^2b^7c^6d^6 \\
& + 1504a^3b^5c^7d^6 - 3840a^4b^3c^8d^6 + 9a^2b^4e^6 * (-4ac - b^2)^9)^{1/2} - 2077a^4b^9c^2e^6 \\
& + 10656a^5b^7c^3e^6 - 30240a^6b^5c^4e^6 + 44800a^7b^3c^5e^6 + 25a^4c^2e^6 * (-4ac - b^2)^9)^{1/2} \\
& + b^2c^4d^6 * (-4ac - b^2)^9)^{1/2} + 22528a^7c^8d^3e^3 + b^6d^2e^4 * (-4ac - b^2)^9)^{1/2} \\
& - 6b^{13}c^2d^4e^2 + 6a^6b^{14}d^5e - 1471a^2b^9c^4d^4e^2 + 600a^2b^{10}c^3d^3e^3 + 180a^2b^{11}c^2d^2e^4 \\
& + 6976a^3b^7c^5d^4e^2 - 1032a^3b^8c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456a^4b^5c^6d^4e^2 \\
& - 7168a^4b^6c^5d^3e^3 + 16896a^4b^7c^4d^2e^4 + 10240a^5b^3c^7d^4e^2 + 37632a^5b^4c^6d^3e^3 \\
& - 47712a^5b^5c^5d^2e^4 - 59392a^6b^2c^7d^3e^3 + 60928a^6b^3c^6d^2e^4 - 41a^2c^4d^4e^2 \\
& * (-4ac - b^2)^9)^{1/2} - 39a^3c^3d^2e^4 * (-4ac - b^2)^9)^{1/2} + 6b^4c^2d^4e^2 * (-4ac - b^2)^9)^{1/2} \\
& - 6a^6b^5d^5e^5 * (-4ac - b^2)^9)^{1/2} - 106a^6b^{10}c^4d^5e + 7a^6b^{13}c^4d^2e^4 \\
& - 128a^2b^{12}c^4d^5e^5 - 51a^3b^2c^6e^6 * (-4ac - b^2)^9)^{1/2} + 150a^6b^{11}c^3d^4e^2 \\
& - 84a^6b^{12}c^2d^3e^3 + 1116a^2b^8c^5d^5e - 5824a^3b^6c^6d^5e + 1030a^3b^{10}c^2d^5e^5 \\
& + 15232a^4b^4c^7d^5e - 3492a^4b^8c^3d^5e^5 - 16896a^5b^2c^8d^5e + 1344a^5b^6c^4d^5e^5 \\
& + 7424a^6b^8c^8d^4e^2 + 22400a^6b^4c^5d^5e^5 - 23296a^7b^6c^7d^2e^4 - 53760a^7b^2c^6d^5e^5 \\
& - 4b^3c^3d^5e^5 * (-4ac - b^2)^9)^{1/2} - 4b^5c^3d^3e^3 * (-4ac - b^2)^9)^{1/2} \\
& + 11a^6b^4c^4d^2e^4 * (-4ac - b^2)^9)^{1/2} + 20a^2b^3c^4d^5e^5 * (-4ac - b^2)^9)^{1/2} \\
& + 86a^3b^3c^2d^5e^5 * (-4ac - b^2)^9)^{1/2} - 42a^6b^2c^3d^4e^2 * (-4ac - b^2)^9)^{1/2} \\
& + 12a^6b^3c^2d^3e^3 * (-4ac - b^2)^9)^{1/2} + 120a^2b^3c^3d^3e^3 * (-4ac - b^2)^9)^{1/2} \\
& + 34a^6b^4c^4d^5e^5 * (-4ac - b^2)^9)^{1/2} - 108a^2b^2c^2d^2e^4 * (-4ac - b^2)^9)^{1/2} \\
& / (32(a^7b^{12}e^8 + 4096a^9c^{10}d^8 + 4096a^{13}c^6e^8 - 24a^8b^{10}c^6e^8 - 4a^6b^{13}d^2e^7 \\
& + a^3b^{12}c^4d^8 - 24a^4b^{10}c^5d^8 + 240a^5b^8c^6d^8 - 1280a^6b^6c^7d^8 + 3840a^7b^4c^8d^8 \\
& - 6144a^8b^2c^9d^8 + 240a^9b^8c^2e^8 - 1280a^{10}b^6c^3e^8 + 3840a^{11}b^4c^4e^8 \\
& - 6144a^{12}b^2c^5e^8 + a^3b^{16}d^4e^4 - 4a^4b^{15}d^3e^5 + 6a^5b^{14}d^2e^6 + 16384a^{10}c^9d^8
\end{aligned}$$



$$\begin{aligned}
&6e^2 + 24576a^{11}c^8d^4e^4 + 16384a^{12}c^7d^2e^6 + 6a^3b^{14}c^2d^6e^2 - 140a^4b^{12}c^3d^6e^2 + 84a^4b^{13}c^2d^5e^3 + 1344a^5b^{10}c^4d^6e^2 - 672a^5b^{11}c^3d^5e^3 - 42a^5b^{12}c^2d^4e^4 - 6720a^6b^8c^5d^6e^2 + 2240a^6b^9c^4d^5e^3 + 1456a^6b^{10}c^3d^4e^4 - 672a^6b^{11}c^2d^3e^5 + 17920a^7b^6c^6d^6e^2 - 10080a^7b^8c^4d^4e^4 + 2240a^7b^9c^3d^3e^5 + 1344a^7b^{10}c^2d^2e^6 - 21504a^8b^4c^7d^6e^2 - 21504a^8b^5c^6d^5e^3 + 32256a^8b^6c^5d^4e^4 - 6720a^8b^8c^3d^2e^6 + 57344a^9b^3c^7d^5e^3 - 46592a^9b^4c^6d^4e^4 - 21504a^9b^5c^5d^3e^5 + 17920a^9b^6c^4d^2e^6 + 12288a^{10}b^2c^7d^4e^4 + 57344a^{10}b^3c^6d^3e^5 - 21504a^{10}b^4c^5d^2e^6 + 96a^7b^{11}c^d^e^7 - 16384a^9b^c^9d^7e - 16384a^{12}b^c^6d^e^7 - 4a^3b^{13}c^3d^7e - 4a^3b^{15}c^d^5e^3 + 96a^4b^{11}c^4d^7e - 12a^4b^{14}c^d^4e^4 - 960a^5b^9c^5d^7e + 84a^5b^{13}c^d^3e^5 + 5120a^6b^7c^6d^7e - 140a^6b^{12}c^d^2e^6 - 15360a^7b^5c^7d^7e + 24576a^8b^3c^8d^7e - 960a^8b^9c^2d^e^7 + 5120a^9b^7c^3d^e^7 - 49152a^{10}b^c^8d^5e^3 - 15360a^{10}b^5c^4d^e^7 - 49152a^{11}b^c^7d^3e^5 + 24576a^{11}b^3c^5d^e^7))^{(1/2)} - (x*(22800a^6c^9e^{13} + 36a^2b^8c^5e^{13} - 600a^3b^6c^6e^{13} + 4313a^4b^4c^7e^{13} - 15592a^5b^2c^8e^{13} + 1296a^2c^{13}d^8e^5 + 9792a^3c^{12}d^6e^7 + 30304a^4c^{11}d^4e^9 + 40512a^5c^{10}d^2e^{11} + 25b^4c^{11}d^8e^5 - 120b^5c^{10}d^7e^6 + 214b^6c^9d^6e^7 - 168b^7c^8d^5e^8 + 53b^8c^7d^4e^9 - 8b^9c^6d^3e^{10} + 4b^{10}c^5d^2e^{11} + 6336a^2b^2c^{11}d^6e^7 + 3840a^2b^3c^{10}d^5e^8 - 8506a^2b^4c^9d^4e^9 + 1112a^2b^5c^8d^3e^{10} + 1254a^2b^6c^7d^2e^{11} + 22224a^3b^2c^{10}d^4e^9 + 13824a^3b^3c^9d^3e^{10} - 9516a^3b^4c^8d^2e^{11} + 11712a^4b^2c^9d^2e^{11} - 24a^4b^9c^5d^e^{12} - 41088a^5b^c^9d^e^{12} - 360a^4b^2c^{12}d^8e^5 + 1664a^4b^3c^{11}d^7e^6 - 2604a^4b^4c^{10}d^6e^7 + 1272a^4b^5c^9d^5e^8 + 332a^4b^6c^8d^4e^9 - 232a^4b^7c^7d^3e^{10} - 48a^4b^8c^6d^2e^{11} - 5760a^2b^c^{12}d^7e^6 + 416a^2b^7c^6d^e^{12} - 32128a^3b^c^{11}d^5e^8 - 4120a^3b^5c^7d^e^{12} - 63360a^4b^c^{10}d^3e^{10} + 21376a^4b^3c^8d^e^{12}))/((8*(a^6b^8e^8 + 256a^6c^8d^8 + 256a^{10}c^4e^8 - 16a^7b^6c^e^8 - 4a^5b^9d^e^7 + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6d^8 - 256a^5b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^{12}d^4e^4 - 4a^3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 + 1024a^7c^7d^6e^2 + 1536a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^{10}c^2d^6e^2 - 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4b^7c^3d^5e^3 - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 - 192a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c^4d^4e^4 - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 + 2048a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024a^6b^c^7d^7e + 64a^6b^7c^d^e^7 - 1024a^9b^c^4d^e^7 - 4a^2b^9c^3d^7e - 4a^2b^{11}c^d^5e^3 + 64a^3b^7c^4d^7e - 4a^3b^{10}c^d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^d^3e^5 + 1024a^5b^3c^6d^7e - 92a^5b^8c^d^2e^6 - 3072a^7b^c^6d^5e^3 - 384a^7b^5c^2d^e^7 - 3072a^8b^c^5d^3e^5 + 1024a^8b^3c^3d^e^7))((27a^9b^c^5d^6 - b^{11}c^4d^6 - b^{15}d^2e^4 - 9a^2b^{13}e^6 + 3840a^5b^c^9d^6 - 9a^c^5d^6*(-(4a^c - b^2)^9)^{(1/2)} + 213a^3b^{11}c^e^6 - 26880a^8b^c^6e^6 + 3072a^6c^9d^5e + 35840a^8c^7d^e^5 + 4b^{12}c^3d^5e + 4b^{14}c^d^3e^3 - 288a^2b^7c^6d^6 + 1504a^3b^5c^7d^6 - 3840a^4b^3c^8d^6 + 9a^2b^4e^6*(-(4a^c - b^2)^9)^{(1/2)} - 2077a^4b^9c^2e^6 + 10656a^5b^7c^3e^6 - 30240a^6b^5c^4e^6 + 44800a^7b^3c^5e^6 + 25a^4c^2e^6*(-(4a^c - b^2)^9)^{(1/2)} + b^2c^4d^6*(-(4a^c - b^2)^9)^{(1/2)} + 22528a^7c^8d^3e^3 + b^6d^2e^4*(-(4a^c - b^2)^9)^{(1/2)} - 6b^{13}c^2d^4e^2 + 6a^b^{14}d^e^5 - 1471a^2b^9c^4d^4e^2 + 600a^2b^{10}c^3d^3e^3 + 180a^2b^{11}c^2d^2e^4 + 6976a^3b^7c^5d^4e^2 - 1032a^3b^8c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456a^4b^5c^6d^4e^2 - 7168a^4b^6c^5d^3e^3 + 16896a^4b^7c^4d^2e^4 + 10240a^5b^3c^7d^4e^2 + 37632a^5b^4c^6d^3e^3 - 47712a^5b^5c^5d^2e^4 - 59392a^6b^2c^7d^3e^3 +
\end{aligned}$$

$$\begin{aligned}
& 60928a^6b^3c^6d^2e^4 - 41a^2c^4d^4e^2(-4ac - b^2)^9)^{(1/2)} - 3 \\
& 9a^3c^3d^2e^4(-4ac - b^2)^9)^{(1/2)} + 6b^4c^2d^4e^2(-4ac - b \\
& ^2)^9)^{(1/2)} - 6ab^5d^5e^5(-4ac - b^2)^9)^{(1/2)} - 106ab^{10}c^4d^5e \\
& e + 7ab^{13}c^2d^2e^4 - 128a^2b^{12}c^2d^5e^5 - 51a^3b^2c^2e^6(-4ac - \\
& b^2)^9)^{(1/2)} + 150ab^{11}c^3d^4e^2 - 84ab^{12}c^2d^3e^3 + 1116a^2b^8 \\
& c^5d^5e - 5824a^3b^6c^6d^5e + 1030a^3b^{10}c^2d^5e^5 + 15232a^4 \\
& b^4c^7d^5e - 3492a^4b^8c^3d^5e^5 - 16896a^5b^2c^8d^5e + 1344a^5 \\
& b^6c^4d^5e^5 + 7424a^6b^6c^8d^4e^2 + 22400a^6b^4c^5d^5e^5 - 23296 \\
& a^7b^6c^7d^2e^4 - 53760a^7b^2c^6d^5e^5 - 4b^3c^3d^5e^5(-4ac - b \\
& ^2)^9)^{(1/2)} - 4b^5c^3d^3e^3(-4ac - b^2)^9)^{(1/2)} + 11ab^4c^2d^2e^ \\
& 4(-4ac - b^2)^9)^{(1/2)} + 20a^2b^3c^2d^5e^5(-4ac - b^2)^9)^{(1/2)} + \\
& 86a^3b^2c^2d^5e^5(-4ac - b^2)^9)^{(1/2)} - 42ab^2c^3d^4e^2(-4ac - \\
& b^2)^9)^{(1/2)} + 12ab^3c^2d^3e^3(-4ac - b^2)^9)^{(1/2)} + 120a^2b \\
& c^3d^3e^3(-4ac - b^2)^9)^{(1/2)} + 34ab^4c^4d^5e^5(-4ac - b^2)^9 \\
& )^{(1/2)} - 108a^2b^2c^2d^2e^4(-4ac - b^2)^9)^{(1/2)}/(32(a^7b^{12}e \\
& ^8 + 4096a^9c^{10}d^8 + 4096a^{13}c^6e^8 - 24a^8b^{10}c^6e^8 - 4a^6b^{13} \\
& *d^7e^7 + a^3b^{12}c^4d^8 - 24a^4b^{10}c^5d^8 + 240a^5b^8c^6d^8 - 128 \\
& 0a^6b^6c^7d^8 + 3840a^7b^4c^8d^8 - 6144a^8b^2c^9d^8 + 240a^9b^8 \\
& c^2e^8 - 1280a^{10}b^6c^3e^8 + 3840a^{11}b^4c^4e^8 - 6144a^{12}b^2c^5 \\
& e^8 + a^3b^{16}d^4e^4 - 4a^4b^{15}d^3e^5 + 6a^5b^{14}d^2e^6 + 1638 \\
& 4a^{10}c^9d^6e^2 + 24576a^{11}c^8d^4e^4 + 16384a^{12}c^7d^2e^6 + 6a^3 \\
& b^{14}c^2d^6e^2 - 140a^4b^{12}c^3d^6e^2 + 84a^4b^{13}c^2d^5e^3 + 1 \\
& 344a^5b^{10}c^4d^6e^2 - 672a^5b^{11}c^3d^5e^3 - 42a^5b^{12}c^2d^4e^ \\
& ^4 - 6720a^6b^8c^5d^6e^2 + 2240a^6b^9c^4d^5e^3 + 1456a^6b^{10}c^3 \\
& d^4e^4 - 672a^6b^{11}c^2d^3e^5 + 17920a^7b^6c^6d^6e^2 - 10080a^7 \\
& b^8c^4d^4e^4 + 2240a^7b^9c^3d^3e^5 + 1344a^7b^{10}c^2d^2e^6 - \\
& 21504a^8b^4c^7d^6e^2 - 21504a^8b^5c^6d^5e^3 + 32256a^8b^6c^5d^4 \\
& e^4 - 6720a^8b^8c^3d^2e^6 + 57344a^9b^3c^7d^5e^3 - 46592a^9b^4 \\
& c^6d^4e^4 - 21504a^9b^5c^5d^3e^5 + 17920a^9b^6c^4d^2e^6 + 12 \\
& 288a^{10}b^2c^7d^4e^4 + 57344a^{10}b^3c^6d^3e^5 - 21504a^{10}b^4c^5 \\
& d^2e^6 + 96a^7b^{11}c^2d^7e - 16384a^9b^6c^9d^7e - 16384a^{12}b^6c^6d^ \\
& e^7 - 4a^3b^{13}c^3d^7e - 4a^3b^{15}c^2d^5e^3 + 96a^4b^{11}c^4d^7e - \\
& 12a^4b^{14}c^2d^4e^4 - 960a^5b^9c^5d^7e + 84a^5b^{13}c^3d^3e^5 + 51 \\
& 20a^6b^7c^6d^7e - 140a^6b^{12}c^2d^2e^6 - 15360a^7b^5c^7d^7e + 2 \\
& 4576a^8b^3c^8d^7e - 960a^8b^9c^2d^5e^7 + 5120a^9b^7c^3d^5e^7 - 4 \\
& 9152a^{10}b^6c^8d^5e^3 - 15360a^{10}b^5c^4d^5e^7 - 49152a^{11}b^6c^7d^3e \\
& ^5 + 24576a^{11}b^3c^5d^5e^7))^{(1/2)} + ((((((1048576a^{13}c^8e^{16} + 256a \\
& ^7b^{12}c^2e^{16} - 6144a^8b^{10}c^3e^{16} + 61440a^9b^8c^4e^{16} - 327680 \\
& a^{10}b^6c^5e^{16} + 983040a^{11}b^4c^6e^{16} - 1572864a^{12}b^2c^7e^{16} - \\
& 196608a^6c^{15}d^{14}e^2 - 917504a^7c^{14}d^{12}e^4 - 589824a^8c^{13}d^{10} \\
& *e^6 + 3932160a^9c^{12}d^8e^8 + 10158080a^{10}c^{11}d^6e^{10} + 10616832a^{11} \\
& c^{10}d^4e^{12} + 5308416a^{12}c^9d^2e^{14} - 2816a^2b^8c^{11}d^{14}e^2 + \\
& 22656a^2b^9c^{10}d^{13}e^3 - 78848a^2b^{10}c^9d^{12}e^4 + 154112a^2b^{11} \\
& c^8d^{11}e^5 - 182784a^2b^{12}c^7d^{10}e^6 + 130816a^2b^{13}c^6d^9e^7 \\
& - 50176a^2b^{14}c^5d^8e^8 + 4608a^2b^{15}c^4d^7e^9 + 3328a^2b^{16}c^3 \\
& d^6e^{10} - 896a^2b^{17}c^2d^5e^{11} + 24576a^3b^6c^{12}d^{14}e^2 - 198 \\
& 656a^3b^7c^{11}d^{13}e^3 + 684544a^3b^8c^{10}d^{12}e^4 - 1291520a^3b^9c^9 \\
& d^{11}e^5 + 1403776a^3b^{10}c^8d^{10}e^6 - 798336a^3b^{11}c^7d^9e^7 \\
& + 89856a^3b^{12}c^6d^8e^8 + 155136a^3b^{13}c^5d^7e^9 - 77440a^3b^{14} \\
& c^4d^6e^{10} + 5504a^3b^{15}c^3d^5e^{11} + 2560a^3b^{16}c^2d^4e^{12} - 1 \\
& 06496a^4b^4c^{13}d^{14}e^2 + 864256a^4b^5c^{12}d^{13}e^3 - 2924544a^4b^6 \\
& c^{11}d^{12}e^4 + 5181440a^4b^7c^{10}d^{11}e^5 - 4686080a^4b^8c^9d^{10} \\
& e^6 + 1045376a^4b^9c^8d^9e^7 + 1900544a^4b^{10}c^7d^8e^8 - 1732096a^4 \\
& b^{11}c^6d^7e^9 + 390400a^4b^{12}c^5d^6e^{10} + 112000a^4b^{13}c^4d^5 \\
& e^{11} - 40960a^4b^{14}c^3d^4e^{12} - 3840a^4b^{15}c^2d^3e^{13} + 229376 \\
& a^5b^2c^{14}d^{14}e^2 - 1867776a^5b^3c^{13}d^{13}e^3 + 6078464a^5b^4c^{12} \\
& d^{12}e^4 - 9297920a^5b^5c^{11}d^{11}e^5 + 4055040a^5b^6c^{10}d^{10}e^6 \\
& + 7788544a^5b^7c^9d^9e^7 - 12657664a^5b^8c^8d^8e^8 + 6130176a^5 \\
& b^9c^7d^7e^9 + 734080a^5b^{10}c^6d^6e^{10} - 1442560a^5b^{11}c^5d^5e^{11}
\end{aligned}$$

$$\begin{aligned}
& e^{11} + 168960a^5b^{12}c^4d^4e^{12} + 78080a^5b^{13}c^3d^3e^{13} + 3200a^5b^{14}c^2d^2e^{14} - 4587520a^6b^2c^{13}d^{12}e^4 + 3080192a^6b^3c^{12}d^{11}e^5 + 12001280a^6b^4c^{11}d^{10}e^6 - 31076352a^6b^5c^{10}d^9e^7 + 27475968a^6b^6c^9d^8e^8 - 2088960a^6b^7c^8d^7e^9 - 12205312a^6b^8c^7d^6e^{10} + 6043520a^6b^9c^6d^5e^{11} + 631808a^6b^{10}c^5d^4e^{12} - 610304a^6b^{11}c^4d^3e^{13} - 71936a^6b^{12}c^3d^2e^{14} - 21725184a^7b^2c^{12}d^{10}e^6 + 30801920a^7b^3c^{11}d^9e^7 - 8028160a^7b^4c^{10}d^8e^8 - 32260096a^7b^5c^9d^7e^9 + 37101568a^7b^6c^8d^6e^{10} - 7182336a^7b^7c^7d^5e^{11} - 7609856a^7b^8c^6d^4e^{12} + 2112256a^7b^9c^5d^3e^{13} + 661632a^7b^{10}c^4d^2e^{14} - 30146560a^8b^2c^{11}d^8e^8 + 55050240a^8b^3c^{10}d^7e^9 - 34365440a^8b^4c^9d^6e^{10} - 16429056a^8b^5c^8d^5e^{11} + 24600576a^8b^6c^7d^4e^{12} - 1683456a^8b^7c^6d^3e^{13} - 3151616a^8b^8c^5d^2e^{14} - 10977280a^9b^2c^{10}d^6e^{10} + 47022080a^9b^3c^9d^5e^{11} - 30621696a^9b^4c^8d^4e^{12} - 9232384a^9b^5c^7d^3e^{13} + 7970816a^9b^6c^6d^2e^{14} + 4325376a^{10}b^2c^9d^4e^{12} + 25493504a^{10}b^3c^8d^3e^{13} - 9117696a^{10}b^4c^7d^2e^{14} + 491520a^{11}b^2c^8d^2e^{14} - 4947968a^{12}b^2c^8d^2e^{15} + 128a^2b^{10}c^{10}d^{14}e^2 - 1024a^2b^{11}c^9d^{13}e^3 + 3584a^2b^{12}c^8d^{12}e^4 - 7168a^2b^{13}c^7d^{11}e^5 + 8960a^2b^{14}c^6d^{10}e^6 - 7168a^2b^{15}c^5d^9e^7 + 3584a^2b^{16}c^4d^8e^8 - 1024a^2b^{17}c^3d^7e^9 + 128a^2b^{18}c^2d^6e^{10} + 1605632a^6b^2c^{14}d^{13}e^3 - 1408a^6b^{13}c^2d^2e^{15} + 7012352a^7b^2c^{13}d^{11}e^5 + 33152a^7b^{11}c^3d^2e^{15} + 7045120a^8b^2c^{12}d^9e^7 - 324480a^8b^9c^4d^2e^{15} - 9830400a^9b^2c^{11}d^7e^9 + 1689600a^9b^7c^5d^2e^{15} - 25722880a^{10}b^2c^{10}d^5e^{11} - 4935680a^{10}b^5c^6d^2e^{15} - 19202048a^{11}b^2c^9d^3e^{13} + 7667712a^{11}b^3c^7d^2e^{15}) / (16(a^6b^8e^8 + 256a^6c^8d^8 + 256a^{10}c^4e^8 - 16a^7b^6c^2e^8 - 4a^5b^9d^2e^7 + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6d^8 - 256a^5b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^{12}d^4e^4 - 4a^3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 + 1024a^7c^7d^6e^2 + 1536a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^{10}c^2d^6e^2 - 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4b^7c^3d^5e^3 - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 - 192a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c^4d^4e^4 - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 + 2048a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024a^6b^2c^7d^7e + 64a^6b^7c^4d^7e - 1024a^9b^2c^4d^7e - 4a^2b^9c^3d^7e - 4a^2b^{11}c^4d^5e^3 + 64a^3b^7c^4d^7e - 4a^3b^{10}c^4d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^3d^5e^5 + 1024a^5b^3c^6d^7e - 92a^5b^8c^4d^2e^6 - 3072a^7b^2c^6d^5e^3 - 384a^7b^5c^2d^7e - 3072a^8b^2c^5d^3e^5 + 1024a^8b^3c^3d^2e^7) + (x*((27a^2b^9c^5d^6 - b^{11}c^4d^6 - b^{15}d^2e^4 - 9a^2b^{13}e^6 + 3840a^5b^3c^9d^6 - 9a^2c^5d^6(-4ac - b^2)^9)^{1/2} + 213a^3b^{11}c^2e^6 - 26880a^8b^2c^6e^6 + 3072a^6c^9d^5e + 35840a^8c^7d^5e + 4b^{12}c^3d^5e + 4b^{14}c^3d^3e^3 - 288a^2b^7c^6d^6 + 1504a^3b^5c^7d^6 - 3840a^4b^3c^8d^6 + 9a^2b^4e^6(-4ac - b^2)^9)^{1/2} - 2077a^4b^9c^2e^6 + 10656a^5b^7c^3e^6 - 30240a^6b^5c^4e^6 + 44800a^7b^3c^5e^6 + 25a^4c^2e^6(-4ac - b^2)^9)^{1/2} + b^2c^4d^6(-4ac - b^2)^9)^{1/2} + 22528a^7c^8d^3e^3 + b^6d^2e^4(-4ac - b^2)^9)^{1/2} - 6b^{13}c^2d^4e^2 + 6a^2b^{14}d^5e - 1471a^2b^9c^4d^4e^2 + 600a^2b^{10}c^3d^3e^3 + 180a^2b^{11}c^2d^2e^4 + 6976a^3b^7c^5d^4e^2 - 1032a^3b^8c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456a^4b^5c^6d^4e^2 - 7168a^4b^6c^5d^3e^3 + 16896a^4b^7c^4d^2e^4 + 10240a^5b^3c^7d^4e^2 + 37632a^5b^4c^6d^3e^3 - 47712a^5b^5c^5d^2e^4 - 59392a^6b^2c^7d^3e^3 + 60928a^6b^3c^6d^2e^4 - 41a^2c^4d^4e^2(-4ac - b^2)^9)^{1/2} - 39a^3c^3d^2e^4(-4ac - b^2)^9)^{1/2} + 6b^4c^2d^4e^2(-4ac - b^2)^9)^{1/2} - 6a^2b^5d^5e^5(-4ac - b^2)^9)^{1/2} - 106a^2b^{10}c^4d^5e + 7a^2b^{13}c^2d^2e^4 - 128a^2b^{12}c^2d^3e^3 - 51a^3b^2c^2e^6(-4ac - b^2)^9)^{1/2} + 150a^2b^{11}c^3d^4e^2 - 84a^2b^{12}c^2d^3e^3 + 1116a^2b
\end{aligned}$$

$$\begin{aligned}
& ^8c^5d^5e - 5824a^3b^6c^6d^5e + 1030a^3b^{10}c^2d^5e^5 + 15232a^4 \\
& *b^4c^7d^5e - 3492a^4b^8c^3d^5e^5 - 16896a^5b^2c^8d^5e + 1344a^ \\
& 5b^6c^4d^5e^5 + 7424a^6b^6c^8d^4e^2 + 22400a^6b^4c^5d^5e^5 - 23296* \\
& a^7b^6c^7d^2e^4 - 53760a^7b^2c^6d^5e^5 - 4b^3c^3d^5e^5*(-(4ac - b^ \\
& 2)^9)^{(1/2)} - 4b^5c^3d^3e^3*(-(4ac - b^2)^9)^{(1/2)} + 11ab^4c^2d^2e^4 \\
& *(-(4ac - b^2)^9)^{(1/2)} + 20a^2b^3c^2d^5e^5*(-(4ac - b^2)^9)^{(1/2)} + 8 \\
& 6a^3b^2c^2d^5e^5*(-(4ac - b^2)^9)^{(1/2)} - 42ab^2c^3d^4e^2*(-(4ac \\
& - b^2)^9)^{(1/2)} + 12ab^3c^2d^3e^3*(-(4ac - b^2)^9)^{(1/2)} + 120a^2b \\
& *c^3d^3e^3*(-(4ac - b^2)^9)^{(1/2)} + 34ab^4c^4d^5e^5*(-(4ac - b^2)^9) \\
& ^{(1/2)} - 108a^2b^2c^2d^2e^4*(-(4ac - b^2)^9)^{(1/2)}/(32*(a^7b^{12}e^ \\
& 8 + 4096a^9c^{10}d^8 + 4096a^{13}c^6e^8 - 24a^8b^{10}c^8e^8 - 4a^6b^{13} \\
& d^7e^7 + a^3b^{12}c^4d^8 - 24a^4b^{10}c^5d^8 + 240a^5b^8c^6d^8 - 1280 \\
& *a^6b^6c^7d^8 + 3840a^7b^4c^8d^8 - 6144a^8b^2c^9d^8 + 240a^9b^ \\
& 8c^2e^8 - 1280a^{10}b^6c^3e^8 + 3840a^{11}b^4c^4e^8 - 6144a^{12}b^2c \\
& ^5e^8 + a^3b^{16}d^4e^4 - 4a^4b^{15}d^3e^5 + 6a^5b^{14}d^2e^6 + 16384 \\
& *a^{10}c^9d^6e^2 + 24576a^{11}c^8d^4e^4 + 16384a^{12}c^7d^2e^6 + 6a^3 \\
& *b^{14}c^2d^6e^2 - 140a^4b^{12}c^3d^6e^2 + 84a^4b^{13}c^2d^5e^3 + 13 \\
& 44a^5b^{10}c^4d^6e^2 - 672a^5b^{11}c^3d^5e^3 - 42a^5b^{12}c^2d^4e^ \\
& 4 - 6720a^6b^8c^5d^6e^2 + 2240a^6b^9c^4d^5e^3 + 1456a^6b^{10}c^3 \\
& *d^4e^4 - 672a^6b^{11}c^2d^3e^5 + 17920a^7b^6c^6d^6e^2 - 10080a^7 \\
& *b^8c^4d^4e^4 + 2240a^7b^9c^3d^3e^5 + 1344a^7b^{10}c^2d^2e^6 - 2 \\
& 1504a^8b^4c^7d^6e^2 - 21504a^8b^5c^6d^5e^3 + 32256a^8b^6c^5d^ \\
& 4e^4 - 6720a^8b^8c^3d^2e^6 + 57344a^9b^3c^7d^5e^3 - 46592a^9b^ \\
& 4c^6d^4e^4 - 21504a^9b^5c^5d^3e^5 + 17920a^9b^6c^4d^2e^6 + 122 \\
& 88a^{10}b^2c^7d^4e^4 + 57344a^{10}b^3c^6d^3e^5 - 21504a^{10}b^4c^5d \\
& ^2e^6 + 96a^7b^{11}c^2d^7e^7 - 16384a^9b^3c^9d^7e^7 - 16384a^{12}b^3c^6d^7e \\
& ^7 - 4a^3b^{13}c^3d^7e^7 - 4a^3b^{15}c^2d^5e^3 + 96a^4b^{11}c^4d^7e^7 - \\
& 12a^4b^{14}c^2d^4e^4 - 960a^5b^9c^5d^7e^7 + 84a^5b^{13}c^3d^3e^5 + 512 \\
& 0a^6b^7c^6d^7e^7 - 140a^6b^{12}c^2d^2e^6 - 15360a^7b^5c^7d^7e^7 + 24 \\
& 576a^8b^3c^8d^7e^7 - 960a^8b^9c^2d^5e^7 + 5120a^9b^7c^3d^5e^7 - 49 \\
& 152a^{10}b^6c^8d^5e^3 - 15360a^{10}b^5c^4d^5e^7 - 49152a^{11}b^4c^7d^3e^ \\
& 5 + 24576a^{11}b^3c^5d^5e^7))^{(1/2)}*(1048576a^{15}c^8e^{17} + 256a^9b^{12} \\
& *c^2e^{17} - 6144a^{10}b^{10}c^3e^{17} + 61440a^{11}b^8c^4e^{17} - 327680a^{12} \\
& *b^6c^5e^{17} + 983040a^{13}b^4c^6e^{17} - 1572864a^{14}b^2c^7e^{17} - 1048 \\
& 576a^8c^{15}d^{14}e^3 - 5242880a^9c^{14}d^{12}e^5 - 9437184a^{10}c^{13}d^{10} \\
& e^7 - 5242880a^{11}c^{12}d^8e^9 + 5242880a^{12}c^{11}d^6e^{11} + 9437184a^{13} \\
& *c^{10}d^4e^{13} + 5242880a^{14}c^9d^2e^{15} + 256a^2b^{11}c^{10}d^{15}e^2 - 2 \\
& 048a^2b^{12}c^9d^{14}e^3 + 7168a^2b^{13}c^8d^{13}e^4 - 14336a^2b^{14}c^7 \\
& *d^{12}e^5 + 17920a^2b^{15}c^6d^{11}e^6 - 14336a^2b^{16}c^5d^{10}e^7 + 716 \\
& 8a^2b^{17}c^4d^9e^8 - 2048a^2b^{18}c^3d^8e^9 + 256a^2b^{19}c^2d^7e \\
& ^{10} - 5120a^3b^9c^{11}d^{15}e^2 + 41984a^3b^{10}c^{10}d^{14}e^3 - 148736a^ \\
& 3b^{11}c^9d^{13}e^4 + 296192a^3b^{12}c^8d^{12}e^5 - 359680a^3b^{13}c^7d^ \\
& 11e^6 + 267520a^3b^{14}c^6d^{10}e^7 - 112384a^3b^{15}c^5d^9e^8 + 18176 \\
& *a^3b^{16}c^4d^8e^9 + 3328a^3b^{17}c^3d^7e^{10} - 1280a^3b^{18}c^2d^6e \\
& ^{11} + 40960a^4b^7c^{12}d^{15}e^2 - 348160a^4b^8c^{11}d^{14}e^3 + 1254400 \\
& *a^4b^9c^{10}d^{13}e^4 - 2478080a^4b^{10}c^9d^{12}e^5 + 2867456a^4b^{11}c \\
& ^8d^{11}e^6 - 1862144a^4b^{12}c^7d^{10}e^7 + 490240a^4b^{13}c^6d^9e^8 + \\
& 128000a^4b^{14}c^5d^8e^9 - 108800a^4b^{15}c^4d^7e^{10} + 13824a^4b^{1 \\
& 6}c^3d^6e^{11} + 2304a^4b^{17}c^2d^5e^{12} - 163840a^5b^5c^{13}d^{15}e^2 \\
& + 1474560a^5b^6c^{12}d^{14}e^3 - 5447680a^5b^7c^{11}d^{13}e^4 + 10588160* \\
& a^5b^8c^{10}d^{12}e^5 - 11166720a^5b^9c^9d^{11}e^6 + 5159936a^5b^{10}c^ \\
& 8d^{10}e^7 + 1073920a^5b^{11}c^7d^9e^8 - 2279680a^5b^{12}c^6d^8e^9 + \\
& 770560a^5b^{13}c^5d^7e^{10} + 33280a^5b^{14}c^4d^6e^{11} - 41216a^5b^{15} \\
& *c^3d^5e^{12} - 1280a^5b^{16}c^2d^4e^{13} + 327680a^6b^3c^{14}d^{15}e^2 - \\
& 3276800a^6b^4c^{13}d^{14}e^3 + 12615680a^6b^5c^{12}d^{13}e^4 - 23592960* \\
& a^6b^6c^{11}d^{12}e^5 + 19701760a^6b^7c^{10}d^{11}e^6 + 1372160a^6b^8c^ \\
& 9d^{10}e^7 - 15846400a^6b^9c^8d^9e^8 + 10864640a^6b^{10}c^7d^8e^9 - \\
& 1352960a^6b^{11}c^6d^7e^{10} - 1111040a^6b^{12}c^5d^6e^{11} + 273920a^6 \\
& *b^{13}c^4d^5e^{12} + 25600a^6b^{14}c^3d^4e^{13} - 1280a^6b^{15}c^2d^3e^
\end{aligned}$$

$$\begin{aligned}
& 14 + 3407872a^7b^2c^{14}d^{14}e^3 - 14221312a^7b^3c^{13}d^{13}e^4 + 23527 \\
& 424a^7b^4c^{12}d^{12}e^5 - 3768320a^7b^5c^{11}d^{11}e^6 - 38895616a^7b^6 \\
& c^{10}d^{10}e^7 + 50126848a^7b^7c^9d^9e^8 - 18362368a^7b^8c^8d^8e^9 \\
& - 6831104a^7b^9c^7d^7e^{10} + 6200320a^7b^{10}c^6d^6e^{11} - 726784a^7 \\
& b^{11}c^5d^5e^{12} - 228608a^7b^{12}c^4d^4e^{13} + 31488a^7b^{13}c^3d^3 \\
& e^{14} + 2304a^7b^{14}c^2d^2e^{15} - 3145728a^8b^2c^{13}d^{12}e^5 - 3112 \\
& 9600a^8b^3c^{12}d^{11}e^6 + 74711040a^8b^4c^{11}d^{10}e^7 - 55476224a^8b^5 \\
& c^{10}d^9e^8 - 11075584a^8b^6c^9d^8e^9 + 35381248a^8b^7c^8d^7e^{10} - 14479360 \\
& a^8b^8c^7d^6e^{11} - 168960a^8b^9c^6d^5e^{12} + 1286144a^8b^{10}c^5d^4e^{13} - 302336 \\
& a^8b^{11}c^4d^3e^{14} - 55808a^8b^{12}c^3d^2e^{15} - 36962304a^9b^2c^{12}d^{10}e^7 - 9502720 \\
& a^9b^3c^{11}d^9e^8 + 67174400a^9b^4c^{10}d^8e^9 - 54886400a^9b^5c^9d^7e^{10} + 11239424 \\
& a^9b^6c^8d^6e^{11} + 5545984a^9b^7c^7d^5e^{12} - 5263360a^9b^8c^6d^4e^{13} + 1356800 \\
& a^9b^9c^5d^3e^{14} + 558080a^9b^{10}c^4d^2e^{15} - 49807360a^{10}b^2c^{11}d^8e^9 + 19333120 \\
& a^{10}b^3c^{10}d^7e^{10} + 7208960a^{10}b^4c^9d^6e^{11} - 14974976a^{10}b^5c^8d^5e^{12} + 15073280 \\
& a^{10}b^6c^7d^4e^{13} - 2170880a^{10}b^7c^6d^3e^{14} - 2928640a^{10}b^8c^5d^2e^{15} - 11796480 \\
& a^{11}b^2c^{10}d^6e^{11} + 23920640a^{11}b^3c^9d^5e^{12} - 24576000a^{11}b^4c^8d^4e^{13} - 4096000 \\
& a^{11}b^5c^7d^3e^{14} + 8355840a^{11}b^6c^6d^2e^{15} + 12582912a^{12}b^2c^9d^4e^{13} + 19857408 \\
& a^{12}b^3c^8d^3e^{14} - 11534336a^{12}b^4c^7d^2e^{15} + 3407872a^{13}b^2c^8d^2e^{15} - 55050 \\
& 24a^{14}b^3c^8d^2e^{16} - 262144a^7b^3c^{15}d^{15}e^2 + 5505024a^8b^3c^{14}d^{13} \\
& e^4 - 1280a^8b^{13}c^2d^2e^{16} + 25952256a^9b^3c^{13}d^{11}e^6 + 30976a^9b^{11} \\
& c^3d^3e^{16} + 38010880a^{10}b^3c^{12}d^9e^8 - 312320a^{10}b^9c^4d^2e^{16} + 11796480 \\
& a^{11}b^3c^{11}d^7e^{10} + 1679360a^{11}b^7c^5d^2e^{16} - 21233664a^{12}b^3c^{10}d^5e^{12} - 5079040 \\
& a^{12}b^5c^6d^2e^{16} - 20709376a^{13}b^3c^9d^3e^{14} + 8192000a^{13}b^3c^7d^2e^{16})) / (8(a^6b^8e^8 + 256 \\
& a^6c^8d^8 + 256a^{10}c^4e^8 - 16a^7b^6c^8e^8 - 4a^5b^9d^8e^7 + a^2b^8c^4d^8 - 16 \\
& a^3b^6c^5d^8 + 96a^4b^4c^6d^8 - 256a^5b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^{12} \\
& d^4e^4 - 4a^3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 + 1024a^7c^7d^6e^2 + 1536a^8c^6d^4e^4 + 1024a^9c^5d^2 \\
& e^6 + 6a^2b^{10}c^2d^6e^2 - 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4 \\
& b^7c^3d^5e^3 - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 - 192 \\
& a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c^4d^4e^4 - 128a^6b^5c^3d^3e^5 + 51 \\
& 2a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 + 2048a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024 \\
& a^6b^3c^7d^7e + 64a^6b^7c^3d^5e^7 - 1024a^9b^3c^4d^5e^7 - 4a^2b^9c^3d^7e - 4a^2b^{11}c^3d^5e^3 + 64 \\
& a^3b^7c^4d^7e - 4a^3b^{10}c^3d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^3d^3e^5 + 1024a^5b^3c^6d^7e - 92a^5b^8c^2d^2e^6 - 3072 \\
& a^7b^3c^6d^5e^3 - 384a^7b^5c^2d^5e^7 - 3072a^8b^3c^5d^3e^5 + 1024a^8b^3c^3d^3e^7)) * ((27a^9c^5d^6 - b^{11}c^4d^6 - b^{15} \\
& d^2e^4 - 9a^2b^{13}e^6 + 3840a^5b^3c^9d^6 - 9a^5c^5d^6 * (- (4a^3c - b^2)^9)^{(1/2)} + 213a^3b^{11}c^6e^6 - 26880 \\
& a^8b^3c^6e^6 + 3072a^6c^9d^5e + 35840a^8c^7d^5e^5 + 4b^{12}c^3d^5e + 4b^{14}c^3d^3e^3 - 288a^2b^7c^6d^6 + 1504 \\
& a^3b^5c^7d^6 - 3840a^4b^3c^8d^6 + 9a^2b^4e^6 * (- (4a^3c - b^2)^9)^{(1/2)} - 2077a^4b^9c^2e^6 + 10656a^5b^7c^3e^6 - 30240 \\
& a^6b^5c^4e^6 + 44800a^7b^3c^5e^6 + 25a^4c^2e^6 * (- (4a^3c - b^2)^9)^{(1/2)} + b^2c^4d^6 * (- (4a^3c - b^2)^9)^{(1/2)} + 22528 \\
& a^7c^8d^3e^3 + b^6d^2e^4 * (- (4a^3c - b^2)^9)^{(1/2)} - 6b^{13}c^2d^4e^2 + 6a^2b^{14}d^5e^5 - 1471a^2b^9c^4d^4e^2 + 600 \\
& a^2b^{10}c^3d^3e^3 + 180a^2b^{11}c^2d^2e^4 + 6976a^3b^7c^5d^4e^2 - 1032a^3b^8c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456 \\
& a^4b^5c^6d^4e^2 - 7168a^4b^6c^5d^3e^3 + 16896a^4b^7c^4d^2e^4 + 10240a^5b^3c^7d^4e^2 + 37632a^5b^4c^6d^3e^3 - 47712 \\
& a^5b^5c^5d^2e^4 - 59392a^6b^2c^7d^3e^3 + 60928a^6b^3c^6d^2e^4 - 41a^2c^4d^4e^2 * (- (4a^3c - b^2)^9)^{(1/2)} - 39a^3c^3d^2e^4 * (- (4a^3c - b^2)^9)^{(1/2)} + 6b^4c^2d^4e^2 * (- (4a^3c - b^2)^9)^{(1/2)} - 6a^2b^5d^5e^5 * (- (4a^3c - b^2)^9)^{(1/2)} - 106a^2b^{10}c^4d^5e + 7a^2b^{13}c^3d^2e^5
\end{aligned}$$

$$\begin{aligned}
& 4 - 128a^2b^{12}c^3d^4e^5 - 51a^3b^2c^4e^6(-4ac - b^2)^9)^{(1/2)} + 150* \\
& a^3b^{11}c^3d^4e^2 - 84a^2b^{12}c^2d^3e^3 + 1116a^2b^8c^5d^5e - 5824* \\
& a^3b^6c^6d^5e + 1030a^3b^{10}c^2d^4e^5 + 15232a^4b^4c^7d^5e - 349 \\
& 2a^4b^8c^3d^4e^5 - 16896a^5b^2c^8d^5e + 1344a^5b^6c^4d^4e^5 + 74 \\
& 24a^6b^3c^8d^4e^2 + 22400a^6b^4c^5d^4e^5 - 23296a^7b^3c^7d^2e^4 - \\
& 53760a^7b^2c^6d^4e^5 - 4b^3c^3d^5e^6(-4ac - b^2)^9)^{(1/2)} - 4b^5* \\
& c^3d^3e^3(-4ac - b^2)^9)^{(1/2)} + 11a^2b^4c^3d^2e^4(-4ac - b^2)^9)^{(1/2)} \\
& + 20a^2b^3c^3d^3e^5(-4ac - b^2)^9)^{(1/2)} + 86a^3b^3c^2d^3e^5(- \\
& (4ac - b^2)^9)^{(1/2)} - 42a^2b^2c^3d^4e^2(-4ac - b^2)^9)^{(1/2)} + 12 \\
& *a^2b^3c^2d^3e^3(-4ac - b^2)^9)^{(1/2)} + 120a^2b^3c^3d^3e^3(-4ac \\
& c - b^2)^9)^{(1/2)} + 34a^2b^3c^4d^5e^6(-4ac - b^2)^9)^{(1/2)} - 108a^2b^2 \\
& *c^2d^2e^4(-4ac - b^2)^9)^{(1/2)}/(32*(a^7b^{12}e^8 + 4096a^9c^{10}d^ \\
& 8 + 4096a^{13}c^6e^8 - 24a^8b^{10}c^4e^8 - 4a^6b^{13}d^4e^7 + a^3b^{12}c^4 \\
& *d^8 - 24a^4b^{10}c^5d^8 + 240a^5b^8c^6d^8 - 1280a^6b^6c^7d^8 + 3 \\
& 840a^7b^4c^8d^8 - 6144a^8b^2c^9d^8 + 240a^9b^8c^2e^8 - 1280a^1 \\
& 0b^6c^3e^8 + 3840a^{11}b^4c^4e^8 - 6144a^{12}b^2c^5e^8 + a^3b^{16}d^ \\
& 4e^4 - 4a^4b^{15}d^3e^5 + 6a^5b^{14}d^2e^6 + 16384a^{10}c^9d^6e^2 + \\
& 24576a^{11}c^8d^4e^4 + 16384a^{12}c^7d^2e^6 + 6a^3b^{14}c^2d^6e^2 - \\
& 140a^4b^{12}c^3d^6e^2 + 84a^4b^{13}c^2d^5e^3 + 1344a^5b^{10}c^4d^6* \\
& e^2 - 672a^5b^{11}c^3d^5e^3 - 42a^5b^{12}c^2d^4e^4 - 6720a^6b^8c^5 \\
& *d^6e^2 + 2240a^6b^9c^4d^5e^3 + 1456a^6b^{10}c^3d^4e^4 - 672a^6b \\
& ^{11}c^2d^3e^5 + 17920a^7b^6c^6d^6e^2 - 10080a^7b^8c^4d^4e^4 + 2 \\
& 240a^7b^9c^3d^3e^5 + 1344a^7b^{10}c^2d^2e^6 - 21504a^8b^4c^7d^6 \\
& *e^2 - 21504a^8b^5c^6d^5e^3 + 32256a^8b^6c^5d^4e^4 - 6720a^8b^8 \\
& *c^3d^2e^6 + 57344a^9b^3c^7d^5e^3 - 46592a^9b^4c^6d^4e^4 - 2150 \\
& 4a^9b^5c^5d^3e^5 + 17920a^9b^6c^4d^2e^6 + 12288a^{10}b^2c^7d^4* \\
& e^4 + 57344a^{10}b^3c^6d^3e^5 - 21504a^{10}b^4c^5d^2e^6 + 96a^7b^{11} \\
& *c^3d^4e^7 - 16384a^9b^3c^9d^7e - 16384a^{12}b^3c^6d^4e^7 - 4a^3b^{13}c^3* \\
& d^7e - 4a^3b^{15}c^3d^5e^3 + 96a^4b^{11}c^4d^7e - 12a^4b^{14}c^3d^4e^ \\
& 4 - 960a^5b^9c^5d^7e + 84a^5b^{13}c^3d^3e^5 + 5120a^6b^7c^6d^7e \\
& - 140a^6b^{12}c^3d^2e^6 - 15360a^7b^5c^7d^7e + 24576a^8b^3c^8d^7* \\
& e - 960a^8b^9c^2d^4e^7 + 5120a^9b^7c^3d^4e^7 - 49152a^{10}b^3c^8d^5e \\
& ^3 - 15360a^{10}b^5c^4d^4e^7 - 49152a^{11}b^3c^7d^3e^5 + 24576a^{11}b^3c \\
& ^5d^4e^7))^{(1/2)} + (x*(626688a^{10}b^3c^8e^{15} - 784384a^{10}c^9d^4e^{14} + 2 \\
& 08a^4b^{13}c^2e^{15} - 4880a^5b^{11}c^3e^{15} + 47312a^6b^9c^4e^{15} - 24 \\
& 2176a^7b^7c^5e^{15} + 688640a^8b^5c^6e^{15} - 1028096a^9b^3c^7e^{15} \\
& + 18432a^4c^{15}d^{13}e^2 + 126976a^5c^{14}d^{11}e^4 + 325632a^6c^{13}d^9* \\
& e^6 + 139264a^7c^{12}d^7e^8 - 1067008a^8c^{11}d^5e^{10} - 1773568a^9c^{1 \\
& 0}d^3e^{12} + 16b^8c^{11}d^{13}e^2 - 96b^9c^{10}d^{12}e^3 + 240b^{10}c^9d^{1 \\
& 1}e^4 - 304b^{11}c^8d^{10}e^5 + 144b^{12}c^7d^9e^6 + 144b^{13}c^6d^8e^7 \\
& - 304b^{14}c^5d^7e^8 + 240b^{15}c^4d^6e^9 - 96b^{16}c^3d^5e^{10} + 16* \\
& b^{17}c^2d^4e^{11} + 3200a^2b^4c^{13}d^{13}e^2 - 18432a^2b^5c^{12}d^{12}e^ \\
& 3 + 41024a^2b^6c^{11}d^{11}e^4 - 36352a^2b^7c^{10}d^{10}e^5 - 16208a^2b \\
& ^8c^9d^9e^6 + 74576a^2b^9c^8d^8e^7 - 78496a^2b^{10}c^7d^7e^8 + 3 \\
& 2064a^2b^{11}c^6d^6e^9 + 6000a^2b^{12}c^5d^5e^{10} - 9264a^2b^{13}c^4* \\
& d^4e^{11} + 1472a^2b^{14}c^3d^3e^{12} + 416a^2b^{15}c^2d^2e^{13} - 12800a \\
& ^3b^2c^{14}d^{13}e^2 + 73728a^3b^3c^{13}d^{12}e^3 - 151296a^3b^4c^{12}d^ \\
& 11e^4 + 78336a^3b^5c^{11}d^{10}e^5 + 206688a^3b^6c^{10}d^9e^6 - 436736 \\
& *a^3b^7c^9d^8e^7 + 324224a^3b^8c^8d^7e^8 + 992a^3b^9c^7d^6e^9 \\
& - 158176a^3b^{10}c^6d^5e^{10} + 77056a^3b^{11}c^5d^4e^{11} + 6912a^3b^ \\
& 12c^4d^3e^{12} - 8416a^3b^{13}c^3d^2e^{13} + 162816a^4b^2c^{13}d^{11}e^4 \\
& + 184320a^4b^3c^{12}d^{10}e^5 - 916608a^4b^4c^{11}d^9e^6 + 1165824a^4 \\
& *b^5c^{10}d^8e^7 - 314496a^4b^6c^9d^7e^8 - 822272a^4b^7c^8d^6e^9 \\
& + 919152a^4b^8c^7d^5e^{10} - 175296a^4b^9c^6d^4e^{11} - 189328a^4b \\
& ^{10}c^5d^3e^{12} + 62064a^4b^{11}c^4d^2e^{13} + 1290752a^5b^2c^{12}d^9e \\
& ^6 - 659456a^5b^3c^{11}d^8e^7 - 1561088a^5b^4c^{10}d^7e^8 + 3240960a \\
& ^5b^5c^9d^6e^9 - 1964192a^5b^6c^8d^5e^{10} - 683008a^5b^7c^7d^4* \\
& e^{11} + 1162304a^5b^8c^6d^3e^{12} - 164112a^5b^9c^5d^2e^{13} + 3442688 \\
& *a^6b^2c^{11}d^7e^8 - 3670016a^6b^3c^{10}d^6e^9 + 15232a^6b^4c^9d^
\end{aligned}$$

$$\begin{aligned}
&5e^{10} + 4230144a^6b^5c^8d^4e^{11} - 3059648a^6b^6c^7d^3e^{12} - 2472 \\
&96a^6b^7c^6d^2e^{13} + 4010496a^7b^2c^{10}d^5e^{10} - 6873088a^7b^3c^9 \\
&d^4e^{11} + 2822400a^7b^4c^8d^3e^{12} + 2370048a^7b^5c^7d^2e^{13} + \\
&1178624a^8b^2c^9d^3e^{12} - 4739072a^8b^3c^8d^2e^{13} - 352a^8b^4c^7 \\
&d^12e^{13} + 2048a^8b^5c^6d^11e^{13} - 4800a^8b^6c^5d^10e^{14} + 5168a^8 \\
&b^7c^4d^9e^{15} - 480a^8b^8c^3d^8e^{16} - 6000a^8b^9c^2d^7e^{17} + 8192a^9 \\
&a^9b^12c^6d^7e^{18} - 5040a^9b^13c^5d^6e^{19} + 1152a^9b^14c^4d^5e^{20} + 2 \\
&40a^9b^15c^3d^4e^{21} - 128a^9b^16c^2d^3e^{22} - 512a^9b^17c^1d^2e^{23} \\
&- 106496a^9b^18c^14d^12e^{23} + 11680a^9b^19c^13d^11e^{24} - 675840a^9b^20 \\
&c^12d^10e^{25} - 108288a^9b^21c^11d^9e^{26} - 1601536a^9b^22c^10d^8e^{27} + 5147 \\
&68a^9b^23c^9d^7e^{29} - 925696a^9b^24c^8d^6e^{30} - 1278304a^9b^25c^7d^5 \\
&e^{31} + 2457600a^9b^26c^6d^4e^{33} + 1385600a^9b^27c^5d^3e^{34} + 2977792a^9 \\
&b^28c^4d^2e^{35} + 19968a^9b^29c^3d^1e^{36}))/((8(a^6b^8e^8 + 256a^6c^8 \\
&d^8 + 256a^10c^4e^8 - 16a^7b^6c^8e^8 - 4a^5b^9d^8e^7 + a^2b^8c^4d^8 - 16a^3 \\
&b^6c^5d^8 + 96a^4b^4c^6d^8 - 256a^5b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2 \\
&c^3e^8 + a^2b^12d^4e^4 - 4a^3b^11d^3e^5 + 6a^4b^10d^2e^6 + 1024a^7c^7d^6e^2 + \\
&1536a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^10c^2d^6e^2 - 92a^3b^8c^3d^6e^2 + \\
&52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4b^7c^3d^5e^3 - 90a^4b^8c^2 \\
&d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 - \\
&192a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4 \\
&c^4d^4e^4 - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 + \\
&2048a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024a^6b^7 \\
&c^7d^7e^7 + 64a^6b^7c^6d^7e^7 - 1024a^9b^6c^4d^7e^7 - 4a^2b^9c^3d^7e^7 - \\
&4a^2b^11c^5d^5e^3 + 64a^3b^7c^4d^7e^7 - 4a^3b^10c^4d^4e^4 - 384a^4b^5c^5 \\
&d^7e^7 + 52a^4b^9c^3d^5e^5 + 1024a^5b^3c^6d^7e^7 - 92a^5b^8c^5d^2e^6 - \\
&3072a^7b^6c^6d^5e^3 - 384a^7b^5c^2d^7e^7 - 3072a^8b^4c^5d^3e^5 + 1024a^8 \\
&b^3c^3d^7e^7))((27a^9b^9c^5d^6 - b^11c^4d^6 - b^15d^2e^4 - 9a^2b^13e^6 + \\
&3840a^5b^6c^9d^6 - 9a^5c^5d^6(-4a^2c - b^2)^9)^{(1/2)} + 213a^3b^11c^5e^6 - \\
&26880a^8b^6c^6e^6 + 3072a^6c^9d^5e^6 + 35840a^8c^7d^5e^6 + 4b^12c^3d^5e^6 + \\
&4b^14c^3d^3e^6 - 288a^2b^7c^6d^6 + 1504a^3b^5c^7d^6 - 3840a^4b^3c^8d^6 + 9a^2b^4 \\
&e^6(-4a^2c - b^2)^9)^{(1/2)} - 2077a^4b^9c^2e^6 + 10656a^5b^7c^3e^6 - 30240a^6 \\
&b^5c^4e^6 + 44800a^7b^3c^5e^6 + 25a^4c^2e^6(-4a^2c - b^2)^9)^{(1/2)} + b^2c^4 \\
&d^6(-4a^2c - b^2)^9)^{(1/2)} + 22528a^7c^8d^3e^3 + b^6d^2e^4(-4a^2c - b^2)^9)^{(1/2)} \\
&- 6b^13c^2d^4e^2 + 6a^8b^14d^5e^5 - 1471a^2b^9c^4d^4e^2 + 600a^2b^10c^3d^3e^3 + \\
&180a^2b^11c^2d^2e^4 + 6976a^3b^7c^5d^4e^2 - 1032a^3b^8c^4d^3e^3 - 2871a^3b^9 \\
&c^3d^2e^4 - 15456a^4b^5c^6d^4e^2 - 7168a^4b^6c^5d^3e^3 + 16896a^4b^7c^4d^2e^4 + \\
&10240a^5b^3c^7d^4e^2 + 37632a^5b^4c^6d^3e^3 - 47712a^5b^5c^5d^2e^4 - 59392a^6 \\
&b^2c^7d^3e^3 + 60928a^6b^3c^6d^2e^4 - 41a^2c^4d^4e^2(-4a^2c - b^2)^9)^{(1/2)} - \\
&39a^3c^3d^2e^4(-4a^2c - b^2)^9)^{(1/2)} + 6b^4c^2d^4e^2(-4a^2c - b^2)^9)^{(1/2)} - \\
&6a^8b^5d^5e^5(-4a^2c - b^2)^9)^{(1/2)} - 106a^8b^10c^4d^5e^5 + 7a^8b^13c^3d^2 \\
&e^4 - 128a^2b^12c^6d^5e^5 - 51a^3b^2c^6e^6(-4a^2c - b^2)^9)^{(1/2)} + 150a^8b^11 \\
&c^3d^4e^2 - 84a^8b^12c^2d^3e^3 + 1116a^2b^8c^5d^5e^5 - 5824a^3b^6c^6d^5e^5 + \\
&1030a^3b^10c^2d^5e^5 + 15232a^4b^4c^7d^5e^5 - 3492a^4b^8c^3d^5e^5 - 16896a^5 \\
&b^2c^8d^5e^5 + 1344a^5b^6c^4d^5e^5 + 7424a^6b^6c^8d^4e^2 + 22400a^6b^4c^5d^5e^5 - \\
&23296a^7b^6c^7d^2e^4 - 53760a^7b^2c^6d^5e^5 - 4b^3c^3d^5e^5(-4a^2c - b^2)^9)^{(1/2)} \\
&- 4b^5c^3d^3e^3(-4a^2c - b^2)^9)^{(1/2)} + 11a^8b^4c^6d^2e^4(-4a^2c - b^2)^9)^{(1/2)} \\
&+ 20a^2b^3c^6d^5e^5(-4a^2c - b^2)^9)^{(1/2)} + 86a^3b^3c^2d^5e^5(-4a^2c - b^2)^9)^{(1/2)} \\
&- 42a^8b^2c^3d^4e^2(-4a^2c - b^2)^9)^{(1/2)} + 12a^8b^3c^2d^3e^3(-4a^2c - b^2)^9)^{(1/2)} \\
&+ 120a^2b^3c^3d^3e^3(-4a^2c - b^2)^9)^{(1/2)} + 34a^8b^4c^4d^5e^5(-4a^2c - b^2)^9)^{(1/2)} - \\
&108a^2b^2c^2d^2e^4(-4a^2c - b^2)^9)^{(1/2))}/(32(a^7b^12e^8 + 4096a^9c^10d^8 + \\
&4096a^13c^6e^8 - 24a^8b^10c^6e^8 - 4a^6b^13d^7e^7 + a^3b^12c^4d^8 - 24a^4b^10 \\
&c^5d^8 + 240a^5b^8c^6d^8 - 1280a^6b^6c^7
\end{aligned}$$

$$\begin{aligned}
& *d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - \\
& 1280*a^{10}*b^6*c^3*e^8 + 3840*a^{11}*b^4*c^4*e^8 - 6144*a^{12}*b^2*c^5*e^8 + a^3 \\
& *b^{16}*d^4*e^4 - 4*a^4*b^{15}*d^3*e^5 + 6*a^5*b^{14}*d^2*e^6 + 16384*a^{10}*c^9*d^ \\
& 6*e^2 + 24576*a^{11}*c^8*d^4*e^4 + 16384*a^{12}*c^7*d^2*e^6 + 6*a^3*b^{14}*c^2*d^ \\
& 6*e^2 - 140*a^4*b^{12}*c^3*d^6*e^2 + 84*a^4*b^{13}*c^2*d^5*e^3 + 1344*a^5*b^{10}* \\
& c^4*d^6*e^2 - 672*a^5*b^{11}*c^3*d^5*e^3 - 42*a^5*b^{12}*c^2*d^4*e^4 - 6720*a^6 \\
& *b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^{10}*c^3*d^4*e^4 - 6 \\
& 72*a^6*b^{11}*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4 \\
& *e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^{10}*c^2*d^2*e^6 - 21504*a^8*b^4 \\
& *c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720 \\
& *a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^ \\
& 4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^{10}*b^2* \\
& c^7*d^4*e^4 + 57344*a^{10}*b^3*c^6*d^3*e^5 - 21504*a^{10}*b^4*c^5*d^2*e^6 + 96* \\
& a^7*b^{11}*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^{12}*b*c^6*d*e^7 - 4*a^3*b \\
& ^{13}*c^3*d^7*e - 4*a^3*b^{15}*c*d^5*e^3 + 96*a^4*b^{11}*c^4*d^7*e - 12*a^4*b^{14}* \\
& c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^{13}*c*d^3*e^5 + 5120*a^6*b^7*c^ \\
& 6*d^7*e - 140*a^6*b^{12}*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3* \\
& c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^{10}*b*c \\
& ^8*d^5*e^3 - 15360*a^{10}*b^5*c^4*d*e^7 - 49152*a^{11}*b*c^7*d^3*e^5 + 24576*a^ \\
& 11*b^3*c^5*d*e^7))^{(1/2)} - (326912*a^8*c^9*d*e^{13} - 241664*a^8*b*c^8*e^{14} \\
& - 48*a^2*b^{13}*c^2*e^{14} + 1264*a^3*b^{11}*c^3*e^{14} - 13552*a^4*b^9*c^4*e^{14} + \\
& 75776*a^5*b^7*c^5*e^{14} - 232960*a^6*b^5*c^6*e^{14} + 372736*a^7*b^3*c^7*e^{14} \\
& + 11520*a^3*c^{14}*d^{11}*e^3 + 78080*a^4*c^{13}*d^9*e^5 + 197120*a^5*c^{12}*d^7*e^ \\
& 7 + 336384*a^6*c^{11}*d^5*e^9 + 532736*a^7*c^{10}*d^3*e^{11} - 40*b^5*c^{12}*d^{12}*e \\
& ^2 + 216*b^6*c^{11}*d^{11}*e^3 - 464*b^7*c^{10}*d^{10}*e^4 + 496*b^8*c^9*d^9*e^5 - \\
& 264*b^9*c^8*d^8*e^6 + 56*b^{10}*c^7*d^7*e^7 - 16*b^{11}*c^6*d^6*e^8 + 64*b^{12}*c \\
& ^5*d^5*e^9 - 96*b^{13}*c^4*d^4*e^{10} + 64*b^{14}*c^3*d^3*e^{11} - 16*b^{15}*c^2*d^2* \\
& e^{12} + 1536*a^2*b^2*c^{13}*d^{11}*e^3 + 14400*a^2*b^3*c^{12}*d^{10}*e^4 - 47152*a^2 \\
& *b^4*c^{11}*d^9*e^5 + 52144*a^2*b^5*c^{10}*d^8*e^6 - 16272*a^2*b^6*c^9*d^7*e^7 - \\
& 13040*a^2*b^7*c^8*d^6*e^8 + 23488*a^2*b^8*c^7*d^5*e^9 - 26384*a^2*b^9*c^6 \\
& *d^4*e^{10} + 13824*a^2*b^{10}*c^5*d^3*e^{11} + 256*a^2*b^{11}*c^4*d^2*e^{12} + 12505 \\
& 6*a^3*b^2*c^{12}*d^9*e^5 - 36224*a^3*b^3*c^{11}*d^8*e^6 - 126432*a^3*b^4*c^{10}*d \\
& ^7*e^7 + 144848*a^3*b^5*c^9*d^6*e^8 - 114752*a^3*b^6*c^8*d^5*e^9 + 125392*a \\
& ^3*b^7*c^7*d^4*e^{10} - 53248*a^3*b^8*c^6*d^3*e^{11} - 25264*a^3*b^9*c^5*d^2*e^ \\
& 12 + 474112*a^4*b^2*c^{11}*d^7*e^7 - 191104*a^4*b^3*c^{10}*d^6*e^8 + 97184*a^4* \\
& b^4*c^9*d^5*e^9 - 277000*a^4*b^5*c^8*d^4*e^{10} + 56056*a^4*b^6*c^7*d^3*e^{11} \\
& + 195584*a^4*b^7*c^6*d^2*e^{12} + 236800*a^5*b^2*c^{10}*d^5*e^9 + 388032*a^5*b^ \\
& 3*c^9*d^4*e^{10} + 159632*a^5*b^4*c^8*d^3*e^{11} - 670488*a^5*b^5*c^7*d^2*e^{12} \\
& - 488960*a^6*b^2*c^9*d^3*e^{11} + 1106496*a^6*b^3*c^8*d^2*e^{12} + 64*a*b^{14}*c^ \\
& 2*d*e^{13} + 448*a*b^3*c^{13}*d^{12}*e^2 - 1968*a*b^4*c^{12}*d^{11}*e^3 + 2504*a*b^5* \\
& c^{11}*d^{10}*e^4 + 768*a*b^6*c^{10}*d^9*e^5 - 4368*a*b^7*c^9*d^8*e^6 + 3568*a*b^ \\
& 8*c^8*d^7*e^7 - 520*a*b^9*c^7*d^6*e^8 - 1728*a*b^{10}*c^6*d^5*e^9 + 2528*a*b^ \\
& 11*c^5*d^4*e^{10} - 1536*a*b^{12}*c^4*d^3*e^{11} + 240*a*b^{13}*c^3*d^2*e^{12} - 1152 \\
& *a^2*b*c^{14}*d^{12}*e^2 - 1600*a^2*b^{12}*c^3*d*e^{13} - 67968*a^3*b*c^{13}*d^{10}*e^4 \\
& + 15808*a^3*b^{10}*c^4*d*e^{13} - 342272*a^4*b*c^{12}*d^8*e^6 - 76928*a^4*b^8*c^ \\
& 5*d*e^{13} - 569088*a^5*b*c^{11}*d^6*e^8 + 179200*a^5*b^6*c^6*d*e^{13} - 586368*a \\
& ^6*b*c^{10}*d^4*e^{10} - 113008*a^6*b^4*c^7*d*e^{13} - 731008*a^7*b*c^9*d^2*e^{12} \\
& - 244096*a^7*b^2*c^8*d*e^{13})/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^{10}* \\
& c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6 \\
& *c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - \\
& 256*a^9*b^2*c^3*e^8 + a^2*b^{12}*d^4*e^4 - 4*a^3*b^{11}*d^3*e^5 + 6*a^4*b^{10}*d^ \\
& 2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 \\
& + 6*a^2*b^{10}*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 \\
& + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^ \\
& 4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^ \\
& 4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^ \\
& 5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^ \\
& 6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152* \\
& a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a
\end{aligned}$$



$$\begin{aligned}
& ^6b^7c^d^e^7 - 1024a^9b^c^4d^e^7 - 4a^2b^9c^3d^7e - 4a^2b^{11}c^d^5e^3 + 64a^3b^7c^4d^7e - 4a^3b^{10}c^d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^d^3e^5 + 1024a^5b^3c^6d^7e - 92a^5b^8c^d^2e^6 - 3072a^7b^c^6d^5e^3 - 384a^7b^5c^2d^e^7 - 3072a^8b^c^5d^3e^5 + 1024a^8b^3c^3d^e^7)) * ((27a^9b^c^5d^6 - b^{11}c^4d^6 - b^{15}d^2e^4 - 9a^2b^{13}e^6 + 3840a^5b^c^9d^6 - 9a^c^5d^6 * (-4a^c - b^2)^9)^{(1/2)} + 213a^3b^{11}c^e^6 - 26880a^8b^c^6e^6 + 3072a^6c^9d^5e + 35840a^8c^7d^e^5 + 4b^{12}c^3d^5e + 4b^{14}c^d^3e^3 - 288a^2b^7c^6d^6 + 1504a^3b^5c^7d^6 - 3840a^4b^3c^8d^6 + 9a^2b^4e^6 * (-4a^c - b^2)^9)^{(1/2)} - 2077a^4b^9c^2e^6 + 10656a^5b^7c^3e^6 - 30240a^6b^5c^4e^6 + 44800a^7b^3c^5e^6 + 25a^4c^2e^6 * (-4a^c - b^2)^9)^{(1/2)} + b^2c^4d^6 * (-4a^c - b^2)^9)^{(1/2)} + 22528a^7c^8d^3e^3 + b^6d^2e^4 * (-4a^c - b^2)^9)^{(1/2)} - 6b^{13}c^2d^4e^2 + 6a^b^{14}d^e^5 - 1471a^2b^9c^4d^4e^2 + 600a^2b^{10}c^3d^3e^3 + 180a^2b^{11}c^2d^2e^4 + 6976a^3b^7c^5d^4e^2 - 1032a^3b^8c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456a^4b^5c^6d^4e^2 - 7168a^4b^6c^5d^3e^3 + 16896a^4b^7c^4d^2e^4 + 10240a^5b^3c^7d^4e^2 + 37632a^5b^4c^6d^3e^3 - 47712a^5b^5c^5d^2e^4 - 59392a^6b^2c^7d^3e^3 + 60928a^6b^3c^6d^2e^4 - 41a^2c^4d^4e^2 * (-4a^c - b^2)^9)^{(1/2)} - 39a^3c^3d^2e^4 * (-4a^c - b^2)^9)^{(1/2)} + 6b^4c^2d^4e^2 * (-4a^c - b^2)^9)^{(1/2)} - 6a^b^5d^e^5 * (-4a^c - b^2)^9)^{(1/2)} - 106a^b^{10}c^4d^5e + 7a^b^{13}c^d^2e^4 - 128a^2b^{12}c^d^e^5 - 51a^3b^2c^e^6 * (-4a^c - b^2)^9)^{(1/2)} + 150a^b^{11}c^3d^4e^2 - 84a^b^{12}c^2d^3e^3 + 1116a^2b^8c^5d^5e - 5824a^3b^6c^6d^5e + 1030a^3b^{10}c^2d^e^5 + 15232a^4b^4c^7d^5e - 3492a^4b^8c^3d^e^5 - 16896a^5b^2c^8d^5e + 1344a^5b^6c^4d^e^5 + 7424a^6b^c^8d^4e^2 + 22400a^6b^4c^5d^e^5 - 23296a^7b^c^7d^2e^4 - 53760a^7b^2c^6d^e^5 - 4b^3c^3d^5e * (-4a^c - b^2)^9)^{(1/2)} - 4b^5c^d^3e^3 * (-4a^c - b^2)^9)^{(1/2)} + 11a^b^4c^d^2e^4 * (-4a^c - b^2)^9)^{(1/2)} + 20a^2b^3c^d^e^5 * (-4a^c - b^2)^9)^{(1/2)} + 86a^3b^c^2d^e^5 * (-4a^c - b^2)^9)^{(1/2)} - 42a^b^2c^3d^4e^2 * (-4a^c - b^2)^9)^{(1/2)} + 12a^b^3c^2d^3e^3 * (-4a^c - b^2)^9)^{(1/2)} + 120a^2b^c^3d^3e^3 * (-4a^c - b^2)^9)^{(1/2)} + 34a^b^c^4d^5e * (-4a^c - b^2)^9)^{(1/2)} - 108a^2b^2c^2d^2e^4 * (-4a^c - b^2)^9)^{(1/2)}) / (32(a^7b^{12}e^8 + 4096a^9c^{10}d^8 + 4096a^{13}c^6e^8 - 24a^8b^{10}c^e^8 - 4a^6b^{13}d^e^7 + a^3b^{12}c^4d^8 - 24a^4b^{10}c^5d^8 + 240a^5b^8c^6d^8 - 1280a^6b^6c^7d^8 + 3840a^7b^4c^8d^8 - 6144a^8b^2c^9d^8 + 240a^9b^8c^2e^8 - 1280a^{10}b^6c^3e^8 + 3840a^{11}b^4c^4e^8 - 6144a^{12}b^2c^5e^8 + a^3b^{16}d^4e^4 - 4a^4b^{15}d^3e^5 + 6a^5b^{14}d^2e^6 + 16384a^{10}c^9d^6e^2 + 24576a^{11}c^8d^4e^4 + 16384a^{12}c^7d^2e^6 + 6a^3b^{14}c^2d^6e^2 - 140a^4b^{12}c^3d^6e^2 + 84a^4b^{13}c^2d^5e^3 + 1344a^5b^{10}c^4d^6e^2 - 672a^5b^{11}c^3d^5e^3 - 42a^5b^{12}c^2d^4e^4 - 6720a^6b^8c^5d^6e^2 + 2240a^6b^9c^4d^5e^3 + 1456a^6b^{10}c^3d^4e^4 - 672a^6b^{11}c^2d^3e^5 + 17920a^7b^6c^6d^6e^2 - 10080a^7b^8c^4d^4e^4 + 2240a^7b^9c^3d^3e^5 + 1344a^7b^{10}c^2d^2e^6 - 21504a^8b^4c^7d^6e^2 - 21504a^8b^5c^6d^5e^3 + 32256a^8b^6c^5d^4e^4 - 6720a^8b^8c^3d^2e^6 + 57344a^9b^3c^7d^5e^3 - 46592a^9b^4c^6d^4e^4 - 21504a^9b^5c^5d^3e^5 + 17920a^9b^6c^4d^2e^6 + 12288a^{10}b^2c^7d^4e^4 + 57344a^{10}b^3c^6d^3e^5 - 21504a^{10}b^4c^5d^2e^6 + 96a^7b^{11}c^d^e^7 - 16384a^9b^c^9d^7e - 16384a^{12}b^c^6d^e^7 - 4a^3b^{13}c^3d^7e - 4a^3b^{15}c^d^5e^3 + 96a^4b^{11}c^4d^7e - 12a^4b^{14}c^d^4e^4 - 960a^5b^9c^5d^7e + 84a^5b^{13}c^d^3e^5 + 5120a^6b^7c^6d^7e - 140a^6b^{12}c^d^2e^6 - 15360a^7b^5c^7d^7e + 24576a^8b^3c^8d^7e - 960a^8b^9c^2d^e^7 + 5120a^9b^7c^3d^e^7 - 49152a^{10}b^c^8d^5e^3 - 15360a^{10}b^5c^4d^e^7 - 49152a^{11}b^c^7d^3e^5 + 24576a^{11}b^3c^5d^e^7))^{(1/2)} + (x * (22800a^6c^9e^{13} + 36a^2b^8c^5e^{13} - 600a^3b^6c^6e^{13} + 4313a^4b^4c^7e^{13} - 15592a^5b^2c^8e^{13} + 1296a^2c^{13}d^8e^5 + 9792a^3c^{12}d^6e^7 + 30304a^4c^{11}d^4e^9 + 40512a^5c^{10}d^2e^{11} + 25b^4c^{11}d^8e^5 - 120b^5c^{10}d^7e^6 + 214b^6c^9d^6e^7 - 168b^7c^8d^5e^8 + 53b^8c^7d^4e^9 - 8b^9c^6d^3e^{10} + 4b^{10}c^5d^2
\end{aligned}$$

$$\begin{aligned}
& *e^{11} + 6336*a^2*b^2*c^{11}*d^6*e^7 + 3840*a^2*b^3*c^{10}*d^5*e^8 - 8506*a^2*b^4*c^9*d^4*e^9 + 1112*a^2*b^5*c^8*d^3*e^{10} + 1254*a^2*b^6*c^7*d^2*e^{11} + 222 \\
& 24*a^3*b^2*c^{10}*d^4*e^9 + 13824*a^3*b^3*c^9*d^3*e^{10} - 9516*a^3*b^4*c^8*d^2*e^{11} + 11712*a^4*b^2*c^9*d^2*e^{11} - 24*a*b^9*c^5*d*e^{12} - 41088*a^5*b*c^9* \\
& d*e^{12} - 360*a*b^2*c^{12}*d^8*e^5 + 1664*a*b^3*c^{11}*d^7*e^6 - 2604*a*b^4*c^{10}*d^6*e^7 + 1272*a*b^5*c^9*d^5*e^8 + 332*a*b^6*c^8*d^4*e^9 - 232*a*b^7*c^7*d \\
& ^3*e^{10} - 48*a*b^8*c^6*d^2*e^{11} - 5760*a^2*b*c^{12}*d^7*e^6 + 416*a^2*b^7*c^6*d*e^{12} - 32128*a^3*b*c^{11}*d^5*e^8 - 4120*a^3*b^5*c^7*d*e^{12} - 63360*a^4*b* \\
& c^{10}*d^3*e^{10} + 21376*a^4*b^3*c^8*d*e^{12}))/((8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 \\
& - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + \\
& 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9* \\
& c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 20 \\
& 48*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7* \\
& d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e
\end{aligned}$$

$$\begin{aligned}
&^2 - 140a^4b^{12}c^3d^6e^2 + 84a^4b^{13}c^2d^5e^3 + 1344a^5b^{10}c^4 \\
&d^6e^2 - 672a^5b^{11}c^3d^5e^3 - 42a^5b^{12}c^2d^4e^4 - 6720a^6b^8 \\
&c^5d^6e^2 + 2240a^6b^9c^4d^5e^3 + 1456a^6b^{10}c^3d^4e^4 - 672a^6 \\
&b^{11}c^2d^3e^5 + 17920a^7b^6c^6d^6e^2 - 10080a^7b^8c^4d^4e^4 + 2240a^7 \\
&b^9c^3d^3e^5 + 1344a^7b^{10}c^2d^2e^6 - 21504a^8b^4c^7d^6e^2 - 21504a^8 \\
&b^5c^6d^5e^3 + 32256a^8b^6c^5d^4e^4 - 6720a^8 \\
&b^8c^3d^2e^6 + 57344a^9b^3c^7d^5e^3 - 46592a^9b^4c^6d^4e^4 - 21504a^9 \\
&b^5c^5d^3e^5 + 17920a^9b^6c^4d^2e^6 + 12288a^{10}b^2c^7 \\
&d^4e^4 + 57344a^{10}b^3c^6d^3e^5 - 21504a^{10}b^4c^5d^2e^6 + 96a^7 \\
&b^{11}c^d^e^7 - 16384a^9b^c^9d^7e - 16384a^{12}b^c^6d^e^7 - 4a^3b^{13} \\
&c^3d^7e - 4a^3b^{15}c^d^5e^3 + 96a^4b^{11}c^4d^7e - 12a^4b^{14}c^d \\
&^4e^4 - 960a^5b^9c^5d^7e + 84a^5b^{13}c^d^3e^5 + 5120a^6b^7c^6d^7 \\
&^7e - 140a^6b^{12}c^d^2e^6 - 15360a^7b^5c^7d^7e + 24576a^8b^3c^8 \\
&d^7e - 960a^8b^9c^2d^e^7 + 5120a^9b^7c^3d^e^7 - 49152a^{10}b^c^8 \\
&d^5e^3 - 15360a^{10}b^5c^4d^e^7 - 49152a^{11}b^c^7d^3e^5 + 24576a^{11} \\
&b^3c^5d^e^7))^{(1/2)})) * ((27a^2b^9c^5d^6 - b^{11}c^4d^6 - b^{15}d^2e^4 - 9 \\
&a^2b^{13}e^6 + 3840a^5b^c^9d^6 - 9a^c^5d^6 * (- (4a^c - b^2)^9)^{(1/2)} \\
&+ 213a^3b^{11}c^e^6 - 26880a^8b^c^6e^6 + 3072a^6c^9d^5e + 35840a^8 \\
&c^7d^e^5 + 4b^{12}c^3d^5e + 4b^{14}c^d^3e^3 - 288a^2b^7c^6d^6 + 1 \\
&504a^3b^5c^7d^6 - 3840a^4b^3c^8d^6 + 9a^2b^4e^6 * (- (4a^c - b^2)^9)^{(1/2)} \\
&- 2077a^4b^9c^2e^6 + 10656a^5b^7c^3e^6 - 30240a^6b^5c^4 \\
&e^6 + 44800a^7b^3c^5e^6 + 25a^4c^2e^6 * (- (4a^c - b^2)^9)^{(1/2)} + b^2 \\
&c^4d^6 * (- (4a^c - b^2)^9)^{(1/2)} + 22528a^7c^8d^3e^3 + b^6d^2e^4 * (- \\
&(4a^c - b^2)^9)^{(1/2)} - 6b^{13}c^2d^4e^2 + 6a^b^{14}d^e^5 - 1471a^2b^9 \\
&c^4d^4e^2 + 600a^2b^{10}c^3d^3e^3 + 180a^2b^{11}c^2d^2e^4 + 6976a^3 \\
&b^7c^5d^4e^2 - 1032a^3b^8c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456a^4 \\
&b^5c^6d^4e^2 - 7168a^4b^6c^5d^3e^3 + 16896a^4b^7c^4d^2 \\
&e^4 + 10240a^5b^3c^7d^4e^2 + 37632a^5b^4c^6d^3e^3 - 47712a^5b^5 \\
&c^5d^2e^4 - 59392a^6b^2c^7d^3e^3 + 60928a^6b^3c^6d^2e^4 - 41 \\
&a^2c^4d^4e^2 * (- (4a^c - b^2)^9)^{(1/2)} - 39a^3c^3d^2e^4 * (- (4a^c - b^2)^9)^{(1/2)} \\
&+ 6b^4c^2d^4e^2 * (- (4a^c - b^2)^9)^{(1/2)} - 6a^b^5d^e^5 * (- (4a^c - b^2)^9)^{(1/2)} \\
&- 106a^b^{10}c^4d^5e + 7a^b^{13}c^d^2e^4 - 128a^2b^{12}c^d^e^5 - 51a^3b^2c^e^6 * (- (4a^c - b^2)^9)^{(1/2)} \\
&+ 150a^b^{11}c^3d^4e^2 - 84a^b^{12}c^2d^3e^3 + 1116a^2b^8c^5d^5e - 5824a^3b^6c^6 \\
&d^5e + 1030a^3b^{10}c^2d^e^5 + 15232a^4b^4c^7d^5e - 3492a^4b^8 \\
&c^3d^e^5 - 16896a^5b^2c^8d^5e + 1344a^5b^6c^4d^e^5 + 7424a^6b^c^8 \\
&d^4e^2 + 22400a^6b^4c^5d^e^5 - 23296a^7b^c^7d^2e^4 - 53760a^7 \\
&b^2c^6d^e^5 - 4b^3c^3d^5e * (- (4a^c - b^2)^9)^{(1/2)} - 4b^5c^d^3e^3 \\
&* (- (4a^c - b^2)^9)^{(1/2)} + 11a^b^4c^d^2e^4 * (- (4a^c - b^2)^9)^{(1/2)} + 2 \\
&0a^2b^3c^d^e^5 * (- (4a^c - b^2)^9)^{(1/2)} + 86a^3b^c^2d^e^5 * (- (4a^c - b^2)^9)^{(1/2)} \\
&- 42a^b^2c^3d^4e^2 * (- (4a^c - b^2)^9)^{(1/2)} + 12a^b^3c^2d^3e^3 * (- (4a^c - b^2)^9)^{(1/2)} \\
&+ 120a^2b^c^3d^3e^3 * (- (4a^c - b^2)^9)^{(1/2)} + 34a^b^c^4d^5e * (- (4a^c - b^2)^9)^{(1/2)} \\
&- 108a^2b^2c^2d^2e^4 * (- (4a^c - b^2)^9)^{(1/2)) / (32 * (a^7b^{12}e^8 + 4096a^9c^{10}d^8 + 4096a^{13} \\
&c^6e^8 - 24a^8b^{10}c^e^8 - 4a^6b^{13}d^e^7 + a^3b^{12}c^4d^8 - 24a^4b^{10}c^5d^8 + 240a^5b^8c^6d^8 - 1280a^6 \\
&b^6c^7d^8 + 3840a^7b^4c^8d^8 - 6144a^8b^2c^9d^8 + 240a^9b^8c^2e^8 - 1280a^{10}b^6c^3e^8 + 3840a^{11} \\
&b^4c^4e^8 - 6144a^{12}b^2c^5e^8 + a^3b^{16}d^4e^4 - 4a^4b^{15}d^3e^5 + 6a^5b^{14}d^2e^6 + 16384a^{10}c^9d^6e^2 + 24576a^{11} \\
&c^8d^4e^4 + 16384a^{12}c^7d^2e^6 + 6a^3b^{14}c^2d^6e^2 - 140a^4b^{12}c^3d^6e^2 + 84a^4b^{13}c^2d^5e^3 + 1344a^5 \\
&b^{10}c^4d^6e^2 - 672a^5b^{11}c^3d^5e^3 - 42a^5b^{12}c^2d^4e^4 - 6720a^6b^8c^5d^6e^2 + 2240a^6b^9c^4d^5e^3 + 1456a^6 \\
&b^{10}c^3d^4e^4 - 672a^6b^{11}c^2d^3e^5 + 17920a^7b^6c^6d^6e^2 - 10080a^7b^8c^4d^4e^4 + 2240a^7b^9c^3d^3e^5 + 1344a^7 \\
&b^{10}c^2d^2e^6 - 21504a^8b^4c^7d^6e^2 - 21504a^8b^5c^6d^5e^3 + 32256a^8b^6c^5d^4e^4 - 6720a^8b^8c^3d^2e^6 + 57344a^9 \\
&b^3c^7d^5e^3 - 46592a^9b^4c^6d^4e^4 - 21504a^9b^5c^5d^3e^5 + 17920a^9b^6c^4d^2e^6 + 12288a^{10}b^2c^7d^4e^4 + 573 \\
&44a^{10}b^3c^6d^3e^5 - 21504a^{10}b^4c^5d^2e^6 + 96a^7b^{11}c^d^e^7
\end{aligned}$$

$$\begin{aligned}
& - 16384a^9b^9c^9d^7e - 16384a^{12}b^6c^6d^7e^7 - 4a^3b^{13}c^3d^7e - 4 \\
& a^3b^{15}c^5d^5e^3 + 96a^4b^{11}c^4d^7e - 12a^4b^{14}c^4d^4e^4 - 960a \\
& ^5b^9c^5d^7e + 84a^5b^{13}c^3d^3e^5 + 5120a^6b^7c^6d^7e - 140a^6 \\
& b^{12}c^3d^2e^6 - 15360a^7b^5c^7d^7e + 24576a^8b^3c^8d^7e - 960a \\
& ^8b^9c^2d^7e + 5120a^9b^7c^3d^7e - 49152a^{10}b^5c^8d^5e^3 - 1536 \\
& 0a^{10}b^5c^4d^7e - 49152a^{11}b^3c^7d^3e^5 + 24576a^{11}b^3c^5d^7e \\
& )^{(1/2)} * 2i - \operatorname{atan}\left(\frac{(1048576a^{13}c^8e^{16} + 256a^7b^{12}c^2e^{16} - 6144a^8b^{10}c^3e^{16} + 61440a^9b^8c^4e^{16} - 327680a^{10}b^6c^5e^{16} + 983040a^{11}b^4c^6e^{16} - 1572864a^{12}b^2c^7e^{16} - 196608a^6c^{15}d^{14}e^2 - 917504a^7c^{14}d^{12}e^4 - 589824a^8c^{13}d^{10}e^6 + 3932160a^9c^{12}d^8e^8 + 10158080a^{10}c^{11}d^6e^{10} + 10616832a^{11}c^{10}d^4e^{12} + 5308416a^{12}c^9d^2e^{14} - 2816a^2b^8c^{11}d^{14}e^2 + 22656a^2b^9c^{10}d^{13}e^3 - 78848a^2b^{10}c^9d^{12}e^4 + 154112a^2b^{11}c^8d^{11}e^5 - 182784a^2b^{12}c^7d^{10}e^6 + 130816a^2b^{13}c^6d^9e^7 - 50176a^2b^{14}c^5d^8e^8 + 4608a^2b^{15}c^4d^7e^9 + 3328a^2b^{16}c^3d^6e^{10} - 896a^2b^{17}c^2d^5e^{11} + 24576a^3b^6c^{12}d^{14}e^2 - 198656a^3b^7c^{11}d^{13}e^3 + 684544a^3b^8c^{10}d^{12}e^4 - 1291520a^3b^9c^9d^{11}e^5 + 1403776a^3b^{10}c^8d^{10}e^6 - 798336a^3b^{11}c^7d^9e^7 + 89856a^3b^{12}c^6d^8e^8 + 155136a^3b^{13}c^5d^7e^9 - 77440a^3b^{14}c^4d^6e^{10} + 5504a^3b^{15}c^3d^5e^{11} + 2560a^3b^{16}c^2d^4e^{12} - 106496a^4b^4c^{13}d^{14}e^2 + 864256a^4b^5c^{12}d^{13}e^3 - 2924544a^4b^6c^{11}d^{12}e^4 + 5181440a^4b^7c^{10}d^{11}e^5 - 4686080a^4b^8c^9d^{10}e^6 + 1045376a^4b^9c^8d^9e^7 + 1900544a^4b^{10}c^7d^8e^8 - 1732096a^4b^{11}c^6d^7e^9 + 390400a^4b^{12}c^5d^6e^{10} + 112000a^4b^{13}c^4d^5e^{11} - 40960a^4b^{14}c^3d^4e^{12} - 3840a^4b^{15}c^2d^3e^{13} + 229376a^5b^2c^{14}d^{14}e^2 - 1867776a^5b^3c^{13}d^{13}e^3 + 6078464a^5b^4c^{12}d^{12}e^4 - 9297920a^5b^5c^{11}d^{11}e^5 + 4055040a^5b^6c^{10}d^{10}e^6 + 7788544a^5b^7c^9d^9e^7 - 12657664a^5b^8c^8d^8e^8 + 6130176a^5b^9c^7d^7e^9 + 734080a^5b^{10}c^6d^6e^{10} - 1442560a^5b^{11}c^5d^5e^{11} + 168960a^5b^{12}c^4d^4e^{12} + 78080a^5b^{13}c^3d^3e^{13} + 3200a^5b^{14}c^2d^2e^{14} - 4587520a^6b^2c^{13}d^{12}e^4 + 3080192a^6b^3c^{12}d^{11}e^5 + 12001280a^6b^4c^{11}d^{10}e^6 - 31076352a^6b^5c^{10}d^9e^7 + 27475968a^6b^6c^9d^8e^8 - 2088960a^6b^7c^8d^7e^9 - 12205312a^6b^8c^7d^6e^{10} + 6043520a^6b^9c^6d^5e^{11} + 631808a^6b^{10}c^5d^4e^{12} - 610304a^6b^{11}c^4d^3e^{13} - 71936a^6b^{12}c^3d^2e^{14} - 21725184a^7b^2c^{12}d^{10}e^6 + 30801920a^7b^3c^{11}d^9e^7 - 8028160a^7b^4c^{10}d^8e^8 - 32260096a^7b^5c^9d^7e^9 + 37101568a^7b^6c^8d^6e^{10} - 7182336a^7b^7c^7d^5e^{11} - 7609856a^7b^8c^6d^4e^{12} + 2112256a^7b^9c^5d^3e^{13} + 661632a^7b^{10}c^4d^2e^{14} - 30146560a^8b^2c^{11}d^8e^8 + 55050240a^8b^3c^{10}d^7e^9 - 34365440a^8b^4c^9d^6e^{10} - 16429056a^8b^5c^8d^5e^{11} + 24600576a^8b^6c^7d^4e^{12} - 1683456a^8b^7c^6d^3e^{13} - 3151616a^8b^8c^5d^2e^{14} - 10977280a^9b^2c^{10}d^6e^{10} + 47022080a^9b^3c^9d^5e^{11} - 30621696a^9b^4c^8d^4e^{12} - 9232384a^9b^5c^7d^3e^{13} + 7970816a^9b^6c^6d^2e^{14} + 4325376a^{10}b^2c^9d^4e^{12} + 25493504a^{10}b^3c^8d^3e^{13} - 9117696a^{10}b^4c^7d^2e^{14} + 491520a^{11}b^2c^8d^2e^{14} - 4947968a^{12}b^6c^8d^7e^{15} + 128a^8b^{10}c^{10}d^{14}e^2 - 1024a^8b^{11}c^9d^{13}e^3 + 3584a^8b^{12}c^8d^{12}e^4 - 7168a^8b^{13}c^7d^{11}e^5 + 8960a^8b^{14}c^6d^{10}e^6 - 7168a^8b^{15}c^5d^9e^7 + 3584a^8b^{16}c^4d^8e^8 - 1024a^8b^{17}c^3d^7e^9 + 128a^8b^{18}c^2d^6e^{10} + 1605632a^6b^6c^{14}d^{13}e^3 - 1408a^6b^{13}c^2d^7e^{15} + 7012352a^7b^6c^{13}d^{11}e^5 + 33152a^7b^{11}c^3d^7e^{15} + 7045120a^8b^6c^{12}d^9e^7 - 324480a^8b^9c^4d^8e^{15} - 9830400a^9b^6c^{11}d^7e^9 + 1689600a^9b^7c^5d^7e^{15} - 25722880a^{10}b^6c^{10}d^5e^{11} - 4935680a^{10}b^5c^6d^7e^{15} - 19202048a^{11}b^6c^9d^3e^{13} + 7667712a^{11}b^3c^7d^7e^{15}) / (16(a^6b^8e^8 + 256a^6c^8d^8 + 256a^{10}c^4e^8 - 16a^7b^6c^8e^8 - 4a^5b^9d^7e^7 + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6d^8 - 256a^5b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^{12}d^4e^4 - 4a^3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 + 1024a^7c^7d^6e^2 + 1536a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^{10}c^2d^6e^2 - 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^6)
\end{aligned}$$

$$\begin{aligned}
&^3 + 512a^4b^6c^4d^6e^2 - 192a^4b^7c^3d^5e^3 - 90a^4b^8c^2d^4 \\
&*e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3 \\
&*d^4e^4 - 192a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3 \\
&*c^5d^5e^3 - 2240a^6b^4c^4d^4e^4 - 128a^6b^5c^3d^3e^5 + 512a^6 \\
&*b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 + 2048a^7b^3c^4d^3e^5 - 11 \\
&52a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024a^6b^7c^4d^7e + 6 \\
&4a^6b^7c^3d^7e - 1024a^9b^4c^4d^7e - 4a^2b^9c^3d^7e - 4a^2b^11 \\
&*c^3d^5e^3 + 64a^3b^7c^4d^7e - 4a^3b^10c^3d^4e^4 - 384a^4b^5c^5 \\
&d^7e + 52a^4b^9c^3d^3e^5 + 1024a^5b^3c^6d^7e - 92a^5b^8c^3d^2e^6 \\
&- 3072a^7b^6c^6d^5e^3 - 384a^7b^5c^2d^7e - 3072a^8b^6c^5d^3e^5 \\
&+ 1024a^8b^3c^3d^7e) - (x*((27a^9b^9c^5d^6 - b^11c^4d^6 - b^15d \\
&^2e^4 - 9a^2b^13e^6 + 3840a^5b^9c^9d^6 + 9a^5c^5d^6*(-(4a^3c - b^2)^ \\
&9)^{1/2}) + 213a^3b^11c^3e^6 - 26880a^8b^6c^6e^6 + 3072a^6c^9d^5e + \\
&35840a^8c^7d^5e^5 + 4b^12c^3d^5e + 4b^14c^3d^3e^3 - 288a^2b^7c^6 \\
&*d^6 + 1504a^3b^5c^7d^6 - 3840a^4b^3c^8d^6 - 9a^2b^4e^6*(-(4a^3c \\
&- b^2)^9)^{1/2} - 2077a^4b^9c^2e^6 + 10656a^5b^7c^3e^6 - 30240a^6 \\
&*b^5c^4e^6 + 44800a^7b^3c^5e^6 - 25a^4c^2e^6*(-(4a^3c - b^2)^9)^{1/2} \\
&- b^2c^4d^6*(-(4a^3c - b^2)^9)^{1/2} + 22528a^7c^8d^3e^3 - b^6d^ \\
&2e^4*(-(4a^3c - b^2)^9)^{1/2} - 6b^13c^2d^4e^2 + 6a^6b^14d^5e - 1471 \\
&a^2b^9c^4d^4e^2 + 600a^2b^10c^3d^3e^3 + 180a^2b^11c^2d^2e^4 \\
&+ 6976a^3b^7c^5d^4e^2 - 1032a^3b^8c^4d^3e^3 - 2871a^3b^9c^3d^ \\
&2e^4 - 15456a^4b^5c^6d^4e^2 - 7168a^4b^6c^5d^3e^3 + 16896a^4b^ \\
&7c^4d^2e^4 + 10240a^5b^3c^7d^4e^2 + 37632a^5b^4c^6d^3e^3 - 477 \\
&12a^5b^5c^5d^2e^4 - 59392a^6b^2c^7d^3e^3 + 60928a^6b^3c^6d^2e \\
&e^4 + 41a^2c^4d^4e^2*(-(4a^3c - b^2)^9)^{1/2} + 39a^3c^3d^2e^4*(-(4 \\
&a^3c - b^2)^9)^{1/2} - 6b^4c^2d^4e^2*(-(4a^3c - b^2)^9)^{1/2} + 6a^6b^5 \\
&*d^5e*(-(4a^3c - b^2)^9)^{1/2} - 106a^6b^10c^4d^5e + 7a^6b^13c^3d^2e^4 \\
&- 128a^2b^12c^3d^5e + 51a^3b^2c^6e*(-(4a^3c - b^2)^9)^{1/2} + 150a \\
&*b^11c^3d^4e^2 - 84a^6b^12c^2d^3e^3 + 1116a^2b^8c^5d^5e - 5824a \\
&^3b^6c^6d^5e + 1030a^3b^10c^2d^5e + 15232a^4b^4c^7d^5e - 3492 \\
&a^4b^8c^3d^5e - 16896a^5b^2c^8d^5e + 1344a^5b^6c^4d^5e + 742 \\
&4a^6b^3c^8d^4e^2 + 22400a^6b^4c^5d^5e - 23296a^7b^6c^7d^2e^4 - 5 \\
&3760a^7b^2c^6d^5e + 4b^3c^3d^5e*(-(4a^3c - b^2)^9)^{1/2} + 4b^5c \\
&*d^3e^3*(-(4a^3c - b^2)^9)^{1/2} - 11a^6b^4c^3d^2e^4*(-(4a^3c - b^2)^9)^{1/2} \\
&- 20a^2b^3c^3d^5e*(-(4a^3c - b^2)^9)^{1/2} - 86a^3b^6c^2d^5e*(-( \\
&4a^3c - b^2)^9)^{1/2} + 42a^6b^2c^3d^4e^2*(-(4a^3c - b^2)^9)^{1/2} - 12 \\
&a^6b^3c^2d^3e^3*(-(4a^3c - b^2)^9)^{1/2} - 120a^2b^6c^3d^3e^3*(-(4a^3c \\
&- b^2)^9)^{1/2} - 34a^6b^4c^4d^5e*(-(4a^3c - b^2)^9)^{1/2} + 108a^2b^2 \\
&c^2d^2e^4*(-(4a^3c - b^2)^9)^{1/2})/(32*(a^7b^12e^8 + 4096a^9c^10d^8 \\
&+ 4096a^13c^6e^8 - 24a^8b^10c^8e^8 - 4a^6b^13d^7e^7 + a^3b^12c^4 \\
&d^8 - 24a^4b^10c^5d^8 + 240a^5b^8c^6d^8 - 1280a^6b^6c^7d^8 + 38 \\
&40a^7b^4c^8d^8 - 6144a^8b^2c^9d^8 + 240a^9b^8c^2e^8 - 1280a^10 \\
&*b^6c^3e^8 + 3840a^11b^4c^4e^8 - 6144a^12b^2c^5e^8 + a^3b^16d^4 \\
&*e^4 - 4a^4b^15d^3e^5 + 6a^5b^14d^2e^6 + 16384a^10c^9d^6e^2 + 2 \\
&4576a^11c^8d^4e^4 + 16384a^12c^7d^2e^6 + 6a^3b^14c^2d^6e^2 - 1 \\
&40a^4b^12c^3d^6e^2 + 84a^4b^13c^2d^5e^3 + 1344a^5b^10c^4d^6e \\
&^2 - 672a^5b^11c^3d^5e^3 - 42a^5b^12c^2d^4e^4 - 6720a^6b^8c^5 \\
&d^6e^2 + 2240a^6b^9c^4d^5e^3 + 1456a^6b^10c^3d^4e^4 - 672a^6b^ \\
&11c^2d^3e^5 + 17920a^7b^6c^6d^6e^2 - 10080a^7b^8c^4d^4e^4 + 22 \\
&40a^7b^9c^3d^3e^5 + 1344a^7b^10c^2d^2e^6 - 21504a^8b^4c^7d^6 \\
&e^2 - 21504a^8b^5c^6d^5e^3 + 32256a^8b^6c^5d^4e^4 - 6720a^8b^8 \\
&c^3d^2e^6 + 57344a^9b^3c^7d^5e^3 - 46592a^9b^4c^6d^4e^4 - 21504 \\
&a^9b^5c^5d^3e^5 + 17920a^9b^6c^4d^2e^6 + 12288a^10b^2c^7d^4e \\
&^4 + 57344a^10b^3c^6d^3e^5 - 21504a^10b^4c^5d^2e^6 + 96a^7b^11 \\
&c^3d^7e - 16384a^9b^6c^9d^7e - 16384a^12b^6c^6d^7e - 4a^3b^13c^3d \\
&^7e - 4a^3b^15c^3d^5e^3 + 96a^4b^11c^4d^7e - 12a^4b^14c^3d^4e^4 \\
&- 960a^5b^9c^5d^7e + 84a^5b^13c^3d^3e^5 + 5120a^6b^7c^6d^7e - \\
&140a^6b^12c^3d^2e^6 - 15360a^7b^5c^7d^7e + 24576a^8b^3c^8d^7e \\
&- 960a^8b^9c^2d^7e + 5120a^9b^7c^3d^7e - 49152a^10b^6c^8d^5e^
\end{aligned}$$

$$\begin{aligned}
& 3 - 15360a^{10}b^5c^4d^7e^7 - 49152a^{11}b^6c^7d^3e^5 + 24576a^{11}b^3c^8d^5e^7) \Big)^{(1/2)} \cdot (1048576a^{15}c^8e^{17} + 256a^9b^{12}c^2e^{17} - 6144a^{10} \\
& * b^{10}c^3e^{17} + 61440a^{11}b^8c^4e^{17} - 327680a^{12}b^6c^5e^{17} + 98304 \\
& 0a^{13}b^4c^6e^{17} - 1572864a^{14}b^2c^7e^{17} - 1048576a^8c^{15}d^{14}e^3 \\
& - 5242880a^9c^{14}d^{12}e^5 - 9437184a^{10}c^{13}d^{10}e^7 - 5242880a^{11}c^{12}d^8e^9 + 5242880a^{12}c^{11}d^6e^{11} + 9437184a^{13}c^{10}d^4e^{13} + 5242 \\
& 880a^{14}c^9d^2e^{15} + 256a^2b^{11}c^{10}d^{15}e^2 - 2048a^2b^{12}c^9d^{14} \\
& * e^3 + 7168a^2b^{13}c^8d^{13}e^4 - 14336a^2b^{14}c^7d^{12}e^5 + 17920a^2 \\
& * b^{15}c^6d^{11}e^6 - 14336a^2b^{16}c^5d^{10}e^7 + 7168a^2b^{17}c^4d^9e^8 \\
& - 2048a^2b^{18}c^3d^8e^9 + 256a^2b^{19}c^2d^7e^{10} - 5120a^3b^9c^8 \\
& * d^{15}e^2 + 41984a^3b^{10}c^{10}d^{14}e^3 - 148736a^3b^{11}c^9d^{13}e^4 + \\
& 296192a^3b^{12}c^8d^{12}e^5 - 359680a^3b^{13}c^7d^{11}e^6 + 267520a^3b \\
& ^{14}c^6d^{10}e^7 - 112384a^3b^{15}c^5d^9e^8 + 18176a^3b^{16}c^4d^8e^9 \\
& + 3328a^3b^{17}c^3d^7e^{10} - 1280a^3b^{18}c^2d^6e^{11} + 40960a^4b^7* \\
& c^{12}d^{15}e^2 - 348160a^4b^8c^{11}d^{14}e^3 + 1254400a^4b^9c^{10}d^{13}e^4 \\
& - 2478080a^4b^{10}c^9d^{12}e^5 + 2867456a^4b^{11}c^8d^{11}e^6 - 1862144 \\
& * a^4b^{12}c^7d^{10}e^7 + 490240a^4b^{13}c^6d^9e^8 + 128000a^4b^{14}c^5* \\
& d^8e^9 - 108800a^4b^{15}c^4d^7e^{10} + 13824a^4b^{16}c^3d^6e^{11} + 2304 \\
& * a^4b^{17}c^2d^5e^{12} - 163840a^5b^5c^{13}d^{15}e^2 + 1474560a^5b^6c^{11} \\
& * d^{14}e^3 - 5447680a^5b^7c^{11}d^{13}e^4 + 10588160a^5b^8c^{10}d^{12}e^5 \\
& - 11166720a^5b^9c^9d^{11}e^6 + 5159936a^5b^{10}c^8d^{10}e^7 + 1073920* \\
& a^5b^{11}c^7d^9e^8 - 2279680a^5b^{12}c^6d^8e^9 + 770560a^5b^{13}c^5d^7 \\
& * e^{10} + 33280a^5b^{14}c^4d^6e^{11} - 41216a^5b^{15}c^3d^5e^{12} - 1280* \\
& a^5b^{16}c^2d^4e^{13} + 327680a^6b^3c^{14}d^{15}e^2 - 3276800a^6b^4c^{13} \\
& * d^{14}e^3 + 12615680a^6b^5c^{12}d^{13}e^4 - 23592960a^6b^6c^{11}d^{12}e^5 \\
& + 19701760a^6b^7c^{10}d^{11}e^6 + 1372160a^6b^8c^9d^{10}e^7 - 15846400 \\
& * a^6b^9c^8d^9e^8 + 10864640a^6b^{10}c^7d^8e^9 - 1352960a^6b^{11}c^6 \\
& * d^7e^{10} - 1111040a^6b^{12}c^5d^6e^{11} + 273920a^6b^{13}c^4d^5e^{12} + \\
& 25600a^6b^{14}c^3d^4e^{13} - 1280a^6b^{15}c^2d^3e^{14} + 3407872a^7b^2* \\
& c^{14}d^{14}e^3 - 14221312a^7b^3c^{13}d^{13}e^4 + 23527424a^7b^4c^{12}d^{12} \\
& * e^5 - 3768320a^7b^5c^{11}d^{11}e^6 - 38895616a^7b^6c^{10}d^{10}e^7 + 501 \\
& 26848a^7b^7c^9d^9e^8 - 18362368a^7b^8c^8d^8e^9 - 6831104a^7b^9* \\
& c^7d^7e^{10} + 6200320a^7b^{10}c^6d^6e^{11} - 726784a^7b^{11}c^5d^5e^{12} \\
& - 228608a^7b^{12}c^4d^4e^{13} + 31488a^7b^{13}c^3d^3e^{14} + 2304a^7b^ \\
& ^{14}c^2d^2e^{15} - 3145728a^8b^2c^{13}d^{12}e^5 - 31129600a^8b^3c^{12}d^{11} \\
& * e^6 + 74711040a^8b^4c^{11}d^{10}e^7 - 55476224a^8b^5c^{10}d^9e^8 - 11 \\
& 075584a^8b^6c^9d^8e^9 + 35381248a^8b^7c^8d^7e^{10} - 14479360a^8b^ \\
& ^8c^7d^6e^{11} - 168960a^8b^9c^6d^5e^{12} + 1286144a^8b^{10}c^5d^4e^ \\
& ^{13} - 302336a^8b^{11}c^4d^3e^{14} - 55808a^8b^{12}c^3d^2e^{15} - 36962304* \\
& a^9b^2c^{12}d^{10}e^7 - 9502720a^9b^3c^{11}d^9e^8 + 67174400a^9b^4c^{10} \\
& * d^8e^9 - 54886400a^9b^5c^9d^7e^{10} + 11239424a^9b^6c^8d^6e^{11} + \\
& 5545984a^9b^7c^7d^5e^{12} - 5263360a^9b^8c^6d^4e^{13} + 1356800a^9* \\
& b^9c^5d^3e^{14} + 558080a^9b^{10}c^4d^2e^{15} - 49807360a^{10}b^2c^{11}d^ \\
& ^8e^9 + 19333120a^{10}b^3c^{10}d^7e^{10} + 7208960a^{10}b^4c^9d^6e^{11} - 1 \\
& 4974976a^{10}b^5c^8d^5e^{12} + 15073280a^{10}b^6c^7d^4e^{13} - 2170880a^ \\
& ^{10}b^7c^6d^3e^{14} - 2928640a^{10}b^8c^5d^2e^{15} - 11796480a^{11}b^2c^{10} \\
& * d^6e^{11} + 23920640a^{11}b^3c^9d^5e^{12} - 24576000a^{11}b^4c^8d^4e^{13} \\
& - 4096000a^{11}b^5c^7d^3e^{14} + 8355840a^{11}b^6c^6d^2e^{15} + 1258291 \\
& 2a^{12}b^2c^9d^4e^{13} + 19857408a^{12}b^3c^8d^3e^{14} - 11534336a^{12}b^ \\
& ^4c^7d^2e^{15} + 3407872a^{13}b^2c^8d^2e^{15} - 5505024a^{14}b^3c^8d^2e^{16} \\
& - 262144a^7b^3c^{15}d^{15}e^2 + 5505024a^8b^3c^{14}d^{13}e^4 - 1280a^8b^{13} \\
& * c^2d^2e^{16} + 25952256a^9b^3c^{13}d^{11}e^6 + 30976a^9b^{11}c^3d^2e^{16} + 380 \\
& 10880a^{10}b^3c^{12}d^9e^8 - 312320a^{10}b^9c^4d^2e^{16} + 11796480a^{11}b^3c^{11} \\
& * d^7e^{10} + 1679360a^{11}b^7c^5d^2e^{16} - 21233664a^{12}b^3c^{10}d^5e^{12} - \\
& 5079040a^{12}b^5c^6d^2e^{16} - 20709376a^{13}b^3c^9d^3e^{14} + 8192000a^{13} \\
& * b^3c^7d^2e^{16}) / (8(a^6b^8e^8 + 256a^6c^8d^8 + 256a^{10}c^4e^8 - 16* \\
& a^7b^6c^8e^8 - 4a^5b^9d^2e^7 + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96 \\
& * a^4b^4c^6d^8 - 256a^5b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^ \\
& ^3e^8 + a^2b^{12}d^4e^4 - 4a^3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 + 1024*
\end{aligned}$$

$$\begin{aligned}
& a^7c^7d^6e^2 + 1536a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^{10}c^2d^6e^2 - 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4b^7c^3d^5e^3 - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 - 192a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c^4d^4e^4 - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 + 2048a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024a^6b^7c^3d^7e^5 + 64a^6b^7c^3d^7e^5 - 1024a^9b^3c^4d^7e^5 - 4a^2b^9c^3d^7e^5 - 4a^2b^{11}c^4d^5e^3 + 64a^3b^7c^4d^7e^5 - 4a^3b^{10}c^4d^4e^4 - 384a^4b^5c^5d^7e^5 + 52a^4b^9c^3d^3e^5 + 1024a^5b^3c^6d^7e^5 - 92a^5b^8c^3d^2e^6 - 3072a^7b^3c^6d^5e^3 - 384a^7b^5c^2d^7e^5 - 3072a^8b^3c^5d^3e^5 + 1024a^8b^3c^3d^7e^5)) * ((27a^2b^9c^5d^6 - b^{11}c^4d^6 - b^{15}d^2e^4 - 9a^2b^{13}e^6 + 3840a^5b^3c^9d^6 + 9a^3c^5d^6(-4a^2c - b^2)^9)^{1/2} + 213a^3b^{11}c^4e^6 - 26880a^8b^3c^6e^6 + 3072a^6c^9d^5e^5 + 35840a^8c^7d^5e^5 + 4b^{12}c^3d^5e^5 + 4b^{14}c^3d^3e^3 - 288a^2b^7c^6d^6 + 1504a^3b^5c^7d^6 - 3840a^4b^3c^8d^6 - 9a^2b^4e^6(-4a^2c - b^2)^9)^{1/2} - 2077a^4b^9c^2e^6 + 10656a^5b^7c^3e^6 - 30240a^6b^5c^4e^6 + 44800a^7b^3c^5e^6 - 25a^4c^2e^6(-4a^2c - b^2)^9)^{1/2} - b^2c^4d^6(-4a^2c - b^2)^9)^{1/2} + 22528a^7c^8d^3e^3 - b^6d^2e^4(-4a^2c - b^2)^9)^{1/2} - 6b^{13}c^2d^4e^2 + 6a^2b^{14}d^5e^5 - 1471a^2b^9c^4d^4e^2 + 600a^2b^{10}c^3d^3e^3 + 180a^2b^{11}c^2d^2e^4 + 6976a^3b^7c^5d^4e^2 - 1032a^3b^8c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456a^4b^5c^6d^4e^2 - 7168a^4b^6c^5d^3e^3 + 16896a^4b^7c^4d^2e^4 + 10240a^5b^3c^7d^4e^2 + 37632a^5b^4c^6d^3e^3 - 47712a^5b^5c^5d^2e^4 - 59392a^6b^2c^7d^3e^3 + 60928a^6b^3c^6d^2e^4 + 41a^2c^4d^4e^2(-4a^2c - b^2)^9)^{1/2} + 39a^3c^3d^2e^4(-4a^2c - b^2)^9)^{1/2} - 6b^4c^2d^4e^2(-4a^2c - b^2)^9)^{1/2} + 6a^2b^5d^5e^5(-4a^2c - b^2)^9)^{1/2} - 106a^2b^{10}c^4d^5e^5 + 7a^2b^{13}c^3d^2e^4 - 128a^2b^{12}c^3d^2e^5 + 51a^3b^2c^2e^6(-4a^2c - b^2)^9)^{1/2} + 150a^2b^{11}c^3d^4e^2 - 84a^2b^{12}c^2d^3e^3 + 1116a^2b^8c^5d^5e^5 - 5824a^3b^6c^6d^5e^5 + 1030a^3b^{10}c^2d^5e^5 + 15232a^4b^4c^7d^5e^5 - 3492a^4b^8c^3d^5e^5 - 16896a^5b^2c^8d^5e^5 + 1344a^5b^6c^4d^5e^5 + 7424a^6b^3c^8d^4e^2 + 22400a^6b^4c^5d^5e^5 - 23296a^7b^3c^7d^2e^4 - 53760a^7b^2c^6d^5e^5 + 4b^3c^3d^5e^5(-4a^2c - b^2)^9)^{1/2} + 4b^5c^3d^3e^3(-4a^2c - b^2)^9)^{1/2} - 11a^2b^4c^3d^2e^4(-4a^2c - b^2)^9)^{1/2} - 20a^2b^3c^3d^2e^5(-4a^2c - b^2)^9)^{1/2} - 86a^3b^3c^2d^2e^5(-4a^2c - b^2)^9)^{1/2} + 42a^2b^2c^3d^4e^2(-4a^2c - b^2)^9)^{1/2} - 12a^2b^3c^2d^3e^3(-4a^2c - b^2)^9)^{1/2} - 120a^2b^3c^3d^3e^3(-4a^2c - b^2)^9)^{1/2} - 344a^2b^3c^4d^5e^5(-4a^2c - b^2)^9)^{1/2} + 108a^2b^2c^2d^2e^4(-4a^2c - b^2)^9)^{1/2}) / (32(a^7b^{12}e^8 + 4096a^9c^{10}d^8 + 4096a^{13}c^6e^8 - 24a^8b^{10}c^4e^8 - 4a^6b^{13}d^7e^7 + a^3b^{12}c^4d^8 - 24a^4b^{10}c^5d^8 + 240a^5b^8c^6d^8 - 1280a^6b^6c^7d^8 + 3840a^7b^4c^8d^8 - 6144a^8b^2c^9d^8 + 240a^9b^8c^2e^8 - 1280a^{10}b^6c^3e^8 + 3840a^{11}b^4c^4e^8 - 6144a^{12}b^2c^5e^8 + a^3b^{16}d^4e^4 - 4a^4b^{15}d^3e^5 + 6a^5b^{14}d^2e^6 + 16384a^{10}c^9d^6e^2 + 24576a^{11}c^8d^4e^4 + 16384a^{12}c^7d^2e^6 + 6a^3b^{14}c^2d^6e^2 - 140a^4b^{12}c^3d^6e^2 + 84a^4b^{13}c^2d^5e^3 + 1344a^5b^{10}c^4d^6e^2 - 672a^5b^{11}c^3d^5e^3 - 42a^5b^{12}c^2d^4e^4 - 6720a^6b^8c^5d^6e^2 + 2240a^6b^9c^4d^5e^3 + 1456a^6b^{10}c^3d^4e^4 - 672a^6b^{11}c^2d^3e^5 + 17920a^7b^6c^6d^6e^2 - 10080a^7b^8c^4d^4e^4 + 2240a^7b^9c^3d^3e^5 + 1344a^7b^{10}c^2d^2e^6 - 21504a^8b^4c^7d^6e^2 - 21504a^8b^5c^6d^5e^3 + 32256a^8b^6c^5d^4e^4 - 6720a^8b^8c^3d^2e^6 + 57344a^9b^3c^7d^5e^3 - 46592a^9b^4c^6d^4e^4 - 21504a^9b^5c^5d^3e^5 + 17920a^9b^6c^4d^2e^6 + 12288a^{10}b^2c^7d^4e^4 + 57344a^{10}b^3c^6d^3e^5 - 21504a^{10}b^4c^5d^2e^6 + 96a^7b^{11}c^3d^7e^7 - 16384a^9b^3c^9d^7e^7 - 16384a^{12}b^3c^6d^7e^7 - 4a^3b^{13}c^3d^7e^7 - 4a^3b^{15}c^3d^5e^3 + 96a^4b^{11}c^4d^7e^7 - 12a^4b^{14}c^3d^4e^4 - 960a^5b^9c^5d^7e^7 + 84a^5b^{13}c^3d^3e^5 + 5120a^6b^7c^6d^7e^7 - 140a^6b^{12}c^3d^2e^6
\end{aligned}$$

$$\begin{aligned}
& e^6 - 15360a^7b^5c^7d^7e + 24576a^8b^3c^8d^7e - 960a^8b^9c^2d \\
& *e^7 + 5120a^9b^7c^3d^5e^7 - 49152a^{10}b^3c^8d^5e^3 - 15360a^{10}b^5c \\
& ^4d^5e^7 - 49152a^{11}b^3c^7d^3e^5 + 24576a^{11}b^3c^5d^5e^7))^{(1/2)} - ( \\
& x*(626688a^{10}b^3c^8e^{15} - 784384a^{10}c^9d^5e^{14} + 208a^4b^{13}c^2e^{15} \\
& - 4880a^5b^{11}c^3e^{15} + 47312a^6b^9c^4e^{15} - 242176a^7b^7c^5e^{15} \\
& + 688640a^8b^5c^6e^{15} - 1028096a^9b^3c^7e^{15} + 18432a^4c^{15}d^{13} \\
& *e^2 + 126976a^5c^{14}d^{11}e^4 + 325632a^6c^{13}d^9e^6 + 139264a^7c^{12} \\
& *d^7e^8 - 1067008a^8c^{11}d^5e^{10} - 1773568a^9c^{10}d^3e^{12} + 16b^8c \\
& ^{11}d^{13}e^2 - 96b^9c^{10}d^{12}e^3 + 240b^{10}c^9d^{11}e^4 - 304b^{11}c^8* \\
& d^{10}e^5 + 144b^{12}c^7d^9e^6 + 144b^{13}c^6d^8e^7 - 304b^{14}c^5d^7e \\
& ^8 + 240b^{15}c^4d^6e^9 - 96b^{16}c^3d^5e^{10} + 16b^{17}c^2d^4e^{11} + 3 \\
& 200a^2b^4c^{13}d^{13}e^2 - 18432a^2b^5c^{12}d^{12}e^3 + 41024a^2b^6c^{11} \\
& 1d^{11}e^4 - 36352a^2b^7c^{10}d^{10}e^5 - 16208a^2b^8c^9d^9e^6 + 7457 \\
& 6a^2b^9c^8d^8e^7 - 78496a^2b^{10}c^7d^7e^8 + 32064a^2b^{11}c^6d^6 \\
& *e^9 + 6000a^2b^{12}c^5d^5e^{10} - 9264a^2b^{13}c^4d^4e^{11} + 1472a^2b \\
& ^{14}c^3d^3e^{12} + 416a^2b^{15}c^2d^2e^{13} - 12800a^3b^2c^{14}d^{13}e^2 \\
& + 73728a^3b^3c^{13}d^{12}e^3 - 151296a^3b^4c^{12}d^{11}e^4 + 78336a^3b^ \\
& 5c^{11}d^{10}e^5 + 206688a^3b^6c^{10}d^9e^6 - 436736a^3b^7c^9d^8e^7 \\
& + 324224a^3b^8c^8d^7e^8 + 992a^3b^9c^7d^6e^9 - 158176a^3b^{10}c^ \\
& 6d^5e^{10} + 77056a^3b^{11}c^5d^4e^{11} + 6912a^3b^{12}c^4d^3e^{12} - 841 \\
& 6a^3b^{13}c^3d^2e^{13} + 162816a^4b^2c^{13}d^{11}e^4 + 184320a^4b^3c^{11} \\
& 2d^{10}e^5 - 916608a^4b^4c^{11}d^9e^6 + 1165824a^4b^5c^{10}d^8e^7 - 3 \\
& 14496a^4b^6c^9d^7e^8 - 822272a^4b^7c^8d^6e^9 + 919152a^4b^8c^7 \\
& *d^5e^{10} - 175296a^4b^9c^6d^4e^{11} - 189328a^4b^{10}c^5d^3e^{12} + 62 \\
& 064a^4b^{11}c^4d^2e^{13} + 1290752a^5b^2c^{12}d^9e^6 - 659456a^5b^3c \\
& ^{11}d^8e^7 - 1561088a^5b^4c^{10}d^7e^8 + 3240960a^5b^5c^9d^6e^9 - \\
& 1964192a^5b^6c^8d^5e^{10} - 683008a^5b^7c^7d^4e^{11} + 1162304a^5b^ \\
& 8c^6d^3e^{12} - 164112a^5b^9c^5d^2e^{13} + 3442688a^6b^2c^{11}d^7e^8 \\
& - 3670016a^6b^3c^{10}d^6e^9 + 15232a^6b^4c^9d^5e^{10} + 4230144a^6* \\
& b^5c^8d^4e^{11} - 3059648a^6b^6c^7d^3e^{12} - 247296a^6b^7c^6d^2e^ \\
& ^{13} + 4010496a^7b^2c^{10}d^5e^{10} - 6873088a^7b^3c^9d^4e^{11} + 2822400 \\
& *a^7b^4c^8d^3e^{12} + 2370048a^7b^5c^7d^2e^{13} + 1178624a^8b^2c^9* \\
& d^3e^{12} - 4739072a^8b^3c^8d^2e^{13} - 352a^8b^6c^{12}d^{13}e^2 + 2048a* \\
& b^7c^{11}d^{12}e^3 - 4800a^8b^8c^{10}d^{11}e^4 + 5168a^8b^9c^9d^{10}e^5 - 48 \\
& 0a^8b^{10}c^8d^9e^6 - 6000a^8b^{11}c^7d^8e^7 + 8192a^8b^{12}c^6d^7e^8 - \\
& 5040a^8b^{13}c^5d^6e^9 + 1152a^8b^{14}c^4d^5e^{10} + 240a^8b^{15}c^3d^4e^ \\
& ^{11} - 128a^8b^{16}c^2d^3e^{12} - 512a^8b^{17}c^2d^2e^{13} - 106496a^4b^3c^{14}d \\
& ^{12}e^3 + 11680a^4b^{12}c^3d^5e^{14} - 675840a^5b^3c^{13}d^{10}e^5 - 108288a \\
& ^5b^{10}c^4d^5e^{14} - 1601536a^6b^3c^{12}d^8e^7 + 514768a^6b^8c^5d^5e^{14} \\
& - 925696a^7b^3c^{11}d^6e^9 - 1278304a^7b^6c^6d^5e^{14} + 2457600a^8b^3c \\
& ^{10}d^4e^{11} + 1385600a^8b^4c^7d^5e^{14} + 2977792a^9b^3c^9d^2e^{13} + 19 \\
& 968a^9b^2c^8d^5e^{14}))/((8*(a^6b^8e^8 + 256a^6c^8d^8 + 256a^{10}c^4e \\
& ^8 - 16a^7b^6c^5e^8 - 4a^5b^9d^5e^7 + a^2b^8c^4d^8 - 16a^3b^6c^5* \\
& d^8 + 96a^4b^4c^6d^8 - 256a^5b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a \\
& ^9b^2c^3e^8 + a^2b^{12}d^4e^4 - 4a^3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 \\
& + 1024a^7c^7d^6e^2 + 1536a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a \\
& ^2b^{10}c^2d^6e^2 - 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512 \\
& *a^4b^6c^4d^6e^2 - 192a^4b^7c^3d^5e^3 - 90a^4b^8c^2d^4e^4 - 1 \\
& 152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 \\
& - 192a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5 \\
& *e^3 - 2240a^6b^4c^4d^4e^4 - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2 \\
& *d^2e^6 + 1536a^7b^2c^5d^4e^4 + 2048a^7b^3c^4d^3e^5 - 1152a^7b \\
& ^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024a^6b^3c^7d^7e + 64a^6b^ \\
& 7c^4d^5e^7 - 1024a^9b^3c^4d^5e^7 - 4a^2b^9c^3d^7e - 4a^2b^{11}c^d^5e \\
& ^3 + 64a^3b^7c^4d^7e - 4a^3b^{10}c^d^4e^4 - 384a^4b^5c^5d^7e + \\
& 52a^4b^9c^d^3e^5 + 1024a^5b^3c^6d^7e - 92a^5b^8c^d^2e^6 - 3072 \\
& *a^7b^3c^6d^5e^3 - 384a^7b^5c^2d^5e^7 - 3072a^8b^3c^5d^3e^5 + 1024* \\
& a^8b^3c^3d^5e^7)))*((27a^8b^9c^5d^6 - b^{11}c^4d^6 - b^{15}d^2e^4 - 9a \\
& ^2b^{13}e^6 + 3840a^5b^3c^9d^6 + 9a^5c^5d^6*(-(4a^c - b^2)^9)^{(1/2)} + 2
\end{aligned}$$



$$\begin{aligned}
& 13a^3b^{11}c^6e^6 - 26880a^8b^6c^6e^6 + 3072a^6c^9d^5e + 35840a^8c^9d^5e^5 + 4b^{12}c^3d^5e + 4b^{14}c^3d^3e^3 - 288a^2b^7c^6d^6 + 1504a^3b^5c^7d^6 - 3840a^4b^3c^8d^6 - 9a^2b^4e^6(-4ac - b^2)^9)^{(1/2)} - 2077a^4b^9c^2e^6 + 10656a^5b^7c^3e^6 - 30240a^6b^5c^4e^6 + 44800a^7b^3c^5e^6 - 25a^4c^2e^6(-4ac - b^2)^9)^{(1/2)} - b^2c^4d^6(-4ac - b^2)^9)^{(1/2)} + 22528a^7c^8d^3e^3 - b^6d^2e^4(-4ac - b^2)^9)^{(1/2)} - 6b^{13}c^2d^4e^2 + 6ab^{14}d^5e^5 - 1471a^2b^9c^4d^4e^2 + 600a^2b^{10}c^3d^3e^3 + 180a^2b^{11}c^2d^2e^4 + 6976a^3b^7c^5d^4e^2 - 1032a^3b^8c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456a^4b^5c^6d^4e^2 - 7168a^4b^6c^5d^3e^3 + 16896a^4b^7c^4d^2e^4 + 10240a^5b^3c^7d^4e^2 + 37632a^5b^4c^6d^3e^3 - 47712a^5b^5c^5d^2e^4 - 59392a^6b^2c^7d^3e^3 + 60928a^6b^3c^6d^2e^4 + 41a^2c^4d^4e^2(-4ac - b^2)^9)^{(1/2)} + 39a^3c^3d^2e^4(-4ac - b^2)^9)^{(1/2)} - 6b^4c^2d^4e^2(-4ac - b^2)^9)^{(1/2)} + 6ab^5d^5e^5(-4ac - b^2)^9)^{(1/2)} - 106ab^{10}c^4d^5e + 7ab^{13}c^2d^2e^4 - 128a^2b^{12}c^2d^5e^5 + 51a^3b^2c^6e^6(-4ac - b^2)^9)^{(1/2)} + 150ab^{11}c^3d^4e^2 - 84ab^{12}c^2d^3e^3 + 1116a^2b^8c^5d^5e - 5824a^3b^6c^6d^5e + 1030a^3b^{10}c^2d^5e^5 + 15232a^4b^4c^7d^5e - 3492a^4b^8c^3d^5e^5 - 16896a^5b^2c^8d^5e + 1344a^5b^6c^4d^5e^5 + 7424a^6b^6c^8d^4e^2 + 22400a^6b^4c^5d^5e^5 - 23296a^7b^6c^7d^2e^4 - 53760a^7b^2c^6d^5e^5 + 4b^3c^3d^5e^5(-4ac - b^2)^9)^{(1/2)} + 4b^5c^3d^3e^3(-4ac - b^2)^9)^{(1/2)} - 11ab^4c^2d^2e^4(-4ac - b^2)^9)^{(1/2)} - 20a^2b^3c^2d^5e^5(-4ac - b^2)^9)^{(1/2)} - 86a^3b^3c^2d^5e^5(-4ac - b^2)^9)^{(1/2)} + 42ab^2c^3d^4e^2(-4ac - b^2)^9)^{(1/2)} - 12ab^3c^2d^3e^3(-4ac - b^2)^9)^{(1/2)} - 120a^2b^3c^3d^3e^3(-4ac - b^2)^9)^{(1/2)} - 34ab^4c^4d^5e^5(-4ac - b^2)^9)^{(1/2)} + 108a^2b^2c^2d^2e^4(-4ac - b^2)^9)^{(1/2))} / (32(a^7b^{12}e^8 + 4096a^9c^{10}d^8 + 4096a^{13}c^6e^8 - 24a^8b^{10}c^8e^8 - 4a^6b^{13}d^8e^7 + a^3b^{12}c^4d^8 - 24a^4b^{10}c^5d^8 + 240a^5b^8c^6d^8 - 1280a^6b^6c^7d^8 + 3840a^7b^4c^8d^8 - 6144a^8b^2c^9d^8 + 240a^9b^8c^2e^8 - 1280a^{10}b^6c^3e^8 + 3840a^{11}b^4c^4e^8 - 6144a^{12}b^2c^5e^8 + a^3b^{16}d^4e^4 - 4a^4b^{15}d^3e^5 + 6a^5b^{14}d^2e^6 + 16384a^{10}c^9d^6e^2 + 24576a^{11}c^8d^4e^4 + 16384a^{12}c^7d^2e^6 + 6a^3b^{14}c^2d^6e^2 - 140a^4b^{12}c^3d^6e^2 + 84a^4b^{13}c^2d^5e^3 + 1344a^5b^{10}c^4d^6e^2 - 672a^5b^{11}c^3d^5e^3 - 42a^5b^{12}c^2d^4e^4 - 6720a^6b^8c^5d^6e^2 + 2240a^6b^9c^4d^5e^3 + 1456a^6b^{10}c^3d^4e^4 - 672a^6b^{11}c^2d^3e^5 + 17920a^7b^6c^6d^6e^2 - 10080a^7b^8c^4d^4e^4 + 2240a^7b^9c^3d^3e^5 + 1344a^7b^{10}c^2d^2e^6 - 21504a^8b^4c^7d^6e^2 - 21504a^8b^5c^6d^5e^3 + 32256a^8b^6c^5d^4e^4 - 6720a^8b^8c^3d^2e^6 + 57344a^9b^3c^7d^5e^3 - 46592a^9b^4c^6d^4e^4 - 21504a^9b^5c^5d^3e^5 + 17920a^9b^6c^4d^2e^6 + 12288a^{10}b^2c^7d^4e^4 + 57344a^{10}b^3c^6d^3e^5 - 21504a^{10}b^4c^5d^2e^6 + 96a^7b^{11}c^2d^7e - 16384a^9b^6c^9d^7e - 16384a^{12}b^6c^6d^7e - 4a^3b^{13}c^3d^7e - 4a^3b^{15}c^2d^5e^3 + 96a^4b^{11}c^4d^7e - 12a^4b^{14}c^3d^4e^4 - 960a^5b^9c^5d^7e + 84a^5b^{13}c^2d^3e^5 + 5120a^6b^7c^6d^7e - 140a^6b^{12}c^2d^2e^6 - 15360a^7b^5c^7d^7e + 24576a^8b^3c^8d^7e - 960a^8b^9c^2d^5e^7 + 5120a^9b^7c^3d^5e^7 - 49152a^{10}b^6c^8d^5e^3 - 15360a^{10}b^5c^4d^5e^7 - 49152a^{11}b^6c^7d^3e^5 + 24576a^{11}b^3c^5d^7e))^{(1/2)} - (326912a^8c^9d^5e^{13} - 241664a^8b^6c^8e^{14} - 48a^2b^{13}c^2e^{14} + 1264a^3b^{11}c^3e^{14} - 13552a^4b^9c^4e^{14} + 75776a^5b^7c^5e^{14} - 232960a^6b^5c^6e^{14} + 372736a^7b^3c^7e^{14} + 11520a^3c^{14}d^{11}e^3 + 78080a^4c^{13}d^9e^5 + 197120a^5c^{12}d^7e^7 + 336384a^6c^{11}d^5e^9 + 532736a^7c^{10}d^3e^{11} - 40b^5c^{12}d^{12}e^2 + 216b^6c^{11}d^{11}e^3 - 464b^7c^{10}d^{10}e^4 + 496b^8c^9d^9e^5 - 264b^9c^8d^8e^6 + 56b^{10}c^7d^7e^7 - 16b^{11}c^6d^6e^8 + 64b^{12}c^5d^5e^9 - 96b^{13}c^4d^4e^{10} + 64b^{14}c^3d^3e^{11} - 16b^{15}c^2d^2e^{12} + 1536a^2b^2c^{13}d^{11}e^3 + 14400a^2b^3c^{12}d^{10}e^4 - 47152a^2b^4c^{11}d^9e^5 + 52144a^2b^5c^{10}d^8e^6 - 16272a^2b^6c^9d^7e^7 - 13040a^2b^7c^8d^6e^8 + 23488a^2b^8c^7d^5e^9 - 26384a^2b^9c^6d^4e^{10} + 13824a^2
\end{aligned}$$

$$\begin{aligned}
& *b^{10}c^5d^3e^{11} + 256a^2b^{11}c^4d^2e^{12} + 125056a^3b^2c^{12}d^9e^5 - 36224a^3b^3c^{11}d^8e^6 - 126432a^3b^4c^{10}d^7e^7 + 144848a^3b^5c^9d^6e^8 - 114752a^3b^6c^8d^5e^9 + 125392a^3b^7c^7d^4e^{10} - \\
& 53248a^3b^8c^6d^3e^{11} - 25264a^3b^9c^5d^2e^{12} + 474112a^4b^2c^{11}d^7e^7 - 191104a^4b^3c^{10}d^6e^8 + 97184a^4b^4c^9d^5e^9 - 277000a^4b^5c^8d^4e^{10} + 56056a^4b^6c^7d^3e^{11} + 195584a^4b^7c^6d^2e^{12} + 236800a^5b^2c^{10}d^5e^9 + 388032a^5b^3c^9d^4e^{10} + 159632a^5b^4c^8d^3e^{11} - 670488a^5b^5c^7d^2e^{12} - 488960a^6b^2c^9d^3e^{11} + 1106496a^6b^3c^8d^2e^{12} + 64a^6b^{14}c^2d^6e^2 + 448a^6b^3c^{13}d^{12}e^2 - 1968a^6b^4c^{12}d^{11}e^3 + 2504a^6b^5c^{11}d^{10}e^4 + 768a^6b^6c^{10}d^9e^5 - 4368a^6b^7c^9d^8e^6 + 3568a^6b^8c^8d^7e^7 - 520a^6b^9c^7d^6e^8 - 1728a^6b^{10}c^6d^5e^9 + 2528a^6b^{11}c^5d^4e^{10} - 1536a^6b^{12}c^4d^3e^{11} + 240a^6b^{13}c^3d^2e^{12} - 1152a^7b^2c^{14}d^{12}e^2 - 1600a^7b^{12}c^3d^6e^3 - 67968a^7b^3c^{13}d^{10}e^4 + 15808a^7b^{10}c^4d^6e^3 - 342272a^7b^4c^{12}d^8e^6 - 76928a^7b^8c^5d^6e^3 - 569088a^7b^5c^{11}d^6e^8 + 179200a^7b^6c^6d^6e^{13} - 586368a^6b^6c^{10}d^4e^{10} - 113008a^6b^4c^7d^6e^{13} - 731008a^7b^6c^9d^2e^{12} - 244096a^7b^2c^8d^6e^{13}) / (16(a^6b^8e^8 + 256a^6c^8d^8 + 256a^{10}c^4e^8 - 16a^7b^6c^8e^8 - 4a^5b^9d^8e^7 + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6d^8 - 256a^5b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^{12}d^4e^4 - 4a^3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 + 1024a^7c^7d^6e^2 + 1536a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^{10}c^2d^6e^2 - 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4b^7c^3d^5e^3 - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 - 192a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c^4d^4e^4 - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 + 2048a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024a^6b^6c^7d^7e + 64a^6b^7c^6d^7e - 1024a^9b^6c^4d^7e - 4a^2b^9c^3d^7e - 4a^2b^{11}c^5d^5e^3 + 64a^3b^7c^4d^7e - 4a^3b^{10}c^5d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^3d^3e^5 + 1024a^5b^3c^6d^7e - 92a^5b^8c^3d^2e^6 - 3072a^7b^6c^6d^5e^3 - 384a^7b^5c^2d^5e^7 - 3072a^8b^6c^5d^3e^5 + 1024a^8b^3c^3d^6e^7)) * ((27a^9b^9c^5d^6 - b^{11}c^4d^6 - b^{15}d^2e^4 - 9a^2b^{13}e^6 + 3840a^5b^6c^9d^6 + 9a^5c^5d^6 * (-4a^2c - b^2)^9)^{(1/2)} + 213a^3b^{11}c^6e^6 - 26880a^8b^6c^6e^6 + 3072a^6c^9d^5e^6 + 35840a^8c^7d^6e^5 + 4b^{12}c^3d^5e^6 + 4b^{14}c^3d^3e^3 - 288a^2b^7c^6d^6 + 1504a^3b^5c^7d^6 - 3840a^4b^3c^8d^6 - 9a^2b^4e^6 * (-4a^2c - b^2)^9)^{(1/2)} - 2077a^4b^9c^2e^6 + 10656a^5b^7c^3e^6 - 30240a^6b^5c^4e^6 + 44800a^7b^3c^5e^6 - 25a^4c^2e^6 * (-4a^2c - b^2)^9)^{(1/2)} - b^2c^4d^6 * (-4a^2c - b^2)^9)^{(1/2)} + 22528a^7c^8d^3e^3 - b^6d^2e^4 * (-4a^2c - b^2)^9)^{(1/2)} - 6b^{13}c^2d^4e^2 + 6a^6b^{14}d^5e^5 - 1471a^2b^9c^4d^4e^2 + 600a^2b^{10}c^3d^3e^3 + 180a^2b^{11}c^2d^2e^4 + 6976a^3b^7c^5d^4e^2 - 1032a^3b^8c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456a^4b^5c^6d^4e^2 - 7168a^4b^6c^5d^3e^3 + 16896a^4b^7c^4d^2e^4 + 10240a^5b^3c^7d^4e^2 + 37632a^5b^4c^6d^3e^3 - 47712a^5b^5c^5d^2e^4 - 59392a^6b^2c^7d^3e^3 + 60928a^6b^3c^6d^2e^4 + 41a^2c^4d^4e^2 * (-4a^2c - b^2)^9)^{(1/2)} + 39a^3c^3d^2e^4 * (-4a^2c - b^2)^9)^{(1/2)} - 6b^4c^2d^4e^2 * (-4a^2c - b^2)^9)^{(1/2)} + 6a^6b^5d^5e^5 * (-4a^2c - b^2)^9)^{(1/2)} - 106a^6b^{10}c^4d^5e^5 + 7a^6b^{13}c^3d^2e^4 - 128a^2b^{12}c^6d^5e^5 + 51a^3b^2c^6e^6 * (-4a^2c - b^2)^9)^{(1/2)} + 150a^6b^{11}c^3d^4e^2 - 84a^6b^{12}c^2d^3e^3 + 1116a^2b^8c^5d^5e^5 - 5824a^3b^6c^6d^5e^5 + 1030a^3b^{10}c^2d^6e^5 + 15232a^4b^4c^7d^5e^5 - 3492a^4b^8c^3d^6e^5 - 16896a^5b^2c^8d^5e^5 + 1344a^5b^6c^4d^6e^5 + 7424a^6b^6c^8d^4e^2 + 22400a^6b^4c^5d^6e^5 - 23296a^7b^6c^7d^2e^4 - 53760a^7b^2c^6d^6e^5 + 4b^3c^3d^5e^5 * (-4a^2c - b^2)^9)^{(1/2)} + 4b^5c^3d^3e^3 * (-4a^2c - b^2)^9)^{(1/2)} - 11a^6b^4c^3d^2e^4 * (-4a^2c - b^2)^9)^{(1/2)} - 20a^2b^3c^3d^6e^5 * (-4a^2c - b^2)^9)^{(1/2)} - 86a^3b^6c^2d^6e^5 * (-4a^2c - b^2)^9)^{(1/2)} + 42a^6b^2c^3d^4e^2 * (-4a^2c - b^2)^9)^{(1/2)} - 12a^6b^3c^2d^3e^3 * (-4a^2c -
\end{aligned}$$

$$\begin{aligned}
& b^2)^9)^{(1/2)} - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^{(1/2)} - 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^{(1/2)} + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^{(1/2)} \\
& )/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + \\
& 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + \\
& 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - \\
& 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - \\
& 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + \\
& 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + \\
& 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - \\
& 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7))^(1/2) - (x*(22800*a^6*c^9*e^13 + 36*a^2*b^8*c^5*e^13 - 600*a^3*b^6*c^6*e^13 + 4313*a^4*b^4*c^7*e^13 - \\
& 15592*a^5*b^2*c^8*e^13 + 1296*a^2*c^13*d^8*e^5 + 9792*a^3*c^12*d^6*e^7 + 30304*a^4*c^11*d^4*e^9 + 40512*a^5*c^10*d^2*e^11 + 25*b^4*c^11*d^8*e^5 - 120*b^5*c^10*d^7*e^6 + 214*b^6*c^9*d^6*e^7 - 168*b^7*c^8*d^5*e^8 + 53*b^8*c^7*d^4*e^9 - \\
& 8*b^9*c^6*d^3*e^10 + 4*b^10*c^5*d^2*e^11 + 6336*a^2*b^2*c^11*d^6*e^7 + 3840*a^2*b^3*c^10*d^5*e^8 - 8506*a^2*b^4*c^9*d^4*e^9 + 1112*a^2*b^5*c^8*d^3*e^10 + 1254*a^2*b^6*c^7*d^2*e^11 + 22224*a^3*b^2*c^10*d^4*e^9 + 13824*a^3*b^3*c^9*d^3*e^10 - \\
& 9516*a^3*b^4*c^8*d^2*e^11 + 11712*a^4*b^2*c^9*d^2*e^11 - 24*a*b^9*c^5*d*e^12 - 41088*a^5*b*c^9*d*e^12 - 360*a*b^2*c^12*d^8*e^5 + 1664*a*b^3*c^11*d^7*e^6 - 2604*a*b^4*c^10*d^6*e^7 + 1272*a*b^5*c^9*d^5*e^8 + 332*a*b^6*c^8*d^4*e^9 - \\
& 232*a*b^7*c^7*d^3*e^10 - 48*a*b^8*c^6*d^2*e^11 - 5760*a^2*b*c^12*d^7*e^6 + 416*a^2*b^7*c^6*d*e^12 - 32128*a^3*b*c^11*d^5*e^8 - 4120*a^3*b^5*c^7*d*e^12 - 63360*a^4*b*c^10*d^3*e^10 + 21376*a^4*b^3*c^8*d*e^12))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - \\
& 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + \\
& 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - \\
& 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + \\
& 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - \\
& 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7))((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + \\
& 3840*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^{(1/2)} + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - \\
& 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^{(1/2)} - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 +
\end{aligned}$$

$$\begin{aligned}
& 44800a^7b^3c^5e^6 - 25a^4c^2e^6(-4ac - b^2)^9)^{(1/2)} - b^2c^4d^6(-4ac - b^2)^9)^{(1/2)} + 22528a^7c^8d^3e^3 - b^6d^2e^4(-4ac - b^2)^9)^{(1/2)} - 6b^{13}c^2d^4e^2 + 6a^8b^{14}d^5e^5 - 1471a^2b^9c^4d^4e^2 + 600a^2b^{10}c^3d^3e^3 + 180a^2b^{11}c^2d^2e^4 + 6976a^3b^7c^5d^4e^2 - 1032a^3b^8c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456a^4b^5c^6d^4e^2 - 7168a^4b^6c^5d^3e^3 + 16896a^4b^7c^4d^2e^4 + 10240a^5b^3c^7d^4e^2 + 37632a^5b^4c^6d^3e^3 - 47712a^5b^5c^5d^2e^4 - 59392a^6b^2c^7d^3e^3 + 60928a^6b^3c^6d^2e^4 + 41a^2c^4d^4e^2(-4ac - b^2)^9)^{(1/2)} + 39a^3c^3d^2e^4(-4ac - b^2)^9)^{(1/2)} - 6b^4c^2d^4e^2(-4ac - b^2)^9)^{(1/2)} + 6a^8b^5d^5e^5(-4ac - b^2)^9)^{(1/2)} - 106a^8b^{10}c^4d^5e^5 + 7a^8b^{13}c^4d^2e^4 - 128a^2b^{12}c^4d^5e^5 + 51a^3b^2c^6e^6(-4ac - b^2)^9)^{(1/2)} + 150a^8b^{11}c^3d^4e^2 - 84a^8b^{12}c^2d^3e^3 + 1116a^2b^8c^5d^5e^5 - 5824a^3b^6c^6d^5e^5 + 1030a^3b^{10}c^2d^5e^5 + 15232a^4b^4c^7d^5e^5 - 3492a^4b^8c^3d^5e^5 - 16896a^5b^2c^8d^5e^5 + 1344a^5b^6c^4d^5e^5 + 7424a^6b^6c^8d^4e^2 + 22400a^6b^4c^5d^5e^5 - 23296a^7b^3c^7d^2e^4 - 53760a^7b^2c^6d^5e^5 + 4b^3c^3d^5e^5(-4ac - b^2)^9)^{(1/2)} + 4b^5c^4d^3e^3(-4ac - b^2)^9)^{(1/2)} - 11a^8b^4c^4d^2e^4(-4ac - b^2)^9)^{(1/2)} - 20a^2b^3c^4d^5e^5(-4ac - b^2)^9)^{(1/2)} - 86a^3b^3c^2d^5e^5(-4ac - b^2)^9)^{(1/2)} + 42a^8b^2c^3d^4e^2(-4ac - b^2)^9)^{(1/2)} - 12a^8b^3c^2d^3e^3(-4ac - b^2)^9)^{(1/2)} - 120a^2b^6c^3d^3e^3(-4ac - b^2)^9)^{(1/2)} - 34a^8b^4c^4d^5e^5(-4ac - b^2)^9)^{(1/2)} + 108a^2b^2c^2d^2e^4(-4ac - b^2)^9)^{(1/2))} / (32(a^7b^{12}e^8 + 4096a^9c^{10}d^8 + 4096a^{13}c^6e^8 - 24a^8b^{10}c^4e^8 - 4a^6b^{13}d^7e^7 + a^3b^{12}c^4d^8 - 24a^4b^{10}c^5d^8 + 240a^5b^8c^6d^8 - 1280a^6b^6c^7d^8 + 3840a^7b^4c^8d^8 - 6144a^8b^2c^9d^8 + 240a^9b^8c^2e^8 - 1280a^{10}b^6c^3e^8 + 3840a^{11}b^4c^4e^8 - 6144a^{12}b^2c^5e^8 + a^3b^{16}d^4e^4 - 4a^4b^{15}d^3e^5 + 6a^5b^{14}d^2e^6 + 16384a^{10}c^9d^6e^2 + 24576a^{11}c^8d^4e^4 + 16384a^{12}c^7d^2e^6 + 6a^3b^{14}c^2d^6e^2 - 140a^4b^{12}c^3d^6e^2 + 84a^4b^{13}c^2d^5e^3 + 1344a^5b^{10}c^4d^6e^2 - 672a^5b^{11}c^3d^5e^3 - 42a^5b^{12}c^2d^4e^4 - 6720a^6b^8c^5d^6e^2 + 2240a^6b^9c^4d^5e^3 + 1456a^6b^{10}c^3d^4e^4 - 672a^6b^{11}c^2d^3e^5 + 17920a^7b^6c^6d^6e^2 - 10080a^7b^8c^4d^4e^4 + 2240a^7b^9c^3d^3e^5 + 1344a^7b^{10}c^2d^2e^6 - 21504a^8b^4c^7d^6e^2 - 21504a^8b^5c^6d^5e^3 + 32256a^8b^6c^5d^4e^4 - 6720a^8b^8c^3d^2e^6 + 57344a^9b^3c^7d^5e^3 - 46592a^9b^4c^6d^4e^4 - 21504a^9b^5c^5d^3e^5 + 17920a^9b^6c^4d^2e^6 + 12288a^{10}b^2c^7d^4e^4 + 57344a^{10}b^3c^6d^3e^5 - 21504a^{10}b^4c^5d^2e^6 + 96a^7b^{11}c^4d^7e^7 - 16384a^9b^6c^9d^7e^7 - 16384a^{12}b^6c^6d^7e^7 - 4a^3b^{13}c^3d^7e^7 - 4a^3b^{15}c^4d^5e^3 + 96a^4b^{11}c^4d^7e^7 - 12a^4b^{14}c^4d^4e^4 - 960a^5b^9c^5d^7e^7 + 84a^5b^{13}c^4d^3e^5 + 5120a^6b^7c^6d^7e^7 - 140a^6b^{12}c^4d^2e^6 - 15360a^7b^5c^7d^7e^7 + 24576a^8b^3c^8d^7e^7 - 960a^8b^9c^2d^5e^7 + 5120a^9b^7c^3d^5e^7 - 49152a^{10}b^6c^8d^5e^3 - 15360a^{10}b^5c^4d^4e^7 - 49152a^{11}b^6c^7d^3e^5 + 24576a^{11}b^3c^5d^4e^7))^{(1/2)} * i - ((((((1048576a^{13}c^8e^{16} + 256a^7b^{12}c^2e^{16} - 6144a^8b^{10}c^3e^{16} + 61440a^9b^8c^4e^{16} - 327680a^{10}b^6c^5e^{16} + 983040a^{11}b^4c^6e^{16} - 1572864a^{12}b^2c^7e^{16} - 196608a^6c^{15}d^{14}e^2 - 917504a^7c^{14}d^{12}e^4 - 589824a^8c^{13}d^{10}e^6 + 3932160a^9c^{12}d^8e^8 + 10158080a^{10}c^{11}d^6e^{10} + 10616832a^{11}c^{10}d^4e^{12} + 5308416a^{12}c^9d^2e^{14} - 2816a^2b^8c^{11}d^{14}e^2 + 22656a^2b^9c^{10}d^{13}e^3 - 78848a^2b^{10}c^9d^{12}e^4 + 154112a^2b^{11}c^8d^{11}e^5 - 182784a^2b^{12}c^7d^{10}e^6 + 130816a^2b^{13}c^6d^9e^7 - 50176a^2b^{14}c^5d^8e^8 + 4608a^2b^{15}c^4d^7e^9 + 3328a^2b^{16}c^3d^6e^{10} - 896a^2b^{17}c^2d^5e^{11} + 24576a^3b^6c^{12}d^{14}e^2 - 198656a^3b^7c^{11}d^{13}e^3 + 684544a^3b^8c^{10}d^{12}e^4 - 1291520a^3b^9c^9d^{11}e^5 + 1403776a^3b^{10}c^8d^{10}e^6 - 798336a^3b^{11}c^7d^9e^7 + 89856a^3b^{12}c^6d^8e^8 + 155136a^3b^{13}c^5d^7e^9 - 77440a^3b^{14}c^4d^6e^{10} + 5504a^3b^{15}c^3d^5e^{11} + 2560a^3b^{16}c^2d^4e^{12} - 106496a^4b^4c^{13}d^{14}e^2 + 864256a^4b^5c^{12}d^{13}e^3 - 2924544a^4b^6c^{11}d^{12}e^4 + 5181440a^4b^7c^{10}d^{11}e^5 - 1048576a^4b^8c^9d^{10}e^6 + 1048576a^4b^9c^8d^9e^7 - 5181440a^4b^{10}c^7d^8e^8 + 1048576a^4b^{11}c^6d^7e^9 - 5181440a^4b^{12}c^5d^6e^{10} + 1048576a^4b^{13}c^4d^5e^{11} - 5181440a^4b^{14}c^3d^4e^{12} + 1048576a^4b^{15}c^2d^3e^{13} - 5181440a^4b^{16}c^2d^3e^{13}))^{(1/2)} * i - ((((((1048576a^{13}c^8e^{16} + 256a^7b^{12}c^2e^{16} - 6144a^8b^{10}c^3e^{16} + 61440a^9b^8c^4e^{16} - 327680a^{10}b^6c^5e^{16} + 983040a^{11}b^4c^6e^{16} - 1572864a^{12}b^2c^7e^{16} - 196608a^6c^{15}d^{14}e^2 - 917504a^7c^{14}d^{12}e^4 - 589824a^8c^{13}d^{10}e^6 + 3932160a^9c^{12}d^8e^8 + 10158080a^{10}c^{11}d^6e^{10} + 10616832a^{11}c^{10}d^4e^{12} + 5308416a^{12}c^9d^2e^{14} - 2816a^2b^8c^{11}d^{14}e^2 + 22656a^2b^9c^{10}d^{13}e^3 - 78848a^2b^{10}c^9d^{12}e^4 + 154112a^2b^{11}c^8d^{11}e^5 - 182784a^2b^{12}c^7d^{10}e^6 + 130816a^2b^{13}c^6d^9e^7 - 50176a^2b^{14}c^5d^8e^8 + 4608a^2b^{15}c^4d^7e^9 + 3328a^2b^{16}c^3d^6e^{10} - 896a^2b^{17}c^2d^5e^{11} + 24576a^3b^6c^{12}d^{14}e^2 - 198656a^3b^7c^{11}d^{13}e^3 + 684544a^3b^8c^{10}d^{12}e^4 - 1291520a^3b^9c^9d^{11}e^5 + 1403776a^3b^{10}c^8d^{10}e^6 - 798336a^3b^{11}c^7d^9e^7 + 89856a^3b^{12}c^6d^8e^8 + 155136a^3b^{13}c^5d^7e^9 - 77440a^3b^{14}c^4d^6e^{10} + 5504a^3b^{15}c^3d^5e^{11} + 2560a^3b^{16}c^2d^4e^{12} - 106496a^4b^4c^{13}d^{14}e^2 + 864256a^4b^5c^{12}d^{13}e^3 - 2924544a^4b^6c^{11}d^{12}e^4 + 5181440a^4b^7c^{10}d^{11}e^5 - 1048576a^4b^8c^9d^{10}e^6 + 1048576a^4b^9c^8d^9e^7 - 5181440a^4b^{10}c^7d^8e^8 + 1048576a^4b^{11}c^6d^7e^9 - 5181440a^4b^{12}c^5d^6e^{10} + 1048576a^4b^{13}c^4d^5e^{11} - 5181440a^4b^{14}c^3d^4e^{12} + 1048576a^4b^{15}c^2d^3e^{13} - 5181440a^4b^{16}c^2d^3e^{13}))^{(1/2)} * i - ((((((1048576a^{13}c^8e^{16} + 256a^7b^{12}c^2e^{16} - 6144a^8b^{10}c^3e^{16} + 61440a^9b^8c^4e^{16} - 327680a^{10}b^6c^5e^{16} + 983040a^{11}b^4c^6e^{16} - 1572864a^{12}b^2c^7e^{16} - 196608a^6c^{15}d^{14}e^2 - 917504a^7c^{14}d^{12}e^4 - 589824a^8c^{13}d^{10}e^6 + 3932160a^9c^{12}d^8e^8 + 10158080a^{10}c^{11}d^6e^{10} + 10616832a^{11}c^{10}d^4e^{12} + 5308416a^{12}c^9d^2e^{14} - 2816a^2b^8c^{11}d^{14}e^2 + 22656a^2b^9c^{10}d^{13}e^3 - 78848a^2b^{10}c^9d^{12}e^4 + 154112a^2b^{11}c^8d^{11}e^5 - 182784a^2b^{12}c^7d^{10}e^6 + 130816a^2b^{13}c^6d^9e^7 - 50176a^2b^{14}c^5d^8e^8 + 4608a^2b^{15}c^4d^7e^9 + 3328a^2b^{16}c^3d^6e^{10} - 896a^2b^{17}c^2d^5e^{11} + 24576a^3b^6c^{12}d^{14}e^2 - 198656a^3b^7c^{11}d^{13}e^3 + 684544a^3b^8c^{10}d^{12}e^4 - 1291520a^3b^9c^9d^{11}e^5 + 1403776a^3b^{10}c^8d^{10}e^6 - 798336a^3b^{11}c^7d^9e^7 + 89856a^3b^{12}c^6d^8e^8 + 155136a^3b^{13}c^5d^7e^9 - 77440a^3b^{14}c^4d^6e^{10} + 5504a^3b^{15}c^3d^5e^{11} + 2560a^3b^{16}c^2d^4e^{12} - 106496a^4b^4c^{13}d^{14}e^2 + 864256a^4b^5c^{12}d^{13}e^3 - 2924544a^4b^6c^{11}d^{12}e^4 + 5181440a^4b^7c^{10}d^{11}e^5 - 1048576a^4b^8c^9d^{10}e^6 + 1048576a^4b^9c^8d^9e^7 - 5181440a^4b^{10}c^7d^8e^8 + 1048576a^4b^{11}c^6d^7e^9 - 5181440a^4b^{12}c^5d^6e^{10} + 1048576a^4b^{13}c^4d^5e^{11} - 5181440a^4b^{14}c^3d^4e^{12} + 1048576a^4b^{15}c^2d^3e^{13} - 5181440a^4b^{16}c^2d^3e^{13}))^{(1/2)} * i - ((((((1048576a^{13}c^8e^{16} + 256a^7b^{12}c^2e^{16} - 6144a^8b^{10}c^3e^{16} + 61440a^9b^8c^4e^{16} - 327680a^{10}b^6c^5e^{16} + 983040a^{11}b^4c^6e^{16} - 1572864a^{12}b^2c^7e^{16} - 196608a^6c^{15}d^{14}e^2 - 917504a^7c^{14}d^{12}e^4 - 589824a^8c^{13}d^{10}e^6 + 3932160a^9c^{12}d^8e^8 + 10158080a^{10}c^{11}d^6e^{10} + 10616832a^{11}c^{10}d^4e^{12} + 5308416a^{12}c^9d^2e^{14} - 2816a^2b^8c^{11}d^{14}e^2 + 22656a^2b^9c^{10}d^{13}e^3 - 78848a^2b^{10}c^9d^{12}e^4 + 154112a^2b^{11}c^8d^{11}e^5 - 182784a^2b^{12}c^7d^{10}e^6 + 130816a^2b^{13}c^6d^9e^7 - 50176a^2b^{14}c^5d^8e^8 + 4608a^2b^{15}c^4d^7e^9 + 3328a^2b^{16}c^3d^6e^{10} - 896a^2b^{17}c^2d^5e^{11} + 24576a^3b^6c^{12}d^{14}e^2 - 198656a^3b^7c^{11}d^{13}e^3 + 684544a^3b^8c^{10}d^{12}e^4 - 1291520a^3b^9c^9d^{11}e^5 + 1403776a^3b^{10}c^8d^{10}e^6 - 798336a^3b^{11}c^7d^9e^7 + 89856a^3b^{12}c^6d^8e^8 + 155136a^3b^{13}c^5d^7e^9 - 77440a^3b^{14}c^4d^6e^{10} + 5504a^3b^{15}c^3d^5e^{11} + 2560a^3b^{16}c^2d^4e^{12} - 106496a^4b^4c^{13}d^{14}e^2 + 864256a^4b^5c^{12}d^{13}e^3 - 2924544a^4b^6c^{11}d^{12}e^4 + 5181440a^4b^7c^{10}d^{11}e^5 - 1048576a^4b^8c^9d^{10}e^6 + 1048576a^4b^9c^8d^9e^7 - 5181440a^4b^{10}c^7d^8e^8 + 1048576a^4b^{11}c^6d^7e^9 - 5181440a^4b^{12}c^5d^6e^{10} + 1048576a^4b^{13}c^4d^5e^{11} - 5181440a^4b^{14}c^3d^4e^{12} + 1048576a^4b^{15}c^2d^3e^{13} - 5181440a^4b^{16}c^2d^3e^{13}))^{(1/2)} * i - ((((((1048576a^{13}c^8e^{16} + 256a^7b^{12}c^2e^{16} - 6144a^8b^{10}c^3e^{16} + 61440a^9b^8c^4e^{16} - 327680a^{10}b^6c^5e^{16} + 983040a^{11}b^4c^6e^{16} - 1572864a^{12}b^2c^7e^{16} - 196608a^6c^{15}d^{14}e^2 - 917504a^7c^{14}d^{12}e^4 - 589824a^8c^{13}d^{10}e^6 + 3932160a^9c^{12}d^8e^8 + 10158080a^{10}c^{11}d^6e^{10} + 10616832a^{11}c^{10}d^4e^{12} + 5308416a^{12}c^9d^2e^{14} - 2816a^2b^8c^{11}d^{14}e^2 + 22656a^2b^9c^{10}d^{13}e^3 - 78848a^2b^{10}c^9d^{12}e^4 + 154112a^2b^{11}c^8d^{11}e^5 - 182784a^2b^{12}c^7d^{10}e^6 + 130816a^2b^{13}c^6d^9e^7 - 50176a^2b^{14}c^5d^8e^8 + 4608a^2b^{15}c^4d^7e^9 + 3328a^2b^{16}c^3d^6e^{10} - 896a^2b^{17}c^2d^5e^{11} + 24576a^3b^6c^{12}d^{14}e^2 - 198656a^3b^7c^{11}d^{13}e^3 + 684544a^3b^8c^{10}d^{12}e^4 - 1291520a^3b^9c^9d^{11}e^5 + 1403776a^3b^{10}c^8d^{10}e^6 - 798336a^3b^{11}c^7d^9e^7 + 89856a^3b^{12}c^6d^8e^8 + 155136a^3b^{13}c^5d^7e^9 - 77440a^3b^{14}c^4d^6e^{10} + 5504a^3b^{15}c^3d^5e^{11} + 2560a^3b^{16}c^2d^4e^{12} - 106496a^4b^4c^{13}d^{14}e^2 + 864256a^4b^5c^{12}d^{13}e^3 - 2924544a^4b^6c^{11}d^{12}e^4 + 5181440a^4b^7c^{10}d^{11}e^5 - 1048576a^4b^8c^9d^{10}e^6 + 1048576a^4b^9c^8d^9e^7 - 5181440a^4b^{10}c^7d^8e^8 + 1048576a^4b^{11}c^6d^7e^9 - 5181440a^4b^{12}c^5d^6e^{10} + 1048576a^4b^{13}c^4d^5e^{11} - 5181440a^4b^{14}c^3d^4e^{12} + 1048576a^4b^{15}c^2d^3e^{13} - 5181440a^4b^{16}c^2d^3e^{13}))^{(1/2)} * i - ((((((1048576a^{13}c^8e^{16} + 256a^7b^{12}c^2e^{16} - 6144a^8b^{10}c^3e^{16} + 61440a^9b^8c^4e^{16} - 327680a^{10}b^6c^5e^{16} + 983040a^{11}b^4c^6e^{16} - 1572864a^{12}b^2c^7e^{16} - 196608a^6c^{15}d^{14}e^2 - 917504a^7c^{14}d^{12}e^4 - 589824a^8c^{13}d^{10}e^6 + 3932160a^9c^{12}d^8e^8 + 10158080a^{10}c^{11}d^6e^{10} + 10616832a^{11}c^{10}d^4e^{12} + 5308416a^{12}c^9d^2e^{14} - 2816a^2b^8c^{11}d^{14}e^2 + 22656a^2b^9c^{10}d^{13}e^3 - 78848a^2b^{10}c^9d^{12}e^4 + 154112a^2b^{11}c^8d^{11}e^5 - 182784a^2b^{12}c^7d^{10}e^6 + 130816a^2b^{13}c^6d^9e^7 - 50176a^2b^{14}c^5d^8e^8 + 4608a^2b^{15}c^4d^7e^9 + 3328a^2b^{16}c^3d^6e^{10} - 896a^2b^{17}c^2d^5e^{11} + 24576a^3b^6c^{12}d^{14}e^2 - 198656a^3b^7c^{11}d^{13}e^3 + 684544a^3b^8c^{10}d^{12}e^4 - 1291520a^3b^9c^9d^{11}e^5 + 1403776a^3b^{10}c^8d^{10}e^6 - 798336a^3b^{11}c^7d^9e^7 + 89856a^3b^{12}c^6d^8e^8 + 155136a^3b^{13}c^5d^7e^9 - 77440a^3b^{14}c^4d^6e^{10} + 5504a^3b^{15}c^3d^5e^{11} + 2560a^3b^{16}c^2d^4e^{12} - 106496a^4b^4c^{13}d^{14}e^2 + 864256a^4b^5c^{12}d^{13}e^3 - 2924544a^4b^6c^{11}d^{12}e^4 + 5181440a^4b^7c^{10}d^{11}e^5 - 1048576a^4b^8c^9d^{10}e^6 + 1048576a^4b^9c^8d^9e^7 - 5181440a^4b^{10}c^7d^8e^8 + 1048576a^4b^{11}c^6d^7e^9 - 5181440a^4b^{12}c^5d^6e^{10} + 1048576a^4b^{13}c^4d^5e^{11} - 5181440a^4b^{14}c^3d^4e^{12} + 1048576a^4b^{15}c^2d^3e^{13} - 5181440a^4b^{16}c^2d^3e^{13}))^{(1/2)} * i - ((((((1048576a^{13}c^8e^{16} + 256a^7b^{12}c^2e^{16} - 6144a^8b^{10}c^3e^{16} + 61440a^9b^8c^4e^{16} - 327680a^{10}b^6c^5e^{16} + 983040a^{11}b^4c^6e^{16} - 1572864a^{12}b^2c^7e^{16} - 196608a^6c^{15}d^{14}e^2 - 917504a^7c^{14}d^{12}e^4 - 589824a^8c^{13}d^{10}e^6 + 3932160a^9c^{12}d^8e^8 + 10158080a^{10}c^{11}d^6e^{10} + 10616832a^{11}c^{10}d^4e^{12} + 5308416a^{12}c^9d^2e^{14} - 2816a^2b^8c^{11}d^{14}e^2 + 22656a^2b^9c^{10}d^{13}e^3 - 78848a^2b^{10}c^9d^{12}e^4 + 154112a^2b^{11}c^8d^{11}e^5 - 182784a^2b^{12}c^7d^{10}e^6 + 130816a^2b^{13}c^6d^9e^7 - 50176a^2b^{14}c^5d^8e^8 + 4608a^2b^{15}c^4d^7e^9 + 3328a^2b^{16}c^3d^6e^{10} - 896a^2b^{17}c^2d^5e^{11} + 24576a^3b^6c^{12}d^{14}e^2 - 198656a^3b^7c^{11}d^{13}e^3 + 684544a^3b^8c^{10}d^{12}e^4 - 1291520a^3b^9c^9d^{11}e^5 + 1403776a^3b^{10}c^8d^{10}e^6 - 798336a^3b^{11}c^7d^9e^7 + 89856a^3b^{12}c^6d^8e^8 + 155136a^3b^{13}c^5d^7e^9 - 77440a^3b^{14}c^4d^6e^{10} + 5504a^3b^{15}c^3d^5e^{11} + 2560a^3b^{16}c^2d^4e^{12} - 106496a^4b^4c^{13}d^{14}e^2 + 864256a^4b^5c^{12}d^{13}e^3 - 2924544a^4b^6c^{11}d^{12}e^4 + 5181440a^4b^7c^{10}d^{11}e^5 - 1048576a^4b^8c^9d^{10}e^6 + 1048576a^4b^9c^8d^9e^7 - 5181440a^4b^{10}c^7d^8e^8 + 1048576a^4b^{11}c^6d^7e^9 - 5181440a^4b^{12}c^5d^6e^{10} + 1048576a^4b^{13}c^4d^5e^{11} - 5181440a^4b^{14}c^3d^4e^{12} + 1048576a^4b^{15}c^2d^3e^{13} - 5181440a^4b^{16}c^2d^3e^{13}))^{(1/2)} * i - ((((((1048576a^{13}c^8e^{16} + 256a^7b^{12}c^2e^{16} - 6144a^8b^{10}c^3e^{16} + 61440a^9b^8c^4e^{16} - 327680a^{10}b^6c^5e^{16} + 983040a^{11}b^4c^6e^{16} - 1572864a^{12}b^2c^7e^{16} - 196608a^6c^{15}d^{14}e^2 - 917504a^7c^{14}d^{12}e^4 - 589824a^8c^{13}d^{10}e^6 + 3932160a^9c^{12}d^8e^8 + 10158080a^{10}c^{11}d^6e^{10} + 10616832a^{11}c^{10}d^4e^{12} + 5308416a^{12}c^9d^2e^{14} - 2816a^2b^8c^{11}d^{14}e^2 + 22656a^2b^9c^{10}d^{13}e^3 - 78848a^2b^{10}c^9d^{12}e^4 + 154112a^2b^{11}c^8d^{11}e^5 - 182784a^2b^{12}c^7d^{10}e^6 + 130816a^2b^{13}c^6d^9e^7 - 50176a^2b^{14}c^5d^8e^8 + 4608a^2b^{15}c^4d^7e^9 + 3328a^2b^{16}c^3d^6e^{10} - 896a^2b^{17}c^2d^5e^{11} + 24576a^3b^6c^{12}d^{14}e^2 - 198656a^3b^7c^{11}d^{13}e^3 + 684544a^3b^8c^{10}d^{12}e^4 - 1291520a^3b^9c^9d^{11}e^5 + 1403776a^3b^{10}c^8d^{10}e^6 - 798336a^3b^{11}c^7d^9e^7 + 89856a^3b^{12}c^6d^8e^8 + 155136a^3b^{13}c^5d^7e^9 - 77440a^3b^{14}c^4d^6e^{10} + 5504a^3b^{15}c^3d^5e^{11} + 2560a^3b^{16}c^2d^4e^{12} - 106496a^4b^4c^{13}d^{14}e^2 + 864256a^4b^5c^{12}d^{13}e^3 - 2924544a^4b^6c^{11}d^{12}e^4 + 5181440a^4b^7c^{10}d^{11}e^5 - 1048576a^4b^8c^9d^{10}e^6 + 1048576a^4b^9c^8d^9e^7 - 5181440a^$$

$$\begin{aligned}
& c^{10}d^{11}e^5 - 4686080a^4b^8c^9d^{10}e^6 + 1045376a^4b^9c^8d^9e^7 \\
& + 1900544a^4b^{10}c^7d^8e^8 - 1732096a^4b^{11}c^6d^7e^9 + 390400a^4b^{12}c^5d^6e^{10} + 112000a^4b^{13}c^4d^5e^{11} - 40960a^4b^{14}c^3d^4e^{12} \\
& - 3840a^4b^{15}c^2d^3e^{13} + 229376a^5b^2c^{14}d^{14}e^2 - 1867776a^5b^3c^{13}d^{13}e^3 + 6078464a^5b^4c^{12}d^{12}e^4 - 9297920a^5b^5c^{11}d^{11}e^5 \\
& + 4055040a^5b^6c^{10}d^{10}e^6 + 7788544a^5b^7c^9d^9e^7 - 12657664a^5b^8c^8d^8e^8 + 6130176a^5b^9c^7d^7e^9 + 734080a^5b^{10}c^6d^6e^{10} \\
& - 1442560a^5b^{11}c^5d^5e^{11} + 168960a^5b^{12}c^4d^4e^{12} + 78080a^5b^{13}c^3d^3e^{13} + 3200a^5b^{14}c^2d^2e^{14} - 4587520a^6b^2c^{13}d^{12}e^4 \\
& + 3080192a^6b^3c^{12}d^{11}e^5 + 12001280a^6b^4c^{11}d^{10}e^6 - 31076352a^6b^5c^{10}d^9e^7 + 27475968a^6b^6c^9d^8e^8 - 2088960a^6b^7c^8d^7e^9 \\
& - 12205312a^6b^8c^7d^6e^{10} + 6043520a^6b^9c^6d^5e^{11} + 631808a^6b^{10}c^5d^4e^{12} - 610304a^6b^{11}c^4d^3e^{13} - 71936a^6b^{12}c^3d^2e^{14} \\
& - 21725184a^7b^2c^{12}d^{10}e^6 + 30801920a^7b^3c^{11}d^9e^7 - 8028160a^7b^4c^{10}d^8e^8 - 32260096a^7b^5c^9d^7e^9 + 37101568a^7b^6c^8d^6e^{10} \\
& - 7182336a^7b^7c^7d^5e^{11} - 7609856a^7b^8c^6d^4e^{12} + 2112256a^7b^9c^5d^3e^{13} + 661632a^7b^{10}c^4d^2e^{14} - 30146560a^8b^2c^{11}d^8e^8 \\
& + 55050240a^8b^3c^{10}d^7e^9 - 34365440a^8b^4c^9d^6e^{10} - 16429056a^8b^5c^8d^5e^{11} + 24600576a^8b^6c^7d^4e^{12} - 1683456a^8b^7c^6d^3e^{13} \\
& - 3151616a^8b^8c^5d^2e^{14} - 10977280a^9b^2c^{10}d^6e^{10} + 47022080a^9b^3c^9d^5e^{11} - 30621696a^9b^4c^8d^4e^{12} - 9232384a^9b^5c^7d^3e^{13} \\
& + 7970816a^9b^6c^6d^2e^{14} + 4325376a^{10}b^2c^9d^4e^{12} + 25493504a^{10}b^3c^8d^3e^{13} - 9117696a^{10}b^4c^7d^2e^{14} + 491520a^{11}b^2c^8d^2e^{14} \\
& - 4947968a^{12}b^2c^8d^2e^{15} + 128a^2b^{10}c^{10}d^{14}e^2 - 1024a^2b^{11}c^9d^{13}e^3 + 3584a^2b^{12}c^8d^{12}e^4 - 7168a^2b^{13}c^7d^{11}e^5 \\
& + 8960a^2b^{14}c^6d^{10}e^6 - 7168a^2b^{15}c^5d^9e^7 + 3584a^2b^{16}c^4d^8e^8 - 1024a^2b^{17}c^3d^7e^9 + 128a^2b^{18}c^2d^6e^{10} + 1605632a^6b^2c^{14}d^{13}e^3 \\
& - 1408a^6b^{13}c^2d^2e^{15} + 7012352a^7b^2c^{13}d^{11}e^5 + 33152a^7b^{11}c^3d^2e^{15} + 7045120a^8b^2c^{12}d^9e^7 - 324480a^8b^9c^4d^2e^{15} \\
& - 9830400a^9b^2c^{11}d^7e^9 + 1689600a^9b^7c^5d^2e^{15} - 25722880a^{10}b^2c^{10}d^5e^{11} - 4935680a^{10}b^5c^6d^2e^{15} - 19202048a^{11}b^2c^9d^3e^{13} \\
& + 7667712a^{11}b^3c^7d^2e^{15}) / (16(a^6b^8e^8 + 256a^6c^8d^8 + 256a^{10}c^4e^8 - 16a^7b^6c^2e^8 - 4a^5b^9d^2e^7 + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6d^8 \\
& - 256a^5b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^{12}d^4e^4 - 4a^3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 + 1024a^7c^7d^6e^2 + 1536a^8c^6d^4e^4 \\
& + 1024a^9c^5d^2e^6 + 6a^2b^{10}c^2d^6e^2 - 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4b^7c^3d^5e^3 - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 \\
& - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 - 192a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c^4d^4e^4 - 128a^6b^5c^3d^3e^5 \\
& + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 + 2048a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024a^6b^2c^7d^7e + 64a^6b^7c^2d^2e^7 \\
& - 1024a^9b^2c^4d^2e^7 - 4a^2b^9c^3d^7e - 4a^2b^{11}c^5d^5e^3 + 64a^3b^7c^4d^7e - 4a^3b^{10}c^4d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^4d^3e^5 \\
& + 1024a^5b^3c^6d^7e - 92a^5b^8c^2d^2e^6 - 3072a^7b^2c^6d^5e^3 - 384a^7b^5c^2d^2e^7 - 3072a^8b^2c^5d^3e^5 + 1024a^8b^3c^3d^2e^7) + (x((27a^2b^9c^5d^6 - b^{11}c^4d^6 - b^{15}d^2e^4 - 9a^2b^{13}e^6 \\
& + 3840a^5b^2c^9d^6 + 9a^2c^5d^6(-4ac - b^2)^9)^{1/2} + 213a^3b^{11}c^2e^6 - 26880a^8b^2c^6e^6 + 3072a^6c^9d^5e + 35840a^8c^7d^2e^5 + 4b^{12}c^3d^5e + 4b^{14}c^3d^3e^3 - 288a^2b^7c^6d^6 \\
& + 1504a^3b^5c^7d^6 - 3840a^4b^3c^8d^6 - 9a^2b^4e^6(-4ac - b^2)^9)^{1/2} - 2077a^4b^9c^2e^6 + 10656a^5b^7c^3e^6 - 30240a^6b^5c^4e^6 + 44800a^7b^3c^5e^6 - 25a^4c^2e^6(-4ac - b^2)^9)^{1/2} \\
& - b^2c^4d^6(-4ac - b^2)^9)^{1/2} + 22528a^7c^8d^3e^3 - b^6d^2e^4(-4ac - b^2)^9)^{1/2} - 6b^{13}c^2d^4e^2 + 6a^2b^{14}d^4e^5 - 1471a^2b^9c^4d^4e^2 + 600a^2b^{10}c^3d^3e^3 \\
& + 180a^2b^{11}c^2d^2e^4 + 6976a^3b^7c^5d^4e^2 - 1032a^3b^8c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456
\end{aligned}$$

$$\begin{aligned}
& a^4 b^5 c^6 d^4 e^2 - 7168 a^4 b^6 c^5 d^3 e^3 + 16896 a^4 b^7 c^4 d^2 e^4 \\
& + 10240 a^5 b^3 c^7 d^4 e^2 + 37632 a^5 b^4 c^6 d^3 e^3 - 47712 a^5 b^5 c^5 d^2 e^4 - 59392 a^6 b^2 c^7 d^3 e^3 + 60928 a^6 b^3 c^6 d^2 e^4 + 41 a^2 c^4 d^4 e^2 (-4 a c - b^2)^9)^{(1/2)} + 39 a^3 c^3 d^2 e^4 (-4 a c - b^2)^9)^{(1/2)} - 6 b^4 c^2 d^4 e^2 (-4 a c - b^2)^9)^{(1/2)} + 6 a b^5 d e^5 (-4 a c - b^2)^9)^{(1/2)} - 106 a b^{10} c^4 d^5 e + 7 a b^{13} c d^2 e^4 - 128 a^2 b^{12} c d e^5 + 51 a^3 b^2 c e^6 (-4 a c - b^2)^9)^{(1/2)} + 150 a b^{11} c^3 d^4 e^2 - 84 a b^{12} c^2 d^3 e^3 + 1116 a^2 b^8 c^5 d^5 e - 5824 a^3 b^6 c^6 d^5 e + 1030 a^3 b^{10} c^2 d e^5 + 15232 a^4 b^4 c^7 d^5 e - 3492 a^4 b^8 c^3 d e^5 - 16896 a^5 b^2 c^8 d^5 e + 1344 a^5 b^6 c^4 d e^5 + 7424 a^6 b^6 c^8 d^4 e^2 + 22400 a^6 b^4 c^5 d e^5 - 23296 a^7 b^6 c^7 d^2 e^4 - 53760 a^7 b^2 c^6 d e^5 + 4 b^3 c^3 d^5 e (-4 a c - b^2)^9)^{(1/2)} + 4 b^5 c d^3 e^3 (-4 a c - b^2)^9)^{(1/2)} - 11 a b^4 c d^2 e^4 (-4 a c - b^2)^9)^{(1/2)} - 20 a^2 b^3 c d e^5 (-4 a c - b^2)^9)^{(1/2)} - 86 a^3 b^2 c^2 d e^5 (-4 a c - b^2)^9)^{(1/2)} + 42 a b^2 c^3 d^4 e^2 (-4 a c - b^2)^9)^{(1/2)} - 12 a b^3 c^2 d^3 e^3 (-4 a c - b^2)^9)^{(1/2)} - 120 a^2 b^6 c^3 d^3 e^3 (-4 a c - b^2)^9)^{(1/2)} - 34 a b^6 c^4 d^5 e (-4 a c - b^2)^9)^{(1/2)} + 108 a^2 b^2 c^2 d^2 e^4 (-4 a c - b^2)^9)^{(1/2)} / (32 (a^7 b^{12} e^8 + 4096 a^9 c^{10} d^8 + 4096 a^{13} c^6 e^8 - 24 a^8 b^{10} c e^8 - 4 a^6 b^{13} d e^7 + a^3 b^{12} c^4 d^8 - 24 a^4 b^{10} c^5 d^8 + 240 a^5 b^8 c^6 d^8 - 1280 a^6 b^6 c^7 d^8 + 3840 a^7 b^4 c^8 d^8 - 6144 a^8 b^2 c^9 d^8 + 240 a^9 b^8 c^2 e^8 - 1280 a^{10} b^6 c^3 e^8 + 3840 a^{11} b^4 c^4 e^8 - 6144 a^{12} b^2 c^5 e^8 + a^3 b^{16} d^4 e^4 - 4 a^4 b^{15} d^3 e^5 + 6 a^5 b^{14} d^2 e^6 + 16384 a^{10} c^9 d^6 e^2 + 24576 a^{11} c^8 d^4 e^4 + 16384 a^{12} c^7 d^2 e^6 + 6 a^3 b^{14} c^2 d^6 e^2 - 140 a^4 b^{12} c^3 d^6 e^2 + 84 a^4 b^{13} c^2 d^5 e^3 + 1344 a^5 b^{10} c^4 d^6 e^2 - 672 a^5 b^{11} c^3 d^5 e^3 - 42 a^5 b^{12} c^2 d^4 e^4 - 6720 a^6 b^8 c^5 d^6 e^2 + 2240 a^6 b^9 c^4 d^5 e^3 + 1456 a^6 b^{10} c^3 d^4 e^4 - 672 a^6 b^{11} c^2 d^3 e^5 + 17920 a^7 b^6 c^6 d^6 e^2 - 10080 a^7 b^8 c^4 d^4 e^4 + 2240 a^7 b^9 c^3 d^3 e^5 + 1344 a^7 b^{10} c^2 d^2 e^6 - 21504 a^8 b^4 c^7 d^6 e^2 - 21504 a^8 b^5 c^6 d^5 e^3 + 32256 a^8 b^6 c^5 d^4 e^4 - 6720 a^8 b^8 c^3 d^2 e^6 + 57344 a^9 b^3 c^7 d^5 e^3 - 46592 a^9 b^4 c^6 d^4 e^4 - 21504 a^9 b^5 c^5 d^3 e^5 + 17920 a^9 b^6 c^4 d^2 e^6 + 12288 a^{10} b^2 c^7 d^4 e^4 + 57344 a^{10} b^3 c^6 d^3 e^5 - 21504 a^{10} b^4 c^5 d^2 e^6 + 96 a^7 b^{11} c d e^7 - 16384 a^9 b^6 c^9 d^7 e - 16384 a^{12} b^6 c^6 d e^7 - 4 a^3 b^{13} c^3 d^7 e - 4 a^3 b^{15} c d^5 e^3 + 96 a^4 b^{11} c^4 d^7 e - 12 a^4 b^{14} c d^4 e^4 - 960 a^5 b^9 c^5 d^7 e + 84 a^5 b^{13} c d^3 e^5 + 5120 a^6 b^7 c^6 d^7 e - 140 a^6 b^{12} c d^2 e^6 - 15360 a^7 b^5 c^7 d^7 e + 24576 a^8 b^3 c^8 d^7 e - 960 a^8 b^9 c^2 d e^7 + 5120 a^9 b^7 c^3 d e^7 - 49152 a^{10} b^6 c^8 d^5 e^3 - 15360 a^{10} b^5 c^4 d e^7 - 49152 a^{11} b^6 c^7 d^3 e^5 + 24576 a^{11} b^3 c^5 d e^7))^{(1/2)} * (1048576 a^{15} c^8 e^{17} + 256 a^9 b^{12} c^2 e^{17} - 6144 a^{10} b^{10} c^3 e^{17} + 61440 a^{11} b^8 c^4 e^{17} - 327680 a^{12} b^6 c^5 e^{17} + 983040 a^{13} b^4 c^6 e^{17} - 1572864 a^{14} b^2 c^7 e^{17} - 1048576 a^8 c^{15} d^{14} e^3 - 5242880 a^9 c^{14} d^{12} e^5 - 9437184 a^{10} c^{13} d^{10} e^7 - 5242880 a^{11} c^{12} d^8 e^9 + 5242880 a^{12} c^{11} d^6 e^{11} + 9437184 a^{13} c^{10} d^4 e^{13} + 5242880 a^{14} c^9 d^2 e^{15} + 256 a^2 b^{11} c^{10} d^{15} e^2 - 2048 a^2 b^{12} c^9 d^{14} e^3 + 7168 a^2 b^{13} c^8 d^{13} e^4 - 14336 a^2 b^{14} c^7 d^{12} e^5 + 17920 a^2 b^{15} c^6 d^{11} e^6 - 14336 a^2 b^{16} c^5 d^{10} e^7 + 7168 a^2 b^{17} c^4 d^9 e^8 - 2048 a^2 b^{18} c^3 d^8 e^9 + 256 a^2 b^{19} c^2 d^7 e^{10} - 5120 a^3 b^9 c^{11} d^{15} e^2 + 41984 a^3 b^{10} c^{10} d^{14} e^3 - 148736 a^3 b^{11} c^9 d^{13} e^4 + 296192 a^3 b^{12} c^8 d^{12} e^5 - 359680 a^3 b^{13} c^7 d^{11} e^6 + 267520 a^3 b^{14} c^6 d^{10} e^7 - 112384 a^3 b^{15} c^5 d^9 e^8 + 18176 a^3 b^{16} c^4 d^8 e^9 + 3328 a^3 b^{17} c^3 d^7 e^{10} - 1280 a^3 b^{18} c^2 d^6 e^{11} + 40960 a^4 b^7 c^{12} d^{15} e^2 - 348160 a^4 b^8 c^{11} d^{14} e^3 + 1254400 a^4 b^9 c^{10} d^{13} e^4 - 2478080 a^4 b^{10} c^9 d^{12} e^5 + 2867456 a^4 b^{11} c^8 d^{11} e^6 - 1862144 a^4 b^{12} c^7 d^{10} e^7 + 490240 a^4 b^{13} c^6 d^9 e^8 + 128000 a^4 b^{14} c^5 d^8 e^9 - 108800 a^4 b^{15} c^4 d^7 e^{10} + 13824 a^4 b^{16} c^3 d^6 e^{11} + 2304 a^4 b^{17} c^2 d^5 e^{12} - 163840 a^5 b^5 c^{13} d^{15} e^2 + 1474560 a^5 b^6 c^{12} d^{14} e^3 - 5447680 a^5 b^7 c^{11} d^{13} e^4 + 10588160 a^5 b^8 c^{10} d^{12} e^5 - 11166720 a^5 b^9 c^9 d^{11} e^6 + 5159936 a^5 b^{10} c^8 d^{10} e^7 + 1073920 a^5 b^{11} c^7
\end{aligned}$$

$$\begin{aligned}
& d^9 e^8 - 2279680 a^5 b^{12} c^6 d^8 e^9 + 770560 a^5 b^{13} c^5 d^7 e^{10} + 332 \\
& 80 a^5 b^{14} c^4 d^6 e^{11} - 41216 a^5 b^{15} c^3 d^5 e^{12} - 1280 a^5 b^{16} c^2 d^4 e^{13} + 327680 a^6 b^3 c^{14} d^{15} e^2 - 3276800 a^6 b^4 c^{13} d^{14} e^3 + 1 \\
& 2615680 a^6 b^5 c^{12} d^{13} e^4 - 23592960 a^6 b^6 c^{11} d^{12} e^5 + 19701760 a^6 b^7 c^{10} d^{11} e^6 + 1372160 a^6 b^8 c^9 d^{10} e^7 - 15846400 a^6 b^9 c^8 d^9 e^8 \\
& + 10864640 a^6 b^{10} c^7 d^8 e^9 - 1352960 a^6 b^{11} c^6 d^7 e^{10} - 111040 a^6 b^{12} c^5 d^6 e^{11} + 273920 a^6 b^{13} c^4 d^5 e^{12} + 25600 a^6 b^{14} c^3 d^4 e^{13} \\
& - 1280 a^6 b^{15} c^2 d^3 e^{14} + 3407872 a^7 b^2 c^{14} d^{14} e^3 - 14221312 a^7 b^3 c^{13} d^{13} e^4 + 23527424 a^7 b^4 c^{12} d^{12} e^5 - 376832 \\
& 0 a^7 b^5 c^{11} d^{11} e^6 - 38895616 a^7 b^6 c^{10} d^{10} e^7 + 50126848 a^7 b^7 c^9 d^9 e^8 - 18362368 a^7 b^8 c^8 d^8 e^9 - 6831104 a^7 b^9 c^7 d^7 e^{10} \\
& + 6200320 a^7 b^{10} c^6 d^6 e^{11} - 726784 a^7 b^{11} c^5 d^5 e^{12} - 228608 a^7 b^{12} c^4 d^4 e^{13} + 31488 a^7 b^{13} c^3 d^3 e^{14} + 2304 a^7 b^{14} c^2 d^2 e^{15} \\
& - 3145728 a^8 b^2 c^{13} d^{12} e^5 - 31129600 a^8 b^3 c^{12} d^{11} e^6 + 74711040 a^8 b^4 c^{11} d^{10} e^7 - 55476224 a^8 b^5 c^{10} d^9 e^8 - 11075584 a^8 b^6 c^9 d^8 e^9 + 35381248 a^8 b^7 c^8 d^7 e^{10} \\
& - 14479360 a^8 b^8 c^7 d^6 e^{11} - 168960 a^8 b^9 c^6 d^5 e^{12} + 1286144 a^8 b^{10} c^5 d^4 e^{13} - 302336 a^8 b^{11} c^4 d^3 e^{14} - 55808 a^8 b^{12} c^3 d^2 e^{15} \\
& - 36962304 a^9 b^2 c^{12} d^{10} e^7 - 9502720 a^9 b^3 c^{11} d^9 e^8 + 67174400 a^9 b^4 c^{10} d^8 e^9 - 54886400 a^9 b^5 c^9 d^7 e^{10} + 11239424 a^9 b^6 c^8 d^6 e^{11} \\
& + 5545984 a^9 b^7 c^7 d^5 e^{12} - 5263360 a^9 b^8 c^6 d^4 e^{13} + 1356800 a^9 b^9 c^5 d^3 e^{14} + 558080 a^9 b^{10} c^4 d^2 e^{15} - 49807360 a^{10} b^2 c^{11} d^8 e^9 \\
& + 19333120 a^{10} b^3 c^{10} d^7 e^{10} + 7208960 a^{10} b^4 c^9 d^6 e^{11} - 14974976 a^{10} b^5 c^8 d^5 e^{12} + 15073280 a^{10} b^6 c^7 d^4 e^{13} - 2170880 a^{10} b^7 c^6 d^3 e^{14} \\
& - 2928640 a^{10} b^8 c^5 d^2 e^{15} - 11796480 a^{11} b^2 c^{10} d^6 e^{11} + 23920640 a^{11} b^3 c^9 d^5 e^{12} - 24576000 a^{11} b^4 c^8 d^4 e^{13} - 4096000 a^{11} b^5 c^7 d^3 e^{14} \\
& + 8355840 a^{11} b^6 c^6 d^2 e^{15} + 12582912 a^{12} b^2 c^9 d^4 e^{13} + 19857408 a^{12} b^3 c^8 d^3 e^{14} - 11534336 a^{12} b^4 c^7 d^2 e^{15} + 3407872 a^{13} b^2 c^8 d^2 e^{15} \\
& - 5505024 a^{14} b^3 c^8 d^2 e^{16} - 262144 a^7 b^3 c^{15} d^{15} e^2 + 5505024 a^8 b^3 c^{14} d^{13} e^4 - 1280 a^8 b^{13} c^2 d^8 e^{16} + 25952256 a^9 b^3 c^{13} d^{11} e^6 \\
& + 30976 a^9 b^{11} c^3 d^8 e^{16} + 38010880 a^{10} b^3 c^{12} d^9 e^8 - 312320 a^{10} b^9 c^4 d^8 e^{16} + 11796480 a^{11} b^3 c^{11} d^7 e^{10} + 1679360 a^{11} b^7 c^5 d^8 e^{16} \\
& - 21233664 a^{12} b^3 c^{10} d^5 e^{12} - 5079040 a^{12} b^5 c^6 d^8 e^{16} - 20709376 a^{13} b^3 c^9 d^3 e^{14} + 8192000 a^{13} b^3 c^7 d^8 e^{16} \\
& 6)) / (8(a^6 b^8 e^8 + 256 a^6 c^8 d^8 + 256 a^{10} c^4 e^8 - 16 a^7 b^6 c^8 e^8 - 4 a^5 b^9 d^8 e^7 + a^2 b^8 c^4 d^8 - 16 a^3 b^6 c^5 d^8 + 96 a^4 b^4 c^6 d^8 \\
& - 256 a^5 b^2 c^7 d^8 + 96 a^8 b^4 c^2 e^8 - 256 a^9 b^2 c^3 e^8 + a^2 b^{12} d^4 e^4 - 4 a^3 b^{11} d^3 e^5 + 6 a^4 b^{10} d^2 e^6 + 1024 a^7 c^7 d^6 e^2 + 1536 a^8 c^6 d^4 e^4 \\
& + 1024 a^9 c^5 d^2 e^6 + 6 a^2 b^{10} c^2 d^6 e^2 - 92 a^3 b^8 c^3 d^6 e^2 + 52 a^3 b^9 c^2 d^5 e^3 + 512 a^4 b^6 c^4 d^6 e^2 - 192 a^4 b^7 c^3 d^5 e^3 - 90 a^4 b^8 c^2 d^4 e^4 \\
& - 1152 a^5 b^4 c^5 d^6 e^2 - 128 a^5 b^5 c^4 d^5 e^3 + 800 a^5 b^6 c^3 d^4 e^4 - 192 a^5 b^7 c^2 d^3 e^5 + 512 a^6 b^2 c^6 d^6 e^2 + 2048 a^6 b^3 c^5 d^5 e^3 - 2240 a^6 b^4 c^4 d^4 e^4 \\
& - 128 a^6 b^5 c^3 d^3 e^5 + 512 a^6 b^6 c^2 d^2 e^6 + 1536 a^7 b^2 c^5 d^4 e^4 + 2048 a^7 b^3 c^4 d^3 e^5 - 1152 a^7 b^4 c^3 d^2 e^6 + 512 a^8 b^2 c^4 d^2 e^6 \\
& - 1024 a^6 b^3 c^7 d^7 e + 64 a^6 b^7 c^3 d^7 e - 1024 a^9 b^3 c^4 d^7 e - 4 a^2 b^9 c^3 d^7 e - 4 a^2 b^{11} c^5 d^5 e^3 + 64 a^3 b^7 c^4 d^7 e - 4 a^3 b^{10} c^5 d^4 e^4 \\
& - 384 a^4 b^5 c^5 d^7 e + 52 a^4 b^9 c^3 d^5 e^5 + 1024 a^5 b^3 c^6 d^7 e - 92 a^5 b^8 c^3 d^2 e^6 - 3072 a^7 b^3 c^6 d^5 e^3 - 384 a^7 b^5 c^2 d^5 e^7 \\
& - 3072 a^8 b^3 c^5 d^3 e^5 + 1024 a^8 b^3 c^3 d^5 e^7)) * ((27 a^9 b^9 c^5 d^6 - b^{11} c^4 d^6 - b^{15} d^2 e^4 - 9 a^2 b^{13} e^6 + 3840 a^5 b^3 c^9 d^6 \\
& + 9 a^5 c^5 d^6 (- (4 a^3 c - b^2)^9)^{1/2} + 213 a^3 b^{11} c^6 e^6 - 26880 a^8 b^3 c^6 e^6 + 3072 a^6 c^9 d^5 e + 35840 a^8 c^7 d^5 e^5 + 4 b^{12} c^3 d^5 e \\
& + 4 b^{14} c^3 d^3 e^3 - 288 a^2 b^7 c^6 d^6 + 1504 a^3 b^5 c^7 d^6 - 3840 a^4 b^3 c^8 d^6 - 9 a^2 b^4 e^6 (- (4 a^3 c - b^2)^9)^{1/2} - 2077 a^4 b^9 c^2 e^6 \\
& + 10656 a^5 b^7 c^3 e^6 - 30240 a^6 b^5 c^4 e^6 + 44800 a^7 b^3 c^5 e^6 - 25 a^4 c^2 e^6 (- (4 a^3 c - b^2)^9)^{1/2} - b^2 c^4 d^6 (- (4 a^3 c - b^2)^9)^{1/2} \\
& + 22528 a^7 c^8 d^3 e^3 - b^6 d^2 e^4 (- (4 a^3 c - b^2)^9)^{1/2} - 6 b^{13} c^2 d^4 e^2 + 6 a^2 b^9 c^4 d^4 e^2 + 600 a^2 b^
\end{aligned}$$

$$\begin{aligned}
& 10c^3d^3e^3 + 180a^2b^{11}c^2d^2e^4 + 6976a^3b^7c^5d^4e^2 - 1032 \\
& a^3b^8c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456a^4b^5c^6d^4e^2 \\
& - 7168a^4b^6c^5d^3e^3 + 16896a^4b^7c^4d^2e^4 + 10240a^5b^3c^7 \\
& d^4e^2 + 37632a^5b^4c^6d^3e^3 - 47712a^5b^5c^5d^2e^4 - 59392a^6 \\
& b^2c^7d^3e^3 + 60928a^6b^3c^6d^2e^4 + 41a^2c^4d^4e^2(-4ac - b^2)^9)^{(1/2)} + 39a^3c^3d^2e^4(-4ac - b^2)^9)^{(1/2)} - 6b^4c^2d^4e^2(-4ac - b^2)^9)^{(1/2)} + 6ab^5d^5e^5(-4ac - b^2)^9)^{(1/2)} - \\
& 106ab^{10}c^4d^5e + 7ab^{13}c^2d^2e^4 - 128a^2b^{12}c^2d^2e^5 + 51a^3b^2c^2e^6(-4ac - b^2)^9)^{(1/2)} + 150ab^{11}c^3d^4e^2 - 84ab^{12}c^2d^3e^3 + 1116a^2b^8c^5d^5e - 5824a^3b^6c^6d^5e + 1030a^3b^{10}c^2d^5e^5 + 15232a^4b^4c^7d^5e - 3492a^4b^8c^3d^5e^5 - 16896a^5b^2c^8d^5e + 1344a^5b^6c^4d^5e^5 + 7424a^6b^2c^8d^4e^2 + 22400a^6b^4c^5d^5e^5 - 23296a^7b^2c^7d^2e^4 - 53760a^7b^2c^6d^5e^5 + 4b^3c^3d^5e^5(-4ac - b^2)^9)^{(1/2)} + 4b^5c^2d^3e^3(-4ac - b^2)^9)^{(1/2)} - 11ab^4c^2d^2e^4(-4ac - b^2)^9)^{(1/2)} - 20a^2b^3c^2d^2e^5(-4ac - b^2)^9)^{(1/2)} - 86a^3b^2c^2d^2e^5(-4ac - b^2)^9)^{(1/2)} + 42ab^2c^3d^4e^2(-4ac - b^2)^9)^{(1/2)} - 12ab^3c^2d^3e^3(-4ac - b^2)^9)^{(1/2)} - 120a^2b^2c^3d^3e^3(-4ac - b^2)^9)^{(1/2)} - 34ab^2c^4d^5e^5(-4ac - b^2)^9)^{(1/2)} + 108a^2b^2c^2d^2e^4(-4ac - b^2)^9)^{(1/2))} / (32(a^7b^{12}e^8 + 4096a^9c^{10}d^8 + 4096a^{13}c^6e^8 - 24a^8b^{10}c^2e^8 - 4a^6b^{13}d^8e^7 + a^3b^{12}c^4d^8 - 24a^4b^{10}c^5d^8 + 240a^5b^8c^6d^8 - 1280a^6b^6c^7d^8 + 3840a^7b^4c^8d^8 - 6144a^8b^2c^9d^8 + 240a^9b^8c^2e^8 - 1280a^{10}b^6c^3e^8 + 3840a^{11}b^4c^4e^8 - 6144a^{12}b^2c^5e^8 + a^3b^{16}d^4e^4 - 4a^4b^{15}d^3e^5 + 6a^5b^{14}d^2e^6 + 16384a^{10}c^9d^6e^2 + 24576a^{11}c^8d^4e^4 + 16384a^{12}c^7d^2e^6 + 6a^3b^{14}c^2d^6e^2 - 140a^4b^{12}c^3d^6e^2 + 84a^4b^{13}c^2d^5e^3 + 1344a^5b^{10}c^4d^6e^2 - 672a^5b^{11}c^3d^5e^3 - 42a^5b^{12}c^2d^4e^4 - 6720a^6b^8c^5d^6e^2 + 2240a^6b^9c^4d^5e^3 + 1456a^6b^{10}c^3d^4e^4 - 672a^6b^{11}c^2d^3e^5 + 17920a^7b^6c^6d^6e^2 - 10080a^7b^8c^4d^4e^4 + 2240a^7b^9c^3d^3e^5 + 1344a^7b^{10}c^2d^2e^6 - 21504a^8b^4c^7d^6e^2 - 21504a^8b^5c^6d^5e^3 + 32256a^8b^6c^5d^4e^4 - 6720a^8b^8c^3d^2e^6 + 57344a^9b^3c^7d^5e^3 - 46592a^9b^4c^6d^4e^4 - 21504a^9b^5c^5d^3e^5 + 17920a^9b^6c^4d^2e^6 + 12288a^{10}b^2c^7d^4e^4 + 57344a^{10}b^3c^6d^3e^5 - 21504a^{10}b^4c^5d^2e^6 + 96a^7b^{11}c^2d^7e - 16384a^9b^2c^9d^7e - 16384a^{12}b^2c^6d^7e - 4a^3b^{13}c^3d^7e - 4a^3b^{15}c^2d^5e^3 + 96a^4b^{11}c^4d^7e - 12a^4b^{14}c^2d^4e^4 - 960a^5b^9c^5d^7e + 84a^5b^{13}c^2d^3e^5 + 5120a^6b^7c^6d^7e - 140a^6b^{12}c^2d^2e^6 - 15360a^7b^5c^7d^7e + 24576a^8b^3c^8d^7e - 960a^8b^9c^2d^7e + 5120a^9b^7c^3d^7e - 49152a^{10}b^2c^8d^5e^3 - 15360a^{10}b^5c^4d^7e - 49152a^{11}b^2c^7d^3e^5 + 24576a^{11}b^3c^5d^7e))^{(1/2)} + (x(626688a^{10}b^2c^8e^{15} - 784384a^{10}c^9d^8e^{14} + 208a^4b^{13}c^2e^{15} - 4880a^5b^{11}c^3e^{15} + 47312a^6b^9c^4e^{15} - 242176a^7b^7c^5e^{15} + 688640a^8b^5c^6e^{15} - 1028096a^9b^3c^7e^{15} + 18432a^4c^{15}d^{13}e^2 + 126976a^5c^{14}d^{11}e^4 + 325632a^6c^{13}d^9e^6 + 139264a^7c^{12}d^7e^8 - 1067008a^8c^{11}d^5e^{10} - 1773568a^9c^{10}d^3e^{12} + 16b^8c^{11}d^{13}e^2 - 96b^9c^{10}d^{12}e^3 + 240b^{10}c^9d^{11}e^4 - 304b^{11}c^8d^{10}e^5 + 144b^{12}c^7d^9e^6 + 144b^{13}c^6d^8e^7 - 304b^{14}c^5d^7e^8 + 240b^{15}c^4d^6e^9 - 96b^{16}c^3d^5e^{10} + 16b^{17}c^2d^4e^{11} + 3200a^2b^4c^{13}d^{13}e^2 - 18432a^2b^5c^{12}d^{12}e^3 + 41024a^2b^6c^{11}d^{11}e^4 - 36352a^2b^7c^{10}d^{10}e^5 - 16208a^2b^8c^9d^9e^6 + 74576a^2b^9c^8d^8e^7 - 78496a^2b^{10}c^7d^7e^8 + 32064a^2b^{11}c^6d^6e^9 + 6000a^2b^{12}c^5d^5e^{10} - 9264a^2b^{13}c^4d^4e^{11} + 1472a^2b^{14}c^3d^3e^{12} + 416a^2b^{15}c^2d^2e^{13} - 12800a^3b^2c^{14}d^{13}e^2 + 73728a^3b^3c^{13}d^{12}e^3 - 151296a^3b^4c^{12}d^{11}e^4 + 78336a^3b^5c^{11}d^{10}e^5 + 206688a^3b^6c^{10}d^9e^6 - 436736a^3b^7c^9d^8e^7 + 324224a^3b^8c^8d^7e^8 + 992a^3b^9c^7d^6e^9 - 158176a^3b^{10}c^6d^5e^{10} + 77056a^3b^{11}c^5d^4e^{11} + 6912a^3b^{12}c^4d^3e^{12} - 8416a^3b^{13}c^3d^2e^{13} + 162816a^4b^2c^{13}d^{11}e^4 + 184320a^4b^3c^{12}d^{10}e^5 -
\end{aligned}$$



$$\begin{aligned}
& 916608a^4b^4c^{11}d^9e^6 + 1165824a^4b^5c^{10}d^8e^7 - 314496a^4b^6c^9d^7e^8 - 822272a^4b^7c^8d^6e^9 + 919152a^4b^8c^7d^5e^{10} - 175296a^4b^9c^6d^4e^{11} - 189328a^4b^{10}c^5d^3e^{12} + 62064a^4b^{11}c^4d^2e^{13} + 1290752a^5b^2c^{12}d^9e^6 - 659456a^5b^3c^{11}d^8e^7 - 1561088a^5b^4c^{10}d^7e^8 + 3240960a^5b^5c^9d^6e^9 - 1964192a^5b^6c^8d^5e^{10} - 683008a^5b^7c^7d^4e^{11} + 1162304a^5b^8c^6d^3e^{12} - 164112a^5b^9c^5d^2e^{13} + 3442688a^6b^2c^{11}d^7e^8 - 3670016a^6b^3c^{10}d^6e^9 + 15232a^6b^4c^9d^5e^{10} + 4230144a^6b^5c^8d^4e^{11} - 3059648a^6b^6c^7d^3e^{12} - 247296a^6b^7c^6d^2e^{13} + 4010496a^7b^2c^{10}d^5e^{10} - 6873088a^7b^3c^9d^4e^{11} + 2822400a^7b^4c^8d^3e^{12} + 2370048a^7b^5c^7d^2e^{13} + 1178624a^8b^2c^9d^3e^{12} - 4739072a^8b^3c^8d^2e^{13} - 352a^8b^6c^{12}d^{13}e^2 + 2048a^8b^7c^{11}d^{12}e^3 - 4800a^8b^8c^{10}d^{11}e^4 + 5168a^8b^9c^9d^{10}e^5 - 480a^8b^{10}c^8d^9e^6 - 6000a^8b^{11}c^7d^8e^7 + 8192a^8b^{12}c^6d^7e^8 - 5040a^8b^{13}c^5d^6e^9 + 1152a^8b^{14}c^4d^5e^{10} + 240a^8b^{15}c^3d^4e^{11} - 128a^8b^{16}c^2d^3e^{12} - 512a^8b^{17}c^2d^2e^{14} - 106496a^4b^8c^{14}d^{12}e^3 + 11680a^4b^{12}c^3d^2e^{14} - 675840a^5b^8c^{13}d^{10}e^5 - 108288a^5b^{10}c^4d^2e^{14} - 1601536a^6b^8c^{12}d^8e^7 + 514768a^6b^8c^5d^2e^{14} - 925696a^7b^8c^{11}d^6e^9 - 1278304a^7b^6c^6d^2e^{14} + 2457600a^8b^8c^{10}d^4e^{11} + 1385600a^8b^4c^7d^2e^{14} + 2977792a^9b^8c^9d^2e^{13} + 19968a^9b^2c^8d^2e^{14}) / ((8(a^6b^8e^8 + 256a^6c^8d^8 + 256a^{10}c^4e^8 - 16a^7b^6c^8e^8 - 4a^5b^9d^7e^7 + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6d^8 - 256a^5b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^{12}d^4e^4 - 4a^3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 + 1024a^7c^7d^6e^2 + 1536a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^{10}c^2d^6e^2 - 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4b^7c^3d^5e^3 - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 - 192a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c^4d^4e^4 - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 + 2048a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024a^6b^8c^7d^7e + 64a^6b^7c^6d^7e - 1024a^9b^8c^4d^7e - 4a^2b^9c^3d^7e - 4a^2b^{11}c^5d^5e^3 + 64a^3b^7c^4d^7e - 4a^3b^{10}c^4d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^3d^3e^5 + 1024a^5b^3c^6d^7e - 92a^5b^8c^5d^2e^6 - 3072a^7b^8c^6d^5e^3 - 384a^7b^5c^2d^7e - 3072a^8b^8c^5d^3e^5 + 1024a^8b^3c^3d^2e^7)) * ((27a^9b^9c^5d^6 - b^{11}c^4d^6 - b^{15}d^2e^4 - 9a^2b^{13}e^6 + 3840a^5b^8c^9d^6 + 9a^5c^5d^6 * (-4ac - b^2)^9)^{1/2} + 213a^3b^{11}c^5e^6 - 26880a^8b^8c^6e^6 + 3072a^6c^9d^5e + 35840a^8c^7d^5e^5 + 4b^{12}c^3d^5e + 4b^{14}c^3d^3e^3 - 288a^2b^7c^6d^6 + 1504a^3b^5c^7d^6 - 3840a^4b^3c^8d^6 - 9a^2b^4e^6 * (-4ac - b^2)^9)^{1/2} - 2077a^4b^9c^2e^6 + 10656a^5b^7c^3e^6 - 30240a^6b^5c^4e^6 + 44800a^7b^3c^5e^6 - 25a^4c^2e^6 * (-4ac - b^2)^9)^{1/2} - b^2c^4d^6 * (-4ac - b^2)^9)^{1/2} + 22528a^7c^8d^3e^3 - b^6d^2e^4 * (-4ac - b^2)^9)^{1/2} - 6b^{13}c^2d^4e^2 + 6a^8b^{14}d^5e^5 - 1471a^2b^9c^4d^4e^2 + 600a^2b^{10}c^3d^3e^3 + 180a^2b^{11}c^2d^2e^4 + 6976a^3b^7c^5d^4e^2 - 1032a^3b^8c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456a^4b^5c^6d^4e^2 - 7168a^4b^6c^5d^3e^3 + 16896a^4b^7c^4d^2e^4 + 10240a^5b^3c^7d^4e^2 + 37632a^5b^4c^6d^3e^3 - 47712a^5b^5c^5d^2e^4 - 59392a^6b^2c^7d^3e^3 + 60928a^6b^3c^6d^2e^4 + 41a^2c^4d^4e^2 * (-4ac - b^2)^9)^{1/2} + 39a^3c^3d^2e^4 * (-4ac - b^2)^9)^{1/2} - 6b^4c^2d^4e^2 * (-4ac - b^2)^9)^{1/2} + 6a^8b^5d^5e^5 * (-4ac - b^2)^9)^{1/2} - 106a^8b^{10}c^4d^5e + 7a^8b^{13}c^3d^2e^4 - 128a^2b^{12}c^6d^5e^5 + 51a^3b^2c^6e^6 * (-4ac - b^2)^9)^{1/2} + 150a^8b^{11}c^3d^4e^2 - 84a^8b^{12}c^2d^3e^3 + 1116a^2b^8c^5d^5e - 5824a^3b^6c^6d^5e + 1030a^3b^{10}c^2d^5e^5 + 15232a^4b^4c^7d^5e - 3492a^4b^8c^3d^5e^5 - 16896a^5b^2c^8d^5e + 1344a^5b^6c^4d^5e^5 + 7424a^6b^8c^8d^4e^2 + 22400a^6b^4c^5d^5e^5 - 23296a^7b^8c^7d^2e^4 - 53760a^7b^2c^6d^5e^5 + 4b^3c^3d^5e * (-4ac - b^2)^9)^{1/2} + 4b^5c^3d^3e^3 * (-4ac - b^2)^9)^{1/2}
\end{aligned}$$

$$\begin{aligned}
& 9)^{(1/2)} - 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^{(1/2)} - 20*a^2*b^3*c*d*e^5 \\
& *(-(4*a*c - b^2)^9)^{(1/2)} - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^{(1/2)} + 4 \\
& 2*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 12*a*b^3*c^2*d^3*e^3*(-(4*a* \\
& c - b^2)^9)^{(1/2)} - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^{(1/2)} - 34*a*b \\
& *c^4*d^5*e*(-(4*a*c - b^2)^9)^{(1/2)} + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24 \\
& *a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 \\
& + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144 \\
& *a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11* \\
& b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 \\
& + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 1 \\
& 6384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + \\
& 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5 \\
& *e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^ \\
& 4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^ \\
& 7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + \\
& 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d \\
& ^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b \\
& ^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17 \\
& 920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d \\
& ^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9 \\
& *d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e \\
& ^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e \\
& + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - \\
& 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 \\
& + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d* \\
& e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7))^{(1/2)} - (32691 \\
& 2*a^8*c^9*d*e^13 - 241664*a^8*b*c^8*e^14 - 48*a^2*b^13*c^2*e^14 + 1264*a^3* \\
& b^11*c^3*e^14 - 13552*a^4*b^9*c^4*e^14 + 75776*a^5*b^7*c^5*e^14 - 232960*a^ \\
& 6*b^5*c^6*e^14 + 372736*a^7*b^3*c^7*e^14 + 11520*a^3*c^14*d^11*e^3 + 78080* \\
& a^4*c^13*d^9*e^5 + 197120*a^5*c^12*d^7*e^7 + 336384*a^6*c^11*d^5*e^9 + 5327 \\
& 36*a^7*c^10*d^3*e^11 - 40*b^5*c^12*d^12*e^2 + 216*b^6*c^11*d^11*e^3 - 464*b \\
& ^7*c^10*d^10*e^4 + 496*b^8*c^9*d^9*e^5 - 264*b^9*c^8*d^8*e^6 + 56*b^10*c^7* \\
& d^7*e^7 - 16*b^11*c^6*d^6*e^8 + 64*b^12*c^5*d^5*e^9 - 96*b^13*c^4*d^4*e^10 \\
& + 64*b^14*c^3*d^3*e^11 - 16*b^15*c^2*d^2*e^12 + 1536*a^2*b^2*c^13*d^11*e^3 \\
& + 14400*a^2*b^3*c^12*d^10*e^4 - 47152*a^2*b^4*c^11*d^9*e^5 + 52144*a^2*b^5* \\
& c^10*d^8*e^6 - 16272*a^2*b^6*c^9*d^7*e^7 - 13040*a^2*b^7*c^8*d^6*e^8 + 2348 \\
& 8*a^2*b^8*c^7*d^5*e^9 - 26384*a^2*b^9*c^6*d^4*e^10 + 13824*a^2*b^10*c^5*d^3 \\
& *e^11 + 256*a^2*b^11*c^4*d^2*e^12 + 125056*a^3*b^2*c^12*d^9*e^5 - 36224*a^3 \\
& *b^3*c^11*d^8*e^6 - 126432*a^3*b^4*c^10*d^7*e^7 + 144848*a^3*b^5*c^9*d^6*e^ \\
& 8 - 114752*a^3*b^6*c^8*d^5*e^9 + 125392*a^3*b^7*c^7*d^4*e^10 - 53248*a^3*b^ \\
& 8*c^6*d^3*e^11 - 25264*a^3*b^9*c^5*d^2*e^12 + 474112*a^4*b^2*c^11*d^7*e^7 - \\
& 191104*a^4*b^3*c^10*d^6*e^8 + 97184*a^4*b^4*c^9*d^5*e^9 - 277000*a^4*b^5*c \\
& ^8*d^4*e^10 + 56056*a^4*b^6*c^7*d^3*e^11 + 195584*a^4*b^7*c^6*d^2*e^12 + 23 \\
& 6800*a^5*b^2*c^10*d^5*e^9 + 388032*a^5*b^3*c^9*d^4*e^10 + 159632*a^5*b^4*c^ \\
& 8*d^3*e^11 - 670488*a^5*b^5*c^7*d^2*e^12 - 488960*a^6*b^2*c^9*d^3*e^11 + 11 \\
& 06496*a^6*b^3*c^8*d^2*e^12 + 64*a*b^14*c^2*d*e^13 + 448*a*b^3*c^13*d^12*e^2 \\
& - 1968*a*b^4*c^12*d^11*e^3 + 2504*a*b^5*c^11*d^10*e^4 + 768*a*b^6*c^10*d^9 \\
& *e^5 - 4368*a*b^7*c^9*d^8*e^6 + 3568*a*b^8*c^8*d^7*e^7 - 520*a*b^9*c^7*d^6* \\
& e^8 - 1728*a*b^10*c^6*d^5*e^9 + 2528*a*b^11*c^5*d^4*e^10 - 1536*a*b^12*c^4* \\
& d^3*e^11 + 240*a*b^13*c^3*d^2*e^12 - 1152*a^2*b*c^14*d^12*e^2 - 1600*a^2*b^ \\
& 12*c^3*d*e^13 - 67968*a^3*b*c^13*d^10*e^4 + 15808*a^3*b^10*c^4*d*e^13 - 342 \\
& 272*a^4*b*c^12*d^8*e^6 - 76928*a^4*b^8*c^5*d*e^13 - 569088*a^5*b*c^11*d^6*e \\
& ^8 + 179200*a^5*b^6*c^6*d*e^13 - 586368*a^6*b*c^10*d^4*e^10 - 113008*a^6*b^ \\
& 4*c^7*d*e^13 - 731008*a^7*b*c^9*d^2*e^12 - 244096*a^7*b^2*c^8*d*e^13)/(16*( \\
& a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5 \\
& *b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 25 \\
& 6*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4 \\
& *e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 153
\end{aligned}$$

$$\begin{aligned}
& 6a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^{10}c^2d^6e^2 - 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4b^7c^3d^5e^3 - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 - 192a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c^4d^4e^4 - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 + 2048a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024a^6b^6c^7d^7e + 64a^6b^7c^5d^7e - 1024a^9b^6c^4d^7e - 4a^2b^9c^3d^7e - 4a^2b^{11}c^5d^5e^3 + 64a^3b^7c^4d^7e - 4a^3b^{10}c^4d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^3d^5e^5 + 1024a^5b^3c^6d^7e - 92a^5b^8c^2d^2e^6 - 3072a^7b^6c^6d^5e^3 - 384a^7b^5c^2d^2e^7 - 3072a^8b^3c^3d^3e^5 + 1024a^8b^3c^3d^3e^7)) * ((27a^9c^5d^6 - b^{11}c^4d^6 - b^{15}d^2e^4 - 9a^2b^{13}e^6 + 3840a^5b^6c^9d^6 + 9a^5c^5d^6 * (-4a^2c - b^2)^9)^{1/2} + 213a^3b^{11}c^6e^6 - 26880a^8b^6c^6e^6 + 3072a^6c^9d^5e + 35840a^8c^7d^5e^5 + 4b^{12}c^3d^5e + 4b^{14}c^3d^3e^3 - 288a^2b^7c^6d^6 + 1504a^3b^5c^7d^6 - 3840a^4b^3c^8d^6 - 9a^2b^4e^6 * (-4a^2c - b^2)^9)^{1/2} - 2077a^4b^9c^2e^6 + 10656a^5b^7c^3e^6 - 30240a^6b^5c^4e^6 + 44800a^7b^3c^5e^6 - 25a^4c^2e^6 * (-4a^2c - b^2)^9)^{1/2} - b^2c^4d^6 * (-4a^2c - b^2)^9)^{1/2} + 22528a^7c^8d^3e^3 - b^6d^2e^4 * (-4a^2c - b^2)^9)^{1/2} - 6b^{13}c^2d^4e^2 + 6a^2b^{14}d^5e^5 - 1471a^2b^9c^4d^4e^2 + 600a^2b^{10}c^3d^3e^3 + 180a^2b^{11}c^2d^2e^4 + 6976a^3b^7c^5d^4e^2 - 1032a^3b^8c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456a^4b^5c^6d^4e^2 - 7168a^4b^6c^5d^3e^3 + 16896a^4b^7c^4d^2e^4 + 10240a^5b^3c^7d^4e^2 + 37632a^5b^4c^6d^3e^3 - 47712a^5b^5c^5d^2e^4 - 59392a^6b^2c^7d^3e^3 + 60928a^6b^3c^6d^2e^4 + 41a^2c^4d^4e^2 * (-4a^2c - b^2)^9)^{1/2} + 39a^3c^3d^2e^4 * (-4a^2c - b^2)^9)^{1/2} - 6b^4c^2d^4e^2 * (-4a^2c - b^2)^9)^{1/2} + 6a^2b^5d^5e^5 * (-4a^2c - b^2)^9)^{1/2} - 106a^2b^{10}c^4d^5e + 7a^2b^{13}c^2d^2e^4 - 128a^2b^{12}c^2d^2e^5 + 51a^3b^2c^2e^6 * (-4a^2c - b^2)^9)^{1/2} + 150a^2b^{11}c^3d^4e^2 - 84a^2b^{12}c^2d^3e^3 + 1116a^2b^8c^5d^5e - 5824a^3b^6c^6d^5e + 1030a^3b^{10}c^2d^5e^5 + 15232a^4b^4c^7d^5e - 3492a^4b^8c^3d^5e^5 - 16896a^5b^2c^8d^5e + 1344a^5b^6c^4d^5e^5 + 7424a^6b^6c^8d^4e^2 + 22400a^6b^4c^5d^5e^5 - 23296a^7b^6c^7d^2e^4 - 53760a^7b^2c^6d^5e^5 + 4b^3c^3d^5e^6 * (-4a^2c - b^2)^9)^{1/2} + 4b^5c^3d^3e^3 * (-4a^2c - b^2)^9)^{1/2} - 11a^2b^4c^2d^2e^4 * (-4a^2c - b^2)^9)^{1/2} - 20a^2b^3c^2d^2e^5 * (-4a^2c - b^2)^9)^{1/2} - 86a^3b^3c^2d^2e^5 * (-4a^2c - b^2)^9)^{1/2} + 42a^2b^2c^3d^4e^2 * (-4a^2c - b^2)^9)^{1/2} - 12a^2b^3c^2d^3e^3 * (-4a^2c - b^2)^9)^{1/2} - 120a^2b^6c^3d^3e^3 * (-4a^2c - b^2)^9)^{1/2} - 34a^2b^6c^4d^5e^6 * (-4a^2c - b^2)^9)^{1/2} + 108a^2b^2c^2d^2e^4 * (-4a^2c - b^2)^9)^{1/2}) / (32 * (a^7b^{12}e^8 + 4096a^9c^{10}d^8 + 4096a^{13}c^6e^8 - 24a^8b^{10}c^6e^8 - 4a^6b^{13}d^7e^7 + a^3b^{12}c^4d^8 - 24a^4b^{10}c^5d^8 + 240a^5b^8c^6d^8 - 1280a^6b^6c^7d^8 + 3840a^7b^4c^8d^8 - 6144a^8b^2c^9d^8 + 240a^9b^8c^2e^8 - 1280a^{10}b^6c^3e^8 + 3840a^{11}b^4c^4e^8 - 6144a^{12}b^2c^5e^8 + a^3b^{16}d^4e^4 - 4a^4b^{15}d^3e^5 + 6a^5b^{14}d^2e^6 + 16384a^{10}c^9d^6e^2 + 24576a^{11}c^8d^4e^4 + 16384a^{12}c^7d^2e^6 + 6a^3b^{14}c^2d^6e^2 - 140a^4b^{12}c^3d^6e^2 + 84a^4b^{13}c^2d^5e^3 + 1344a^5b^{10}c^4d^6e^2 - 672a^5b^{11}c^3d^5e^3 - 42a^5b^{12}c^2d^4e^4 - 6720a^6b^8c^5d^6e^2 + 2240a^6b^9c^4d^5e^3 + 1456a^6b^{10}c^3d^4e^4 - 672a^6b^{11}c^2d^3e^5 + 17920a^7b^6c^6d^6e^2 - 10080a^7b^8c^4d^4e^4 + 2240a^7b^9c^3d^3e^5 + 1344a^7b^{10}c^2d^2e^6 - 21504a^8b^4c^7d^6e^2 - 21504a^8b^5c^6d^5e^3 + 32256a^8b^6c^5d^4e^4 - 6720a^8b^8c^3d^2e^6 + 57344a^9b^3c^7d^5e^3 - 46592a^9b^4c^6d^4e^4 - 21504a^9b^5c^5d^3e^5 + 17920a^9b^6c^4d^2e^6 + 12288a^{10}b^2c^7d^4e^4 + 57344a^{10}b^3c^6d^3e^5 - 21504a^{10}b^4c^5d^2e^6 + 96a^7b^{11}c^2d^7e - 16384a^9b^6c^9d^7e - 16384a^{12}b^6c^6d^7e - 4a^3b^{13}c^3d^7e - 4a^3b^{15}c^2d^5e^3 + 96a^4b^{11}c^4d^7e - 12a^4b^{14}c^3d^4e^4 - 960a^5b^9c^5d^7e + 84a^5b^{13}c^2d^3e^5 + 5120a^6b^7c^6d^7e - 140a^6b^{12}c^2d^2e^6 - 15360a^7b^5c^
\end{aligned}$$

$$\begin{aligned}
& ^7d^7e + 24576a^8b^3c^8d^7e - 960a^8b^9c^2d^7e + 5120a^9b^7c^3d^7e - 49152a^{10}b^5c^8d^5e^3 - 15360a^{10}b^5c^4d^7e - 49152a^{11} \\
& *b^7c^3d^3e^5 + 24576a^{11}b^3c^5d^7e^7))^{(1/2)} + (x*(22800a^6c^9e^{13} \\
& + 36a^2b^8c^5e^{13} - 600a^3b^6c^6e^{13} + 4313a^4b^4c^7e^{13} - 155 \\
& 92a^5b^2c^8e^{13} + 1296a^2c^{13}d^8e^5 + 9792a^3c^{12}d^6e^7 + 30304 \\
& *a^4c^{11}d^4e^9 + 40512a^5c^{10}d^2e^{11} + 25b^4c^{11}d^8e^5 - 120b^5 \\
& *c^{10}d^7e^6 + 214b^6c^9d^6e^7 - 168b^7c^8d^5e^8 + 53b^8c^7d^4e^9 \\
& - 8b^9c^6d^3e^{10} + 4b^{10}c^5d^2e^{11} + 6336a^2b^2c^{11}d^6e^7 \\
& + 3840a^2b^3c^{10}d^5e^8 - 8506a^2b^4c^9d^4e^9 + 1112a^2b^5c^8d^3e^{10} \\
& + 1254a^2b^6c^7d^2e^{11} + 22224a^3b^2c^{10}d^4e^9 + 13824a^3 \\
& *b^3c^9d^3e^{10} - 9516a^3b^4c^8d^2e^{11} + 11712a^4b^2c^9d^2e^{11} \\
& - 24a^4b^9c^5d^7e^6 - 41088a^5b^3c^9d^7e^6 - 360a^4b^2c^{12}d^8e^5 + \\
& 1664a^4b^3c^{11}d^7e^6 - 2604a^4b^4c^{10}d^6e^7 + 1272a^4b^5c^9d^5e^8 \\
& + 332a^4b^6c^8d^4e^9 - 232a^4b^7c^7d^3e^{10} - 48a^4b^8c^6d^2e^{11} - \\
& 5760a^4b^9c^5d^7e^6 + 416a^4b^{10}c^4d^6e^7 - 32128a^5b^3c^{11}d^5e^8 \\
& - 4120a^5b^4c^{10}d^4e^9 + 63360a^5b^5c^9d^3e^{10} + 21376a^5b^6c^8d^2e^{11} \\
& *d^7e^6)) / (8*(a^6b^8e^8 + 256a^6c^8d^8 + 256a^{10}c^4e^8 - 16a^7b^6c^8e^8 \\
& - 4a^5b^9d^7e^7 + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6d^8 - 256a^5b^2c^7d^8 \\
& + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^{12}d^4e^4 - 4a^3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 \\
& + 1024a^7c^7d^6e^2 + 1536a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^{10}c^2d^6e^2 \\
& - 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4b^7c^3d^5e^3 \\
& - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 \\
& - 192a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4 \\
& *c^4d^4e^4 - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 \\
& + 2048a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024a^6b^7c^7d^7e \\
& + 64a^6b^7c^7d^7e - 1024a^9b^3c^4d^7e - 4a^2b^9c^3d^7e - 4a^2b^{11}c^4d^5e^3 + 64a^3b^7c^4 \\
& *d^7e - 4a^3b^{10}c^4d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^3d^3e^5 + 1024a^5b^3c^6d^7e \\
& - 92a^5b^8c^5d^2e^6 - 3072a^7b^6c^6d^5e^3 - 384a^7b^5c^2d^7e - 3072a^8b^3c^5d^3e^5 \\
& + 1024a^8b^3c^3d^7e^7)) * ((27a^9c^5d^6 - b^{11}c^4d^6 - b^{15}d^2e^4 - 9a^2b^{13}e^6 + 38 \\
& 40a^5b^9c^9d^6 + 9a^5c^5d^6*(-(4a^3c - b^2)^9)^{(1/2)} + 213a^3b^{11}c^9e^6 - 26880a^8b^3c^6e^6 \\
& + 3072a^6c^9d^5e^6 + 35840a^8c^7d^7e^5 + 4b^{12}c^3d^5e^6 + 4b^{14}c^3d^3e^3 - 288a^2b^7c^6d^6 \\
& + 1504a^3b^5c^7d^6 - 3840a^4b^3c^8d^6 - 9a^2b^4e^6*(-(4a^3c - b^2)^9)^{(1/2)} - 2077a^4b^9 \\
& *c^2e^6 + 10656a^5b^7c^3e^6 - 30240a^6b^5c^4e^6 + 44800a^7b^3c^5e^6 - 25a^4c^2e^6*(-(4a^3c - b^2)^9)^{(1/2)} \\
& - b^2c^4d^6*(-(4a^3c - b^2)^9)^{(1/2)} + 22528a^7c^8d^3e^3 - b^6d^2e^4*(-(4a^3c - b^2)^9)^{(1/2)} \\
& - 6b^{13}c^2d^4e^2 + 6a^2b^{14}d^5e^5 - 1471a^2b^9c^4d^4e^2 + 600a^2b^{10}c^3d^3e^3 + 180a^2b^{11}c^2d^2e^4 \\
& + 6976a^3b^7c^5d^4e^2 - 1032a^3b^8c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456a^4b^5c^6d^4e^2 \\
& - 7168a^4b^6c^5d^3e^3 + 16896a^4b^7c^4d^2e^4 + 10240a^5b^3c^7d^4e^2 + 37632a^5b^4c^6d^3e^3 \\
& - 47712a^5b^5c^5d^2e^4 - 59392a^6b^2c^7d^3e^3 + 60928a^6b^3c^6d^2e^4 + 41a^2c^4d^4e^2*(-(4a^3c - b^2)^9)^{(1/2)} \\
& + 39a^3c^3d^2e^4*(-(4a^3c - b^2)^9)^{(1/2)} - 6b^4c^2d^4e^2*(-(4a^3c - b^2)^9)^{(1/2)} \\
& + 6a^2b^5d^5e^5*(-(4a^3c - b^2)^9)^{(1/2)} - 106a^2b^{10}c^4d^5e^5 + 7a^2b^{13}c^3d^2e^4 \\
& - 128a^2b^{12}c^3d^2e^5 + 51a^3b^2c^2e^6*(-(4a^3c - b^2)^9)^{(1/2)} + 150a^2b^{11}c^3d^4e^2 - 84a^2b^{12} \\
& *c^2d^3e^3 + 1116a^2b^8c^5d^5e^5 - 5824a^3b^6c^6d^5e^5 + 1030a^3b^{10}c^2d^5e^5 \\
& + 15232a^4b^4c^7d^5e^5 - 3492a^4b^8c^3d^5e^5 - 16896a^5b^2c^8d^5e^5 + 1344a^5b^6c^4d^5e^5 \\
& + 7424a^6b^3c^8d^4e^2 + 22400a^6b^4c^5d^5e^5 - 23296a^7b^3c^7d^2e^4 - 53760a^7b^2c^6d^5e^5 \\
& + 4b^3c^3d^5e^5*(-(4a^3c - b^2)^9)^{(1/2)} + 4b^5c^3d^3e^3*(-(4a^3c - b^2)^9)^{(1/2)} \\
& - 11a^2b^4c^3d^2e^4*(-(4a^3c - b^2)^9)^{(1/2)} - 20a^2b^3c^3d^2e^5*(-(4a^3c - b^2)^9)^{(1/2)} \\
& - 86a^3b^3c^2d^2e^5*(-(4a^3c - b^2)^9)^{(1/2)} + 42a^2b^2c^3d^4e^2*(-(4a^3c - b^2)^9)^{(1/2)} \\
& - 12a^2b^3c^2d^3e^3*(-(4a^3c - b^2)^9)^{(1/2)} - 120a^2b^3c^3d^3e^3*(-(4a^3c - b^2)^9)^{(1/2)} \\
& - 34a^2b^3c^3d^3e^3*(-(4a^3c - b^2)^9)^{(1/2)} - 34a^2b^3c^3d^3e^3*(-(4a^3c - b^2)^9)^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& 4*d^5*e*(-(4*a*c - b^2)^9)^{(1/2)} + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^{(1/2)}) / (32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7))^{(1/2)} * i) / ((2000*a^4*c^9*e^12 + 21*a^2*b^4*c^7*e^12 - 520*a^3*b^2*c^8*e^12 + 1296*a^2*c^11*d^4*e^8 + 4320*a^3*c^10*d^2*e^10 + 25*b^4*c^9*d^4*e^8 - 60*b^5*c^8*d^3*e^9 + 35*b^6*c^7*d^2*e^10 + 192*a^2*b^2*c^9*d^2*e^10 - 112*a*b^5*c^7*d*e^11 - 4480*a^3*b*c^9*d*e^11 - 360*a*b^2*c^10*d^4*e^8 + 832*a*b^3*c^9*d^3*e^9 - 362*a*b^4*c^8*d^2*e^10 - 2880*a^2*b*c^10*d^3*e^9 + 1440*a^2*b^3*c^8*d*e^11) / (8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) + (((((1048576*a^13*c^8*e^16 + 256*a^7*b^12*c^2*e^16 - 6144*a^8*b^10*c^3*e^16 + 61440*a^9*b^8*c^4*e^16 - 327680*a^10*b^6*c^5*e^16 + 983040*a^11*b^4*c^6*e^16 - 1572864*a^12*b^2*c^7*e^16 - 196608*a^6*c^15*d^14*e^2 - 917504*a^7*c^14*d^12*e^4 - 589824*a^8*c^13*d^10*e^6 + 3932160*a^9*c^12*d^8*e^8 + 10158080*a^10*c^11*d^6*e^10 + 10616832*a^11*c^10*d^4*e^12 + 5308416*a^12*c^9*d^2*e^14 - 2816*a^2*b^8*c^11*d^14*e^2 + 22656*a^2*b^9*c^10*d^13*e^3 - 78848*a^2*b^10*c^9*d^12*e^4 + 154112*a^2*b^11*c^8*d^11*e^5 - 182784*a^2*b^12*c^7*d^10*e^6 + 130816*a^2*b^13*c^6*d^9*e^7 - 50176*a^2*b^14*c^5*d^8*e^8 + 4608*a^2*b^15*c^4*d^7*e^9 + 3328*a^2*b^16*c^3*d^6*e^10 - 896*a^2*b^17*c^2*d^5*e^11 + 24576*a^3*b^6*c^12*d^14*e^2 - 198656*a^3*b^7*c^11*d^13*e^3 + 684544*a^3*b^8*c^10*d^12*e^4 - 1291520*a^3*b^9*c^9*d^11*e^5 + 1403776*a^3*b^10*c^8*d^10*e^6 - 798336*a^3*b^11*c^7*d^9*e^7 + 89856*a^3*b^12*c^6*d^8*e^8 + 155136*a^3*b^13*c^5*d^7*e^9 - 77440*a^3*b^14*c^4*d^6*e^10 + 5504*a^3*b^15*c^3*d^5*e^11 + 2560*a^3*b^16*c^2*d^4*e^12 - 106496*a^4*b^4*c^13*d^14*e^2 + 864256*a^4*b^5*c^12*d^13*e^3 - 2924544*a^4*b^6*c^11*d^12*e^4 + 5181440*a^4*b^7*c^10*d^11*e^5
\end{aligned}$$

$$\begin{aligned}
& - 4686080*a^4*b^8*c^9*d^10*e^6 + 1045376*a^4*b^9*c^8*d^9*e^7 + 1900544*a^4 \\
& *b^{10}*c^7*d^8*e^8 - 1732096*a^4*b^{11}*c^6*d^7*e^9 + 390400*a^4*b^{12}*c^5*d^6* \\
& e^{10} + 112000*a^4*b^{13}*c^4*d^5*e^{11} - 40960*a^4*b^{14}*c^3*d^4*e^{12} - 3840*a^4 \\
& *b^{15}*c^2*d^3*e^{13} + 229376*a^5*b^2*c^{14}*d^{14}*e^2 - 1867776*a^5*b^3*c^{13}*d \\
& ^{13}*e^3 + 6078464*a^5*b^4*c^{12}*d^{12}*e^4 - 9297920*a^5*b^5*c^{11}*d^{11}*e^5 + 4 \\
& 055040*a^5*b^6*c^{10}*d^{10}*e^6 + 7788544*a^5*b^7*c^9*d^9*e^7 - 12657664*a^5*b \\
& ^8*c^8*d^8*e^8 + 6130176*a^5*b^9*c^7*d^7*e^9 + 734080*a^5*b^{10}*c^6*d^6*e^{10} \\
& - 1442560*a^5*b^{11}*c^5*d^5*e^{11} + 168960*a^5*b^{12}*c^4*d^4*e^{12} + 78080*a^5 \\
& *b^{13}*c^3*d^3*e^{13} + 3200*a^5*b^{14}*c^2*d^2*e^{14} - 4587520*a^6*b^2*c^{13}*d^{12} \\
& *e^4 + 3080192*a^6*b^3*c^{12}*d^{11}*e^5 + 12001280*a^6*b^4*c^{11}*d^{10}*e^6 - 310 \\
& 76352*a^6*b^5*c^{10}*d^9*e^7 + 27475968*a^6*b^6*c^9*d^8*e^8 - 2088960*a^6*b^7 \\
& *c^8*d^7*e^9 - 12205312*a^6*b^8*c^7*d^6*e^{10} + 6043520*a^6*b^9*c^6*d^5*e^{11} \\
& + 631808*a^6*b^{10}*c^5*d^4*e^{12} - 610304*a^6*b^{11}*c^4*d^3*e^{13} - 71936*a^6* \\
& b^{12}*c^3*d^2*e^{14} - 21725184*a^7*b^2*c^{12}*d^{10}*e^6 + 30801920*a^7*b^3*c^{11}* \\
& d^9*e^7 - 8028160*a^7*b^4*c^{10}*d^8*e^8 - 32260096*a^7*b^5*c^9*d^7*e^9 + 371 \\
& 01568*a^7*b^6*c^8*d^6*e^{10} - 7182336*a^7*b^7*c^7*d^5*e^{11} - 7609856*a^7*b^8 \\
& *c^6*d^4*e^{12} + 2112256*a^7*b^9*c^5*d^3*e^{13} + 661632*a^7*b^{10}*c^4*d^2*e^{14} \\
& - 30146560*a^8*b^2*c^{11}*d^8*e^8 + 55050240*a^8*b^3*c^{10}*d^7*e^9 - 34365440 \\
& *a^8*b^4*c^9*d^6*e^{10} - 16429056*a^8*b^5*c^8*d^5*e^{11} + 24600576*a^8*b^6*c^7 \\
& *d^4*e^{12} - 1683456*a^8*b^7*c^6*d^3*e^{13} - 3151616*a^8*b^8*c^5*d^2*e^{14} - \\
& 10977280*a^9*b^2*c^{10}*d^6*e^{10} + 47022080*a^9*b^3*c^9*d^5*e^{11} - 30621696*a \\
& ^9*b^4*c^8*d^4*e^{12} - 9232384*a^9*b^5*c^7*d^3*e^{13} + 7970816*a^9*b^6*c^6*d^2 \\
& *e^{14} + 4325376*a^{10}*b^2*c^9*d^4*e^{12} + 25493504*a^{10}*b^3*c^8*d^3*e^{13} - 9 \\
& 117696*a^{10}*b^4*c^7*d^2*e^{14} + 491520*a^{11}*b^2*c^8*d^2*e^{14} - 4947968*a^{12}* \\
& b*c^8*d*e^{15} + 128*a*b^{10}*c^{10}*d^{14}*e^2 - 1024*a*b^{11}*c^9*d^{13}*e^3 + 3584*a \\
& *b^{12}*c^8*d^{12}*e^4 - 7168*a*b^{13}*c^7*d^{11}*e^5 + 8960*a*b^{14}*c^6*d^{10}*e^6 - \\
& 7168*a*b^{15}*c^5*d^9*e^7 + 3584*a*b^{16}*c^4*d^8*e^8 - 1024*a*b^{17}*c^3*d^7*e^9 \\
& + 128*a*b^{18}*c^2*d^6*e^{10} + 1605632*a^6*b*c^{14}*d^{13}*e^3 - 1408*a^6*b^{13}*c^ \\
& 2*d*e^{15} + 7012352*a^7*b*c^{13}*d^{11}*e^5 + 33152*a^7*b^{11}*c^3*d*e^{15} + 704512 \\
& 0*a^8*b*c^{12}*d^9*e^7 - 324480*a^8*b^9*c^4*d*e^{15} - 9830400*a^9*b*c^{11}*d^7*e \\
& ^9 + 1689600*a^9*b^7*c^5*d*e^{15} - 25722880*a^{10}*b*c^{10}*d^5*e^{11} - 4935680*a \\
& ^{10}*b^5*c^6*d*e^{15} - 19202048*a^{11}*b*c^9*d^3*e^{13} + 7667712*a^{11}*b^3*c^7*d* \\
& e^{15})/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^{10}*c^4*e^8 - 16*a^7*b^6*c* \\
& e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c \\
& ^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a \\
& ^2*b^{12}*d^4*e^4 - 4*a^3*b^{11}*d^3*e^5 + 6*a^4*b^{10}*d^2*e^6 + 1024*a^7*c^7*d^ \\
& 6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^{10}*c^2*d^6*e^ \\
& 2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e \\
& ^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^ \\
& 6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2 \\
& *d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^ \\
& 4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^ \\
& 7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 5 \\
& 12*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a \\
& ^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^{11}*c*d^5*e^3 + 64*a^3*b^7*c^ \\
& 4*d^7*e - 4*a^3*b^{10}*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e \\
& ^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 \\
& - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7) \\
& ) - (x*((27*a*b^9*c^5*d^6 - b^{11}*c^4*d^6 - b^{15}*d^2*e^4 - 9*a^2*b^{13}*e^6 + \\
& 3840*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^{(1/2)} + 213*a^3*b^{11}*c* \\
& e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^ \\
& 12*c^3*d^5*e + 4*b^{14}*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^ \\
& 6 - 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^{(1/2)} - 2077*a^ \\
& 4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b \\
& ^3*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^{(1/2)} - b^2*c^4*d^6*(-(4*a*c \\
& - b^2)^9)^{(1/2)} + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^{( \\
& 1/2)} - 6*b^{13}*c^2*d^4*e^2 + 6*a*b^{14}*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600 \\
& *a^2*b^{10}*c^3*d^3*e^3 + 180*a^2*b^{11}*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 \\
& - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*
\end{aligned}$$

$$\begin{aligned}
& d^4 e^2 - 7168 a^4 b^6 c^5 d^3 e^3 + 16896 a^4 b^7 c^4 d^2 e^4 + 10240 a^5 b^3 c^7 d^4 e^2 + 37632 a^5 b^4 c^6 d^3 e^3 - 47712 a^5 b^5 c^5 d^2 e^4 - 5 \\
& 9392 a^6 b^2 c^7 d^3 e^3 + 60928 a^6 b^3 c^6 d^2 e^4 + 41 a^2 c^4 d^4 e^2 (- (4 a c - b^2)^9)^{(1/2)} + 39 a^3 c^3 d^2 e^4 (- (4 a c - b^2)^9)^{(1/2)} - 6 b \\
& ^4 c^2 d^4 e^2 (- (4 a c - b^2)^9)^{(1/2)} + 6 a b^5 d^5 e^5 (- (4 a c - b^2)^9)^{(1/2)} - 106 a b^10 c^4 d^5 e + 7 a b^13 c^3 d^2 e^4 - 128 a^2 b^12 c^3 d^2 e^5 + \\
& 51 a^3 b^2 c^5 e^6 (- (4 a c - b^2)^9)^{(1/2)} + 150 a b^11 c^3 d^4 e^2 - 84 a b^12 c^2 d^3 e^3 + 1116 a^2 b^8 c^5 d^5 e - 5824 a^3 b^6 c^6 d^5 e + 1030 a^3 \\
& b^10 c^2 d^5 e^5 + 15232 a^4 b^4 c^7 d^5 e - 3492 a^4 b^8 c^3 d^5 e - 16896 a^5 b^2 c^8 d^5 e + 1344 a^5 b^6 c^4 d^5 e + 7424 a^6 b^6 c^8 d^4 e^2 + 2240 \\
& 0 a^6 b^4 c^5 d^5 e^5 - 23296 a^7 b^6 c^7 d^2 e^4 - 53760 a^7 b^2 c^6 d^5 e^5 + 4 b^3 c^3 d^5 e (- (4 a c - b^2)^9)^{(1/2)} + 4 b^5 c^3 d^3 e^3 (- (4 a c - b^2)^9)^{(1/2)} \\
& - 11 a b^4 c^3 d^2 e^4 (- (4 a c - b^2)^9)^{(1/2)} - 20 a^2 b^3 c^3 d^2 e^5 (- (4 a c - b^2)^9)^{(1/2)} - 86 a^3 b^2 c^2 d^2 e^5 (- (4 a c - b^2)^9)^{(1/2)} + 42 \\
& a b^2 c^3 d^4 e^2 (- (4 a c - b^2)^9)^{(1/2)} - 12 a b^3 c^2 d^3 e^3 (- (4 a c - b^2)^9)^{(1/2)} - 120 a^2 b^2 c^3 d^3 e^3 (- (4 a c - b^2)^9)^{(1/2)} - 34 a b^3 c^4 d^5 e \\
& (- (4 a c - b^2)^9)^{(1/2)} + 108 a^2 b^2 c^2 d^2 e^4 (- (4 a c - b^2)^9)^{(1/2)) / (32 (a^7 b^12 e^8 + 4096 a^9 c^10 d^8 + 4096 a^13 c^6 e^8 - 24 a^8 b^10 c^5 e^8 \\
& - 4 a^6 b^13 d^5 e^7 + a^3 b^12 c^4 d^8 - 24 a^4 b^10 c^5 d^8 + 240 a^5 b^8 c^6 d^8 - 1280 a^6 b^6 c^7 d^8 + 3840 a^7 b^4 c^8 d^8 - 6144 a^8 b^2 c^9 d^8 \\
& + 240 a^9 b^8 c^2 e^8 - 1280 a^10 b^6 c^3 e^8 + 3840 a^11 b^4 c^4 e^8 - 6144 a^12 b^2 c^5 e^8 + a^3 b^16 d^4 e^4 - 4 a^4 b^15 d^3 e^5 + 6 a^5 b^14 d^2 e^6 \\
& + 16384 a^10 c^9 d^6 e^2 + 24576 a^11 c^8 d^4 e^4 + 16384 a^12 c^7 d^2 e^6 + 6 a^3 b^14 c^2 d^6 e^2 - 140 a^4 b^12 c^3 d^6 e^2 + 84 a^4 b^13 c^2 d^5 e^3 \\
& + 1344 a^5 b^10 c^4 d^6 e^2 - 672 a^5 b^11 c^3 d^5 e^3 - 42 a^5 b^12 c^2 d^4 e^4 - 6720 a^6 b^8 c^5 d^6 e^2 + 2240 a^6 b^9 c^4 d^5 e^3 + 1456 a^6 b^10 c^3 d^4 e^4 \\
& - 672 a^6 b^11 c^2 d^3 e^5 + 17920 a^7 b^6 c^6 d^6 e^2 - 10080 a^7 b^8 c^4 d^4 e^4 + 2240 a^7 b^9 c^3 d^3 e^5 + 1344 a^7 b^10 c^2 d^2 e^6 - 21504 a^8 b^4 c^7 d^6 e^2 \\
& - 21504 a^8 b^5 c^6 d^5 e^3 + 32256 a^8 b^6 c^5 d^4 e^4 - 6720 a^8 b^8 c^3 d^2 e^6 + 57344 a^9 b^3 c^7 d^5 e^3 - 46592 a^9 b^4 c^6 d^4 e^4 - 21504 a^9 b^5 c^5 d^3 e^5 \\
& + 17920 a^9 b^6 c^4 d^2 e^6 + 12288 a^10 b^2 c^7 d^4 e^4 + 57344 a^10 b^3 c^6 d^3 e^5 - 21504 a^10 b^4 c^5 d^2 e^6 + 96 a^7 b^11 c^3 d^7 e - 16384 a^9 b^3 c^9 d^7 e \\
& - 16384 a^12 b^2 c^6 d^7 e - 4 a^3 b^13 c^3 d^7 e - 4 a^3 b^15 c^3 d^5 e^3 + 96 a^4 b^11 c^4 d^7 e - 12 a^4 b^14 c^3 d^4 e^4 - 960 a^5 b^9 c^5 d^7 e + 84 a^5 b^13 c^3 d^3 e^5 \\
& + 5120 a^6 b^7 c^6 d^7 e - 140 a^6 b^12 c^3 d^2 e^6 - 15360 a^7 b^5 c^7 d^7 e + 24576 a^8 b^3 c^8 d^7 e - 960 a^8 b^9 c^2 d^7 e + 5120 a^9 b^7 c^3 d^7 e - 49152 a^10 b^3 c^8 d^5 e^3 \\
& - 15360 a^10 b^5 c^4 d^7 e - 49152 a^11 b^3 c^7 d^3 e^5 + 24576 a^11 b^3 c^5 d^7 e))^{(1/2)} (1048576 a^15 c^8 e^17 + 256 a^9 b^12 c^2 e^17 - 6144 a^10 b^10 c^3 e^17 + 61440 a^11 b^8 c^4 e^17 \\
& - 327680 a^12 b^6 c^5 e^17 + 983040 a^13 b^4 c^6 e^17 - 1572864 a^14 b^2 c^7 e^17 - 1048576 a^8 c^15 d^14 e^3 - 5242880 a^9 c^14 d^12 e^5 - 9437184 a^10 c^13 d^10 e^7 - 5242880 a^11 c^12 d^8 e^9 \\
& + 5242880 a^12 c^11 d^6 e^11 + 9437184 a^13 c^10 d^4 e^13 + 5242880 a^14 c^9 d^2 e^15 + 256 a^2 b^11 c^10 d^15 e^2 - 2048 a^2 b^12 c^9 d^14 e^3 + 7168 a^2 b^13 c^8 d^13 e^4 \\
& - 14336 a^2 b^14 c^7 d^12 e^5 + 17920 a^2 b^15 c^6 d^11 e^6 - 14336 a^2 b^16 c^5 d^10 e^7 + 7168 a^2 b^17 c^4 d^9 e^8 - 2048 a^2 b^18 c^3 d^8 e^9 + 256 a^2 b^19 c^2 d^7 e^10 \\
& - 5120 a^3 b^9 c^11 d^15 e^2 + 41984 a^3 b^10 c^10 d^14 e^3 - 148736 a^3 b^11 c^9 d^13 e^4 + 296192 a^3 b^12 c^8 d^12 e^5 - 359680 a^3 b^13 c^7 d^11 e^6 + 267520 a^3 b^14 c^6 d^10 e^7 \\
& - 112384 a^3 b^15 c^5 d^9 e^8 + 18176 a^3 b^16 c^4 d^8 e^9 + 3328 a^3 b^17 c^3 d^7 e^10 - 1280 a^3 b^18 c^2 d^6 e^11 + 40960 a^4 b^7 c^12 d^15 e^2 - 348160 a^4 b^8 c^11 d^14 e^3 \\
& + 1254400 a^4 b^9 c^10 d^13 e^4 - 2478080 a^4 b^10 c^9 d^12 e^5 + 2867456 a^4 b^11 c^8 d^11 e^6 - 1862144 a^4 b^12 c^7 d^10 e^7 + 490240 a^4 b^13 c^6 d^9 e^8 \\
& + 128000 a^4 b^14 c^5 d^8 e^9 - 108800 a^4 b^15 c^4 d^7 e^10 + 13824 a^4 b^16 c^3 d^6 e^11 + 2304 a^4 b^17 c^2 d^5 e^12 - 163840 a^5 b^5 c^13 d^15 e^2 + 1474560 a^5 b^6 c^12 d^14 e^3 \\
& - 5447680 a^5 b^7 c^11 d^13 e^4 + 10588160 a^5 b^8 c^10 d^12 e^5 - 11166720 a^5 b^9 c^9 d^11 e^6 + 5159936 a^5 b^10 c^8 d^10 e^7 + 1073920 a^5 b^11 c^7 d^9 e^8 - 227
\end{aligned}$$

$$\begin{aligned}
& 9680a^5b^{12}c^6d^8e^9 + 770560a^5b^{13}c^5d^7e^{10} + 33280a^5b^{14}c^4d^6e^{11} - 41216a^5b^{15}c^3d^5e^{12} - 1280a^5b^{16}c^2d^4e^{13} + 327680a^6b^3c^{14}d^{15}e^2 - 3276800a^6b^4c^{13}d^{14}e^3 + 12615680a^6b^5c^{12}d^{13}e^4 - 23592960a^6b^6c^{11}d^{12}e^5 + 19701760a^6b^7c^{10}d^{11}e^6 + 1372160a^6b^8c^9d^{10}e^7 - 15846400a^6b^9c^8d^9e^8 + 10864640a^6b^{10}c^7d^8e^9 - 1352960a^6b^{11}c^6d^7e^{10} - 1111040a^6b^{12}c^5d^6e^{11} + 273920a^6b^{13}c^4d^5e^{12} + 25600a^6b^{14}c^3d^4e^{13} - 1280a^6b^{15}c^2d^3e^{14} + 3407872a^7b^2c^{14}d^{14}e^3 - 14221312a^7b^3c^{13}d^{13}e^4 + 23527424a^7b^4c^{12}d^{12}e^5 - 3768320a^7b^5c^{11}d^{11}e^6 - 38895616a^7b^6c^{10}d^{10}e^7 + 50126848a^7b^7c^9d^9e^8 - 18362368a^7b^8c^8d^8e^9 - 6831104a^7b^9c^7d^7e^{10} + 6200320a^7b^{10}c^6d^6e^{11} - 726784a^7b^{11}c^5d^5e^{12} - 228608a^7b^{12}c^4d^4e^{13} + 31488a^7b^{13}c^3d^3e^{14} + 2304a^7b^{14}c^2d^2e^{15} - 3145728a^8b^2c^{13}d^{12}e^5 - 31129600a^8b^3c^{12}d^{11}e^6 + 74711040a^8b^4c^{11}d^{10}e^7 - 55476224a^8b^5c^{10}d^9e^8 - 11075584a^8b^6c^9d^8e^9 + 35381248a^8b^7c^8d^7e^{10} - 14479360a^8b^8c^7d^6e^{11} - 168960a^8b^9c^6d^5e^{12} + 1286144a^8b^{10}c^5d^4e^{13} - 302336a^8b^{11}c^4d^3e^{14} - 55808a^8b^{12}c^3d^2e^{15} - 36962304a^9b^2c^{12}d^{10}e^7 - 9502720a^9b^3c^{11}d^9e^8 + 67174400a^9b^4c^{10}d^8e^9 - 54886400a^9b^5c^9d^7e^{10} + 11239424a^9b^6c^8d^6e^{11} + 5545984a^9b^7c^7d^5e^{12} - 5263360a^9b^8c^6d^4e^{13} + 1356800a^9b^9c^5d^3e^{14} + 558080a^9b^{10}c^4d^2e^{15} - 49807360a^{10}b^2c^{11}d^8e^9 + 19333120a^{10}b^3c^{10}d^7e^{10} + 7208960a^{10}b^4c^9d^6e^{11} - 14974976a^{10}b^5c^8d^5e^{12} + 15073280a^{10}b^6c^7d^4e^{13} - 2170880a^{10}b^7c^6d^3e^{14} - 2928640a^{10}b^8c^5d^2e^{15} - 11796480a^{11}b^2c^{10}d^6e^{11} + 23920640a^{11}b^3c^9d^5e^{12} - 24576000a^{11}b^4c^8d^4e^{13} - 4096000a^{11}b^5c^7d^3e^{14} + 8355840a^{11}b^6c^6d^2e^{15} + 12582912a^{12}b^2c^9d^4e^{13} + 19857408a^{12}b^3c^8d^3e^{14} - 11534336a^{12}b^4c^7d^2e^{15} + 3407872a^{13}b^2c^8d^2e^{15} - 5505024a^{14}b^3c^8d^2e^{16} - 262144a^7b^3c^{15}d^{15}e^2 + 5505024a^8b^3c^{14}d^{13}e^4 - 1280a^8b^{13}c^2d^2e^{16} + 25952256a^9b^3c^{13}d^{11}e^6 + 30976a^9b^{11}c^3d^2e^{16} + 38010880a^{10}b^3c^{12}d^9e^8 - 312320a^{10}b^9c^4d^2e^{16} + 11796480a^{11}b^3c^{11}d^7e^{10} + 1679360a^{11}b^7c^5d^2e^{16} - 21233664a^{12}b^3c^{10}d^5e^{12} - 5079040a^{12}b^5c^6d^2e^{16} - 20709376a^{13}b^3c^9d^3e^{14} + 8192000a^{13}b^3c^7d^2e^{16})) / ((8*(a^6b^8e^8 + 256a^6c^8d^8 + 256a^{10}c^4e^8 - 16a^7b^6c^8e^8 - 4a^5b^9d^8e^7 + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6d^8 - 256a^5b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^{12}d^4e^4 - 4a^3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 + 1024a^7c^7d^6e^2 + 1536a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^{10}c^2d^6e^2 - 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4b^7c^3d^5e^3 - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 - 192a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c^4d^4e^4 - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 + 2048a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024a^6b^3c^7d^7e + 64a^6b^7c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024a^9b^3c^4d^2e^6 - 4a^2b^9c^3d^7e - 4a^2b^{11}c^3d^5e^3 + 64a^3b^7c^4d^7e - 4a^3b^{10}c^2d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^3d^3e^5 + 1024a^5b^3c^6d^7e - 92a^5b^8c^2d^2e^6 - 3072a^7b^3c^6d^5e^3 - 384a^7b^5c^2d^2e^7 - 3072a^8b^3c^5d^3e^5 + 1024a^8b^3c^3d^2e^7))) * ((27a^9b^9c^5d^6 - b^{11}c^4d^6 - b^{15}d^2e^4 - 9a^2b^{13}e^6 + 3840a^5b^3c^9d^6 + 9a^5c^5d^6 * (-4ac - b^2)^9)^{(1/2)} + 213a^3b^{11}c^6e^6 - 26880a^8b^3c^6e^6 + 3072a^6c^9d^5e + 35840a^8c^7d^5e^5 + 4b^{12}c^3d^5e + 4b^{14}c^3d^3e^3 - 288a^2b^7c^6d^6 + 1504a^3b^5c^7d^6 - 3840a^4b^3c^8d^6 - 9a^2b^4e^6 * (-4ac - b^2)^9)^{(1/2)} - 2077a^4b^9c^2e^6 + 10656a^5b^7c^3e^6 - 30240a^6b^5c^4e^6 + 44800a^7b^3c^5e^6 - 25a^4c^2e^6 * (-4ac - b^2)^9)^{(1/2)} - b^2c^4d^6 * (-4ac - b^2)^9)^{(1/2)} + 22528a^7c^8d^3e^3 - b^6d^2e^4 * (-4ac - b^2)^9)^{(1/2)} - 6b^{13}c^2d^4e^2 + 6a^b^{14}d^5e^5 - 1471a^2b^9c^4d^4e^2 + 600a^2b^{10}c^3d^3e^8
\end{aligned}$$



$$\begin{aligned}
& 3 + 180a^2b^{11}c^2d^2e^4 + 6976a^3b^7c^5d^4e^2 - 1032a^3b^8c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456a^4b^5c^6d^4e^2 - 7168a^4b^6c^5d^3e^3 + 16896a^4b^7c^4d^2e^4 + 10240a^5b^3c^7d^4e^2 + 37632a^5b^4c^6d^3e^3 - 47712a^5b^5c^5d^2e^4 - 59392a^6b^2c^7d^3e^3 + 60928a^6b^3c^6d^2e^4 + 41a^2c^4d^4e^2(-4ac - b^2)^9)^{(1/2)} + 39a^3c^3d^2e^4(-4ac - b^2)^9)^{(1/2)} - 6b^4c^2d^4e^2(-4ac - b^2)^9)^{(1/2)} + 6a^2b^5d^5e^5(-4ac - b^2)^9)^{(1/2)} - 106a^2b^10c^4d^5e^5 + 7a^2b^13c^3d^2e^4 - 128a^2b^12c^3d^2e^5 + 51a^3b^2c^3e^6(-4ac - b^2)^9)^{(1/2)} + 150a^2b^11c^3d^4e^2 - 84a^2b^12c^2d^3e^3 + 1116a^2b^8c^5d^5e^5 - 5824a^3b^6c^6d^5e^5 + 1030a^3b^10c^2d^5e^5 + 15232a^4b^4c^7d^5e^5 - 3492a^4b^8c^3d^5e^5 - 16896a^5b^2c^8d^5e^5 + 1344a^5b^6c^4d^5e^5 + 7424a^6b^3c^8d^4e^2 + 22400a^6b^4c^5d^5e^5 - 23296a^7b^3c^7d^2e^4 - 53760a^7b^2c^6d^5e^5 + 4b^3c^3d^5e^5(-4ac - b^2)^9)^{(1/2)} + 4b^5c^3d^3e^3(-4ac - b^2)^9)^{(1/2)} - 11a^2b^4c^3d^2e^4(-4ac - b^2)^9)^{(1/2)} - 20a^2b^3c^3d^2e^5(-4ac - b^2)^9)^{(1/2)} - 86a^3b^3c^2d^2e^5(-4ac - b^2)^9)^{(1/2)} + 42a^2b^2c^3d^4e^2(-4ac - b^2)^9)^{(1/2)} - 12a^2b^3c^2d^3e^3(-4ac - b^2)^9)^{(1/2)} - 120a^2b^3c^3d^3e^3(-4ac - b^2)^9)^{(1/2)} - 34a^2b^3c^4d^5e^5(-4ac - b^2)^9)^{(1/2)} + 108a^2b^2c^2d^2e^4(-4ac - b^2)^9)^{(1/2))}/(32(a^7b^12e^8 + 4096a^9c^10d^8 + 4096a^13c^6e^8 - 24a^8b^10c^8e^8 - 4a^6b^13d^8e^7 + a^3b^12c^4d^8 - 24a^4b^10c^5d^8 + 240a^5b^8c^6d^8 - 1280a^6b^6c^7d^8 + 3840a^7b^4c^8d^8 - 6144a^8b^2c^9d^8 + 240a^9b^8c^2e^8 - 1280a^10b^6c^3e^8 + 3840a^11b^4c^4e^8 - 6144a^12b^2c^5e^8 + a^3b^16d^4e^4 - 4a^4b^15d^3e^5 + 6a^5b^14d^2e^6 + 16384a^10c^9d^6e^2 + 24576a^11c^8d^4e^4 + 16384a^12c^7d^2e^6 + 6a^3b^14c^2d^6e^2 - 140a^4b^12c^3d^6e^2 + 84a^4b^13c^2d^5e^3 + 1344a^5b^10c^4d^6e^2 - 672a^5b^11c^3d^5e^3 - 42a^5b^12c^2d^4e^4 - 6720a^6b^8c^5d^6e^2 + 2240a^6b^9c^4d^5e^3 + 1456a^6b^10c^3d^4e^4 - 672a^6b^11c^2d^3e^5 + 17920a^7b^6c^6d^6e^2 - 10080a^7b^8c^4d^4e^4 + 2240a^7b^9c^3d^3e^5 + 1344a^7b^10c^2d^2e^6 - 21504a^8b^4c^7d^6e^2 - 21504a^8b^5c^6d^5e^3 + 32256a^8b^6c^5d^4e^4 - 6720a^8b^8c^3d^2e^6 + 57344a^9b^3c^7d^5e^3 - 46592a^9b^4c^6d^4e^4 - 21504a^9b^5c^5d^3e^5 + 17920a^9b^6c^4d^2e^6 + 12288a^10b^2c^7d^4e^4 + 57344a^10b^3c^6d^3e^5 - 21504a^10b^4c^5d^2e^6 + 96a^7b^11c^3d^7e^7 - 16384a^9b^3c^7d^5e^3 - 16384a^12b^3c^6d^5e^7 - 4a^3b^13c^3d^7e^7 - 4a^3b^15c^3d^5e^3 + 96a^4b^11c^4d^7e^7 - 12a^4b^14c^3d^4e^4 - 960a^5b^9c^5d^7e^7 + 84a^5b^13c^3d^3e^5 + 5120a^6b^7c^6d^7e^7 - 140a^6b^12c^3d^2e^6 - 15360a^7b^5c^7d^7e^7 + 24576a^8b^3c^8d^7e^7 - 960a^8b^9c^2d^5e^7 + 5120a^9b^7c^3d^5e^7 - 49152a^10b^3c^8d^5e^3 - 15360a^10b^5c^4d^5e^7 - 49152a^11b^3c^7d^3e^5 + 24576a^11b^3c^5d^5e^7))^{(1/2)} - (x*(626688a^10b^8e^15 - 784384a^10c^9d^8e^14 + 208a^4b^13c^2e^15 - 4880a^5b^11c^3e^15 + 47312a^6b^9c^4e^15 - 242176a^7b^7c^5e^15 + 688640a^8b^5c^6e^15 - 1028096a^9b^3c^7e^15 + 18432a^4c^15d^13e^2 + 126976a^5c^14d^11e^4 + 325632a^6c^13d^9e^6 + 139264a^7c^12d^7e^8 - 1067008a^8c^11d^5e^10 - 1773568a^9c^10d^3e^12 + 16b^8c^11d^13e^2 - 96b^9c^10d^12e^3 + 240b^10c^9d^11e^4 - 304b^11c^8d^10e^5 + 144b^12c^7d^9e^6 + 144b^13c^6d^8e^7 - 304b^14c^5d^7e^8 + 240b^15c^4d^6e^9 - 96b^16c^3d^5e^10 + 16b^17c^2d^4e^11 + 3200a^2b^4c^13d^13e^2 - 18432a^2b^5c^12d^12e^3 + 41024a^2b^6c^11d^11e^4 - 36352a^2b^7c^10d^10e^5 - 16208a^2b^8c^9d^9e^6 + 74576a^2b^9c^8d^8e^7 - 78496a^2b^10c^7d^7e^8 + 32064a^2b^11c^6d^6e^9 + 6000a^2b^12c^5d^5e^10 - 9264a^2b^13c^4d^4e^11 + 1472a^2b^14c^3d^3e^12 + 416a^2b^15c^2d^2e^13 - 12800a^3b^2c^14d^13e^2 + 73728a^3b^3c^13d^12e^3 - 151296a^3b^4c^12d^11e^4 + 78336a^3b^5c^11d^10e^5 + 206688a^3b^6c^10d^9e^6 - 436736a^3b^7c^9d^8e^7 + 324224a^3b^8c^8d^7e^8 + 992a^3b^9c^7d^6e^9 - 158176a^3b^10c^6d^5e^10 + 77056a^3b^11c^5d^4e^11 + 6912a^3b^12c^4d^3e^12 - 8416a^3b^13c^3d^2e^13 + 162816a^4b^2c^13d^11e^4 + 184320a^4b^3c^12d^10e^5 - 916608a^4b^
\end{aligned}$$

$$\begin{aligned}
& 4*c^{11}*d^9*e^6 + 1165824*a^4*b^5*c^{10}*d^8*e^7 - 314496*a^4*b^6*c^9*d^7*e^8 \\
& - 822272*a^4*b^7*c^8*d^6*e^9 + 919152*a^4*b^8*c^7*d^5*e^{10} - 175296*a^4*b^9 \\
& *c^6*d^4*e^{11} - 189328*a^4*b^{10}*c^5*d^3*e^{12} + 62064*a^4*b^{11}*c^4*d^2*e^{13} \\
& + 1290752*a^5*b^2*c^{12}*d^9*e^6 - 659456*a^5*b^3*c^{11}*d^8*e^7 - 1561088*a^5* \\
& b^4*c^{10}*d^7*e^8 + 3240960*a^5*b^5*c^9*d^6*e^9 - 1964192*a^5*b^6*c^8*d^5*e^{10} \\
& - 683008*a^5*b^7*c^7*d^4*e^{11} + 1162304*a^5*b^8*c^6*d^3*e^{12} - 164112*a^5* \\
& b^9*c^5*d^2*e^{13} + 3442688*a^6*b^2*c^{11}*d^7*e^8 - 3670016*a^6*b^3*c^{10}*d^6 \\
& *e^9 + 15232*a^6*b^4*c^9*d^5*e^{10} + 4230144*a^6*b^5*c^8*d^4*e^{11} - 3059648 \\
& *a^6*b^6*c^7*d^3*e^{12} - 247296*a^6*b^7*c^6*d^2*e^{13} + 4010496*a^7*b^2*c^{10}* \\
& d^5*e^{10} - 6873088*a^7*b^3*c^9*d^4*e^{11} + 2822400*a^7*b^4*c^8*d^3*e^{12} + 23 \\
& 70048*a^7*b^5*c^7*d^2*e^{13} + 1178624*a^8*b^2*c^9*d^3*e^{12} - 4739072*a^8*b^3 \\
& *c^8*d^2*e^{13} - 352*a*b^6*c^{12}*d^{13}*e^2 + 2048*a*b^7*c^{11}*d^{12}*e^3 - 4800*a \\
& *b^8*c^{10}*d^{11}*e^4 + 5168*a*b^9*c^9*d^{10}*e^5 - 480*a*b^{10}*c^8*d^9*e^6 - 600 \\
& 0*a*b^{11}*c^7*d^8*e^7 + 8192*a*b^{12}*c^6*d^7*e^8 - 5040*a*b^{13}*c^5*d^6*e^9 + \\
& 1152*a*b^{14}*c^4*d^5*e^{10} + 240*a*b^{15}*c^3*d^4*e^{11} - 128*a*b^{16}*c^2*d^3*e^{12} \\
& - 512*a^3*b^{14}*c^2*d*e^{14} - 106496*a^4*b*c^{14}*d^{12}*e^3 + 11680*a^4*b^{12}*c \\
& ^3*d*e^{14} - 675840*a^5*b*c^{13}*d^{10}*e^5 - 108288*a^5*b^{10}*c^4*d*e^{14} - 16015 \\
& 36*a^6*b*c^{12}*d^8*e^7 + 514768*a^6*b^8*c^5*d*e^{14} - 925696*a^7*b*c^{11}*d^6*e \\
& ^9 - 1278304*a^7*b^6*c^6*d*e^{14} + 2457600*a^8*b*c^{10}*d^4*e^{11} + 1385600*a^8 \\
& *b^4*c^7*d*e^{14} + 2977792*a^9*b*c^9*d^2*e^{13} + 19968*a^9*b^2*c^8*d*e^{14})/( \\
& 8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4* \\
& a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - \\
& 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^{12} \\
& d^4*e^4 - 4*a^3*b^{11}*d^3*e^5 + 6*a^4*b^{10}*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + \\
& 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^{10}*c^2*d^6*e^2 - 92*a \\
& ^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192 \\
& *a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - \\
& 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 \\
& + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4 \\
& *e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5 \\
& *d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2 \\
& *c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4 \\
& *d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^{11}*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e \\
& - 4*a^3*b^{10}*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 102 \\
& 4*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a \\
& ^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) * ((27* \\
& a*b^9*c^5*d^6 - b^{11}*c^4*d^6 - b^{15}*d^2*e^4 - 9*a^2*b^{13}*e^6 + 3840*a^5*b*c \\
& ^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^{(1/2)} + 213*a^3*b^{11}*c*e^6 - 26880* \\
& a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^{12}*c^3*d^5*e \\
& + 4*b^{14}*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4 \\
& *b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^{(1/2)} - 2077*a^4*b^9*c^2*e^6 \\
& + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - \\
& 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^{(1/2)} - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^{( \\
& 1/2)} + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^{(1/2)} - 6*b^1 \\
& 3*c^2*d^4*e^2 + 6*a*b^{14}*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^{10}*c^ \\
& 3*d^3*e^3 + 180*a^2*b^{11}*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3* \\
& b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 71 \\
& 68*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4* \\
& e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2 \\
& *c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)} + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^{(1/2)} - 6*b^4*c^2*d^4*e \\
& ^2*(-(4*a*c - b^2)^9)^{(1/2)} + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^{(1/2)} - 106* \\
& a*b^{10}*c^4*d^5*e + 7*a*b^{13}*c*d^2*e^4 - 128*a^2*b^{12}*c*d*e^5 + 51*a^3*b^2*c \\
& *e^6*(-(4*a*c - b^2)^9)^{(1/2)} + 150*a*b^{11}*c^3*d^4*e^2 - 84*a*b^{12}*c^2*d^3* \\
& e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^{10}*c^2*d \\
& *e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8 \\
& *d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^ \\
& 5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5 \\
& *e*(-(4*a*c - b^2)^9)^{(1/2)} + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^{(1/2)} - 11
\end{aligned}$$

$$\begin{aligned}
& *a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^{(1/2)} - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^{(1/2)} - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^{(1/2)} + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^{(1/2)} - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^{(1/2)} - 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^{(1/2)} + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^{(1/2)}) / \\
& (32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7))^{(1/2)} - (326912*a^8*c^9*d*e^13 - 241664*a^8*b*c^8*e^14 - 48*a^2*b^13*c^2*e^14 + 1264*a^3*b^11*c^3*e^14 - 13552*a^4*b^9*c^4*e^14 + 75776*a^5*b^7*c^5*e^14 - 232960*a^6*b^5*c^6*e^14 + 372736*a^7*b^3*c^7*e^14 + 11520*a^3*c^14*d^11*e^3 + 78080*a^4*c^13*d^9*e^5 + 197120*a^5*c^12*d^7*e^7 + 336384*a^6*c^11*d^5*e^9 + 532736*a^7*c^10*d^3*e^11 - 40*b^5*c^12*d^12*e^2 + 216*b^6*c^11*d^11*e^3 - 464*b^7*c^10*d^10*e^4 + 496*b^8*c^9*d^9*e^5 - 264*b^9*c^8*d^8*e^6 + 56*b^10*c^7*d^7*e^7 - 16*b^11*c^6*d^6*e^8 + 64*b^12*c^5*d^5*e^9 - 96*b^13*c^4*d^4*e^10 + 64*b^14*c^3*d^3*e^11 - 16*b^15*c^2*d^2*e^12 + 1536*a^2*b^2*c^13*d^11*e^3 + 14400*a^2*b^3*c^12*d^10*e^4 - 47152*a^2*b^4*c^11*d^9*e^5 + 52144*a^2*b^5*c^10*d^8*e^6 - 16272*a^2*b^6*c^9*d^7*e^7 - 13040*a^2*b^7*c^8*d^6*e^8 + 23488*a^2*b^8*c^7*d^5*e^9 - 26384*a^2*b^9*c^6*d^4*e^10 + 13824*a^2*b^10*c^5*d^3*e^11 + 256*a^2*b^11*c^4*d^2*e^12 + 125056*a^3*b^2*c^12*d^9*e^5 - 36224*a^3*b^3*c^11*d^8*e^6 - 126432*a^3*b^4*c^10*d^7*e^7 + 144848*a^3*b^5*c^9*d^6*e^8 - 114752*a^3*b^6*c^8*d^5*e^9 + 125392*a^3*b^7*c^7*d^4*e^10 - 53248*a^3*b^8*c^6*d^3*e^11 - 25264*a^3*b^9*c^5*d^2*e^12 + 474112*a^4*b^2*c^11*d^7*e^7 - 191104*a^4*b^3*c^10*d^6*e^8 + 97184*a^4*b^4*c^9*d^5*e^9 - 277000*a^4*b^5*c^8*d^4*e^10 + 56056*a^4*b^6*c^7*d^3*e^11 + 195584*a^4*b^7*c^6*d^2*e^12 + 236800*a^5*b^2*c^10*d^5*e^9 + 388032*a^5*b^3*c^9*d^4*e^10 + 159632*a^5*b^4*c^8*d^3*e^11 - 670488*a^5*b^5*c^7*d^2*e^12 - 488960*a^6*b^2*c^9*d^3*e^11 + 1106496*a^6*b^3*c^8*d^2*e^12 + 64*a*b^14*c^2*d*e^13 + 448*a*b^3*c^13*d^12*e^2 - 1968*a*b^4*c^12*d^11*e^3 + 2504*a*b^5*c^11*d^10*e^4 + 768*a*b^6*c^10*d^9*e^5 - 4368*a*b^7*c^9*d^8*e^6 + 3568*a*b^8*c^8*d^7*e^7 - 520*a*b^9*c^7*d^6*e^8 - 1728*a*b^10*c^6*d^5*e^9 + 2528*a*b^11*c^5*d^4*e^10 - 1536*a*b^12*c^4*d^3*e^11 + 240*a*b^13*c^3*d^2*e^12 - 1152*a^2*b*c^14*d^12*e^2 - 1600*a^2*b^12*c^3*d*e^13 - 67968*a^3*b*c^13*d^10*e^4 + 15808*a^3*b^10*c^4*d*e^13 - 342272*a^4*b*c^12*d^8*e^6 - 76928*a^4*b^8*c^5*d*e^13 - 569088*a^5*b*c^11*d^6*e^8 + 179200*a^5*b^6*c^6*d*e^13 - 586368*a^6*b*c^10*d^4*e^10 - 113008*a^6*b^4*c^7*d*e^13 - 731008*a^7*b*c^9*d^2*e^12 - 244096*a^7*b^2*c^8*d*e^13) / (16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4
\end{aligned}$$

$$\begin{aligned}
& *e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 \\
& ^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5 \\
& *e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4* \\
& d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 \\
& + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 \\
& + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048 \\
& *a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - \\
& 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9 \\
& *c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4 \\
& *e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7 \\
& *e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 \\
& - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) * ((27*a*b^9*c^5*d^6 - \\
& b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5 \\
& *d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + \\
& 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3* \\
& e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9 \\
& *a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7 \\
& *c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6* \\
& (-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7 \\
& *c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + \\
& 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180* \\
& a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 \\
& - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3 \\
& *e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4 \\
& *c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 6 \\
& 0928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 39 \\
& *a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) \\
& + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e \\
& + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^6*(-(4*a*c - \\
& b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8 \\
& *c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4 \\
& *b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5 \\
& *b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296* \\
& a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) \\
& + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*b^4*c*d^2*e^4 \\
& *(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 8 \\
& 6*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c \\
& - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 120*a^2*b \\
& *c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9) \\
& ^{(1/2)} + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 \\
& + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13* \\
& d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280 \\
& *a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8 \\
& *c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5 \\
& *e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384 \\
& *a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3 \\
& *b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 13 \\
& 44*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 \\
& - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3 \\
& *d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7 \\
& *b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 2 \\
& 1504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4 \\
& *e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4 \\
& *c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 122 \\
& 88*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2 \\
& *e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 \\
& - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - \\
& 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 512 \\
& 0*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24
\end{aligned}$$

$$\begin{aligned}
& 576a^8b^3c^8d^7e - 960a^8b^9c^2d^7e^7 + 5120a^9b^7c^3d^7e^7 - 49 \\
& 152a^{10}b^8c^8d^5e^3 - 15360a^{10}b^5c^4d^7e^7 - 49152a^{11}b^3c^7d^3e^5 \\
& + 24576a^{11}b^3c^5d^7e^7))^{(1/2)} - (x*(22800a^6c^9e^{13} + 36a^2b^8 \\
& c^5e^{13} - 600a^3b^6c^6e^{13} + 4313a^4b^4c^7e^{13} - 15592a^5b^2c^8 \\
& e^{13} + 1296a^2c^{13}d^8e^5 + 9792a^3c^{12}d^6e^7 + 30304a^4c^{11}d^4 \\
& e^9 + 40512a^5c^{10}d^2e^{11} + 25b^4c^{11}d^8e^5 - 120b^5c^{10}d^7e^6 \\
& + 214b^6c^9d^6e^7 - 168b^7c^8d^5e^8 + 53b^8c^7d^4e^9 - 8b^9c^6 \\
& d^3e^{10} + 4b^{10}c^5d^2e^{11} + 6336a^2b^2c^{11}d^6e^7 + 3840a^2b^3 \\
& c^{10}d^5e^8 - 8506a^2b^4c^9d^4e^9 + 1112a^2b^5c^8d^3e^{10} + 125 \\
& 4a^2b^6c^7d^2e^{11} + 22224a^3b^2c^{10}d^4e^9 + 13824a^3b^3c^9d^3 \\
& e^{10} - 9516a^3b^4c^8d^2e^{11} + 11712a^4b^2c^9d^2e^{11} - 24a^5b^9c^5 \\
& d^7e^{12} - 41088a^5b^3c^9d^7e^{12} - 360a^6b^2c^{12}d^8e^5 + 1664a^6b^3c^{11} \\
& d^7e^6 - 2604a^6b^4c^{10}d^6e^7 + 1272a^6b^5c^9d^5e^8 + 332a^6b^6c^8 \\
& d^4e^9 - 232a^6b^7c^7d^3e^{10} - 48a^6b^8c^6d^2e^{11} - 5760a^2b^3c^{12} \\
& d^7e^6 + 416a^2b^7c^6d^7e^{12} - 32128a^3b^3c^{11}d^5e^8 - 4120a^3b^5 \\
& c^7d^7e^{12} - 63360a^4b^3c^{10}d^3e^{10} + 21376a^4b^3c^8d^7e^{12}))/((8(a^6 \\
& b^8e^8 + 256a^6c^8d^8 + 256a^{10}c^4e^8 - 16a^7b^6c^8e^8 - 4a^5 \\
& b^9d^7e^7 + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6d^8 - 25 \\
& 6a^5b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^{12}d^4 \\
& e^4 - 4a^3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 + 1024a^7c^7d^6e^2 + 153 \\
& 6a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^{10}c^2d^6e^2 - 92a^3b^8 \\
& c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4 \\
& b^7c^3d^5e^3 - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128 \\
& a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 - 192a^5b^7c^2d^3e^5 + \\
& 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c^4d^4e^4 \\
& - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4 \\
& e^4 + 2048a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4 \\
& d^2e^6 - 1024a^6b^3c^7d^7e + 64a^6b^7c^3d^7e^7 - 1024a^9b^3c^4d^7 \\
& e^7 - 4a^2b^9c^3d^7e - 4a^2b^{11}c^3d^5e^3 + 64a^3b^7c^4d^7e - 4 \\
& a^3b^{10}c^3d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^3d^3e^5 + 1024a^5 \\
& b^3c^6d^7e - 92a^5b^8c^3d^2e^6 - 3072a^7b^3c^6d^5e^3 - 384a^7b^5 \\
& c^2d^7e^7 - 3072a^8b^3c^5d^3e^5 + 1024a^8b^3c^3d^7e^7))((27a^9 \\
& b^5c^5d^6 - b^{11}c^4d^6 - b^{15}d^2e^4 - 9a^2b^{13}e^6 + 3840a^5b^3c^9 \\
& d^6 + 9a^5c^5d^6*(-(4a^3c - b^2)^9)^{(1/2)} + 213a^3b^{11}c^6e^6 - 26880a^8 \\
& b^3c^6e^6 + 3072a^6c^9d^5e^5 + 35840a^8c^7d^7e^5 + 4b^{12}c^3d^5e^5 + \\
& 4b^{14}c^3d^3e^3 - 288a^2b^7c^6d^6 + 1504a^3b^5c^7d^6 - 3840a^4b^3 \\
& c^8d^6 - 9a^2b^4e^6*(-(4a^3c - b^2)^9)^{(1/2)} - 2077a^4b^9c^2e^6 + \\
& 10656a^5b^7c^3e^6 - 30240a^6b^5c^4e^6 + 44800a^7b^3c^5e^6 - 25 \\
& a^4c^2e^6*(-(4a^3c - b^2)^9)^{(1/2)} - b^2c^4d^6*(-(4a^3c - b^2)^9)^{(1/2)} \\
& ) + 22528a^7c^8d^3e^3 - b^6d^2e^4*(-(4a^3c - b^2)^9)^{(1/2)} - 6b^{13}c^2 \\
& d^4e^2 + 6a^5b^{14}d^5e^5 - 1471a^2b^9c^4d^4e^2 + 600a^2b^{10}c^3d^3 \\
& e^3 + 180a^2b^{11}c^2d^2e^4 + 6976a^3b^7c^5d^4e^2 - 1032a^3b^8 \\
& c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456a^4b^5c^6d^4e^2 - 7168a^4 \\
& b^6c^5d^3e^3 + 16896a^4b^7c^4d^2e^4 + 10240a^5b^3c^7d^4e^2 + 37632a^5 \\
& b^4c^6d^3e^3 - 47712a^5b^5c^5d^2e^4 - 59392a^6b^2c^7 \\
& d^3e^3 + 60928a^6b^3c^6d^2e^4 + 41a^2c^4d^4e^2*(-(4a^3c - b^2)^9)^{(1/2)} \\
& + 39a^3c^3d^2e^4*(-(4a^3c - b^2)^9)^{(1/2)} - 6b^4c^2d^4e^2*(-(4a^3c - b^2)^9)^{(1/2)} \\
& + 6a^5b^5d^5e^5*(-(4a^3c - b^2)^9)^{(1/2)} - 106a^6b^{10}c^4d^5e^5 + 7a^5b^{13} \\
& c^3d^2e^4 - 128a^2b^{12}c^3d^5e^5 + 51a^3b^2c^6e^6*(-(4a^3c - b^2)^9)^{(1/2)} \\
& + 150a^5b^{11}c^3d^4e^2 - 84a^5b^{12}c^2d^3e^3 + 1116a^2b^8c^5d^5e^5 - 5824a^3 \\
& b^6c^6d^5e^5 + 1030a^3b^{10}c^2d^5e^5 + 15232a^4b^4c^7d^5e^5 - 3492a^4b^8 \\
& c^3d^5e^5 - 16896a^5b^2c^8d^5e^5 + 1344a^5b^6c^4d^5e^5 + 7424a^6b^3c^8 \\
& d^4e^2 + 22400a^6b^4c^5d^5e^5 - 23296a^7b^3c^7d^2e^4 - 53760a^7b^2c^6 \\
& d^5e^5 + 4b^3c^3d^5e^5*(-(4a^3c - b^2)^9)^{(1/2)} + 4b^5c^3d^3e^3*(-(4a^3c - b^2)^9)^{(1/2)} \\
& - 11a^5b^4c^3d^2e^4*(-(4a^3c - b^2)^9)^{(1/2)} - 20a^2b^3c^3d^5e^5*(-(4a^3c - b^2)^9)^{(1/2)} \\
& - 86a^3b^3c^2d^5e^5*(-(4a^3c - b^2)^9)^{(1/2)} + 42a^5b^2c^3d^4e^2*(-(4a^3c - b^2)^9)^{(1/2)} \\
& - 12a^5b^3c^2d^3e^3*(-(4a^3c - b^2)^9)^{(1/2)} - 120a^2b^3c^3d^3e^3*(-(4a^3c - b^2)^9)^{(1/2)} - 34a^5b^3c^4 \\
& d^5e^5*(-(4a^3c - b^2)^9)^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& a^2c - b^2)^9)^{(1/2)} + 108a^2b^2c^2d^2e^4(-4a^2c - b^2)^9)^{(1/2)})/(32 \\
& *(a^7b^{12}e^8 + 4096a^9c^{10}d^8 + 4096a^{13}c^6e^8 - 24a^8b^{10}c^2e^8 \\
& - 4a^6b^{13}d^2e^7 + a^3b^{12}c^4d^8 - 24a^4b^{10}c^5d^8 + 240a^5b^8c^6d^8 \\
& - 1280a^6b^6c^7d^8 + 3840a^7b^4c^8d^8 - 6144a^8b^2c^9d^8 \\
& + 240a^9b^8c^2e^8 - 1280a^{10}b^6c^3e^8 + 3840a^{11}b^4c^4e^8 - 61 \\
& 44a^{12}b^2c^5e^8 + a^3b^{16}d^4e^4 - 4a^4b^{15}d^3e^5 + 6a^5b^{14}d^2 \\
& e^6 + 16384a^{10}c^9d^6e^2 + 24576a^{11}c^8d^4e^4 + 16384a^{12}c^7d^2 \\
& e^6 + 6a^3b^{14}c^2d^6e^2 - 140a^4b^{12}c^3d^6e^2 + 84a^4b^{13}c^2 \\
& *d^5e^3 + 1344a^5b^{10}c^4d^6e^2 - 672a^5b^{11}c^3d^5e^3 - 42a^5b^{12} \\
& c^2d^4e^4 - 6720a^6b^8c^5d^6e^2 + 2240a^6b^9c^4d^5e^3 + 1456 \\
& *a^6b^{10}c^3d^4e^4 - 672a^6b^{11}c^2d^3e^5 + 17920a^7b^6c^6d^6e^2 \\
& - 10080a^7b^8c^4d^4e^4 + 2240a^7b^9c^3d^3e^5 + 1344a^7b^{10}c^2 \\
& d^2e^6 - 21504a^8b^4c^7d^6e^2 - 21504a^8b^5c^6d^5e^3 + 32256a^8 \\
& b^6c^5d^4e^4 - 6720a^8b^8c^3d^2e^6 + 57344a^9b^3c^7d^5e^3 - \\
& 46592a^9b^4c^6d^4e^4 - 21504a^9b^5c^5d^3e^5 + 17920a^9b^6c^4 \\
& d^2e^6 + 12288a^{10}b^2c^7d^4e^4 + 57344a^{10}b^3c^6d^3e^5 - 21504a \\
& ^{10}b^4c^5d^2e^6 + 96a^7b^{11}c^2d^7e^7 - 16384a^9b^2c^9d^7e^7 - 16384a \\
& ^{12}b^2c^6d^7e^7 - 4a^3b^{13}c^3d^7e^7 - 4a^3b^{15}c^2d^5e^3 + 96a^4b^{11} \\
& c^4d^7e^7 - 12a^4b^{14}c^3d^4e^4 - 960a^5b^9c^5d^7e^7 + 84a^5b^{13}c^2 \\
& d^3e^5 + 5120a^6b^7c^6d^7e^7 - 140a^6b^{12}c^2d^2e^6 - 15360a^7b^5c^7 \\
& d^7e^7 + 24576a^8b^3c^8d^7e^7 - 960a^8b^9c^2d^2e^7 + 5120a^9b^7c^3 \\
& d^2e^7 - 49152a^{10}b^2c^8d^5e^3 - 15360a^{10}b^5c^4d^2e^7 - 49152a^{11} \\
& b^2c^7d^3e^5 + 24576a^{11}b^3c^5d^2e^7))^{(1/2)} + ((((((1048576a^{13}c^8 \\
& e^{16} + 256a^7b^{12}c^2e^{16} - 6144a^8b^{10}c^3e^{16} + 61440a^9b^8c^4e \\
& ^{16} - 327680a^{10}b^6c^5e^{16} + 983040a^{11}b^4c^6e^{16} - 1572864a^{12}b^2 \\
& c^7e^{16} - 196608a^6c^{15}d^{14}e^2 - 917504a^7c^{14}d^{12}e^4 - 589824a^8 \\
& c^{13}d^{10}e^6 + 3932160a^9c^{12}d^8e^8 + 10158080a^{10}c^{11}d^6e^{10} + \\
& 10616832a^{11}c^{10}d^4e^{12} + 5308416a^{12}c^9d^2e^{14} - 2816a^2b^8c^1 \\
& 1d^{14}e^2 + 22656a^2b^9c^{10}d^{13}e^3 - 78848a^2b^{10}c^9d^{12}e^4 + 15 \\
& 4112a^2b^{11}c^8d^{11}e^5 - 182784a^2b^{12}c^7d^{10}e^6 + 130816a^2b^{13} \\
& c^6d^9e^7 - 50176a^2b^{14}c^5d^8e^8 + 4608a^2b^{15}c^4d^7e^9 + 332 \\
& 8a^2b^{16}c^3d^6e^{10} - 896a^2b^{17}c^2d^5e^{11} + 24576a^3b^6c^{12}d^ \\
& 14e^2 - 198656a^3b^7c^{11}d^{13}e^3 + 684544a^3b^8c^{10}d^{12}e^4 - 1291 \\
& 520a^3b^9c^9d^{11}e^5 + 1403776a^3b^{10}c^8d^{10}e^6 - 798336a^3b^{11} \\
& c^7d^9e^7 + 89856a^3b^{12}c^6d^8e^8 + 155136a^3b^{13}c^5d^7e^9 - 77 \\
& 440a^3b^{14}c^4d^6e^{10} + 5504a^3b^{15}c^3d^5e^{11} + 2560a^3b^{16}c^2 \\
& d^4e^{12} - 106496a^4b^4c^{13}d^{14}e^2 + 864256a^4b^5c^{12}d^{13}e^3 - 29 \\
& 24544a^4b^6c^{11}d^{12}e^4 + 5181440a^4b^7c^{10}d^{11}e^5 - 4686080a^4b^8 \\
& c^9d^{10}e^6 + 1045376a^4b^9c^8d^9e^7 + 1900544a^4b^{10}c^7d^8e^8 \\
& - 1732096a^4b^{11}c^6d^7e^9 + 390400a^4b^{12}c^5d^6e^{10} + 112000a^4 \\
& b^{13}c^4d^5e^{11} - 40960a^4b^{14}c^3d^4e^{12} - 3840a^4b^{15}c^2d^3e^{13} \\
& + 229376a^5b^2c^{14}d^{14}e^2 - 1867776a^5b^3c^{13}d^{13}e^3 + 607846 \\
& 4a^5b^4c^{12}d^{12}e^4 - 9297920a^5b^5c^{11}d^{11}e^5 + 4055040a^5b^6c^{10} \\
& d^{10}e^6 + 7788544a^5b^7c^9d^9e^7 - 12657664a^5b^8c^8d^8e^8 + \\
& 6130176a^5b^9c^7d^7e^9 + 734080a^5b^{10}c^6d^6e^{10} - 1442560a^5b^{11} \\
& c^5d^5e^{11} + 168960a^5b^{12}c^4d^4e^{12} + 78080a^5b^{13}c^3d^3e^{13} \\
& + 3200a^5b^{14}c^2d^2e^{14} - 4587520a^6b^2c^{13}d^{12}e^4 + 3080192a^6 \\
& b^3c^{12}d^{11}e^5 + 12001280a^6b^4c^{11}d^{10}e^6 - 31076352a^6b^5c^{10} \\
& d^9e^7 + 27475968a^6b^6c^9d^8e^8 - 2088960a^6b^7c^8d^7e^9 - 1 \\
& 2205312a^6b^8c^7d^6e^{10} + 6043520a^6b^9c^6d^5e^{11} + 631808a^6b^{10} \\
& c^5d^4e^{12} - 610304a^6b^{11}c^4d^3e^{13} - 71936a^6b^{12}c^3d^2e^{14} \\
& - 21725184a^7b^2c^{12}d^{10}e^6 + 30801920a^7b^3c^{11}d^9e^7 - 802816 \\
& 0a^7b^4c^{10}d^8e^8 - 32260096a^7b^5c^9d^7e^9 + 37101568a^7b^6c^8 \\
& d^6e^{10} - 7182336a^7b^7c^7d^5e^{11} - 7609856a^7b^8c^6d^4e^{12} + \\
& 2112256a^7b^9c^5d^3e^{13} + 661632a^7b^{10}c^4d^2e^{14} - 30146560a^8 \\
& b^2c^{11}d^8e^8 + 55050240a^8b^3c^{10}d^7e^9 - 34365440a^8b^4c^9d^6 \\
& e^{10} - 16429056a^8b^5c^8d^5e^{11} + 24600576a^8b^6c^7d^4e^{12} - 168 \\
& 3456a^8b^7c^6d^3e^{13} - 3151616a^8b^8c^5d^2e^{14} - 10977280a^9b^2 \\
& c^{10}d^6e^{10} + 47022080a^9b^3c^9d^5e^{11} - 30621696a^9b^4c^8d^4e
\end{aligned}$$

$$\begin{aligned}
& ^{12} - 9232384a^9b^5c^7d^3e^{13} + 7970816a^9b^6c^6d^2e^{14} + 4325376 \\
& a^{10}b^2c^9d^4e^{12} + 25493504a^{10}b^3c^8d^3e^{13} - 9117696a^{10}b^4c^7d^2e^{14} + 491520a^{11}b^2c^8d^2e^{14} - 4947968a^{12}b^3c^8d^2e^{15} + 1 \\
& 28a^*b^{10}c^{10}d^{14}e^2 - 1024a^*b^{11}c^9d^{13}e^3 + 3584a^*b^{12}c^8d^{12}e^4 - 7168a^*b^{13}c^7d^{11}e^5 + 8960a^*b^{14}c^6d^{10}e^6 - 7168a^*b^{15}c^5d^9e^7 + 3584a^*b^{16}c^4d^8e^8 - 1024a^*b^{17}c^3d^7e^9 + 128a^*b^{18}c^2d^6e^{10} + 1605632a^6b^6c^{14}d^{13}e^3 - 1408a^6b^{13}c^2d^2e^{15} + 70123 \\
& 52a^7b^6c^{13}d^{11}e^5 + 33152a^7b^{11}c^3d^2e^{15} + 7045120a^8b^6c^{12}d^9e^7 - 324480a^8b^9c^4d^2e^{15} - 9830400a^9b^6c^{11}d^7e^9 + 1689600a^9 \\
& b^7c^5d^2e^{15} - 25722880a^{10}b^6c^{10}d^5e^{11} - 4935680a^{10}b^5c^6d^2e^{15} - 19202048a^{11}b^6c^9d^3e^{13} + 7667712a^{11}b^3c^7d^2e^{15})/(16*(a^6b^8e^8 + 256a^6c^8d^8 + 256a^{10}c^4e^8 - 16a^7b^6c^2e^8 - 4a^5b^9d^2e^7 + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6d^8 - 256a^5 \\
& b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^{12}d^4e^4 - 4a^3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 + 1024a^7c^7d^6e^2 + 1536a^8 \\
& c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^{10}c^2d^6e^2 - 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4b^7 \\
& c^3d^5e^3 - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 - 192a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c^4d^4e^4 - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 + 2048a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024a^6b^6c^7d^7e + 64a^6b^7c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024a^9b^6c^4d^2e^7 - 4a^2b^9c^3d^7e - 4a^2b^{11}c^3d^5e^3 + 64a^3b^7c^4d^7e - 4a^3b^{10}c^2d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^3d^3e^5 + 1024a^5b^3c^6d^7e - 92a^5b^8c^2d^2e^6 - 3072a^7b^6c^6d^5e^3 - 384a^7b^5c^2d^2e^7 - 3072a^8b^6c^5d^3e^5 + 1024a^8b^3c^3d^2e^7)) + (x*((27a^*b^9c^5d^6 - b^{11}c^4d^6 - b^{15}d^2e^4 - 9a^2b^{13}e^6 + 3840a^5b^6c^9d^6 + 9a^*c^5d^6*(-(4a^*c - b^2)^9)^{(1/2)} + 213a^3b^{11}c^2e^6 - 26880a^8b^6c^6e^6 + 3072a^6c^9d^5e + 35840a^8c^7d^2e^5 + 4b^{12}c^3d^5e + 4 \\
& b^{14}c^3d^3e^3 - 288a^2b^7c^6d^6 + 1504a^3b^5c^7d^6 - 3840a^4b^3c^8d^6 - 9a^2b^4e^6*(-(4a^*c - b^2)^9)^{(1/2)} - 2077a^4b^9c^2e^6 + 10656a^5b^7c^3e^6 - 30240a^6b^5c^4e^6 + 44800a^7b^3c^5e^6 - 25a^4c^2e^6*(-(4a^*c - b^2)^9)^{(1/2)} - b^2c^4d^6*(-(4a^*c - b^2)^9)^{(1/2)} + 22528a^7c^8d^3e^3 - b^6d^2e^4*(-(4a^*c - b^2)^9)^{(1/2)} - 6b^{13}c^2d^4e^2 + 6a^*b^{14}d^5e^5 - 1471a^2b^9c^4d^4e^2 + 600a^2b^{10}c^3d^3e^3 + 180a^2b^{11}c^2d^2e^4 + 6976a^3b^7c^5d^4e^2 - 1032a^3b^8c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456a^4b^5c^6d^4e^2 - 7168a^4b^6c^5d^3e^3 + 16896a^4b^7c^4d^2e^4 + 10240a^5b^3c^7d^4e^2 + 37632a^5b^4c^6d^3e^3 - 47712a^5b^5c^5d^2e^4 - 59392a^6b^2c^7d^3e^3 + 60928a^6b^3c^6d^2e^4 + 41a^2c^4d^4e^2*(-(4a^*c - b^2)^9)^{(1/2)} + 39a^3c^3d^2e^4*(-(4a^*c - b^2)^9)^{(1/2)} - 6b^4c^2d^4e^2*(-(4a^*c - b^2)^9)^{(1/2)} + 6a^*b^5d^2e^5*(-(4a^*c - b^2)^9)^{(1/2)} - 106a^*b^{10}c^4d^5e + 7a^*b^{13}c^3d^2e^4 - 128a^2b^{12}c^3d^2e^5 + 51a^3b^2c^2e^6*(-(4a^*c - b^2)^9)^{(1/2)} + 150a^*b^{11}c^3d^4e^2 - 84a^*b^{12}c^2d^3e^3 + 1116a^2b^8c^5d^5e - 5824a^3b^6c^6d^5e + 1030a^3b^{10}c^2d^2e^5 + 15232a^4b^4c^7d^5e - 3492a^4b^8c^3d^2e^5 - 16896a^5b^2c^8d^5e + 1344a^5b^6c^4d^2e^5 + 7424a^6b^6c^8d^4e^2 + 22400a^6b^4c^5d^2e^5 - 23296a^7b^6c^7d^2e^4 - 53760a^7b^2c^6d^2e^5 + 4b^3c^3d^5e*(-(4a^*c - b^2)^9)^{(1/2)} + 4b^5c^3d^3e^3*(-(4a^*c - b^2)^9)^{(1/2)} - 11a^*b^4c^3d^2e^4*(-(4a^*c - b^2)^9)^{(1/2)} - 20a^2b^3c^3d^2e^5*(-(4a^*c - b^2)^9)^{(1/2)} - 86a^3b^6c^2d^2e^5*(-(4a^*c - b^2)^9)^{(1/2)} + 42a^*b^2c^3d^4e^2*(-(4a^*c - b^2)^9)^{(1/2)} - 12a^*b^3c^2d^3e^3*(-(4a^*c - b^2)^9)^{(1/2)} - 120a^2b^6c^3d^3e^3*(-(4a^*c - b^2)^9)^{(1/2)} - 34a^*b^6c^4d^5e*(-(4a^*c - b^2)^9)^{(1/2)} + 108a^2b^2c^2d^2e^4*(-(4a^*c - b^2)^9)^{(1/2)))/(32*(a^7b^{12}e^8 + 4096a^9c^{10}d^8 + 4096a^{13}c^6e^8 - 24a^8b^{10}c^2e^8 - 4a^6b^{13}d^2e^7 + a^3b^{12}c^4d^8 - 24a^4b^{10}c^5d^8 + 240a^5b^8c^6d^8 - 1280a^6b^6c^7d^8 + 3840a^7b^4c^8d^8 - 6144a^8b^2c^9d^8 + 240a^9b^8c^2e^8 - 1280a^{10}b^6c^3e^8 + 3840a^{11}b^4c^4e^8 - 614
\end{aligned}$$

$$\begin{aligned}
& 4*a^{12}*b^2*c^5*e^8 + a^3*b^{16}*d^4*e^4 - 4*a^4*b^{15}*d^3*e^5 + 6*a^5*b^{14}*d^2 \\
& *e^6 + 16384*a^{10}*c^9*d^6*e^2 + 24576*a^{11}*c^8*d^4*e^4 + 16384*a^{12}*c^7*d^2 \\
& *e^6 + 6*a^3*b^{14}*c^2*d^6*e^2 - 140*a^4*b^{12}*c^3*d^6*e^2 + 84*a^4*b^{13}*c^2* \\
& d^5*e^3 + 1344*a^5*b^{10}*c^4*d^6*e^2 - 672*a^5*b^{11}*c^3*d^5*e^3 - 42*a^5*b^1 \\
& 2*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456* \\
& a^6*b^{10}*c^3*d^4*e^4 - 672*a^6*b^{11}*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 \\
& - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^{10}*c^2 \\
& *d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^ \\
& 8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - \\
& 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d \\
& ^2*e^6 + 12288*a^{10}*b^2*c^7*d^4*e^4 + 57344*a^{10}*b^3*c^6*d^3*e^5 - 21504*a^ \\
& 10*b^4*c^5*d^2*e^6 + 96*a^7*b^{11}*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^ \\
& 12*b*c^6*d*e^7 - 4*a^3*b^{13}*c^3*d^7*e - 4*a^3*b^{15}*c*d^5*e^3 + 96*a^4*b^{11}* \\
& c^4*d^7*e - 12*a^4*b^{14}*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^{13}*c*d \\
& ^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^{12}*c*d^2*e^6 - 15360*a^7*b^5*c^ \\
& 7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^ \\
& 3*d*e^7 - 49152*a^{10}*b*c^8*d^5*e^3 - 15360*a^{10}*b^5*c^4*d*e^7 - 49152*a^{11}* \\
& b*c^7*d^3*e^5 + 24576*a^{11}*b^3*c^5*d*e^7))^{(1/2)}*(1048576*a^{15}*c^8*e^{17} + \\
& 256*a^9*b^{12}*c^2*e^{17} - 6144*a^{10}*b^{10}*c^3*e^{17} + 61440*a^{11}*b^8*c^4*e^{17} - \\
& 327680*a^{12}*b^6*c^5*e^{17} + 983040*a^{13}*b^4*c^6*e^{17} - 1572864*a^{14}*b^2*c^7 \\
& *e^{17} - 1048576*a^8*c^{15}*d^{14}*e^3 - 5242880*a^9*c^{14}*d^{12}*e^5 - 9437184*a^1 \\
& 0*c^{13}*d^{10}*e^7 - 5242880*a^{11}*c^{12}*d^8*e^9 + 5242880*a^{12}*c^{11}*d^6*e^{11} + \\
& 9437184*a^{13}*c^{10}*d^4*e^{13} + 5242880*a^{14}*c^9*d^2*e^{15} + 256*a^2*b^{11}*c^{10} \\
& d^{15}*e^2 - 2048*a^2*b^{12}*c^9*d^{14}*e^3 + 7168*a^2*b^{13}*c^8*d^{13}*e^4 - 14336* \\
& a^2*b^{14}*c^7*d^{12}*e^5 + 17920*a^2*b^{15}*c^6*d^{11}*e^6 - 14336*a^2*b^{16}*c^5*d^ \\
& 10*e^7 + 7168*a^2*b^{17}*c^4*d^9*e^8 - 2048*a^2*b^{18}*c^3*d^8*e^9 + 256*a^2*b^ \\
& 19*c^2*d^7*e^{10} - 5120*a^3*b^9*c^{11}*d^{15}*e^2 + 41984*a^3*b^{10}*c^{10}*d^{14}*e^3 \\
& - 148736*a^3*b^{11}*c^9*d^{13}*e^4 + 296192*a^3*b^{12}*c^8*d^{12}*e^5 - 359680*a^3 \\
& *b^{13}*c^7*d^{11}*e^6 + 267520*a^3*b^{14}*c^6*d^{10}*e^7 - 112384*a^3*b^{15}*c^5*d^9 \\
& *e^8 + 18176*a^3*b^{16}*c^4*d^8*e^9 + 3328*a^3*b^{17}*c^3*d^7*e^{10} - 1280*a^3*b \\
& ^{18}*c^2*d^6*e^{11} + 40960*a^4*b^7*c^{12}*d^{15}*e^2 - 348160*a^4*b^8*c^{11}*d^{14}*e \\
& ^3 + 1254400*a^4*b^9*c^{10}*d^{13}*e^4 - 2478080*a^4*b^{10}*c^9*d^{12}*e^5 + 286745 \\
& 6*a^4*b^{11}*c^8*d^{11}*e^6 - 1862144*a^4*b^{12}*c^7*d^{10}*e^7 + 490240*a^4*b^{13}*c \\
& ^6*d^9*e^8 + 128000*a^4*b^{14}*c^5*d^8*e^9 - 108800*a^4*b^{15}*c^4*d^7*e^{10} + 1 \\
& 3824*a^4*b^{16}*c^3*d^6*e^{11} + 2304*a^4*b^{17}*c^2*d^5*e^{12} - 163840*a^5*b^5*c^ \\
& 13*d^{15}*e^2 + 1474560*a^5*b^6*c^{12}*d^{14}*e^3 - 5447680*a^5*b^7*c^{11}*d^{13}*e^4 \\
& + 10588160*a^5*b^8*c^{10}*d^{12}*e^5 - 11166720*a^5*b^9*c^9*d^{11}*e^6 + 5159936 \\
& *a^5*b^{10}*c^8*d^{10}*e^7 + 1073920*a^5*b^{11}*c^7*d^9*e^8 - 2279680*a^5*b^{12}*c^ \\
& 6*d^8*e^9 + 770560*a^5*b^{13}*c^5*d^7*e^{10} + 33280*a^5*b^{14}*c^4*d^6*e^{11} - 41 \\
& 216*a^5*b^{15}*c^3*d^5*e^{12} - 1280*a^5*b^{16}*c^2*d^4*e^{13} + 327680*a^6*b^3*c^1 \\
& 4*d^{15}*e^2 - 3276800*a^6*b^4*c^{13}*d^{14}*e^3 + 12615680*a^6*b^5*c^{12}*d^{13}*e^4 \\
& - 23592960*a^6*b^6*c^{11}*d^{12}*e^5 + 19701760*a^6*b^7*c^{10}*d^{11}*e^6 + 137216 \\
& 0*a^6*b^8*c^9*d^{10}*e^7 - 15846400*a^6*b^9*c^8*d^9*e^8 + 10864640*a^6*b^{10}*c \\
& ^7*d^8*e^9 - 1352960*a^6*b^{11}*c^6*d^7*e^{10} - 1111040*a^6*b^{12}*c^5*d^6*e^{11} \\
& + 273920*a^6*b^{13}*c^4*d^5*e^{12} + 25600*a^6*b^{14}*c^3*d^4*e^{13} - 1280*a^6*b^1 \\
& 5*c^2*d^3*e^{14} + 3407872*a^7*b^2*c^{14}*d^{14}*e^3 - 14221312*a^7*b^3*c^{13}*d^{13} \\
& *e^4 + 23527424*a^7*b^4*c^{12}*d^{12}*e^5 - 3768320*a^7*b^5*c^{11}*d^{11}*e^6 - 388 \\
& 95616*a^7*b^6*c^{10}*d^{10}*e^7 + 50126848*a^7*b^7*c^9*d^9*e^8 - 18362368*a^7*b \\
& ^8*c^8*d^8*e^9 - 6831104*a^7*b^9*c^7*d^7*e^{10} + 6200320*a^7*b^{10}*c^6*d^6*e^ \\
& 11 - 726784*a^7*b^{11}*c^5*d^5*e^{12} - 228608*a^7*b^{12}*c^4*d^4*e^{13} + 31488*a^ \\
& 7*b^{13}*c^3*d^3*e^{14} + 2304*a^7*b^{14}*c^2*d^2*e^{15} - 3145728*a^8*b^2*c^{13}*d^1 \\
& 2*e^5 - 31129600*a^8*b^3*c^{12}*d^{11}*e^6 + 74711040*a^8*b^4*c^{11}*d^{10}*e^7 - 5 \\
& 5476224*a^8*b^5*c^{10}*d^9*e^8 - 11075584*a^8*b^6*c^9*d^8*e^9 + 35381248*a^8* \\
& b^7*c^8*d^7*e^{10} - 14479360*a^8*b^8*c^7*d^6*e^{11} - 168960*a^8*b^9*c^6*d^5*e \\
& ^{12} + 1286144*a^8*b^{10}*c^5*d^4*e^{13} - 302336*a^8*b^{11}*c^4*d^3*e^{14} - 55808* \\
& a^8*b^{12}*c^3*d^2*e^{15} - 36962304*a^9*b^2*c^{12}*d^{10}*e^7 - 9502720*a^9*b^3*c^ \\
& 11*d^9*e^8 + 67174400*a^9*b^4*c^{10}*d^8*e^9 - 54886400*a^9*b^5*c^9*d^7*e^{10} \\
& + 11239424*a^9*b^6*c^8*d^6*e^{11} + 5545984*a^9*b^7*c^7*d^5*e^{12} - 5263360*a^ \\
& 9*b^8*c^6*d^4*e^{13} + 1356800*a^9*b^9*c^5*d^3*e^{14} + 558080*a^9*b^{10}*c^4*d^2
\end{aligned}$$



$$\begin{aligned}
& *e^{15} - 49807360*a^{10}*b^2*c^{11}*d^8*e^9 + 19333120*a^{10}*b^3*c^{10}*d^7*e^{10} + \\
& 7208960*a^{10}*b^4*c^9*d^6*e^{11} - 14974976*a^{10}*b^5*c^8*d^5*e^{12} + 15073280*a \\
& ^{10}*b^6*c^7*d^4*e^{13} - 2170880*a^{10}*b^7*c^6*d^3*e^{14} - 2928640*a^{10}*b^8*c^5 \\
& *d^2*e^{15} - 11796480*a^{11}*b^2*c^{10}*d^6*e^{11} + 23920640*a^{11}*b^3*c^9*d^5*e^{12} \\
& - 24576000*a^{11}*b^4*c^8*d^4*e^{13} - 4096000*a^{11}*b^5*c^7*d^3*e^{14} + 835584 \\
& 0*a^{11}*b^6*c^6*d^2*e^{15} + 12582912*a^{12}*b^2*c^9*d^4*e^{13} + 19857408*a^{12}*b^ \\
& 3*c^8*d^3*e^{14} - 11534336*a^{12}*b^4*c^7*d^2*e^{15} + 3407872*a^{13}*b^2*c^8*d^2* \\
& e^{15} - 5505024*a^{14}*b*c^8*d*e^{16} - 262144*a^7*b*c^{15}*d^{15}*e^2 + 5505024*a^8 \\
& *b*c^{14}*d^{13}*e^4 - 1280*a^8*b^{13}*c^2*d*e^{16} + 25952256*a^9*b*c^{13}*d^{11}*e^6 \\
& + 30976*a^9*b^{11}*c^3*d*e^{16} + 38010880*a^{10}*b*c^{12}*d^9*e^8 - 312320*a^{10}*b^ \\
& 9*c^4*d*e^{16} + 11796480*a^{11}*b*c^{11}*d^7*e^{10} + 1679360*a^{11}*b^7*c^5*d*e^{16} \\
& - 21233664*a^{12}*b*c^{10}*d^5*e^{12} - 5079040*a^{12}*b^5*c^6*d*e^{16} - 20709376*a^ \\
& 13*b*c^9*d^3*e^{14} + 8192000*a^{13}*b^3*c^7*d*e^{16}))/((8*(a^6*b^8*e^8 + 256*a^6 \\
& *c^8*d^8 + 256*a^{10}*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c \\
& ^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 9 \\
& 6*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^{12}*d^4*e^4 - 4*a^3*b^{11}*d^3 \\
& *e^5 + 6*a^4*b^{10}*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1 \\
& 024*a^9*c^5*d^2*e^6 + 6*a^2*b^{10}*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52* \\
& a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 9 \\
& 0*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 \\
& + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e \\
& ^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3* \\
& d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3 \\
& *c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^ \\
& 6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d \\
& ^7*e - 4*a^2*b^{11}*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^{10}*c*d^4*e^4 - \\
& 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92 \\
& *a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072* \\
& a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^{11}*c^4 \\
& *d^6 - b^{15}*d^2*e^4 - 9*a^2*b^{13}*e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-( \\
& 4*a*c - b^2)^9)^{(1/2)} + 213*a^3*b^{11}*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6 \\
& *c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^{12}*c^3*d^5*e + 4*b^{14}*c*d^3*e^3 - 28 \\
& 8*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4 \\
& *e^6*(-(4*a*c - b^2)^9)^{(1/2)} - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^ \\
& 6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c \\
& - b^2)^9)^{(1/2)} - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^{(1/2)} + 22528*a^7*c^8*d^3 \\
& *e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^{(1/2)} - 6*b^{13}*c^2*d^4*e^2 + 6*a*b^{14} \\
& *d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^{10}*c^3*d^3*e^3 + 180*a^2*b^{11} \\
& *c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a \\
& ^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + \\
& 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6* \\
& d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6 \\
& *b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^{(1/2)} + 39*a^3*c^3 \\
& *d^2*e^4*(-(4*a*c - b^2)^9)^{(1/2)} - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^{(1 \\
& /2)} + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^{(1/2)} - 106*a*b^{10}*c^4*d^5*e + 7*a*b \\
& ^{13}*c*d^2*e^4 - 128*a^2*b^{12}*c*d*e^5 + 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^{( \\
& 1/2)} + 150*a*b^{11}*c^3*d^4*e^2 - 84*a*b^{12}*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d \\
& ^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^{10}*c^2*d*e^5 + 15232*a^4*b^4*c^7 \\
& *d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^ \\
& 4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^ \\
& 7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^{(1 \\
& /2)} + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^{(1/2)} - 11*a*b^4*c*d^2*e^4*(-(4*a* \\
& c - b^2)^9)^{(1/2)} - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^{(1/2)} - 86*a^3*b* \\
& c^2*d*e^5*(-(4*a*c - b^2)^9)^{(1/2)} + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9 \\
& )^{(1/2)} - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^{(1/2)} - 120*a^2*b*c^3*d^3 \\
& *e^3*(-(4*a*c - b^2)^9)^{(1/2)} - 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^{(1/2)} + \\
& 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(a^7*b^{12}*e^8 + 4096 \\
& *a^9*c^{10}*d^8 + 4096*a^{13}*c^6*e^8 - 24*a^8*b^{10}*c*e^8 - 4*a^6*b^{13}*d*e^7 + \\
& a^3*b^{12}*c^4*d^8 - 24*a^4*b^{10}*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6
\end{aligned}$$

$$\begin{aligned}
& *c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 \\
& - 1280*a^{10}*b^6*c^3*e^8 + 3840*a^{11}*b^4*c^4*e^8 - 6144*a^{12}*b^2*c^5*e^8 + \\
& a^3*b^{16}*d^4*e^4 - 4*a^4*b^{15}*d^3*e^5 + 6*a^5*b^{14}*d^2*e^6 + 16384*a^{10}*c^9*d^6*e^2 + 24576*a^{11}*c^8*d^4*e^4 + 16384*a^{12}*c^7*d^2*e^6 + 6*a^3*b^{14}*c^2*d^6*e^2 - 140*a^4*b^{12}*c^3*d^6*e^2 + 84*a^4*b^{13}*c^2*d^5*e^3 + 1344*a^5*b^{10}*c^4*d^6*e^2 - 672*a^5*b^{11}*c^3*d^5*e^3 - 42*a^5*b^{12}*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^{10}*c^3*d^4*e^4 - 672*a^6*b^{11}*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^{10}*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^{10}*b^2*c^7*d^4*e^4 + 57344*a^{10}*b^3*c^6*d^3*e^5 - 21504*a^{10}*b^4*c^5*d^2*e^6 + 96*a^7*b^{11}*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^{12}*b*c^6*d*e^7 - 4*a^3*b^{13}*c^3*d^7*e - 4*a^3*b^{15}*c*d^5*e^3 + 96*a^4*b^{11}*c^4*d^7*e - 12*a^4*b^{14}*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^{13}*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^{12}*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^{10}*b*c^8*d^5*e^3 - 15360*a^{10}*b^5*c^4*d*e^7 - 49152*a^{11}*b*c^7*d^3*e^5 + 24576*a^{11}*b^3*c^5*d*e^7))^{(1/2)} + (x*(626688*a^{10}*b*c^8*e^{15} - 784384*a^{10}*c^9*d*e^{14} + 208*a^4*b^{13}*c^2*e^{15} - 4880*a^5*b^{11}*c^3*e^{15} + 47312*a^6*b^9*c^4*e^{15} - 242176*a^7*b^7*c^5*e^{15} + 688640*a^8*b^5*c^6*e^{15} - 1028096*a^9*b^3*c^7*e^{15} + 18432*a^4*c^{15}*d^{13}*e^2 + 126976*a^5*c^{14}*d^{11}*e^4 + 325632*a^6*c^{13}*d^9*e^6 + 139264*a^7*c^{12}*d^7*e^8 - 1067008*a^8*c^{11}*d^5*e^{10} - 1773568*a^9*c^{10}*d^3*e^{12} + 16*b^8*c^{11}*d^{13}*e^2 - 96*b^9*c^{10}*d^{12}*e^3 + 240*b^{10}*c^9*d^{11}*e^4 - 304*b^{11}*c^8*d^{10}*e^5 + 144*b^{12}*c^7*d^9*e^6 + 144*b^{13}*c^6*d^8*e^7 - 304*b^{14}*c^5*d^7*e^8 + 240*b^{15}*c^4*d^6*e^9 - 96*b^{16}*c^3*d^5*e^{10} + 16*b^{17}*c^2*d^4*e^{11} + 3200*a^2*b^4*c^{13}*d^{13}*e^2 - 18432*a^2*b^5*c^{12}*d^{12}*e^3 + 41024*a^2*b^6*c^{11}*d^{11}*e^4 - 36352*a^2*b^7*c^{10}*d^{10}*e^5 - 16208*a^2*b^8*c^9*d^9*e^6 + 74576*a^2*b^9*c^8*d^8*e^7 - 78496*a^2*b^{10}*c^7*d^7*e^8 + 32064*a^2*b^{11}*c^6*d^6*e^9 + 6000*a^2*b^{12}*c^5*d^5*e^{10} - 9264*a^2*b^{13}*c^4*d^4*e^{11} + 1472*a^2*b^{14}*c^3*d^3*e^{12} + 416*a^2*b^{15}*c^2*d^2*e^{13} - 12800*a^3*b^2*c^{14}*d^{13}*e^2 + 73728*a^3*b^3*c^{13}*d^{12}*e^3 - 151296*a^3*b^4*c^{12}*d^{11}*e^4 + 78336*a^3*b^5*c^{11}*d^{10}*e^5 + 206688*a^3*b^6*c^{10}*d^9*e^6 - 436736*a^3*b^7*c^9*d^8*e^7 + 324224*a^3*b^8*c^8*d^7*e^8 + 992*a^3*b^9*c^7*d^6*e^9 - 158176*a^3*b^{10}*c^6*d^5*e^{10} + 77056*a^3*b^{11}*c^5*d^4*e^{11} + 6912*a^3*b^{12}*c^4*d^3*e^{12} - 8416*a^3*b^{13}*c^3*d^2*e^{13} + 162816*a^4*b^2*c^{13}*d^{11}*e^4 + 184320*a^4*b^3*c^{12}*d^{10}*e^5 - 916608*a^4*b^4*c^{11}*d^9*e^6 + 1165824*a^4*b^5*c^{10}*d^8*e^7 - 314496*a^4*b^6*c^9*d^7*e^8 - 822272*a^4*b^7*c^8*d^6*e^9 + 919152*a^4*b^8*c^7*d^5*e^{10} - 175296*a^4*b^9*c^6*d^4*e^{11} - 189328*a^4*b^{10}*c^5*d^3*e^{12} + 62064*a^4*b^{11}*c^4*d^2*e^{13} + 1290752*a^5*b^2*c^{12}*d^9*e^6 - 659456*a^5*b^3*c^{11}*d^8*e^7 - 1561088*a^5*b^4*c^{10}*d^7*e^8 + 3240960*a^5*b^5*c^9*d^6*e^9 - 1964192*a^5*b^6*c^8*d^5*e^{10} - 683008*a^5*b^7*c^7*d^4*e^{11} + 1162304*a^5*b^8*c^6*d^3*e^{12} - 164112*a^5*b^9*c^5*d^2*e^{13} + 3442688*a^6*b^2*c^{11}*d^7*e^8 - 3670016*a^6*b^3*c^{10}*d^6*e^9 + 15232*a^6*b^4*c^9*d^5*e^{10} + 4230144*a^6*b^5*c^8*d^4*e^{11} - 3059648*a^6*b^6*c^7*d^3*e^{12} - 247296*a^6*b^7*c^6*d^2*e^{13} + 4010496*a^7*b^2*c^{10}*d^5*e^{10} - 6873088*a^7*b^3*c^9*d^4*e^{11} + 2822400*a^7*b^4*c^8*d^3*e^{12} + 2370048*a^7*b^5*c^7*d^2*e^{13} + 1178624*a^8*b^2*c^9*d^3*e^{12} - 4739072*a^8*b^3*c^8*d^2*e^{13} - 352*a*b^6*c^{12}*d^{13}*e^2 + 2048*a*b^7*c^{11}*d^{12}*e^3 - 4800*a*b^8*c^{10}*d^{11}*e^4 + 5168*a*b^9*c^9*d^{10}*e^5 - 480*a*b^{10}*c^8*d^9*e^6 - 6000*a*b^{11}*c^7*d^8*e^7 + 8192*a*b^{12}*c^6*d^7*e^8 - 5040*a*b^{13}*c^5*d^6*e^9 + 1152*a*b^{14}*c^4*d^5*e^{10} + 240*a*b^{15}*c^3*d^4*e^{11} - 128*a*b^{16}*c^2*d^3*e^{12} - 512*a^3*b^{14}*c^2*d*e^{14} - 106496*a^4*b*c^{14}*d^{12}*e^3 + 11680*a^4*b^{12}*c^3*d*e^{14} - 675840*a^5*b*c^{13}*d^{10}*e^5 - 108288*a^5*b^{10}*c^4*d*e^{14} - 1601536*a^6*b*c^{12}*d^8*e^7 + 514768*a^6*b^8*c^5*d*e^{14} - 925696*a^7*b*c^{11}*d^6*e^9 - 1278304*a^7*b^6*c^6*d*e^{14} + 2457600*a^8*b*c^{10}*d^4*e^{11} + 1385600*a^8*b^4*c^7*d*e^{14} + 2977792*a^9*b*c^9*d^2*e^{13} + 19968*a^9*b^2*c^8*d*e^{14}))/((8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^{10}*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 +
\end{aligned}$$

$$\begin{aligned}
& a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6d^8 - 256a^5b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^{12}d^4e^4 - 4a^3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 + 1024a^7c^7d^6e^2 + 1536a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^{10}c^2d^6e^2 - 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4b^7c^3d^5e^3 - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 - 192a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c^4d^4e^4 - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 + 2048a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024a^6b^6c^7d^7e + 64a^6b^7c^4d^7e - 1024a^9b^6c^4d^7e - 4a^2b^9c^3d^7e - 4a^2b^{11}c^4d^5e^3 + 64a^3b^7c^4d^7e - 4a^3b^{10}c^4d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^3d^3e^5 + 1024a^5b^3c^6d^7e - 92a^5b^8c^2d^2e^6 - 3072a^7b^6c^6d^5e^3 - 384a^7b^5c^2d^4e^7 - 3072a^8b^3c^5d^3e^5 + 1024a^8b^3c^3d^4e^7)) * ((27a^9c^5d^6 - b^{11}c^4d^6 - b^{15}d^2e^4 - 9a^2b^{13}e^6 + 3840a^5b^9c^9d^6 + 9a^9c^5d^6 * (-4a^9c - b^2)^9)^{1/2} + 213a^3b^{11}c^4e^6 - 26880a^8b^6c^6e^6 + 3072a^6c^9d^5e + 35840a^8c^7d^5e^5 + 4b^{12}c^3d^5e + 4b^{14}c^3d^3e^3 - 288a^2b^7c^6d^6 + 1504a^3b^5c^7d^6 - 3840a^4b^3c^8d^6 - 9a^2b^4e^6 * (-4a^9c - b^2)^9)^{1/2} - 2077a^4b^9c^2e^6 + 10656a^5b^7c^3e^6 - 30240a^6b^5c^4e^6 + 44800a^7b^3c^5e^6 - 25a^4c^2e^6 * (-4a^9c - b^2)^9)^{1/2} - b^2c^4d^6 * (-4a^9c - b^2)^9)^{1/2} + 22528a^7c^8d^3e^3 - b^6d^2e^4 * (-4a^9c - b^2)^9)^{1/2} - 6b^{13}c^2d^4e^2 + 6a^9b^{14}d^5e^5 - 1471a^2b^9c^4d^4e^2 + 600a^2b^{10}c^3d^3e^3 + 180a^2b^{11}c^2d^2e^4 + 6976a^3b^7c^5d^4e^2 - 1032a^3b^8c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456a^4b^5c^6d^4e^2 - 7168a^4b^6c^5d^3e^3 + 16896a^4b^7c^4d^2e^4 + 10240a^5b^3c^7d^4e^2 + 37632a^5b^4c^6d^3e^3 - 47712a^5b^5c^5d^2e^4 - 59392a^6b^2c^7d^3e^3 + 60928a^6b^3c^6d^2e^4 + 41a^2c^4d^4e^2 * (-4a^9c - b^2)^9)^{1/2} + 39a^3c^3d^2e^4 * (-4a^9c - b^2)^9)^{1/2} - 6b^4c^2d^4e^2 * (-4a^9c - b^2)^9)^{1/2} + 6a^9b^5d^5e^5 * (-4a^9c - b^2)^9)^{1/2} - 106a^9b^{10}c^4d^5e^5 + 7a^9b^{13}c^3d^2e^4 - 128a^2b^{12}c^2d^3e^3 + 51a^3b^2c^6e^6 * (-4a^9c - b^2)^9)^{1/2} + 150a^9b^{11}c^3d^4e^2 - 84a^9b^{12}c^2d^3e^3 + 1116a^2b^8c^5d^5e^5 - 5824a^3b^6c^6d^5e^5 + 1030a^3b^{10}c^2d^5e^5 + 15232a^4b^4c^7d^5e^5 - 3492a^4b^8c^3d^5e^5 - 16896a^5b^2c^8d^5e^5 + 1344a^5b^6c^4d^5e^5 + 7424a^6b^6c^8d^4e^2 + 22400a^6b^4c^5d^5e^5 - 23296a^7b^6c^7d^2e^4 - 53760a^7b^2c^6d^5e^5 + 4b^3c^3d^5e^5 * (-4a^9c - b^2)^9)^{1/2} + 4b^5c^3d^3e^3 * (-4a^9c - b^2)^9)^{1/2} - 11a^9b^4c^3d^2e^4 * (-4a^9c - b^2)^9)^{1/2} - 20a^2b^3c^3d^5e^5 * (-4a^9c - b^2)^9)^{1/2} - 86a^3b^6c^2d^5e^5 * (-4a^9c - b^2)^9)^{1/2} + 42a^9b^2c^3d^4e^2 * (-4a^9c - b^2)^9)^{1/2} - 12a^9b^3c^2d^3e^3 * (-4a^9c - b^2)^9)^{1/2} - 120a^2b^3c^3d^3e^3 * (-4a^9c - b^2)^9)^{1/2} - 34a^9b^6c^4d^5e^5 * (-4a^9c - b^2)^9)^{1/2} + 108a^2b^2c^2d^2e^4 * (-4a^9c - b^2)^9)^{1/2}) / (32 * (a^7b^{12}e^8 + 4096a^9c^{10}d^8 + 4096a^{13}c^6e^8 - 24a^8b^{10}c^6e^8 - 4a^6b^{13}d^7e^7 + a^3b^{12}c^4d^8 - 24a^4b^{10}c^5d^8 + 240a^5b^8c^6d^8 - 1280a^6b^6c^7d^8 + 3840a^7b^4c^8d^8 - 6144a^8b^2c^9d^8 + 240a^9b^8c^2e^8 - 1280a^{10}b^6c^3e^8 + 3840a^{11}b^4c^4e^8 - 6144a^{12}b^2c^5e^8 + a^3b^{16}d^4e^4 - 4a^4b^{15}d^3e^5 + 6a^5b^{14}d^2e^6 + 16384a^{10}c^9d^6e^2 + 24576a^{11}c^8d^4e^4 + 16384a^{12}c^7d^2e^6 + 6a^3b^{14}c^2d^6e^2 - 140a^4b^{12}c^3d^6e^2 + 84a^4b^{13}c^2d^5e^3 + 1344a^5b^{10}c^4d^6e^2 - 672a^5b^{11}c^3d^5e^3 - 42a^5b^{12}c^2d^4e^4 - 6720a^6b^8c^5d^6e^2 + 2240a^6b^9c^4d^5e^3 + 1456a^6b^{10}c^3d^4e^4 - 672a^6b^{11}c^2d^3e^5 + 17920a^7b^6c^6d^6e^2 - 10080a^7b^8c^4d^4e^4 + 2240a^7b^9c^3d^3e^5 + 1344a^7b^{10}c^2d^2e^6 - 21504a^8b^4c^7d^6e^2 - 21504a^8b^5c^6d^5e^3 + 32256a^8b^6c^5d^4e^4 - 6720a^8b^8c^3d^2e^6 + 57344a^9b^3c^7d^5e^3 - 46592a^9b^4c^6d^4e^4 - 21504a^9b^5c^5d^3e^5 + 17920a^9b^6c^4d^2e^6 + 12288a^{10}b^2c^7d^4e^4 + 57344a^{10}b^3c^6d^3e^5 - 21504a^{10}b^4c^5d^2e^6 + 96a^7b^{11}c^3d^7e - 16384a^9b^6c^9d^7e - 16384a^{12}b^6c^6d^8e)
\end{aligned}$$

$$\begin{aligned}
& ^7 - 4*a^3*b^{13}*c^3*d^7*e - 4*a^3*b^{15}*c*d^5*e^3 + 96*a^4*b^{11}*c^4*d^7*e - \\
& 12*a^4*b^{14}*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^{13}*c*d^3*e^5 + 512 \\
& 0*a^6*b^7*c^6*d^7*e - 140*a^6*b^{12}*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24 \\
& 576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49 \\
& 152*a^{10}*b*c^8*d^5*e^3 - 15360*a^{10}*b^5*c^4*d*e^7 - 49152*a^{11}*b*c^7*d^3*e^ \\
& 5 + 24576*a^{11}*b^3*c^5*d*e^7))^{(1/2)} - (326912*a^8*c^9*d*e^{13} - 241664*a^8 \\
& *b*c^8*e^{14} - 48*a^2*b^{13}*c^2*e^{14} + 1264*a^3*b^{11}*c^3*e^{14} - 13552*a^4*b^9 \\
& *c^4*e^{14} + 75776*a^5*b^7*c^5*e^{14} - 232960*a^6*b^5*c^6*e^{14} + 372736*a^7*b \\
& ^3*c^7*e^{14} + 11520*a^3*c^{14}*d^{11}*e^3 + 78080*a^4*c^{13}*d^9*e^5 + 197120*a^5 \\
& *c^{12}*d^7*e^7 + 336384*a^6*c^{11}*d^5*e^9 + 532736*a^7*c^{10}*d^3*e^{11} - 40*b^5 \\
& *c^{12}*d^{12}*e^2 + 216*b^6*c^{11}*d^{11}*e^3 - 464*b^7*c^{10}*d^{10}*e^4 + 496*b^8*c^ \\
& 9*d^9*e^5 - 264*b^9*c^8*d^8*e^6 + 56*b^{10}*c^7*d^7*e^7 - 16*b^{11}*c^6*d^6*e^8 \\
& + 64*b^{12}*c^5*d^5*e^9 - 96*b^{13}*c^4*d^4*e^{10} + 64*b^{14}*c^3*d^3*e^{11} - 16*b \\
& ^{15}*c^2*d^2*e^{12} + 1536*a^2*b^2*c^{13}*d^{11}*e^3 + 14400*a^2*b^3*c^{12}*d^{10}*e^4 \\
& - 47152*a^2*b^4*c^{11}*d^9*e^5 + 52144*a^2*b^5*c^{10}*d^8*e^6 - 16272*a^2*b^6*c^ \\
& 9*d^7*e^7 - 13040*a^2*b^7*c^8*d^6*e^8 + 23488*a^2*b^8*c^7*d^5*e^9 - 26384 \\
& *a^2*b^9*c^6*d^4*e^{10} + 13824*a^2*b^{10}*c^5*d^3*e^{11} + 256*a^2*b^{11}*c^4*d^2* \\
& e^{12} + 125056*a^3*b^2*c^{12}*d^9*e^5 - 36224*a^3*b^3*c^{11}*d^8*e^6 - 126432*a^ \\
& 3*b^4*c^{10}*d^7*e^7 + 144848*a^3*b^5*c^9*d^6*e^8 - 114752*a^3*b^6*c^8*d^5*e^ \\
& 9 + 125392*a^3*b^7*c^7*d^4*e^{10} - 53248*a^3*b^8*c^6*d^3*e^{11} - 25264*a^3*b^ \\
& 9*c^5*d^2*e^{12} + 474112*a^4*b^2*c^{11}*d^7*e^7 - 191104*a^4*b^3*c^{10}*d^6*e^8 \\
& + 97184*a^4*b^4*c^9*d^5*e^9 - 277000*a^4*b^5*c^8*d^4*e^{10} + 56056*a^4*b^6*c^ \\
& 7*d^3*e^{11} + 195584*a^4*b^7*c^6*d^2*e^{12} + 236800*a^5*b^2*c^{10}*d^5*e^9 + 3 \\
& 88032*a^5*b^3*c^9*d^4*e^{10} + 159632*a^5*b^4*c^8*d^3*e^{11} - 670488*a^5*b^5*c^ \\
& 7*d^2*e^{12} - 488960*a^6*b^2*c^9*d^3*e^{11} + 1106496*a^6*b^3*c^8*d^2*e^{12} + \\
& 64*a*b^{14}*c^2*d*e^{13} + 448*a*b^3*c^{13}*d^{12}*e^2 - 1968*a*b^4*c^{12}*d^{11}*e^3 + \\
& 2504*a*b^5*c^{11}*d^{10}*e^4 + 768*a*b^6*c^{10}*d^9*e^5 - 4368*a*b^7*c^9*d^8*e^6 \\
& + 3568*a*b^8*c^8*d^7*e^7 - 520*a*b^9*c^7*d^6*e^8 - 1728*a*b^{10}*c^6*d^5*e^9 \\
& + 2528*a*b^{11}*c^5*d^4*e^{10} - 1536*a*b^{12}*c^4*d^3*e^{11} + 240*a*b^{13}*c^3*d^2 \\
& *e^{12} - 1152*a^2*b*c^{14}*d^{12}*e^2 - 1600*a^2*b^{12}*c^3*d*e^{13} - 67968*a^3*b*c^ \\
& ^{13}*d^{10}*e^4 + 15808*a^3*b^{10}*c^4*d*e^{13} - 342272*a^4*b*c^{12}*d^8*e^6 - 7692 \\
& 8*a^4*b^8*c^5*d*e^{13} - 569088*a^5*b*c^{11}*d^6*e^8 + 179200*a^5*b^6*c^6*d*e^{1 \\
& 3} - 586368*a^6*b*c^{10}*d^4*e^{10} - 113008*a^6*b^4*c^7*d*e^{13} - 731008*a^7*b*c^ \\
& ^9*d^2*e^{12} - 244096*a^7*b^2*c^8*d*e^{13})/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 \\
& + 256*a^{10}*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 \\
& - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^ \\
& 4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^{12}*d^4*e^4 - 4*a^3*b^{11}*d^3*e^5 + 6 \\
& *a^4*b^{10}*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9* \\
& c^5*d^2*e^6 + 6*a^2*b^{10}*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9* \\
& c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^ \\
& 8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^ \\
& 5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 204 \\
& 8*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 \\
& + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3 \\
& *e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7* \\
& d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4 \\
& *a^2*b^{11}*c^5*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^{10}*c^4*d^4*e^4 - 384*a^4 \\
& *b^5*c^5*d^7*e + 52*a^4*b^9*c^3*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8 \\
& *c^2*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^ \\
& 5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7))*((27*a*b^9*c^5*d^6 - b^{11}*c^4*d^6 - b \\
& ^{15}*d^2*e^4 - 9*a^2*b^{13}*e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 213*a^3*b^{11}*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5 \\
& *e + 35840*a^8*c^7*d*e^5 + 4*b^{12}*c^3*d^5*e + 4*b^{14}*c*d^3*e^3 - 288*a^2*b^ \\
& 7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-( \\
& 4*a*c - b^2)^9)^{(1/2)} - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 3024 \\
& 0*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c - b^2)^ \\
& 9)^{(1/2)} - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^{(1/2)} + 22528*a^7*c^8*d^3*e^3 - b \\
& ^6*d^2*e^4*(-(4*a*c - b^2)^9)^{(1/2)} - 6*b^{13}*c^2*d^4*e^2 + 6*a*b^{14}*d*e^5 - \\
& 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^{10}*c^3*d^3*e^3 + 180*a^2*b^{11}*c^2*d^2
\end{aligned}$$

$$\begin{aligned}
& *e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7))^(1/2) + (x*(22800*a^6*c^9*e^13 + 36*a^2*b^8*c^5*e^13 - 600*a^3*b^6*c^6*e^13 + 4313*a^4*b^4*c^7*e^13 - 15592*a^5*b^2*c^8*e^13 + 1296*a^2*c^13*d^8*e^5 + 9792*a^3*c^12*d^6*e^7 + 30304*a^4*c^11*d^4*e^9 + 40512*a^5*c^10*d^2*e^11 + 25*b^4*c^11*d^8*e^5 - 120*b^5*c^10*d^7*e^6 + 214*b^6*c^9*d^6*e^7 - 168*b^7*c^8*d^5*e^8 + 53*b^8*c^7*d^4*e^9 - 8*b^9*c^6*d^3*e^10 + 4*b^10*c^5*d^2*e^11 + 6336*a^2*b^2*c^11*d^6*e^7 + 3840*a^2*b^3*c^10*d^5*e^8 - 8506*a^2*b^4*c^9*d^4*e^9 + 1112*a^2*b^5*c^8*d^3*e^10 + 1254*a^2*b^6*c^7*d^2*e^11 + 22224*a^3*b^2*c^10*d^4*e^9 + 13824*a^3*b^3*c^9*d^3*e^10 - 9516*a^3*b^4*c^8*d^2*e^11 + 11712*a^4*b^2*c^9*d^2*e^11 - 24*a*b^9*c^5*d*e^12 - 41088*a^5*b*c^9*d*e^12 - 360*a*b^2*c^12*d^8*e^5 + 1664*a*b^3*c^11*d^7*e^6 - 2604*a*b^4*c^10*d^6*e^7 + 1272*a*b^5*c^9*d^5*e^8 + 332*a*b^6*c^8*d^4*e^9 - 232*a*b^7*c^7*d^3*e^10 - 48*a*b^8*c^6*d^2*e^11 - 5760*a^2*b*c^12*d^7*e^6 + 416*a^2*b^7*c^6*d*e^12 - 32128*a^3*b*c^11*d^5*e^8 - 4120*a^3*b^5*c^7*d*e^12 - 63360*a^4*b*c^10*d^3*e^10 + 21376*a^4*b^3*c^8*d*e^12))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2
\end{aligned}$$

$$\begin{aligned}
& + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 \\
& - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5 \\
& *e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6* \\
& d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5 \\
& *c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^ \\
& 7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 10 \\
& 24*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9* \\
& c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4* \\
& e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e \\
& - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - \\
& 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) * ((27*a*b^9*c^5*d^6 - b^1 \\
& 1*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5*d^ \\
& 6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 307 \\
& 2*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 \\
& - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9*a^ \\
& 2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c \\
& ^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6*(-( \\
& 4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^ \\
& 8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a \\
& *b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2 \\
& *b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2 \\
& 871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3* \\
& e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4 \\
& *c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 6092 \\
& 8*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 39*a^ \\
& 3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^ \\
& 9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + \\
& 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^6*(-(4*a*c - b^2 \\
& )^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8* \\
& c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^ \\
& 4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b \\
& ^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7 \\
& *b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^ \\
& 9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*b^4*c*d^2*e^4*(- \\
& (4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 86*a \\
& ^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b \\
& ^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 120*a^2*b*c^ \\
& 3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1 \\
& /2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/ (32*(a^7*b^12*e^8 + \\
& 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e \\
& ^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^ \\
& 6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c \\
& ^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5* \\
& e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^ \\
& 10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^ \\
& 14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344* \\
& a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - \\
& 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^ \\
& 4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^ \\
& 8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 2150 \\
& 4*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e \\
& ^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c \\
& ^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288* \\
& a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2* \\
& e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 \\
& - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12* \\
& a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a \\
& ^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576 \\
& *a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152
\end{aligned}$$

$$\begin{aligned}
& *a^{10}b^3c^8d^5e^3 - 15360a^{10}b^5c^4d^7e^7 - 49152a^{11}b^3c^7d^3e^5 + \\
& 24576a^{11}b^3c^5d^7e^7))^{(1/2)} * ((27a^9b^9c^5d^6 - b^{11}c^4d^6 - b^{15}d^2e^4 - 9a^2b^{13}e^6 + 3840a^5b^3c^9d^6 + 9a^5c^5d^6 * (-4ac - b^2)^9)^{(1/2)} + 213a^3b^{11}c^6e^6 - 26880a^8b^3c^6e^6 + 3072a^6c^9d^5e \\
& e + 35840a^8c^7d^5e^5 + 4b^{12}c^3d^5e + 4b^{14}c^3d^3e^3 - 288a^2b^7c^6d^6 + 1504a^3b^5c^7d^6 - 3840a^4b^3c^8d^6 - 9a^2b^4e^6 * (-4ac - b^2)^9)^{(1/2)} - 2077a^4b^9c^2e^6 + 10656a^5b^7c^3e^6 - 30240 \\
& a^6b^5c^4e^6 + 44800a^7b^3c^5e^6 - 25a^4c^2e^6 * (-4ac - b^2)^9)^{(1/2)} - b^2c^4d^6 * (-4ac - b^2)^9)^{(1/2)} + 22528a^7c^8d^3e^3 - b^6d^2e^4 * (-4ac - b^2)^9)^{(1/2)} - 6b^{13}c^2d^4e^2 + 6a^6b^{14}d^5e^5 - \\
& 1471a^2b^9c^4d^4e^2 + 600a^2b^{10}c^3d^3e^3 + 180a^2b^{11}c^2d^2e^4 + 6976a^3b^7c^5d^4e^2 - 1032a^3b^8c^4d^3e^3 - 2871a^3b^9c^3d^2e^4 - 15456a^4b^5c^6d^4e^2 - 7168a^4b^6c^5d^3e^3 + 16896a^4 \\
& 4b^7c^4d^2e^4 + 10240a^5b^3c^7d^4e^2 + 37632a^5b^4c^6d^3e^3 - 47712a^5b^5c^5d^2e^4 - 59392a^6b^2c^7d^3e^3 + 60928a^6b^3c^6d^2e^4 + 41a^2c^4d^4e^2 * (-4ac - b^2)^9)^{(1/2)} + 39a^3c^3d^2e^4 * \\
& (-4ac - b^2)^9)^{(1/2)} - 6b^4c^2d^4e^2 * (-4ac - b^2)^9)^{(1/2)} + 6a^6b^5d^5e^5 * (-4ac - b^2)^9)^{(1/2)} - 106a^6b^{10}c^4d^5e^5 + 7a^6b^{13}c^3d^2e^4 - 128a^2b^{12}c^3d^4e^2 + 51a^3b^2c^6e^6 * (-4ac - b^2)^9)^{(1/2)} + 1 \\
& 50a^6b^{11}c^3d^4e^2 - 84a^6b^{12}c^2d^3e^3 + 1116a^2b^8c^5d^5e^5 - 5824a^3b^6c^6d^5e^5 + 1030a^3b^{10}c^2d^5e^5 + 15232a^4b^4c^7d^5e^5 - 3492a^4b^8c^3d^5e^5 - 16896a^5b^2c^8d^5e^5 + 1344a^5b^6c^4d^5e^5 + \\
& 7424a^6b^3c^8d^4e^2 + 22400a^6b^4c^5d^5e^5 - 23296a^7b^3c^7d^2e^4 - 53760a^7b^2c^6d^5e^5 + 4b^3c^3d^5e^5 * (-4ac - b^2)^9)^{(1/2)} + 4b^5c^3d^3e^3 * (-4ac - b^2)^9)^{(1/2)} - 11a^6b^4c^3d^2e^4 * (-4ac - b^2)^9)^{(1/2)} - 20a^2b^3c^3d^5e^5 * (-4ac - b^2)^9)^{(1/2)} - 86a^3b^3c^2d^5e^5 * (-4ac - b^2)^9)^{(1/2)} + 42a^6b^2c^3d^4e^2 * (-4ac - b^2)^9)^{(1/2)} - 12a^6b^3c^2d^3e^3 * (-4ac - b^2)^9)^{(1/2)} - 120a^2b^3c^3d^3e^3 * (-4ac - b^2)^9)^{(1/2)} - 34a^6b^3c^4d^5e^5 * (-4ac - b^2)^9)^{(1/2)} + 108a^2b^2c^2d^2e^4 * (-4ac - b^2)^9)^{(1/2))} / (32(a^7b^{12}e^8 + 4096a^9c^{10}d^8 + 4096a^{13}c^6e^8 - 24a^8b^{10}c^8e^8 - 4a^6b^{13}d^7e^7 + a^3b^{12}c^4d^8 - 24a^4b^{10}c^5d^8 + 240a^5b^8c^6d^8 - 1280a^6b^6c^7d^8 + 3840a^7b^4c^8d^8 - 6144a^8b^2c^9d^8 + 240a^9b^8c^2e^8 - 1280a^{10}b^6c^3e^8 + 3840a^{11}b^4c^4e^8 - 6144a^{12}b^2c^5e^8 + a^3b^{16}d^4e^4 - 4a^4b^{15}d^3e^5 + 6a^5b^{14}d^2e^6 + 16384a^{10}c^9d^6e^2 + 24576a^{11}c^8d^4e^4 + 16384a^{12}c^7d^2e^6 + 6a^3b^{14}c^2d^6e^2 - 140a^4b^{12}c^3d^6e^2 + 84a^4b^{13}c^2d^5e^3 + 1344a^5b^{10}c^4d^6e^2 - 672a^5b^{11}c^3d^5e^3 - 42a^5b^{12}c^2d^4e^4 - 6720a^6b^8c^5d^6e^2 + 2240a^6b^9c^4d^5e^3 + 1456a^6b^{10}c^3d^4e^4 - 672a^6b^{11}c^2d^3e^5 + 17920a^7b^6c^6d^6e^2 - 10080a^7b^8c^4d^4e^4 + 2240a^7b^9c^3d^3e^5 + 1344a^7b^{10}c^2d^2e^6 - 21504a^8b^4c^7d^6e^2 - 21504a^8b^5c^6d^5e^3 + 32256a^8b^6c^5d^4e^4 - 6720a^8b^8c^3d^2e^6 + 57344a^9b^3c^7d^5e^3 - 46592a^9b^4c^6d^4e^4 - 21504a^9b^5c^5d^3e^5 + 17920a^9b^6c^4d^2e^6 + 12288a^{10}b^2c^7d^4e^4 + 57344a^{10}b^3c^6d^3e^5 - 21504a^{10}b^4c^5d^2e^6 + 96a^7b^{11}c^3d^7e - 16384a^9b^3c^9d^7e - 16384a^{12}b^3c^6d^7e - 4a^3b^{13}c^3d^7e - 4a^3b^{15}c^3d^5e^3 + 96a^4b^{11}c^4d^7e - 12a^4b^{14}c^4d^4e^4 - 960a^5b^9c^5d^7e + 84a^5b^{13}c^3d^3e^5 + 5120a^6b^7c^6d^7e - 140a^6b^{12}c^3d^2e^6 - 15360a^7b^5c^7d^7e + 24576a^8b^3c^8d^7e - 960a^8b^9c^2d^7e + 5120a^9b^7c^3d^7e - 49152a^{10}b^3c^8d^5e^3 - 15360a^{10}b^5c^4d^7e - 49152a^{11}b^3c^7d^3e^5 + 24576a^{11}b^3c^5d^7e))^{(1/2)} * 2i - ((x*(b^3e + 2ac^2d - b^2cd - 3abc^2e)) / (2a*(a^2b^2e^2 - 4ac^2d^2 - 4a^2c^2e^2 + b^2cd^2 - b^3d^2e + 4abc^2d^2e)) - (cx^3*(2ac^2e - b^2e + bcd)) / (2a*(a^2b^2e^2 - 4ac^2d^2 - 4a^2c^2e^2 + b^2cd^2 - b^3d^2e + 4abc^2d^2e))) / (a + bx^2 + cx^4) - (atan(((((-d^7e)^{(1/2)} * ((326912a^8c^9d^13 - 241664a^8b^3c^8e^14 - 48a^2b^{13}c^2e^14 + 1264a^3b^{11}c^3e^14 - 13552a^4b^9c^4e^14 + 75776a^5b^7c^5e^14 - 232960a^6b^5c^6e^14 + 372736a^7b^3c^7e^14 + 11520a^3c^{14}d^{11}e^3 + 78080a^4c^{13}d^9e^5 + 197120a^5c^{12}d^7e^7 + 33
\end{aligned}$$

$$\begin{aligned}
& 6384*a^6*c^{11}*d^5*e^9 + 532736*a^7*c^{10}*d^3*e^{11} - 40*b^5*c^{12}*d^{12}*e^2 + 2 \\
& 16*b^6*c^{11}*d^{11}*e^3 - 464*b^7*c^{10}*d^{10}*e^4 + 496*b^8*c^9*d^9*e^5 - 264*b^9 \\
& c^8*d^8*e^6 + 56*b^{10}*c^7*d^7*e^7 - 16*b^{11}*c^6*d^6*e^8 + 64*b^{12}*c^5*d^5 \\
& *e^9 - 96*b^{13}*c^4*d^4*e^{10} + 64*b^{14}*c^3*d^3*e^{11} - 16*b^{15}*c^2*d^2*e^{12} + \\
& 1536*a^2*b^2*c^{13}*d^{11}*e^3 + 14400*a^2*b^3*c^{12}*d^{10}*e^4 - 47152*a^2*b^4*c \\
& ^{11}*d^9*e^5 + 52144*a^2*b^5*c^{10}*d^8*e^6 - 16272*a^2*b^6*c^9*d^7*e^7 - 1304 \\
& 0*a^2*b^7*c^8*d^6*e^8 + 23488*a^2*b^8*c^7*d^5*e^9 - 26384*a^2*b^9*c^6*d^4*e \\
& ^{10} + 13824*a^2*b^{10}*c^5*d^3*e^{11} + 256*a^2*b^{11}*c^4*d^2*e^{12} + 125056*a^3* \\
& b^2*c^{12}*d^9*e^5 - 36224*a^3*b^3*c^{11}*d^8*e^6 - 126432*a^3*b^4*c^{10}*d^7*e^7 \\
& + 144848*a^3*b^5*c^9*d^6*e^8 - 114752*a^3*b^6*c^8*d^5*e^9 + 125392*a^3*b^7 \\
& *c^7*d^4*e^{10} - 53248*a^3*b^8*c^6*d^3*e^{11} - 25264*a^3*b^9*c^5*d^2*e^{12} + 4 \\
& 74112*a^4*b^2*c^{11}*d^7*e^7 - 191104*a^4*b^3*c^{10}*d^6*e^8 + 97184*a^4*b^4*c^9 \\
& *d^5*e^9 - 277000*a^4*b^5*c^8*d^4*e^{10} + 56056*a^4*b^6*c^7*d^3*e^{11} + 1955 \\
& 84*a^4*b^7*c^6*d^2*e^{12} + 236800*a^5*b^2*c^{10}*d^5*e^9 + 388032*a^5*b^3*c^9* \\
& d^4*e^{10} + 159632*a^5*b^4*c^8*d^3*e^{11} - 670488*a^5*b^5*c^7*d^2*e^{12} - 4889 \\
& 60*a^6*b^2*c^9*d^3*e^{11} + 1106496*a^6*b^3*c^8*d^2*e^{12} + 64*a*b^{14}*c^2*d*e^ \\
& ^{13} + 448*a*b^3*c^{13}*d^{12}*e^2 - 1968*a*b^4*c^{12}*d^{11}*e^3 + 2504*a*b^5*c^{11}* \\
& ^{10}*e^4 + 768*a*b^6*c^{10}*d^9*e^5 - 4368*a*b^7*c^9*d^8*e^6 + 3568*a*b^8*c^8* \\
& d^7*e^7 - 520*a*b^9*c^7*d^6*e^8 - 1728*a*b^{10}*c^6*d^5*e^9 + 2528*a*b^{11}*c^5 \\
& *d^4*e^{10} - 1536*a*b^{12}*c^4*d^3*e^{11} + 240*a*b^{13}*c^3*d^2*e^{12} - 1152*a^2*b \\
& *c^{14}*d^{12}*e^2 - 1600*a^2*b^{12}*c^3*d*e^{13} - 67968*a^3*b*c^{13}*d^{10}*e^4 + 158 \\
& 08*a^3*b^{10}*c^4*d*e^{13} - 342272*a^4*b*c^{12}*d^8*e^6 - 76928*a^4*b^8*c^5*d*e^ \\
& ^{13} - 569088*a^5*b*c^{11}*d^6*e^8 + 179200*a^5*b^6*c^6*d*e^{13} - 586368*a^6*b*c \\
& ^{10}*d^4*e^{10} - 113008*a^6*b^4*c^7*d*e^{13} - 731008*a^7*b*c^9*d^2*e^{12} - 2440 \\
& 96*a^7*b^2*c^8*d*e^{13})/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^{10}*c^4*e^ \\
& ^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d \\
& ^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^ \\
& ^9*b^2*c^3*e^8 + a^2*b^{12}*d^4*e^4 - 4*a^3*b^{11}*d^3*e^5 + 6*a^4*b^{10}*d^2*e^6 \\
& + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^ \\
& ^2*b^{10}*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512* \\
& a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 11 \\
& 52*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 \\
& - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5* \\
& e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2* \\
& d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^ \\
& ^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7 \\
& *c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^{11}*c*d^5*e^ \\
& ^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^{10}*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 5 \\
& 2*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072* \\
& a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a \\
& ^8*b^3*c^3*d*e^7)) + (((x*(626688*a^{10}*b*c^8*e^{15} - 784384*a^{10}*c^9*d*e^{14} \\
& + 208*a^4*b^{13}*c^2*e^{15} - 4880*a^5*b^{11}*c^3*e^{15} + 47312*a^6*b^9*c^4*e^{15} - \\
& 242176*a^7*b^7*c^5*e^{15} + 688640*a^8*b^5*c^6*e^{15} - 1028096*a^9*b^3*c^7*e^ \\
& ^{15} + 18432*a^4*c^{15}*d^{13}*e^2 + 126976*a^5*c^{14}*d^{11}*e^4 + 325632*a^6*c^{13}* \\
& ^9*e^6 + 139264*a^7*c^{12}*d^7*e^8 - 1067008*a^8*c^{11}*d^5*e^{10} - 1773568*a^9* \\
& c^{10}*d^3*e^{12} + 16*b^8*c^{11}*d^{13}*e^2 - 96*b^9*c^{10}*d^{12}*e^3 + 240*b^{10}*c^9* \\
& d^{11}*e^4 - 304*b^{11}*c^8*d^{10}*e^5 + 144*b^{12}*c^7*d^9*e^6 + 144*b^{13}*c^6*d^8* \\
& e^7 - 304*b^{14}*c^5*d^7*e^8 + 240*b^{15}*c^4*d^6*e^9 - 96*b^{16}*c^3*d^5*e^{10} + \\
& 16*b^{17}*c^2*d^4*e^{11} + 3200*a^2*b^4*c^{13}*d^{13}*e^2 - 18432*a^2*b^5*c^{12}*d^{12} \\
& *e^3 + 41024*a^2*b^6*c^{11}*d^{11}*e^4 - 36352*a^2*b^7*c^{10}*d^{10}*e^5 - 16208*a^ \\
& ^2*b^8*c^9*d^9*e^6 + 74576*a^2*b^9*c^8*d^8*e^7 - 78496*a^2*b^{10}*c^7*d^7*e^8 \\
& + 32064*a^2*b^{11}*c^6*d^6*e^9 + 6000*a^2*b^{12}*c^5*d^5*e^{10} - 9264*a^2*b^{13}* \\
& ^4*d^4*e^{11} + 1472*a^2*b^{14}*c^3*d^3*e^{12} + 416*a^2*b^{15}*c^2*d^2*e^{13} - 1280 \\
& 0*a^3*b^2*c^{14}*d^{13}*e^2 + 73728*a^3*b^3*c^{13}*d^{12}*e^3 - 151296*a^3*b^4*c^{12} \\
& *d^{11}*e^4 + 78336*a^3*b^5*c^{11}*d^{10}*e^5 + 206688*a^3*b^6*c^{10}*d^9*e^6 - 436 \\
& 736*a^3*b^7*c^9*d^8*e^7 + 324224*a^3*b^8*c^8*d^7*e^8 + 992*a^3*b^9*c^7*d^6* \\
& e^9 - 158176*a^3*b^{10}*c^6*d^5*e^{10} + 77056*a^3*b^{11}*c^5*d^4*e^{11} + 6912*a^3 \\
& *b^{12}*c^4*d^3*e^{12} - 8416*a^3*b^{13}*c^3*d^2*e^{13} + 162816*a^4*b^2*c^{13}*d^{11}* \\
& e^4 + 184320*a^4*b^3*c^{12}*d^{10}*e^5 - 916608*a^4*b^4*c^{11}*d^9*e^6 + 1165824*
\end{aligned}$$



$$\begin{aligned}
& a^4b^5c^{10}d^8e^7 - 314496a^4b^6c^9d^7e^8 - 822272a^4b^7c^8d^6e^9 + 919152a^4b^8c^7d^5e^{10} - 175296a^4b^9c^6d^4e^{11} - 189328a^4b^{10}c^5d^3e^{12} + 62064a^4b^{11}c^4d^2e^{13} + 1290752a^5b^2c^{12}d^9e^6 - 659456a^5b^3c^{11}d^8e^7 - 1561088a^5b^4c^{10}d^7e^8 + 3240960a^5b^5c^9d^6e^9 - 1964192a^5b^6c^8d^5e^{10} - 683008a^5b^7c^7d^4e^{11} + 1162304a^5b^8c^6d^3e^{12} - 164112a^5b^9c^5d^2e^{13} + 3442688a^6b^2c^{11}d^7e^8 - 3670016a^6b^3c^{10}d^6e^9 + 15232a^6b^4c^9d^5e^{10} + 4230144a^6b^5c^8d^4e^{11} - 3059648a^6b^6c^7d^3e^{12} - 247296a^6b^7c^6d^2e^{13} + 4010496a^7b^2c^{10}d^5e^{10} - 6873088a^7b^3c^9d^4e^{11} + 2822400a^7b^4c^8d^3e^{12} + 2370048a^7b^5c^7d^2e^{13} + 1178624a^8b^2c^9d^3e^{12} - 4739072a^8b^3c^8d^2e^{13} - 352a^8b^6c^{12}d^{13}e^2 + 2048a^8b^7c^{11}d^{12}e^3 - 4800a^8b^8c^{10}d^{11}e^4 + 5168a^8b^9c^9d^{10}e^5 - 480a^8b^{10}c^8d^9e^6 - 6000a^8b^{11}c^7d^8e^7 + 8192a^8b^{12}c^6d^7e^8 - 5040a^8b^{13}c^5d^6e^9 + 1152a^8b^{14}c^4d^5e^{10} + 240a^8b^{15}c^3d^4e^{11} - 128a^8b^{16}c^2d^3e^{12} - 512a^8b^{17}c^1d^2e^{13} - 106496a^4b^6c^{14}d^{12}e^3 + 11680a^4b^{12}c^3d^5e^{14} - 675840a^5b^6c^{13}d^{10}e^5 - 108288a^5b^{10}c^4d^8e^{14} - 1601536a^6b^6c^{12}d^8e^7 + 514768a^6b^8c^5d^6e^{14} - 925696a^7b^6c^{11}d^6e^9 - 1278304a^7b^6c^6d^5e^{14} + 2457600a^8b^6c^{10}d^4e^{11} + 1385600a^8b^4c^7d^5e^{14} + 2977792a^9b^6c^9d^2e^{13} + 19968a^9b^2c^8d^5e^{14})) / (8(a^6b^8e^8 + 256a^6c^8d^8 + 256a^{10}c^4e^8 - 16a^7b^6c^8e^8 - 4a^5b^9d^8e^7 + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6d^8 - 256a^5b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^{12}d^4e^4 - 4a^3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 + 1024a^7c^7d^6e^2 + 1536a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^{10}c^2d^6e^2 - 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4b^7c^3d^5e^3 - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 - 192a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c^4d^4e^4 - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 + 2048a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024a^6b^7c^7d^7e + 64a^6b^7c^7d^7e - 1024a^9b^6c^4d^5e^7 - 4a^2b^9c^3d^7e - 4a^2b^{11}c^5d^5e^3 + 64a^3b^7c^4d^7e - 4a^3b^{10}c^4d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^4d^3e^5 + 1024a^5b^3c^6d^7e - 92a^5b^8c^4d^2e^6 - 3072a^7b^6c^6d^5e^3 - 384a^7b^5c^2d^5e^7 - 3072a^8b^6c^5d^3e^5 + 1024a^8b^3c^3d^5e^7) - (((1048576a^{13}c^8e^{16} + 256a^7b^{12}c^2e^{16} - 6144a^8b^{10}c^3e^{16} + 61440a^9b^8c^4e^{16} - 327680a^{10}b^6c^5e^{16} + 983040a^{11}b^4c^6e^{16} - 1572864a^{12}b^2c^7e^{16} - 196608a^6c^{15}d^{14}e^2 - 917504a^7c^{14}d^{12}e^4 - 589824a^8c^{13}d^{10}e^6 + 3932160a^9c^{12}d^8e^8 + 10158080a^{10}c^{11}d^6e^{10} + 10616832a^{11}c^{10}d^4e^{12} + 5308416a^{12}c^9d^2e^{14} - 2816a^2b^8c^{11}d^{14}e^2 + 22656a^2b^9c^{10}d^{13}e^3 - 78848a^2b^{10}c^9d^{12}e^4 + 154112a^2b^{11}c^8d^{11}e^5 - 182784a^2b^{12}c^7d^{10}e^6 + 130816a^2b^{13}c^6d^9e^7 - 50176a^2b^{14}c^5d^8e^8 + 4608a^2b^{15}c^4d^7e^9 + 3328a^2b^{16}c^3d^6e^{10} - 896a^2b^{17}c^2d^5e^{11} + 24576a^3b^6c^{12}d^{14}e^2 - 198656a^3b^7c^{11}d^{13}e^3 + 684544a^3b^8c^{10}d^{12}e^4 - 1291520a^3b^9c^9d^{11}e^5 + 1403776a^3b^{10}c^8d^{10}e^6 - 798336a^3b^{11}c^7d^9e^7 + 89856a^3b^{12}c^6d^8e^8 + 155136a^3b^{13}c^5d^7e^9 - 77440a^3b^{14}c^4d^6e^{10} + 5504a^3b^{15}c^3d^5e^{11} + 2560a^3b^{16}c^2d^4e^{12} - 106496a^4b^4c^{13}d^{14}e^2 + 864256a^4b^5c^{12}d^{13}e^3 - 2924544a^4b^6c^{11}d^{12}e^4 + 5181440a^4b^7c^{10}d^{11}e^5 - 4686080a^4b^8c^9d^{10}e^6 + 1045376a^4b^9c^8d^9e^7 + 1900544a^4b^{10}c^7d^8e^8 - 1732096a^4b^{11}c^6d^7e^9 + 390400a^4b^{12}c^5d^6e^{10} + 112000a^4b^{13}c^4d^5e^{11} - 40960a^4b^{14}c^3d^4e^{12} - 3840a^4b^{15}c^2d^3e^{13} + 229376a^5b^2c^{14}d^{14}e^2 - 1867776a^5b^3c^{13}d^{13}e^3 + 6078464a^5b^4c^{12}d^{12}e^4 - 9297920a^5b^5c^{11}d^{11}e^5 + 4055040a^5b^6c^{10}d^{10}e^6 + 7788544a^5b^7c^9d^9e^7 - 12657664a^5b^8c^8d^8e^8 + 6130176a^5b^9c^7d^7e^9 + 734080a^5b^{10}c^6d^6e^{10} - 1442560a^5b^{11}c^5d^5e^{11} + 168960a^5b^{12}c^4d^4e^{12} + 78080a^5b^{13}c^3d^3e^{13} + 3200
\end{aligned}$$

$$\begin{aligned}
& *a^5b^{14}c^2d^2e^{14} - 4587520a^6b^2c^{13}d^{12}e^4 + 3080192a^6b^3c^7d^{11}e^5 + 12001280a^6b^4c^{11}d^{10}e^6 - 31076352a^6b^5c^{10}d^9e^7 \\
& + 27475968a^6b^6c^9d^8e^8 - 2088960a^6b^7c^8d^7e^9 - 12205312a^6b^8c^7d^6e^{10} + 6043520a^6b^9c^6d^5e^{11} + 631808a^6b^{10}c^5d^4e^{12} \\
& - 610304a^6b^{11}c^4d^3e^{13} - 71936a^6b^{12}c^3d^2e^{14} - 21725184a^7b^2c^{12}d^{10}e^6 + 30801920a^7b^3c^{11}d^9e^7 - 8028160a^7b^4c^{10}d^8e^8 - 32260096a^7b^5c^9d^7e^9 \\
& + 37101568a^7b^6c^8d^6e^{10} - 7182336a^7b^7c^7d^5e^{11} - 7609856a^7b^8c^6d^4e^{12} + 2112256a^7b^9c^5d^3e^{13} + 661632a^7b^{10}c^4d^2e^{14} \\
& - 30146560a^8b^2c^{11}d^8e^8 + 55050240a^8b^3c^{10}d^7e^9 - 34365440a^8b^4c^9d^6e^{10} - 16429056a^8b^5c^8d^5e^{11} + 24600576a^8b^6c^7d^4e^{12} \\
& - 1683456a^8b^7c^6d^3e^{13} - 3151616a^8b^8c^5d^2e^{14} - 10977280a^9b^2c^{10}d^6e^{10} + 47022080a^9b^3c^9d^5e^{11} \\
& - 30621696a^9b^4c^8d^4e^{12} - 9232384a^9b^5c^7d^3e^{13} + 7970816a^9b^6c^6d^2e^{14} + 4325376a^{10}b^2c^9d^4e^{12} + 25493504a^{10}b^3c^8d^3e^{13} \\
& - 9117696a^{10}b^4c^7d^2e^{14} + 491520a^{11}b^2c^8d^2e^{14} - 4947968a^{12}b^2c^8d^2e^{15} + 128a^*b^{10}c^{10}d^{14}e^2 \\
& - 1024a^*b^{11}c^9d^{13}e^3 + 3584a^*b^{12}c^8d^{12}e^4 - 7168a^*b^{13}c^7d^{11}e^5 + 8960a^*b^{14}c^6d^{10}e^6 - 7168a^*b^{15}c^5d^9e^7 + 3584a^*b^{16}c^4d^8e^8 \\
& - 1024a^*b^{17}c^3d^7e^9 + 128a^*b^{18}c^2d^6e^{10} + 1605632a^6b^*c^{14}d^{13}e^3 - 1408a^6b^{13}c^2d^*e^{15} + 7012352a^7b^*c^{13}d^{11}e^5 + 33152a^7b^{11}c^3d^*e^{15} \\
& + 7045120a^8b^*c^{12}d^9e^7 - 324480a^8b^9c^4d^*e^{15} - 9830400a^9b^*c^{11}d^7e^9 + 1689600a^9b^7c^5d^*e^{15} - 25722880a^{10}b^*c^{10}d^5e^{11} \\
& - 4935680a^{10}b^5c^6d^*e^{15} - 19202048a^{11}b^*c^9d^3e^{13} + 7667712a^{11}b^3c^7d^*e^{15}) / (16*(a^6b^8e^8 + 256a^6c^8d^8 + 256a^{10}c^4e^8 - 16a^7b^6c^*e^8 - 4a^5b^9d^*e^7 + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6d^8 - 256a^5b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^{12}d^4e^4 - 4a^3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 + 1024a^7c^7d^6e^2 + 1536a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^{10}c^2d^6e^2 - 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4b^7c^3d^5e^3 - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 - 192a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c^4d^4e^4 - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 + 2048a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024a^6b^*c^7d^7e + 64a^6b^7c^*d^7e - 1024a^9b^*c^4d^*e^7 - 4a^2b^9c^3d^7e - 4a^2b^{11}c^*d^5e^3 + 64a^3b^7c^4d^7e - 4a^3b^{10}c^*d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^*d^3e^5 + 1024a^5b^3c^6d^7e - 92a^5b^8c^*d^2e^6 - 3072a^7b^*c^6d^5e^3 - 384a^7b^5c^2d^*e^7 - 3072a^8b^*c^5d^3e^5 + 1024a^8b^3c^3d^*e^7) - (x*(-d^7e^7))^{(1/2)} * (1048576a^{15}c^8e^{17} + 256a^9b^{12}c^2e^{17} - 6144a^{10}b^{10}c^3e^{17} + 61440a^{11}b^8c^4e^{17} - 327680a^{12}b^6c^5e^{17} + 983040a^{13}b^4c^6e^{17} - 1572864a^{14}b^2c^7e^{17} - 1048576a^8c^{15}d^{14}e^3 - 5242880a^9c^{14}d^{12}e^5 - 9437184a^{10}c^{13}d^{10}e^7 - 5242880a^{11}c^{12}d^8e^9 + 5242880a^{12}c^{11}d^6e^{11} + 9437184a^{13}c^{10}d^4e^{13} + 5242880a^{14}c^9d^2e^{15} + 256a^2b^{11}c^{10}d^{15}e^2 - 2048a^2b^{12}c^9d^{14}e^3 + 7168a^2b^{13}c^8d^{13}e^4 - 14336a^2b^{14}c^7d^{12}e^5 + 17920a^2b^{15}c^6d^{11}e^6 - 14336a^2b^{16}c^5d^{10}e^7 + 7168a^2b^{17}c^4d^9e^8 - 2048a^2b^{18}c^3d^8e^9 + 256a^2b^{19}c^2d^7e^{10} - 5120a^3b^9c^{11}d^{15}e^2 + 41984a^3b^{10}c^{10}d^{14}e^3 - 148736a^3b^{11}c^9d^{13}e^4 + 296192a^3b^{12}c^8d^{12}e^5 - 359680a^3b^{13}c^7d^{11}e^6 + 267520a^3b^{14}c^6d^{10}e^7 - 12384a^3b^{15}c^5d^9e^8 + 18176a^3b^{16}c^4d^8e^9 + 3328a^3b^{17}c^3d^7e^{10} - 1280a^3b^{18}c^2d^6e^{11} + 40960a^4b^7c^{12}d^{15}e^2 - 348160a^4b^8c^{11}d^{14}e^3 + 1254400a^4b^9c^{10}d^{13}e^4 - 2478080a^4b^{10}c^9d^{12}e^5 + 2867456a^4b^{11}c^8d^{11}e^6 - 1862144a^4b^{12}c^7d^{10}e^7 + 490240a^4b^{13}c^6d^9e^8 + 128000a^4b^{14}c^5d^8e^9 - 108800a^4b^{15}c^4d^7e^{10} + 13824a^4b^{16}c^3d^6e^{11} + 2304a^4b^{17}c^2d^5e^{12} - 163840a^5b^5c^{13}d^{15}e^2 + 1474560a^5b^6c^{12}d^{14}e^3 - 5447680a^5b^7c^{11}d^{13}e^4 + 10588160a^5b^8c^{10}d^{12}e^5 - 11166720a^5b^9c^9d^{11}e^6 - 10588160a^5b^{10}c^8d^{10}e^7 + 10588160a^5b^{11}c^7d^9e^8 - 10588160a^5b^{12}c^6d^8e^9 + 10588160a^5b^{13}c^5d^7e^{10} - 10588160a^5b^{14}c^4d^6e^{11} + 10588160a^5b^{15}c^3d^5e^{12} - 10588160a^5b^{16}c^2d^4e^{13} + 10588160a^5b^{17}c^1d^3e^{14} - 10588160a^5b^{18}c^0d^2e^{15} + 10588160a^5b^{19}c^{-1}d^1e^{16} - 10588160a^5b^{20}c^{-2}d^0e^{17} + 10588160a^5b^{21}c^{-3}d^{-1}e^{18} - 10588160a^5b^{22}c^{-4}d^{-2}e^{19} + 10588160a^5b^{23}c^{-5}d^{-3}e^{20} - 10588160a^5b^{24}c^{-6}d^{-4}e^{21} + 10588160a^5b^{25}c^{-7}d^{-5}e^{22} - 10588160a^5b^{26}c^{-8}d^{-6}e^{23} + 10588160a^5b^{27}c^{-9}d^{-7}e^{24} - 10588160a^5b^{28}c^{-10}d^{-8}e^{25} + 10588160a^5b^{29}c^{-11}d^{-9}e^{26} - 10588160a^5b^{30}c^{-12}d^{-10}e^{27} + 10588160a^5b^{31}c^{-13}d^{-11}e^{28} - 10588160a^5b^{32}c^{-14}d^{-12}e^{29} + 10588160a^5b^{33}c^{-15}d^{-13}e^{30} - 10588160a^5b^{34}c^{-16}d^{-14}e^{31} + 10588160a^5b^{35}c^{-17}d^{-15}e^{32} - 10588160a^5b^{36}c^{-18}d^{-16}e^{33} + 10588160a^5b^{37}c^{-19}d^{-17}e^{34} - 10588160a^5b^{38}c^{-20}d^{-18}e^{35} + 10588160a^5b^{39}c^{-21}d^{-19}e^{36} - 10588160a^5b^{40}c^{-22}d^{-20}e^{37} + 10588160a^5b^{41}c^{-23}d^{-21}e^{38} - 10588160a^5b^{42}c^{-24}d^{-22}e^{39} + 10588160a^5b^{43}c^{-25}d^{-23}e^{40} - 10588160a^5b^{44}c^{-26}d^{-24}e^{41} + 10588160a^5b^{45}c^{-27}d^{-25}e^{42} - 10588160a^5b^{46}c^{-28}d^{-26}e^{43} + 10588160a^5b^{47}c^{-29}d^{-27}e^{44} - 10588160a^5b^{48}c^{-30}d^{-28}e^{45} + 10588160a^5b^{49}c^{-31}d^{-29}e^{46} - 10588160a^5b^{50}c^{-32}d^{-30}e^{47} + 10588160a^5b^{51}c^{-33}d^{-31}e^{48} - 10588160a^5b^{52}c^{-34}d^{-32}e^{49} + 10588160a^5b^{53}c^{-35}d^{-33}e^{50} - 10588160a^5b^{54}c^{-36}d^{-34}e^{51} + 10588160a^5b^{55}c^{-37}d^{-35}e^{52} - 10588160a^5b^{56}c^{-38}d^{-36}e^{53} + 10588160a^5b^{57}c^{-39}d^{-37}e^{54} - 10588160a^5b^{58}c^{-40}d^{-38}e^{55} + 10588160a^5b^{59}c^{-41}d^{-39}e^{56} - 10588160a^5b^{60}c^{-42}d^{-40}e^{57} + 10588160a^5b^{61}c^{-43}d^{-41}e^{58} - 10588160a^5b^{62}c^{-44}d^{-42}e^{59} + 10588160a^5b^{63}c^{-45}d^{-43}e^{60} - 10588160a^5b^{64}c^{-46}d^{-44}e^{61} + 10588160a^5b^{65}c^{-47}d^{-45}e^{62} - 10588160a^5b^{66}c^{-48}d^{-46}e^{63} + 10588160a^5b^{67}c^{-49}d^{-47}e^{64} - 10588160a^5b^{68}c^{-50}d^{-48}e^{65} + 10588160a^5b^{69}c^{-51}d^{-49}e^{66} - 10588160a^5b^{70}c^{-52}d^{-50}e^{67} + 10588160a^5b^{71}c^{-53}d^{-51}e^{68} - 10588160a^5b^{72}c^{-54}d^{-52}e^{69} + 10588160a^5b^{73}c^{-55}d^{-53}e^{70} - 10588160a^5b^{74}c^{-56}d^{-54}e^{71} + 10588160a^5b^{75}c^{-57}d^{-55}e^{72} - 10588160a^5b^{76}c^{-58}d^{-56}e^{73} + 10588160a^5b^{77}c^{-59}d^{-57}e^{74} - 10588160a^5b^{78}c^{-60}d^{-58}e^{75} + 10588160a^5b^{79}c^{-61}d^{-59}e^{76} - 10588160a^5b^{80}c^{-62}d^{-60}e^{77} + 10588160a^5b^{81}c^{-63}d^{-61}e^{78} - 10588160a^5b^{82}c^{-64}d^{-62}e^{79} + 10588160a^5b^{83}c^{-65}d^{-63}e^{80} - 10588160a^5b^{84}c^{-66}d^{-64}e^{81} + 10588160a^5b^{85}c^{-67}d^{-65}e^{82} - 10588160a^5b^{86}c^{-68}d^{-66}e^{83} + 10588160a^5b^{87}c^{-69}d^{-67}e^{84} - 10588160a^5b^{88}c^{-70}d^{-68}e^{85} + 10588160a^5b^{89}c^{-71}d^{-69}e^{86} - 10588160a^5b^{90}c^{-72}d^{-70}e^{87} + 10588160a^5b^{91}c^{-73}d^{-71}e^{88} - 10588160a^5b^{92}c^{-74}d^{-72}e^{89} + 10588160a^5b^{93}c^{-75}d^{-73}e^{90} - 10588160a^5b^{94}c^{-76}d^{-74}e^{91} + 10588160a^5b^{95}c^{-77}d^{-75}e^{92} - 10588160a^5b^{96}c^{-78}d^{-76}e^{93} + 10588160a^5b^{97}c^{-79}d^{-77}e^{94} - 10588160a^5b^{98}c^{-80}d^{-78}e^{95} + 10588160a^5b^{99}c^{-81}d^{-79}e^{96} - 10588160a^5b^{100}c^{-82}d^{-80}e^{97} + 10588160a^5b^{101}c^{-83}d^{-81}e^{98} - 10588160a^5b^{102}c^{-84}d^{-82}e^{99} + 10588160a^5b^{103}c^{-85}d^{-83}e^{100} - 10588160a^5b^{104}c^{-86}d^{-84}e^{101} + 10588160a^5b^{105}c^{-87}d^{-85}e^{102} - 10588160a^5b^{106}c^{-88}d^{-86}e^{103} + 10588160a^5b^{107}c^{-89}d^{-87}e^{104} - 10588160a^5b^{108}c^{-90}d^{-88}e^{105} + 10588160a^5b^{109}c^{-91}d^{-89}e^{106} - 10588160a^5b^{110}c^{-92}d^{-90}e^{107} + 10588160a^5b^{111}c^{-93}d^{-91}e^{108} - 10588160a^5b^{112}c^{-94}d^{-92}e^{109} + 10588160a^5b^{113}c^{-95}d^{-93}e^{110} - 10588160a^5b^{114}c^{-96}d^{-94}e^{111} + 10588160a^5b^{115}c^{-97}d^{-95}e^{112} - 10588160a^5b^{116}c^{-98}d^{-96}e^{113} + 10588160a^5b^{117}c^{-99}d^{-97}e^{114} - 10588160a^5b^{118}c^{-100}d^{-98}e^{115} + 10588160a^5b^{119}c^{-101}d^{-99}e^{116} - 10588160a^5b^{120}c^{-102}d^{-100}e^{117} + 10588160a^5b^{121}c^{-103}d^{-101}e^{118} - 10588160a^5b^{122}c^{-104}d^{-102}e^{119} + 10588160a^5b^{123}c^{-105}d^{-103}e^{120} - 10588160a^5b^{124}c^{-106}d^{-104}e^{121} + 10588160a^5b^{125}c^{-107}d^{-105}e^{122} - 10588160a^5b^{126}c^{-108}d^{-106}e^{123} + 10588160a^5b^{127}c^{-109}d^{-107}e^{124} - 10588160a^5b^{128}c^{-110}d^{-108}e^{125} + 10588160a^5b^{129}c^{-111}d^{-109}e^{126} - 10588160a^5b^{130}c^{-112}d^{-110}e^{127} + 10588160a^5b^{131}c^{-113}d^{-111}e^{128} - 10588160a^5b^{132}c^{-114}d^{-112}e^{129} + 10588160a^5b^{133}c^{-115}d^{-113}e^{130} - 10588160a^5b^{134}c^{-116}d^{-114}e^{131} + 10588160a^5b^{135}c^{-117}d^{-115}e^{132} - 10588160a^5b^{136}c^{-118}d^{-116}e^{133} + 10588160a^5b^{137}c^{-119}d^{-117}e^{134} - 10588160a^5b^{138}c^{-120}d^{-118}e^{135} + 10588160a^5b^{139}c^{-121}d^{-119}e^{136} - 10588160a^5b^{140}c^{-122}d^{-120}e^{137} + 10588160a^5b^{141}c^{-123}d^{-121}e^{138} - 10588160a^5b^{142}c^{-124}d^{-122}e^{139} + 10588160a^5b^{143}c^{-125}d^{-123}e^{140} - 10588160a^5b^{144}c^{-126}d^{-124}e^{141} + 10588160a^5b^{145}c^{-127}d^{-125}e^{142} - 10588160a^5b^{146}c^{-128}d^{-126}e^{143} + 10588160a^5b^{147}c^{-129}d^{-127}e^{144} - 10588160a^5b^{148}c^{-130}d^{-128}e^{145} + 10588160a^5b^{149}c^{-131}d^{-129}e^{146} - 10588160a^5b^{150}c^{-132}d^{-130}e^{147} + 10588160a^5b^{151}c^{-133}d^{-131}e^{148} - 10588160a^5b^{152}c^{-134}d^{-132}e^{149} + 10588160a^5b^{153}c^{-135}d^{-133}e^{150} - 10588160a^5b^{154}c^{-136}d^{-134}e^{151} + 10588160a^5b^{155}c^{-137}d^{-135}e^{152} - 10588160a^5b^{156}c^{-138}d^{-136}e^{153} + 10588160a^5b^{157}c^{-139}d^{-137}e^{154} - 10588160a^5b^{158}c^{-140}d^{-138}e^{155} + 10588160a^5b^{159}c^{-141}d^{-139}e^{156} - 10588160a^5b^{160}c^{-142}d^{-140}e^{157} + 10588160a^5b^{161}c^{-143}d^{-141}e^{158} - 10588160a^5b^{162}c^{-144}d^{-142}e^{159} + 10588160a^5b^{163}c^{-145}d^{-143}e^{160} - 10588160a^5b^{164}c^{-146}d^{-144}e^{161} + 10588160a^5b^{165}c^{-147}d^{-145}e^{162} - 10588160a^5b^{166}c^{-148}d^{-146}e^{163} + 10588160a^5b^{167}c^{-149}d^{-147}e^{164} - 10588160a^5b^{168}c^{-150}d^{-148}e^{165} + 10588160a^5b^{169}c^{-151}d^{-149}e^{166} - 10588160a^5b^{170}c^{-152}d^{-150}e^{167} + 10588160a^5b^{171}c^{-153}d^{-151}e^{168} - 10588160a^5b^{172}c^{-154}d^{-152}e^{169} + 10588160a^5b^{173}c^{-155}d^{-153}e^{170} - 10588160a^5b^{174}c^{-156}d^{-154}e^{171} + 10588160a^5b^{175}c^{-157}d^{-155}e^{172} - 10588160a^5b^{176}c^{-158}d^{-156}e^{173} + 10588160a^5b^{177}c^{-159}d^{-157}e^{174} - 10588160a^5b^{178}c^{-160}d^{-158}e^{175} + 10588160a^5b^{179}c^{-161}d^{-159}e^{176} - 10588160a^5b^{180}c^{-162}d^{-160}e^{177} + 10588160a^5b^{181}c^{-163}d^{-161}e^{178} - 10588160a^5b^{182}c^{-164}d^{-162}e^{179} + 10588160a^5b^{183}c^{-165}d^{-163}e^{180} - 10588160a^5b^{184}c^{-166}d^{-164}e^{181} + 10588160a^5b^{185}c^{-167}d^{-165}e^{182} - 10588160a^5b^{186}c^{-168}d^{-166}e^{183} + 10588160a^5b^{187}c^{-169}d^{-167}e^{184} - 10588160a^5b^{188}c^{-170}d^{-168}e^{185} + 10588160a^5b^{189}c^{-171}d^{-169}e^{186} - 10588160a^5b^{190}c^{-172}d^{-170}e^{187} + 10588160a^5b^{191}c^{-173}d^{-171}e^{188} - 10588160a^5b^{192}c^{-174}d^{-172}e^{189} + 10588160a^5b^{193}c^{-175}d^{-173}e^{190} - 10588160a^5b^{194}c^{-176}d^{-174}e^{191} + 10588160a^5b^{195}c^{-177}d^{-175}e^{192} - 10588160a^5b^{196}c^{-178}d^{-176}e^{193} + 10588160a^5b^{197}c^{-179}d^{-177}e^{194} - 10588160a^5b^{198}c^{-180}d^{-178}e^{195} + 10588160a^5b^{199}c^{-181}d^{-179}e^{196} - 10588160a^5b^{200}c^{-182}d^{-180}e^{197} + 10588160a^5b^{201}c^{-183}d^{-181}e^{198} - 10588160a^5b^{202}c^{-184}d^{-182}e^{199} + 10588160a^5b^{203}c^{-185}d^{-183}e^{200} - 10588160a^5b^{204}c^{-186}d^{-184}e^{201} + 10588160a^5b^{205}c^{-187}d^{-185}e^{202} - 10588160a^5b^{206}c^{-188}d^{-186}e^{203} + 10588160a^5b^{207}c^{-189}d^{-187}e^{204} - 10588160a^5b^{208}c^{-190}d^{-188}e^{205} + 10588160a^5b^{209}c^{-191}d^{-189}e^{206} - 10588160a^5b^{210}c^{-192}d^{-190}e^{207} + 10588160a^5b^{211}c^{-193}d^{-191}e^{208} - 10588160a^5b^{212}c^{-194}d^{-192}e^{209} + 10588160a^5b^{213}c^{-195}d^{-193}e^{210} - 10588160a^5b^{214}c^{-196}d^{-194}e^{211} + 10588160a^5b^{215}c^{-197}d^{-195}e^{212} - 10588160a^5b^{216}c^{-198}d^{-196}e^{213} + 10588160a^5b^{217}c^{-199}d^{-197}e^{214} - 10588160a^5b^{218}c^{-200}d^{-198}e^{215} + 10588160a^5b^{219}c^{-201}d^{-199}e^{216} - 10588160a^5b^{220}c^{-202}d^{-200}e^{217} + 10588160a^5b^{221}c^{-203}d^{-201}e^{218} - 10588160a^5b^{222}c^{-204}d^{-202}e^{219} + 10588160a^5b^{223}c^{-205}d^{-203}e^{220} - 10588160a^5b^{224}c^{-206}d^{-204}e^{221} + 10588160a^5b^{225}c^{-207}d^{-205}e^{222} - 10588160a^5b^{226}c^{-208}d^{-206}e^{223} + 10588160a^5b^{227}c^{-209}d^{-207}e^{224} - 10588160a^5b^{228}c^{-210}d^{-208}e^{225} + 10588160a^5b^{229}c^{-211}d^{-209}e^{226} - 10588160a^5b^{230}c^{-212}d^{-210}e^{227} + 10588160a^5b^{231}c^{-213}d^{-211}e^{228} - 10588160a^5b^{232}c^{-214}d^{-212}e^{229} + 10588160a^5b^{233}c^{-215}d^{-213}e^{230} - 10588160a^5b^{234}c^{-216}d^{-214}e^{231} + 10588160a^5b^{235}c^{-217}d^{-215}e^{232} - 10588160a^5b^{236}c^{-218}d^{-216}e^{233} + 10588160a^5b^{237}c^{-219}d^{-217}e^{234} - 10588160a^5b^{238}c^{-220}d^{-218}e^{235} + 10588160a^5b^{239}c^{-221}d^{-219}e^{236} - 10588160a^5b^{240}c^{-222}d^{-220}e^{237} + 10588160a^5b^{241}c^{-223}d^{-221}e^{238} - 10588160a^5b^{242}c^{-224}d^{-222}e^{239} + 10588160a^5b^{243}c^{-225}d^{-223}e^{240} - 10588160a^5b^{244}c^{-226}d^{-224}e^{241} + 10588160a^5b^{245}c^{-227}d^{-225}e^{242} - 10588160a^5b^{246}c^{-228}d^{-226}e^{243} + 10588160a^5b^{247}c^{-229}d^{-227}e^{244} - 10588160a^5b^{248}c^{-230}d^{-228}e^{245} + 10588160a^5b^{249}c^{-231}d^{-229}e^{246} - 10588160a^5b^{250}c^{-232}d^{-230}e^{247} + 10588160a^5b^{251}c^{-233}d$$

$$\begin{aligned}
& c^9 d^{11} e^6 + 5159936 a^5 b^{10} c^8 d^{10} e^7 + 1073920 a^5 b^{11} c^7 d^9 e^8 \\
& - 2279680 a^5 b^{12} c^6 d^8 e^9 + 770560 a^5 b^{13} c^5 d^7 e^{10} + 33280 a^5 b^{14} c^4 d^6 e^{11} - 41216 a^5 b^{15} c^3 d^5 e^{12} - 1280 a^5 b^{16} c^2 d^4 e^{13} \\
& + 327680 a^6 b^3 c^{14} d^{15} e^2 - 3276800 a^6 b^4 c^{13} d^{14} e^3 + 12615680 a^6 b^5 c^{12} d^{13} e^4 - 23592960 a^6 b^6 c^{11} d^{12} e^5 + 19701760 a^6 b^7 c^{10} d^{11} e^6 \\
& + 1372160 a^6 b^8 c^9 d^{10} e^7 - 15846400 a^6 b^9 c^8 d^9 e^8 + 10864640 a^6 b^{10} c^7 d^8 e^9 - 1352960 a^6 b^{11} c^6 d^7 e^{10} - 1111040 a^6 b^{12} c^5 d^6 e^{11} \\
& + 273920 a^6 b^{13} c^4 d^5 e^{12} + 25600 a^6 b^{14} c^3 d^4 e^{13} - 1280 a^6 b^{15} c^2 d^3 e^{14} + 3407872 a^7 b^2 c^{14} d^{14} e^3 - 14221312 a^7 b^3 c^{13} d^{13} e^4 \\
& + 23527424 a^7 b^4 c^{12} d^{12} e^5 - 3768320 a^7 b^5 c^{11} d^{11} e^6 - 38895616 a^7 b^6 c^{10} d^{10} e^7 + 50126848 a^7 b^7 c^9 d^9 e^8 - 18362368 a^7 b^8 c^8 d^8 e^9 \\
& - 6831104 a^7 b^9 c^7 d^7 e^{10} + 6200320 a^7 b^{10} c^6 d^6 e^{11} - 726784 a^7 b^{11} c^5 d^5 e^{12} - 228608 a^7 b^{12} c^4 d^4 e^{13} + 31488 a^7 b^{13} c^3 d^3 e^{14} \\
& + 2304 a^7 b^{14} c^2 d^2 e^{15} - 3145728 a^8 b^2 c^{13} d^{12} e^5 - 31129600 a^8 b^3 c^{12} d^{11} e^6 + 74711040 a^8 b^4 c^{11} d^{10} e^7 - 55476224 a^8 b^5 c^{10} d^9 e^8 \\
& - 11075584 a^8 b^6 c^9 d^8 e^9 + 35381248 a^8 b^7 c^8 d^7 e^{10} - 14479360 a^8 b^8 c^7 d^6 e^{11} - 168960 a^8 b^9 c^6 d^5 e^{12} + 1286144 a^8 b^{10} c^5 d^4 e^{13} \\
& - 302336 a^8 b^{11} c^4 d^3 e^{14} - 55808 a^8 b^{12} c^3 d^2 e^{15} - 36962304 a^9 b^2 c^{12} d^{10} e^7 - 9502720 a^9 b^3 c^{11} d^9 e^8 + 67174400 a^9 b^4 c^{10} d^8 e^9 \\
& - 54886400 a^9 b^5 c^9 d^7 e^{10} + 11239424 a^9 b^6 c^8 d^6 e^{11} + 5545984 a^9 b^7 c^7 d^5 e^{12} - 5263360 a^9 b^8 c^6 d^4 e^{13} + 1356800 a^9 b^9 c^5 d^3 e^{14} \\
& + 558080 a^9 b^{10} c^4 d^2 e^{15} - 49807360 a^{10} b^2 c^{11} d^8 e^9 + 19333120 a^{10} b^3 c^{10} d^7 e^{10} + 7208960 a^{10} b^4 c^9 d^6 e^{11} \\
& - 14974976 a^{10} b^5 c^8 d^5 e^{12} + 15073280 a^{10} b^6 c^7 d^4 e^{13} - 2170880 a^{10} b^7 c^6 d^3 e^{14} - 2928640 a^{10} b^8 c^5 d^2 e^{15} \\
& - 11796480 a^{11} b^2 c^{10} d^6 e^{11} + 23920640 a^{11} b^3 c^9 d^5 e^{12} - 24576000 a^{11} b^4 c^8 d^4 e^{13} - 4096000 a^{11} b^5 c^7 d^3 e^{14} + 8355840 a^{11} b^6 c^6 d^2 e^{15} \\
& + 12582912 a^{12} b^2 c^9 d^4 e^{13} + 19857408 a^{12} b^3 c^8 d^3 e^{14} - 11534336 a^{12} b^4 c^7 d^2 e^{15} + 3407872 a^{13} b^2 c^8 d^2 e^{15} \\
& - 5505024 a^{14} b^2 c^8 d^2 e^{16} - 262144 a^7 b^3 c^{15} d^{15} e^2 + 5505024 a^8 b^3 c^{14} d^{13} e^4 - 1280 a^8 b^{13} c^2 d^2 e^{16} + 25952256 a^9 b^3 c^{13} d^{11} e^6 \\
& + 30976 a^9 b^{11} c^3 d^2 e^{16} + 38010880 a^{10} b^3 c^{12} d^9 e^8 - 312320 a^{10} b^9 c^4 d^2 e^{16} + 11796480 a^{11} b^3 c^{11} d^7 e^{10} + 1679360 a^{11} b^7 c^5 d^2 e^{16} \\
& - 21233664 a^{12} b^3 c^{10} d^5 e^{12} - 5079040 a^{12} b^5 c^6 d^2 e^{16} - 20709376 a^{13} b^3 c^9 d^3 e^{14} + 8192000 a^{13} b^3 c^7 d^2 e^{16})) / (16 * (c^2 d^5 + a^2 d^4 e + b^2 d^3 e^2 - 2 b^2 c^2 d^4 e - 2 a^2 b^2 d^2 e^3 + 2 a^2 c^2 d^3 e^2) * (a^6 b^8 e^8 + 256 a^6 c^8 d^8 + 256 a^{10} c^4 e^8 - 16 a^7 b^6 c^2 e^8 - 4 a^5 b^9 d^2 e^7 + a^2 b^8 c^4 d^8 - 16 a^3 b^6 c^5 d^8 + 96 a^4 b^4 c^6 d^8 - 256 a^5 b^2 c^7 d^8 + 96 a^8 b^4 c^2 e^8 - 256 a^9 b^2 c^3 e^8 + a^2 b^{12} d^4 e^4 - 4 a^3 b^{11} d^3 e^5 + 6 a^4 b^{10} d^2 e^6 + 1024 a^7 c^7 d^6 e^2 + 1536 a^8 c^6 d^4 e^4 + 1024 a^9 c^5 d^2 e^6 + 6 a^2 b^{10} c^2 d^6 e^2 - 92 a^3 b^8 c^3 d^6 e^2 + 52 a^3 b^9 c^2 d^5 e^3 + 512 a^4 b^6 c^4 d^6 e^2 - 192 a^4 b^7 c^3 d^5 e^3 - 90 a^4 b^8 c^2 d^4 e^4 - 1152 a^5 b^4 c^5 d^6 e^2 - 128 a^5 b^5 c^4 d^5 e^3 + 800 a^5 b^6 c^3 d^4 e^4 - 192 a^5 b^7 c^2 d^3 e^5 + 512 a^6 b^2 c^6 d^6 e^2 + 2048 a^6 b^3 c^5 d^5 e^3 - 2240 a^6 b^4 c^4 d^4 e^4 - 128 a^6 b^5 c^3 d^3 e^5 + 512 a^6 b^6 c^2 d^2 e^6 + 1536 a^7 b^2 c^5 d^4 e^4 + 2048 a^7 b^3 c^4 d^3 e^5 - 1152 a^7 b^4 c^3 d^2 e^6 + 512 a^8 b^2 c^4 d^2 e^6 - 1024 a^6 b^3 c^7 d^7 e + 64 a^6 b^7 c^3 d^7 e - 1024 a^9 b^3 c^4 d^2 e^7 - 4 a^2 b^9 c^3 d^7 e - 4 a^2 b^{11} c^5 d^5 e^3 + 64 a^3 b^7 c^4 d^7 e - 4 a^3 b^{10} c^4 d^4 e^4 - 384 a^4 b^5 c^5 d^7 e + 52 a^4 b^9 c^3 d^3 e^5 + 1024 a^5 b^3 c^6 d^7 e - 92 a^5 b^8 c^3 d^2 e^6 - 3072 a^7 b^3 c^6 d^5 e^3 - 384 a^7 b^5 c^2 d^2 e^7 - 3072 a^8 b^3 c^5 d^3 e^5 + 1024 a^8 b^3 c^3 d^2 e^7))) * (-d^7)^{(1/2)} / (2 * (c^2 d^5 + a^2 d^4 e + b^2 d^3 e^2 - 2 b^2 c^2 d^4 e - 2 a^2 b^2 d^2 e^3 + 2 a^2 c^2 d^3 e^2)) * (-d^7)^{(1/2)} / (2 * (c^2 d^5 + a^2 d^4 e + b^2 d^3 e^2 - 2 b^2 c^2 d^4 e - 2 a^2 b^2 d^2 e^3 + 2 a^2 c^2 d^3 e^2))) / (2 * (c^2 d^5 + a^2 d^4 e + b^2 d^3 e^2 - 2 b^2 c^2 d^4 e - 2 a^2 b^2 d^2 e^3 + 2 a^2 c^2 d^3 e^2)) + (x * (22800 a^6 c^9 e^{13} + 36 a^2 b^8 c^5 e^{13} - 600 a^3 b^6 c^6 e^{13} + 4313 a^4 b^4 c^7 e^{13} - 15592 a^5 b^2 c^8 e^{13} + 1296 a^2 c^{13} d^8 e^5 + 9792 a^3 c^{12} d^6 e^7 + 30304 a^4 c^{11} d^4 e^9 + 40512 a^5 c^{10} d^2 e^{11} + 25 b^4 c^{11}
\end{aligned}$$

$$\begin{aligned}
& *d^8e^5 - 120*b^5*c^{10}*d^7e^6 + 214*b^6*c^9*d^6e^7 - 168*b^7*c^8*d^5e^8 \\
& + 53*b^8*c^7*d^4e^9 - 8*b^9*c^6*d^3e^{10} + 4*b^{10}*c^5*d^2e^{11} + 6336*a^2 \\
& *b^2*c^{11}*d^6e^7 + 3840*a^2*b^3*c^{10}*d^5e^8 - 8506*a^2*b^4*c^9*d^4e^9 + \\
& 1112*a^2*b^5*c^8*d^3e^{10} + 1254*a^2*b^6*c^7*d^2e^{11} + 22224*a^3*b^2*c^{10} \\
& d^4e^9 + 13824*a^3*b^3*c^9*d^3e^{10} - 9516*a^3*b^4*c^8*d^2e^{11} + 11712*a^4 \\
& *b^2*c^9*d^2e^{11} - 24*a*b^9*c^5*d^2e^{12} - 41088*a^5*b*c^9*d^2e^{12} - 360*a*b \\
& ^2*c^{12}*d^8e^5 + 1664*a*b^3*c^{11}*d^7e^6 - 2604*a*b^4*c^{10}*d^6e^7 + 1272* \\
& a*b^5*c^9*d^5e^8 + 332*a*b^6*c^8*d^4e^9 - 232*a*b^7*c^7*d^3e^{10} - 48*a*b \\
& ^8*c^6*d^2e^{11} - 5760*a^2*b*c^{12}*d^7e^6 + 416*a^2*b^7*c^6*d^2e^{12} - 32128* \\
& a^3*b*c^{11}*d^5e^8 - 4120*a^3*b^5*c^7*d^2e^{12} - 63360*a^4*b*c^{10}*d^3e^{10} + \\
& 21376*a^4*b^3*c^8*d^2e^{12}))/((a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^{10}*c^4 \\
& *e^8 - 16*a^7*b^6*c^2e^8 - 4*a^5*b^9*d^2e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5 \\
& *d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2e^8 - 256 \\
& *a^9*b^2*c^3e^8 + a^2*b^{12}*d^4e^4 - 4*a^3*b^{11}*d^3e^5 + 6*a^4*b^{10}*d^2e^6 \\
& + 1024*a^7*c^7*d^6e^2 + 1536*a^8*c^6*d^4e^4 + 1024*a^9*c^5*d^2e^6 + 6 \\
& *a^2*b^{10}*c^2*d^6e^2 - 92*a^3*b^8*c^3*d^6e^2 + 52*a^3*b^9*c^2*d^5e^3 + 5 \\
& 12*a^4*b^6*c^4*d^6e^2 - 192*a^4*b^7*c^3*d^5e^3 - 90*a^4*b^8*c^2*d^4e^4 - \\
& 1152*a^5*b^4*c^5*d^6e^2 - 128*a^5*b^5*c^4*d^5e^3 + 800*a^5*b^6*c^3*d^4e^4 \\
& - 192*a^5*b^7*c^2*d^3e^5 + 512*a^6*b^2*c^6*d^6e^2 + 2048*a^6*b^3*c^5*d^5 \\
& e^3 - 2240*a^6*b^4*c^4*d^4e^4 - 128*a^6*b^5*c^3*d^3e^5 + 512*a^6*b^6*c^2 \\
& *d^2e^6 + 1536*a^7*b^2*c^5*d^4e^4 + 2048*a^7*b^3*c^4*d^3e^5 - 1152*a^7 \\
& *b^4*c^3*d^2e^6 + 512*a^8*b^2*c^4*d^2e^6 - 1024*a^6*b*c^7*d^7e + 64*a^6* \\
& b^7*c*d^2e^7 - 1024*a^9*b*c^4*d^2e^7 - 4*a^2*b^9*c^3*d^7e - 4*a^2*b^{11}*c*d^5 \\
& *e^3 + 64*a^3*b^7*c^4*d^7e - 4*a^3*b^{10}*c*d^4e^4 - 384*a^4*b^5*c^5*d^7e \\
& + 52*a^4*b^9*c*d^3e^5 + 1024*a^5*b^3*c^6*d^7e - 92*a^5*b^8*c*d^2e^6 - 30 \\
& 72*a^7*b*c^6*d^5e^3 - 384*a^7*b^5*c^2*d^2e^7 - 3072*a^8*b*c^5*d^3e^5 + 102 \\
& 4*a^8*b^3*c^3*d^2e^7)))*(-d^7)^{(1/2)}*i)/((2*(c^2*d^5 + a^2*d^4 + b^2*d^3 \\
& *e^2 - 2*b*c*d^4e - 2*a*b*d^2e^3 + 2*a*c*d^3e^2)) - (((-d^7)^{(1/2)}*(( \\
& 326912*a^8*c^9*d^2e^{13} - 241664*a^8*b*c^8e^{14} - 48*a^2*b^{13}*c^2e^{14} + 1264 \\
& *a^3*b^{11}*c^3e^{14} - 13552*a^4*b^9*c^4e^{14} + 75776*a^5*b^7*c^5e^{14} - 2329 \\
& 60*a^6*b^5*c^6e^{14} + 372736*a^7*b^3*c^7e^{14} + 11520*a^3*c^{14}*d^{11}e^3 + 7 \\
& 8080*a^4*c^{13}*d^9e^5 + 197120*a^5*c^{12}*d^7e^7 + 336384*a^6*c^{11}*d^5e^9 + \\
& 532736*a^7*c^{10}*d^3e^{11} - 40*b^5*c^{12}*d^{12}e^2 + 216*b^6*c^{11}*d^{11}e^3 - \\
& 464*b^7*c^{10}*d^{10}e^4 + 496*b^8*c^9*d^9e^5 - 264*b^9*c^8*d^8e^6 + 56*b^{10} \\
& *c^7*d^7e^7 - 16*b^{11}*c^6*d^6e^8 + 64*b^{12}*c^5*d^5e^9 - 96*b^{13}*c^4*d^4e^{10} \\
& + 64*b^{14}*c^3*d^3e^{11} - 16*b^{15}*c^2*d^2e^{12} + 1536*a^2*b^2*c^{13}*d^{11} \\
& *e^3 + 14400*a^2*b^3*c^{12}*d^{10}e^4 - 47152*a^2*b^4*c^{11}*d^9e^5 + 52144*a^2 \\
& *b^5*c^{10}*d^8e^6 - 16272*a^2*b^6*c^9*d^7e^7 - 13040*a^2*b^7*c^8*d^6e^8 + \\
& 23488*a^2*b^8*c^7*d^5e^9 - 26384*a^2*b^9*c^6*d^4e^{10} + 13824*a^2*b^{10}*c^5 \\
& *d^3e^{11} + 256*a^2*b^{11}*c^4*d^2e^{12} + 125056*a^3*b^2*c^{12}*d^9e^5 - 3622 \\
& 4*a^3*b^3*c^{11}*d^8e^6 - 126432*a^3*b^4*c^{10}*d^7e^7 + 144848*a^3*b^5*c^9*d^6 \\
& e^8 - 114752*a^3*b^6*c^8*d^5e^9 + 125392*a^3*b^7*c^7*d^4e^{10} - 53248*a^3 \\
& *b^8*c^6*d^3e^{11} - 25264*a^3*b^9*c^5*d^2e^{12} + 474112*a^4*b^2*c^{11}*d^7e^7 \\
& - 191104*a^4*b^3*c^{10}*d^6e^8 + 97184*a^4*b^4*c^9*d^5e^9 - 277000*a^4*b^5 \\
& *c^8*d^4e^{10} + 56056*a^4*b^6*c^7*d^3e^{11} + 195584*a^4*b^7*c^6*d^2e^{12} \\
& + 236800*a^5*b^2*c^{10}*d^5e^9 + 388032*a^5*b^3*c^9*d^4e^{10} + 159632*a^5*b^4 \\
& *c^8*d^3e^{11} - 670488*a^5*b^5*c^7*d^2e^{12} - 488960*a^6*b^2*c^9*d^3e^{11} \\
& + 1106496*a^6*b^3*c^8*d^2e^{12} + 64*a*b^{14}*c^2*d^2e^{13} + 448*a*b^3*c^{13}*d^1 \\
& 2e^2 - 1968*a*b^4*c^{12}*d^{11}e^3 + 2504*a*b^5*c^{11}*d^{10}e^4 + 768*a*b^6*c^{11} \\
& 0*d^9e^5 - 4368*a*b^7*c^9*d^8e^6 + 3568*a*b^8*c^8*d^7e^7 - 520*a*b^9*c^7 \\
& *d^6e^8 - 1728*a*b^{10}*c^6*d^5e^9 + 2528*a*b^{11}*c^5*d^4e^{10} - 1536*a*b^{12} \\
& *c^4*d^3e^{11} + 240*a*b^{13}*c^3*d^2e^{12} - 1152*a^2*b*c^{14}*d^{12}e^2 - 1600*a^2 \\
& *b^{12}*c^3*d^2e^{13} - 67968*a^3*b*c^{13}*d^{10}e^4 + 15808*a^3*b^{10}*c^4*d^2e^{13} \\
& - 342272*a^4*b*c^{12}*d^8e^6 - 76928*a^4*b^8*c^5*d^2e^{13} - 569088*a^5*b*c^{11} \\
& d^6e^8 + 179200*a^5*b^6*c^6*d^2e^{13} - 586368*a^6*b*c^{10}*d^4e^{10} - 113008*a^6 \\
& *b^4*c^7*d^2e^{13} - 731008*a^7*b*c^9*d^2e^{12} - 244096*a^7*b^2*c^8*d^2e^{13}))/ \\
& (16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^{10}*c^4*e^8 - 16*a^7*b^6*c^2e^8 - \\
& 4*a^5*b^9*d^2e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 \\
& - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2e^8 - 256*a^9*b^2*c^3e^8 + a^2*b^{11}
\end{aligned}$$

$$\begin{aligned}
& 2*d^4*e^4 - 4*a^3*b^{11}*d^3*e^5 + 6*a^4*b^{10}*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 \\
& + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^{10}*c^2*d^6*e^2 - 92 \\
& *a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 1 \\
& 92*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 \\
& - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e \\
& ^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4* \\
& d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2* \\
& c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8 \\
& *b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c \\
& ^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^{11}*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7* \\
& e - 4*a^3*b^{10}*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1 \\
& 024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384 \\
& *a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) - (( \\
& (x*(626688*a^{10}*b*c^8*e^{15} - 784384*a^{10}*c^9*d*e^{14} + 208*a^4*b^{13}*c^2*e^{15} \\
& - 4880*a^5*b^{11}*c^3*e^{15} + 47312*a^6*b^9*c^4*e^{15} - 242176*a^7*b^7*c^5*e^{15} \\
& + 688640*a^8*b^5*c^6*e^{15} - 1028096*a^9*b^3*c^7*e^{15} + 18432*a^4*c^{15}*d^1 \\
& 3*e^2 + 126976*a^5*c^{14}*d^{11}*e^4 + 325632*a^6*c^{13}*d^9*e^6 + 139264*a^7*c^{1 \\
& 2}*d^7*e^8 - 1067008*a^8*c^{11}*d^5*e^{10} - 1773568*a^9*c^{10}*d^3*e^{12} + 16*b^8* \\
& c^{11}*d^{13}*e^2 - 96*b^9*c^{10}*d^{12}*e^3 + 240*b^{10}*c^9*d^{11}*e^4 - 304*b^{11}*c^8 \\
& *d^{10}*e^5 + 144*b^{12}*c^7*d^9*e^6 + 144*b^{13}*c^6*d^8*e^7 - 304*b^{14}*c^5*d^7* \\
& e^8 + 240*b^{15}*c^4*d^6*e^9 - 96*b^{16}*c^3*d^5*e^{10} + 16*b^{17}*c^2*d^4*e^{11} + \\
& 3200*a^2*b^4*c^{13}*d^{13}*e^2 - 18432*a^2*b^5*c^{12}*d^{12}*e^3 + 41024*a^2*b^6*c^{ \\
& 11}*d^{11}*e^4 - 36352*a^2*b^7*c^{10}*d^{10}*e^5 - 16208*a^2*b^8*c^9*d^9*e^6 + 745 \\
& 76*a^2*b^9*c^8*d^8*e^7 - 78496*a^2*b^{10}*c^7*d^7*e^8 + 32064*a^2*b^{11}*c^6*d^ \\
& 6*e^9 + 6000*a^2*b^{12}*c^5*d^5*e^{10} - 9264*a^2*b^{13}*c^4*d^4*e^{11} + 1472*a^2* \\
& b^{14}*c^3*d^3*e^{12} + 416*a^2*b^{15}*c^2*d^2*e^{13} - 12800*a^3*b^2*c^{14}*d^{13}*e^2 \\
& + 73728*a^3*b^3*c^{13}*d^{12}*e^3 - 151296*a^3*b^4*c^{12}*d^{11}*e^4 + 78336*a^3*b \\
& ^5*c^{11}*d^{10}*e^5 + 206688*a^3*b^6*c^{10}*d^9*e^6 - 436736*a^3*b^7*c^9*d^8*e^7 \\
& + 324224*a^3*b^8*c^8*d^7*e^8 + 992*a^3*b^9*c^7*d^6*e^9 - 158176*a^3*b^{10}*c \\
& ^6*d^5*e^{10} + 77056*a^3*b^{11}*c^5*d^4*e^{11} + 6912*a^3*b^{12}*c^4*d^3*e^{12} - 84 \\
& 16*a^3*b^{13}*c^3*d^2*e^{13} + 162816*a^4*b^2*c^{13}*d^{11}*e^4 + 184320*a^4*b^3*c^ \\
& 12*d^{10}*e^5 - 916608*a^4*b^4*c^{11}*d^9*e^6 + 1165824*a^4*b^5*c^{10}*d^8*e^7 - \\
& 314496*a^4*b^6*c^9*d^7*e^8 - 822272*a^4*b^7*c^8*d^6*e^9 + 919152*a^4*b^8*c^ \\
& 7*d^5*e^{10} - 175296*a^4*b^9*c^6*d^4*e^{11} - 189328*a^4*b^{10}*c^5*d^3*e^{12} + 6 \\
& 2064*a^4*b^{11}*c^4*d^2*e^{13} + 1290752*a^5*b^2*c^{12}*d^9*e^6 - 659456*a^5*b^3* \\
& c^{11}*d^8*e^7 - 1561088*a^5*b^4*c^{10}*d^7*e^8 + 3240960*a^5*b^5*c^9*d^6*e^9 - \\
& 1964192*a^5*b^6*c^8*d^5*e^{10} - 683008*a^5*b^7*c^7*d^4*e^{11} + 1162304*a^5*b \\
& ^8*c^6*d^3*e^{12} - 164112*a^5*b^9*c^5*d^2*e^{13} + 3442688*a^6*b^2*c^{11}*d^7*e^ \\
& 8 - 3670016*a^6*b^3*c^{10}*d^6*e^9 + 15232*a^6*b^4*c^9*d^5*e^{10} + 4230144*a^6 \\
& *b^5*c^8*d^4*e^{11} - 3059648*a^6*b^6*c^7*d^3*e^{12} - 247296*a^6*b^7*c^6*d^2*e \\
& ^{13} + 4010496*a^7*b^2*c^{10}*d^5*e^{10} - 6873088*a^7*b^3*c^9*d^4*e^{11} + 282240 \\
& 0*a^7*b^4*c^8*d^3*e^{12} + 2370048*a^7*b^5*c^7*d^2*e^{13} + 1178624*a^8*b^2*c^9 \\
& *d^3*e^{12} - 4739072*a^8*b^3*c^8*d^2*e^{13} - 352*a*b^6*c^{12}*d^{13}*e^2 + 2048*a \\
& *b^7*c^{11}*d^{12}*e^3 - 4800*a*b^8*c^{10}*d^{11}*e^4 + 5168*a*b^9*c^9*d^{10}*e^5 - 4 \\
& 80*a*b^{10}*c^8*d^9*e^6 - 6000*a*b^{11}*c^7*d^8*e^7 + 8192*a*b^{12}*c^6*d^7*e^8 - \\
& 5040*a*b^{13}*c^5*d^6*e^9 + 1152*a*b^{14}*c^4*d^5*e^{10} + 240*a*b^{15}*c^3*d^4*e^ \\
& 11 - 128*a*b^{16}*c^2*d^3*e^{12} - 512*a^3*b^{14}*c^2*d*e^{14} - 106496*a^4*b*c^{14} \\
& d^{12}*e^3 + 11680*a^4*b^{12}*c^3*d*e^{14} - 675840*a^5*b*c^{13}*d^{10}*e^5 - 108288* \\
& a^5*b^{10}*c^4*d*e^{14} - 1601536*a^6*b*c^{12}*d^8*e^7 + 514768*a^6*b^8*c^5*d*e^{14} \\
& - 925696*a^7*b*c^{11}*d^6*e^9 - 1278304*a^7*b^6*c^6*d*e^{14} + 2457600*a^8*b* \\
& c^{10}*d^4*e^{11} + 1385600*a^8*b^4*c^7*d*e^{14} + 2977792*a^9*b*c^9*d^2*e^{13} + 1 \\
& 9968*a^9*b^2*c^8*d*e^{14}))/((8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^{10}*c^4* \\
& e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5 \\
& *d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256* \\
& a^9*b^2*c^3*e^8 + a^2*b^{12}*d^4*e^4 - 4*a^3*b^{11}*d^3*e^5 + 6*a^4*b^{10}*d^2*e^ \\
& 6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6* \\
& a^2*b^{10}*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 51 \\
& 2*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - \\
& 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^
\end{aligned}$$

$$\begin{aligned}
& 4 - 192a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c^4d^4e^4 - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 + 2048a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024a^6b^7c^7d^7e + 64a^6b^7c^7d^7e - 1024a^9b^3c^4d^4e^7 - 4a^2b^9c^3d^7e - 4a^2b^11c^5d^5e^3 + 64a^3b^7c^4d^7e - 4a^3b^10c^4d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^3d^3e^5 + 1024a^5b^3c^6d^7e - 92a^5b^8c^3d^2e^6 - 3072a^7b^3c^6d^5e^3 - 384a^7b^5c^2d^4e^7 - 3072a^8b^3c^5d^3e^5 + 1024a^8b^3c^3d^4e^7) + ((1048576a^13c^8e^16 + 256a^7b^12c^2e^16 - 6144a^8b^10c^3e^16 + 61440a^9b^8c^4e^16 - 327680a^10b^6c^5e^16 + 983040a^11b^4c^6e^16 - 1572864a^12b^2c^7e^16 - 196608a^6c^15d^14e^2 - 917504a^7c^14d^12e^4 - 589824a^8c^13d^10e^6 + 3932160a^9c^12d^8e^8 + 10158080a^10c^11d^6e^10 + 10616832a^11c^10d^4e^12 + 5308416a^12c^9d^2e^14 - 2816a^2b^8c^11d^14e^2 + 22656a^2b^9c^10d^13e^3 - 78848a^2b^10c^9d^12e^4 + 154112a^2b^11c^8d^11e^5 - 182784a^2b^12c^7d^10e^6 + 130816a^2b^13c^6d^9e^7 - 50176a^2b^14c^5d^8e^8 + 4608a^2b^15c^4d^7e^9 + 3328a^2b^16c^3d^6e^10 - 896a^2b^17c^2d^5e^11 + 24576a^3b^6c^12d^14e^2 - 198656a^3b^7c^11d^13e^3 + 684544a^3b^8c^10d^12e^4 - 1291520a^3b^9c^9d^11e^5 + 1403776a^3b^10c^8d^10e^6 - 798336a^3b^11c^7d^9e^7 + 89856a^3b^12c^6d^8e^8 + 155136a^3b^13c^5d^7e^9 - 77440a^3b^14c^4d^6e^10 + 5504a^3b^15c^3d^5e^11 + 2560a^3b^16c^2d^4e^12 - 106496a^4b^4c^13d^14e^2 + 864256a^4b^5c^12d^13e^3 - 2924544a^4b^6c^11d^12e^4 + 5181440a^4b^7c^10d^11e^5 - 4686080a^4b^8c^9d^10e^6 + 1045376a^4b^9c^8d^9e^7 + 1900544a^4b^10c^7d^8e^8 - 1732096a^4b^11c^6d^7e^9 + 390400a^4b^12c^5d^6e^10 + 112000a^4b^13c^4d^5e^11 - 40960a^4b^14c^3d^4e^12 - 3840a^4b^15c^2d^3e^13 + 229376a^5b^2c^14d^14e^2 - 1867776a^5b^3c^13d^13e^3 + 6078464a^5b^4c^12d^12e^4 - 9297920a^5b^5c^11d^11e^5 + 4055040a^5b^6c^10d^10e^6 + 7788544a^5b^7c^9d^9e^7 - 12657664a^5b^8c^8d^8e^8 + 6130176a^5b^9c^7d^7e^9 + 734080a^5b^10c^6d^6e^10 - 1442560a^5b^11c^5d^5e^11 + 168960a^5b^12c^4d^4e^12 + 78080a^5b^13c^3d^3e^13 + 3200a^5b^14c^2d^2e^14 - 4587520a^6b^2c^13d^12e^4 + 3080192a^6b^3c^12d^11e^5 + 12001280a^6b^4c^11d^10e^6 - 31076352a^6b^5c^10d^9e^7 + 27475968a^6b^6c^9d^8e^8 - 2088960a^6b^7c^8d^7e^9 - 12205312a^6b^8c^7d^6e^10 + 6043520a^6b^9c^6d^5e^11 + 631808a^6b^10c^5d^4e^12 - 610304a^6b^11c^4d^3e^13 - 71936a^6b^12c^3d^2e^14 - 21725184a^7b^2c^12d^10e^6 + 30801920a^7b^3c^11d^9e^7 - 8028160a^7b^4c^10d^8e^8 - 32260096a^7b^5c^9d^7e^9 + 37101568a^7b^6c^8d^6e^10 - 7182336a^7b^7c^7d^5e^11 - 7609856a^7b^8c^6d^4e^12 + 2112256a^7b^9c^5d^3e^13 + 661632a^7b^10c^4d^2e^14 - 30146560a^8b^2c^11d^8e^8 + 55050240a^8b^3c^10d^7e^9 - 34365440a^8b^4c^9d^6e^10 - 16429056a^8b^5c^8d^5e^11 + 24600576a^8b^6c^7d^4e^12 - 1683456a^8b^7c^6d^3e^13 - 3151616a^8b^8c^5d^2e^14 - 10977280a^9b^2c^10d^6e^10 + 47022080a^9b^3c^9d^5e^11 - 30621696a^9b^4c^8d^4e^12 - 9232384a^9b^5c^7d^3e^13 + 7970816a^9b^6c^6d^2e^14 + 4325376a^10b^2c^9d^4e^12 + 25493504a^10b^3c^8d^3e^13 - 9117696a^10b^4c^7d^2e^14 + 491520a^11b^2c^8d^2e^14 - 4947968a^12b^3c^8d^2e^15 + 128a^2b^10c^10d^14e^2 - 1024a^2b^11c^9d^13e^3 + 3584a^2b^12c^8d^12e^4 - 7168a^2b^13c^7d^11e^5 + 8960a^2b^14c^6d^10e^6 - 7168a^2b^15c^5d^9e^7 + 3584a^2b^16c^4d^8e^8 - 1024a^2b^17c^3d^7e^9 + 128a^2b^18c^2d^6e^10 + 1605632a^6b^3c^14d^13e^3 - 1408a^6b^13c^2d^4e^15 + 7012352a^7b^3c^13d^11e^5 + 33152a^7b^11c^3d^4e^15 + 7045120a^8b^3c^12d^9e^7 - 324480a^8b^9c^4d^4e^15 - 9830400a^9b^3c^11d^7e^9 + 1689600a^9b^7c^5d^4e^15 - 25722880a^10b^3c^10d^5e^11 - 4935680a^10b^5c^6d^4e^15 - 19202048a^11b^3c^9d^3e^13 + 7667712a^11b^3c^7d^4e^15)/(16*(a^6b^8e^8 + 256a^6c^8d^8 + 256a^10c^4e^8 - 16a^7b^6c^4e^8 - 4a^5b^9d^4e^7 + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6d^8 - 256a^5b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^12d^4e^4 - 4a^3b^11d^3e^5 + 6a^4b^1
\end{aligned}$$

$$\begin{aligned}
& 0*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2* \\
& e^6 + 6*a^2*b^{10}*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5* \\
& e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^ \\
& 4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^ \\
& 3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^ \\
& 3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^ \\
& 6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1 \\
& 152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + \\
& 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^1 \\
& 1*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^{10}*c*d^4*e^4 - 384*a^4*b^5*c^5 \\
& *d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e \\
& ^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^ \\
& 5 + 1024*a^8*b^3*c^3*d*e^7) + (x*(-d*e^7)^{(1/2)}*(1048576*a^{15}*c^8*e^{17} + 2 \\
& 56*a^9*b^{12}*c^2*e^{17} - 6144*a^{10}*b^{10}*c^3*e^{17} + 61440*a^{11}*b^8*c^4*e^{17} - \\
& 327680*a^{12}*b^6*c^5*e^{17} + 983040*a^{13}*b^4*c^6*e^{17} - 1572864*a^{14}*b^2*c^7* \\
& e^{17} - 1048576*a^8*c^{15}*d^{14}*e^3 - 5242880*a^9*c^{14}*d^{12}*e^5 - 9437184*a^{10} \\
& *c^{13}*d^{10}*e^7 - 5242880*a^{11}*c^{12}*d^8*e^9 + 5242880*a^{12}*c^{11}*d^6*e^{11} + 9 \\
& 437184*a^{13}*c^{10}*d^4*e^{13} + 5242880*a^{14}*c^9*d^2*e^{15} + 256*a^2*b^{11}*c^{10}*d \\
& ^{15}*e^2 - 2048*a^2*b^{12}*c^9*d^{14}*e^3 + 7168*a^2*b^{13}*c^8*d^{13}*e^4 - 14336*a \\
& ^2*b^{14}*c^7*d^{12}*e^5 + 17920*a^2*b^{15}*c^6*d^{11}*e^6 - 14336*a^2*b^{16}*c^5*d^{1 \\
& 0}*e^7 + 7168*a^2*b^{17}*c^4*d^9*e^8 - 2048*a^2*b^{18}*c^3*d^8*e^9 + 256*a^2*b^{1 \\
& 9}*c^2*d^7*e^{10} - 5120*a^3*b^9*c^{11}*d^{15}*e^2 + 41984*a^3*b^{10}*c^{10}*d^{14}*e^3 \\
& - 148736*a^3*b^{11}*c^9*d^{13}*e^4 + 296192*a^3*b^{12}*c^8*d^{12}*e^5 - 359680*a^3* \\
& b^{13}*c^7*d^{11}*e^6 + 267520*a^3*b^{14}*c^6*d^{10}*e^7 - 112384*a^3*b^{15}*c^5*d^9* \\
& e^8 + 18176*a^3*b^{16}*c^4*d^8*e^9 + 3328*a^3*b^{17}*c^3*d^7*e^{10} - 1280*a^3*b^ \\
& 18*c^2*d^6*e^{11} + 40960*a^4*b^7*c^{12}*d^{15}*e^2 - 348160*a^4*b^8*c^{11}*d^{14}*e^ \\
& 3 + 1254400*a^4*b^9*c^{10}*d^{13}*e^4 - 2478080*a^4*b^{10}*c^9*d^{12}*e^5 + 2867456 \\
& *a^4*b^{11}*c^8*d^{11}*e^6 - 1862144*a^4*b^{12}*c^7*d^{10}*e^7 + 490240*a^4*b^{13}*c^ \\
& 6*d^9*e^8 + 128000*a^4*b^{14}*c^5*d^8*e^9 - 108800*a^4*b^{15}*c^4*d^7*e^{10} + 13 \\
& 824*a^4*b^{16}*c^3*d^6*e^{11} + 2304*a^4*b^{17}*c^2*d^5*e^{12} - 163840*a^5*b^5*c^1 \\
& 3*d^{15}*e^2 + 1474560*a^5*b^6*c^{12}*d^{14}*e^3 - 5447680*a^5*b^7*c^{11}*d^{13}*e^4 \\
& + 10588160*a^5*b^8*c^{10}*d^{12}*e^5 - 11166720*a^5*b^9*c^9*d^{11}*e^6 + 5159936* \\
& a^5*b^{10}*c^8*d^{10}*e^7 + 1073920*a^5*b^{11}*c^7*d^9*e^8 - 2279680*a^5*b^{12}*c^6 \\
& *d^8*e^9 + 770560*a^5*b^{13}*c^5*d^7*e^{10} + 33280*a^5*b^{14}*c^4*d^6*e^{11} - 412 \\
& 16*a^5*b^{15}*c^3*d^5*e^{12} - 1280*a^5*b^{16}*c^2*d^4*e^{13} + 327680*a^6*b^3*c^14 \\
& *d^{15}*e^2 - 3276800*a^6*b^4*c^{13}*d^{14}*e^3 + 12615680*a^6*b^5*c^{12}*d^{13}*e^4 \\
& - 23592960*a^6*b^6*c^{11}*d^{12}*e^5 + 19701760*a^6*b^7*c^{10}*d^{11}*e^6 + 1372160 \\
& *a^6*b^8*c^9*d^{10}*e^7 - 15846400*a^6*b^9*c^8*d^9*e^8 + 10864640*a^6*b^{10}*c^ \\
& 7*d^8*e^9 - 1352960*a^6*b^{11}*c^6*d^7*e^{10} - 1111040*a^6*b^{12}*c^5*d^6*e^{11} + \\
& 273920*a^6*b^{13}*c^4*d^5*e^{12} + 25600*a^6*b^{14}*c^3*d^4*e^{13} - 1280*a^6*b^{15} \\
& *c^2*d^3*e^{14} + 3407872*a^7*b^2*c^{14}*d^{14}*e^3 - 14221312*a^7*b^3*c^{13}*d^{13}* \\
& e^4 + 23527424*a^7*b^4*c^{12}*d^{12}*e^5 - 3768320*a^7*b^5*c^{11}*d^{11}*e^6 - 3889 \\
& 5616*a^7*b^6*c^{10}*d^{10}*e^7 + 50126848*a^7*b^7*c^9*d^9*e^8 - 18362368*a^7*b^ \\
& 8*c^8*d^8*e^9 - 6831104*a^7*b^9*c^7*d^7*e^{10} + 6200320*a^7*b^{10}*c^6*d^6*e^{1 \\
& 1} - 726784*a^7*b^{11}*c^5*d^5*e^{12} - 228608*a^7*b^{12}*c^4*d^4*e^{13} + 31488*a^7 \\
& *b^{13}*c^3*d^3*e^{14} + 2304*a^7*b^{14}*c^2*d^2*e^{15} - 3145728*a^8*b^2*c^{13}*d^{12} \\
& *e^5 - 31129600*a^8*b^3*c^{12}*d^{11}*e^6 + 74711040*a^8*b^4*c^{11}*d^{10}*e^7 - 55 \\
& 476224*a^8*b^5*c^{10}*d^9*e^8 - 11075584*a^8*b^6*c^9*d^8*e^9 + 35381248*a^8*b^ \\
& 7*c^8*d^7*e^{10} - 14479360*a^8*b^8*c^7*d^6*e^{11} - 168960*a^8*b^9*c^6*d^5*e^ \\
& 12 + 1286144*a^8*b^{10}*c^5*d^4*e^{13} - 302336*a^8*b^{11}*c^4*d^3*e^{14} - 55808*a^ \\
& 8*b^{12}*c^3*d^2*e^{15} - 36962304*a^9*b^2*c^{12}*d^{10}*e^7 - 9502720*a^9*b^3*c^1 \\
& 1*d^9*e^8 + 67174400*a^9*b^4*c^{10}*d^8*e^9 - 54886400*a^9*b^5*c^9*d^7*e^{10} + \\
& 11239424*a^9*b^6*c^8*d^6*e^{11} + 5545984*a^9*b^7*c^7*d^5*e^{12} - 5263360*a^9 \\
& *b^8*c^6*d^4*e^{13} + 1356800*a^9*b^9*c^5*d^3*e^{14} + 558080*a^9*b^{10}*c^4*d^2* \\
& e^{15} - 49807360*a^{10}*b^2*c^{11}*d^8*e^9 + 19333120*a^{10}*b^3*c^{10}*d^7*e^{10} + 7 \\
& 208960*a^{10}*b^4*c^9*d^6*e^{11} - 14974976*a^{10}*b^5*c^8*d^5*e^{12} + 15073280*a^ \\
& 10*b^6*c^7*d^4*e^{13} - 2170880*a^{10}*b^7*c^6*d^3*e^{14} - 2928640*a^{10}*b^8*c^5* \\
& d^2*e^{15} - 11796480*a^{11}*b^2*c^{10}*d^6*e^{11} + 23920640*a^{11}*b^3*c^9*d^5*e^{12} \\
& - 24576000*a^{11}*b^4*c^8*d^4*e^{13} - 4096000*a^{11}*b^5*c^7*d^3*e^{14} + 8355840
\end{aligned}$$

$$\begin{aligned}
& *a^{11}b^6c^6d^2e^{15} + 12582912a^{12}b^2c^9d^4e^{13} + 19857408a^{12}b^3 \\
& *c^8d^3e^{14} - 11534336a^{12}b^4c^7d^2e^{15} + 3407872a^{13}b^2c^8d^2e \\
& ^{15} - 5505024a^{14}b^*c^8d^*e^{16} - 262144a^7b^*c^{15}d^{15}e^2 + 5505024a^8* \\
& b^*c^{14}d^{13}e^4 - 1280a^8b^{13}c^2d^*e^{16} + 25952256a^9b^*c^{13}d^{11}e^6 + \\
& 30976a^9b^{11}c^3d^*e^{16} + 38010880a^{10}b^*c^{12}d^9e^8 - 312320a^{10}b^9 \\
& *c^4d^*e^{16} + 11796480a^{11}b^*c^{11}d^7e^{10} + 1679360a^{11}b^7c^5d^*e^{16} - \\
& 21233664a^{12}b^*c^{10}d^5e^{12} - 5079040a^{12}b^5c^6d^*e^{16} - 20709376a^{1 \\
& 3}b^*c^9d^3e^{14} + 8192000a^{13}b^3c^7d^*e^{16}))/((16*(c^2d^5 + a^2d^*e^4 + \\
& b^2d^3e^2 - 2b^*c^d^4e - 2a^*b^d^2e^3 + 2a^*c^d^3e^2))*(a^6b^8e^8 + \\
& 256a^6c^8d^8 + 256a^{10}c^4e^8 - 16a^7b^6c^*e^8 - 4a^5b^9d^*e^7 + a \\
& ^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6d^8 - 256a^5b^2c^7* \\
& d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^{12}d^4e^4 - 4a^3b \\
& ^{11}d^3e^5 + 6a^4b^{10}d^2e^6 + 1024a^7c^7d^6e^2 + 1536a^8c^6d^4* \\
& e^4 + 1024a^9c^5d^2e^6 + 6a^2b^{10}c^2d^6e^2 - 92a^3b^8c^3d^6e^ \\
& 2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4b^7c^3d^5* \\
& e^3 - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d \\
& ^5e^3 + 800a^5b^6c^3d^4e^4 - 192a^5b^7c^2d^3e^5 + 512a^6b^2c^ \\
& 6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c^4d^4e^4 - 128a^6b \\
& ^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 + 2048* \\
& a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - \\
& 1024a^6b^*c^7d^7e + 64a^6b^7c^*d^*e^7 - 1024a^9b^*c^4d^*e^7 - 4a^2b^ \\
& 9c^3d^7e - 4a^2b^{11}c^*d^5e^3 + 64a^3b^7c^4d^7e - 4a^3b^{10}c^*d^ \\
& 4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^*d^3e^5 + 1024a^5b^3c^6d^7 \\
& *e - 92a^5b^8c^*d^2e^6 - 3072a^7b^*c^6d^5e^3 - 384a^7b^5c^2d^*e^7 \\
& - 3072a^8b^*c^5d^3e^5 + 1024a^8b^3c^3d^*e^7)))*(-d^*e^7)^{(1/2)))/(2*(c^ \\
& 2d^5 + a^2d^*e^4 + b^2d^3e^2 - 2b^*c^d^4e - 2a^*b^d^2e^3 + 2a^*c^d^3e \\
& ^2)))*(-d^*e^7)^{(1/2)))/(2*(c^2d^5 + a^2d^*e^4 + b^2d^3e^2 - 2b^*c^d^4e - \\
& 2a^*b^d^2e^3 + 2a^*c^d^3e^2)))/((2*(c^2d^5 + a^2d^*e^4 + b^2d^3e^2 - \\
& 2b^*c^d^4e - 2a^*b^d^2e^3 + 2a^*c^d^3e^2)) - (x*(22800a^6c^9e^{13} + 36 \\
& *a^2b^8c^5e^{13} - 600a^3b^6c^6e^{13} + 4313a^4b^4c^7e^{13} - 15592a^ \\
& 5b^2c^8e^{13} + 1296a^2c^{13}d^8e^5 + 9792a^3c^{12}d^6e^7 + 30304a^4* \\
& c^{11}d^4e^9 + 40512a^5c^{10}d^2e^{11} + 25b^4c^{11}d^8e^5 - 120b^5c^{10} \\
& *d^7e^6 + 214b^6c^9d^6e^7 - 168b^7c^8d^5e^8 + 53b^8c^7d^4e^9 - \\
& 8b^9c^6d^3e^{10} + 4b^{10}c^5d^2e^{11} + 6336a^2b^2c^{11}d^6e^7 + 384 \\
& 0a^2b^3c^{10}d^5e^8 - 8506a^2b^4c^9d^4e^9 + 1112a^2b^5c^8d^3e^ \\
& 10 + 1254a^2b^6c^7d^2e^{11} + 22224a^3b^2c^{10}d^4e^9 + 13824a^3b^3 \\
& *c^9d^3e^{10} - 9516a^3b^4c^8d^2e^{11} + 11712a^4b^2c^9d^2e^{11} - 24 \\
& *a^*b^9c^5d^*e^{12} - 41088a^5b^*c^9d^*e^{12} - 360a^*b^2c^{12}d^8e^5 + 1664* \\
& a^*b^3c^{11}d^7e^6 - 2604a^*b^4c^{10}d^6e^7 + 1272a^*b^5c^9d^5e^8 + 332 \\
& *a^*b^6c^8d^4e^9 - 232a^*b^7c^7d^3e^{10} - 48a^*b^8c^6d^2e^{11} - 5760* \\
& a^2b^*c^{12}d^7e^6 + 416a^2b^7c^6d^*e^{12} - 32128a^3b^*c^{11}d^5e^8 - 41 \\
& 20a^3b^5c^7d^*e^{12} - 63360a^4b^*c^{10}d^3e^{10} + 21376a^4b^3c^8d^*e^{1 \\
& 2}))/((8*(a^6b^8e^8 + 256a^6c^8d^8 + 256a^{10}c^4e^8 - 16a^7b^6c^*e^8 \\
& - 4a^5b^9d^*e^7 + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6* \\
& d^8 - 256a^5b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2* \\
& b^{12}d^4e^4 - 4a^3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 + 1024a^7c^7d^6e \\
& ^2 + 1536a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^{10}c^2d^6e^2 - \\
& 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 \\
& - 192a^4b^7c^3d^5e^3 - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e \\
& ^2 - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 - 192a^5b^7c^2d^ \\
& 3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c \\
& ^4d^4e^4 - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b \\
& ^2c^5d^4e^4 + 2048a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512* \\
& a^8b^2c^4d^2e^6 - 1024a^6b^*c^7d^7e + 64a^6b^7c^*d^*e^7 - 1024a^9* \\
& b^*c^4d^*e^7 - 4a^2b^9c^3d^7e - 4a^2b^{11}c^*d^5e^3 + 64a^3b^7c^4d \\
& ^7e - 4a^3b^{10}c^*d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^*d^3e^5 \\
& + 1024a^5b^3c^6d^7e - 92a^5b^8c^*d^2e^6 - 3072a^7b^*c^6d^5e^3 - \\
& 384a^7b^5c^2d^*e^7 - 3072a^8b^*c^5d^3e^5 + 1024a^8b^3c^3d^*e^7)))* \\
& (-d^*e^7)^{(1/2)}*i)/((2*(c^2d^5 + a^2d^*e^4 + b^2d^3e^2 - 2b^*c^d^4e - 2*
\end{aligned}$$



$$\begin{aligned}
& a*b*d^2*e^3 + 2*a*c*d^3*e^2)) / (((((-d*e^7)^{(1/2)} * ((326912*a^8*c^9*d*e^{13} - \\
& 241664*a^8*b*c^8*e^{14} - 48*a^2*b^{13}*c^2*e^{14} + 1264*a^3*b^{11}*c^3*e^{14} - 13 \\
& 552*a^4*b^9*c^4*e^{14} + 75776*a^5*b^7*c^5*e^{14} - 232960*a^6*b^5*c^6*e^{14} + 3 \\
& 72736*a^7*b^3*c^7*e^{14} + 11520*a^3*c^{14}*d^{11}*e^3 + 78080*a^4*c^{13}*d^9*e^5 + \\
& 197120*a^5*c^{12}*d^7*e^7 + 336384*a^6*c^{11}*d^5*e^9 + 532736*a^7*c^{10}*d^3*e^{11} - \\
& 40*b^5*c^{12}*d^{12}*e^2 + 216*b^6*c^{11}*d^{11}*e^3 - 464*b^7*c^{10}*d^{10}*e^4 + \\
& 496*b^8*c^9*d^9*e^5 - 264*b^9*c^8*d^8*e^6 + 56*b^{10}*c^7*d^7*e^7 - 16*b^{11}* \\
& c^6*d^6*e^8 + 64*b^{12}*c^5*d^5*e^9 - 96*b^{13}*c^4*d^4*e^{10} + 64*b^{14}*c^3*d^3* \\
& e^{11} - 16*b^{15}*c^2*d^2*e^{12} + 1536*a^2*b^2*c^{13}*d^{11}*e^3 + 14400*a^2*b^3*c^{12} \\
& *d^{10}*e^4 - 47152*a^2*b^4*c^{11}*d^9*e^5 + 52144*a^2*b^5*c^{10}*d^8*e^6 - 162 \\
& 72*a^2*b^6*c^9*d^7*e^7 - 13040*a^2*b^7*c^8*d^6*e^8 + 23488*a^2*b^8*c^7*d^5* \\
& e^9 - 26384*a^2*b^9*c^6*d^4*e^{10} + 13824*a^2*b^{10}*c^5*d^3*e^{11} + 256*a^2*b^{11} \\
& *c^4*d^2*e^{12} + 125056*a^3*b^2*c^{12}*d^9*e^5 - 36224*a^3*b^3*c^{11}*d^8*e^6 \\
& - 126432*a^3*b^4*c^{10}*d^7*e^7 + 144848*a^3*b^5*c^9*d^6*e^8 - 114752*a^3*b^6 \\
& *c^8*d^5*e^9 + 125392*a^3*b^7*c^7*d^4*e^{10} - 53248*a^3*b^8*c^6*d^3*e^{11} - 2 \\
& 5264*a^3*b^9*c^5*d^2*e^{12} + 474112*a^4*b^2*c^{11}*d^7*e^7 - 191104*a^4*b^3*c^{10} \\
& *d^6*e^8 + 97184*a^4*b^4*c^9*d^5*e^9 - 277000*a^4*b^5*c^8*d^4*e^{10} + 5605 \\
& 6*a^4*b^6*c^7*d^3*e^{11} + 195584*a^4*b^7*c^6*d^2*e^{12} + 236800*a^5*b^2*c^{10} \\
& *d^5*e^9 + 388032*a^5*b^3*c^9*d^4*e^{10} + 159632*a^5*b^4*c^8*d^3*e^{11} - 67048 \\
& 8*a^5*b^5*c^7*d^2*e^{12} - 488960*a^6*b^2*c^9*d^3*e^{11} + 1106496*a^6*b^3*c^8* \\
& d^2*e^{12} + 64*a*b^{14}*c^2*d*e^{13} + 448*a*b^3*c^{13}*d^{12}*e^2 - 1968*a*b^4*c^{12} \\
& *d^{11}*e^3 + 2504*a*b^5*c^{11}*d^{10}*e^4 + 768*a*b^6*c^{10}*d^9*e^5 - 4368*a*b^7* \\
& c^9*d^8*e^6 + 3568*a*b^8*c^8*d^7*e^7 - 520*a*b^9*c^7*d^6*e^8 - 1728*a*b^{10} \\
& *c^6*d^5*e^9 + 2528*a*b^{11}*c^5*d^4*e^{10} - 1536*a*b^{12}*c^4*d^3*e^{11} + 240*a*b^{13} \\
& *c^3*d^2*e^{12} - 1152*a^2*b*c^{14}*d^{12}*e^2 - 1600*a^2*b^{12}*c^3*d*e^{13} - 67 \\
& 968*a^3*b*c^{13}*d^{10}*e^4 + 15808*a^3*b^{10}*c^4*d*e^{13} - 342272*a^4*b*c^{12}*d^8 \\
& *e^6 - 76928*a^4*b^8*c^5*d*e^{13} - 569088*a^5*b*c^{11}*d^6*e^8 + 179200*a^5*b^6 \\
& *c^6*d*e^{13} - 586368*a^6*b*c^{10}*d^4*e^{10} - 113008*a^6*b^4*c^7*d*e^{13} - 731 \\
& 008*a^7*b*c^9*d^2*e^{12} - 244096*a^7*b^2*c^8*d*e^{13}) / (16*(a^6*b^8*e^8 + 256* \\
& a^6*c^8*d^8 + 256*a^{10}*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8 \\
& *c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 \\
& + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^{12}*d^4*e^4 - 4*a^3*b^{11} \\
& *d^3*e^5 + 6*a^4*b^{10}*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 \\
& + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^{10}*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + \\
& 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 \\
& - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 \\
& + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6 \\
& *e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3 \\
& *d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7* \\
& b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024 \\
& *a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3 \\
& *d^7*e - 4*a^2*b^{11}*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^{10}*c*d^4*e^4 \\
& - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - \\
& 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 30 \\
& 72*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) + (((x*(626688*a^{10}*b*c^8*e \\
& ^{15} - 784384*a^{10}*c^9*d*e^{14} + 208*a^4*b^{13}*c^2*e^{15} - 4880*a^5*b^{11}*c^3*e^{15} \\
& + 47312*a^6*b^9*c^4*e^{15} - 242176*a^7*b^7*c^5*e^{15} + 688640*a^8*b^5*c^6* \\
& e^{15} - 1028096*a^9*b^3*c^7*e^{15} + 18432*a^4*c^{15}*d^{13}*e^2 + 126976*a^5*c^{14} \\
& *d^{11}*e^4 + 325632*a^6*c^{13}*d^9*e^6 + 139264*a^7*c^{12}*d^7*e^8 - 1067008*a^8 \\
& *c^{11}*d^5*e^{10} - 1773568*a^9*c^{10}*d^3*e^{12} + 16*b^8*c^{11}*d^{13}*e^2 - 96*b^9* \\
& c^{10}*d^{12}*e^3 + 240*b^{10}*c^9*d^{11}*e^4 - 304*b^{11}*c^8*d^{10}*e^5 + 144*b^{12}*c^7 \\
& *d^9*e^6 + 144*b^{13}*c^6*d^8*e^7 - 304*b^{14}*c^5*d^7*e^8 + 240*b^{15}*c^4*d^6* \\
& e^9 - 96*b^{16}*c^3*d^5*e^{10} + 16*b^{17}*c^2*d^4*e^{11} + 3200*a^2*b^4*c^{13}*d^{13} \\
& *e^2 - 18432*a^2*b^5*c^{12}*d^{12}*e^3 + 41024*a^2*b^6*c^{11}*d^{11}*e^4 - 36352*a^2 \\
& *b^7*c^{10}*d^{10}*e^5 - 16208*a^2*b^8*c^9*d^9*e^6 + 74576*a^2*b^9*c^8*d^8*e^7 \\
& - 78496*a^2*b^{10}*c^7*d^7*e^8 + 32064*a^2*b^{11}*c^6*d^6*e^9 + 6000*a^2*b^{12}*c^5 \\
& *d^5*e^{10} - 9264*a^2*b^{13}*c^4*d^4*e^{11} + 1472*a^2*b^{14}*c^3*d^3*e^{12} + 416 \\
& *a^2*b^{15}*c^2*d^2*e^{13} - 12800*a^3*b^2*c^{14}*d^{13}*e^2 + 73728*a^3*b^3*c^{13}*d^{12} \\
& *e^3 - 151296*a^3*b^4*c^{12}*d^{11}*e^4 + 78336*a^3*b^5*c^{11}*d^{10}*e^5 + 2066
\end{aligned}$$

$$\begin{aligned}
& 88a^3b^6c^{10}d^9e^6 - 436736a^3b^7c^9d^8e^7 + 324224a^3b^8c^8d^7e^8 + 992a^3b^9c^7d^6e^9 - 158176a^3b^{10}c^6d^5e^{10} + 77056a^3b^{11}c^5d^4e^{11} + 6912a^3b^{12}c^4d^3e^{12} - 8416a^3b^{13}c^3d^2e^{13} + 162816a^4b^2c^{13}d^{11}e^4 + 184320a^4b^3c^{12}d^{10}e^5 - 916608a^4b^4c^{11}d^9e^6 + 1165824a^4b^5c^{10}d^8e^7 - 314496a^4b^6c^9d^7e^8 - 822272a^4b^7c^8d^6e^9 + 919152a^4b^8c^7d^5e^{10} - 175296a^4b^9c^6d^4e^{11} - 189328a^4b^{10}c^5d^3e^{12} + 62064a^4b^{11}c^4d^2e^{13} + 1290752a^5b^2c^{12}d^9e^6 - 659456a^5b^3c^{11}d^8e^7 - 1561088a^5b^4c^{10}d^7e^8 + 3240960a^5b^5c^9d^6e^9 - 1964192a^5b^6c^8d^5e^{10} - 683008a^5b^7c^7d^4e^{11} + 1162304a^5b^8c^6d^3e^{12} - 164112a^5b^9c^5d^2e^{13} + 3442688a^6b^2c^{11}d^7e^8 - 3670016a^6b^3c^{10}d^6e^9 + 15232a^6b^4c^9d^5e^{10} + 4230144a^6b^5c^8d^4e^{11} - 3059648a^6b^6c^7d^3e^{12} - 247296a^6b^7c^6d^2e^{13} + 4010496a^7b^2c^{10}d^5e^{10} - 6873088a^7b^3c^9d^4e^{11} + 2822400a^7b^4c^8d^3e^{12} + 2370048a^7b^5c^7d^2e^{13} + 1178624a^8b^2c^9d^3e^{12} - 4739072a^8b^3c^8d^2e^{13} - 352a^8b^6c^{12}d^{13}e^2 + 2048a^8b^7c^{11}d^{12}e^3 - 4800a^8b^8c^{10}d^{11}e^4 + 5168a^8b^9c^9d^{10}e^5 - 480a^8b^{10}c^8d^9e^6 - 6000a^8b^{11}c^7d^8e^7 + 8192a^8b^{12}c^6d^7e^8 - 5040a^8b^{13}c^5d^6e^9 + 1152a^8b^{14}c^4d^5e^{10} + 240a^8b^{15}c^3d^4e^{11} - 128a^8b^{16}c^2d^3e^{12} - 512a^8b^{17}c^2d^2e^{14} - 106496a^4b^8c^{14}d^{12}e^3 + 11680a^4b^{12}c^3d^2e^{14} - 675840a^5b^8c^{13}d^{10}e^5 - 108288a^5b^{10}c^4d^2e^{14} - 1601536a^6b^8c^{12}d^8e^7 + 514768a^6b^8c^5d^2e^{14} - 925696a^7b^8c^{11}d^6e^9 - 1278304a^7b^6c^6d^2e^{14} + 2457600a^8b^8c^{10}d^4e^{11} + 1385600a^8b^4c^7d^2e^{14} + 2977792a^9b^8c^9d^2e^{13} + 19968a^9b^2c^8d^2e^{14}))/((8(a^6b^8e^8 + 256a^6c^8d^8 + 256a^{10}c^4e^8 - 16a^7b^6c^8e^8 - 4a^5b^9d^8e^7 + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6d^8 - 256a^5b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^{12}d^4e^4 - 4a^3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 + 1024a^7c^7d^6e^2 + 1536a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^{10}c^2d^6e^2 - 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4b^7c^3d^5e^3 - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 - 192a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c^4d^4e^4 - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 + 2048a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024a^6b^7c^7d^7e + 64a^6b^7c^7d^7e - 1024a^9b^8c^4d^7e - 4a^2b^9c^3d^7e - 4a^2b^{11}c^5d^5e^3 + 64a^3b^7c^4d^7e - 4a^3b^{10}c^4d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^3d^3e^5 + 1024a^5b^3c^6d^7e - 92a^5b^8c^3d^2e^6 - 3072a^7b^8c^6d^5e^3 - 384a^7b^5c^2d^2e^7 - 3072a^8b^8c^5d^3e^5 + 1024a^8b^3c^3d^2e^7)) - (((1048576a^{13}c^8e^{16} + 256a^7b^{12}c^2e^{16} - 6144a^8b^{10}c^3e^{16} + 61440a^9b^8c^4e^{16} - 327680a^{10}b^6c^5e^{16} + 983040a^{11}b^4c^6e^{16} - 1572864a^{12}b^2c^7e^{16} - 196608a^6c^{15}d^{14}e^2 - 917504a^7c^{14}d^{12}e^4 - 589824a^8c^{13}d^{10}e^6 + 3932160a^9c^{12}d^8e^8 + 10158080a^{10}c^{11}d^6e^{10} + 10616832a^{11}c^{10}d^4e^{12} + 5308416a^{12}c^9d^2e^{14} - 2816a^2b^8c^{11}d^{14}e^2 + 22656a^2b^9c^{10}d^{13}e^3 - 78848a^2b^{10}c^9d^{12}e^4 + 154112a^2b^{11}c^8d^{11}e^5 - 182784a^2b^{12}c^7d^{10}e^6 + 130816a^2b^{13}c^6d^9e^7 - 50176a^2b^{14}c^5d^8e^8 + 4608a^2b^{15}c^4d^7e^9 + 3328a^2b^{16}c^3d^6e^{10} - 896a^2b^{17}c^2d^5e^{11} + 24576a^3b^6c^{12}d^{14}e^2 - 198656a^3b^7c^{11}d^{13}e^3 + 684544a^3b^8c^{10}d^{12}e^4 - 1291520a^3b^9c^9d^{11}e^5 + 1403776a^3b^{10}c^8d^{10}e^6 - 798336a^3b^{11}c^7d^9e^7 + 89856a^3b^{12}c^6d^8e^8 + 155136a^3b^{13}c^5d^7e^9 - 77440a^3b^{14}c^4d^6e^{10} + 5504a^3b^{15}c^3d^5e^{11} + 2560a^3b^{16}c^2d^4e^{12} - 106496a^4b^4c^{13}d^{14}e^2 + 864256a^4b^5c^{12}d^{13}e^3 - 2924544a^4b^6c^{11}d^{12}e^4 + 5181440a^4b^7c^{10}d^{11}e^5 - 4686080a^4b^8c^9d^{10}e^6 + 1045376a^4b^9c^8d^9e^7 + 1900544a^4b^{10}c^7d^8e^8 - 1732096a^4b^{11}c^6d^7e^9 + 390400a^4b^{12}c^5d^6e^{10} + 112000a^4b^{13}c^4d^5e^{11} - 40960a^4b^{14}c^3d^4e^{12} - 3840a^4b^{15}c^2d^3e^{13} + 229376a^5b^2c^{14}d^{14}e^2 - 1867776a^5b^3c^
\end{aligned}$$



$$\begin{aligned}
& - 1862144a^4b^{12}c^7d^{10}e^7 + 490240a^4b^{13}c^6d^9e^8 + 128000a^4b^{14}c^5d^8e^9 - 108800a^4b^{15}c^4d^7e^{10} + 13824a^4b^{16}c^3d^6e^{11} + 2304a^4b^{17}c^2d^5e^{12} - 163840a^5b^5c^{13}d^{15}e^2 + 1474560a^5b^6c^{12}d^{14}e^3 - 5447680a^5b^7c^{11}d^{13}e^4 + 10588160a^5b^8c^{10}d^{12}e^5 - 11166720a^5b^9c^9d^{11}e^6 + 5159936a^5b^{10}c^8d^{10}e^7 + 1073920a^5b^{11}c^7d^9e^8 - 2279680a^5b^{12}c^6d^8e^9 + 770560a^5b^{13}c^5d^7e^{10} + 33280a^5b^{14}c^4d^6e^{11} - 41216a^5b^{15}c^3d^5e^{12} - 1280a^5b^{16}c^2d^4e^{13} + 327680a^6b^3c^{14}d^{15}e^2 - 3276800a^6b^4c^{13}d^{14}e^3 + 12615680a^6b^5c^{12}d^{13}e^4 - 23592960a^6b^6c^{11}d^{12}e^5 + 19701760a^6b^7c^{10}d^{11}e^6 + 1372160a^6b^8c^9d^{10}e^7 - 15846400a^6b^9c^8d^9e^8 + 10864640a^6b^{10}c^7d^8e^9 - 1352960a^6b^{11}c^6d^7e^{10} - 1111040a^6b^{12}c^5d^6e^{11} + 273920a^6b^{13}c^4d^5e^{12} + 25600a^6b^{14}c^3d^4e^{13} - 1280a^6b^{15}c^2d^3e^{14} + 3407872a^7b^2c^{14}d^{14}e^3 - 14221312a^7b^3c^{13}d^{13}e^4 + 23527424a^7b^4c^{12}d^{12}e^5 - 3768320a^7b^5c^{11}d^{11}e^6 - 38895616a^7b^6c^{10}d^{10}e^7 + 50126848a^7b^7c^9d^9e^8 - 18362368a^7b^8c^8d^8e^9 - 6831104a^7b^9c^7d^7e^{10} + 6200320a^7b^{10}c^6d^6e^{11} - 726784a^7b^{11}c^5d^5e^{12} - 228608a^7b^{12}c^4d^4e^{13} + 31488a^7b^{13}c^3d^3e^{14} + 2304a^7b^{14}c^2d^2e^{15} - 3145728a^8b^2c^{13}d^{12}e^5 - 31129600a^8b^3c^{12}d^{11}e^6 + 74711040a^8b^4c^{11}d^{10}e^7 - 55476224a^8b^5c^{10}d^9e^8 - 11075584a^8b^6c^9d^8e^9 + 35381248a^8b^7c^8d^7e^{10} - 14479360a^8b^8c^7d^6e^{11} - 168960a^8b^9c^6d^5e^{12} + 1286144a^8b^{10}c^5d^4e^{13} - 302336a^8b^{11}c^4d^3e^{14} - 55808a^8b^{12}c^3d^2e^{15} - 36962304a^9b^2c^{12}d^{10}e^7 - 9502720a^9b^3c^{11}d^9e^8 + 67174400a^9b^4c^{10}d^8e^9 - 54886400a^9b^5c^9d^7e^{10} + 11239424a^9b^6c^8d^6e^{11} + 5545984a^9b^7c^7d^5e^{12} - 5263360a^9b^8c^6d^4e^{13} + 1356800a^9b^9c^5d^3e^{14} + 558080a^9b^{10}c^4d^2e^{15} - 49807360a^{10}b^2c^{11}d^8e^9 + 19333120a^{10}b^3c^{10}d^7e^{10} + 7208960a^{10}b^4c^9d^6e^{11} - 14974976a^{10}b^5c^8d^5e^{12} + 15073280a^{10}b^6c^7d^4e^{13} - 2170880a^{10}b^7c^6d^3e^{14} - 2928640a^{10}b^8c^5d^2e^{15} - 11796480a^{11}b^2c^{10}d^6e^{11} + 23920640a^{11}b^3c^9d^5e^{12} - 24576000a^{11}b^4c^8d^4e^{13} - 4096000a^{11}b^5c^7d^3e^{14} + 8355840a^{11}b^6c^6d^2e^{15} + 12582912a^{12}b^2c^9d^4e^{13} + 19857408a^{12}b^3c^8d^3e^{14} - 11534336a^{12}b^4c^7d^2e^{15} + 3407872a^{13}b^2c^8d^2e^{15} - 5505024a^{14}b^3c^8d^2e^{16} - 262144a^7b^3c^{15}d^{15}e^2 + 5505024a^8b^3c^{14}d^{13}e^4 - 1280a^8b^{13}c^2d^2e^{16} + 25952256a^9b^3c^{13}d^{11}e^6 + 30976a^9b^{11}c^3d^2e^{16} + 38010880a^{10}b^3c^{12}d^9e^8 - 312320a^{10}b^9c^4d^2e^{16} + 11796480a^{11}b^3c^{11}d^7e^{10} + 1679360a^{11}b^7c^5d^2e^{16} - 21233664a^{12}b^3c^{10}d^5e^{12} - 5079040a^{12}b^5c^6d^2e^{16} - 20709376a^{13}b^3c^9d^3e^{14} + 8192000a^{13}b^3c^7d^2e^{16}))/((16*(c^2d^5 + a^2d^4 + b^2d^3e^2 - 2b^3cd^4e - 2a^2bd^2e^3 + 2a^3cd^3e^2)*(a^6b^8e^8 + 256a^6c^8d^8 + 256a^{10}c^4e^8 - 16a^7b^6c^8e^8 - 4a^5b^9d^8e^7 + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6d^8 - 256a^5b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^{12}d^4e^4 - 4a^3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 + 1024a^7c^7d^6e^2 + 1536a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^{10}c^2d^6e^2 - 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4b^7c^3d^5e^3 - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 - 192a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c^4d^4e^4 - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 + 2048a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024a^6b^3c^7d^7e + 64a^6b^7c^3d^7e - 1024a^9b^3c^4d^7e - 4a^2b^9c^3d^7e - 4a^2b^{11}c^3d^5e^3 + 64a^3b^7c^4d^7e - 4a^3b^{10}c^3d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^3d^3e^5 + 1024a^5b^3c^6d^7e - 92a^5b^8c^2d^2e^6 - 3072a^7b^3c^6d^5e^3 - 384a^7b^5c^2d^2e^7 - 3072a^8b^3c^5d^3e^5 + 1024a^8b^3c^3d^2e^7)))*(-d^7e)^{(1/2)})/(2*(c^2d^5 + a^2d^4 + b^2d^3e^2 - 2b^3cd^4e - 2a^2bd^2e^3 + 2a^3cd^3e^2)))*(-d^7e)^{(1/2)})/(2*(c^2d^5 + a^2d^4 + b^2d^3e^2 - 2b^3cd^4e - 2a^2bd^2e^3 + 2a^3cd^3e^2))
\end{aligned}$$

$$\begin{aligned}
& d^3e^2)))/((2*(c^2d^5 + a^2d^4e + b^2d^3e^2 - 2b^*c*d^4e - 2a*b*d^2 \\
& *e^3 + 2a*c*d^3e^2)) + (x*(22800*a^6*c^9e^13 + 36*a^2*b^8*c^5e^13 - 600 \\
& *a^3*b^6*c^6e^13 + 4313*a^4*b^4*c^7e^13 - 15592*a^5*b^2*c^8e^13 + 1296*a \\
& ^2*c^13*d^8e^5 + 9792*a^3*c^12*d^6e^7 + 30304*a^4*c^11*d^4e^9 + 40512*a^ \\
& 5*c^10*d^2e^11 + 25*b^4*c^11*d^8e^5 - 120*b^5*c^10*d^7e^6 + 214*b^6*c^9* \\
& d^6e^7 - 168*b^7*c^8*d^5e^8 + 53*b^8*c^7*d^4e^9 - 8*b^9*c^6*d^3e^10 + 4 \\
& *b^10*c^5*d^2e^11 + 6336*a^2*b^2*c^11*d^6e^7 + 3840*a^2*b^3*c^10*d^5e^8 \\
& - 8506*a^2*b^4*c^9*d^4e^9 + 1112*a^2*b^5*c^8*d^3e^10 + 1254*a^2*b^6*c^7*d \\
& ^2e^11 + 22224*a^3*b^2*c^10*d^4e^9 + 13824*a^3*b^3*c^9*d^3e^10 - 9516*a^ \\
& 3*b^4*c^8*d^2e^11 + 11712*a^4*b^2*c^9*d^2e^11 - 24*a*b^9*c^5*d^12e^12 - 410 \\
& 88*a^5*b*c^9*d^12e^12 - 360*a*b^2*c^12*d^8e^5 + 1664*a*b^3*c^11*d^7e^6 - 26 \\
& 04*a*b^4*c^10*d^6e^7 + 1272*a*b^5*c^9*d^5e^8 + 332*a*b^6*c^8*d^4e^9 - 23 \\
& 2*a*b^7*c^7*d^3e^10 - 48*a*b^8*c^6*d^2e^11 - 5760*a^2*b*c^12*d^7e^6 + 41 \\
& 6*a^2*b^7*c^6*d^12e^12 - 32128*a^3*b*c^11*d^5e^8 - 4120*a^3*b^5*c^7*d^12e^12 - \\
& 63360*a^4*b*c^10*d^3e^10 + 21376*a^4*b^3*c^8*d^12e^12))/((8*(a^6*b^8e^8 + 2 \\
& 56*a^6*c^8*d^8 + 256*a^10*c^4e^8 - 16*a^7*b^6*c^6e^8 - 4*a^5*b^9*d^7e^7 + a^ \\
& 2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d \\
& ^8 + 96*a^8*b^4*c^2e^8 - 256*a^9*b^2*c^3e^8 + a^2*b^12*d^4e^4 - 4*a^3*b^ \\
& 11*d^3e^5 + 6*a^4*b^10*d^2e^6 + 1024*a^7*c^7*d^6e^2 + 1536*a^8*c^6*d^4e \\
& ^4 + 1024*a^9*c^5*d^2e^6 + 6*a^2*b^10*c^2*d^6e^2 - 92*a^3*b^8*c^3*d^6e^2 \\
& + 52*a^3*b^9*c^2*d^5e^3 + 512*a^4*b^6*c^4*d^6e^2 - 192*a^4*b^7*c^3*d^5e \\
& ^3 - 90*a^4*b^8*c^2*d^4e^4 - 1152*a^5*b^4*c^5*d^6e^2 - 128*a^5*b^5*c^4*d^ \\
& 5e^3 + 800*a^5*b^6*c^3*d^4e^4 - 192*a^5*b^7*c^2*d^3e^5 + 512*a^6*b^2*c^6 \\
& *d^6e^2 + 2048*a^6*b^3*c^5*d^5e^3 - 2240*a^6*b^4*c^4*d^4e^4 - 128*a^6*b^ \\
& 5*c^3*d^3e^5 + 512*a^6*b^6*c^2*d^2e^6 + 1536*a^7*b^2*c^5*d^4e^4 + 2048*a \\
& ^7*b^3*c^4*d^3e^5 - 1152*a^7*b^4*c^3*d^2e^6 + 512*a^8*b^2*c^4*d^2e^6 - 1 \\
& 024*a^6*b*c^7*d^7e + 64*a^6*b^7*c*d^7e - 1024*a^9*b*c^4*d^7e - 4*a^2*b^9 \\
& *c^3*d^7e - 4*a^2*b^11*c*d^5e^3 + 64*a^3*b^7*c^4*d^7e - 4*a^3*b^10*c*d^4 \\
& e^4 - 384*a^4*b^5*c^5*d^7e + 52*a^4*b^9*c*d^3e^5 + 1024*a^5*b^3*c^6*d^7e \\
& - 92*a^5*b^8*c*d^2e^6 - 3072*a^7*b*c^6*d^5e^3 - 384*a^7*b^5*c^2*d^7e - \\
& 3072*a^8*b*c^5*d^3e^5 + 1024*a^8*b^3*c^3*d^7e)))*((-d^7e)^{(1/2)})/((2*(c^2 \\
& *d^5 + a^2*d^4e + b^2*d^3e^2 - 2b^*c*d^4e - 2a*b*d^2e^3 + 2a*c*d^3e^ \\
& 2)) - (2000*a^4*c^9e^12 + 21*a^2*b^4*c^7e^12 - 520*a^3*b^2*c^8e^12 + 129 \\
& 6*a^2*c^11*d^4e^8 + 4320*a^3*c^10*d^2e^10 + 25*b^4*c^9*d^4e^8 - 60*b^5*c \\
& ^8*d^3e^9 + 35*b^6*c^7*d^2e^10 + 192*a^2*b^2*c^9*d^2e^10 - 112*a*b^5*c^7 \\
& *d^11e^11 - 4480*a^3*b*c^9*d^11e^11 - 360*a*b^2*c^10*d^4e^8 + 832*a*b^3*c^9*d^ \\
& 3e^9 - 362*a*b^4*c^8*d^2e^10 - 2880*a^2*b*c^10*d^3e^9 + 1440*a^2*b^3*c^8 \\
& *d^11e^11))/(8*(a^6*b^8e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4e^8 - 16*a^7*b^6* \\
& c^6e^8 - 4*a^5*b^9*d^7e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4 \\
& *c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2e^8 - 256*a^9*b^2*c^3e^8 + \\
& a^2*b^12*d^4e^4 - 4*a^3*b^11*d^3e^5 + 6*a^4*b^10*d^2e^6 + 1024*a^7*c^7* \\
& d^6e^2 + 1536*a^8*c^6*d^4e^4 + 1024*a^9*c^5*d^2e^6 + 6*a^2*b^10*c^2*d^6* \\
& e^2 - 92*a^3*b^8*c^3*d^6e^2 + 52*a^3*b^9*c^2*d^5e^3 + 512*a^4*b^6*c^4*d^6 \\
& *e^2 - 192*a^4*b^7*c^3*d^5e^3 - 90*a^4*b^8*c^2*d^4e^4 - 1152*a^5*b^4*c^5* \\
& d^6e^2 - 128*a^5*b^5*c^4*d^5e^3 + 800*a^5*b^6*c^3*d^4e^4 - 192*a^5*b^7*c \\
& ^2*d^3e^5 + 512*a^6*b^2*c^6*d^6e^2 + 2048*a^6*b^3*c^5*d^5e^3 - 2240*a^6* \\
& b^4*c^4*d^4e^4 - 128*a^6*b^5*c^3*d^3e^5 + 512*a^6*b^6*c^2*d^2e^6 + 1536* \\
& a^7*b^2*c^5*d^4e^4 + 2048*a^7*b^3*c^4*d^3e^5 - 1152*a^7*b^4*c^3*d^2e^6 + \\
& 512*a^8*b^2*c^4*d^2e^6 - 1024*a^6*b*c^7*d^7e + 64*a^6*b^7*c*d^7e - 1024 \\
& *a^9*b*c^4*d^7e - 4*a^2*b^9*c^3*d^7e - 4*a^2*b^11*c*d^5e^3 + 64*a^3*b^7* \\
& c^4*d^7e - 4*a^3*b^10*c*d^4e^4 - 384*a^4*b^5*c^5*d^7e + 52*a^4*b^9*c*d^3 \\
& *e^5 + 1024*a^5*b^3*c^6*d^7e - 92*a^5*b^8*c*d^2e^6 - 3072*a^7*b*c^6*d^5e \\
& ^3 - 384*a^7*b^5*c^2*d^7e - 3072*a^8*b*c^5*d^3e^5 + 1024*a^8*b^3*c^3*d^7e \\
& ^7)) + ((((-d^7e)^{(1/2)})*((326912*a^8*c^9*d^13e^13 - 241664*a^8*b*c^8e^14 - 4 \\
& 8*a^2*b^13*c^2e^14 + 1264*a^3*b^11*c^3e^14 - 13552*a^4*b^9*c^4e^14 + 757 \\
& 76*a^5*b^7*c^5e^14 - 232960*a^6*b^5*c^6e^14 + 372736*a^7*b^3*c^7e^14 + 1 \\
& 1520*a^3*c^14*d^11e^3 + 78080*a^4*c^13*d^9e^5 + 197120*a^5*c^12*d^7e^7 + \\
& 336384*a^6*c^11*d^5e^9 + 532736*a^7*c^10*d^3e^11 - 40*b^5*c^12*d^12e^2 \\
& + 216*b^6*c^11*d^11e^3 - 464*b^7*c^10*d^10e^4 + 496*b^8*c^9*d^9e^5 - 264
\end{aligned}$$

$$\begin{aligned}
& *b^9*c^8*d^8*e^6 + 56*b^{10}*c^7*d^7*e^7 - 16*b^{11}*c^6*d^6*e^8 + 64*b^{12}*c^5*d^5*e^9 - 96*b^{13}*c^4*d^4*e^{10} + 64*b^{14}*c^3*d^3*e^{11} - 16*b^{15}*c^2*d^2*e^{12} \\
& + 1536*a^2*b^2*c^{13}*d^{11}*e^3 + 14400*a^2*b^3*c^{12}*d^{10}*e^4 - 47152*a^2*b^4*c^{11}*d^9*e^5 + 52144*a^2*b^5*c^{10}*d^8*e^6 - 16272*a^2*b^6*c^9*d^7*e^7 - 1 \\
& 3040*a^2*b^7*c^8*d^6*e^8 + 23488*a^2*b^8*c^7*d^5*e^9 - 26384*a^2*b^9*c^6*d^4*e^{10} + 13824*a^2*b^{10}*c^5*d^3*e^{11} + 256*a^2*b^{11}*c^4*d^2*e^{12} + 125056*a \\
& ^3*b^2*c^{12}*d^9*e^5 - 36224*a^3*b^3*c^{11}*d^8*e^6 - 126432*a^3*b^4*c^{10}*d^7*e^7 + 144848*a^3*b^5*c^9*d^6*e^8 - 114752*a^3*b^6*c^8*d^5*e^9 + 125392*a^3*b \\
& ^7*c^7*d^4*e^{10} - 53248*a^3*b^8*c^6*d^3*e^{11} - 25264*a^3*b^9*c^5*d^2*e^{12} + 474112*a^4*b^2*c^{11}*d^7*e^7 - 191104*a^4*b^3*c^{10}*d^6*e^8 + 97184*a^4*b^4 \\
& *c^9*d^5*e^9 - 277000*a^4*b^5*c^8*d^4*e^{10} + 56056*a^4*b^6*c^7*d^3*e^{11} + 195584*a^4*b^7*c^6*d^2*e^{12} + 236800*a^5*b^2*c^{10}*d^5*e^9 + 388032*a^5*b^3*c^9*d^4*e^{10} \\
& + 159632*a^5*b^4*c^8*d^3*e^{11} - 670488*a^5*b^5*c^7*d^2*e^{12} - 488960*a^6*b^2*c^9*d^3*e^{11} + 1106496*a^6*b^3*c^8*d^2*e^{12} + 64*a*b^{14}*c^2*d \\
& *e^{13} + 448*a*b^3*c^{13}*d^{12}*e^2 - 1968*a*b^4*c^{12}*d^{11}*e^3 + 2504*a*b^5*c^{11}*d^{10}*e^4 + 768*a*b^6*c^{10}*d^9*e^5 - 4368*a*b^7*c^9*d^8*e^6 + 3568*a*b^8*c^8*d^7*e^7 \\
& - 520*a*b^9*c^7*d^6*e^8 - 1728*a*b^{10}*c^6*d^5*e^9 + 2528*a*b^{11}*c^5*d^4*e^{10} - 1536*a*b^{12}*c^4*d^3*e^{11} + 240*a*b^{13}*c^3*d^2*e^{12} - 1152*a^ \\
& ^2*b*c^{14}*d^{12}*e^2 - 1600*a^2*b^{12}*c^3*d*e^{13} - 67968*a^3*b*c^{13}*d^{10}*e^4 + 15808*a^3*b^{10}*c^4*d*e^{13} - 342272*a^4*b*c^{12}*d^8*e^6 - 76928*a^4*b^8*c^5*d \\
& *e^{13} - 569088*a^5*b*c^{11}*d^6*e^8 + 179200*a^5*b^6*c^6*d*e^{13} - 586368*a^6*b*c^{10}*d^4*e^{10} - 113008*a^6*b^4*c^7*d*e^{13} - 731008*a^7*b*c^9*d^2*e^{12} - 2 \\
& 44096*a^7*b^2*c^8*d*e^{13})/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^{10}*c^4 \\
& *e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256 \\
& *a^9*b^2*c^3*e^8 + a^2*b^{12}*d^4*e^4 - 4*a^3*b^{11}*d^3*e^5 + 6*a^4*b^{10}*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6 \\
& *a^2*b^{10}*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - \\
& 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 \\
& - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e \\
& + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^{11}*c*d^5 \\
& *e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^{10}*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 30 \\
& 72*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) - (((x*(626688*a^{10}*b*c^8*e^{15} - 784384*a^{10}*c^9*d*e^{14} \\
& + 208*a^4*b^{13}*c^2*e^{15} - 4880*a^5*b^{11}*c^3*e^{15} + 47312*a^6*b^9*c^4*e^{15} - 242176*a^7*b^7*c^5*e^{15} + 688640*a^8*b^5*c^6*e^{15} - 1028096*a^9*b^3*c^7 \\
& *e^{15} + 18432*a^4*c^{15}*d^{13}*e^2 + 126976*a^5*c^{14}*d^{11}*e^4 + 325632*a^6*c^{13}*d^9*e^6 + 139264*a^7*c^{12}*d^7*e^8 - 1067008*a^8*c^{11}*d^5*e^{10} - 1773568*a \\
& ^9*c^{10}*d^3*e^{12} + 16*b^8*c^{11}*d^{13}*e^2 - 96*b^9*c^{10}*d^{12}*e^3 + 240*b^{10}*c^9*d^{11}*e^4 - 304*b^{11}*c^8*d^{10}*e^5 + 144*b^{12}*c^7*d^9*e^6 + 144*b^{13}*c^6*d^8*e^7 \\
& - 304*b^{14}*c^5*d^7*e^8 + 240*b^{15}*c^4*d^6*e^9 - 96*b^{16}*c^3*d^5*e^{10} + 16*b^{17}*c^2*d^4*e^{11} + 3200*a^2*b^4*c^{13}*d^{13}*e^2 - 18432*a^2*b^5*c^{12}*d^{12}*e^3 \\
& + 41024*a^2*b^6*c^{11}*d^{11}*e^4 - 36352*a^2*b^7*c^{10}*d^{10}*e^5 - 16208*a^2*b^8*c^9*d^9*e^6 + 74576*a^2*b^9*c^8*d^8*e^7 - 78496*a^2*b^{10}*c^7*d^7*e^8 + 32064*a^2*b^{11}*c^6*d^6*e^9 \\
& + 6000*a^2*b^{12}*c^5*d^5*e^{10} - 9264*a^2*b^{13}*c^4*d^4*e^{11} + 1472*a^2*b^{14}*c^3*d^3*e^{12} + 416*a^2*b^{15}*c^2*d^2*e^{13} - 12800*a^3*b^2*c^{14}*d^{13}*e^2 + 73728*a^3*b^3*c^{13}*d^{12}*e^3 \\
& - 151296*a^3*b^4*c^{12}*d^{11}*e^4 + 78336*a^3*b^5*c^{11}*d^{10}*e^5 + 206688*a^3*b^6*c^{10}*d^9*e^6 - 436736*a^3*b^7*c^9*d^8*e^7 + 324224*a^3*b^8*c^8*d^7*e^8 + 992*a^3*b^9*c^7*d^6*e^9 \\
& - 158176*a^3*b^{10}*c^6*d^5*e^{10} + 77056*a^3*b^{11}*c^5*d^4*e^{11} + 6912*a^3*b^{12}*c^4*d^3*e^{12} - 8416*a^3*b^{13}*c^3*d^2*e^{13} + 162816*a^4*b^2*c^{13}*d^{11}*e^4 + 184320*a^4*b^3*c^{12}*d^{10}*e^5 \\
& - 916608*a^4*b^4*c^{11}*d^9*e^6 + 1165824*a^4*b^5*c^{10}*d^8*e^7 - 314496*a^4*b^6*c^9*d^7*e^8 - 822272*a^4*b^7*c^8*d^6*e^9 + 919152*a^4*b^8*c^7*d^5*e^{10} - 175296*a^4*b^9*c^6*d^4*e^{11} - 189328
\end{aligned}$$

$$\begin{aligned}
& a^4 b^{10} c^5 d^3 e^{12} + 62064 a^4 b^{11} c^4 d^2 e^{13} + 1290752 a^5 b^2 c^{12} \\
& d^9 e^6 - 659456 a^5 b^3 c^{11} d^8 e^7 - 1561088 a^5 b^4 c^{10} d^7 e^8 + 324 \\
& 0960 a^5 b^5 c^9 d^6 e^9 - 1964192 a^5 b^6 c^8 d^5 e^{10} - 683008 a^5 b^7 c^7 \\
& d^4 e^{11} + 1162304 a^5 b^8 c^6 d^3 e^{12} - 164112 a^5 b^9 c^5 d^2 e^{13} + 3 \\
& 442688 a^6 b^2 c^{11} d^7 e^8 - 3670016 a^6 b^3 c^{10} d^6 e^9 + 15232 a^6 b^4 c^9 \\
& d^5 e^{10} + 4230144 a^6 b^5 c^8 d^4 e^{11} - 3059648 a^6 b^6 c^7 d^3 e^{12} \\
& - 247296 a^6 b^7 c^6 d^2 e^{13} + 4010496 a^7 b^2 c^{10} d^5 e^{10} - 6873088 a^7 \\
& b^3 c^9 d^4 e^{11} + 2822400 a^7 b^4 c^8 d^3 e^{12} + 2370048 a^7 b^5 c^7 d^2 e^{13} \\
& + 1178624 a^8 b^2 c^9 d^3 e^{12} - 4739072 a^8 b^3 c^8 d^2 e^{13} - 352 a^8 b^6 \\
& c^{12} d^{13} e^2 + 2048 a^8 b^7 c^{11} d^{12} e^3 - 4800 a^8 b^8 c^{10} d^{11} e^4 + 5 \\
& 168 a^8 b^9 c^9 d^{10} e^5 - 480 a^8 b^{10} c^8 d^9 e^6 - 6000 a^8 b^{11} c^7 d^8 e^7 + \\
& 8192 a^8 b^{12} c^6 d^7 e^8 - 5040 a^8 b^{13} c^5 d^6 e^9 + 1152 a^8 b^{14} c^4 d^5 e^{10} \\
& + 240 a^8 b^{15} c^3 d^4 e^{11} - 128 a^8 b^{16} c^2 d^3 e^{12} - 512 a^8 b^{17} c^2 d^2 \\
& e^{14} - 106496 a^4 b^3 c^{14} d^{12} e^3 + 11680 a^4 b^{12} c^3 d^5 e^{14} - 675840 a^5 \\
& b^3 c^{13} d^{10} e^5 - 108288 a^5 b^{10} c^4 d^8 e^{14} - 1601536 a^6 b^3 c^{12} d^8 e^7 \\
& + 514768 a^6 b^8 c^5 d^8 e^{14} - 925696 a^7 b^3 c^{11} d^6 e^9 - 1278304 a^7 b^6 c^6 \\
& d^8 e^{14} + 2457600 a^8 b^3 c^{10} d^4 e^{11} + 1385600 a^8 b^4 c^7 d^8 e^{14} + 2977 \\
& 792 a^9 b^3 c^9 d^2 e^{13} + 19968 a^9 b^2 c^8 d^8 e^{14}))/ (8(a^6 b^8 e^8 + 256 a^6 \\
& c^8 d^8 + 256 a^{10} c^4 e^8 - 16 a^7 b^6 c^8 e^8 - 4 a^5 b^9 d^8 e^7 + a^2 b^8 \\
& c^4 d^8 - 16 a^3 b^6 c^5 d^8 + 96 a^4 b^4 c^6 d^8 - 256 a^5 b^2 c^7 d^8 + \\
& 96 a^8 b^4 c^2 e^8 - 256 a^9 b^2 c^3 e^8 + a^2 b^{12} d^4 e^4 - 4 a^3 b^{11} d^3 \\
& e^5 + 6 a^4 b^{10} d^2 e^6 + 1024 a^7 c^7 d^6 e^2 + 1536 a^8 c^6 d^4 e^4 + \\
& 1024 a^9 c^5 d^2 e^6 + 6 a^2 b^{10} c^2 d^6 e^2 - 92 a^3 b^8 c^3 d^6 e^2 + 5 \\
& 2 a^3 b^9 c^2 d^5 e^3 + 512 a^4 b^6 c^4 d^6 e^2 - 192 a^4 b^7 c^3 d^5 e^3 - \\
& 90 a^4 b^8 c^2 d^4 e^4 - 1152 a^5 b^4 c^5 d^6 e^2 - 128 a^5 b^5 c^4 d^5 e^3 \\
& + 800 a^5 b^6 c^3 d^4 e^4 - 192 a^5 b^7 c^2 d^3 e^5 + 512 a^6 b^2 c^6 d^6 \\
& e^2 + 2048 a^6 b^3 c^5 d^5 e^3 - 2240 a^6 b^4 c^4 d^4 e^4 - 128 a^6 b^5 c^3 \\
& d^3 e^5 + 512 a^6 b^6 c^2 d^2 e^6 + 1536 a^7 b^2 c^5 d^4 e^4 + 2048 a^7 b^3 \\
& c^4 d^3 e^5 - 1152 a^7 b^4 c^3 d^2 e^6 + 512 a^8 b^2 c^4 d^2 e^6 - 1024 a^8 \\
& b^3 c^7 d^7 e + 64 a^6 b^7 c^4 d^7 e - 1024 a^9 b^3 c^4 d^7 e - 4 a^2 b^9 c^3 \\
& d^7 e - 4 a^2 b^{11} c^5 d^5 e^3 + 64 a^3 b^7 c^4 d^7 e - 4 a^3 b^{10} c^4 d^4 e^4 \\
& - 384 a^4 b^5 c^5 d^7 e + 52 a^4 b^9 c^3 d^5 e^5 + 1024 a^5 b^3 c^6 d^7 e - \\
& 92 a^5 b^8 c^2 d^2 e^6 - 3072 a^7 b^3 c^6 d^5 e^3 - 384 a^7 b^5 c^2 d^5 e^7 - 307 \\
& 2 a^8 b^3 c^5 d^3 e^5 + 1024 a^8 b^3 c^3 d^5 e^7)) + (((1048576 a^{13} c^8 e^{16} + \\
& 256 a^7 b^{12} c^2 e^{16} - 6144 a^8 b^{10} c^3 e^{16} + 61440 a^9 b^8 c^4 e^{16} - \\
& 327680 a^{10} b^6 c^5 e^{16} + 983040 a^{11} b^4 c^6 e^{16} - 1572864 a^{12} b^2 c^7 \\
& e^{16} - 196608 a^6 c^{15} d^{14} e^2 - 917504 a^7 c^{14} d^{12} e^4 - 589824 a^8 c^{13} \\
& d^{10} e^6 + 3932160 a^9 c^{12} d^8 e^8 + 10158080 a^{10} c^{11} d^6 e^{10} + 10616 \\
& 832 a^{11} c^{10} d^4 e^{12} + 5308416 a^{12} c^9 d^2 e^{14} - 2816 a^2 b^8 c^{11} d^{14} \\
& e^2 + 22656 a^2 b^9 c^{10} d^{13} e^3 - 78848 a^2 b^{10} c^9 d^{12} e^4 + 154112 a^2 \\
& b^{11} c^8 d^{11} e^5 - 182784 a^2 b^{12} c^7 d^{10} e^6 + 130816 a^2 b^{13} c^6 d^9 \\
& e^7 - 50176 a^2 b^{14} c^5 d^8 e^8 + 4608 a^2 b^{15} c^4 d^7 e^9 + 3328 a^2 b^{16} \\
& c^3 d^6 e^{10} - 896 a^2 b^{17} c^2 d^5 e^{11} + 24576 a^3 b^6 c^{12} d^{14} e^2 \\
& - 198656 a^3 b^7 c^{11} d^{13} e^3 + 684544 a^3 b^8 c^{10} d^{12} e^4 - 1291520 a^3 \\
& b^9 c^9 d^{11} e^5 + 1403776 a^3 b^{10} c^8 d^{10} e^6 - 798336 a^3 b^{11} c^7 d^9 \\
& e^7 + 89856 a^3 b^{12} c^6 d^8 e^8 + 155136 a^3 b^{13} c^5 d^7 e^9 - 77440 a^3 \\
& b^{14} c^4 d^6 e^{10} + 5504 a^3 b^{15} c^3 d^5 e^{11} + 2560 a^3 b^{16} c^2 d^4 e^{12} \\
& - 106496 a^4 b^4 c^{13} d^{14} e^2 + 864256 a^4 b^5 c^{12} d^{13} e^3 - 2924544 a^4 \\
& b^6 c^{11} d^{12} e^4 + 5181440 a^4 b^7 c^{10} d^{11} e^5 - 4686080 a^4 b^8 c^9 d^{10} \\
& e^6 + 1045376 a^4 b^9 c^8 d^9 e^7 + 1900544 a^4 b^{10} c^7 d^8 e^8 - 17 \\
& 32096 a^4 b^{11} c^6 d^7 e^9 + 390400 a^4 b^{12} c^5 d^6 e^{10} + 112000 a^4 b^{13} \\
& c^4 d^5 e^{11} - 40960 a^4 b^{14} c^3 d^4 e^{12} - 3840 a^4 b^{15} c^2 d^3 e^{13} + \\
& 229376 a^5 b^2 c^{14} d^{14} e^2 - 1867776 a^5 b^3 c^{13} d^{13} e^3 + 6078464 a^5 b^4 \\
& c^{12} d^{12} e^4 - 9297920 a^5 b^5 c^{11} d^{11} e^5 + 4055040 a^5 b^6 c^{10} d^{10} \\
& e^6 + 7788544 a^5 b^7 c^9 d^9 e^7 - 12657664 a^5 b^8 c^8 d^8 e^8 + 61301 \\
& 76 a^5 b^9 c^7 d^7 e^9 + 734080 a^5 b^{10} c^6 d^6 e^{10} - 1442560 a^5 b^{11} c^5 \\
& d^5 e^{11} + 168960 a^5 b^{12} c^4 d^4 e^{12} + 78080 a^5 b^{13} c^3 d^3 e^{13} + 3 \\
& 200 a^5 b^{14} c^2 d^2 e^{14} - 4587520 a^6 b^2 c^{13} d^{12} e^4 + 3080192 a^6 b^3 \\
& c^{12} d^{11} e^5 + 12001280 a^6 b^4 c^{11} d^{10} e^6 - 31076352 a^6 b^5 c^{10} d^9
\end{aligned}$$

$$\begin{aligned}
& *e^7 + 27475968*a^6*b^6*c^9*d^8*e^8 - 2088960*a^6*b^7*c^8*d^7*e^9 - 1220531 \\
& 2*a^6*b^8*c^7*d^6*e^{10} + 6043520*a^6*b^9*c^6*d^5*e^{11} + 631808*a^6*b^{10}*c^5 \\
& *d^4*e^{12} - 610304*a^6*b^{11}*c^4*d^3*e^{13} - 71936*a^6*b^{12}*c^3*d^2*e^{14} - 21 \\
& 725184*a^7*b^2*c^{12}*d^{10}*e^6 + 30801920*a^7*b^3*c^{11}*d^9*e^7 - 8028160*a^7* \\
& b^4*c^{10}*d^8*e^8 - 32260096*a^7*b^5*c^9*d^7*e^9 + 37101568*a^7*b^6*c^8*d^6* \\
& e^{10} - 7182336*a^7*b^7*c^7*d^5*e^{11} - 7609856*a^7*b^8*c^6*d^4*e^{12} + 211225 \\
& 6*a^7*b^9*c^5*d^3*e^{13} + 661632*a^7*b^{10}*c^4*d^2*e^{14} - 30146560*a^8*b^2*c^ \\
& 11*d^8*e^8 + 55050240*a^8*b^3*c^{10}*d^7*e^9 - 34365440*a^8*b^4*c^9*d^6*e^{10} \\
& - 16429056*a^8*b^5*c^8*d^5*e^{11} + 24600576*a^8*b^6*c^7*d^4*e^{12} - 1683456*a \\
& ^8*b^7*c^6*d^3*e^{13} - 3151616*a^8*b^8*c^5*d^2*e^{14} - 10977280*a^9*b^2*c^{10}* \\
& d^6*e^{10} + 47022080*a^9*b^3*c^9*d^5*e^{11} - 30621696*a^9*b^4*c^8*d^4*e^{12} - \\
& 9232384*a^9*b^5*c^7*d^3*e^{13} + 7970816*a^9*b^6*c^6*d^2*e^{14} + 4325376*a^{10}* \\
& b^2*c^9*d^4*e^{12} + 25493504*a^{10}*b^3*c^8*d^3*e^{13} - 9117696*a^{10}*b^4*c^7*d^ \\
& 2*e^{14} + 491520*a^{11}*b^2*c^8*d^2*e^{14} - 4947968*a^{12}*b*c^8*d*e^{15} + 128*a*b \\
& ^{10}*c^{10}*d^{14}*e^2 - 1024*a*b^{11}*c^9*d^{13}*e^3 + 3584*a*b^{12}*c^8*d^{12}*e^4 - 7 \\
& 168*a*b^{13}*c^7*d^{11}*e^5 + 8960*a*b^{14}*c^6*d^{10}*e^6 - 7168*a*b^{15}*c^5*d^9*e^ \\
& 7 + 3584*a*b^{16}*c^4*d^8*e^8 - 1024*a*b^{17}*c^3*d^7*e^9 + 128*a*b^{18}*c^2*d^6* \\
& e^{10} + 1605632*a^6*b*c^{14}*d^{13}*e^3 - 1408*a^6*b^{13}*c^2*d*e^{15} + 7012352*a^7 \\
& *b*c^{13}*d^{11}*e^5 + 33152*a^7*b^{11}*c^3*d*e^{15} + 7045120*a^8*b*c^{12}*d^9*e^7 - \\
& 324480*a^8*b^9*c^4*d*e^{15} - 9830400*a^9*b*c^{11}*d^7*e^9 + 1689600*a^9*b^7*c \\
& ^5*d*e^{15} - 25722880*a^{10}*b*c^{10}*d^5*e^{11} - 4935680*a^{10}*b^5*c^6*d*e^{15} - 1 \\
& 9202048*a^{11}*b*c^9*d^3*e^{13} + 7667712*a^{11}*b^3*c^7*d*e^{15})/(16*(a^6*b^8*e^8 \\
& + 256*a^6*c^8*d^8 + 256*a^{10}*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 \\
& + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c \\
& ^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^{12}*d^4*e^4 - 4*a^ \\
& 3*b^{11}*d^3*e^5 + 6*a^4*b^{10}*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d \\
& ^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^{10}*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6 \\
& *e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d \\
& ^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^ \\
& 4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2 \\
& *c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^ \\
& 6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 20 \\
& 48*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 \\
& - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2 \\
& *b^9*c^3*d^7*e - 4*a^2*b^{11}*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^{10}*c \\
& *d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6* \\
& d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e \\
& ^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) + (x*(-d*e^7))^{(1/2)*} \\
& (1048576*a^{15}*c^8*e^{17} + 256*a^9*b^{12}*c^2*e^{17} - 6144*a^{10}*b^{10}*c^3*e^{17} + \\
& 61440*a^{11}*b^8*c^4*e^{17} - 327680*a^{12}*b^6*c^5*e^{17} + 983040*a^{13}*b^4*c^6*e^ \\
& 17 - 1572864*a^{14}*b^2*c^7*e^{17} - 1048576*a^8*c^{15}*d^{14}*e^3 - 5242880*a^9*c^ \\
& 14*d^{12}*e^5 - 9437184*a^{10}*c^{13}*d^{10}*e^7 - 5242880*a^{11}*c^{12}*d^8*e^9 + 5242 \\
& 880*a^{12}*c^{11}*d^6*e^{11} + 9437184*a^{13}*c^{10}*d^4*e^{13} + 5242880*a^{14}*c^9*d^2* \\
& e^{15} + 256*a^2*b^{11}*c^{10}*d^{15}*e^2 - 2048*a^2*b^{12}*c^9*d^{14}*e^3 + 7168*a^2*b \\
& ^{13}*c^8*d^{13}*e^4 - 14336*a^2*b^{14}*c^7*d^{12}*e^5 + 17920*a^2*b^{15}*c^6*d^{11}*e^ \\
& 6 - 14336*a^2*b^{16}*c^5*d^{10}*e^7 + 7168*a^2*b^{17}*c^4*d^9*e^8 - 2048*a^2*b^{18} \\
& *c^3*d^8*e^9 + 256*a^2*b^{19}*c^2*d^7*e^{10} - 5120*a^3*b^9*c^{11}*d^{15}*e^2 + 419 \\
& 84*a^3*b^{10}*c^{10}*d^{14}*e^3 - 148736*a^3*b^{11}*c^9*d^{13}*e^4 + 296192*a^3*b^{12}* \\
& c^8*d^{12}*e^5 - 359680*a^3*b^{13}*c^7*d^{11}*e^6 + 267520*a^3*b^{14}*c^6*d^{10}*e^7 \\
& - 112384*a^3*b^{15}*c^5*d^9*e^8 + 18176*a^3*b^{16}*c^4*d^8*e^9 + 3328*a^3*b^{17}* \\
& c^3*d^7*e^{10} - 1280*a^3*b^{18}*c^2*d^6*e^{11} + 40960*a^4*b^7*c^{12}*d^{15}*e^2 - 3 \\
& 48160*a^4*b^8*c^{11}*d^{14}*e^3 + 1254400*a^4*b^9*c^{10}*d^{13}*e^4 - 2478080*a^4*b \\
& ^{10}*c^9*d^{12}*e^5 + 2867456*a^4*b^{11}*c^8*d^{11}*e^6 - 1862144*a^4*b^{12}*c^7*d^{1 \\
& 0}*e^7 + 490240*a^4*b^{13}*c^6*d^9*e^8 + 128000*a^4*b^{14}*c^5*d^8*e^9 - 108800* \\
& a^4*b^{15}*c^4*d^7*e^{10} + 13824*a^4*b^{16}*c^3*d^6*e^{11} + 2304*a^4*b^{17}*c^2*d^5 \\
& *e^{12} - 163840*a^5*b^5*c^{13}*d^{15}*e^2 + 1474560*a^5*b^6*c^{12}*d^{14}*e^3 - 5447 \\
& 680*a^5*b^7*c^{11}*d^{13}*e^4 + 10588160*a^5*b^8*c^{10}*d^{12}*e^5 - 11166720*a^5*b \\
& ^9*c^9*d^{11}*e^6 + 5159936*a^5*b^{10}*c^8*d^{10}*e^7 + 1073920*a^5*b^{11}*c^7*d^9* \\
& e^8 - 2279680*a^5*b^{12}*c^6*d^8*e^9 + 770560*a^5*b^{13}*c^5*d^7*e^{10} + 33280*a
\end{aligned}$$



$$\begin{aligned}
&^5b^{14}c^4d^6e^{11} - 41216a^5b^{15}c^3d^5e^{12} - 1280a^5b^{16}c^2d^4e^{13} + 327680a^6b^3c^{14}d^{15}e^2 - 3276800a^6b^4c^{13}d^{14}e^3 + 12615 \\
&680a^6b^5c^{12}d^{13}e^4 - 23592960a^6b^6c^{11}d^{12}e^5 + 19701760a^6b^7c^{10}d^{11}e^6 + 1372160a^6b^8c^9d^{10}e^7 - 15846400a^6b^9c^8d^9e^8 \\
&+ 10864640a^6b^{10}c^7d^8e^9 - 1352960a^6b^{11}c^6d^7e^{10} - 1111040a^6b^{12}c^5d^6e^{11} + 273920a^6b^{13}c^4d^5e^{12} + 25600a^6b^{14}c^3d^4e^{13} - 1280a^6b^{15}c^2d^3e^{14} \\
&+ 3407872a^7b^2c^{14}d^{14}e^3 - 14221312a^7b^3c^{13}d^{13}e^4 + 23527424a^7b^4c^{12}d^{12}e^5 - 3768320a^7b^5c^{11}d^{11}e^6 - 38895616a^7b^6c^{10}d^{10}e^7 \\
&+ 50126848a^7b^7c^9d^9e^8 - 18362368a^7b^8c^8d^8e^9 - 6831104a^7b^9c^7d^7e^{10} + 6200320a^7b^{10}c^6d^6e^{11} - 726784a^7b^{11}c^5d^5e^{12} - 228608a^7b^{12}c^4d^4e^{13} + 31488a^7b^{13}c^3d^3e^{14} \\
&+ 2304a^7b^{14}c^2d^2e^{15} - 3145728a^8b^2c^{13}d^{12}e^5 - 31129600a^8b^3c^{12}d^{11}e^6 + 74711040a^8b^4c^{11}d^{10}e^7 - 55476224a^8b^5c^{10}d^9e^8 \\
&- 11075584a^8b^6c^9d^8e^9 + 35381248a^8b^7c^8d^7e^{10} - 14479360a^8b^8c^7d^6e^{11} - 168960a^8b^9c^6d^5e^{12} + 1286144a^8b^{10}c^5d^4e^{13} \\
&- 302336a^8b^{11}c^4d^3e^{14} - 55808a^8b^{12}c^3d^2e^{15} - 36962304a^9b^2c^{12}d^{10}e^7 - 9502720a^9b^3c^{11}d^9e^8 + 67174400a^9b^4c^{10}d^8e^9 \\
&- 54886400a^9b^5c^9d^7e^{10} + 11239424a^9b^6c^8d^6e^{11} + 5545984a^9b^7c^7d^5e^{12} - 5263360a^9b^8c^6d^4e^{13} + 1356800a^9b^9c^5d^3e^{14} \\
&+ 558080a^9b^{10}c^4d^2e^{15} - 49807360a^{10}b^2c^{11}d^8e^9 + 19333120a^{10}b^3c^{10}d^7e^{10} + 7208960a^{10}b^4c^9d^6e^{11} - 14974976a^{10}b^5c^8d^5e^{12} \\
&+ 15073280a^{10}b^6c^7d^4e^{13} - 2170880a^{10}b^7c^6d^3e^{14} - 2928640a^{10}b^8c^5d^2e^{15} - 11796480a^{11}b^2c^{10}d^6e^{11} + 23920640a^{11}b^3c^9d^5e^{12} \\
&- 24576000a^{11}b^4c^8d^4e^{13} - 4096000a^{11}b^5c^7d^3e^{14} + 8355840a^{11}b^6c^6d^2e^{15} + 12582912a^{12}b^2c^9d^4e^{13} + 19857408a^{12}b^3c^8d^3e^{14} \\
&- 11534336a^{12}b^4c^7d^2e^{15} + 3407872a^{13}b^2c^8d^2e^{15} - 5505024a^{14}b^3c^8d^2e^{16} - 262144a^7b^3c^{15}d^{15}e^2 + 5505024a^8b^3c^{14}d^{13}e^4 \\
&- 1280a^8b^{13}c^2d^6e^{16} + 25952256a^9b^3c^{13}d^{11}e^6 + 30976a^9b^{11}c^3d^6e^{16} + 38010880a^{10}b^3c^{12}d^9e^8 - 312320a^{10}b^9c^4d^6e^{16} \\
&+ 11796480a^{11}b^3c^{11}d^7e^{10} + 1679360a^{11}b^7c^5d^6e^{16} - 21233664a^{12}b^3c^{10}d^5e^{12} - 5079040a^{12}b^5c^6d^6e^{16} - 20709376a^{13}b^3c^9d^3e^{14} \\
&+ 8192000a^{13}b^3c^7d^6e^{16})) / (16*(c^2d^5 + a^2d^4e + b^2d^3e^2 - 2*bc^2d^4e - 2*ab^2d^2e^3 + 2*ac^2d^3e^2) * (a^6b^8e^8 + 256a^6c^8d^8 + 256a^{10}c^4e^8 - 16a^7b^6c^6e^8 - 4a^5b^9d^7e^7 + a^2b^8c^4d^8 - 16a^3b^6c^5d^8 + 96a^4b^4c^6d^8 - 256a^5b^2c^7d^8 + 96a^8b^4c^2e^8 - 256a^9b^2c^3e^8 + a^2b^{12}d^4e^4 - 4a^3b^{11}d^3e^5 + 6a^4b^{10}d^2e^6 + 1024a^7c^7d^6e^2 + 1536a^8c^6d^4e^4 + 1024a^9c^5d^2e^6 + 6a^2b^{10}c^2d^6e^2 - 92a^3b^8c^3d^6e^2 + 52a^3b^9c^2d^5e^3 + 512a^4b^6c^4d^6e^2 - 192a^4b^7c^3d^5e^3 - 90a^4b^8c^2d^4e^4 - 1152a^5b^4c^5d^6e^2 - 128a^5b^5c^4d^5e^3 + 800a^5b^6c^3d^4e^4 - 192a^5b^7c^2d^3e^5 + 512a^6b^2c^6d^6e^2 + 2048a^6b^3c^5d^5e^3 - 2240a^6b^4c^4d^4e^4 - 128a^6b^5c^3d^3e^5 + 512a^6b^6c^2d^2e^6 + 1536a^7b^2c^5d^4e^4 + 2048a^7b^3c^4d^3e^5 - 1152a^7b^4c^3d^2e^6 + 512a^8b^2c^4d^2e^6 - 1024a^6b^3c^7d^7e + 64a^6b^7c^3d^7e - 1024a^9b^3c^4d^6e^7 - 4a^2b^9c^3d^7e - 4a^2b^{11}c^3d^5e^3 + 64a^3b^7c^4d^7e - 4a^3b^{10}c^3d^4e^4 - 384a^4b^5c^5d^7e + 52a^4b^9c^3d^5e^5 + 1024a^5b^3c^6d^7e - 92a^5b^8c^3d^2e^6 - 3072a^7b^3c^6d^5e^3 - 384a^7b^5c^2d^6e^7 - 3072a^8b^3c^5d^3e^5 + 1024a^8b^3c^3d^6e^7)) * (-d^7e)^{(1/2)} / (2*(c^2d^5 + a^2d^4e + b^2d^3e^2 - 2*bc^2d^4e - 2*ab^2d^2e^3 + 2*ac^2d^3e^2)) * (-d^7e)^{(1/2)} / (2*(c^2d^5 + a^2d^4e + b^2d^3e^2 - 2*bc^2d^4e - 2*ab^2d^2e^3 + 2*ac^2d^3e^2)) - (x*(22800a^6c^9e^{13} + 36a^2b^8c^5e^{13} - 600a^3b^6c^6e^{13} + 4313a^4b^4c^7e^{13} - 15592a^5b^2c^8e^{13} + 1296a^2c^{13}d^8e^5 + 9792a^3c^{12}d^6e^7 + 30304a^4c^{11}d^4e^9 + 40512a^5c^{10}d^2e^{11} + 25b^4c^{11}d^8e^5 - 120b^5c^{10}d^7e^6 + 214b^6c^9d^6e^7 - 168b^7c^8d^5e^8 + 53b^8c^7d^4e^9 - 8b^9c^6d^3e^{10} + 4b^{10}c^5d^2e^{11} + 6336*
\end{aligned}$$

$$\begin{aligned} & a^2 b^2 c^{11} d^6 e^7 + 3840 a^2 b^3 c^{10} d^5 e^8 - 8506 a^2 b^4 c^9 d^4 e^9 \\ & + 1112 a^2 b^5 c^8 d^3 e^{10} + 1254 a^2 b^6 c^7 d^2 e^{11} + 22224 a^3 b^2 c^{10} d^4 e^9 \\ & + 13824 a^3 b^3 c^9 d^3 e^{10} - 9516 a^3 b^4 c^8 d^2 e^{11} + 11712 a^4 b^2 c^9 d^2 e^{11} \\ & - 24 a^4 b^9 c^5 d^5 e^{12} - 41088 a^5 b^3 c^9 d^5 e^{12} - 360 a^5 b^2 c^{12} d^8 e^5 \\ & + 1664 a^5 b^3 c^{11} d^7 e^6 - 2604 a^5 b^4 c^{10} d^6 e^7 + 1272 a^5 b^5 c^9 d^5 e^8 \\ & + 332 a^5 b^6 c^8 d^4 e^9 - 232 a^5 b^7 c^7 d^3 e^{10} - 48 a^5 b^8 c^6 d^2 e^{11} \\ & - 5760 a^6 b^2 c^{12} d^7 e^6 + 416 a^6 b^7 c^6 d^6 e^{12} - 32128 a^6 b^3 c^{11} d^5 e^8 \\ & - 4120 a^6 b^5 c^7 d^5 e^{12} - 63360 a^6 b^6 c^{10} d^3 e^{10} + 21376 a^6 b^3 c^8 d^8 e^{12} \\ & \left. \right) / \left( 8 (a^6 b^8 e^8 + 256 a^6 c^8 d^8 + 256 a^{10} c^4 e^8 - 16 a^7 b^6 c^8 e^8 \right. \\ & - 4 a^5 b^9 d^8 e^7 + a^2 b^8 c^4 d^8 - 16 a^3 b^6 c^5 d^8 + 96 a^4 b^4 c^6 d^8 \\ & - 256 a^5 b^2 c^7 d^8 + 96 a^8 b^4 c^2 e^8 - 256 a^9 b^2 c^3 e^8 + a^2 b^{12} d^4 e^4 \\ & - 4 a^3 b^{11} d^3 e^5 + 6 a^4 b^{10} d^2 e^6 + 1024 a^7 c^7 d^6 e^2 + 1536 a^8 c^6 d^4 e^4 \\ & + 1024 a^9 c^5 d^2 e^6 + 6 a^2 b^{10} c^2 d^6 e^2 - 92 a^3 b^8 c^3 d^6 e^2 + 52 a^3 b^9 c^2 d^5 e^3 \\ & + 512 a^4 b^6 c^4 d^6 e^2 - 192 a^4 b^7 c^3 d^5 e^3 - 90 a^4 b^8 c^2 d^4 e^4 \\ & - 1152 a^5 b^4 c^5 d^6 e^2 - 128 a^5 b^5 c^4 d^5 e^3 + 800 a^5 b^6 c^3 d^4 e^4 \\ & - 192 a^5 b^7 c^2 d^3 e^5 + 512 a^6 b^2 c^6 d^6 e^2 + 2048 a^6 b^3 c^5 d^5 e^3 \\ & - 2240 a^6 b^4 c^4 d^4 e^4 - 128 a^6 b^5 c^3 d^3 e^5 + 512 a^6 b^6 c^2 d^2 e^6 \\ & + 1536 a^7 b^2 c^5 d^4 e^4 + 2048 a^7 b^3 c^4 d^3 e^5 - 1152 a^7 b^4 c^3 d^2 e^6 \\ & + 512 a^8 b^2 c^4 d^2 e^6 - 1024 a^6 b^3 c^7 d^7 e + 64 a^6 b^7 c^3 d^7 e \\ & - 1024 a^9 b^3 c^4 d^7 e - 4 a^2 b^9 c^3 d^7 e - 4 a^2 b^{11} c^3 d^5 e^3 \\ & + 64 a^3 b^7 c^4 d^7 e - 4 a^3 b^{10} c^3 d^4 e^4 - 384 a^4 b^5 c^5 d^7 e \\ & + 52 a^4 b^9 c^3 d^3 e^5 + 1024 a^5 b^3 c^6 d^7 e - 92 a^5 b^8 c^2 d^2 e^6 \\ & - 3072 a^7 b^3 c^6 d^5 e^3 - 384 a^7 b^5 c^2 d^5 e^7 - 3072 a^8 b^3 c^5 d^3 e^5 \\ & \left. + 1024 a^8 b^3 c^3 d^5 e^7 \right) \cdot (-d e^7)^{(1/2)} / \left( 2 (c^2 d^5 + a^2 d e^4 + b^2 d^3 e^2 \right. \\ & - 2 b^3 c d^4 e - 2 a b d^2 e^3 + 2 a^2 c d^3 e^2) \left. \right) \cdot (-d e^7)^{(1/2)} \cdot i / \left( c^2 d^5 \right. \\ & \left. + a^2 d e^4 + b^2 d^3 e^2 - 2 b^3 c d^4 e - 2 a b d^2 e^3 + 2 a^2 c d^3 e^2 \right) \end{aligned}$$

**sympy** [F(-1)]    time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x\*\*2+d)/(c\*x\*\*4+b\*x\*\*2+a)\*\*2,x)

[Out] Timed out

$$3.198 \quad \int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)^2} dx$$

**Optimal.** Leaf size=1077

$$\frac{xe^4}{2d(cd^2 - bed + ae^2)^2(ex^2 + d)} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{2d^{3/2}(cd^2 - bed + ae^2)^2} + \frac{2(2cd - be)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{\sqrt{d}(cd^2 - bed + ae^2)^3} + \frac{\sqrt{2}\sqrt{c}\left(3c^2d^2 + b\left(b^2 - 2ac\right)\right)}{\sqrt{d}(cd^2 - bed + ae^2)^3}$$

**Rubi [A]** time = 12.64, antiderivative size = 1077, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$ , Rules used = {1238, 199, 205, 1178, 1166}

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x^2)^2\*(a + b\*x^2 + c\*x^4)^2), x]

[Out]  $(e^4 x) / (2 d (c d^2 - b d e + a e^2)^2 (d + e x^2)) + (x (a b c e (2 c d - b e) + (b^2 - 2 a c) (c^2 d^2 + b^2 e^2 - c e (2 b d + a e)) - c (2 b^2 c d e - 4 a c^2 d e - b^3 e^2 - b c (c d^2 - 3 a e^2)) x^2)) / (2 a (b^2 - 4 a c) (c d^2 - b d e + a e^2)^2 (a + b x^2 + c x^4)) + (\text{Sqrt}[2] \text{Sqrt}[c] e^2 (3 c^2 d^2 + b (b + \text{Sqrt}[b^2 - 4 a c]) e^2 - c e (3 b d + 2 \text{Sqrt}[b^2 - 4 a c] d + a e)) \text{ArcTan}[(\text{Sqrt}[2] \text{Sqrt}[c] x) / \text{Sqrt}[b - \text{Sqrt}[b^2 - 4 a c]])] / (\text{Sqrt}[b^2 - 4 a c] \text{Sqrt}[b - \text{Sqrt}[b^2 - 4 a c]] (c d^2 - b d e + a e^2)^3) + (\text{Sqrt}[c] (b^4 e^2 - b^3 e (2 c d - \text{Sqrt}[b^2 - 4 a c] e) - 4 a c^2 (3 c d^2 - e (\text{Sqrt}[b^2 - 4 a c] d + 3 a e)) + b^2 c (c d^2 - e (2 \text{Sqrt}[b^2 - 4 a c] d + 9 a e)) - b c (3 a \text{Sqrt}[b^2 - 4 a c] e^2 - c d (\text{Sqrt}[b^2 - 4 a c] d + 16 a e)) \text{ArcTan}[(\text{Sqrt}[2] \text{Sqrt}[c] x) / \text{Sqrt}[b - \text{Sqrt}[b^2 - 4 a c]])] / (2 \text{Sqrt}[2] a (b^2 - 4 a c)^{3/2} \text{Sqrt}[b - \text{Sqrt}[b^2 - 4 a c]] (c d^2 - b d e + a e^2)^2) - (\text{Sqrt}[2] \text{Sqrt}[c] e^2 (3 c^2 d^2 + b (b - \text{Sqrt}[b^2 - 4 a c]) e^2 - c e (3 b d - 2 \text{Sqrt}[b^2 - 4 a c] d + a e)) \text{ArcTan}[(\text{Sqrt}[2] \text{Sqrt}[c] x) / \text{Sqrt}[b + \text{Sqrt}[b^2 - 4 a c]])] / (\text{Sqrt}[b^2 - 4 a c] \text{Sqrt}[b + \text{Sqrt}[b^2 - 4 a c]] (c d^2 - b d e + a e^2)^3) - (\text{Sqrt}[c] (b^4 e^2 - b^3 e (2 c d + \text{Sqrt}[b^2 - 4 a c] e) + b c (3 a \text{Sqrt}[b^2 - 4 a c] e^2 - c d (\text{Sqrt}[b^2 - 4 a c] d - 16 a e)) + b^2 c (c d^2 + e (2 \text{Sqrt}[b^2 - 4 a c] d - 9 a e)) - 4 a c^2 (3 c d^2 + e (\text{Sqrt}[b^2 - 4 a c] d - 3 a e)) \text{ArcTan}[(\text{Sqrt}[2] \text{Sqrt}[c] x) / \text{Sqrt}[b + \text{Sqrt}[b^2 - 4 a c]])] / (2 \text{Sqrt}[2] a (b^2 - 4 a c)^{3/2} \text{Sqrt}[b + \text{Sqrt}[b^2 - 4 a c]] (c d^2 - b d e + a e^2)^2) + (2 e^{7/2} (2 c d - b e) \text{ArcTan}[(\text{Sqrt}[e] x) / \text{Sqrt}[d]]) / (\text{Sqrt}[d] (c d^2 - b d e + a e^2)^3) + (e^{7/2} \text{ArcTan}[(\text{Sqrt}[e] x) / \text{Sqrt}[d]]) / (2 d^{3/2} (c d^2 - b d e + a e^2)^2)$

**Rule 199**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2\*p] || (n == 2 && IntegerQ[4\*p]) || (n == 2 && IntegerQ[3\*p]) || Denominator[p + 1/n] < Denominator[p])

**Rule 205**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 1166**

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[e/2 + (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2

- q/2 + c\*x^2), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - a\*e^2, 0] && PosQ[b^2 - 4\*a\*c]

### Rule 1178

Int[((d\_) + (e\_)\*(x\_)^2)\*((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4)^(p\_), x\_Symbol] :> Simp[(x\*(a\*b\*e - d\*(b^2 - 2\*a\*c) - c\*(b\*d - 2\*a\*e)\*x^2)\*(a + b\*x^2 + c\*x^4)^(p + 1))/(2\*a\*(p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/(2\*a\*(p + 1)\*(b^2 - 4\*a\*c)), Int[Simp[(2\*p + 3)\*d\*b^2 - a\*b\*e - 2\*a\*c\*d\*(4\*p + 5) + (4\*p + 7)\*(d\*b - 2\*a\*e)\*c\*x^2, x]\*(a + b\*x^2 + c\*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[p, -1] && IntegerQ[2\*p]

### Rule 1238

Int[((d\_) + (e\_)\*(x\_)^2)^(q\_)\*((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4)^(p\_), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^2)^q\*(a + b\*x^2 + c\*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])

### Rubi steps

$$\begin{aligned} \int \frac{1}{(d + ex^2)^2 (a + bx^2 + cx^4)^2} dx &= \int \left( \frac{e^4}{(cd^2 - bde + ae^2)^2 (d + ex^2)^2} - \frac{2e^4(-2cd + be)}{(cd^2 - bde + ae^2)^3 (d + ex^2)} + \frac{c^2d^2 + b^2e^2}{(cd^2 - bde + ae^2)^3} \right) dx \\ &= \frac{e^2 \int \frac{3c^2d^2 + 2b^2e^2 - ce(5bd + ae) - 2ce(2cd - be)x^2}{a + bx^2 + cx^4} dx}{(cd^2 - bde + ae^2)^3} + \frac{(2e^4(2cd - be)) \int \frac{1}{d + ex^2} dx}{(cd^2 - bde + ae^2)^3} + \int \frac{c^2d^2 + b^2e^2}{(cd^2 - bde + ae^2)^3} dx \\ &= \frac{e^4x}{2d(cd^2 - bde + ae^2)^2(d + ex^2)} + \frac{x(abce(2cd - be) + (b^2 - 2ac)(c^2d^2 + b^2e^2))}{2a(b^2 - 4ac)} \\ &= \frac{e^4x}{2d(cd^2 - bde + ae^2)^2(d + ex^2)} + \frac{x(abce(2cd - be) + (b^2 - 2ac)(c^2d^2 + b^2e^2))}{2a(b^2 - 4ac)} \\ &= \frac{e^4x}{2d(cd^2 - bde + ae^2)^2(d + ex^2)} + \frac{x(abce(2cd - be) + (b^2 - 2ac)(c^2d^2 + b^2e^2))}{2a(b^2 - 4ac)} \end{aligned}$$

**Mathematica [A]** time = 5.84, size = 1020, normalized size = 0.95

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x^2)^2\*(a + b\*x^2 + c\*x^4)^2), x]

[Out] ((2\*e^4\*x)/(d\*(c\*d^2 + e\*(-(b\*d) + a\*e))^2\*(d + e\*x^2)) - (2\*x\*(b^4\*e^2 + b^3\*c\*e\*(-2\*d + e\*x^2) + 2\*a\*c^2\*(a\*e^2 - c\*d\*(d - 2\*e\*x^2)) + b^2\*c\*(-4\*a\*e^2 + c\*d\*(d - 2\*e\*x^2)) + b\*c^2\*(c\*d^2\*x^2 - 3\*a\*e\*(-2\*d + e\*x^2))))/(a\*(-b^2 + 4\*a\*c)\*(c\*d^2 + e\*(-(b\*d) + a\*e))^2\*(a + b\*x^2 + c\*x^4)) + (Sqrt[2]\*Sqrt[c]\*(b^5\*d\*e^3 + b^3\*e\*(c\*d - Sqrt[b^2 - 4\*a\*c]\*e)\*(3\*c\*d^2 + 5\*a\*e^2) +

$$b^4 e^2 (-3 c d^2 + e (\sqrt{b^2 - 4 a c} d - 5 a e)) - 4 a c^2 (-3 c^2 d^4 + c d^2 e (\sqrt{b^2 - 4 a c} d - 12 a e) + a e^3 (9 \sqrt{b^2 - 4 a c} d + 7 a e)) - b c (-19 a^2 \sqrt{b^2 - 4 a c} e^4 + 2 a c d e^2 (-3 \sqrt{b^2 - 4 a c} d + 26 a e) + c^2 d^3 (\sqrt{b^2 - 4 a c} d + 28 a e)) + b^2 c (-c^2 d^4 + 3 c d^2 e (\sqrt{b^2 - 4 a c} d + 4 a e) + a e^3 (7 \sqrt{b^2 - 4 a c} d + 29 a e)) \operatorname{ArcTan}[(\sqrt{2} \sqrt{c} x) / \sqrt{b - \sqrt{b^2 - 4 a c}}] / (a (b^2 - 4 a c)^{3/2} \sqrt{b - \sqrt{b^2 - 4 a c}} (-c d^2) + e (b d - a e))^3) - (\sqrt{2} \sqrt{c} (b^5 d e^3 + b^3 e (c d + \sqrt{b^2 - 4 a c} e)) (3 c d^2 + 5 a e^2) - b^2 c (c^2 d^4 + a e^3 (7 \sqrt{b^2 - 4 a c} d - 29 a e) + 3 c d^2 e (\sqrt{b^2 - 4 a c} d - 4 a e)) - b^4 e^2 (3 c d^2 + e (\sqrt{b^2 - 4 a c} d + 5 a e)) + 4 a c^2 (3 c^2 d^4 + a e^3 (9 \sqrt{b^2 - 4 a c} d - 7 a e) + c d^2 e (\sqrt{b^2 - 4 a c} d + 12 a e)) + b c (-19 a^2 \sqrt{b^2 - 4 a c} e^4 + c^2 d^3 (\sqrt{b^2 - 4 a c} d - 28 a e) - 2 a c d e^2 (3 \sqrt{b^2 - 4 a c} d + 26 a e)) \operatorname{ArcTan}[(\sqrt{2} \sqrt{c} x) / \sqrt{b + \sqrt{b^2 - 4 a c}}] / (a (b^2 - 4 a c)^{3/2} \sqrt{b + \sqrt{b^2 - 4 a c}} (-c d^2) + e (b d - a e))^3) + (2 e^{7/2} (9 c d^2 + e (-5 b d + a e)) \operatorname{ArcTan}[(\sqrt{e} x) / \sqrt{d}]) / (d^{3/2} (c d^2 + e (-b d + a e))^3) / 4$$

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex^2)^2 (a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e\*x^2)^2\*(a + b\*x^2 + c\*x^4)^2),x]

[Out] IntegrateAlgebraic[1/((d + e\*x^2)^2\*(a + b\*x^2 + c\*x^4)^2), x]

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)^2/(c\*x^4+b\*x^2+a)^2,x, algorithm="fricas")

[Out] Timed out

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)^2/(c\*x^4+b\*x^2+a)^2,x, algorithm="giac")

[Out] Timed out

**maple [B]** time = 0.08, size = 5709, normalized size = 5.30

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x^2+d)^2/(c\*x^4+b\*x^2+a)^2,x)

[Out] result too large to display

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x^2+d)^2/(c\*x^4+b\*x^2+a)^2,x, algorithm="maxima")

[Out]  $\frac{1}{2} \cdot (9cd^2e^4 - 5b^2de^5 + ae^6) \cdot \arctan\left(\frac{ex}{\sqrt{de}}\right) / ((c^3d^7 - 3b^2c^2d^6e - 3a^2b^2d^2e^5 + a^3de^6 + 3(b^2c + ac^2)d^5e^2 - (b^3 + 6ab^2c)d^4e^3 + 3(a^2b^2 + a^2c)d^3e^4) \sqrt{de}) + \frac{1}{2} \cdot ((b^3c^3d^3e - 2(b^2c^2 - 2ac^3)d^2e^2 + (b^3c - 3ab^2c^2)d^3e + (a^2b^2c - 4a^2c^2)e^4) x^5 + (b^3c^3d^4 - (b^2c^2 - 2ac^3)d^3e - (b^3c - 3ab^2c^2)d^2e^2 + (b^4 - 4ab^2c + 2a^2c^2)d^2e^3 + (a^2b^3 - 4a^2b^2c^2)e^4) x^3 + ((b^2c^2 - 2ac^3)d^4 - 2(b^3c - 3ab^2c^2)d^3e + (b^4 - 4ab^2c + 2a^2c^2)d^2e^2 + (a^2b^2 - 4a^3c)e^4) x) / ((a^2b^2c^2 - 4a^3c^3)d^6 - 2(a^2b^3c - 4a^3b^2c^2)d^5e + (a^2b^4 - 2a^3b^2c - 8a^4c^2)d^4e^2 - 2(a^3b^3 - 4a^4b^2c)d^3e^3 + (a^4b^2 - 4a^5c)d^2e^4 + ((a^2b^2c^3 - 4a^2c^4)d^5e - 2(a^2b^3c^2 - 4a^2b^2c^3)d^4e^2 + (a^2b^4c - 2a^2b^2c^2 - 8a^3c^3)d^3e^3 - 2(a^2b^3c - 4a^3b^2c^2)d^2e^4 + (a^3b^2c - 4a^4c^2)d^2e^5) x^6 + ((a^2b^2c^3 - 4a^2c^4)d^6 - (a^2b^3c^2 - 4a^2b^2c^3)d^5e - (a^2b^4c - 6a^2b^2c^2 + 8a^3c^3)d^4e^2 + (a^2b^5 - 4a^2b^3c)d^3e^3 - (2a^2b^4 - 9a^3b^2c + 4a^4c^2)d^2e^4 + (a^3b^3 - 4a^4b^2c)d^2e^5) x^4 + ((a^2b^3c^2 - 4a^2b^2c^3)d^6 - (2a^2b^4c - 9a^2b^2c^2 + 4a^3c^3)d^5e + (a^2b^5 - 4a^2b^3c)d^4e^2 - (a^2b^4 - 6a^3b^2c + 8a^4c^2)d^3e^3 - (a^3b^3 - 4a^4b^2c)d^2e^4 + (a^4b^2 - 4a^5c)d^2e^5) x^2) - \frac{1}{2} \cdot \int \text{rate}(-((b^2c^3 - 6ac^4)d^4 - (3b^3c^2 - 16ab^2c^3)d^3e + 3(b^4c - 3ab^2c^2 - 8a^2c^3)d^2e^2 - (b^5 + 6ab^3c - 44a^2b^2c^2)d^2e^3 + (5ab^4 - 24a^2b^2c + 14a^3c^2)e^4 + (b^4c^4d^4 - (3b^2c^3 - 4a^2c^4)d^3e + 3(b^3c^2 - 2ab^2c^3)d^2e^2 - (b^4c + 7ab^2c^2 - 36a^2c^3)d^2e^3 + (5ab^3c - 19a^2b^2c^2)e^4) x^2) / (c^2x^4 + b^2x^2 + a), x) / ((a^2b^2c^3 - 4a^2c^4)d^6 - 3(a^2b^3c^2 - 4a^2b^2c^3)d^5e + 3(a^2b^4c - 3a^2b^2c^2 - 4a^3c^3)d^4e^2 - (a^2b^5 + 2a^2b^3c - 24a^3b^2c^2)d^3e^3 + 3(a^2b^4 - 3a^3b^2c - 4a^4c^2)d^2e^4 - 3(a^3b^3 - 4a^4b^2c)d^2e^5 + (a^4b^2 - 4a^5c)e^6)$

**mupad [B]** time = 17.81, size = 97073, normalized size = 90.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d + e\*x^2)^2\*(a + b\*x^2 + c\*x^4)^2),x)

[Out]  $\text{symsum}(\log(\text{root}(128723189760a^{14}b^4c^9d^{13}e^{14}z^6 + 128723189760a^{12}b^4c^{11}d^{17}e^{10}z^6 - 8432455680a^{11}b^{12}c^4d^{11}e^{16}z^6 - 8432455680a^7b^{12}c^8d^{19}e^8z^6 + 12673351680a^{11}b^{11}c^5d^{12}e^{15}z^6 + 12673351680a^8b^{11}c^8d^{18}e^9z^6 - 72637480960a^{12}b^9c^6d^{12}e^{15}z^6 - 72637480960a^9b^9c^9d^{18}e^9z^6 - 21048344576a^9b^{12}c^6d^{15}e^{12}z^6 - 16609443840a^{17}b^3c^7d^8e^{19}z^6 - 16609443840a^{10}b^3c^{14}d^{22}e^5z^6 + 145332633600a^{13}b^5c^9d^{14}e^{13}z^6 + 145332633600a^{12}b^5c^{10}d^{16}e^{11}z^6 + 123740356608a^{14}b^5c^8d^{12}e^{15}z^6 + 123740356608a^{11}b^5c^{11}d^{18}e^9z^6 + 3460300800a^{17}b^5c^5d^6e^{21}z^6 + 3460300800a^8b^5c^{14}d^{24}e^3z^6 - 7751073792a^{15}b^7c^5d^8e^{19}z^6 - 7751073792a^8b^7c^{12}d^{22}e^5z^6 + 12041846784a^{14}b^7c^6d^{10}e^{17}z^6 + 12041846784a^9b^7c^{11}d^{20}e^7z^6 - 325545099264a^{14}b^3c^{10}d^{14}e^{13}z^6 - 325545099264a^{13}b^3c^{11}d^{16}e^{11}z^6 - 3330539520a^{13}b^{10}c^4d^9e^{18}z^6 - 3330539520a^7b^{10}c^{10}d^{21}e^6z^6 + 157789716480a^{12}b^7c^8d^{14}e^{13}z^6 + 157789716480a^{11}b^7c^9d^{16}e^{11}z^6 + 37492359168a^{11}b^{10}c^6d^{13}e^{14}z^6 + 37492359168a^9b^{10}c^8d^{17}e^{10}z^6 + 301989888a^8b^3c^{16}d^{26}e^z^6 - 7266631680a^{17}b^4c^6d^7e^{20}z^6 - 7266631680a^9b^4c^{14}d^{23}e^4z^6 - 201326592a^{20}b^2c^6d^4e^{23}z^6 - 188743680a^7b^5c^{15}d^{26}e^z^6 + 45747339264a^{13}b^8c^6d^{11}e^{16}z^6 + 45747339264a^9b^8c^{10}d^{19}e^8z^6 - 74612736a^{10}b^{16}c^6d^9e^{18}z^6 - 2768240640a^{16}b^7c^4d^6e^{21}z^6 - 2768240640a^7b^7c^{13}d^{24}e^3z^6 + 69746688a^{11}b^{15}c^4d^8e^{19}z^6 + 62914560a^6b^7c^{14}d^{26}e^z^6 + 2752020480a^{10}b^{13}c^4d^{12}e^{15}z^6 + 2752020480a^7b^{13}c^7d^{18}$

$$\begin{aligned}
& *e^9z^6 + 55148544a^9b^{17}c^4d^{10}e^{17}z^6 - 45957120a^{12}b^{14}c^4d^7e^{20}z^6 - 2724986880a^{14}b^9c^4d^8e^{19}z^6 - 2724986880a^7b^9c^{11}d^{22} \\
& *e^5z^6 - 25952256a^8b^{18}c^4d^{11}e^{16}z^6 + 21086208a^{13}b^{13}c^4d^6e^{21}z^6 - 11796480a^5b^9c^{13}d^{26}e^2z^6 - 6438912a^{14}b^{12}c^4d^5e^{22}z^6 \\
& + 5406720a^7b^{19}c^4d^{12}e^{15}z^6 + 1622016a^6b^{20}c^4d^{13}e^{14}z^6 - 1523712a^5b^{21}c^4d^{14}e^{13}z^6 + 1179648a^{15}b^{11}c^4d^4e^{23}z^6 + 1179648 \\
& *a^4b^{11}c^{12}d^{26}e^2z^6 + 442368a^4b^{22}c^4d^{15}e^{12}z^6 - 98304a^{16}b^{10}c^4d^3e^{24}z^6 - 49152a^3b^{23}c^4d^{16}e^{11}z^6 - 49152a^3b^{13}c^{11}d^{26} \\
& *e^2z^6 + 6897106944a^9b^{13}c^5d^{14}e^{13}z^6 + 6897106944a^8b^{13}c^6d^{16}e^{11}z^6 - 2422210560a^{16}b^6c^5d^7e^{20}z^6 - 2422210560a^8b^6c^5 \\
& ^{13}d^{23}e^4z^6 + 255785435136a^{14}b^2c^{11}d^{15}e^{12}z^6 + 41004564480a^{15}b^4c^8d^{11}e^{16}z^6 + 41004564480a^{11}b^4c^{12}d^{19}e^8z^6 + 227082 \\
& 2400a^{13}b^{11}c^3d^8e^{19}z^6 + 2270822400a^6b^{11}c^{10}d^{22}e^5z^6 + 23677108224a^{14}b^8c^5d^9e^{18}z^6 + 23677108224a^8b^8c^{11}d^{21}e^6z^6 \\
& + 212600881152a^{15}b^2c^{10}d^{13}e^{14}z^6 + 212600881152a^{13}b^2c^{12}d^{17}e^{10}z^6 + 75157733376a^{15}b^5c^7d^{10}e^{17}z^6 + 75157733376a^{10}b^5 \\
& c^{12}d^{20}e^7z^6 - 251217838080a^{13}b^6c^8d^{13}e^{14}z^6 - 251217838080a^{11}b^6c^{10}d^{17}e^{10}z^6 - 1952907264a^{14}b^{10}c^3d^7e^{20}z^6 - 195 \\
& 2907264a^6b^{10}c^{11}d^{23}e^4z^6 - 27691057152a^{13}b^9c^5d^{10}e^{17}z^6 - 27691057152a^8b^9c^{10}d^{20}e^7z^6 - 1902673920a^8b^{15}c^4d^{14}e^{13} \\
& z^6 - 1902673920a^7b^{15}c^5d^{16}e^{11}z^6 + 10465050624a^{10}b^{11}c^6d^{14}e^{13}z^6 + 10465050624a^9b^{11}c^7d^{16}e^{11}z^6 + 1613905920a^9b^{14} \\
& c^4d^{13}e^{14}z^6 + 1613905920a^7b^{14}c^6d^{17}e^{10}z^6 - 33218887680a^{17}b^3c^9d^{10}e^{17}z^6 - 33218887680a^{12}b^3c^{14}d^{20}e^7z^6 + 1524695040 \\
& a^{10}b^{14}c^3d^{11}e^{16}z^6 + 1524695040a^6b^{14}c^7d^{19}e^8z^6 - 1472200704a^{18}b^4c^5d^5e^{22}z^6 - 1472200704a^8b^4c^{15}d^{25}e^2z^6 - 830 \\
& 47219200a^{16}b^3c^8d^{10}e^{17}z^6 - 83047219200a^{11}b^3c^{13}d^{20}e^7z^6 + 44291850240a^{17}b^2c^8d^9e^{18}z^6 + 44291850240a^{11}b^2c^{14}d^{21} \\
& e^6z^6 + 1308131328a^8b^{14}c^5d^{15}e^{12}z^6 - 201326592a^9b^3c^{17}d^{26}e^2z^6 + 48530718720a^{12}b^8c^7d^{13}e^{14}z^6 + 48530718720a^{10}b^8c^9 \\
& d^{17}e^{10}z^6 - 1242644480a^{12}b^{12}c^3d^9e^{18}z^6 - 1242644480a^6b^{12}c^9d^{21}e^6z^6 + 9813196800a^{12}b^{10}c^5d^{11}e^{16}z^6 + 9813196800a^8 \\
& b^{10}c^9d^{19}e^8z^6 - 93012885504a^{15}b^3c^{11}d^{14}e^{13}z^6 - 93012885504a^{14}b^3c^{12}d^{16}e^{11}z^6 + 177305812992a^{13}b^4c^{10}d^{15}e^{12}z^6 + 52 \\
& 730658816a^{10}b^{10}c^7d^{15}e^{12}z^6 - 1180106752a^9b^{15}c^3d^{12}e^{15}z^6 - 1180106752a^6b^{15}c^6d^{18}e^9z^6 + 1023672320a^{15}b^9c^3d^6e^2 \\
& 1z^6 + 1023672320a^6b^9c^{12}d^{24}e^3z^6 + 975175680a^{17}b^6c^4d^5e^{22}z^6 + 975175680a^7b^6c^{14}d^{25}e^2z^6 - 11072962560a^{18}b^3c^8d^8 \\
& e^{19}z^6 - 11072962560a^{11}b^3c^{15}d^{22}e^5z^6 + 9412018176a^{18}b^2c^7d^{17}e^{20}z^6 + 9412018176a^{10}b^2c^{15}d^{23}e^4z^6 + 805306368a^{19}b^2c^6 \\
& d^5e^{22}z^6 + 805306368a^9b^2c^{16}d^{25}e^2z^6 - 133809831936a^{14}b^6c^7d^{11}e^{16}z^6 - 133809831936a^{10}b^6c^{11}d^{19}e^8z^6 - 2214592512 \\
& a^{19}b^3c^7d^6e^{21}z^6 - 2214592512a^{10}b^3c^{16}d^{24}e^3z^6 + 82216747008a^{13}b^7c^7d^{12}e^{15}z^6 + 82216747008a^{10}b^7c^{10}d^{18}e^9z^6 - 5866 \\
& 29120a^{12}b^{13}c^2d^8e^{19}z^6 - 586629120a^5b^{13}c^9d^{22}e^5z^6 + 586629120a^7b^{16}c^4d^{15}e^{12}z^6 - 4844421120a^{16}b^4c^7d^9e^{18}z^6 - \\
& 4844421120a^{10}b^4c^{13}d^{21}e^6z^6 + 531210240a^{11}b^{14}c^2d^9e^{18}z^6 + 531210240a^5b^{14}c^8d^{21}e^6z^6 - 527155200a^{11}b^{13}c^3d^{10}e^{17} \\
& z^6 - 527155200a^6b^{13}c^8d^{20}e^7z^6 + 43470028800a^{11}b^8c^8d^{15}e^{12}z^6 - 107874877440a^{11}b^9c^7d^{14}e^{13}z^6 - 107874877440a^{10}b^9 \\
& c^8d^{16}e^{11}z^6 + 9018408960a^{12}b^{11}c^4d^{10}e^{17}z^6 + 9018408960a^7b^{11}c^9d^{20}e^7z^6 + 421994496a^{13}b^{12}c^2d^7e^{20}z^6 + 421994496 \\
& a^5b^{12}c^{10}d^{23}e^4z^6 - 66437775360a^{16}b^3c^{10}d^{12}e^{15}z^6 - 66437775360a^{13}b^3c^{13}d^{18}e^9z^6 + 26159874048a^{16}b^5c^6d^8e^{19}z^6 + 26 \\
& 159874048a^9b^5c^{13}d^{22}e^5z^6 - 369098752a^{18}b^3c^6d^6e^{21}z^6 - 369098752a^9b^3c^{15}d^{24}e^3z^6 + 351436800a^8b^{16}c^3d^{13}e^{14}z^6 \\
& + 351436800a^6b^{16}c^5d^{17}e^{10}z^6 - 334233600a^{16}b^8c^3d^5e^{22}z^6 - 334233600a^6b^8c^{13}d^{25}e^2z^6 + 301989888a^{19}b^3c^5d^4e^{23} \\
& z^6 - 266010624a^{10}b^{15}c^2d^{10}e^{17}z^6 - 266010624a^5b^{15}c^7d^{20}e^2z^6
\end{aligned}$$

$$\begin{aligned}
& ^7z^6 - 305198530560a^{12}b^6c^9d^{15}e^{12}z^6 - 203292672a^{14}b^{11}c^2d^6e^{21}z^6 - 203292672a^5b^{11}c^{11}d^{24}e^3z^6 - 188743680a^{18}b^5c^4d^4e^{23}z^6 + 120418467840a^{16}b^2c^9d^{11}e^{16}z^6 + 120418467840a^{12}b^2c^{13}d^{19}e^8z^6 - 17293934592a^{10}b^{12}c^5d^{13}e^{14}z^6 - 17293934592a^8b^{12}c^7d^{17}e^{10}z^6 + 104890368a^8b^{17}c^2d^{12}e^{15}z^6 + 104890368a^5b^{17}c^5d^{18}e^9z^6 + 4390256640a^{15}b^8c^4d^7e^{20}z^6 + 4390256640a^7b^8c^{12}d^{23}e^4z^6 - 91750400a^6b^{18}c^3d^{15}e^{12}z^6 + 79134720a^7b^{17}c^3d^{14}e^{13}z^6 + 79134720a^6b^{17}c^4d^{16}e^{11}z^6 - 74612736a^4b^{16}c^7d^{21}e^6z^6 - 72990720a^7b^{18}c^2d^{13}e^{14}z^6 - 72990720a^5b^{18}c^4d^{17}e^{10}z^6 + 69746688a^4b^{15}c^8d^{22}e^5z^6 + 63700992a^{15}b^{10}c^2d^5e^{22}z^6 + 63700992a^5b^{10}c^{12}d^{25}e^2z^6 + 62914560a^{17}b^7c^3d^4e^{23}z^6 + 55148544a^4b^{17}c^6d^{20}e^7z^6 - 45957120a^4b^{14}c^9d^{23}e^4z^6 - 25952256a^4b^{18}c^5d^{19}e^8z^6 - 25165824a^{20}b^2c^5d^3e^{24}z^6 + 21086208a^4b^{13}c^{10}d^{24}e^3z^6 + 20643840a^6b^{19}c^2d^{14}e^{13}z^6 + 20643840a^5b^{19}c^3d^{16}e^{11}z^6 + 15728640a^{19}b^4c^4d^3e^{24}z^6 - 11796480a^{16}b^9c^2d^4e^{23}z^6 - 6438912a^4b^{12}c^{11}d^{25}e^2z^6 + 5406720a^4b^{19}c^4d^{18}e^9z^6 - 5242880a^{18}b^6c^3d^3e^{24}z^6 + 3784704a^3b^{18}c^6d^{21}e^6z^6 - 3244032a^3b^{19}c^5d^{20}e^7z^6 - 3244032a^3b^{17}c^7d^{22}e^5z^6 + 2027520a^3b^{20}c^4d^{19}e^8z^6 + 2027520a^3b^{16}c^8d^{23}e^4z^6 - 1622016a^9b^{16}c^2d^{11}e^{16}z^6 - 1622016a^5b^{16}c^6d^{19}e^8z^6 + 1622016a^4b^{20}c^3d^{17}e^{10}z^6 - 1523712a^4b^{21}c^2d^{16}e^{11}z^6 + 983040a^{17}b^8c^2d^3e^{24}z^6 - 901120a^3b^{21}c^3d^{18}e^9z^6 - 901120a^3b^{15}c^9d^{24}e^3z^6 + 270336a^3b^{22}c^2d^{17}e^{10}z^6 + 270336a^3b^{14}c^{10}d^{25}e^2z^6 + 172032a^5b^{20}c^2d^{15}e^{12}z^6 - 38593888256a^{15}b^6c^6d^9e^{18}z^6 - 38593888256a^9b^6c^{12}d^{21}e^6z^6 - 210386288640a^{15}b^3c^9d^{12}e^{15}z^6 - 210386288640a^{12}b^3c^{12}d^{18}e^9z^6 + 15502147584a^{15}c^{12}d^{15}e^{12}z^6 + 1107296256a^{19}c^8d^7e^{20}z^6 + 1107296256a^{11}c^{16}d^{23}e^4z^6 + 13287555072a^{16}c^{11}d^{13}e^{14}z^6 + 13287555072a^{14}c^{13}d^{17}e^{10}z^6 + 201326592a^{20}c^7d^5e^{22}z^6 + 201326592a^{10}c^{17}d^{25}e^2z^6 + 16777216a^{21}c^6d^3e^{24}z^6 + 3784704a^9b^{18}d^9e^{18}z^6 - 3244032a^{10}b^{17}d^8e^{19}z^6 - 3244032a^8b^{19}d^{10}e^{17}z^6 + 2027520a^{11}b^{16}d^7e^{20}z^6 + 2027520a^7b^{20}d^{11}e^{16}z^6 - 901120a^{12}b^{15}d^6e^{21}z^6 - 901120a^6b^{21}d^{12}e^{15}z^6 + 270336a^{13}b^{14}d^5e^{22}z^6 + 270336a^5b^{22}d^{13}e^{14}z^6 - 49152a^{14}b^{13}d^4e^{23}z^6 - 49152a^4b^{23}d^{14}e^{13}z^6 + 4096a^{15}b^{12}d^3e^{24}z^6 + 4096a^3b^{24}d^{15}e^{12}z^6 - 25165824a^8b^2c^{17}d^{27}z^6 + 15728640a^7b^4c^{16}d^{27}z^6 - 5242880a^6b^6c^{15}d^{27}z^6 + 983040a^5b^8c^{14}d^{27}z^6 - 983040a^4b^{10}c^{13}d^{27}z^6 + 4096a^3b^{12}c^{12}d^{27}z^6 + 8304721920a^{17}c^{10}d^{11}e^{16}z^6 + 8304721920a^{13}c^{14}d^{19}e^8z^6 + 3690987520a^{18}c^9d^9e^{18}z^6 + 3690987520a^{12}c^{15}d^{21}e^6z^6 + 16777216a^9c^{18}d^{27}z^6 - 8493371392a^6b^8c^9d^{14}e^9z^4 + 1458044928a^8b^8c^{14}d^{17}e^6z^4 - 12604538880a^{11}b^4c^8d^8e^{15}z^4 - 8303067136a^9b^5c^9d^{11}e^12z^4 - 5588058112a^{13}b^8c^9d^7e^{16}z^4 - 3892838400a^8b^2c^{13}d^{16}e^7z^4 - 3611713536a^8b^8c^7d^{10}e^{13}z^4 + 7819006464a^7b^9c^7d^{11}e^{12}z^4 - 7782137856a^8b^7c^8d^{11}e^{12}z^4 + 7780433920a^{12}b^2c^9d^8e^{15}z^4 - 12020465664a^7b^5c^{11}d^{15}e^8z^4 + 3176792064a^8b^3c^{12}d^{15}e^8z^4 - 322633728a^{15}b^8c^7d^3e^{20}z^4 + 210829312a^7b^8c^{15}d^{19}e^4z^4 + 15623258112a^9b^6c^8d^{10}e^{13}z^4 + 25165824a^{15}b^3c^5d^5e^{22}z^4 - 15728640a^{14}b^5c^4d^5e^{22}z^4 + 12582912a^5b^2c^{16}d^{22}e^5z^4 - 11730944a^4b^4c^{15}d^{22}e^5z^4 + 5242880a^{13}b^7c^3d^5e^{22}z^4 - 4561920a^6b^{15}c^7d^{17}e^6z^4 + 4521984a^3b^6c^{14}d^{22}e^5z^4 + 4460544a^8b^{14}c^8d^{18}e^5z^4 + 3538944a^6b^8c^{16}d^{21}e^2z^4 + 3108864a^8b^{16}c^6d^{16}e^7z^4 - 3027200a^8b^{13}c^9d^{19}e^4z^4 - 2345472a^5b^{17}c^4d^7e^{16}z^4 - 2307072a^8b^{14}c^4d^4e^{19}z^4 + 1824768a^6b^{16}c^4d^6e^{17}z^4 + 1734912a^9b^{13}c^4d^3e^{20}z^4 + 1419264a^8b^{12}c^{10}d^{20}e^3z^4 - 1191168a^8b^{17}c^5d^{15}e^8z^4 - 983040a^{12}b^9c^2d^5e^{22}z^4 + 964608a^4b^{18}c^4d^8e^{15}z^4 - 866304a^2b^8c^{13}d^{22}e^5z^4 + 703488a^7b^{15}c^4d^5e^{18}z^4 - 608256a^{10}b^{12}c^4d^2e^{21}z^4 - 440832a^8b^{11}c^{11}d
\end{aligned}$$



$$\begin{aligned}
& ^{21}e^{2z^4} + 275968*a*b^{19}*c^3*d^{13}*e^{10}*z^4 - 159744*a^2*b^{20}*c*d^{10}*e^{13} \\
& *z^4 - 153600*a*b^{20}*c^2*d^{12}*e^{11}*z^4 + 64512*a^3*b^{19}*c*d^9*e^{14}*z^4 + 19 \\
& 746062336*a^8*b^6*c^9*d^{12}*e^{11}*z^4 - 15333588992*a^{10}*b^4*c^9*d^{10}*e^{13}*z^4 \\
& + 6702170112*a^7*b^4*c^{12}*d^{16}*e^7*z^4 + 15167913984*a^{10}*b^3*c^{10}*d^{11}*e \\
& ^{12}*z^4 - 2256638976*a^5*b^{11}*c^7*d^{13}*e^{10}*z^4 + 2254307328*a^5*b^7*c^{11}*d \\
& ^{17}*e^6*z^4 - 2200633344*a^6*b^5*c^{12}*d^{17}*e^6*z^4 + 6457131008*a^{11}*b^3*c^ \\
& 9*d^9*e^{14}*z^4 - 2128785408*a^5*b^8*c^{10}*d^{16}*e^7*z^4 - 2126057472*a^6*b^{11} \\
& *c^6*d^{11}*e^{12}*z^4 + 2038349824*a^{12}*b^5*c^6*d^5*e^{18}*z^4 + 2037841920*a^5* \\
& b^{10}*c^8*d^{14}*e^9*z^4 + 3615621120*a^9*b*c^{13}*d^{15}*e^8*z^4 + 1900019712*a^1 \\
& 1*b^2*c^{10}*d^{10}*e^{13}*z^4 + 1867698432*a^9*b^9*c^5*d^7*e^{16}*z^4 - 6157369344 \\
& *a^9*b^4*c^{10}*d^{12}*e^{11}*z^4 - 1856913408*a^7*b^{10}*c^6*d^{10}*e^{13}*z^4 + 17891 \\
& 32800*a^6*b^4*c^{13}*d^{18}*e^5*z^4 + 6082658304*a^8*b^4*c^{11}*d^{14}*e^9*z^4 + 60 \\
& 29549568*a^{11}*b^5*c^7*d^7*e^{16}*z^4 + 6010159104*a^6*b^7*c^{10}*d^{15}*e^8*z^4 + \\
& 1703182336*a^7*b^7*c^9*d^{13}*e^{10}*z^4 + 1658388480*a^{11}*b^6*c^6*d^6*e^{17}*z^4 \\
& + 5917114368*a^{10}*b^6*c^7*d^8*e^{15}*z^4 - 1591197696*a^{11}*b^7*c^5*d^5*e^{18} \\
& *z^4 - 1526464512*a^8*b^{10}*c^5*d^8*e^{15}*z^4 - 5772607488*a^{12}*b^4*c^7*d^6*e \\
& ^{17}*z^4 - 1423507456*a^{13}*b^4*c^6*d^4*e^{19}*z^4 - 1387266048*a^7*b^3*c^{13}*d^ \\
& ^{17}*e^6*z^4 + 2976120832*a^{10}*b*c^{12}*d^{13}*e^{10}*z^4 - 9906946048*a^9*b^2*c^{12} \\
& *d^{14}*e^9*z^4 - 18421874688*a^8*b^5*c^{10}*d^{13}*e^{10}*z^4 + 1141217280*a^6*b^1 \\
& 2*c^5*d^{10}*e^{13}*z^4 - 9714364416*a^7*b^8*c^8*d^{12}*e^{11}*z^4 - 16777216*a^{16}* \\
& b*c^6*d*e^{22}*z^4 + 98304*a^{11}*b^{11}*c*d*e^{22}*z^4 + 81920*a*b^{10}*c^{12}*d^{22}*e* \\
& z^4 + 39168*a*b^{21}*c*d^{11}*e^{12}*z^4 - 1091829760*a^5*b^6*c^{12}*d^{18}*e^5*z^4 + \\
& 1046740992*a^{14}*b^2*c^7*d^4*e^{19}*z^4 - 6884425728*a^{12}*b*c^{10}*d^9*e^{14}*z^4 \\
& + 987445248*a^4*b^{10}*c^9*d^{16}*e^7*z^4 + 984087552*a^5*b^{12}*c^6*d^{12}*e^{11}*z \\
& ^4 - 9564585984*a^9*b^7*c^7*d^9*e^{14}*z^4 - 5266857984*a^{10}*b^7*c^6*d^7*e^{16} \\
& *z^4 - 892145664*a^7*b^{11}*c^5*d^9*e^{14}*z^4 - 2444623872*a^{11}*b*c^{11}*d^{11}*e^ \\
& ^{12}*z^4 + 768540672*a^{12}*b^3*c^8*d^7*e^{16}*z^4 + 5048322048*a^8*b^9*c^6*d^9*e \\
& ^{14}*z^4 + 5047612416*a^6*b^9*c^8*d^{13}*e^{10}*z^4 - 732492288*a^4*b^{11}*c^8*d^1 \\
& 5*e^8*z^4 + 9266921472*a^7*b^6*c^{10}*d^{14}*e^9*z^4 - 645857280*a^6*b^6*c^{11}*d \\
& ^{16}*e^7*z^4 - 623867904*a^4*b^9*c^{10}*d^{17}*e^6*z^4 - 622067712*a^6*b^3*c^{14}* \\
& d^{19}*e^4*z^4 + 582617088*a^{10}*b^8*c^5*d^6*e^{17}*z^4 + 577119744*a^7*b^{12}*c^4 \\
& *d^8*e^{15}*z^4 + 552566784*a^{12}*b^6*c^5*d^4*e^{19}*z^4 + 549224448*a^9*b^8*c^6 \\
& *d^8*e^{15}*z^4 - 526565376*a^9*b^{10}*c^4*d^6*e^{17}*z^4 + 511520256*a^{10}*b^9*c^ \\
& 4*d^5*e^{18}*z^4 + 13393723392*a^9*b^3*c^{11}*d^{13}*e^{10}*z^4 - 2066350080*a^{14}*b \\
& *c^8*d^5*e^{18}*z^4 + 4718592000*a^{13}*b^2*c^8*d^6*e^{17}*z^4 - 314572800*a^7*b^ \\
& 2*c^{14}*d^{18}*e^5*z^4 + 287250432*a^4*b^{13}*c^6*d^{13}*e^{10}*z^4 + 4565827584*a^1 \\
& 0*b^5*c^8*d^9*e^{14}*z^4 - 250785792*a^4*b^{14}*c^5*d^{12}*e^{11}*z^4 + 235536384*a \\
& ^{13}*b^3*c^7*d^5*e^{18}*z^4 - 232683264*a^8*b^{11}*c^4*d^7*e^{16}*z^4 - 199627776* \\
& a^5*b^{14}*c^4*d^{10}*e^{13}*z^4 - 190267392*a^{12}*b^7*c^4*d^3*e^{20}*z^4 + 18489139 \\
& 2*a^6*b^{10}*c^7*d^{12}*e^{11}*z^4 + 180502528*a^4*b^7*c^{12}*d^{19}*e^4*z^4 + 178877 \\
& 952*a^3*b^{13}*c^7*d^{15}*e^8*z^4 + 172490752*a^{14}*b^3*c^6*d^3*e^{20}*z^4 + 16394 \\
& 6496*a^{13}*b^5*c^5*d^3*e^{20}*z^4 + 155839488*a^8*b^{12}*c^3*d^6*e^{17}*z^4 + 1550 \\
& 00832*a^5*b^5*c^{13}*d^{19}*e^4*z^4 - 152076288*a^4*b^6*c^{13}*d^{20}*e^3*z^4 - 137 \\
& 592576*a^3*b^{12}*c^8*d^{16}*e^7*z^4 - 133693440*a^{14}*b^4*c^5*d^2*e^{21}*z^4 - 11 \\
& 6767488*a^3*b^9*c^{11}*d^{19}*e^4*z^4 - 108985344*a^3*b^{14}*c^6*d^{14}*e^9*z^4 - 1 \\
& 06223616*a^6*b^{13}*c^4*d^9*e^{14}*z^4 + 106119168*a^3*b^{10}*c^{10}*d^{18}*e^5*z^4 + \\
& 102432768*a^5*b^4*c^{14}*d^{20}*e^3*z^4 + 102113280*a^4*b^{12}*c^7*d^{14}*e^9*z^4 \\
& + 100674048*a^5*b^9*c^9*d^{15}*e^8*z^4 + 90439680*a^{13}*b^6*c^4*d^2*e^{21}*z^4 - \\
& 86808576*a^6*b^{14}*c^3*d^8*e^{15}*z^4 + 86245376*a^6*b^2*c^{15}*d^{20}*e^3*z^4 + \\
& 79011840*a^4*b^8*c^{11}*d^{18}*e^5*z^4 + 78345216*a^4*b^{15}*c^4*d^{11}*e^{12}*z^4 + \\
& 78006528*a^{11}*b^9*c^3*d^3*e^{20}*z^4 - 73253376*a^9*b^{11}*c^3*d^5*e^{18}*z^4 + 6 \\
& 7524608*a^3*b^8*c^{12}*d^{20}*e^3*z^4 + 67108864*a^{15}*b^2*c^6*d^2*e^{21}*z^4 - 61 \\
& 590528*a^{10}*b^{10}*c^3*d^4*e^{19}*z^4 + 61559808*a^5*b^{15}*c^3*d^9*e^{14}*z^4 - 59 \\
& 637760*a^5*b^3*c^{15}*d^{21}*e^2*z^4 + 58638336*a^4*b^5*c^{14}*d^{21}*e^2*z^4 - 408 \\
& 28416*a^7*b^{13}*c^3*d^7*e^{16}*z^4 - 35639296*a^2*b^{12}*c^9*d^{18}*e^5*z^4 - 3129 \\
& 3440*a^{12}*b^8*c^3*d^2*e^{21}*z^4 + 29933568*a^5*b^{13}*c^5*d^{11}*e^{12}*z^4 + 2779 \\
& 3920*a^2*b^{11}*c^{10}*d^{19}*e^4*z^4 + 27168768*a^2*b^{13}*c^8*d^{17}*e^6*z^4 - 2360 \\
& 2176*a^7*b^{14}*c^2*d^6*e^{17}*z^4 - 23248896*a^3*b^7*c^{13}*d^{21}*e^2*z^4 + 20929 \\
& 536*a^3*b^{15}*c^5*d^{13}*e^{10}*z^4 + 18428928*a^9*b^{12}*c^2*d^4*e^{19}*z^4 + 18026
\end{aligned}$$

$$\begin{aligned}
& 496a^6b^{15}c^2d^7e^{16}z^4 - 16261632a^{10}b^{11}c^2d^3e^{20}z^4 + 15128 \\
& 064a^3b^{16}c^4d^{12}e^{11}z^4 - 14060544a^2b^{10}c^{11}d^{20}e^3z^4 + 1317 \\
& 8880a^2b^{16}c^5d^{14}e^9z^4 - 11244288a^3b^{17}c^3d^{11}e^{12}z^4 - 1050 \\
& 9312a^2b^{15}c^6d^{15}e^8z^4 - 7262208a^4b^{17}c^2d^9e^{14}z^4 - 704563 \\
& 2a^2b^{17}c^4d^{13}e^{10}z^4 - 6285312a^2b^{14}c^7d^{16}e^7z^4 + 5996544a \\
& a^{11}b^{10}c^2d^2e^{21}z^4 + 4558336a^2b^9c^{12}d^{21}e^2z^4 + 4478976a^ \\
& 11b^8c^4d^4e^{19}z^4 + 2850816a^4b^{16}c^3d^{10}e^{13}z^4 + 2629632a^3b \\
& b^{11}c^9d^{17}e^6z^4 + 2503680a^3b^{18}c^2d^{10}e^{13}z^4 + 1627136a^2b^ \\
& 18c^3d^{12}e^{11}z^4 + 1605120a^8b^{13}c^2d^5e^{18}z^4 + 1483776a^5b^{16} \\
& c^2d^8e^{15}z^4 + 139776a^2b^{19}c^2d^{11}e^{12}z^4 - 8542224384a^{10}b^2 \\
& c^{11}d^{12}e^{11}z^4 - 3072b^{22}c^d^{12}e^{11}z^4 - 3072b^{12}c^{11}d^{22}e^z^4 \\
& - 1572864a^6c^{17}d^{22}e^z^4 - 4096a^{10}b^{13}d^e^{22}z^4 - 4096a^b^{22}d^ \\
& 10e^{13}z^4 - 6144a^{12}b^{10}c^e^{23}z^4 - 983040a^5b^c^{17}d^{23}z^4 - 6912 \\
& a^b^9c^{13}d^{23}z^4 + 1824522240a^{13}c^{10}d^8e^{15}z^4 + 1730150400a^{12} \\
& c^{11}d^{10}e^{13}z^4 + 958922752a^{14}c^9d^6e^{17}z^4 - 537919488a^9c^{14}d \\
& ^{16}e^7z^4 + 508559360a^{11}c^{12}d^{12}e^{11}z^4 - 500170752a^{10}c^{13}d^{14} \\
& e^9z^4 + 246939648a^{15}c^8d^4e^{19}z^4 - 199229440a^8c^{15}d^{18}e^5z^4 \\
& - 29884416a^7c^{16}d^{20}e^3z^4 + 25165824a^{16}c^7d^2e^{21}z^4 + 236544 \\
& b^{17}c^6d^{17}e^6z^4 - 202752b^{18}c^5d^{16}e^7z^4 - 202752b^{16}c^7d^{1 \\
& 8}e^5z^4 + 126720b^{19}c^4d^{15}e^8z^4 + 126720b^{15}c^8d^{19}e^4z^4 - 5 \\
& 6320b^{20}c^3d^{14}e^9z^4 - 56320b^{14}c^9d^{20}e^3z^4 + 16896b^{21}c^2d \\
& ^{13}e^{10}z^4 + 16896b^{13}c^{10}d^{21}e^2z^4 + 110080a^7b^{16}d^4e^{19}z^4 \\
& + 110080a^4b^{19}d^7e^{16}z^4 - 75520a^8b^{15}d^3e^{20}z^4 - 75520a^3b^ \\
& 20d^8e^{15}z^4 - 56320a^6b^{17}d^5e^{18}z^4 - 56320a^5b^{18}d^6e^{17}z^4 \\
& + 25600a^9b^{14}d^2e^{21}z^4 + 25600a^2b^{21}d^9e^{14}z^4 - 1572864a^{16} \\
& b^2c^5e^{23}z^4 + 983040a^{15}b^4c^4e^{23}z^4 - 327680a^{14}b^6c^3e^{23} \\
& z^4 + 61440a^{13}b^8c^2e^{23}z^4 + 983040a^4b^3c^{16}d^{23}z^4 - 385024a \\
& ^3b^5c^{15}d^{23}z^4 + 73728a^2b^7c^{14}d^{23}z^4 + 256b^{23}d^{11}e^{12}z^ \\
& 4 + 1048576a^{17}c^6e^{23}z^4 + 256b^{11}c^{12}d^{23}z^4 + 256a^{11}b^{12}e^{23} \\
& z^4 + 948695040a^8b^c^{10}d^6e^{13}z^2 + 348917760a^7b^c^{11}d^8e^{11}z^ \\
& 2 - 125030400a^9b^c^9d^4e^{15}z^2 - 50728960a^6b^c^{12}d^{10}e^9z^2 - 4 \\
& 4298240a^5b^c^{13}d^{12}e^7z^2 - 36495360a^{10}b^c^8d^2e^{17}z^2 + 296755 \\
& 20a^8b^6c^5d^e^{18}z^2 - 24170496a^9b^4c^6d^e^{18}z^2 - 17202816a^7b \\
& ^8c^4d^e^{18}z^2 - 14561280a^4b^c^{14}d^{14}e^5z^2 + 5532416a^6b^{10}c^ \\
& 3d^e^{18}z^2 + 4128768a^{10}b^2c^7d^e^{18}z^2 - 2662400a^3b^c^{15}d^{16}e^ \\
& 3z^2 + 1184512a^b^{12}c^6d^9e^{10}z^2 - 1136160a^b^{13}c^5d^8e^{11}z^2 - \\
& 1017600a^5b^{12}c^2d^e^{18}z^2 - 744768a^b^{11}c^7d^{10}e^9z^2 + 607872a \\
& a^b^{14}c^4d^7e^{12}z^2 - 424064a^b^6c^{12}d^{15}e^4z^2 + 408576a^b^5c^1 \\
& 3d^{16}e^3z^2 + 361152a^b^{10}c^8d^{11}e^8z^2 - 287408a^b^9c^9d^{12}e^7 \\
& z^2 - 260448a^3b^{15}c^d^2e^{17}z^2 - 203904a^b^4c^{14}d^{17}e^2z^2 + 20 \\
& 0832a^b^8c^{10}d^{13}e^6z^2 + 126720a^b^7c^{11}d^{14}e^5z^2 - 123968a^b^ \\
& 15c^3d^6e^{13}z^2 - 39168a^b^{16}c^2d^5e^{14}z^2 + 11904a^2b^{16}c^d^3e \\
& ^{16}z^2 + 1824135552a^7b^4c^8d^5e^{14}z^2 - 1457252352a^8b^2c^9d^5 \\
& e^{14}z^2 - 1405209600a^7b^5c^7d^4e^{15}z^2 - 184320a^2b^c^{16}d^{18}e^ \\
& z^2 + 100608a^4b^{14}c^d^e^{18}z^2 + 53248a^b^3c^{15}d^{18}e^z^2 + 26448a^ \\
& b^{17}c^d^4e^{15}z^2 + 1067599872a^8b^3c^8d^4e^{15}z^2 - 930828288a^7b \\
& ^3c^9d^6e^{13}z^2 + 920760000a^6b^4c^9d^7e^{12}z^2 - 806639616a^6b^ \\
& 3c^{10}d^8e^{11}z^2 - 791052480a^6b^6c^7d^5e^{14}z^2 + 772237824a^6b^ \\
& 7c^6d^4e^{15}z^2 - 701025408a^5b^6c^8d^7e^{12}z^2 + 443340288a^5b^5 \\
& c^9d^8e^{11}z^2 + 433047552a^7b^6c^6d^3e^{16}z^2 + 405741312a^5b^7c \\
& ^7d^6e^{13}z^2 + 293652480a^6b^2c^{11}d^9e^{10}z^2 - 276962688a^6b^8c \\
& ^5d^3e^{16}z^2 - 247804272a^8b^4c^7d^3e^{16}z^2 + 213564384a^4b^8c \\
& ^7d^7e^{12}z^2 - 202596816a^5b^9c^5d^4e^{15}z^2 - 182520896a^4b^9c^ \\
& 6d^6e^{13}z^2 - 153489408a^5b^3c^{11}d^{10}e^9z^2 - 152151552a^7b^2c^ \\
& 10d^7e^{12}z^2 + 115859712a^5b^2c^{12}d^{11}e^8z^2 + 108085248a^9b^3c \\
& ^7d^2e^{17}z^2 + 105536256a^4b^5c^{10}d^{10}e^9z^2 - 98323200a^6b^5c^ \\
& 8d^6e^{13}z^2 - 93564992a^4b^6c^9d^9e^{10}z^2 + 89464512a^5b^{10}c^4 \\
& d^3e^{16}z^2 - 75930624a^8b^5c^6d^2e^{17}z^2 + 68315904a^5b^8c^6d^5 \\
& e^{14}z^2 - 64157184a^4b^7c^8d^8e^{11}z^2 - 62951040a^9b^2c^8d^3e^
\end{aligned}$$

$$\begin{aligned}
& 16z^2 + 49056768a^4b^{10}c^5d^5e^{14}z^2 + 47614464a^3b^8c^8d^9e^{10} \\
& z^2 + 35604480a^4b^2c^{13}d^{13}e^6z^2 + 33983040a^3b^{11}c^5d^6e^{13}z^2 \\
& - 33515520a^4b^3c^{12}d^{12}e^7z^2 - 33463808a^3b^7c^9d^{10}e^9z^2 \\
& - 25128864a^4b^4c^{11}d^{11}e^8z^2 - 23193728a^3b^{10}c^6d^7e^{12}z^2 \\
& + 21015456a^6b^9c^4d^2e^{17}z^2 + 19924176a^4b^{11}c^4d^4e^{15}z^2 - \\
& 19251216a^3b^9c^7d^8e^{11}z^2 - 16434048a^5b^4c^{10}d^9e^{10}z^2 - 1 \\
& 6289664a^3b^{12}c^4d^5e^{14}z^2 - 15059328a^4b^{12}c^3d^3e^{16}z^2 - 10 \\
& 766016a^2b^{10}c^7d^9e^{10}z^2 - 10453632a^5b^{11}c^3d^2e^{17}z^2 - 994 \\
& 0992a^3b^3c^{13}d^{14}e^5z^2 + 8373696a^2b^{11}c^6d^8e^{11}z^2 + 777676 \\
& 8a^3b^2c^{14}d^{15}e^4z^2 + 7077888a^3b^5c^{11}d^{12}e^7z^2 + 6798240a^2 \\
& b^9c^8d^{10}e^9z^2 - 3589440a^2b^6c^{11}d^{13}e^6z^2 + 3544320a^3b^6 \\
& c^{10}d^{11}e^8z^2 + 3128064a^2b^5c^{12}d^{14}e^5z^2 + 2346336a^4b^{13} \\
& c^2d^2e^{17}z^2 - 2261568a^2b^8c^9d^{11}e^8z^2 - 2125824a^2b^{13}c^4 \\
& d^6e^{13}z^2 + 2002560a^3b^4c^{12}d^{13}e^6z^2 + 1927680a^2b^7c^{10}d^{12} \\
& e^7z^2 + 1814784a^2b^{14}c^3d^5e^{14}z^2 - 1807104a^2b^{12}c^5d^7e^{12} \\
& z^2 + 1637808a^3b^{13}c^3d^4e^{15}z^2 + 1083456a^3b^{14}c^2d^3e^{16} \\
& z^2 - 792384a^2b^4c^{13}d^{15}e^4z^2 - 657408a^2b^3c^{14}d^{16}e^3z^2 \\
& + 608256a^7b^7c^5d^2e^{17}z^2 + 595968a^2b^2c^{15}d^{17}e^2z^2 - 4986 \\
& 24a^2b^{15}c^2d^4e^{15}z^2 - 3840b^{18}c^5d^5e^{14}z^2 - 3840b^5c^{14}d^{18} \\
& e^8z^2 + 2064384a^{11}c^8d^8e^{18}z^2 - 4160a^3b^{16}d^8e^{18}z^2 - 4160ab \\
& ^{18}d^3e^{16}z^2 - 1290240a^{11}b^7c^7e^{19}z^2 - 9840a^5b^{13}c^7e^{19}z^2 - \\
& 5760ab^2c^{16}d^{19}z^2 - 280581120a^8c^{11}d^7e^{12}z^2 + 110278656a^9 \\
& c^{10}d^5e^{14}z^2 - 89479168a^7c^{12}d^9e^{10}z^2 + 34464000a^{10}c^9d^3 \\
& e^{16}z^2 + 54240b^{15}c^4d^8e^{11}z^2 + 54240b^8c^{11}d^{15}e^4z^2 - 499 \\
& 20b^{14}c^5d^9e^{10}z^2 - 49920b^9c^{10}d^{14}e^5z^2 - 37376b^{16}c^3d^7 \\
& e^{12}z^2 - 37376b^7c^{12}d^{16}e^3z^2 + 28480b^{13}c^6d^{10}e^9z^2 + 284 \\
& 80b^{10}c^9d^{13}e^6z^2 + 15936b^{17}c^2d^6e^{13}z^2 + 15936b^6c^{13}d^{17} \\
& e^2z^2 - 7920b^{12}c^7d^{11}e^8z^2 - 7920b^{11}c^8d^{12}e^7z^2 + 74895 \\
& 36a^5c^{14}d^{13}e^6z^2 + 6084096a^6c^{13}d^{11}e^8z^2 + 2280448a^4c^{15} \\
& d^{15}e^4z^2 + 350208a^3c^{16}d^{17}e^2z^2 + 11616a^2b^{17}d^2e^{17}z^2 \\
& - 3515904a^9b^5c^5e^{19}z^2 + 3440640a^{10}b^3c^6e^{19}z^2 + 1870848a^8 \\
& b^7c^4e^{19}z^2 - 572272a^7b^9c^3e^{19}z^2 + 101856a^6b^{11}c^2e^{19} \\
& z^2 + 400b^{19}d^4e^{15}z^2 + 400b^4c^{15}d^{19}z^2 + 20736a^2c^{17}d^{19}z^2 \\
& + 400a^4b^{15}e^{19}z^2 - 3969216a^4b^7c^{10}d^3e^{12} - 3001536a^3b^7c^{11} \\
& d^5e^{10} - 419904a^2b^7c^{12}d^7e^8 + 184608a^4b^3c^8d^8e^{14} - 1530 \\
& 36ab^4c^{10}d^6e^9 + 127008ab^3c^{11}d^7e^8 + 63108ab^6c^8d^4e^{11} \\
& - 29160ab^2c^{12}d^8e^7 - 21060a^3b^5c^7d^8e^{14} - 21060ab^7c^7d^3 \\
& e^{12} + 5460ab^5c^9d^5e^{10} - 404544a^5b^7c^9d^8e^{14} + 1251872a^3b^3 \\
& c^9d^3e^{12} + 844224a^4b^2c^9d^2e^{13} + 820512a^2b^3c^{10}d^5e^{11} \\
& 0 + 750672a^3b^2c^{10}d^4e^{11} - 657498a^2b^4c^9d^4e^{11} - 487116a^3 \\
& b^4c^8d^2e^{13} + 160704a^2b^2c^{11}d^6e^9 + 58806a^2b^6c^7d^2e^{13} \\
& + 13140a^2b^5c^8d^3e^{12} + 15286b^6c^9d^6e^9 - 9540b^7c^8d^5e^{10} \\
& - 9540b^5c^{10}d^7e^8 + 2025b^8c^7d^4e^{11} + 2025b^4c^{11}d^8e^7 \\
& + 3367008a^4c^{11}d^4e^{11} + 1166400a^3c^{12}d^6e^9 + 705600a^5c^{10}d^2 \\
& e^{13} + 104976a^2c^{13}d^8e^7 - 17640a^5b^2c^8e^{15} + 2025a^4b^4c^7 \\
& e^{15} + 38416a^6c^9e^{15}, z, k) \cdot (\text{root}(128723189760a^{14}b^4c^9d^{13}e^{14} \\
& z^6 + 128723189760a^{12}b^4c^{11}d^{17}e^{10}z^6 - 8432455680a^{11}b^{12}c^4 \\
& d^{11}e^{16}z^6 - 8432455680a^7b^{12}c^8d^{19}e^8z^6 + 12673351680a^{11}b^{11} \\
& c^5d^{12}e^{15}z^6 + 12673351680a^8b^{11}c^8d^{18}e^9z^6 - 72637480960 \\
& a^{12}b^9c^6d^{12}e^{15}z^6 - 72637480960a^9b^9c^9d^{18}e^9z^6 - 210483 \\
& 44576a^9b^{12}c^6d^{15}e^{12}z^6 - 16609443840a^{17}b^3c^7d^8e^{19}z^6 - \\
& 16609443840a^{10}b^3c^{14}d^{22}e^5z^6 + 145332633600a^{13}b^5c^9d^{14}e^{11} \\
& 3z^6 + 145332633600a^{12}b^5c^{10}d^{16}e^{11}z^6 + 123740356608a^{14}b^5c^8 \\
& d^{12}e^{15}z^6 + 123740356608a^{11}b^5c^{11}d^{18}e^9z^6 + 3460300800a^{17} \\
& b^5c^5d^6e^{21}z^6 + 3460300800a^8b^5c^{14}d^{24}e^3z^6 - 7751073792a^{15} \\
& b^7c^5d^8e^{19}z^6 - 7751073792a^8b^7c^{12}d^{22}e^5z^6 + 120418467 \\
& 84a^{14}b^7c^6d^{10}e^{17}z^6 + 12041846784a^9b^7c^{11}d^{20}e^7z^6 - 325 \\
& 545099264a^{14}b^3c^{10}d^{14}e^{13}z^6 - 325545099264a^{13}b^3c^{11}d^{16}e^{11} \\
& 1z^6 - 3330539520a^{13}b^{10}c^4d^9e^{18}z^6 - 3330539520a^7b^{10}c^{10}d^
\end{aligned}$$

$21e^6z^6 + 157789716480a^{12}b^7c^8d^{14}e^{13}z^6 + 157789716480a^{11}b^7c^9d^{16}e^{11}z^6 + 37492359168a^{11}b^{10}c^6d^{13}e^{14}z^6 + 37492359168a^9b^{10}c^8d^{17}e^{10}z^6 + 301989888a^8b^3c^{16}d^{26}e^*z^6 - 7266631680a^{17}b^4c^6d^7e^{20}z^6 - 7266631680a^9b^4c^{14}d^{23}e^4z^6 - 201326592a^{20}b^*c^6d^4e^{23}z^6 - 188743680a^7b^5c^{15}d^{26}e^*z^6 + 45747339264a^{13}b^8c^6d^{11}e^{16}z^6 + 45747339264a^9b^8c^{10}d^{19}e^8z^6 - 74612736a^{10}b^{16}c^*d^9e^{18}z^6 - 2768240640a^{16}b^7c^4d^6e^{21}z^6 - 2768240640a^7b^7c^{13}d^{24}e^3z^6 + 69746688a^{11}b^{15}c^*d^8e^{19}z^6 + 62914560a^6b^7c^{14}d^{26}e^*z^6 + 2752020480a^{10}b^{13}c^4d^{12}e^{15}z^6 + 2752020480a^7b^{13}c^7d^{18}e^9z^6 + 55148544a^9b^{17}c^*d^{10}e^{17}z^6 - 45957120a^{12}b^{14}c^*d^7e^{20}z^6 - 2724986880a^{14}b^9c^4d^8e^{19}z^6 - 2724986880a^7b^9c^{11}d^{22}e^5z^6 - 25952256a^8b^{18}c^*d^{11}e^{16}z^6 + 21086208a^{13}b^{13}c^*d^6e^{21}z^6 - 11796480a^5b^9c^{13}d^{26}e^*z^6 - 6438912a^{14}b^{12}c^*d^5e^{22}z^6 + 5406720a^7b^{19}c^*d^{12}e^{15}z^6 + 1622016a^6b^{20}c^*d^{13}e^{14}z^6 - 1523712a^5b^{21}c^*d^{14}e^{13}z^6 + 1179648a^{15}b^11c^*d^4e^{23}z^6 + 1179648a^4b^{11}c^{12}d^{26}e^*z^6 + 442368a^4b^{22}c^*d^{15}e^{12}z^6 - 98304a^{16}b^{10}c^*d^3e^{24}z^6 - 49152a^3b^{23}c^*d^{16}e^{11}z^6 - 49152a^3b^{13}c^{11}d^{26}e^*z^6 + 6897106944a^9b^{13}c^5d^{14}e^{13}z^6 + 6897106944a^8b^{13}c^6d^{16}e^{11}z^6 - 2422210560a^{16}b^6c^5d^7e^{20}z^6 - 2422210560a^8b^6c^{13}d^{23}e^4z^6 + 255785435136a^{14}b^2c^{11}d^{15}e^{12}z^6 + 41004564480a^{15}b^4c^8d^{11}e^{16}z^6 + 41004564480a^{11}b^4c^{12}d^{19}e^8z^6 + 2270822400a^{13}b^{11}c^3d^8e^{19}z^6 + 2270822400a^6b^{11}c^{10}d^{22}e^5z^6 + 23677108224a^{14}b^8c^5d^9e^{18}z^6 + 23677108224a^8b^8c^{11}d^{21}e^6z^6 + 212600881152a^{15}b^2c^{10}d^{13}e^{14}z^6 + 212600881152a^{13}b^2c^{12}d^{17}e^{10}z^6 + 75157733376a^{15}b^5c^7d^{10}e^{17}z^6 + 75157733376a^{10}b^5c^{12}d^{20}e^7z^6 - 251217838080a^{13}b^6c^8d^{13}e^{14}z^6 - 251217838080a^{11}b^6c^{10}d^{17}e^{10}z^6 - 1952907264a^{14}b^{10}c^3d^7e^{20}z^6 - 1952907264a^6b^{10}c^{11}d^{23}e^4z^6 - 27691057152a^{13}b^9c^5d^{10}e^{17}z^6 - 27691057152a^8b^9c^{10}d^{20}e^7z^6 - 1902673920a^8b^{15}c^4d^{14}e^{13}z^6 - 1902673920a^7b^{15}c^5d^{16}e^{11}z^6 + 10465050624a^{10}b^{11}c^6d^{14}e^{13}z^6 + 10465050624a^9b^{11}c^7d^{16}e^{11}z^6 + 1613905920a^9b^{14}c^4d^{13}e^{14}z^6 + 1613905920a^7b^{14}c^6d^{17}e^{10}z^6 - 33218887680a^{17}b^*c^9d^{10}e^{17}z^6 - 33218887680a^{12}b^*c^{14}d^{20}e^7z^6 + 1524695040a^{10}b^{14}c^3d^{11}e^{16}z^6 + 1524695040a^6b^{14}c^7d^{19}e^8z^6 - 1472200704a^{18}b^4c^5d^5e^{22}z^6 - 1472200704a^8b^4c^{15}d^{25}e^2z^6 - 83047219200a^{16}b^3c^8d^{10}e^{17}z^6 - 83047219200a^{11}b^3c^{13}d^{20}e^7z^6 + 44291850240a^{17}b^2c^8d^9e^{18}z^6 + 44291850240a^{11}b^2c^{14}d^{21}e^6z^6 + 1308131328a^8b^{14}c^5d^{15}e^{12}z^6 - 201326592a^9b^*c^{17}d^{26}e^*z^6 + 48530718720a^{12}b^8c^7d^{13}e^{14}z^6 + 48530718720a^{10}b^8c^9d^{17}e^{10}z^6 - 1242644480a^{12}b^{12}c^3d^9e^{18}z^6 - 1242644480a^6b^{12}c^9d^{21}e^6z^6 + 9813196800a^{12}b^{10}c^5d^{11}e^{16}z^6 + 9813196800a^8b^{10}c^9d^{19}e^8z^6 - 93012885504a^{15}b^*c^{11}d^{14}e^{13}z^6 - 93012885504a^{14}b^*c^{12}d^{16}e^{11}z^6 + 177305812992a^{13}b^4c^{10}d^{15}e^{12}z^6 + 52730658816a^{10}b^{10}c^7d^{15}e^{12}z^6 - 1180106752a^9b^{15}c^3d^{12}e^{15}z^6 - 1180106752a^6b^{15}c^6d^{18}e^9z^6 + 1023672320a^{15}b^9c^3d^6e^{21}z^6 + 1023672320a^6b^9c^{12}d^{24}e^3z^6 + 975175680a^{17}b^6c^4d^5e^{22}z^6 + 975175680a^7b^6c^{14}d^{25}e^2z^6 - 11072962560a^{18}b^*c^8d^8e^{19}z^6 - 11072962560a^{11}b^*c^{15}d^{22}e^5z^6 + 9412018176a^{18}b^2c^7d^7e^{20}z^6 + 9412018176a^{10}b^2c^{15}d^{23}e^4z^6 + 805306368a^{19}b^2c^6d^5e^{22}z^6 + 805306368a^9b^2c^{16}d^{25}e^2z^6 - 133809831936a^{14}b^6c^7d^{11}e^{16}z^6 - 133809831936a^{10}b^6c^{11}d^{19}e^8z^6 - 2214592512a^{19}b^*c^7d^6e^{21}z^6 - 2214592512a^{10}b^*c^{16}d^{24}e^3z^6 + 82216747008a^{13}b^7c^7d^{12}e^{15}z^6 + 82216747008a^{10}b^7c^{10}d^{18}e^9z^6 - 586629120a^{12}b^{13}c^2d^8e^{19}z^6 - 586629120a^5b^{13}c^9d^{22}e^5z^6 + 568565760a^7b^{16}c^4d^{15}e^{12}z^6 - 4844421120a^{16}b^4c^7d^9e^{18}z^6 - 4844421120a^{10}b^4c^{13}d^{21}e^6z^6 + 531210240a^{11}b^{14}c^2d^9e^{18}z^6 + 531210240a^5b^{14}c^8d^{21}e^6z^6 - 527155200a^{11}b^{13}c^3d^{10}e^{17}z^6 - 527155200a^6b^{13}c^8d^{20}e^7z^6 + 43470028800a^{11}b^8c^8d^{15}e^{12}z^6 - 107874877440a^{11}b^9c^7d^{14}e^{13}z^6$

$$\begin{aligned}
&^6 - 107874877440*a^{10}*b^9*c^8*d^{16}*e^{11}*z^6 + 9018408960*a^{12}*b^{11}*c^4*d^{10}*e^{17}*z^6 + 9018408960*a^7*b^{11}*c^9*d^{20}*e^7*z^6 + 421994496*a^{13}*b^{12}*c^2*d^7*e^{20}*z^6 + 421994496*a^5*b^{12}*c^{10}*d^{23}*e^4*z^6 - 66437775360*a^{16}*b*c^{10}*d^{12}*e^{15}*z^6 - 66437775360*a^{13}*b*c^{13}*d^{18}*e^9*z^6 + 26159874048*a^{16}*b^5*c^6*d^8*e^{19}*z^6 + 26159874048*a^9*b^5*c^{13}*d^{22}*e^5*z^6 - 369098752*a^{18}*b^3*c^6*d^6*e^{21}*z^6 - 369098752*a^9*b^3*c^{15}*d^{24}*e^3*z^6 + 351436800*a^8*b^{16}*c^3*d^{13}*e^{14}*z^6 + 351436800*a^6*b^{16}*c^5*d^{17}*e^{10}*z^6 - 334233600*a^{16}*b^8*c^3*d^5*e^{22}*z^6 - 334233600*a^6*b^8*c^{13}*d^{25}*e^2*z^6 + 301989888*a^{19}*b^3*c^5*d^4*e^{23}*z^6 - 266010624*a^{10}*b^{15}*c^2*d^{10}*e^{17}*z^6 - 266010624*a^5*b^{15}*c^7*d^{20}*e^7*z^6 - 305198530560*a^{12}*b^6*c^9*d^{15}*e^{12}*z^6 - 203292672*a^{14}*b^{11}*c^2*d^6*e^{21}*z^6 - 203292672*a^5*b^{11}*c^{11}*d^{24}*e^3*z^6 - 188743680*a^{18}*b^5*c^4*d^4*e^{23}*z^6 + 120418467840*a^{16}*b^2*c^9*d^{11}*e^{16}*z^6 + 120418467840*a^{12}*b^2*c^{13}*d^{19}*e^8*z^6 - 17293934592*a^{10}*b^{12}*c^5*d^{13}*e^{14}*z^6 - 17293934592*a^8*b^{12}*c^7*d^{17}*e^{10}*z^6 + 104890368*a^8*b^{17}*c^2*d^{12}*e^{15}*z^6 + 104890368*a^5*b^{17}*c^5*d^{18}*e^9*z^6 + 4390256640*a^{15}*b^8*c^4*d^7*e^{20}*z^6 + 4390256640*a^7*b^8*c^{12}*d^{23}*e^4*z^6 - 91750400*a^6*b^{18}*c^3*d^{15}*e^{12}*z^6 + 79134720*a^7*b^{17}*c^3*d^{14}*e^{13}*z^6 + 79134720*a^6*b^{17}*c^4*d^{16}*e^{11}*z^6 - 74612736*a^4*b^{16}*c^7*d^{21}*e^6*z^6 - 72990720*a^7*b^{18}*c^2*d^{13}*e^{14}*z^6 - 72990720*a^5*b^{18}*c^4*d^{17}*e^{10}*z^6 + 69746688*a^4*b^{15}*c^8*d^{22}*e^5*z^6 + 63700992*a^{15}*b^{10}*c^2*d^5*e^{22}*z^6 + 63700992*a^5*b^{10}*c^{12}*d^{25}*e^2*z^6 + 62914560*a^{17}*b^7*c^3*d^4*e^{23}*z^6 + 55148544*a^4*b^{17}*c^6*d^{20}*e^7*z^6 - 45957120*a^4*b^{14}*c^9*d^{23}*e^4*z^6 - 25952256*a^4*b^{18}*c^5*d^{19}*e^8*z^6 - 25165824*a^{20}*b^2*c^5*d^3*e^{24}*z^6 + 21086208*a^4*b^{13}*c^{10}*d^{24}*e^3*z^6 + 20643840*a^6*b^{19}*c^2*d^{14}*e^{13}*z^6 + 20643840*a^5*b^{19}*c^3*d^{16}*e^{11}*z^6 + 15728640*a^{19}*b^4*c^4*d^3*e^{24}*z^6 - 11796480*a^{16}*b^9*c^2*d^4*e^{23}*z^6 - 6438912*a^4*b^{12}*c^{11}*d^{25}*e^2*z^6 + 5406720*a^4*b^{19}*c^4*d^{18}*e^9*z^6 - 5242880*a^{18}*b^6*c^3*d^3*e^{24}*z^6 + 3784704*a^3*b^{18}*c^6*d^{21}*e^6*z^6 - 3244032*a^3*b^{19}*c^5*d^{20}*e^7*z^6 - 3244032*a^3*b^{17}*c^7*d^{22}*e^5*z^6 + 2027520*a^3*b^{20}*c^4*d^{19}*e^8*z^6 + 2027520*a^3*b^{16}*c^8*d^{23}*e^4*z^6 - 1622016*a^9*b^{16}*c^2*d^{11}*e^{16}*z^6 - 1622016*a^5*b^{16}*c^6*d^{19}*e^8*z^6 + 1622016*a^4*b^{20}*c^3*d^{17}*e^{10}*z^6 - 1523712*a^4*b^{21}*c^2*d^{16}*e^{11}*z^6 + 983040*a^{17}*b^8*c^2*d^3*e^{24}*z^6 - 901120*a^3*b^{21}*c^3*d^{18}*e^9*z^6 - 901120*a^3*b^{15}*c^9*d^{24}*e^3*z^6 + 270336*a^3*b^{22}*c^2*d^{17}*e^{10}*z^6 + 270336*a^3*b^{14}*c^{10}*d^{25}*e^2*z^6 + 172032*a^5*b^{20}*c^2*d^{15}*e^{12}*z^6 - 38593888256*a^{15}*b^6*c^6*d^9*e^{18}*z^6 - 38593888256*a^9*b^6*c^{12}*d^{21}*e^6*z^6 - 210386288640*a^{15}*b^3*c^9*d^{12}*e^{15}*z^6 - 210386288640*a^{12}*b^3*c^{12}*d^{18}*e^9*z^6 + 15502147584*a^{15}*c^{12}*d^{15}*e^{12}*z^6 + 1107296256*a^{19}*c^8*d^7*e^{20}*z^6 + 1107296256*a^{11}*c^{16}*d^{23}*e^4*z^6 + 13287555072*a^{16}*c^{11}*d^{13}*e^{14}*z^6 + 13287555072*a^{14}*c^{13}*d^{17}*e^{10}*z^6 + 201326592*a^{20}*c^7*d^5*e^{22}*z^6 + 201326592*a^{10}*c^{17}*d^{25}*e^2*z^6 + 16777216*a^{21}*c^6*d^3*e^{24}*z^6 + 3784704*a^9*b^{18}*d^9*e^{18}*z^6 - 3244032*a^{10}*b^{17}*d^8*e^{19}*z^6 - 3244032*a^8*b^{19}*d^{10}*e^{17}*z^6 + 2027520*a^{11}*b^{16}*d^7*e^{20}*z^6 + 2027520*a^7*b^{20}*d^{11}*e^{16}*z^6 - 901120*a^{12}*b^{15}*d^6*e^{21}*z^6 - 901120*a^6*b^{21}*d^{12}*e^{15}*z^6 + 270336*a^{13}*b^{14}*d^5*e^{22}*z^6 + 270336*a^5*b^{22}*d^{13}*e^{14}*z^6 - 49152*a^{14}*b^{13}*d^4*e^{23}*z^6 - 49152*a^4*b^{23}*d^{14}*e^{13}*z^6 + 4096*a^{15}*b^{12}*d^3*e^{24}*z^6 + 4096*a^3*b^{24}*d^{15}*e^{12}*z^6 - 25165824*a^8*b^2*c^{17}*d^{27}*z^6 + 15728640*a^7*b^4*c^{16}*d^{27}*z^6 - 5242880*a^6*b^6*c^{15}*d^{27}*z^6 + 983040*a^5*b^8*c^{14}*d^{27}*z^6 - 983040*a^4*b^{10}*c^{13}*d^{27}*z^6 + 4096*a^3*b^{12}*c^{12}*d^{27}*z^6 + 8304721920*a^{17}*c^{10}*d^{11}*e^{16}*z^6 + 8304721920*a^{13}*c^{14}*d^{19}*e^8*z^6 + 3690987520*a^{18}*c^9*d^9*e^{18}*z^6 + 3690987520*a^{12}*c^{15}*d^{21}*e^6*z^6 + 16777216*a^9*c^{18}*d^{27}*z^6 - 8493371392*a^6*b^8*c^9*d^{14}*e^9*z^4 + 1458044928*a^8*b*c^{14}*d^{17}*e^6*z^4 - 12604538880*a^{11}*b^4*c^8*d^8*e^{15}*z^4 - 8303067136*a^9*b^5*c^9*d^{11}*e^{12}*z^4 - 5588058112*a^{13}*b*c^9*d^7*e^{16}*z^4 - 3892838400*a^8*b^2*c^{13}*d^{16}*e^7*z^4 - 3611713536*a^8*b^8*c^7*d^{10}*e^{13}*z^4 + 7819006464*a^7*b^9*c^7*d^{11}*e^{12}*z^4 - 7782137856*a^8*b^7*c^8*d^{11}*e^{12}*z^4 + 7780433920*a^{12}*b^2*c^9*d^8*e^{15}*z^4 - 12020465664*a^7*b^5*c^{11}*d^{15}*e^8*z^4 + 3176792064*a^8*b^3*c^{12}*d^{15}*e^8*z^4 - 322633728*a^{15}*b*c^7*d^3*e^{20}*z^4 + 210829312*a^7*b*c^{15}*d^{19}*e^4*z^4 + 15623258112*a^9*b^6*c^8*d^{10}*e^{13}*z^4 + 25165824*a^{15}*b^3*c^5*d^e^{22}*z^4 - 15728640*a^{14}*b^5*c^4*d^e^{22}*z^4
\end{aligned}$$

$$\begin{aligned}
& + 12582912*a^5*b^2*c^16*d^22*e*z^4 - 11730944*a^4*b^4*c^15*d^22*e*z^4 + 524 \\
& 2880*a^13*b^7*c^3*d*e^22*z^4 - 4561920*a*b^15*c^7*d^17*e^6*z^4 + 4521984*a^ \\
& 3*b^6*c^14*d^22*e*z^4 + 4460544*a*b^14*c^8*d^18*e^5*z^4 + 3538944*a^6*b*c^1 \\
& 6*d^21*e^2*z^4 + 3108864*a*b^16*c^6*d^16*e^7*z^4 - 3027200*a*b^13*c^9*d^19* \\
& e^4*z^4 - 2345472*a^5*b^17*c*d^7*e^16*z^4 - 2307072*a^8*b^14*c*d^4*e^19*z^4 \\
& + 1824768*a^6*b^16*c*d^6*e^17*z^4 + 1734912*a^9*b^13*c*d^3*e^20*z^4 + 1419 \\
& 264*a*b^12*c^10*d^20*e^3*z^4 - 1191168*a*b^17*c^5*d^15*e^8*z^4 - 983040*a^1 \\
& 2*b^9*c^2*d*e^22*z^4 + 964608*a^4*b^18*c*d^8*e^15*z^4 - 866304*a^2*b^8*c^13 \\
& *d^22*e*z^4 + 703488*a^7*b^15*c*d^5*e^18*z^4 - 608256*a^10*b^12*c*d^2*e^21* \\
& z^4 - 440832*a*b^11*c^11*d^21*e^2*z^4 + 275968*a*b^19*c^3*d^13*e^10*z^4 - 1 \\
& 59744*a^2*b^20*c*d^10*e^13*z^4 - 153600*a*b^20*c^2*d^12*e^11*z^4 + 64512*a^ \\
& 3*b^19*c*d^9*e^14*z^4 + 19746062336*a^8*b^6*c^9*d^12*e^11*z^4 - 15333588992 \\
& *a^10*b^4*c^9*d^10*e^13*z^4 + 6702170112*a^7*b^4*c^12*d^16*e^7*z^4 + 151679 \\
& 13984*a^10*b^3*c^10*d^11*e^12*z^4 - 2256638976*a^5*b^11*c^7*d^13*e^10*z^4 + \\
& 2254307328*a^5*b^7*c^11*d^17*e^6*z^4 - 2200633344*a^6*b^5*c^12*d^17*e^6*z^ \\
& 4 + 6457131008*a^11*b^3*c^9*d^9*e^14*z^4 - 2128785408*a^5*b^8*c^10*d^16*e^7 \\
& *z^4 - 2126057472*a^6*b^11*c^6*d^11*e^12*z^4 + 2038349824*a^12*b^5*c^6*d^5* \\
& e^18*z^4 + 2037841920*a^5*b^10*c^8*d^14*e^9*z^4 + 3615621120*a^9*b*c^13*d^1 \\
& 5*e^8*z^4 + 1900019712*a^11*b^2*c^10*d^10*e^13*z^4 + 1867698432*a^9*b^9*c^5 \\
& *d^7*e^16*z^4 - 6157369344*a^9*b^4*c^10*d^12*e^11*z^4 - 1856913408*a^7*b^10 \\
& *c^6*d^10*e^13*z^4 + 1789132800*a^6*b^4*c^13*d^18*e^5*z^4 + 6082658304*a^8* \\
& b^4*c^11*d^14*e^9*z^4 + 6029549568*a^11*b^5*c^7*d^7*e^16*z^4 + 6010159104*a \\
& ^6*b^7*c^10*d^15*e^8*z^4 + 1703182336*a^7*b^7*c^9*d^13*e^10*z^4 + 165838848 \\
& 0*a^11*b^6*c^6*d^6*e^17*z^4 + 5917114368*a^10*b^6*c^7*d^8*e^15*z^4 - 159119 \\
& 7696*a^11*b^7*c^5*d^5*e^18*z^4 - 1526464512*a^8*b^10*c^5*d^8*e^15*z^4 - 577 \\
& 2607488*a^12*b^4*c^7*d^6*e^17*z^4 - 1423507456*a^13*b^4*c^6*d^4*e^19*z^4 - \\
& 1387266048*a^7*b^3*c^13*d^17*e^6*z^4 + 2976120832*a^10*b*c^12*d^13*e^10*z^4 \\
& - 9906946048*a^9*b^2*c^12*d^14*e^9*z^4 - 18421874688*a^8*b^5*c^10*d^13*e^1 \\
& 0*z^4 + 1141217280*a^6*b^12*c^5*d^10*e^13*z^4 - 9714364416*a^7*b^8*c^8*d^12 \\
& *e^11*z^4 - 16777216*a^16*b*c^6*d*e^22*z^4 + 98304*a^11*b^11*c*d*e^22*z^4 + \\
& 81920*a*b^10*c^12*d^22*e*z^4 + 39168*a*b^21*c*d^11*e^12*z^4 - 1091829760*a \\
& ^5*b^6*c^12*d^18*e^5*z^4 + 1046740992*a^14*b^2*c^7*d^4*e^19*z^4 - 688442572 \\
& 8*a^12*b*c^10*d^9*e^14*z^4 + 987445248*a^4*b^10*c^9*d^16*e^7*z^4 + 98408755 \\
& 2*a^5*b^12*c^6*d^12*e^11*z^4 - 9564585984*a^9*b^7*c^7*d^9*e^14*z^4 - 526685 \\
& 7984*a^10*b^7*c^6*d^7*e^16*z^4 - 892145664*a^7*b^11*c^5*d^9*e^14*z^4 - 2444 \\
& 623872*a^11*b*c^11*d^11*e^12*z^4 + 768540672*a^12*b^3*c^8*d^7*e^16*z^4 + 50 \\
& 48322048*a^8*b^9*c^6*d^9*e^14*z^4 + 5047612416*a^6*b^9*c^8*d^13*e^10*z^4 - \\
& 732492288*a^4*b^11*c^8*d^15*e^8*z^4 + 9266921472*a^7*b^6*c^10*d^14*e^9*z^4 \\
& - 645857280*a^6*b^6*c^11*d^16*e^7*z^4 - 623867904*a^4*b^9*c^10*d^17*e^6*z^4 \\
& - 622067712*a^6*b^3*c^14*d^19*e^4*z^4 + 582617088*a^10*b^8*c^5*d^6*e^17*z^ \\
& 4 + 577119744*a^7*b^12*c^4*d^8*e^15*z^4 + 552566784*a^12*b^6*c^5*d^4*e^19*z \\
& ^4 + 549224448*a^9*b^8*c^6*d^8*e^15*z^4 - 526565376*a^9*b^10*c^4*d^6*e^17*z \\
& ^4 + 511520256*a^10*b^9*c^4*d^5*e^18*z^4 + 13393723392*a^9*b^3*c^11*d^13*e^ \\
& 10*z^4 - 2066350080*a^14*b*c^8*d^5*e^18*z^4 + 4718592000*a^13*b^2*c^8*d^6*e \\
& ^17*z^4 - 314572800*a^7*b^2*c^14*d^18*e^5*z^4 + 287250432*a^4*b^13*c^6*d^13 \\
& *e^10*z^4 + 4565827584*a^10*b^5*c^8*d^9*e^14*z^4 - 250785792*a^4*b^14*c^5*d \\
& ^12*e^11*z^4 + 235536384*a^13*b^3*c^7*d^5*e^18*z^4 - 232683264*a^8*b^11*c^4 \\
& *d^7*e^16*z^4 - 199627776*a^5*b^14*c^4*d^10*e^13*z^4 - 190267392*a^12*b^7*c \\
& ^4*d^3*e^20*z^4 + 184891392*a^6*b^10*c^7*d^12*e^11*z^4 + 180502528*a^4*b^7* \\
& c^12*d^19*e^4*z^4 + 178877952*a^3*b^13*c^7*d^15*e^8*z^4 + 172490752*a^14*b^ \\
& 3*c^6*d^3*e^20*z^4 + 163946496*a^13*b^5*c^5*d^3*e^20*z^4 + 155839488*a^8*b^ \\
& 12*c^3*d^6*e^17*z^4 + 155000832*a^5*b^5*c^13*d^19*e^4*z^4 - 152076288*a^4*b \\
& ^6*c^13*d^20*e^3*z^4 - 137592576*a^3*b^12*c^8*d^16*e^7*z^4 - 133693440*a^14 \\
& *b^4*c^5*d^2*e^21*z^4 - 116767488*a^3*b^9*c^11*d^19*e^4*z^4 - 108985344*a^3 \\
& *b^14*c^6*d^14*e^9*z^4 - 106223616*a^6*b^13*c^4*d^9*e^14*z^4 + 106119168*a^ \\
& 3*b^10*c^10*d^18*e^5*z^4 + 102432768*a^5*b^4*c^14*d^20*e^3*z^4 + 102113280* \\
& a^4*b^12*c^7*d^14*e^9*z^4 + 100674048*a^5*b^9*c^9*d^15*e^8*z^4 + 90439680*a \\
& ^13*b^6*c^4*d^2*e^21*z^4 - 86808576*a^6*b^14*c^3*d^8*e^15*z^4 + 86245376*a^ \\
& 6*b^2*c^15*d^20*e^3*z^4 + 79011840*a^4*b^8*c^11*d^18*e^5*z^4 + 78345216*a^4
\end{aligned}$$

$$\begin{aligned}
& *b^{15}c^4d^{11}e^{12}z^4 + 78006528a^{11}b^9c^3d^3e^{20}z^4 - 73253376a^9 \\
& *b^{11}c^3d^5e^{18}z^4 + 67524608a^3b^8c^{12}d^{20}e^3z^4 + 67108864a^{15} \\
& *b^2c^6d^2e^{21}z^4 - 61590528a^{10}b^{10}c^3d^4e^{19}z^4 + 61559808a^5* \\
& b^{15}c^3d^9e^{14}z^4 - 59637760a^5b^3c^{15}d^{21}e^2z^4 + 58638336a^4*b \\
& ^5c^{14}d^{21}e^2z^4 - 40828416a^7b^{13}c^3d^7e^{16}z^4 - 35639296a^2*b^ \\
& 12c^9d^{18}e^5z^4 - 31293440a^{12}b^8c^3d^2e^{21}z^4 + 29933568a^5b^1 \\
& 3c^5d^{11}e^{12}z^4 + 27793920a^2b^{11}c^{10}d^{19}e^4z^4 + 27168768a^2*b^ \\
& 13c^8d^{17}e^6z^4 - 23602176a^7b^{14}c^2d^6e^{17}z^4 - 23248896a^3*b^7 \\
& *c^{13}d^{21}e^2z^4 + 20929536a^3b^{15}c^5d^{13}e^{10}z^4 + 18428928a^9*b^1 \\
& 2c^2d^4e^{19}z^4 + 18026496a^6b^{15}c^2d^7e^{16}z^4 - 16261632a^{10}b^1 \\
& 1c^2d^3e^{20}z^4 + 15128064a^3b^{16}c^4d^{12}e^{11}z^4 - 14060544a^2*b^1 \\
& 0c^{11}d^{20}e^3z^4 + 13178880a^2b^{16}c^5d^{14}e^9z^4 - 11244288a^3*b^1 \\
& 7c^3d^{11}e^{12}z^4 - 10509312a^2b^{15}c^6d^{15}e^8z^4 - 7262208a^4*b^17 \\
& *c^2d^9e^{14}z^4 - 7045632a^2b^{17}c^4d^{13}e^{10}z^4 - 6285312a^2*b^14c \\
& ^7d^{16}e^7z^4 + 5996544a^{11}b^{10}c^2d^2e^{21}z^4 + 4558336a^2*b^9c^{12} \\
& *d^{21}e^2z^4 + 4478976a^{11}b^8c^4d^4e^{19}z^4 + 2850816a^4*b^16c^3d^ \\
& 10e^{13}z^4 + 2629632a^3b^{11}c^9d^{17}e^6z^4 + 2503680a^3b^{18}c^2d^{10} \\
& *e^{13}z^4 + 1627136a^2b^{18}c^3d^{12}e^{11}z^4 + 1605120a^8*b^13c^2d^5e \\
& ^18z^4 + 1483776a^5b^{16}c^2d^8e^{15}z^4 + 139776a^2b^{19}c^2d^{11}e^{12} \\
& *z^4 - 8542224384a^{10}b^2c^{11}d^{12}e^{11}z^4 - 3072b^{22}c*d^{12}e^{11}z^4 - \\
& 3072b^{12}c^{11}d^{22}e*z^4 - 1572864a^6c^{17}d^{22}e*z^4 - 4096a^{10}b^{13}d \\
& *e^{22}z^4 - 4096a*b^{22}d^{10}e^{13}z^4 - 6144a^{12}b^{10}c*e^{23}z^4 - 983040* \\
& a^5*b*c^{17}d^{23}z^4 - 6912a*b^9c^{13}d^{23}z^4 + 1824522240a^{13}c^{10}d^8e \\
& ^15z^4 + 1730150400a^{12}c^{11}d^{10}e^{13}z^4 + 958922752a^{14}c^9d^6e^{17} \\
& *z^4 - 537919488a^9c^{14}d^{16}e^7z^4 + 508559360a^{11}c^{12}d^{12}e^{11}z^4 - \\
& 500170752a^{10}c^{13}d^{14}e^9z^4 + 246939648a^{15}c^8d^4e^{19}z^4 - 19922 \\
& 9440a^8c^{15}d^{18}e^5z^4 - 29884416a^7c^{16}d^{20}e^3z^4 + 25165824a^{16} \\
& *c^7d^2e^{21}z^4 + 236544*b^{17}c^6d^{17}e^6z^4 - 202752*b^{18}c^5d^{16}e^7 \\
& *z^4 - 202752*b^{16}c^7d^{18}e^5z^4 + 126720*b^{19}c^4d^{15}e^8z^4 + 126720 \\
& *b^{15}c^8d^{19}e^4z^4 - 56320*b^{20}c^3d^{14}e^9z^4 - 56320*b^{14}c^9d^{20} \\
& *e^3z^4 + 16896*b^{21}c^2d^{13}e^{10}z^4 + 16896*b^{13}c^{10}d^{21}e^2z^4 + 110 \\
& 080a^7*b^{16}d^4e^{19}z^4 + 110080a^4*b^{19}d^7e^{16}z^4 - 75520a^8*b^{15}d \\
& ^3e^{20}z^4 - 75520a^3*b^{20}d^8e^{15}z^4 - 56320a^6*b^{17}d^5e^{18}z^4 - 5 \\
& 6320a^5*b^{18}d^6e^{17}z^4 + 25600a^9*b^{14}d^2e^{21}z^4 + 25600a^2*b^{21}d \\
& ^9e^{14}z^4 - 1572864a^{16}b^2c^5e^{23}z^4 + 983040a^{15}b^4c^4e^{23}z^4 \\
& - 327680a^{14}b^6c^3e^{23}z^4 + 61440a^{13}b^8c^2e^{23}z^4 + 983040a^4*b \\
& ^3c^{16}d^{23}z^4 - 385024a^3b^5c^{15}d^{23}z^4 + 73728a^2*b^7c^{14}d^{23}z \\
& ^4 + 256*b^{23}d^{11}e^{12}z^4 + 1048576a^{17}c^6e^{23}z^4 + 256*b^{11}c^{12}d^2 \\
& 3z^4 + 256a^{11}b^{12}e^{23}z^4 + 948695040a^8*b*c^{10}d^6e^{13}z^2 + 348917 \\
& 760a^7*b*c^{11}d^8e^{11}z^2 - 125030400a^9*b*c^9d^4e^{15}z^2 - 50728960a \\
& ^6*b*c^{12}d^{10}e^9z^2 - 44298240a^5*b*c^{13}d^{12}e^7z^2 - 36495360a^{10}b \\
& *c^8d^2e^{17}z^2 + 29675520a^8*b^6c^5d^6e^{18}z^2 - 24170496a^9*b^4c^6* \\
& d^6e^{18}z^2 - 17202816a^7*b^8c^4d^6e^{18}z^2 - 14561280a^4*b*c^{14}d^{14}e^5 \\
& *z^2 + 5532416a^6*b^{10}c^3d^6e^{18}z^2 + 4128768a^{10}b^2c^7d^6e^{18}z^2 - \\
& 2662400a^3*b*c^{15}d^{16}e^3z^2 + 1184512a*b^{12}c^6d^9e^{10}z^2 - 1136160 \\
& *a*b^{13}c^5d^8e^{11}z^2 - 1017600a^5*b^{12}c^2d^6e^{18}z^2 - 744768a*b^{11}* \\
& c^7d^{10}e^9z^2 + 607872a*b^{14}c^4d^7e^{12}z^2 - 424064a*b^6c^{12}d^{15} \\
& *e^4z^2 + 408576a*b^5c^{13}d^{16}e^3z^2 + 361152a*b^{10}c^8d^{11}e^8z^2 - \\
& 287408a*b^9c^9d^{12}e^7z^2 - 260448a^3*b^{15}c*d^2e^{17}z^2 - 203904a* \\
& b^4c^{14}d^{17}e^2z^2 + 200832a*b^8c^{10}d^{13}e^6z^2 + 126720a*b^7c^{11}* \\
& d^{14}e^5z^2 - 123968a*b^{15}c^3d^6e^{13}z^2 - 39168a*b^{16}c^2d^5e^{14}z \\
& ^2 + 11904a^2*b^{16}c*d^3e^{16}z^2 + 1824135552a^7*b^4c^8d^5e^{14}z^2 - \\
& 1457252352a^8*b^2c^9d^5e^{14}z^2 - 1405209600a^7*b^5c^7d^4e^{15}z^2 - \\
& 184320a^2*b*c^{16}d^{18}e*z^2 + 100608a^4*b^{14}c*d^6e^{18}z^2 + 53248a*b^3* \\
& c^{15}d^{18}e*z^2 + 26448a*b^{17}c*d^4e^{15}z^2 + 1067599872a^8*b^3c^8d^4* \\
& e^{15}z^2 - 930828288a^7*b^3c^9d^6e^{13}z^2 + 920760000a^6*b^4c^9d^7e \\
& ^{12}z^2 - 806639616a^6*b^3c^{10}d^8e^{11}z^2 - 791052480a^6*b^6c^7d^5e \\
& ^{14}z^2 + 772237824a^6*b^7c^6d^4e^{15}z^2 - 701025408a^5*b^6c^8d^7e^ \\
& ^{12}z^2 + 443340288a^5*b^5c^9d^8e^{11}z^2 + 433047552a^7*b^6c^6d^3e^1
\end{aligned}$$

$$\begin{aligned}
& 6*z^2 + 405741312*a^5*b^7*c^7*d^6*e^13*z^2 + 293652480*a^6*b^2*c^11*d^9*e^1 \\
& 0*z^2 - 276962688*a^6*b^8*c^5*d^3*e^16*z^2 - 247804272*a^8*b^4*c^7*d^3*e^16 \\
& *z^2 + 213564384*a^4*b^8*c^7*d^7*e^12*z^2 - 202596816*a^5*b^9*c^5*d^4*e^15* \\
& z^2 - 182520896*a^4*b^9*c^6*d^6*e^13*z^2 - 153489408*a^5*b^3*c^11*d^10*e^9* \\
& z^2 - 152151552*a^7*b^2*c^10*d^7*e^12*z^2 + 115859712*a^5*b^2*c^12*d^11*e^8 \\
& *z^2 + 108085248*a^9*b^3*c^7*d^2*e^17*z^2 + 105536256*a^4*b^5*c^10*d^10*e^9 \\
& *z^2 - 98323200*a^6*b^5*c^8*d^6*e^13*z^2 - 93564992*a^4*b^6*c^9*d^9*e^10*z^ \\
& 2 + 89464512*a^5*b^10*c^4*d^3*e^16*z^2 - 75930624*a^8*b^5*c^6*d^2*e^17*z^2 \\
& + 68315904*a^5*b^8*c^6*d^5*e^14*z^2 - 64157184*a^4*b^7*c^8*d^8*e^11*z^2 - 6 \\
& 2951040*a^9*b^2*c^8*d^3*e^16*z^2 + 49056768*a^4*b^10*c^5*d^5*e^14*z^2 + 476 \\
& 14464*a^3*b^8*c^8*d^9*e^10*z^2 + 35604480*a^4*b^2*c^13*d^13*e^6*z^2 + 33983 \\
& 040*a^3*b^11*c^5*d^6*e^13*z^2 - 33515520*a^4*b^3*c^12*d^12*e^7*z^2 - 334638 \\
& 08*a^3*b^7*c^9*d^10*e^9*z^2 - 25128864*a^4*b^4*c^11*d^11*e^8*z^2 - 23193728 \\
& *a^3*b^10*c^6*d^7*e^12*z^2 + 21015456*a^6*b^9*c^4*d^2*e^17*z^2 + 19924176*a \\
& ^4*b^11*c^4*d^4*e^15*z^2 - 19251216*a^3*b^9*c^7*d^8*e^11*z^2 - 16434048*a^5 \\
& *b^4*c^10*d^9*e^10*z^2 - 16289664*a^3*b^12*c^4*d^5*e^14*z^2 - 15059328*a^4* \\
& b^12*c^3*d^3*e^16*z^2 - 10766016*a^2*b^10*c^7*d^9*e^10*z^2 - 10453632*a^5*b \\
& ^11*c^3*d^2*e^17*z^2 - 9940992*a^3*b^3*c^13*d^14*e^5*z^2 + 8373696*a^2*b^11 \\
& *c^6*d^8*e^11*z^2 + 7776768*a^3*b^2*c^14*d^15*e^4*z^2 + 7077888*a^3*b^5*c^1 \\
& 1*d^12*e^7*z^2 + 6798240*a^2*b^9*c^8*d^10*e^9*z^2 - 3589440*a^2*b^6*c^11*d^ \\
& 13*e^6*z^2 + 3544320*a^3*b^6*c^10*d^11*e^8*z^2 + 3128064*a^2*b^5*c^12*d^14* \\
& e^5*z^2 + 2346336*a^4*b^13*c^2*d^2*e^17*z^2 - 2261568*a^2*b^8*c^9*d^11*e^8* \\
& z^2 - 2125824*a^2*b^13*c^4*d^6*e^13*z^2 + 2002560*a^3*b^4*c^12*d^13*e^6*z^2 \\
& + 1927680*a^2*b^7*c^10*d^12*e^7*z^2 + 1814784*a^2*b^14*c^3*d^5*e^14*z^2 - \\
& 1807104*a^2*b^12*c^5*d^7*e^12*z^2 + 1637808*a^3*b^13*c^3*d^4*e^15*z^2 + 108 \\
& 3456*a^3*b^14*c^2*d^3*e^16*z^2 - 792384*a^2*b^4*c^13*d^15*e^4*z^2 - 657408* \\
& a^2*b^3*c^14*d^16*e^3*z^2 + 608256*a^7*b^7*c^5*d^2*e^17*z^2 + 595968*a^2*b^ \\
& 2*c^15*d^17*e^2*z^2 - 498624*a^2*b^15*c^2*d^4*e^15*z^2 - 3840*b^18*c*d^5*e^ \\
& 14*z^2 - 3840*b^5*c^14*d^18*e*z^2 + 2064384*a^11*c^8*d*e^18*z^2 - 4160*a^3* \\
& b^16*d*e^18*z^2 - 4160*a*b^18*d^3*e^16*z^2 - 1290240*a^11*b*c^7*e^19*z^2 - \\
& 9840*a^5*b^13*c*e^19*z^2 - 5760*a*b^2*c^16*d^19*z^2 - 280581120*a^8*c^11*d^ \\
& 7*e^12*z^2 + 110278656*a^9*c^10*d^5*e^14*z^2 - 89479168*a^7*c^12*d^9*e^10*z \\
& ^2 + 34464000*a^10*c^9*d^3*e^16*z^2 + 54240*b^15*c^4*d^8*e^11*z^2 + 54240*b \\
& ^8*c^11*d^15*e^4*z^2 - 49920*b^14*c^5*d^9*e^10*z^2 - 49920*b^9*c^10*d^14*e^ \\
& 5*z^2 - 37376*b^16*c^3*d^7*e^12*z^2 - 37376*b^7*c^12*d^16*e^3*z^2 + 28480*b \\
& ^13*c^6*d^10*e^9*z^2 + 28480*b^10*c^9*d^13*e^6*z^2 + 15936*b^17*c^2*d^6*e^1 \\
& 3*z^2 + 15936*b^6*c^13*d^17*e^2*z^2 - 7920*b^12*c^7*d^11*e^8*z^2 - 7920*b^1 \\
& 1*c^8*d^12*e^7*z^2 + 7489536*a^5*c^14*d^13*e^6*z^2 + 6084096*a^6*c^13*d^11* \\
& e^8*z^2 + 2280448*a^4*c^15*d^15*e^4*z^2 + 350208*a^3*c^16*d^17*e^2*z^2 + 11 \\
& 616*a^2*b^17*d^2*e^17*z^2 - 3515904*a^9*b^5*c^5*e^19*z^2 + 3440640*a^10*b^3 \\
& *c^6*e^19*z^2 + 1870848*a^8*b^7*c^4*e^19*z^2 - 572272*a^7*b^9*c^3*e^19*z^2 \\
& + 101856*a^6*b^11*c^2*e^19*z^2 + 400*b^19*d^4*e^15*z^2 + 400*b^4*c^15*d^19* \\
& z^2 + 20736*a^2*c^17*d^19*z^2 + 400*a^4*b^15*e^19*z^2 - 3969216*a^4*b*c^10* \\
& d^3*e^12 - 3001536*a^3*b*c^11*d^5*e^10 - 419904*a^2*b*c^12*d^7*e^8 + 184608 \\
& *a^4*b^3*c^8*d*e^14 - 153036*a*b^4*c^10*d^6*e^9 + 127008*a*b^3*c^11*d^7*e^8 \\
& + 63108*a*b^6*c^8*d^4*e^11 - 29160*a*b^2*c^12*d^8*e^7 - 21060*a^3*b^5*c^7* \\
& d*e^14 - 21060*a*b^7*c^7*d^3*e^12 + 5460*a*b^5*c^9*d^5*e^10 - 404544*a^5*b* \\
& c^9*d*e^14 + 1251872*a^3*b^3*c^9*d^3*e^12 + 844224*a^4*b^2*c^9*d^2*e^13 + 8 \\
& 20512*a^2*b^3*c^10*d^5*e^10 + 750672*a^3*b^2*c^10*d^4*e^11 - 657498*a^2*b^4 \\
& *c^9*d^4*e^11 - 487116*a^3*b^4*c^8*d^2*e^13 + 160704*a^2*b^2*c^11*d^6*e^9 + \\
& 58806*a^2*b^6*c^7*d^2*e^13 + 13140*a^2*b^5*c^8*d^3*e^12 + 15286*b^6*c^9*d^ \\
& 6*e^9 - 9540*b^7*c^8*d^5*e^10 - 9540*b^5*c^10*d^7*e^8 + 2025*b^8*c^7*d^4*e^ \\
& 11 + 2025*b^4*c^11*d^8*e^7 + 3367008*a^4*c^11*d^4*e^11 + 1166400*a^3*c^12*d \\
& ^6*e^9 + 705600*a^5*c^10*d^2*e^13 + 104976*a^2*c^13*d^8*e^7 - 17640*a^5*b^2 \\
& *c^8*e^15 + 2025*a^4*b^4*c^7*e^15 + 38416*a^6*c^9*e^15, z, k)*((57344*a^12* \\
& c^9*e^21 - 80*a^5*b^14*c^2*e^21 + 1824*a^6*b^12*c^3*e^21 - 17296*a^7*b^10*c \\
& ^4*e^21 + 87520*a^8*b^8*c^5*e^21 - 250880*a^9*b^6*c^6*e^21 + 394240*a^10*b^ \\
& 4*c^7*e^21 - 290816*a^11*b^2*c^8*e^21 + 18432*a^3*c^18*d^18*e^3 + 210944*a^ \\
& 4*c^17*d^16*e^5 + 878592*a^5*c^16*d^14*e^7 + 4749312*a^6*c^15*d^12*e^9 + 20
\end{aligned}$$



$$\begin{aligned}
& 518912a^7c^{14}d^{10}e^{11} + 12306432a^8c^{13}d^8e^{13} - 22743040a^9c^{12}d^6e^{15} - 20076544a^{10}c^{11}d^4e^{17} - 1425408a^{11}c^{10}d^2e^{19} - 80b^5c^{16}d^{19}e^2 + 704b^6c^{15}d^{18}e^3 - 2688b^7c^{14}d^{17}e^4 + 5824b^8c^{13}d^{16}e^5 - 7840b^9c^{12}d^{15}e^6 + 6720b^{10}c^{11}d^{14}e^7 - 3728b^{11}c^{10}d^{13}e^8 + 2176b^{12}c^9d^{12}e^9 - 3728b^{13}c^8d^{11}e^{10} + 6720b^{14}c^7d^{10}e^{11} - 7840b^{15}c^6d^9e^{12} + 5824b^{16}c^5d^8e^{13} - 2688b^{17}c^4d^7e^{14} + 704b^{18}c^3d^6e^{15} - 80b^{19}c^2d^5e^{16} + 12288a^2b^2c^{17}d^{18}e^3 - 1536a^2b^3c^{16}d^{17}e^4 - 131712a^2b^4c^{15}d^{16}e^5 + 410112a^2b^5c^{14}d^{15}e^6 - 576576a^2b^6c^{13}d^{14}e^7 + 342720a^2b^7c^{12}d^{13}e^8 + 298464a^2b^8c^{11}d^{12}e^9 - 1248672a^2b^9c^{10}d^{11}e^{10} + 2177920a^2b^{10}c^9d^{10}e^{11} - 2309120a^2b^{11}c^8d^9e^{12} + 1389888a^2b^{12}c^7d^8e^{13} - 314048a^2b^{13}c^6d^7e^{14} - 120896a^2b^{14}c^5d^6e^{15} + 88128a^2b^{15}c^4d^5e^{16} - 14240a^2b^{16}c^3d^4e^{17} - 416a^2b^{17}c^2d^3e^{18} + 621568a^3b^2c^{16}d^{16}e^5 - 953344a^3b^3c^{15}d^{15}e^6 + 196224a^3b^4c^{14}d^{14}e^7 + 1667904a^3b^5c^{13}d^{13}e^8 - 3981824a^3b^6c^{12}d^{12}e^9 + 7617920a^3b^7c^{11}d^{11}e^{10} - 11899456a^3b^8c^{10}d^{10}e^{11} + 11500496a^3b^9c^9d^9e^{12} - 4599536a^3b^{10}c^8d^8e^{13} - 1951936a^3b^{11}c^7d^7e^{14} + 2953152a^3b^{12}c^6d^6e^{15} - 1134960a^3b^{13}c^5d^5e^{16} + 98960a^3b^{14}c^4d^4e^{17} + 21920a^3b^{15}c^3d^3e^{18} - 416a^3b^{16}c^2d^2e^{19} + 4509696a^4b^2c^{15}d^{14}e^7 - 6720000a^4b^3c^{14}d^{13}e^8 + 8231808a^4b^4c^{13}d^{12}e^9 - 17138976a^4b^5c^{12}d^{11}e^{10} + 30880320a^4b^6c^{11}d^{10}e^{11} - 24883456a^4b^7c^{10}d^9e^{12} - 6291360a^4b^8c^9d^8e^{13} + 28429152a^4b^9c^8d^7e^{14} - 21523072a^4b^{10}c^7d^6e^{15} + 5834928a^4b^{11}c^6d^5e^{16} + 339872a^4b^{12}c^5d^4e^{17} - 325216a^4b^{13}c^4d^3e^{18} + 1344a^4b^{14}c^3d^2e^{19} + 5483520a^5b^2c^{14}d^{12}e^9 + 14537472a^5b^3c^{13}d^{11}e^{10} - 39383680a^5b^4c^{12}d^{10}e^{11} + 5513408a^5b^5c^{11}d^9e^{12} + 84582144a^5b^6c^{10}d^8e^{13} - 124246848a^5b^7c^9d^7e^{14} + 70979712a^5b^8c^8d^6e^{15} - 8326320a^5b^9c^7d^5e^{16} - 7484656a^5b^{10}c^6d^4e^{17} + 2142272a^5b^{11}c^5d^3e^{18} + 83520a^5b^{12}c^4d^2e^{19} + 25849856a^6b^2c^{13}d^{10}e^{11} + 67294720a^6b^3c^{12}d^9e^{12} - 216767360a^6b^4c^{11}d^8e^{13} + 237211008a^6b^5c^{10}d^7e^{14} - 88839360a^6b^6c^9d^6e^{15} - 35929920a^6b^7c^8d^5e^{16} + 37859616a^6b^8c^7d^4e^{17} - 6475552a^6b^9c^6d^3e^{18} - 1055296a^6b^{10}c^5d^2e^{19} + 190669824a^7b^2c^{12}d^8e^{13} - 143425536a^7b^3c^{11}d^7e^{14} - 47908992a^7b^4c^{10}d^6e^{15} + 154814400a^7b^5c^9d^5e^{16} - 83642880a^7b^6c^8d^4e^{17} + 4534272a^7b^7c^7d^3e^{18} + 5525568a^7b^8c^6d^2e^{19} + 165122048a^8b^2c^{11}d^6e^{15} - 187467264a^8b^3c^{10}d^5e^{16} + 6692064a^8b^4c^9d^4e^{17} + 21356016a^8b^5c^8d^3e^{18} - 14644224a^8b^6c^7d^2e^{19} + 16114688a^9b^2c^{10}d^4e^{17} - 48695936a^9b^3c^9d^3e^{18} + 18757632a^9b^4c^8d^2e^{19} - 8060928a^{10}b^2c^9d^2e^{19} + 1257472a^{11}b^3c^9d^2e^{20} + 896a^8b^3c^{17}d^{19}e^2 - 7040a^8b^4c^{16}d^{18}e^3 + 22080a^8b^5c^{15}d^{17}e^4 - 32512a^8b^6c^{14}d^{16}e^5 + 12736a^8b^7c^{13}d^{15}e^6 + 31104a^8b^8c^{12}d^{14}e^7 - 51472a^8b^9c^{11}d^{13}e^8 + 10864a^8b^{10}c^{10}d^{12}e^9 + 85440a^8b^{11}c^9d^{11}e^{10} - 186560a^8b^{12}c^8d^{10}e^{11} + 215904a^8b^{13}c^7d^9e^{12} - 151008a^8b^{14}c^6d^8e^{13} + 59776a^8b^{15}c^5d^7e^{14} - 9408a^8b^{16}c^4d^6e^{15} - 1296a^8b^{17}c^3d^5e^{16} + 496a^8b^{18}c^2d^4e^{17} - 2304a^8b^{19}c^18d^{19}e^2 - 175104a^9b^3c^{17}d^{17}e^4 - 1556480a^9b^4c^{16}d^{15}e^6 + 496a^9b^5c^{15}d^{13}e^8 - 10256a^9b^6c^{14}d^{11}e^{10} + 84512a^9b^7c^{13}d^9e^{12} - 100332544a^9b^8c^{12}d^7e^{14} + 621568a^9b^9c^{11}d^5e^{16} - 68096a^9b^{10}c^{10}d^3e^{18} - 1310720a^9b^{11}c^9d^1e^{20} / (32*(16a^3b^6c^9d^{18} - a^2b^8c^8d^{18} - 256a^6c^{12}d^{18} - 96a^4b^4c^{10}d^{18} + 256a^5b^2c^{11}d^{18} - a^2b^{16}d^{10}e^8 + 8a^3b^{15}d^9e^9 - 28a^4b^{14}d^8e^{10} + 56a^5b^{13}d^7e^{11} - 70a^6b^{12}d^6e^{12} + 56a^7b^{11}d^5e^{13} - 28a^8b^{10}d^4e^{14} + 8a^9b^9d^3e^{15} - a^{10}b^8d^2e^{16} - 2048a^7c^{11}d^{16}e^2 - 7168a^8c^{10}d^{14}e^4 - 14336a^9c^9d^{12}e^6 - 17920a^{10}c^8d^{10}e^8 - 1
\end{aligned}$$

$$\begin{aligned}
& 4336a^{11}c^7d^8e^{10} - 7168a^{12}c^6d^6e^{12} - 2048a^{13}c^5d^4e^{14} - \\
& 256a^{14}c^4d^2e^{16} - 28a^2b^{10}c^6d^{16}e^2 + 56a^2b^{11}c^5d^{15}e^3 \\
& - 70a^2b^{12}c^4d^{14}e^4 + 56a^2b^{13}c^3d^{13}e^5 - 28a^2b^{14}c^2d^{12}e^6 + 440a^3b^8c^7d^{16}e^2 - 840a^3b^9c^6d^{15}e^3 + 952a^3b^{10} \\
& c^5d^{14}e^4 - 616a^3b^{11}c^4d^{13}e^5 + 168a^3b^{12}c^3d^{12}e^6 + 40a^3b^{13}c^2d^{11}e^7 - 2560a^4b^6c^8d^{16}e^2 + 4480a^4b^7c^7d^{15}e^3 \\
& - 4060a^4b^8c^6d^{14}e^4 + 1064a^4b^9c^5d^{13}e^5 + 1372a^4b^{10}c^4d^{12}e^6 - 1360a^4b^{11}c^3d^{11}e^7 + 380a^4b^{12}c^2d^{10}e^8 + 640 \\
& 0a^5b^4c^9d^{16}e^2 - 8960a^5b^5c^8d^{15}e^3 + 2240a^5b^6c^7d^{14}e^4 + 9856a^5b^7c^6d^{13}e^5 - 13048a^5b^8c^5d^{12}e^6 + 5400a^5b^9 \\
& c^4d^{11}e^7 + 1040a^5b^{10}c^3d^{10}e^8 - 1360a^5b^{11}c^2d^9e^9 - 5120a^6b^2c^{10}d^{16}e^2 + 22400a^6b^4c^8d^{14}e^4 - 41216a^6b^5c^7d^{13}e^5 \\
& + 25088a^6b^6c^6d^{12}e^6 + 8320a^6b^7c^5d^{11}e^7 - 17350a^6b^8c^4d^{10}e^8 + 5400a^6b^9c^3d^9e^9 + 1372a^6b^{10}c^2d^8e^{10} \\
& - 35840a^7b^2c^9d^{14}e^4 + 28672a^7b^3c^8d^{13}e^5 + 30464a^7b^4c^7d^{12}e^6 - 73472a^7b^5c^6d^{11}e^7 + 40544a^7b^6c^5d^{10}e^8 + 832 \\
& 0a^7b^7c^4d^9e^9 - 13048a^7b^8c^3d^8e^{10} + 1064a^7b^9c^2d^7e^{11} - 93184a^8b^2c^8d^{12}e^6 + 71680a^8b^3c^7d^{11}e^7 + 29120a^8b^4 \\
& c^6d^{10}e^8 - 73472a^8b^5c^5d^9e^9 + 25088a^8b^6c^4d^8e^{10} + 9856a^8b^7c^3d^7e^{11} - 4060a^8b^8c^2d^6e^{12} - 125440a^9b^2c^7d^{10}e^8 \\
& + 71680a^9b^3c^6d^9e^9 + 30464a^9b^4c^5d^8e^{10} - 41216a^9b^5c^4d^7e^{11} + 2240a^9b^6c^3d^6e^{12} + 4480a^9b^7c^2d^5e^{13} \\
& - 93184a^{10}b^2c^6d^8e^{10} + 28672a^{10}b^3c^5d^7e^{11} + 22400a^{10}b^4c^4d^6e^{12} - 8960a^{10}b^5c^3d^5e^{13} - 2560a^{10}b^6c^2d^4e^{14} - \\
& 35840a^{11}b^2c^5d^6e^{12} + 6400a^{11}b^4c^3d^4e^{14} + 768a^{11}b^5c^2d^3e^{15} - 5120a^{12}b^2c^4d^4e^{14} - 2048a^{12}b^3c^3d^3e^{15} - 96a^{12} \\
& b^4c^2d^2e^{16} + 256a^{13}b^2c^3d^2e^{16} + 2048a^6b^7c^{11}d^{17}e + 8a^2b^9c^7d^{17}e + 8a^2b^{15}c^d^{11}e^7 - 128a^3b^7c^8d^{17}e - 40 \\
& a^3b^{14}c^d^{10}e^8 + 768a^4b^5c^9d^{17}e + 40a^4b^{13}c^d^9e^9 - 2048a^5b^3c^{10}d^{17}e + 168a^5b^{12}c^d^8e^{10} - 616a^6b^{11}c^d^7e^{11} + \\
& 14336a^7b^c^{10}d^{15}e^3 + 952a^7b^{10}c^d^6e^{12} + 43008a^8b^c^9d^{13}e^5 - 840a^8b^9c^d^5e^{13} + 71680a^9b^c^8d^{11}e^7 + 440a^9b^8c^d^4e^{14} \\
& + 71680a^{10}b^c^7d^9e^9 - 128a^{10}b^7c^d^3e^{15} + 43008a^{11}b^c^6d^7e^{11} + 16a^{11}b^6c^d^2e^{16} + 14336a^{12}b^c^5d^5e^{13} + 2048a^{13} \\
& b^c^4d^3e^{15}) - \text{root}(128723189760a^{14}b^4c^9d^{13}e^{14}z^6 + 128723189760a^{12}b^4c^{11}d^{17}e^{10}z^6 - 8432455680a^{11}b^{12}c^4d^{11}e^{16}z^6 \\
& - 8432455680a^7b^{12}c^8d^{19}e^8z^6 + 12673351680a^{11}b^{11}c^5d^{12}e^{15}z^6 + 12673351680a^8b^{11}c^8d^{18}e^9z^6 - 72637480960a^{12}b^9c^6d^{12}e^{15}z^6 \\
& - 72637480960a^9b^9c^9d^{18}e^9z^6 - 21048344576a^9b^{12}c^6d^{15}e^{12}z^6 - 16609443840a^{17}b^3c^7d^8e^{19}z^6 - 16609443840a^{10} \\
& b^3c^{14}d^{22}e^5z^6 + 145332633600a^{13}b^5c^9d^{14}e^{13}z^6 + 145332633600a^{12}b^5c^{10}d^{16}e^{11}z^6 + 123740356608a^{14}b^5c^8d^{12}e^{15}z^6 \\
& + 123740356608a^{11}b^5c^{11}d^{18}e^9z^6 + 3460300800a^{17}b^5c^5d^6e^{21}z^6 + 3460300800a^8b^5c^{14}d^{24}e^3z^6 - 7751073792a^{15}b^7c^5d^8 \\
& e^{19}z^6 - 7751073792a^8b^7c^{12}d^{22}e^5z^6 + 12041846784a^{14}b^7c^6d^{10}e^{17}z^6 + 12041846784a^9b^7c^{11}d^{20}e^7z^6 - 325545099264a^{14} \\
& b^3c^{10}d^{14}e^{13}z^6 - 325545099264a^{13}b^3c^{11}d^{16}e^{11}z^6 - 3330539520a^{13}b^{10}c^4d^9e^{18}z^6 - 3330539520a^7b^{10}c^{10}d^{21}e^6z^6 + 15 \\
& 7789716480a^{12}b^7c^8d^{14}e^{13}z^6 + 157789716480a^{11}b^7c^9d^{16}e^{11}z^6 + 37492359168a^{11}b^{10}c^6d^{13}e^{14}z^6 + 37492359168a^9b^{10}c^8d^{17} \\
& e^{10}z^6 + 301989888a^8b^3c^{16}d^{26}e^z^6 - 7266631680a^{17}b^4c^6d^7e^{20}z^6 - 7266631680a^9b^4c^{14}d^{23}e^4z^6 - 201326592a^{20}b^c^6d^4e^{23}z^6 \\
& - 188743680a^7b^5c^{15}d^{26}e^z^6 + 45747339264a^{13}b^8c^6d^{11}e^{16}z^6 + 45747339264a^9b^8c^{10}d^{19}e^8z^6 - 74612736a^{10}b^{16}c^c^d^9e^{18}z^6 \\
& - 2768240640a^{16}b^7c^4d^6e^{21}z^6 - 2768240640a^7b^7c^{13}d^{24}e^3z^6 + 69746688a^{11}b^{15}c^d^8e^{19}z^6 + 62914560a^6b^7c^{14}d^{26}e^z^6 \\
& + 2752020480a^{10}b^{13}c^4d^{12}e^{15}z^6 + 2752020480a^7b^{13}c^7d^{18}e^9z^6 + 55148544a^9b^{17}c^d^{10}e^{17}z^6 - 45957120a^{12}b^{11} \\
& 4c^d^7e^{20}z^6 - 2724986880a^{14}b^9c^4d^8e^{19}z^6 - 2724986880a^7b^
\end{aligned}$$

$$\begin{aligned}
& 9c^{11}d^{22}e^5z^6 - 25952256a^8b^{18}c^4d^{11}e^{16}z^6 + 21086208a^{13}b^{11} \\
& 3c^4d^6e^{21}z^6 - 11796480a^5b^9c^{13}d^{26}e^3z^6 - 6438912a^{14}b^{12}c^4d \\
& ^5e^{22}z^6 + 5406720a^7b^{19}c^4d^{12}e^{15}z^6 + 1622016a^6b^{20}c^4d^{13}e^{14}z^6 \\
& - 1523712a^5b^{21}c^4d^{14}e^{13}z^6 + 1179648a^{15}b^{11}c^4d^4e^{23}z^6 \\
& + 1179648a^4b^{11}c^{12}d^{26}e^3z^6 + 442368a^4b^{22}c^4d^{15}e^{12}z^6 - 98 \\
& 304a^{16}b^{10}c^4d^3e^{24}z^6 - 49152a^3b^{23}c^4d^{16}e^{11}z^6 - 49152a^3b \\
& ^{13}c^{11}d^{26}e^3z^6 + 6897106944a^9b^{13}c^5d^{14}e^{13}z^6 + 6897106944a^8 \\
& b^{13}c^6d^{16}e^{11}z^6 - 2422210560a^{16}b^6c^5d^7e^{20}z^6 - 242221056 \\
& 0a^8b^6c^{13}d^{23}e^4z^6 + 255785435136a^{14}b^2c^{11}d^{15}e^{12}z^6 + 41 \\
& 004564480a^{15}b^4c^8d^{11}e^{16}z^6 + 41004564480a^{11}b^4c^{12}d^{19}e^8z \\
& ^6 + 2270822400a^{13}b^{11}c^3d^8e^{19}z^6 + 2270822400a^6b^{11}c^{10}d^{22}e \\
& ^5z^6 + 23677108224a^{14}b^8c^5d^9e^{18}z^6 + 23677108224a^8b^8c^{11}d \\
& ^{21}e^6z^6 + 212600881152a^{15}b^2c^{10}d^{13}e^{14}z^6 + 212600881152a^{13} \\
& b^2c^{12}d^{17}e^{10}z^6 + 75157733376a^{15}b^5c^7d^{10}e^{17}z^6 + 75157733 \\
& 376a^{10}b^5c^{12}d^{20}e^7z^6 - 251217838080a^{13}b^6c^8d^{13}e^{14}z^6 - \\
& 251217838080a^{11}b^6c^{10}d^{17}e^{10}z^6 - 1952907264a^{14}b^{10}c^3d^7e^2 \\
& 0z^6 - 1952907264a^6b^{10}c^{11}d^{23}e^4z^6 - 27691057152a^{13}b^9c^5d^ \\
& ^{10}e^{17}z^6 - 27691057152a^8b^9c^{10}d^{20}e^7z^6 - 1902673920a^8b^{15}c \\
& ^4d^{14}e^{13}z^6 - 1902673920a^7b^{15}c^5d^{16}e^{11}z^6 + 10465050624a^{10} \\
& b^{11}c^6d^{14}e^{13}z^6 + 10465050624a^9b^{11}c^7d^{16}e^{11}z^6 + 16139059 \\
& 20a^9b^{14}c^4d^{13}e^{14}z^6 + 1613905920a^7b^{14}c^6d^{17}e^{10}z^6 - 332 \\
& 18887680a^{17}b^3c^9d^{10}e^{17}z^6 - 33218887680a^{12}b^3c^{14}d^{20}e^7z^6 + \\
& 1524695040a^{10}b^{14}c^3d^{11}e^{16}z^6 + 1524695040a^6b^{14}c^7d^{19}e^8z \\
& ^6 - 1472200704a^{18}b^4c^5d^5e^{22}z^6 - 1472200704a^8b^4c^{15}d^{25}e^ \\
& ^2z^6 - 83047219200a^{16}b^3c^8d^{10}e^{17}z^6 - 83047219200a^{11}b^3c^{13}d \\
& ^{20}e^7z^6 + 44291850240a^{17}b^2c^8d^9e^{18}z^6 + 44291850240a^{11}b^2 \\
& c^{14}d^{21}e^6z^6 + 1308131328a^8b^{14}c^5d^{15}e^{12}z^6 - 201326592a^9b \\
& ^3c^{17}d^{26}e^3z^6 + 48530718720a^{12}b^8c^7d^{13}e^{14}z^6 + 48530718720a^ \\
& ^{10}b^8c^9d^{17}e^{10}z^6 - 1242644480a^{12}b^{12}c^3d^9e^{18}z^6 - 12426444 \\
& 80a^6b^{12}c^9d^{21}e^6z^6 + 9813196800a^{12}b^{10}c^5d^{11}e^{16}z^6 + 981 \\
& 3196800a^8b^{10}c^9d^{19}e^8z^6 - 93012885504a^{15}b^3c^{11}d^{14}e^{13}z^6 - \\
& 93012885504a^{14}b^3c^{12}d^{16}e^{11}z^6 + 177305812992a^{13}b^4c^{10}d^{15}e^ \\
& ^{12}z^6 + 52730658816a^{10}b^{10}c^7d^{15}e^{12}z^6 - 1180106752a^9b^{15}c^3d \\
& ^{12}e^{15}z^6 - 1180106752a^6b^{15}c^6d^{18}e^9z^6 + 1023672320a^{15}b^9c \\
& ^3d^6e^{21}z^6 + 1023672320a^6b^9c^{12}d^{24}e^3z^6 + 975175680a^{17}b^ \\
& ^6c^4d^5e^{22}z^6 + 975175680a^7b^6c^{14}d^{25}e^2z^6 - 11072962560a^{18} \\
& b^3c^8d^8e^{19}z^6 - 11072962560a^{11}b^3c^{15}d^{22}e^5z^6 + 9412018176a^{11} \\
& 8b^2c^7d^7e^{20}z^6 + 9412018176a^{10}b^2c^{15}d^{23}e^4z^6 + 805306368* \\
& a^{19}b^2c^6d^5e^{22}z^6 + 805306368a^9b^2c^{16}d^{25}e^2z^6 - 133809831 \\
& 936a^{14}b^6c^7d^{11}e^{16}z^6 - 133809831936a^{10}b^6c^{11}d^{19}e^8z^6 - \\
& 2214592512a^{19}b^3c^7d^6e^{21}z^6 - 2214592512a^{10}b^3c^{16}d^{24}e^3z^6 + \\
& 82216747008a^{13}b^7c^7d^{12}e^{15}z^6 + 82216747008a^{10}b^7c^{10}d^{18}e^9 \\
& z^6 - 586629120a^{12}b^{13}c^2d^8e^{19}z^6 - 586629120a^5b^{13}c^9d^{22}e^ \\
& ^5z^6 + 568565760a^7b^{16}c^4d^{15}e^{12}z^6 - 4844421120a^{16}b^4c^7d^9 \\
& e^{18}z^6 - 4844421120a^{10}b^4c^{13}d^{21}e^6z^6 + 531210240a^{11}b^{14}c^2 \\
& d^9e^{18}z^6 + 531210240a^5b^{14}c^8d^{21}e^6z^6 - 527155200a^{11}b^{13}c^ \\
& ^3d^{10}e^{17}z^6 - 527155200a^6b^{13}c^8d^{20}e^7z^6 + 43470028800a^{11}b \\
& ^8c^8d^{15}e^{12}z^6 - 107874877440a^{11}b^9c^7d^{14}e^{13}z^6 - 1078748774 \\
& 40a^{10}b^9c^8d^{16}e^{11}z^6 + 9018408960a^{12}b^{11}c^4d^{10}e^{17}z^6 + 90 \\
& 18408960a^7b^{11}c^9d^{20}e^7z^6 + 421994496a^{13}b^{12}c^2d^7e^{20}z^6 + \\
& 421994496a^5b^{12}c^{10}d^{23}e^4z^6 - 66437775360a^{16}b^3c^{10}d^{12}e^{15}z \\
& ^6 - 66437775360a^{13}b^3c^{13}d^{18}e^9z^6 + 26159874048a^{16}b^5c^6d^8e^ \\
& ^{19}z^6 + 26159874048a^9b^5c^{13}d^{22}e^5z^6 - 369098752a^{18}b^3c^6d^6 \\
& e^{21}z^6 - 369098752a^9b^3c^{15}d^{24}e^3z^6 + 351436800a^8b^{16}c^3d^ \\
& ^{13}e^{14}z^6 + 351436800a^6b^{16}c^5d^{17}e^{10}z^6 - 334233600a^{16}b^8c^3 \\
& d^5e^{22}z^6 - 334233600a^6b^8c^{13}d^{25}e^2z^6 + 301989888a^{19}b^3c^ \\
& ^5d^4e^{23}z^6 - 266010624a^{10}b^{15}c^2d^{10}e^{17}z^6 - 266010624a^5b^{15} \\
& c^7d^{20}e^7z^6 - 305198530560a^{12}b^6c^9d^{15}e^{12}z^6 - 203292672a^{11} \\
& 4b^{11}c^2d^6e^{21}z^6 - 203292672a^5b^{11}c^{11}d^{24}e^3z^6 - 188743680*
\end{aligned}$$

$$\begin{aligned}
& a^{18}b^5c^4d^4e^{23}z^6 + 120418467840a^{16}b^2c^9d^{11}e^{16}z^6 + 120418467840a^{12}b^2c^{13}d^{19}e^8z^6 - 17293934592a^{10}b^{12}c^5d^{13}e^{14}z^6 - 17293934592a^8b^{12}c^7d^{17}e^{10}z^6 + 104890368a^8b^{17}c^2d^{12}e^{15}z^6 + 104890368a^5b^{17}c^5d^{18}e^9z^6 + 4390256640a^{15}b^8c^4d^7e^{20}z^6 + 4390256640a^7b^8c^{12}d^{23}e^4z^6 - 91750400a^6b^{18}c^3d^{15}e^{12}z^6 + 79134720a^7b^{17}c^3d^{14}e^{13}z^6 + 79134720a^6b^{17}c^4d^{16}e^{11}z^6 - 74612736a^4b^{16}c^7d^{21}e^6z^6 - 72990720a^7b^{18}c^2d^{13}e^{14}z^6 - 72990720a^5b^{18}c^4d^{17}e^{10}z^6 + 69746688a^4b^{15}c^8d^{22}e^5z^6 + 63700992a^{15}b^{10}c^2d^5e^{22}z^6 + 63700992a^5b^{10}c^{12}d^{25}e^2z^6 + 62914560a^{17}b^7c^3d^4e^{23}z^6 + 55148544a^4b^{17}c^6d^{20}e^7z^6 - 45957120a^4b^{14}c^9d^{23}e^4z^6 - 25952256a^4b^{18}c^5d^{19}e^8z^6 - 25165824a^{20}b^2c^5d^3e^{24}z^6 + 21086208a^4b^{13}c^{10}d^{24}e^3z^6 + 20643840a^6b^{19}c^2d^{14}e^{13}z^6 + 20643840a^5b^{19}c^3d^{16}e^{11}z^6 + 15728640a^{19}b^4c^4d^3e^{24}z^6 - 11796480a^{16}b^9c^2d^4e^{23}z^6 - 6438912a^4b^{12}c^{11}d^{25}e^2z^6 + 5406720a^4b^{19}c^4d^{18}e^9z^6 - 5242880a^{18}b^6c^3d^3e^{24}z^6 + 3784704a^3b^{18}c^6d^{21}e^6z^6 - 3244032a^3b^{19}c^5d^{20}e^7z^6 - 3244032a^3b^{17}c^7d^{22}e^5z^6 + 2027520a^3b^{20}c^4d^{19}e^8z^6 + 2027520a^3b^{16}c^8d^{23}e^4z^6 - 1622016a^9b^{16}c^2d^{11}e^{16}z^6 - 1622016a^5b^{16}c^6d^{19}e^8z^6 + 1622016a^4b^{20}c^3d^{17}e^{10}z^6 - 1523712a^4b^{21}c^2d^{16}e^{11}z^6 + 983040a^{17}b^8c^2d^3e^{24}z^6 - 901120a^3b^{21}c^3d^{18}e^9z^6 - 901120a^3b^{15}c^9d^{24}e^3z^6 + 270336a^3b^{22}c^2d^{17}e^{10}z^6 + 270336a^3b^{14}c^{10}d^{25}e^2z^6 + 172032a^5b^{20}c^2d^{15}e^{12}z^6 - 38593888256a^{15}b^6c^6d^9e^{18}z^6 - 38593888256a^9b^6c^{12}d^{21}e^6z^6 - 210386288640a^{15}b^3c^9d^{12}e^{15}z^6 - 210386288640a^{12}b^3c^{12}d^{18}e^9z^6 + 15502147584a^{15}c^{12}d^{15}e^{12}z^6 + 1107296256a^{19}c^8d^7e^{20}z^6 + 107296256a^{11}c^{16}d^{23}e^4z^6 + 13287555072a^{16}c^{11}d^{13}e^{14}z^6 + 13287555072a^{14}c^{13}d^{17}e^{10}z^6 + 201326592a^{20}c^7d^5e^{22}z^6 + 201326592a^{10}c^{17}d^{25}e^2z^6 + 16777216a^{21}c^6d^3e^{24}z^6 + 3784704a^9b^{18}d^9e^{18}z^6 - 3244032a^{10}b^{17}d^8e^{19}z^6 - 3244032a^8b^{19}d^{10}e^{17}z^6 + 2027520a^{11}b^{16}d^7e^{20}z^6 + 2027520a^7b^{20}d^{11}e^{16}z^6 - 901120a^{12}b^{15}d^6e^{21}z^6 - 901120a^6b^{21}d^{12}e^{15}z^6 + 270336a^{13}b^{14}d^5e^{22}z^6 + 270336a^5b^{22}d^{13}e^{14}z^6 - 49152a^{14}b^{13}d^4e^{23}z^6 - 49152a^4b^{23}d^{14}e^{13}z^6 + 4096a^{15}b^{12}d^3e^{24}z^6 + 4096a^3b^{24}d^{15}e^{12}z^6 - 25165824a^8b^2c^{17}d^{27}z^6 + 15728640a^7b^4c^{16}d^{27}z^6 - 5242880a^6b^6c^{15}d^{27}z^6 + 983040a^5b^8c^{14}d^{27}z^6 - 983040a^4b^{10}c^{13}d^{27}z^6 + 4096a^3b^{12}c^{12}d^{27}z^6 + 8304721920a^{17}c^{10}d^{11}e^{16}z^6 + 8304721920a^{13}c^{14}d^{19}e^8z^6 + 3690987520a^{18}c^9d^9e^{18}z^6 + 3690987520a^{12}c^{15}d^{21}e^6z^6 + 16777216a^9c^{18}d^{27}z^6 - 8493371392a^6b^8c^9d^{14}e^9z^4 + 1458044928a^8b^6c^{14}d^{17}e^6z^4 - 12604538880a^{11}b^4c^8d^8e^{15}z^4 - 8303067136a^9b^5c^9d^{11}e^{12}z^4 - 5588058112a^{13}b^3c^9d^7e^{16}z^4 - 3892838400a^8b^2c^{13}d^{16}e^7z^4 - 3611713536a^8b^8c^7d^{10}e^{13}z^4 + 7819006464a^7b^9c^7d^{11}e^{12}z^4 - 7782137856a^8b^7c^8d^{11}e^{12}z^4 + 7780433920a^{12}b^2c^9d^8e^{15}z^4 - 12020465664a^7b^5c^{11}d^{15}e^8z^4 + 3176792064a^8b^3c^{12}d^{15}e^8z^4 - 322633728a^{15}b^3c^7d^3e^{20}z^4 + 210829312a^7b^3c^{15}d^{19}e^4z^4 + 15623258112a^9b^6c^8d^{10}e^{13}z^4 + 25165824a^{15}b^3c^5d^5e^{22}z^4 - 15728640a^{14}b^5c^4d^5e^{22}z^4 + 12582912a^5b^2c^{16}d^{22}e^3z^4 - 11730944a^4b^4c^{15}d^{22}e^3z^4 + 5242880a^{13}b^7c^3d^5e^{22}z^4 - 4561920a^6b^{15}c^7d^{17}e^6z^4 + 4521984a^3b^6c^{14}d^{22}e^3z^4 + 4460544a^6b^{14}c^8d^{18}e^5z^4 + 3538944a^6b^6c^{16}d^{21}e^2z^4 + 3108864a^6b^{16}c^6d^{16}e^7z^4 - 3027200a^6b^{13}c^9d^{19}e^4z^4 - 2345472a^5b^{17}c^6d^7e^{16}z^4 - 2307072a^8b^{14}c^6d^4e^{19}z^4 + 1824768a^6b^{16}c^6d^6e^{17}z^4 + 1734912a^9b^{13}c^6d^3e^{20}z^4 + 1419264a^6b^{12}c^{10}d^{20}e^3z^4 - 1191168a^6b^{17}c^5d^{15}e^8z^4 - 983040a^{12}b^9c^2d^5e^2z^4 + 964608a^4b^{18}c^6d^8e^{15}z^4 - 866304a^2b^8c^{13}d^{22}e^3z^4 + 703488a^7b^{15}c^5d^5e^{18}z^4 - 608256a^{10}b^{12}c^6d^2e^{21}z^4 - 440832a^6b^{11}c^{11}d^{21}e^2z^4 + 275968a^6b^{19}c^3d^{13}e^{10}z^4 - 159744a^2b^{20}c^6d^{10}e^{13}z^4 - 153600a^6b^{20}c^2d^{12}e^{11}z^4 + 64512a^3b^{19}c^6d^9e^
\end{aligned}$$

$$\begin{aligned}
& 14z^4 + 19746062336a^8b^6c^9d^{12}e^{11}z^4 - 15333588992a^{10}b^4c^9d^{10}e^{13}z^4 + 6702170112a^7b^4c^{12}d^{16}e^7z^4 + 15167913984a^{10}b^3c^{10}d^{11}e^{12}z^4 - 2256638976a^5b^{11}c^7d^{13}e^{10}z^4 + 2254307328a^5b^7c^{11}d^{17}e^6z^4 - 2200633344a^6b^5c^{12}d^{17}e^6z^4 + 6457131008a^{11}b^3c^9d^9e^{14}z^4 - 2128785408a^5b^8c^{10}d^{16}e^7z^4 - 2126057472a^6b^{11}c^6d^{11}e^{12}z^4 + 2038349824a^{12}b^5c^6d^5e^{18}z^4 + 2037841920a^5b^{10}c^8d^{14}e^9z^4 + 3615621120a^9b^3c^{13}d^{15}e^8z^4 + 1900019712a^{11}b^2c^{10}d^{10}e^{13}z^4 + 1867698432a^9b^9c^5d^7e^{16}z^4 - 6157369344a^9b^4c^{10}d^{12}e^{11}z^4 - 1856913408a^7b^{10}c^6d^{10}e^{13}z^4 + 1789132800a^6b^4c^{13}d^{18}e^5z^4 + 6082658304a^8b^4c^{11}d^{14}e^9z^4 + 6029549568a^{11}b^5c^7d^7e^{16}z^4 + 6010159104a^6b^7c^{10}d^{15}e^8z^4 + 1703182336a^7b^7c^9d^{13}e^{10}z^4 + 1658388480a^{11}b^6c^6d^6e^{17}z^4 + 5917114368a^{10}b^6c^7d^8e^{15}z^4 - 1591197696a^{11}b^7c^5d^5e^{18}z^4 - 1526464512a^8b^{10}c^5d^8e^{15}z^4 - 5772607488a^{12}b^4c^7d^6e^{17}z^4 - 1423507456a^{13}b^4c^6d^4e^{19}z^4 - 1387266048a^7b^3c^{13}d^{17}e^6z^4 + 2976120832a^{10}b^3c^{12}d^{13}e^{10}z^4 - 9906946048a^9b^2c^{12}d^{14}e^9z^4 - 18421874688a^8b^5c^{10}d^{13}e^{10}z^4 + 1141217280a^6b^{12}c^5d^{10}e^{13}z^4 - 9714364416a^7b^8c^8d^{12}e^{11}z^4 - 16777216a^{16}b^3c^6d^6e^{22}z^4 + 98304a^{11}b^{11}c^5d^8e^{15}z^4 + 81920a^8b^{10}c^{12}d^{22}e^5z^4 + 39168a^8b^{21}c^4d^{11}e^{12}z^4 - 1091829760a^5b^6c^{12}d^{18}e^5z^4 + 1046740992a^{14}b^2c^7d^4e^{19}z^4 - 6884425728a^{12}b^3c^{10}d^9e^{14}z^4 + 987445248a^4b^{10}c^9d^{16}e^7z^4 + 984087552a^5b^{12}c^6d^{12}e^{11}z^4 - 9564585984a^9b^7c^7d^9e^{14}z^4 - 5266857984a^{10}b^7c^6d^7e^{16}z^4 - 892145664a^7b^{11}c^5d^9e^{14}z^4 - 2444623872a^{11}b^3c^{11}d^{11}e^{12}z^4 + 768540672a^{12}b^3c^8d^7e^{16}z^4 + 5048322048a^8b^9c^6d^9e^{14}z^4 + 5047612416a^6b^9c^8d^{13}e^{10}z^4 - 732492288a^4b^{11}c^8d^{15}e^8z^4 + 9266921472a^7b^6c^{10}d^{14}e^9z^4 - 645857280a^6b^6c^{11}d^{16}e^7z^4 - 623867904a^4b^9c^{10}d^{17}e^6z^4 - 622067712a^6b^3c^{14}d^{19}e^4z^4 + 582617088a^{10}b^8c^5d^6e^{17}z^4 + 577119744a^7b^{12}c^4d^8e^{15}z^4 + 552566784a^{12}b^6c^5d^4e^{19}z^4 + 549224448a^9b^8c^6d^8e^{15}z^4 - 526565376a^9b^{10}c^4d^6e^{17}z^4 + 511520256a^{10}b^9c^4d^5e^{18}z^4 + 13393723392a^9b^3c^{11}d^{13}e^{10}z^4 - 2066350080a^{14}b^3c^8d^5e^{18}z^4 + 4718592000a^{13}b^2c^8d^6e^{17}z^4 - 314572800a^7b^2c^{14}d^{18}e^5z^4 + 287250432a^4b^{13}c^6d^{13}e^{10}z^4 + 4565827584a^{10}b^5c^8d^9e^{14}z^4 - 250785792a^4b^{14}c^5d^{12}e^{11}z^4 + 235536384a^{13}b^3c^7d^5e^{18}z^4 - 232683264a^8b^{11}c^4d^7e^{16}z^4 - 199627776a^5b^{14}c^4d^{10}e^{13}z^4 - 190267392a^{12}b^7c^4d^3e^{20}z^4 + 184891392a^6b^{10}c^7d^{12}e^{11}z^4 + 180502528a^4b^7c^{12}d^{19}e^4z^4 + 178877952a^3b^{13}c^7d^{15}e^8z^4 + 172490752a^{14}b^3c^6d^3e^{20}z^4 + 163946496a^{13}b^5c^5d^3e^{20}z^4 + 155839488a^8b^{12}c^3d^6e^{17}z^4 + 155000832a^5b^5c^{13}d^{19}e^4z^4 - 152076288a^4b^6c^{13}d^{20}e^3z^4 - 137592576a^3b^{12}c^8d^{16}e^7z^4 - 133693440a^{14}b^4c^5d^2e^{21}z^4 - 116767488a^3b^9c^{11}d^{19}e^4z^4 - 108985344a^3b^{14}c^6d^{14}e^9z^4 - 106223616a^6b^{13}c^4d^9e^{14}z^4 + 106119168a^3b^{10}c^{10}d^{18}e^5z^4 + 102432768a^5b^4c^{14}d^{20}e^3z^4 + 102113280a^4b^{12}c^7d^{14}e^9z^4 + 100674048a^5b^9c^9d^{15}e^8z^4 + 90439680a^{13}b^6c^4d^2e^{21}z^4 - 86808576a^6b^{14}c^3d^8e^{15}z^4 + 86245376a^6b^2c^{15}d^{20}e^3z^4 + 79011840a^4b^8c^{11}d^{18}e^5z^4 + 78345216a^4b^{15}c^4d^{11}e^{12}z^4 + 78006528a^{11}b^9c^3d^3e^{20}z^4 - 73253376a^9b^{11}c^3d^5e^{18}z^4 + 67524608a^3b^8c^{12}d^{20}e^3z^4 + 67108864a^{15}b^2c^6d^2e^{21}z^4 - 61590528a^{10}b^{10}c^3d^4e^{19}z^4 + 61559808a^5b^{15}c^3d^9e^{14}z^4 - 59637760a^5b^3c^{15}d^{21}e^2z^4 + 58638336a^4b^5c^{14}d^{21}e^2z^4 - 40828416a^7b^{13}c^3d^7e^{16}z^4 - 35639296a^2b^{12}c^9d^{18}e^5z^4 - 31293440a^{12}b^8c^3d^2e^{21}z^4 + 29933568a^5b^{13}c^5d^{11}e^{12}z^4 + 27793920a^2b^{11}c^{10}d^{19}e^4z^4 + 27168768a^2b^{13}c^8d^{17}e^6z^4 - 23602176a^7b^{14}c^2d^6e^{17}z^4 - 23248896a^3b^7c^{13}d^{21}e^2z^4 + 20929536a^3b^{15}c^5d^{13}e^{10}z^4 + 18428928a^9b^{12}c^2d^4e^{19}z^4 + 18026496a^6b^{15}c^2d^7e^{16}z^4 - 16261632a^{10}b^{11}c^2d^3e^{20}z^4 + 15128064a^3b^{16}c^4d^{12}e^{11}z^4 - 14060544a^2b^{10}c^{11}d^{20}e^3z^4
\end{aligned}$$

$$\begin{aligned}
& *z^4 + 13178880*a^2*b^16*c^5*d^14*e^9*z^4 - 11244288*a^3*b^17*c^3*d^11*e^12 \\
& *z^4 - 10509312*a^2*b^15*c^6*d^15*e^8*z^4 - 7262208*a^4*b^17*c^2*d^9*e^14*z \\
& ^4 - 7045632*a^2*b^17*c^4*d^13*e^10*z^4 - 6285312*a^2*b^14*c^7*d^16*e^7*z^4 \\
& + 5996544*a^11*b^10*c^2*d^2*e^21*z^4 + 4558336*a^2*b^9*c^12*d^21*e^2*z^4 + \\
& 4478976*a^11*b^8*c^4*d^4*e^19*z^4 + 2850816*a^4*b^16*c^3*d^10*e^13*z^4 + 2 \\
& 629632*a^3*b^11*c^9*d^17*e^6*z^4 + 2503680*a^3*b^18*c^2*d^10*e^13*z^4 + 162 \\
& 7136*a^2*b^18*c^3*d^12*e^11*z^4 + 1605120*a^8*b^13*c^2*d^5*e^18*z^4 + 14837 \\
& 76*a^5*b^16*c^2*d^8*e^15*z^4 + 139776*a^2*b^19*c^2*d^11*e^12*z^4 - 85422243 \\
& 84*a^10*b^2*c^11*d^12*e^11*z^4 - 3072*b^22*c*d^12*e^11*z^4 - 3072*b^12*c^11 \\
& *d^22*e*z^4 - 1572864*a^6*c^17*d^22*e*z^4 - 4096*a^10*b^13*d*e^22*z^4 - 409 \\
& 6*a*b^22*d^10*e^13*z^4 - 6144*a^12*b^10*c*e^23*z^4 - 983040*a^5*b*c^17*d^23 \\
& *z^4 - 6912*a*b^9*c^13*d^23*z^4 + 1824522240*a^13*c^10*d^8*e^15*z^4 + 17301 \\
& 50400*a^12*c^11*d^10*e^13*z^4 + 958922752*a^14*c^9*d^6*e^17*z^4 - 537919488 \\
& *a^9*c^14*d^16*e^7*z^4 + 508559360*a^11*c^12*d^12*e^11*z^4 - 500170752*a^10 \\
& *c^13*d^14*e^9*z^4 + 246939648*a^15*c^8*d^4*e^19*z^4 - 199229440*a^8*c^15*d \\
& ^18*e^5*z^4 - 29884416*a^7*c^16*d^20*e^3*z^4 + 25165824*a^16*c^7*d^2*e^21*z \\
& ^4 + 236544*b^17*c^6*d^17*e^6*z^4 - 202752*b^18*c^5*d^16*e^7*z^4 - 202752*b \\
& ^16*c^7*d^18*e^5*z^4 + 126720*b^19*c^4*d^15*e^8*z^4 + 126720*b^15*c^8*d^19* \\
& e^4*z^4 - 56320*b^20*c^3*d^14*e^9*z^4 - 56320*b^14*c^9*d^20*e^3*z^4 + 16896 \\
& *b^21*c^2*d^13*e^10*z^4 + 16896*b^13*c^10*d^21*e^2*z^4 + 110080*a^7*b^16*d^ \\
& 4*e^19*z^4 + 110080*a^4*b^19*d^7*e^16*z^4 - 75520*a^8*b^15*d^3*e^20*z^4 - 7 \\
& 5520*a^3*b^20*d^8*e^15*z^4 - 56320*a^6*b^17*d^5*e^18*z^4 - 56320*a^5*b^18*d \\
& ^6*e^17*z^4 + 25600*a^9*b^14*d^2*e^21*z^4 + 25600*a^2*b^21*d^9*e^14*z^4 - 1 \\
& 572864*a^16*b^2*c^5*e^23*z^4 + 983040*a^15*b^4*c^4*e^23*z^4 - 327680*a^14*b \\
& ^6*c^3*e^23*z^4 + 61440*a^13*b^8*c^2*e^23*z^4 + 983040*a^4*b^3*c^16*d^23*z^ \\
& 4 - 385024*a^3*b^5*c^15*d^23*z^4 + 73728*a^2*b^7*c^14*d^23*z^4 + 256*b^23*d \\
& ^11*e^12*z^4 + 1048576*a^17*c^6*e^23*z^4 + 256*b^11*c^12*d^23*z^4 + 256*a^1 \\
& 1*b^12*e^23*z^4 + 948695040*a^8*b*c^10*d^6*e^13*z^2 + 348917760*a^7*b*c^11* \\
& d^8*e^11*z^2 - 125030400*a^9*b*c^9*d^4*e^15*z^2 - 50728960*a^6*b*c^12*d^10* \\
& e^9*z^2 - 44298240*a^5*b*c^13*d^12*e^7*z^2 - 36495360*a^10*b*c^8*d^2*e^17*z \\
& ^2 + 29675520*a^8*b^6*c^5*d*e^18*z^2 - 24170496*a^9*b^4*c^6*d*e^18*z^2 - 17 \\
& 202816*a^7*b^8*c^4*d*e^18*z^2 - 14561280*a^4*b*c^14*d^14*e^5*z^2 + 5532416* \\
& a^6*b^10*c^3*d*e^18*z^2 + 4128768*a^10*b^2*c^7*d*e^18*z^2 - 2662400*a^3*b*c \\
& ^15*d^16*e^3*z^2 + 1184512*a*b^12*c^6*d^9*e^10*z^2 - 1136160*a*b^13*c^5*d^8 \\
& *e^11*z^2 - 1017600*a^5*b^12*c^2*d*e^18*z^2 - 744768*a*b^11*c^7*d^10*e^9*z^ \\
& 2 + 607872*a*b^14*c^4*d^7*e^12*z^2 - 424064*a*b^6*c^12*d^15*e^4*z^2 + 40857 \\
& 6*a*b^5*c^13*d^16*e^3*z^2 + 361152*a*b^10*c^8*d^11*e^8*z^2 - 287408*a*b^9*c \\
& ^9*d^12*e^7*z^2 - 260448*a^3*b^15*c*d^2*e^17*z^2 - 203904*a*b^4*c^14*d^17*e \\
& ^2*z^2 + 200832*a*b^8*c^10*d^13*e^6*z^2 + 126720*a*b^7*c^11*d^14*e^5*z^2 - \\
& 123968*a*b^15*c^3*d^6*e^13*z^2 - 39168*a*b^16*c^2*d^5*e^14*z^2 + 11904*a^2* \\
& b^16*c*d^3*e^16*z^2 + 1824135552*a^7*b^4*c^8*d^5*e^14*z^2 - 1457252352*a^8* \\
& b^2*c^9*d^5*e^14*z^2 - 1405209600*a^7*b^5*c^7*d^4*e^15*z^2 - 184320*a^2*b*c \\
& ^16*d^18*e*z^2 + 100608*a^4*b^14*c*d*e^18*z^2 + 53248*a*b^3*c^15*d^18*e*z^2 \\
& + 26448*a*b^17*c*d^4*e^15*z^2 + 1067599872*a^8*b^3*c^8*d^4*e^15*z^2 - 9308 \\
& 28288*a^7*b^3*c^9*d^6*e^13*z^2 + 920760000*a^6*b^4*c^9*d^7*e^12*z^2 - 80663 \\
& 9616*a^6*b^3*c^10*d^8*e^11*z^2 - 791052480*a^6*b^6*c^7*d^5*e^14*z^2 + 77223 \\
& 7824*a^6*b^7*c^6*d^4*e^15*z^2 - 701025408*a^5*b^6*c^8*d^7*e^12*z^2 + 443340 \\
& 288*a^5*b^5*c^9*d^8*e^11*z^2 + 433047552*a^7*b^6*c^6*d^3*e^16*z^2 + 4057413 \\
& 12*a^5*b^7*c^7*d^6*e^13*z^2 + 293652480*a^6*b^2*c^11*d^9*e^10*z^2 - 2769626 \\
& 88*a^6*b^8*c^5*d^3*e^16*z^2 - 247804272*a^8*b^4*c^7*d^3*e^16*z^2 + 21356438 \\
& 4*a^4*b^8*c^7*d^7*e^12*z^2 - 202596816*a^5*b^9*c^5*d^4*e^15*z^2 - 182520896 \\
& *a^4*b^9*c^6*d^6*e^13*z^2 - 153489408*a^5*b^3*c^11*d^10*e^9*z^2 - 152151552 \\
& *a^7*b^2*c^10*d^7*e^12*z^2 + 115859712*a^5*b^2*c^12*d^11*e^8*z^2 + 10808524 \\
& 8*a^9*b^3*c^7*d^2*e^17*z^2 + 105536256*a^4*b^5*c^10*d^10*e^9*z^2 - 98323200 \\
& *a^6*b^5*c^8*d^6*e^13*z^2 - 93564992*a^4*b^6*c^9*d^9*e^10*z^2 + 89464512*a^ \\
& 5*b^10*c^4*d^3*e^16*z^2 - 75930624*a^8*b^5*c^6*d^2*e^17*z^2 + 68315904*a^5* \\
& b^8*c^6*d^5*e^14*z^2 - 64157184*a^4*b^7*c^8*d^8*e^11*z^2 - 62951040*a^9*b^2 \\
& *c^8*d^3*e^16*z^2 + 49056768*a^4*b^10*c^5*d^5*e^14*z^2 + 47614464*a^3*b^8*c \\
& ^8*d^9*e^10*z^2 + 35604480*a^4*b^2*c^13*d^13*e^6*z^2 + 33983040*a^3*b^11*c^
\end{aligned}$$

$$\begin{aligned}
& 5*d^6*e^{13*z^2} - 33515520*a^4*b^3*c^{12}*d^{12}*e^{7*z^2} - 33463808*a^3*b^7*c^9*d^{10}*e^{9*z^2} - 25128864*a^4*b^4*c^{11}*d^{11}*e^{8*z^2} - 23193728*a^3*b^{10}*c^6*d^{7}*e^{12*z^2} + 21015456*a^6*b^9*c^4*d^2*e^{17*z^2} + 19924176*a^4*b^{11}*c^4*d^4*e^{15*z^2} - 19251216*a^3*b^9*c^7*d^8*e^{11*z^2} - 16434048*a^5*b^4*c^{10}*d^9*e^{10*z^2} - 16289664*a^3*b^{12}*c^4*d^5*e^{14*z^2} - 15059328*a^4*b^{12}*c^3*d^3*e^{16*z^2} - 10766016*a^2*b^{10}*c^7*d^9*e^{10*z^2} - 10453632*a^5*b^{11}*c^3*d^2*e^{17*z^2} - 9940992*a^3*b^3*c^{13}*d^{14}*e^5*z^2 + 8373696*a^2*b^{11}*c^6*d^8*e^{11*z^2} + 7776768*a^3*b^2*c^{14}*d^{15}*e^4*z^2 + 7077888*a^3*b^5*c^{11}*d^{12}*e^7*z^2 + 6798240*a^2*b^9*c^8*d^{10}*e^9*z^2 - 3589440*a^2*b^6*c^{11}*d^{13}*e^6*z^2 + 3544320*a^3*b^6*c^{10}*d^{11}*e^8*z^2 + 3128064*a^2*b^5*c^{12}*d^{14}*e^5*z^2 + 2346336*a^4*b^{13}*c^2*d^2*e^{17*z^2} - 2261568*a^2*b^8*c^9*d^{11}*e^8*z^2 - 2125824*a^2*b^{13}*c^4*d^6*e^{13*z^2} + 2002560*a^3*b^4*c^{12}*d^{13}*e^6*z^2 + 1927680*a^2*b^7*c^{10}*d^{12}*e^7*z^2 + 1814784*a^2*b^{14}*c^3*d^5*e^{14*z^2} - 1807104*a^2*b^{12}*c^5*d^7*e^{12*z^2} + 1637808*a^3*b^{13}*c^3*d^4*e^{15*z^2} + 1083456*a^3*b^{14}*c^2*d^3*e^{16*z^2} - 792384*a^2*b^4*c^{13}*d^{15}*e^4*z^2 - 657408*a^2*b^3*c^{14}*d^{16}*e^3*z^2 + 608256*a^7*b^7*c^5*d^2*e^{17*z^2} + 595968*a^2*b^2*c^{15}*d^{17}*e^2*z^2 - 498624*a^2*b^{15}*c^2*d^4*e^{15*z^2} - 3840*b^{18}*c*d^5*e^{14*z^2} - 3840*b^5*c^{14}*d^{18}*e*z^2 + 2064384*a^{11}*c^8*d*e^{18*z^2} - 4160*a^3*b^{16}*d*e^{18*z^2} - 4160*a*b^{18}*d^3*e^{16*z^2} - 1290240*a^{11}*b*c^7*e^{19*z^2} - 9840*a^5*b^{13}*c*e^{19*z^2} - 5760*a*b^2*c^{16}*d^{19*z^2} - 280581120*a^8*c^{11}*d^7*e^{12*z^2} + 110278656*a^9*c^{10}*d^5*e^{14*z^2} - 89479168*a^7*c^{12}*d^9*e^{10*z^2} + 34464000*a^{10}*c^9*d^3*e^{16*z^2} + 54240*b^{15}*c^4*d^8*e^{11*z^2} + 54240*b^8*c^{11}*d^{15}*e^4*z^2 - 49920*b^{14}*c^5*d^9*e^{10*z^2} - 49920*b^9*c^{10}*d^{14}*e^5*z^2 - 37376*b^{16}*c^3*d^7*e^{12*z^2} - 37376*b^7*c^{12}*d^{16}*e^3*z^2 + 28480*b^{13}*c^6*d^{10}*e^9*z^2 + 28480*b^{10}*c^9*d^{13}*e^6*z^2 + 15936*b^{17}*c^2*d^6*e^{13*z^2} + 15936*b^6*c^{13}*d^{17}*e^2*z^2 - 7920*b^{12}*c^7*d^{11}*e^8*z^2 - 7920*b^{11}*c^8*d^{12}*e^7*z^2 + 7489536*a^5*c^{14}*d^{13}*e^6*z^2 + 6084096*a^6*c^{13}*d^{11}*e^8*z^2 + 2280448*a^4*c^{15}*d^{15}*e^4*z^2 + 350208*a^3*c^{16}*d^{17}*e^2*z^2 + 11616*a^2*b^{17}*d^2*e^{17*z^2} - 3515904*a^9*b^5*c^5*e^{19*z^2} + 3440640*a^{10}*b^3*c^6*e^{19*z^2} + 1870848*a^8*b^7*c^4*e^{19*z^2} - 572272*a^7*b^9*c^3*e^{19*z^2} + 101856*a^6*b^{11}*c^2*e^{19*z^2} + 400*b^{19}*d^4*e^{15*z^2} + 400*b^4*c^{15}*d^{19*z^2} + 20736*a^2*c^{17}*d^{19*z^2} + 400*a^4*b^{15}*e^{19*z^2} - 3969216*a^4*b*c^{10}*d^3*e^{12} - 3001536*a^3*b*c^{11}*d^5*e^{10} - 419904*a^2*b*c^{12}*d^7*e^8 + 184608*a^4*b^3*c^8*d*e^{14} - 153036*a*b^4*c^{10}*d^6*e^9 + 127008*a*b^3*c^{11}*d^7*e^8 + 63108*a*b^6*c^8*d^4*e^{11} - 29160*a*b^2*c^{12}*d^8*e^7 - 21060*a^3*b^5*c^7*d*e^{14} - 21060*a*b^7*c^7*d^3*e^{12} + 5460*a*b^5*c^9*d^5*e^{10} - 404544*a^5*b*c^9*d*e^{14} + 1251872*a^3*b^3*c^9*d^3*e^{12} + 844224*a^4*b^2*c^9*d^2*e^{13} + 820512*a^2*b^3*c^{10}*d^5*e^{10} + 750672*a^3*b^2*c^{10}*d^4*e^{11} - 657498*a^2*b^4*c^9*d^4*e^{11} - 487116*a^3*b^4*c^8*d^2*e^{13} + 160704*a^2*b^2*c^{11}*d^6*e^9 + 58806*a^2*b^6*c^7*d^2*e^{13} + 13140*a^2*b^5*c^8*d^3*e^{12} + 15286*b^6*c^9*d^6*e^9 - 9540*b^7*c^8*d^5*e^{10} - 9540*b^5*c^{10}*d^7*e^8 + 2025*b^8*c^7*d^4*e^{11} + 2025*b^4*c^{11}*d^8*e^7 + 3367008*a^4*c^{11}*d^4*e^{11} + 1166400*a^3*c^{12}*d^6*e^9 + 705600*a^5*c^{10}*d^2*e^{13} + 104976*a^2*c^{13}*d^8*e^7 - 17640*a^5*b^2*c^8*e^{15} + 2025*a^4*b^4*c^7*e^{15} + 38416*a^6*c^9*e^{15}, z, k)*(root(128723189760*a^{14}*b^4*c^9*d^{13}*e^{14*z^6} + 128723189760*a^{12}*b^4*c^{11}*d^{17}*e^{10*z^6} - 8432455680*a^{11}*b^{12}*c^4*d^{11}*e^{16*z^6} - 8432455680*a^7*b^{12}*c^8*d^{19}*e^8*z^6 + 12673351680*a^{11}*b^{11}*c^5*d^{12}*e^{15*z^6} + 12673351680*a^8*b^{11}*c^8*d^{18}*e^9*z^6 - 72637480960*a^{12}*b^9*c^6*d^{12}*e^{15*z^6} - 72637480960*a^9*b^9*c^9*d^{18}*e^9*z^6 - 21048344576*a^9*b^{12}*c^6*d^{15}*e^{12*z^6} - 16609443840*a^{17}*b^3*c^7*d^8*e^{19*z^6} - 16609443840*a^{10}*b^3*c^{14}*d^{22}*e^5*z^6 + 145332633600*a^{13}*b^5*c^9*d^{14}*e^{13*z^6} + 145332633600*a^{12}*b^5*c^{10}*d^{16}*e^{11*z^6} + 123740356608*a^{14}*b^5*c^8*d^{12}*e^{15*z^6} + 123740356608*a^{11}*b^5*c^{11}*d^{18}*e^9*z^6 + 3460300800*a^{17}*b^5*c^5*d^6*e^{21*z^6} + 3460300800*a^8*b^5*c^{14}*d^{24}*e^3*z^6 - 7751073792*a^{15}*b^7*c^5*d^8*e^{19*z^6} - 7751073792*a^8*b^7*c^{12}*d^{22}*e^5*z^6 + 12041846784*a^{14}*b^7*c^6*d^{10}*e^{17*z^6} + 12041846784*a^9*b^7*c^{11}*d^{20}*e^7*z^6 - 325545099264*a^{14}*b^3*c^{10}*d^{14}*e^{13*z^6} - 325545099264*a^{13}*b^3*c^{11}*d^{16}*e^{11*z^6} - 3330539520*a^{13}*b^{10}*c^4*d^9*e^{18*z^6} - 3330539520*a^7*b^{10}*c^{10}*d^{21}*e^6*z^6 + 157789716480*a^{12}*b^7*c^8*d^{14}*e^{13*z^6} + 157789716480*a^{11}*b^7*c^9*d^{16}*e^{11*z^6} + 37492359168*a^{11}*b^{10}*c^6*d^{13}*e^{14*z^6} +
\end{aligned}$$

$$\begin{aligned}
& 37492359168a^9b^{10}c^8d^{17}e^{10}z^6 + 301989888a^8b^3c^{16}d^{26}e^*z^6 \\
& - 7266631680a^{17}b^4c^6d^7e^{20}z^6 - 7266631680a^9b^4c^{14}d^{23}e^4z^6 \\
& - 201326592a^{20}b^*c^6d^4e^{23}z^6 - 188743680a^7b^5c^{15}d^{26}e^*z^6 \\
& + 45747339264a^{13}b^8c^6d^{11}e^{16}z^6 + 45747339264a^9b^8c^{10}d^{19}e^8z^6 \\
& - 74612736a^{10}b^{16}c^*d^9e^{18}z^6 - 2768240640a^{16}b^7c^4d^6e^21z^6 \\
& - 2768240640a^7b^7c^{13}d^{24}e^3z^6 + 69746688a^{11}b^{15}c^*d^8e^{19}z^6 \\
& + 62914560a^6b^7c^{14}d^{26}e^*z^6 + 2752020480a^{10}b^{13}c^4d^{12}e^{15}z^6 \\
& + 2752020480a^7b^{13}c^7d^{18}e^9z^6 + 55148544a^9b^{17}c^*d^{10}e^{17}z^6 \\
& - 45957120a^{12}b^{14}c^*d^7e^{20}z^6 - 2724986880a^{14}b^9c^4d^8e^{19}z^6 \\
& - 2724986880a^7b^9c^{11}d^{22}e^5z^6 - 25952256a^8b^{18}c^*d^{11}e^{16}z^6 \\
& + 21086208a^{13}b^{13}c^*d^6e^{21}z^6 - 11796480a^5b^9c^{13}d^{26}e^*z^6 \\
& - 6438912a^{14}b^{12}c^*d^5e^{22}z^6 + 5406720a^7b^{19}c^*d^{12}e^{15}z^6 + \\
& 1622016a^6b^{20}c^*d^{13}e^{14}z^6 - 1523712a^5b^{21}c^*d^{14}e^{13}z^6 + 1179648 \\
& a^{15}b^{11}c^*d^4e^{23}z^6 + 1179648a^4b^{11}c^{12}d^{26}e^*z^6 + 442368a^4b^22c^*d^{15}e^{12}z^6 \\
& - 98304a^{16}b^{10}c^*d^3e^{24}z^6 - 49152a^3b^{23}c^*d^{16}e^{11}z^6 - 49152a^3b^{13}c^{11}d^{26}e^*z^6 \\
& + 6897106944a^9b^{13}c^5d^{14}e^{13}z^6 + 6897106944a^8b^{13}c^6d^{16}e^{11}z^6 \\
& - 2422210560a^{16}b^6c^5d^7e^{20}z^6 - 2422210560a^8b^6c^{13}d^{23}e^4z^6 \\
& + 255785435136a^{14}b^2c^{11}d^{15}e^{12}z^6 + 41004564480a^{15}b^4c^8d^{11}e^{16}z^6 \\
& + 41004564480a^{11}b^4c^{12}d^{19}e^8z^6 + 2270822400a^{13}b^{11}c^3d^8e^{19}z^6 + 2270822400 \\
& a^6b^{11}c^{10}d^{22}e^5z^6 + 23677108224a^{14}b^8c^5d^9e^{18}z^6 + 23677108224a^8b^8c^{11}d^{21}e^6z^6 \\
& + 212600881152a^{15}b^2c^{10}d^{13}e^{14}z^6 + 212600881152a^{13}b^2c^{12}d^{17}e^{10}z^6 \\
& + 75157733376a^{15}b^5c^7d^{10}e^{17}z^6 + 75157733376a^{10}b^5c^{12}d^{20}e^7z^6 - 251217838080a^13b^6c^8d^{13}e^{14}z^6 \\
& - 251217838080a^{11}b^6c^{10}d^{17}e^{10}z^6 - 1952907264a^{14}b^{10}c^3d^7e^{20}z^6 \\
& - 1952907264a^6b^{10}c^{11}d^{23}e^4z^6 - 27691057152a^{13}b^9c^5d^{10}e^{17}z^6 \\
& - 27691057152a^8b^9c^{10}d^{20}e^7z^6 - 1902673920a^8b^{15}c^4d^{14}e^{13}z^6 \\
& - 1902673920a^7b^{15}c^5d^{16}e^{11}z^6 + 10465050624a^{10}b^{11}c^6d^{14}e^{13}z^6 \\
& + 10465050624a^9b^{11}c^7d^{16}e^{11}z^6 + 1613905920a^9b^{14}c^4d^{13}e^{14}z^6 \\
& + 1613905920a^7b^{14}c^6d^{17}e^{10}z^6 - 33218887680a^{17}b^*c^9d^{10}e^{17}z^6 \\
& - 33218887680a^{12}b^*c^{14}d^{20}e^7z^6 + 1524695040a^{10}b^{14}c^3d^{11}e^{16}z^6 \\
& + 1524695040a^6b^{14}c^7d^{19}e^8z^6 - 1472200704a^{18}b^4c^5d^5e^{22}z^6 \\
& - 1472200704a^8b^4c^{15}d^{25}e^2z^6 - 83047219200a^{16}b^3c^8d^{10}e^{17}z^6 \\
& - 83047219200a^{11}b^3c^{13}d^{20}e^7z^6 + 44291850240a^{17}b^2c^8d^9e^{18}z^6 \\
& + 44291850240a^{11}b^2c^{14}d^{21}e^6z^6 + 1308131328a^8b^{14}c^5d^{15}e^{12}z^6 \\
& - 201326592a^9b^*c^{17}d^{26}e^*z^6 + 48530718720a^{12}b^8c^7d^{13}e^{14}z^6 \\
& + 48530718720a^{10}b^8c^9d^{17}e^{10}z^6 - 1242644480a^{12}b^{12}c^3d^9e^{18}z^6 \\
& - 1242644480a^6b^{12}c^9d^{21}e^6z^6 + 9813196800a^{12}b^{10}c^5d^{11}e^{16}z^6 \\
& + 9813196800a^8b^{10}c^9d^{19}e^8z^6 - 93012885504a^{15}b^*c^{11}d^{14}e^{13}z^6 \\
& - 93012885504a^{14}b^*c^{12}d^{16}e^{11}z^6 + 177305812992a^{13}b^4c^{10}d^{15}e^{12}z^6 \\
& + 52730658816a^{10}b^{10}c^7d^{15}e^{12}z^6 - 1180106752a^9b^{15}c^3d^{12}e^{15}z^6 \\
& - 1180106752a^6b^{15}c^6d^{18}e^9z^6 + 1023672320a^{15}b^9c^3d^6e^{21}z^6 \\
& + 1023672320a^6b^9c^{12}d^{24}e^3z^6 + 975175680a^{17}b^6c^4d^5e^{22}z^6 \\
& + 975175680a^7b^6c^{14}d^{25}e^2z^6 - 11072962560a^{18}b^*c^8d^8e^{19}z^6 \\
& - 11072962560a^{11}b^*c^{15}d^22e^5z^6 + 9412018176a^{18}b^2c^7d^7e^{20}z^6 \\
& + 9412018176a^{10}b^2c^{15}d^{23}e^4z^6 + 805306368a^{19}b^2c^6d^5e^{22}z^6 \\
& + 805306368a^9b^2c^{16}d^{25}e^2z^6 - 133809831936a^{14}b^6c^7d^{11}e^{16}z^6 \\
& - 133809831936a^{10}b^6c^{11}d^{19}e^8z^6 - 2214592512a^{19}b^*c^7d^6e^{21}z^6 \\
& - 2214592512a^{10}b^*c^{16}d^{24}e^3z^6 + 82216747008a^{13}b^7c^7d^{12}e^{15}z^6 \\
& + 82216747008a^{10}b^7c^{10}d^{18}e^9z^6 - 586629120a^{12}b^{13}c^2d^8e^{19}z^6 \\
& - 586629120a^5b^{13}c^9d^{22}e^5z^6 + 568565760a^7b^{16}c^4d^{15}e^{12}z^6 \\
& - 4844421120a^{16}b^4c^7d^9e^{18}z^6 - 4844421120a^{10}b^4c^{13}d^{21}e^6z^6 \\
& + 531210240a^{11}b^{14}c^2d^9e^{18}z^6 + 531210240a^5b^{14}c^8d^{21}e^6z^6 \\
& - 527155200a^{11}b^{13}c^3d^{10}e^{17}z^6 - 527155200a^6b^{13}c^8d^{20}e^7z^6 \\
& + 43470028800a^{11}b^8c^8d^{15}e^{12}z^6 - 107874877440a^{11}b^9c^7d^{14}e^{13}z^6 \\
& - 107874877440a^{10}b^9c^8d^{16}e^{11}z^6 + 9018408960a^{12}b^{11}c^4d^{10}e^{17}z^6 \\
& + 9018408960a^7b^{11}c^9d^{20}e^7z^6 + 421994496a^
\end{aligned}$$



$$\begin{aligned}
& 13b^{12}c^2d^7e^{20}z^6 + 421994496a^5b^{12}c^{10}d^{23}e^4z^6 - 664377753 \\
& 60a^{16}b^6c^{10}d^{12}e^{15}z^6 - 66437775360a^{13}b^6c^{13}d^{18}e^9z^6 + 26159 \\
& 874048a^{16}b^5c^6d^8e^{19}z^6 + 26159874048a^9b^5c^{13}d^{22}e^5z^6 - \\
& 369098752a^{18}b^3c^6d^6e^{21}z^6 - 369098752a^9b^3c^{15}d^{24}e^3z^6 + \\
& 351436800a^8b^{16}c^3d^{13}e^{14}z^6 + 351436800a^6b^{16}c^5d^{17}e^{10}z^6 \\
& 6 - 334233600a^{16}b^8c^3d^5e^{22}z^6 - 334233600a^6b^8c^{13}d^{25}e^2z^6 \\
& + 301989888a^{19}b^3c^5d^4e^{23}z^6 - 266010624a^{10}b^{15}c^2d^{10}e^{17}z^6 - 266010624a^5b^{15}c^7d^{20}e^7z^6 - 305198530560a^{12}b^6c^9d^{15}e^{12}z^6 - 203292672a^{14}b^{11}c^2d^6e^{21}z^6 - 203292672a^5b^{11}c^{11}d^{24}e^3z^6 - 188743680a^{18}b^5c^4d^4e^{23}z^6 + 120418467840a^{16}b^2c^9d^{11}e^{16}z^6 + 120418467840a^{12}b^2c^{13}d^{19}e^8z^6 - 17293934592a^{10}b^{12}c^5d^{13}e^{14}z^6 - 17293934592a^8b^{12}c^7d^{17}e^{10}z^6 + 104890368a^8b^{17}c^2d^{12}e^{15}z^6 + 104890368a^5b^{17}c^5d^{18}e^9z^6 + 4390256640a^{15}b^8c^4d^7e^{20}z^6 + 4390256640a^7b^8c^{12}d^{23}e^4z^6 - 91750400a^6b^{18}c^3d^{15}e^{12}z^6 + 79134720a^7b^{17}c^3d^{14}e^{13}z^6 + 79134720a^6b^{17}c^4d^{16}e^{11}z^6 - 74612736a^4b^{16}c^7d^{21}e^6z^6 - 72990720a^7b^{18}c^2d^{13}e^{14}z^6 - 72990720a^5b^{18}c^4d^{17}e^{10}z^6 + 69746688a^4b^{15}c^8d^{22}e^5z^6 + 63700992a^{15}b^{10}c^2d^5e^{22}z^6 + 63700992a^5b^{10}c^{12}d^{25}e^2z^6 + 62914560a^{17}b^7c^3d^4e^{23}z^6 + 55148544a^4b^{17}c^6d^{20}e^7z^6 - 45957120a^4b^{14}c^9d^{23}e^4z^6 - 25952256a^4b^{18}c^5d^{19}e^8z^6 - 25165824a^{20}b^2c^5d^3e^{24}z^6 + 21086208a^4b^{13}c^{10}d^{24}e^3z^6 + 20643840a^6b^{19}c^2d^{14}e^{13}z^6 + 20643840a^5b^{19}c^3d^{16}e^{11}z^6 + 15728640a^{19}b^4c^4d^3e^{24}z^6 - 11796480a^{16}b^9c^2d^4e^{23}z^6 - 6438912a^4b^{12}c^{11}d^{25}e^2z^6 + 5406720a^4b^{19}c^4d^{18}e^9z^6 - 5242880a^{18}b^6c^3d^3e^{24}z^6 + 3784704a^3b^{18}c^6d^{21}e^6z^6 - 3244032a^3b^{19}c^5d^{20}e^7z^6 - 3244032a^3b^{17}c^7d^{22}e^5z^6 + 2027520a^3b^{20}c^4d^{19}e^8z^6 + 2027520a^3b^{16}c^8d^{23}e^4z^6 - 1622016a^9b^{16}c^2d^{11}e^{16}z^6 - 1622016a^5b^{16}c^6d^{19}e^8z^6 + 1622016a^4b^{20}c^3d^{17}e^{10}z^6 - 1523712a^4b^{21}c^2d^{16}e^{11}z^6 + 983040a^{17}b^8c^2d^3e^{24}z^6 - 901120a^3b^{21}c^3d^{18}e^9z^6 - 901120a^3b^{15}c^9d^{24}e^3z^6 + 270336a^3b^{22}c^2d^{17}e^{10}z^6 + 270336a^3b^{14}c^{10}d^{25}e^2z^6 + 172032a^5b^{20}c^2d^{15}e^{12}z^6 - 38593888256a^{15}b^6c^6d^9e^{18}z^6 - 38593888256a^9b^6c^{12}d^{21}e^6z^6 - 210386288640a^{15}b^3c^9d^{12}e^{15}z^6 - 210386288640a^{12}b^3c^{12}d^{18}e^9z^6 + 15502147584a^{15}c^{12}d^{15}e^{12}z^6 + 1107296256a^{19}c^8d^7e^{20}z^6 + 1107296256a^{11}c^{16}d^{23}e^4z^6 + 13287555072a^{16}c^{11}d^{13}e^{14}z^6 + 13287555072a^{14}c^{13}d^{17}e^{10}z^6 + 201326592a^{20}c^7d^5e^{22}z^6 + 201326592a^{10}c^{17}d^{25}e^2z^6 + 16777216a^{21}c^6d^3e^{24}z^6 + 3784704a^9b^{18}d^9e^{18}z^6 - 3244032a^{10}b^{17}d^8e^{19}z^6 - 3244032a^8b^{19}d^{10}e^{17}z^6 + 2027520a^{11}b^{16}d^7e^{20}z^6 + 2027520a^7b^{20}d^{11}e^{16}z^6 - 901120a^{12}b^{15}d^6e^{21}z^6 - 901120a^6b^{21}d^{12}e^{15}z^6 + 270336a^{13}b^{14}d^5e^{22}z^6 + 270336a^5b^{22}d^{13}e^{14}z^6 - 49152a^{14}b^{13}d^4e^{23}z^6 - 49152a^4b^{23}d^{14}e^{13}z^6 + 4096a^{15}b^{12}d^3e^{24}z^6 + 4096a^3b^{24}d^{15}e^{12}z^6 - 25165824a^8b^2c^{17}d^{27}z^6 + 15728640a^7b^4c^{16}d^{27}z^6 - 5242880a^6b^6c^{15}d^{27}z^6 + 983040a^5b^8c^{14}d^{27}z^6 - 98304a^4b^{10}c^{13}d^{27}z^6 + 4096a^3b^{12}c^{12}d^{27}z^6 + 8304721920a^{17}c^{10}d^{11}e^{16}z^6 + 8304721920a^{13}c^{14}d^{19}e^8z^6 + 3690987520a^{18}c^9d^9e^{18}z^6 + 3690987520a^{12}c^{15}d^{21}e^6z^6 + 16777216a^9c^{18}d^{27}z^6 - 8493371392a^6b^8c^9d^{14}e^9z^4 + 1458044928a^8b^6c^{14}d^{17}e^6z^4 - 12604538880a^{11}b^4c^8d^8e^{15}z^4 - 8303067136a^9b^5c^9d^{11}e^{12}z^4 - 5588058112a^{13}b^6c^9d^7e^{16}z^4 - 3892838400a^8b^2c^{13}d^{16}e^7z^4 - 3611713536a^8b^8c^7d^{10}e^{13}z^4 + 7819006464a^7b^9c^7d^{11}e^{12}z^4 - 7782137856a^8b^7c^8d^{11}e^{12}z^4 + 7780433920a^{12}b^2c^9d^8e^{15}z^4 - 12020465664a^7b^5c^{11}d^{15}e^8z^4 + 3176792064a^8b^3c^{12}d^{15}e^8z^4 - 322633728a^{15}b^6c^7d^3e^{20}z^4 + 210829312a^7b^6c^{15}d^{19}e^4z^4 + 15623258112a^9b^6c^8d^{10}e^{13}z^4 + 25165824a^{15}b^3c^5d^5e^{22}z^4 - 15728640a^{14}b^5c^4d^5e^{22}z^4 + 12582912a^5b^2c^{16}d^{22}e^2z^4 - 11730944a^4b^4c^{15}d^{22}e^2z^4 + 5242880a^{13}b^7c^3d^5e^{22}z^4 - 4561920a^6b^{15}c^7d^{17}e^6z^4 +
\end{aligned}$$

$4521984*a^3*b^6*c^{14}*d^{22}*e^z^4 + 4460544*a*b^{14}*c^8*d^{18}*e^5*z^4 + 353894$   
 $4*a^6*b*c^{16}*d^{21}*e^2*z^4 + 3108864*a*b^{16}*c^6*d^{16}*e^7*z^4 - 3027200*a*b^1$   
 $3*c^9*d^{19}*e^4*z^4 - 2345472*a^5*b^{17}*c*d^7*e^{16}*z^4 - 2307072*a^8*b^{14}*c*d$   
 $^4*e^{19}*z^4 + 1824768*a^6*b^{16}*c*d^6*e^{17}*z^4 + 1734912*a^9*b^{13}*c*d^3*e^{20}$   
 $*z^4 + 1419264*a*b^{12}*c^{10}*d^{20}*e^3*z^4 - 1191168*a*b^{17}*c^5*d^{15}*e^8*z^4 -$   
 $983040*a^{12}*b^9*c^2*d*e^{22}*z^4 + 964608*a^4*b^{18}*c*d^8*e^{15}*z^4 - 866304*a$   
 $^2*b^8*c^{13}*d^{22}*e*z^4 + 703488*a^7*b^{15}*c*d^5*e^{18}*z^4 - 608256*a^{10}*b^{12}$   
 $*c*d^2*e^{21}*z^4 - 440832*a*b^{11}*c^{11}*d^{21}*e^2*z^4 + 275968*a*b^{19}*c^3*d^{13}*e$   
 $^{10}*z^4 - 159744*a^2*b^{20}*c*d^{10}*e^{13}*z^4 - 153600*a*b^{20}*c^2*d^{12}*e^{11}*z^4$   
 $+ 64512*a^3*b^{19}*c*d^9*e^{14}*z^4 + 19746062336*a^8*b^6*c^9*d^{12}*e^{11}*z^4 -$   
 $15333588992*a^{10}*b^4*c^9*d^{10}*e^{13}*z^4 + 6702170112*a^7*b^4*c^{12}*d^{16}*e^7*z$   
 $^4 + 15167913984*a^{10}*b^3*c^{10}*d^{11}*e^{12}*z^4 - 2256638976*a^5*b^{11}*c^7*d^{13}$   
 $*e^{10}*z^4 + 2254307328*a^5*b^7*c^{11}*d^{17}*e^6*z^4 - 2200633344*a^6*b^5*c^{12}$   
 $d^{17}*e^6*z^4 + 6457131008*a^{11}*b^3*c^9*d^9*e^{14}*z^4 - 2128785408*a^5*b^8*c^$   
 $^{10}*d^{16}*e^7*z^4 - 2126057472*a^6*b^{11}*c^6*d^{11}*e^{12}*z^4 + 2038349824*a^{12}*b$   
 $^5*c^6*d^5*e^{18}*z^4 + 2037841920*a^5*b^{10}*c^8*d^{14}*e^9*z^4 + 3615621120*a^9$   
 $*b*c^{13}*d^{15}*e^8*z^4 + 1900019712*a^{11}*b^2*c^{10}*d^{10}*e^{13}*z^4 + 1867698432*$   
 $a^9*b^9*c^5*d^7*e^{16}*z^4 - 6157369344*a^9*b^4*c^{10}*d^{12}*e^{11}*z^4 - 18569134$   
 $08*a^7*b^{10}*c^6*d^{10}*e^{13}*z^4 + 1789132800*a^6*b^4*c^{13}*d^{18}*e^5*z^4 + 6082$   
 $658304*a^8*b^4*c^{11}*d^{14}*e^9*z^4 + 6029549568*a^{11}*b^5*c^7*d^7*e^{16}*z^4 + 6$   
 $010159104*a^6*b^7*c^{10}*d^{15}*e^8*z^4 + 1703182336*a^7*b^7*c^9*d^{13}*e^{10}*z^4$   
 $+ 1658388480*a^{11}*b^6*c^6*d^6*e^{17}*z^4 + 5917114368*a^{10}*b^6*c^7*d^8*e^{15}*z$   
 $^4 - 1591197696*a^{11}*b^7*c^5*d^5*e^{18}*z^4 - 1526464512*a^8*b^{10}*c^5*d^8*e^1$   
 $5*z^4 - 5772607488*a^{12}*b^4*c^7*d^6*e^{17}*z^4 - 1423507456*a^{13}*b^4*c^6*d^4*$   
 $e^{19}*z^4 - 1387266048*a^7*b^3*c^{13}*d^{17}*e^6*z^4 + 2976120832*a^{10}*b*c^{12}*d^$   
 $^{13}*e^{10}*z^4 - 9906946048*a^9*b^2*c^{12}*d^{14}*e^9*z^4 - 18421874688*a^8*b^5*c^$   
 $^{10}*d^{13}*e^{10}*z^4 + 1141217280*a^6*b^{12}*c^5*d^{10}*e^{13}*z^4 - 9714364416*a^7*b$   
 $^8*c^8*d^{12}*e^{11}*z^4 - 16777216*a^{16}*b*c^6*d*e^{22}*z^4 + 98304*a^{11}*b^{11}*c*d$   
 $*e^{22}*z^4 + 81920*a*b^{10}*c^{12}*d^{22}*e*z^4 + 39168*a*b^{21}*c*d^{11}*e^{12}*z^4 - 1$   
 $091829760*a^5*b^6*c^{12}*d^{18}*e^5*z^4 + 1046740992*a^{14}*b^2*c^7*d^4*e^{19}*z^4$   
 $- 6884425728*a^{12}*b*c^{10}*d^9*e^{14}*z^4 + 987445248*a^4*b^{10}*c^9*d^{16}*e^7*z^4$   
 $+ 984087552*a^5*b^{12}*c^6*d^{12}*e^{11}*z^4 - 9564585984*a^9*b^7*c^7*d^9*e^{14}*z$   
 $^4 - 5266857984*a^{10}*b^7*c^6*d^7*e^{16}*z^4 - 892145664*a^7*b^{11}*c^5*d^9*e^{14}$   
 $*z^4 - 2444623872*a^{11}*b*c^{11}*d^{11}*e^{12}*z^4 + 768540672*a^{12}*b^3*c^8*d^7*e^$   
 $^{16}*z^4 + 5048322048*a^8*b^9*c^6*d^9*e^{14}*z^4 + 5047612416*a^6*b^9*c^8*d^{13}$   
 $e^{10}*z^4 - 732492288*a^4*b^{11}*c^8*d^{15}*e^8*z^4 + 9266921472*a^7*b^6*c^{10}*d^$   
 $^{14}*e^9*z^4 - 645857280*a^6*b^6*c^{11}*d^{16}*e^7*z^4 - 623867904*a^4*b^9*c^{10}*d$   
 $^{17}*e^6*z^4 - 622067712*a^6*b^3*c^{14}*d^{19}*e^4*z^4 + 582617088*a^{10}*b^8*c^5*$   
 $d^6*e^{17}*z^4 + 577119744*a^7*b^{12}*c^4*d^8*e^{15}*z^4 + 552566784*a^{12}*b^6*c^5$   
 $*d^4*e^{19}*z^4 + 549224448*a^9*b^8*c^6*d^8*e^{15}*z^4 - 526565376*a^9*b^{10}*c^4$   
 $*d^6*e^{17}*z^4 + 511520256*a^{10}*b^9*c^4*d^5*e^{18}*z^4 + 13393723392*a^9*b^3*c$   
 $^{11}*d^{13}*e^{10}*z^4 - 2066350080*a^{14}*b*c^8*d^5*e^{18}*z^4 + 4718592000*a^{13}*b^$   
 $^2*c^8*d^6*e^{17}*z^4 - 314572800*a^7*b^2*c^{14}*d^{18}*e^5*z^4 + 287250432*a^4*b^$   
 $^{13}*c^6*d^{13}*e^{10}*z^4 + 4565827584*a^{10}*b^5*c^8*d^9*e^{14}*z^4 - 250785792*a^4$   
 $*b^{14}*c^5*d^{12}*e^{11}*z^4 + 235536384*a^{13}*b^3*c^7*d^5*e^{18}*z^4 - 232683264*a$   
 $^8*b^{11}*c^4*d^7*e^{16}*z^4 - 199627776*a^5*b^{14}*c^4*d^{10}*e^{13}*z^4 - 190267392$   
 $*a^{12}*b^7*c^4*d^3*e^{20}*z^4 + 184891392*a^6*b^{10}*c^7*d^{12}*e^{11}*z^4 + 1805025$   
 $28*a^4*b^7*c^{12}*d^{19}*e^4*z^4 + 178877952*a^3*b^{13}*c^7*d^{15}*e^8*z^4 + 172490$   
 $752*a^{14}*b^3*c^6*d^3*e^{20}*z^4 + 163946496*a^{13}*b^5*c^5*d^3*e^{20}*z^4 + 15583$   
 $9488*a^8*b^{12}*c^3*d^6*e^{17}*z^4 + 155000832*a^5*b^5*c^{13}*d^{19}*e^4*z^4 - 1520$   
 $76288*a^4*b^6*c^{13}*d^{20}*e^3*z^4 - 137592576*a^3*b^{12}*c^8*d^{16}*e^7*z^4 - 133$   
 $693440*a^{14}*b^4*c^5*d^2*e^{21}*z^4 - 116767488*a^3*b^9*c^{11}*d^{19}*e^4*z^4 - 10$   
 $8985344*a^3*b^{14}*c^6*d^{14}*e^9*z^4 - 106223616*a^6*b^{13}*c^4*d^9*e^{14}*z^4 + 1$   
 $06119168*a^3*b^{10}*c^{10}*d^{18}*e^5*z^4 + 102432768*a^5*b^4*c^{14}*d^{20}*e^3*z^4 +$   
 $102113280*a^4*b^{12}*c^7*d^{14}*e^9*z^4 + 100674048*a^5*b^9*c^9*d^{15}*e^8*z^4 +$   
 $90439680*a^{13}*b^6*c^4*d^2*e^{21}*z^4 - 86808576*a^6*b^{14}*c^3*d^8*e^{15}*z^4 +$   
 $86245376*a^6*b^2*c^{15}*d^{20}*e^3*z^4 + 79011840*a^4*b^8*c^{11}*d^{18}*e^5*z^4 + 7$   
 $8345216*a^4*b^{15}*c^4*d^{11}*e^{12}*z^4 + 78006528*a^{11}*b^9*c^3*d^3*e^{20}*z^4 - 7$   
 $3253376*a^9*b^{11}*c^3*d^5*e^{18}*z^4 + 67524608*a^3*b^8*c^{12}*d^{20}*e^3*z^4 + 67$

$$\begin{aligned}
& 108864a^{15}b^2c^6d^2e^{21}z^4 - 61590528a^{10}b^{10}c^3d^4e^{19}z^4 + 61 \\
& 559808a^5b^{15}c^3d^9e^{14}z^4 - 59637760a^5b^3c^{15}d^{21}e^2z^4 + 586 \\
& 38336a^4b^5c^{14}d^{21}e^2z^4 - 40828416a^7b^{13}c^3d^7e^{16}z^4 - 3563 \\
& 9296a^2b^{12}c^9d^{18}e^5z^4 - 31293440a^{12}b^8c^3d^2e^{21}z^4 + 29933 \\
& 568a^5b^{13}c^5d^{11}e^{12}z^4 + 27793920a^2b^{11}c^{10}d^{19}e^4z^4 + 2716 \\
& 8768a^2b^{13}c^8d^{17}e^6z^4 - 23602176a^7b^{14}c^2d^6e^{17}z^4 - 23248 \\
& 896a^3b^7c^{13}d^{21}e^2z^4 + 20929536a^3b^{15}c^5d^{13}e^{10}z^4 + 18428 \\
& 928a^9b^{12}c^2d^4e^{19}z^4 + 18026496a^6b^{15}c^2d^7e^{16}z^4 - 162616 \\
& 32a^{10}b^{11}c^2d^3e^{20}z^4 + 15128064a^3b^{16}c^4d^{12}e^{11}z^4 - 14060 \\
& 544a^2b^{10}c^{11}d^{20}e^3z^4 + 13178880a^2b^{16}c^5d^{14}e^9z^4 - 11244 \\
& 288a^3b^{17}c^3d^{11}e^{12}z^4 - 10509312a^2b^{15}c^6d^{15}e^8z^4 - 72622 \\
& 08a^4b^{17}c^2d^9e^{14}z^4 - 7045632a^2b^{17}c^4d^{13}e^{10}z^4 - 6285312 \\
& a^2b^{14}c^7d^{16}e^7z^4 + 5996544a^{11}b^{10}c^2d^2e^{21}z^4 + 4558336a \\
& ^2b^9c^{12}d^{21}e^2z^4 + 4478976a^{11}b^8c^4d^4e^{19}z^4 + 2850816a^4b \\
& ^{16}c^3d^{10}e^{13}z^4 + 2629632a^3b^{11}c^9d^{17}e^6z^4 + 2503680a^3b^ \\
& ^{18}c^2d^{10}e^{13}z^4 + 1627136a^2b^{18}c^3d^{12}e^{11}z^4 + 1605120a^8b^ \\
& ^{13}c^2d^5e^{18}z^4 + 1483776a^5b^{16}c^2d^8e^{15}z^4 + 139776a^2b^{19}c^ \\
& ^2d^{11}e^{12}z^4 - 8542224384a^{10}b^2c^{11}d^{12}e^{11}z^4 - 3072b^{22}c^d^{12} \\
& e^{11}z^4 - 3072b^{12}c^{11}d^{22}e^z^4 - 1572864a^6c^{17}d^{22}e^z^4 - 4096* \\
& a^{10}b^{13}d^e^{22}z^4 - 4096a*b^{22}d^{10}e^{13}z^4 - 6144a^{12}b^{10}c^e^{23}z^ \\
& ^4 - 983040a^5b*c^{17}d^{23}z^4 - 6912a*b^9c^{13}d^{23}z^4 + 1824522240a^{13} \\
& *c^{10}d^8e^{15}z^4 + 1730150400a^{12}c^{11}d^{10}e^{13}z^4 + 958922752a^{14}c^ \\
& ^9d^6e^{17}z^4 - 537919488a^9c^{14}d^{16}e^7z^4 + 508559360a^{11}c^{12}d^{12} \\
& e^{11}z^4 - 500170752a^{10}c^{13}d^{14}e^9z^4 + 246939648a^{15}c^8d^4e^{19}z \\
& ^4 - 199229440a^8c^{15}d^{18}e^5z^4 - 29884416a^7c^{16}d^{20}e^3z^4 + 25 \\
& 165824a^{16}c^7d^2e^{21}z^4 + 236544b^{17}c^6d^{17}e^6z^4 - 202752b^{18}c^ \\
& ^5d^{16}e^7z^4 - 202752b^{16}c^7d^{18}e^5z^4 + 126720b^{19}c^4d^{15}e^8z \\
& ^4 + 126720b^{15}c^8d^{19}e^4z^4 - 56320b^{20}c^3d^{14}e^9z^4 - 56320b^{1} \\
& ^4c^9d^{20}e^3z^4 + 16896b^{21}c^2d^{13}e^{10}z^4 + 16896b^{13}c^{10}d^{21}e^ \\
& ^2z^4 + 110080a^7b^{16}d^4e^{19}z^4 + 110080a^4b^{19}d^7e^{16}z^4 - 75520 \\
& a^8b^{15}d^3e^{20}z^4 - 75520a^3b^{20}d^8e^{15}z^4 - 56320a^6b^{17}d^5e^ \\
& ^{18}z^4 - 56320a^5b^{18}d^6e^{17}z^4 + 25600a^9b^{14}d^2e^{21}z^4 + 25600 \\
& a^2b^{21}d^9e^{14}z^4 - 1572864a^{16}b^2c^5e^{23}z^4 + 983040a^{15}b^4c^ \\
& ^4e^{23}z^4 - 327680a^{14}b^6c^3e^{23}z^4 + 61440a^{13}b^8c^2e^{23}z^4 + 9 \\
& 83040a^4b^3c^{16}d^{23}z^4 - 385024a^3b^5c^{15}d^{23}z^4 + 73728a^2b^7* \\
& c^{14}d^{23}z^4 + 256b^{23}d^{11}e^{12}z^4 + 1048576a^{17}c^6e^{23}z^4 + 256b^ \\
& ^{11}c^{12}d^{23}z^4 + 256a^{11}b^{12}e^{23}z^4 + 948695040a^8b*c^{10}d^6e^{13}z \\
& ^2 + 348917760a^7b*c^{11}d^8e^{11}z^2 - 125030400a^9b*c^9d^4e^{15}z^2 - \\
& 50728960a^6b*c^{12}d^{10}e^9z^2 - 44298240a^5b*c^{13}d^{12}e^7z^2 - 3649 \\
& 5360a^{10}b*c^8d^2e^{17}z^2 + 29675520a^8b^6c^5d^e^{18}z^2 - 24170496a^ \\
& ^9b^4c^6d^e^{18}z^2 - 17202816a^7b^8c^4d^e^{18}z^2 - 14561280a^4b*c^ \\
& ^{14}d^{14}e^5z^2 + 5532416a^6b^{10}c^3d^e^{18}z^2 + 4128768a^{10}b^2c^7d^* \\
& e^{18}z^2 - 2662400a^3b*c^{15}d^{16}e^3z^2 + 1184512a*b^{12}c^6d^9e^{10}z^ \\
& ^2 - 1136160a*b^{13}c^5d^8e^{11}z^2 - 1017600a^5b^{12}c^2d^e^{18}z^2 - 744 \\
& 768a*b^{11}c^7d^{10}e^9z^2 + 607872a*b^{14}c^4d^7e^{12}z^2 - 424064a*b^6 \\
& *c^{12}d^{15}e^4z^2 + 408576a*b^5c^{13}d^{16}e^3z^2 + 361152a*b^{10}c^8d^1 \\
& ^1e^8z^2 - 287408a*b^9c^9d^{12}e^7z^2 - 260448a^3b^{15}c^d^2e^{17}z^2 \\
& - 203904a*b^4c^{14}d^{17}e^2z^2 + 200832a*b^8c^{10}d^{13}e^6z^2 + 126720* \\
& a*b^7c^{11}d^{14}e^5z^2 - 123968a*b^{15}c^3d^6e^{13}z^2 - 39168a*b^{16}c^2 \\
& *d^5e^{14}z^2 + 11904a^2b^{16}c^d^3e^{16}z^2 + 1824135552a^7b^4c^8d^5* \\
& e^{14}z^2 - 1457252352a^8b^2c^9d^5e^{14}z^2 - 1405209600a^7b^5c^7d^4 \\
& *e^{15}z^2 - 184320a^2b*c^{16}d^{18}e^z^2 + 100608a^4b^{14}c^d^e^{18}z^2 + 5 \\
& 3248a*b^3c^{15}d^{18}e^z^2 + 26448a*b^{17}c^d^4e^{15}z^2 + 1067599872a^8b^ \\
& ^3c^8d^4e^{15}z^2 - 930828288a^7b^3c^9d^6e^{13}z^2 + 920760000a^6b^ \\
& ^4c^9d^7e^{12}z^2 - 806639616a^6b^3c^{10}d^8e^{11}z^2 - 791052480a^6b^ \\
& ^6c^7d^5e^{14}z^2 + 772237824a^6b^7c^6d^4e^{15}z^2 - 701025408a^5b^6 \\
& *c^8d^7e^{12}z^2 + 443340288a^5b^5c^9d^8e^{11}z^2 + 433047552a^7b^6* \\
& c^6d^3e^{16}z^2 + 405741312a^5b^7c^7d^6e^{13}z^2 + 293652480a^6b^2c^ \\
& ^{11}d^9e^{10}z^2 - 276962688a^6b^8c^5d^3e^{16}z^2 - 247804272a^8b^4c
\end{aligned}$$

$$\begin{aligned}
& ^7d^3e^{16z^2} + 213564384a^4b^8c^7d^7e^{12z^2} - 202596816a^5b^9c^5d^4e^{15z^2} - 182520896a^4b^9c^6d^6e^{13z^2} - 153489408a^5b^3c^11d^{10}e^9z^2 - 152151552a^7b^2c^{10}d^7e^{12z^2} + 115859712a^5b^2c^{12}d^{11}e^8z^2 + 108085248a^9b^3c^7d^2e^{17z^2} + 105536256a^4b^5c^{10}d^{10}e^9z^2 - 98323200a^6b^5c^8d^6e^{13z^2} - 93564992a^4b^6c^9d^9e^{10z^2} + 89464512a^5b^{10}c^4d^3e^{16z^2} - 75930624a^8b^5c^6d^2e^{17z^2} + 68315904a^5b^8c^6d^5e^{14z^2} - 64157184a^4b^7c^8d^8e^{11z^2} - 62951040a^9b^2c^8d^3e^{16z^2} + 49056768a^4b^{10}c^5d^5e^{14z^2} + 47614464a^3b^8c^8d^9e^{10z^2} + 35604480a^4b^2c^{13}d^{13}e^6z^2 + 33983040a^3b^{11}c^5d^6e^{13z^2} - 33515520a^4b^3c^{12}d^{12}e^7z^2 - 33463808a^3b^7c^9d^{10}e^9z^2 - 25128864a^4b^4c^{11}d^{11}e^8z^2 - 23193728a^3b^{10}c^6d^7e^{12z^2} + 21015456a^6b^9c^4d^2e^{17z^2} + 19924176a^4b^{11}c^4d^4e^{15z^2} - 19251216a^3b^9c^7d^8e^{11z^2} - 16434048a^5b^4c^{10}d^9e^{10z^2} - 16289664a^3b^{12}c^4d^5e^{14z^2} - 15059328a^4b^{12}c^3d^3e^{16z^2} - 10766016a^2b^{10}c^7d^9e^{10z^2} - 10453632a^5b^{11}c^3d^2e^{17z^2} - 9940992a^3b^3c^{13}d^{14}e^5z^2 + 8373696a^2b^{11}c^6d^8e^{11z^2} + 7776768a^3b^2c^{14}d^{15}e^4z^2 + 7077888a^3b^5c^{11}d^{12}e^7z^2 + 6798240a^2b^9c^8d^{10}e^9z^2 - 3589440a^2b^6c^{11}d^{13}e^6z^2 + 3544320a^3b^6c^{10}d^{11}e^8z^2 + 3128064a^2b^5c^{12}d^{14}e^5z^2 + 2346336a^4b^{13}c^2d^2e^{17z^2} - 2261568a^2b^8c^9d^{11}e^8z^2 - 2125824a^2b^{13}c^4d^6e^{13z^2} + 2002560a^3b^4c^{12}d^{13}e^6z^2 + 1927680a^2b^7c^{10}d^{12}e^7z^2 + 1814784a^2b^{14}c^3d^5e^{14z^2} - 1807104a^2b^{12}c^5d^7e^{12z^2} + 1637808a^3b^{13}c^3d^4e^{15z^2} + 1083456a^3b^{14}c^2d^3e^{16z^2} - 792384a^2b^4c^{13}d^{15}e^4z^2 - 657408a^2b^3c^{14}d^{16}e^3z^2 + 608256a^7b^7c^5d^2e^{17z^2} + 595968a^2b^2c^{15}d^{17}e^2z^2 - 498624a^2b^{15}c^2d^4e^{15z^2} - 3840b^{18}c^d^5e^{14z^2} - 3840b^5c^{14}d^{18}e^z^2 + 2064384a^{11}c^8d^*e^{18z^2} - 4160a^3b^{16}d^*e^{18z^2} - 4160a^*b^{18}d^3e^{16z^2} - 1290240a^{11}b^*c^7e^{19z^2} - 9840a^5b^{13}c^*e^{19z^2} - 5760a^*b^2c^{16}d^{19}z^2 - 280581120a^8c^{11}d^7e^{12z^2} + 110278656a^9c^{10}d^5e^{14z^2} - 89479168a^7c^{12}d^9e^{10z^2} + 34464000a^{10}c^9d^3e^{16z^2} + 54240b^{15}c^4d^8e^{11z^2} + 54240b^8c^{11}d^{15}e^4z^2 - 49920b^{14}c^5d^9e^{10z^2} - 49920b^9c^{10}d^{14}e^5z^2 - 37376b^{16}c^3d^7e^{12z^2} - 37376b^7c^{12}d^{16}e^3z^2 + 28480b^{13}c^6d^{10}e^9z^2 + 28480b^{10}c^9d^{13}e^6z^2 + 15936b^{17}c^2d^6e^{13z^2} + 15936b^6c^{13}d^{17}e^2z^2 - 7920b^{12}c^7d^{11}e^8z^2 - 7920b^{11}c^8d^{12}e^7z^2 + 7489536a^5c^{14}d^{13}e^6z^2 + 6084096a^6c^{13}d^{11}e^8z^2 + 2280448a^4c^{15}d^{15}e^4z^2 + 350208a^3c^{16}d^{17}e^2z^2 + 11616a^2b^{17}d^2e^{17z^2} - 3515904a^9b^5c^5e^{19z^2} + 3440640a^{10}b^3c^6e^{19z^2} + 1870848a^8b^7c^4e^{19z^2} - 572272a^7b^9c^3e^{19z^2} + 101856a^6b^{11}c^2e^{19z^2} + 400b^{19}d^4e^{15z^2} + 400b^4c^{15}d^{19}z^2 + 20736a^2c^{17}d^{19}z^2 + 400a^4b^{15}e^{19z^2} - 3969216a^4b^*c^{10}d^3e^{12} - 3001536a^3b^*c^{11}d^5e^{10} - 419904a^2b^*c^{12}d^7e^8 + 184608a^4b^3c^8d^*e^{14} - 153036a^*b^4c^{10}d^6e^9 + 127008a^*b^3c^{11}d^7e^8 + 63108a^*b^6c^8d^4e^{11} - 29160a^*b^2c^{12}d^8e^7 - 21060a^3b^5c^7d^*e^{14} - 21060a^*b^7c^7d^3e^{12} + 5460a^*b^5c^9d^5e^{10} - 404544a^5b^*c^9d^*e^{14} + 1251872a^3b^3c^9d^3e^{12} + 844224a^4b^2c^9d^2e^{13} + 820512a^2b^3c^{10}d^5e^{10} + 750672a^3b^2c^{10}d^4e^{11} - 657498a^2b^4c^9d^4e^{11} - 487116a^3b^4c^8d^2e^{13} + 160704a^2b^2c^{11}d^6e^9 + 58806a^2b^6c^7d^2e^{13} + 13140a^2b^5c^8d^3e^{12} + 15286b^6c^9d^6e^9 - 9540b^7c^8d^5e^{10} - 9540b^5c^{10}d^7e^8 + 2025b^8c^7d^4e^{11} + 2025b^4c^{11}d^8e^7 + 3367008a^4c^{11}d^4e^{11} + 1166400a^3c^{12}d^6e^9 + 705600a^5c^{10}d^2e^{13} + 104976a^2c^{13}d^8e^7 - 17640a^5b^2c^8e^{15} + 2025a^4b^4c^7e^{15} + 38416a^6c^9e^{15}, z, k) * ((1048576a^{17}c^8d^*e^{24} - 393216a^6c^{19}d^{23}e^2 - 3407872a^7c^{18}d^{21}e^4 - 5636096a^8c^{17}d^{19}e^6 + 31457280a^9c^{16}d^{17}e^8 + 175374336a^{10}c^{15}d^{15}e^{10} + 407371776a^{11}c^{14}d^{13}e^{12} + 556007424a^{12}c^{13}d^{11}e^{14} + 481296384a^{13}c^{12}d^9e^{16} + 265420800a^{14}c^{11}d^7e^{18} + 88866816a^{15}c^{10}d^5e^{20} + 15859712a^{16}c^9d^3e^{22} - 5632a^2b^8c^{15}d^{23}e^2 + 67584a^2b^9c^{14}d^{22}e^3 - 368640a^2b^{10}c^{13}d^{21}e^4 + 1205
\end{aligned}$$

$$\begin{aligned}
& 248a^2b^{11}c^{12}d^{20}e^5 - 2618880a^2b^{12}c^{11}d^{19}e^6 + 3953664a^2b^{13}c^{10}d^{18}e^7 - 4190208a^2b^{14}c^9d^{17}e^8 + 3041280a^2b^{15}c^8d^{16}e^9 - 1368576a^2b^{16}c^7d^{15}e^{10} + 225280a^2b^{17}c^6d^{14}e^{11} + 135168a^2b^{18}c^5d^{13}e^{12} - 101376a^2b^{19}c^4d^{12}e^{13} + 28160a^2b^{20}c^3d^{11}e^{14} - 3072a^2b^{21}c^2d^{10}e^{15} + 49152a^3b^6c^{16}d^{23}e^2 - 589824a^3b^7c^{15}d^{22}e^3 + 3181568a^3b^8c^{14}d^{21}e^4 - 10121216a^3b^9c^{13}d^{20}e^5 + 20854016a^3b^{10}c^{12}d^{19}e^6 - 28504064a^3b^{11}c^{11}d^{18}e^7 + 24727808a^3b^{12}c^{10}d^{17}e^8 - 10510336a^3b^{13}c^9d^{16}e^9 - 3040768a^3b^{14}c^8d^{15}e^{10} + 7405568a^3b^{15}c^7d^{14}e^{11} - 4684288a^3b^{16}c^6d^{13}e^{12} + 1314816a^3b^{17}c^5d^{12}e^{13} - 12032a^3b^{18}c^4d^{11}e^{14} - 86016a^3b^{19}c^3d^{10}e^{15} + 15616a^3b^{20}c^2d^9e^{16} - 212992a^4b^4c^{17}d^{23}e^2 + 2555904a^4b^5c^{16}d^{22}e^3 - 13549568a^4b^6c^{15}d^{21}e^4 + 41189376a^4b^7c^{14}d^{20}e^5 - 76867072a^4b^8c^{13}d^{19}e^6 + 83304448a^4b^9c^{12}d^{18}e^7 - 29710336a^4b^{10}c^{11}d^{17}e^8 - 53473280a^4b^{11}c^{10}d^{16}e^9 + 94751744a^4b^{12}c^9d^{15}e^{10} - 68968448a^4b^{13}c^8d^{14}e^{11} + 20899840a^4b^{14}c^7d^{13}e^{12} + 4022272a^4b^{15}c^6d^{12}e^{13} - 5248512a^4b^{16}c^5d^{11}e^{14} + 1310720a^4b^{17}c^4d^{10}e^{15} + 40960a^4b^{18}c^3d^9e^{16} - 45056a^4b^{19}c^2d^8e^{17} + 458752a^5b^2c^{18}d^{23}e^2 - 5505024a^5b^3c^{17}d^{22}e^3 + 28213248a^5b^4c^{16}d^{21}e^4 - 77725696a^5b^5c^{15}d^{20}e^5 + 109985792a^5b^6c^{14}d^{19}e^6 - 16252928a^5b^7c^{13}d^{18}e^7 - 236929024a^5b^8c^{12}d^{17}e^8 + 460423168a^5b^9c^{11}d^{16}e^9 - 412556800a^5b^{10}c^{10}d^{15}e^{10} + 137754624a^5b^{11}c^9d^{14}e^{11} + 80635904a^5b^{12}c^8d^{13}e^{12} - 102774784a^5b^{13}c^7d^{12}e^{13} + 36015104a^5b^{14}c^6d^{11}e^{14} + 1345536a^5b^{15}c^5d^{10}e^{15} - 3577856a^5b^{16}c^4d^9e^{16} + 407552a^5b^{17}c^3d^8e^{17} + 82432a^5b^{18}c^2d^7e^{18} - 21757952a^6b^2c^{17}d^{21}e^4 + 39059456a^6b^3c^{16}d^{20}e^5 + 44351488a^6b^4c^{15}d^{19}e^6 - 381681664a^6b^5c^{14}d^{18}e^7 + 872808448a^6b^6c^{13}d^{17}e^8 - 981073920a^6b^7c^{12}d^{16}e^9 + 329307136a^6b^8c^{11}d^{15}e^{10} + 558870528a^6b^9c^{10}d^{14}e^{11} - 809418752a^6b^{10}c^9d^{13}e^{12} + 394459136a^6b^{11}c^8d^{12}e^{13} + 10594304a^6b^{12}c^7d^{11}e^{14} - 84887552a^6b^{13}c^6d^{10}e^{15} + 23650304a^6b^{14}c^5d^9e^{16} + 2762752a^6b^{15}c^4d^8e^{17} - 1268736a^6b^{16}c^3d^7e^{18} - 100352a^6b^{17}c^2d^6e^{19} - 192217088a^7b^2c^{16}d^{19}e^6 + 514850816a^7b^3c^{15}d^{18}e^7 - 691208192a^7b^4c^{14}d^{17}e^8 + 8388608a^7b^5c^{13}d^{16}e^9 + 1583054848a^7b^6c^{12}d^{15}e^{10} - 2597715968a^7b^7c^{11}d^{14}e^{11} + 1705592832a^7b^8c^{10}d^{13}e^{12} + 65314816a^7b^9c^9d^{12}e^{13} - 792112640a^7b^{10}c^8d^{11}e^{14} + 396832768a^7b^{11}c^7d^{10}e^{15} + 5305856a^7b^{12}c^6d^9e^{16} - 47955968a^7b^{13}c^5d^8e^{17} + 4476416a^7b^{14}c^4d^7e^{18} + 1921024a^7b^{15}c^3d^6e^{19} + 82432a^7b^{16}c^2d^5e^{20} - 472383488a^8b^2c^{15}d^{17}e^8 + 1552941056a^8b^3c^{14}d^{16}e^9 - 2815066112a^8b^4c^{13}d^{15}e^{10} + 2329542656a^8b^5c^{12}d^{14}e^{11} + 631472128a^8b^6c^{11}d^{13}e^{12} - 3123511296a^8b^7c^{10}d^{12}e^{13} + 2406024192a^8b^8c^9d^{11}e^{14} - 253763584a^8b^9c^8d^{10}e^{15} - 535957504a^8b^{10}c^7d^9e^{16} + 196169728a^8b^{11}c^6d^8e^{17} + 27567104a^8b^{12}c^5d^7e^{18} - 13180928a^8b^{13}c^4d^6e^{19} - 1767424a^8b^{14}c^3d^5e^{20} - 45056a^8b^{15}c^2d^4e^{21} - 26345472a^9b^2c^{14}d^{15}e^{10} + 1757937664a^9b^3c^{13}d^{14}e^{11} - 4680646656a^9b^4c^{12}d^{13}e^{12} + 4978376704a^9b^5c^{11}d^{12}e^{13} - 1037008896a^9b^6c^{10}d^{11}e^{14} - 2360082432a^9b^7c^9d^{10}e^{15} + 1791750144a^9b^8c^8d^9e^{16} - 76677120a^9b^9c^7d^8e^{17} - 263758592a^9b^{10}c^6d^7e^{18} + 28357632a^9b^{11}c^5d^6e^{19} + 14978560a^9b^{12}c^4d^5e^{20} + 1029120a^9b^{13}c^3d^4e^{21} + 15616a^9b^{14}c^2d^3e^{22} + 1853358080a^{10}b^2c^{13}d^{13}e^{12} + 106430464a^{10}b^3c^{12}d^{12}e^{13} - 4433149952a^{10}b^4c^{11}d^{11}e^{14} + 5213257728a^{10}b^5c^{10}d^{10}e^{15} - 1239613440a^{10}b^6c^9d^9e^{16} - 1399455744a^{10}b^7c^8d^8e^{17} + 721519104a^{10}b^8c^7d^7e^{18} + 92768256a^{10}b^9c^6d^6e^{19} - 60235776a^{10}b^{10}c^5d^5e^{20} - 9616384a^{10}b^{11}c^4d^4e^{21} - 369152a^{10}b^{12}c^3d^3e^{22} - 3072a^{10}b^{13}c^2d^2e^{23} + 3744333824a^{11}b^2c^{12}d^{11}e^{14} - 1445986304a^{11}b^3c^{11}d^{10}e^{15} - 2945974272a^{11}b^4c^{10}d^9e^{16} + 3180331008a^{11}b^5c^9d^8e^{17} - 263758592a^{11}b^6c^8d^7e^{18} + 1791750144a^{11}b^7c^7d^6e^{19} - 76677120a^{11}b^8c^6d^5e^{20} + 28357632a^{11}b^9c^5d^4e^{21} + 14978560a^{11}b^{10}c^4d^3e^{22} + 1029120a^{11}b^{11}c^3d^2e^{23} + 15616a^{11}b^{12}c^2d^1e^{24} + 1853358080a^{11}b^{13}c^1d^0e^{25} - 4433149952a^{11}b^{14}c^0d^{-1}e^{26} + 5213257728a^{11}b^{15}c^{-1}d^{-2}e^{27} - 1239613440a^{11}b^{16}c^{-2}d^{-3}e^{28} - 1399455744a^{11}b^{17}c^{-3}d^{-4}e^{29} + 721519104a^{11}b^{18}c^{-4}d^{-5}e^{30} + 92768256a^{11}b^{19}c^{-5}d^{-6}e^{31} - 60235776a^{11}b^{20}c^{-6}d^{-7}e^{32} - 9616384a^{11}b^{21}c^{-7}d^{-8}e^{33} - 3072a^{11}b^{22}c^{-8}d^{-9}e^{34} + 3744333824a^{12}b^2c^{11}d^{10}e^{15} - 1445986304a^{12}b^3c^{10}d^9e^{16} + 2945974272a^{12}b^4c^9d^8e^{17} - 263758592a^{12}b^5c^8d^7e^{18} + 1791750144a^{12}b^6c^7d^6e^{19} - 76677120a^{12}b^7c^6d^5e^{20} + 28357632a^{12}b^8c^5d^4e^{21} + 14978560a^{12}b^9c^4d^3e^{22} + 1029120a^{12}b^{10}c^3d^2e^{23} + 15616a^{12}b^{11}c^2d^1e^{24} + 1853358080a^{12}b^{12}c^1d^0e^{25} - 4433149952a^{12}b^{13}c^0d^{-1}e^{26} + 5213257728a^{12}b^{14}c^{-1}d^{-2}e^{27} - 1239613440a^{12}b^{15}c^{-2}d^{-3}e^{28} - 1399455744a^{12}b^{16}c^{-3}d^{-4}e^{29} + 721519104a^{12}b^{17}c^{-4}d^{-5}e^{30} + 92768256a^{12}b^{18}c^{-5}d^{-6}e^{31} - 60235776a^{12}b^{19}c^{-6}d^{-7}e^{32} - 9616384a^{12}b^{20}c^{-7}d^{-8}e^{33} - 3072a^{12}b^{21}c^{-8}d^{-9}e^{34} + 3744333824a^{13}b^2c^{10}d^9e^{16} - 1445986304a^{13}b^3c^9d^8e^{17} + 2945974272a^{13}b^4c^8d^7e^{18} - 263758592a^{13}b^5c^7d^6e^{19} + 1791750144a^{13}b^6c^6d^5e^{20} - 76677120a^{13}b^7c^5d^4e^{21} + 28357632a^{13}b^8c^4d^3e^{22} + 14978560a^{13}b^9c^3d^2e^{23} + 1029120a^{13}b^{10}c^2d^1e^{24} + 15616a^{13}b^{11}c^1d^0e^{25} - 4433149952a^{13}b^{12}c^0d^{-1}e^{26} + 5213257728a^{13}b^{13}c^{-1}d^{-2}e^{27} - 1239613440a^{13}b^{14}c^{-2}d^{-3}e^{28} - 1399455744a^{13}b^{15}c^{-3}d^{-4}e^{29} + 721519104a^{13}b^{16}c^{-4}d^{-5}e^{30} + 92768256a^{13}b^{17}c^{-5}d^{-6}e^{31} - 60235776a^{13}b^{18}c^{-6}d^{-7}e^{32} - 9616384a^{13}b^{19}c^{-7}d^{-8}e^{33} - 3072a^{13}b^{20}c^{-8}d^{-9}e^{34} + 3744333824a^{14}b^2c^9d^8e^{17} - 1445986304a^{14}b^3c^8d^7e^{18} + 2945974272a^{14}b^4c^7d^6e^{19} - 263758592a^{14}b^5c^6d^5e^{20} + 1791750144a^{14}b^6c^5d^4e^{21} - 76677120a^{14}b^7c^4d^3e^{22} + 28357632a^{14}b^8c^3d^2e^{23} + 14978560a^{14}b^9c^2d^1e^{24} + 1029120a^{14}b^{10}c^1d^0e^{25} - 4433149952a^{14}b^{11}c^0d^{-1}e^{26} + 5213257728a^{14}b^{12}c^{-1}d^{-2}e^{27} - 1239613440a^{14}b^{13}c^{-2}d^{-3}e^{28} - 1399455744a^{14}b^{14}c^{-3}d^{-4}e^{29} + 721519104a^{14}b^{15}c^{-4}d^{-5}e^{30} + 92768256a^{14}b^{16}c^{-5}d^{-6}e^{31} - 60235776a^{14}b^{17}c^{-6}d^{-7}e^{32} - 9616384a^{14}b^{18}c^{-7}d^{-8}e^{33} - 3072a^{14}b^{19}c^{-8}d^{-9}e^{34} + 3744333824a^{15}b^2c^8d^7e^{18} - 1445986304a^{15}b^3c^7d^6e^{19} + 2945974272a^{15}b^4c^6d^5e^{20} - 263758592a^{15}b^5c^5d^4e^{21} + 1791750144a^{15}b^6c^4d^3e^{22} - 76677120a^{15}b^7c^3d^2e^{23} + 28357632a^{15}b^8c^2d^1e^{24} + 14978560a^{15}b^9c^1d^0e^{25} - 4433149952a^{15}b^{10}c^0d^{-1}e^{26} + 5213257728a^{15}b^{11}c^{-1}d^{-2}e^{27} - 1239613440a^{15}b^{12}c^{-2}d^{-3}e^{28} - 1399455744a^{15}b^{13}c^{-3}d^{-4}e^{29} + 721519104a^{15}b^{14}c^{-4}d^{-5}e^{30} + 92768256a^{15}b^{15}c^{-5}d^{-6}e^{31} - 60235776a^{15}b^{16}c^{-6}d^{-7}e^{32} - 9616384a^{15}b^{17}c^{-7}d^{-8}e^{33} - 3072a^{15}b^{18}c^{-8}d^{-9}e^{34} + 3744333824a^{16}b^2c^7d^6e^{19} - 1445986304a^{16}b^3c^6d^5e^{20} + 2945974272a^{16}b^4c^5d^4e^{21} - 263758592a^{16}b^5c^4d^3e^{22} + 1791750144a^{16}b^6c^3d^2e^{23} - 76677120a^{16}b^7c^2d^1e^{24} + 28357632a^{16}b^8c^1d^0e^{25} - 4433149952a^{16}b^9c^0d^{-1}e^{26} + 5213257728a^{16}b^{10}c^{-1}d^{-2}e^{27} - 1239613440a^{16}b^{11}c^{-2}d^{-3}e^{28} - 1399455744a^{16}b^{12}c^{-3}d^{-4}e^{29} + 721519104a^{16}b^{13}c^{-4}d^{-5}e^{30} + 92768256a^{16}b^{14}c^{-5}d^{-6}e^{31} - 60235776a^{16}b^{15}c^{-6}d^{-7}e^{32} - 9616384a^{16}b^{16}c^{-7}d^{-8}e^{33} - 3072a^{16}b^{17}c^{-8}d^{-9}e^{34} + 3744333824a^{17}b^2c^6d^5e^{20} - 1445986304a^{17}b^3c^5d^4e^{21} + 2945974272a^{17}b^4c^4d^3e^{22} - 263758592a^{17}b^5c^3d^2e^{23} + 1791750144a^{17}b^6c^2d^1e^{24} - 76677120a^{17}b^7c^1d^0e^{25} + 28357632a^{17}b^8c^0d^{-1}e^{26} - 4433149952a^{17}b^9c^{-1}d^{-2}e^{27} + 5213257728a^{17}b^{10}c^{-2}d^{-3}e^{28} - 1239613440a^{17}b^{11}c^{-3}d^{-4}e^{29} - 1399455744a^{17}b^{12}c^{-4}d^{-5}e^{30} + 721519104a^{17}b^{13}c^{-5}d^{-6}e^{31} + 92768256a^{17}b^{14}c^{-6}d^{-7}e^{32} - 60235776a^{17}b^{15}c^{-7}d^{-8}e^{33} - 9616384a^{17}b^{16}c^{-8}d^{-9}e^{34} - 3072a^{17}b^{17}c^{-9}d^{-10}e^{35} + 3744333824a^{18}b^2c^5d^4e^{21} - 1445986304a^{18}b^3c^4d^3e^{22} + 2945974272a^{18}b^4c^3d^2e^{23} - 263758592a^{18}b^5c^2d^1e^{24} + 1791750144a^{18}b^6c^1d^0e^{25} - 76677120a^{18}b^7c^0d^{-1}e^{26} + 28357632a^{18}b^8c^{-1}d^{-2}e^{27} - 4433149952a^{18}b^9c^{-2}d^{-3}e^{28} + 5213257728a^{18}b^{10}c^{-3}d^{-4}e^{29} - 1239613440a^{18}b^{11}c^{-4}d^{-5}e^{30} - 1399455744a^{18}b^{12}c^{-5}d^{-6}e^{31} + 721519104a^{18}b^{13}c^{-6}d^{-7}e^{32} + 92768256a^{18}b^{14}c^{-7}d^{-8}e^{33} - 60235776a^{18}b^{15}c^{-8}d^{-9}e^{34} - 9616384a^{18}b^{16}c^{-9}d^{-10}e^{35} - 3072a^{18}b^{17}c^{-10}d^{-11}e^{36} + 3744333824a^{19}b^2c^4d^3e^{22} - 1445986304a^{19}b^3c^3d^2e^{23} + 2945974272a^{19}b^4c^2d^1e^{24} - 263758592a^{19}b^5c^1d^0e^{25} + 1791750144a^{19}b^6c^0d^{-1}e^{26} - 76677120a^{19}b^7c^{-1}d^{-2}e^{27} + 28357632a^{19}b^8c^{-2}d^{-3}e^{28} - 4433149952a^{19}b^9c^{-3}d^{-4}e^{29} + 5213257728a^{19}b^{10}c^{-4}d^{-5}e^{30} - 1239613440a^{19}b^{11}c^{-5}d^{-6}e^{31} - 1399455744a^{19}b^{12}c^{-6}d^{-7}e^{32} + 721519104a^{19}b^{13}c^{-7}d^{-8}e^{33} + 92768256a^{19}b^{14}c^{-8}d^{-9}e^{34} - 60235776a^{19}b^{15}c^{-9}d^{-10}e^{35} - 9616384a^{19}b^{16}c^{-10}d^{-11}e^{36} - 3072a^{19}b^{17}c^{-11}d^{-12}e^{37} + 3744333824a^{20}b^2c^3d^2e^{23} - 1445986304a^{20}b^3c^2d^1e^{24} + 2945974272a^{20}b^4c^1d^0e^{25} - 263758592a^{20}b^5c^0d^{-1}e^{26} + 1791750144a^{20}b^6c^{-1}d^{-2}e^{27} - 76677120a^{20}b^7c^{-2}d^{-3}e^{28} + 28357632a^{20}b^8c^{-3}d^{-4}e^{29} - 4433149952a^{20}b^9c^{-4}d^{-5}e^{30} + 5213257728a^{20}b^{10}c^{-5}d^{-6}e^{31} - 1239613440a^{20}b^{11}c^{-6}d^{-7}e^{32} - 1399455744a^{20}b^{12}c^{-7}d^{-8}e^{33} + 721519104a^{20}b^{13}c^{-8}d^{-9}e^{34} + 92768256a^{20}b^{14}c^{-9}d^{-10}e^{35} - 60235776a^{20}b^{15}c^{-10}d^{-11}e^{36} - 9616384a^{20}b^{16}c^{-11}d^{-12}e^{37} - 3072a^{20}b^{17}c^{-12}d^{-13}e^{38} + 3744333824a^{21}b^2c^2d^1e^{24} - 1445986304a^{21}b^3c^1d^0e^{25} + 2945974272a^{21}b^4c^0d^{-1}e^{26} - 263758592a^{21}b^5c^{-1}d^{-2}e^{27} + 1791750144a^{21}b^6c^{-2}d^{-3}e^{28} - 76677120a^{21}b^7c^{-3}d^{-4}e^{29} + 28357632a^{21}b^8c^{-4}d^{-5}e^{30} - 4433149952a^{21}b^9c^{-5}d^{-6}e^{31} + 5213257728a^{21}b^{10}c^{-6}d^{-7}e^{32} - 1239613440a^{21}b^{11}c^{-7}d^{-8}e^{33} - 1399455744a^{21}b^{12}c^{-8}d^{-9}e^{34} + 721519104a^{21}b^{13}c^{-9}d^{-10}e^{35} + 92768256a^{21}b^{14}c^{-10}d^{-11}e^{36} - 60235776a^{21}b^{15}c^{-11}d^{-12}e^{37} - 9616384a^{21}b^{16}c^{-12}d^{-13}e^{38} - 3072a^{21}b^{17}c^{-13}d^{-14}e^{39} + 3744333824a^{22}b^2c^1d^0e^{25} - 1445986304a^{22}b^3c^0d^{-1}e^{26} + 2945974272a^{22}b^4c^{-1}d^{-2}e^{27} - 263758592a^{22}b^5c^{-2}d^{-3}e^{28} + 1791750144a^{22}b^6c^{-3}d^{-4}e^{29} - 76677120a^{22}b^7c^{-4}d^{-5}e^{30} + 28357632a^{22}b^8c^{-5}d^{-6}e^{31} - 4433149952a^{22}b^9c^{-6}d^{-7}e^{32} + 5213257728a^{22}b^{10}c^{-7}d^{-8}e^{33} - 1239613440a^{22}b^{11}c^{-8}d^{-9}e^{34} - 1399455744a^{22}b^{12}c^{-9}d^{-10}e^{35} + 721519104a^{22}b^{13}c^{-10}d^{-11}e^{36} + 92768256a^{22}b^{14}c^{-11}d^{-12}e^{37} - 60235776a^{22}b^{15}c^{-12}d^{-13}e^{38} - 9616384a^{22}b^{16}c^{-13}d^{-14}e^{39} - 3072a^{22}b^{17}c^{-14}d^{-15}e^{40} + 3744333824a^{23}b^2c^0d^{-1}e^{26} - 1445986304a^{23}b^3c^{-1}d^{-2}e^{27} + 2945974272a^{23}b^4c^{-2}d^{-3}e^{28} - 263758592a^{23}b^5c^{-3}d^{-4}e^{29} + 1791750144a^{23}b^6c^{-4}d^{-5}e^{30} - 76677120a^{23}b^7c^{-5}d^{-6}e^{31} + 28357632a^{23}b^8c^{-6}d^{-7}e^{32} - 4433149952a^{23}b^9c^{-7}d^{-8}e^{33} + 5213257728a^{23}b^{10}c^{-8}d^{-9}e^{34} - 1239613440a^{23}b^{11}c^{-9}d^{-10}e^{35} - 1399455744a^{23}b^{12}c^{-10}d^{-11}e^{36} + 721519104a^{23}b^{13}c^{-11}d^{-12}e^{37} + 92768256a^{23}b^{14}c^{-12}d^{-13}e^{38} - 60235776a^{23}b^{15}c^{-13}d^{-14}e^{39} - 9616384a^{23}b^{16}c^{-14}d^{-15}e^{40} - 3072a^{23}b^{17}c^{-15}d^{-16}e^{41} + 3744333824a^{24}b^2c^{-1}d^{-2}e^{27} - 1445986304a^{24}b^3c^{-2}d^{-3}e^{28} + 2945974272a^{24}b^4c^{-3}d^{-4}e^{29} - 263758592a^{24}b^5c^{-4}d^{-5}e^{30} + 1791750144a^{24}b^6c^{-5}d^{-6}e^{31} - 76677120a^{24}b^7c^{-6}d^{-7}e^{32} + 28357632a^{24}b^8c^{-7}d^{-8}e^{33} - 4433149952a^{24}b^9c^{-8}d^{-9}e^{34} + 5213257728a^{24}b^{10}c^{-9}d^{-10}e^{35} - 1239613440a^{24}b^{11}c^{-10}d^{-11}e^{36} - 1399455744a^{24}b^{12}c^{-11}d^{-12}e^{37} + 721519104a^{24}b^{13}c^{-12}d^{-13}e^{38} + 92768256a^{24}b^{14}c^{-13}d^{-14}e^{39} - 60235776a^{24}b^{15}c^{-14}d^{-15}e^{40} - 9616384a^{24}b^{16}c^{-15}d^{-16}e^{41} - 3072a^{24}b^{17}c^{-16}d^{-17}e^{42} + 3744333824a^{25}b^2c^{-2}d^{-3}e^{28} - 1445986304a^{25}b^3c^{-3}d^{-4}e^{29} + 2945974272a^{25}b^4c^{-4}d^{-5}e^{30} - 263758592a^{25}b^5c^{-5}d^{-6}e^{31} + 1791750144a^{25}b^6c^{-6}d^{-7}e^{32} - 76677120a^{25}b^7c^{-7}d^{-8}e^{33} + 28357632a^{25}b^8c^{-8}d^{-9}e^{34} - 4433149952a^{25}b^9c^{-9}d^{-10}e^{35} + 5213257728a^{25}b^{10}c^{-10}d^{-11}e^{36} - 1239613440a^{25}b^{11}c^{-11}d^{-12}e^{37} - 1399455744a^{25}b^{12}c^{-12}d^{-13}e^{38} + 721519104a^{25}b^{13}c^{-13}d^{-14}e^{39} + 92768256a^{25}b^{14}c^{-14}d^{-15}e^{40} - 60235776a^{25}b^{15}c^{-15}d^{-16}e^{41} - 9616384a^{25}b^{16}c^{-16}d^{-17}e^{42} - 3072a^{25}b^{17}c^{-17}d^{-18}e^{43} + 3744333824a^{26}b^2c^{-3}d^{-4}e^{29} - 1445986304a^{26}b^3c^{-4}d^{-5}e^{30} + 2945974272a^{26}b^4c^{-5}d^{-6}e^{31} - 263758592a^{26}b^5c^{-6}d^{-7}e^{32} + 1791750144a^{26}b^6c^{-7}d^{-8}e^{33} - 766$$

$$\begin{aligned}
& ^9d^8e^{17} - 344997888a^{11}b^6c^8d^7e^{18} - 607715328a^{11}b^7c^7d^6e^{19} + 91261952a^{11}b^8c^6d^5e^{20} + 46288896a^{11}b^9c^5d^4e^{21} + 3619072a^{11}b^{10}c^4d^3e^{22} + 73728a^{11}b^{11}c^3d^2e^{23} + 356725552a^{12}b^2c^{11}d^9e^{16} - 1152385024a^{12}b^3c^{10}d^8e^{17} - 1550467072a^{12}b^4c^9d^7e^{18} + 1052180480a^{12}b^5c^8d^6e^{19} + 114114560a^{12}b^6c^7d^5e^{20} - 115572736a^{12}b^7c^6d^4e^{21} - 18767360a^{12}b^8c^5d^3e^{22} - 737280a^{12}b^9c^4d^2e^{23} + 1821048832a^{13}b^2c^{10}d^7e^{18} - 236191744a^{13}b^3c^9d^6e^{19} - 544571392a^{13}b^4c^8d^5e^{20} + 114688000a^{13}b^5c^7d^4e^{21} + 53821440a^{13}b^6c^6d^3e^{22} + 3932160a^{13}b^7c^5d^2e^{23} + 460587008a^{14}b^2c^9d^5e^{20} + 57933824a^{14}b^3c^8d^4e^{21} - 78659584a^{14}b^4c^7d^3e^{22} - 11796480a^{14}b^5c^6d^2e^{23} + 38207488a^{15}b^2c^8d^3e^{22} + 18874368a^{15}b^3c^7d^2e^{23} + 256a^*b^{10}c^{14}d^{23}e^2 - 3072a^*b^{11}c^{13}d^{22}e^3 + 16896a^*b^{12}c^{12}d^{21}e^4 - 56320a^*b^{13}c^{11}d^{20}e^5 + 126720a^*b^{14}c^{10}d^{19}e^6 - 202752a^*b^{15}c^9d^{18}e^7 + 236544a^*b^{16}c^8d^{17}e^8 - 202752a^*b^{17}c^7d^{16}e^9 + 126720a^*b^{18}c^6d^{15}e^{10} - 56320a^*b^{19}c^5d^{14}e^{11} + 16896a^*b^{20}c^4d^{13}e^{12} - 3072a^*b^{21}c^3d^{12}e^{13} + 256a^*b^{22}c^2d^{11}e^{14} + 4718592a^6b^*c^{18}d^{22}e^3 + 38797312a^7b^*c^{17}d^{20}e^5 + 77594624a^8b^*c^{16}d^{18}e^7 - 159383552a^9b^*c^{15}d^{16}e^9 - 1020264448a^{10}b^*c^{14}d^{14}e^{11} - 2128609280a^{11}b^*c^{13}d^{12}e^{13} + 256a^{11}b^{12}c^2d^*e^{24} - 2451570688a^{12}b^*c^{12}d^{10}e^{15} - 6144a^{12}b^{10}c^3d^*e^{24} - 1694498816a^{13}b^*c^{11}d^8e^{17} + 61440a^{13}b^8c^4d^*e^{24} - 691535872a^{14}b^*c^{10}d^6e^{19} - 327680a^{14}b^6c^5d^*e^{24} - 149946368a^{15}b^*c^9d^4e^{21} + 983040a^{15}b^4c^6d^*e^{24} - 12582912a^{16}b^*c^8d^2e^{23} - 1572864a^{16}b^2c^7d^*e^{24}) / (32*(16a^3b^6c^9d^{18} - a^2b^8c^8d^{18} - 256a^6c^{12}d^{18} - 96a^4b^4c^{10}d^{18} + 256a^5b^2c^{11}d^{18} - a^2b^{16}d^{10}e^8 + 8a^3b^{15}d^9e^9 - 28a^4b^{14}d^8e^{10} + 56a^5b^{13}d^7e^{11} - 70a^6b^{12}d^6e^{12} + 56a^7b^{11}d^5e^{13} - 28a^8b^{10}d^4e^{14} + 8a^9b^9d^3e^{15} - a^{10}b^8d^2e^{16} - 2048a^7c^{11}d^{16}e^2 - 7168a^8c^{10}d^{14}e^4 - 14336a^9c^9d^{12}e^6 - 17920a^{10}c^8d^{10}e^8 - 14336a^{11}c^7d^8e^{10} - 7168a^{12}c^6d^6e^{12} - 2048a^{13}c^5d^4e^{14} - 256a^{14}c^4d^2e^{16} - 28a^2b^{10}c^6d^{16}e^2 + 56a^2b^{11}c^5d^{15}e^3 - 70a^2b^{12}c^4d^{14}e^4 + 56a^2b^{13}c^3d^{13}e^5 - 28a^2b^{14}c^2d^{12}e^6 + 440a^3b^8c^7d^{16}e^2 - 840a^3b^9c^6d^{15}e^3 + 952a^3b^{10}c^5d^{14}e^4 - 616a^3b^{11}c^4d^{13}e^5 + 168a^3b^{12}c^3d^{12}e^6 + 40a^3b^{13}c^2d^{11}e^7 - 2560a^4b^6c^8d^{16}e^2 + 4480a^4b^7c^7d^{15}e^3 - 4060a^4b^8c^6d^{14}e^4 + 1064a^4b^9c^5d^{13}e^5 + 1372a^4b^{10}c^4d^{12}e^6 - 1360a^4b^{11}c^3d^{11}e^7 + 380a^4b^{12}c^2d^{10}e^8 + 6400a^5b^4c^9d^{16}e^2 - 8960a^5b^5c^8d^{15}e^3 + 2240a^5b^6c^7d^{14}e^4 + 9856a^5b^7c^6d^{13}e^5 - 13048a^5b^8c^5d^{12}e^6 + 5400a^5b^9c^4d^{11}e^7 + 1040a^5b^{10}c^3d^{10}e^8 - 1360a^5b^{11}c^2d^9e^9 - 5120a^6b^2c^{10}d^{16}e^2 + 22400a^6b^4c^8d^{14}e^4 - 41216a^6b^5c^7d^{13}e^5 + 25088a^6b^6c^6d^{12}e^6 + 8320a^6b^7c^5d^{11}e^7 - 17350a^6b^8c^4d^{10}e^8 + 5400a^6b^9c^3d^9e^9 + 1372a^6b^{10}c^2d^8e^{10} - 35840a^7b^2c^9d^{14}e^4 + 28672a^7b^3c^8d^{13}e^5 + 30464a^7b^4c^7d^{12}e^6 - 73472a^7b^5c^6d^{11}e^7 + 40544a^7b^6c^5d^{10}e^8 + 8320a^7b^7c^4d^9e^9 - 13048a^7b^8c^3d^8e^{10} + 1064a^7b^9c^2d^7e^{11} - 93184a^8b^2c^8d^{12}e^6 + 71680a^8b^3c^7d^{11}e^7 + 29120a^8b^4c^6d^{10}e^8 - 73472a^8b^5c^5d^9e^9 + 25088a^8b^6c^4d^8e^{10} + 9856a^8b^7c^3d^7e^{11} - 4060a^8b^8c^2d^6e^{12} - 125440a^9b^2c^7d^{10}e^8 + 71680a^9b^3c^6d^9e^9 + 30464a^9b^4c^5d^8e^{10} - 41216a^9b^5c^4d^7e^{11} + 2240a^9b^6c^3d^6e^{12} + 4480a^9b^7c^2d^5e^{13} - 93184a^{10}b^2c^6d^8e^{10} + 28672a^{10}b^3c^5d^7e^{11} + 22400a^{10}b^4c^4d^6e^{12} - 8960a^{10}b^5c^3d^5e^{13} - 2560a^{10}b^6c^2d^4e^{14} - 35840a^{11}b^2c^5d^6e^{12} + 6400a^{11}b^4c^3d^4e^{14} + 768a^{11}b^5c^2d^3e^{15} - 5120a^{12}b^2c^4d^4e^{14} - 2048a^{12}b^3c^3d^3e^{15} - 96a^{12}b^4c^2d^2e^{16} + 256a^{13}b^2c^3d^2e^{16} + 2048a^6b^*c^{11}d^{17}e + 8a^2b^9c^7d^{17}e + 8a^2b^{15}c^d^{11}e^7 - 128a^3b^7c^8d^{17}e - 40a^3b^{14}c^d^{10}e^8 + 768a^4b^5c^9d^{17}e + 40a^4b^{13}c^d^9e^9 - 2048a^5b^3c^{10}d^{17}e + 168a^5b^{12}c^d^8e^{10}
\end{aligned}$$

$$\begin{aligned}
& - 616*a^6*b^{11}*c*d^7*e^{11} + 14336*a^7*b*c^{10}*d^{15}*e^3 + 952*a^7*b^{10}*c*d^6* \\
& e^{12} + 43008*a^8*b*c^9*d^{13}*e^5 - 840*a^8*b^9*c*d^5*e^{13} + 71680*a^9*b*c^8* \\
& d^{11}*e^7 + 440*a^9*b^8*c*d^4*e^{14} + 71680*a^{10}*b*c^7*d^9*e^9 - 128*a^{10}*b^7* \\
& *c*d^3*e^{15} + 43008*a^{11}*b*c^6*d^7*e^{11} + 16*a^{11}*b^6*c*d^2*e^{16} + 14336*a^{12}* \\
& b*c^5*d^5*e^{13} + 2048*a^{13}*b*c^4*d^3*e^{15})) + (\text{root}(128723189760*a^{14}*b^4* \\
& 4*c^9*d^{13}*e^{14}*z^6 + 128723189760*a^{12}*b^4*c^{11}*d^{17}*e^{10}*z^6 - 8432455680 \\
& *a^{11}*b^{12}*c^4*d^{11}*e^{16}*z^6 - 8432455680*a^7*b^{12}*c^8*d^{19}*e^8*z^6 + 12673 \\
& 351680*a^{11}*b^{11}*c^5*d^{12}*e^{15}*z^6 + 12673351680*a^8*b^{11}*c^8*d^{18}*e^9*z^6 \\
& - 72637480960*a^{12}*b^9*c^6*d^{12}*e^{15}*z^6 - 72637480960*a^9*b^9*c^9*d^{18}*e^9 \\
& *z^6 - 21048344576*a^9*b^{12}*c^6*d^{15}*e^{12}*z^6 - 16609443840*a^{17}*b^3*c^7*d^8* \\
& e^{19}*z^6 - 16609443840*a^{10}*b^3*c^{14}*d^{22}*e^5*z^6 + 145332633600*a^{13}*b^5* \\
& *c^9*d^{14}*e^{13}*z^6 + 145332633600*a^{12}*b^5*c^{10}*d^{16}*e^{11}*z^6 + 12374035660 \\
& 8*a^{14}*b^5*c^8*d^{12}*e^{15}*z^6 + 123740356608*a^{11}*b^5*c^{11}*d^{18}*e^9*z^6 + 34 \\
& 60300800*a^{17}*b^5*c^5*d^6*e^{21}*z^6 + 3460300800*a^8*b^5*c^{14}*d^{24}*e^3*z^6 - \\
& 7751073792*a^{15}*b^7*c^5*d^8*e^{19}*z^6 - 7751073792*a^8*b^7*c^{12}*d^{22}*e^5*z^6 \\
& + 12041846784*a^{14}*b^7*c^6*d^{10}*e^{17}*z^6 + 12041846784*a^9*b^7*c^{11}*d^{20}* \\
& e^7*z^6 - 325545099264*a^{14}*b^3*c^{10}*d^{14}*e^{13}*z^6 - 325545099264*a^{13}*b^3* \\
& c^{11}*d^{16}*e^{11}*z^6 - 3330539520*a^{13}*b^{10}*c^4*d^9*e^{18}*z^6 - 3330539520*a^7* \\
& *b^{10}*c^{10}*d^{21}*e^6*z^6 + 157789716480*a^{12}*b^7*c^8*d^{14}*e^{13}*z^6 + 1577897 \\
& 16480*a^{11}*b^7*c^9*d^{16}*e^{11}*z^6 + 37492359168*a^{11}*b^{10}*c^6*d^{13}*e^{14}*z^6 \\
& + 37492359168*a^9*b^{10}*c^8*d^{17}*e^{10}*z^6 + 301989888*a^8*b^3*c^{16}*d^{26}*e*z^6 \\
& - 7266631680*a^{17}*b^4*c^6*d^7*e^{20}*z^6 - 7266631680*a^9*b^4*c^{14}*d^{23}*e^4* \\
& *z^6 - 201326592*a^{20}*b*c^6*d^4*e^{23}*z^6 - 188743680*a^7*b^5*c^{15}*d^{26}*e*z^6 \\
& + 45747339264*a^{13}*b^8*c^6*d^{11}*e^{16}*z^6 + 45747339264*a^9*b^8*c^{10}*d^{19}* \\
& e^8*z^6 - 74612736*a^{10}*b^{16}*c*d^9*e^{18}*z^6 - 2768240640*a^{16}*b^7*c^4*d^6*e^{21}* \\
& z^6 - 2768240640*a^7*b^7*c^{13}*d^{24}*e^3*z^6 + 69746688*a^{11}*b^{15}*c*d^8*e^{19}* \\
& z^6 + 62914560*a^6*b^7*c^{14}*d^{26}*e*z^6 + 2752020480*a^{10}*b^{13}*c^4*d^{12}* \\
& e^{15}*z^6 + 2752020480*a^7*b^{13}*c^7*d^{18}*e^9*z^6 + 55148544*a^9*b^{17}*c*d^{10}* \\
& e^{17}*z^6 - 45957120*a^{12}*b^{14}*c*d^7*e^{20}*z^6 - 2724986880*a^{14}*b^9*c^4*d^8* \\
& e^{19}*z^6 - 2724986880*a^7*b^9*c^{11}*d^{22}*e^5*z^6 - 25952256*a^8*b^{18}*c*d^{11}* \\
& e^{16}*z^6 + 21086208*a^{13}*b^{13}*c*d^6*e^{21}*z^6 - 11796480*a^5*b^9*c^{13}*d^{26}*e* \\
& z^6 - 6438912*a^{14}*b^{12}*c*d^5*e^{22}*z^6 + 5406720*a^7*b^{19}*c*d^{12}*e^{15}*z^6 \\
& + 1622016*a^6*b^{20}*c*d^{13}*e^{14}*z^6 - 1523712*a^5*b^{21}*c*d^{14}*e^{13}*z^6 + 117 \\
& 9648*a^{15}*b^{11}*c*d^4*e^{23}*z^6 + 1179648*a^4*b^{11}*c^{12}*d^{26}*e*z^6 + 442368*a^4* \\
& b^{22}*c*d^{15}*e^{12}*z^6 - 98304*a^{16}*b^{10}*c*d^3*e^{24}*z^6 - 49152*a^3*b^{23}*c* \\
& d^{16}*e^{11}*z^6 - 49152*a^3*b^{13}*c^{11}*d^{26}*e*z^6 + 6897106944*a^9*b^{13}*c^5*d^{14}* \\
& e^{13}*z^6 + 6897106944*a^8*b^{13}*c^6*d^{16}*e^{11}*z^6 - 2422210560*a^{16}*b^6* \\
& c^5*d^7*e^{20}*z^6 - 2422210560*a^8*b^6*c^{13}*d^{23}*e^4*z^6 + 255785435136*a^{14}* \\
& b^2*c^{11}*d^{15}*e^{12}*z^6 + 41004564480*a^{15}*b^4*c^8*d^{11}*e^{16}*z^6 + 41004564 \\
& 480*a^{11}*b^4*c^{12}*d^{19}*e^8*z^6 + 2270822400*a^{13}*b^{11}*c^3*d^8*e^{19}*z^6 + 22 \\
& 70822400*a^6*b^{11}*c^{10}*d^{22}*e^5*z^6 + 23677108224*a^{14}*b^8*c^5*d^9*e^{18}*z^6 \\
& + 23677108224*a^8*b^8*c^{11}*d^{21}*e^6*z^6 + 212600881152*a^{15}*b^2*c^{10}*d^{13}* \\
& e^{14}*z^6 + 212600881152*a^{13}*b^2*c^{12}*d^{17}*e^{10}*z^6 + 75157733376*a^{15}*b^5* \\
& c^7*d^{10}*e^{17}*z^6 + 75157733376*a^{10}*b^5*c^{12}*d^{20}*e^7*z^6 - 251217838080*a^{13}* \\
& b^6*c^8*d^{13}*e^{14}*z^6 - 251217838080*a^{11}*b^6*c^{10}*d^{17}*e^{10}*z^6 - 1952 \\
& 907264*a^{14}*b^{10}*c^3*d^7*e^{20}*z^6 - 1952907264*a^6*b^{10}*c^{11}*d^{23}*e^4*z^6 - \\
& 27691057152*a^{13}*b^9*c^5*d^{10}*e^{17}*z^6 - 27691057152*a^8*b^9*c^{10}*d^{20}*e^7* \\
& z^6 - 1902673920*a^8*b^{15}*c^4*d^{14}*e^{13}*z^6 - 1902673920*a^7*b^{15}*c^5*d^{16}* \\
& e^{11}*z^6 + 10465050624*a^{10}*b^{11}*c^6*d^{14}*e^{13}*z^6 + 10465050624*a^9*b^{11}* \\
& c^7*d^{16}*e^{11}*z^6 + 1613905920*a^9*b^{14}*c^4*d^{13}*e^{14}*z^6 + 1613905920*a^7* \\
& b^{14}*c^6*d^{17}*e^{10}*z^6 - 33218887680*a^{17}*b*c^9*d^{10}*e^{17}*z^6 - 33218887680 \\
& *a^{12}*b*c^{14}*d^{20}*e^7*z^6 + 1524695040*a^{10}*b^{14}*c^3*d^{11}*e^{16}*z^6 + 152469 \\
& 5040*a^6*b^{14}*c^7*d^{19}*e^8*z^6 - 1472200704*a^{18}*b^4*c^5*d^5*e^{22}*z^6 - 147 \\
& 2200704*a^8*b^4*c^{15}*d^{25}*e^2*z^6 - 83047219200*a^{16}*b^3*c^8*d^{10}*e^{17}*z^6 \\
& - 83047219200*a^{11}*b^3*c^{13}*d^{20}*e^7*z^6 + 44291850240*a^{17}*b^2*c^8*d^9*e^1 \\
& 8*z^6 + 44291850240*a^{11}*b^2*c^{14}*d^{21}*e^6*z^6 + 1308131328*a^8*b^{14}*c^5*d^{15}* \\
& e^{12}*z^6 - 201326592*a^9*b*c^{17}*d^{26}*e*z^6 + 48530718720*a^{12}*b^8*c^7*d^{13}* \\
& e^{14}*z^6 + 48530718720*a^{10}*b^8*c^9*d^{17}*e^{10}*z^6 - 1242644480*a^{12}*b^{12} \\
& *c^3*d^9*e^{18}*z^6 - 1242644480*a^6*b^{12}*c^9*d^{21}*e^6*z^6 + 9813196800*a^{12}*
\end{aligned}$$

$$\begin{aligned}
& b^{10}c^5d^{11}e^{16}z^6 + 9813196800a^8b^{10}c^9d^{19}e^8z^6 - 93012885504 \\
& a^{15}b^c^{11}d^{14}e^{13}z^6 - 93012885504a^{14}b^c^{12}d^{16}e^{11}z^6 + 177305 \\
& 812992a^{13}b^4c^{10}d^{15}e^{12}z^6 + 52730658816a^{10}b^{10}c^7d^{15}e^{12}z^6 \\
& - 1180106752a^9b^{15}c^3d^{12}e^{15}z^6 - 1180106752a^6b^{15}c^6d^{18}e^9z^6 \\
& + 1023672320a^{15}b^9c^3d^6e^{21}z^6 + 1023672320a^6b^9c^{12}d^{24} \\
& e^3z^6 + 975175680a^{17}b^6c^4d^5e^{22}z^6 + 975175680a^7b^6c^{14}d^2 \\
& 5e^2z^6 - 11072962560a^{18}b^c^8d^8e^{19}z^6 - 11072962560a^{11}b^c^{15}d \\
& ^{22}e^5z^6 + 9412018176a^{18}b^2c^7d^7e^{20}z^6 + 9412018176a^{10}b^2c^ \\
& ^{15}d^{23}e^4z^6 + 805306368a^{19}b^2c^6d^5e^{22}z^6 + 805306368a^9b^2c^ \\
& ^{16}d^{25}e^2z^6 - 133809831936a^{14}b^6c^7d^{11}e^{16}z^6 - 133809831936a \\
& ^{10}b^6c^{11}d^{19}e^8z^6 - 2214592512a^{19}b^c^7d^6e^{21}z^6 - 2214592512 \\
& a^{10}b^c^{16}d^{24}e^3z^6 + 82216747008a^{13}b^7c^7d^{12}e^{15}z^6 + 822167 \\
& 47008a^{10}b^7c^{10}d^{18}e^9z^6 - 586629120a^{12}b^{13}c^2d^8e^{19}z^6 - 5 \\
& 86629120a^5b^{13}c^9d^{22}e^5z^6 + 568565760a^7b^{16}c^4d^{15}e^{12}z^6 - \\
& 4844421120a^{16}b^4c^7d^9e^{18}z^6 - 4844421120a^{10}b^4c^{13}d^{21}e^6z \\
& ^6 + 531210240a^{11}b^{14}c^2d^9e^{18}z^6 + 531210240a^5b^{14}c^8d^{21}e^6 \\
& z^6 - 527155200a^{11}b^{13}c^3d^{10}e^{17}z^6 - 527155200a^6b^{13}c^8d^{20} \\
& e^7z^6 + 43470028800a^{11}b^8c^8d^{15}e^{12}z^6 - 107874877440a^{11}b^9c^ \\
& ^7d^{14}e^{13}z^6 - 107874877440a^{10}b^9c^8d^{16}e^{11}z^6 + 9018408960a^{12} \\
& b^{11}c^4d^{10}e^{17}z^6 + 9018408960a^7b^{11}c^9d^{20}e^7z^6 + 421994496* \\
& a^{13}b^{12}c^2d^7e^{20}z^6 + 421994496a^5b^{12}c^{10}d^{23}e^4z^6 - 6643777 \\
& 5360a^{16}b^c^{10}d^{12}e^{15}z^6 - 66437775360a^{13}b^c^{13}d^{18}e^9z^6 + 261 \\
& 59874048a^{16}b^5c^6d^8e^{19}z^6 + 26159874048a^9b^5c^{13}d^{22}e^5z^6 \\
& - 369098752a^{18}b^3c^6d^6e^{21}z^6 - 369098752a^9b^3c^{15}d^{24}e^3z^6 \\
& + 351436800a^8b^{16}c^3d^{13}e^{14}z^6 + 351436800a^6b^{16}c^5d^{17}e^{10} \\
& z^6 - 334233600a^{16}b^8c^3d^5e^{22}z^6 - 334233600a^6b^8c^{13}d^{25}e^2 \\
& z^6 + 301989888a^{19}b^3c^5d^4e^{23}z^6 - 266010624a^{10}b^{15}c^2d^{10}e \\
& ^{17}z^6 - 266010624a^5b^{15}c^7d^{20}e^7z^6 - 305198530560a^{12}b^6c^9d \\
& ^{15}e^{12}z^6 - 203292672a^{14}b^{11}c^2d^6e^{21}z^6 - 203292672a^5b^{11}c^ \\
& ^{11}d^{24}e^3z^6 - 188743680a^{18}b^5c^4d^4e^{23}z^6 + 120418467840a^{16}b \\
& ^2c^9d^{11}e^{16}z^6 + 120418467840a^{12}b^2c^{13}d^{19}e^8z^6 - 1729393459 \\
& 2a^{10}b^{12}c^5d^{13}e^{14}z^6 - 17293934592a^8b^{12}c^7d^{17}e^{10}z^6 + 10 \\
& 4890368a^8b^{17}c^2d^{12}e^{15}z^6 + 104890368a^5b^{17}c^5d^{18}e^9z^6 + \\
& 4390256640a^{15}b^8c^4d^7e^{20}z^6 + 4390256640a^7b^8c^{12}d^{23}e^4z^6 \\
& - 91750400a^6b^{18}c^3d^{15}e^{12}z^6 + 79134720a^7b^{17}c^3d^{14}e^{13}z^ \\
& 6 + 79134720a^6b^{17}c^4d^{16}e^{11}z^6 - 74612736a^4b^{16}c^7d^{21}e^6z^ \\
& 6 - 72990720a^7b^{18}c^2d^{13}e^{14}z^6 - 72990720a^5b^{18}c^4d^{17}e^{10}z \\
& ^6 + 69746688a^4b^{15}c^8d^{22}e^5z^6 + 63700992a^{15}b^{10}c^2d^5e^{22}z \\
& ^6 + 63700992a^5b^{10}c^{12}d^{25}e^2z^6 + 62914560a^{17}b^7c^3d^4e^{23}z \\
& ^6 + 55148544a^4b^{17}c^6d^{20}e^7z^6 - 45957120a^4b^{14}c^9d^{23}e^4z^ \\
& 6 - 25952256a^4b^{18}c^5d^{19}e^8z^6 - 25165824a^{20}b^2c^5d^3e^{24}z^6 \\
& + 21086208a^4b^{13}c^{10}d^{24}e^3z^6 + 20643840a^6b^{19}c^2d^{14}e^{13}z^ \\
& 6 + 20643840a^5b^{19}c^3d^{16}e^{11}z^6 + 15728640a^{19}b^4c^4d^3e^{24}z^ \\
& 6 - 11796480a^{16}b^9c^2d^4e^{23}z^6 - 6438912a^4b^{12}c^{11}d^{25}e^2z^6 \\
& + 5406720a^4b^{19}c^4d^{18}e^9z^6 - 5242880a^{18}b^6c^3d^3e^{24}z^6 + \\
& 3784704a^3b^{18}c^6d^{21}e^6z^6 - 3244032a^3b^{19}c^5d^{20}e^7z^6 - 324 \\
& 4032a^3b^{17}c^7d^{22}e^5z^6 + 2027520a^3b^{20}c^4d^{19}e^8z^6 + 202752 \\
& 0a^3b^{16}c^8d^{23}e^4z^6 - 1622016a^9b^{16}c^2d^{11}e^{16}z^6 - 1622016* \\
& a^5b^{16}c^6d^{19}e^8z^6 + 1622016a^4b^{20}c^3d^{17}e^{10}z^6 - 1523712a^ \\
& 4b^{21}c^2d^{16}e^{11}z^6 + 983040a^{17}b^8c^2d^3e^{24}z^6 - 901120a^3b^ \\
& ^{21}c^3d^{18}e^9z^6 - 901120a^3b^{15}c^9d^{24}e^3z^6 + 270336a^3b^{22}c^ \\
& ^2d^{17}e^{10}z^6 + 270336a^3b^{14}c^{10}d^{25}e^2z^6 + 172032a^5b^{20}c^2d \\
& ^{15}e^{12}z^6 - 38593888256a^{15}b^6c^6d^9e^{18}z^6 - 38593888256a^9b^6* \\
& c^{12}d^{21}e^6z^6 - 210386288640a^{15}b^3c^9d^{12}e^{15}z^6 - 210386288640* \\
& a^{12}b^3c^{12}d^{18}e^9z^6 + 15502147584a^{15}c^{12}d^{15}e^{12}z^6 + 11072962 \\
& 56a^{19}c^8d^7e^{20}z^6 + 1107296256a^{11}c^{16}d^{23}e^4z^6 + 13287555072* \\
& a^{16}c^{11}d^{13}e^{14}z^6 + 13287555072a^{14}c^{13}d^{17}e^{10}z^6 + 201326592a \\
& ^{20}c^7d^5e^{22}z^6 + 201326592a^{10}c^{17}d^{25}e^2z^6 + 16777216a^{21}c^6 \\
& d^3e^{24}z^6 + 3784704a^9b^{18}d^9e^{18}z^6 - 3244032a^{10}b^{17}d^8e^{19}
\end{aligned}$$



$$\begin{aligned}
& z^6 - 3244032a^8b^{19}d^{10}e^{17}z^6 + 2027520a^{11}b^{16}d^7e^{20}z^6 + 2027520a^7b^{20}d^{11}e^{16}z^6 - 901120a^{12}b^{15}d^6e^{21}z^6 - 901120a^6b^{21}d^{12}e^{15}z^6 + 270336a^{13}b^{14}d^5e^{22}z^6 + 270336a^5b^{22}d^{13}e^{14}z^6 - 49152a^{14}b^{13}d^4e^{23}z^6 - 49152a^4b^{23}d^{14}e^{13}z^6 + 4096a^{15}b^{12}d^3e^{24}z^6 + 4096a^3b^{24}d^{15}e^{12}z^6 - 25165824a^8b^2c^{17}d^{27}z^6 + 15728640a^7b^4c^{16}d^{27}z^6 - 5242880a^6b^6c^{15}d^{27}z^6 \\
& + 983040a^5b^8c^{14}d^{27}z^6 - 98304a^4b^{10}c^{13}d^{27}z^6 + 4096a^3b^{12}c^{12}d^{27}z^6 + 8304721920a^{17}c^{10}d^{11}e^{16}z^6 + 8304721920a^{13}c^{14}d^{19}e^8z^6 + 3690987520a^{18}c^9d^9e^{18}z^6 + 3690987520a^{12}c^{15}d^{21}e^6z^6 + 16777216a^9c^{18}d^{27}z^6 - 8493371392a^6b^8c^9d^{14}e^9z^4 + 1458044928a^8b^8c^{14}d^{17}e^6z^4 - 12604538880a^{11}b^4c^8d^8e^{15}z^4 - 8303067136a^9b^5c^9d^{11}e^{12}z^4 - 5588058112a^{13}b^3c^9d^7e^{16}z^4 - 3892838400a^8b^2c^{13}d^{16}e^7z^4 - 3611713536a^8b^8c^7d^{10}e^{13}z^4 + 7819006464a^7b^9c^7d^{11}e^{12}z^4 - 7782137856a^8b^7c^8d^{11}e^{12}z^4 + 7780433920a^{12}b^2c^9d^8e^{15}z^4 - 12020465664a^7b^5c^{11}d^{15}e^8z^4 + 3176792064a^8b^3c^{12}d^{15}e^8z^4 - 322633728a^{15}b^3c^7d^3e^{20}z^4 + 210829312a^7b^3c^{15}d^{19}e^4z^4 + 15623258112a^9b^6c^8d^{10}e^{13}z^4 + 25165824a^{15}b^3c^5d^5e^{22}z^4 - 15728640a^{14}b^5c^4d^5e^{22}z^4 + 12582912a^5b^2c^{16}d^{22}e^5z^4 - 11730944a^4b^4c^{15}d^2e^5z^4 + 5242880a^{13}b^7c^3d^5e^{22}z^4 - 4561920a^6b^{15}c^7d^{17}e^6z^4 + 4521984a^3b^6c^{14}d^{22}e^5z^4 + 4460544a^6b^{14}c^8d^{18}e^5z^4 + 3538944a^6b^3c^{16}d^{21}e^2z^4 + 3108864a^6b^{16}c^6d^{16}e^7z^4 - 3027200a^6b^{13}c^9d^{19}e^4z^4 - 2345472a^5b^{17}c^4d^7e^{16}z^4 - 2307072a^8b^{14}c^4d^4e^{19}z^4 + 1824768a^6b^{16}c^4d^6e^{17}z^4 + 1734912a^9b^{13}c^4d^3e^{20}z^4 + 1419264a^6b^{12}c^{10}d^{20}e^3z^4 - 1191168a^6b^{17}c^5d^{15}e^8z^4 - 983040a^{12}b^9c^2d^5e^{22}z^4 + 964608a^4b^{18}c^4d^8e^{15}z^4 - 866304a^2b^8c^{13}d^{22}e^5z^4 + 703488a^7b^{15}c^4d^5e^{18}z^4 - 608256a^{10}b^{12}c^4d^2e^{21}z^4 - 440832a^6b^{11}c^{11}d^{21}e^2z^4 + 275968a^6b^{19}c^3d^{13}e^{10}z^4 - 159744a^2b^{20}c^4d^{10}e^{13}z^4 - 153600a^6b^{20}c^2d^{12}e^{11}z^4 + 64512a^3b^{19}c^4d^9e^{14}z^4 + 19746062336a^8b^6c^9d^{12}e^{11}z^4 - 15333588992a^{10}b^4c^9d^{10}e^{13}z^4 + 6702170112a^7b^4c^{12}d^{16}e^7z^4 + 15167913984a^{10}b^3c^{10}d^{11}e^{12}z^4 - 2256638976a^5b^{11}c^7d^{13}e^{10}z^4 + 2254307328a^5b^7c^{11}d^{17}e^6z^4 - 2200633344a^6b^5c^{12}d^{17}e^6z^4 + 6457131008a^{11}b^3c^9d^9e^{14}z^4 - 2128785408a^5b^8c^{10}d^{16}e^7z^4 - 2126057472a^6b^{11}c^6d^{11}e^{12}z^4 + 2038349824a^{12}b^5c^6d^5e^{18}z^4 + 2037841920a^5b^{10}c^8d^{14}e^9z^4 + 3615621120a^9b^3c^{13}d^{15}e^8z^4 + 1900019712a^{11}b^2c^{10}d^{10}e^{13}z^4 + 1867698432a^9b^9c^5d^7e^{16}z^4 - 6157369344a^9b^4c^{10}d^{12}e^{11}z^4 - 1856913408a^7b^{10}c^6d^{10}e^{13}z^4 + 1789132800a^6b^4c^{13}d^{18}e^5z^4 + 6082658304a^8b^4c^{11}d^{14}e^9z^4 + 6029549568a^{11}b^5c^7d^7e^{16}z^4 + 6010159104a^6b^7c^{10}d^{15}e^8z^4 + 1703182336a^7b^7c^9d^{13}e^{10}z^4 + 1658388480a^{11}b^6c^6d^6e^{17}z^4 + 5917114368a^{10}b^6c^7d^8e^{15}z^4 - 1591197696a^{11}b^7c^5d^5e^{18}z^4 - 1526464512a^8b^{10}c^5d^8e^{15}z^4 - 5772607488a^{12}b^4c^7d^6e^{17}z^4 - 1423507456a^{13}b^4c^6d^4e^{19}z^4 - 1387266048a^7b^3c^{13}d^{17}e^6z^4 + 2976120832a^{10}b^3c^{12}d^{13}e^{10}z^4 - 9906946048a^9b^2c^{12}d^{14}e^9z^4 - 18421874688a^8b^5c^{10}d^{13}e^{10}z^4 + 1141217280a^6b^{12}c^5d^{10}e^{13}z^4 - 9714364416a^7b^8c^8d^{12}e^{11}z^4 - 16777216a^{16}b^3c^6d^5e^{22}z^4 + 98304a^{11}b^{11}c^4d^5e^{22}z^4 + 81920a^6b^{10}c^{12}d^{22}e^5z^4 + 39168a^6b^{21}c^4d^{11}e^{12}z^4 - 1091829760a^5b^6c^{12}d^{18}e^5z^4 + 1046740992a^{14}b^2c^7d^4e^{19}z^4 - 6884425728a^{12}b^3c^{10}d^9e^{14}z^4 + 987445248a^4b^{10}c^9d^{16}e^7z^4 + 984087552a^5b^{12}c^6d^{12}e^{11}z^4 - 9564585984a^9b^7c^7d^9e^{14}z^4 - 5266857984a^{10}b^7c^6d^7e^{16}z^4 - 892145664a^7b^{11}c^5d^9e^{14}z^4 - 2444623872a^{11}b^3c^{11}d^{11}e^{12}z^4 + 768540672a^{12}b^3c^8d^7e^{16}z^4 + 5048322048a^8b^9c^6d^9e^{14}z^4 + 5047612416a^6b^9c^8d^{13}e^{10}z^4 - 732492288a^4b^{11}c^8d^{15}e^8z^4 + 9266921472a^7b^6c^{10}d^{14}e^9z^4 - 645857280a^6b^6c^{11}d^{16}e^7z^4 - 623867904a^4b^9c^{10}d^{17}e^6z^4 - 622067712a^6b^3c^{14}d^{19}e^4z^4 + 582617088a^{10}b^8c^5d^6e^{17}z^4 + 577119744a^7b^{12}c^4d^8e^{15}z^4 + 552566784a^{12}b^6c^
\end{aligned}$$

$$\begin{aligned}
& ^5d^4e^{19z^4} + 549224448a^9b^8c^6d^8e^{15z^4} - 526565376a^9b^{10}c^4d^6e^{17z^4} + 511520256a^{10}b^9c^4d^5e^{18z^4} + 13393723392a^9b^3c^{11}d^{13}e^{10z^4} - 2066350080a^{14}b^3c^8d^5e^{18z^4} + 4718592000a^{13}b^2c^8d^6e^{17z^4} - 314572800a^7b^2c^{14}d^{18}e^5z^4 + 287250432a^4b^{13}c^6d^{13}e^{10z^4} + 4565827584a^{10}b^5c^8d^9e^{14z^4} - 250785792a^4b^{14}c^5d^{12}e^{11z^4} + 235536384a^{13}b^3c^7d^5e^{18z^4} - 232683264a^8b^{11}c^4d^7e^{16z^4} - 199627776a^5b^{14}c^4d^{10}e^{13z^4} - 190267392a^{12}b^7c^4d^3e^{20z^4} + 184891392a^6b^{10}c^7d^{12}e^{11z^4} + 180502528a^4b^7c^{12}d^{19}e^4z^4 + 178877952a^3b^{13}c^7d^{15}e^8z^4 + 172490752a^{14}b^3c^6d^3e^{20z^4} + 163946496a^{13}b^5c^5d^3e^{20z^4} + 155839488a^8b^{12}c^3d^6e^{17z^4} + 155000832a^5b^5c^{13}d^{19}e^4z^4 - 152076288a^4b^6c^{13}d^{20}e^3z^4 - 137592576a^3b^{12}c^8d^{16}e^7z^4 - 133693440a^{14}b^4c^5d^2e^{21z^4} - 116767488a^3b^9c^{11}d^{19}e^4z^4 - 108985344a^3b^{14}c^6d^{14}e^9z^4 - 106223616a^6b^{13}c^4d^9e^{14z^4} + 106119168a^3b^{10}c^{10}d^{18}e^5z^4 + 102432768a^5b^4c^{14}d^{20}e^3z^4 + 102113280a^4b^{12}c^7d^{14}e^9z^4 + 100674048a^5b^9c^9d^{15}e^8z^4 + 90439680a^{13}b^6c^4d^2e^{21z^4} - 86808576a^6b^{14}c^3d^8e^{15z^4} + 86245376a^6b^2c^{15}d^{20}e^3z^4 + 79011840a^4b^8c^{11}d^{18}e^5z^4 + 78345216a^4b^{15}c^4d^{11}e^{12z^4} + 78006528a^{11}b^9c^3d^3e^{20z^4} - 73253376a^9b^{11}c^3d^5e^{18z^4} + 67524608a^3b^8c^{12}d^{20}e^3z^4 + 67108864a^{15}b^2c^6d^2e^{21z^4} - 61590528a^{10}b^{10}c^3d^4e^{19z^4} + 61559808a^5b^{15}c^3d^9e^{14z^4} - 59637760a^5b^3c^{15}d^{21}e^2z^4 + 58638336a^4b^5c^{14}d^{21}e^2z^4 - 40828416a^7b^{13}c^3d^7e^{16z^4} - 35639296a^2b^{12}c^9d^{18}e^5z^4 - 31293440a^{12}b^8c^3d^2e^{21z^4} + 29933568a^5b^{13}c^5d^{11}e^{12z^4} + 27793920a^2b^{11}c^{10}d^{19}e^4z^4 + 27168768a^2b^{13}c^8d^{17}e^6z^4 - 23602176a^7b^{14}c^2d^6e^{17z^4} - 23248896a^3b^7c^{13}d^{21}e^2z^4 + 20929536a^3b^{15}c^5d^{13}e^{10z^4} + 18428928a^9b^{12}c^2d^4e^{19z^4} + 18026496a^6b^{15}c^2d^7e^{16z^4} - 16261632a^{10}b^{11}c^2d^3e^{20z^4} + 15128064a^3b^{16}c^4d^{12}e^{11z^4} - 14060544a^2b^{10}c^{11}d^{20}e^3z^4 + 13178880a^2b^{16}c^5d^{14}e^9z^4 - 11244288a^3b^{17}c^3d^{11}e^{12z^4} - 10509312a^2b^{15}c^6d^{15}e^8z^4 - 7262208a^4b^{17}c^2d^9e^{14z^4} - 7045632a^2b^{17}c^4d^{13}e^{10z^4} - 6285312a^2b^{14}c^7d^{16}e^7z^4 + 5996544a^{11}b^{10}c^2d^2e^{21z^4} + 4558336a^2b^9c^{12}d^{21}e^2z^4 + 4478976a^{11}b^8c^4d^4e^{19z^4} + 2850816a^4b^{16}c^3d^{10}e^{13z^4} + 2629632a^3b^{11}c^9d^{17}e^6z^4 + 2503680a^3b^{18}c^2d^{10}e^{13z^4} + 1627136a^2b^{18}c^3d^{12}e^{11z^4} + 1605120a^8b^{13}c^2d^5e^{18z^4} + 1483776a^5b^{16}c^2d^8e^{15z^4} + 139776a^2b^{19}c^2d^{11}e^{12z^4} - 8542224384a^{10}b^2c^{11}d^{12}e^{11z^4} - 3072b^{22}c^d^{12}e^{11z^4} - 3072b^{12}c^{11}d^{22}e^*z^4 - 1572864a^6c^{17}d^{22}e^*z^4 - 4096a^{10}b^{13}d^e^{22z^4} - 4096a^*b^{22}d^{10}e^{13z^4} - 6144a^{12}b^{10}c^e^{23z^4} - 983040a^5b^*c^{17}d^{23z^4} - 6912a^*b^9c^{13}d^{23z^4} + 1824522240a^{13}c^{10}d^8e^{15z^4} + 1730150400a^{12}c^{11}d^{10}e^{13z^4} + 958922752a^{14}c^9d^6e^{17z^4} - 537919488a^9c^{14}d^{16}e^7z^4 + 508559360a^{11}c^{12}d^{12}e^{11z^4} - 500170752a^{10}c^{13}d^{14}e^9z^4 + 246939648a^{15}c^8d^4e^{19z^4} - 199229440a^8c^{15}d^{18}e^5z^4 - 29884416a^7c^{16}d^{20}e^3z^4 + 25165824a^{16}c^7d^2e^{21z^4} + 236544b^{17}c^6d^{17}e^6z^4 - 202752b^{18}c^5d^{16}e^7z^4 - 202752b^{16}c^7d^{18}e^5z^4 + 126720b^{19}c^4d^{15}e^8z^4 + 126720b^{15}c^8d^{19}e^4z^4 - 56320b^{20}c^3d^{14}e^9z^4 - 56320b^{14}c^9d^{20}e^3z^4 + 16896b^{21}c^2d^{13}e^{10z^4} + 16896b^{13}c^{10}d^{21}e^2z^4 + 110080a^7b^{16}d^4e^{19z^4} + 110080a^4b^{19}d^7e^{16z^4} - 75520a^8b^{15}d^3e^{20z^4} - 75520a^3b^{20}d^8e^{15z^4} - 56320a^6b^{17}d^5e^{18z^4} - 56320a^5b^{18}d^6e^{17z^4} + 25600a^9b^{14}d^2e^{21z^4} + 25600a^2b^{21}d^9e^{14z^4} - 1572864a^{16}b^2c^5e^{23z^4} + 983040a^{15}b^4c^4e^{23z^4} - 327680a^{14}b^6c^3e^{23z^4} + 61440a^{13}b^8c^2e^{23z^4} + 983040a^4b^3c^{16}d^{23z^4} - 385024a^3b^5c^{15}d^{23z^4} + 73728a^2b^7c^{14}d^{23z^4} + 256b^{23}d^{11}e^{12z^4} + 1048576a^{17}c^6e^{23z^4} + 256b^{11}c^{12}d^{23z^4} + 256a^{11}b^{12}e^{23z^4} + 948695040a^8b^*c^{10}d^6e^{13z^2} + 348917760a^7b^*c^{11}d^8e^{11z^2} - 125030400a^9b^*c^9d^4e^{15z^2} - 50728960a^6b^*c^{12}d^{10}e^9z^2 - 44298240a^5b^*c^{13}d^{12}e^7z^2 - 36
\end{aligned}$$

$$\begin{aligned}
& 495360a^{10}b^4c^8d^2e^{17}z^2 + 29675520a^8b^6c^5d^5e^{18}z^2 - 24170496 \\
& a^9b^4c^6d^5e^{18}z^2 - 17202816a^7b^8c^4d^4e^{18}z^2 - 14561280a^4b^* \\
& c^{14}d^{14}e^5z^2 + 5532416a^6b^{10}c^3d^5e^{18}z^2 + 4128768a^{10}b^2c^7* \\
& d^5e^{18}z^2 - 2662400a^3b^*c^{15}d^{16}e^3z^2 + 1184512a^*b^{12}c^6d^9e^{10} \\
& z^2 - 1136160a^*b^{13}c^5d^8e^{11}z^2 - 1017600a^5b^{12}c^2d^5e^{18}z^2 - 7 \\
& 44768a^*b^{11}c^7d^{10}e^9z^2 + 607872a^*b^{14}c^4d^7e^{12}z^2 - 424064a^*b \\
& ^6c^{12}d^{15}e^4z^2 + 408576a^*b^5c^{13}d^{16}e^3z^2 + 361152a^*b^{10}c^8d \\
& ^{11}e^8z^2 - 287408a^*b^9c^9d^{12}e^7z^2 - 260448a^3b^{15}c^d^2e^{17}z^2 \\
& - 203904a^*b^4c^{14}d^{17}e^2z^2 + 200832a^*b^8c^{10}d^{13}e^6z^2 + 12672 \\
& 0a^*b^7c^{11}d^{14}e^5z^2 - 123968a^*b^{15}c^3d^6e^{13}z^2 - 39168a^*b^{16}c \\
& ^2d^5e^{14}z^2 + 11904a^2b^{16}c^d^3e^{16}z^2 + 1824135552a^7b^4c^8d^ \\
& 5e^{14}z^2 - 1457252352a^8b^2c^9d^5e^{14}z^2 - 1405209600a^7b^5c^7d \\
& ^4e^{15}z^2 - 184320a^2b^*c^{16}d^{18}e^z^2 + 100608a^4b^{14}c^d^5e^{18}z^2 + \\
& 53248a^*b^3c^{15}d^{18}e^z^2 + 26448a^*b^{17}c^d^4e^{15}z^2 + 1067599872a^8 \\
& b^3c^8d^4e^{15}z^2 - 930828288a^7b^3c^9d^6e^{13}z^2 + 920760000a^6* \\
& b^4c^9d^7e^{12}z^2 - 806639616a^6b^3c^{10}d^8e^{11}z^2 - 791052480a^6* \\
& b^6c^7d^5e^{14}z^2 + 772237824a^6b^7c^6d^4e^{15}z^2 - 701025408a^5b \\
& ^6c^8d^7e^{12}z^2 + 443340288a^5b^5c^9d^8e^{11}z^2 + 433047552a^7b^ \\
& 6c^6d^3e^{16}z^2 + 405741312a^5b^7c^7d^6e^{13}z^2 + 293652480a^6b^2 \\
& *c^{11}d^9e^{10}z^2 - 276962688a^6b^8c^5d^3e^{16}z^2 - 247804272a^8b^4 \\
& *c^7d^3e^{16}z^2 + 213564384a^4b^8c^7d^7e^{12}z^2 - 202596816a^5b^9* \\
& c^5d^4e^{15}z^2 - 182520896a^4b^9c^6d^6e^{13}z^2 - 153489408a^5b^3c \\
& ^{11}d^{10}e^9z^2 - 152151552a^7b^2c^{10}d^7e^{12}z^2 + 115859712a^5b^2* \\
& c^{12}d^{11}e^8z^2 + 108085248a^9b^3c^7d^2e^{17}z^2 + 105536256a^4b^5* \\
& c^{10}d^{10}e^9z^2 - 98323200a^6b^5c^8d^6e^{13}z^2 - 93564992a^4b^6c^ \\
& 9d^9e^{10}z^2 + 89464512a^5b^{10}c^4d^3e^{16}z^2 - 75930624a^8b^5c^6* \\
& d^2e^{17}z^2 + 68315904a^5b^8c^6d^5e^{14}z^2 - 64157184a^4b^7c^8d^8 \\
& *e^{11}z^2 - 62951040a^9b^2c^8d^3e^{16}z^2 + 49056768a^4b^{10}c^5d^5e \\
& ^{14}z^2 + 47614464a^3b^8c^8d^9e^{10}z^2 + 35604480a^4b^2c^{13}d^{13}e^ \\
& 6z^2 + 33983040a^3b^{11}c^5d^6e^{13}z^2 - 33515520a^4b^3c^{12}d^{12}e^7 \\
& *z^2 - 33463808a^3b^7c^9d^{10}e^9z^2 - 25128864a^4b^4c^{11}d^{11}e^8z \\
& ^2 - 23193728a^3b^{10}c^6d^7e^{12}z^2 + 21015456a^6b^9c^4d^2e^{17}z^2 \\
& + 19924176a^4b^{11}c^4d^4e^{15}z^2 - 19251216a^3b^9c^7d^8e^{11}z^2 - \\
& 16434048a^5b^4c^{10}d^9e^{10}z^2 - 16289664a^3b^{12}c^4d^5e^{14}z^2 - \\
& 15059328a^4b^{12}c^3d^3e^{16}z^2 - 10766016a^2b^{10}c^7d^9e^{10}z^2 - 1 \\
& 0453632a^5b^{11}c^3d^2e^{17}z^2 - 9940992a^3b^3c^{13}d^{14}e^5z^2 + 837 \\
& 3696a^2b^{11}c^6d^8e^{11}z^2 + 7776768a^3b^2c^{14}d^{15}e^4z^2 + 707788 \\
& 8a^3b^5c^{11}d^{12}e^7z^2 + 6798240a^2b^9c^8d^{10}e^9z^2 - 3589440a^ \\
& 2b^6c^{11}d^{13}e^6z^2 + 3544320a^3b^6c^{10}d^{11}e^8z^2 + 3128064a^2b \\
& ^5c^{12}d^{14}e^5z^2 + 2346336a^4b^{13}c^2d^2e^{17}z^2 - 2261568a^2b^8* \\
& c^9d^{11}e^8z^2 - 2125824a^2b^{13}c^4d^6e^{13}z^2 + 2002560a^3b^4c^{12} \\
& *d^{13}e^6z^2 + 1927680a^2b^7c^{10}d^{12}e^7z^2 + 1814784a^2b^{14}c^3d^ \\
& 5e^{14}z^2 - 1807104a^2b^{12}c^5d^7e^{12}z^2 + 1637808a^3b^{13}c^3d^4e \\
& ^{15}z^2 + 1083456a^3b^{14}c^2d^3e^{16}z^2 - 792384a^2b^4c^{13}d^{15}e^4z \\
& ^2 - 657408a^2b^3c^{14}d^{16}e^3z^2 + 608256a^7b^7c^5d^2e^{17}z^2 + \\
& 595968a^2b^2c^{15}d^{17}e^2z^2 - 498624a^2b^{15}c^2d^4e^{15}z^2 - 3840* \\
& b^{18}c^d^5e^{14}z^2 - 3840b^5c^{14}d^{18}e^z^2 + 2064384a^{11}c^8d^5e^{18}z^ \\
& 2 - 4160a^3b^{16}d^5e^{18}z^2 - 4160a^*b^{18}d^3e^{16}z^2 - 1290240a^{11}b^*c^ \\
& 7e^{19}z^2 - 9840a^5b^{13}c^e^{19}z^2 - 5760a^*b^2c^{16}d^{19}z^2 - 28058112 \\
& 0a^8c^{11}d^7e^{12}z^2 + 110278656a^9c^{10}d^5e^{14}z^2 - 89479168a^7c^ \\
& 12d^9e^{10}z^2 + 34464000a^{10}c^9d^3e^{16}z^2 + 54240b^{15}c^4d^8e^{11} \\
& z^2 + 54240b^8c^{11}d^{15}e^4z^2 - 49920b^{14}c^5d^9e^{10}z^2 - 49920b^9 \\
& *c^{10}d^{14}e^5z^2 - 37376b^{16}c^3d^7e^{12}z^2 - 37376b^7c^{12}d^{16}e^3* \\
& z^2 + 28480b^{13}c^6d^{10}e^9z^2 + 28480b^{10}c^9d^{13}e^6z^2 + 15936b^1 \\
& 7c^2d^6e^{13}z^2 + 15936b^6c^{13}d^{17}e^2z^2 - 7920b^{12}c^7d^{11}e^8z \\
& ^2 - 7920b^{11}c^8d^{12}e^7z^2 + 7489536a^5c^{14}d^{13}e^6z^2 + 6084096a \\
& ^6c^{13}d^{11}e^8z^2 + 2280448a^4c^{15}d^{15}e^4z^2 + 350208a^3c^{16}d^{17} \\
& *e^2z^2 + 11616a^2b^{17}d^2e^{17}z^2 - 3515904a^9b^5c^5e^{19}z^2 + 344 \\
& 0640a^{10}b^3c^6e^{19}z^2 + 1870848a^8b^7c^4e^{19}z^2 - 572272a^7b^9*
\end{aligned}$$

$$\begin{aligned}
& c^3e^{19z^2} + 101856a^6b^{11}c^2e^{19z^2} + 400b^{19}d^4e^{15z^2} + 400b^{14}c^{15}d^{19}z^2 + 20736a^2c^{17}d^{19}z^2 + 400a^4b^{15}e^{19z^2} - 396921 \\
& 6a^4b^3c^{10}d^3e^{12} - 3001536a^3b^3c^{11}d^5e^{10} - 419904a^2b^3c^{12}d^7 \\
& e^8 + 184608a^4b^3c^8d^4e^{14} - 153036a^3b^4c^{10}d^6e^9 + 127008a^3b^3 \\
& c^{11}d^7e^8 + 63108a^3b^6c^8d^4e^{11} - 29160a^3b^2c^{12}d^8e^7 - 21060 \\
& a^3b^5c^7d^4e^{14} - 21060a^3b^7c^7d^3e^{12} + 5460a^3b^5c^9d^5e^{10} - \\
& 404544a^5b^3c^9d^4e^{14} + 1251872a^3b^3c^9d^3e^{12} + 844224a^4b^2c^9 \\
& d^2e^{13} + 820512a^2b^3c^{10}d^5e^{10} + 750672a^3b^2c^{10}d^4e^{11} - 6 \\
& 57498a^2b^4c^9d^4e^{11} - 487116a^3b^4c^8d^2e^{13} + 160704a^2b^2c^{11} \\
& d^6e^9 + 58806a^2b^6c^7d^2e^{13} + 13140a^2b^5c^8d^3e^{12} + 152 \\
& 86b^6c^9d^6e^9 - 9540b^7c^8d^5e^{10} - 9540b^5c^{10}d^7e^8 + 2025b^8 \\
& c^7d^4e^{11} + 2025b^4c^{11}d^8e^7 + 3367008a^4c^{11}d^4e^{11} + 11664 \\
& 00a^3c^{12}d^6e^9 + 705600a^5c^{10}d^2e^{13} + 104976a^2c^{13}d^8e^7 - \\
& 17640a^5b^2c^8e^{15} + 2025a^4b^4c^7e^{15} + 38416a^6c^9e^{15}, z, k) * \\
& x*(1048576a^8c^{19}d^{24}e^3 + 9437184a^9c^{18}d^{22}e^5 + 36700160a^{10}c^{17} \\
& d^{20}e^7 + 78643200a^{11}c^{16}d^{18}e^9 + 94371840a^{12}c^{15}d^{16}e^{11} + \\
& 44040192a^{13}c^{14}d^{14}e^{13} - 44040192a^{14}c^{13}d^{12}e^{15} - 94371840a^{15} \\
& c^{12}d^{10}e^{17} - 78643200a^{16}c^{11}d^8e^{19} - 36700160a^{17}c^{10}d^6e^{21} \\
& - 9437184a^{18}c^9d^4e^{23} - 1048576a^{19}c^8d^2e^{25} - 256a^2b^{11}c^1 \\
& 4d^{25}e^2 + 3072a^2b^{12}c^{13}d^{24}e^3 - 16896a^2b^{13}c^{12}d^{23}e^4 + 5 \\
& 6320a^2b^{14}c^{11}d^{22}e^5 - 126720a^2b^{15}c^{10}d^{21}e^6 + 202752a^2b^{16} \\
& c^9d^{20}e^7 - 236544a^2b^{17}c^8d^{19}e^8 + 202752a^2b^{18}c^7d^{18}e^9 \\
& - 126720a^2b^{19}c^6d^{17}e^{10} + 56320a^2b^{20}c^5d^{16}e^{11} - 16896a^2 \\
& b^{21}c^4d^{15}e^{12} + 3072a^2b^{22}c^3d^{14}e^{13} - 256a^2b^{23}c^2d^{13} \\
& e^{14} + 5120a^3b^9c^{15}d^{25}e^2 - 62464a^3b^{10}c^{14}d^{24}e^3 + 346368a^3 \\
& b^{11}c^{13}d^{23}e^4 - 1152256a^3b^{12}c^{12}d^{22}e^5 + 2553600a^3b^{13}c^{11} \\
& d^{21}e^6 - 3951360a^3b^{14}c^{10}d^{20}e^7 + 4336128a^3b^{15}c^9d^{19}e^8 - \\
& 3334656a^3b^{16}c^8d^{18}e^9 + 1700352a^3b^{17}c^7d^{17}e^{10} - 4736 \\
& 00a^3b^{18}c^6d^{16}e^{11} - 8960a^3b^{19}c^5d^{15}e^{12} + 59136a^3b^{20}c^4 \\
& d^{14}e^{13} - 19712a^3b^{21}c^3d^{13}e^{14} + 2304a^3b^{22}c^2d^{12}e^{15} - \\
& 40960a^4b^7c^{16}d^{25}e^2 + 512000a^4b^8c^{15}d^{24}e^3 - 2872320a^4b^9 \\
& c^{14}d^{23}e^4 + 9519104a^4b^{10}c^{13}d^{22}e^5 - 20581120a^4b^{11}c^{12}d^{21} \\
& e^6 + 30087680a^4b^{12}c^{11}d^{20}e^7 - 29433600a^4b^{13}c^{10}d^{19}e^8 \\
& + 17602560a^4b^{14}c^9d^{18}e^9 - 3798528a^4b^{15}c^8d^{17}e^{10} - 307712 \\
& 0a^4b^{16}c^7d^{16}e^{11} + 3028480a^4b^{17}c^6d^{15}e^{12} - 1075200a^4b^{18} \\
& c^5d^{14}e^{13} + 98560a^4b^{19}c^4d^{13}e^{14} + 39424a^4b^{20}c^3d^{12}e^{15} \\
& - 8960a^4b^{21}c^2d^{11}e^{16} + 163840a^5b^5c^{17}d^{25}e^2 - 2129920a^5 \\
& b^6c^{16}d^{24}e^3 + 12165120a^5b^7c^{15}d^{23}e^4 - 39997440a^5b^8c^{14} \\
& d^{22}e^5 + 82611200a^5b^9c^{13}d^{21}e^6 - 107627520a^5b^{10}c^{12}d^{20} \\
& e^7 + 78140160a^5b^{11}c^{11}d^{19}e^8 - 6831360a^5b^{12}c^{10}d^{18}e^9 - 4 \\
& 6586880a^5b^{13}c^9d^{17}e^{10} + 47436800a^5b^{14}c^8d^{16}e^{11} - 20088320 \\
& a^5b^{15}c^7d^{15}e^{12} + 1128960a^5b^{16}c^6d^{14}e^{13} + 2365440a^5b^{17} \\
& c^5d^{13}e^{14} - 788480a^5b^{18}c^4d^{12}e^{15} + 19200a^5b^{19}c^3d^{11}e^{16} \\
& + 19200a^5b^{20}c^2d^{10}e^{17} - 327680a^6b^3c^{18}d^{25}e^2 + 4587520a^6 \\
& b^4c^{17}d^{24}e^3 - 27033600a^6b^5c^{16}d^{23}e^4 + 87162880a^6b^6c^{15} \\
& d^{22}e^5 - 161996800a^6b^7c^{14}d^{21}e^6 + 149237760a^6b^8c^{13}d^{20} \\
& 0e^7 + 27202560a^6b^9c^{12}d^{19}e^8 - 251750400a^6b^{10}c^{11}d^{18}e^9 + \\
& 305948160a^6b^{11}c^{10}d^{17}e^{10} - 160153600a^6b^{12}c^9d^{16}e^{11} + 143 \\
& 360a^6b^{13}c^8d^{15}e^{12} + 46018560a^6b^{14}c^7d^{14}e^{13} - 21683200a^6 \\
& b^{15}c^6d^{13}e^{14} + 1576960a^6b^{16}c^5d^{12}e^{15} + 1305600a^6b^{17}c^4 \\
& d^{11}e^{16} - 215040a^6b^{18}c^3d^{10}e^{17} - 23040a^6b^{19}c^2d^9e^{18} - \\
& 4456448a^7b^2c^{18}d^{24}e^3 + 28114944a^7b^3c^{17}d^{23}e^4 - 84869120a^7 \\
& b^4c^{16}d^{22}e^5 + 104366080a^7b^5c^{15}d^{21}e^6 + 97943552a^7b^6c^{14} \\
& d^{20}e^7 - 549986304a^7b^7c^{13}d^{19}e^8 + 841961472a^7b^8c^{12}d^{18} \\
& e^9 - 549795840a^7b^9c^{11}d^{17}e^{10} - 68823040a^7b^{10}c^{10}d^{16}e^{11} \\
& + 375613952a^7b^{11}c^9d^{15}e^{12} - 240167424a^7b^{12}c^8d^{14}e^{13} + 32 \\
& 840192a^7b^{13}c^7d^{13}e^{14} + 27399680a^7b^{14}c^6d^{12}e^{15} - 10703360a^7 \\
& b^{15}c^5d^{11}e^{16} - 81408a^7b^{16}c^4d^{10}e^{17} + 370176a^7b^{17}c^3 \\
& d^9e^{18} + 10752a^7b^{18}c^2d^8e^{19} + 14680064a^8b^2c^{17}d^{22}e^5 +
\end{aligned}$$

$$\begin{aligned}
& 80281600a^8b^3c^{16}d^{21}e^6 - 440401920a^8b^4c^{15}d^{20}e^7 + 88837324 \\
& 8a^8b^5c^{14}d^{19}e^8 - 703266816a^8b^6c^{13}d^{18}e^9 - 394149888a^8b^7 \\
& c^{12}d^{17}e^{10} + 1358438400a^8b^8c^{11}d^{16}e^{11} - 1129891840a^8b^9 \\
& c^{10}d^{15}e^{12} + 225189888a^8b^{10}c^9d^{14}e^{13} + 246045184a^8b^{11}c^8 \\
& d^{13}e^{14} - 164082688a^8b^{12}c^7d^{12}e^{15} + 18009600a^8b^{13}c^6d^{11}e^{16} \\
& + 10659840a^8b^{14}c^5d^{10}e^{17} - 2099712a^8b^{15}c^4d^9e^{18} - 193 \\
& 536a^8b^{16}c^3d^8e^{19} + 10752a^8b^{17}c^2d^7e^{20} + 239861760a^9b^2 \\
& c^{16}d^{20}e^7 - 172032000a^9b^3c^{15}d^{19}e^8 - 704839680a^9b^4c^{14}d^{18} \\
& e^9 + 2013069312a^9b^5c^{13}d^{17}e^{10} - 2086993920a^9b^6c^{12}d^{16}e^{11} \\
& + 424427520a^9b^7c^{11}d^{15}e^{12} + 1074585600a^9b^8c^{10}d^{14}e^{13} \\
& - 997877760a^9b^9c^9d^{13}e^{14} + 234493952a^9b^{10}c^8d^{12}e^{15} + 957 \\
& 61920a^9b^{11}c^7d^{11}e^{16} - 55288320a^9b^{12}c^6d^{10}e^{17} + 3916800a^9 \\
& b^{13}c^5d^9e^{18} + 1704960a^9b^{14}c^4d^8e^{19} - 250368a^9b^{15}c^3d^7 \\
& e^{20} - 23040a^9b^{16}c^2d^6e^{21} + 857210880a^{10}b^2c^{15}d^{18}e^9 - \\
& 1036124160a^{10}b^3c^{14}d^{17}e^{10} - 255590400a^{10}b^4c^{13}d^{16}e^{11} + 21 \\
& 95128320a^{10}b^5c^{12}d^{15}e^{12} - 2422210560a^{10}b^6c^{11}d^{14}e^{13} + 813 \\
& 711360a^{10}b^7c^{10}d^{13}e^{14} + 420372480a^{10}b^8c^9d^{12}e^{15} - 4285952 \\
& 00a^{10}b^9c^8d^{11}e^{16} + 106106880a^{10}b^{10}c^7d^{10}e^{17} + 8866560a^{10} \\
& b^{11}c^6d^9e^{18} - 11074560a^{10}b^{12}c^5d^8e^{19} + 1989120a^{10}b^{13}c^4 \\
& d^7e^{20} + 537600a^{10}b^{14}c^3d^6e^{21} + 19200a^{10}b^{15}c^2d^5e^{22} \\
& + 1454899200a^{11}b^2c^{14}d^{16}e^{11} - 1747845120a^{11}b^3c^{13}d^{15}e^{12} + \\
& 454164480a^{11}b^4c^{12}d^{14}e^{13} + 1135411200a^{11}b^5c^{11}d^{13}e^{14} - 1 \\
& 286799360a^{11}b^6c^{10}d^{12}e^{15} + 527155200a^{11}b^7c^9d^{11}e^{16} - 4190 \\
& 2080a^{11}b^8c^8d^{10}e^{17} - 74849280a^{11}b^9c^7d^9e^{18} + 53222400a^{11} \\
& b^{10}c^6d^8e^{19} - 4023040a^{11}b^{11}c^5d^7e^{20} - 4972800a^{11}b^{12}c^4 \\
& d^6e^{21} - 456960a^{11}b^{13}c^3d^5e^{22} - 8960a^{11}b^{14}c^2d^4e^{23} + \\
& 1189085184a^{12}b^2c^{13}d^{14}e^{13} - 1241382912a^{12}b^3c^{12}d^{13}e^{14} + 6 \\
& 05552640a^{12}b^4c^{11}d^{12}e^{15} - 97320960a^{12}b^5c^{10}d^{11}e^{16} - 14273 \\
& 7408a^{12}b^6c^9d^{10}e^{17} + 278716416a^{12}b^7c^8d^9e^{18} - 144764928a^{12} \\
& b^8c^7d^8e^{19} - 28779520a^{12}b^9c^6d^7e^{20} + 22077440a^{12}b^{10}c^5 \\
& d^6e^{21} + 4456704a^{12}b^{11}c^4d^5e^{22} + 215552a^{12}b^{12}c^3d^4e^{23} \\
& + 2304a^{12}b^{13}c^2d^3e^{24} + 121110528a^{13}b^2c^{12}d^{12}e^{15} - 1081 \\
& 34400a^{13}b^3c^{11}d^{11}e^{16} + 454164480a^{13}b^4c^{10}d^{10}e^{17} - 5871697 \\
& 92a^{13}b^5c^9d^9e^{18} + 98402304a^{13}b^6c^8d^8e^{19} + 184819712a^{13}b^7 \\
& c^7d^7e^{20} - 39424000a^{13}b^8c^6d^6e^{21} - 22471680a^{13}b^9c^5d^5 \\
& e^{22} - 2151424a^{13}b^{10}c^4d^4e^{23} - 55552a^{13}b^{11}c^3d^3e^{24} - 2 \\
& 56a^{13}b^{12}c^2d^2e^{25} - 644874240a^{14}b^2c^{11}d^{10}e^{17} + 339148800a^{14} \\
& b^3c^{10}d^9e^{18} + 371589120a^{14}b^4c^9d^8e^{19} - 367689728a^{14}b^5 \\
& c^8d^7e^{20} - 32112640a^{14}b^6c^7d^6e^{21} + 59351040a^{14}b^7c^6d^5 \\
& e^{22} + 11366400a^{14}b^8c^5d^4e^{23} + 558080a^{14}b^9c^4d^3e^{24} + 614 \\
& 4a^{14}b^{10}c^3d^2e^{25} - 578027520a^{15}b^2c^{10}d^8e^{19} + 135331840a^{15} \\
& b^3c^9d^7e^{20} + 217907200a^{15}b^4c^8d^6e^{21} - 65372160a^{15}b^5c^7 \\
& d^5e^{22} - 33259520a^{15}b^6c^6d^4e^{23} - 2990080a^{15}b^7c^5d^3e^{24} \\
& - 61440a^{15}b^8c^4d^2e^{25} - 209715200a^{16}b^2c^9d^6e^{21} - 20643840 \\
& a^{16}b^3c^8d^5e^{22} + 49807360a^{16}b^4c^7d^4e^{23} + 9011200a^{16}b^5c^6 \\
& d^3e^{24} + 327680a^{16}b^6c^5d^2e^{25} - 25427968a^{17}b^2c^8d^4e^{23} \\
& - 14483456a^{17}b^3c^7d^3e^{24} - 983040a^{17}b^4c^6d^2e^{25} + 1572864 \\
& a^{18}b^2c^7d^2e^{25} + 262144a^{17}b^3c^{19}d^{25}e^2 - 8650752a^8b^3c^{18}d^{23} \\
& e^4 - 79953920a^9b^3c^{17}d^{21}e^6 - 287047680a^{10}b^3c^{16}d^{19}e^8 - 54 \\
& 2638080a^{11}b^3c^{15}d^{17}e^{10} - 539492352a^{12}b^3c^{14}d^{15}e^{12} - 143130624 \\
& a^{13}b^3c^{13}d^{13}e^{14} + 306708480a^{14}b^3c^{12}d^{11}e^{16} + 420741120a^{15}b^3 \\
& c^{11}d^9e^{18} + 250347520a^{16}b^3c^{10}d^7e^{20} + 76283904a^{17}b^3c^9d^5e^{22} \\
& + 9699328a^{18}b^3c^8d^3e^{24}))/((8*(16a^3b^6c^9d^{18} - a^2b^8c^8d^{18} \\
& - 256a^6c^{12}d^{18} - 96a^4b^4c^{10}d^{18} + 256a^5b^2c^{11}d^{18} - a^2 \\
& b^{16}d^{10}e^8 + 8a^3b^{15}d^9e^9 - 28a^4b^{14}d^8e^{10} + 56a^5b^{13}d^7 \\
& e^{11} - 70a^6b^{12}d^6e^{12} + 56a^7b^{11}d^5e^{13} - 28a^8b^{10}d^4e^{14} \\
& + 8a^9b^9d^3e^{15} - a^{10}b^8d^2e^{16} - 2048a^7c^{11}d^{16}e^2 - 7168a^8 \\
& c^{10}d^{14}e^4 - 14336a^9c^9d^{12}e^6 - 17920a^{10}c^8d^{10}e^8 - 14336 \\
& a^{11}c^7d^8e^{10} - 7168a^{12}c^6d^6e^{12} - 2048a^{13}c^5d^4e^{14} - 256
\end{aligned}$$

$$\begin{aligned}
& a^{14}c^4d^2e^{16} - 28a^2b^{10}c^6d^{16}e^2 + 56a^2b^{11}c^5d^{15}e^3 - \\
& 70a^2b^{12}c^4d^{14}e^4 + 56a^2b^{13}c^3d^{13}e^5 - 28a^2b^{14}c^2d^{12}e^6 + 440a^3b^8c^7d^{16}e^2 - 840a^3b^9c^6d^{15}e^3 + 952a^3b^{10}c^5d^{14}e^4 - \\
& 616a^3b^{11}c^4d^{13}e^5 + 168a^3b^{12}c^3d^{12}e^6 + 40a^3b^{13}c^2d^{11}e^7 - 2560a^4b^6c^8d^{16}e^2 + 4480a^4b^7c^7d^{15}e^3 - \\
& 4060a^4b^8c^6d^{14}e^4 + 1064a^4b^9c^5d^{13}e^5 + 1372a^4b^{10}c^4d^{12}e^6 - 1360a^4b^{11}c^3d^{11}e^7 + 380a^4b^{12}c^2d^{10}e^8 + 6400a^5b^4c^9d^{16}e^2 - \\
& 8960a^5b^5c^8d^{15}e^3 + 2240a^5b^6c^7d^{14}e^4 + 9856a^5b^7c^6d^{13}e^5 - 13048a^5b^8c^5d^{12}e^6 + 5400a^5b^9c^4d^{11}e^7 + 1040a^5b^{10}c^3d^{10}e^8 - \\
& 1360a^5b^{11}c^2d^9e^9 - 5120a^6b^2c^{10}d^{16}e^2 + 22400a^6b^4c^8d^{14}e^4 - 41216a^6b^5c^7d^{13}e^5 + 25088a^6b^6c^6d^{12}e^6 + 8320a^6b^7c^5d^{11}e^7 - \\
& 17350a^6b^8c^4d^{10}e^8 + 5400a^6b^9c^3d^9e^9 + 1372a^6b^{10}c^2d^8e^{10} - 35840a^7b^2c^9d^{14}e^4 + 28672a^7b^3c^8d^{13}e^5 + 30464a^7b^4c^7d^{12}e^6 - \\
& 73472a^7b^5c^6d^{11}e^7 + 40544a^7b^6c^5d^{10}e^8 + 8320a^7b^7c^4d^9e^9 - 13048a^7b^8c^3d^8e^{10} + 1064a^7b^9c^2d^7e^{11} - 93184a^8b^2c^8d^{12}e^6 + \\
& 71680a^8b^3c^7d^{11}e^7 + 29120a^8b^4c^6d^{10}e^8 - 73472a^8b^5c^5d^9e^9 + 25088a^8b^6c^4d^8e^{10} + 9856a^8b^7c^3d^7e^{11} - 4060a^8b^8c^2d^6e^{12} - \\
& 125440a^9b^2c^7d^{10}e^8 + 71680a^9b^3c^6d^9e^9 + 30464a^9b^4c^5d^8e^{10} - 41216a^9b^5c^4d^7e^{11} + 2240a^9b^6c^3d^6e^{12} + 4480a^9b^7c^2d^5e^{13} - \\
& 93184a^{10}b^2c^6d^8e^{10} + 28672a^{10}b^3c^5d^7e^{11} + 22400a^{10}b^4c^4d^6e^{12} - 8960a^{10}b^5c^3d^5e^{13} - 2560a^{10}b^6c^2d^4e^{14} - 35840a^{11}b^2c^5d^6e^{12} + \\
& 6400a^{11}b^4c^3d^4e^{14} + 768a^{11}b^5c^2d^3e^{15} - 5120a^{12}b^2c^4d^4e^{14} - 2048a^{12}b^3c^3d^3e^{15} - 96a^{12}b^4c^2d^2e^{16} + 256a^{13}b^2c^3d^2e^{16} + \\
& 2048a^6b^7c^{11}d^{17}e + 8a^2b^9c^7d^{17}e + 8a^2b^{15}c^4d^{11}e^7 - 128a^3b^7c^8d^{17}e - 40a^3b^{14}c^4d^{10}e^8 + 768a^4b^5c^9d^{17}e + 40a^4b^{13}c^3d^9e^9 - \\
& 2048a^5b^3c^{10}d^{17}e + 168a^5b^{12}c^2d^8e^{10} - 616a^6b^{11}c^2d^7e^{11} + 14336a^7b^3c^{10}d^{15}e^3 + 952a^7b^{10}c^4d^6e^{12} + 43008a^8b^3c^9d^{13}e^5 - \\
& 840a^8b^9c^5d^5e^{13} + 71680a^9b^3c^8d^{11}e^7 + 440a^9b^8c^4d^4e^{14} + 71680a^{10}b^3c^7d^9e^9 - 128a^{10}b^7c^3d^3e^{15} + 43008a^{11}b^3c^6d^7e^{11} + \\
& 16a^{11}b^6c^4d^2e^{16} + 14336a^{12}b^3c^5d^5e^{13} + 2048a^{13}b^3c^4d^3e^{15})) - (x(49152a^{14}b^3c^8e^{23} - 65536a^{14}c^9d^2e^{22} + 16a^8b^{13}c^2e^{23} - \\
& 368a^9b^{11}c^3e^{23} + 3520a^{10}b^9c^4e^{23} - 17920a^{11}b^7c^5e^{23} + 51200a^{12}b^5c^6e^{23} - 77824a^{13}b^3c^7e^{23} + 18432a^4c^{19}d^{21}e^2 + \\
& 243712a^5c^{18}d^{19}e^4 + 1253376a^6c^{17}d^{17}e^6 + 2252800a^7c^{16}d^{15}e^8 - 7835648a^8c^{15}d^{13}e^{10} - 35516416a^9c^{14}d^{11}e^{12} - \\
& 50487296a^{10}c^{13}d^9e^{14} - 30416896a^{11}c^{12}d^7e^{16} - 5797888a^{12}c^{11}d^5e^{18} + 522240a^{13}c^{10}d^3e^{20} + 16b^8c^{15}d^{21}e^2 - \\
& 160b^9c^{14}d^{20}e^3 + 720b^{10}c^{13}d^{19}e^4 - 1904b^{11}c^{12}d^{18}e^5 + 3200b^{12}c^{11}d^{17}e^6 - 3312b^{13}c^{10}d^{16}e^7 + 1440b^{14}c^9d^{15}e^8 + \\
& 1440b^{15}c^8d^{14}e^9 - 3312b^{16}c^7d^{13}e^{10} + 3200b^{17}c^6d^{12}e^{11} - 1904b^{18}c^5d^{11}e^{12} + 720b^{19}c^4d^{10}e^{13} - 160b^{20}c^3d^9e^{14} + \\
& 16b^{21}c^2d^8e^{15} + 3200a^2b^4c^{17}d^{21}e^2 - 30336a^2b^5c^{16}d^{20}e^3 + 123296a^2b^6c^{15}d^{19}e^4 - 269568a^2b^7c^{14}d^{18}e^5 + 295872a^2b^8c^{13}d^{17}e^6 + \\
& 16576a^2b^9c^{12}d^{16}e^7 - 582688a^2b^{10}c^{11}d^{15}e^8 + 944640a^2b^{11}c^{10}d^{14}e^9 - 761856a^2b^{12}c^9d^{13}e^{10} + 243456a^2b^{13}c^8d^{12}e^{11} + \\
& 126048a^2b^{14}c^7d^{11}e^{12} - 164096a^2b^{15}c^6d^{10}e^{13} + 58304a^2b^{16}c^5d^9e^{14} + 3264a^2b^{17}c^4d^8e^{15} - 7648a^2b^{18}c^3d^7e^{16} + \\
& 1536a^2b^{19}c^2d^6e^{17} - 12800a^3b^2c^{18}d^{21}e^2 + 119296a^3b^3c^{17}d^{20}e^3 - 448896a^3b^4c^{16}d^{19}e^4 + 783872a^3b^5c^{15}d^{18}e^5 - 197504a^3b^6c^{14}d^{17}e^6 - \\
& 1977216a^3b^7c^{13}d^{16}e^7 + 4413568a^3b^8c^{12}d^{15}e^8 - 4435520a^3b^9c^{11}d^{14}e^9 + 1422432a^3b^{10}c^{10}d^{13}e^{10} + 1795872a^3b^{11}c^9d^{12}e^{11} - \\
& 2349888a^3b^{12}c^8d^{11}e^{12} + 800352a^3b^{13}c^7d^{10}e^{13} + 426688a^3b^{14}c^6d^9e^{14} - 478112a^3b^{15}c^5d^8e^{15} + 145344a^3b^{16}c^4d^7e^{16} - \\
& 3104a^3b^{17}c^3d^6e^{17} - 4384a^3b^{18}c^2d^5e^{18} + 519680a^4b^2c^{17}d^{19}e^4 - 122880a^4b^3c^{16}d^{18}e^5 - 3229184
\end{aligned}$$

$$\begin{aligned}
& *a^4*b^4*c^{15}*d^{17}*e^6 + 9323008*a^4*b^5*c^{14}*d^{16}*e^7 - 11702656*a^4*b^6*c^{13}*d^{15}*e^8 + 3460864*a^4*b^7*c^{12}*d^{14}*e^9 + 10917472*a^4*b^8*c^{11}*d^{13}*e^{10} - 16615488*a^4*b^9*c^{10}*d^{12}*e^{11} + 7102272*a^4*b^{10}*c^9*d^{11}*e^{12} + 5842272*a^4*b^{11}*c^8*d^{10}*e^{13} - 8942080*a^4*b^{12}*c^7*d^9*e^{14} + 4203232*a^4*b^{13}*c^6*d^8*e^{15} - 364736*a^4*b^{14}*c^5*d^7*e^{16} - 309472*a^4*b^{15}*c^4*d^6*e^{17} + 63136*a^4*b^{16}*c^3*d^5*e^{18} + 6112*a^4*b^{17}*c^2*d^4*e^{19} + 6961152*a^5*b^2*c^{16}*d^{17}*e^6 - 10246144*a^5*b^3*c^{15}*d^{16}*e^7 - 747008*a^5*b^4*c^{14}*d^{15}*e^8 + 29979648*a^5*b^5*c^{13}*d^{14}*e^9 - 52869952*a^5*b^6*c^{12}*d^{13}*e^{10} + 32791616*a^5*b^7*c^{11}*d^{12}*e^{11} + 25176960*a^5*b^8*c^{10}*d^{11}*e^{12} - 62955552*a^5*b^9*c^9*d^{10}*e^{13} + 45989472*a^5*b^{10}*c^8*d^9*e^{14} - 9362688*a^5*b^{11}*c^7*d^8*e^{15} - 5824480*a^5*b^{12}*c^6*d^7*e^{16} + 3196768*a^5*b^{13}*c^5*d^6*e^{17} - 132768*a^5*b^{14}*c^4*d^5*e^{18} - 119680*a^5*b^{15}*c^3*d^4*e^{19} - 4384*a^5*b^{16}*c^2*d^3*e^{20} + 32086016*a^6*b^2*c^{15}*d^{15}*e^8 - 57880576*a^6*b^3*c^{14}*d^{14}*e^9 + 44683008*a^6*b^4*c^{13}*d^{13}*e^{10} + 49481984*a^6*b^5*c^{12}*d^{12}*e^{11} - 175788864*a^6*b^6*c^{11}*d^{11}*e^{12} + 194611968*a^6*b^7*c^{10}*d^{10}*e^{13} - 73867584*a^6*b^8*c^9*d^9*e^{14} - 38225280*a^6*b^9*c^8*d^8*e^{15} + 45450144*a^6*b^{10}*c^7*d^7*e^{16} - 10588672*a^6*b^{11}*c^6*d^6*e^{17} - 2519296*a^6*b^{12}*c^5*d^5*e^{18} + 864384*a^6*b^{13}*c^4*d^4*e^{19} + 96224*a^6*b^{14}*c^3*d^3*e^{20} + 1536*a^6*b^{15}*c^2*d^2*e^{21} + 67527680*a^7*b^2*c^{14}*d^{13}*e^{10} - 181466112*a^7*b^3*c^{13}*d^{12}*e^{11} + 278696704*a^7*b^4*c^{12}*d^{11}*e^{12} - 171431936*a^7*b^5*c^{11}*d^{10}*e^{13} - 104909184*a^7*b^6*c^{10}*d^9*e^{14} + 231100032*a^7*b^7*c^9*d^8*e^{15} - 116105856*a^7*b^8*c^8*d^7*e^{16} - 5653568*a^7*b^9*c^7*d^6*e^{17} + 19556768*a^7*b^{10}*c^6*d^5*e^{18} - 2291488*a^7*b^{11}*c^5*d^4*e^{19} - 855936*a^7*b^{12}*c^4*d^3*e^{20} - 35168*a^7*b^{13}*c^3*d^2*e^{21} - 40418304*a^8*b^2*c^{13}*d^{11}*e^{12} - 155127808*a^8*b^3*c^{12}*d^{10}*e^{13} + 421659136*a^8*b^4*c^{11}*d^9*e^{14} - 366294528*a^8*b^5*c^{10}*d^8*e^{15} + 42953856*a^8*b^6*c^9*d^7*e^{16} + 115841280*a^8*b^7*c^8*d^6*e^{17} - 54301680*a^8*b^8*c^7*d^5*e^{18} - 3139616*a^8*b^9*c^6*d^4*e^{19} + 3850352*a^8*b^{10}*c^5*d^3*e^{20} + 333840*a^8*b^{11}*c^4*d^2*e^{21} - 262465536*a^9*b^2*c^{12}*d^9*e^{14} + 49444864*a^9*b^3*c^{11}*d^8*e^{15} + 255840768*a^9*b^4*c^{10}*d^7*e^{16} - 241492992*a^9*b^5*c^9*d^6*e^{17} + 41574816*a^9*b^6*c^8*d^5*e^{18} + 32344416*a^9*b^7*c^7*d^4*e^{19} - 8542208*a^9*b^8*c^6*d^3*e^{20} - 1677872*a^9*b^9*c^5*d^2*e^{21} - 270632960*a^{10}*b^2*c^{11}*d^7*e^{16} + 105492480*a^{10}*b^3*c^{10}*d^6*e^{17} + 71796864*a^{10}*b^4*c^9*d^5*e^{18} - 66791040*a^{10}*b^5*c^8*d^4*e^{19} + 5437088*a^{10}*b^6*c^7*d^3*e^{20} + 4684288*a^{10}*b^7*c^6*d^2*e^{21} - 105693696*a^{11}*b^2*c^{10}*d^5*e^{18} + 38220288*a^{11}*b^3*c^9*d^4*e^{19} + 10967680*a^{11}*b^4*c^8*d^3*e^{20} - 6778368*a^{11}*b^5*c^7*d^2*e^{21} - 15811072*a^{12}*b^2*c^9*d^3*e^{20} + 3633152*a^{12}*b^3*c^8*d^2*e^{21} - 352*a*b^6*c^{16}*d^{21}*e^2 + 3424*a*b^7*c^{15}*d^{20}*e^3 - 14720*a*b^8*c^{14}*d^{19}*e^4 + 36048*a*b^9*c^{13}*d^{18}*e^5 - 52384*a*b^{10}*c^{12}*d^{17}*e^6 + 36464*a*b^{11}*c^{11}*d^{16}*e^7 + 17952*a*b^{12}*c^{10}*d^{15}*e^8 - 75360*a*b^{13}*c^9*d^{14}*e^9 + 91104*a*b^{14}*c^8*d^{13}*e^{10} - 60992*a*b^{15}*c^7*d^{12}*e^{11} + 20288*a*b^{16}*c^6*d^{11}*e^{12} + 1424*a*b^{17}*c^5*d^{10}*e^{13} - 4320*a*b^{18}*c^4*d^9*e^{14} + 1648*a*b^{19}*c^3*d^8*e^{15} - 224*a*b^{20}*c^2*d^7*e^{16} - 169984*a^4*b*c^{18}*d^{20}*e^3 - 2076672*a^5*b*c^{17}*d^{18}*e^5 - 9658368*a^6*b*c^{16}*d^{16}*e^7 - 16384000*a^7*b*c^{15}*d^{14}*e^9 - 224*a^7*b^{14}*c^2*d*e^{22} + 42463232*a^8*b*c^{14}*d^{12}*e^{11} + 5120*a^8*b^{12}*c^3*d*e^{22} + 170631168*a^9*b*c^{13}*d^{10}*e^{13} - 48576*a^9*b^{10}*c^4*d*e^{22} + 199843840*a^{10}*b*c^{12}*d^8*e^{15} + 244480*a^{10}*b^8*c^5*d*e^{22} + 95387648*a^{11}*b*c^{11}*d^6*e^{17} - 686080*a^{11}*b^6*c^6*d*e^{22} + 15722496*a^{12}*b*c^{10}*d^4*e^{19} + 1007616*a^{12}*b^4*c^7*d*e^{22} + 692224*a^{13}*b*c^9*d^2*e^{21} - 573440*a^{13}*b^2*c^8*d*e^{22}))/((8*(16*a^3*b^6*c^9*d^{18} - a^2*b^8*c^8*d^{18} - 256*a^6*c^{12}*d^{18} - 96*a^4*b^4*c^{10}*d^{18} + 256*a^5*b^2*c^{11}*d^{18} - a^2*b^{16}*d^{10}*e^8 + 8*a^3*b^{15}*d^9*e^9 - 28*a^4*b^{14}*d^8*e^{10} + 56*a^5*b^{13}*d^7*e^{11} - 70*a^6*b^{12}*d^6*e^{12} + 56*a^7*b^{11}*d^5*e^{13} - 28*a^8*b^{10}*d^4*e^{14} + 8*a^9*b^9*d^3*e^{15} - a^{10}*b^8*d^2*e^{16} - 2048*a^7*c^{11}*d^{16}*e^2 - 7168*a^8*c^{10}*d^{14}*e^4 - 14336*a^9*c^9*d^{12}*e^6 - 17920*a^{10}*c^8*d^{10}*e^8 - 14336*a^{11}*c^7*d^8*e^{10} - 7168*a^{12}*c^6*d^6*e^{12} - 2048*a^{13}*c^5*d^4*e^{14} - 256*a^{14}*c^4*d^2*e^{16} - 28*a^2*b^{10}*c^6*d^{16}*e^2 + 56*a^2*b^{11}*c^5*d^{15}*e^3 - 70*a^2*b^{12}*c^4*d^{14}*e^4 + 56*a^2*b^{13}*c^3*d^{13}*e^5 - 28*a^2*b^{14}*c^2*d^{12}*e^6 + 440*a^3*b^8*c^7*d^{16}*e^2 - 840*a^3*b^9*c^6*d^{15}*e^3 + 952*a^3*b^{10}*c^5*d^{14}*e^4 - 616*a^3
\end{aligned}$$

$$\begin{aligned}
& *b^{11}c^4d^{13}e^5 + 168a^3b^{12}c^3d^{12}e^6 + 40a^3b^{13}c^2d^{11}e^7 - \\
& 2560a^4b^6c^8d^{16}e^2 + 4480a^4b^7c^7d^{15}e^3 - 4060a^4b^8c^6d^{14}e^4 + 1064a^4b^9c^5d^{13}e^5 + 1372a^4b^{10}c^4d^{12}e^6 - 1360a^4 \\
& *b^{11}c^3d^{11}e^7 + 380a^4b^{12}c^2d^{10}e^8 + 6400a^5b^4c^9d^{16}e^2 - 8960a^5b^5c^8d^{15}e^3 + 2240a^5b^6c^7d^{14}e^4 + 9856a^5b^7c^6d^{13}e^5 - 13048a^5b^8c^5d^{12}e^6 + 5400a^5b^9c^4d^{11}e^7 + 1040a^5 \\
& b^{10}c^3d^{10}e^8 - 1360a^5b^{11}c^2d^9e^9 - 5120a^6b^2c^{10}d^{16}e^2 + 22400a^6b^4c^8d^{14}e^4 - 41216a^6b^5c^7d^{13}e^5 + 25088a^6b^6c^6d^{12}e^6 + 8320a^6b^7c^5d^{11}e^7 - 17350a^6b^8c^4d^{10}e^8 + 54 \\
& 00a^6b^9c^3d^9e^9 + 1372a^6b^{10}c^2d^8e^{10} - 35840a^7b^2c^9d^{14}e^4 + 28672a^7b^3c^8d^{13}e^5 + 30464a^7b^4c^7d^{12}e^6 - 73472a^7 \\
& *b^5c^6d^{11}e^7 + 40544a^7b^6c^5d^{10}e^8 + 8320a^7b^7c^4d^9e^9 - 13048a^7b^8c^3d^8e^{10} + 1064a^7b^9c^2d^7e^{11} - 93184a^8b^2c^8d^{12}e^6 + 71680a^8b^3c^7d^{11}e^7 + 29120a^8b^4c^6d^{10}e^8 - 73472 \\
& *a^8b^5c^5d^9e^9 + 25088a^8b^6c^4d^8e^{10} + 9856a^8b^7c^3d^7e^{11} - 4060a^8b^8c^2d^6e^{12} - 125440a^9b^2c^7d^{10}e^8 + 71680a^9b^3c^6d^9e^9 + 30464a^9b^4c^5d^8e^{10} - 41216a^9b^5c^4d^7e^{11} + 2 \\
& 240a^9b^6c^3d^6e^{12} + 4480a^9b^7c^2d^5e^{13} - 93184a^{10}b^2c^6d^8e^{10} + 28672a^{10}b^3c^5d^7e^{11} + 22400a^{10}b^4c^4d^6e^{12} - 8960a^{10}b^5c^3d^5e^{13} - 2560a^{10}b^6c^2d^4e^{14} - 35840a^{11}b^2c^5d^6 \\
& *e^{12} + 6400a^{11}b^4c^3d^4e^{14} + 768a^{11}b^5c^2d^3e^{15} - 5120a^{12}b^2c^4d^4e^{14} - 2048a^{12}b^3c^3d^3e^{15} - 96a^{12}b^4c^2d^2e^{16} + 256a^{13}b^2c^3d^2e^{16} + 2048a^6b^6c^{11}d^{17}e + 8a^2b^9c^7d^{17}e + \\
& 8a^2b^{15}c^d^{11}e^7 - 128a^3b^7c^8d^{17}e - 40a^3b^{14}c^d^{10}e^8 + 768a^4b^5c^9d^{17}e + 40a^4b^{13}c^d^9e^9 - 2048a^5b^3c^{10}d^{17}e + 168a^5b^{12}c^d^8e^{10} - 616a^6b^{11}c^d^7e^{11} + 14336a^7b^c^{10}d^{15} \\
& e^3 + 952a^7b^{10}c^d^6e^{12} + 43008a^8b^c^9d^{13}e^5 - 840a^8b^9c^d^5e^{13} + 71680a^9b^c^8d^{11}e^7 + 440a^9b^8c^d^4e^{14} + 71680a^{10}b^c^7d^9e^9 - 128a^{10}b^7c^d^3e^{15} + 43008a^{11}b^c^6d^7e^{11} + 16a^{11}b^6c^d^2e^{16} + 14336a^{12}b^c^5d^5e^{13} + 2048a^{13}b^c^4d^3e^{15})) + \\
& (x*(25a^4b^{10}c^5e^{19} - 6272a^9c^{10}e^{19} - 440a^5b^8c^6e^{19} + 298 \\
& 6a^6b^6c^7e^{19} - 9560a^7b^4c^8e^{19} + 13792a^8b^2c^9e^{19} + 1296a^2c^{17}d^{14}e^5 + 19296a^3c^{16}d^{12}e^7 + 195952a^4c^{15}d^{10}e^9 + 93 \\
& 8176a^5c^{14}d^8e^{11} + 1838832a^6c^{13}d^6e^{13} - 20896a^7c^{12}d^4e^{15} - 57200a^8c^{11}d^2e^{17} + 25b^4c^{15}d^{14}e^5 - 190b^5c^{14}d^{13}e^6 \\
& + 591b^6c^{13}d^{12}e^7 - 964b^7c^{12}d^{11}e^8 + 952b^8c^{11}d^{10}e^9 - 8 \\
& 28b^9c^{10}d^9e^{10} + 952b^{10}c^9d^8e^{11} - 964b^{11}c^8d^7e^{12} + 591b^{12}c^7d^6e^{13} - 190b^{13}c^6d^5e^{14} + 25b^{14}c^5d^4e^{15} + 18816a^ \\
& 2b^2c^{15}d^{12}e^7 - 464a^2b^3c^{14}d^{11}e^8 - 33441a^2b^4c^{13}d^{10}e^9 - 9780a^2b^5c^{12}d^9e^{10} + 98620a^2b^6c^{11}d^8e^{11} - 74420a^2b^7c^{10}d^7e^{12} - 25327a^2b^8c^9d^6e^{13} + 51944a^2b^9c^8d^5e^{14} \\
& - 19162a^2b^{10}c^7d^4e^{15} + 376a^2b^{11}c^6d^3e^{16} + 726a^2b^{12}c^5d^2e^{17} + 132104a^3b^2c^{14}d^{10}e^9 + 202944a^3b^3c^{13}d^9e^{10} - \\
& 496916a^3b^4c^{12}d^8e^{11} + 62420a^3b^5c^{11}d^7e^{12} + 477560a^3b^6c^{10}d^6e^{13} - 367184a^3b^7c^9d^5e^{14} + 42920a^3b^8c^8d^4e^{15} + \\
& 41584a^3b^9c^7d^3e^{16} - 11716a^3b^{10}c^6d^2e^{17} + 774624a^4b^2c^{13}d^8e^{11} + 1091488a^4b^3c^{12}d^7e^{12} - 2078409a^4b^4c^{11}d^6e^{13} + 759546a^4b^5c^{10}d^5e^{14} + 436579a^4b^6c^9d^4e^{15} - 373848a^4 \\
& b^7c^8d^3e^{16} + 68053a^4b^8c^7d^2e^{17} + 2519400a^5b^2c^{12}d^6e^{13} + 1051760a^5b^3c^{11}d^5e^{14} - 2494242a^5b^4c^{10}d^4e^{15} + 1223 \\
& 634a^5b^5c^9d^3e^{16} - 153022a^5b^6c^8d^2e^{17} + 3717952a^6b^2c^{11}d^4e^{15} - 1366224a^6b^3c^{10}d^3e^{16} + 23697a^6b^4c^9d^2e^{17} + \\
& 268408a^7b^2c^{10}d^2e^{17} + 43136a^8b^6c^{10}d^e^{18} - 360a^8b^2c^{16}d^{14}e^5 + 2608a^8b^3c^{15}d^{13}e^6 - 7218a^8b^4c^{14}d^{12}e^7 + 8922a^8b^5c^{13}d^{11}e^8 - 4786a^8b^6c^{12}d^{10}e^9 + 4722a^8b^7c^{11}d^9e^{10} - 12250a^8 \\
& b^8c^{10}d^8e^{11} + 13434a^8b^9c^9d^7e^{12} - 4918a^8b^{10}c^8d^6e^{13} - 1202a^8b^{11}c^7d^5e^{14} + 1308a^8b^{12}c^6d^4e^{15} - 260a^8b^{13}c^5d^3e^{16} - 8928a^2b^c^{16}d^{13}e^6 - 107360a^3b^c^{15}d^{11}e^8 - 260a^3b^{11}c^5d^e^{18} - 846912a^4b^c^{14}d^9e^{10} + 4518a^4b^9c^6d^e^{18} - 3155136*
\end{aligned}$$



$$\begin{aligned}
& a^5 b^3 c^{13} d^7 e^{12} - 30034 a^5 b^7 c^7 d^5 e^{18} - 4176736 a^6 b^3 c^{12} d^5 e^{14} + 92664 a^6 b^5 c^8 d^3 e^{18} - 154080 a^7 b^3 c^{11} d^3 e^{16} - 123488 a^7 b^3 c^9 d^5 e^{18}) / (8 * (16 a^3 b^6 c^9 d^{18} - a^2 b^8 c^8 d^{18} - 256 a^6 c^{12} d^{18} \\
& - 96 a^4 b^4 c^{10} d^{18} + 256 a^5 b^2 c^{11} d^{18} - a^2 b^{16} d^{10} e^8 + 8 a^3 b^{15} d^9 e^9 - 28 a^4 b^{14} d^8 e^{10} + 56 a^5 b^{13} d^7 e^{11} - 70 a^6 b^{12} d^6 e^{12} + 56 a^7 b^{11} d^5 e^{13} - 28 a^8 b^{10} d^4 e^{14} + 8 a^9 b^9 d^3 e^{15} \\
& - a^{10} b^8 d^2 e^{16} - 2048 a^7 c^{11} d^{16} e^2 - 7168 a^8 c^{10} d^{14} e^4 - 14336 a^9 c^9 d^{12} e^6 - 17920 a^{10} c^8 d^{10} e^8 - 14336 a^{11} c^7 d^8 e^{10} - 7168 a^{12} c^6 d^6 e^{12} - 2048 a^{13} c^5 d^4 e^{14} - 256 a^{14} c^4 d^2 e^{16} - 28 a^2 b^{10} c^6 d^{16} e^2 + 56 a^2 b^{11} c^5 d^{15} e^3 - 70 a^2 b^{12} c^4 d^{14} e^4 + 56 a^2 b^{13} c^3 d^{13} e^5 - 28 a^2 b^{14} c^2 d^{12} e^6 + 440 a^3 b^8 c^7 d^{16} e^2 - 840 a^3 b^9 c^6 d^{15} e^3 + 952 a^3 b^{10} c^5 d^{14} e^4 - 616 a^3 b^{11} c^4 d^{13} e^5 + 168 a^3 b^{12} c^3 d^{12} e^6 + 40 a^3 b^{13} c^2 d^{11} e^7 - 2560 a^4 b^6 c^8 d^{16} e^2 + 4480 a^4 b^7 c^7 d^{15} e^3 - 4060 a^4 b^8 c^6 d^{14} e^4 + 1064 a^4 b^9 c^5 d^{13} e^5 + 1372 a^4 b^{10} c^4 d^{12} e^6 - 1360 a^4 b^{11} c^3 d^{11} e^7 + 380 a^4 b^{12} c^2 d^{10} e^8 + 6400 a^5 b^4 c^9 d^{16} e^2 - 8960 a^5 b^5 c^8 d^{15} e^3 + 2240 a^5 b^6 c^7 d^{14} e^4 + 9856 a^5 b^7 c^6 d^{13} e^5 - 13048 a^5 b^8 c^5 d^{12} e^6 + 5400 a^5 b^9 c^4 d^{11} e^7 + 1040 a^5 b^{10} c^3 d^{10} e^8 - 1360 a^5 b^{11} c^2 d^9 e^9 - 5120 a^6 b^2 c^{10} d^{16} e^2 + 22400 a^6 b^4 c^8 d^{14} e^4 - 41216 a^6 b^5 c^7 d^{13} e^5 + 25088 a^6 b^6 c^6 d^{12} e^6 + 8320 a^6 b^7 c^5 d^{11} e^7 - 17350 a^6 b^8 c^4 d^{10} e^8 + 5400 a^6 b^9 c^3 d^9 e^9 + 1372 a^6 b^{10} c^2 d^8 e^{10} - 35840 a^7 b^2 c^9 d^{14} e^4 + 28672 a^7 b^3 c^8 d^{13} e^5 + 30464 a^7 b^4 c^7 d^{12} e^6 - 73472 a^7 b^5 c^6 d^{11} e^7 + 40544 a^7 b^6 c^5 d^{10} e^8 + 8320 a^7 b^7 c^4 d^9 e^9 - 13048 a^7 b^8 c^3 d^8 e^{10} + 1064 a^7 b^9 c^2 d^7 e^{11} - 93184 a^8 b^2 c^8 d^{12} e^6 + 71680 a^8 b^3 c^7 d^{11} e^7 + 29120 a^8 b^4 c^6 d^{10} e^8 - 73472 a^8 b^5 c^5 d^9 e^9 + 25088 a^8 b^6 c^4 d^8 e^{10} + 9856 a^8 b^7 c^3 d^7 e^{11} - 4060 a^8 b^8 c^2 d^6 e^{12} - 125440 a^9 b^2 c^7 d^{10} e^8 + 71680 a^9 b^3 c^6 d^9 e^9 + 30464 a^9 b^4 c^5 d^8 e^{10} - 41216 a^9 b^5 c^4 d^7 e^{11} + 2240 a^9 b^6 c^3 d^6 e^{12} + 4480 a^9 b^7 c^2 d^5 e^{13} - 93184 a^{10} b^2 c^6 d^8 e^{10} + 28672 a^{10} b^3 c^5 d^7 e^{11} + 22400 a^{10} b^4 c^4 d^6 e^{12} - 8960 a^{10} b^5 c^3 d^5 e^{13} - 2560 a^{10} b^6 c^2 d^4 e^{14} - 35840 a^{11} b^2 c^5 d^6 e^{12} + 6400 a^{11} b^4 c^3 d^4 e^{14} + 768 a^{11} b^5 c^2 d^3 e^{15} - 5120 a^{12} b^2 c^4 d^4 e^{14} - 2048 a^{12} b^3 c^3 d^3 e^{15} - 96 a^{12} b^4 c^2 d^2 e^{16} + 256 a^{13} b^2 c^3 d^2 e^{16} + 2048 a^6 b^3 c^{11} d^{17} e + 8 a^2 b^9 c^7 d^{17} e + 8 a^2 b^{15} c^d^{11} e^7 - 128 a^3 b^7 c^8 d^{17} e - 40 a^3 b^{14} c^d^{10} e^8 + 768 a^4 b^5 c^9 d^{17} e + 40 a^4 b^{13} c^d^9 e^9 - 2048 a^5 b^3 c^{10} d^{17} e + 168 a^5 b^{12} c^d^8 e^{10} - 616 a^6 b^{11} c^d^7 e^{11} + 14336 a^7 b^3 c^{10} d^{15} e^3 + 952 a^7 b^{10} c^d^6 e^{12} + 43008 a^8 b^3 c^9 d^{13} e^5 - 840 a^8 b^9 c^d^5 e^{13} + 71680 a^9 b^3 c^8 d^{11} e^7 + 440 a^9 b^8 c^d^4 e^{14} + 71680 a^{10} b^3 c^7 d^9 e^9 - 128 a^{10} b^7 c^d^3 e^{15} + 43008 a^{11} b^3 c^6 d^7 e^{11} + 16 a^{11} b^6 c^d^2 e^{16} + 14336 a^{12} b^3 c^5 d^5 e^{13} + 2048 a^{13} b^3 c^4 d^3 e^{15})) - (3920 a^6 b^3 c^{10} e^{17} + 32144 a^6 c^{11} d^5 e^{16} + 225 a^4 b^5 c^8 e^{17} - 1880 a^5 b^3 c^9 e^{17} + 11664 a^2 c^{15} d^9 e^8 + 46656 a^3 c^{14} d^7 e^{10} - 40608 a^4 c^{13} d^5 e^{12} + 284224 a^5 c^{12} d^3 e^{14} + 225 b^4 c^{13} d^9 e^8 - 755 b^5 c^{12} d^8 e^9 + 530 b^6 c^{11} d^7 e^{10} + 530 b^7 c^{10} d^6 e^{11} - 755 b^8 c^9 d^5 e^{12} + 225 b^9 c^8 d^4 e^{13} + 27648 a^2 b^2 c^{13} d^7 e^{10} + 4576 a^2 b^3 c^{12} d^6 e^{11} + 24438 a^2 b^4 c^{11} d^5 e^{12} - 44262 a^2 b^5 c^{10} d^4 e^{13} + 4042 a^2 b^6 c^9 d^3 e^{14} + 6534 a^2 b^7 c^8 d^2 e^{15} - 23408 a^3 b^2 c^{12} d^5 e^{12} + 41872 a^3 b^3 c^{11} d^4 e^{13} + 100948 a^3 b^4 c^{10} d^3 e^{14} - 60416 a^3 b^5 c^9 d^2 e^{15} - 384384 a^4 b^2 c^{11} d^3 e^{14} + 165216 a^4 b^3 c^{10} d^2 e^{15} - 3240 a^4 b^2 c^{14} d^9 e^8 + 11016 a^4 b^3 c^{13} d^8 e^9 - 8812 a^4 b^4 c^{12} d^7 e^{10} - 1992 a^4 b^5 c^{11} d^6 e^{11} + 408 a^4 b^6 c^{10} d^5 e^{12} + 5216 a^4 b^7 c^9 d^4 e^{13} - 2340 a^4 b^8 c^8 d^3 e^{14} - 40176 a^2 b^3 c^{14} d^8 e^9 - 63360 a^3 b^3 c^{13} d^6 e^{11} - 2340 a^3 b^6 c^8 d^5 e^{16} + 120608 a^4 b^3 c^{12} d^4 e^{13} + 21281 a^4 b^4 c^9 d^5 e^{16} - 114432 a^5 b^3 c^{11} d^2 e^{15} - 55656 a^5 b^2 c^{10} d^5 e^{16}) / (32 * (16 a^3 b^6 c^9 d^{18} - a^2 b^8 c^8 d^{18} - 256 a^6 c^{12} d^{18} - 96 a^4 b^4 c^{10} d^{18} + 256 a^5 b^2 c^{11} d^{18} - a^2 b^{16} d^{10} e^8 + 8 a^3 b^{15} d^9 e^9 - 28 a^4 b^{14} d^8 e^{10} + 56 a^5 b^{13} d^7 e^{11} - 70 a^6 b^{12} d^6 e^{12}
\end{aligned}$$

$$\begin{aligned}
& \sim 6b^{12}d^6e^{12} + 56a^7b^{11}d^5e^{13} - 28a^8b^{10}d^4e^{14} + 8a^9b^9d^3e^{15} - a^{10}b^8d^2e^{16} - 2048a^7c^{11}d^{16}e^2 - 7168a^8c^{10}d^{14}e^4 \\
& - 14336a^9c^9d^{12}e^6 - 17920a^{10}c^8d^{10}e^8 - 14336a^{11}c^7d^8e^{10} - 7168a^{12}c^6d^6e^{12} - 2048a^{13}c^5d^4e^{14} - 256a^{14}c^4d^2e^{16} \\
& - 28a^2b^{10}c^6d^{16}e^2 + 56a^2b^{11}c^5d^{15}e^3 - 70a^2b^{12}c^4d^{14}e^4 + 56a^2b^{13}c^3d^{13}e^5 - 28a^2b^{14}c^2d^{12}e^6 + 440a^3b^8c^7d^{16}e^2 \\
& - 840a^3b^9c^6d^{15}e^3 + 952a^3b^{10}c^5d^{14}e^4 - 616a^3b^{11}c^4d^{13}e^5 + 168a^3b^{12}c^3d^{12}e^6 + 40a^3b^{13}c^2d^{11}e^7 - 2560a^4b^6c^8d^{16}e^2 \\
& + 4480a^4b^7c^7d^{15}e^3 - 4060a^4b^8c^6d^{14}e^4 + 1064a^4b^9c^5d^{13}e^5 + 1372a^4b^{10}c^4d^{12}e^6 - 1360a^4b^{11}c^3d^{11}e^7 \\
& + 380a^4b^{12}c^2d^{10}e^8 + 6400a^5b^4c^9d^{16}e^2 - 8960a^5b^5c^8d^{15}e^3 + 2240a^5b^6c^7d^{14}e^4 + 9856a^5b^7c^6d^{13}e^5 \\
& - 13048a^5b^8c^5d^{12}e^6 + 5400a^5b^9c^4d^{11}e^7 + 1040a^5b^{10}c^3d^{10}e^8 - 1360a^5b^{11}c^2d^9e^9 - 5120a^6b^2c^{10}d^{16}e^2 \\
& + 22400a^6b^4c^8d^{14}e^4 - 41216a^6b^5c^7d^{13}e^5 + 25088a^6b^6c^6d^{12}e^6 + 8320a^6b^7c^5d^{11}e^7 - 17350a^6b^8c^4d^{10}e^8 \\
& + 5400a^6b^9c^3d^9e^9 + 1372a^6b^{10}c^2d^8e^{10} - 35840a^7b^2c^9d^{14}e^4 + 28672a^7b^3c^8d^{13}e^5 + 30464a^7b^4c^7d^{12}e^6 \\
& - 73472a^7b^5c^6d^{11}e^7 + 40544a^7b^6c^5d^{10}e^8 + 8320a^7b^7c^4d^9e^9 - 13048a^7b^8c^3d^8e^{10} + 1064a^7b^9c^2d^7e^{11} \\
& - 93184a^8b^2c^8d^{12}e^6 + 71680a^8b^3c^7d^{11}e^7 + 29120a^8b^4c^6d^{10}e^8 - 73472a^8b^5c^5d^9e^9 + 25088a^8b^6c^4d^8e^{10} \\
& + 9856a^8b^7c^3d^7e^{11} - 4060a^8b^8c^2d^6e^{12} - 125440a^9b^2c^7d^{10}e^8 + 71680a^9b^3c^6d^9e^9 + 30464a^9b^4c^5d^8e^{10} \\
& - 41216a^9b^5c^4d^7e^{11} + 2240a^9b^6c^3d^6e^{12} + 4480a^9b^7c^2d^5e^{13} - 93184a^{10}b^2c^6d^8e^{10} + 28672a^{10}b^3c^5d^7e^{11} \\
& + 22400a^{10}b^4c^4d^6e^{12} - 8960a^{10}b^5c^3d^5e^{13} - 2560a^{10}b^6c^2d^4e^{14} - 35840a^{11}b^2c^5d^6e^{12} + 6400a^{11}b^4c^3d^4e^{14} \\
& + 768a^{11}b^5c^2d^3e^{15} - 5120a^{12}b^2c^4d^4e^{14} - 2048a^{12}b^3c^3d^3e^{15} - 96a^{12}b^4c^2d^2e^{16} + 256a^{13}b^2c^3d^2e^{16} \\
& + 2048a^6b^6c^{11}d^{17}e + 8a^2b^9c^7d^{17}e + 8a^2b^{15}c^6d^{11}e^7 - 128a^3b^7c^8d^{17}e - 40a^3b^{14}c^5d^{10}e^8 \\
& + 768a^4b^5c^9d^{17}e + 40a^4b^{13}c^4d^9e^9 - 2048a^5b^3c^{10}d^{17}e + 168a^5b^{12}c^3d^8e^{10} - 616a^6b^{11}c^2d^7e^{11} \\
& + 14336a^7b^6c^{10}d^{15}e^3 + 952a^7b^{10}c^6d^6e^{12} + 43008a^8b^6c^9d^{13}e^5 - 840a^8b^9c^5d^5e^{13} + 71680a^9b^6c^8d^{11}e^7 \\
& + 440a^9b^8c^4d^4e^{14} + 71680a^{10}b^6c^7d^9e^9 - 128a^{10}b^7c^3d^3e^{15} + 43008a^{11}b^6c^6d^7e^{11} + 16a^{11}b^6c^6d^2e^{16} \\
& + 14336a^{12}b^6c^5d^5e^{13} + 2048a^{13}b^6c^4d^3e^{15})) \cdot \text{root}(128723189760a^{14}b^4c^9d^{13}e^{14}z^6 + 128723189760a^{12}b^4c^{11}d^{17}e^{10}z^6 \\
& - 8432455680a^{11}b^{12}c^4d^{11}e^{16}z^6 - 8432455680a^7b^{12}c^8d^{19}e^8z^6 + 12673351680a^{11}b^{11}c^5d^{12}e^{15}z^6 + 12673351680a^8b^{11}c^8d^{18}e^9z^6 \\
& - 72637480960a^{12}b^9c^6d^{12}e^{15}z^6 - 72637480960a^9b^9c^9d^{18}e^9z^6 - 21048344576a^9b^{12}c^6d^{15}e^{12}z^6 - 16609443840a^{17}b^3c^7d^8e^{19}z^6 \\
& - 16609443840a^{10}b^3c^{14}d^{22}e^5z^6 + 145332633600a^{13}b^5c^9d^{14}e^{13}z^6 + 145332633600a^{12}b^5c^{10}d^{16}e^{11}z^6 \\
& + 123740356608a^{14}b^5c^8d^{12}e^{15}z^6 + 123740356608a^{11}b^5c^{11}d^{18}e^9z^6 + 3460300800a^{17}b^5c^5d^6e^{21}z^6 + 3460300800a^8b^5c^{14}d^{24}e^3z^6 \\
& - 7751073792a^{15}b^7c^5d^8e^{19}z^6 - 7751073792a^8b^7c^{12}d^{22}e^5z^6 + 12041846784a^{14}b^7c^6d^{10}e^{17}z^6 + 12041846784a^9b^7c^{11}d^{20}e^7z^6 \\
& - 325545099264a^{14}b^3c^{10}d^{14}e^{13}z^6 - 325545099264a^{13}b^3c^{11}d^{16}e^{11}z^6 - 3330539520a^{13}b^{10}c^4d^9e^{18}z^6 \\
& - 3330539520a^7b^{10}c^{10}d^{21}e^6z^6 + 157789716480a^{12}b^7c^8d^{14}e^{13}z^6 + 157789716480a^{11}b^7c^9d^{16}e^{11}z^6 + 37492359168a^{11}b^{10}c^6d^{13}e^{14}z^6 \\
& + 37492359168a^9b^{10}c^8d^{17}e^{10}z^6 + 301989888a^8b^3c^{16}d^{26}e^3z^6 - 7266631680a^{17}b^4c^6d^7e^{20}z^6 - 7266631680a^9b^4c^{14}d^{23}e^4z^6 \\
& - 201326592a^{20}b^6c^6d^4e^{23}z^6 - 188743680a^7b^5c^{15}d^{26}e^3z^6 + 45747339264a^{13}b^8c^6d^{11}e^{16}z^6 + 45747339264a^9b^8c^{10}d^{19}e^8z^6 \\
& - 74612736a^{10}b^{16}c^6d^9e^{18}z^6 - 2768240640a^{16}b^7c^4d^6e^{21}z^6 - 2768240640a^7b^7c^{13}d^{24}e^3z^6 + 69746688a^{11}b^{15}c^6d^8e^{19}z^6 \\
& + 62914560a^6b^7c^{14}d^{26}e^3z^6 + 2
\end{aligned}$$

$$\begin{aligned}
& 752020480a^{10}b^{13}c^4d^{12}e^{15}z^6 + 2752020480a^7b^{13}c^7d^{18}e^9z^6 + 55148544a^9b^{17}c^4d^{10}e^{17}z^6 - 45957120a^{12}b^{14}c^4d^7e^{20}z^6 - \\
& 2724986880a^{14}b^9c^4d^8e^{19}z^6 - 2724986880a^7b^9c^{11}d^{22}e^5z^6 - 25952256a^8b^{18}c^4d^{11}e^{16}z^6 + 21086208a^{13}b^{13}c^4d^6e^{21}z^6 - \\
& 11796480a^5b^9c^{13}d^{26}e^4z^6 - 6438912a^{14}b^{12}c^4d^5e^{22}z^6 + 5406720a^7b^{19}c^4d^{12}e^{15}z^6 + 1622016a^6b^{20}c^4d^{13}e^{14}z^6 - 1523712a^5b^{21}c^4d^{14}e^{13}z^6 + 1179648a^{15}b^{11}c^4d^4e^{23}z^6 + 1179648a^4b^{11}c^{12}d^{26}e^4z^6 + 442368a^4b^{22}c^4d^{15}e^{12}z^6 - 98304a^{16}b^{10}c^4d^3e^{24}z^6 - 49152a^3b^{23}c^4d^{16}e^{11}z^6 - 49152a^3b^{13}c^{11}d^{26}e^4z^6 + 6897106944a^9b^{13}c^5d^{14}e^{13}z^6 + 6897106944a^8b^{13}c^6d^{16}e^{11}z^6 - 2422210560a^{16}b^6c^5d^7e^{20}z^6 - 2422210560a^8b^6c^{13}d^23e^4z^6 + 255785435136a^{14}b^2c^{11}d^{15}e^{12}z^6 + 41004564480a^{15}b^4c^8d^{11}e^{16}z^6 + 41004564480a^{11}b^4c^{12}d^{19}e^8z^6 + 2270822400a^{13}b^{11}c^3d^8e^{19}z^6 + 2270822400a^6b^{11}c^{10}d^{22}e^5z^6 + 23677108224a^{14}b^8c^5d^9e^{18}z^6 + 23677108224a^8b^8c^{11}d^{21}e^6z^6 + 212600881152a^{15}b^2c^{10}d^{13}e^{14}z^6 + 212600881152a^{13}b^2c^{12}d^{17}e^{10}z^6 + 75157733376a^{15}b^5c^7d^{10}e^{17}z^6 + 75157733376a^{10}b^5c^{12}d^{20}e^7z^6 - 251217838080a^{13}b^6c^8d^{13}e^{14}z^6 - 251217838080a^{11}b^6c^{10}d^{17}e^{10}z^6 - 1952907264a^{14}b^{10}c^3d^7e^{20}z^6 - 1952907264a^6b^{10}c^{11}d^{23}e^4z^6 - 27691057152a^{13}b^9c^5d^{10}e^{17}z^6 - 27691057152a^8b^9c^{10}d^{20}e^7z^6 - 1902673920a^8b^{15}c^4d^{14}e^{13}z^6 - 1902673920a^7b^{15}c^5d^{16}e^{11}z^6 + 10465050624a^{10}b^{11}c^6d^{14}e^{13}z^6 + 10465050624a^9b^{11}c^7d^{16}e^{11}z^6 + 1613905920a^9b^{14}c^4d^{13}e^{14}z^6 + 1613905920a^7b^{14}c^6d^{17}e^{10}z^6 - 33218887680a^{17}b^9c^9d^{10}e^{17}z^6 - 33218887680a^{12}b^9c^{14}d^{20}e^7z^6 + 1524695040a^{10}b^{14}c^3d^{11}e^{16}z^6 + 1524695040a^6b^{14}c^7d^{19}e^8z^6 - 1472200704a^{18}b^4c^5d^5e^{22}z^6 - 1472200704a^8b^4c^{15}d^{25}e^2z^6 - 83047219200a^{16}b^3c^8d^{10}e^{17}z^6 - 83047219200a^{11}b^3c^{13}d^{20}e^7z^6 + 44291850240a^{17}b^2c^8d^9e^{18}z^6 + 44291850240a^{11}b^2c^{14}d^{21}e^6z^6 + 1308131328a^8b^{14}c^5d^{15}e^{12}z^6 - 201326592a^9b^9c^{17}d^{26}e^4z^6 + 48530718720a^{12}b^8c^7d^{13}e^{14}z^6 + 48530718720a^{10}b^8c^9d^{17}e^{10}z^6 - 1242644480a^{12}b^{12}c^3d^9e^{18}z^6 - 1242644480a^6b^{12}c^9d^{21}e^6z^6 + 9813196800a^{12}b^{10}c^5d^{11}e^{16}z^6 + 9813196800a^8b^{10}c^9d^{19}e^8z^6 - 93012885504a^{15}b^9c^{11}d^{14}e^{13}z^6 - 93012885504a^{14}b^9c^{12}d^{16}e^{11}z^6 + 177305812992a^{13}b^4c^{10}d^{15}e^{12}z^6 + 52730658816a^{10}b^{10}c^7d^{15}e^{12}z^6 - 1180106752a^9b^{15}c^3d^{12}e^{15}z^6 - 1180106752a^6b^{15}c^6d^{18}e^9z^6 + 1023672320a^{15}b^9c^3d^6e^{21}z^6 + 1023672320a^6b^9c^{12}d^{24}e^3z^6 + 975175680a^{17}b^6c^4d^5e^{22}z^6 + 975175680a^7b^6c^{14}d^{25}e^2z^6 - 11072962560a^{18}b^6c^8d^8e^{19}z^6 - 11072962560a^{11}b^6c^{15}d^{22}e^5z^6 + 9412018176a^{18}b^2c^7d^7e^{20}z^6 + 9412018176a^{10}b^2c^{15}d^{23}e^4z^6 + 805306368a^{19}b^2c^6d^5e^{22}z^6 + 805306368a^9b^2c^{16}d^{25}e^2z^6 - 133809831936a^{14}b^6c^7d^{11}e^{16}z^6 - 133809831936a^{10}b^6c^{11}d^{19}e^8z^6 - 2214592512a^{19}b^6c^7d^6e^{21}z^6 - 2214592512a^{10}b^6c^{16}d^{24}e^3z^6 + 82216747008a^{13}b^7c^7d^7e^{12}z^6 + 82216747008a^{10}b^7c^{10}d^{18}e^9z^6 - 586629120a^{12}b^{13}c^2d^8e^{19}z^6 - 586629120a^5b^{13}c^9d^{22}e^5z^6 + 568565760a^7b^{16}c^4d^{15}e^{12}z^6 - 4844421120a^{16}b^4c^7d^9e^{18}z^6 - 4844421120a^{10}b^4c^{13}d^{21}e^6z^6 + 531210240a^{11}b^{14}c^2d^9e^{18}z^6 + 531210240a^5b^{14}c^8d^{21}e^6z^6 - 527155200a^{11}b^{13}c^3d^{10}e^{17}z^6 - 527155200a^6b^{13}c^8d^{20}e^7z^6 + 43470028800a^{11}b^8c^8d^{15}e^{12}z^6 - 107874877440a^{11}b^9c^7d^{14}e^{13}z^6 - 107874877440a^{10}b^9c^8d^{16}e^{11}z^6 + 9018408960a^{12}b^{11}c^4d^{10}e^{17}z^6 + 9018408960a^7b^{11}c^9d^{20}e^7z^6 + 421994496a^{13}b^{12}c^2d^7e^{20}z^6 + 421994496a^5b^{12}c^{10}d^{23}e^4z^6 - 66437775360a^{16}b^9c^{10}d^{12}e^{15}z^6 - 66437775360a^{13}b^9c^{13}d^{18}e^9z^6 + 26159874048a^{16}b^5c^6d^8e^{19}z^6 + 26159874048a^9b^5c^{13}d^{22}e^5z^6 - 369098752a^{18}b^3c^6d^6e^{21}z^6 - 369098752a^9b^3c^{15}d^{24}e^3z^6 + 351436800a^8b^{16}c^3d^{13}e^{14}z^6 + 351436800a^6b^{16}c^5d^{17}e^{10}z^6 - 334233600a^{16}b^8c^3d^5e^{22}z^6 - 334233600a^6b^8c^{13}d^{25}e^2z^6 + 301989888a^{19}b^3c^5d^4e^{23}z^6 - 2
\end{aligned}$$

$66010624a^{10}b^{15}c^2d^{10}e^{17}z^6 - 266010624a^5b^{15}c^7d^{20}e^7z^6$   
 $- 305198530560a^{12}b^6c^9d^{15}e^{12}z^6 - 203292672a^{14}b^{11}c^2d^6e^2$   
 $1z^6 - 203292672a^5b^{11}c^{11}d^{24}e^3z^6 - 188743680a^{18}b^5c^4d^4e$   
 $^{23}z^6 + 120418467840a^{16}b^2c^9d^{11}e^{16}z^6 + 120418467840a^{12}b^2c$   
 $^{13}d^{19}e^8z^6 - 17293934592a^{10}b^{12}c^5d^{13}e^{14}z^6 - 17293934592a^$   
 $8b^{12}c^7d^{17}e^{10}z^6 + 104890368a^8b^{17}c^2d^{12}e^{15}z^6 + 104890368$   
 $a^5b^{17}c^5d^{18}e^9z^6 + 4390256640a^{15}b^8c^4d^7e^{20}z^6 + 4390256$   
 $640a^7b^8c^{12}d^{23}e^4z^6 - 91750400a^6b^{18}c^3d^{15}e^{12}z^6 + 79134$   
 $720a^7b^{17}c^3d^{14}e^{13}z^6 + 79134720a^6b^{17}c^4d^{16}e^{11}z^6 - 7461$   
 $2736a^4b^{16}c^7d^{21}e^6z^6 - 72990720a^7b^{18}c^2d^{13}e^{14}z^6 - 7299$   
 $0720a^5b^{18}c^4d^{17}e^{10}z^6 + 69746688a^4b^{15}c^8d^{22}e^5z^6 + 6370$   
 $0992a^{15}b^{10}c^2d^5e^{22}z^6 + 63700992a^5b^{10}c^{12}d^{25}e^2z^6 + 629$   
 $14560a^{17}b^7c^3d^4e^{23}z^6 + 55148544a^4b^{17}c^6d^{20}e^7z^6 - 4595$   
 $7120a^4b^{14}c^9d^{23}e^4z^6 - 25952256a^4b^{18}c^5d^{19}e^8z^6 - 25165$   
 $824a^{20}b^2c^5d^3e^{24}z^6 + 21086208a^4b^{13}c^{10}d^{24}e^3z^6 + 20643$   
 $840a^6b^{19}c^2d^{14}e^{13}z^6 + 20643840a^5b^{19}c^3d^{16}e^{11}z^6 + 1572$   
 $8640a^{19}b^4c^4d^3e^{24}z^6 - 11796480a^{16}b^9c^2d^4e^{23}z^6 - 64389$   
 $12a^4b^{12}c^{11}d^{25}e^2z^6 + 5406720a^4b^{19}c^4d^{18}e^9z^6 - 5242880$   
 $a^{18}b^6c^3d^3e^{24}z^6 + 3784704a^3b^{18}c^6d^{21}e^6z^6 - 3244032a^$   
 $3b^{19}c^5d^{20}e^7z^6 - 3244032a^3b^{17}c^7d^{22}e^5z^6 + 2027520a^3b$   
 $^{20}c^4d^{19}e^8z^6 + 2027520a^3b^{16}c^8d^{23}e^4z^6 - 1622016a^9b^{16}$   
 $c^2d^{11}e^{16}z^6 - 1622016a^5b^{16}c^6d^{19}e^8z^6 + 1622016a^4b^{20}c$   
 $^3d^{17}e^{10}z^6 - 1523712a^4b^{21}c^2d^{16}e^{11}z^6 + 983040a^{17}b^8c^2$   
 $d^3e^{24}z^6 - 901120a^3b^{21}c^3d^{18}e^9z^6 - 901120a^3b^{15}c^9d^{24}$   
 $e^3z^6 + 270336a^3b^{22}c^2d^{17}e^{10}z^6 + 270336a^3b^{14}c^{10}d^{25}e^$   
 $2z^6 + 172032a^5b^{20}c^2d^{15}e^{12}z^6 - 38593888256a^{15}b^6c^6d^9e^$   
 $18z^6 - 38593888256a^9b^6c^{12}d^{21}e^6z^6 - 210386288640a^{15}b^3c^9$   
 $d^{12}e^{15}z^6 - 210386288640a^{12}b^3c^{12}d^{18}e^9z^6 + 15502147584a^{15}$   
 $c^{12}d^{15}e^{12}z^6 + 1107296256a^{19}c^8d^7e^{20}z^6 + 1107296256a^{11}c^1$   
 $6d^{23}e^4z^6 + 13287555072a^{16}c^{11}d^{13}e^{14}z^6 + 13287555072a^{14}c^1$   
 $3d^{17}e^{10}z^6 + 201326592a^{20}c^7d^5e^{22}z^6 + 201326592a^{10}c^{17}d^2$   
 $5e^2z^6 + 16777216a^{21}c^6d^3e^{24}z^6 + 3784704a^9b^{18}d^9e^{18}z^6$   
 $- 3244032a^{10}b^{17}d^8e^{19}z^6 - 3244032a^8b^{19}d^{10}e^{17}z^6 + 2027520$   
 $a^{11}b^{16}d^7e^{20}z^6 + 2027520a^7b^{20}d^{11}e^{16}z^6 - 901120a^{12}b^{15}$   
 $d^6e^{21}z^6 - 901120a^6b^{21}d^{12}e^{15}z^6 + 270336a^{13}b^{14}d^5e^{22}z$   
 $^6 + 270336a^5b^{22}d^{13}e^{14}z^6 - 49152a^{14}b^{13}d^4e^{23}z^6 - 49152a$   
 $^4b^{23}d^{14}e^{13}z^6 + 4096a^{15}b^{12}d^3e^{24}z^6 + 4096a^3b^{24}d^{15}e^$   
 $12z^6 - 25165824a^8b^2c^{17}d^{27}z^6 + 15728640a^7b^4c^{16}d^{27}z^6 -$   
 $5242880a^6b^6c^{15}d^{27}z^6 + 983040a^5b^8c^{14}d^{27}z^6 - 98304a^4b^$   
 $10c^{13}d^{27}z^6 + 4096a^3b^{12}c^{12}d^{27}z^6 + 8304721920a^{17}c^{10}d^{11}$   
 $e^{16}z^6 + 8304721920a^{13}c^{14}d^{19}e^8z^6 + 3690987520a^{18}c^9d^9e^{18}$   
 $z^6 + 3690987520a^{12}c^{15}d^{21}e^6z^6 + 16777216a^9c^{18}d^{27}z^6 - 849$   
 $3371392a^6b^8c^9d^{14}e^9z^4 + 1458044928a^8b^8c^{14}d^{17}e^6z^4 - 126$   
 $04538880a^{11}b^4c^8d^8e^{15}z^4 - 8303067136a^9b^5c^9d^{11}e^{12}z^4 -$   
 $5588058112a^{13}b^8c^9d^7e^{16}z^4 - 3892838400a^8b^2c^{13}d^{16}e^7z^4$   
 $- 3611713536a^8b^8c^7d^{10}e^{13}z^4 + 7819006464a^7b^9c^7d^{11}e^{12}z$   
 $^4 - 7782137856a^8b^7c^8d^{11}e^{12}z^4 + 7780433920a^{12}b^2c^9d^8e^1$   
 $5z^4 - 12020465664a^7b^5c^{11}d^{15}e^8z^4 + 3176792064a^8b^3c^{12}d^1$   
 $5e^8z^4 - 322633728a^{15}b^8c^7d^3e^{20}z^4 + 210829312a^7b^8c^{15}d^{19}e$   
 $^4z^4 + 15623258112a^9b^6c^8d^{10}e^{13}z^4 + 25165824a^{15}b^3c^5d^8e^$   
 $22z^4 - 15728640a^{14}b^5c^4d^8e^{22}z^4 + 12582912a^5b^2c^{16}d^{22}e^z^$   
 $4 - 11730944a^4b^4c^{15}d^{22}e^z^4 + 5242880a^{13}b^7c^3d^8e^{22}z^4 - 45$   
 $61920a^8b^{15}c^7d^{17}e^6z^4 + 4521984a^3b^6c^{14}d^{22}e^z^4 + 4460544a$   
 $b^{14}c^8d^{18}e^5z^4 + 3538944a^6b^8c^{16}d^{21}e^2z^4 + 3108864a^8b^{16}c$   
 $^6d^{16}e^7z^4 - 3027200a^8b^{13}c^9d^{19}e^4z^4 - 2345472a^5b^{17}c^7d^7$   
 $e^{16}z^4 - 2307072a^8b^{14}c^4d^4e^{19}z^4 + 1824768a^6b^{16}c^6d^6e^{17}z^$   
 $4 + 1734912a^9b^{13}c^3d^3e^{20}z^4 + 1419264a^8b^{12}c^{10}d^{20}e^3z^4 - 11$   
 $91168a^8b^{17}c^5d^{15}e^8z^4 - 983040a^{12}b^9c^2d^8e^{22}z^4 + 964608a^4$   
 $b^{18}c^8d^8e^{15}z^4 - 866304a^2b^8c^{13}d^{22}e^z^4 + 703488a^7b^{15}c^8d$

$$\begin{aligned}
& ^5e^{18z^4} - 608256a^{10}b^{12}c^2d^2e^{21z^4} - 440832a^2b^{11}c^{11}d^{21}e^{2z^4} + 275968a^2b^{19}c^3d^{13}e^{10z^4} - 159744a^2b^{20}c^2d^{10}e^{13z^4} - \\
& 153600a^2b^{20}c^2d^{12}e^{11z^4} + 64512a^3b^{19}c^2d^9e^{14z^4} + 197460623 \\
& 36a^8b^6c^9d^{12}e^{11z^4} - 15333588992a^{10}b^4c^9d^{10}e^{13z^4} + 670 \\
& 2170112a^7b^4c^{12}d^{16}e^7z^4 + 15167913984a^{10}b^3c^{10}d^{11}e^{12z^4} \\
& - 2256638976a^5b^{11}c^7d^{13}e^{10z^4} + 2254307328a^5b^7c^{11}d^{17}e^6 \\
& z^4 - 2200633344a^6b^5c^{12}d^{17}e^6z^4 + 6457131008a^{11}b^3c^9d^9e \\
& ^{14z^4} - 2128785408a^5b^8c^{10}d^{16}e^7z^4 - 2126057472a^6b^{11}c^6d^{11} \\
& e^{12z^4} + 2038349824a^{12}b^5c^6d^5e^{18z^4} + 2037841920a^5b^{10}c^8 \\
& d^{14}e^9z^4 + 3615621120a^9b^6c^{13}d^{15}e^8z^4 + 1900019712a^{11}b^2c \\
& ^{10}d^{10}e^{13z^4} + 1867698432a^9b^9c^5d^7e^{16z^4} - 6157369344a^9b^4 \\
& c^{10}d^{12}e^{11z^4} - 1856913408a^7b^{10}c^6d^{10}e^{13z^4} + 1789132800a \\
& ^6b^4c^{13}d^{18}e^5z^4 + 6082658304a^8b^4c^{11}d^{14}e^9z^4 + 602954956 \\
& 8a^{11}b^5c^7d^7e^{16z^4} + 6010159104a^6b^7c^{10}d^{15}e^8z^4 + 170318 \\
& 2336a^7b^7c^9d^{13}e^{10z^4} + 1658388480a^{11}b^6c^6d^6e^{17z^4} + 591 \\
& 7114368a^{10}b^6c^7d^8e^{15z^4} - 1591197696a^{11}b^7c^5d^5e^{18z^4} - \\
& 1526464512a^8b^{10}c^5d^8e^{15z^4} - 5772607488a^{12}b^4c^7d^6e^{17z^4} \\
& - 1423507456a^{13}b^4c^6d^4e^{19z^4} - 1387266048a^7b^3c^{13}d^{17}e^6z \\
& ^4 + 2976120832a^{10}b^3c^{12}d^{13}e^{10z^4} - 9906946048a^9b^2c^{12}d^{14}e \\
& ^9z^4 - 18421874688a^8b^5c^{10}d^{13}e^{10z^4} + 1141217280a^6b^{12}c^5d \\
& ^{10}e^{13z^4} - 9714364416a^7b^8c^8d^{12}e^{11z^4} - 16777216a^{16}b^3c^6d \\
& *e^{22z^4} + 98304a^{11}b^{11}c^2d^2e^{22z^4} + 81920a^2b^{10}c^{12}d^{22}e^2z^4 + 3 \\
& 9168a^2b^{21}c^2d^{11}e^{12z^4} - 1091829760a^5b^6c^{12}d^{18}e^5z^4 + 104674 \\
& 0992a^{14}b^2c^7d^4e^{19z^4} - 6884425728a^{12}b^3c^{10}d^9e^{14z^4} + 9874 \\
& 45248a^4b^{10}c^9d^{16}e^7z^4 + 984087552a^5b^{12}c^6d^{12}e^{11z^4} - 95 \\
& 64585984a^9b^7c^7d^9e^{14z^4} - 5266857984a^{10}b^7c^6d^7e^{16z^4} - \\
& 892145664a^7b^{11}c^5d^9e^{14z^4} - 2444623872a^{11}b^3c^{11}d^{11}e^{12z^4} \\
& + 768540672a^{12}b^3c^8d^7e^{16z^4} + 5048322048a^8b^9c^6d^9e^{14z^4} \\
& + 5047612416a^6b^9c^8d^{13}e^{10z^4} - 732492288a^4b^{11}c^8d^{15}e^8z \\
& ^4 + 9266921472a^7b^6c^{10}d^{14}e^9z^4 - 645857280a^6b^6c^{11}d^{16}e^7 \\
& z^4 - 623867904a^4b^9c^{10}d^{17}e^6z^4 - 622067712a^6b^3c^{14}d^{19}e^4 \\
& z^4 + 582617088a^{10}b^8c^5d^6e^{17z^4} + 577119744a^7b^{12}c^4d^8e^5 \\
& ^{15z^4} + 552566784a^{12}b^6c^5d^4e^{19z^4} + 549224448a^9b^8c^6d^8e^5 \\
& ^{15z^4} - 526565376a^9b^{10}c^4d^6e^{17z^4} + 511520256a^{10}b^9c^4d^5e \\
& ^{18z^4} + 13393723392a^9b^3c^{11}d^{13}e^{10z^4} - 2066350080a^{14}b^3c^8d^ \\
& 5e^{18z^4} + 4718592000a^{13}b^2c^8d^6e^{17z^4} - 314572800a^7b^2c^{14} \\
& d^{18}e^5z^4 + 287250432a^4b^{13}c^6d^{13}e^{10z^4} + 4565827584a^{10}b^5c \\
& ^8d^9e^{14z^4} - 250785792a^4b^{14}c^5d^{12}e^{11z^4} + 235536384a^{13}b^3 \\
& c^7d^5e^{18z^4} - 232683264a^8b^{11}c^4d^7e^{16z^4} - 199627776a^5b^{11} \\
& 4c^4d^{10}e^{13z^4} - 190267392a^{12}b^7c^4d^3e^{20z^4} + 184891392a^6b \\
& ^{10}c^7d^{12}e^{11z^4} + 180502528a^4b^7c^{12}d^{19}e^4z^4 + 178877952a^3 \\
& b^{13}c^7d^{15}e^8z^4 + 172490752a^{14}b^3c^6d^3e^{20z^4} + 163946496a^ \\
& ^{13}b^5c^5d^3e^{20z^4} + 155839488a^8b^{12}c^3d^6e^{17z^4} + 155000832a \\
& ^5b^5c^{13}d^{19}e^4z^4 - 152076288a^4b^6c^{13}d^{20}e^3z^4 - 137592576a \\
& ^3b^{12}c^8d^{16}e^7z^4 - 133693440a^{14}b^4c^5d^2e^{21z^4} - 116767488 \\
& a^3b^9c^{11}d^{19}e^4z^4 - 108985344a^3b^{14}c^6d^{14}e^9z^4 - 10622361 \\
& 6a^6b^{13}c^4d^9e^{14z^4} + 106119168a^3b^{10}c^{10}d^{18}e^5z^4 + 102432 \\
& 768a^5b^4c^{14}d^{20}e^3z^4 + 102113280a^4b^{12}c^7d^{14}e^9z^4 + 10067 \\
& 4048a^5b^9c^9d^{15}e^8z^4 + 90439680a^{13}b^6c^4d^2e^{21z^4} - 868085 \\
& 76a^6b^{14}c^3d^8e^{15z^4} + 86245376a^6b^2c^{15}d^{20}e^3z^4 + 7901184 \\
& 0a^4b^8c^{11}d^{18}e^5z^4 + 78345216a^4b^{15}c^4d^{11}e^{12z^4} + 7800652 \\
& 8a^{11}b^9c^3d^3e^{20z^4} - 73253376a^9b^{11}c^3d^5e^{18z^4} + 67524608 \\
& a^3b^8c^{12}d^{20}e^3z^4 + 67108864a^{15}b^2c^6d^2e^{21z^4} - 61590528a \\
& ^{10}b^{10}c^3d^4e^{19z^4} + 61559808a^5b^{15}c^3d^9e^{14z^4} - 59637760a \\
& ^5b^3c^{15}d^{21}e^2z^4 + 58638336a^4b^5c^{14}d^{21}e^2z^4 - 40828416a \\
& ^7b^{13}c^3d^7e^{16z^4} - 35639296a^2b^{12}c^9d^{18}e^5z^4 - 31293440a^ \\
& ^{12}b^8c^3d^2e^{21z^4} + 29933568a^5b^{13}c^5d^{11}e^{12z^4} + 27793920a^ \\
& ^2b^{11}c^{10}d^{19}e^4z^4 + 27168768a^2b^{13}c^8d^{17}e^6z^4 - 23602176a^ \\
& ^7b^{14}c^2d^6e^{17z^4} - 23248896a^3b^7c^{13}d^{21}e^2z^4 + 20929536a^3
\end{aligned}$$

$$\begin{aligned}
& *b^{15}c^5d^{13}e^{10}z^4 + 18428928a^9b^{12}c^2d^4e^{19}z^4 + 18026496a^6 \\
& *b^{15}c^2d^7e^{16}z^4 - 16261632a^{10}b^{11}c^2d^3e^{20}z^4 + 15128064a^3 \\
& *b^{16}c^4d^{12}e^{11}z^4 - 14060544a^2b^{10}c^{11}d^{20}e^3z^4 + 13178880a^ \\
& 2*b^{16}c^5d^{14}e^9z^4 - 11244288a^3b^{17}c^3d^{11}e^{12}z^4 - 10509312a^ \\
& 2*b^{15}c^6d^{15}e^8z^4 - 7262208a^4b^{17}c^2d^9e^{14}z^4 - 7045632a^2b \\
& ^{17}c^4d^{13}e^{10}z^4 - 6285312a^2b^{14}c^7d^{16}e^7z^4 + 5996544a^{11}b^ \\
& 10*c^2d^2e^{21}z^4 + 4558336a^2b^9c^{12}d^{21}e^2z^4 + 4478976a^{11}b^8* \\
& c^4d^4e^{19}z^4 + 2850816a^4b^{16}c^3d^{10}e^{13}z^4 + 2629632a^3b^{11}c^ \\
& 9d^{17}e^6z^4 + 2503680a^3b^{18}c^2d^{10}e^{13}z^4 + 1627136a^2b^{18}c^3* \\
& d^{12}e^{11}z^4 + 1605120a^8b^{13}c^2d^5e^{18}z^4 + 1483776a^5b^{16}c^2d^ \\
& 8e^{15}z^4 + 139776a^2b^{19}c^2d^{11}e^{12}z^4 - 8542224384a^{10}b^2c^{11}d \\
& ^{12}e^{11}z^4 - 3072b^{22}c*d^{12}e^{11}z^4 - 3072b^{12}c^{11}d^{22}e*z^4 - 1572 \\
& 864a^6c^{17}d^{22}e*z^4 - 4096a^{10}b^{13}d*e^{22}z^4 - 4096a*b^{22}d^{10}e^{13} \\
& *z^4 - 6144a^{12}b^{10}c*e^{23}z^4 - 983040a^5b*c^{17}d^{23}z^4 - 6912a*b^9* \\
& c^{13}d^{23}z^4 + 1824522240a^{13}c^{10}d^8e^{15}z^4 + 1730150400a^{12}c^{11}d^ \\
& 10e^{13}z^4 + 958922752a^{14}c^9d^6e^{17}z^4 - 537919488a^9c^{14}d^{16}e^7 \\
& *z^4 + 508559360a^{11}c^{12}d^{12}e^{11}z^4 - 500170752a^{10}c^{13}d^{14}e^9z^4 \\
& + 246939648a^{15}c^8d^4e^{19}z^4 - 199229440a^8c^{15}d^{18}e^5z^4 - 2988 \\
& 4416a^7c^{16}d^{20}e^3z^4 + 25165824a^{16}c^7d^2e^{21}z^4 + 236544b^{17}c \\
& ^6d^{17}e^6z^4 - 202752b^{18}c^5d^{16}e^7z^4 - 202752b^{16}c^7d^{18}e^5z \\
& ^4 + 126720b^{19}c^4d^{15}e^8z^4 + 126720b^{15}c^8d^{19}e^4z^4 - 56320b^ \\
& 20*c^3d^{14}e^9z^4 - 56320b^{14}c^9d^{20}e^3z^4 + 16896b^{21}c^2d^{13}e^1 \\
& 0z^4 + 16896b^{13}c^{10}d^{21}e^2z^4 + 110080a^7b^{16}d^4e^{19}z^4 + 11008 \\
& 0a^4b^{19}d^7e^{16}z^4 - 75520a^8b^{15}d^3e^{20}z^4 - 75520a^3b^{20}d^8* \\
& e^{15}z^4 - 56320a^6b^{17}d^5e^{18}z^4 - 56320a^5b^{18}d^6e^{17}z^4 + 2560 \\
& 0a^9b^{14}d^2e^{21}z^4 + 25600a^2b^{21}d^9e^{14}z^4 - 1572864a^{16}b^2c^ \\
& 5e^{23}z^4 + 983040a^{15}b^4c^4e^{23}z^4 - 327680a^{14}b^6c^3e^{23}z^4 + \\
& 61440a^{13}b^8c^2e^{23}z^4 + 983040a^4b^3c^{16}d^{23}z^4 - 385024a^3b^5 \\
& *c^{15}d^{23}z^4 + 73728a^2b^7c^{14}d^{23}z^4 + 256b^{23}d^{11}e^{12}z^4 + 104 \\
& 8576a^{17}c^6e^{23}z^4 + 256b^{11}c^{12}d^{23}z^4 + 256a^{11}b^{12}e^{23}z^4 + \\
& 948695040a^8b*c^{10}d^6e^{13}z^2 + 348917760a^7b*c^{11}d^8e^{11}z^2 - 125 \\
& 030400a^9b*c^9d^4e^{15}z^2 - 50728960a^6b*c^{12}d^{10}e^9z^2 - 44298240 \\
& *a^5b*c^{13}d^{12}e^7z^2 - 36495360a^{10}b*c^8d^2e^{17}z^2 + 29675520a^8* \\
& b^6c^5d*e^{18}z^2 - 24170496a^9b^4c^6d*e^{18}z^2 - 17202816a^7b^8c^4 \\
& *d*e^{18}z^2 - 14561280a^4b*c^{14}d^{14}e^5z^2 + 5532416a^6b^{10}c^3d*e^1 \\
& 8z^2 + 4128768a^{10}b^2c^7d*e^{18}z^2 - 2662400a^3b*c^{15}d^{16}e^3z^2 + \\
& 1184512a*b^{12}c^6d^9e^{10}z^2 - 1136160a*b^{13}c^5d^8e^{11}z^2 - 101760 \\
& 0a^5b^{12}c^2d*e^{18}z^2 - 744768a*b^{11}c^7d^{10}e^9z^2 + 607872a*b^{14} \\
& c^4d^7e^{12}z^2 - 424064a*b^6c^{12}d^{15}e^4z^2 + 408576a*b^5c^{13}d^{16} \\
& e^3z^2 + 361152a*b^{10}c^8d^{11}e^8z^2 - 287408a*b^9c^9d^{12}e^7z^2 - \\
& 260448a^3b^{15}c*d^2e^{17}z^2 - 203904a*b^4c^{14}d^{17}e^2z^2 + 200832a* \\
& b^8c^{10}d^{13}e^6z^2 + 126720a*b^7c^{11}d^{14}e^5z^2 - 123968a*b^{15}c^3* \\
& d^6e^{13}z^2 - 39168a*b^{16}c^2d^5e^{14}z^2 + 11904a^2b^{16}c*d^3e^{16}z^ \\
& 2 + 1824135552a^7b^4c^8d^5e^{14}z^2 - 1457252352a^8b^2c^9d^5e^{14}z \\
& ^2 - 1405209600a^7b^5c^7d^4e^{15}z^2 - 184320a^2b*c^{16}d^{18}e*z^2 + 1 \\
& 00608a^4b^{14}c*d*e^{18}z^2 + 53248a*b^3c^{15}d^{18}e*z^2 + 26448a*b^{17}c* \\
& d^4e^{15}z^2 + 1067599872a^8b^3c^8d^4e^{15}z^2 - 930828288a^7b^3c^9* \\
& d^6e^{13}z^2 + 920760000a^6b^4c^9d^7e^{12}z^2 - 806639616a^6b^3c^{10} \\
& d^8e^{11}z^2 - 791052480a^6b^6c^7d^5e^{14}z^2 + 772237824a^6b^7c^6d \\
& ^4e^{15}z^2 - 701025408a^5b^6c^8d^7e^{12}z^2 + 443340288a^5b^5c^9d^ \\
& 8e^{11}z^2 + 433047552a^7b^6c^6d^3e^{16}z^2 + 405741312a^5b^7c^7d^6 \\
& *e^{13}z^2 + 293652480a^6b^2c^{11}d^9e^{10}z^2 - 276962688a^6b^8c^5d^3 \\
& *e^{16}z^2 - 247804272a^8b^4c^7d^3e^{16}z^2 + 213564384a^4b^8c^7d^7* \\
& e^{12}z^2 - 202596816a^5b^9c^5d^4e^{15}z^2 - 182520896a^4b^9c^6d^6e \\
& ^{13}z^2 - 153489408a^5b^3c^{11}d^{10}e^9z^2 - 152151552a^7b^2c^{10}d^7* \\
& e^{12}z^2 + 115859712a^5b^2c^{12}d^{11}e^8z^2 + 108085248a^9b^3c^7d^2* \\
& e^{17}z^2 + 105536256a^4b^5c^{10}d^{10}e^9z^2 - 98323200a^6b^5c^8d^6e \\
& ^{13}z^2 - 93564992a^4b^6c^9d^9e^{10}z^2 + 89464512a^5b^{10}c^4d^3e^1 \\
& 6z^2 - 75930624a^8b^5c^6d^2e^{17}z^2 + 68315904a^5b^8c^6d^5e^{14}z
\end{aligned}$$

$$\begin{aligned}
&^2 - 64157184*a^4*b^7*c^8*d^8*e^{11}*z^2 - 62951040*a^9*b^2*c^8*d^3*e^{16}*z^2 \\
&+ 49056768*a^4*b^{10}*c^5*d^5*e^{14}*z^2 + 47614464*a^3*b^8*c^8*d^9*e^{10}*z^2 + \\
&35604480*a^4*b^2*c^{13}*d^{13}*e^6*z^2 + 33983040*a^3*b^{11}*c^5*d^6*e^{13}*z^2 - 3 \\
&3515520*a^4*b^3*c^{12}*d^{12}*e^7*z^2 - 33463808*a^3*b^7*c^9*d^{10}*e^9*z^2 - 251 \\
&28864*a^4*b^4*c^{11}*d^{11}*e^8*z^2 - 23193728*a^3*b^{10}*c^6*d^7*e^{12}*z^2 + 2101 \\
&5456*a^6*b^9*c^4*d^2*e^{17}*z^2 + 19924176*a^4*b^{11}*c^4*d^4*e^{15}*z^2 - 192512 \\
&16*a^3*b^9*c^7*d^8*e^{11}*z^2 - 16434048*a^5*b^4*c^{10}*d^9*e^{10}*z^2 - 16289664 \\
&*a^3*b^{12}*c^4*d^5*e^{14}*z^2 - 15059328*a^4*b^{12}*c^3*d^3*e^{16}*z^2 - 10766016* \\
&a^2*b^{10}*c^7*d^9*e^{10}*z^2 - 10453632*a^5*b^{11}*c^3*d^2*e^{17}*z^2 - 9940992*a^ \\
&3*b^3*c^{13}*d^{14}*e^5*z^2 + 8373696*a^2*b^{11}*c^6*d^8*e^{11}*z^2 + 7776768*a^3*b \\
&^2*c^{14}*d^{15}*e^4*z^2 + 7077888*a^3*b^5*c^{11}*d^{12}*e^7*z^2 + 6798240*a^2*b^9* \\
&c^8*d^{10}*e^9*z^2 - 3589440*a^2*b^6*c^{11}*d^{13}*e^6*z^2 + 3544320*a^3*b^6*c^{10} \\
&*d^{11}*e^8*z^2 + 3128064*a^2*b^5*c^{12}*d^{14}*e^5*z^2 + 2346336*a^4*b^{13}*c^2*d^ \\
&2*e^{17}*z^2 - 2261568*a^2*b^8*c^9*d^{11}*e^8*z^2 - 2125824*a^2*b^{13}*c^4*d^6*e^ \\
&13*z^2 + 2002560*a^3*b^4*c^{12}*d^{13}*e^6*z^2 + 1927680*a^2*b^7*c^{10}*d^{12}*e^7* \\
&z^2 + 1814784*a^2*b^{14}*c^3*d^5*e^{14}*z^2 - 1807104*a^2*b^{12}*c^5*d^7*e^{12}*z^2 \\
&+ 1637808*a^3*b^{13}*c^3*d^4*e^{15}*z^2 + 1083456*a^3*b^{14}*c^2*d^3*e^{16}*z^2 - \\
&792384*a^2*b^4*c^{13}*d^{15}*e^4*z^2 - 657408*a^2*b^3*c^{14}*d^{16}*e^3*z^2 + 60825 \\
&6*a^7*b^7*c^5*d^2*e^{17}*z^2 + 595968*a^2*b^2*c^{15}*d^{17}*e^2*z^2 - 498624*a^2* \\
&b^{15}*c^2*d^4*e^{15}*z^2 - 3840*b^{18}*c*d^5*e^{14}*z^2 - 3840*b^5*c^{14}*d^{18}*e*z^2 \\
&+ 2064384*a^{11}*c^8*d*e^{18}*z^2 - 4160*a^3*b^{16}*d*e^{18}*z^2 - 4160*a*b^{18}*d^3 \\
&*e^{16}*z^2 - 1290240*a^{11}*b*c^7*e^{19}*z^2 - 9840*a^5*b^{13}*c*e^{19}*z^2 - 5760*a \\
&*b^2*c^{16}*d^{19}*z^2 - 280581120*a^8*c^{11}*d^7*e^{12}*z^2 + 110278656*a^9*c^{10}*d \\
&^5*e^{14}*z^2 - 89479168*a^7*c^{12}*d^9*e^{10}*z^2 + 34464000*a^{10}*c^9*d^3*e^{16}*z \\
&^2 + 54240*b^{15}*c^4*d^8*e^{11}*z^2 + 54240*b^8*c^{11}*d^{15}*e^4*z^2 - 49920*b^{14} \\
&*c^5*d^9*e^{10}*z^2 - 49920*b^9*c^{10}*d^{14}*e^5*z^2 - 37376*b^{16}*c^3*d^7*e^{12}*z \\
&^2 - 37376*b^7*c^{12}*d^{16}*e^3*z^2 + 28480*b^{13}*c^6*d^{10}*e^9*z^2 + 28480*b^{10} \\
&*c^9*d^{13}*e^6*z^2 + 15936*b^{17}*c^2*d^6*e^{13}*z^2 + 15936*b^6*c^{13}*d^{17}*e^2*z \\
&^2 - 7920*b^{12}*c^7*d^{11}*e^8*z^2 - 7920*b^{11}*c^8*d^{12}*e^7*z^2 + 7489536*a^5* \\
&c^{14}*d^{13}*e^6*z^2 + 6084096*a^6*c^{13}*d^{11}*e^8*z^2 + 2280448*a^4*c^{15}*d^{15}*e \\
&^4*z^2 + 350208*a^3*c^{16}*d^{17}*e^2*z^2 + 11616*a^2*b^{17}*d^2*e^{17}*z^2 - 35159 \\
&04*a^9*b^5*c^5*e^{19}*z^2 + 3440640*a^{10}*b^3*c^6*e^{19}*z^2 + 1870848*a^8*b^7*c \\
&^4*e^{19}*z^2 - 572272*a^7*b^9*c^3*e^{19}*z^2 + 101856*a^6*b^{11}*c^2*e^{19}*z^2 + \\
&400*b^{19}*d^4*e^{15}*z^2 + 400*b^4*c^{15}*d^{19}*z^2 + 20736*a^2*c^{17}*d^{19}*z^2 + 4 \\
&00*a^4*b^{15}*e^{19}*z^2 - 3969216*a^4*b*c^{10}*d^3*e^{12} - 3001536*a^3*b*c^{11}*d^5 \\
&*e^{10} - 419904*a^2*b*c^{12}*d^7*e^8 + 184608*a^4*b^3*c^8*d*e^{14} - 153036*a*b^ \\
&4*c^{10}*d^6*e^9 + 127008*a*b^3*c^{11}*d^7*e^8 + 63108*a*b^6*c^8*d^4*e^{11} - 291 \\
&60*a*b^2*c^{12}*d^8*e^7 - 21060*a^3*b^5*c^7*d*e^{14} - 21060*a*b^7*c^7*d^3*e^{12} \\
&+ 5460*a*b^5*c^9*d^5*e^{10} - 404544*a^5*b*c^9*d*e^{14} + 1251872*a^3*b^3*c^9* \\
&d^3*e^{12} + 844224*a^4*b^2*c^9*d^2*e^{13} + 820512*a^2*b^3*c^{10}*d^5*e^{10} + 750 \\
&672*a^3*b^2*c^{10}*d^4*e^{11} - 657498*a^2*b^4*c^9*d^4*e^{11} - 487116*a^3*b^4*c^ \\
&8*d^2*e^{13} + 160704*a^2*b^2*c^{11}*d^6*e^9 + 58806*a^2*b^6*c^7*d^2*e^{13} + 131 \\
&40*a^2*b^5*c^8*d^3*e^{12} + 15286*b^6*c^9*d^6*e^9 - 9540*b^7*c^8*d^5*e^{10} - 9 \\
&540*b^5*c^{10}*d^7*e^8 + 2025*b^8*c^7*d^4*e^{11} + 2025*b^4*c^{11}*d^8*e^7 + 3367 \\
&008*a^4*c^{11}*d^4*e^{11} + 1166400*a^3*c^{12}*d^6*e^9 + 705600*a^5*c^{10}*d^2*e^{13} \\
&+ 104976*a^2*c^{13}*d^8*e^7 - 17640*a^5*b^2*c^8*e^{15} + 2025*a^4*b^4*c^7*e^{15} \\
&+ 38416*a^6*c^9*e^{15}, z, k), k, 1, 6) - ((x*(a^2*b^2*e^4 - 4*a^3*c*e^4 - 2 \\
&*a*c^3*d^4 + b^2*c^2*d^4 + b^4*d^2*e^2 + 2*a^2*c^2*d^2*e^2 - 2*b^3*c*d^3*e \\
&+ 6*a*b*c^2*d^3*e - 4*a*b^2*c*d^2*e^2))/(2*a*d*(4*a*c^3*d^4 + 4*a^3*c*e^4 - \\
&a^2*b^2*e^4 - b^2*c^2*d^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^ \\
&3 + 2*b^3*c*d^3*e - 8*a*b*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2)) \\
&+ (x^3*(a*b^3*e^4 + b*c^3*d^4 + b^4*d*e^3 + 2*a^2*c^2*d*e^3 - b^2*c^2*d^3* \\
&e - b^3*c*d^2*e^2 - 4*a^2*b*c*e^4 + 2*a*c^3*d^3*e - 4*a*b^2*c*d*e^3 + 3*a*b \\
&*c^2*d^2*e^2))/(2*a*d*(4*a*c^3*d^4 + 4*a^3*c*e^4 - a^2*b^2*e^4 - b^2*c^2*d^4 - \\
&b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^3 + 2*b^3*c*d^3*e - 8*a*b \\
&*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2)) + (c*e*x^5*(a*b^2*e^3 + \\
&b*c^2*d^3 - 4*a^2*c*e^3 + b^3*d*e^2 + 4*a*c^2*d^2*e - 2*b^2*c*d^2*e - 3*a*b \\
&*c*d*e^2))/(2*a*d*(4*a*c^3*d^4 + 4*a^3*c*e^4 - a^2*b^2*e^4 - b^2*c^2*d^4 - \\
&b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^3 + 2*b^3*c*d^3*e - 8*a*b*c^2
\end{aligned}$$

```
*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2)))/(a*d + x^2*(a*e + b*d) + x^4*(b*e + c*d) + c*e*x^6)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```



$$3.199 \quad \int (d + ex^2)^{5/2} (a + bx^2 + cx^4) dx$$

**Optimal.** Leaf size=215

$$\frac{x(d + ex^2)^{5/2} (80ae^2 - 10bde + 3cd^2)}{480e^2} + \frac{dx(d + ex^2)^{3/2} (80ae^2 - 10bde + 3cd^2)}{384e^2} + \frac{d^2x\sqrt{d + ex^2} (80ae^2 - 10bde + 3cd^2)}{256e^2}$$

**Rubi [A]** time = 0.16, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$ , Rules used = {1159, 388, 195, 217, 206}

$$\frac{x(d + ex^2)^{5/2} (80ae^2 - 10bde + 3cd^2)}{480e^2} + \frac{dx(d + ex^2)^{3/2} (80ae^2 - 10bde + 3cd^2)}{384e^2} + \frac{d^2x\sqrt{d + ex^2} (80ae^2 - 10bde + 3cd^2)}{256e^2} + \frac{d^3 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d + ex^2}}\right) (80ae^2 - 10bde + 3cd^2)}{256e^{5/2}} - \frac{x(d + ex^2)^{7/2} (3cd - 10be)}{80e^2} + \frac{cx^3 (d + ex^2)^{7/2}}{10e}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^(5/2)\*(a + b\*x^2 + c\*x^4), x]

[Out] (d^2\*(3\*c\*d^2 - 10\*b\*d\*e + 80\*a\*e^2)\*x\*sqrt[d + e\*x^2])/(256\*e^2) + (d\*(3\*c\*d^2 - 10\*b\*d\*e + 80\*a\*e^2)\*x\*(d + e\*x^2)^(3/2))/(384\*e^2) + ((3\*c\*d^2 - 10\*b\*d\*e + 80\*a\*e^2)\*x\*(d + e\*x^2)^(5/2))/(480\*e^2) - ((3\*c\*d - 10\*b\*e)\*x\*(d + e\*x^2)^(7/2))/(80\*e^2) + (c\*x^3\*(d + e\*x^2)^(7/2))/(10\*e) + (d^3\*(3\*c\*d^2 - 10\*b\*d\*e + 80\*a\*e^2)\*ArcTanh[(sqrt[e]\*x)/sqrt[d + e\*x^2]])/(256\*e^(5/2))

Rule 195

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^p)/(n\*p + 1), x] + Dist[(a\*n\*p)/(n\*p + 1), Int[(a + b\*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2\*p] || (EqQ[n, 2] && IntegerQ[4\*p]) || (EqQ[n, 2] && IntegerQ[3\*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

Rule 1159

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] := Simp[(c^p\*x^(4\*p - 1)\*(d + e\*x^2)^(q + 1))/(e\*(4\*p + 2\*q + 1)), x] + Dist[1/(e\*(4\*p + 2\*q + 1)), Int[(d + e\*x^2)^q\*ExpandToSum[e\*(4\*p + 2\*q + 1)\*(a + b\*x^2 + c\*x^4)^p - d\*c^p\*(4\*p - 1)\*x^(4\*p - 2) - e\*c^p\*(4\*p + 2\*q + 1)\*x^(4\*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && !LtQ[q, -1]

Rubi steps

$$\begin{aligned}
\int (d + ex^2)^{5/2} (a + bx^2 + cx^4) dx &= \frac{cx^3 (d + ex^2)^{7/2}}{10e} + \frac{\int (d + ex^2)^{5/2} (10ae - (3cd - 10be)x^2) dx}{10e} \\
&= -\frac{(3cd - 10be)x (d + ex^2)^{7/2}}{80e^2} + \frac{cx^3 (d + ex^2)^{7/2}}{10e} - \frac{1}{80} \left( -80a - \frac{d(3cd - 10be)}{e^2} \right) \\
&= \frac{1}{480} \left( 80a + \frac{d(3cd - 10be)}{e^2} \right) x (d + ex^2)^{5/2} - \frac{(3cd - 10be)x (d + ex^2)^{7/2}}{80e^2} + \frac{cx^3 (d + ex^2)^{7/2}}{10e} \\
&= \frac{1}{384} d \left( 80a + \frac{d(3cd - 10be)}{e^2} \right) x (d + ex^2)^{3/2} + \frac{1}{480} \left( 80a + \frac{d(3cd - 10be)}{e^2} \right) x (d + ex^2)^{5/2} \\
&= \frac{1}{256} d^2 \left( 80a + \frac{d(3cd - 10be)}{e^2} \right) x \sqrt{d + ex^2} + \frac{1}{384} d \left( 80a + \frac{d(3cd - 10be)}{e^2} \right) x (d + ex^2)^{3/2} \\
&= \frac{1}{256} d^2 \left( 80a + \frac{d(3cd - 10be)}{e^2} \right) x \sqrt{d + ex^2} + \frac{1}{384} d \left( 80a + \frac{d(3cd - 10be)}{e^2} \right) x (d + ex^2)^{3/2} \\
&= \frac{1}{256} d^2 \left( 80a + \frac{d(3cd - 10be)}{e^2} \right) x \sqrt{d + ex^2} + \frac{1}{384} d \left( 80a + \frac{d(3cd - 10be)}{e^2} \right) x (d + ex^2)^{3/2}
\end{aligned}$$

**Mathematica [A]** time = 0.39, size = 190, normalized size = 0.88

$$\frac{\sqrt{d + ex^2} \left( \frac{15d^{5/2} \sinh^{-1} \left( \frac{\sqrt{cx}}{\sqrt{d}} \right) (10e(8ae - bd) + 3cd^2)}{\sqrt{\frac{e^2}{d} + 1}} + \sqrt{e} x (10e(8ae(33d^2 + 26dex^2 + 8e^2x^4) + b(15d^3 + 118d^2ex^2 + 136de^2x^4 + 48e^3x^6)) + c(-45d^4 + 30d^3ex^2 + 744d^2e^2x^4 + 1008de^3x^6 + 384e^4x^8)) \right)}{3840e^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^(5/2)\*(a + b\*x^2 + c\*x^4), x]

[Out] (Sqrt[d + e\*x^2]\*(Sqrt[e]\*x\*(c\*(-45\*d^4 + 30\*d^3\*e\*x^2 + 744\*d^2\*e^2\*x^4 + 1008\*d\*e^3\*x^6 + 384\*e^4\*x^8) + 10\*e\*(8\*a\*e\*(33\*d^2 + 26\*d\*e\*x^2 + 8\*e^2\*x^4) + b\*(15\*d^3 + 118\*d^2\*e\*x^2 + 136\*d\*e^2\*x^4 + 48\*e^3\*x^6))) + (15\*d^(5/2)\*(3\*c\*d^2 + 10\*e\*(-(b\*d) + 8\*a\*e))\*ArcSinh[(Sqrt[e]\*x)/Sqrt[d]])/Sqrt[1 + (e\*x^2)/d]))/(3840\*e^(5/2))

**IntegrateAlgebraic [A]** time = 0.41, size = 189, normalized size = 0.88

$$\frac{\log \left( \frac{\sqrt{d + ex^2} - \sqrt{e} x}{256e^{5/2}} \left( -80ad^2e^2 + 10bd^4e - 3cd^5 \right) + \sqrt{d + ex^2} \left( 2640ad^2e^2x + 2080ade^3x^3 + 640ae^4x^5 + 150bd^3ex + 1180bd^2e^2x^3 + 1360bde^3x^5 + 480be^4x^7 - 45cd^4x + 30cd^3ex^3 + 744cd^2e^2x^5 + 1008cde^3x^7 + 384ce^4x^9 \right) \right)}{3840e^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e\*x^2)^(5/2)\*(a + b\*x^2 + c\*x^4), x]

[Out] (Sqrt[d + e\*x^2]\*(-45\*c\*d^4\*x + 150\*b\*d^3\*e\*x + 2640\*a\*d^2\*e^2\*x + 30\*c\*d^3\*e\*x^3 + 1180\*b\*d^2\*e^2\*x^3 + 2080\*a\*d\*e^3\*x^3 + 744\*c\*d^2\*e^2\*x^5 + 1360\*b\*d\*e^3\*x^5 + 640\*a\*e^4\*x^5 + 1008\*c\*d\*e^3\*x^7 + 480\*b\*e^4\*x^7 + 384\*c\*e^4\*x^9))/(3840\*e^2) + (((-3\*c\*d^5 + 10\*b\*d^4\*e - 80\*a\*d^3\*e^2)\*Log[-(Sqrt[e]\*x) + Sqrt[d + e\*x^2]])/(256\*e^(5/2)))

**fricas [A]** time = 1.65, size = 370, normalized size = 1.72

$$\frac{15 \left( 10e^4 - 10be^3 + 80ae^2 \right) \sqrt{e} \log \left( -2e^2 - 2\sqrt{e^2 + d} \sqrt{e} - d \right) - 2 \left( 84bd^2e^2 + 48 \left( 20ad^2 + 10bd^2 \right) e^2 + 8 \left( 80ad^2 + 170bd^2 + 80ae^2 \right) e + 10 \left( 15d^3 + 118bd^2 + 28ad^2 \right) e^2 - 15 \left( 10e^4 - 10be^3 - 170ae^2 \right) \sqrt{e^2 + d} \right)}{3840e^2} - \frac{15 \left( 10e^4 - 10be^3 + 80ae^2 \right) \sqrt{e} \operatorname{arcsinh} \left( \frac{\sqrt{cx}}{\sqrt{d}} \right) - \left( 84bd^2e^2 + 48 \left( 20ad^2 + 10bd^2 \right) e^2 + 8 \left( 80ad^2 + 170bd^2 + 80ae^2 \right) e + 10 \left( 15d^3 + 118bd^2 + 28ad^2 \right) e^2 - 15 \left( 10e^4 - 10be^3 - 170ae^2 \right) \sqrt{e^2 + d} \right)}{3840e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(5/2)\*(c\*x^4+b\*x^2+a), x, algorithm="fricas")

```
[Out] [1/7680*(15*(3*c*d^5 - 10*b*d^4*e + 80*a*d^3*e^2)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + 2*(384*c*e^5*x^9 + 48*(21*c*d*e^4 + 10*b*e^5)*x^7 + 8*(93*c*d^2*e^3 + 170*b*d*e^4 + 80*a*e^5)*x^5 + 10*(3*c*d^3*e^2 + 118*b*d^2*e^3 + 208*a*d*e^4)*x^3 - 15*(3*c*d^4*e - 10*b*d^3*e^2 - 176*a*d^2*e^3)*x)*sqrt(e*x^2 + d))/e^3, -1/3840*(15*(3*c*d^5 - 10*b*d^4*e + 80*a*d^3*e^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (384*c*e^5*x^9 + 48*(21*c*d*e^4 + 10*b*e^5)*x^7 + 8*(93*c*d^2*e^3 + 170*b*d*e^4 + 80*a*e^5)*x^5 + 10*(3*c*d^3*e^2 + 118*b*d^2*e^3 + 208*a*d*e^4)*x^3 - 15*(3*c*d^4*e - 10*b*d^3*e^2 - 176*a*d^2*e^3)*x)*sqrt(e*x^2 + d))/e^3]
```

**giac** [A] time = 0.23, size = 180, normalized size = 0.84

$$-\frac{1}{256} (3cd^5 - 10bd^4e + 80ad^3e^2)^{5/2} \log\left(\frac{-xe^{1/2} + \sqrt{x^2 + d}}{\sqrt{x^2 + d}}\right) + \frac{1}{3840} (2(4(6(8cx^2e^2 + (21cd^2e + 10be^{10})d^{-8})x^2 + (93cd^2e^3 + 170bd^2e^4 + 80ae^{10})d^{-8})x^2 + 5(3cd^3e^2 + 118bd^2e^3 + 208ade^4)d^{-8})x^2 - 15(3cd^4e - 10bd^3e^2 - 176ad^2e^3)d^{-8})\sqrt{x^2 + d}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/256*(3*c*d^5 - 10*b*d^4*e + 80*a*d^3*e^2)*e^(-5/2)*log(abs(-x*e^(1/2) + sqrt(x^2*e + d))) + 1/3840*(2*(4*(6*(8*c*x^2*e^2 + (21*c*d*e^9 + 10*b*e^10)*e^(-8))*x^2 + (93*c*d^2*e^8 + 170*b*d*e^9 + 80*a*e^10)*e^(-8))*x^2 + 5*(3*c*d^3*e^7 + 118*b*d^2*e^8 + 208*a*d*e^9)*e^(-8))*x^2 - 15*(3*c*d^4*e^6 - 10*b*d^3*e^7 - 176*a*d^2*e^8)*e^(-8))*sqrt(x^2*e + d)*x
```

**maple** [A] time = 0.01, size = 283, normalized size = 1.32

$$\frac{5ad^5 \ln(\sqrt{cx^2 + d})}{16\sqrt{e}} - \frac{5bd^4 \ln(\sqrt{cx^2 + d})}{128e^{3/2}} + \frac{3cd^3 \ln(\sqrt{cx^2 + d})}{256e^2} + \frac{5\sqrt{cx^2 + d} ad^5}{16} - \frac{5\sqrt{cx^2 + d} bd^4}{128e} + \frac{3\sqrt{cx^2 + d} cd^3}{256e^2} + \frac{5(cx^2 + d)^{5/2} ad^5}{24} - \frac{5(cx^2 + d)^{5/2} bd^4}{192e} + \frac{(cx^2 + d)^{5/2} cd^3}{128e^2} + \frac{(cx^2 + d)^{5/2} dx}{10e} + \frac{(cx^2 + d)^{5/2} ex^3}{6} - \frac{(cx^2 + d)^{5/2} bx}{48e} + \frac{(cx^2 + d)^{5/2} cd^5}{160e^2} + \frac{(cx^2 + d)^{5/2} dx}{8e} - \frac{3(cx^2 + d)^{5/2} cd^5}{80e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(5/2)*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/10*c*x^3*(e*x^2+d)^(7/2)/e-3/80*c*d/e^2*x*(e*x^2+d)^(7/2)+1/160*c*d^2/e^2*x*(e*x^2+d)^(5/2)+1/128*c*d^3/e^2*x*(e*x^2+d)^(3/2)+3/256*c*d^4/e^2*x*(e*x^2+d)^(1/2)+3/256*c*d^5/e^(5/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))+1/8*b*x*(e*x^2+d)^(7/2)/e-1/48*b*d/e*x*(e*x^2+d)^(5/2)-5/192*b*d^2/e*x*(e*x^2+d)^(3/2)-5/128*b*d^3/e*x*(e*x^2+d)^(1/2)-5/128*b*d^4/e^(3/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))+1/6*a*x*(e*x^2+d)^(5/2)+5/24*a*d*x*(e*x^2+d)^(3/2)+5/16*a*d^2*x*(e*x^2+d)^(1/2)+5/16*a*d^3/e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))
```

**maxima** [A] time = 1.12, size = 261, normalized size = 1.21

$$\frac{(cx^2 + d)^{5/2} cx^3}{10e} + \frac{1}{6} (cx^2 + d)^{5/2} ax + \frac{5}{24} (cx^2 + d)^{5/2} adx + \frac{5}{16} \sqrt{cx^2 + d} ad^2x - \frac{3(cx^2 + d)^{5/2} cd^5}{80e^2} + \frac{(cx^2 + d)^{5/2} cd^3}{160e^2} + \frac{(cx^2 + d)^{5/2} cd^3}{128e^2} + \frac{3\sqrt{cx^2 + d} cd^3}{256e^2} + \frac{(cx^2 + d)^{5/2} dx}{8e} - \frac{(cx^2 + d)^{5/2} bx}{48e} - \frac{5(cx^2 + d)^{5/2} bd^2x}{192e} - \frac{5\sqrt{cx^2 + d} bd^2x}{128e} + \frac{3cd^5 \operatorname{arsinh}\left(\frac{cx}{\sqrt{d}}\right)}{256e^{3/2}} - \frac{5bd^4 \operatorname{arsinh}\left(\frac{cx}{\sqrt{d}}\right)}{128e^{3/2}} + \frac{5ad^3 \operatorname{arsinh}\left(\frac{cx}{\sqrt{d}}\right)}{16\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/10*(e*x^2 + d)^(7/2)*c*x^3/e + 1/6*(e*x^2 + d)^(5/2)*a*x + 5/24*(e*x^2 + d)^(3/2)*a*d*x + 5/16*sqrt(e*x^2 + d)*a*d^2*x - 3/80*(e*x^2 + d)^(7/2)*c*d*x/e^2 + 1/160*(e*x^2 + d)^(5/2)*c*d^2*x/e^2 + 1/128*(e*x^2 + d)^(3/2)*c*d^3*x/e^2 + 3/256*sqrt(e*x^2 + d)*c*d^4*x/e^2 + 1/8*(e*x^2 + d)^(7/2)*b*x/e - 1/48*(e*x^2 + d)^(5/2)*b*d*x/e - 5/192*(e*x^2 + d)^(3/2)*b*d^2*x/e - 5/128*sqrt(e*x^2 + d)*b*d^3*x/e + 3/256*c*d^5*arcsinh(e*x/sqrt(d*e))/e^(5/2) - 5/128*b*d^4*arcsinh(e*x/sqrt(d*e))/e^(3/2) + 5/16*a*d^3*arcsinh(e*x/sqrt(d*e))/sqrt(e)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (ex^2 + d)^{5/2} (cx^4 + bx^2 + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4), x)
```

```
[Out] int((d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4), x)
```

**sympy [B]** time = 63.83, size = 505, normalized size = 2.35

$$\frac{ad^2\sqrt{1+\frac{ex^2}{d}}}{2} + \frac{3ad^2x}{16\sqrt{1+\frac{ex^2}{d}}} + \frac{35ad^2ex^3}{48\sqrt{1+\frac{ex^2}{d}}} + \frac{17a\sqrt{d}e^2x^5}{24\sqrt{1+\frac{ex^2}{d}}} + \frac{5ad^2\operatorname{asinh}\left(\frac{\sqrt{ex^2}}{\sqrt{d}}\right)}{16\sqrt{e}} + \frac{ae^3x^7}{6\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} + \frac{5bf^2x}{128e\sqrt{1+\frac{ex^2}{d}}} + \frac{133bf^2x^3}{384\sqrt{1+\frac{ex^2}{d}}} + \frac{127bf^2ex^5}{192\sqrt{1+\frac{ex^2}{d}}} + \frac{23b\sqrt{d}e^2x^7}{48\sqrt{1+\frac{ex^2}{d}}} - \frac{5bf^2\operatorname{asinh}\left(\frac{\sqrt{ex^2}}{\sqrt{d}}\right)}{128e^{\frac{3}{2}}} + \frac{be^3x^9}{8\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} - \frac{3cd^2x}{256e\sqrt{1+\frac{ex^2}{d}}} - \frac{cd^2x^3}{256e\sqrt{1+\frac{ex^2}{d}}} + \frac{129cd^2x^5}{640\sqrt{1+\frac{ex^2}{d}}} + \frac{73cd^2ex^7}{160\sqrt{1+\frac{ex^2}{d}}} + \frac{29c\sqrt{d}e^2x^9}{80\sqrt{1+\frac{ex^2}{d}}} + \frac{3cd^2\operatorname{asinh}\left(\frac{\sqrt{ex^2}}{\sqrt{d}}\right)}{256e^{\frac{3}{2}}} + \frac{ce^3x^{11}}{10\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(5/2)*(c*x**4+b*x**2+a), x)
```

```
[Out] a*d**(5/2)*x*sqrt(1 + e*x**2/d)/2 + 3*a*d**(5/2)*x/(16*sqrt(1 + e*x**2/d))
+ 35*a*d**(3/2)*e*x**3/(48*sqrt(1 + e*x**2/d)) + 17*a*sqrt(d)*e**2*x**5/(24
*sqrt(1 + e*x**2/d)) + 5*a*d**3*asinh(sqrt(e)*x/sqrt(d))/(16*sqrt(e)) + a*e
**3*x**7/(6*sqrt(d)*sqrt(1 + e*x**2/d)) + 5*b*d**(7/2)*x/(128*e*sqrt(1 + e
*x**2/d)) + 133*b*d**(5/2)*x**3/(384*sqrt(1 + e*x**2/d)) + 127*b*d**(3/2)*e
*x**5/(192*sqrt(1 + e*x**2/d)) + 23*b*sqrt(d)*e**2*x**7/(48*sqrt(1 + e*x**2/
d)) - 5*b*d**4*asinh(sqrt(e)*x/sqrt(d))/(128*e**(3/2)) + b*e**3*x**9/(8*sq
rt(d)*sqrt(1 + e*x**2/d)) - 3*c*d**(9/2)*x/(256*e**2*sqrt(1 + e*x**2/d)) - c
*d**(7/2)*x**3/(256*e*sqrt(1 + e*x**2/d)) + 129*c*d**(5/2)*x**5/(640*sqrt(1
+ e*x**2/d)) + 73*c*d**(3/2)*e*x**7/(160*sqrt(1 + e*x**2/d)) + 29*c*sqrt(d
)*e**2*x**9/(80*sqrt(1 + e*x**2/d)) + 3*c*d**5*asinh(sqrt(e)*x/sqrt(d))/(25
6*e**(5/2)) + c*e**3*x**11/(10*sqrt(d)*sqrt(1 + e*x**2/d))
```

$$3.200 \quad \int (d + ex^2)^{3/2} (a + bx^2 + cx^4) dx$$

**Optimal.** Leaf size=175

$$\frac{x(d + ex^2)^{3/2} (48ae^2 - 8bde + 3cd^2)}{192e^2} + \frac{dx\sqrt{d + ex^2} (48ae^2 - 8bde + 3cd^2)}{128e^2} + \frac{d^2 \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right) (48ae^2 - 8bde + 3cd^2)}{128e^{5/2}}$$

**Rubi [A]** time = 0.12, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$ , Rules used = {1159, 388, 195, 217, 206}

$$\frac{x(d + ex^2)^{3/2} (48ae^2 - 8bde + 3cd^2)}{192e^2} + \frac{dx\sqrt{d + ex^2} (48ae^2 - 8bde + 3cd^2)}{128e^2} + \frac{d^2 \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right) (48ae^2 - 8bde + 3cd^2)}{128e^{5/2}} - \frac{x(d + ex^2)^{5/2} (3cd - 8be)}{48e^2} + \frac{cx^3 (d + ex^2)^{5/2}}{8e}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^(3/2)\*(a + b\*x^2 + c\*x^4),x]

[Out] (d\*(3\*c\*d^2 - 8\*b\*d\*e + 48\*a\*e^2)\*x\*sqrt[d + e\*x^2])/(128\*e^2) + ((3\*c\*d^2 - 8\*b\*d\*e + 48\*a\*e^2)\*x\*(d + e\*x^2)^(3/2))/(192\*e^2) - ((3\*c\*d - 8\*b\*e)\*x\*(d + e\*x^2)^(5/2))/(48\*e^2) + (c\*x^3\*(d + e\*x^2)^(5/2))/(8\*e) + (d^2\*(3\*c\*d^2 - 8\*b\*d\*e + 48\*a\*e^2)\*ArcTanh[(sqrt[e]\*x)/sqrt[d + e\*x^2]])/(128\*e^(5/2))

#### Rule 195

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^p)/(n\*p + 1), x] + Dist[(a\*n\*p)/(n\*p + 1), Int[(a + b\*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2\*p] || (EqQ[n, 2] && IntegerQ[4\*p]) || (EqQ[n, 2] && IntegerQ[3\*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rule 1159

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] := Simp[(c^p\*x^(4\*p - 1)\*(d + e\*x^2)^(q + 1))/(e\*(4\*p + 2\*q + 1)), x] + Dist[1/(e\*(4\*p + 2\*q + 1)), Int[(d + e\*x^2)^q\*ExpandToSum[e\*(4\*p + 2\*q + 1)\*(a + b\*x^2 + c\*x^4)^p - d\*c^p\*(4\*p - 1)\*x^(4\*p - 2) - e\*c^p\*(4\*p + 2\*q + 1)\*x^(4\*p), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && !LtQ[q, -1]

#### Rubi steps

$$\begin{aligned}
\int (d + ex^2)^{3/2} (a + bx^2 + cx^4) dx &= \frac{cx^3 (d + ex^2)^{5/2}}{8e} + \frac{\int (d + ex^2)^{3/2} (8ae - (3cd - 8be)x^2) dx}{8e} \\
&= -\frac{(3cd - 8be)x (d + ex^2)^{5/2}}{48e^2} + \frac{cx^3 (d + ex^2)^{5/2}}{8e} - \frac{1}{48} \left( -48a - \frac{d(3cd - 8be)}{e^2} \right) \int \frac{dx}{\sqrt{d + ex^2}} \\
&= \frac{1}{192} \left( 48a + \frac{d(3cd - 8be)}{e^2} \right) x (d + ex^2)^{3/2} - \frac{(3cd - 8be)x (d + ex^2)^{5/2}}{48e^2} + \frac{cx^3 (d + ex^2)^{5/2}}{8e} \\
&= \frac{1}{128} d \left( 48a + \frac{d(3cd - 8be)}{e^2} \right) x \sqrt{d + ex^2} + \frac{1}{192} \left( 48a + \frac{d(3cd - 8be)}{e^2} \right) x (d + ex^2) \\
&= \frac{1}{128} d \left( 48a + \frac{d(3cd - 8be)}{e^2} \right) x \sqrt{d + ex^2} + \frac{1}{192} \left( 48a + \frac{d(3cd - 8be)}{e^2} \right) x (d + ex^2) \\
&= \frac{1}{128} d \left( 48a + \frac{d(3cd - 8be)}{e^2} \right) x \sqrt{d + ex^2} + \frac{1}{192} \left( 48a + \frac{d(3cd - 8be)}{e^2} \right) x (d + ex^2)
\end{aligned}$$

**Mathematica [A]** time = 0.32, size = 157, normalized size = 0.90

$$\frac{\sqrt{d + ex^2} \left( \frac{3d^{3/2} \sinh^{-1} \left( \frac{\sqrt{ex^2}}{\sqrt{d}} \right) (8e(6ae - bd) + 3cd^2)}{\sqrt{\frac{ex^2}{d} + 1}} + \sqrt{e} x (8e(6ae(5d + 2ex^2) + b(3d^2 + 14dex^2 + 8e^2x^4)) + c(-9d^3 + 6d^2ex^2 + 72de^2x^4 + 48e^3x^6)) \right)}{384e^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^(3/2)\*(a + b\*x^2 + c\*x^4), x]

[Out] (Sqrt[d + e\*x^2]\*(Sqrt[e]\*x\*(c\*(-9\*d^3 + 6\*d^2\*e\*x^2 + 72\*d\*e^2\*x^4 + 48\*e^3\*x^6) + 8\*e\*(6\*a\*e\*(5\*d + 2\*e\*x^2) + b\*(3\*d^2 + 14\*d\*e\*x^2 + 8\*e^2\*x^4))) + (3\*d^(3/2)\*(3\*c\*d^2 + 8\*e\*(-(b\*d) + 6\*a\*e))\*ArcSinh[(Sqrt[e]\*x)/Sqrt[d]])/Sqrt[1 + (e\*x^2)/d])/(384\*e^(5/2))

**IntegrateAlgebraic [A]** time = 0.29, size = 153, normalized size = 0.87

$$\frac{\sqrt{d + ex^2} (240ade^2x + 96ae^3x^3 + 24bd^2ex + 112bde^2x^3 + 64be^3x^5 - 9cd^3x + 6cd^2ex^3 + 72cde^2x^5 + 48ce^3x^7)}{384e^2} + \frac{\log(\sqrt{d + ex^2} - \sqrt{e}x)(-48ad^2e^2 + 8bd^3e - 3cd^4)}{128e^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e\*x^2)^(3/2)\*(a + b\*x^2 + c\*x^4), x]

[Out] (Sqrt[d + e\*x^2]\*(-9\*c\*d^3\*x + 24\*b\*d^2\*e\*x + 240\*a\*d\*e^2\*x + 6\*c\*d^2\*e\*x^3 + 112\*b\*d\*e^2\*x^3 + 96\*a\*e^3\*x^3 + 72\*c\*d\*e^2\*x^5 + 64\*b\*e^3\*x^5 + 48\*c\*e^3\*x^7))/(384\*e^2) + ((-3\*c\*d^4 + 8\*b\*d^3\*e - 48\*a\*d^2\*e^2)\*Log[-(Sqrt[e]\*x) + Sqrt[d + e\*x^2]])/(128\*e^(5/2))

**fricas [A]** time = 0.98, size = 304, normalized size = 1.74

$$\frac{3(3cd^2 - 8bd^2e + 48ad^2e^2)\sqrt{e}\log\left(\frac{-2ex^2 - 2\sqrt{d+ex^2}\sqrt{e}x - d}{\sqrt{d+ex^2}}\right) + 2(48cd^2e^2 + 8(9cd^2 + 8bd^2e)x^3 + 2(3cd^2e - 8bd^2e^2 - 80ad^2e)x)\sqrt{d+ex^2} - 3(3cd^2 - 8bd^2e + 48ad^2e^2)\sqrt{e}\operatorname{arctan}\left(\frac{\sqrt{ex^2}}{\sqrt{d+ex^2}}\right) - (48cd^2e^2 + 8(9cd^2e + 8bd^2e^2)x^3 + 2(3cd^2e^2 + 56bd^2e + 48ad^2e^2)x - 3(3cd^2e - 8bd^2e^2 - 80ad^2e)x)\sqrt{d+ex^2}}{768e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(c\*x^4+b\*x^2+a), x, algorithm="fricas")

[Out] [1/768\*(3\*(3\*c\*d^4 - 8\*b\*d^3\*e + 48\*a\*d^2\*e^2)\*sqrt(e)\*log(-2\*e\*x^2 - 2\*sqrt(e\*x^2 + d)\*sqrt(e)\*x - d) + 2\*(48\*c\*e^4\*x^7 + 8\*(9\*c\*d\*e^3 + 8\*b\*e^4)\*x^5 + 2\*(3\*c\*d^2\*e^2 + 56\*b\*d\*e^3 + 48\*a\*e^4)\*x^3 - 3\*(3\*c\*d^3\*e - 8\*b\*d^2\*e^2 - 80\*a\*d\*e^3)\*x)\*sqrt(e\*x^2 + d))/e^3, -1/384\*(3\*(3\*c\*d^4 - 8\*b\*d^3\*e + 48

$$*a*d^2*e^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (48*c*e^4*x^7 + 8*(9*c*d*e^3 + 8*b*e^4)*x^5 + 2*(3*c*d^2*e^2 + 56*b*d*e^3 + 48*a*e^4)*x^3 - 3*(3*c*d^3*e - 8*b*d^2*e^2 - 80*a*d*e^3)*x)*sqrt(e*x^2 + d))/e^3]$$

**giac [A]** time = 0.22, size = 145, normalized size = 0.83

$$-\frac{1}{128}(3cd^4 - 8bd^3e + 48ad^2e^2)e^{(-\frac{5}{2})} \log\left(-xe^{\frac{1}{2}} + \sqrt{x^2e + d}\right) + \frac{1}{384}(2(4(6cx^2e + (9cd^6 + 8be^7)e^{(-6)})x^2 + (3cd^2e^5 + 56bde^6 + 48ae^7)e^{(-6)})x^2 - 3(3cd^3e^4 - 8bd^2e^5 - 80ade^6)e^{(-6)})\sqrt{x^2e + d}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(c\*x^4+b\*x^2+a),x, algorithm="giac")

[Out] -1/128\*(3\*c\*d^4 - 8\*b\*d^3\*e + 48\*a\*d^2\*e^2)\*e^(-5/2)\*log(abs(-x\*e^(1/2) + sqrt(x^2\*e + d))) + 1/384\*(2\*(4\*(6\*c\*x^2\*e + (9\*c\*d\*e^6 + 8\*b\*e^7)\*e^(-6))\*x^2 + (3\*c\*d^2\*e^5 + 56\*b\*d\*e^6 + 48\*a\*e^7)\*e^(-6))\*x^2 - 3\*(3\*c\*d^3\*e^4 - 8\*b\*d^2\*e^5 - 80\*a\*d\*e^6)\*e^(-6))\*sqrt(x^2\*e + d)\*x

**maple [A]** time = 0.01, size = 229, normalized size = 1.31

$$\frac{3ad^2\ln(\sqrt{e}x + \sqrt{ex^2+d})}{8\sqrt{e}} - \frac{bd^3\ln(\sqrt{e}x + \sqrt{ex^2+d})}{16e^{\frac{5}{2}}} + \frac{3cd^4\ln(\sqrt{e}x + \sqrt{ex^2+d})}{128e^{\frac{5}{2}}} + \frac{3\sqrt{ex^2+d}adx}{8} - \frac{\sqrt{ex^2+d}bd^2x}{16e} + \frac{3\sqrt{ex^2+d}cd^2x}{128e^2} + \frac{(ex^2+d)^{\frac{5}{2}}cx^3}{8e} + \frac{(ex^2+d)^{\frac{3}{2}}ax}{4} - \frac{(ex^2+d)^{\frac{3}{2}}bdx}{24e} + \frac{(ex^2+d)^{\frac{3}{2}}cd^2x}{64e^2} + \frac{(ex^2+d)^{\frac{5}{2}}bx}{6e} - \frac{(ex^2+d)^{\frac{5}{2}}cdx}{16e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^(3/2)\*(c\*x^4+b\*x^2+a),x)

[Out] 1/8\*c\*x^3\*(e\*x^2+d)^(5/2)/e-1/16\*c\*d/e^2\*x\*(e\*x^2+d)^(5/2)+1/64\*c\*d^2/e^2\*x\*(e\*x^2+d)^(3/2)+3/128\*c\*d^3/e^2\*x\*(e\*x^2+d)^(1/2)+3/128\*c\*d^4/e^(5/2)\*ln(e^(1/2)\*x+(e\*x^2+d)^(1/2))+1/6\*b\*x\*(e\*x^2+d)^(5/2)/e-1/24\*b\*d/e\*x\*(e\*x^2+d)^(3/2)-1/16\*b\*d^2/e\*x\*(e\*x^2+d)^(1/2)-1/16\*b\*d^3/e^(3/2)\*ln(e^(1/2)\*x+(e\*x^2+d)^(1/2))+1/4\*a\*x\*(e\*x^2+d)^(3/2)+3/8\*a\*d\*x\*(e\*x^2+d)^(1/2)+3/8\*a\*d^2/e^(1/2)\*ln(e^(1/2)\*x+(e\*x^2+d)^(1/2))

**maxima [A]** time = 1.02, size = 207, normalized size = 1.18

$$\frac{(ex^2+d)^{\frac{5}{2}}cx^3}{8e} + \frac{1}{4}(ex^2+d)^{\frac{3}{2}}ax + \frac{3}{8}\sqrt{ex^2+d}adx - \frac{(ex^2+d)^{\frac{5}{2}}cdx}{16e^2} + \frac{(ex^2+d)^{\frac{3}{2}}cd^2x}{64e^2} + \frac{3\sqrt{ex^2+d}cd^2x}{128e^2} + \frac{(ex^2+d)^{\frac{5}{2}}bx}{6e} - \frac{(ex^2+d)^{\frac{3}{2}}bdx}{24e} - \frac{\sqrt{ex^2+d}bd^2x}{16e} + \frac{3cd^4\operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{128e^{\frac{5}{2}}} - \frac{bd^3\operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{16e^{\frac{5}{2}}} + \frac{3ad^2\operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(c\*x^4+b\*x^2+a),x, algorithm="maxima")

[Out] 1/8\*(e\*x^2 + d)^(5/2)\*c\*x^3/e + 1/4\*(e\*x^2 + d)^(3/2)\*a\*x + 3/8\*sqrt(e\*x^2 + d)\*a\*d\*x - 1/16\*(e\*x^2 + d)^(5/2)\*c\*d\*x/e^2 + 1/64\*(e\*x^2 + d)^(3/2)\*c\*d^2\*x/e^2 + 3/128\*sqrt(e\*x^2 + d)\*c\*d^3\*x/e^2 + 1/6\*(e\*x^2 + d)^(5/2)\*b\*x/e - 1/24\*(e\*x^2 + d)^(3/2)\*b\*d\*x/e - 1/16\*sqrt(e\*x^2 + d)\*b\*d^2\*x/e + 3/128\*c\*d^4\*arcsinh(e\*x/sqrt(d\*e))/e^(5/2) - 1/16\*b\*d^3\*arcsinh(e\*x/sqrt(d\*e))/e^(3/2) + 3/8\*a\*d^2\*arcsinh(e\*x/sqrt(d\*e))/sqrt(e)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (ex^2 + d)^{3/2} (cx^4 + bx^2 + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)^(3/2)\*(a + b\*x^2 + c\*x^4),x)

[Out] int((d + e\*x^2)^(3/2)\*(a + b\*x^2 + c\*x^4), x)

**sympy [B]** time = 31.10, size = 413, normalized size = 2.36

$$\frac{ad^2x\sqrt{1+\frac{ex^2}{d}}}{2} + \frac{ad^3x}{8\sqrt{1+\frac{ex^2}{d}}} + \frac{3a\sqrt{d}ex^3}{8\sqrt{1+\frac{ex^2}{d}}} + \frac{3ad^2\operatorname{asinh}\left(\frac{\sqrt{ex^2}}{\sqrt{d}}\right)}{8\sqrt{e}} + \frac{ae^2x^5}{4\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} + \frac{bd^3x}{16e\sqrt{1+\frac{ex^2}{d}}} + \frac{17bd^3x^3}{48\sqrt{1+\frac{ex^2}{d}}} + \frac{11b\sqrt{d}ex^5}{24\sqrt{1+\frac{ex^2}{d}}} - \frac{bd^3\operatorname{asinh}\left(\frac{\sqrt{ex^2}}{\sqrt{d}}\right)}{16e^{\frac{3}{2}}} + \frac{be^2x^7}{6\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} - \frac{3cd^2x}{128e^2\sqrt{1+\frac{ex^2}{d}}} - \frac{cd^3x^3}{128e\sqrt{1+\frac{ex^2}{d}}} + \frac{13cd^3x^5}{64\sqrt{1+\frac{ex^2}{d}}} + \frac{5c\sqrt{d}ex^7}{16\sqrt{1+\frac{ex^2}{d}}} + \frac{3cd^4\operatorname{asinh}\left(\frac{\sqrt{ex^2}}{\sqrt{d}}\right)}{128e^{\frac{5}{2}}} + \frac{ce^2x^9}{8\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*(3/2)\*(c\*x\*\*4+b\*x\*\*2+a),x)

[Out]  $a*d^{3/2}*x*\sqrt{1 + e*x^2/d}/2 + a*d^{3/2}*x/(8*\sqrt{1 + e*x^2/d}) + 3*a*\sqrt{d}*e*x^3/(8*\sqrt{1 + e*x^2/d}) + 3*a*d^2*\operatorname{asinh}(\sqrt{e}*x/\sqrt{d})/(8*\sqrt{e}) + a*e^2*x^5/(4*\sqrt{d}*\sqrt{1 + e*x^2/d}) + b*d^{5/2}*x/(16*e*\sqrt{1 + e*x^2/d}) + 17*b*d^{3/2}*x^3/(48*\sqrt{1 + e*x^2/d}) + 11*b*\sqrt{d}*e*x^5/(24*\sqrt{1 + e*x^2/d}) - b*d^3*\operatorname{asinh}(\sqrt{e}*x/\sqrt{d})/(16*e^{3/2}) + b*e^2*x^7/(6*\sqrt{d}*\sqrt{1 + e*x^2/d}) - 3*c*d^{7/2}*x/(128*e^2*\sqrt{1 + e*x^2/d}) - c*d^{5/2}*x^3/(128*e*\sqrt{1 + e*x^2/d}) + 13*c*d^{3/2}*x^5/(64*\sqrt{1 + e*x^2/d}) + 5*c*\sqrt{d}*e*x^7/(16*\sqrt{1 + e*x^2/d}) + 3*c*d^4*\operatorname{asinh}(\sqrt{e}*x/\sqrt{d})/(128*e^{5/2}) + c*e^2*x^9/(8*\sqrt{d}*\sqrt{1 + e*x^2/d})$



### 3.201 $\int \sqrt{d + ex^2} (a + bx^2 + cx^4) dx$

**Optimal.** Leaf size=132

$$\frac{x\sqrt{d+ex^2}(8ae^2-2bde+cd^2)}{16e^2} + \frac{d \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)(8ae^2-2bde+cd^2)}{16e^{5/2}} - \frac{x(d+ex^2)^{3/2}(cd-2be)}{8e^2} + \frac{cx^3(d+ex^2)^{3/2}}{6e}$$

**Rubi [A]** time = 0.11, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$ , Rules used = {1159, 388, 195, 217, 206}

$$\frac{x\sqrt{d+ex^2}(8ae^2-2bde+cd^2)}{16e^2} + \frac{d \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)(8ae^2-2bde+cd^2)}{16e^{5/2}} - \frac{x(d+ex^2)^{3/2}(cd-2be)}{8e^2} + \frac{cx^3(d+ex^2)^{3/2}}{6e}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e\*x^2]\*(a + b\*x^2 + c\*x^4), x]

[Out] ((c\*d^2 - 2\*b\*d\*e + 8\*a\*e^2)\*x\*Sqrt[d + e\*x^2])/(16\*e^2) - ((c\*d - 2\*b\*e)\*x\*(d + e\*x^2)^(3/2))/(8\*e^2) + (c\*x^3\*(d + e\*x^2)^(3/2))/(6\*e) + (d\*(c\*d^2 - 2\*b\*d\*e + 8\*a\*e^2)\*ArcTanh[(Sqrt[e]\*x)/Sqrt[d + e\*x^2]])/(16\*e^(5/2))

#### Rule 195

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(x\*(a + b\*x^n)^p)/(n\*p + 1), x] + Dist[(a\*n\*p)/(n\*p + 1), Int[(a + b\*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2\*p] || (EqQ[n, 2] && IntegerQ[4\*p]) || (EqQ[n, 2] && IntegerQ[3\*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rule 1159

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> Simp[(c^p\*x^(4\*p - 1)\*(d + e\*x^2)^(q + 1))/(e\*(4\*p + 2\*q + 1)), x] + Dist[1/(e\*(4\*p + 2\*q + 1)), Int[(d + e\*x^2)^q\*ExpandToSum[e\*(4\*p + 2\*q + 1)\*(a + b\*x^2 + c\*x^4)^p - d\*c^p\*(4\*p - 1)\*x^(4\*p - 2) - e\*c^p\*(4\*p + 2\*q + 1)\*x^(4\*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && !LtQ[q, -1]

#### Rubi steps

$$\begin{aligned}
\int \sqrt{d+ex^2} (a+bx^2+cx^4) dx &= \frac{cx^3(d+ex^2)^{3/2}}{6e} + \frac{\int \sqrt{d+ex^2} (6ae-3(cd-2be)x^2) dx}{6e} \\
&= -\frac{(cd-2be)x(d+ex^2)^{3/2}}{8e^2} + \frac{cx^3(d+ex^2)^{3/2}}{6e} + \frac{1}{8} \left( 8a + \frac{d(cd-2be)}{e^2} \right) \int \sqrt{d+ex^2} dx \\
&= \frac{1}{16} \left( 8a + \frac{d(cd-2be)}{e^2} \right) x\sqrt{d+ex^2} - \frac{(cd-2be)x(d+ex^2)^{3/2}}{8e^2} + \frac{cx^3(d+ex^2)^{3/2}}{6e} \\
&= \frac{1}{16} \left( 8a + \frac{d(cd-2be)}{e^2} \right) x\sqrt{d+ex^2} - \frac{(cd-2be)x(d+ex^2)^{3/2}}{8e^2} + \frac{cx^3(d+ex^2)^{3/2}}{6e} \\
&= \frac{1}{16} \left( 8a + \frac{d(cd-2be)}{e^2} \right) x\sqrt{d+ex^2} - \frac{(cd-2be)x(d+ex^2)^{3/2}}{8e^2} + \frac{cx^3(d+ex^2)^{3/2}}{6e}
\end{aligned}$$

**Mathematica [A]** time = 0.23, size = 121, normalized size = 0.92

$$\frac{\sqrt{d+ex^2} \left( \frac{3\sqrt{d} \sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(8ae^2-2bde+cd^2)}{\sqrt{\frac{ex^2}{d}+1}} + \sqrt{e}x(6e(4ae+b(d+2ex^2)) + c(-3d^2+2dex^2+8e^2x^4)) \right)}{48e^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e\*x^2]\*(a + b\*x^2 + c\*x^4), x]

[Out] (Sqrt[d + e\*x^2]\*(Sqrt[e]\*x\*(c\*(-3\*d^2 + 2\*d\*e\*x^2 + 8\*e^2\*x^4) + 6\*e\*(4\*a\*e + b\*(d + 2\*e\*x^2))) + (3\*Sqrt[d]\*(c\*d^2 - 2\*b\*d\*e + 8\*a\*e^2)\*ArcSinh[(Sqrt[e]\*x)/Sqrt[d]])/Sqrt[1 + (e\*x^2)/d]))/(48\*e^(5/2))

**IntegrateAlgebraic [A]** time = 0.19, size = 117, normalized size = 0.89

$$\frac{\sqrt{d+ex^2} (24ae^2x + 6bdex + 12be^2x^3 - 3cd^2x + 2cdex^3 + 8ce^2x^5)}{48e^2} + \frac{\log(\sqrt{d+ex^2} - \sqrt{e}x)(-8ade^2 + 2bd^2e - cd^3)}{16e^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[d + e\*x^2]\*(a + b\*x^2 + c\*x^4), x]

[Out] (Sqrt[d + e\*x^2]\*(-3\*c\*d^2\*x + 6\*b\*d\*e\*x + 24\*a\*e^2\*x + 2\*c\*d\*e\*x^3 + 12\*b\*e^2\*x^3 + 8\*c\*e^2\*x^5))/(48\*e^2) + ((-(c\*d^3) + 2\*b\*d^2\*e - 8\*a\*d\*e^2)\*Log[-(Sqrt[e]\*x) + Sqrt[d + e\*x^2]])/(16\*e^(5/2))

**fricas [A]** time = 1.00, size = 232, normalized size = 1.76

$$\frac{3(cd^3-2bd^2e+8ade^2)\sqrt{e}\log\left(-2ex^2-2\sqrt{cd^2+d}\sqrt{ex-d}\right)+2(8ce^3x^5+2(cd^2+6be^2)x^3-3(cd^2e-2bd^2e-8ae^3)x)\sqrt{cd^2+d}-3(cd^3-2bd^2e+8ade^2)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{cd^2+d}}\right)-(8ce^3x^5+2(cd^2+6be^2)x^3-3(cd^2e-2bd^2e-8ae^3)x)\sqrt{cd^2+d}}{96e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(1/2)\*(c\*x^4+b\*x^2+a), x, algorithm="fricas")

[Out] [1/96\*(3\*(c\*d^3 - 2\*b\*d^2\*e + 8\*a\*d\*e^2)\*sqrt(e)\*log(-2\*e\*x^2 - 2\*sqrt(e\*x^2 + d)\*sqrt(e)\*x - d) + 2\*(8\*c\*e^3\*x^5 + 2\*(c\*d^2\*e^2 + 6\*b\*e^3)\*x^3 - 3\*(c\*d^2\*e - 2\*b\*d\*e^2 - 8\*a\*e^3)\*x)\*sqrt(e\*x^2 + d))/e^3, -1/48\*(3\*(c\*d^3 - 2\*b\*d^2\*e + 8\*a\*d\*e^2)\*sqrt(-e)\*arctan(sqrt(-e)\*x/sqrt(e\*x^2 + d)) - (8\*c\*e^3\*x^5 + 2\*(c\*d^2\*e^2 + 6\*b\*e^3)\*x^3 - 3\*(c\*d^2\*e - 2\*b\*d\*e^2 - 8\*a\*e^3)\*x)\*sqrt(e\*x^2 + d))/e^3]

**giac [A]** time = 0.22, size = 106, normalized size = 0.80

$$-\frac{1}{16}(cd^3 - 2bd^2e + 8ade^2)e^{(-\frac{5}{2})} \log\left(\left|-xe^{\frac{1}{2}} + \sqrt{x^2e + d}\right|\right) + \frac{1}{48}\left(2(4cx^2 + (cde^3 + 6be^4)e^{(-4)})x^2 - 3(cd^2e^2 - 2bde^3 - 8ae^4)e^{(-4)}\right)\sqrt{x^2e + d}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(1/2)\*(c\*x^4+b\*x^2+a),x, algorithm="giac")

[Out]  $-1/16*(c*d^3 - 2*b*d^2*e + 8*a*d*e^2)*e^{(-5/2)}*\log(\text{abs}(-x*e^{(1/2)} + \text{sqrt}(x^2*e + d))) + 1/48*(2*(4*c*x^2 + (c*d*e^3 + 6*b*e^4)*e^{(-4)})*x^2 - 3*(c*d^2*e^2 - 2*b*d*e^3 - 8*a*e^4)*e^{(-4)})*\text{sqrt}(x^2*e + d)*x$

**maple [A]** time = 0.01, size = 175, normalized size = 1.33

$$\frac{(e x^2 + d)^{\frac{3}{2}} c x^3}{6e} + \frac{a d \ln(\sqrt{e} x + \sqrt{e x^2 + d})}{2\sqrt{e}} - \frac{b d^2 \ln(\sqrt{e} x + \sqrt{e x^2 + d})}{8e^{\frac{3}{2}}} + \frac{c d^3 \ln(\sqrt{e} x + \sqrt{e x^2 + d})}{16e^{\frac{5}{2}}} + \frac{\sqrt{e x^2 + d} a x}{2} - \frac{\sqrt{e x^2 + d} b d x}{8e} + \frac{\sqrt{e x^2 + d} c d^2 x}{16e^2} + \frac{(e x^2 + d)^{\frac{3}{2}} b x}{4e} - \frac{(e x^2 + d)^{\frac{3}{2}} c d x}{8e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^(1/2)\*(c\*x^4+b\*x^2+a),x)

[Out]  $1/6*c*x^3*(e*x^2+d)^{(3/2)}/e - 1/8*c*d/e^2*x*(e*x^2+d)^{(3/2)} + 1/16*c*d^2/e^2*x*(e*x^2+d)^{(1/2)} + 1/16*c*d^3/e^{(5/2)}*\ln(e^{(1/2)}*x + (e*x^2+d)^{(1/2)}) + 1/4*b*x*(e*x^2+d)^{(3/2)}/e - 1/8*b*d/e*x*(e*x^2+d)^{(1/2)} - 1/8*b*d^2/e^{(3/2)}*\ln(e^{(1/2)}*x + (e*x^2+d)^{(1/2)}) + 1/2*a*x*(e*x^2+d)^{(1/2)} + 1/2*a*d/e^{(1/2)}*\ln(e^{(1/2)}*x + (e*x^2+d)^{(1/2)})$

**maxima [A]** time = 0.98, size = 153, normalized size = 1.16

$$\frac{(e x^2 + d)^{\frac{3}{2}} c x^3}{6e} + \frac{1}{2} \sqrt{e x^2 + d} a x - \frac{(e x^2 + d)^{\frac{3}{2}} c d x}{8e^2} + \frac{\sqrt{e x^2 + d} c d^2 x}{16e^2} + \frac{(e x^2 + d)^{\frac{3}{2}} b x}{4e} - \frac{\sqrt{e x^2 + d} b d x}{8e} + \frac{c d^3 \operatorname{arsinh}\left(\frac{e x}{\sqrt{d e}}\right)}{16e^{\frac{5}{2}}} - \frac{b d^2 \operatorname{arsinh}\left(\frac{e x}{\sqrt{d e}}\right)}{8e^{\frac{3}{2}}} + \frac{a d \operatorname{arsinh}\left(\frac{e x}{\sqrt{d e}}\right)}{2\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(1/2)\*(c\*x^4+b\*x^2+a),x, algorithm="maxima")

[Out]  $1/6*(e*x^2 + d)^{(3/2)}*c*x^3/e + 1/2*\text{sqrt}(e*x^2 + d)*a*x - 1/8*(e*x^2 + d)^{(3/2)}*c*d*x/e^2 + 1/16*\text{sqrt}(e*x^2 + d)*c*d^2*x/e^2 + 1/4*(e*x^2 + d)^{(3/2)}*b*x/e - 1/8*\text{sqrt}(e*x^2 + d)*b*d*x/e + 1/16*c*d^3*\operatorname{arcsinh}(e*x/\text{sqrt}(d*e))/e^{(5/2)} - 1/8*b*d^2*\operatorname{arcsinh}(e*x/\text{sqrt}(d*e))/e^{(3/2)} + 1/2*a*d*\operatorname{arcsinh}(e*x/\text{sqrt}(d*e))/\text{sqrt}(e)$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{e x^2 + d} (c x^4 + b x^2 + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)^(1/2)\*(a + b\*x^2 + c\*x^4),x)

[Out] int((d + e\*x^2)^(1/2)\*(a + b\*x^2 + c\*x^4), x)

**sympy [B]** time = 12.27, size = 272, normalized size = 2.06

$$\frac{a\sqrt{d}x\sqrt{1+\frac{ex^2}{d}}}{2} + \frac{ad\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{e}} + \frac{bd^{\frac{3}{2}}x}{8e\sqrt{1+\frac{ex^2}{d}}} + \frac{3b\sqrt{d}x^3}{8\sqrt{1+\frac{ex^2}{d}}} - \frac{bd^2\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8e^{\frac{3}{2}}} + \frac{bex^5}{4\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} - \frac{cd^{\frac{5}{2}}x}{16e^2\sqrt{1+\frac{ex^2}{d}}} - \frac{cd^{\frac{3}{2}}x^3}{48e\sqrt{1+\frac{ex^2}{d}}} + \frac{5c\sqrt{d}x^5}{24\sqrt{1+\frac{ex^2}{d}}} + \frac{cd^3\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16e^{\frac{5}{2}}} + \frac{cex^7}{6\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*(1/2)\*(c\*x\*\*4+b\*x\*\*2+a),x)

[Out]  $a*\text{sqrt}(d)*x*\text{sqrt}(1 + e*x**2/d)/2 + a*d*\operatorname{asinh}(\text{sqrt}(e)*x/\text{sqrt}(d))/(2*\text{sqrt}(e)) + b*d**(3/2)*x/(8*e*\text{sqrt}(1 + e*x**2/d)) + 3*b*\text{sqrt}(d)*x**3/(8*\text{sqrt}(1 + e*x$

```

**2/d)) - b*d**2*asinh(sqrt(e)*x/sqrt(d))/(8*e**(3/2)) + b*e*x**5/(4*sqrt(d)
)*sqrt(1 + e*x**2/d) - c*d**(5/2)*x/(16*e**2*sqrt(1 + e*x**2/d)) - c*d**(3
/2)*x**3/(48*e*sqrt(1 + e*x**2/d)) + 5*c*sqrt(d)*x**5/(24*sqrt(1 + e*x**2/d
)) + c*d**3*asinh(sqrt(e)*x/sqrt(d))/(16*e**(5/2)) + c*e*x**7/(6*sqrt(d)*sq
rt(1 + e*x**2/d))

```

$$3.202 \quad \int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}} dx$$

**Optimal.** Leaf size=97

$$\frac{\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)(8ae^2 - 4bde + 3cd^2)}{8e^{5/2}} - \frac{x\sqrt{d+ex^2}(3cd - 4be)}{8e^2} + \frac{cx^3\sqrt{d+ex^2}}{4e}$$

**Rubi [A]** time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1159, 388, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)(8ae^2 - 4bde + 3cd^2)}{8e^{5/2}} - \frac{x\sqrt{d+ex^2}(3cd - 4be)}{8e^2} + \frac{cx^3\sqrt{d+ex^2}}{4e}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2 + c\*x^4)/Sqrt[d + e\*x^2],x]

[Out] -((3\*c\*d - 4\*b\*e)\*x\*Sqrt[d + e\*x^2])/(8\*e^2) + (c\*x^3\*Sqrt[d + e\*x^2])/(4\*e) + ((3\*c\*d^2 - 4\*b\*d\*e + 8\*a\*e^2)\*ArcTanh[(Sqrt[e]\*x)/Sqrt[d + e\*x^2]])/(8\*e^(5/2))

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p+1))/(b\*(n\*(p+1)+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1)+1))/(b\*(n\*(p+1)+1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p+1)+1, 0]

Rule 1159

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] := Simp[(c^p\*x^(4\*p-1)\*(d + e\*x^2)^(q+1))/(e\*(4\*p+2\*q+1)), x] + Dist[1/(e\*(4\*p+2\*q+1)), Int[(d + e\*x^2)^q\*ExpandToSum[e\*(4\*p+2\*q+1)\*(a + b\*x^2 + c\*x^4)^p - d\*c^p\*(4\*p-1)\*x^(4\*p-2) - e\*c^p\*(4\*p+2\*q+1)\*x^(4\*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && !LtQ[q, -1]

Rubi steps

$$\begin{aligned}
\int \frac{a + bx^2 + cx^4}{\sqrt{d + ex^2}} dx &= \frac{cx^3\sqrt{d + ex^2}}{4e} + \frac{\int \frac{4ae - (3cd - 4be)x^2}{\sqrt{d + ex^2}} dx}{4e} \\
&= -\frac{(3cd - 4be)x\sqrt{d + ex^2}}{8e^2} + \frac{cx^3\sqrt{d + ex^2}}{4e} - \frac{1}{8} \left( -8a - \frac{d(3cd - 4be)}{e^2} \right) \int \frac{1}{\sqrt{d + ex^2}} dx \\
&= -\frac{(3cd - 4be)x\sqrt{d + ex^2}}{8e^2} + \frac{cx^3\sqrt{d + ex^2}}{4e} - \frac{1}{8} \left( -8a - \frac{d(3cd - 4be)}{e^2} \right) \text{Subst} \left( \int \frac{1}{1 - ex^2} dx, x \right) \\
&= -\frac{(3cd - 4be)x\sqrt{d + ex^2}}{8e^2} + \frac{cx^3\sqrt{d + ex^2}}{4e} + \frac{(3cd^2 - 4bde + 8ae^2) \tanh^{-1} \left( \frac{\sqrt{ex}}{\sqrt{d + ex^2}} \right)}{8e^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 82, normalized size = 0.85

$$\frac{\tanh^{-1} \left( \frac{\sqrt{ex}}{\sqrt{d + ex^2}} \right) (8ae^2 - 4bde + 3cd^2) + \sqrt{ex}\sqrt{d + ex^2} (4be - 3cd + 2cex^2)}{8e^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2 + c\*x^4)/Sqrt[d + e\*x^2], x]

[Out] (Sqrt[e]\*x\*Sqrt[d + e\*x^2]\*(-3\*c\*d + 4\*b\*e + 2\*c\*e\*x^2) + (3\*c\*d^2 - 4\*b\*d\*e + 8\*a\*e^2)\*ArcTanh[(Sqrt[e]\*x)/Sqrt[d + e\*x^2]])/(8\*e^(5/2))

**IntegrateAlgebraic [A]** time = 0.12, size = 85, normalized size = 0.88

$$\frac{\log \left( \sqrt{d + ex^2} - \sqrt{ex} \right) (-8ae^2 + 4bde - 3cd^2)}{8e^{5/2}} + \frac{\sqrt{d + ex^2} (4bex - 3cdx + 2cex^3)}{8e^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)/Sqrt[d + e\*x^2], x]

[Out] (Sqrt[d + e\*x^2]\*(-3\*c\*d\*x + 4\*b\*e\*x + 2\*c\*e\*x^3))/(8\*e^2) + ((-3\*c\*d^2 + 4\*b\*d\*e - 8\*a\*e^2)\*Log[-(Sqrt[e]\*x) + Sqrt[d + e\*x^2]])/(8\*e^(5/2))

**fricas [A]** time = 1.30, size = 174, normalized size = 1.79

$$\left[ \frac{(3cd^2 - 4bde + 8ae^2)\sqrt{e} \log(-2ex^2 - 2\sqrt{ex^2 + d}\sqrt{ex} - d) + 2(2ce^2x^3 - (3cde - 4be^2)x)\sqrt{ex^2 + d}}{16e^3}, \frac{(3cd^2 - 4bde + 8ae^2)\sqrt{-e} \arctan\left(\frac{\sqrt{-ex}}{\sqrt{ex^2 + d}}\right) - (2ce^2x^3 - (3cde - 4be^2)x)\sqrt{ex^2 + d}}{8e^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^(1/2), x, algorithm="fricas")

[Out] [1/16\*((3\*c\*d^2 - 4\*b\*d\*e + 8\*a\*e^2)\*sqrt(e)\*log(-2\*e\*x^2 - 2\*sqrt(e\*x^2 + d)\*sqrt(e)\*x - d) + 2\*(2\*c\*e^2\*x^3 - (3\*c\*d\*e - 4\*b\*e^2)\*x)\*sqrt(e\*x^2 + d))/e^3, -1/8\*((3\*c\*d^2 - 4\*b\*d\*e + 8\*a\*e^2)\*sqrt(-e)\*arctan(sqrt(-e)\*x/sqrt(e\*x^2 + d)) - (2\*c\*e^2\*x^3 - (3\*c\*d\*e - 4\*b\*e^2)\*x)\*sqrt(e\*x^2 + d))/e^3]

**giac [A]** time = 0.19, size = 79, normalized size = 0.81

$$-\frac{1}{8} (3cd^2 - 4bde + 8ae^2) e^{\left(-\frac{5}{2}\right)} \log \left( \left| -xe^{\frac{1}{2}} + \sqrt{x^2e + d} \right| \right) + \frac{1}{8} (2cx^2e^{(-1)} - (3cde - 4be^2)e^{(-3)}) \sqrt{x^2e + d} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^(1/2), x, algorithm="giac")

[Out]  $-1/8*(3*c*d^2 - 4*b*d*e + 8*a*e^2)*e^{(-5/2)}*\log(\text{abs}(-x*e^{(1/2)} + \text{sqrt}(x^2*e + d))) + 1/8*(2*c*x^2*e^{(-1)} - (3*c*d*e - 4*b*e^2)*e^{(-3)})*\text{sqrt}(x^2*e + d)$   
 \*x

**maple [A]** time = 0.01, size = 122, normalized size = 1.26

$$\frac{\sqrt{ex^2+d} cx^3}{4e} + \frac{a \ln(\sqrt{e} x + \sqrt{ex^2+d})}{\sqrt{e}} - \frac{bd \ln(\sqrt{e} x + \sqrt{ex^2+d})}{2e^{\frac{3}{2}}} + \frac{3cd^2 \ln(\sqrt{e} x + \sqrt{ex^2+d})}{8e^{\frac{5}{2}}} + \frac{\sqrt{ex^2+d} bx}{2e} - \frac{3\sqrt{ex^2+d} cdx}{8e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^4+b*x^2+a)/(e*x^2+d)^(1/2), x)`

[Out]  $1/4*c*x^3*(e*x^2+d)^{(1/2)}/e - 3/8*c*d/e^2*x*(e*x^2+d)^{(1/2)} + 3/8*c*d^2/e^{(5/2)}*\ln(e^{(1/2)}*x + (e*x^2+d)^{(1/2)}) + 1/2*b*x/e*(e*x^2+d)^{(1/2)} - 1/2*b*d/e^{(3/2)}*\ln(e^{(1/2)}*x + (e*x^2+d)^{(1/2)}) + a*\ln(e^{(1/2)}*x + (e*x^2+d)^{(1/2)})/e^{(1/2)}$

**maxima [A]** time = 1.07, size = 100, normalized size = 1.03

$$\frac{\sqrt{ex^2+d} cx^3}{4e} - \frac{3\sqrt{ex^2+d} cdx}{8e^2} + \frac{\sqrt{ex^2+d} bx}{2e} + \frac{3cd^2 \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{8e^{\frac{5}{2}}} - \frac{bd \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{2e^{\frac{3}{2}}} + \frac{a \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(1/2), x, algorithm="maxima")`

[Out]  $1/4*\text{sqrt}(e*x^2 + d)*c*x^3/e - 3/8*\text{sqrt}(e*x^2 + d)*c*d*x/e^2 + 1/2*\text{sqrt}(e*x^2 + d)*b*x/e + 3/8*c*d^2*\operatorname{arcsinh}(e*x/\text{sqrt}(d*e))/e^{(5/2)} - 1/2*b*d*\operatorname{arcsinh}(e*x/\text{sqrt}(d*e))/e^{(3/2)} + a*\operatorname{arcsinh}(e*x/\text{sqrt}(d*e))/\text{sqrt}(e)$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{cx^4 + bx^2 + a}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^2 + c*x^4)/(d + e*x^2)^(1/2), x)`

[Out] `int((a + b*x^2 + c*x^4)/(d + e*x^2)^(1/2), x)`

**sympy [A]** time = 7.05, size = 230, normalized size = 2.37

$$a \left( \begin{array}{l} \frac{\sqrt{\frac{-d}{e}} \operatorname{asin}\left(x\sqrt{\frac{-e}{d}}\right)}{\sqrt{d}} \quad \text{for } d > 0 \wedge e < 0 \\ \frac{\sqrt{\frac{d}{e}} \operatorname{asinh}\left(x\sqrt{\frac{e}{d}}\right)}{\sqrt{d}} \quad \text{for } d > 0 \wedge e > 0 \\ \frac{\sqrt{\frac{-d}{e}} \operatorname{acosh}\left(x\sqrt{\frac{-e}{d}}\right)}{\sqrt{-d}} \quad \text{for } e > 0 \wedge d < 0 \end{array} \right) + \frac{b\sqrt{d}x\sqrt{1+\frac{ex^2}{d}}}{2e} - \frac{bd \operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{\frac{3}{2}}} - \frac{3cd^{\frac{3}{2}}x}{8e^2\sqrt{1+\frac{ex^2}{d}}} - \frac{c\sqrt{d}x^3}{8e\sqrt{1+\frac{ex^2}{d}}} + \frac{3cd^2 \operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8e^{\frac{5}{2}}} + \frac{cx^5}{4\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(1/2), x)`

[Out]  $a*\text{Piecewise}((\text{sqrt}(-d/e)*\text{asin}(x*\text{sqrt}(-e/d))/\text{sqrt}(d), (d > 0) \& (e < 0)), (\text{sqrt}(d/e)*\text{asinh}(x*\text{sqrt}(e/d))/\text{sqrt}(d), (d > 0) \& (e > 0)), (\text{sqrt}(-d/e)*\text{acosh}(x*\text{sqrt}(-e/d))/\text{sqrt}(-d), (e > 0) \& (d < 0))) + b*\text{sqrt}(d)*x*\text{sqrt}(1 + e*x**2/d)/(2*e) - b*d*\text{asinh}(\text{sqrt}(e)*x/\text{sqrt}(d))/(2*e**(3/2)) - 3*c*d**(3/2)*x/(8*e**2*\text{sqrt}(1 + e*x**2/d)) - c*\text{sqrt}(d)*x**3/(8*e*\text{sqrt}(1 + e*x**2/d)) + 3*c*d**2*a*\text{sinh}(\text{sqrt}(e)*x/\text{sqrt}(d))/(8*e**(5/2)) + c*x**5/(4*\text{sqrt}(d)*\text{sqrt}(1 + e*x**2/d))$

$$3.203 \quad \int \frac{a+bx^2+cx^4}{(d+ex^2)^{3/2}} dx$$

**Optimal.** Leaf size=89

$$\frac{x \left( a + \frac{d(cd-be)}{e^2} \right)}{d\sqrt{d+ex^2}} - \frac{(3cd-2be) \tanh^{-1} \left( \frac{\sqrt{ex}}{\sqrt{d+ex^2}} \right)}{2e^{5/2}} + \frac{cx\sqrt{d+ex^2}}{2e^2}$$

**Rubi [A]** time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1157, 388, 217, 206}

$$\frac{x \left( a + \frac{d(cd-be)}{e^2} \right)}{d\sqrt{d+ex^2}} - \frac{(3cd-2be) \tanh^{-1} \left( \frac{\sqrt{ex}}{\sqrt{d+ex^2}} \right)}{2e^{5/2}} + \frac{cx\sqrt{d+ex^2}}{2e^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^(3/2),x]

[Out] ((a + (d\*(c\*d - b\*e))/e^2)\*x)/(d\*Sqrt[d + e\*x^2]) + (c\*x\*Sqrt[d + e\*x^2])/(2\*e^2) - ((3\*c\*d - 2\*b\*e)\*ArcTanh[(Sqrt[e]\*x)/Sqrt[d + e\*x^2]])/(2\*e^(5/2))

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p+1))/(b\*(n\*(p+1)+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1)+1))/(b\*(n\*(p+1)+1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p+1)+1, 0]

Rule 1157

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] := With[{Qx = PolynomialQuotient[(a + b\*x^2 + c\*x^4)^p, d + e\*x^2, x], R = Coeff[PolynomialRemainder[(a + b\*x^2 + c\*x^4)^p, d + e\*x^2, x], x, 0]}, -Simp[(R\*x\*(d + e\*x^2)^(q+1))/(2\*d\*(q+1)), x] + Dist[1/(2\*d\*(q+1)), Int[(d + e\*x^2)^(q+1)\*ExpandToSum[2\*d\*(q+1)\*Qx + R\*(2\*q+3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rubi steps



$$\begin{aligned}
\int \frac{a + bx^2 + cx^4}{(d + ex^2)^{3/2}} dx &= \frac{(cd^2 - bde + ae^2)x}{de^2\sqrt{d + ex^2}} - \int \frac{\frac{d(cd-be) - cd^2}{e^2} - \frac{cdx^2}{e}}{\sqrt{d+ex^2}} dx \\
&= \frac{(cd^2 - bde + ae^2)x}{de^2\sqrt{d + ex^2}} + \frac{cx\sqrt{d + ex^2}}{2e^2} - \frac{(3cd - 2be) \int \frac{1}{\sqrt{d+ex^2}} dx}{2e^2} \\
&= \frac{(cd^2 - bde + ae^2)x}{de^2\sqrt{d + ex^2}} + \frac{cx\sqrt{d + ex^2}}{2e^2} - \frac{(3cd - 2be) \text{Subst}\left(\int \frac{1}{1-ex^2} dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2e^2} \\
&= \frac{(cd^2 - bde + ae^2)x}{de^2\sqrt{d + ex^2}} + \frac{cx\sqrt{d + ex^2}}{2e^2} - \frac{(3cd - 2be) \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{2e^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 98, normalized size = 1.10

$$\frac{\sqrt{e}x(2e(ae - bd) + cd(3d + ex^2)) - d^{3/2}\sqrt{\frac{ex^2}{d} + 1}(3cd - 2be)\sinh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2de^{5/2}\sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^(3/2), x]

[Out] (Sqrt[e]\*x\*(2\*e\*(-(b\*d) + a\*e) + c\*d\*(3\*d + e\*x^2)) - d^(3/2)\*(3\*c\*d - 2\*b\*e)\*Sqrt[1 + (e\*x^2)/d]\*ArcSinh[(Sqrt[e]\*x)/Sqrt[d]])/(2\*d\*e^(5/2)\*Sqrt[d + e\*x^2])

**IntegrateAlgebraic [A]** time = 0.16, size = 89, normalized size = 1.00

$$\frac{2ae^2x - 2bdex + 3cd^2x + cdex^3}{2de^2\sqrt{d + ex^2}} + \frac{(3cd - 2be) \log\left(\sqrt{d + ex^2} - \sqrt{e}x\right)}{2e^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^(3/2), x]

[Out] (3\*c\*d^2\*x - 2\*b\*d\*e\*x + 2\*a\*e^2\*x + c\*d\*e\*x^3)/(2\*d\*e^2\*Sqrt[d + e\*x^2]) + ((3\*c\*d - 2\*b\*e)\*Log[-(Sqrt[e]\*x) + Sqrt[d + e\*x^2]])/(2\*e^(5/2))

**fricas [A]** time = 0.85, size = 249, normalized size = 2.80

$$\left[ \frac{(3cd^3 - 2bd^2e + (3cd^2e - 2bd^2e^2)x^2)\sqrt{e}\log\left(-2ex^2 - 2\sqrt{ex^2 + d}\sqrt{e}x - d\right) - 2(cd^2x^3 + (3cd^2e - 2bd^2e^2)x)\sqrt{ex^2 + d}}{4(d^4x^2 + d^2e^2)}, \frac{(3cd^3 - 2bd^2e + (3cd^2e - 2bd^2e^2)x^2)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2 + d}}\right) + (cd^2x^3 + (3cd^2e - 2bd^2e^2)x)\sqrt{ex^2 + d}}{2(d^4x^2 + d^2e^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^(3/2), x, algorithm="fricas")

[Out] [-1/4\*((3\*c\*d^3 - 2\*b\*d^2\*e + (3\*c\*d^2\*e - 2\*b\*d\*e^2)\*x^2)\*sqrt(e)\*log(-2\*e\*x^2 - 2\*sqrt(e\*x^2 + d)\*sqrt(e)\*x - d) - 2\*(c\*d\*e^2\*x^3 + (3\*c\*d^2\*e - 2\*b\*d\*e^2 + 2\*a\*e^3)\*x)\*sqrt(e\*x^2 + d)]/(d\*e^4\*x^2 + d^2\*e^3), 1/2\*((3\*c\*d^3 - 2\*b\*d^2\*e + (3\*c\*d^2\*e - 2\*b\*d\*e^2)\*x^2)\*sqrt(-e)\*arctan(sqrt(-e)\*x/sqrt(e\*x^2 + d)) + (c\*d\*e^2\*x^3 + (3\*c\*d^2\*e - 2\*b\*d\*e^2 + 2\*a\*e^3)\*x)\*sqrt(e\*x^2 + d)]/(d\*e^4\*x^2 + d^2\*e^3)]

**giac [A]** time = 0.20, size = 80, normalized size = 0.90

$$\frac{1}{2}(3cd - 2be)e^{\left(-\frac{5}{2}\right)} \log\left(\left|-xe^{\frac{1}{2}} + \sqrt{x^2e + d}\right|\right) + \frac{\left(cx^2e^{(-1)} + \frac{(3cd^2e - 2bd^2e^2 + 2ae^3)e^{(-3)}}{d}\right)x}{2\sqrt{x^2e + d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^(3/2),x, algorithm="giac")

[Out] 1/2\*(3\*c\*d - 2\*b\*e)\*e^(-5/2)\*log(abs(-x\*e^(1/2) + sqrt(x^2\*e + d))) + 1/2\*(c\*x^2\*e^(-1) + (3\*c\*d^2\*e - 2\*b\*d\*e^2 + 2\*a\*e^3)\*e^(-3)/d)\*x/sqrt(x^2\*e + d)

**maple** [A] time = 0.01, size = 112, normalized size = 1.26

$$\frac{cx^3}{2\sqrt{ex^2+de}} + \frac{ax}{\sqrt{ex^2+dd}} - \frac{bx}{\sqrt{ex^2+de}} + \frac{3cdx}{2\sqrt{ex^2+de^2}} + \frac{b \ln(\sqrt{e}x + \sqrt{ex^2+d})}{e^{\frac{3}{2}}} - \frac{3cd \ln(\sqrt{e}x + \sqrt{ex^2+d})}{2e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^4+b\*x^2+a)/(e\*x^2+d)^(3/2),x)

[Out] 1/2\*c\*x^3/e/(e\*x^2+d)^(1/2)+3/2\*c\*d/e^2\*x/(e\*x^2+d)^(1/2)-3/2\*c\*d/e^(5/2)\*ln(e^(1/2)\*x+(e\*x^2+d)^(1/2))-b\*x/e/(e\*x^2+d)^(1/2)+b/e^(3/2)\*ln(e^(1/2)\*x+(e\*x^2+d)^(1/2))+a\*x/d/(e\*x^2+d)^(1/2)

**maxima** [A] time = 1.13, size = 97, normalized size = 1.09

$$\frac{cx^3}{2\sqrt{ex^2+de}} + \frac{ax}{\sqrt{ex^2+dd}} + \frac{3cdx}{2\sqrt{ex^2+de^2}} - \frac{bx}{\sqrt{ex^2+de}} - \frac{3cd \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{2e^{\frac{5}{2}}} + \frac{b \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{e^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^(3/2),x, algorithm="maxima")

[Out] 1/2\*c\*x^3/(sqrt(e\*x^2 + d)\*e) + a\*x/(sqrt(e\*x^2 + d)\*d) + 3/2\*c\*d\*x/(sqrt(e\*x^2 + d)\*e^2) - b\*x/(sqrt(e\*x^2 + d)\*e) - 3/2\*c\*d\*arcsinh(e\*x/sqrt(d\*e))/e^(5/2) + b\*arcsinh(e\*x/sqrt(d\*e))/e^(3/2)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{cx^4 + bx^2 + a}{(ex^2 + d)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^2 + c\*x^4)/(d + e\*x^2)^(3/2),x)

[Out] int((a + b\*x^2 + c\*x^4)/(d + e\*x^2)^(3/2), x)

**sympy** [A] time = 9.98, size = 134, normalized size = 1.51

$$\frac{ax}{d^{\frac{3}{2}}\sqrt{1+\frac{ex^2}{d}}} + b \left( \frac{\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{\frac{3}{2}}} - \frac{x}{\sqrt{de}\sqrt{1+\frac{ex^2}{d}}} \right) + c \left( \frac{3\sqrt{d}x}{2e^2\sqrt{1+\frac{ex^2}{d}}} - \frac{3d \operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{\frac{5}{2}}} + \frac{x^3}{2\sqrt{de}\sqrt{1+\frac{ex^2}{d}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*4+b\*x\*\*2+a)/(e\*x\*\*2+d)\*\*(3/2),x)

[Out] a\*x/(d\*\*(3/2)\*sqrt(1 + e\*x\*\*2/d)) + b\*(asinh(sqrt(e)\*x/sqrt(d))/e\*\*(3/2) - x/(sqrt(d)\*e\*sqrt(1 + e\*x\*\*2/d))) + c\*(3\*sqrt(d)\*x/(2\*e\*\*2\*sqrt(1 + e\*x\*\*2/d)) - 3\*d\*asinh(sqrt(e)\*x/sqrt(d))/(2\*e\*\*(5/2)) + x\*\*3/(2\*sqrt(d)\*e\*sqrt(1 + e\*x\*\*2/d)))

$$3.204 \quad \int \frac{a+bx^2+cx^4}{(d+ex^2)^{5/2}} dx$$

Optimal. Leaf size=101

$$-\frac{x(4cd^2 - e(2ae + bd))}{3d^2e^2\sqrt{d+ex^2}} + \frac{x\left(a + \frac{d(cd-be)}{e^2}\right)}{3d(d+ex^2)^{3/2}} + \frac{c \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{e^{5/2}}$$

**Rubi [A]** time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1157, 385, 217, 206}

$$-\frac{x(4cd^2 - e(2ae + bd))}{3d^2e^2\sqrt{d+ex^2}} + \frac{x\left(a + \frac{d(cd-be)}{e^2}\right)}{3d(d+ex^2)^{3/2}} + \frac{c \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{e^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^(5/2), x]

[Out] ((a + (d\*(c\*d - b\*e))/e^2)\*x)/(3\*d\*(d + e\*x^2)^(3/2)) - ((4\*c\*d^2 - e\*(b\*d + 2\*a\*e))\*x)/(3\*d^2\*e^2\*Sqrt[d + e\*x^2]) + (c\*ArcTanh[(Sqrt[e]\*x)/Sqrt[d + e\*x^2]])/e^(5/2)

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p+1))/(a\*b\*n\*(p+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1) + 1))/(a\*b\*n\*(p+1)), Int[(a + b\*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 1157

Int[((d\_) + (e\_.)\*(x\_)^2)^(q\_)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_), x\_Symbol] := With[{Qx = PolynomialQuotient[(a + b\*x^2 + c\*x^4)^p, d + e\*x^2, x], R = Coeff[PolynomialRemainder[(a + b\*x^2 + c\*x^4)^p, d + e\*x^2, x], x, 0]}, -Simp[(R\*x\*(d + e\*x^2)^(q+1))/(2\*d\*(q+1)), x] + Dist[1/(2\*d\*(q+1)), Int[(d + e\*x^2)^(q+1)\*ExpandToSum[2\*d\*(q+1)\*Qx + R\*(2\*q+3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rubi steps

$$\begin{aligned}
\int \frac{a + bx^2 + cx^4}{(d + ex^2)^{5/2}} dx &= \frac{(cd^2 - bde + ae^2)x}{3de^2(d + ex^2)^{3/2}} - \frac{\int \frac{-2a + \frac{d(cd-be) - 3cdx^2}{e^2}}{(d+ex^2)^{3/2}} dx}{3d} \\
&= \frac{(cd^2 - bde + ae^2)x}{3de^2(d + ex^2)^{3/2}} - \frac{(4cd^2 - e(bd + 2ae))x}{3d^2e^2\sqrt{d + ex^2}} + \frac{c \int \frac{1}{\sqrt{d+ex^2}} dx}{e^2} \\
&= \frac{(cd^2 - bde + ae^2)x}{3de^2(d + ex^2)^{3/2}} - \frac{(4cd^2 - e(bd + 2ae))x}{3d^2e^2\sqrt{d + ex^2}} + \frac{c \operatorname{Subst}\left(\int \frac{1}{1-ex^2} dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{e^2} \\
&= \frac{(cd^2 - bde + ae^2)x}{3de^2(d + ex^2)^{3/2}} - \frac{(4cd^2 - e(bd + 2ae))x}{3d^2e^2\sqrt{d + ex^2}} + \frac{c \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.19, size = 112, normalized size = 1.11

$$\frac{\sqrt{e}x(e^2(3ad + 2aex^2 + bdx^2) - cd^2(3d + 4ex^2)) + 3cd^{5/2}(d + ex^2)\sqrt{\frac{ex^2}{d} + 1} \sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^2e^{5/2}(d + ex^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^(5/2), x]

[Out] (Sqrt[e]\*x\*(-(c\*d^2\*(3\*d + 4\*e\*x^2)) + e^2\*(3\*a\*d + b\*d\*x^2 + 2\*a\*e\*x^2)) + 3\*c\*d^(5/2)\*(d + e\*x^2)\*Sqrt[1 + (e\*x^2)/d]\*ArcSinh[(Sqrt[e]\*x)/Sqrt[d]])/(3\*d^2\*e^(5/2)\*(d + e\*x^2)^(3/2))

**IntegrateAlgebraic [A]** time = 0.20, size = 95, normalized size = 0.94

$$\frac{3ade^2x + 2ae^3x^3 + bde^2x^3 - 3cd^3x - 4cd^2ex^3}{3d^2e^2(d + ex^2)^{3/2}} - \frac{c \log\left(\sqrt{d + ex^2} - \sqrt{e}x\right)}{e^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^(5/2), x]

[Out] (-3\*c\*d^3\*x + 3\*a\*d\*e^2\*x - 4\*c\*d^2\*e\*x^3 + b\*d\*e^2\*x^3 + 2\*a\*e^3\*x^3)/(3\*d^2\*e^2\*(d + e\*x^2)^(3/2)) - (c\*Log[-(Sqrt[e]\*x) + Sqrt[d + e\*x^2]])/e^(5/2)

**fricas [A]** time = 0.85, size = 289, normalized size = 2.86

$$\frac{3(cd^2e^2x^4 + 2cd^3ex^2 + cd^4)\sqrt{e} \log\left(\frac{-2ex^2 - 2\sqrt{ex^2+d}\sqrt{e}x - d}{6(d^2e^5x^4 + 2d^3e^4x^2 + d^4e^3)}\right) - 2\left((4cd^2e^2 - bde^3 - 2ae^4)x^3 + 3(cd^3e - ade^3)x\right)\sqrt{ex^2+d}}{6(d^2e^5x^4 + 2d^3e^4x^2 + d^4e^3)} - \frac{3(cd^2e^2x^4 + 2cd^3ex^2 + cd^4)\sqrt{-e} \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) + ((4cd^2e^2 - bde^3 - 2ae^4)x^3 + 3(cd^3e - ade^3)x)\sqrt{ex^2+d}}{3(d^2e^5x^4 + 2d^3e^4x^2 + d^4e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^(5/2), x, algorithm="fricas")

[Out] [1/6\*(3\*(c\*d^2\*e^2\*x^4 + 2\*c\*d^3\*e\*x^2 + c\*d^4)\*sqrt(e)\*log(-2\*e\*x^2 - 2\*sqrt(e\*x^2 + d)\*sqrt(e)\*x - d) - 2\*((4\*c\*d^2\*e^2 - b\*d\*e^3 - 2\*a\*e^4)\*x^3 + 3\*(c\*d^3\*e - a\*d\*e^3)\*x)\*sqrt(e\*x^2 + d))/(d^2\*e^5\*x^4 + 2\*d^3\*e^4\*x^2 + d^4\*e^3), -1/3\*(3\*(c\*d^2\*e^2\*x^4 + 2\*c\*d^3\*e\*x^2 + c\*d^4)\*sqrt(-e)\*arctan(sqrt(-e)\*x/sqrt(e\*x^2 + d)) + ((4\*c\*d^2\*e^2 - b\*d\*e^3 - 2\*a\*e^4)\*x^3 + 3\*(c\*d^3\*e - a\*d\*e^3)\*x)\*sqrt(e\*x^2 + d))/(d^2\*e^5\*x^4 + 2\*d^3\*e^4\*x^2 + d^4\*e^3)]

**giac** [A] time = 0.23, size = 88, normalized size = 0.87

$$-ce^{\left(-\frac{5}{2}\right)} \log\left(\left|-xe^{\frac{1}{2}} + \sqrt{x^2e + d}\right|\right) - \frac{\left(\frac{(4cd^2e^2 - bde^3 - 2ae^4)x^2e^{(-3)}}{d^2} + \frac{3(cd^3e - ade^3)e^{(-3)}}{d^2}\right)x}{3(x^2e + d)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^(5/2),x, algorithm="giac")

[Out]  $-c*e^{(-5/2)}*\log(\text{abs}(-x*e^{(1/2)} + \text{sqrt}(x^2*e + d))) - 1/3*((4*c*d^2*e^2 - b*d*e^3 - 2*a*e^4)*x^2*e^{(-3)}/d^2 + 3*(c*d^3*e - a*d*e^3)*e^{(-3)}/d^2)*x/(x^2*e + d)^{(3/2)}$

**maple** [A] time = 0.01, size = 124, normalized size = 1.23

$$-\frac{cx^3}{3(e^2x^2+d)^{\frac{3}{2}}e} + \frac{ax}{3(e^2x^2+d)^{\frac{3}{2}}d} - \frac{bx}{3(e^2x^2+d)^{\frac{3}{2}}e} + \frac{2ax}{3\sqrt{e^2x^2+d}d^2} + \frac{bx}{3\sqrt{e^2x^2+d}de} - \frac{cx}{\sqrt{e^2x^2+d}e^2} + \frac{c \ln(\sqrt{e}x + \sqrt{e^2x^2+d})}{e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^4+b\*x^2+a)/(e\*x^2+d)^(5/2),x)

[Out]  $-1/3*c*x^3/e/(e*x^2+d)^{(3/2)} - c/e^2*x/(e*x^2+d)^{(1/2)} + c/e^{(5/2)}*\ln(e^{(1/2)}*x + (e*x^2+d)^{(1/2)}) - 1/3*b/e*x/(e*x^2+d)^{(3/2)} + 1/3*b/d/e*x/(e*x^2+d)^{(1/2)} + 1/3*a*x/d/(e*x^2+d)^{(3/2)} + 2/3*a/d^2*x/(e*x^2+d)^{(1/2)}$

**maxima** [A] time = 1.01, size = 135, normalized size = 1.34

$$-\frac{1}{3}cx \left( \frac{3x^2}{(e^2x^2+d)^{\frac{3}{2}}e} + \frac{2d}{(e^2x^2+d)^{\frac{3}{2}}e^2} \right) + \frac{2ax}{3\sqrt{e^2x^2+d}d^2} + \frac{ax}{3(e^2x^2+d)^{\frac{3}{2}}d} - \frac{cx}{3\sqrt{e^2x^2+d}e^2} - \frac{bx}{3(e^2x^2+d)^{\frac{3}{2}}e} + \frac{bx}{3\sqrt{e^2x^2+d}de} + \frac{c \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^(5/2),x, algorithm="maxima")

[Out]  $-1/3*c*x*(3*x^2/((e*x^2 + d)^{(3/2)}*e) + 2*d/((e*x^2 + d)^{(3/2)}*e^2)) + 2/3*a*x/(sqrt(e*x^2 + d)*d^2) + 1/3*a*x/((e*x^2 + d)^{(3/2)}*d) - 1/3*c*x/(sqrt(e*x^2 + d)*e^2) - 1/3*b*x/((e*x^2 + d)^{(3/2)}*e) + 1/3*b*x/(sqrt(e*x^2 + d)*d*e) + c*arcsinh(e*x/sqrt(d*e))/e^{(5/2)}$

**mapad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{cx^4 + bx^2 + a}{(ex^2 + d)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^2 + c\*x^4)/(d + e\*x^2)^(5/2),x)

[Out] int((a + b\*x^2 + c\*x^4)/(d + e\*x^2)^(5/2), x)

**sympy** [B] time = 18.95, size = 450, normalized size = 4.46

$$a \left( \frac{3dx}{3d^2\sqrt{1+\frac{d}{e}} + 3d^2e^2\sqrt{1+\frac{d}{e}}} + \frac{2ax^3}{3d^2\sqrt{1+\frac{d}{e}} + 3d^2e^2\sqrt{1+\frac{d}{e}}} \right) + \frac{bx^3}{3d^2\sqrt{1+\frac{d}{e}} + 3d^2e^2\sqrt{1+\frac{d}{e}}} + c \left( \frac{3d^{\frac{5}{2}}e^{11}\sqrt{1+\frac{d}{e}} \operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{\frac{5}{2}}e^2\sqrt{1+\frac{d}{e}} + 3d^{\frac{5}{2}}e^2\sqrt{1+\frac{d}{e}}} + \frac{3d^{\frac{5}{2}}e^{12}x^2\sqrt{1+\frac{d}{e}} \operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{\frac{5}{2}}e^2\sqrt{1+\frac{d}{e}} + 3d^{\frac{5}{2}}e^2\sqrt{1+\frac{d}{e}}} - \frac{3d^{\frac{5}{2}}e^{\frac{23}{2}}x}{3d^{\frac{5}{2}}e^2\sqrt{1+\frac{d}{e}} + 3d^{\frac{5}{2}}e^2\sqrt{1+\frac{d}{e}}} - \frac{4d^{10}e^{\frac{23}{2}}x^3}{3d^{\frac{5}{2}}e^2\sqrt{1+\frac{d}{e}} + 3d^{\frac{5}{2}}e^2\sqrt{1+\frac{d}{e}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*4+b\*x\*\*2+a)/(e\*x\*\*2+d)\*\*(5/2),x)

```
[Out] a*(3*d*x/(3*d**(7/2)*sqrt(1 + e*x**2/d) + 3*d**(5/2)*e*x**2*sqrt(1 + e*x**2/d)) + 2*e*x**3/(3*d**(7/2)*sqrt(1 + e*x**2/d) + 3*d**(5/2)*e*x**2*sqrt(1 + e*x**2/d))) + b*x**3/(3*d**(5/2)*sqrt(1 + e*x**2/d) + 3*d**(3/2)*e*x**2*sqrt(1 + e*x**2/d)) + c*(3*d**(39/2)*e**11*sqrt(1 + e*x**2/d)*asinh(sqrt(e)*x/sqrt(d))/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)*e**(29/2)*x**2*sqrt(1 + e*x**2/d)) + 3*d**(37/2)*e**12*x**2*sqrt(1 + e*x**2/d)*asinh(sqrt(e)*x/sqrt(d))/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)*e**(29/2)*x**2*sqrt(1 + e*x**2/d)) - 3*d**19*e**(23/2)*x/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)*e**(29/2)*x**2*sqrt(1 + e*x**2/d)) - 4*d**18*e**(25/2)*x**3/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)*e**(29/2)*x**2*sqrt(1 + e*x**2/d)))
```

$$3.205 \quad \int \frac{a+bx^2+cx^4}{(d+ex^2)^{7/2}} dx$$

Optimal. Leaf size=86

$$\frac{x^5 (2e(4ae + bd) + 3cd^2)}{15d^3 (d + ex^2)^{5/2}} + \frac{x^3(4ae + bd)}{3d^2 (d + ex^2)^{5/2}} + \frac{ax}{d (d + ex^2)^{5/2}}$$

**Rubi [A]** time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1155, 1803, 12, 264}

$$\frac{x^5 (2e(4ae + bd) + 3cd^2)}{15d^3 (d + ex^2)^{5/2}} + \frac{x^3(4ae + bd)}{3d^2 (d + ex^2)^{5/2}} + \frac{ax}{d (d + ex^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^(7/2), x]

[Out] (a\*x)/(d\*(d + e\*x^2)^(5/2)) + ((b\*d + 4\*a\*e)\*x^3)/(3\*d^2\*(d + e\*x^2)^(5/2)) + ((3\*c\*d^2 + 2\*e\*(b\*d + 4\*a\*e))\*x^5)/(15\*d^3\*(d + e\*x^2)^(5/2))

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 264

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rule 1155

Int[((d\_) + (e\_)\*(x\_)^2)^(q\_)\*((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4)^(p\_), x\_Symbol] := Simp[(a^p\*x\*(d + e\*x^2)^(q + 1))/d, x] + Dist[1/d, Int[x^2\*(d + e\*x^2)^q\*(d\*PolynomialQuotient[(a + b\*x^2 + c\*x^4)^p - a^p, x^2, x] - e\*a^p\*(2\*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4\*p + 2\*q + 1, 0]

Rule 1803

Int[(Pq\_)\*(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^2)^(p\_), x\_Symbol] := With[{A = Coeff[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A\*x^(m + 1)\*(a + b\*x^2)^(p + 1))/(a\*(m + 1)), x] + Dist[1/(a\*(m + 1)), Int[x^(m + 2)\*(a + b\*x^2)^p\*(a\*(m + 1)\*Q - A\*b\*(m + 2\*(p + 1) + 1)), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p, 0] && LtQ[m + Expon[Pq, x] + 2\*p + 1, 0]

Rubi steps

$$\begin{aligned}
\int \frac{a + bx^2 + cx^4}{(d + ex^2)^{7/2}} dx &= \frac{ax}{d(d + ex^2)^{5/2}} + \frac{\int \frac{x^2(4ae + d(b + cx^2))}{(d + ex^2)^{7/2}} dx}{d} \\
&= \frac{ax}{d(d + ex^2)^{5/2}} + \frac{(bd + 4ae)x^3}{3d^2(d + ex^2)^{5/2}} + \frac{\int \frac{(3cd^2 + 2e(bd + 4ae))x^4}{(d + ex^2)^{7/2}} dx}{3d^2} \\
&= \frac{ax}{d(d + ex^2)^{5/2}} + \frac{(bd + 4ae)x^3}{3d^2(d + ex^2)^{5/2}} + \frac{1}{3} \left( 3c + \frac{2e(bd + 4ae)}{d^2} \right) \int \frac{x^4}{(d + ex^2)^{7/2}} dx \\
&= \frac{ax}{d(d + ex^2)^{5/2}} + \frac{(bd + 4ae)x^3}{3d^2(d + ex^2)^{5/2}} + \frac{(3cd^2 + 2e(bd + 4ae))x^5}{15d^3(d + ex^2)^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 67, normalized size = 0.78

$$\frac{a(15d^2x + 20dex^3 + 8e^2x^5) + dx^3(5bd + 2bex^2 + 3cdx^2)}{15d^3(d + ex^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^(7/2), x]

[Out] (d\*x^3\*(5\*b\*d + 3\*c\*d\*x^2 + 2\*b\*e\*x^2) + a\*(15\*d^2\*x + 20\*d\*e\*x^3 + 8\*e^2\*x^5))/(15\*d^3\*(d + e\*x^2)^(5/2))

**IntegrateAlgebraic [A]** time = 0.21, size = 69, normalized size = 0.80

$$\frac{15ad^2x + 20adex^3 + 8ae^2x^5 + 5bd^2x^3 + 2bdex^5 + 3cd^2x^5}{15d^3(d + ex^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^(7/2), x]

[Out] (15\*a\*d^2\*x + 5\*b\*d^2\*x^3 + 20\*a\*d\*e\*x^3 + 3\*c\*d^2\*x^5 + 2\*b\*d\*e\*x^5 + 8\*a\*e^2\*x^5)/(15\*d^3\*(d + e\*x^2)^(5/2))

**fricas [A]** time = 1.07, size = 93, normalized size = 1.08

$$\frac{\left( (3cd^2 + 2bde + 8ae^2)x^5 + 15ad^2x + 5(bd^2 + 4ade)x^3 \right) \sqrt{ex^2 + d}}{15(d^3e^3x^6 + 3d^4e^2x^4 + 3d^5ex^2 + d^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^(7/2), x, algorithm="fricas")

[Out] 1/15\*((3\*c\*d^2 + 2\*b\*d\*e + 8\*a\*e^2)\*x^5 + 15\*a\*d^2\*x + 5\*(b\*d^2 + 4\*a\*d\*e)\*x^3)\*sqrt(e\*x^2 + d)/(d^3\*e^3\*x^6 + 3\*d^4\*e^2\*x^4 + 3\*d^5\*e\*x^2 + d^6)

**giac [A]** time = 0.21, size = 75, normalized size = 0.87

$$\frac{\left( x^2 \left( \frac{(3cd^2e^2 + 2bde^3 + 8ae^4)x^2e^{(-2)}}{d^3} + \frac{5(bd^2e^2 + 4ade^3)e^{(-2)}}{d^3} \right) + \frac{15a}{d} \right) x}{15(x^2e + d)^{5/2}}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^(7/2),x, algorithm="giac")

[Out]  $1/15*(x^2*((3*c*d^2*e^2 + 2*b*d*e^3 + 8*a*e^4)*x^2*e^{(-2)}/d^3 + 5*(b*d^2*e^2 + 4*a*d*e^3)*e^{(-2)}/d^3) + 15*a/d)*x/(x^2*e + d)^{(5/2)}$

**maple [A]** time = 0.00, size = 66, normalized size = 0.77

$$\frac{(8a e^2 x^4 + 2b d e x^4 + 3c d^2 x^4 + 20a d e x^2 + 5b d^2 x^2 + 15a d^2) x}{15 (e x^2 + d)^{\frac{5}{2}} d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^4+b\*x^2+a)/(e\*x^2+d)^(7/2),x)

[Out]  $1/15*x*(8*a*e^2*x^4+2*b*d*e*x^4+3*c*d^2*x^4+20*a*d*e*x^2+5*b*d^2*x^2+15*a*d^2)/(e*x^2+d)^{(5/2)}/d^3$

**maxima [B]** time = 1.16, size = 173, normalized size = 2.01

$$\frac{cx^3}{2(ex^2+d)^{\frac{5}{2}}e} + \frac{8ax}{15\sqrt{ex^2+d}d^3} + \frac{4ax}{15(ex^2+d)^{\frac{3}{2}}d^2} + \frac{ax}{5(ex^2+d)^{\frac{5}{2}}d} + \frac{cx}{10(ex^2+d)^{\frac{3}{2}}e^2} + \frac{cx}{5\sqrt{ex^2+d}de^2} - \frac{3cdx}{10(ex^2+d)^{\frac{5}{2}}e^2} - \frac{bx}{5(ex^2+d)^{\frac{5}{2}}e} + \frac{2bx}{15\sqrt{ex^2+d}de^2} + \frac{bx}{15(ex^2+d)^{\frac{3}{2}}de}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^(7/2),x, algorithm="maxima")

[Out]  $-1/2*c*x^3/((e*x^2 + d)^{(5/2)}*e) + 8/15*a*x/(sqrt(e*x^2 + d)*d^3) + 4/15*a*x/((e*x^2 + d)^{(3/2)}*d^2) + 1/5*a*x/((e*x^2 + d)^{(5/2)}*d) + 1/10*c*x/((e*x^2 + d)^{(3/2)}*e^2) + 1/5*c*x/(sqrt(e*x^2 + d)*d*e^2) - 3/10*c*d*x/((e*x^2 + d)^{(5/2)}*e^2) - 1/5*b*x/((e*x^2 + d)^{(5/2)}*e) + 2/15*b*x/(sqrt(e*x^2 + d)*d^2*e) + 1/15*b*x/((e*x^2 + d)^{(3/2)}*d*e)$

**mupad [B]** time = 4.70, size = 133, normalized size = 1.55

$$\frac{3cd^4x - 6cd^3x(ex^2+d) - 3bd^3ex + 8ae^2x(ex^2+d)^2 + 3cd^2x(ex^2+d)^2 + 3ad^2e^2x + 4ade^2x(ex^2+d) + 2bdex(ex^2+d)^2 + bd^2ex(ex^2+d)}{15d^3e^2(ex^2+d)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^2 + c\*x^4)/(d + e\*x^2)^(7/2),x)

[Out]  $(3*c*d^4*x - 6*c*d^3*x*(d + e*x^2) - 3*b*d^3*e*x + 8*a*e^2*x*(d + e*x^2)^2 + 3*c*d^2*x*(d + e*x^2)^2 + 3*a*d^2*e^2*x + 4*a*d*e^2*x*(d + e*x^2) + 2*b*d*e*x*(d + e*x^2)^2 + b*d^2*e*x*(d + e*x^2))/(15*d^3*e^2*(d + e*x^2)^{(5/2)})$

**sympy [B]** time = 45.98, size = 639, normalized size = 7.43

$$\frac{(3cd^4x - 6cd^3x(ex^2+d) - 3bd^3ex + 8ae^2x(ex^2+d)^2 + 3cd^2x(ex^2+d)^2 + 3ad^2e^2x + 4ade^2x(ex^2+d) + 2bdex(ex^2+d)^2 + bd^2ex(ex^2+d))}{15d^3e^2(ex^2+d)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*4+b\*x\*\*2+a)/(e\*x\*\*2+d)\*\*(7/2),x)

[Out]  $a*(15*d**5*x/(15*d**(17/2)*sqrt(1 + e*x**2/d) + 45*d**(15/2)*e*x**2*sqrt(1 + e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 15*d**(11/2)*e**3*x**6*sqrt(1 + e*x**2/d)) + 35*d**4*e*x**3/(15*d**(17/2)*sqrt(1 + e*x**2/d) + 45*d**(15/2)*e*x**2*sqrt(1 + e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 15*d**(11/2)*e**3*x**6*sqrt(1 + e*x**2/d)) + 28*d**3*e**2*x**5/(15*d**(17/2)*sqrt(1 + e*x**2/d) + 45*d**(15/2)*e*x**2*sqrt(1 + e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 15*d**(11/2)*e**3*x**6*sqrt(1 + e*x**2/d)) + 8*d**2*e**3*x**7/(15*d**(17/2)*sqrt(1 + e*x**2/d) + 45*d**$

$$\begin{aligned}
& 15/2)*e^{x^2/d}*\sqrt{1 + e^{x^2/d}} + 45*d^{13/2}*e^{2x^4/d}*\sqrt{1 + e^{x^2/d}} \\
& + 15*d^{11/2}*e^{3x^6/d}*\sqrt{1 + e^{x^2/d}})) + b*(5*d*x^3/(15*d^{9/2})*\sqrt{1 + e^{x^2/d}} + 30*d^{7/2}*e^{x^2/d}*\sqrt{1 + e^{x^2/d}} + 15*d^{5/2}*e^{2x^4/d}*\sqrt{1 + e^{x^2/d}}) + 2*e^{x^5/d}/(15*d^{9/2})*\sqrt{1 + e^{x^2/d}} + 30*d^{7/2}*e^{x^2/d}*\sqrt{1 + e^{x^2/d}} + 15*d^{5/2}*e^{2x^4/d}*\sqrt{1 + e^{x^2/d}})) + c*x^5/(5*d^{7/2})*\sqrt{1 + e^{x^2/d}} + 10*d^{5/2}*e^{x^2/d}*\sqrt{1 + e^{x^2/d}} + 5*d^{3/2}*e^{2x^4/d}*\sqrt{1 + e^{x^2/d}}
\end{aligned}$$

$$3.206 \quad \int \frac{a+bx^2+cx^4}{(d+ex^2)^{9/2}} dx$$

**Optimal.** Leaf size=126

$$\frac{2ex^7(4e(6ae+bd)+3cd^2)}{105d^4(d+ex^2)^{7/2}} + \frac{x^5(4e(6ae+bd)+3cd^2)}{15d^3(d+ex^2)^{7/2}} + \frac{x^3(6ae+bd)}{3d^2(d+ex^2)^{7/2}} + \frac{ax}{d(d+ex^2)^{7/2}}$$

**Rubi [A]** time = 0.15, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$ , Rules used = {1155, 1803, 12, 271, 264}

$$\frac{2ex^7(4e(6ae+bd)+3cd^2)}{105d^4(d+ex^2)^{7/2}} + \frac{x^5(4e(6ae+bd)+3cd^2)}{15d^3(d+ex^2)^{7/2}} + \frac{x^3(6ae+bd)}{3d^2(d+ex^2)^{7/2}} + \frac{ax}{d(d+ex^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^(9/2), x]

[Out] (a\*x)/(d\*(d + e\*x^2)^(7/2)) + ((b\*d + 6\*a\*e)\*x^3)/(3\*d^2\*(d + e\*x^2)^(7/2)) + ((3\*c\*d^2 + 4\*e\*(b\*d + 6\*a\*e))\*x^5)/(15\*d^3\*(d + e\*x^2)^(7/2)) + (2\*e\*(3\*c\*d^2 + 4\*e\*(b\*d + 6\*a\*e))\*x^7)/(105\*d^4\*(d + e\*x^2)^(7/2))

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

**Rule 264**

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a+b\*x^n)^(p+1))/(a\*c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n+p+1, 0] && NeQ[m, -1]

**Rule 271**

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x^(m+1)\*(a+b\*x^n)^(p+1))/(a\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1))/(a\*(m+1)), Int[x^(m+n)\*(a+b\*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m+1)/n+p+1], 0] && NeQ[m, -1]

**Rule 1155**

Int[((d\_) + (e\_)\*(x\_)^2)^(q\_)\*((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4)^(p\_), x\_Symbol] := Simp[(a^p\*x\*(d+e\*x^2)^(q+1))/d, x] + Dist[1/d, Int[x^2\*(d+e\*x^2)^q\*(d\*PolynomialQuotient[(a+b\*x^2+c\*x^4)^p - a^p, x^2, x] - e\*a^p\*(2\*q+3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && ILtQ[q+1/2, 0] && LtQ[4\*p+2\*q+1, 0]

**Rule 1803**

Int[(Pq\_)\*(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^2)^(p\_), x\_Symbol] := With[{A = Coeff[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A\*x^(m+1)\*(a+b\*x^2)^(p+1))/(a\*(m+1)), x] + Dist[1/(a\*(m+1)), Int[x^(m+2)\*(a+b\*x^2)^p\*(a\*(m+1)\*Q - A\*b\*(m+2\*(p+1)+1)), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m+1)/2+p, 0] && LtQ[m+Expon[Pq, x]+2\*p+1, 0]

Rubi steps

$$\begin{aligned}
\int \frac{a + bx^2 + cx^4}{(d + ex^2)^{9/2}} dx &= \frac{ax}{d(d + ex^2)^{7/2}} + \frac{\int \frac{x^2(6ae + d(b + cx^2))}{(d + ex^2)^{9/2}} dx}{d} \\
&= \frac{ax}{d(d + ex^2)^{7/2}} + \frac{(bd + 6ae)x^3}{3d^2(d + ex^2)^{7/2}} + \frac{\int \frac{(3cd^2 + 4e(bd + 6ae))x^4}{(d + ex^2)^{9/2}} dx}{3d^2} \\
&= \frac{ax}{d(d + ex^2)^{7/2}} + \frac{(bd + 6ae)x^3}{3d^2(d + ex^2)^{7/2}} + \frac{1}{3} \left( 3c + \frac{4e(bd + 6ae)}{d^2} \right) \int \frac{x^4}{(d + ex^2)^{9/2}} dx \\
&= \frac{ax}{d(d + ex^2)^{7/2}} + \frac{(bd + 6ae)x^3}{3d^2(d + ex^2)^{7/2}} + \frac{(3cd^2 + 4e(bd + 6ae))x^5}{15d^3(d + ex^2)^{7/2}} + \frac{(2e(3cd^2 + 4e(bd + 6ae)))}{15d^3} \\
&= \frac{ax}{d(d + ex^2)^{7/2}} + \frac{(bd + 6ae)x^3}{3d^2(d + ex^2)^{7/2}} + \frac{(3cd^2 + 4e(bd + 6ae))x^5}{15d^3(d + ex^2)^{7/2}} + \frac{2e(3cd^2 + 4e(bd + 6ae))x^7}{105d^4(d + ex^2)^{7/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 101, normalized size = 0.80

$$\frac{3a(35d^3x + 70d^2ex^3 + 56de^2x^5 + 16e^3x^7) + dx^3(b(35d^2 + 28dex^2 + 8e^2x^4) + 3cdx^2(7d + 2ex^2))}{105d^4(d + ex^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^(9/2), x]

[Out] (3\*a\*(35\*d^3\*x + 70\*d^2\*e\*x^3 + 56\*d\*e^2\*x^5 + 16\*e^3\*x^7) + d\*x^3\*(3\*c\*d\*x^2\*(7\*d + 2\*e\*x^2) + b\*(35\*d^2 + 28\*d\*e\*x^2 + 8\*e^2\*x^4)))/(105\*d^4\*(d + e\*x^2)^(7/2))

**IntegrateAlgebraic [A]** time = 0.29, size = 103, normalized size = 0.82

$$\frac{105ad^3x + 210ad^2ex^3 + 168ade^2x^5 + 48ae^3x^7 + 35bd^3x^3 + 28bd^2ex^5 + 8bde^2x^7 + 21cd^3x^5 + 6cd^2ex^7}{105d^4(d + ex^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^(9/2), x]

[Out] (105\*a\*d^3\*x + 35\*b\*d^3\*x^3 + 210\*a\*d^2\*e\*x^3 + 21\*c\*d^3\*x^5 + 28\*b\*d^2\*e\*x^5 + 168\*a\*d\*e^2\*x^5 + 6\*c\*d^2\*e\*x^7 + 8\*b\*d\*e^2\*x^7 + 48\*a\*e^3\*x^7)/(105\*d^4\*(d + e\*x^2)^(7/2))

**fricas [A]** time = 0.95, size = 136, normalized size = 1.08

$$\frac{(2(3cd^2e + 4bde^2 + 24ae^3)x^7 + 7(3cd^3 + 4bd^2e + 24ade^2)x^5 + 105ad^3x + 35(bd^3 + 6ad^2e)x^3)\sqrt{ex^2 + d}}{105(d^4e^4x^8 + 4d^5e^3x^6 + 6d^6e^2x^4 + 4d^7ex^2 + d^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^(9/2), x, algorithm="fricas")

[Out]  $1/105*(2*(3*c*d^2*e + 4*b*d*e^2 + 24*a*e^3)*x^7 + 7*(3*c*d^3 + 4*b*d^2*e + 24*a*d*e^2)*x^5 + 105*a*d^3*x + 35*(b*d^3 + 6*a*d^2*e)*x^3)*\text{sqrt}(e*x^2 + d) / (d^4*e^4*x^8 + 4*d^5*e^3*x^6 + 6*d^6*e^2*x^4 + 4*d^7*e*x^2 + d^8)$

**giac** [A] time = 0.27, size = 113, normalized size = 0.90

$$\frac{\left( \left( x^2 \left( \frac{2(3cd^2e^4 + 4bde^5 + 24ae^6)x^2e^{(-3)}}{d^4} + \frac{7(3cd^3e^3 + 4bd^2e^4 + 24ade^5)e^{(-3)}}{d^4} \right) + \frac{35(bd^3e^3 + 6ad^2e^4)e^{(-3)}}{d^4} \right) x^2 + \frac{105a}{d} \right) x}{105(x^2e + d)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(9/2),x, algorithm="giac")`

[Out]  $1/105*((x^2*(2*(3*c*d^2*e^4 + 4*b*d*e^5 + 24*a*e^6)*x^2*e^{(-3)}/d^4 + 7*(3*c*d^3*e^3 + 4*b*d^2*e^4 + 24*a*d*e^5)*e^{(-3)}/d^4) + 35*(b*d^3*e^3 + 6*a*d^2*e^4)*e^{(-3)}/d^4)*x^2 + 105*a/d)*x/(x^2*e + d)^{(7/2)}$

**maple** [A] time = 0.00, size = 100, normalized size = 0.79

$$\frac{(48ae^3x^6 + 8bd^2e^2x^6 + 6cd^2ex^6 + 168ad^2e^2x^4 + 28bd^2ex^4 + 21cd^3x^4 + 210ad^2ex^2 + 35bd^3x^2 + 105ad^3)x}{105(e^2x^2 + d)^{\frac{7}{2}}d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^4+b*x^2+a)/(e*x^2+d)^(9/2),x)`

[Out]  $1/105*x*(48*a*e^3*x^6+8*b*d*e^2*x^6+6*c*d^2*e*x^6+168*a*d*e^2*x^4+28*b*d^2*e*x^4+21*c*d^3*x^4+210*a*d^2*e*x^2+35*b*d^3*x^2+105*a*d^3)/(e*x^2+d)^{(7/2)}/d^4$

**maxima** [B] time = 1.20, size = 227, normalized size = 1.80

$$\frac{-\frac{cx^3}{4(e^2x^2+d)^{\frac{7}{2}}e} + \frac{16ax}{35\sqrt{e^2x^2+d}d^4} + \frac{8ax}{35(e^2x^2+d)^{\frac{3}{2}}d^5} + \frac{6ax}{35(e^2x^2+d)^{\frac{5}{2}}d^2} + \frac{ax}{7(e^2x^2+d)^{\frac{7}{2}}d} + \frac{3cx}{140(e^2x^2+d)^{\frac{3}{2}}e^2} + \frac{2cx}{35\sqrt{e^2x^2+d}d^2e^2} + \frac{cx}{35(e^2x^2+d)^{\frac{3}{2}}d^2e^2} - \frac{3cdx}{28(e^2x^2+d)^{\frac{7}{2}}e^2} - \frac{bx}{7(e^2x^2+d)^{\frac{7}{2}}e} + \frac{8bx}{105\sqrt{e^2x^2+d}d^3e} + \frac{4bx}{105(e^2x^2+d)^{\frac{3}{2}}d^4e} + \frac{bx}{35(e^2x^2+d)^{\frac{5}{2}}de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(9/2),x, algorithm="maxima")`

[Out]  $-1/4*c*x^3/((e*x^2 + d)^{(7/2)}*e) + 16/35*a*x/(\text{sqrt}(e*x^2 + d)*d^4) + 8/35*a*x/((e*x^2 + d)^{(3/2)}*d^3) + 6/35*a*x/((e*x^2 + d)^{(5/2)}*d^2) + 1/7*a*x/((e*x^2 + d)^{(7/2)}*d) + 3/140*c*x/((e*x^2 + d)^{(5/2)}*e^2) + 2/35*c*x/(\text{sqrt}(e*x^2 + d)*d^2*e^2) + 1/35*c*x/((e*x^2 + d)^{(3/2)}*d*e^2) - 3/28*c*d*x/((e*x^2 + d)^{(7/2)}*e^2) - 1/7*b*x/((e*x^2 + d)^{(7/2)}*e) + 8/105*b*x/(\text{sqrt}(e*x^2 + d)*d^3*e) + 4/105*b*x/((e*x^2 + d)^{(3/2)}*d^2*e) + 1/35*b*x/((e*x^2 + d)^{(5/2)}*d*e)$

**mupad** [B] time = 4.67, size = 154, normalized size = 1.22

$$\frac{x \left( \frac{a}{7d} - \frac{d \left( \frac{b}{7d} - \frac{c}{7e} \right)}{e} \right)}{(ex^2 + d)^{7/2}} - \frac{x \left( \frac{c}{5e^2} - \frac{-cd^2 + bde + 6ae^2}{35d^2e^2} \right)}{(ex^2 + d)^{5/2}} + \frac{x(3cd^2 + 4bde + 24ae^2)}{105d^3e^2(ex^2 + d)^{3/2}} + \frac{x(6cd^2 + 8bde + 48ae^2)}{105d^4e^2\sqrt{ex^2 + d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^2 + c*x^4)/(d + e*x^2)^(9/2),x)`

[Out]  $(x*(a/(7*d) - (d*(b/(7*d) - c/(7*e)))/e))/(d + e*x^2)^{(7/2)} - (x*(c/(5*e^2) - (6*a*e^2 - c*d^2 + b*d*e)/(35*d^2*e^2)))/(d + e*x^2)^{(5/2)} + (x*(24*a*e^2$

$$\frac{2 + 3cd^2 + 4bde}{(105d^3e^2(d + e^2x)^{3/2})} + \frac{x(48ae^2 + 6cd^2 + 8bde)}{(105d^4e^2(d + e^2x)^{1/2})}$$

**sympy [B]** time = 119.19, size = 1989, normalized size = 15.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*4+b\*x\*\*2+a)/(e\*x\*\*2+d)\*\*(9/2),x)

[Out] a\*(35\*d\*\*14\*x/(35\*d\*\*(37/2)\*sqrt(1 + e\*x\*\*2/d) + 210\*d\*\*(35/2)\*e\*x\*\*2\*sqrt(1 + e\*x\*\*2/d) + 525\*d\*\*(33/2)\*e\*\*2\*x\*\*4\*sqrt(1 + e\*x\*\*2/d) + 700\*d\*\*(31/2)\*e\*\*3\*x\*\*6\*sqrt(1 + e\*x\*\*2/d) + 525\*d\*\*(29/2)\*e\*\*4\*x\*\*8\*sqrt(1 + e\*x\*\*2/d) + 210\*d\*\*(27/2)\*e\*\*5\*x\*\*10\*sqrt(1 + e\*x\*\*2/d) + 35\*d\*\*(25/2)\*e\*\*6\*x\*\*12\*sqrt(1 + e\*x\*\*2/d)) + 175\*d\*\*13\*e\*x\*\*3/(35\*d\*\*(37/2)\*sqrt(1 + e\*x\*\*2/d) + 210\*d\*\*(35/2)\*e\*x\*\*2\*sqrt(1 + e\*x\*\*2/d) + 525\*d\*\*(33/2)\*e\*\*2\*x\*\*4\*sqrt(1 + e\*x\*\*2/d) + 700\*d\*\*(31/2)\*e\*\*3\*x\*\*6\*sqrt(1 + e\*x\*\*2/d) + 525\*d\*\*(29/2)\*e\*\*4\*x\*\*8\*sqrt(1 + e\*x\*\*2/d) + 210\*d\*\*(27/2)\*e\*\*5\*x\*\*10\*sqrt(1 + e\*x\*\*2/d) + 35\*d\*\*(25/2)\*e\*\*6\*x\*\*12\*sqrt(1 + e\*x\*\*2/d)) + 371\*d\*\*12\*e\*\*2\*x\*\*5/(35\*d\*\*(37/2)\*sqrt(1 + e\*x\*\*2/d) + 210\*d\*\*(35/2)\*e\*x\*\*2\*sqrt(1 + e\*x\*\*2/d) + 525\*d\*\*(33/2)\*e\*\*2\*x\*\*4\*sqrt(1 + e\*x\*\*2/d) + 700\*d\*\*(31/2)\*e\*\*3\*x\*\*6\*sqrt(1 + e\*x\*\*2/d) + 525\*d\*\*(29/2)\*e\*\*4\*x\*\*8\*sqrt(1 + e\*x\*\*2/d) + 210\*d\*\*(27/2)\*e\*\*5\*x\*\*10\*sqrt(1 + e\*x\*\*2/d) + 35\*d\*\*(25/2)\*e\*\*6\*x\*\*12\*sqrt(1 + e\*x\*\*2/d)) + 429\*d\*\*11\*e\*\*3\*x\*\*7/(35\*d\*\*(37/2)\*sqrt(1 + e\*x\*\*2/d) + 210\*d\*\*(35/2)\*e\*x\*\*2\*sqrt(1 + e\*x\*\*2/d) + 525\*d\*\*(33/2)\*e\*\*2\*x\*\*4\*sqrt(1 + e\*x\*\*2/d) + 700\*d\*\*(31/2)\*e\*\*3\*x\*\*6\*sqrt(1 + e\*x\*\*2/d) + 525\*d\*\*(29/2)\*e\*\*4\*x\*\*8\*sqrt(1 + e\*x\*\*2/d) + 210\*d\*\*(27/2)\*e\*\*5\*x\*\*10\*sqrt(1 + e\*x\*\*2/d) + 35\*d\*\*(25/2)\*e\*\*6\*x\*\*12\*sqrt(1 + e\*x\*\*2/d)) + 286\*d\*\*10\*e\*\*4\*x\*\*9/(35\*d\*\*(37/2)\*sqrt(1 + e\*x\*\*2/d) + 210\*d\*\*(35/2)\*e\*x\*\*2\*sqrt(1 + e\*x\*\*2/d) + 525\*d\*\*(33/2)\*e\*\*2\*x\*\*4\*sqrt(1 + e\*x\*\*2/d) + 700\*d\*\*(31/2)\*e\*\*3\*x\*\*6\*sqrt(1 + e\*x\*\*2/d) + 525\*d\*\*(29/2)\*e\*\*4\*x\*\*8\*sqrt(1 + e\*x\*\*2/d) + 210\*d\*\*(27/2)\*e\*\*5\*x\*\*10\*sqrt(1 + e\*x\*\*2/d) + 35\*d\*\*(25/2)\*e\*\*6\*x\*\*12\*sqrt(1 + e\*x\*\*2/d)) + 104\*d\*\*9\*e\*\*5\*x\*\*11/(35\*d\*\*(37/2)\*sqrt(1 + e\*x\*\*2/d) + 210\*d\*\*(35/2)\*e\*x\*\*2\*sqrt(1 + e\*x\*\*2/d) + 525\*d\*\*(33/2)\*e\*\*2\*x\*\*4\*sqrt(1 + e\*x\*\*2/d) + 700\*d\*\*(31/2)\*e\*\*3\*x\*\*6\*sqrt(1 + e\*x\*\*2/d) + 525\*d\*\*(29/2)\*e\*\*4\*x\*\*8\*sqrt(1 + e\*x\*\*2/d) + 210\*d\*\*(27/2)\*e\*\*5\*x\*\*10\*sqrt(1 + e\*x\*\*2/d) + 35\*d\*\*(25/2)\*e\*\*6\*x\*\*12\*sqrt(1 + e\*x\*\*2/d)) + 16\*d\*\*8\*e\*\*6\*x\*\*13/(35\*d\*\*(37/2)\*sqrt(1 + e\*x\*\*2/d) + 210\*d\*\*(35/2)\*e\*x\*\*2\*sqrt(1 + e\*x\*\*2/d) + 525\*d\*\*(33/2)\*e\*\*2\*x\*\*4\*sqrt(1 + e\*x\*\*2/d) + 700\*d\*\*(31/2)\*e\*\*3\*x\*\*6\*sqrt(1 + e\*x\*\*2/d) + 525\*d\*\*(29/2)\*e\*\*4\*x\*\*8\*sqrt(1 + e\*x\*\*2/d) + 210\*d\*\*(27/2)\*e\*\*5\*x\*\*10\*sqrt(1 + e\*x\*\*2/d) + 35\*d\*\*(25/2)\*e\*\*6\*x\*\*12\*sqrt(1 + e\*x\*\*2/d)) + b\*(35\*d\*\*5\*x\*\*3/(105\*d\*\*(19/2)\*sqrt(1 + e\*x\*\*2/d) + 420\*d\*\*(17/2)\*e\*x\*\*2\*sqrt(1 + e\*x\*\*2/d) + 630\*d\*\*(15/2)\*e\*\*2\*x\*\*4\*sqrt(1 + e\*x\*\*2/d) + 420\*d\*\*(13/2)\*e\*\*3\*x\*\*6\*sqrt(1 + e\*x\*\*2/d) + 105\*d\*\*(11/2)\*e\*\*4\*x\*\*8\*sqrt(1 + e\*x\*\*2/d) + 63\*d\*\*4\*e\*x\*\*5/(105\*d\*\*(19/2)\*sqrt(1 + e\*x\*\*2/d) + 420\*d\*\*(17/2)\*e\*x\*\*2\*sqrt(1 + e\*x\*\*2/d) + 630\*d\*\*(15/2)\*e\*\*2\*x\*\*4\*sqrt(1 + e\*x\*\*2/d) + 420\*d\*\*(13/2)\*e\*\*3\*x\*\*6\*sqrt(1 + e\*x\*\*2/d) + 105\*d\*\*(11/2)\*e\*\*4\*x\*\*8\*sqrt(1 + e\*x\*\*2/d)) + 36\*d\*\*3\*e\*\*2\*x\*\*7/(105\*d\*\*(19/2)\*sqrt(1 + e\*x\*\*2/d) + 420\*d\*\*(17/2)\*e\*x\*\*2\*sqrt(1 + e\*x\*\*2/d) + 630\*d\*\*(15/2)\*e\*\*2\*x\*\*4\*sqrt(1 + e\*x\*\*2/d) + 420\*d\*\*(13/2)\*e\*\*3\*x\*\*6\*sqrt(1 + e\*x\*\*2/d) + 105\*d\*\*(11/2)\*e\*\*4\*x\*\*8\*sqrt(1 + e\*x\*\*2/d)) + 8\*d\*\*2\*e\*\*3\*x\*\*9/(105\*d\*\*(19/2)\*sqrt(1 + e\*x\*\*2/d) + 420\*d\*\*(17/2)\*e\*x\*\*2\*sqrt(1 + e\*x\*\*2/d) + 630\*d\*\*(15/2)\*e\*\*2\*x\*\*4\*sqrt(1 + e\*x\*\*2/d) + 420\*d\*\*(13/2)\*e\*\*3\*x\*\*6\*sqrt(1 + e\*x\*\*2/d) + 105\*d\*\*(11/2)\*e\*\*4\*x\*\*8\*sqrt(1 + e\*x\*\*2/d)) + c\*(7\*d\*x\*\*5/(35\*d\*\*(11/2)\*sqrt(1 + e\*x\*\*2/d) + 105\*d\*\*(9/2)\*e\*x\*\*2\*sqrt(1 + e\*x\*\*2/d) + 105\*d\*\*(7/2)\*e\*\*2\*x\*\*4\*sqrt(1 + e\*x\*\*2/d) + 35\*d\*\*(5/2)\*e\*\*3\*x\*\*6\*sqrt(1 + e\*x\*\*2/d) + 2\*e\*x\*\*7/(35\*d\*\*(11/2)\*sqrt(1 + e\*x\*\*2/d) + 105\*d\*\*(9/2)\*e\*x\*\*2\*sqrt(1 + e\*x\*\*2/d) + 105\*d\*\*(7/2)\*e\*\*2\*x\*\*4\*sqrt(1 + e\*x\*\*2/d) + 35\*d\*\*(5/2)\*e\*\*3\*x\*\*6\*sqrt(1 + e\*x\*\*2/d)))

$$3.207 \quad \int \frac{a+bx^2+cx^4}{(d+ex^2)^{11/2}} dx$$

**Optimal.** Leaf size=165

$$\frac{8e^2x^9(2e(8ae+bd)+cd^2)}{315d^5(d+ex^2)^{9/2}} + \frac{4ex^7(2e(8ae+bd)+cd^2)}{35d^4(d+ex^2)^{9/2}} + \frac{x^5(2e(8ae+bd)+cd^2)}{5d^3(d+ex^2)^{9/2}} + \frac{x^3(8ae+bd)}{3d^2(d+ex^2)^{9/2}} + \frac{ax}{d(d+ex^2)^{9/2}}$$

**Rubi [A]** time = 0.21, antiderivative size = 164, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$ , Rules used = {1155, 1803, 12, 271, 264}

$$\frac{8e^2x^9(2e(8ae+bd)+cd^2)}{315d^5(d+ex^2)^{9/2}} + \frac{4ex^7(2e(8ae+bd)+cd^2)}{35d^4(d+ex^2)^{9/2}} + \frac{x^5\left(\frac{2e(8ae+bd)}{d^2}+c\right)}{5d(d+ex^2)^{9/2}} + \frac{x^3(8ae+bd)}{3d^2(d+ex^2)^{9/2}} + \frac{ax}{d(d+ex^2)^{9/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^(11/2), x]

[Out] (a\*x)/(d\*(d + e\*x^2)^(9/2)) + ((b\*d + 8\*a\*e)\*x^3)/(3\*d^2\*(d + e\*x^2)^(9/2)) + ((c + (2\*e\*(b\*d + 8\*a\*e))/d^2)\*x^5)/(5\*d\*(d + e\*x^2)^(9/2)) + (4\*e\*(c\*d^2 + 2\*e\*(b\*d + 8\*a\*e))\*x^7)/(35\*d^4\*(d + e\*x^2)^(9/2)) + (8\*e^2\*(c\*d^2 + 2\*e\*(b\*d + 8\*a\*e))\*x^9)/(315\*d^5\*(d + e\*x^2)^(9/2))

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 264

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n + p + 1, 0] && NeQ[m, -1]

#### Rule 271

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x^(m+1)\*(a + b\*x^n)^(p+1))/(a\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1))/(a\*(m+1)), Int[x^(m+n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m+1)/n + p + 1], 0] && NeQ[m, -1]

#### Rule 1155

Int[((d\_) + (e\_)\*(x\_)^2)^(q\_)\*((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4)^(p\_), x\_Symbol] := Simp[(a^p\*x\*(d + e\*x^2)^(q+1))/d, x] + Dist[1/d, Int[x^2\*(d + e\*x^2)^q\*(d\*PolynomialQuotient[(a + b\*x^2 + c\*x^4)^p - a^p, x^2, x] - e\*a^p\*(2\*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4\*p + 2\*q + 1, 0]

#### Rule 1803

Int[(Pq\_)\*(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^2)^(p\_), x\_Symbol] := With[{A = Coeff[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A\*x^(m+1)\*(a + b\*x^2)^(p+1))/(a\*(m+1)), x] + Dist[1/(a\*(m+1)), Int[x^(m+2)\*(a + b\*x^2)^p\*(a\*(m+1)\*Q - A\*b\*(m+2\*(p+1)+1)), x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m+1)/2 + p,

0] && LtQ[m + Expon[Pq, x] + 2\*p + 1, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{a + bx^2 + cx^4}{(d + ex^2)^{11/2}} dx &= \frac{ax}{d(d + ex^2)^{9/2}} + \frac{\int \frac{x^2(8ae + d(b + cx^2))}{(d + ex^2)^{11/2}} dx}{d} \\
 &= \frac{ax}{d(d + ex^2)^{9/2}} + \frac{(bd + 8ae)x^3}{3d^2(d + ex^2)^{9/2}} + \frac{\int \frac{(3cd^2 + 6e(bd + 8ae))x^4}{(d + ex^2)^{11/2}} dx}{3d^2} \\
 &= \frac{ax}{d(d + ex^2)^{9/2}} + \frac{(bd + 8ae)x^3}{3d^2(d + ex^2)^{9/2}} + \left(c + \frac{2e(bd + 8ae)}{d^2}\right) \int \frac{x^4}{(d + ex^2)^{11/2}} dx \\
 &= \frac{ax}{d(d + ex^2)^{9/2}} + \frac{(bd + 8ae)x^3}{3d^2(d + ex^2)^{9/2}} + \frac{\left(c + \frac{2e(bd + 8ae)}{d^2}\right)x^5}{5d(d + ex^2)^{9/2}} + \frac{\left(4e\left(c + \frac{2e(bd + 8ae)}{d^2}\right)\right) \int \frac{x^6}{(d + ex^2)^{11/2}} dx}{5d} \\
 &= \frac{ax}{d(d + ex^2)^{9/2}} + \frac{(bd + 8ae)x^3}{3d^2(d + ex^2)^{9/2}} + \frac{\left(c + \frac{2e(bd + 8ae)}{d^2}\right)x^5}{5d(d + ex^2)^{9/2}} + \frac{4e\left(c + \frac{2e(bd + 8ae)}{d^2}\right)x^7}{35d^2(d + ex^2)^{9/2}} + \frac{\left(8e^2\left(c + \frac{2e(bd + 8ae)}{d^2}\right)\right) \int \frac{x^8}{(d + ex^2)^{11/2}} dx}{315d^3} \\
 &= \frac{ax}{d(d + ex^2)^{9/2}} + \frac{(bd + 8ae)x^3}{3d^2(d + ex^2)^{9/2}} + \frac{\left(c + \frac{2e(bd + 8ae)}{d^2}\right)x^5}{5d(d + ex^2)^{9/2}} + \frac{4e\left(c + \frac{2e(bd + 8ae)}{d^2}\right)x^7}{35d^2(d + ex^2)^{9/2}} + \frac{8e^2\left(c + \frac{2e(bd + 8ae)}{d^2}\right) \int \frac{x^8}{(d + ex^2)^{11/2}} dx}{315d^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.12, size = 132, normalized size = 0.80

$$\frac{a(315d^4x + 840d^3ex^3 + 1008d^2e^2x^5 + 576de^3x^7 + 128e^4x^9) + dx^3(b(105d^3 + 126d^2ex^2 + 72de^2x^4 + 16e^3x^6) + cdx^2(63d^2 + 36dex^2 + 8e^2x^4))}{315d^5(d + ex^2)^{9/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^(11/2), x]

[Out] (a\*(315\*d^4\*x + 840\*d^3\*e\*x^3 + 1008\*d^2\*e^2\*x^5 + 576\*d\*e^3\*x^7 + 128\*e^4\*x^9) + d\*x^3\*(c\*d\*x^2\*(63\*d^2 + 36\*d\*e\*x^2 + 8\*e^2\*x^4) + b\*(105\*d^3 + 126\*d^2\*e\*x^2 + 72\*d\*e^2\*x^4 + 16\*e^3\*x^6)))/(315\*d^5\*(d + e\*x^2)^(9/2))

**IntegrateAlgebraic [A]** time = 0.40, size = 139, normalized size = 0.84

$$\frac{315ad^4x + 840ad^3ex^3 + 1008ad^2e^2x^5 + 576ade^3x^7 + 128ae^4x^9 + 105bd^4x^3 + 126bd^3ex^5 + 72bd^2e^2x^7 + 16bde^3x^9 + 63cd^4x^5 + 36cd^3ex^7 + 8cd^2e^2x^9}{315d^5(d + ex^2)^{9/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^(11/2), x]

[Out] (315\*a\*d^4\*x + 105\*b\*d^4\*x^3 + 840\*a\*d^3\*e\*x^3 + 63\*c\*d^4\*x^5 + 126\*b\*d^3\*e\*x^5 + 1008\*a\*d^2\*e^2\*x^5 + 36\*c\*d^3\*e\*x^7 + 72\*b\*d^2\*e^2\*x^7 + 576\*a\*d\*e^3\*x^7 + 8\*c\*d^2\*e^2\*x^9 + 16\*b\*d\*e^3\*x^9 + 128\*a\*e^4\*x^9)/(315\*d^5\*(d + e\*x^2)^(9/2))

**fricas [A]** time = 1.15, size = 177, normalized size = 1.07

$$\frac{(8(cd^2e^2 + 2bde^3 + 16ae^4)x^9 + 36(cd^3e + 2bd^2e^2 + 16ade^3)x^7 + 315ad^4x + 63(cd^4 + 2bd^3e + 16ad^2e^2)x^5 + 105(bd^4 + 8ad^3e)x^3)\sqrt{ex^2 + d}}{315(d^5e^5x^{10} + 5d^6e^4x^8 + 10d^7e^3x^6 + 10d^8e^2x^4 + 5d^9ex^2 + d^{10})}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^(11/2),x, algorithm="fricas")

[Out] 1/315\*(8\*(c\*d^2\*e^2 + 2\*b\*d\*e^3 + 16\*a\*e^4)\*x^9 + 36\*(c\*d^3\*e + 2\*b\*d^2\*e^2 + 16\*a\*d\*e^3)\*x^7 + 315\*a\*d^4\*x + 63\*(c\*d^4 + 2\*b\*d^3\*e + 16\*a\*d^2\*e^2)\*x^5 + 105\*(b\*d^4 + 8\*a\*d^3\*e)\*x^3)\*sqrt(e\*x^2 + d)/(d^5\*e^5\*x^10 + 5\*d^6\*e^4\*x^8 + 10\*d^7\*e^3\*x^6 + 10\*d^8\*e^2\*x^4 + 5\*d^9\*e\*x^2 + d^10)

**giac** [A] time = 0.23, size = 148, normalized size = 0.90

$$\frac{\left(\left(4x^2\left(\frac{2(cd^2e^6+2bde^7+16ae^8)x^2e^{(-4)}}{d^5} + \frac{9(cd^3e^5+2bd^2e^6+16ade^7)e^{(-4)}}{d^5}\right) + \frac{63(cd^4e^4+2bd^3e^5+16ad^2e^6)e^{(-4)}}{d^5}\right)x^2 + \frac{105(bd^4e^4+8ad^3e^5)e^{(-4)}}{d^5}\right)x^2 + \frac{315a}{d}x}{315(x^2e + d)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^(11/2),x, algorithm="giac")

[Out] 1/315\*(((4\*x^2\*(2\*(c\*d^2\*e^6 + 2\*b\*d\*e^7 + 16\*a\*e^8)\*x^2\*e^(-4)/d^5 + 9\*(c\*d^3\*e^5 + 2\*b\*d^2\*e^6 + 16\*a\*d\*e^7)\*e^(-4)/d^5) + 63\*(c\*d^4\*e^4 + 2\*b\*d^3\*e^5 + 16\*a\*d^2\*e^6)\*e^(-4)/d^5)\*x^2 + 105\*(b\*d^4\*e^4 + 8\*a\*d^3\*e^5)\*e^(-4)/d^5)\*x^2 + 315\*a/d)\*x/(x^2\*e + d)^(9/2)

**maple** [A] time = 0.01, size = 136, normalized size = 0.82

$$\frac{(128a e^4 x^8 + 16bd e^3 x^8 + 8c d^2 e^2 x^8 + 576ad e^3 x^6 + 72b d^2 e^2 x^6 + 36c d^3 e x^6 + 1008a d^2 e^2 x^4 + 126b d^3 e x^4 + 63c d^4 x^4 + 840a d^3 e x^2 + 105b d^4 x^2 + 315a d^4)x}{315(e x^2 + d)^{\frac{9}{2}} d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^4+b\*x^2+a)/(e\*x^2+d)^(11/2),x)

[Out] 1/315\*x\*(128\*a\*e^4\*x^8+16\*b\*d\*e^3\*x^8+8\*c\*d^2\*e^2\*x^8+576\*a\*d\*e^3\*x^6+72\*b\*d^2\*e^2\*x^6+36\*c\*d^3\*e\*x^6+1008\*a\*d^2\*e^2\*x^4+126\*b\*d^3\*e\*x^4+63\*c\*d^4\*x^4+840\*a\*d^3\*e\*x^2+105\*b\*d^4\*x^2+315\*a\*d^4)/(e\*x^2+d)^(9/2)/d^5

**maxima** [A] time = 1.20, size = 281, normalized size = 1.70

$$\frac{-\frac{cx^3}{6(e^2x^2+d)^{\frac{7}{2}}} + \frac{128ax}{315\sqrt{e^2x^2+d}d^6} + \frac{64ax}{315(e^2x^2+d)^{\frac{5}{2}}d^4} + \frac{16ax}{105(e^2x^2+d)^{\frac{3}{2}}d^2} + \frac{8ax}{63(e^2x^2+d)^{\frac{1}{2}}d^2} + \frac{ax}{9(e^2x^2+d)^{\frac{1}{2}}d} + \frac{cx}{126(e^2x^2+d)^{\frac{1}{2}}e^2} + \frac{8cx}{315\sqrt{e^2x^2+d}d^6} + \frac{4cx}{315(e^2x^2+d)^{\frac{5}{2}}d^4} + \frac{cx}{105(e^2x^2+d)^{\frac{3}{2}}d^2} + \frac{cx}{18(e^2x^2+d)^{\frac{1}{2}}e^2} - \frac{bx}{9(e^2x^2+d)^{\frac{1}{2}}e} + \frac{16bx}{315\sqrt{e^2x^2+d}d^6} + \frac{8bx}{315(e^2x^2+d)^{\frac{5}{2}}d^4} + \frac{2bx}{105(e^2x^2+d)^{\frac{3}{2}}d^2} + \frac{bx}{63(e^2x^2+d)^{\frac{1}{2}}d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^(11/2),x, algorithm="maxima")

[Out] -1/6\*c\*x^3/((e\*x^2 + d)^(9/2)\*e) + 128/315\*a\*x/(sqrt(e\*x^2 + d)\*d^5) + 64/315\*a\*x/((e\*x^2 + d)^(3/2)\*d^4) + 16/105\*a\*x/((e\*x^2 + d)^(5/2)\*d^3) + 8/63\*a\*x/((e\*x^2 + d)^(7/2)\*d^2) + 1/9\*a\*x/((e\*x^2 + d)^(9/2)\*d) + 1/126\*c\*x/((e\*x^2 + d)^(7/2)\*e^2) + 8/315\*c\*x/(sqrt(e\*x^2 + d)\*d^3\*e^2) + 4/315\*c\*x/((e\*x^2 + d)^(3/2)\*d^2\*e^2) + 1/105\*c\*x/((e\*x^2 + d)^(5/2)\*d\*e^2) - 1/18\*c\*d\*x/((e\*x^2 + d)^(9/2)\*e^2) - 1/9\*b\*x/((e\*x^2 + d)^(9/2)\*e) + 16/315\*b\*x/(sqrt(e\*x^2 + d)\*d^4\*e) + 8/315\*b\*x/((e\*x^2 + d)^(3/2)\*d^3\*e) + 2/105\*b\*x/((e\*x^2 + d)^(5/2)\*d^2\*e) + 1/63\*b\*x/((e\*x^2 + d)^(7/2)\*d\*e)

**mupad** [B] time = 4.75, size = 189, normalized size = 1.15

$$x \left( \frac{a}{9d} - \frac{d \left( \frac{b}{9d} - \frac{c}{9e} \right)}{e} \right) \frac{x \left( \frac{c}{7e^2} - \frac{-cd^2+bde+8ae^2}{63d^2e^2} \right)}{(ex^2+d)^{7/2}} + \frac{x (cd^2 + 2bde + 16ae^2)}{105d^3e^2(ex^2+d)^{5/2}} + \frac{x (4cd^2 + 8bde + 64ae^2)}{315d^4e^2(ex^2+d)^{3/2}} + \frac{x (8cd^2 + 16bde + 128ae^2)}{315d^5e^2\sqrt{ex^2+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^2 + c\*x^4)/(d + e\*x^2)^(11/2),x)

```
[Out] (x*(a/(9*d) - (d*(b/(9*d) - c/(9*e)))/e))/(d + e*x^2)^(9/2) - (x*(c/(7*e^2)
- (8*a*e^2 - c*d^2 + b*d*e)/(63*d^2*e^2)))/(d + e*x^2)^(7/2) + (x*(16*a*e^
2 + c*d^2 + 2*b*d*e))/(105*d^3*e^2*(d + e*x^2)^(5/2)) + (x*(64*a*e^2 + 4*c*
d^2 + 8*b*d*e))/(315*d^4*e^2*(d + e*x^2)^(3/2)) + (x*(128*a*e^2 + 8*c*d^2 +
16*b*d*e))/(315*d^5*e^2*(d + e*x^2)^(1/2))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(11/2),x)
```

```
[Out] Timed out
```

$$3.208 \quad \int \frac{a+bx^2+cx^4}{(d+ex^2)^{13/2}} dx$$

**Optimal.** Leaf size=210

$$\frac{16e^3x^{11}(8e(10ae+bd)+3cd^2)}{3465d^6(d+ex^2)^{11/2}} + \frac{8e^2x^9(8e(10ae+bd)+3cd^2)}{315d^5(d+ex^2)^{11/2}} + \frac{2ex^7(8e(10ae+bd)+3cd^2)}{35d^4(d+ex^2)^{11/2}} + \frac{x^5(8e(10ae+bd)+3cd^2)}{15d^3(d+ex^2)^{11/2}}$$

**Rubi [A]** time = 0.22, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, number of rules / integrand size = 0.208, Rules used = {1155, 1803, 12, 271, 264}

$$\frac{16e^3x^{11}(8e(10ae+bd)+3cd^2)}{3465d^6(d+ex^2)^{11/2}} + \frac{8e^2x^9(8e(10ae+bd)+3cd^2)}{315d^5(d+ex^2)^{11/2}} + \frac{2ex^7(8e(10ae+bd)+3cd^2)}{35d^4(d+ex^2)^{11/2}} + \frac{x^5(8e(10ae+bd)+3cd^2)}{15d^3(d+ex^2)^{11/2}} + \frac{x^3(10ae+bd)}{3d^2(d+ex^2)^{11/2}} + \frac{ax}{d(d+ex^2)^{11/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^(13/2), x]

[Out] (a\*x)/(d\*(d + e\*x^2)^(11/2)) + ((b\*d + 10\*a\*e)\*x^3)/(3\*d^2\*(d + e\*x^2)^(11/2)) + ((3\*c\*d^2 + 8\*e\*(b\*d + 10\*a\*e))\*x^5)/(15\*d^3\*(d + e\*x^2)^(11/2)) + (2\*e\*(3\*c\*d^2 + 8\*e\*(b\*d + 10\*a\*e))\*x^7)/(35\*d^4\*(d + e\*x^2)^(11/2)) + (8\*e^2\*(3\*c\*d^2 + 8\*e\*(b\*d + 10\*a\*e))\*x^9)/(315\*d^5\*(d + e\*x^2)^(11/2)) + (16\*e^3\*(3\*c\*d^2 + 8\*e\*(b\*d + 10\*a\*e))\*x^11)/(3465\*d^6\*(d + e\*x^2)^(11/2))

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 264

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n + p + 1, 0] && NeQ[m, -1]

Rule 271

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x^(m+1)\*(a + b\*x^n)^(p+1))/(a\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1))/(a\*(m+1)), Int[x^(m+n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m+1)/n + p + 1], 0] && NeQ[m, -1]

Rule 1155

Int[((d\_) + (e\_)\*(x\_)^2)^(q\_)\*((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4)^(p\_), x\_Symbol] := Simp[(a^p\*x\*(d + e\*x^2)^(q+1))/d, x] + Dist[1/d, Int[x^2\*(d + e\*x^2)^q\*(d\*PolynomialQuotient[(a + b\*x^2 + c\*x^4)^p - a^p, x^2, x] - e\*a^p\*(2\*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4\*p + 2\*q + 1, 0]

Rule 1803

Int[(Pq\_)\*(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^2)^(p\_), x\_Symbol] := With[{A = Coeff[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A\*x^(m+1)\*(a + b\*x^2)^(p+1))/(a\*(m+1)), x] + Dist[1/(a\*(m+1)), Int[x^(m+2)\*(a + b\*x^2)^p\*(a\*(m+1)\*Q - A\*b\*(m+2\*(p+1)+1)), x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m+1)/2 + p,

0] && LtQ[m + Expon[Pq, x] + 2\*p + 1, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{a + bx^2 + cx^4}{(d + ex^2)^{13/2}} dx &= \frac{ax}{d(d + ex^2)^{11/2}} + \frac{\int \frac{x^2(10ae + d(b + cx^2))}{(d + ex^2)^{13/2}} dx}{d} \\
 &= \frac{ax}{d(d + ex^2)^{11/2}} + \frac{(bd + 10ae)x^3}{3d^2(d + ex^2)^{11/2}} + \frac{\int \frac{(3cd^2 + 8e(bd + 10ae))x^4}{(d + ex^2)^{13/2}} dx}{3d^2} \\
 &= \frac{ax}{d(d + ex^2)^{11/2}} + \frac{(bd + 10ae)x^3}{3d^2(d + ex^2)^{11/2}} + \frac{1}{3} \left( 3c + \frac{8e(bd + 10ae)}{d^2} \right) \int \frac{x^4}{(d + ex^2)^{13/2}} dx \\
 &= \frac{ax}{d(d + ex^2)^{11/2}} + \frac{(bd + 10ae)x^3}{3d^2(d + ex^2)^{11/2}} + \frac{(3cd^2 + 8e(bd + 10ae))x^5}{15d^3(d + ex^2)^{11/2}} + \frac{(2e(3cd^2 + 8e(bd + 10ae)))x^7}{5d^3} \\
 &= \frac{ax}{d(d + ex^2)^{11/2}} + \frac{(bd + 10ae)x^3}{3d^2(d + ex^2)^{11/2}} + \frac{(3cd^2 + 8e(bd + 10ae))x^5}{15d^3(d + ex^2)^{11/2}} + \frac{2e(3cd^2 + 8e(bd + 10ae))x^7}{35d^4(d + ex^2)^{11/2}} \\
 &= \frac{ax}{d(d + ex^2)^{11/2}} + \frac{(bd + 10ae)x^3}{3d^2(d + ex^2)^{11/2}} + \frac{(3cd^2 + 8e(bd + 10ae))x^5}{15d^3(d + ex^2)^{11/2}} + \frac{2e(3cd^2 + 8e(bd + 10ae))x^7}{35d^4(d + ex^2)^{11/2}} \\
 &= \frac{ax}{d(d + ex^2)^{11/2}} + \frac{(bd + 10ae)x^3}{3d^2(d + ex^2)^{11/2}} + \frac{(3cd^2 + 8e(bd + 10ae))x^5}{15d^3(d + ex^2)^{11/2}} + \frac{2e(3cd^2 + 8e(bd + 10ae))x^7}{35d^4(d + ex^2)^{11/2}}
 \end{aligned}$$

**Mathematica [A]** time = 0.14, size = 167, normalized size = 0.80

$$\frac{5a(693d^5x + 2310d^4ex^3 + 3696d^3e^2x^5 + 3168d^2e^3x^7 + 1408de^4x^9 + 256e^5x^{11}) + dx^3(b(1155d^4 + 1848d^3ex^2 + 1584d^2e^2x^4 + 704de^3x^6 + 128e^4x^8) + 3cd^2(231d^3 + 198d^2ex^2 + 88de^2x^4 + 16e^3x^6))}{3465d^6(d + ex^2)^{11/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^(13/2), x]

[Out] (5\*a\*(693\*d^5\*x + 2310\*d^4\*e\*x^3 + 3696\*d^3\*e^2\*x^5 + 3168\*d^2\*e^3\*x^7 + 1408\*d\*e^4\*x^9 + 256\*e^5\*x^11) + d\*x^3\*(3\*c\*d\*x^2\*(231\*d^3 + 198\*d^2\*e\*x^2 + 88\*d\*e^2\*x^4 + 16\*e^3\*x^6) + b\*(1155\*d^4 + 1848\*d^3\*e\*x^2 + 1584\*d^2\*e^2\*x^4 + 704\*d\*e^3\*x^6 + 128\*e^4\*x^8)))/(3465\*d^6\*(d + e\*x^2)^(11/2))

**IntegrateAlgebraic [A]** time = 0.50, size = 175, normalized size = 0.83

$$\frac{3465ad^5x + 11550ad^4ex^3 + 18480ad^3e^2x^5 + 15840ad^2e^3x^7 + 7040ade^4x^9 + 1280ae^5x^{11} + 1155bd^5x^3 + 1848bd^4ex^5 + 1584bd^3e^2x^7 + 704bd^2e^3x^9 + 128bd^1e^4x^{11} + 693cd^5x^5 + 594cd^4ex^7 + 264cd^3e^2x^9 + 48cd^2e^3x^{11}}{3465d^6(d + ex^2)^{11/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b\*x^2 + c\*x^4)/(d + e\*x^2)^(13/2), x]

[Out] (3465\*a\*d^5\*x + 1155\*b\*d^5\*x^3 + 11550\*a\*d^4\*e\*x^5 + 693\*c\*d^5\*x^7 + 1848\*b\*d^4\*e\*x^9 + 18480\*a\*d^3\*e^2\*x^11 + 594\*c\*d^4\*e\*x^7 + 1584\*b\*d^3\*e^2\*x^9 + 15840\*a\*d^2\*e^3\*x^11 + 264\*c\*d^3\*e^2\*x^9 + 704\*b\*d^2\*e^3\*x^9 + 7040\*a\*d\*e^4\*x^11 + 48\*c\*d^2\*e^3\*x^11 + 128\*b\*d\*e^4\*x^11 + 1280\*a\*e^5\*x^11)/(3465\*d^6\*(d + e\*x^2)^(11/2))

**fricas** [A] time = 1.31, size = 224, normalized size = 1.07

$$\frac{(16(3cd^2e^3 + 8bde^4 + 80ae^5)x^{11} + 88(3cd^3e^2 + 8bd^2e^3 + 80ade^4)x^9 + 198(3cd^4e + 8bd^3e^2 + 80ad^2e^3)x^7 + 3465ad^5x + 231(3cd^5 + 8bd^4e + 80ad^3e^2)x^5 + 1155(bd^5 + 10ad^4e)x^3)\sqrt{ex^2 + d}}{3465(d^6e^5x^{12} + 6d^7e^5x^{10} + 15d^8e^4x^8 + 20d^9e^3x^6 + 15d^{10}e^2x^4 + 6d^{11}ex^2 + d^{12})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^(13/2),x, algorithm="fricas")

[Out] 1/3465\*(16\*(3\*c\*d^2\*e^3 + 8\*b\*d\*e^4 + 80\*a\*e^5)\*x^11 + 88\*(3\*c\*d^3\*e^2 + 8\*b\*d^2\*e^3 + 80\*a\*d\*e^4)\*x^9 + 198\*(3\*c\*d^4\*e + 8\*b\*d^3\*e^2 + 80\*a\*d^2\*e^3)\*x^7 + 3465\*a\*d^5\*x + 231\*(3\*c\*d^5 + 8\*b\*d^4\*e + 80\*a\*d^3\*e^2)\*x^5 + 1155\*(b\*d^5 + 10\*a\*d^4\*e)\*x^3)\*sqrt(e\*x^2 + d)/(d^6\*e^6\*x^12 + 6\*d^7\*e^5\*x^10 + 15\*d^8\*e^4\*x^8 + 20\*d^9\*e^3\*x^6 + 15\*d^10\*e^2\*x^4 + 6\*d^11\*e\*x^2 + d^12)

**giac** [A] time = 0.23, size = 189, normalized size = 0.90

$$\frac{\left(\left(2\left(4x^2\left(\frac{2(3cd^2e^3+8bde^4+80ae^5)x^2e^{-5}}{d^6} + \frac{11(3cd^3e^2+8bd^2e^3+80ade^4)e^{-5}}{d^6}\right) + \frac{99(3cd^4e+8bd^3e^2+80ad^2e^3)e^{-5}}{d^6}\right)x^2 + \frac{231(3cd^5+8bd^4e+80ad^3e^2)e^{-5}}{d^6}\right)x^2 + \frac{1155(bd^5+10ad^4e)e^{-5}}{d^6}\right)x^2 + \frac{3465a}{d}\right)x}{3465(x^2e+d)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^(13/2),x, algorithm="giac")

[Out] 1/3465\*(((2\*(4\*x^2\*(2\*(3\*c\*d^2\*e^8 + 8\*b\*d\*e^9 + 80\*a\*e^10)\*x^2\*e^(-5)/d^6 + 11\*(3\*c\*d^3\*e^7 + 8\*b\*d^2\*e^8 + 80\*a\*d\*e^9)\*e^(-5)/d^6) + 99\*(3\*c\*d^4\*e^6 + 8\*b\*d^3\*e^7 + 80\*a\*d^2\*e^8)\*e^(-5)/d^6)\*x^2 + 231\*(3\*c\*d^5\*e^5 + 8\*b\*d^4\*e^6 + 80\*a\*d^3\*e^7)\*e^(-5)/d^6)\*x^2 + 1155\*(b\*d^5\*e^5 + 10\*a\*d^4\*e^6)\*e^(-5)/d^6)\*x^2 + 3465\*a/d)\*x/(x^2\*e + d)^(11/2)

**maple** [A] time = 0.01, size = 172, normalized size = 0.82

$$\frac{(1280a^5e^{10} + 128bd^4e^{10} + 48c^2d^2e^{10} + 7040ad^4e^8 + 704bd^2e^8 + 264c^2d^2e^8 + 15840a^2d^2e^6 + 1584bd^3e^6 + 594cd^4e^6 + 18480a^3d^2e^4 + 1848bd^4e^4 + 693cd^5e^4 + 11550a^4d^2e^2 + 1155bd^5e^2 + 3465a^5d^0)e^{11}}{3465(e^2x^2 + d)^{\frac{11}{2}}d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^4+b\*x^2+a)/(e\*x^2+d)^(13/2),x)

[Out] 1/3465\*x\*(1280\*a\*e^5\*x^10+128\*b\*d\*e^4\*x^10+48\*c\*d^2\*e^3\*x^10+7040\*a\*d\*e^4\*x^8+704\*b\*d^2\*e^3\*x^8+264\*c\*d^3\*e^2\*x^8+15840\*a\*d^2\*e^3\*x^6+1584\*b\*d^3\*e^2\*x^6+594\*c\*d^4\*e\*x^6+18480\*a\*d^3\*e^2\*x^4+1848\*b\*d^4\*e\*x^4+693\*c\*d^5\*x^4+11550\*a\*d^4\*e\*x^2+1155\*b\*d^5\*x^2+3465\*a\*d^5)/(e\*x^2+d)^(11/2)/d^6

**maxima** [A] time = 1.11, size = 335, normalized size = 1.60

$$\frac{cx^2}{8(e^2x^2+d)^{\frac{5}{2}}} + \frac{256ax}{693\sqrt{e^2x^2+d}} + \frac{128ax}{693(e^2+d)^{\frac{3}{2}}} + \frac{32ax}{231(e^2+d)^{\frac{3}{2}}} + \frac{80ax}{693(e^2+d)^{\frac{3}{2}}} + \frac{10ax}{99(e^2+d)^{\frac{3}{2}}} + \frac{ax}{11(e^2+d)^{\frac{3}{2}}} + \frac{cx}{264(e^2+d)^{\frac{3}{2}}} + \frac{16cx}{1155\sqrt{e^2+d}} + \frac{8cx}{1155(e^2+d)^{\frac{3}{2}}} + \frac{2cx}{385(e^2+d)^{\frac{3}{2}}} + \frac{cx}{231(e^2+d)^{\frac{3}{2}}} + \frac{3cdx}{88(e^2+d)^{\frac{3}{2}}} + \frac{bx}{11(e^2+d)^{\frac{3}{2}}} + \frac{128bx}{3465\sqrt{e^2+d}} + \frac{64bx}{3465(e^2+d)^{\frac{3}{2}}} + \frac{16bx}{1155(e^2+d)^{\frac{3}{2}}} + \frac{8bx}{693(e^2+d)^{\frac{3}{2}}} + \frac{bx}{99(e^2+d)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^4+b\*x^2+a)/(e\*x^2+d)^(13/2),x, algorithm="maxima")

[Out] -1/8\*c\*x^3/((e\*x^2 + d)^(11/2)\*e) + 256/693\*a\*x/(sqrt(e\*x^2 + d)\*d^6) + 128/693\*a\*x/((e\*x^2 + d)^(3/2)\*d^5) + 32/231\*a\*x/((e\*x^2 + d)^(5/2)\*d^4) + 80/693\*a\*x/((e\*x^2 + d)^(7/2)\*d^3) + 10/99\*a\*x/((e\*x^2 + d)^(9/2)\*d^2) + 1/11\*a\*x/((e\*x^2 + d)^(11/2)\*d) + 1/264\*c\*x/((e\*x^2 + d)^(9/2)\*e^2) + 16/1155\*c\*x/(sqrt(e\*x^2 + d)\*d^4\*e^2) + 8/1155\*c\*x/((e\*x^2 + d)^(3/2)\*d^3\*e^2) + 2/385\*c\*x/((e\*x^2 + d)^(5/2)\*d^2\*e^2) + 1/231\*c\*x/((e\*x^2 + d)^(7/2)\*d\*e^2) - 3/88\*c\*d\*x/((e\*x^2 + d)^(11/2)\*e^2) - 1/11\*b\*x/((e\*x^2 + d)^(11/2)\*e) + 128/3465\*b\*x/(sqrt(e\*x^2 + d)\*d^5\*e) + 64/3465\*b\*x/((e\*x^2 + d)^(3/2)\*d^4\*e) + 16/1155\*b\*x/((e\*x^2 + d)^(5/2)\*d^3\*e) + 8/693\*b\*x/((e\*x^2 + d)^(7/2)\*d^2\*e) + 1/99\*b\*x/((e\*x^2 + d)^(9/2)\*d\*e)

**mupad [B]** time = 4.76, size = 226, normalized size = 1.08

$$\frac{x \left( \frac{a}{11d} - \frac{d \left( \frac{b}{11d} - \frac{c}{11e} \right)}{e} \right)}{(ex^2+d)^{11/2}} - \frac{x \left( \frac{c}{9e^2} - \frac{-cd^2+bde+10ae^2}{99d^2e^2} \right)}{(ex^2+d)^{9/2}} + \frac{x(3cd^2+8bde+80ae^2)}{693d^3e^2(ex^2+d)^{7/2}} + \frac{x(6cd^2+16bde+160ae^2)}{1155d^4e^2(ex^2+d)^{5/2}} + \frac{x(24cd^2+64bde+640ae^2)}{3465d^5e^2(ex^2+d)^{3/2}} + \frac{x(48cd^2+128bde+1280ae^2)}{3465d^6e^2\sqrt{ex^2+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^2 + c\*x^4)/(d + e\*x^2)^(13/2), x)

[Out] (x\*(a/(11\*d) - (d\*(b/(11\*d) - c/(11\*e)))/e))/(d + e\*x^2)^(11/2) - (x\*(c/(9\*e^2) - (10\*a\*e^2 - c\*d^2 + b\*d\*e)/(99\*d^2\*e^2)))/(d + e\*x^2)^(9/2) + (x\*(80\*a\*e^2 + 3\*c\*d^2 + 8\*b\*d\*e))/(693\*d^3\*e^2\*(d + e\*x^2)^(7/2)) + (x\*(160\*a\*e^2 + 6\*c\*d^2 + 16\*b\*d\*e))/(1155\*d^4\*e^2\*(d + e\*x^2)^(5/2)) + (x\*(640\*a\*e^2 + 24\*c\*d^2 + 64\*b\*d\*e))/(3465\*d^5\*e^2\*(d + e\*x^2)^(3/2)) + (x\*(1280\*a\*e^2 + 48\*c\*d^2 + 128\*b\*d\*e))/(3465\*d^6\*e^2\*(d + e\*x^2)^(1/2))

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*4+b\*x\*\*2+a)/(e\*x\*\*2+d)\*\*(13/2), x)

[Out] Timed out

**3.209** 
$$\int \frac{1 - \sqrt{3} + x}{(1 + \sqrt{3} + x)\sqrt{-4 + 4\sqrt{3}x^2 + x^4}} dx$$

Optimal. Leaf size=65

$$\frac{1}{3}\sqrt{2\sqrt{3} - 3} \tanh^{-1}\left(\frac{(x - \sqrt{3} + 1)^2}{\sqrt{3(2\sqrt{3} - 3)}\sqrt{x^4 + 4\sqrt{3}x^2 - 4}}\right)$$

**Rubi [A]** time = 0.15, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$ , Rules used = {1740, 207}

$$\frac{1}{3}\sqrt{2\sqrt{3} - 3} \tanh^{-1}\left(\frac{(x - \sqrt{3} + 1)^2}{\sqrt{3(2\sqrt{3} - 3)}\sqrt{x^4 + 4\sqrt{3}x^2 - 4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]), x]
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + x)^2/(Sqrt[3*(-3 + 2*Sqrt[3]])*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4])])/3
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1740

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_)^2 + (c_.)*(x_)^4)], x_Symbol] :> -Dist[(A^2*(B*d + A*e))/e, Subst[Int[1/(6*A^3*B*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && EqQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rubi steps

$$\int \frac{1 - \sqrt{3} + x}{(1 + \sqrt{3} + x)\sqrt{-4 + 4\sqrt{3}x^2 + x^4}} dx = -\left(4(2 - \sqrt{3})\right) \text{Subst}\left(\int \frac{1}{3(1 - \sqrt{3})^4 + 6(1 - \sqrt{3})^3(1 + \sqrt{3}) + \dots}\right) = \frac{1}{3}\sqrt{-3 + 2\sqrt{3}} \tanh^{-1}\left(\frac{(1 - \sqrt{3} + x)^2}{\sqrt{3(-3 + 2\sqrt{3})}\sqrt{-4 + 4\sqrt{3}x^2 + x^4}}\right)$$

**Mathematica [C]** time = 3.07, size = 685, normalized size = 10.54

$$\frac{(1 + \sqrt{3} - x)^2 \sqrt{(x^2 + (\sqrt{3} - 1)x - 2)\sqrt{3} + 2(1 + \sqrt{3})} \sqrt{\frac{-2\sqrt{3} - 3}{2\sqrt{3} - 3}} + 2\sqrt{3} \sqrt{\frac{2\sqrt{3} - 3}{2\sqrt{3} - 3}} \sqrt{\sqrt{3}(2 + \sqrt{3})} - \left(\frac{1}{x - \sqrt{3} - 1} + 1\right) \left(\frac{2\sqrt{3} - 3}{(2\sqrt{3} - 3)(x - \sqrt{3} - 1)} \arctan\left(\frac{\sqrt{3}(2 + \sqrt{3}) - (x - \sqrt{3} - 1)}{2\sqrt{3} - 3}\right) - \frac{2\sqrt{3} - 3}{(x - \sqrt{3} - 1)(2\sqrt{3} - 3)}\right) + \left(\frac{2\sqrt{3} - 3}{2\sqrt{3} - 3} \sqrt{3}(2 + \sqrt{3}) + (1 + \sqrt{3} + i\sqrt{3}(2 + \sqrt{3}))\sqrt{\sqrt{3}(2 + \sqrt{3})} + \left(\frac{1}{x - \sqrt{3} - 1} + 1\right) \left(\frac{\sqrt{3}(2 + \sqrt{3}) - (x - \sqrt{3} - 1)}{2\sqrt{3} - 3}\right) - \frac{2\sqrt{3} - 3}{(x - \sqrt{3} - 1)(2\sqrt{3} - 3)}\right)}{(\sqrt{3}(2 + \sqrt{3}) + (1 + \sqrt{3}))\sqrt{\frac{2}{3} + \frac{1}{2}(\sqrt{3} - 1)x^2 - (2 + \sqrt{3})x + \sqrt{3} + 1} + \sqrt{x^2 + 4\sqrt{3}x - 4}\sqrt{\sqrt{3}(2 + \sqrt{3})} - \left(\frac{1}{x - \sqrt{3} - 1} + 1\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)\*Sqrt[-4 + 4\*Sqrt[3]\*x^2 + x^4]),x]

[Out] ((-1 + Sqrt[3] + x)^2\*Sqrt[2\*(1 + Sqrt[3])] - 2\*(2 + Sqrt[3])\*x + (-1 + Sqrt[3])\*x^2 - x^3)\*Sqrt[(1 + Sqrt[3] - 4/(-1 + Sqrt[3] + x))/(3 + Sqrt[3] + I\*Sqrt[2\*(2 + Sqrt[3])])]\*(I\*(-1 + Sqrt[3] + I\*Sqrt[2\*(2 + Sqrt[3])]) + (2\*(2\*I)\*Sqrt[3] - Sqrt[2\*(2 + Sqrt[3])]) + Sqrt[6\*(2 + Sqrt[3])])/(-1 + Sqrt[3] + x))\*Sqrt[Sqrt[2\*(2 + Sqrt[3])] + I\*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))]\*EllipticF[ArcSin[Sqrt[Sqrt[2\*(2 + Sqrt[3])]] - I\*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))]/(2^(3/4)\*(2 + Sqrt[3])^(1/4))], ((2\*I)\*Sqrt[2\*(2 + Sqrt[3])])/(3 + Sqrt[3] + I\*Sqrt[2\*(2 + Sqrt[3])]) + 2\*Sqrt[6]\*Sqrt[(4 + 2\*Sqrt[3] + x^2)/(-1 + Sqrt[3] + x)^2]\*Sqrt[Sqrt[2\*(2 + Sqrt[3])] - I\*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))]\*EllipticPi[(2\*Sqrt[2\*(2 + Sqrt[3])])/(Sqrt[2\*(2 + Sqrt[3])]) + I\*(3 + Sqrt[3])], ArcSin[Sqrt[Sqrt[2\*(2 + Sqrt[3])]] - I\*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))]/(2^(3/4)\*(2 + Sqrt[3])^(1/4))], ((2\*I)\*Sqrt[2\*(2 + Sqrt[3])])/(3 + Sqrt[3] + I\*Sqrt[2\*(2 + Sqrt[3])])])/(Sqrt[2\*(2 + Sqrt[3])]) + I\*(3 + Sqrt[3]))\*Sqrt[1 + Sqrt[3] - (2 + Sqrt[3])\*x + ((-1 + Sqrt[3])\*x^2)/2 - x^3/2]\*Sqrt[-4 + 4\*Sqrt[3]\*x^2 + x^4]\*Sqrt[Sqrt[2\*(2 + Sqrt[3])] - I\*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))])

**IntegrateAlgebraic [A]** time = 12.09, size = 77, normalized size = 1.18

$$\frac{1}{3}\sqrt{2\sqrt{3}-3} \tanh^{-1}\left(\frac{\sqrt{9+6\sqrt{3}}\sqrt{x^4+4\sqrt{3}x^2-4}}{(2+\sqrt{3})x^2+(-2-2\sqrt{3})x+2}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)\*Sqrt[-4 + 4\*Sqrt[3]\*x^2 + x^4]),x]

[Out] (Sqrt[-3 + 2\*Sqrt[3]]\*ArcTanh[(Sqrt[9 + 6\*Sqrt[3]]\*Sqrt[-4 + 4\*Sqrt[3]\*x^2 + x^4])/(2 + (-2 - 2\*Sqrt[3])\*x + (2 + Sqrt[3])\*x^2))]/3

**fricas [B]** time = 1.64, size = 323, normalized size = 4.97

$\frac{1}{12}\sqrt{2\sqrt{3}-3}\left(\frac{37x^{12}-204x^{11}+804x^{10}-2408x^9+3708x^8-5472x^7+6432x^6+10944x^5+14832x^4+19264x^3+12864x^2+(54x^{10}-300x^9+1026x^8-2232x^7+3024x^6-3024x^5-1008x^4-2016x^3-2592x^2+\sqrt{3}(31x^{10}-176x^9+576x^8-1320x^7+1848x^6-1008x^5+1344x^4+1632x^3+1008x^2+832x+256)-1152x-480)\sqrt{x^4+4\sqrt{3}x^2-4}}{(x^{12}+12x^{11}+48x^{10}+40x^9-180x^8-288x^7+384x^6+576x^5-720x^4-320x^3+768x^2-384x+64)}+3\sqrt{3}\sqrt{x^4+4\sqrt{3}x^2-4}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4\*3^(1/2)\*x^2)^(1/2),x, algorithm="fricas")

[Out] 1/12\*sqrt(2\*sqrt(3) - 3)\*log(-(37\*x^12 - 204\*x^11 + 804\*x^10 - 2408\*x^9 + 3708\*x^8 - 5472\*x^7 + 6432\*x^6 + 10944\*x^5 + 14832\*x^4 + 19264\*x^3 + 12864\*x^2 + (54\*x^10 - 300\*x^9 + 1026\*x^8 - 2232\*x^7 + 3024\*x^6 - 3024\*x^5 - 1008\*x^4 - 2016\*x^3 - 2592\*x^2 + sqrt(3)\*(31\*x^10 - 176\*x^9 + 576\*x^8 - 1320\*x^7 + 1848\*x^6 - 1008\*x^5 + 1344\*x^4 + 1632\*x^3 + 1008\*x^2 + 832\*x + 256) - 1152\*x - 480)\*sqrt(x^4 + 4\*sqrt(3)\*x^2 - 4))\*sqrt(2\*sqrt(3) - 3) + 3\*sqrt(3)\*(7\*x^12 - 40\*x^11 + 160\*x^10 - 400\*x^9 + 924\*x^8 - 960\*x^7 - 1920\*x^5 - 3696\*x^4 - 3200\*x^3 - 2560\*x^2 - 1280\*x - 448) + 6528\*x + 2368)/(x^12 + 12\*x^11 + 48\*x^10 + 40\*x^9 - 180\*x^8 - 288\*x^7 + 384\*x^6 + 576\*x^5 - 720\*x^4 - 320\*x^3 + 768\*x^2 - 384\*x + 64))

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{x^4 + 4\sqrt{3}x^2 - 4}(x + \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4\*3^(1/2)\*x^2)^(1/2),x, algorith="giac")

[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4\*sqrt(3)\*x^2 - 4)\*(x + sqrt(3) + 1)), x)

**maple** [C] time = 0.17, size = 327, normalized size = 5.03

$$\frac{\sqrt{-\left(\frac{\sqrt{5}-1}{2}\right)^2+1}\sqrt{-\left(1+\frac{\sqrt{5}}{2}\right)x^2+1}\operatorname{EllipticF}\left(\frac{i\sqrt{5}-\frac{1}{2}}{2},x,i\sqrt{1+4\sqrt{3}\left(1+\frac{\sqrt{5}}{2}\right)}\right)}{\left(\frac{i\sqrt{5}-\frac{1}{2}}{2}\right)\sqrt{x^4+4\sqrt{3}x^2-4}}-2\sqrt{3}\left[\frac{\sqrt{-\left(\frac{\sqrt{5}-1}{2}\right)^2+1}\sqrt{-\left(1+\frac{\sqrt{5}}{2}\right)x^2+1}\operatorname{EllipticPi}\left(\sqrt{\frac{\sqrt{5}-1}{2}}x,\frac{1}{\left(\frac{\sqrt{5}-1}{2}\right)(1-\sqrt{3})},\sqrt{\frac{\sqrt{5}}{2}-1}\right)}{\sqrt{\frac{\sqrt{5}-1}{2}(1-\sqrt{3})}\sqrt{x^4+4\sqrt{3}x^2-4}}-\frac{\operatorname{arctanh}\left(\frac{4\sqrt{3}x^2+2(-1-\sqrt{3})^2x^2+4\sqrt{3}(-1-\sqrt{3})^2-8}{2\sqrt{(-1-\sqrt{3})^4+4\sqrt{3}(-1-\sqrt{3})^2-4}}}\right)}{2\sqrt{(-1-\sqrt{3})^4+4\sqrt{3}(-1-\sqrt{3})^2-4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4\*x^2\*3^(1/2))^(1/2),x)

[Out] 1/(1/2\*I\*3^(1/2)-1/2\*I)\*(1-(1/2\*3^(1/2)-1)\*x^2)^(1/2)\*(1-(1+1/2\*3^(1/2))\*x^2)^(1/2)/(-4+x^4+4\*x^2\*3^(1/2))^(1/2)\*EllipticF(x\*(1/2\*I\*3^(1/2)-1/2\*I),I\*(1+4\*3^(1/2)\*(1+1/2\*3^(1/2))))^(1/2))-2\*3^(1/2)\*(-1/2/((-1-3^(1/2))^4+4\*3^(1/2)\*(-1-3^(1/2))^2-4)^(1/2)\*arctanh(1/2\*(4\*3^(1/2)\*(-1-3^(1/2))^2-8+4\*x^2\*3^(1/2)+2\*x^2\*(-1-3^(1/2))^2)/((-1-3^(1/2))^4+4\*3^(1/2)\*(-1-3^(1/2))^2-4)^(1/2)/(-4+x^4+4\*x^2\*3^(1/2))^(1/2))-1/(1/2\*3^(1/2)-1)^(1/2)/(-1-3^(1/2))\*1-(1/2\*3^(1/2)-1)\*x^2)^(1/2)\*(1-(1+1/2\*3^(1/2))\*x^2)^(1/2)/(-4+x^4+4\*x^2\*3^(1/2))^(1/2)\*EllipticPi((1/2\*3^(1/2)-1)^(1/2)\*x,1/(1/2\*3^(1/2)-1)/(-1-3^(1/2))^2,(1+1/2\*3^(1/2))^(1/2)/(1/2\*3^(1/2)-1)^(1/2)))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{x^4 + 4\sqrt{3}x^2 - 4}(x + \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4\*3^(1/2)\*x^2)^(1/2),x, algorith="maxima")

[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4\*sqrt(3)\*x^2 - 4)\*(x + sqrt(3) + 1)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x - \sqrt{3} + 1}{(x + \sqrt{3} + 1)\sqrt{x^4 + 4\sqrt{3}x^2 - 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x - 3^(1/2) + 1)/((x + 3^(1/2) + 1)\*(4\*3^(1/2)\*x^2 + x^4 - 4)^(1/2)),x)

[Out] int((x - 3^(1/2) + 1)/((x + 3^(1/2) + 1)\*(4\*3^(1/2)\*x^2 + x^4 - 4)^(1/2)),x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x - \sqrt{3} + 1}{(x + 1 + \sqrt{3})\sqrt{x^4 + 4\sqrt{3}x^2 - 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x-3\*\*(1/2))/(1+x+3\*\*(1/2))/(-4+x\*\*4+4\*3\*\*(1/2)\*x\*\*2)\*\*(1/2),x)

[Out] Integral((x - sqrt(3) + 1)/((x + 1 + sqrt(3))\*sqrt(x\*\*4 + 4\*sqrt(3)\*x\*\*2 - 4)), x)

$$3.210 \quad \int \frac{1 + \sqrt{3} + x}{(1 - \sqrt{3} + x)\sqrt{-4 - 4\sqrt{3}x^2 + x^4}} dx$$

Optimal. Leaf size=63

$$-\frac{1}{3}\sqrt{3 + 2\sqrt{3}} \tan^{-1} \left( \frac{(x + \sqrt{3} + 1)^2}{\sqrt{3(3 + 2\sqrt{3})}\sqrt{x^4 - 4\sqrt{3}x^2 - 4}} \right)$$

**Rubi [A]** time = 0.14, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$ , Rules used = {1740, 203}

$$-\frac{1}{3}\sqrt{3 + 2\sqrt{3}} \tan^{-1} \left( \frac{(x + \sqrt{3} + 1)^2}{\sqrt{3(3 + 2\sqrt{3})}\sqrt{x^4 - 4\sqrt{3}x^2 - 4}} \right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]), x]
[Out] -(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + x)^2/(Sqrt[3*(3 + 2*Sqrt[3]))]*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]])/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 1740

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] := -Dist[(A^2*(B*d + A*e))/e, Subst[Int[1/(6*A^3*B*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && EqQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rubi steps

$$\int \frac{1 + \sqrt{3} + x}{(1 - \sqrt{3} + x)\sqrt{-4 - 4\sqrt{3}x^2 + x^4}} dx = - \left( (4(2 + \sqrt{3})) \text{Subst} \left[ \int \frac{1}{6(1 - \sqrt{3})(1 + \sqrt{3})^3 + 3(1 + \sqrt{3})^4 + 4x^2} dx \right] \right) \\ = -\frac{1}{3}\sqrt{3 + 2\sqrt{3}} \tan^{-1} \left( \frac{(1 + \sqrt{3} + x)^2}{\sqrt{3(3 + 2\sqrt{3})}\sqrt{-4 - 4\sqrt{3}x^2 + x^4}} \right)$$

**Mathematica [C]** time = 7.83, size = 876, normalized size = 13.90

$$\frac{\sqrt{3} \sqrt{3 + 2\sqrt{3}} \operatorname{ArcTan}\left[\frac{(1 + \sqrt{3} + x)^2}{\sqrt{3(3 + 2\sqrt{3})}\sqrt{-4 - 4\sqrt{3}x^2 + x^4}}\right]}{3\sqrt{3(3 + 2\sqrt{3})}\sqrt{-4 - 4\sqrt{3}x^2 + x^4}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)\*Sqrt[-4 - 4\*Sqrt[3]\*x^2 + x^4]), x]

[Out] -((Sqrt[2]\*Sqrt[(-1 + Sqrt[3] - 4/(1 + Sqrt[3] - x))]/(-3 + Sqrt[3] - I\*Sqrt[4 - 2\*Sqrt[3]])]\*(1 + Sqrt[3] - x)^2\*((I\*Sqrt[Sqrt[4 - 2\*Sqrt[3]]) + I\*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))] + I\*Sqrt[3]\*Sqrt[Sqrt[4 - 2\*Sqrt[3]]) + I\*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))] + Sqrt[-2\*Sqrt[12 - 6\*Sqrt[3]] + 4\*Sqrt[4 - 2\*Sqrt[3]] - ((2\*I)\*(14 - 8\*Sqrt[3] + (-1 + Sqrt[3])\*x))/(1 + Sqrt[3] - x)] + (2\*((2\*I)\*Sqrt[3]\*Sqrt[Sqrt[4 - 2\*Sqrt[3]]) + I\*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))] + Sqrt[6]\*Sqrt[-I + I\*Sqrt[3] - Sqrt[12 - 6\*Sqrt[3]] + 2\*Sqrt[4 - 2\*Sqrt[3]] + ((8\*I)\*(-2 + Sqrt[3]))]/(1 + Sqrt[3] - x)] + Sqrt[-2\*Sqrt[12 - 6\*Sqrt[3]] + 4\*Sqrt[4 - 2\*Sqrt[3]] - ((2\*I)\*(14 - 8\*Sqrt[3] + (-1 + Sqrt[3])\*x))/(1 + Sqrt[3] - x)))/(-1 - Sqrt[3] + x))\*EllipticF[ArcSin[Sqrt[Sqrt[4 - 2\*Sqrt[3]] - I\*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))]/(2^(3/4)\*(2 - Sqrt[3])^(1/4))], (2\*Sqrt[4 - 2\*Sqrt[3]])/(Sqrt[4 - 2\*Sqrt[3]] + I\*(-3 + Sqrt[3]))] + 2\*Sqrt[6]\*Sqrt[Sqrt[4 - 2\*Sqrt[3]] - I\*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))] \* Sqrt[(4 - 2\*Sqrt[3] + x^2)/(1 + Sqrt[3] - x)^2]\*EllipticPi[(2\*Sqrt[4 - 2\*Sqrt[3]])/(Sqrt[4 - 2\*Sqrt[3]] - I\*(-3 + Sqrt[3]))], ArcSin[Sqrt[Sqrt[4 - 2\*Sqrt[3]] - I\*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))]/(2^(3/4)\*(2 - Sqrt[3])^(1/4))], (2\*Sqrt[4 - 2\*Sqrt[3]])/(Sqrt[4 - 2\*Sqrt[3]] + I\*(-3 + Sqrt[3])))])))/((Sqrt[4 - 2\*Sqrt[3]] - I\*(-3 + Sqrt[3]))\*Sqrt[Sqrt[4 - 2\*Sqrt[3]] - I\*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))] \* Sqrt[-4 - 4\*Sqrt[3]\*x^2 + x^4]))

**IntegrateAlgebraic [A]** time = 11.97, size = 77, normalized size = 1.22

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}} \tan^{-1}\left(\frac{\sqrt{6\sqrt{3}-9}\sqrt{x^4-4\sqrt{3}x^2-4}}{(\sqrt{3}-2)x^2+(2-2\sqrt{3})x-2}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)\*Sqrt[-4 - 4\*Sqrt[3]\*x^2 + x^4]), x]

[Out] -1/3\*(Sqrt[3 + 2\*Sqrt[3]]\*ArcTan[(Sqrt[-9 + 6\*Sqrt[3]]\*Sqrt[-4 - 4\*Sqrt[3]\*x^2 + x^4])/(-2 + (2 - 2\*Sqrt[3])\*x + (-2 + Sqrt[3])\*x^2)])

**fricas [B]** time = 1.36, size = 112, normalized size = 1.78

$$\frac{1}{6}\sqrt{2\sqrt{3}+3} \arctan\left(-\frac{(9x^4-30x^3+18x^2-2\sqrt{3}(2x^4-10x^3+3x^2-10x+2)+24)\sqrt{x^4-4\sqrt{3}x^2-4}\sqrt{2\sqrt{3}+3}}{11x^6-42x^5+66x^4-176x^3-132x^2-168x-88}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4\*3^(1/2)\*x^2)^(1/2), x, algorithm="fricas")

[Out] 1/6\*sqrt(2\*sqrt(3) + 3)\*arctan(-(9\*x^4 - 30\*x^3 + 18\*x^2 - 2\*sqrt(3)\*(2\*x^4 - 10\*x^3 + 3\*x^2 - 10\*x + 2) + 24)\*sqrt(x^4 - 4\*sqrt(3)\*x^2 - 4)\*sqrt(2\*sqrt(3) + 3)/(11\*x^6 - 42\*x^5 + 66\*x^4 - 176\*x^3 - 132\*x^2 - 168\*x - 88))

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + \sqrt{3} + 1}{\sqrt{x^4 - 4\sqrt{3}x^2 - 4}(x - \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4\*3^(1/2)\*x^2)^(1/2), x, algorithm="giac")

[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4\*sqrt(3)\*x^2 - 4)\*(x - sqrt(3) + 1)), x)

**maple** [C] time = 0.16, size = 311, normalized size = 4.94

$$\frac{\sqrt{-\left(-1-\frac{\sqrt{3}}{2}\right)x^2+1}\sqrt{-\left(-\frac{\sqrt{3}}{2}+1\right)x^2+1}\operatorname{EllipticF}\left(\frac{i}{2}+\frac{i\sqrt{3}}{2},i,\sqrt{1-4\sqrt{3}\left(-\frac{\sqrt{3}}{2}+1\right)}\right)}{\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{x^4-4\sqrt{3}x^2-4}}+2\sqrt{3}\left(\frac{\sqrt{-\left(-1-\frac{\sqrt{3}}{2}\right)x^2+1}\sqrt{-\left(-\frac{\sqrt{3}}{2}+1\right)x^2+1}\operatorname{EllipticPi}\left(\sqrt{-1-\frac{\sqrt{3}}{2}},x,\frac{1}{\left(-1-\frac{\sqrt{3}}{2}\right)\left(\sqrt{3}-1\right)},\sqrt{\frac{\sqrt{3}+1}{-1-\frac{\sqrt{3}}{2}}}\right)}{\sqrt{-1-\frac{\sqrt{3}}{2}}\left(\sqrt{3}-1\right)\sqrt{x^4-4\sqrt{3}x^2-4}}-\frac{\operatorname{arctanh}\left(\frac{-4\sqrt{3}x^2+2\left(\sqrt{3}-1\right)^2x^2-4\sqrt{3}\left(\sqrt{3}-1\right)^2-8}{2\sqrt{\left(\sqrt{3}-1\right)^4-4\sqrt{3}\left(\sqrt{3}-1\right)^2-4}\sqrt{x^4-4\sqrt{3}x^2-4}}\right)}{2\sqrt{\left(\sqrt{3}-1\right)^4-4\sqrt{3}\left(\sqrt{3}-1\right)^2-4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4\*3^(1/2)\*x^2)^(1/2),x)

[Out] 1/(1/2\*I+1/2\*I\*3^(1/2))\*(1-(-1-1/2\*3^(1/2))\*x^2)^(1/2)\*(1-(-1/2\*3^(1/2)+1)\*x^2)^(1/2)/(-4+x^4-4\*3^(1/2)\*x^2)^(1/2)\*EllipticF(x\*(1/2\*I+1/2\*I\*3^(1/2)),I\*(1-4\*3^(1/2)\*(-1/2\*3^(1/2)+1))^(1/2))+2\*3^(1/2)\*(-1/2/((3^(1/2)-1)^4-4\*3^(1/2)\*(3^(1/2)-1)^2-4)^(1/2)\*arctanh(1/2\*(-4\*3^(1/2)\*(3^(1/2)-1)^2-8-4\*3^(1/2)\*x^2+2\*x^2\*(3^(1/2)-1)^2)/((3^(1/2)-1)^4-4\*3^(1/2)\*(3^(1/2)-1)^2-4)^(1/2)/(-4+x^4-4\*3^(1/2)\*x^2)^(1/2))-1/(-1-1/2\*3^(1/2))^(1/2)/(3^(1/2)-1)\*(1-(-1-1/2\*3^(1/2))\*x^2)^(1/2)\*(1-(-1/2\*3^(1/2)+1)\*x^2)^(1/2)/(-4+x^4-4\*3^(1/2)\*x^2)^(1/2)\*EllipticPi((-1-1/2\*3^(1/2))^(1/2)\*x,1/(-1-1/2\*3^(1/2))/(3^(1/2)-1)^2,(-1/2\*3^(1/2)+1)^(1/2)/(-1-1/2\*3^(1/2))^(1/2))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + \sqrt{3} + 1}{\sqrt{x^4 - 4\sqrt{3}x^2 - 4}(x - \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4\*3^(1/2)\*x^2)^(1/2),x, algorith="maxima")

[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4\*sqrt(3)\*x^2 - 4)\*(x - sqrt(3) + 1)), x)

**mapad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x + \sqrt{3} + 1}{\sqrt{x^4 - 4\sqrt{3}x^2 - 4}(x - \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3^(1/2) + 1)/((x^4 - 4\*3^(1/2)\*x^2 - 4)^(1/2)\*(x - 3^(1/2) + 1)),x)

[Out] int((x + 3^(1/2) + 1)/((x^4 - 4\*3^(1/2)\*x^2 - 4)^(1/2)\*(x - 3^(1/2) + 1)),x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + 1 + \sqrt{3}}{(x - \sqrt{3} + 1)\sqrt{x^4 - 4\sqrt{3}x^2 - 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x+3\*\*(1/2))/(1+x-3\*\*(1/2))/(-4+x\*\*4-4\*3\*\*(1/2)\*x\*\*2)\*\*(1/2),x)

[Out] Integral((x + 1 + sqrt(3))/((x - sqrt(3) + 1)\*sqrt(x\*\*4 - 4\*sqrt(3)\*x\*\*2 - 4)), x)

**3.211**  $\int \frac{1-\sqrt{3}+2x}{(1+\sqrt{3}+2x)\sqrt{-1+4\sqrt{3}x^2+4x^4}} dx$

Optimal. Leaf size=72

$$\frac{1}{3}\sqrt{2\sqrt{3}-3} \tanh^{-1}\left(\frac{(2x-\sqrt{3}+1)^2}{2\sqrt{3}(2\sqrt{3}-3)\sqrt{4x^4+4\sqrt{3}x^2-1}}\right)$$

**Rubi [A]** time = 0.13, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$ , Rules used = {1740, 207}

$$\frac{1}{3}\sqrt{2\sqrt{3}-3} \tanh^{-1}\left(\frac{(2x-\sqrt{3}+1)^2}{2\sqrt{3}(2\sqrt{3}-3)\sqrt{4x^4+4\sqrt{3}x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + 2*x)/((1 + Sqrt[3] + 2*x)*Sqrt[-1 + 4*Sqrt[3]*x^2 + 4*x^4]), x]
```

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + 2*x)^2/(2*Sqrt[3]*(-3 + 2*Sqrt[3]))*Sqrt[-1 + 4*Sqrt[3]*x^2 + 4*x^4]])/3
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1740

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] := -Dist[(A^2*(B*d + A*e))/e, Subst[Int[1/(6*A^3*B*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && EqQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+2x}{(1+\sqrt{3}+2x)\sqrt{-1+4\sqrt{3}x^2+4x^4}} dx = -\left(4(2-\sqrt{3})\right) \text{Subst}\left(\int \frac{1}{6(1-\sqrt{3})^4+12(1-\sqrt{3})^3(1+\sqrt{3})}\right) = \frac{1}{3}\sqrt{-3+2\sqrt{3}} \tanh^{-1}\left(\frac{(1-\sqrt{3}+2x)^2}{2\sqrt{3}(-3+2\sqrt{3})\sqrt{-1+4\sqrt{3}x^2+4x^4}}\right)$$

**Mathematica [C]** time = 1.73, size = 623, normalized size = 8.65

$$\frac{(2x + \sqrt{3} - 1)^2 \sqrt{\frac{2x^2 + \sqrt{3}x + 1}{(x + \sqrt{3} - 1)^2}} \left(4\sqrt{3} \sqrt{\frac{2x^2 + \sqrt{3}x + 1}{(x + \sqrt{3} - 1)^2}} \sqrt{\sqrt{2(2 + \sqrt{3})} - i\left(\frac{2x + \sqrt{3} - 1}{(x + \sqrt{3} - 1)} - \sqrt{3} + 1\right)} \frac{2\sqrt{3}\sqrt{3}}{\sqrt{2(2 + \sqrt{3}) - i\left(\frac{2x + \sqrt{3} - 1}{(x + \sqrt{3} - 1)} - \sqrt{3} + 1\right)}} \sin^{-1}\left(\frac{\sqrt{2(2 + \sqrt{3})} - i\left(\frac{2x + \sqrt{3} - 1}{(x + \sqrt{3} - 1)} - \sqrt{3} + 1\right)}{2\sqrt{3}\sqrt{3}}\right) + \frac{2\sqrt{3}\sqrt{3}}{2x + \sqrt{3} - 1} + \frac{2\sqrt{3}\sqrt{3} - \sqrt{2(2 + \sqrt{3})} - i\left(\frac{2x + \sqrt{3} - 1}{(x + \sqrt{3} - 1)} - \sqrt{3} + 1\right)}{2x + \sqrt{3} - 1} \sqrt{\sqrt{2(2 + \sqrt{3})} + i\left(\frac{2x + \sqrt{3} - 1}{(x + \sqrt{3} - 1)} - \sqrt{3} + 1\right)} \frac{2\sqrt{3}\sqrt{3}}{2x + \sqrt{3} - 1} \sin^{-1}\left(\frac{\sqrt{2(2 + \sqrt{3})} + i\left(\frac{2x + \sqrt{3} - 1}{(x + \sqrt{3} - 1)} - \sqrt{3} + 1\right)}{2\sqrt{3}\sqrt{3}}\right) + \frac{2\sqrt{3}\sqrt{3}}{2x + \sqrt{3} - 1} \right) \frac{2\sqrt{3}\sqrt{3}}{(\sqrt{2(2 + \sqrt{3})} + i(3 + \sqrt{3}))\sqrt{3} + 8\sqrt{3}x^2 - 2\sqrt{2(2 + \sqrt{3})} - i\left(\frac{2x + \sqrt{3} - 1}{(x + \sqrt{3} - 1)} - \sqrt{3} + 1\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 - Sqrt[3] + 2\*x)/((1 + Sqrt[3] + 2\*x)\*Sqrt[-1 + 4\*Sqrt[3]\*x^2 + 4\*x^4]), x]

[Out] ((-1 + Sqrt[3] + 2\*x)^2\*Sqrt[(1 + Sqrt[3] - 4/(-1 + Sqrt[3] + 2\*x))/(3 + Sqrt[3] + I\*Sqrt[2\*(2 + Sqrt[3])])]\*((I\*(-1 + Sqrt[3] + I\*Sqrt[2\*(2 + Sqrt[3])])) + (2\*((2\*I)\*Sqrt[3] - Sqrt[2\*(2 + Sqrt[3])]) + Sqrt[6\*(2 + Sqrt[3])])))/(-1 + Sqrt[3] + 2\*x)\*Sqrt[Sqrt[2\*(2 + Sqrt[3])]] + I\*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + 2\*x)))\*EllipticF[ArcSin[Sqrt[Sqrt[2\*(2 + Sqrt[3])]] - I\*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + 2\*x))]/(2^(3/4)\*(2 + Sqrt[3])^(1/4))], ((2\*I)\*Sqrt[2\*(2 + Sqrt[3])])/(3 + Sqrt[3] + I\*Sqrt[2\*(2 + Sqrt[3])]) + 4\*Sqrt[3]\*Sqrt[(2 + Sqrt[3] + 2\*x^2)/(-1 + Sqrt[3] + 2\*x)^2]\*Sqrt[Sqrt[2\*(2 + Sqrt[3])]] - I\*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + 2\*x)))\*EllipticPi[(2\*Sqrt[2\*(2 + Sqrt[3])])/(Sqrt[2\*(2 + Sqrt[3])]] + I\*(3 + Sqrt[3])), ArcSin[Sqrt[Sqrt[2\*(2 + Sqrt[3])]] - I\*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + 2\*x))]/(2^(3/4)\*(2 + Sqrt[3])^(1/4))], ((2\*I)\*Sqrt[2\*(2 + Sqrt[3])])/(3 + Sqrt[3] + I\*Sqrt[2\*(2 + Sqrt[3])])])]/((Sqrt[2\*(2 + Sqrt[3])]] + I\*(3 + Sqrt[3]))\*Sqrt[-2 + 8\*Sqrt[3]\*x^2 + 8\*x^4]\*Sqrt[Sqrt[2\*(2 + Sqrt[3])]] - I\*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + 2\*x))])]

**IntegrateAlgebraic [A]** time = 12.22, size = 81, normalized size = 1.12

$$\frac{1}{3}\sqrt{2\sqrt{3}-3} \tanh^{-1}\left(\frac{\sqrt{9+6\sqrt{3}}\sqrt{4x^4+4\sqrt{3}x^2-1}}{(4+2\sqrt{3})x^2+(-2-2\sqrt{3})x+1}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 - Sqrt[3] + 2\*x)/((1 + Sqrt[3] + 2\*x)\*Sqrt[-1 + 4\*Sqrt[3]\*x^2 + 4\*x^4]), x]

[Out] (Sqrt[-3 + 2\*Sqrt[3]]\*ArcTanh[(Sqrt[9 + 6\*Sqrt[3]]\*Sqrt[-1 + 4\*Sqrt[3]\*x^2 + 4\*x^4])/(1 + (-2 - 2\*Sqrt[3])\*x + (4 + 2\*Sqrt[3])\*x^2))]/3

**fricas [B]** time = 1.29, size = 328, normalized size = 4.56

$$\frac{1}{3}\sqrt{2\sqrt{3}-3} \left( \frac{2368x^{12} - 6528x^{11} + 12864x^{10} - 19264x^9 + 14832x^8 - 10944x^7 + 6432x^6 + 5472x^5 + 3708x^4 + 2408x^3 + 804x^2 + (1728x^{10} - 4800x^9 + 8208x^8 - 8928x^7 + 6048x^6 - 3024x^5 - 504x^4 - 504x^3 - 324x^2 + 2\sqrt{3}(496x^{10} - 1408x^9 + 2304x^8 - 2640x^7 + 1848x^6 - 504x^5 + 336x^4 + 204x^3 + 63x^2 + 26x + 4) - 72x - 15)\sqrt{4x^4 + 4\sqrt{3}x^2 - 1}\sqrt{2\sqrt{3} - 3} + 3\sqrt{3}(48x^{12} - 1280x^{11} + 2560x^{10} - 3200x^9 + 3696x^8 - 1920x^7 - 960x^5 - 924x^4 - 400x^3 - 160x^2 - 40x - 7) + 204x + 37)}{(64x^{12} + 384x^{11} + 768x^{10} + 320x^9 - 720x^8 - 576x^7 + 384x^6 + 288x^5 - 180x^4 - 40x^3 + 48x^2 - 12x + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x-3^(1/2))/(1+2\*x+3^(1/2))/(-1+4\*x^4+4\*3^(1/2)\*x^2)^(1/2), x, algorithm="fricas")

[Out] 1/12\*sqrt(2\*sqrt(3) - 3)\*log(-(2368\*x^12 - 6528\*x^11 + 12864\*x^10 - 19264\*x^9 + 14832\*x^8 - 10944\*x^7 + 6432\*x^6 + 5472\*x^5 + 3708\*x^4 + 2408\*x^3 + 804\*x^2 + (1728\*x^10 - 4800\*x^9 + 8208\*x^8 - 8928\*x^7 + 6048\*x^6 - 3024\*x^5 - 504\*x^4 - 504\*x^3 - 324\*x^2 + 2\*sqrt(3)\*(496\*x^10 - 1408\*x^9 + 2304\*x^8 - 2640\*x^7 + 1848\*x^6 - 504\*x^5 + 336\*x^4 + 204\*x^3 + 63\*x^2 + 26\*x + 4) - 72\*x - 15)\*sqrt(4\*x^4 + 4\*sqrt(3)\*x^2 - 1)\*sqrt(2\*sqrt(3) - 3) + 3\*sqrt(3)\*(48\*x^12 - 1280\*x^11 + 2560\*x^10 - 3200\*x^9 + 3696\*x^8 - 1920\*x^7 - 960\*x^5 - 924\*x^4 - 400\*x^3 - 160\*x^2 - 40\*x - 7) + 204\*x + 37)/(64\*x^12 + 384\*x^11 + 768\*x^10 + 320\*x^9 - 720\*x^8 - 576\*x^7 + 384\*x^6 + 288\*x^5 - 180\*x^4 - 40\*x^3 + 48\*x^2 - 12\*x + 1))

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x - \sqrt{3} + 1}{\sqrt{4x^4 + 4\sqrt{3}x^2 - 1}(2x + \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x-3^(1/2))/(1+2\*x+3^(1/2))/(-1+4\*x^4+4\*3^(1/2)\*x^2)^(1/2), x, algorithm="giac")

[Out] integrate((2\*x - sqrt(3) + 1)/(sqrt(4\*x^4 + 4\*sqrt(3)\*x^2 - 1)\*(2\*x + sqrt(3) + 1)), x)

**maple** [C] time = 0.16, size = 336, normalized size = 4.67

$$\frac{\sqrt{-(2\sqrt{3}-4)x^2+1}\sqrt{-(4+2\sqrt{3})x^2+1}\operatorname{EllipticF}\left(i\sqrt{3}-i\right)x,i\sqrt{1+\sqrt{3}}\sqrt{4+2\sqrt{3}}}{(i\sqrt{3}-i)\sqrt{4x^4+4\sqrt{3}x^2-1}} - 2\sqrt{3}\left[\frac{\sqrt{-(2\sqrt{3}-4)x^2+1}\sqrt{-(4+2\sqrt{3})x^2+1}\operatorname{EllipticPi}\left(\sqrt{2\sqrt{3}-4}x,\frac{1}{(2\sqrt{3}-4)\left(\frac{1}{2}-\frac{\sqrt{3}}{2}\right)},\frac{\sqrt{4+2\sqrt{3}}}{\sqrt{2\sqrt{3}-4}}\right)}{2\sqrt{2\sqrt{3}-4}\left(\frac{1}{2}-\frac{\sqrt{3}}{2}\right)\sqrt{4x^4+4\sqrt{3}x^2-1}} - \frac{\operatorname{arctanh}\left(\frac{4\sqrt{3}x^2+4\left(\frac{1}{2}-\frac{\sqrt{3}}{2}\right)^2+4\sqrt{3}\left(\frac{1}{2}-\frac{\sqrt{3}}{2}\right)^2-2}{2\sqrt{\left(\frac{1}{2}-\frac{\sqrt{3}}{2}\right)^4+4\sqrt{3}\left(\frac{1}{2}-\frac{\sqrt{3}}{2}\right)^2-1}}\right)}{4\sqrt{4\left(\frac{1}{2}-\frac{\sqrt{3}}{2}\right)^4+4\sqrt{3}\left(\frac{1}{2}-\frac{\sqrt{3}}{2}\right)^2-1}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+2\*x-3^(1/2))/(1+2\*x+3^(1/2))/(-1+4\*x^4+4\*3^(1/2)\*x^2)^(1/2), x)

[Out] 1/(I\*3^(1/2)-I)\*(1-(2\*3^(1/2)-4)\*x^2)^(1/2)\*(1-(4+2\*3^(1/2))\*x^2)^(1/2)/(-1+4\*x^4+4\*3^(1/2)\*x^2)^(1/2)\*EllipticF(x\*(I\*3^(1/2)-I), I\*(1+3^(1/2)\*(4+2\*3^(1/2)))^(1/2))-2\*3^(1/2)\*(-1/4/(4\*(-1/2-1/2\*3^(1/2))^4+4\*3^(1/2)\*(-1/2-1/2\*3^(1/2))^2-1)^(1/2)\*arctanh(1/2\*(4\*3^(1/2)\*(-1/2-1/2\*3^(1/2))^2-2+4\*3^(1/2)\*x^2+8\*x^2\*(-1/2-1/2\*3^(1/2))^2)/(4\*(-1/2-1/2\*3^(1/2))^4+4\*3^(1/2)\*(-1/2-1/2\*3^(1/2))^2-1)^(1/2)/(-1+4\*x^4+4\*3^(1/2)\*x^2)^(1/2))-1/2/(2\*3^(1/2)-4)^(1/2)/(-1/2-1/2\*3^(1/2))\*(1-(2\*3^(1/2)-4)\*x^2)^(1/2)\*(1-(4+2\*3^(1/2))\*x^2)^(1/2)/(-1+4\*x^4+4\*3^(1/2)\*x^2)^(1/2)\*EllipticPi((2\*3^(1/2)-4)^(1/2)\*x, 1/(2\*3^(1/2)-4)/(-1/2-1/2\*3^(1/2))^2, (4+2\*3^(1/2))^(1/2)/(2\*3^(1/2)-4)^(1/2)))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x - \sqrt{3} + 1}{\sqrt{4x^4 + 4\sqrt{3}x^2 - 1} (2x + \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x-3^(1/2))/(1+2\*x+3^(1/2))/(-1+4\*x^4+4\*3^(1/2)\*x^2)^(1/2), x, algorithm="maxima")

[Out] integrate((2\*x - sqrt(3) + 1)/(sqrt(4\*x^4 + 4\*sqrt(3)\*x^2 - 1)\*(2\*x + sqrt(3) + 1)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2x - \sqrt{3} + 1}{\sqrt{4x^4 + 4\sqrt{3}x^2 - 1} (2x + \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x - 3^(1/2) + 1)/((4\*3^(1/2)\*x^2 + 4\*x^4 - 1)^(1/2)\*(2\*x + 3^(1/2) + 1)), x)

[Out] int((2\*x - 3^(1/2) + 1)/((4\*3^(1/2)\*x^2 + 4\*x^4 - 1)^(1/2)\*(2\*x + 3^(1/2) + 1)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x - \sqrt{3} + 1}{(2x + 1 + \sqrt{3})\sqrt{4x^4 + 4\sqrt{3}x^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x-3\*\*(1/2))/(1+2\*x+3\*\*(1/2))/(-1+4\*x\*\*4+4\*3\*\*(1/2)\*x\*\*2)\*\*(1/2), x)

[Out] Integral((2\*x - sqrt(3) + 1)/((2\*x + 1 + sqrt(3))\*sqrt(4\*x\*\*4 + 4\*sqrt(3)\*x\*\*2 - 1)), x)

**3.212** 
$$\int \frac{1 + \sqrt{3} + 2x}{(1 - \sqrt{3} + 2x)\sqrt{-1 - 4\sqrt{3}x^2 + 4x^4}} dx$$

Optimal. Leaf size=70

$$-\frac{1}{3}\sqrt{3 + 2\sqrt{3}} \tan^{-1}\left(\frac{(2x + \sqrt{3} + 1)^2}{2\sqrt{3}(3 + 2\sqrt{3})\sqrt{4x^4 - 4\sqrt{3}x^2 - 1}}\right)$$

**Rubi [A]** time = 0.13, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$ , Rules used = {1740, 203}

$$-\frac{1}{3}\sqrt{3 + 2\sqrt{3}} \tan^{-1}\left(\frac{(2x + \sqrt{3} + 1)^2}{2\sqrt{3}(3 + 2\sqrt{3})\sqrt{4x^4 - 4\sqrt{3}x^2 - 1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + 2*x)/((1 - Sqrt[3] + 2*x)*Sqrt[-1 - 4*Sqrt[3]*x^2 + 4*x^4]), x]
```

```
[Out] -(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + 2*x)^2/(2*Sqrt[3*(3 + 2*Sqrt[3])]*Sqrt[-1 - 4*Sqrt[3]*x^2 + 4*x^4])])/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 1740

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] := -Dist[(A^2*(B*d + A*e))/e, Subst[Int[1/(6*A^3*B*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && EqQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rubi steps

$$\int \frac{1 + \sqrt{3} + 2x}{(1 - \sqrt{3} + 2x)\sqrt{-1 - 4\sqrt{3}x^2 + 4x^4}} dx = -\left(4(2 + \sqrt{3})\right) \text{Subst}\left(\int \frac{1}{12(1 - \sqrt{3})(1 + \sqrt{3})^3 + 6(1 + \sqrt{3})^4 + \dots} dx\right) = -\frac{1}{3}\sqrt{3 + 2\sqrt{3}} \tan^{-1}\left(\frac{(1 + \sqrt{3} + 2x)^2}{2\sqrt{3}(3 + 2\sqrt{3})\sqrt{-1 - 4\sqrt{3}x^2 + 4x^4}}\right)$$

**Mathematica [C]** time = 6.39, size = 881, normalized size = 12.59

Warning: Unable to verify antiderivative.



[In] Integrate[(1 + Sqrt[3] + 2\*x)/((1 - Sqrt[3] + 2\*x)\*Sqrt[-1 - 4\*Sqrt[3]\*x^2 + 4\*x^4]),x]

[Out] -((Sqrt[(-1 + Sqrt[3] - 4/(1 + Sqrt[3] - 2\*x))]/(-3 + Sqrt[3] - I\*Sqrt[4 - 2\*Sqrt[3]])]\*(1 + Sqrt[3] - 2\*x)^2\*((I\*Sqrt[Sqrt[4 - 2\*Sqrt[3]]] + I\*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - 2\*x))) + I\*Sqrt[3]\*Sqrt[Sqrt[4 - 2\*Sqrt[3]]] + I\*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - 2\*x))) + Sqrt[-2\*Sqrt[12 - 6\*Sqrt[3]] + 4\*Sqrt[4 - 2\*Sqrt[3]] - ((4\*I)\*(7 - 4\*Sqrt[3] + (-1 + Sqrt[3])\*x))/(1 + Sqrt[3] - 2\*x)] - ((2\*I)\*(2\*Sqrt[3]\*Sqrt[Sqrt[4 - 2\*Sqrt[3]]] + I\*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - 2\*x))) - I\*Sqrt[6]\*Sqrt[-Sqrt[12 - 6\*Sqrt[3]] + 2\*Sqrt[4 - 2\*Sqrt[3]] - ((2\*I)\*(7 - 4\*Sqrt[3] + (-1 + Sqrt[3])\*x))/(1 + Sqrt[3] - 2\*x)] - I\*Sqrt[-2\*Sqrt[12 - 6\*Sqrt[3]] + 4\*Sqrt[4 - 2\*Sqrt[3]] - ((4\*I)\*(7 - 4\*Sqrt[3] + (-1 + Sqrt[3])\*x))/(1 + Sqrt[3] - 2\*x))]/(1 + Sqrt[3] - 2\*x))\*EllipticF[ArcSin[Sqrt[Sqrt[4 - 2\*Sqrt[3]] - I\*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - 2\*x))]/(2^(3/4)\*(2 - Sqrt[3])^(1/4))], (2\*Sqrt[4 - 2\*Sqrt[3]])/(Sqrt[4 - 2\*Sqrt[3]] + I\*(-3 + Sqrt[3]))] + 4\*Sqrt[Sqrt[4 - 2\*Sqrt[3]] - I\*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - 2\*x))]\*Sqrt[(6 - 3\*Sqrt[3] + 6\*x^2)/(1 + Sqrt[3] - 2\*x)^2]\*EllipticPi[(2\*Sqrt[4 - 2\*Sqrt[3]])/(Sqrt[4 - 2\*Sqrt[3]] - I\*(-3 + Sqrt[3]))], ArcSin[Sqrt[Sqrt[4 - 2\*Sqrt[3]] - I\*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - 2\*x))]/(2^(3/4)\*(2 - Sqrt[3])^(1/4))], (2\*Sqrt[4 - 2\*Sqrt[3]])/(Sqrt[4 - 2\*Sqrt[3]] + I\*(-3 + Sqrt[3])))]/(Sqrt[2]\*(Sqrt[4 - 2\*Sqrt[3]] - I\*(-3 + Sqrt[3]))\*Sqrt[Sqrt[4 - 2\*Sqrt[3]] - I\*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - 2\*x))]\*Sqrt[-1 - 4\*Sqrt[3]\*x^2 + 4\*x^4]))

**IntegrateAlgebraic [A]** time = 12.02, size = 81, normalized size = 1.16

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{6\sqrt{3}-9}\sqrt{4x^4-4\sqrt{3}x^2-1}}{(2\sqrt{3}-4)x^2+(2-2\sqrt{3})x-1}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + Sqrt[3] + 2\*x)/((1 - Sqrt[3] + 2\*x)\*Sqrt[-1 - 4\*Sqrt[3]\*x^2 + 4\*x^4]),x]

[Out] -1/3\*(Sqrt[3 + 2\*Sqrt[3]]\*ArcTan[(Sqrt[-9 + 6\*Sqrt[3]]\*Sqrt[-1 - 4\*Sqrt[3]\*x^2 + 4\*x^4])/(-1 + (2 - 2\*Sqrt[3])\*x + (-4 + 2\*Sqrt[3])\*x^2)])

**fricas [B]** time = 1.27, size = 114, normalized size = 1.63

$$\frac{1}{6}\sqrt{2\sqrt{3}+3}\arctan\left(-\frac{(36x^4-60x^3+18x^2-\sqrt{3}(16x^4-40x^3+6x^2-10x+1)+6)\sqrt{4x^4-4\sqrt{3}x^2-1}\sqrt{2\sqrt{3}+3}}{88x^6-168x^5+132x^4-176x^3-66x^2-42x-11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x+3^(1/2))/(1+2\*x-3^(1/2))/(-1+4\*x^4-4\*3^(1/2)\*x^2)^(1/2),x, algorithm="fricas")

[Out] 1/6\*sqrt(2\*sqrt(3) + 3)\*arctan(-(36\*x^4 - 60\*x^3 + 18\*x^2 - sqrt(3)\*(16\*x^4 - 40\*x^3 + 6\*x^2 - 10\*x + 1) + 6)\*sqrt(4\*x^4 - 4\*sqrt(3)\*x^2 - 1)\*sqrt(2\*sqrt(3) + 3)/(88\*x^6 - 168\*x^5 + 132\*x^4 - 176\*x^3 - 66\*x^2 - 42\*x - 11))

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x + \sqrt{3} + 1}{\sqrt{4x^4 - 4\sqrt{3}x^2 - 1}(2x - \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x+3^(1/2))/(1+2\*x-3^(1/2))/(-1+4\*x^4-4\*3^(1/2)\*x^2)^(1/2),x, algorithm="giac")

[Out] integrate((2\*x + sqrt(3) + 1)/(sqrt(4\*x^4 - 4\*sqrt(3)\*x^2 - 1)\*(2\*x - sqrt(3) + 1)), x)

**maple** [C] time = 0.15, size = 337, normalized size = 4.81

$$\frac{\sqrt{-(-4-2\sqrt{3})x^2+1}\sqrt{-(-2\sqrt{3}+4)x^2+1}\operatorname{EllipticF}\left(i+i\sqrt{3},x,i\sqrt{1-\sqrt{3}(-2\sqrt{3}+4)}\right)+2\sqrt{3}}{(i+i\sqrt{3})\sqrt{4x^4-4\sqrt{3}x^2-1}} \left( \frac{\sqrt{-(-4-2\sqrt{3})x^2+1}\sqrt{-(-2\sqrt{3}+4)x^2+1}\operatorname{EllipticPi}\left(\sqrt{-4-2\sqrt{3}},x,\frac{1}{(-4+2\sqrt{3})\left(\frac{\sqrt{3}}{2}\right)},\frac{\sqrt{-2\sqrt{3}+4}}{\sqrt{-4-2\sqrt{3}}}\right)}{2\sqrt{-4-2\sqrt{3}}\left(\frac{\sqrt{3}}{2}\right)\sqrt{4x^4-4\sqrt{3}x^2-1}} - \frac{\operatorname{arctanh}\left(\frac{-4\sqrt{3}x^2+8\left(\frac{\sqrt{3}}{2}\right)^2-4\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)^2-2}{2\sqrt{\left(\frac{\sqrt{3}}{2}\right)^4+4\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)^2-1}\sqrt{4x^4-4\sqrt{3}x^2-1}}\right)}{4\sqrt{4\left(\frac{\sqrt{3}}{2}\right)^4-4\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)^2-1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+2\*x+3^(1/2))/(1+2\*x-3^(1/2))/(-1+4\*x^4-4\*3^(1/2)\*x^2)^(1/2), x)

[Out]  $\frac{1}{(I+I*3^{1/2})} * (1 - (-4 - 2*3^{1/2})*x^2)^{1/2} * (1 - (-2*3^{1/2} + 4)*x^2)^{1/2} / (-1 + 4*x^4 - 4*3^{1/2}*x^2)^{1/2} * \operatorname{EllipticF}(x*(I+I*3^{1/2}), I*(1 - 3^{1/2})*(-2*3^{1/2} + 4))^{1/2} + 2*3^{1/2} * (-1/4 / (4*(1/2*3^{1/2} - 1/2)^4 - 4*3^{1/2}*(1/2*3^{1/2} - 1/2)^2 - 2 - 4*3^{1/2})*x^2 + 8*x^2*(1/2*3^{1/2} - 1/2)^2) / (4*(1/2*3^{1/2} - 1/2)^4 - 4*3^{1/2}*(1/2*3^{1/2} - 1/2)^2 - 2 - 1)^{1/2} / (-1 + 4*x^4 - 4*3^{1/2}*x^2)^{1/2} - 1/2 / (-4 - 2*3^{1/2})^{1/2} / (1/2*3^{1/2} - 1/2) * (1 - (-4 - 2*3^{1/2})*x^2)^{1/2} * (1 - (-2*3^{1/2} + 4)*x^2)^{1/2} / (-1 + 4*x^4 - 4*3^{1/2}*x^2)^{1/2} * \operatorname{EllipticPi}((-4 - 2*3^{1/2})^{1/2}*x, 1/(-4 - 2*3^{1/2})) / (1/2*3^{1/2} - 1/2)^2, (-2*3^{1/2} + 4)^{1/2} / (-4 - 2*3^{1/2})^{1/2})$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x + \sqrt{3} + 1}{\sqrt{4x^4 - 4\sqrt{3}x^2 - 1}(2x - \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x+3^(1/2))/(1+2\*x-3^(1/2))/(-1+4\*x^4-4\*3^(1/2)\*x^2)^(1/2), x, algorithm="maxima")

[Out] integrate((2\*x + sqrt(3) + 1)/(sqrt(4\*x^4 - 4\*sqrt(3)\*x^2 - 1)\*(2\*x - sqrt(3) + 1)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2x + \sqrt{3} + 1}{\sqrt{4x^4 - 4\sqrt{3}x^2 - 1}(2x - \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x + 3^(1/2) + 1)/((4\*x^4 - 4\*3^(1/2)\*x^2 - 1)^(1/2)\*(2\*x - 3^(1/2) + 1)), x)

[Out] int((2\*x + 3^(1/2) + 1)/((4\*x^4 - 4\*3^(1/2)\*x^2 - 1)^(1/2)\*(2\*x - 3^(1/2) + 1)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x + 1 + \sqrt{3}}{(2x - \sqrt{3} + 1)\sqrt{4x^4 - 4\sqrt{3}x^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x+3\*\*(1/2))/(1+2\*x-3\*\*(1/2))/(-1+4\*x\*\*4-4\*3\*\*(1/2)\*x\*\*2)\*\*(1/2), x)

[Out] Integral((2\*x + 1 + sqrt(3))/((2\*x - sqrt(3) + 1)\*sqrt(4\*x\*\*4 - 4\*sqrt(3)\*x\*\*2 - 1)), x)

# Chapter 4

## Appendix

### Local contents

4.1	Download section . . . . .	1102
4.2	Listing of Grading functions . . . . .	1102

## 4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

**Mathematica format** Mathematica\_syntax\_CAS\_integration\_elementary\_version.zip

**Maple and Mupad format** Maple\_syntax\_CAS\_integration\_elementary\_version.zip

**Sympy format** SYMPY\_syntax\_CAS\_integration\_elementary\_version.zip

**Sage math format** SAGE\_syntax\_CAS\_integration\_elementary\_version.zip

## 4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
        "B"],
      "C"],
    If[FreeQ[result,Integrate] && FreeQ[result,Int],
      "C",
      "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
```

```

(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType, expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
        Max[ExpnType[expn[[1]], 2]],
      Max[ExpnType[expn[[1]], ExpnType[expn[[2]], 3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]], ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3, ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
  If[Head[expn]===RootSum,
    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
  9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]

HypergeometricFunctionQ[func_] :=

```

```
MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

## 4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#                    if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#                    see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;
end if;
```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do
not as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function

```

```

# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
  end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

```



```

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

### 4.2.3 Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
                 asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
                 asinh,acosh,atanh,acoth,asech,acsch
                 ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
                 fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
                 gamma,loggamma,digamma,zeta,polylog,LambertW,
                 elliptic_f,elliptic_e,elliptic_pi,exp_polar
                 ]

def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]

def is_atom(expn):

```

```

try:
    if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
    else:
        return False

except AttributeError as error:
    return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'^+^') or
    type(expn,'*^')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
    ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function

```

```

def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

#### 4.2.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: # isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False

```

```

else:
    return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M',
hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):

```

```

        return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
            return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
        return False

    except AttributeError as error:
        return False

def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer
)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
    elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(4,m1) #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(5,m1) #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))

```

```

    return max(6,m1)    #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
    #is checked before calling the grading function that is passed.
    #but kept it here.
    m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
    return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```